Chapter 4

Individual and Market Demand

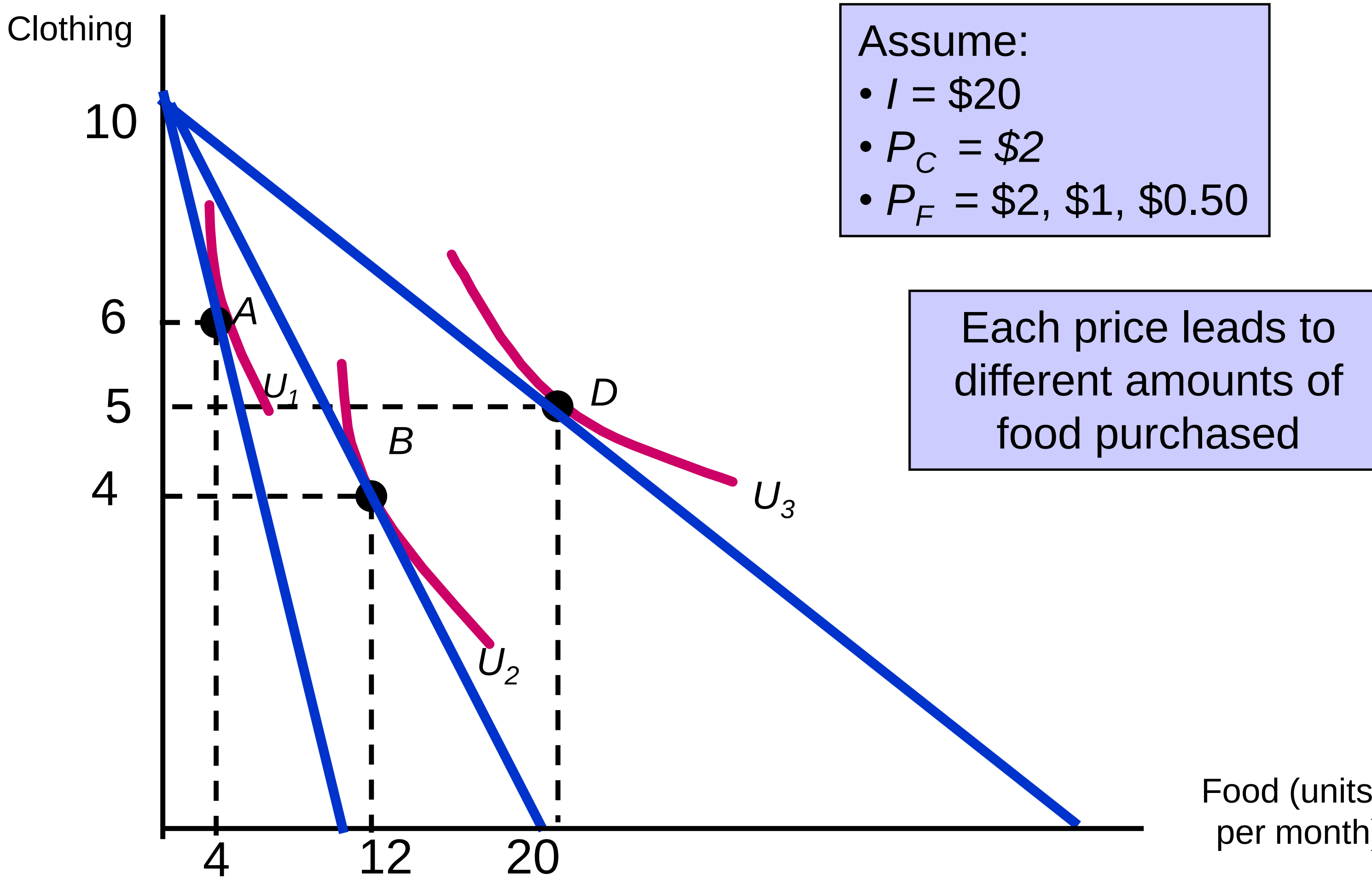
Topics to be Discussed

- Individual Demand
- Income and Substitution Effects
- Market Demand
- Consumer Surplus

Individual Demand

- Price Changes
  - Using the figures developed in the previous chapter, the impact of a change in the price of food can be illustrated using indifference curves
  - For each price change, we can determine how much of the good the individual would purchase given their budget lines and indifference curves

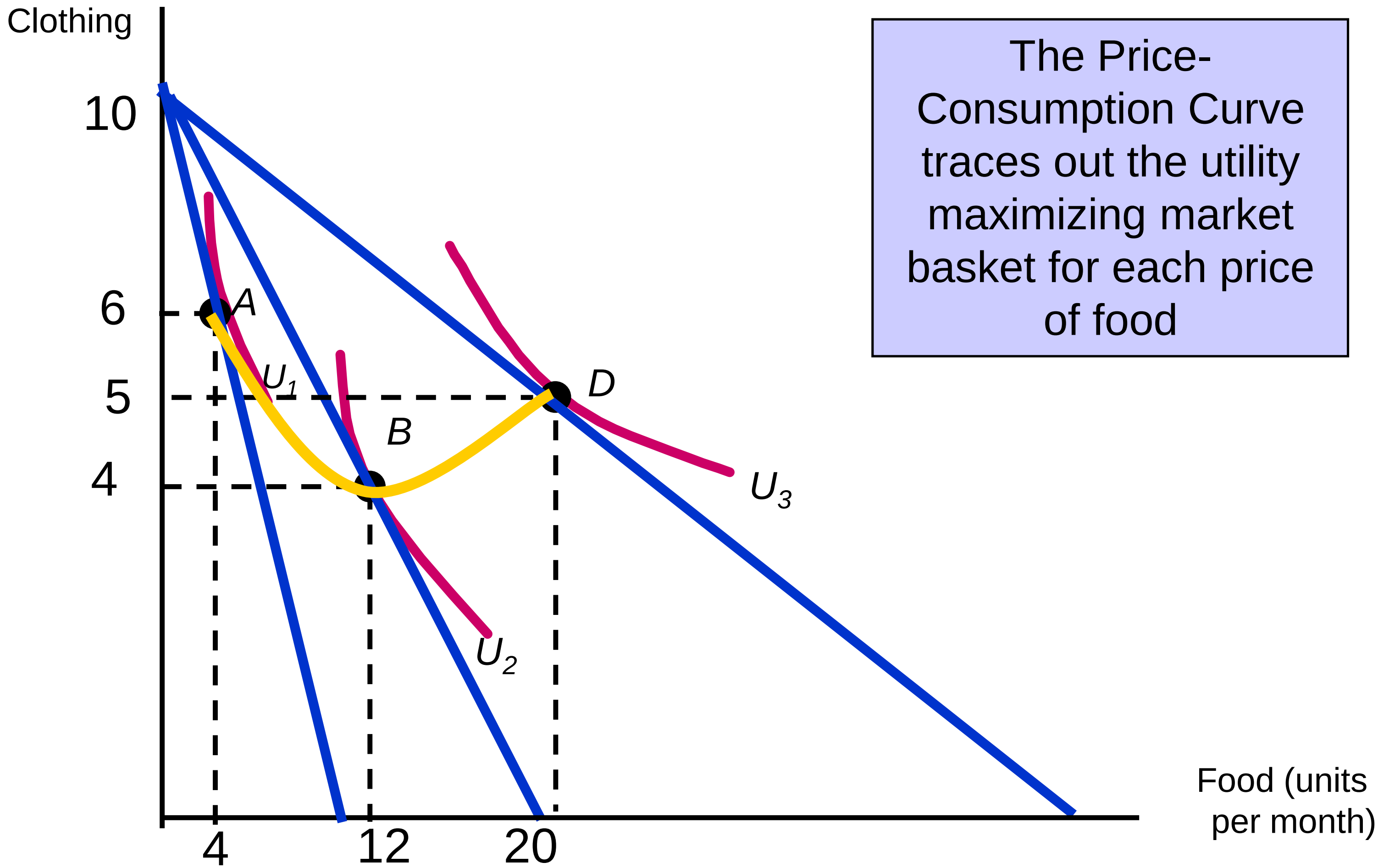
Effect of a Price Change



Assume:
- I = \$20
- P_C = \$2
- P_F = \$2, \$1, \$0.50

Each price leads to different amounts of food purchased

Effect of a Price Change



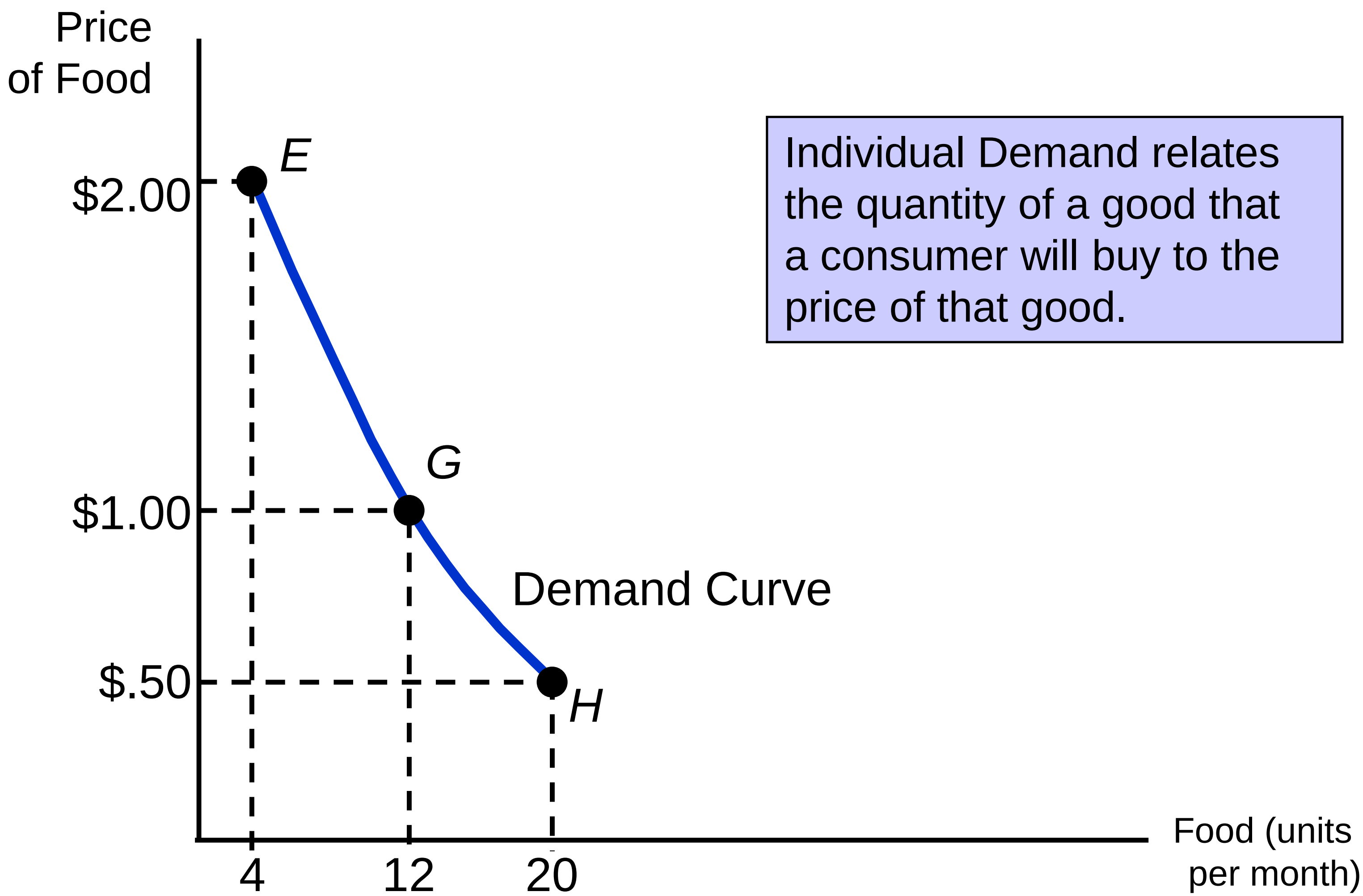
The Price-Consumption Curve traces out the utility maximizing market basket for each price of food

Effect of a Price Change

- By changing prices and showing what the consumer will purchase, we can create a demand schedule and demand curve for the individual
- From the previous example:

Demand Schedule	
P	Q
$2.00	4
$1.00	12
$0.50	20

Effect of a Price Change

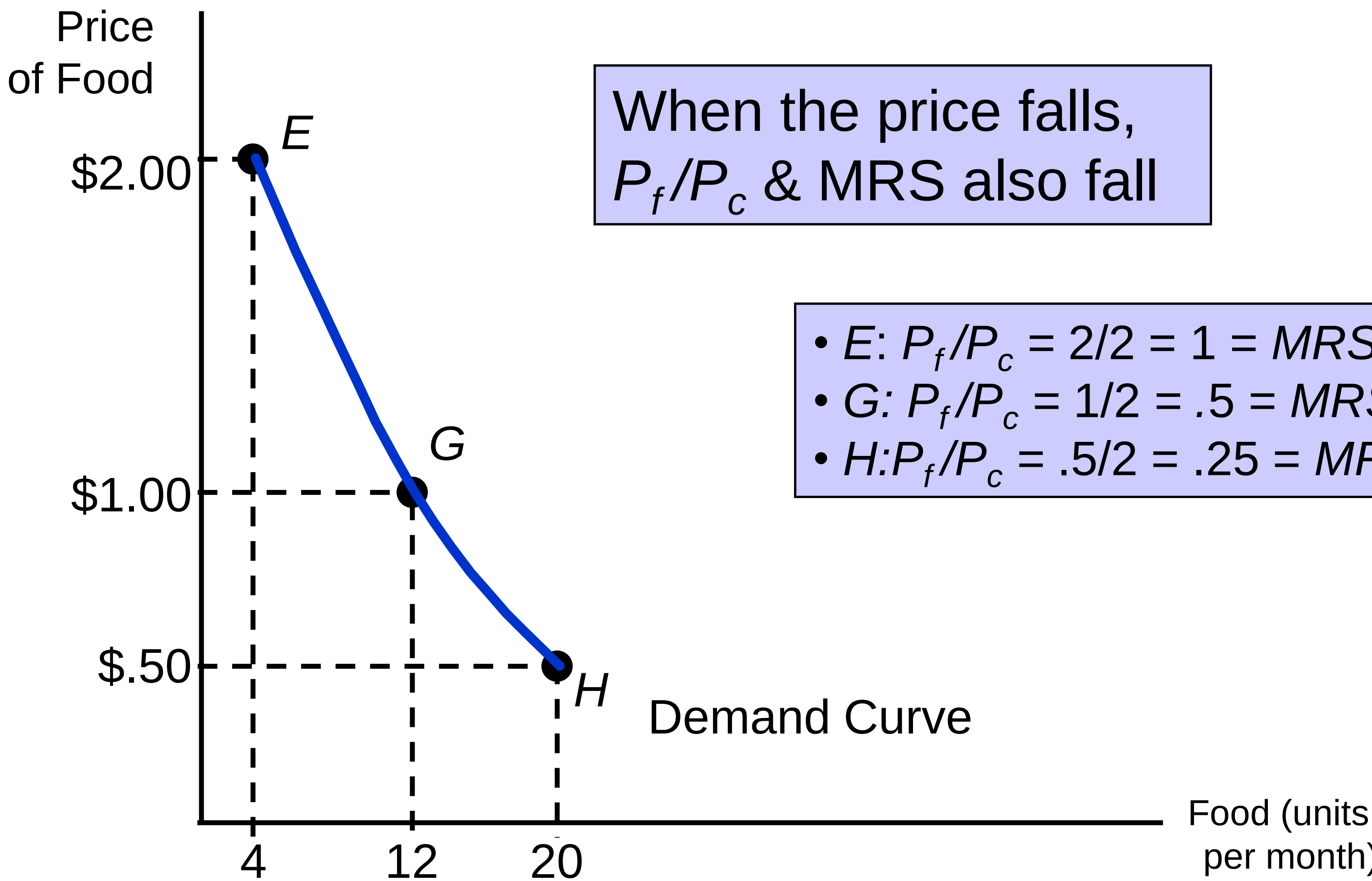


Individual Demand relates the quantity of a good that a consumer will buy to the price of that good.

