#### The Marginal Efficiency of Active Search

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- Consider non-employed workers willing to accept a job... (as in CPS)
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  - ▶ 1/2 of new hires from non-employment come from OLF
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    ightarrow U flows generate recessionary increase in unemployment
- Contemporary literature incorporates non-participation into DMP (including flows from OLF to employment)
  - e.g., Krusell et al (2017), Faberman et al (2022)

Maintained assumption: proportional job-finding rates

- ▶ Document crowding-out of active search during recessions:
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- Show formally: violation of proportional job-finding rates
  - Rejection of DMP w/ active/passive as perfect subst's
  - Consistent with diminishing marginal efficiency of active search
- ► Incorporate diminishing marginal efficiency of active search into three-state DMP, test/estimate, and explore implications:
  - 1. Active search less important during a recession
  - 2. Bailey-Chetty formula prescribes recessionary increase in UI
  - 3. Decentralized allocation not efficient under Hosios condition

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- 5. Use model to develop new restriction & estimation equation
- 6. Fail to reject restriction
- 7. Structurally estimate parameters of CES aggregator

#### Why we should care

- Novel implications of three-state DMP under estimated CES:
  - 1. Active search less important during recessions
  - 2. Disincentive effect of UI less distortionary during recessions
  - 3. Failure of Hosios condition (i.e., scope for policy)
- Implications follow from
  - Inclusion of participation margin
  - Imperfect substitution of active and passive search
- Reinforces importance of three-state DMP model for understanding unemployment dynamics and role of policy

# A general model

#### Goal

- Write down labor-supply block of DMP model incorporating
  - Extensive and intensive margins of active search
  - Curvature in marginal utility of consumption
- Show how job-finding probabilities depend on active search
- Derive theoretical restriction relating
  - Active-passive ratio of job-finding probabilities
  - Average quantity of active search
- Then take restriction to data

#### Setting

- ightharpoonup All jobs generate  $y_t$  units of output (can relax)
- Large measure of firms post  $v_t$  vacancies
- Representative family à la Andolfatto (1995) and Merz (1996)
  - Unit measure of workers indexed by i within each family
  - $u_t$  workers are non-employed and search  $1 u_t$  are employed (Allows for curvature in marginal utility of consumption)
- Search of non-employed can be passive and/or active
- Contacts generated through matching function m<sub>t</sub>
  - ▶ Note: matching efficiency can vary with *t*
- Going forward, focus on labor supply

### Active and passive search

- Non-employed inelastically provide one unit of passive search
- Non-employed workers choose  $s_{i,t}^A$  units of active search, subject to
  - Fixed costs,  $\varsigma_{i,t} \sim \Gamma$  drawn *iid* at rate λ
  - ightharpoonup Convex costs,  $c\left(s_{i,t}^A\right)$
- Flexible to different notions of active search:
  - ► Intensive & extensive margin:  $s_{i,t}^A \in \mathbb{R}_+$  (FMST 2022)
  - Extensive margin only:  $s_{i,t}^A \in \{0,1\}$  (KMRS 2017)

### Matching function and job-finding probabilities

ightharpoonup Job-finding rate,  $f_{i,t}$ 

$$f_{i,t} = \mathbf{s}_{i,t} \cdot \left(\frac{m_t(\mathbf{s}_t, \mathbf{v}_t)}{\mathbf{s}_t}\right) \tag{*}$$

with CRS matching function,  $m_t(s_t, v_t)$ 

► Search efficiency, *s<sub>i,t</sub>* 

$$\mathbf{s}_{i,t} = \omega \cdot \mathbf{s}_{i,t}^{\mathsf{A}} + (1 - \omega) \cdot 1 \tag{**}$$

(Can allow for  $\omega_t$ )

► Aggregate active search, S<sub>A,t</sub>

$$s_{A,t} = \int_i s_{i,t}^A d\Gamma_t^u$$

Aggregate search efficiency, S<sub>t</sub>

$$s_t = \omega \cdot s_{A,t} + (1 - \omega) \cdot u_t$$

### Optimal active search

- Fixed cost of active search is  $\varsigma_{i,t}$
- Can show
  - Active search  $s_{i,t}^A$  increasing in fixed cost  $\varsigma_{i,t}$  up to some  $\zeta_t > 0$
  - ► Workers with  $\varsigma_{i,t} > \check{\varsigma}_t$  set  $s_{i,t}^A = 0$

