# Econ 139 Scribe Notes - Lecture 3

**General Equilibrium: Edgeworth Box, Welfare Theorems, Social Planner's Problem**

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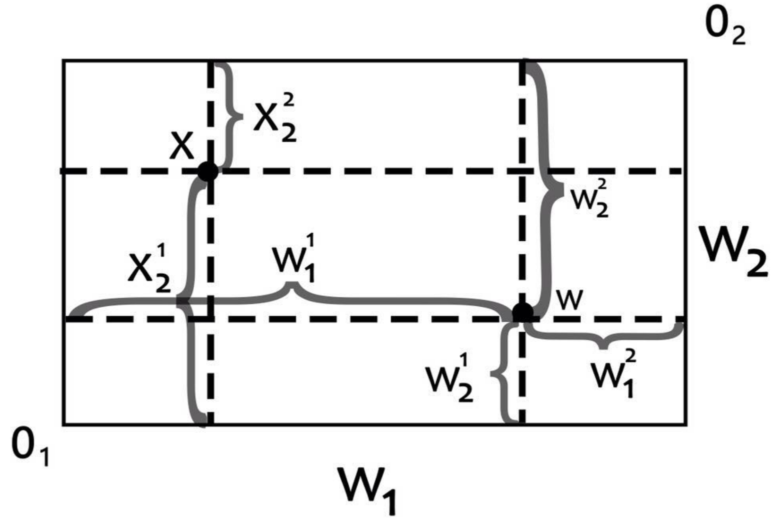
## General Equilibrium

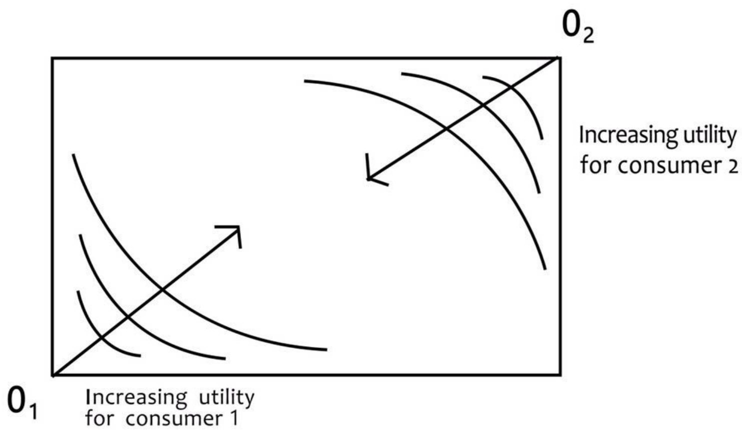
- GE is the theory of the determination of prices and quantities in a system of perfectly competitive markets.
- Restrict ourselves to a pure exchange economy:
    - No production
    - Commodities consumes are those that individuals possess as endowments
    - Agents trade endowments to mutual advantage
- Restrict ourselves to simplest pure exchange economy
    - 2 agents trading
    - 2 commodities

## General Equilibrium -- Edgeworth Box

- 2 consumers: i =1,2
- 2 goods: j =1,2
- Consumers i's consumption vector: $(x_1^i, x_2^i)$
- Consumer i's endowment vector: $\omega^i = (\omega_1^i, \omega_2^i)$
- Endowment vector: $\omega = ((\omega_1^1, \omega_2^1), (\omega_1^2, \omega_2^2))$
- Allocation vector: x = $((x_1^1, x_2^1),(x_1^2, x_2^2))$
- Allocation is feasible of $x_1^1 + x_2^1 \leq \omega_1 = \omega_1^1 + \omega_2^1$, $x_1^2 + x_2^2 \leq \omega_2 = \omega_1^2 + \omega_2^2$

- Allocation called non-wasteful if satisfied with equality:



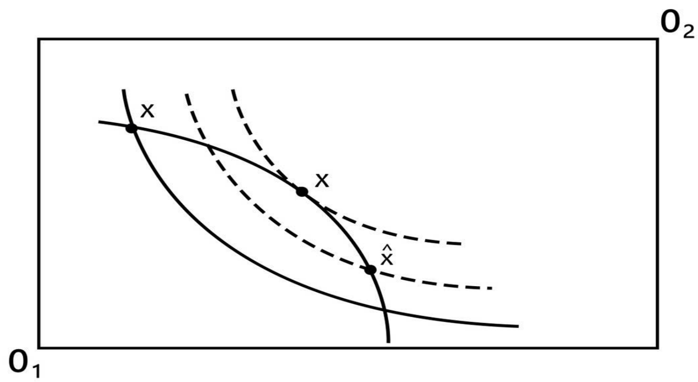



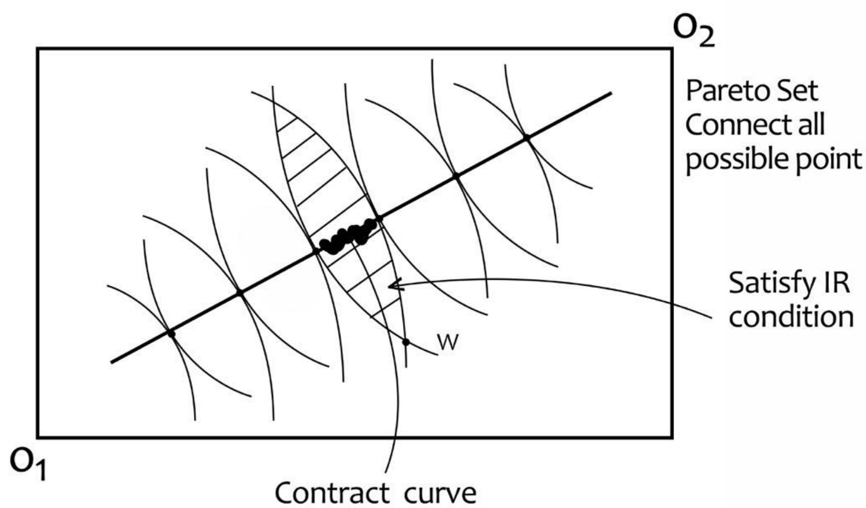

1. Individual rationality (IR)

$$u^i(x_1^i, x_2^i) \geq u^i(\omega_1^i, \omega_2^i),\ \forall_i$$

2. Pareto optimal (PO)

There is no allocation $\hat{x} = ((\hat{x}_1^1, \hat{x}_2^1), (\hat{x}_1^2, \hat{x}_2^2))$ such that $u'(\hat{x}_1^1, \hat{x}_2^1),\ \forall_i$ with strict inequality for at least one agent.



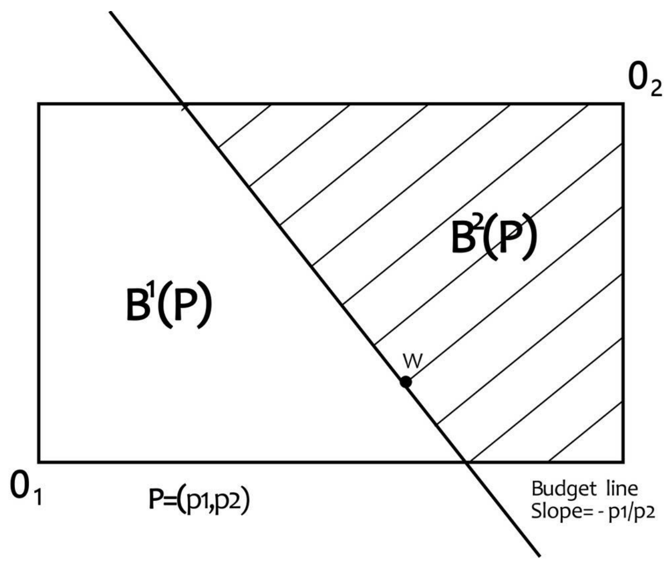



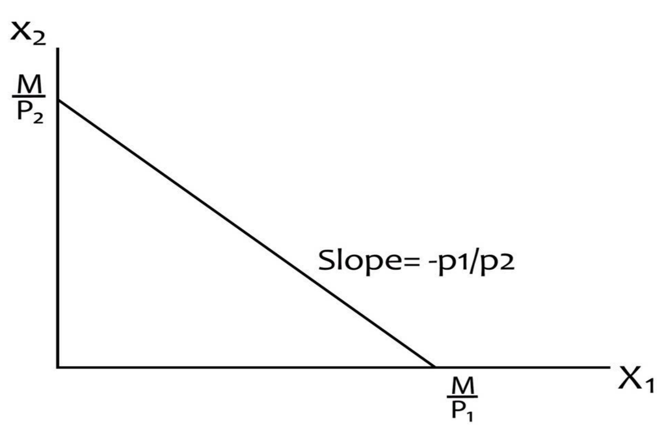

**Walrasian Equilibrium**

- Prices for two goods: P1, P2
- Wealth of consumers:
  - Consumer 1: $P_1\ \omega_1^1 + P_2\ \omega_2^1$

- Consumer 2: $P_1 \ \omega_2^1 + P_2 \ \omega_2^2$

- Consumers face a budget set:
  - Consumer 1: $P_1 \ x_1^1 + P_2 \ x_2^1 \leq P_1 \ \omega_1^1 + P_2 \ \omega_2^1$
  - Consumer 2: $P_1 \ x_2^1 + P_2 \ x_2^2 \leq P_1 \ \omega_2^1 + P_2 \ \omega_2^2$
  - $$\beta^i(P_1, P_2) = \{x^i \in \mathbb{R}_+^2 : P_1 x_1^i + P_2 x_2^i \leq P_1 \omega_1^i + P_2 \omega_2^i\}$$



