JPMorgan Credit Derivatives Conference

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E2C: A Simple Model to Assess Default Probabilities from Equity Markets

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Agenda

- **The model**
- Empirical study
- Risk measurement
- Conclusion

Introduction of E2C

- Equity-to-Credit Model (named “E2C”) is based on market data or observable parameters only
- Simplicity and robustness
- Currently being used to manage High Yield Credit Derivative inventory and monitor the Investment Grade Credit Derivative book

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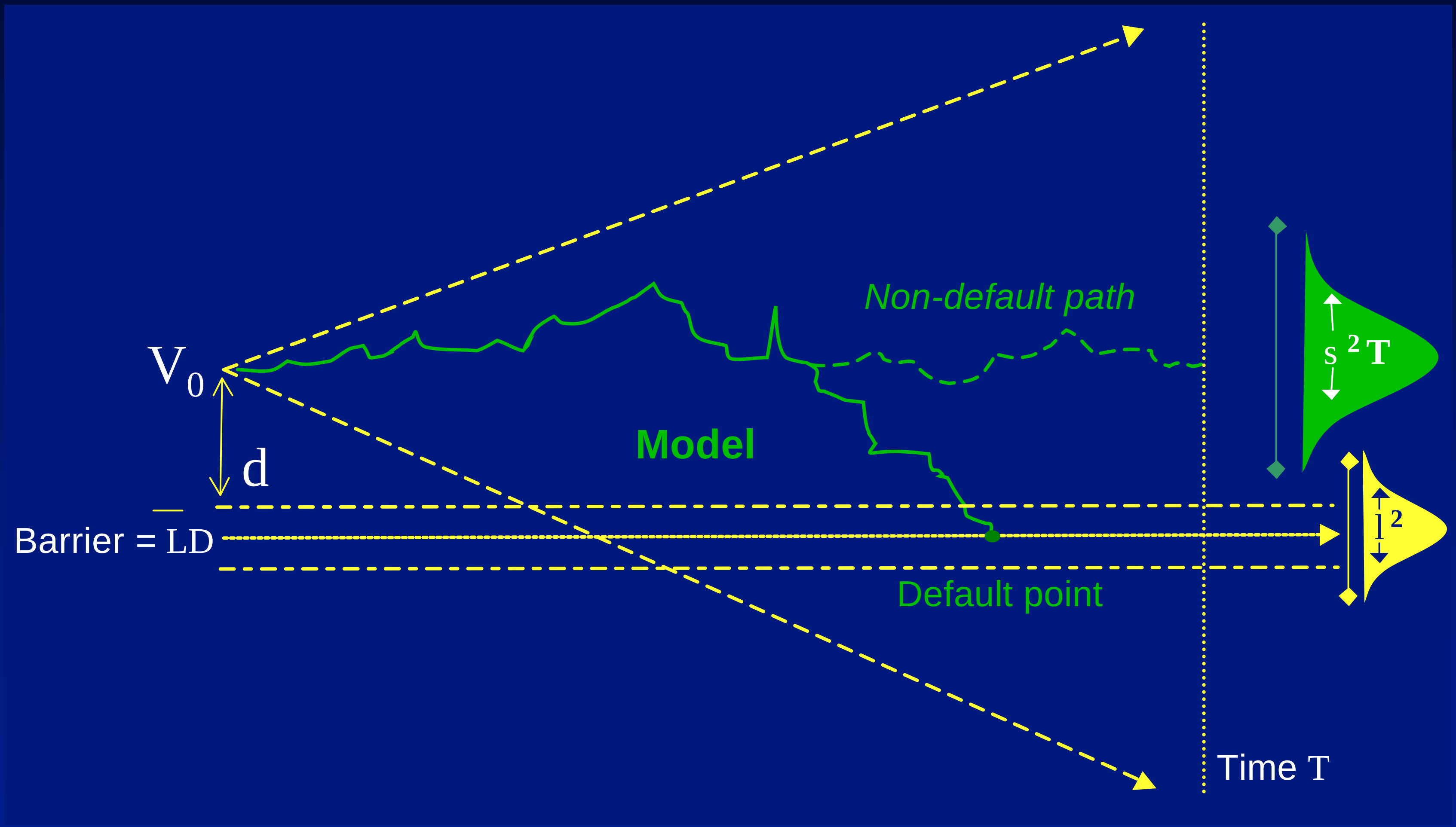
Price process and default

“Merton” framework with a “down-and-out” random barrier



The model

- Asset value follows a log-normal process with zero drift

$$\frac{dV_t}{V_t} = s_a \cdot dW_t$$

  – W_t is Brownian motion, σ_a is asset volatility

- Default occurs when

$$V_t < LD$$

  – D is the debt per share of the firm
  – L is the global recovery of the firm's liabilities $L = \frac{V(default)}{D}$

