The marginal efficiency of active search

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Background

- Two types of non-employed workers willing to accept a job (BLS)
  - Passive searchers: e.g., waits for an employer to contact them
  - Active searchers: e.g., contacts an employer about a position

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  - Marginal efficiency of active and passive search are fixed
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  - Marginal efficiency of active and passive search are fixed
  - Offers justification for abstracting from passive searchers
- This paper:
  - Study standard DMP model with active and passive search
  - Identify restriction implied by perfect substitutability (and reject)
  - Estimate elasticity of substitution < 1, explore implications

What I do, 1/2

(constant marginal efficiency of active search?)

- Formulate standard DMP model w/ active & passive search
  - Active searcher expends effort to find job, passive does not
  - Returns to active and passive search given by fixed parameters
- Derive restriction: active-passive ratio of job-finding probabilities has unit elasticity in average search effort
- Time-series data: elasticity is negative & statistically significant
- Qualitative rejection of standard DMP w/ perfect substitutability
- Show from individual-level data: when aggregate active search is high,
  - Active search effort is less effective
  - Penalty from purely passive search is lower

  Suggests crowding-out or diminishing returns

What I do, 2/2 (diminishing marginal efficiency of active search.)

- Back to theory: allow crowding-out of active search via CES aggregator
  - Relax restriction that elas. of subst. btwn active/passive $= \infty$
  - Marginal efficiency of active search no longer constant
- Establish that CES search aggregator is equivalent to linear aggregator with separate matching functions for active and passive search
- Estimate "new" equation for active-passive ratio from the data
  - Recover parameters of aggregator w/ finite elasticity
  - Elasticity of substitution less than one
- Implications:
  - Diminishing marginal efficiency of active search
  - Vacancy share for active search declining in active search

Application: Unemployment Insurance during a recession

- Active search higher during recessions (among non-employed)
  - Along both extensive and intensive margins
  - See also Shimer (2004), Mukoyama et al. (2018), etc.
- Elast'y of subst. $< 1 \Rightarrow$ active search "less important" during recessions
  - Marginal efficiency of active search falls
  - Larger share of vacancies for passive searchers
- Implies less distortion from UI disincentive
  - Operates through "microelasticity"
  - Independent of surplus sharing mechanism
- Rationalizes estimates from studies of recessionary expansions of UI

Existing literature

1. Endogenous search intensity: Shimer (2004), Faberman and Kudlyak (2016), Mukoyama, Patterson, and Sahin (2018)
2. Active & passive search: Krusell et al. (2017), Faberman et al. (2022)
3. UI expansions (theory): Nakajima (2012), Mitman and Rabinovich (2015, 2020), Landais et al (2018, 2018a)
4. UI expansions (empirics): Krueger and Meyer (2002) Farber and Valletta (2015), Kroft and Notowidigdo (2016), Johnston and Mas (2018), Chodorow-Reich et al (2019)

Theory

Goal

- Write down DMP model incorporating
  - Extensive and intensive margins of active search
  - Curvature in marginal utility of consumption
- Show how job-finding probabilities depend on active search
- Derive theoretical restriction relating
  - Active-passive ratio of job-finding probabilities
  - Average quantity of active search
- Note: focus on equations describing labor supply

Setting

- All jobs generate y_t units of output
- Large measure of firms post v_t vacancies
- Representative family à la Andolfatto (1995) and Merz (1996)
- Unit measure of workers indexed by i within each family
- u_t workers are unemployed, $1 - u_t$ are employed
- Unemployed search for jobs
- Two forms of search: passive and active

(Next: from active and passive search effort to search efficiency

Active and passive search

- Unemployed inelastically provide one unit of passive search
- Unemployed workers choose $s_{i,t}^{A}$ units of active search
  - Fixed costs, $\varsigma_{i,t} \sim \Gamma$ drawn *iid* at rate λ
  - Convex costs, $c\left(s_{i,t}^{A}\right)$
- Flexible to different notions of active search
  - e.g., $s_{i,t}^{A} \in \{0, 1\}$, $s_{i,t}^{A} \in \mathbb{N}_0$, or $s_{i,t}^{A} \in \mathbb{R}_+$
- Can show that only workers with $\varsigma_{i,t} < \check{\varsigma}_t$ actively search, for some $\check{\varsigma}_t$
- $\Gamma_t^u(\check{\varsigma}_t)u_t$ workers supply active search