Thus, flow surplus of employment is increasing in  $\varsigma_{i,t}$  up to  $\zeta_t$ 

- Active search  $s_{i,t}^A$  and threshold  $\zeta_t$  are increasing in  $\omega$
- ▶ Generates endogenous distributions  $\Gamma_t^u$  and  $\Gamma_t^e$  of workers over  $\varsigma_{i,t}$
- ▶ Thus,  $\Gamma_t^u(\zeta_t)$  of non-employed are engaged in active search



# Restriction: active-passive ratio and average active search

▶ Restriction in active-passive ratio:  $\bar{t}_t^{\text{A}}/\bar{t}_t^{\text{P}}$  and  $\bar{s}_A^*$ 

$$\frac{\overline{f}_t^A}{\overline{f}_t^P} - 1 = \frac{\left(\omega \cdot \overline{s}_t^{A,*} + (1 - \omega)\right) \left(\frac{m_t(s_t, v_t)}{s_t}\right)}{(1 - \omega) \left(\frac{m_t(s_t, v_t)}{s_t}\right)} - 1 = \left(\frac{\omega}{1 - \omega}\right) \cdot \overline{s}_t^{A,*}$$

from eqn's (\*) and (\*\*)

- ▶ Unit elasticity in  $\bar{s}_t^{A,*}$  all other quantities drop out!
  - Match efficiency differenced out
  - ▶ Unobserved heterogeneity of non-employed enters through  $\bar{s}_t^{A,*}$
- ▶ Similar restr'n appears in KMRS (2017, AER) & FMST (2022, ECTA) & ...

# the data

Bringing the restriction to

#### CPS, 1996-2019

- Starting in 1996, CPS records following for jobless respondents:
  - Whether the respondent would be willing to accept a job
  - ▶ Whether the worker is engaged in nine methods of active search
  - ► If # search methods = 0, why no active search?
- Non-employed worker willing to accept a job is
  - Active searcher if # search methods > 0
  - Passive searcher: if # search methods = 0 & "able" to accept work
- Time spent searching near linear in # of search methods (Mukoyama, Patterson, and Sahin 2018) ⇒ measure of search effort
- Note: exclude temporary-layoff for practical and conceptual reasons

# The active and passive non-employed

	Active	Passive	A-NE	Avg. # of
	non-employed	non-employed	$\overline{A}$ - $\overline{NE}$ + $\overline{P}$ - $\overline{NE}$	search methods
mean(x)	4.9	1.3	0.79	1.85
std(x)/std(Y)	11.0	5.7	1.50	2.65
corr(x, Y)	-0.89	-0.70	-0.75	-0.64

Note: Data from CPS, 1996-2019. A-NE and P-NE refer to active and passive non-employed Y indicates quarterly GDP. For second and third row, series are taken as (1) quarterly averages of seasonally adjusted monthly series, (2) logged, then (3) HP-filtered with smoothing parameter of 1600

- Both frac. searching & # of search methods is countercyclical
- See also Osberg (1993), Shimer (2004), Faberman and Kudlyak (2016),
   Elsby, Hobijn and Sahin (2015), Mukoyama, Patterson, and Sahin (2018)

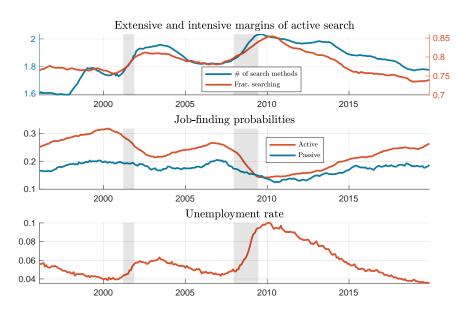
# Job-finding rates of the active and passive non-employed

	$ extit{A-NE}  ightarrow  extit{E}$	$ extit{P-NE}  ightarrow  extit{E}$	A-P
	probability	probability	ratio
mean(x)	0.23	0.17	1/1.32
std(x)/std(Y)	8.67	8.87	9.53
corr(x, Y)	0.85	0.32	0.48