- Default barrier has a log-normal distribution with percentage standard deviation λ

$$E(LD) = \overline{L}D$$

$$Stdev(\ln(LD)) = l$$

Probability of no default

- We are able to calculate the probability of no default B(T) using the known distributions for the first stopping times of Brownian motion

$$B(T) = N\left(\frac{\ln(d)}{A_T} - \frac{A_T}{2}\right) - d \cdot N\left(-\frac{\ln(d)}{A_T} - \frac{A_T}{2}\right)$$

where: $$A_T^{\,2} = s_a^{\,2} T + l^2$$

$$d = \frac{V_0}{LD}\exp(l^2)$$

Note: N: cumulative standard normal distribution function

Determining the values of the expected recovery fraction of the debt L and the uncertainty of the default barrier λ

- $\bar{L}$ and λ are determined thanks to an empirical study of recovery rates in the event of default

- Based on the extensive database of Portfolio Management Data (PMD)/Standard & Poor’s, we assume that: $\bar{L} = 0.5$
  $\lambda = 0.3$

- Tests and comparisons of the formula’s results with the market confirm these assumptions

Note: A uniform (0,1) distribution for L would give $\bar{L}$ = 0.5 and λ = 0.58

Calculating par credit spread

- Using boundary conditions, we can identify the non-observable parameters V_0 and s_a from market observable data

$$V_0 = S_0 + \overline{L}D$$

$$s_a = s_s^* \left(\frac{V}{S^*} \frac{\partial S^*}{\partial V} \right)^{-1} = s_s^* \frac{S^*}{S^* + \overline{L}D}$$

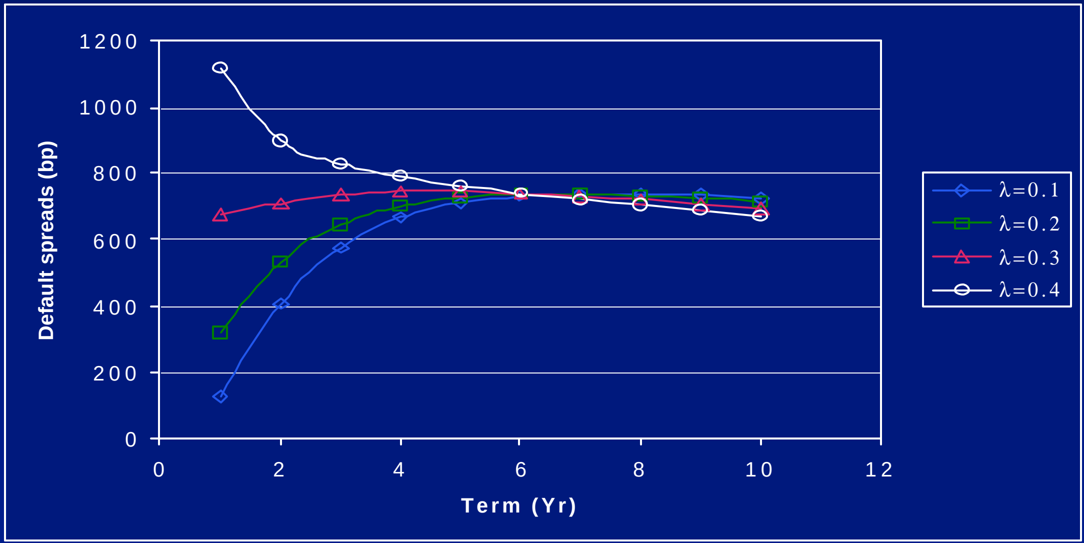
where S^* and s_s^* are reference stock price and stock volatility respectively

- By integrating the default probability density and applying an asset specific recovery assumption R_a, we arrive at a par credit spread Sp

$$Sp(T) = (1 - R_a) \frac{-\int_0^T e^{-rt} dB(t) + 1 - B(0)}{\int_0^T e^{-rt} B(t) dt}$$

Uncertainty of the default barrier (λ)

- Important parameter as you move higher up the credit spectrum
- The main driver of short-term spreads for firms with high distances to default
- The model is capable of producing different shapes for the term structure of probability of default



(S/D=0.5, $\bar{L}$ =0.5, σ_S=50%)