(Next: search efficiency & job finding prob's)

Job-finding probabilities

- Job-finding rate, $f_{i,t}$

$$f_{i,t} = s_{i,t} \cdot \left(\frac{m_t(s_t, v_t)}{s_t} \right) \qquad (*)$$

with CRS matching function, $m_t(s_t, v_t)$

- Search efficiency, $s_{i,t}$

$$s_{i,t} = \alpha_1 \cdot s^A_{i,t} + \alpha_0 \qquad (**)$$

where α_1 is the marginal efficiency of active search

- Aggregate search efficiency, s_t

$$s_t = \int_i s_{i,t} d\Gamma^u_t$$

(Next: optimal active search)

Problem of the unemployed

- Annuity value of unemployment:

$$rU_{i,t} = \max_{s^A_{i,t}} \left\{ \frac{b_t - \varsigma_i \cdot \mathbb{I}\left\{s^A_{i,t} > 0\right\} - c\left(s^A_{i,t}\right)}{\mu_t} + \left(\alpha_0 + \alpha_1 \cdot s^A_{i,t}\right) \cdot \left(\frac{m_t\left(s_t, v_t\right)}{s_t}\right) \cdot \left(V_{i,t} - U_{i,t}\right) - \dot{U}_{i,t} \right\}$$

- Marginal utility of consumption, μ_t
- Flow value of leisure, b_t
- Values of employment and unemployment, $V_{i,t}$ and $U_{i,t}$
- $\dot{U}_t \neq 0$ given jump process for $\varsigma_{i,t}$, etc

(Next: derive $s^{A,*}_{i,t}$ and $\check{\varsigma}_t$)

Optimal active search

- Optimal quantity of active search (intensive margin):

$$s_{i,t}^{A,*} = (c')^{-1}\left(\mu_t \cdot \alpha_1 \cdot \left(\frac{m_t(s_t, v_t)}{s_t}\right)(V_{i,t} - U_{i,t})\right) \quad \text{when } \varsigma_{i,t} < \check{\xi}_t$$

- Optimal participation in active search (extensive margin):

$$\varsigma_{i,t} \leq -c(s_{i,t}^{A,*}) + \alpha_1 \cdot s_{i,t}^{A,*} \cdot \left(\frac{m_t(s_t, \upsilon_t)}{s_t}\right) \cdot \mu_t \cdot (V_{i,t} - U_{i,t}) \qquad (\dagger)$$

where $\check{\xi}_t$ defined by $\varsigma_{i,t}$ s.t. ($\dagger$) holds with equality

(Next: active-passive ratio)

Restriction: active-passive ratio and average active search

- Restriction in active-passive ratio $\bar{f}_t^{\mathrm{A}}/\bar{f}_t^{\mathrm{P}}$ and $\bar{s}_A^*$

$$\frac{\bar{f}_t^A}{\bar{f}_t^P} = \frac{\left(\alpha_1 \cdot \bar{s}_t^{A,*} + \alpha_0\right)\left(\frac{m_t(s_t,v_t)}{s_t}\right)}{\alpha_0\left(\frac{m_t(s_t,v_t)}{s_t}\right)} = \left(\frac{\alpha_1}{\alpha_0}\right) \cdot \bar{s}_t^{A,*} + 1$$

  (from eqn's $*$ and $**$)

- Unit elasticity in $\bar{s}_t^{A,*}$
- All other aggregate quantities drop out!
  - Note: match efficiency differenced out
- Take equation to the data

Empirics

CPS, 1996-2019

- Starting in 1996, CPS records following for jobless respondents:
  - Whether the respondent would be willing to accept a job
  - Whether the worker is engaged in nine methods of active search
  - If # search methods $= 0$, why no active search?
- Non-employed worker willing to accept a job is
  - Active searcher if # search methods > 0
  - Passive searcher: if # search methods $= 0$
- # of search methods highly correlated with time spent searching (Mukoyama, Patterson, and Sahin 2018) $\Rightarrow$ measure of search effort
- Note: excluding temporary-layoff for practical and conceptual reasons

Active and passive searchers: empirical properties (1/2)

Active and passive searchers in non-employment

	Active U	Passive U	$\frac{AU}{AU+PU}$
mean(x)	4.9	1.3	78.6
std(x)/std(Y)	11.0	5.8	1.5
corr(x, Y)	-0.89	-0.69	-0.74

