The Marginal Efficiency of Active Search

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Background

- Two types of non-employed workers willing to accept a job (BLS)
  - Passive searchers: e.g., waits for an employer to contact them
  - Active searchers: e.g., contacts an employer about a position

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- Implied marginal efficiency of active search is constant

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- Existing literature assumes passive and active searchers enter matching function as perfect substitutes, e.g. Blanchard and Diamond (1990)
- Implied marginal efficiency of active search is constant
- This paper:
  - Study standard DMP model with active and passive search
  - Identify restriction implied by perfect substitutability (and reject)
  - Estimate elasticity of substitution < 1, explore implications

What I do, 1/2 (constant marginal efficiency of active search?)

- Formulate standard DMP model w/ active & passive search
  - Active searcher expends effort to find job, passive does not
  - Returns to active and passive search given by fixed parameters
- Derive restriction: active-passive ratio of job-finding probabilities (minus one) has unit elasticity in average search effort
- Time-series data: elasticity is negative & statistically significant
- Qualitative rejection of perfect substitutability in DMP

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- Time-series data: elasticity is negative & statistically significant
- Qualitative rejection of perfect substitutability in DMP
- Show from individual-level data: when aggregate active search is high,
  - Active search effort is less effective
  - Penalty from purely passive search is lower

  Suggestive of crowding-out via diminishing returns

What I do, 2/2 (diminishing marginal efficiency of active search.)

- Back to theory: allow for crowding-out via CES aggregator
  - Relax assumption that elas. of subst. btwn active & passive $= \infty$
  - Marginal efficiency of active search no longer constant
- Formulate "new" equation for active-passive ratio from the data
- Estimate parameters: finite elasticity of substitution
- Lower marginal efficiency of active search during recessions
- Illustrate importance through two applications:
  - Application 1: Optimal policy under Bailey-Chetty formula
  - Application 2: Failure of Hosios condition

A general model

Goal

- Write down DMP model incorporating
  - Extensive and intensive margins of active search
  - Curvature in marginal utility of consumption
- Show how job-finding probabilities depend on active search
- Derive theoretical restriction relating
  - Active-passive ratio of job-finding probabilities
  - Average quantity of active search
- Note: focus on equations describing labor supply

Setting

- All jobs generate y_t units of output (can relax)
- Large measure of firms post v_t vacancies
- Representative family à la Andolfatto (1995) and Merz (1996)
  - Unit measure of workers indexed by i within each family
  - u_t workers are non-employed and search $1 - u_t$ are employed

  (Allows for curvature in marginal utility of consumption)
- Search of non-employed can be passive and/or active
- Contacts generated through matching function m_t
  - Note: matching efficiency can vary with t
- Going forward, focus on labor supply

Active and passive search

- Non-employed inelastically provide one unit of passive search
- Non-employed workers choose $s_{i,t}^{A}$ units of active search, subject to
  - Fixed costs, $\varsigma_{i,t} \sim \Gamma$ drawn *iid* at rate λ
  - Convex costs, $c\left(s_{i,t}^{A}\right)$
- Flexible to different notions of active search:
  - Intensive & extensive margin: $s_{i,t}^{A} \in \mathbb{R}_{+}$ (FMST 2022)
  - Extensive margin only: $s_{i,t}^{A} \in \{0, 1\}$ (KMRS 2017)

Matching function and job-finding probabilities

- Job-finding rate, $f_{i,t}$

$$f_{i,t} = s_{i,t} \cdot \left(\frac{m_t(s_t, v_t)}{s_t} \right) \qquad (*)$$

with CRS matching function, $m_t(s_t, v_t)$

- Search efficiency, $s_{i,t}$

$$s_{i,t} = \alpha_1 \cdot s^A_{i,t} + \alpha_0 \cdot 1 \qquad (**)$$

- Aggregate active search, s^A_t

$$s^A_t = \int_i s^A_{i,t} d\Gamma^u_t$$

- Aggregate search efficiency, s_t

$$s_t = \alpha_1 \cdot s^A_t + \alpha_0 \cdot u_t$$

Optimal active search

- Recall, fixed cost of active search is $\varsigma_{i,t}$
- Can show
  - Active search $s^A_{i,t}$ increasing in fixed cost $\varsigma_{i,t}$ up to some $\check{\varsigma}_t > 0$
  - Workers with $\varsigma_{i,t} > \check{\varsigma}_t$ set $s^A_{i,t} = 0$