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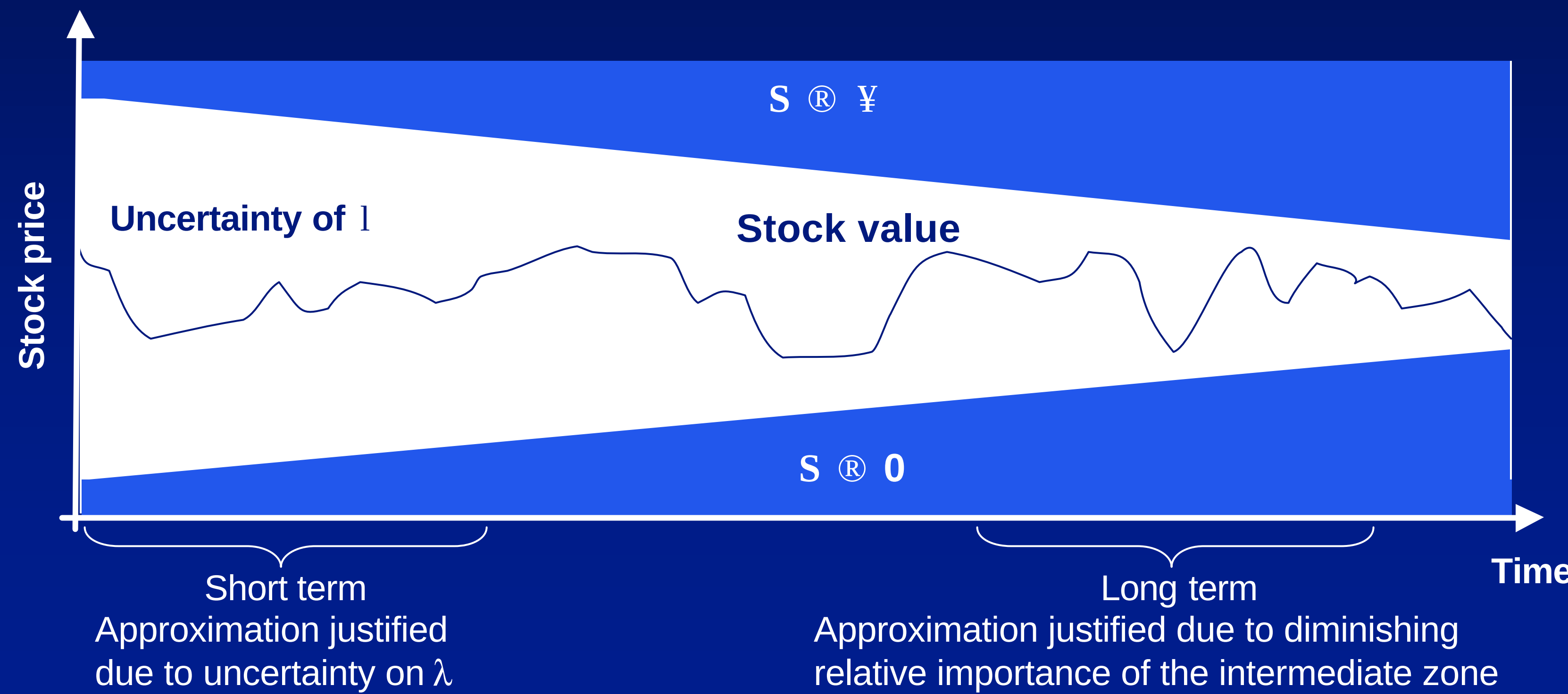
Key assumptions are validated by boundary conditions

- Default occurs for $S \sim 0$
- The simplified assumptions of $V = S_0 + \bar{L}D$ and no asset drift are accurate in the two scenarios that become more and more probable with time ($S \to 0$ and $S \to \infty$)
- In the intermediate zone, this simplification produces only minor inaccuracies



Determining the expressions of the asset value V and its volatility s_a

- The distance R (in a log–normal space) between the assets and default boundary can be expressed as follows:

$$R = \frac{\ln\left(V/LD\right)}{s_a} = \left[\frac{V}{S}\left(\frac{\partial S}{\partial V}\right)\frac{1}{s_s}\right]\ln\left(\frac{V}{LD}\right)$$

  – Near default:

$$V \cong V_{(S=0)} + \frac{\partial V}{\partial S}S + o(S^2) = LD + \frac{\partial V}{\partial S}S + o(S^2)$$

  where $V_{(S=0)} = LD$

  – Substituting into the expression for R gives up to first order:

$$R \cong \frac{1}{s_S}$$

Determining the expressions of the asset value V and its volatility s_a (cont'd)

