Tutorial problems for Lectures 10 though 11. Due Wednesday October 24th.

Problem 1:

Let's get some experience using CMB power spectra and healpy. First, you'll need to get healpy installed on your computers. Something like "pip install healpy –user" will probably work. I've also provided a sample power spectrum in "example_ps.txt". You can read this with e.g. numpy.loadtxt(). WARNING - as pointed out in class, what people usually report is $l(l+1)C_l/2\pi$, which is the case here. If you actually want to get the variance of an a_{lm}, you need to convert to actual C_l.

part a) What *should* the variance in the map be, given the power spectrum? Don't forget that the Y_{lm} also introduce a scaling.

part b) Generate a set of a_{lm} from this power spectrum yourself. You can turn this into a map with the healpix command alm2map, as shown in class. Make an image of the map (healpy.mollview). Does the map look sensible? Does the variance of the map agree with your answer from part a)? If it disagrees, you might want to see if it's an obvious numerical factor (usually consisting of small powers of 2 an π from the Y_{lm}), and go back and adjust your answer to part a.

part c) You could turn your map back into a_{lm}'s with healpy.map2alm. However, you can do that, plus average the squared a_{lm}'s with healpy.anafast. Plot the power spectrum you get here against the input model. Do they agree?

part d) Healpy also provides functionality to generate a_{lm}'s and maps directly, via healpy.synalm and healpy.synfast. Use one of these to repeat the problem, generating a map that you analyze with anafast and compare to the input.

~~**Problem 2**: In this problem, we'll repeat the essentials of Problem 1, but on a flat sky. A~~ perfectly sensible patch size is 20 by 20 degrees, large enough to be well away from the first peak, but small enough to be reasonably flat.

part a) What is the (rough) conversion between k and l for this map? What is the lowest ℓ you could think of going to?

part b) Up to some l_{max} and corresponding k_{max}, how many a_{lm}'s are there in the sky? How many modes up to k_{max}? From this, show how you would convert the usual CMB power spectrum to a flat-sky one.

part c) Given the max l in the sample power spectrum, there should be a maximum pixel size you can pick to capture all the possible information in the power spectrum (for instance, if your power spectrum goes out to the first peak, 2 degree pixels are clearly too large). What is it? Pick a pixel size much smaller than this, and simulate flat-sky CMB maps. This is the nside option?

part d) Show that the variance in your maps agrees with what you expect from the first problem.

Problem 3: In this problem, we'll investigate the ultimate performance of a perfect 150 GHz detector in space, with the only load coming from the CMB.

part a) The units of the Planck function are ergs per second square cm per hertz per steradian. A typical detector will see a solid angle of aroun a steradian, with a collecting area of λ^2. How many ergs per second per Hz. would hit a detector from the 2.725K CMB?

part b) Let's assume we have a 30 GHz bandwidth for our 150 GHz detector. How many photons per second come into the detector (as a sanity check, I get several billion photons per second per detector)? Are we in the shot-noise or continuous signal regime?

part c) From your answer to part b), work out the noise in μK in one second. How does this compare to the best Planck 143 detectors, with noises in the $50\mu\mathrm{K}s^{1/2}$ range? Could a next-generation CMB satellite get to much deeper maps just by improving detectors?

use flux of centre frequency times bandwidth, no need for real integral