- Note: extensive margin of active search is countercyclical!
- Robust to controls for composition (Mukoyama et al 2018)

Active and passive searchers: empirical properties (2/2)

Gross worker flows, 1996:I–2019:IV

From	*To*				
	E	AU	PU	TL	I
E	0.958	0.008	0.003	0.005	0.025
AU	0.310	0.345	0.063	0.024	0.258
PU	0.190	0.142	0.105	0.020	0.543
TL	0.497	0.024	0.041	0.243	0.114
I	0.040	0.258	0.013	0.001	0.932

$$AU = \text{Active } U, \quad PU = \text{Passive } U,$$
$$TL = \text{Temporary Layoff}, \quad I = \text{Inactive}$$

- Evidence of mixing btwn. active & passive
- Higher job-finding prob's from active

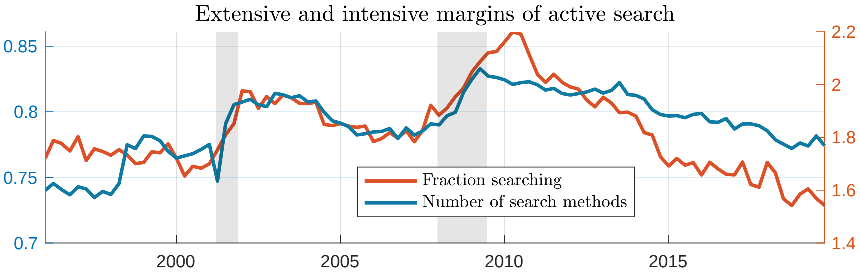
Search and job-finding probabilities

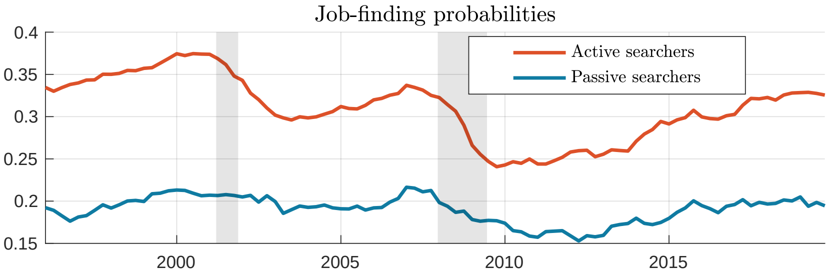
The active-passive ratio of job-finding prob's and aggregate search

	A-P ratio (minus one)	Frac. searching	# search methods
mean(x)	0.6	0.8	1.9
std(x)/std(Y)	21.3	1.5	2.6
corr(x, Y)	0.43	−0.74	−0.64

- Both frac. searching & # of search methods is countercyclical
  - See also Shimer (2004), Faberman and Kudlyak (2016), Mukoyama, Patterson, and Sahin (2018)
- Active-passive ratio is procyclical

Search and job-finding probabilities





Testing the restriction

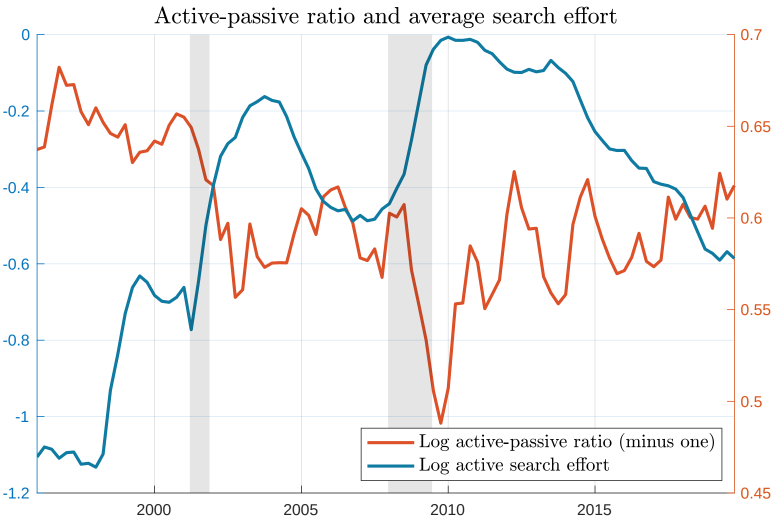
- Recall restriction:

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\alpha_1}{\alpha_0}\right) + \log \bar{s}_t^{A,*}$$

  Theory predicts unit elasticity

- Estimated elasticity from data: -2.50 (SE$=$ 0.334)
- Robust to:
  - Restricting active-searchers to low duration of unemployment
  - Disaggregating by gender, age, education, region, marital status …

Testing the restriction



When is active search most effective?