  Thus, flow surplus of employment is increasing in $\varsigma_{i,t}$ up to $\check{\varsigma}_t$
- Generates endogenous distributions Γ^u_t and Γ^e_t of workers over $\varsigma_{i,t}$
- Thus, $\Gamma^u_t(\check{\varsigma}_t)$ of non-employed are engaged in active search

▸ Worker's problem ▸ Solution

Restriction: active-passive ratio and average active search

- Restriction in active-passive ratio: $\bar{f}_t^{\mathrm{A}} / \bar{f}_t^{\mathrm{P}}$ and $\bar{s}_A^*$

$$\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1 = \frac{\left(\alpha_1 \cdot \bar{s}_t^{A,*} + \alpha_0\right)\left(\frac{m_t(s_t,v_t)}{s_t}\right)}{\alpha_0 \left(\frac{m_t(s_t,v_t)}{s_t}\right)} - 1 = \left(\frac{\alpha_1}{\alpha_0}\right) \cdot \bar{s}_t^{A,*}$$

  from eqn's $(*)$ and $(**)$

- Unit elasticity in $\bar{s}_t^{A,*}$— all other quantities drop out!
  - Match efficiency differenced out
  - Unobserved heterogeneity of non-employed enters through $\bar{s}_t^{A,*}$

- Similar restr'n appears in KMRS (2017, AER) & FMST (2022, ECTA) & …

Bringing the restriction to the data

CPS, 1996-2019

- Starting in 1996, CPS records following for jobless respondents:
  - Whether the respondent would be willing to accept a job
  - Whether the worker is engaged in nine methods of active search
  - If # search methods $= 0$, why no active search?
- Non-employed worker willing to accept a job is
  - Active searcher if # search methods > 0
  - Passive searcher: if # search methods $= 0$ & "able" to accept work
- # of search methods highly correlated with time spent searching (Mukoyama, Patterson, and Sahin 2018) $\Rightarrow$ measure of search effort
- Note: exclude temporary-layoff for practical and conceptual reasons

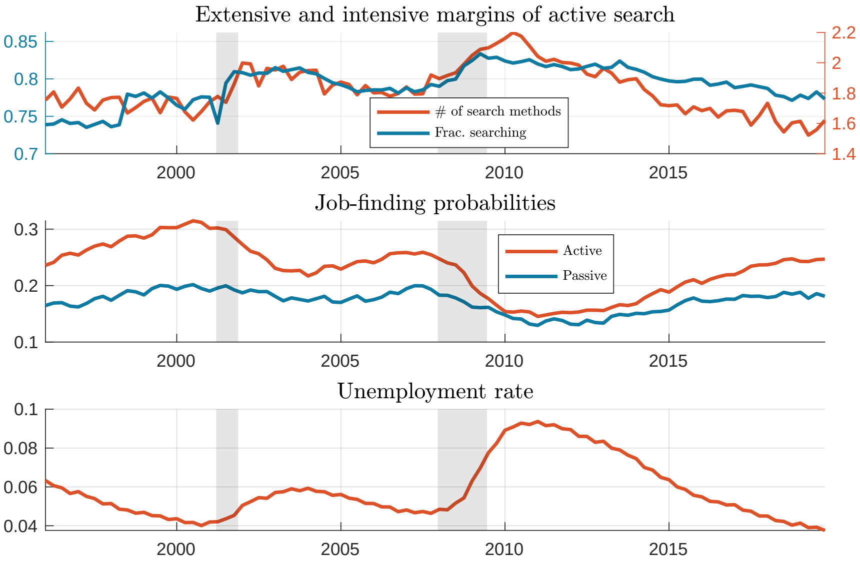
Search and job-finding probabilities

The active-passive ratio of job-finding prob's and aggregate search

	Frac. searching	# search methods	*A*-*P* ratio in JFP's
mean(x)	0.8	1.9	$1/0.75$
std(x)/std(Y)	1.7	2.8	9.2
corr(x, Y)	-0.69	-0.60	0.50