Note: Data from CPS, 1996-2019. A-NE and P-NE refer to active and passive non-employed, "A-P ratio" refers to active-passive ratio of job-finding probabilities, Y indicates quarterly GDP. For second and third row, series are taken as (1) quarterly averages of seasonally adjusted monthly series, (2) logged, then (3) HP-filtered with smoothing parameter of 1600

- Mildy procyclical job-finding probability of passive non-employed
- Highly procyclical job-finding probability of active non-employed
- ► Thus, procyclical active-passive ratio in job-finding probabilities

## Search and job-finding probabilities



Recall restriction:

$$\log\left(\frac{\overline{f}_t^A}{\overline{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1 - \omega}\right) + 1 \cdot \log \overline{s}_t^{A,*}$$

Theory predicts unit elasticity

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- **Estimated elasticity from data:** -5.85 (SE= 0.873)
- Robust to:
  - Inclusion of time trend, cyclical indicator
  - Restricting active searchers to low duration of unemployment
  - Controls for cyclical composition along observable dimensions ...
  - lacktriangle Time-varying  $\omega$  (introduces upward bias into estimated elasticity)

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  - ightharpoonup Time-varying  $\omega$  (introduces upward bias into estimated elasticity)
- Rejection of DMP with active and passive as perfect substitutes

#### When is active search most effective?

- Question: is job-finding probability increasing in active search effort?
  - Not an obvious question given evidence from aggregate data!
- ► Next, look at individual-level data and introduce
  - Time fixed effects
  - Rich individual controls
- Will show that when aggregate active search is high,
  - Active search effort is less effective
  - Penalty from purely passive search is lower

Suggestive of crowding-out via diminishing returns

#### When is active search most effective?

# Dependent variable: Indicator for moving to employment in subsequent period

	(1)	(2)	(3)
$S_{i,t}^A$	0.006*** (0.0003)	-0.002*** (0.0004)	0.028*** (0.0048)
$oldsymbol{\mathcal{S}}_{i,t}^{A}  imes  ext{relative quantity}$ of active search in aggr.	_	-	-0.003*** (0.0005)
$\mathbb{I}\{s_{i,t}^A \neq 0\}$	_	0.040*** (0.0013)	0.202*** (0.0125)
$\mathbb{I}\{s_{i,t}^A \neq 0\} \times \text{relative quantity}$ of active search in aggr.	-	-	-0.016*** (0.0012)
N	865,079	865,079	865,079
Time fixed effects?	Yes	Yes	Yes
Region fixed effects?	Yes	Yes	Yes

Note: Sample of active and passive non-employed, 1996-2019. Includes controls for education, gender, race, marital status, a quartic for age, and fixed-effects for time and region.

# An unrestricted CES search aggregator

#### What went wrong

- Reject restriction from perfect substitution of active/passive
  - ightharpoonup Perfect substitutes  $\iff$  CES with elasticity of subst.  $= \infty$
- Additional findings from micro-level data:
  - Active search less effective when aggregate search is higher
  - Penalty to passive searchers decreasing in aggregate search
- Suggests efficiency of active search diminishing in aggr. active search
  - ightharpoonup w/ CES, requires elasticity of subst.  $< \infty$
- Next: estimate parameters of unrestricted CES

#### CES aggregator for search effort

▶ Aggregate search effort  $s_t$  given by CES aggregator over  $s_{A,t}$  and  $s_{P,t}$ 

$$oldsymbol{s}_t = \left(\omega \ (oldsymbol{z}_t \cdot oldsymbol{s}_{ extsf{A},t})^
ho + (\mathbf{1} - \omega) oldsymbol{s}_{ extsf{P},t}^
ho
ight)^{rac{1}{
ho}}$$

w/ exogenous z<sub>t</sub>

Aggregate active & passive search satisfy

$$oldsymbol{s}_{\mathsf{A},t} = \int^{ec{arsigma}_t} oldsymbol{s}_{i,t}^{\mathsf{A}} d\Gamma_t^{\mathit{u}} = (\Gamma_t^{\mathit{u}}(ec{arsigma}_t) u_t) \cdot ar{\mathbf{s}}_{\mathsf{A},t}^*, \quad oldsymbol{s}_{\mathsf{P},t} = \int d\Gamma_t^{\mathit{u}} = u_t$$