– Away from the boundary:

$$\frac{S}{V} \to 1$$

– So that:

$$R \cong \frac{1}{s_S} \ln\left(\frac{S}{LD}\right)$$

– The simplest expression for R that satisfies both conditions is:

$$R = \frac{(S + LD)}{S s_s} \ln\left(\frac{S + LD}{LD}\right)$$

– We identify asset value and volatility as:

$$V = S + LD$$

$$s_a = s_S \frac{S}{S + LD}$$

Approximating the asset value drift m

- Near default: S = a(V – LD) $\Rightarrow$

$$m\left(LD + S/a\right) = (r - p)S/a \quad \Rightarrow \quad m \xrightarrow[S \to 0]{} 0$$

Note: (r - p): stock drift (r: stock financing rate, p: dividend yield)

- Away from the boundary: we assume more debt would be issued to keep the leverage level steady or pay dividends. So the drift of the assets relative to the default boundary would become 0

$\Rightarrow$ We assume that m ~ 0 is the best approximation

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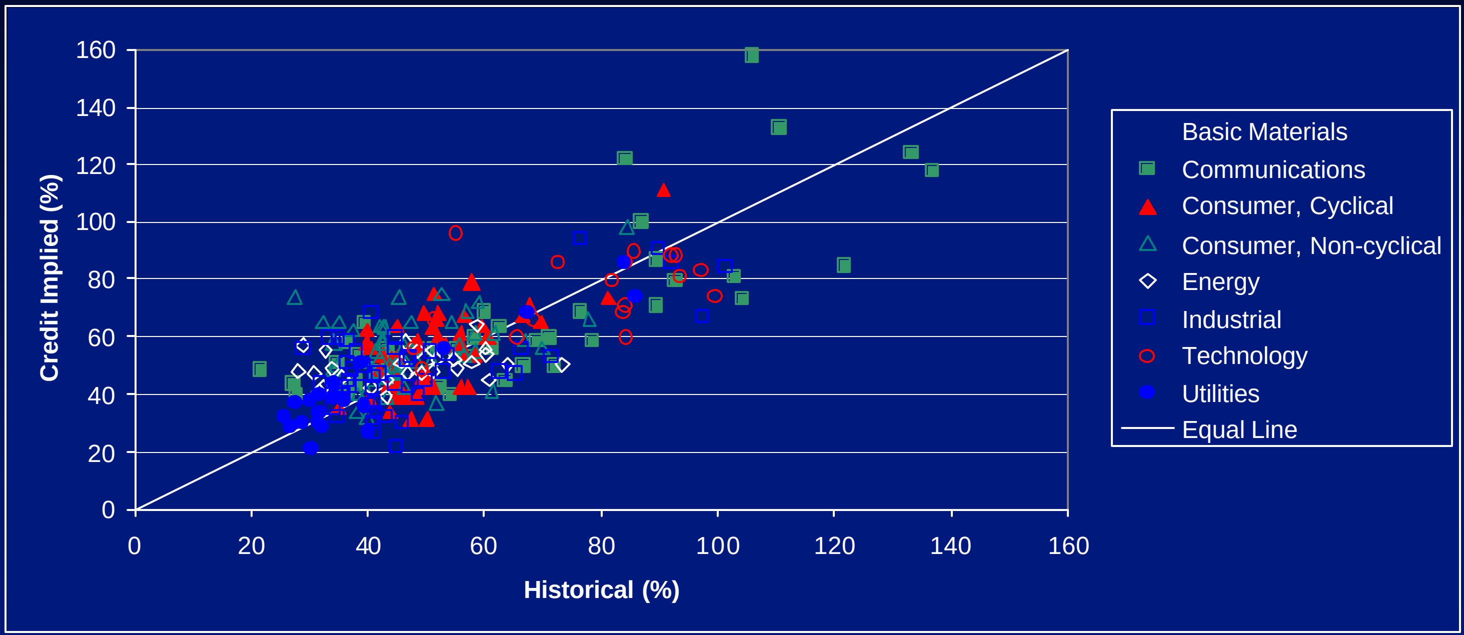
Snapshot analysis of volatility

- 298 U.S. industrial High Yield and High Grade names
- Data is as of July 2nd, 2001
- Solve for implied stock volatility s_s^* that equals three-year Credit Default Swap par spread (using $S^* = S_0$)
- Comparison with 360-day stock volatility time series

$\rightarrow$ Results show significant noise but appears unbiased overall and across industries

Snapshot analysis of volatility (cont'd)



	Historical (%)	Credit implied (%)
Average	51.6	54.2
Stdev	19.0	18.1

Quartiles	Historical (%)	Credit implied (%)
Minimum	21.6	21.0
25%	39.8	42.0
50%	46.5	51.0
75%	57.8	60.8
Maximum	136.7	158.0