- Given countercyclicality of search, would not be surprising if **raw** job-finding prob's negatively correlated with aggregate search
- But active-passive ratio controls for time variation in
  (i) market tightness
  (ii) aggregate match efficiency
- Still highly procyclical
- Next: look at individual-level data and introduce
  - Time fixed effects
  - Rich individual controls
- Ask: when does greater search effort improve search outcomes?

When is active search most effective?

	(1)	(2)	(3)	(4)
	Indicator variable for moving to employment in subsequent period			
# of search methods	-0.002	0.113	0.057	–
	(0.0004)	(0.0058)	(0.0079)	
# of search methods × aggr. active search	–	-0.060	-0.031	–
		(0.0030)	(0.0041)	
$\mathbb{I}\{\text{\# search methods} = 0\}$	-0.040	-0.036	-0.261	-0.414
	(0.0013)	(0.0013)	(0.0192)	(0.0215)
$\mathbb{I}\{\text{\# search methods} = 0\} \times$ aggr. active search	–	–	0.120	0.479
			(0.0101)	(0.0270)
N	865079	865079	865079	865079
Time fixed effects?	Yes	Yes	Yes	Yes
Region fixed effects?	Yes	Yes	Yes	Yes

Sample of active and passive searchers, 1996-2019

Incl. controls for education, quartic for age, gender, race, and marital status

- Search is less effective when aggregate search is higher
- Penalty to to purely passive search lower when aggregate search is higher

Reevaluating the theory

What went wrong

- Reject restriction from perfect substitution of active/passive
  - Perfect substitutes $\iff$ CES with elasticity of subst. $= \infty$
- Show that efficiency of active search diminishing in aggr. active search
  - For CES, requires elasticity of subst. $< \infty$
- Next: estimate parameters of CES over active/passive ratio with unrestricted elasticity

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- Microfoundations?

CES aggregator for search effort

- Aggregate search effort s_t given by CES aggregator over $s_{A,t}$ and $s_{P,t}$

$$s_t = \left(\omega\, {s_{A,t}}^{\rho} + (1-\omega) s_{P,t}^{\rho}\right)^{\frac{1}{\rho}}$$

- Aggregate active & passive search satisfy

$$s_{A,t} = \int^{\check{\varsigma}_t} s_{i,t}^{A} d\Gamma_t^u = (\Gamma_t^u(\check{\varsigma}_t) u_t) \cdot \bar{s}_{A,t}^{*}, \quad s_{P,t} = \int d\Gamma_t^u = u_t$$

- $ME_{A,t}$ and $ME_{P,t}$ are marginal efficiencies of active and passive search

$$ME_{A,t} = \frac{\partial s_t}{\partial s_{A,t}} = \omega \cdot \left(\frac{s_t}{s_{A,t}}\right)^{1-\rho}, \quad ME_{P,t} = \frac{\partial s_t}{\partial s_{P,t}} = (1-\omega) \cdot \left(\frac{s_t}{s_{P,t}}\right)^{1-\rho}$$

What is a CES search aggregator?

- Equivalence: separate submarkets for active and passive search

$$m_t(s_t, v_t) = m_t\left(ME_{A,t} \cdot s_{A,t}, \alpha_t \cdot v_t\right) + m_t\left(ME_{P,t} \cdot s_{P,t}, (1-\alpha_t) \cdot v_t\right)$$

with

$$\alpha_t = \alpha(s_{A,t}/s_{P,t}) = \frac{ME_{A,t} \cdot s_{A,t}}{s_t} = \frac{s_{A,t}^{\rho}}{s_{A,t}^{\rho} + s_{P,t}^{\rho}}, \quad \rho \leq 1$$

- Vacancies distributed across matching functions to equalize tightness

$$\theta_{A,t} = \frac{\alpha\left(s_{A,t}/s_{P,t}\right) \cdot v_t}{ME_{A,t} \cdot s_{A,t}} = \frac{v_t}{s_t} = \theta_t$$