- Both frac. searching & # of search methods is countercyclical
  - See also Shimer (2004), Faberman and Kudlyak (2016), Mukoyama, Patterson, and Sahin (2018)
- Active-passive ratio is procyclical

Search and job-finding probabilities



Testing the restriction

- Recall restriction:

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\alpha_1}{\alpha_0}\right) + 1 \cdot \log \bar{s}_t^{A,*}$$

Theory predicts unit elasticity

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- Estimated elasticity from data: -5.47 (SE$=0.765$)

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- Estimated elasticity from data: -5.47 (SE$=0.765$)
- Robust to:
  - Restricting active searchers to low duration of unemployment
  - Disaggregating by gender, age, education, region, marital status ...
- Rejection of DMP with active and passive as perfect substitutes

When is active search most effective?

- Question: is job-finding probability increasing in active search effort?
  - Not an obvious question given evidence from aggregate data!
- Next, look at individual-level data and introduce
  - Time fixed effects
  - Rich individual controls
- Will show that when aggregate active search is high,
  - Active search effort is less effective
  - Penalty from purely passive search is lower

  Suggestive of crowding-out via diminishing returns
- But diminishing returns are precluded under perfect substitutes

When is active search most effective?

	Indicator variable for moving to employment in subsequent period			
	(1)	(2)	(3)	(4)
# of search methods	-0.002	0.113	0.057	–
	(0.0004)	(0.0058)	(0.0079)	
# of search methods $\times$ aggr. active search	–	-0.060	-0.031	–
		(0.0030)	(0.0041)	
$\mathbb{I}\{\text{\# search methods} = 0\}$	-0.040	-0.036	-0.261	-0.414
	(0.0013)	(0.0013)	(0.0192)	(0.0215)
$\mathbb{I}\{\text{\# search methods} = 0\} \times$ aggr. active search	–	–	0.120	0.479
			(0.0101)	(0.0270)
N	865079	865079	865079	865079
Time fixed effects?	Yes	Yes	Yes	Yes
Region fixed effects?	Yes	Yes	Yes	Yes

Sample of active and passive searchers, 1996-2019

Incl. controls for education, quartic for age, gender, race, and marital status

- Search is less effective when aggregate search is higher
- Penalty to to purely passive search lower when aggregate search is higher

An unrestricted
CES search aggregator

What went wrong

- Reject restriction from perfect substitution of active/passive
  - Perfect substitutes $\iff$ CES with elasticity of subst. $= \infty$
- Additional findings from micro-level data:
  - Active search less effective when aggregate search is higher
  - Penalty to passive searchers decreasing in aggregate search
- Suggests efficiency of active search diminishing in aggr. active search
  - w/ CES, requires elasticity of subst. $< \infty$
- Next: estimate parameters of unrestricted CES

CES aggregator for search effort

- Aggregate search effort s_t given by CES aggregator over $s_{A,t}$ and $s_{P,t}$

$$s_t = \left(\omega\, {s_{A,t}}^{\rho} + (1-\omega)s_{P,t}^{\rho}\right)^{\frac{1}{\rho}}$$

- Aggregate active & passive search satisfy

$$s_{A,t} = \int^{\check{\varsigma}_t} s_{i,t}^{A} d\Gamma_t^u = (\Gamma_t^u(\check{\varsigma}_t)u_t)\cdot \bar{s}_{A,t}^{*}, \quad s_{P,t} = \int d\Gamma_t^u = u_t$$

