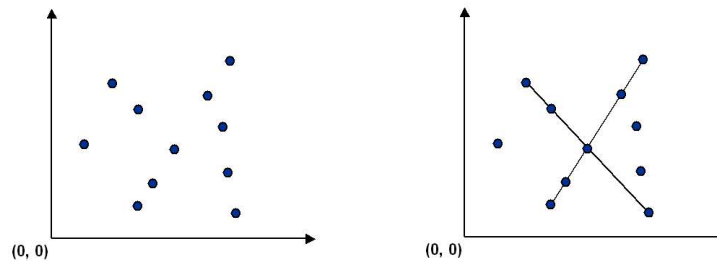




Write a program to recognize line patterns in a given set of points.

Computer vision involves analyzing patterns in visual images and reconstructing the real-world objects that produced them. The process is often broken up into two phases: *feature detection* and *pattern recognition*. Feature detection involves selecting important features of the image; pattern recognition involves discovering patterns in the features. We will investigate a particularly clean pattern recognition problem involving points and line segments. This kind of pattern recognition arises in many other applications such as statistical data analysis.

**The problem.** Given a set of  $n$  distinct points in the plane, find every (maximal) line segment that connects a subset of 4 or more of the points.



**Point data type.** Create an immutable data type `Point` that represents a point in the plane by implementing the following API:

```
public class Point implements Comparable<Point> {
    public Point(int x, int y)                // constructs the point (x, y)

    public void draw()                        // draws this point
    public void drawTo(Point that)            // draws the line segment from this point to that point
    public String toString()                   // string representation

    public int compareTo(Point that)           // compare two points by y-coordinates, breaking ties by x-coordinates
    public double slopeTo(Point that)          // the slope between this point and that point
    public Comparator<Point> slopeOrder()      // compare two points by slopes they make with this point
}
```

To get started, use the data type [Point.java](#), which implements the constructor and the `draw()`, `drawTo()`, and `toString()` methods. Your job is to add the following components.

- The `compareTo()` method should compare points by their  $y$ -coordinates, breaking ties by their  $x$ -coordinates. Formally, the invoking point  $(x_0, y_0)$  is *less than* the argument point  $(x_1, y_1)$  if and only if either  $y_0 < y_1$  or if  $y_0 = y_1$  and  $x_0 < x_1$ .
- The `slopeTo()` method should return the slope between the invoking point  $(x_0, y_0)$  and the argument point  $(x_1, y_1)$ , which is given by the formula  $(y_1 - y_0) / (x_1 - x_0)$ . Treat the slope of a horizontal line segment as positive zero; treat the slope of a vertical line segment as positive infinity; treat the slope of a degenerate line segment (between a point and itself) as negative infinity.
- The `slopeOrder()` method should return a comparator that compares its two argument points by the slopes they make with the invoking point  $(x_0, y_0)$ . Formally, the point  $(x_1, y_1)$  is *less than* the point  $(x_2, y_2)$  if and only if the slope  $(y_1 - y_0) / (x_1 - x_0)$  is less than the slope  $(y_2 - y_0) / (x_2 - x_0)$ . Treat horizontal, vertical, and degenerate line segments as in the `slopeTo()` method.
- Do *not* override the `equals()` or `hashCode()` methods.

**Corner cases.** To avoid potential complications with integer overflow or floating-point precision, you may assume that the constructor arguments  $x$  and  $y$  are each between 0 and 32,767.

**Line segment data type.** To represent line segments in the plane, use the data type [LineSegment.java](#), which has the following API:

```
public class LineSegment {
    public LineSegment(Point p, Point q)    // constructs the line segment between points p and q
    public void draw()                       // draws this line segment
    public String toString()                 // string representation
}
```

**Brute force.** Write a program `BruteCollinearPoints.java` that examines 4 points at a time and checks whether they all lie on the same line segment, returning all such line segments. To check whether the 4 points  $p$ ,  $q$ ,  $r$ , and  $s$  are collinear, check whether the three slopes between  $p$  and  $q$ , between  $p$  and  $r$ , and between  $p$  and  $s$  are all equal.

```
public class BruteCollinearPoints {
    public BruteCollinearPoints(Point[] points) // finds all line segments containing 4 points
    public int numberOfSegments() // the number of line segments
    public LineSegment[] segments() // the line segments
}
```

The method `segments()` should include each line segment containing 4 points exactly once. If 4 points appear on a line segment in the order  $p \rightarrow q \rightarrow r \rightarrow s$ , then you should include either the line segment  $p \rightarrow s$  or  $s \rightarrow p$  (but not both) and you should not include *subsegments* such as  $p \rightarrow r$  or  $q \rightarrow r$ . For simplicity, we will not supply any input to `BruteCollinearPoints` that has 5 or more collinear points.