Demand Curves – Important Properties

- The level of utility that can be attained changes as we move along the curve
- At every point on the demand curve, the consumer is maximizing utility by satisfying the condition that the MRS of food for clothing equals the ratio of the prices of food and clothing

Effect of a Price Change



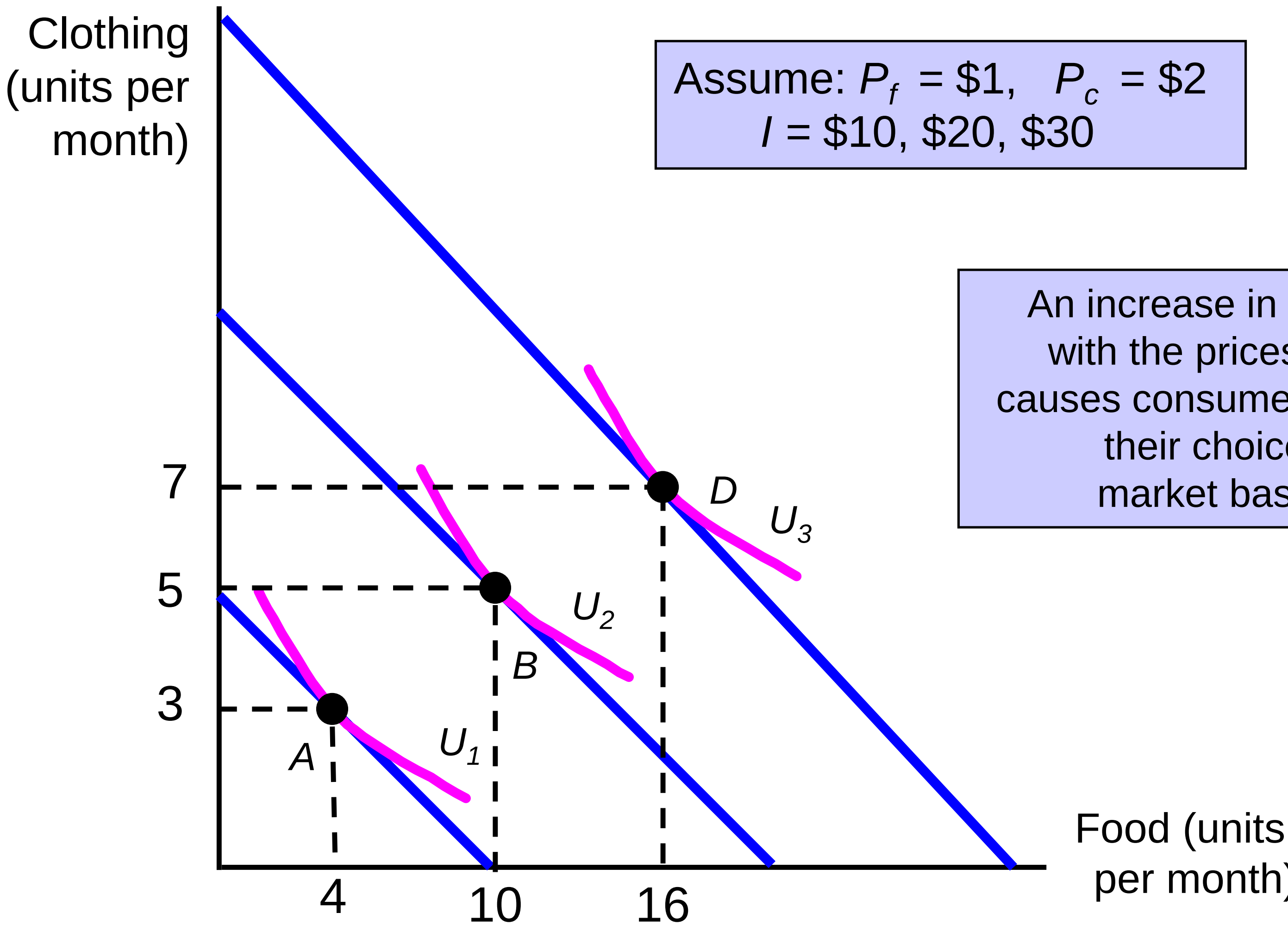
When the price falls, P_f/P_c & MRS also fall

- *E*: $P_f/P_c = 2/2 = 1 = MRS$
- *G*: $P_f/P_c = 1/2 = .5 = MRS$
- *H*: $P_f/P_c = .5/2 = .25 = MRS$

Individual Demand

- Income Changes
  - Using the figures developed in the previous chapter, the impact of a change in the income can be illustrated using indifference curves
  - Changing income, with prices fixed, causes consumers to change their market baskets

Effects of Income Changes



Assume: P_f = \$1, P_c = \$2
I = \$10, \$20, \$30

An increase in income, with the prices fixed, causes consumers to alter their choice of market basket.

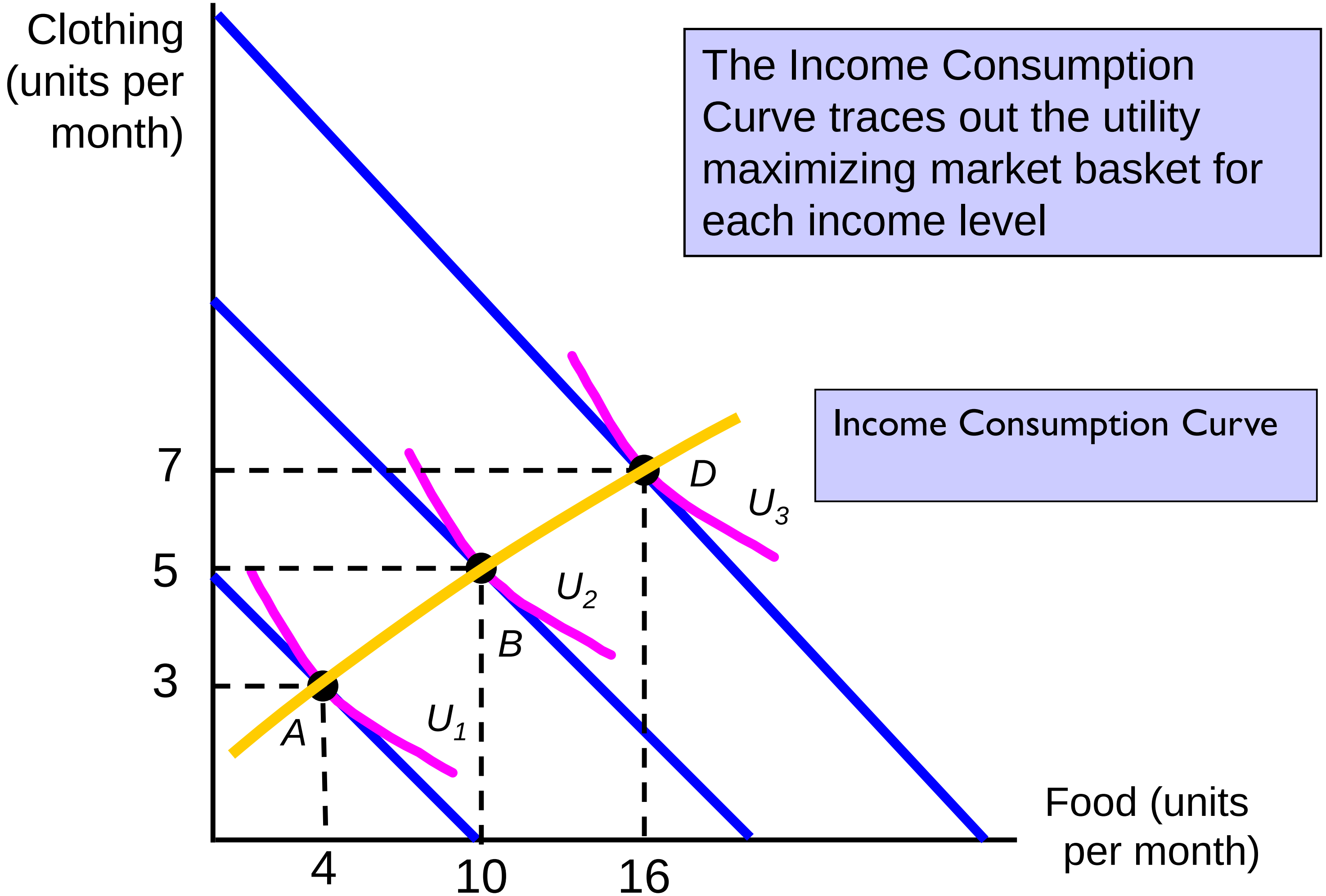
Individual Demand

- Income Changes
  - The income-consumption curve traces out the utility-maximizing combinations of food and clothing associated with every income level

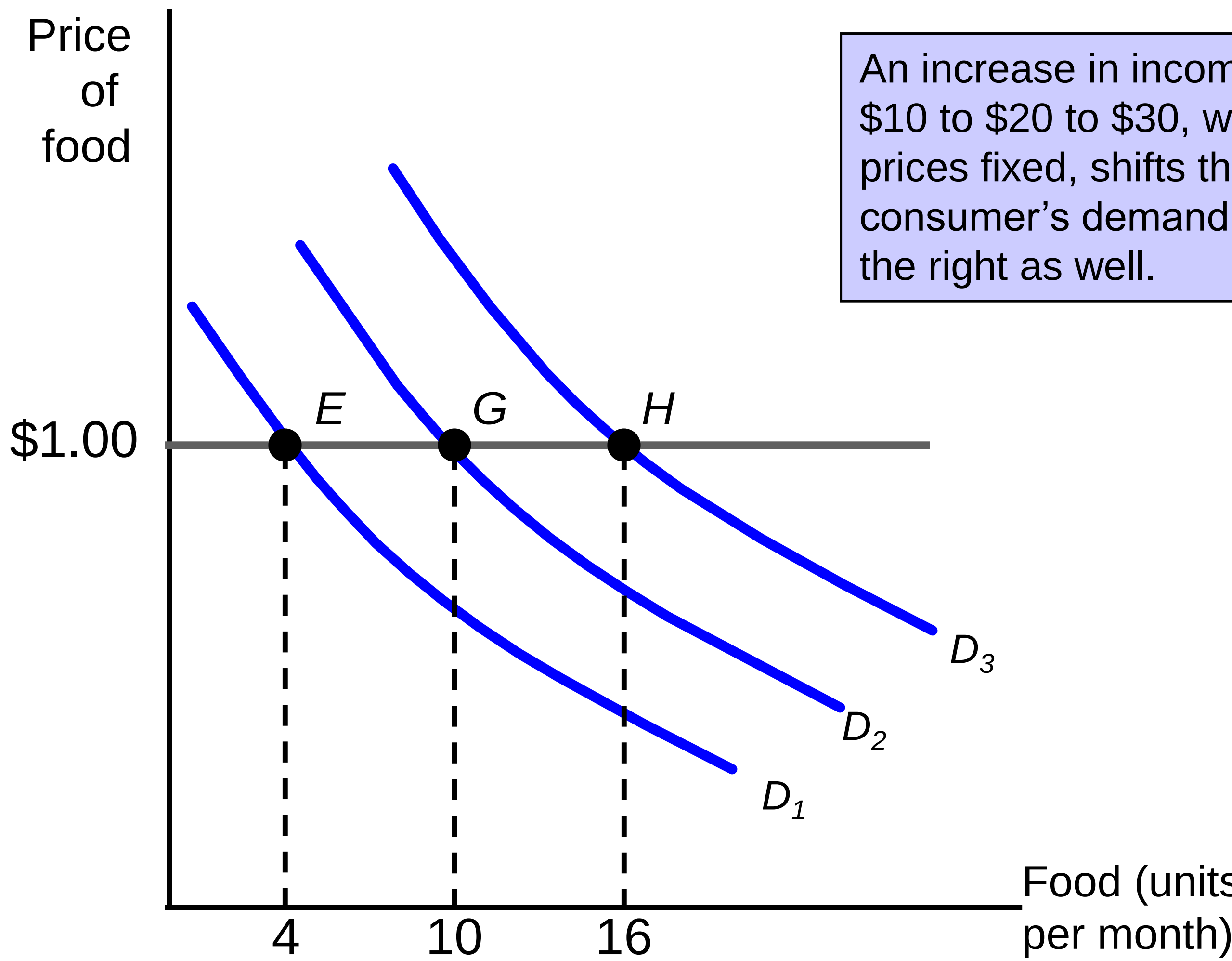
Individual Demand

- Income Changes
  - An increase in income shifts the budget line to the right, increasing consumption along the income-consumption curve
  - Simultaneously, the increase in income shifts the demand curve to the right

Effects of Income Changes



Effects of Income Changes



An increase in income, from $10 to $20 to $30, with the prices fixed, shifts the consumer's demand curve to the right as well.

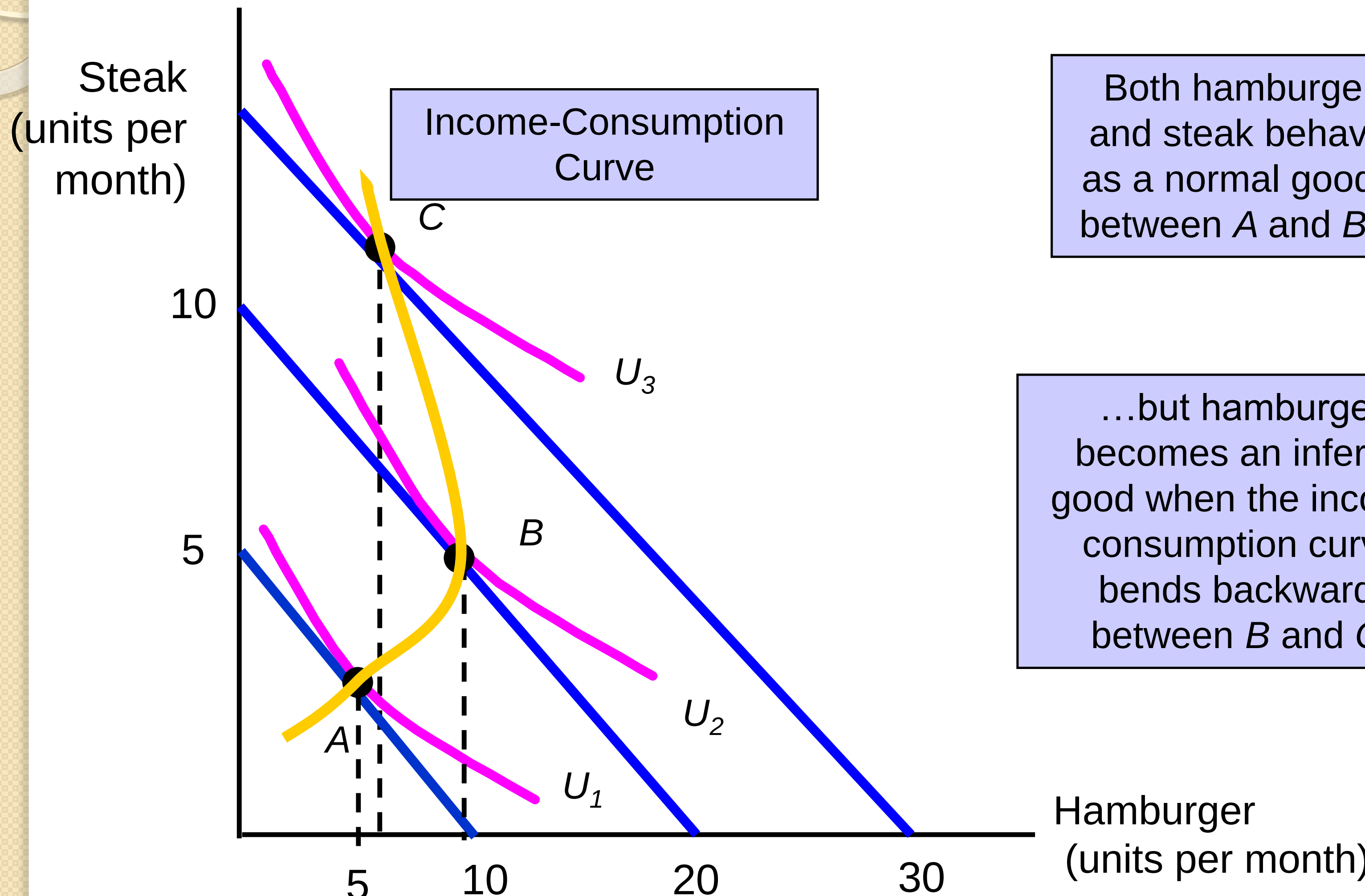
Individual Demand

- Income Changes
  - When the income-consumption curve has a positive slope:
    - The quantity demanded increases with income
    - The income elasticity of demand is positive
    - The good is a **normal good**