Time series analysis of the market and model spreads

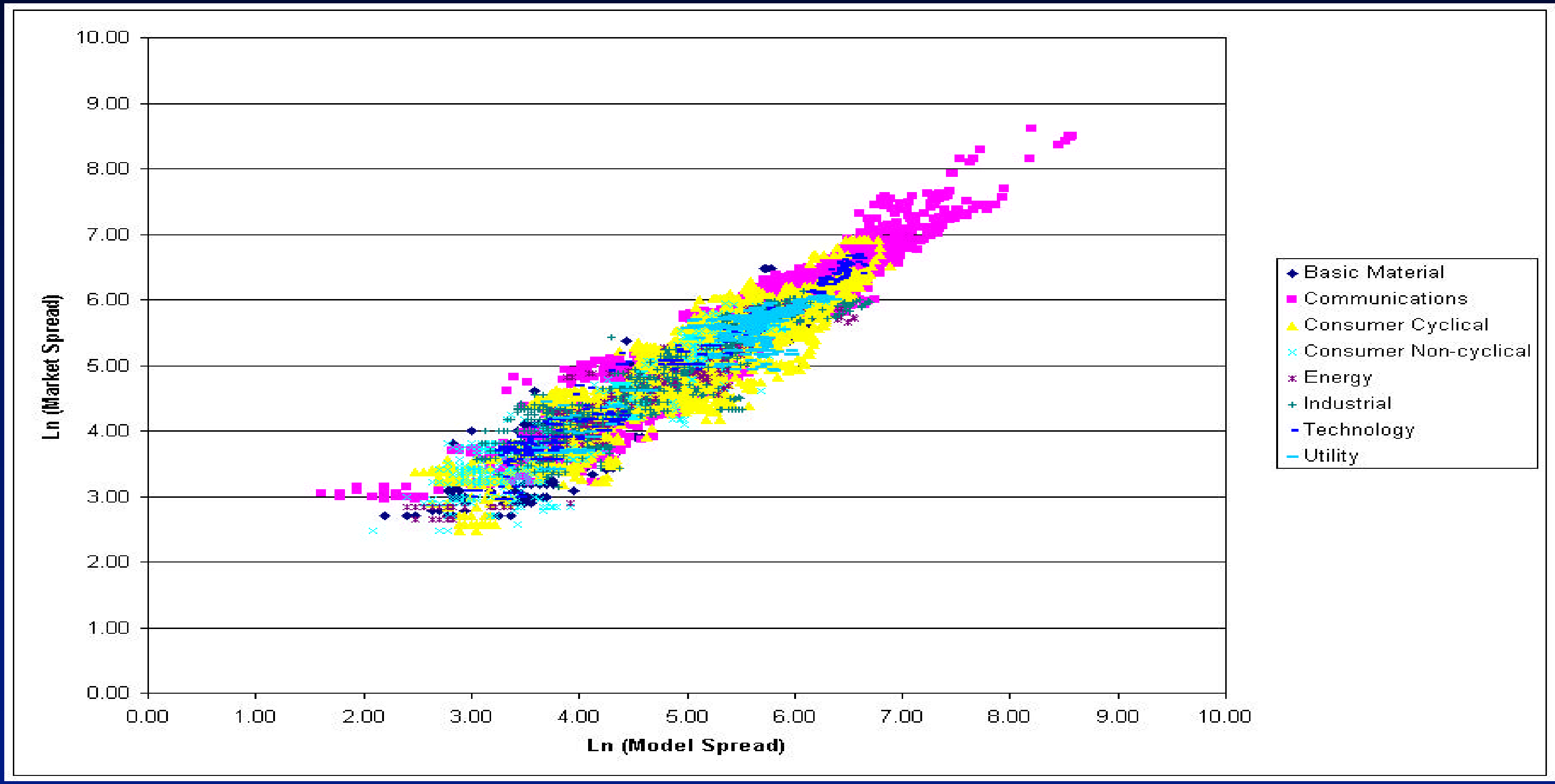
Num of names	Total data points	Live data points	Time period
142	32778	8089	01/06/2000 – 8/31/2001

Industry	Num of names
Basic Materials	14
Communications	24
Consumer Cyclical	30
Consumer non-Cyclical	24
Energy	15
Industrial	19
Technology	7
Utility	7

Rating	Number of names
AAA	1
AA	10
A	42
BBB	64
BB	20
B	5

- Model (E2C) spreads are calculated using a “flat” asset volatility (average implied volatility) for the whole period
- Market data point is called “live” on the day it is updated

Market spread versus model (E2C) spread for industries (daily – live data)