- Vacancy share of active search α_t analogous to factor share

Returns to search

- The job-finding probability $f_{i,t}$ of a worker with search efficiency $s_{i,t}$ is

$$f_{i,t} = s_{i,t} \cdot \left(\frac{m_t \left(s_t, v_t \right)}{s_t} \right)$$

- The search efficiency s_i of a worker supplying $s^A_{i,t}$

$$s_{i,t} = ME_{A,t} \cdot s^A_{i,t} + ME_{P,t} \cdot 1$$

  by linear homogeneity of the CES search aggregator

- Nests prior case when $\rho = 1$:

$$s_{i,t} = \Big(\underbrace{\omega}_{\equiv \alpha_1} s^A_{i,t} + \underbrace{(1 - \omega)}_{\equiv \alpha_0} \Big).$$

Restriction from theory, redux

- Relative job-finding probabilities, active vs. passive search

$$\frac{\bar{f}_t^A}{\bar{f}_t^P} = \frac{\left(ME_{A,t} \cdot \bar{s}_t^{A,*} + ME_{P,t}\right)\left(\frac{m_t(s_t,v_t)}{s_t}\right)}{ME_{P,t}\left(\frac{m_t(s_t,v_t)}{s}\right)}$$

$$= \left(\frac{\omega}{1-\omega}\right)\left(\frac{1}{\Gamma_t^u(\check{\varsigma}_t)\bar{s}_t^{A,*}}\right)^{1-\rho} \cdot \bar{s}_t^{A,*} + 1$$

- Thus,

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1) \cdot \log \Gamma_t^u(\check{\varsigma}_t) + \rho \cdot \log \bar{s}_t^{A,*}$$

- Return to data, estimate ω and ρ, test restriction in ρ

Regression estimates

	(1)	(2)	(3)	(4)	(5)	(6)
Fraction searching	-3.331	-1.119	-2.177	-3.899	-1.555	-2.500
	(0.7571)	(1.0283)	(0.2978)	(1.3595)	(1.5344)	(0.4102)
# of search methods	–	-1.842	-1.177	–	-1.830	-1.500
		(0.5742)	(0.2978)		(0.5757)	(0.4102)
cons	-0.555	0.624	0.145	-0.738	0.481	0.107
	(0.2508)	(0.4292)	(0.1547)	(0.4851)	(0.6072)	(0.1725)
Additional controls	Time trend			Time trend + unempl. rate		
Constrain $\beta_{\text{Frac}} - 1 = \beta_{\#}$?	N/A	No	Yes	N/A	No	Yes
F-test	$p(\rho = 1)$ =0.0000	$p(\beta_{\text{Frac}} + 1 = \beta_{\#})$ =0.2465	$p(\rho = 1)$ =0.0000	$p(\rho = 1)$ =0.0004	$p(\beta_{\text{Frac}} + 1 = \beta_{\#})$ =0.5004	$p(\rho = 1)$ =0.0000
N	286	286	286	286	286	286
Implied ρ	-3.3310	–	-2.1767	-3.8987	–	-2.4995
Implied ω	0.3646		0.5362	0.3646		0.5268

CPS, 1996-20019

Takeaway

$$\log\left(\frac{\bar{f}_t^A}{\bar{f}_t^P} - 1\right) = \log\left(\frac{\omega}{1-\omega}\right) + (\rho - 1)\cdot\log\Gamma_t^u(\check{\varsigma}_t) + \rho\cdot\log\bar{s}_t^{A,*}$$

- Fail to reject restriction $\beta_{\Gamma(\check{\varsigma})} + 1 = \beta_{\bar{s}^{A,*}}$ (i.e., new framework)
- Reject restriction $\rho = 1$ (i.e., existing framework)
- Elasticity of substitution $\frac{1}{1-\rho}$ falls in range $(\frac{1}{5}, \frac{1}{3})$
  - Indicates that active and passive search are complements
  - Substantial scope for diminishing returns

Applications

Applications

1. Cyclicality of active-search vacancy share
   - Active search is countercyclical & elasticity of substitution < 1
   - Thus, marginal efficiency of active search is procyclical, and
   - Vacancy share for active search procyclical
2. Unexplored externality
   - Congestion externality: agents don't internalize impact of active search on job-finding prob.
   - Here: agents don't internalize impact of active search on marginal efficiencies of active and passive search
     - Hosios condition not enough

Appl. 1) Cyclicality of vacancy share

- Diminishing marginal efficiency of active search by $\rho \neq 1$
- But $\rho < 1$ implies "highly" diminishing marginal efficiency
- Recall:

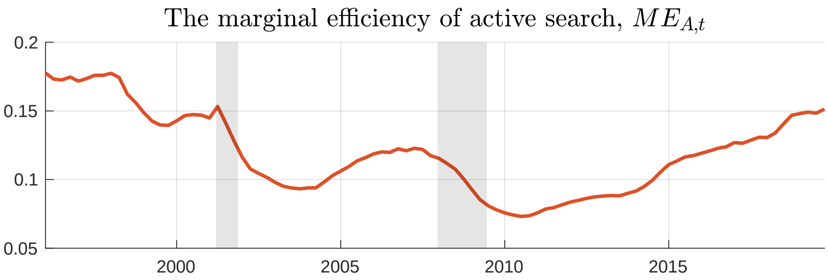
$$m_t(s_t, v_t) = m_t\left(ME_{A,t} \cdot s_{A,t}, \alpha_t \cdot v_t\right) + m_t\left(ME_{P,t} \cdot s_{P,t}, (1-\alpha_t) \cdot v_t\right)$$

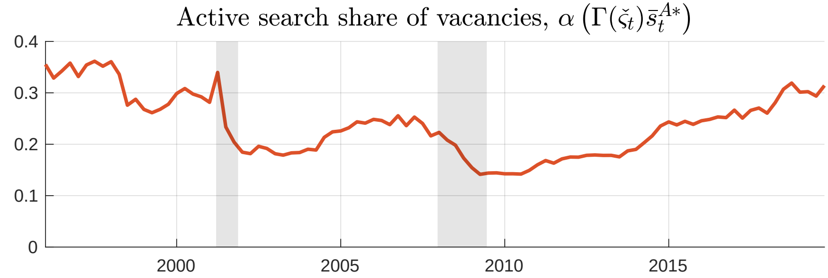
with

$$\alpha_t = \frac{ME_{A,t} \cdot s_{A,t}}{s_t} = \frac{\left(\Gamma_t^u(\check{\varsigma}_t) \cdot \bar{s}_t\right)^\rho}{1 + \left(\Gamma_t^u(\check{\varsigma}_t) \cdot \bar{s}_t\right)^\rho}$$

- Thus, α_t decreasing in $s_{A,t}$ when $\rho < 1$
  - (We estimated $\rho = -2.5$)
- Countercyclical active search $\Rightarrow$ "less important" during recessions

Appl. 1) Cyclicality of vacancy share, cont.





Appl. 2) Unexplored externality

$$rU_i = \max_{s_i^A} \left\{ \frac{b - \varsigma_i \cdot \mathbb{I}\left\{ s_i^A > 0 \right\} - c\left(s_i^A\right)}{\mu} \right.$$
$$\left. + \left(ME_A \cdot s_i^A + ME_P\right) \cdot \left(\frac{m\left(s, v\right)}{s}\right) \cdot \left(V_i - U_i\right) - \dot{U}_i \right\}$$

- Congestion externality: searchers fail to internalize how $s_{A,i}$ affects s
- Here: searchers also fail to internalize how $s_{A,i}$ affects ME_A and ME_P
- $s^*_{A,i} \uparrow \Rightarrow ME_A \downarrow$ and $ME_P \uparrow$

Appl. 2) Unexplored externality, con't

- Optimal search, worker's problem:

$$s^*_{A,i} = (c')^{-1}\left(ME_A \cdot f(\theta) \cdot \lambda_i\right)$$

where λ_i is the marginal value to the HH of having agent i employed

- Optimal search, Planner's problem:

$$s^{SP}_{A,i} = (c')^{-1}\left(ME^{SP}_A \cdot f(\theta^{SP}) \cdot \lambda^{SP}_i + \underbrace{\frac{\partial ME^{SP}_A}{\partial s_A} \cdot cov(s^{SP}_{A,i}, \lambda^{SP}_i)}_{<0}\right)$$

where λ^{SP}_i is the marginal social value of having agent i employed

Conclusion

- Identify restriction from DMP model under perfect substitutability of active and passive search (and reject)
- Evidence of "crowding-out" of active search
  - Active search more effective when aggr. active search lower
  - Formalize with finite elasticity of subst.
- Take model to the data:
  - Formally reject infinite elasticity from data
  - Estimate elasticity < 1
  - Unable to reject unrestricted CES
- Noteworthy implications of diminishing marginal efficiency:
  - Efficiency of active search lower during recessions
  - Unexplored search externality that holds under Hosios condition