- $ME_{A,t}$ and $ME_{P,t}$ are marginal efficiencies of active and passive search

$$ME_{A,t} = \frac{\partial s_t}{\partial s_{A,t}} = \omega \cdot \left(\frac{s_t}{s_{A,t}}\right)^{1-\rho}, \quad ME_{P,t} = \frac{\partial s_t}{\partial s_{P,t}} = (1-\omega)\cdot\left(\frac{s_t}{s_{P,t}}\right)^{1-\rho}$$

Returns to search

- The job-finding probability $f_{i,t}$ of a worker with search efficiency $s_{i,t}$ is

$$f_{i,t} = s_{i,t} \cdot \left(\frac{m_t\left(s_t, v_t\right)}{s_t} \right)$$

- The search efficiency $s_{i,t}$ of a worker supplying $s^A_{i,t}$

$$s_{i,t} = ME_{A,t} \cdot s^A_{i,t} + ME_{P,t} \cdot 1$$

by linear homogeneity of the CES search aggregator

- Nests prior case when $\rho = 1$:

$$s_{i,t} = \Big(\underbrace{\omega}_{\equiv \alpha_1} s^A_{i,t} + \underbrace{(1 - \omega)}_{\equiv \alpha_0} \Big).$$

Restriction from theory, redux

- Relative job-finding probabilities, active vs. passive search

$$\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1 = \frac{\left(ME_{A,t} \cdot \bar{s}_t^{A,*} + ME_{P,t}\right)\left(\frac{m_t(s_t,v_t)}{s_t}\right)}{ME_{P,t}\left(\frac{m_t(s_t,v_t)}{s}\right)} - 1$$

$$= \left(\frac{\omega}{1-\omega}\right)\left(\frac{1}{\Gamma_t^u(\check{\varsigma}_t)\bar{s}_t^{A,*}}\right)^{1-\rho} \cdot \bar{s}_t^{A,*}$$

- Thus,

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1) \cdot \log \Gamma_t^u(\check{\varsigma}_t) + \rho \cdot \log \bar{s}_t^{A,*}$$

- Return to data, estimate ω and ρ, test restriction in ρ

Regression estimates

	(1)	(2)	(3)
Fraction searching	-7.31	-5.281	-3.819
	(1.426)	(1.606)	(0.5549)
# of search methods	–	-1.927	-2.812
		(0.8609)	(0.5549)
Constant	-0.684	0.489	1.140
	(0.4257)	(0.6392)	(0.1547)
Additional controls		Time trend	
Constrain $\beta_{\text{Frac}} - 1 = \beta_{\#}$?	N/A	No	Yes
F-test	$p(\rho = 1)$	$p(\beta_{\text{Frac}} + 1 = \beta_{\#})$	$p(\rho = 1)$
	=0.0000	=0.2799	=0.0000
N	261	261	261
Implied ρ	-8.308	–	-2.819
Implied ω	0.335		0.758
Elasticity of substitution	0.107		0.268

CPS, 1996-2019

What is a CES search aggregator?

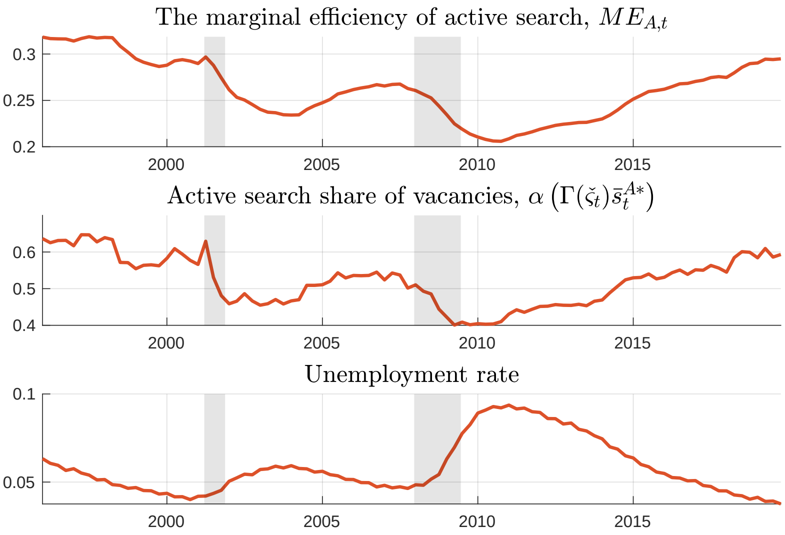
- Equivalence: separate submarkets for active and passive search