S mkt = S model * 0.94 + 12.26
(0.0037) (1.41)
R^2 =89%

Ln (S mkt) = 1.000028 * Ln (S model)
(0.00072)
R^2=90%

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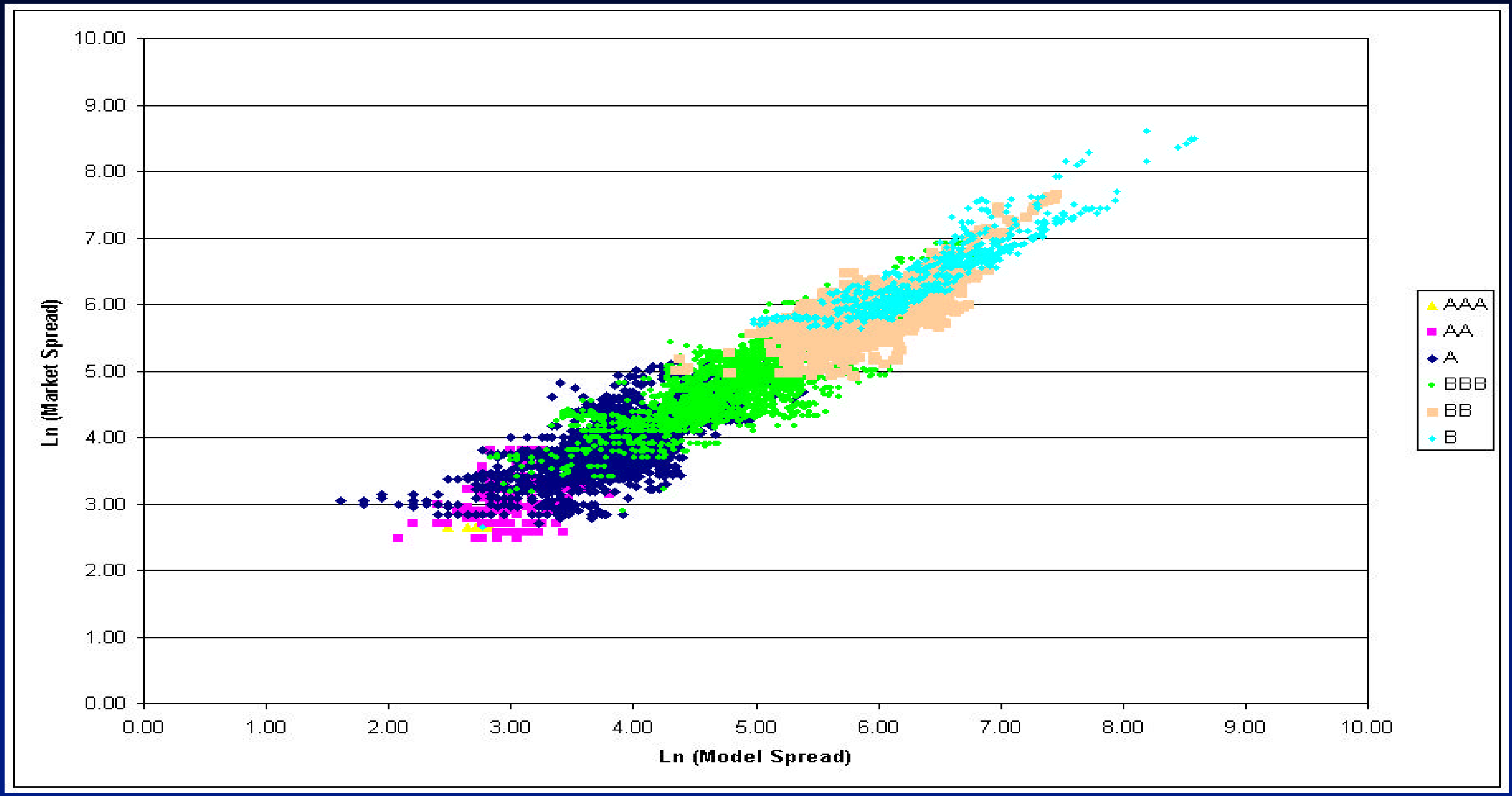
Market spread versus model (E2C) spread for industries (daily – live data) (cont'd)



S mkt = S model * 0.94 + 12.26

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Ln (S mkt) = 1.000028 * Ln (S model)

(0.00072)