 $\blacktriangleright$   $ME_{A,t}$  and  $ME_{P,t}$  are marginal efficiencies of active and passive search

$$ME_{A,t} = \frac{\partial s_t}{\partial s_{A,t}} = \omega \cdot z_t^{\rho} \cdot \left(\frac{s_t}{s_{A,t}}\right)^{1-\rho}, \quad ME_{P,t} = \frac{\partial s_t}{\partial s_{P,t}} = (1-\omega) \cdot \left(\frac{s_t}{s_{P,t}}\right)^{1-\rho}$$

#### Returns to search

▶ The job-finding probability  $f_{i,t}$  of a worker with search efficiency  $s_{i,t}$  is

$$f_{i,t} = \mathbf{s}_{i,t} \cdot \left( \frac{m_t(\mathbf{s}_t, \mathbf{v}_t)}{\mathbf{s}_t} \right)$$

► The search efficiency  $s_{i,t}$  of a worker supplying  $s_{i,t}^A$ 

$$s_{i,t} = ME_{A,t} \cdot s_{i,t}^A + ME_{P,t} \cdot 1$$

by linear homogeneity of the CES search aggregator

Nests prior case when  $\rho = 1 \& z_t = 1$ :

$$s_{i,t} = \omega \cdot s_{i,t}^{A} + (1 - \omega) \cdot 1$$

#### Restriction from theory, redux

▶ Relative job-finding probabilities, active vs. passive search

$$\frac{\bar{f}_{t}^{A}}{\bar{f}_{t}^{P}} - 1 = \frac{\left(ME_{A,t} \cdot \bar{s}_{t}^{A,*} + ME_{P,t}\right) \left(\frac{m_{t}(s_{t},v_{t})}{s_{t}}\right)}{ME_{P,t} \left(\frac{m_{t}(s_{t},v_{t})}{s}\right)} - 1$$

$$= \left(\frac{\omega \cdot Z_{t}^{\rho}}{1 - \omega}\right) \left(\frac{1}{\Gamma_{t}^{U}(\xi_{t})\bar{s}_{t}^{A,*}}\right)^{1-\rho} \cdot \bar{s}_{t}^{A,*}$$

► Thus,

$$\log\left(\frac{\overline{t}_t^A}{\overline{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1 - \omega}\right) + \rho \cdot \log Z_t + (\rho - 1) \cdot \log \Gamma_t^{u}(\zeta_t) + \rho \cdot \log \overline{S}_t^{A,*}$$

**Proof.** Return to data: test restriction in  $\rho$ , estimate  $\omega$  and  $\rho$ 

#### Regression estimates

	(1)	(2)	(3)	(4)	(5)	(6)
$eta_{Frac}$	-5.997** (2.9035)	-3.752*** (0.8383)	-7.767*** (2.8319)	-1.422 (3.1452)	-2.844*** (0.7974)	-4.342 (3.1475)
$\beta_{\#}$	-1.766 (1.1270)	-2.752*** (0.8383)	_	-2.272** (1.1373)	-1.844** (0.7974)	_
$eta_0$	0.121 (1.1943)	1.140*** (0.4423)	-0.807 (1.1468)	1.680 (1.3329)	1.102** (0.4336)	0.282 (1.3093)
Additional controls	Time trend			Time trend + unempl. rate		
Constrain $\beta_{\text{Frac}} + 1 = \beta_{\text{\#}}$ ?	No	Yes	_	No	Yes	_
F-test	$p(\beta_{Frac} + 1 = \beta_{\#})$	$p(\rho = 1)$	$p(\rho = 1)$	$p(\beta_{Frac} + 1 = \beta_{\#})$	$p(\rho = 1)$	$p(\rho = 1)$
	= 0.350	= 0.000	= 0.008	= 0.615	= 0.000	= 0.172
N	204	204	204	204	204	204
Implied $\rho$		-2.752	-8.767	_	-1.844	-5.342
Implied $\omega$	_	0.758	-8.767	_	0.751	0.570