Individual Demand

- Income Changes
  - When the income-consumption curve has a negative slope:
    - The quantity demanded decreases with income
    - The income elasticity of demand is negative
    - The good is an **inferior good**

An Inferior Good



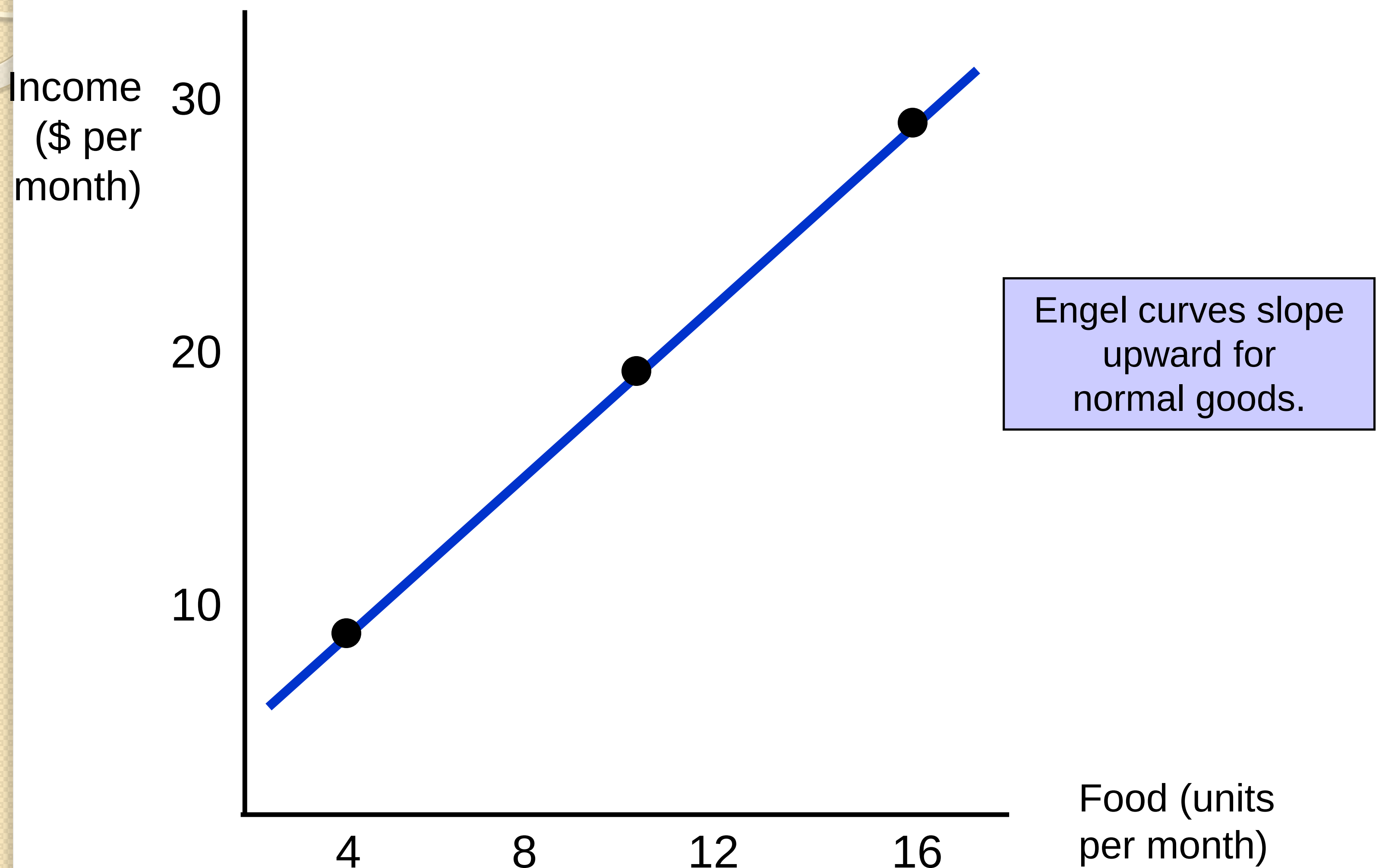
Both hamburger and steak behave as a normal good, between *A* and *B*...

…but hamburger becomes an inferior good when the income consumption curve bends backward between *B* and *C*.

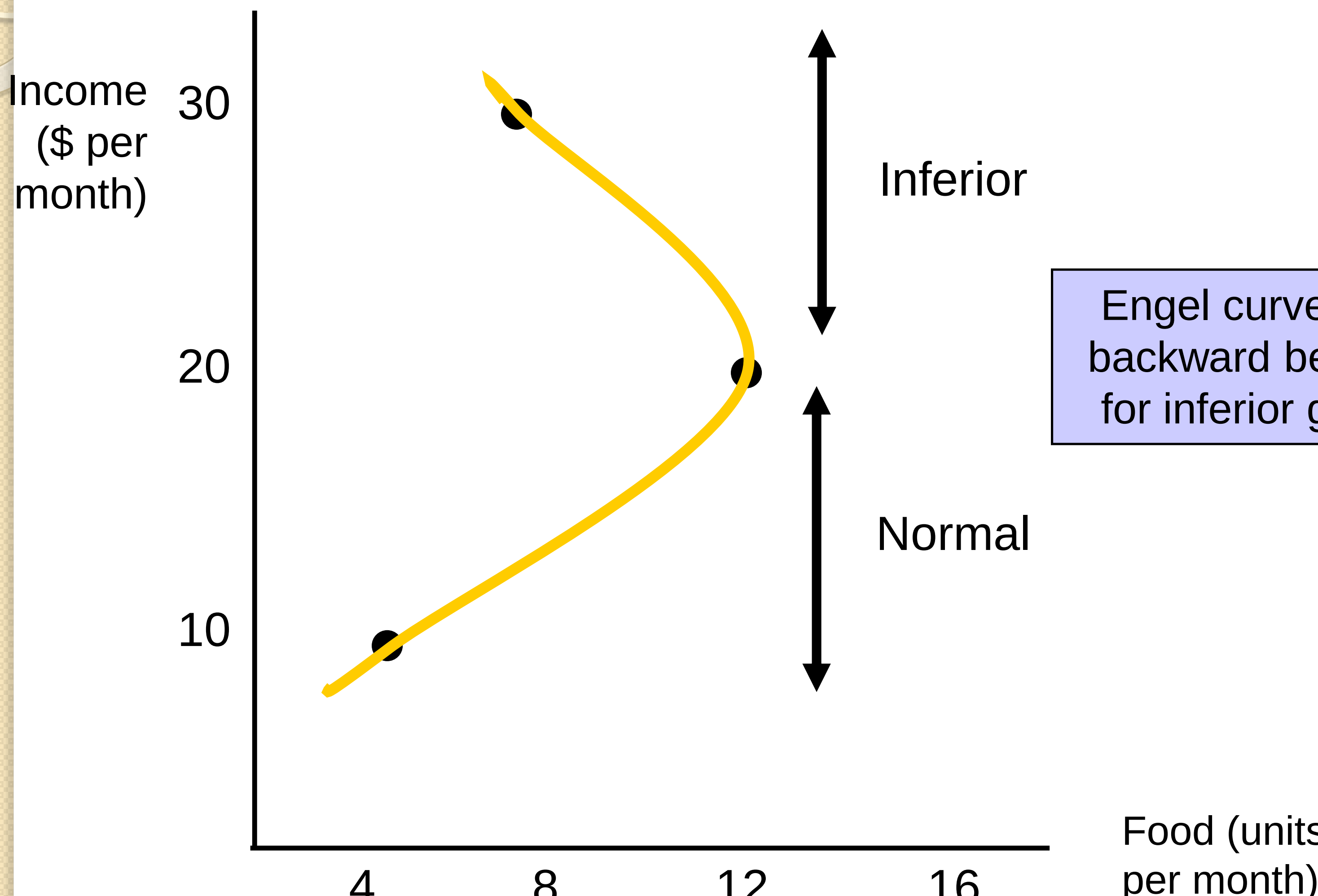
Individual Demand

- Engel Curves
  - Engel curves relate the quantity of good consumed to income
  - If the good is a normal good, the Engel curve is upward sloping
  - If the good is an inferior good, the Engel curve is downward sloping

Engel Curves



Engel Curves



Engel curves are backward bending for inferior goods.

Substitutes & Complements

- Two goods are considered substitutes if an increase (decrease) in the price of one leads to an increase (decrease) in the quantity demanded of the other
  - Ex: movie tickets and video rentals

Substitutes & Complements

- Two goods are considered complements if an increase (decrease) in the price of one leads to a decrease (increase) in the quantity demanded of the other
  - Ex: gasoline and motor oil

Substitutes & Complements

- If two goods are independent, then a change in the price of one good has no effect on the quantity demanded of the other
  - Ex: price of chicken and price of airplane tickets

Income and Substitution Effects

- A change in the price of a good has two effects:
  - Substitution Effect
  - Income Effect

Income and Substitution Effects

- Substitution Effect
  - The substitution effect is the change in an item's consumption associated with a change in the price of the item, with the level of utility held constant
  - When the price of an item declines, the substitution effect always leads to an increase in the quantity demanded of the good
  - ***Why is substitution effect always nonpositive***?

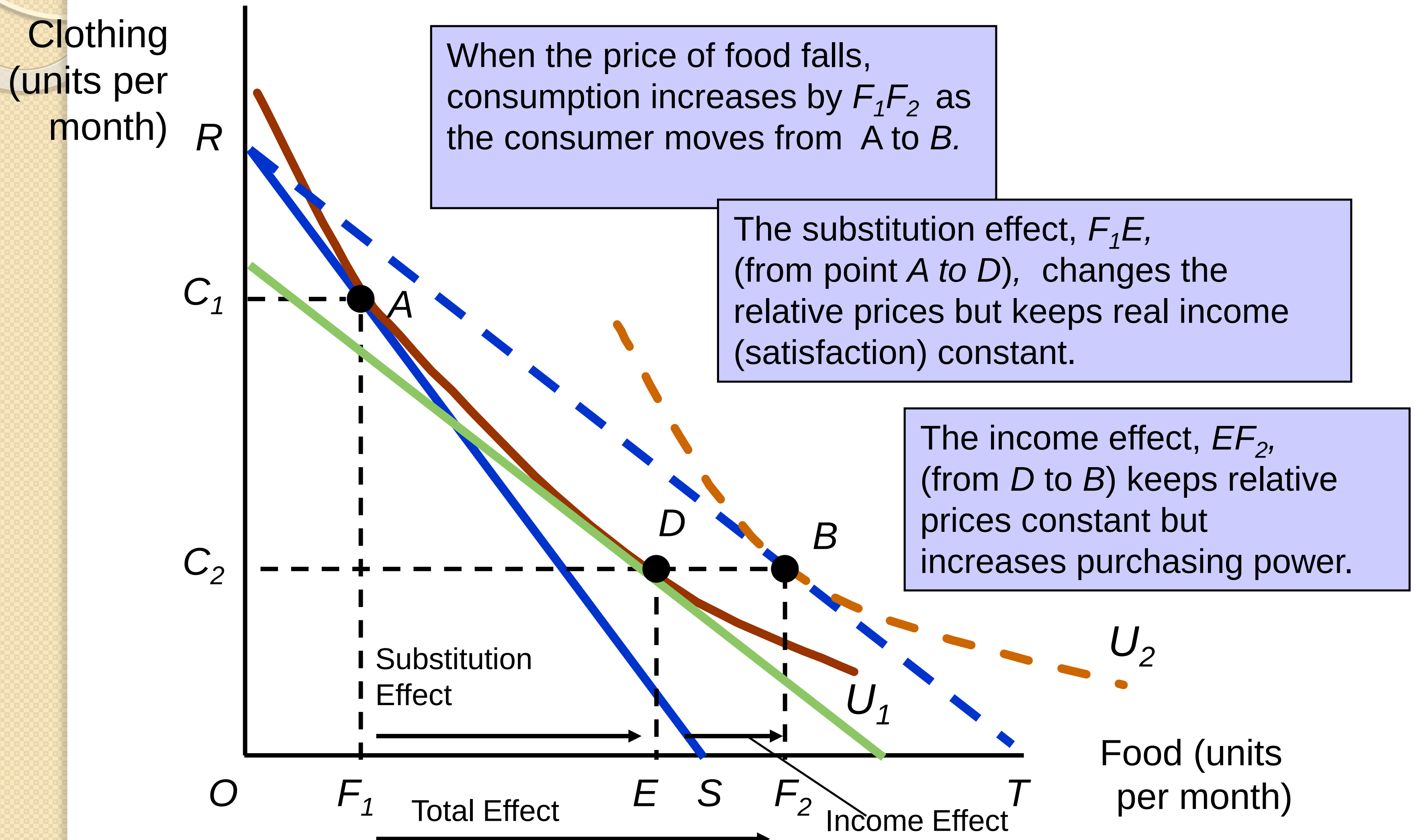
Income and Substitution Effects

- Income Effect
  - The income effect is the change in an item’s consumption brought about by the increase in purchasing power, with the price of the item held constant
  - When a person’s income increases, the quantity demanded for the product may increase or decrease

Income and Substitution Effects

- Income Effect
  - Even with inferior goods, the income effect is rarely large enough to outweigh the substitution effect

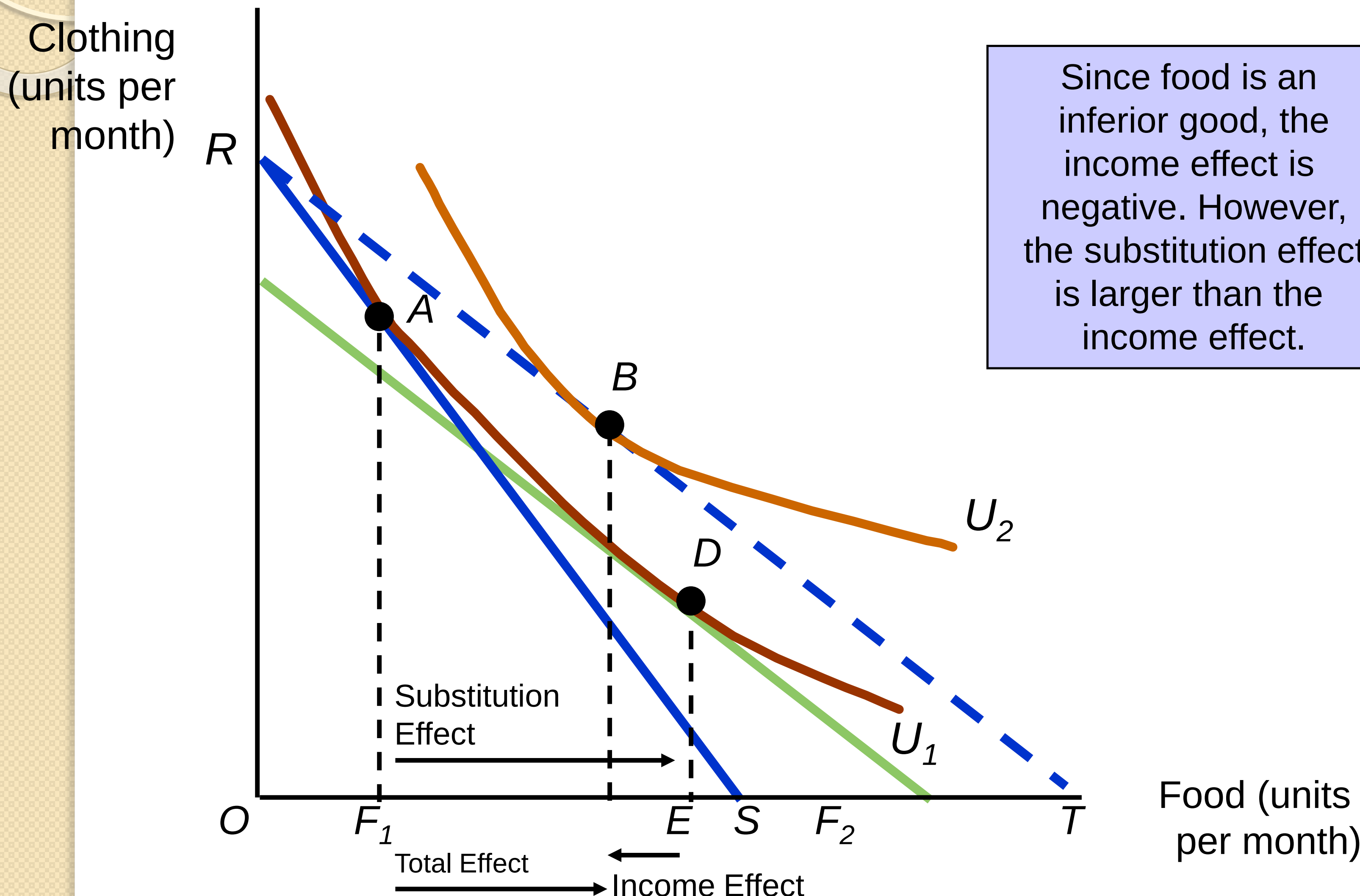
Income and Substitution Effects: Normal Good