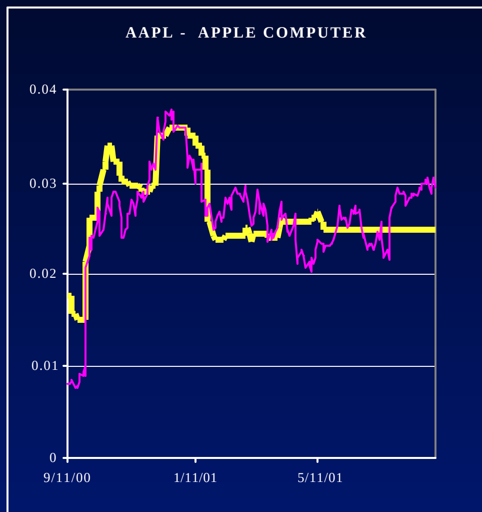
R^2=90%

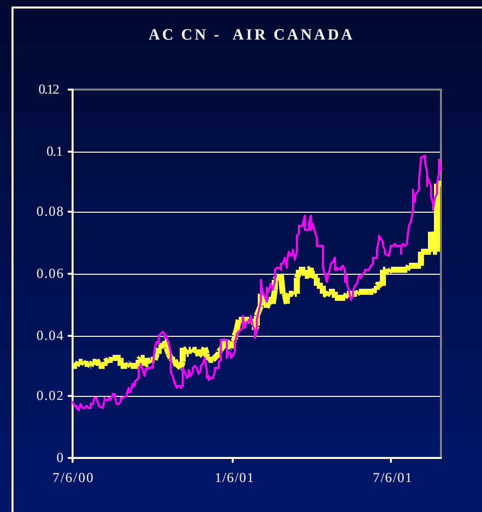
Time series analysis of the market and model (E2C) spreads

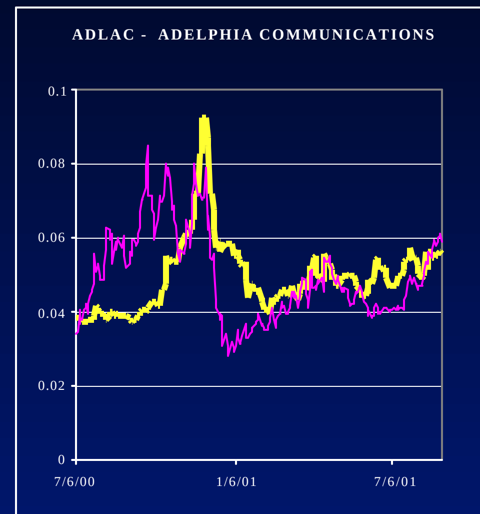
	All data			Live data		
Quartile	**1st Q**	**2nd Q**	**3rd Q**	**1st Q**	**2nd Q**	**3rd Q**
AA	(24%)	0%	16%	(25%)	0%	17%
A	(21%)	0%	18%	(17%)	7%	27%
BBB	(20%)	1%	18%	(22%)	2%	22%
BB	(18%)	2%	16%	(17%)	3%	14%
B	(19%)	(5%)	16%	(17%)	(4%)	16%
Basic materials	(23%)	0%	18%	(14%)	6%	19%
Communications	(22%)	(1%)	16%	(19%)	0%	16%
Consumer cyclical	(23%)	0%	19%	(26%)	2%	22%
Consumer non-cyclical	(21%)	0%	17%	(15%)	11%	29%
Energy	(14%)	4%	18%	(15%)	0%	12%
Industrial	(16%)	2%	18%	(16%)	7%	27%
Technology	(16%)	(2%)	14%	(6%)	8%	20%
Utility	(18%)	(2%)	11%	(19%)	(4%)	9%

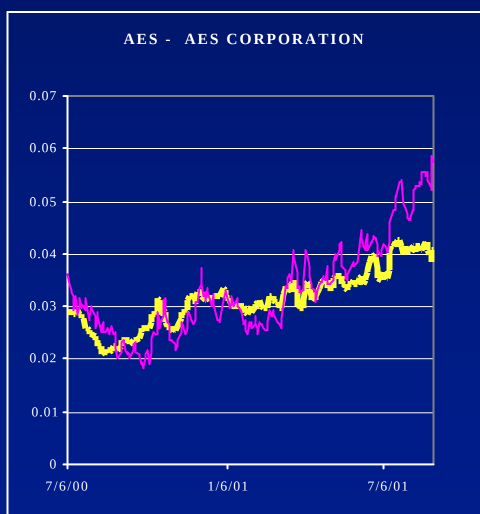
- Relative Basis = $\dfrac{Market\ \ Spread \quad - \quad Model\ \ Spread}{Market\ \ Spread}$
- 50% of the data is between [-20%, 20%] regardless of rating or industries
  $\rightarrow$ The "precision" of the model does not depend on rating or industry
- The results have been obtained with a fixed volatility and debt per share
  $\rightarrow$ Better results would have been obtained by taking a dynamic approach on these parameters

