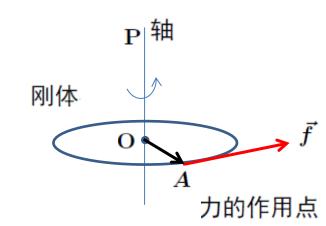
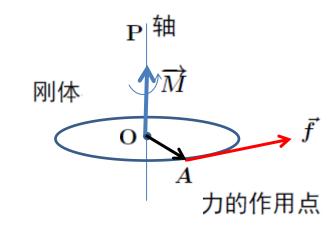
向量的向量积(叉积)



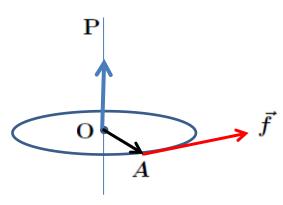
力关于转轴OP的力矩 \overrightarrow{M} 为一个矢量

一个物理问题

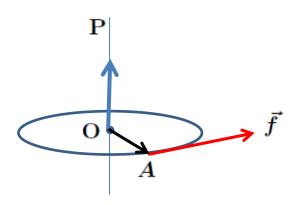
一个物理问题



力关于转轴OP的力矩 \overrightarrow{M} 为一个矢量 其方向垂直于 \overrightarrow{OA} , \overrightarrow{f} 所决定的平面, 并使得 \overrightarrow{OA} , \overrightarrow{f} , \overrightarrow{M} 三者构成右手系, 其大小为 $|\overrightarrow{OA}||\overrightarrow{f}|\sin(\overrightarrow{OA},\overrightarrow{f})$



两个向量产生出第三个与它们都垂直的向量的运算 具有广泛的数学背景和物理背景,所以 定义

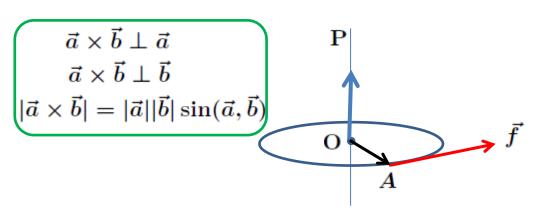


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两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

① $\vec{c} \perp \vec{a}$, $\vec{c} \perp \vec{b}$, ② \vec{a} , \vec{b} , \vec{c} 构成右手系,

 $\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$



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显见向量积具有下述性质: ① \vec{a} , \vec{b} 非零, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$

①
$$\vec{a}$$
, \vec{b} 罪零, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} =$

两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

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$$\vec{a}$$
, \vec{b} 非零, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} =$ ② $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

$$\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$$

显见向量积具有下述性质:

①
$$\vec{a}$$
, \vec{b} 非零, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$

$$2\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$(2)\vec{a} \times b = -b \times \vec{a}$$

$$\label{eq:control_equation} \ensuremath{ \Im } \vec{i} \times \vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{i}, \vec{k} \times \vec{i} = \vec{j}$$

$$\vec{k}$$

定义

两向量
$$\vec{a}$$
, \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

$$\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$$

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显见向量积具有下述性质:

$$\Rightarrow \vec{a} \times \vec{b} =$$

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$$\vec{a}$$
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$$2\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

定义

$$\sqrt{\vec{i}} = \vec{k} \vec{i} \vee \vec{k}$$

$$\vec{3}\vec{i} \times \vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{i}, \vec{k} \times \vec{i} = \vec{j}$$

$$\langle \vec{j} = \vec{k}, \vec{j} \times \vec{k} \rangle$$

$$ec{j}=ec{k},ec{j} imesec{k}$$
 =

$$ec{j}=ec{k},ec{j} imesec{k}=ec{k}$$

 $\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$

$$ec{j}=ec{k},ec{j} imesec{k}$$
 =

$$ec{j}=ec{k},ec{j} imesec{k}=% ec{k} \end{k} \label{k}ec{k}ec{k}ec{k}ec{k}ec{k} \end{k} \label{k} \label{k} \end{k} \label{k} \end{k} \label{k} \end{k} \label{k} \end{k} \label{k} \label{k} \label{k} \label{k} \end{k} \label{k} \la$$

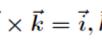
③
$$i \times j = k, j \times k = i, k \times i = j$$

④如图: $S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$

$$j = k, j \times k =$$

$$j = k, j \times k =$$

$$j \times \vec{k} =$$

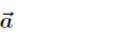








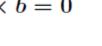


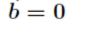


两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足















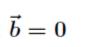




























显见向量积具有下述性质:

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, \vec{b} 非零, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$

$$2\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

定义

$$\vec{3}\vec{i}\times\vec{j}=\vec{k},\vec{j}\times\vec{k}=\vec{i},\vec{k}\times\vec{i}=\vec{j}$$

$$\times j = k, j \times k$$

$$j = k, j \times k =$$

$$\langle j = k, j \times k = 0 \rangle$$

 $\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a},\vec{b})$

$$\vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{k}$$

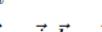
$$j = k, j \times k =$$

③
$$i \times j = k, j \times k = i, k \times i = j$$

④如图: $S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$

$$\vec{j} \times \vec{k} =$$

$$\times \vec{k} =$$



 $(5)\lambda \in \mathbb{R} \Rightarrow (\lambda \vec{a}) \times (\vec{b}) = \lambda (\vec{a} \times \vec{b}) = \vec{a} \times (\lambda \vec{b}),$

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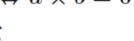
 $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$





























⑥ 三个向量所决定的平行六面体的体积为
$$V=|(\vec{a}\times\vec{b})\cdot\vec{c}|$$
 \vec{c}

两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

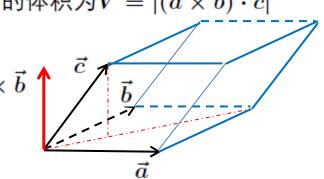
$$\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$$

⑥ 三个向量所决定的平行六面体的体积为
$$V=|(\vec{a}\times\vec{b})\cdot\vec{c}|$$
 $\vec{a}\times\vec{b}$ \vec{c}

两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

$$\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$$

⑥ 三个向量所决定的平行六面体的体积为 $V = |(\vec{a} \times \vec{b}) \cdot \vec{c}|$



三个向量
$$\vec{a}$$
, \vec{b} , \vec{c} 共面的充分必要条件是($\vec{a} \times \vec{b}$) · $\vec{c} = 0$

两向量 \vec{a} , \vec{b} 的向量积 $\vec{a} \times \vec{b}$ 为一个向量 \vec{c} , 它满足

①
$$\vec{c} \perp \vec{a}, \vec{c} \perp \vec{b}$$
 , ② \vec{a} , \vec{b} , \vec{c} 构成右手系,

$$\Im|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}, \vec{b})$$

$$\begin{split} \vec{a} &= \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\} \\ \vec{a} &= a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}, \end{split}$$

$$ec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

 $ec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$\begin{split} \vec{a} &= \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\} \\ \vec{a} &= a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}, \\ \vec{a} &\times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k}) \\ &= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i}) \end{split}$$

$$\vec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

$$ec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \ \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$$

$$\vec{i}$$
 \vec{j}

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{i}) + a_y b_z (-\vec{k}) + a_y b_z \vec{i} + a_z b_y \vec{i} + a_z b_y (-\vec{i})$$

$$= a_{x}b_{y}\vec{k} + a_{x}b_{z}(-\vec{j}) + a_{y}b_{x}(-\vec{k}) + a_{y}b_{z}\vec{i} + a_{z}b_{x}\vec{j} + a_{z}b_{y}(-\vec{i})$$

$$ec{a}=\{a_x,a_y,a_z\},ec{b}=\{b_x,b_y,b_z\}$$

$$\vec{i}$$
 \vec{j}

$$ec{a}=a_xec{i}+a_yec{j}+a_zec{k}, ec{b}=b_xec{i}+b_yec{j}+b_zec{k},$$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i})$$

$$= (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_y)\vec{j} + (a_x b_y - a_y b_x)\vec{k}$$

$$\vec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$$

$$\vec{k}$$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_u \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_u \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i})$$

$$= (a_y b_z - a_z b_y) \vec{i} + (a_z b_x - a_x b_y) \vec{j} + (a_x b_y - a_y b_x) \vec{k}$$

$$= (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_y)\vec{j} + (a_x b_y - a_y b_x)$$

a_1	b_1	c_1
a_2	b_2	c_2
a_3	b_3	c_3

$$=a_1b_2c_3+b_1c_2a_3+c_1a_2b_3-c_1b_2a_3-a_1c_2b_3-b_1a_2c_3$$

$$= a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - c_1b_2a_3 - a_1c_2b_3 - b_1a_2c_3$$

$$= \begin{array}{|c|c|c|c|c|} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{array}$$

$$= a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - c_1b_2a_3 - a_1c_2b_3 - b_1a_2c_3$$