The Law of Demand

- The demand for a normal good (i.e., the income effect is positive) increases when its price decreases

Income and Substitution Effects: Inferior Good

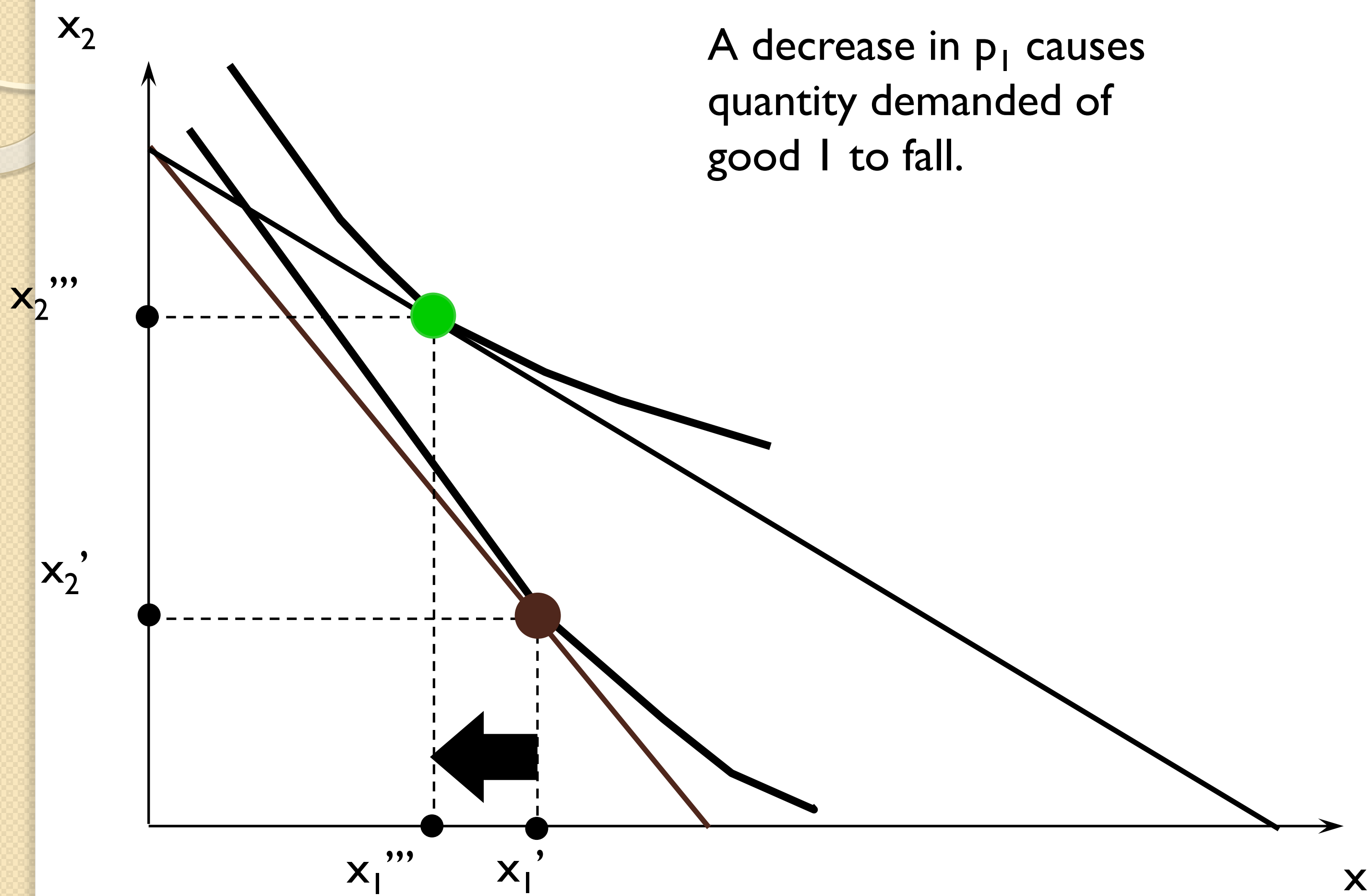


Since food is an inferior good, the income effect is negative. However, the substitution effect is larger than the income effect.

Income and Substitution Effects

- A Special Case: The Giffen Good
  - The income effect may theoretically be large enough to cause the demand curve for a good to slope upward
  - This rarely occurs and is of little practical interest

Giffen Goods



A decrease in p_1 causes quantity demanded of good 1 to fall.

Slutsky Equation

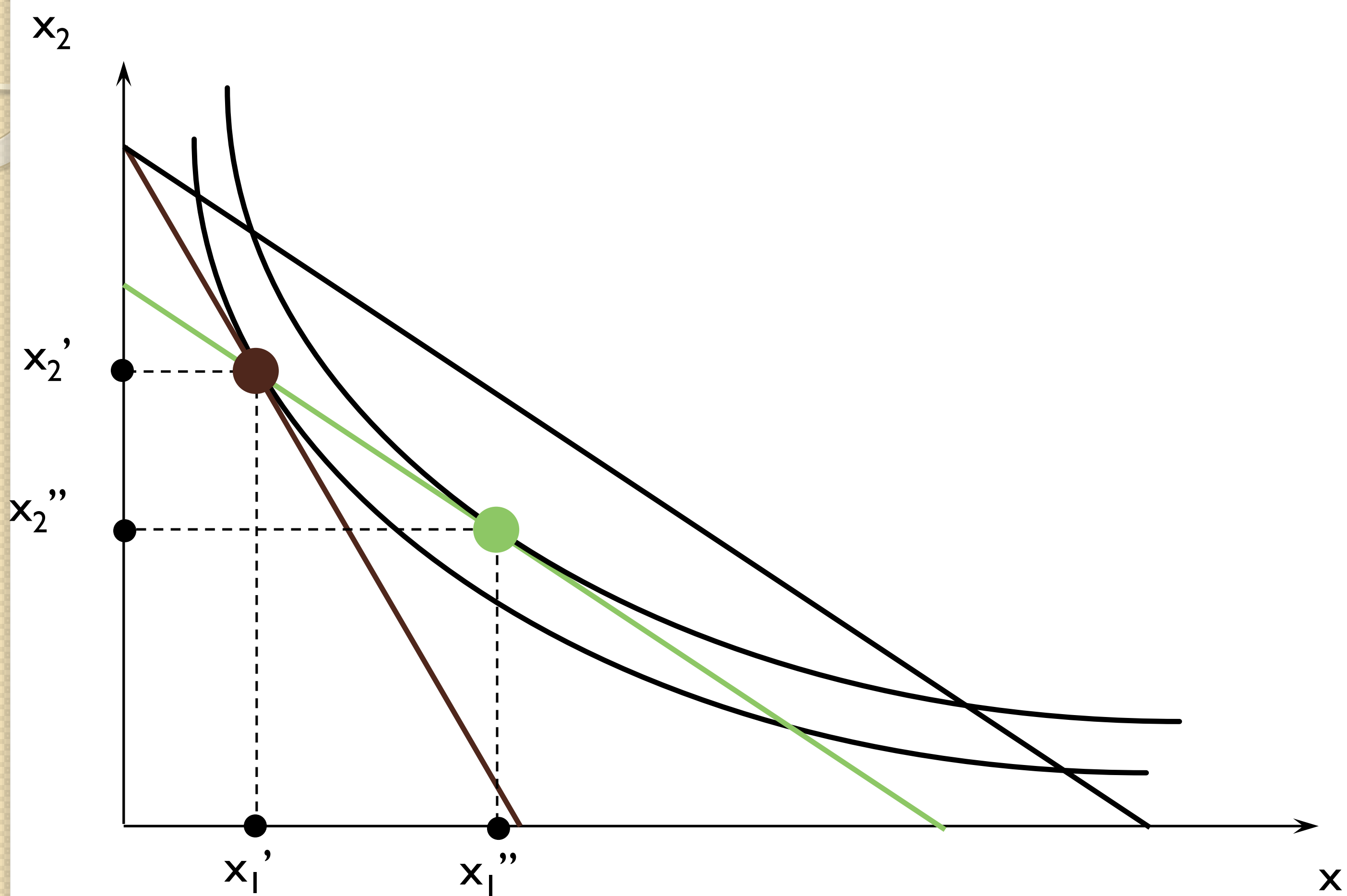
$$\frac{dx}{dp_x} = \left.\frac{\partial x}{\partial p_x}\right|_{u=\bar{u}} - \frac{\partial x}{\partial I} \bullet \frac{\partial I}{\partial p_x}$$

$$-\frac{dx}{dp_x} = -\left.\frac{\partial x}{\partial p_x}\right|_{u=\bar{u}} + \frac{\partial x}{\partial I} \bullet \frac{\partial I}{\partial p_x}$$

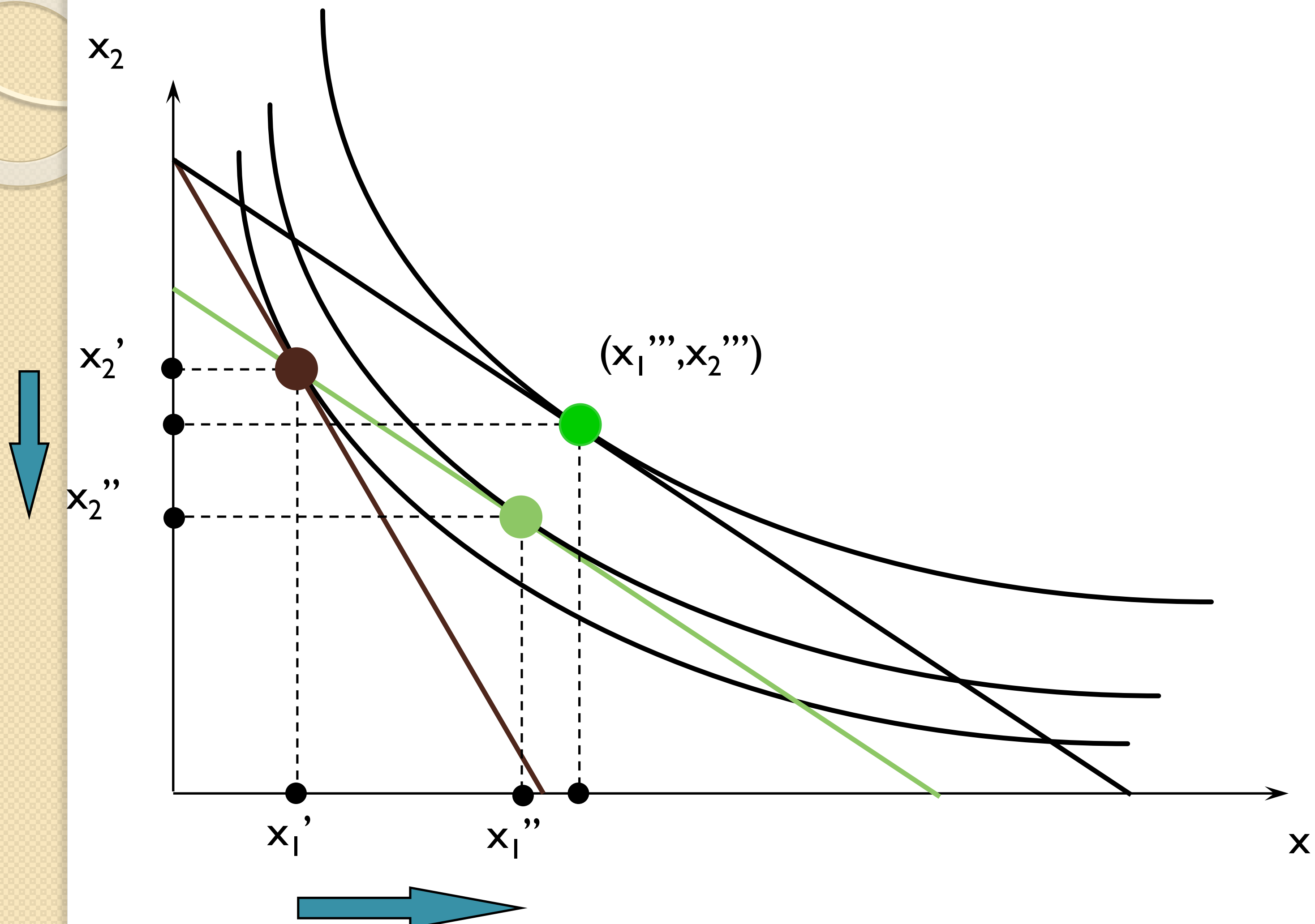
Another Way of Decomposing Price Effect

- Hicksian substitution effect: holding the *utility* constant
- Slutsky substitution effect: holding the *purchasing power* constant

Slutsky Substitution Effect



And Now the Income Effect



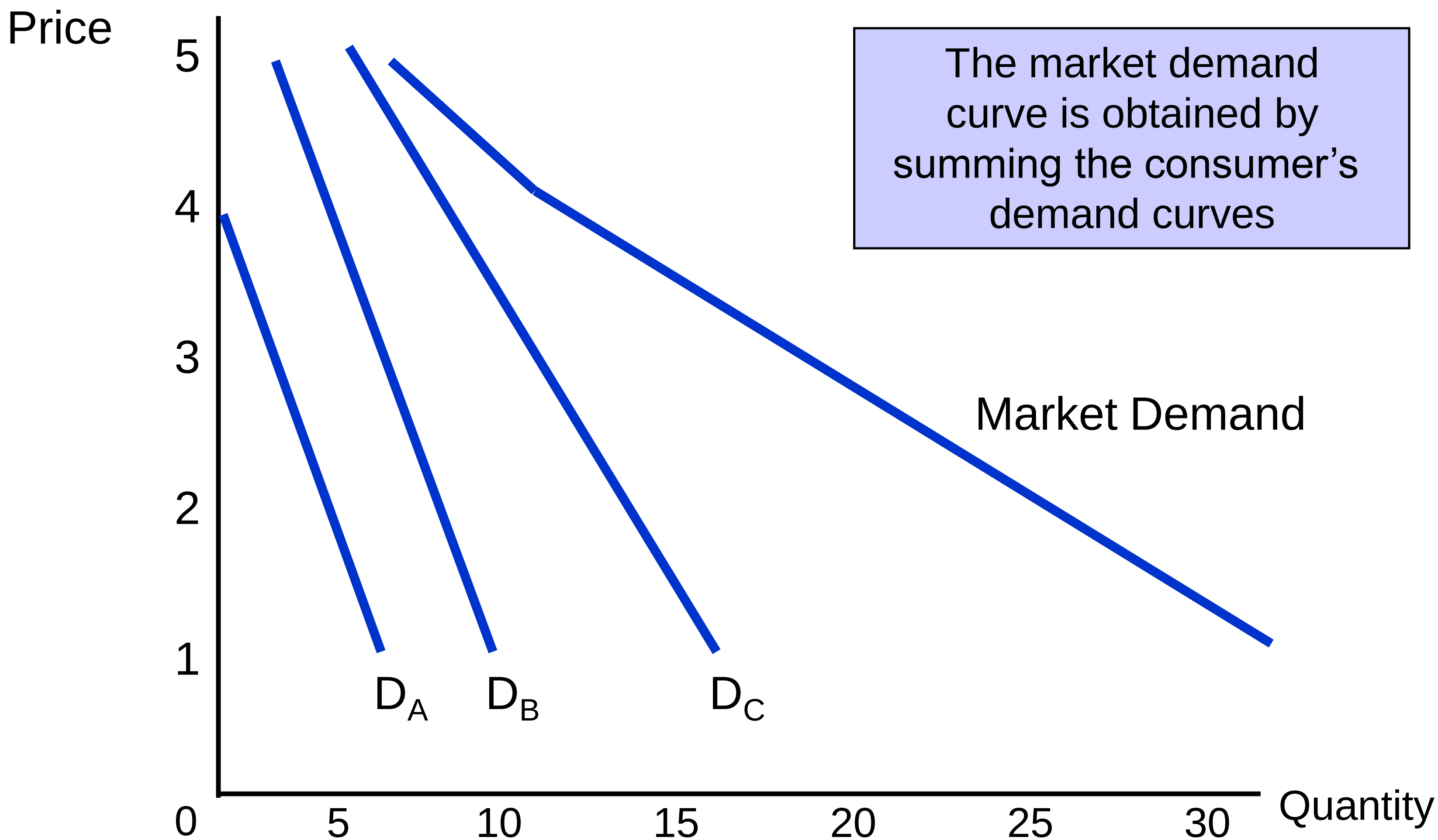
Market Demand

- Market Demand Curves
  - A curve that relates the quantity of a good that all consumers in a market buy to the price of that good
  - The sum of all the individual demand curves in the market

Determining the Market Demand Curve

Price	A	B	C	Market Demand
1	6	10	16	32
2	4	8	13	25
3	2	6	10	18
4	0	4	7	11
5	0	2	4	6

Summing to Obtain a Market Demand Curve



Market Demand

- From this analysis one can see two important points:
  - The market demand will shift to the right as more consumers enter the market
  - Factors that influence the demands of many consumers will also affect the market demand

Market Demand

- Aggregation is important to be able to discuss regarding demand for different groups
  - Households with children
  - Consumers aged 20 – 30, etc.