Note: CPS, 1996-20019

#### Quick takeaway

$$\log\left(\frac{\overline{f}_t^A}{\overline{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1 - \omega}\right) + \rho \cdot \log z_t + (\rho - 1) \cdot \log \Gamma_t^u(\zeta_t) + \rho \cdot \log \overline{s}_t^{A,*}$$

- **Reject** restriction  $\rho = 1$  (i.e., existing framework)
- Fail to reject restriction  $\beta_{\Gamma(\zeta)} + 1 = \beta_{\overline{s}^{A,*}}$  (i.e., unrestricted framework)
  - ▶ (But cannot allow  $z_t = 1$ )
- ► Elasticity of substitution  $\frac{1}{1-\rho}$  is about 1/4
- Easy interpretation (more later):
  - For the worker: active search as a strategic substitute
  - For the firm: stable ratio of hires from referrals to outside applications

Application 1:
The marginal efficiency of active search over the business cycle

#### What is a CES search aggregator?

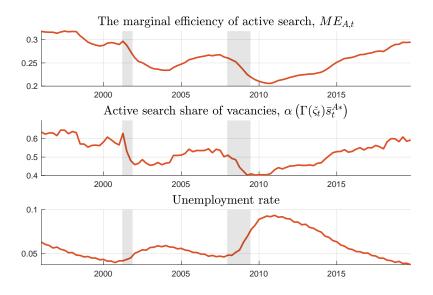
Equivalence: separate submarkets for active and passive search

$$m_t(s_t, v_t) = m_t(ME_{A,t} \cdot s_{A,t}, \alpha_t \cdot v_t) + m_t(ME_{P,t} \cdot s_{P,t}, (1 - \alpha_t) \cdot v_t)$$

with 
$$\alpha_t = \alpha(s_{A,t}/s_{P,t}) = \frac{ME_{A,t} \cdot s_{A,t}}{s_t} = \frac{s_{A,t}^{\rho}}{s_{A,t}^{\rho} + s_{P,t}^{\rho}}, \quad \rho \leq 1$$

- (Obtains through constant returns)
- ▶ Vacancy share of active search  $\alpha_t$  analogous to factor share
  - $ho < 0 \Rightarrow \alpha_t$  decreasing in  $(s_{A,t}/s_{P,t})$
  - ► Countercyclical  $(s_{A,t}/s_{P,t}) \Rightarrow$  Procyclical  $\alpha_t$
- $\blacktriangleright$  ME<sub>A,t</sub> and  $\alpha_t$  both fall during recessions

#### Backing out the marginal efficiency of active search



## Application 2:

**Bailey-Chetty Formula** 

#### Appl. 2) Bailey-Chetty Formula

Optimal UI described by Bailey-Chetty formula:

$$\frac{d \log u}{d \log R} = \underbrace{\left(\frac{U'(c^u)}{U'(c^e)} - 1\right)}_{\text{decreasing in } R}$$
(BC)

where u is unemployment and R is the replacement rate

- ▶ Landais et al. (2018): if wages are perfectly rigid (+ other conditions), (BC) describes optimal replacement rate R
- ▶ Micro-elasticity  $\frac{d \log u}{d \log R}$  typically taken as constant  $\Rightarrow R$  constant
- ▶ But  $\frac{d \log u}{d \log R}$  is proportional to the marginal efficiency of active search...

#### Appl. 2) Bailey-Chetty Formula, cont'd

Write micro-elasticity as

$$\frac{d \log u}{d \log R} = \frac{d \log u}{d \log f} \cdot \frac{d \log f}{d \log R}$$

$$\approx -(1 - \tilde{u}) \cdot \frac{d \log f}{d \log s} \cdot \frac{d \log s}{d \log s_A} \cdot \frac{d \log s_A}{d \log R}$$

$$= -(1 - \tilde{u}) \cdot \sigma \cdot \left[\omega \cdot \left(\frac{s_A}{s}\right)^{\rho}\right] \cdot \frac{d \log s_A}{d \log R}$$