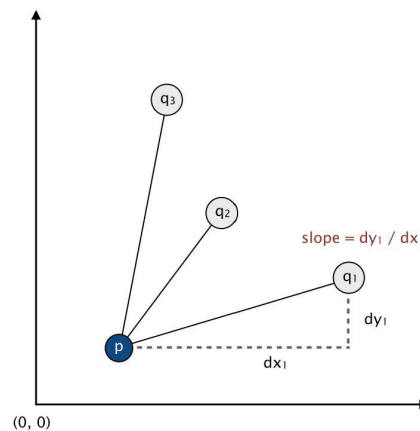
**Corner cases.** Throw an `IllegalArgumentException` if the argument to the constructor is `null`, if any point in the array is `null`, or if the argument to the constructor contains a repeated point.

**Performance requirement.** The order of growth of the running time of your program should be  $n^4$  in the worst case and it should use space proportional to  $n$  plus the number of line segments returned.

**A faster, sorting-based solution.** Remarkably, it is possible to solve the problem much faster than the brute-force solution described above. Given a point  $p$ , the following method determines whether  $p$  participates in a set of 4 or more collinear points.

- Think of  $p$  as the origin.
- For each other point  $q$ , determine the slope it makes with  $p$ .
- Sort the points according to the slopes they makes with  $p$ .
- Check if any 3 (or more) adjacent points in the sorted order have equal slopes with respect to  $p$ . If so, these points, together with  $p$ , are collinear.

Applying this method for each of the  $n$  points in turn yields an efficient algorithm to the problem. The algorithm solves the problem because points that have equal slopes with respect to  $p$  are collinear, and sorting brings such points together. The algorithm is fast because the bottleneck operation is sorting.



Write a program `FastCollinearPoints.java` that implements this algorithm.

```
public class FastCollinearPoints {
    public FastCollinearPoints(Point[] points) // finds all line segments containing 4 or more points
    public int numberOfSegments() // the number of line segments
    public LineSegment[] segments() // the line segments
}
```

The method `segments()` should include each *maximal* line segment containing 4 (or more) points exactly once. For example, if 5 points appear on a line segment in the order  $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$ , then do not include the subsegments  $p \rightarrow s$  or  $q \rightarrow t$ .

**Corner cases.** Throw an `IllegalArgumentException` if the argument to the constructor is `null`, if any point in the array is `null`, or if the argument to the constructor contains a repeated point.

**Performance requirement.** The order of growth of the running time of your program should be  $n^2 \log n$  in the worst case and it should use space proportional to  $n$  plus the number of line segments returned. `FastCollinearPoints` should work properly even if the input has 5 or more collinear points.

**Sample client.** This client program takes the name of an input file as a command-line argument; read the input file (in the format specified below); prints to standard output the line segments that your program discovers, one per line; and draws to standard draw the line segments.

```
public static void main(String[] args) {

    // read the n points from a file
    In in = new In(args[0]);
    int n = in.readInt();
    Point[] points = new Point[n];
    for (int i = 0; i < n; i++) {
        int x = in.readInt();
        int y = in.readInt();
        points[i] = new Point(x, y);
    }

    // draw the points
    StdDraw.enableDoubleBuffering();
    StdDraw.setXscale(0, 32768);
    StdDraw.setYscale(0, 32768);
    for (Point p : points) {
        p.draw();
    }
    StdDraw.show();

    // print and draw the line segments
    FastCollinearPoints collinear = new FastCollinearPoints(points);
    for (LineSegment segment : collinear.segments()) {
        StdOut.println(segment);
        segment.draw();
    }
    StdDraw.show();
}
```

**Input format.** We supply several sample input files (suitable for use with the test client above) in the following format: An integer  $n$ , followed by  $n$  pairs of integers  $(x, y)$ , each between 0 and 32,767. Below are two examples.

% cat input6.txt	% cat input8.txt
6	8
19000 10000	10000 0
18000 10000	0 10000
32000 10000	3000 7000
21000 10000	7000 3000
1234 5678	20000 21000
14000 10000	3000 4000
	14000 15000
	6000 7000

```
% java-algs4 BruteCollinearPoints input8.txt
(10000, 0) -> (0, 10000)
(3000, 4000) -> (20000, 21000)

% java-algs4 FastCollinearPoints input8.txt
(3000, 4000) -> (20000, 21000)
(0, 10000) -> (10000, 0)

% java-algs4 FastCollinearPoints input6.txt
(14000, 10000) -> (32000, 10000)
```

**Web submission.** Submit a .zip file containing only `BruteCollinearPoints.java`, `FastCollinearPoints.java`, and `Point.java`. We will supply `LineSegment.java` and `algs4.jar`. You may not call any library functions except those in `java.lang`, `java.util`, and `algs4.jar`. You may use library functions in `java.util` only if they have already been introduced in the course. For example, you may use `Arrays.sort()`, but not `java.util.HashSet`.