Market Demand

- Price Elasticity of Demand
  - Measures the percentage change in the quantity demanded resulting from a percent change in price

$$E_P = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

Price Elasticity of Demand

- Inelastic Demand
  - E_p is less than 1 in absolute value
  - Quantity demanded is relatively unresponsive to a change in price
  - $|\%\Delta Q| < |\%\Delta P|$
  - Total expenditure (P*Q) increases when price increases

Price Elasticity of Demand

- Elastic Demand
  - E_p is greater than than 1 in absolute value
  - Quantity demanded is relatively responsive to a change in price
  - $|\%\Delta Q| > |\%\Delta P|$
  - Total expenditure (P*Q) decreases when price increases

Price Elasticity and Consumer Expenditure

Demand	*If Price Increases, Expenditures*	*If Price Decreases, Expenditures*
Inelastic	Increase	Decrease
Unit elastic	Are unchanged	Are unchanged
Elastic	Decrease	Increase

Price Elasticity of Demand

- Isoelastic Demand
  - When price elasticity of demand is constant along the entire demand curve
  - Demand curve is bowed inward (not linear)

Revenue and Own-Price Elasticity of Demand

- If raising a commodity's price causes little decrease in quantity demanded, then sellers' revenues rise.
  - Hence own-price inelastic demand causes sellers' revenues to rise as price rises.
- If raising a commodity's price causes a large decrease in quantity demanded, then sellers' revenues fall.
  - Hence own-price elastic demand causes sellers' revenues to fall as price rises.

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is

$$R(p) = p \times X^*(p).$$

Revenue and Own-Price Elasticity of Demand

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So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $$R(p) = p \times X^*(p).$$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is

$$R(p) = p \times X^*(p).$$

So

$$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^{*}(p)\left[1 + \varepsilon\right]$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

so if $\varepsilon = -1$ then $\dfrac{dR}{dp} = 0$

and a change to price does not alter sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

but if $-1 < \varepsilon \le 0$ then $\frac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

And if $\varepsilon < -1$ then $\frac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Marginal Revenue and Own-Price Elasticity of Demand

- A seller's marginal revenue is the rate at which revenue changes with the number of units sold by the seller.

$$MR(q) = \frac{dR(q)}{dq}.$$

Marginal Revenue and Own-Price Elasticity of Demand

p(q) denotes the seller's inverse demand function; i.e. the price at which the seller can sell q units. Then

$$R(q) = p(q) \times q$$

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].$$

and

$$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$ says that the rate

at which a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; *i.e.*, upon the of the own-price elasticity of demand.

Marginal Revenue and Own-Price Elasticity of Demand

$$\mathbf{MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]}$$

If $\varepsilon = -1$ then $\mathbf{MR(q) = 0.}$

If $-1 < \varepsilon \leq 0$ then $\mathbf{MR(q) < 0.}$

If $\varepsilon < -1$ then $\mathbf{MR(q) > 0.}$

Marginal Revenue and Own-Price Elasticity of Demand

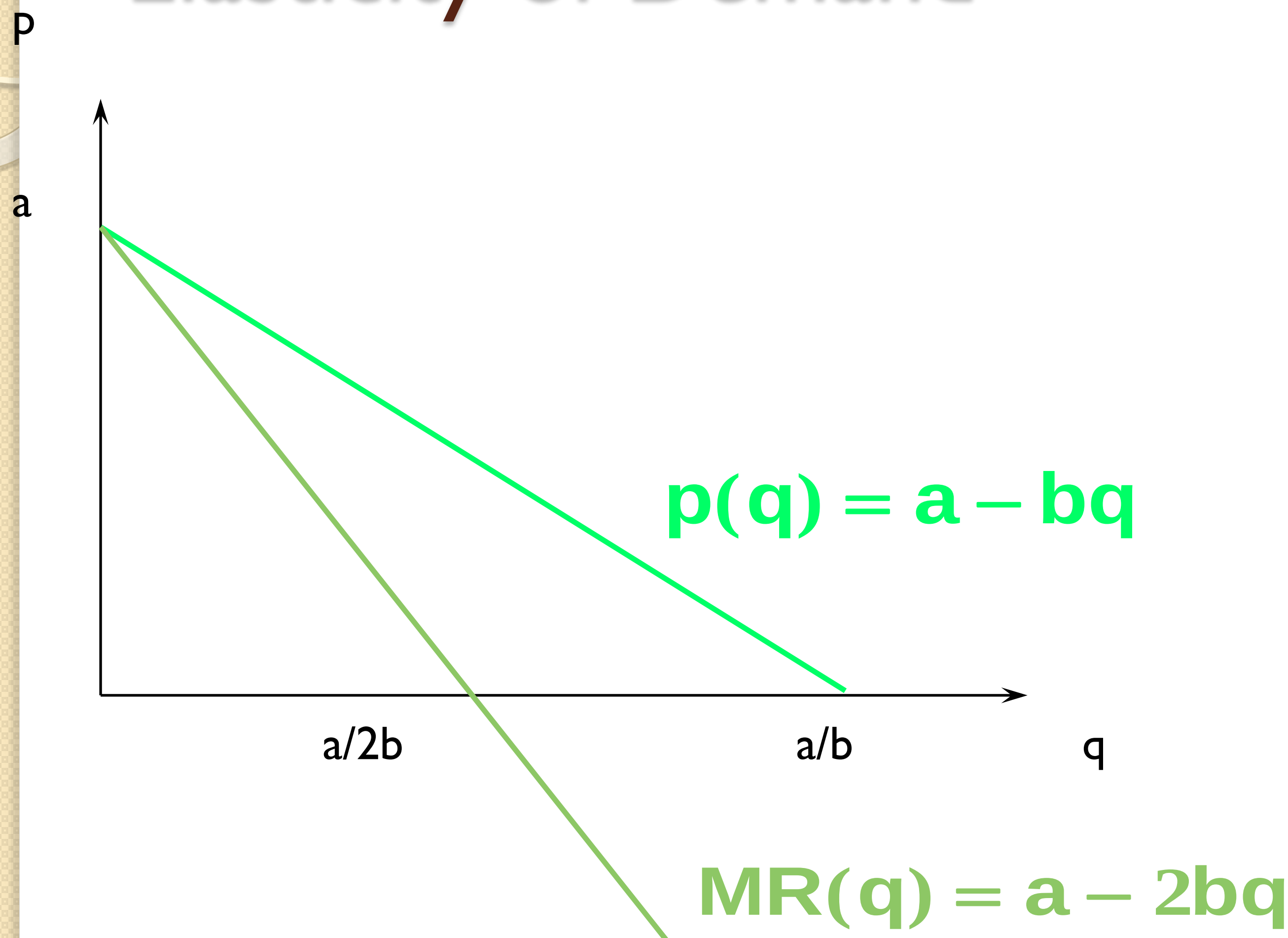
An example with linear inverse demand.

$$p(q) = a - bq.$$

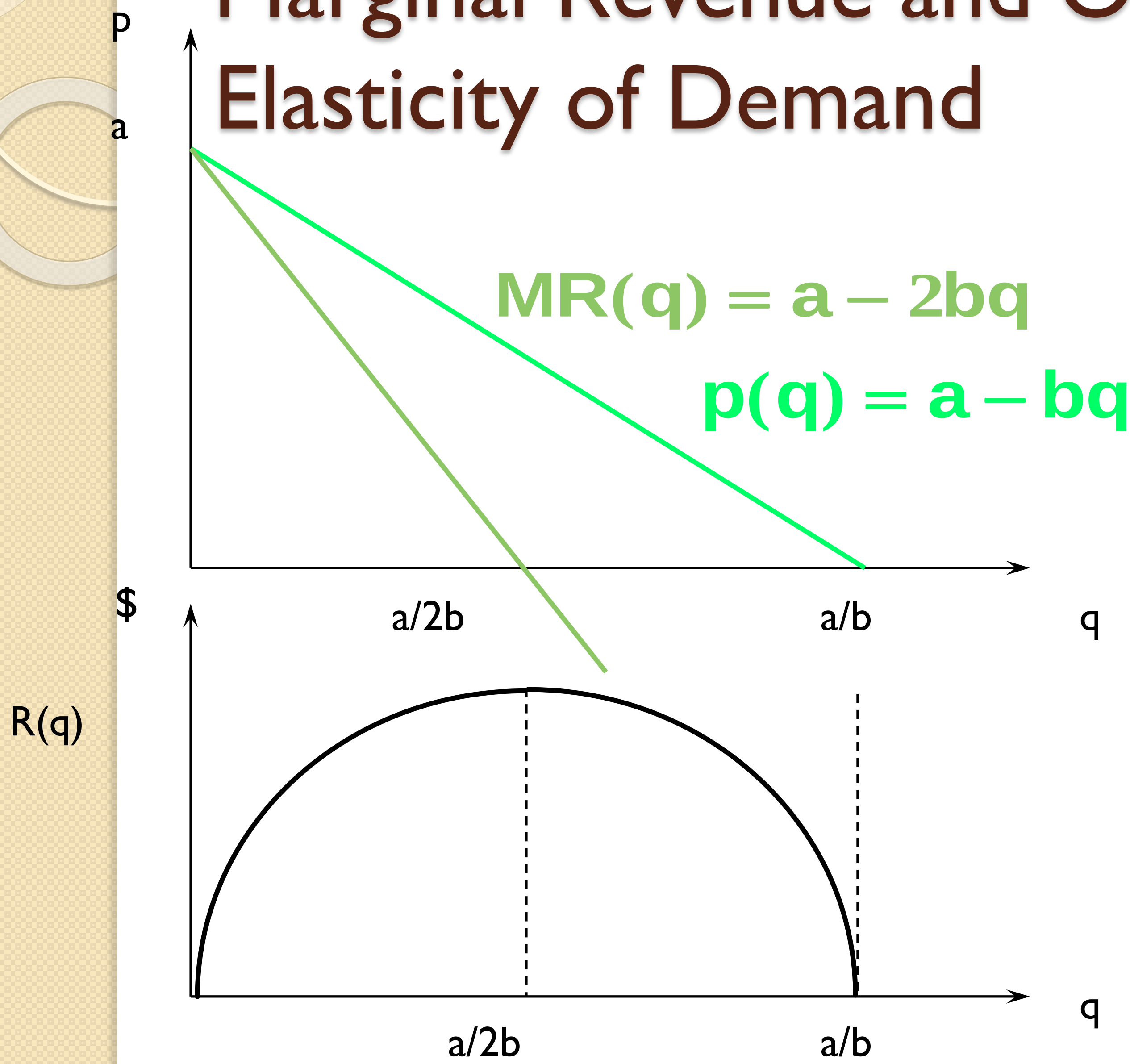
Then $$R(q) = p(q)q = (a - bq)q$$

and $$MR(q) = a - 2bq.$$

Marginal Revenue and Own-Price Elasticity of Demand



Marginal Revenue and Own-Price Elasticity of Demand



Income Elasticity

$$\eta_{X_i^*,m} = \frac{m}{X_i^*} \times \frac{dX_i^*}{dm}$$

- Normal good: $\eta>0$
- Inferior good: $\eta<0$
- Luxury good: $\eta>1$
- Necessary good: $0<\eta<1$

Consumer Surplus

- Consumers buy goods because it makes them better off
- Consumer Surplus measures how much better off they are

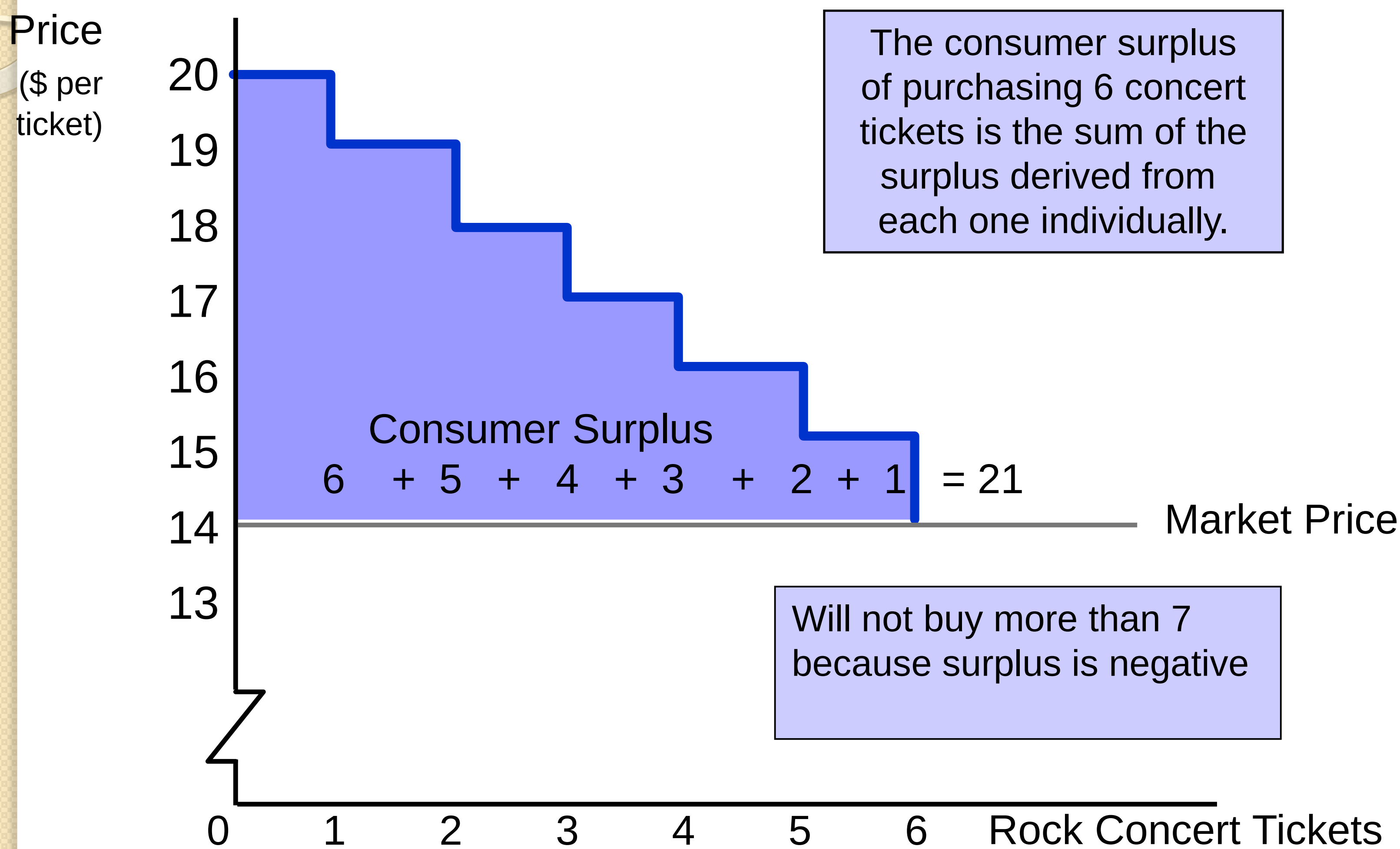
Consumer Surplus

- **Consumer Surplus**
  - The difference between the maximum amount a consumer is willing to pay for a good and the amount actually paid
  - Can calculate consumer surplus from the demand curve