- Note,  $\rho$  < 0, so the elasticity is not constant!
- Next, (i) take avg.  $-\frac{d \log f}{d \log R}$  to be equal to 0.42 (Katz and Meyer, 1990), (ii) compute average  $\frac{d \log s}{d \log s_A}$ , and (iii) solve for  $\frac{d \log s_A}{d \log R}$
- ► Use to obtain time series for  $\frac{d \log u}{d \log R}$

#### Appl. 2) Bailey-Chetty Formula, cont'd



- lacktriangle Define the consumption increase upon employment:  $\Delta_t = (c_t^e/c_t^u) 1$
- ► Assume  $U(c) = \log c$ . Then, (BC)  $\Rightarrow \frac{d \log u}{d \log R} = \Delta_t^*$
- $ightharpoonup \Delta_t^*$  lower during recessions due to marginal efficiency of active search

### **Application 3:**

Failure of Hosios condition

#### Appl. 3) Failure of Hosios condition

$$rU_{i} = \max_{s_{A,i}} \left\{ \frac{b - \varsigma_{i} \cdot \mathbb{I} \left\{ s_{A,i} > 0 \right\} - c \left( s_{A,i} \right)}{\mu} + \left( ME_{A} \cdot s_{i}^{A} + ME_{P} \right) \cdot \left( \frac{m(s, \upsilon)}{s} \right) \cdot \left( V_{i} - U_{i} \right) - \dot{U}_{i} \right\}$$

- $\triangleright$  Congestion externality: searchers fail to internalize how  $S_{A,i}$  affects S
- ► Here: searchers also fail to internalize how  $S_{A,i}$  affects  $ME_A$  and  $ME_P$
- $\triangleright$   $s_{A,i}^* \uparrow \Rightarrow ME_A \downarrow \text{ and } ME_P \uparrow$

#### Appl. 3) Failure of Hosios condition, cont'd

Optimal search, worker's problem:

$$c'(s_{A,i}^*) = ME_A \cdot f(\theta) \cdot \psi_i$$

where  $\psi_i$  is the marginal value to the HH of having agent i employed

#### Appl. 3) Failure of Hosios condition, cont'd

Optimal search, worker's problem:

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where  $\psi_i$  is the marginal value to the HH of having agent i employed

▶ Optimal search, Planner's problem:

$$C'(\boldsymbol{s}_{A,i}^{SP}) = \boldsymbol{ME}_{A}^{SP} \cdot \boldsymbol{f}(\boldsymbol{\theta}^{SP}) \cdot \psi_{i}^{SP} + \underbrace{\frac{\partial \boldsymbol{ME}_{A}^{SP}}{\partial \boldsymbol{s}_{A}} \cdot \boldsymbol{cov}(\boldsymbol{s}_{A,i}^{SP}, \psi_{i}^{SP})}_{<0}$$

where  $\psi_i^{SP}$  is the marginal social value of having agent i employed

#### Appl. 3) Failure of Hosios condition, cont'd

Optimal search, worker's problem:

$$c'(s_{A,i}^*) = ME_A \cdot f(\theta) \cdot \psi_i$$

where  $\psi_i$  is the marginal value to the HH of having agent i employed

Optimal search, Planner's problem:

$$C'(s_{A,i}^{SP}) = ME_A^{SP} \cdot f(\theta^{SP}) \cdot \psi_i^{SP} + \underbrace{\frac{\partial ME_A^{SP}}{\partial s_A} \cdot cov(s_{A,i}^{SP}, \psi_i^{SP})}_{<0}$$

where  $\psi_i^{SP}$  is the marginal social value of having agent *i* employed

- Two allocations only coincide if
  - 1. No persistent heterogeneity in fixed cost of search ( $\lambda \to \infty$ )
  - 2. Constant marginal efficiency of active search,  $ME_A$



#### Conclusion

- Active-passive ratio in job-finding prob's fluctuates over cycle
- During recession,
  - Active search goes up
  - Active-passive ratio in job-finding probabilities goes down Inconsistent with perfect substitutability of active and passive search
- Imperfect substitutability of active and passive search generates crowding-out of active search
- Reinforces importance of participation margin for understanding unemployment dynamics

