Problem Set 4
Due Monday, April 27 at the beginning of lecture
ECON 381-2, Northwestern University, Spring 2015

You may work in groups of 2 or 3 as long as each group member turns in their own handwritten copy and all group members are clearly indicated at the top of every copy. No credit is given if no work is shown. For Stata/R problems include both your code and the output.

1. Suppose that for $t = 1, 2$,

$$Y_{it} = \alpha + \beta X_{it} + A_i + V_{it},$$

   where $(Y_{i1}, Y_{i2}, X_{i1}, X_{i2}, W_i), i = 1, \ldots, N$ is an i.i.d. sample with bounded fourth moments. Assume that $\text{Cov}(X_{i2}, W_i) \neq \text{Cov}(X_{i1}, W_i)$ and that $\mathbb{E}(V_{it} \mid W_i) = 0$ for $t = 1, 2$. Find a consistent estimator of β and justify its consistency.

2. Consider the linear instrumental variables model

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + U_i, \tag{1}$$

   where we observe an i.i.d. sample (Y_i, X_i, W_i, Z_i) with bounded fourth moments, and we assume that $\mathbb{E}[U_i|W_i, Z_i] = 0$.

   (a) Suppose that we want to add an interaction term X_iW_i to the equation in (1) and hence need another excluded instrument. Would Z_iX_i be an appropriate instrument? Would Z_iW_i? Would $Z_iW_i^2$? Of these three instruments, which are you most likely to prefer using, and why?
   Note: I am looking for intuitive arguments here, even if they are not 100% airtight in all cases. Do not attempt to formally verify the rank condition or derive the asymptotic variance, since this will be quite tedious.

   (b) Suppose that we want to add an X_i^2 term to the equation in (1) and hence need another excluded instrument. Would Z_i^2 be an appropriate instrument? Would Z_i^3? Would W_i^2? Of these three instruments, which are you most likely to prefer using, and why? Determine a situation in which you know for sure that none of these proposed instruments would be appropriate.
   Note: Same as in part (a).

3. Suppose that $Z \in \{0, 1\}$ is a binary random variable.

   (a) Show that for any random variables X and Y,

$$\frac{\mathbb{E}(Y \mid Z = 1) - \mathbb{E}(Y \mid Z = 0)}{\mathbb{E}(X \mid Z = 1) - \mathbb{E}(X \mid Z = 0)} = \frac{\text{Cov}(Y, Z)}{\text{Cov}(X, Z)}.$$

(b) Now suppose we have a sample (Y_i, X_i, Z_i) for $i = 1, \ldots, N$. Let $\overline{Y}_z$ be the mean of Y_i for observations with $Z_i = z$ for $z = 0, 1$ and similarly for $\overline{X}_z$. That is,

$$\overline{Y}_1 \equiv \frac{1}{N_1} \sum_{i=1}^{N} Y_i Z_i \qquad \text{and} \qquad \overline{Y}_0 \equiv \frac{1}{N_0} \sum_{i=1}^{N} Y_i (1 - Z_i),$$

where $N_1 = \sum_{i=1}^{N} Z_i$ is the number of observations with $Z_i = 1$ and $N_0 = N - N_1$ is the number of observations with $Z_i = 0$. Let $\widehat{\beta}_{iv}$ be the coefficient on X_i in an instrumental variables regression of Y_i on X_i (and a constant) that uses Z_i as an instrument for X_i. Show that

$$\widehat{\beta}_{iv} = \frac{\overline{Y}_1 - \overline{Y}_0}{\overline{X}_1 - \overline{X}_0}.$$

4. Suppose that firm i has the Cobb-Douglas production function

$$Y_i = A_i L_i^{\beta},$$

where Y_i is output, L_i is labor input, $A_i > 0$ is total factor productivity and $\beta \in (0, 1)$ is the output elasticity of labor. Every firm receives a price of 1 for each unit of Y_i and pays a price P_i for each unit of labor L_i. Assume that the price P_i that firm i pays its workers is independent of total factor productivity A_i.

Suppose that we observe an i.i.d. sample (Y_i, L_i, P_i) for a cross-section of firms $i = 1, \ldots, N$ where all variables have finite fourth moments and strictly positive variances. Assume that firms choose labor L_i to maximize profits. Find a consistent estimator of β and justify its consistency.