Consumer Surplus - Example

- Student wants to buy concert tickets
- Demand curve tells us willingness to pay for each concert ticket
  - 1st ticket worth $20 but price is $14 so student generates $6 worth of surplus
  - Can measure this for each ticket
  - Total surplus is addition of surplus for each ticket purchased

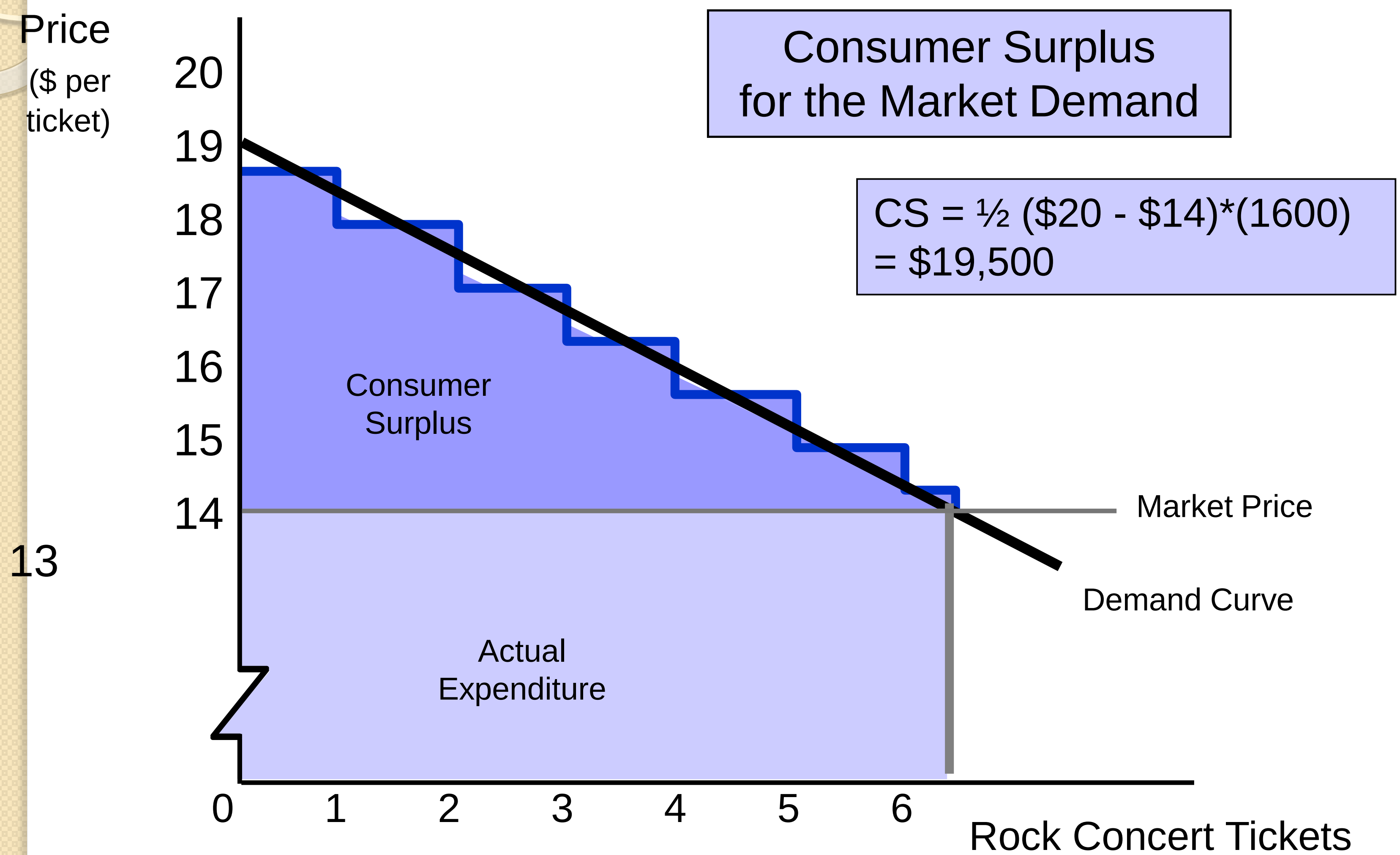
Consumer Surplus - Example



Consumer Surplus

- The stepladder demand curve can be converted into a straight-line demand curve by making the units of the good smaller
- Consumer surplus is the area under the demand curve and above the price

Consumer Surplus



Applying Consumer Surplus

- Combining consumer surplus with the aggregate profits that producers obtain, we can evaluate:
  1. Costs and benefits of different market structures
  2. Public policies that alter the behavior of consumers and firms

Applying Consumer Surplus – An Example

- The Value of Clean Air
  - Air is free in the sense that we don't pay to breathe it
  - The Clean Air Act was amended in 1970
  - Question: Were the benefits of cleaning up the air worth the costs?

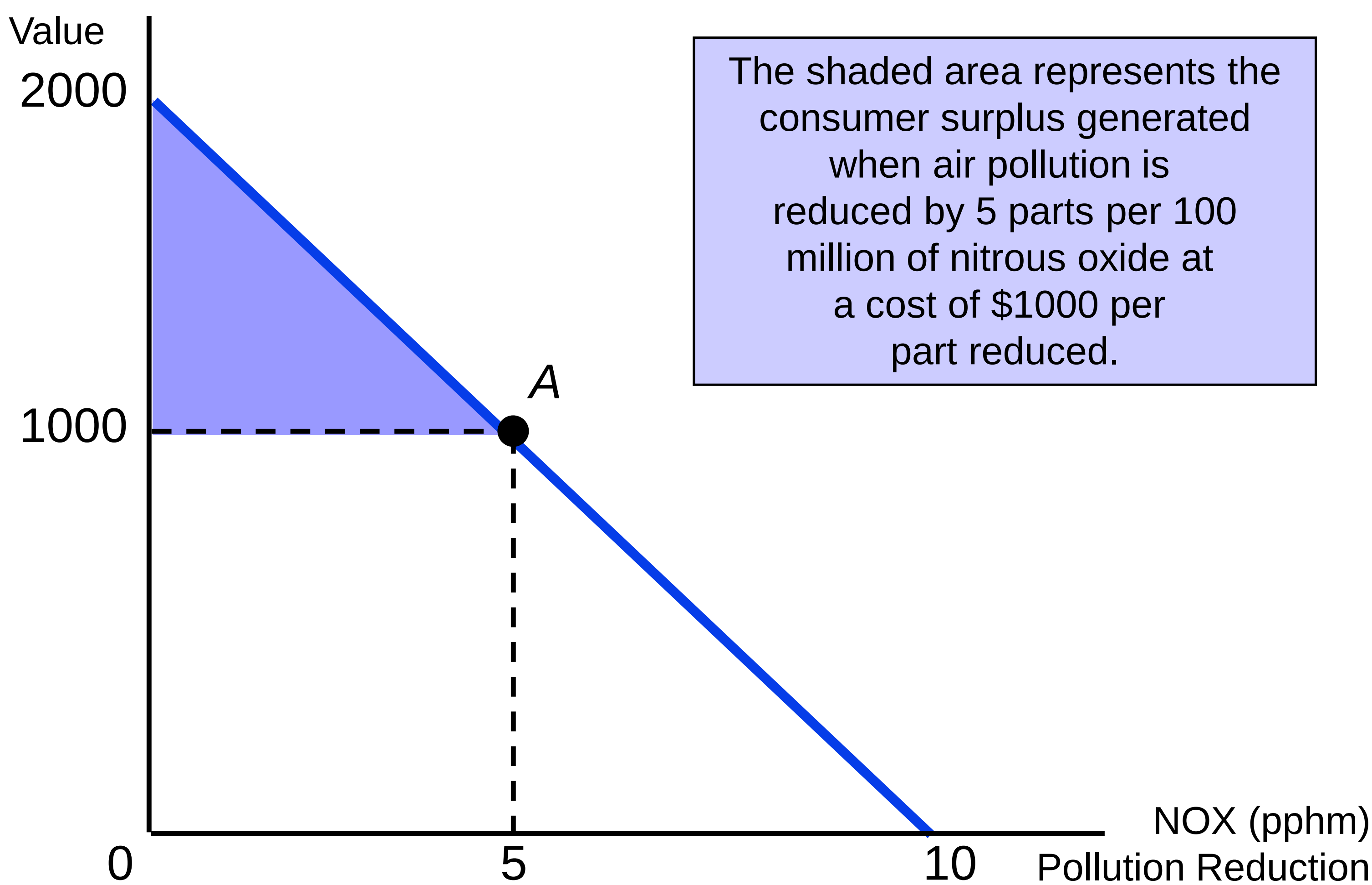
The Value of Clean Air

- Empirical data determined estimates for the demand for clean air
- No market exists for clean air, but can see people are willing to pay for it
  - Ex: People pay more to buy houses where the air is clean

The Value of Cleaner Air

- Using these empirical estimates, we can measure people's consumer surplus for pollution reduction from the demand curve

Valuing Cleaner Air



Value of Cleaner Air

- A full cost-benefit analysis would include total benefit of cleanup
- Total benefits would be compared to total costs to determine if the clean up was worthwhile

Chapter 5

Choice under Uncertainty

Topics to be Discussed

- Describing Risk
- Preferences Toward Risk
- Reducing Risk

Introduction

- Choice with certainty is reasonably straightforward
- How do we make choices when certain variables such as income and prices are uncertain (making choices with risk)?

Describing Risk

- To measure risk we must know:

1. All of the possible outcomes
2. The **probability** or likelihood that each outcome will occur

States of Nature

- Possible states of Nature:
  - "car accident" (a)
  - "no car accident" (na).
- Accident occurs with probability π_a, does not with probability π_{na} ;

$$\pi_a + \pi_{na} = 1.$$

- Accident causes a loss of \$L.

Contingencies

- A contract implemented only when a particular state of Nature occurs is state-contingent.
- E.g. the insurer pays only if there is an accident.

Contingencies

- A state-contingent consumption plan is implemented only when a particular state of Nature occurs.
- E.g. take a vacation only if there is no accident.

Describing Risk

- Interpreting Probability
  1. Objective Interpretation
     - Based on the observed frequency of past events
  2. Subjective Interpretation
     - Based on perception that an outcome will occur

Interpreting Probability

- Subjective Probability
  - Different information or different abilities to process the same information can influence the subjective probability
  - Based on judgment or experience

Describing Risk

- With an interpretation of probability, must determine 2 measures to help describe and compare risky choices
  1. Expected value
  2. Variability

Describing Risk

- Expected Value
  - The weighted average of the payoffs or values resulting from all possible outcomes
    - Expected value measures the central tendency; the payoff or value expected on average

$$\bar{X}_n = EX = \sum_{i=1}^{n} P_i X_i$$

Expected Value – An Example

- Investment in offshore drilling exploration:
- Two outcomes are possible
  - Success – the stock price increases from $30 to $40/share
  - Failure – the stock price falls from $30 to $20/share

Expected Value – An Example

- Objective Probability
  - 100 explorations, 25 successes and 75 failures
  - Probability (Pr) of success = 1/4 and the probability of failure = 3/4

Expected Value – An Example

$$EV = \Pr(\text{success})(\text{value of success}) + \Pr(\text{failure})(\text{value of failure})$$

$$EV = 1/4(\$40/\text{share}) + 3/4(\$20/\text{share})$$

$$EV = \$25/\text{share}$$

Expected Value

- In general, for n possible outcomes:
  - Possible outcomes having payoffs $X_1, X_2, \ldots, X_n$
  - Probabilities of each outcome is given by $Pr_1, Pr_2, \ldots, Pr_n$

$$E(X) = Pr_1 X_1 + Pr_2 X_2 + \ldots + Pr_n X_n$$

Describing Risk

- Variability
  - The extent to which possible outcomes of an uncertain event may differ
  - How much variation exists in the possible choice

Variability – An Example

- Suppose you are choosing between two part-time sales jobs that have the same expected income ($1,500)
- The first job is based entirely on commission
- The second is a salaried position

Variability – An Example

- There are two equally likely outcomes in the first job: \$2,000 for a good sales job and \$1,000 for a modestly successful one
- The second pays \$1,510 most of the time (.99 probability), but you will earn \$510 if the company goes out of business (.01 probability)

Variability – An Example

	Outcome 1		Outcome 2	
	Prob.	**Income**	**Prob.**	**Income**
Job 1: Commission	.5	2000	.5	1000
Job 2: Fixed Salary	.99	1510	.01	510

Variability – An Example

- Income from Possible Sales Job

Job 1 Expected Income

$$E(X_1) = .5(\$2000) + .5(\$1000) = \$1500$$

Job 2 Expected Income

$$E(X_2) = .99(\$1510) + .01(\$510) = \$1500$$

Variability

- While the expected values are the same, the variability is not
- Greater variability from expected values signals greater risk
- Variability comes from **deviations** in payoffs
  - Difference between expected payoff and actual payoff

Variability – An Example

Deviations from Expected Income ($)				
	Outcome 1	**Deviation**	**Outcome 2**	**Deviation**
Job 1	$2000	$500	$1000	-$500
Job 2	1510	10	510	-900

Variability

- Average deviations are always zero so we must adjust for negative numbers
- We can measure variability with **standard deviation**
  - The square root of the average of the squares of the deviations of the payoffs associated with each outcome from their expected value

Variability

- The standard deviation is written:

$$\boldsymbol{\sigma} = \sqrt{\Pr_1\left[\boldsymbol{X}_1 - \boldsymbol{E}(\boldsymbol{X})\right]^2 + \Pr_2\left[\boldsymbol{X}_2 - \boldsymbol{E}(\boldsymbol{X})\right]^2}$$

Variability

- Standard deviation is a measure of risk
  - Measures how variable your payoff will be
  - More variability means more risk
  - Individuals generally prefer less variability – less risk

Standard Deviation – Example 1

Deviations from Expected Income ($)				
	Outcome 1	**Deviation**	**Outcome 2**	**Deviation**
Job 1	$2000	$500	$1000	-$500
Job 2	1510	10	510	-990

Standard Deviation – Example 1

- Standard deviations of the two jobs are:

$$\boldsymbol{\sigma} = \sqrt{\Pr_1\left[\boldsymbol{X}_1 - \boldsymbol{E}(\boldsymbol{X})\right]^2 + \Pr_2\left[\boldsymbol{X}_2 - \boldsymbol{E}(\boldsymbol{X})\right]^2}$$

$$\sigma_1 = \sqrt{0.5(\$250{,}000) + 0.5(\$250{,}000)}$$

$$\sigma_1 = \sqrt{250{,}000} = 500$$

$$\sigma_2 = \sqrt{0.99(\$100) + 0.01(\$980{,}100)}$$

$$\sigma_2 = \sqrt{9{,}900} = 99.50$$

Standard Deviation – Example 1

- Job 1 has a larger standard deviation and therefore it is the riskier alternative
- The standard deviation also can be used when there are many outcomes instead of only two

Preferences Toward Risk

- Can expand evaluation of risky alternative by considering utility that is obtained by risk
  - A consumer gets utility from income
  - Payoff measured in terms of utility

Preferences Toward Risk - Example

- A person is earning $15,000 and receiving 13.5 units of utility from the job
- She is considering a new, but risky job
  - 0.50 chance of $30,000
  - 0.50 chance of $10,000

Preferences Toward Risk - Example

- Utility at $30,000 is 18
- Utility at $10,000 is 10
- Must compare utility from the risky job with current utility of 13.5
- To evaluate the new job, we must calculate the **expected utility** of the risky job

Preferences Under Uncertainty

- 2 states of nature:
  - At probability π_a , consumption is c_a
  - At probability π_{na} , consumption is c_{na}
  - $\pi_a + \pi_{na} = 1$.
- Utility is $U(c_a, c_{na}, \pi_a, \pi_{na})$.

