Poisson Regression with Robust Standard Errors

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The Poisson Distribution

A random variable Y is said to have a Poisson distribution with parameter λ if its probability is given by the probability mass function

$$Pr(Y = y) = \frac{e^{-\lambda}\lambda^{y}}{y!}$$

for $\lambda > 0$ and y = 0, 1, 2, ...

The mean and variance of this distribution can be shown to be

$$E(Y) = Var(Y) = \lambda$$

Introduction to Poisson Regression

In Poisson Regression:

- Model used when the desired response variable, Y_i, is a count (eg. Number of vehicle accidents per year, number of visits to a website over a certain time span, etc)
- We can also have the response variable be Y_i/t, the rate at which the event happens with t being an interval representing time, space, or some other grouping of interest

Introduction to Poisson Regression

- The regression model with the log link function:

$$log(\lambda_i) = \beta_0 + \beta_1 x_{i1} + ... + \beta_p x_{ip} = X_i\beta$$

  where $E(Y_i) = \lambda_i = e^{X_i\beta}$

- Predictor variables are estimated by maximizing the likelihood function:

$$L(\beta) = \prod_{i=1}^{n} f(Y_i) = \prod_{i=1}^{n} \frac{e^{-\lambda_i}\lambda_i^{Y_i}}{Y_i!}$$

Poisson Regression with Robust Standard Errors

Our focus, however, is to discuss **Poisson Regression with Robust Standard Errors**

- Modified Poisson Regression that can work with response variables with binary outcomes
- Addresses problems with overdispersion

Poisson Regression with Robust Standard Errors

Main problem is with the Poisson assumption of

$$E(Y_i) = Var(Y_i)$$

- With binomial data, Poisson regression usually underestimates variance of data
- We need a way to address this problem when making inferences

Poisson Regression with Robust Standard Errors

The only adjustments to make:

- We keep the maximum likelihood estimates of β
- The standard errors, however, are replaced with "robust" standard errors from the sandwich estimator

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