What measure of effect size when comparing two groups based on their means?

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Abstract

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Intro

During decades, researchers in social science (Henson & Smith, 2000) and education (Fan, 2001) have overestimated the ability of the null hypothesis (H0) testing to determine the importance of their results. The standard for researchers in social science is to define H0 as the absence of effect (Meehl, 1990). For example, when comparing the mean of two groups, researchers commonly test the H0 that there is no mean differences between groups (Steyn, 2000). Any effect that is significantly different from zero will be seen as sole support for a theory.

Such an approach has faced many criticisms among which the most relevant to our concern is that the null hypothesis testing highly depends on sample size: for a given alpha level and a given difference between groups, the larger the sample size, the higher the probability of rejecting the null hypothesis (Fan, 2001; Kirk, 2009; Olejnik & Algina, 2000; Sullivan & Feinn, 2012). It implies that even tiny differences could be detected as statistically significant with very large sample sizes (McBride, Loftis, & Adkins, 1993)[1].

Facing this argument, it has become an adviced practice to report the *p*-value assorted by a measure of the effect size, that is, a quantitative measure of the magnitude of the experimental effect (Cohen, 1965; Fan, 2001; Hays, 1963). This practice is also highly endorsed by the American Psychological Association (APA) and the American Educational Research Association (AERA) (American Educational Research Association, 2006; American Psychological Association, 2010). However, limited studies properly report effect size in the

[1] Tiny differences might be due to sampling error, or to other factors than the one of interest: even under the assumption of random assignent (which is a necessary but not sufficient condition), it is almost impossible to be sure that the only difference between two conditions is the one defined by the factor of interest. Other tiny factors of no theoretical interest might slighly influence results, making the probability of getting an actual zero effect very low. This is what Meehl (1990) calls 'systematic noise'

last several decades.

First, there is a high confusion between the effect size and other related concepts such as the clininal significance [SEE NOTE LATER] of a result (i.e. the relevance of an effect in real life). Moreover, there are several situations that call for effect size measures and in the current litterature, it's not always easy to know which measure using in a specific context.

Second, when used for inference, the main measures of effect sizes (i.e. Cohen's d and point-biserial r) are submitted to a range of assumptions (i.e. normality and heteroscedasticity) and these assumptions are known to be unrealistic in many research designs (Cain, Zhang, & Yuan, 2017; Erceg-Hurn & Mirosevich, 2008; Glass, Peckham, & Sanders, 1972; Grissom, 2000; Micceri, 1989; Yuan, Bentler, & Chan, 2004). As consequences many estimations of effect size are inaccurate and alter the robustness of the statistical conclusions. In the context of comparing two groups based on their means, Cohen's d_s is the dominant effect size measure used by researchers (Peng, Chen, Chiang, & Chiang, 2013). We will argue that, like Student's t-test, this measure rely on the often untenable assumptions of normality and homogeneity of variances (Cumming, 2013; Grissom & Kim, 2005; Kelley, 2005; Shieh, 2013). While it is becoming more common in statistical software to present Welch's t-test by default, when performing a t-test (i.e., R, Minitab), Cohen's d_s remains persistent [2].

In sum the aim of this paper is threefold:

1. Clearly define what is (and what is not) a measure of effect size;

2. Listing the different situations that call for effect sizes measure and reviewing which measure is appropriate in which circumstance;

3. Define different properties of a good effect size estimator and discuss the impact of assumptions violations on the robustness of the measures of effect size, based on simulations.

[2] For example, in Jamovi, Cohen's ds is provided, whatever one performs Student's or Welch's t-test

Measure of effect size: what it is, what it is not

The effect size is commonly refered to the practical significance of a test. Grissom and Kim (2005) define the effect size as the extent to which results differ from what is implied by the null hypothesis. In the context of the comparison of two groups based on their mean, depending on the defined null hypothesis (considering the absence of effect as the null hypothesis), we could define the effect size either as the magnitude of differences between parameters of two populations groups are extracted from (e.g. the mean; Peng & Chen, 2014) or as the magnitude of the relation between one dichotomous factor and one dependent variable (American Educational Research Association, 2006). Both definitions refers to as the most famous families of measures of effect sizes [Rosenthal_1994]: respectively the *d*-family and the *r*-family.

Very often, the contribution of the measures of effect size is overestimated.

First, benchmarks about what should be a small, medium or large effect size might have contribued at seeing the effect size as a measure of the importance or the relevance of an effect in real life, but it is not (Stout & Ruble, 1995). The effect size is only a mathematical indicator of the magnitude of a difference, which depends on the way a variable is converted into numerical indicator. In order to assess the meaningfulness of an effect, we should be able to relate this effect with behaviors/meaningful consequences in the real world (Andersen, McCullagh, & Wilson, 2007). For example, let us imagine a sample of students in serious school failure who are randomly divided into two groups: an experimental group following a training program and a control group. At the end of the training, students in the experimental group have on average significantly higher scores on a test than students in the control group, and the difference is large (e.g. 30 percents). Does it mean that students in the experimental condition will be able to pass to the next grade and to continue normal schooling? Whether the computed magnitude of difference is an important, meaningful change in everyday life refers to another construct: the *clinical significance*

(Bothe & Richardson, 2011). [I DON'T LIKE THE WORD "CLINICAL" BECAUSE IT'S NOT GENERAL ENOUGH.A MEANINGFUL SIGNIFICANCE COULD BE SOCIAL, PERSONAL, CLINICAL, PROFESIONNAL... ANY IDEA OF A MORE GENERAL WORD?]. It refers to the interpretation of treatment outcomes and is neither statistical nor mathematical, it is related to underlying theory that posits an empirical hypothesis. In other words, the relation between *practical* and *clinical* significance is more a theoretical argument than a statistical one.

Second, in the context of the comparison of two groups based on their means, it should not replace the null hypothesis testing. Statistical testing allows the researcher to determine whether the oberved departure from H0 occured by chance or not (Stout & Ruble, 1995) while effect size estimators allow to assess the practical signficance of an effect, and as reminds Fan (2001) "a practically meaningful outcome may also have occured by chance, and consequently, is not trustworthy". For this reason, the use of confidence intervals around the effect size estimate is highly recommended (Bothe & Richardson, 2011).

Different goals of measures of effect sizes

Robust measures

Properties of a good effect size estimator

Unbiasedness.

Consistency.

Efficiency.

Interpretability. Interpretability is not a very clear concept to me. Reading about it!

Simulations

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