Preferences Toward Risk

- The **expected utility** of the risky option is the sum of the utilities associated with all her possible incomes weighted by the probability that each income will occur

$$E(u) = (\text{Prob. of Utility 1}) * (\text{Utility 1}) + (\text{Prob. of Utility 2}) * (\text{Utility 2})$$

Preferences Toward Risk – Example

- The expected is:

$$E(u) = (1/2)u(\$10{,}000) + (1/2)u(\$30{,}000)$$
$$= 0.5(10) + 0.5(18)$$
$$= 14$$

  - E(u) of new job is 14, which is greater than the current utility of 13.5 and therefore preferred

Expected Utility Theorem

$$U(x_1, x_2, \ldots, x_N; p_1, p_2, \ldots, p_N)$$

$$= Eu(x) = \sum_{i=1}^{N} p_i u(x_i)$$

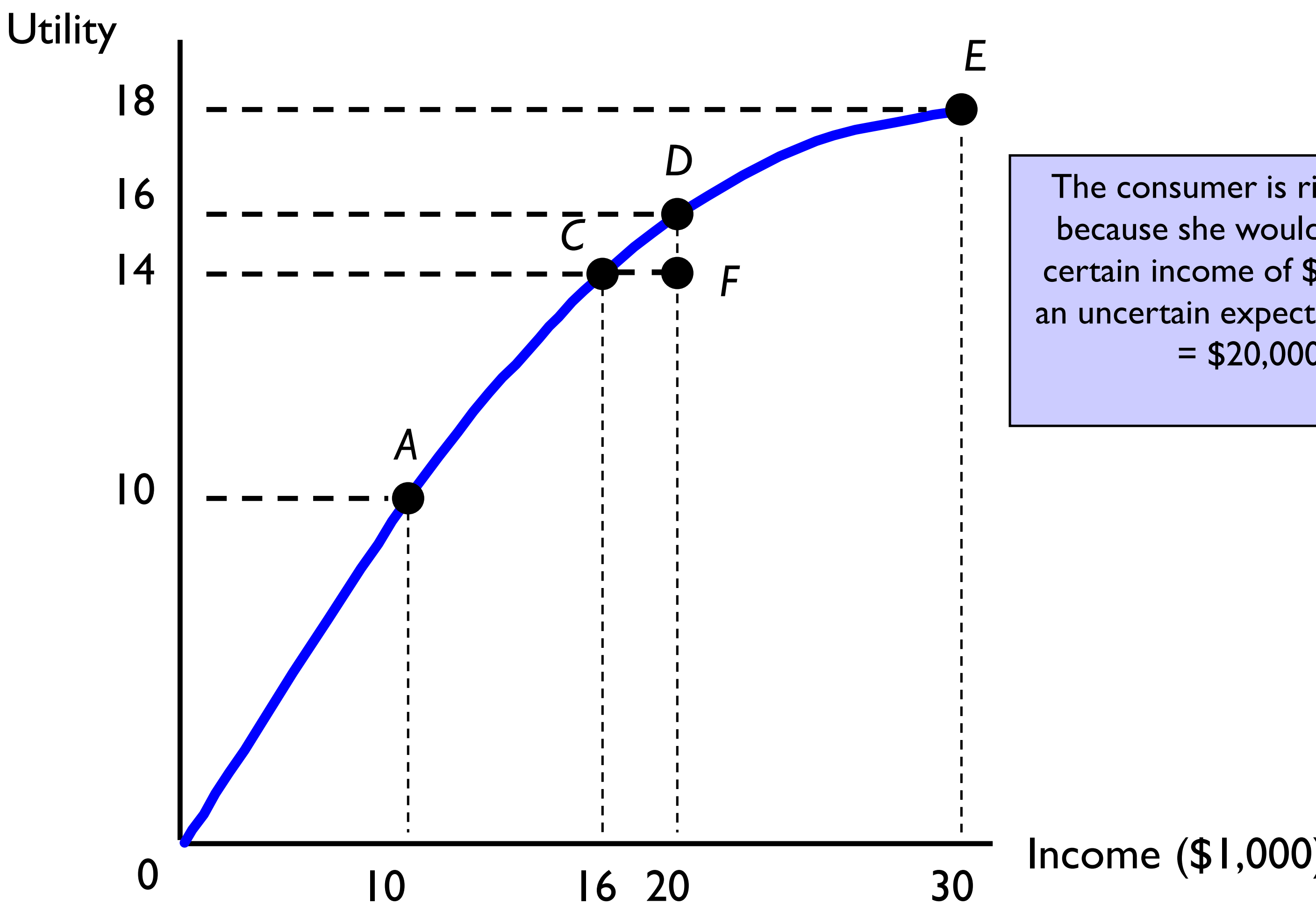
Preferences Toward Risk

- People differ in their preference toward risk
- People can be *risk averse, risk neutral,* or *risk loving*

Preferences Toward Risk

- Risk Averse
  - A person who prefers a certain given income to a risky income with the same expected value
  - $U(EX) > Eu(x)$
  - The person has a diminishing marginal utility of income
  - Most common attitude towards risk
    - Ex: Market for insurance

Risk Averse Utility Function



The consumer is risk averse because she would prefer a certain income of $20,000 to an uncertain expected income = $20,000

Risk Averse - Example

- A person can have a $20,000 job with 100% probability and receive a utility level of 16
- The person could have a job with a 0.5 chance of earning $30,000 and a 0.5 chance of earning $10,000

Risk Averse – Example

- Expected Income of Risky Job

  $E(I) = (0.5)(\$30{,}000) + (0.5)(\$10{,}000)$

  $E(I) = \$20{,}000$

- Expected Utility of Risky Job

  $E(u) = (0.5)(10) + (0.5)(18)$

  $E(u) = 14$

Risk Averse – Example

- Expected income from both jobs is the same – risk averse may choose current job
- Expected utility is greater for certain job
  - Would keep certain job
- Risk averse person's losses (decreased utility) are more important than risky gains

Implication of Risk Aversion

- A risk-averse person will always prefer a *certain* amount of income to a gamble with the same expected value

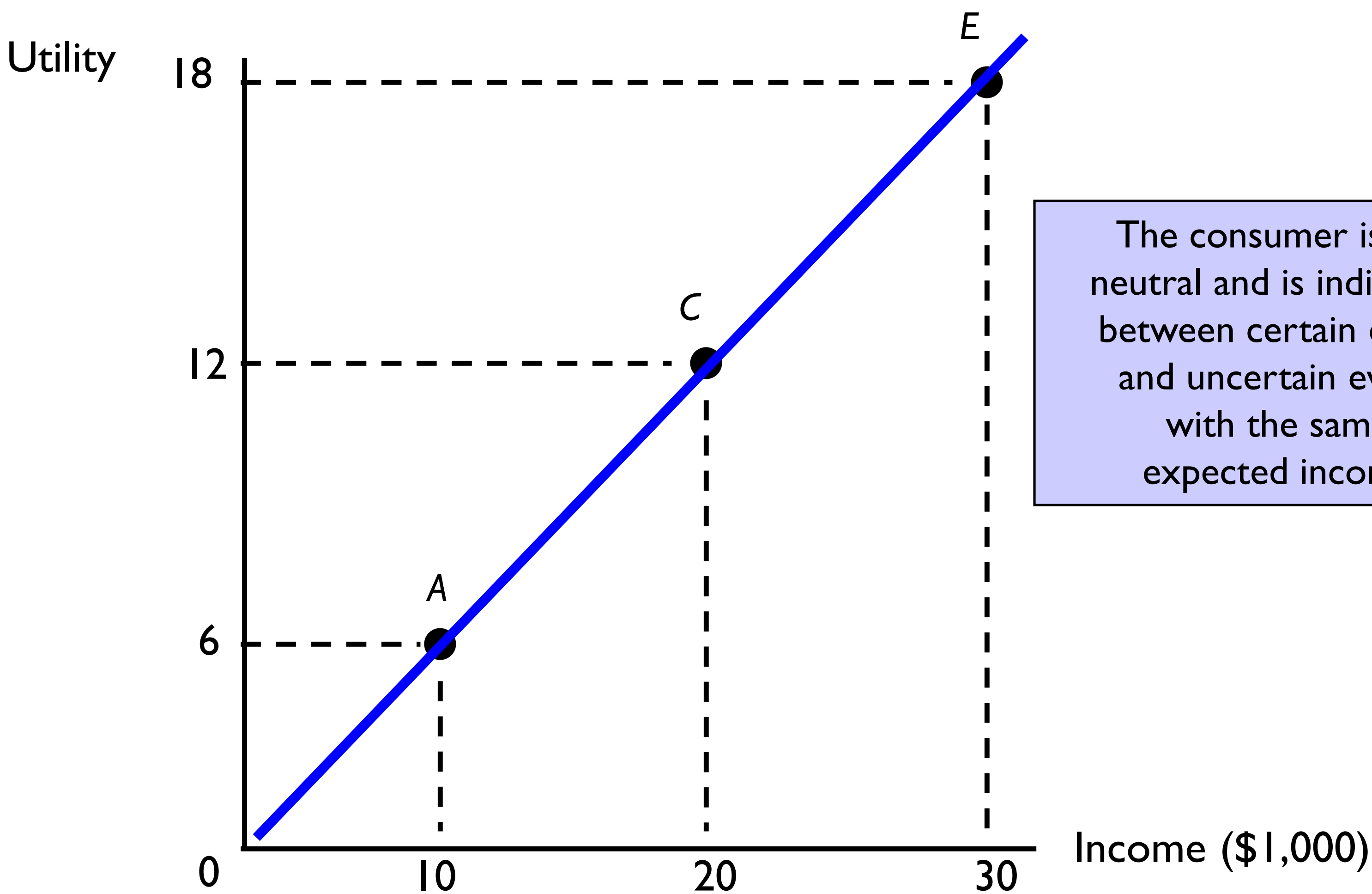
- This is supported by the so-called **Jensen Inequality**: *for any strictly concave function u(.),* $U(EX) > Eu(x)$ *(X is a random variable)*

- A risk averse person will never take a fair gamble (i.e., a game with zero expected income)

Preferences Toward Risk

- A person is said to be **risk neutral** if they show no preference between a certain income, and an uncertain income with the same expected value
- $U(EX) = Eu(x)$
- Constant marginal utility of income

Risk Neutral

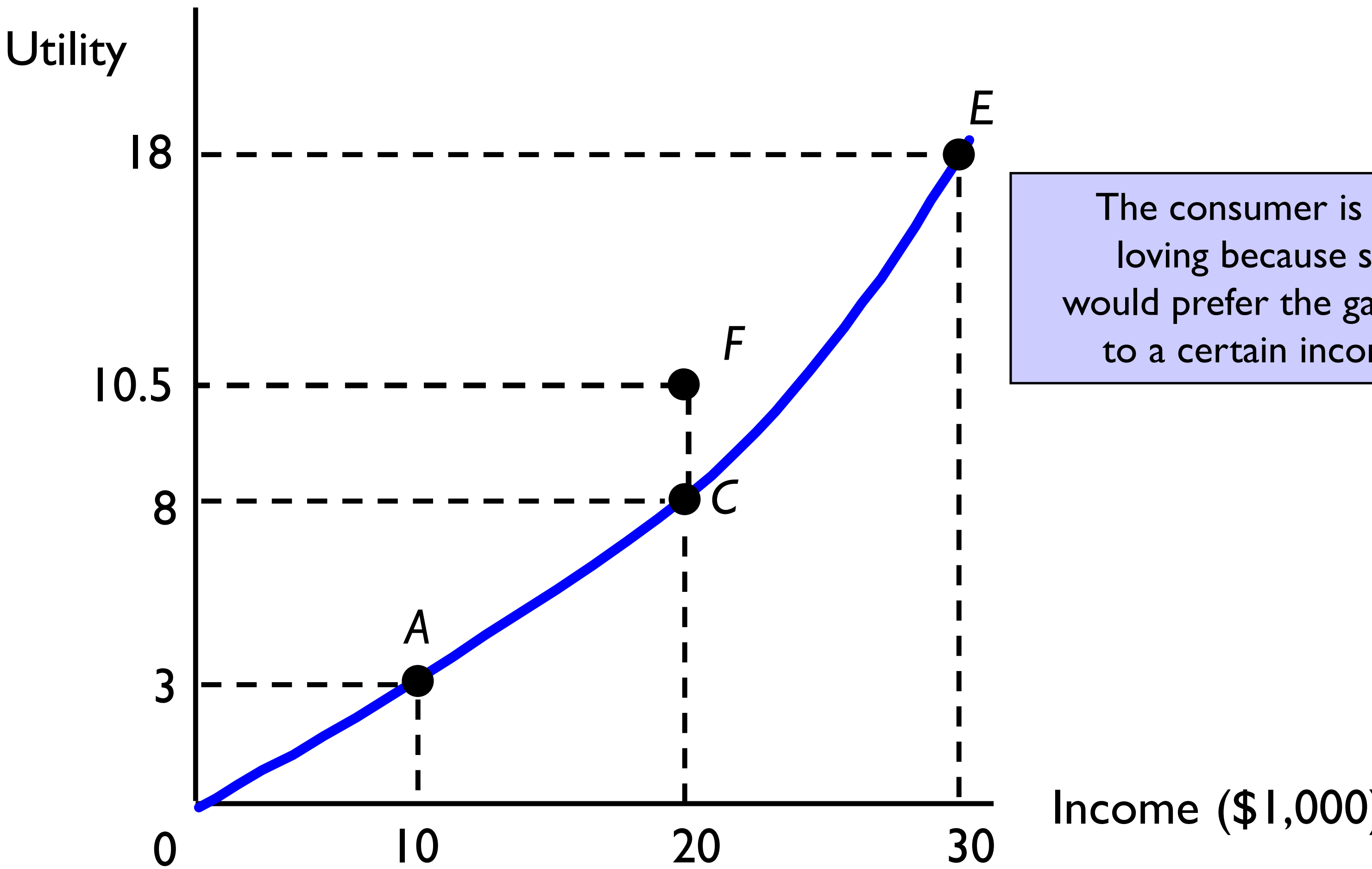


The consumer is risk neutral and is indifferent between certain events and uncertain events with the same expected income.

Preferences Toward Risk

- A person is said to be **risk loving** if they show a preference toward an uncertain income over a certain income with the same expected value
  - Examples: Gambling, some criminal activities
  - $U(EX)<Eu(x)$
- Increasing marginal utility of income

Risk Loving



The consumer is risk loving because she would prefer the gamble to a certain income.