**Walrasian Equilibrium -- WE:**

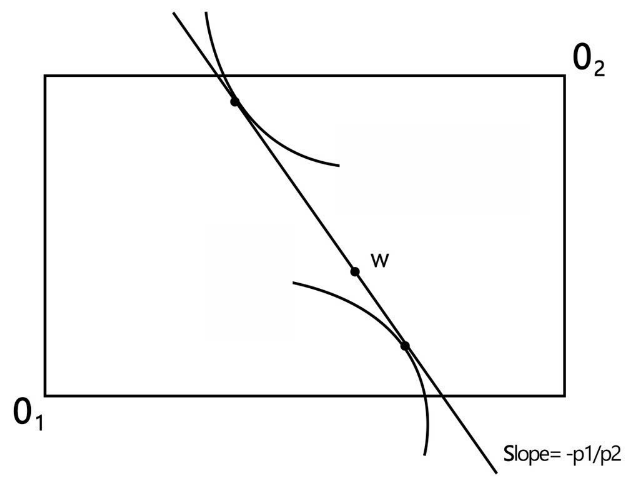
$((x_1^1, x_2^1),(x_1^2, x_2^2),\ P_1,\ P_2)$ is a WE if:

(1) Each agent maximizes utility s.t.a budget constraint

$$(x_1^i, x_2^i) = argmax\ U^{i}(x_2^1, x_2^1)\ s.t.(\bar{x_2^i}, x_2^i) \in B^i(P_1, P_2)\ \forall i$$

(2) All markets clear:

$$x_j^1 + x_j^2 = \omega_j^1, \omega_j^2\ \forall j\ \ (j = 1, 2)$$




$-x_2^1 + x_2^2 > \omega_2^1 + \omega_2^2$ Excess demand for good 2

$x_1^1 + x_2^1 > \omega_1^1 + \omega_2^1$ Excess supply for good 1

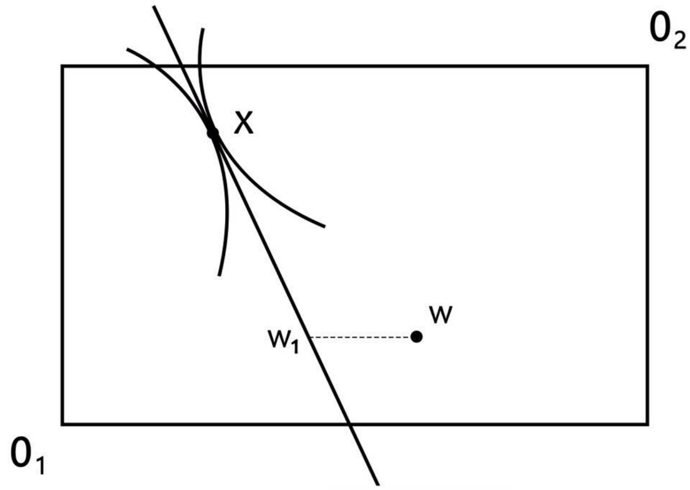
=> price of good 1 relative to good 2 will drop

**First Fundamental Theorem of Welfare Economics**

- All Walrasian Equilibrium belong to the Pareto Set
- All Walrasian Equilibrium are om the contract curve

**Second Fundamental Theorem of Welfare Economics**

- Given convex indifference curves, a planner can achieve any Pareto efficient allocation by redistributing endowment “letting the market work”




- Consider the following constrained optimization:

$$\max_{x^1, x^2} \quad u^1(x_1^1, x_2^1)$$

S.t. $\quad u^2(x_1^2, x_2^2) = \bar{u}$

$$x_1^1 + x_2^1 = w_1 \quad x_1^2 + x_2^2 = w_2$$

$$\max_{x_1^1, x_2^1} \quad u^1(x_1^1, x_2^1)$$

s.t. $\quad u^2 = (w_1 - x_1^1,\ w_2 - x_2^1)$

$$(x_1^1, x_2^1,\ \lambda) = u^1(x_1^1, x_2^1) - \lambda(u^2(w_1 - x_1^1,\ w_2 - x_2^1) - \bar{u})$$

$$\frac{dL}{dx_1^1} = u_1^1 - \lambda\ u_2^1 = 0$$

$$\frac{dL}{dx_2^1} = u_2^1 - \lambda\ u_2^2 = 0$$

$$\frac{dL}{d\lambda} = u^2(w_1 - x_1^1,\ w_2 - x_2^1) - \bar{u} = 0$$

$$\lambda = \frac{u_1^1}{u_2^1} = \frac{u_1^2}{u_2^2} \quad => \quad -\frac{u_1^1}{u_2^1} = -\frac{u_1^2}{u_2^2}$$