Introduction to Poisson Regression with Robust Standard Errors - Part 4

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11/29/2018

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Objectives of the Video

- Discuss the motivation for the Poisson Regression model with Robust Standard Errors
- Understand the difference between this model and the "regular" Poisson Regression model
- Understand the interpretation of the model's parameters

Intro to Poisson Regression with Robust Standard Errors

What is Poisson Regression with **Robust Standard Errors**?

$\rightarrow$ Modified Poisson Regression model that can work with outcomes that are binary

The Motivation

Many real-world scenarios can be modeled with binary regression

- Health-related problems such as association of disease and certain factors
- Widely used in fields such as Epidemiology and Public Health
- Allows us to estimate the "Relative Risk"

The Framework of the Model

The framework for the regression model is exactly the same as the "regular" Poisson Regression Model:

$$log[E(Y_i)] = \beta_0 + \beta_1 x_{i1} + ... + \beta_p x_{ip} = X_i \beta$$

- Log link function for our GLM
- β's are our predictor variables
- $E(Y_i) = \lambda_i$, the expected value of our output variable

Adjustments to Make

Why do we need to modify the model to work with binary data?

Main problem is with the Poisson assumption of

$$E(Y_i) = Var(Y_i)$$

- With binomial data, Poisson regression usually overestimates the variance of data
- For binomial case, the mean is usually greater than the variance

How the Variance is Overestimated

- For the binomial distribution:

$$E(X) = np, Var(X) = np(1-p)$$
$$\implies E(X) > Var(X)$$

- Under the Poisson Regression model we assume:

$$Var(Y_i) = E(Y_i)$$

$\implies$ The model overestimates the true variance

The Robust Standard Errors Estimator

The only adjustment to make:

- The standard errors of the estimated predictors, $\hat{\beta}$, are replaced with "robust" standard errors (also known as Huber-White Standard Errors)

$$Var(\hat{\beta}) = (X^T X)^{-1} X^T \Sigma X (X^T X)^{-1}$$

where Σ is the covariance matrix of the residuals

Implementing the Robust Standard Errors

$$Var(\hat{\beta}) = (X^T X)^{-1} X^T \Sigma X (X^T X)^{-1}$$

- The "sandwich" name comes from the appearance of the equation
- Simple to implement/acquire through software such as R with the "sandwich" library

Differences in the Models

The estimates of the predictor variables are acquired in the same way with the Likelihood Function. As a result:

- The values of the $\hat{\beta}$'s are the exact same
- Their estimated standard errors will differ due to the use of the Sandwich Estimator
- Their interpretations change to match the data being analyzed

Interpretation of the Predictors

Note the derivation is exactly the same:

$$
\begin{gathered}
log[E(Y_i|x+1)] - log[E(Y_i|x)] \\
= \beta_0 + \beta_1(x+1) - (\beta_0 + \beta_1 x) \\
= \beta_1 \\
\implies \frac{E(Y_i|x+1)}{E(Y_i|x)} = e^{\beta_1}
\end{gathered}
$$

Interpretation of the Predictors

But we are now working with binomial data

- The expected value of the output is a probability
- The expected value of "regular" Poisson Regression outputs is a count

Parameter Interpretation Example

Ex. Assume our model gives:

$$\frac{E(Y_i|x+1)}{E(Y_i|x)} = 1.15$$

- An increase in x by 1 unit, with every other predictor held constant, increases the **probability of the outcome** by 15 percent

Compare this with the interpretation for the "regular" Poisson Regression model

In the next video

In the next video:

- Tutorial of how to apply the Poisson Regression model with Robust Standard Errors to real data with binary outcomes in R
- Working example of interpreting the output in terms of the theory just discussed