# Extra slides

#### Problem of the unemployed

Annuity value of unemployment:

$$rU_{i,t} = \max_{\boldsymbol{s}_{i,t}^{A}} \left\{ \frac{\gamma - \varsigma_{i} \cdot \mathbb{I}\left\{\boldsymbol{s}_{i,t}^{A} > 0\right\} - c\left(\boldsymbol{s}_{i,t}^{A}\right)}{\mu_{t}} + \left(\omega_{0} + \omega_{1} \cdot \boldsymbol{s}_{i,t}^{A}\right) \cdot \left(\frac{m_{t}\left(\boldsymbol{s}_{t}, \boldsymbol{v}_{t}\right)}{\boldsymbol{s}_{t}}\right) \cdot \left(\boldsymbol{V}_{i,t} - \boldsymbol{U}_{i,t}\right) - \dot{\boldsymbol{U}}_{i,t} \right\}$$

- Marginal utility of consumption,  $\mu_t$
- Flow value of leisure,  $\gamma$
- $\triangleright$  Values of employment and unemployment,  $V_{i,t}$  and  $U_{i,t}$
- $\dot{U}_t \neq 0$  given jump process for  $\varsigma_{i,t}$ , etc

#### Optimal active search

Optimal quantity of active search (intensive margin):

$$\mathbf{s}_{i,t}^{\mathbf{A},*} = (\mathbf{c}')^{-1} \left( \mu_t \cdot \omega_1 \cdot \left( \frac{m_t(\mathbf{s}_t, \mathbf{v}_t)}{\mathbf{s}_t} \right) (V_{i,t} - U_{i,t}) \right) \quad \text{ when } \varsigma_{i,t} < \zeta_t$$

Optimal participation in active search (extensive margin):

$$\varsigma_{i,t} \leq -c(\mathbf{s}_{i,t}^{A,*}) + \omega_1 \cdot \mathbf{s}_{i,t}^{A,*} \cdot \left(\frac{m_t(\mathbf{s}_t, \upsilon_t)}{\mathbf{s}_t}\right) \cdot \mu_t \cdot (V_{i,t} - U_{i,t})$$
 (†)

where  $\zeta_t$  defined by  $\varsigma_{i,t}$  s.t. (†) holds with equality

#### Time spent searching (MPS 2018)

198

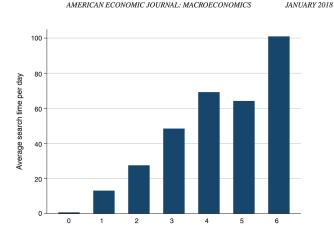


Figure 1. The Average Minutes  $(per\ day)$  Spent on Job Search Activities by the Number of Search Methods

Notes: Each bin reflects the average search time in minutes per day by the number of search methods that the individual reports using in the previous month. Data is pooled from 2003–2014 and observations are weighted by the individual sample weight.

#### Definitions of job search (MPS 2018)

#### TABLE 2—DEFINITIONS OF JOB SEARCH METHODS IN CPS AND ATUS

Contacting an employer directly or having a job interview

Contacting a public employment agency

Contacting a private employment agency

Contacting friends or relatives

Contacting a school or university employment center

Checking union or professional registers

Sending out resumes or filling out applications

Placing or answering advertisements

Other means of active job search

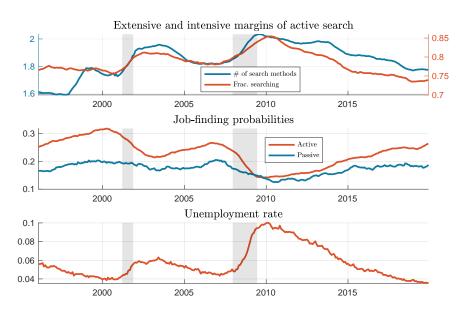
Reading about job openings that are posted in newspapers or on the internet

Attending job training program or course

Other means of passive job search

Note: The first nine are active, the last three are passive.