$$m_t(s_t, v_t) = m_t\left(ME_{A,t} \cdot s_{A,t}, \alpha_t \cdot v_t\right) + m_t\left(ME_{P,t} \cdot s_{P,t}, (1-\alpha_t) \cdot v_t\right)$$

$$\text{with} \quad \alpha_t = \alpha(s_{A,t}/s_{P,t}) = \frac{ME_{A,t} \cdot s_{A,t}}{s_t} = \frac{s_{A,t}^{\rho}}{s_{A,t}^{\rho} + s_{P,t}^{\rho}}, \quad \rho \leq 1$$

- (Obtains through constant returns)
- Vacancy share of active search α_t analogous to factor share

Backing out the marginal efficiency of active search



▸ Extensive and intensive margins of active search

Quick takeaway

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1)\cdot\log\Gamma_t^u(\check{\varsigma}_t) + \rho\cdot\log\bar{s}_t^{A,*}$$

- Reject restriction $\rho = 1$ (i.e., existing framework)
- Fail to reject restriction $\beta_{\Gamma(\check{\varsigma})} + 1 = \beta_{\bar{s}^{A,*}}$ (i.e., unrestricted framework)
- Elasticity of substitution $\frac{1}{1-\rho}$ falls in range $(\frac{1}{10}, \frac{1}{4})$
  - Indicates that active and passive search are "complements"
  - Thus active search vacancy share declining in active search
- Active search "less important" during recessions due to crowding-out
  - Marginal efficiency of active search falls
  - Active search vacancy share declines

Application 1:
Bailey-Chetty Formula

Appl. 1) Bailey-Chetty Formula

- Optimal UI described by Bailey-Chetty formula:

$$\underbrace{\frac{d\log u}{d\log R}}_{\text{increasing in } R} = \underbrace{\left(\frac{U'(c^u)}{U'(c^e)} - 1\right)}_{\text{decreasing in } R} \tag{BC}$$

  where u is unemployment and R is the replacement rate

- Landais et al. (2018): if wages are perfectly rigid (+ other conditions), (BC) describes optimal replacement rate R
- Micro-elasticity $\frac{d\log u}{d\log R}$ typically taken as constant $\Rightarrow$ R constant
- But $\frac{d\log u}{d\log R}$ is proportional to the marginal efficiency of active search...