$$= \begin{array}{|c|c|c|c|c|} \hline a_1 & b_1 & c_1 \\ \hline a_2 & b_2 & c_2 \\ \hline a_3 & b_3 & c_3 \\ \hline \end{array}$$

$$\vec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$$

$$\vec{k}$$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_u \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_u \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i})$$

$$= (a_y b_z - a_z b_y) \vec{i} + (a_z b_x - a_x b_y) \vec{j} + (a_x b_y - a_y b_x) \vec{k}$$

$$= (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_y)\vec{j} + (a_x b_y - a_y b_x)$$

$$\vec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i})$$

$$=(a_yb_z-a_zb_y)\vec{i}+(a_zb_x-a_xb_y)\vec{j}+(a_xb_y-a_yb_x)\vec{k}$$

这正是如下三阶行列式的值
$$ec{a} imesec{b}=egin{bmatrix}ec{i}&ec{j}&ec{k}&ec{i}\ a_x&a_y&a_z&a_x\ b_x&b_y&b_z&b_x \end{bmatrix}$$

$$\vec{i}$$
 \vec{j}

$$\vec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i})$$

$$= a_x b_y k + a_x b_z (-j) + a_y b_x (-k) + a_y b_z i + a_z b_x j + a_z b_y (-k)$$

$$= (a_y b_z - a_z b_y) \vec{i} + (a_z b_x - a_x b_y) \vec{j} + (a_x b_y - a_y b_x) \vec{k}$$

$$= (a_y b_z - a_z b_y)i + (a_z b_x - a_x b_y)j + (a_x b_y - a_y b_x)i$$

这正是如下三阶行列式的值

$$\vec{a} = \{a_x, a_y, a_z\}, \vec{b} = \{b_x, b_y, b_z\}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k},$$

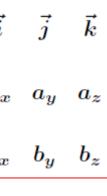
$$\vec{i}$$
 \vec{j}

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$= a_x b_y \vec{k} + a_x b_z (-\vec{j}) + a_y b_x (-\vec{k}) + a_y b_z \vec{i} + a_z b_x \vec{j} + a_z b_y (-\vec{i})$$

$$= (a_{y}b_{z} - a_{z}b_{y})\vec{i} + (a_{z}b_{x} - a_{x}b_{y})\vec{j} + (a_{x}b_{y} - a_{y}b_{x})\vec{k}$$

这正是如下三阶行列式的值
$$ec{i}$$
 $ec{j}$



三阶行列式的性质:

①行列式行列互换(转置), 其值不变,

三阶行列式的性质:

①行列式行列互换(转置), 其值不变,

②行列式两行(列)互换时, 行列式的值变号,

即
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = - \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$$
 $\begin{vmatrix} a_3 & b_3 & c_3 \end{vmatrix}$

例
$$ec{a} = \{2,5,7\}, ec{b} = \{1,2,4\},$$
 则 $ec{a} imes ec{b} =$

例
$$ec{a} = \{2,5,7\}, ec{b} = \{1,2,4\},$$
则 $ec{a} imes ec{b} = egin{bmatrix} ec{i} & ec{j} & ec{k} \\ 2 & 5 & 7 \\ 1 & 2 & 4 \end{bmatrix}$

例
$$ec{a} = \{2, 5, 7\}, ec{b} = \{1, 2, 4\},$$

则
$$ec{a} imesec{b}=egin{bmatrix} ec{i} & ec{j} & ec{k} \ 2 & 5 & 7 \ 1 & 2 & 4 \ \end{bmatrix}$$

$$=20\vec{i}+4\vec{k}+7\vec{j}-5\vec{k}-8\vec{j}-14\vec{i}$$

例
$$ec{a} = \{2,5,7\}, ec{b} = \{1,2,4\},$$

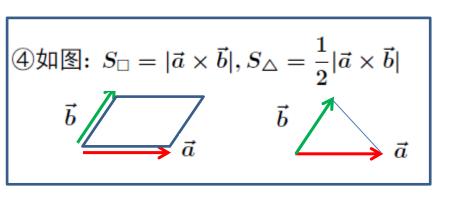
则
$$ec{a} imesec{b}=egin{bmatrix} ec{i} & ec{j} & ec{k} \ & & \ & 2 & 5 & 7 \ & & 1 & 2 & 4 \ \end{pmatrix}$$

$$=20\vec{i}+4\vec{k}+7\vec{j}-5\vec{k}-8\vec{j}-14\vec{i}$$

$$=6\vec{i}-\vec{j}-\vec{k}$$

解:

$$\overrightarrow{AB} = \{2, 3, -1\}, \overrightarrow{AC} = \{-3, -1, 1\},$$



$$\overrightarrow{AB} = \{2, 3, -1\}, \overrightarrow{AC} = \{-3, -1, 1\},$$

$$\overrightarrow{AB} = \{2, 3, -1\}, \overrightarrow{AC} = \{-3, -1, 1\},$$

$$4$$
如图: $S_{\square} = |\overrightarrow{a} \times \overrightarrow{b}|, S_{\triangle} = \frac{1}{2}|\overrightarrow{a} \times \overrightarrow{b}|$
 \overrightarrow{b}
 \overrightarrow{a}
 \overrightarrow{a}

$$\overrightarrow{AB} imes \overrightarrow{AC} = egin{bmatrix} c & J & \kappa \ & & & & & & \ & 2 & 3 & -1 \ & & & & & \ & & & & \ & & & & \ & & & & \ & & & & \ & & & \ & & & \ & & & \ & & \ & & & \ & & \ & & \ & & \ & & \ & & \ & & \ & & \ & & \ & \ & & \ & & \ &$$

$$\overrightarrow{AB} = \{2,3,-1\}, \overrightarrow{AC} = \{-3,-1,1\},$$
 $\overrightarrow{AB} imes \overrightarrow{AC} = egin{bmatrix} i & j & \kappa \ 2 & 3 & -1 \end{bmatrix}$

④如图:
$$S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$\vec{b} = \vec{a}$$

$$\begin{vmatrix} -3 & -1 & 1 \\ -3\vec{i} & 0\vec{k} + 2\vec{k} + 0\vec{k} & 0\vec{k} & \vec{k} \end{vmatrix}$$

$$=3\vec{i}-2\vec{k}+3\vec{j}+9\vec{k}-2\vec{j}-\vec{i}$$

$$\overrightarrow{AB} = \{2,3,-1\}, \overrightarrow{AC} = \{-3,-1,1\},$$
 $\overrightarrow{AB} imes \overrightarrow{AC} = \begin{bmatrix} \imath & \jmath & \kappa \\ 2 & 3 & -1 \end{bmatrix}$

④如图:
$$S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$\vec{b} = \vec{a} \times \vec{b}$$

$$\overrightarrow{AB} imes \overrightarrow{AC} = egin{bmatrix} 2 & 3 & -1 \ -3 & -1 & 1 \end{bmatrix}$$
 $= 3\vec{i} - 2\vec{k} + 3\vec{j} + 9\vec{k} - 2\vec{j} - \vec{i}$

 $= 2\vec{i} + \vec{j} + 7\vec{k}$

$$\overrightarrow{AB} = \{2,3,-1\}, \overrightarrow{AC} = \{-3,-1,1\},$$
 $\overrightarrow{AB} imes \overrightarrow{AC} = \{-3,-1,1\},$

④如图:
$$S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$\vec{b} \qquad \vec{a} \qquad \vec{a}$$

$$\overrightarrow{AB} imes \overrightarrow{AC} = egin{bmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \ & & & & \ & 2 & 3 & -1 \ & & -3 & -1 & 1 \ \end{pmatrix}$$

$$=3\vec{i}-2\vec{k}+3\vec{j}+9\vec{k}-2\vec{j}-\vec{i}$$
 $=2\vec{i}+\vec{j}+7\vec{k}$

$$\overrightarrow{AC} = \sqrt{4 + 1 + 49} = \sqrt{54}$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{4+1+49} = \sqrt{54},$$

解:
$$\overrightarrow{AB} = \{2,3,-1\}, \overrightarrow{AC} = \{-3,-1,1\},$$
 $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \end{bmatrix}$ 到图: $S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$ $-3 \quad -1 \quad 1$

④如图:
$$S_{\square} = |\vec{a} \times \vec{b}|, S_{\triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$\vec{b} / \vec{a}$$

$$\begin{vmatrix} 2 & 3 & -1 \\ -3 & -1 & 1 \end{vmatrix}$$

$$= 3\vec{i} - 2\vec{k} + 3\vec{j} + 9\vec{k} - 2\vec{j} - \vec{i}$$

$$= 2\vec{i} + \vec{j} + 7\vec{k}$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{4+1+49} = \sqrt{54}, \qquad \therefore S_{\triangle} = \frac{1}{2}\sqrt{54}$$

$$=\frac{1}{2}\sqrt{54}$$