Preferences Toward Risk

- The **risk premium** is the maximum amount of money that a risk-averse person would pay to avoid taking a risk
- The risk premium depends on the risky alternatives the person faces

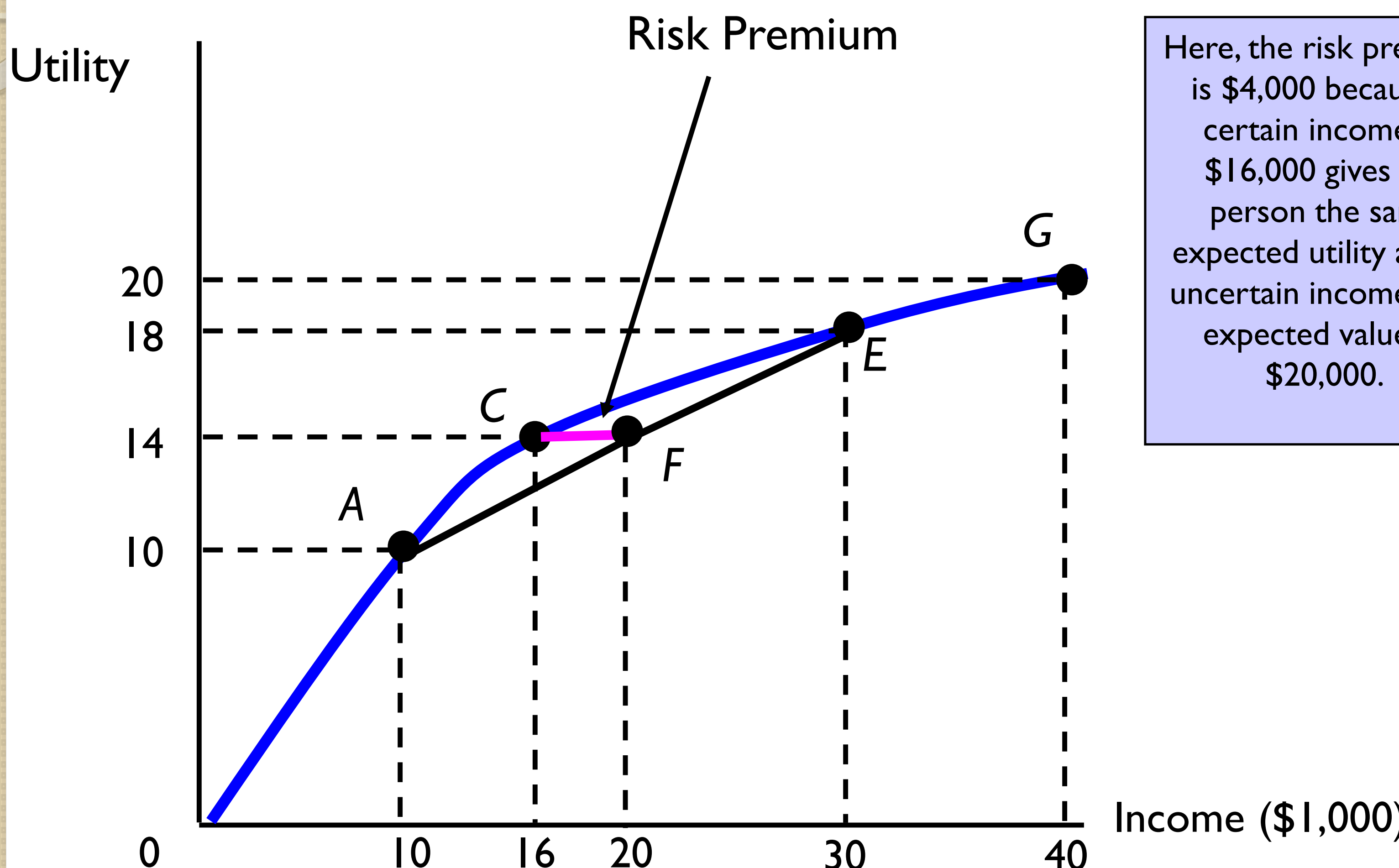
Risk Premium – Example

- From the previous example
  - A person has a .5 probability of earning \$30,000 and a .5 probability of earning \$10,000
  - The expected income is \$20,000 with expected utility of 14

Risk Premium – Example

- Point F shows the risky scenario – the utility of 14 can also be obtained with certain income of $16,000
- This person would be willing to pay up to $4000 (20,000 – 16,000) to avoid the risk of uncertain income
- Can show this graphically by drawing a straight line between the two points – line CF

Risk Premium – Example



Here, the risk premium is $4,000 because a certain income of $16,000 gives the person the same expected utility as the uncertain income with expected value of $20,000.

Risk Aversion and Income

- Variability in potential payoffs increases the risk premium
- Example:
    - A job has a .5 probability of paying $40,000 (utility of 20) and a .5 chance of paying 0 (utility of 0).

Risk Aversion and Income

- Example (cont.):
  - The expected income is still \$20,000, but the expected utility falls to 10
    - $E(u) = (0.5)u(\$0) + (0.5)u(\$40{,}000)$

      $= 0 + .5(20) = 10$
  - The certain income of \$20,000 has utility of 16
  - If person must take new job, their utility will fall by 6

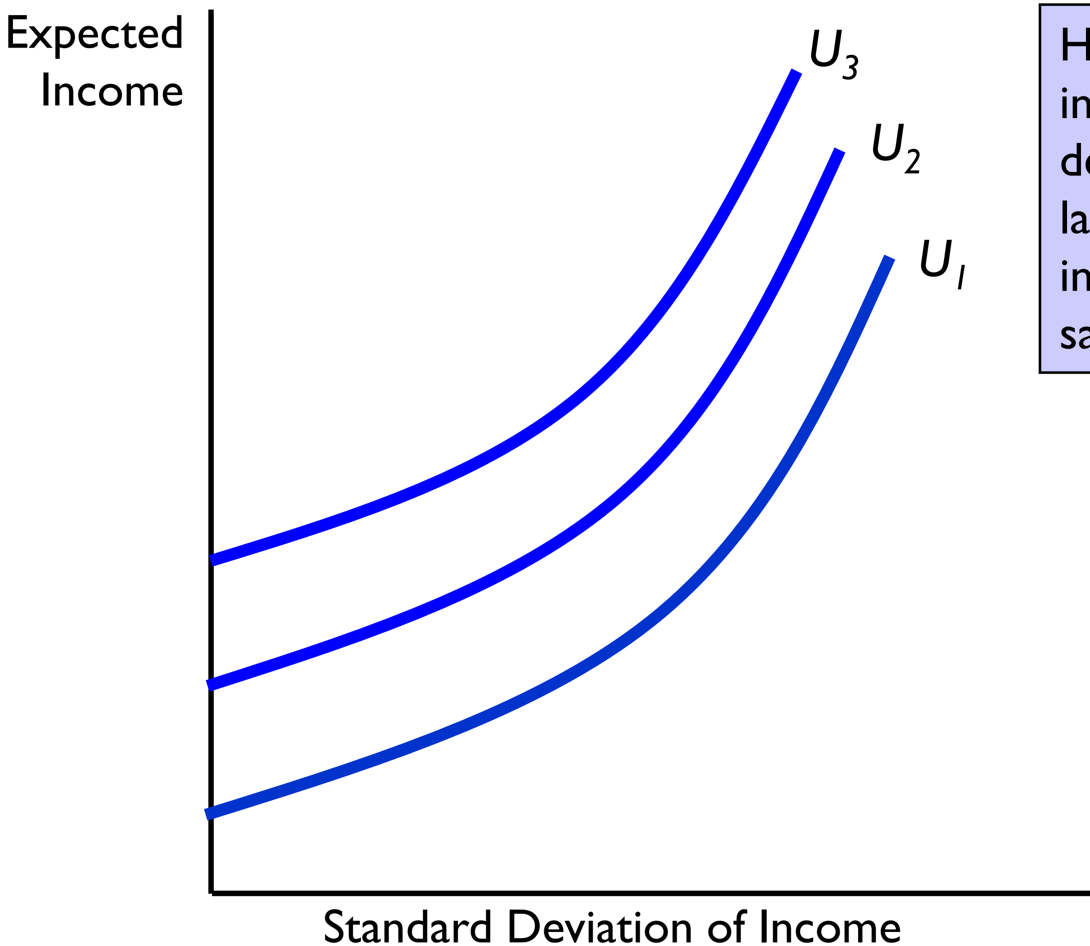
Risk Aversion and Income

- Example (cont.):
  - They can get 10 units of utility by taking a certain job paying $10,000
  - The risk premium, therefore, is $10,000 (i.e. they would be willing to give up $10,000 of the $20,000 and have the same E(u) as the risky job

Risk Aversion and Indifference Curves

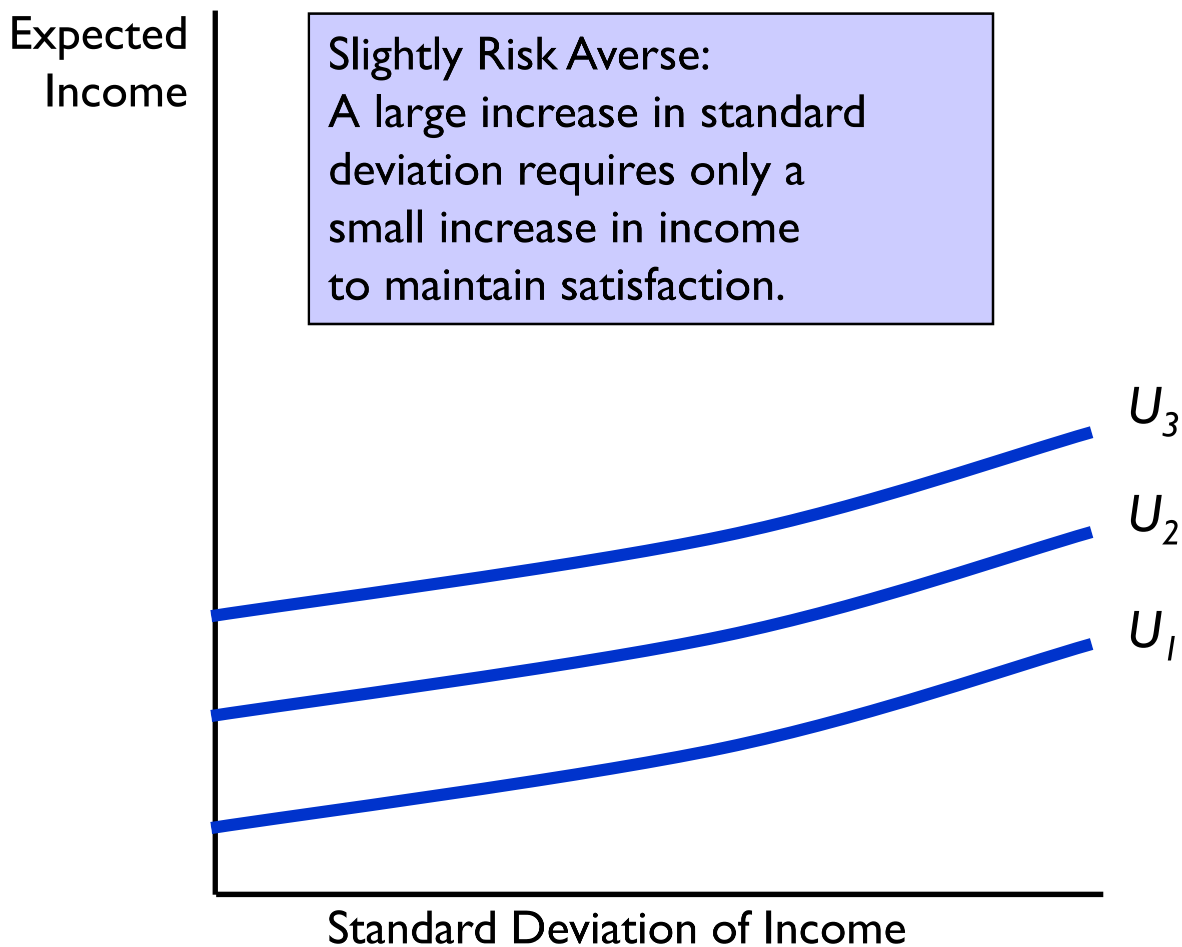
- Can describe a person's risk aversion using indifference curves that relate expected income to variability of income (standard deviation)
- Since risk is undesirable, greater risk requires greater expected income to make the person equally well off
- Indifference curves are therefore upward sloping

Risk Aversion and Indifference Curves



Highly Risk Averse: An increase in standard deviation requires a large increase in income to maintain satisfaction.

Risk Aversion and Indifference Curves



Risk and Crime Deterrence

- Attitudes toward risk affect willingness to break the law
- Suppose a city wants to deter people from double parking
- Monetary fines may be better than jail time

Risk and Crime Deterrence

- Costs of apprehending criminals are not zero, therefore
  - Fines must be higher than the costs to society
  - Probability of apprehension is actually less than one

Risk and Crime Deterrence - Example

- Assumptions:
  1. Double-parking saves a person $5 in terms of time spent searching for a parking space
  2. The driver is risk neutral
  3. Cost of apprehension is zero

Risk and Crime Deterrence - Example

- A fine greater than $5.00 would deter the driver from double parking
  - Benefit of double parking ($5) is less than the cost ($6.00) equals a net benefit that is negative
  - If the value of double parking is greater than $5.00, then the person would still break the law

Risk and Crime Deterrence - Example

- The same deterrence effect is obtained by either
  - A $50 fine with a 0.1 probability of being caught resulting in an expected penalty of $5

    or
  - A $500 fine with a 0.01 probability of being caught resulting in an expected penalty of $5

Risk and Crime Deterrence - Example

- Assumptions:
  1. The crime brings R to the criminal
  2. He is facing a penalty of C if being caught
  3. The probability of catching the criminal is p
  4. The criminal is risk neutral

Risk and Crime Deterrence - Example

- A rational criminal will not commit a crime if

  $$(1-p)\,R < pC \text{ or } R/C < p/(1-p)$$

- So raising p or raising C will get the same deterrence effect

- A similar result will be obtained if assuming risk aversion

Risk and Crime Deterrence - Example

- Enforcement costs are reduced with high fine and low probability
- Most effective if criminals don't like to take risks
- Questions:
  - Why not set an excessively high penalty?
  - Why is the penalty so low in China?

《韩非子·内储说上》

- 殷之法，刑弃灰于街者。子贡以为重，问之仲尼。仲尼曰：“知治之道也，夫弃灰于街必掩人，掩人，人必怒，怒则斗，斗必三族相残也；此残三族之道也，虽刑之可也。且夫重刑者，人之所恶也；而无弃灰，人之所易也。使人行其所易而无离其所恶，此治之道。”
-
- 公孙鞅之法也重轻罪。重罪者，人之所难犯也；而小过者，人之所易去也。使人去其所易，无离其所难，此治之道。**夫小过不生，大罪不至，是人无罪而乱不生也。**

《韩非子·内储说上》

- 荆南之地，丽水之中生金，人多窃采金。采金之禁：得而辄辜磔于市。甚众，壅离其水也，而人窃金不止。大罪莫重辜磔于市，犹不止者，不必得也。故今有于此，曰："予汝天下而杀汝身。"庸人不为也。夫有天下，大利也，犹不为者，知必死。故不必得也，则虽辜磔，窃金不止；知必死，则有天下不为也。

Reducing Risk

- Diversification
  - Reducing risk by allocating resources to a variety of activities whose outcomes are not closely related
- Example:
  - Suppose a firm has a choice of selling air conditioners, heaters, or both
  - The probability of it being hot or cold is 0.5
  - How does a firm decide what to sell?

Income from Sales of Appliances

	Hot Weather	Cold Weather
Air conditioner sales	$30,000	$12,000
Heater sales	12,000	30,000

Diversification – Example

- If the firm sells only heaters or air conditioners their income will be either \$12,000 or \$30,000
- Their expected income would be:
  - 1/2(\$12,000) + 1/2(\$30,000) = \$21,000

Diversification – Example

- If the firm divides their time evenly between appliances, their air conditioning and heating sales would be half their original values
- If it were hot, their expected income would be $15,000 from air conditioners and $6,000 from heaters, or $21,000
- If it were cold, their expected income would be $6,000 from air conditioners and $15,000 from heaters, or $21,000

Diversification – Example

- With diversification, expected income is $21,000 with no risk
- Better off diversifying to minimize risk
- Firms can reduce risk by diversifying among a variety of activities that are not closely related

Reducing Risk – The Stock Market

- If invest all money in one stock, then take on a lot of risk
  - If that stock loses value, you lose all your investment value
- Can spread risk out by investing in many different stocks or investments
  - Ex: Mutual funds

Reducing Risk – Insurance

- Risk averse are willing to pay to avoid risk
- If the cost of insurance equals the expected loss, risk averse people will buy enough insurance to recover fully from a potential financial loss

The Decision to Insure

Insurance	Burglary (Pr = .1)	No Burglary (Pr = .9)	Expected Wealth	Standard Deviation
No	40,000	50,000	49,000	3000
Yes	49,000	49,000	49,000	0

Reducing Risk – Insurance

- For the risk averse consumer, guarantee of same income regardless of outcome has higher utility than facing the probability of risk
- Expected utility with insurance is higher than without

The Law of Large Numbers

- Insurance companies know that although single events are random and largely unpredictable, the average outcome of many similar events can be predicted
- When insurance companies sell many policies, they face relatively little risk
- Why insurance companies generally don't insure earthquake and war?
- Why nationwide compulsory health insurance?

The Law of Large Numbers: An Example

- Why pooling individual and independent random outcomes will reduce risk substantially?
- Suppose N farmers and each faces a random yearly output; they agree to pool their outputs and each gets the mean

$$X_i \sim N(\mu, \sigma^2), \quad X_i \ are\ i.i.d$$

Risk Spreading

$$\bar{X}_N = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$\bar{X}_N \to \mu$$

$$Var(\bar{X}_N) = \frac{1}{N}\sigma^2 \to 0$$

How Insurance Works

- Insurance companies can be sure total premiums paid will equal total money paid out
- Companies set the premiums so money received will be enough to pay *expected* losses

How Insurance Works

- Some events with very little probability of occurrence such as floods and earthquakes are no longer insured privately
  - Cannot calculate true expected values and expected losses
  - Governments have had to create insurance for these types of events
    - Ex: National Flood Insurance Program