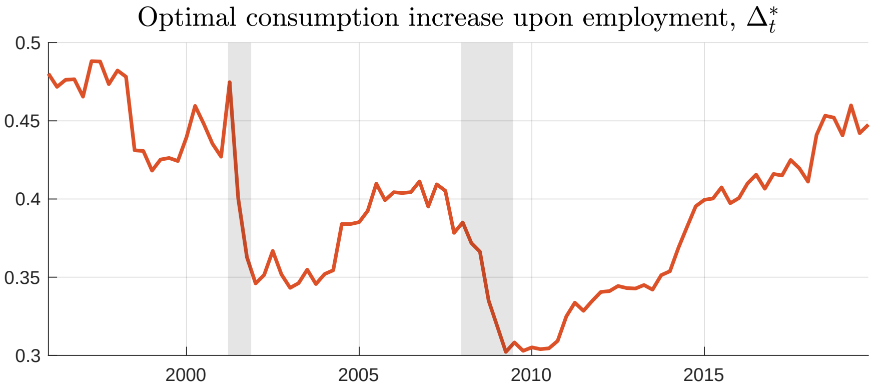
Appl. 1) Bailey-Chetty Formula, cont'd

- Write micro-elasticity as

$$
\begin{aligned}
\frac{d\log u}{d\log R} &= \frac{d\log u}{d\log f}\cdot\frac{d\log f}{d\log R} \\
&\approx -(1-\tilde{u})\cdot\frac{d\log f}{d\log s}\cdot\frac{d\log s}{d\log s_A}\cdot\frac{d\log s_A}{d\log R} \\
&= -(1-\tilde{u})\cdot\sigma\cdot\left[\omega\cdot\left(\frac{s_A}{s}\right)^{\rho}\right]\cdot\frac{d\log s_A}{d\log R}
\end{aligned}
$$

- Note, $\rho < 0$, so the elasticity is not constant!
- Next, (*i*) take avg. $-\frac{d\log f}{d\log R}$ to be equal to 0.42 (Katz and Meyer, 1990), (*ii*) compute average $\frac{d\log s}{d\log s_A}$, and (*iii*) solve for $\frac{d\log s_A}{d\log R}$
- Use to obtain time series for $\frac{d\log u}{d\log R}$

Appl. 1) Bailey-Chetty Formula, cont'd



- Define the *consumption increase upon employment*: $\Delta_t = (c_t^e / c_t^u) - 1$
- Assume $U(c) = \log c$. Then, (BC) $\Rightarrow \frac{d \log u}{d \log R} = \Delta_t^*$
- Δ_t^* lower during recessions due to marginal efficiency of active search

Application 2:
Failure of Hosios condition

Appl. 2) Failure of Hosios condition

$$rU_i = \max_{s_{A,i}} \Bigg\{ \frac{b - \varsigma_i \cdot \mathbb{I}\{s_{A,i} > 0\} - c(s_{A,i})}{\mu} + (ME_A \cdot s_i^A + ME_P) \cdot \left(\frac{m(s, \upsilon)}{s}\right) \cdot (V_i - U_i) - \dot{U}_i \Bigg\}$$

- Congestion externality: searchers fail to internalize how $s_{A,i}$ affects s
- Here: searchers also fail to internalize how $s_{A,i}$ affects ME_A and ME_P
- $s_{A,i}^* \uparrow \Rightarrow ME_A \downarrow$ and $ME_P \uparrow$

Appl. 2) Failure of Hosios condition, cont'd

- Optimal search, worker's problem:

$$c'(s^*_{A,i}) = ME_A \cdot f(\theta) \cdot \psi_i$$

where ψ_i is the marginal value to the HH of having agent i employed

Appl. 2) Failure of Hosios condition, cont'd

- Optimal search, worker's problem:

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where ψ_i is the marginal value to the HH of having agent i employed

- Optimal search, Planner's problem:

$$c'(s^{SP}_{A,i}) = ME^{SP}_A \cdot f(\theta^{SP}) \cdot \psi^{SP}_i + \underbrace{\frac{\partial ME^{SP}_A}{\partial s_A} \cdot cov(s^{SP}_{A,i}, \psi^{SP}_i)}_{<0}$$

where ψ^{SP}_i is the marginal social value of having agent i employed

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- Two allocations only coincide if
  1. No persistent heterogeneity in fixed cost of search ($\lambda \to \infty$)
  2. Constant marginal efficiency of active search, ME_A

Concluding remarks

Conclusion

- Finite elasticity of substitution between active and passive search
- Thus, dynamics of unemployment and job-finding rates depend on aggregate composition of active/passive search
- Reinforces implicit message of Elsby, Hobijn, and Sahin (2015), Krusell et al. (2017), Faberman et al. (2022), and more:

  We need to incorporate non-participation & passive search into more of our models to better understand unemployment