#### Search and job-finding probabilities



#### Elasticity of active-passive ratio in job-finding probabilities

Dependent variable: Log active-passive ratio in					
in job-finding probabilities (minus one)					
	(1)	(2)	(3)		
Log # of search methods		-5.017***	-2.977***		
	(0.8729)	(0.9411)	(0.9620)		
Additional controls	None	Time trend	Time trend + unempl. rate		
$p(eta_\#=0)$	0.000	0.000	0.003		
$p(\beta_{\#}=1)$	0.000	0.000	0.000		
N	264	264	264		
CPS, 1996-2019					

#### Elasticity of the active-passive ratio: duration dependence

Dependent va	riable: Log acti	ve-passive ratio	o in		
in job-finding probabilities (minus one)					
	(1)	(2)	(3)		
Log # of search methods	-2.410***	-2.259***	-2.431***		
	(0.5178)	(0.6372)	(0.7242)		
Additional controls	None	Time trend	Time trend		
	None	Time trend	+ unempl. rate		
$p(\beta_{\#}=0)$	0.000	0.001	0.001		
$p(\beta_{\#}=1)$	0.000	0.000	0.000		
N	287	287	287		
CPS 1996-2019					

#### Elasticity of active-passive ratio: cyclical composition

	riable: Log activ	ve-passive ratio es (minus one)	in
	None	Time trend	Time trend + unempl. rate
		1. Gender	
Log # of search methods	-6.447*** (0.9040)	-5.760*** (1.0593)	-3.004*** (0.9476)
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000
N	266	266	266
		2. Race	
Log # of search methods	-6.150*** (0.7947)	-5.355*** (0.9732)	-3.439*** (1.0255)
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000
N	265	265	265
		3. Age	
Log # of search methods	-6.211*** (0.8260)	-4.998*** (0.9519)	-2.117*** (0.7850)
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000
N	267	267	267

	riable: Log activ	re-passive rations es (minus one)	in	
	None	Time trend	Time trend + unempl. rate	
	4. Mo	artial status (b	y gender)	
Log # of search methods	-6.126*** -5.465*** (0.7903) (0.9173)		-2.520*** (0.8010)	
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000	
N	265	265	265	
	5. Education			
Log # of search methods	-5.744***	-4.961***	-3.458***	
	(0.9564)	(1.0153)	(1.1548)	
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000	
N	223	223	223	
		6. Region		
Log # of search methods	-5.870***	-4.910***	-2.659***	
	(0.8166)	(0.9365)	(0.9044)	
$p(\beta_{\log \#} = 1)$	0.000	0.000	0.000	
N	265	265	265	

Table 5. Sumary statistics: cyclical composition

Subgroup	$ar{\omega}_j^A$	$ar{f}_j^A$	$\bar{s}_{j,t}^{A,*}$	$ar{\omega}_j^P$	$ar{f}_{j,t}^P$
1. Gender					
Female	0.46	0.22	1.84	0.46	0.16
Male	0.54	0.24	1.86	0.54	0.19
2. Race					
Black	0.08	0.21	1.81	0.09	0.16
White	0.70	0.25	1.86	0.70	0.18
Other non-white	0.22	0.18	1.84	0.21	0.14
3. Age					
16-24	0.32	0.26	1.69	0.24	0.19
25-55	0.57	0.22	1.95	0.47	0.20
56-64	0.08	0.16	1.92	0.13	0.13
65+	0.03	0.13	1.60	0.16	0.08
4. Marital status (male/female)					
Married	0.17/0.15	0.24/0.21	2.00/1.85	0.19/0.19	0.20/0.16
Previously married	0.07/0.09	0.22/0.20	1.92/1.95	0.08/0.11	0.15/0.12
Never married	0.30/0.22	0.24/0.23	1.77/1.78	0.27/0.16	0.19/0.18
5. Education					
Less than high school diploma (or equivalent)	0.11	0.21	1.71	0.13	0.16
High school diploma (or equivalent)	0.23	0.21	1.85	0.25	0.16
Some college	0.19	0.22	1.99	0.19	0.16
BA/ Four years of college	0.11	0.23	2.14	0.12	0.19
BA+/ More than four years of college	0.04	0.22	2.18	0.06	0.19
6. Region					
Northeast	0.18	0.21	1.88	0.20	0.17
	0.21	0.23	1.80	0.20	0.17
Midwest					
Midwest	0.36	0.23	1.84	0.36	0.17

Note: