Linear Systems

1 Basics

Consider the system of,

$$y_1 = 2x_1 + x_2 \tag{1}$$
$$y_2 = 2x_2 \tag{2}$$

This can be written as,

$$y = Ax \tag{3}$$
$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \tag{4}$$

2 Differential Equations

Consider the system of,

$$\dot{x} = y \tag{5}$$
$$\dot{y} = -x \tag{6}$$

which as above is (writing x as a vector now, where $x_1 = x$ and $x_2 = y$ above),

$$\dot{x} = Ax \tag{7}$$
$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{8}$$

The solution to differential equations of the form $\dot{x} = Ax$ is $x = e^{At}$. This is true even when A is a matrix. However, the value of this exponent is found from the sum of an infinite power series.

$$e^A = \sum_{k=0}^{\infty} \frac{1}{k!} A^k \tag{9}$$

This reduces to the expected e^a when A is a 1×1 matrix with value a. See the exponential function.

$$e^a = \sum_{k=0}^{\infty} \frac{a^k}{k!} = 1 + a + \frac{a^2}{2} + \frac{a^3}{6} + \dots \tag{10}$$