Extra slides

Problem of the unemployed

- Annuity value of unemployment:

$$rU_{i,t} = \max_{s^A_{i,t}} \left\{ \frac{b_t - \varsigma_i \cdot \mathbb{I}\left\{s^A_{i,t} > 0\right\} - c\left(s^A_{i,t}\right)}{\mu_t} \right.$$
$$\left. + \left(\alpha_0 + \alpha_1 \cdot s^A_{i,t}\right) \cdot \left(\frac{m_t\left(s_t, v_t\right)}{s_t}\right) \cdot \left(V_{i,t} - U_{i,t}\right) - \dot{U}_{i,t} \right\}$$

- Marginal utility of consumption, μ_t
- Flow value of leisure, b_t
- Values of employment and unemployment, $V_{i,t}$ and $U_{i,t}$
- $\dot{U}_t \neq 0$ given jump process for $\varsigma_{i,t}$, etc

Optimal active search

- Optimal quantity of active search (intensive margin):

$$s_{i,t}^{A,*} = (c')^{-1}\left(\mu_t \cdot \alpha_1 \cdot \left(\frac{m_t(s_t, v_t)}{s_t}\right)(V_{i,t} - U_{i,t})\right) \quad \text{when } \varsigma_{i,t} < \check{\varsigma}_t$$

- Optimal participation in active search (extensive margin):

$$\varsigma_{i,t} \leq -c(s_{i,t}^{A,*}) + \alpha_1 \cdot s_{i,t}^{A,*} \cdot \left(\frac{m_t(s_t, \upsilon_t)}{s_t}\right) \cdot \mu_t \cdot (V_{i,t} - U_{i,t}) \qquad (\dagger)$$

where $\check{\varsigma}_t$ defined by $\varsigma_{i,t}$ s.t. ($\dagger$) holds with equality

Time spent searching (MPS 2018)

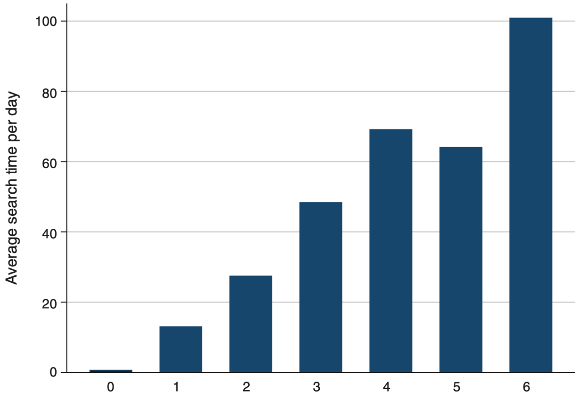

FIGURE 1. THE AVERAGE MINUTES (*per day*) SPENT ON JOB SEARCH ACTIVITIES BY THE NUMBER OF SEARCH METHODS

Notes: Each bin reflects the average search time in minutes per day by the number of search methods that the individual reports using in the previous month. Data is pooled from 2003–2014 and observations are weighted by the individual sample weight.

Definitions of job search (MPS 2018)

Table 2—Definitions of Job Search Methods in CPS and ATUS

Contacting an employer directly or having a job interview
Contacting a public employment agency
Contacting a private employment agency
Contacting friends or relatives
Contacting a school or university employment center
Checking union or professional registers
Sending out resumes or filling out applications
Placing or answering advertisements
Other means of active job search
Reading about job openings that are posted in newspapers or on the internet
Attending job training program or course
Other means of passive job search

Note: The first nine are active, the last three are passive.

Lalive, Landais, and Zweimüller (2016)

- Regional Extend Benefit Program in Austria, 1988-1993
  - Increase in benefit durations from 52 to 209 weeks
  - Only launched in select regions
- Finding: ineligible workers in treated regions have significantly lower unemployment durations
- Consistent with “positive elasticity wedge” (Landais, Michaillat, and Saez, 2018)
- Also consistent with crowding-out of active search

Search and job-finding probabilities