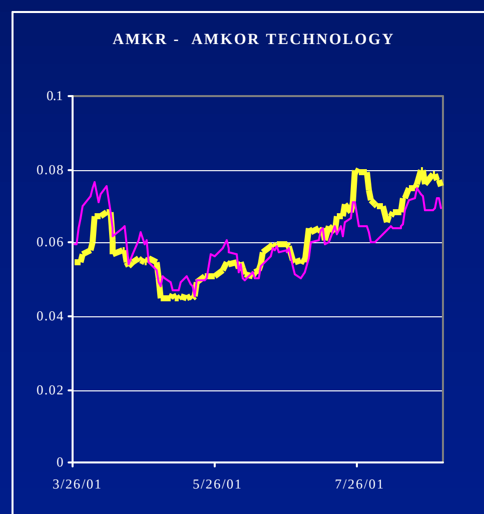
Market and model spreads (examples)





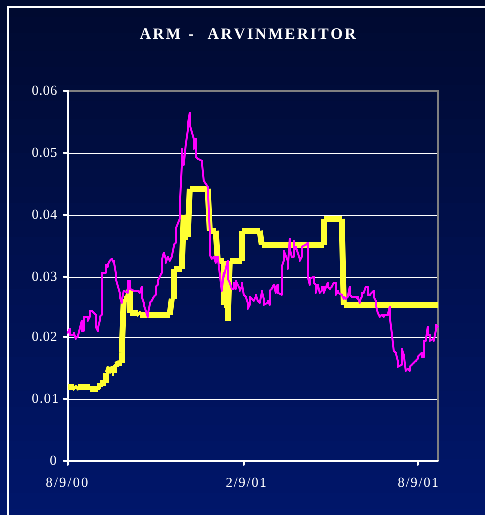


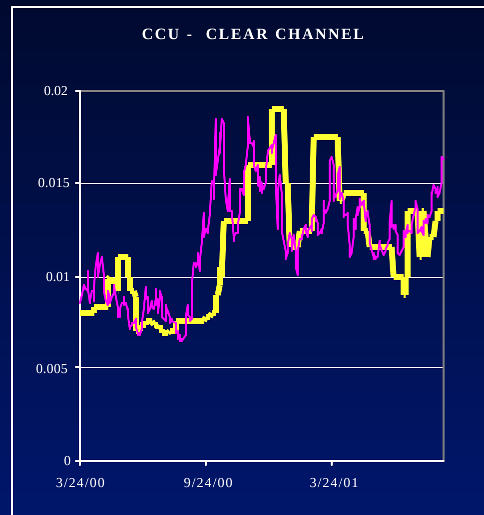


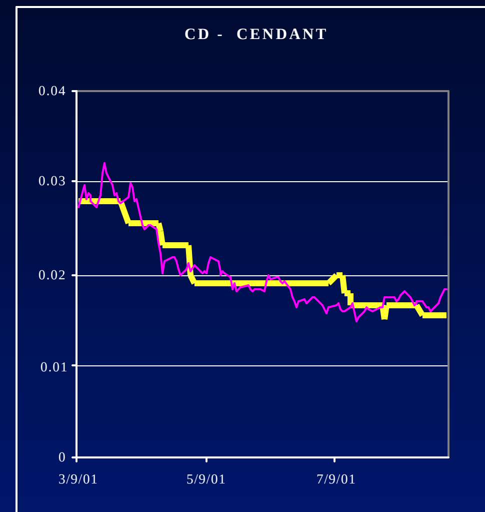


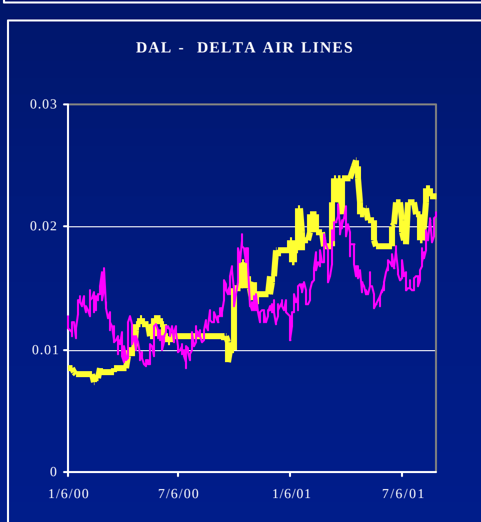
Name	AAPL	AC CN	ADLAC	AES	AMKR	AMT
Correlation of monthly variation	0.81	0.66	0.47	0.55	0.89	0.91

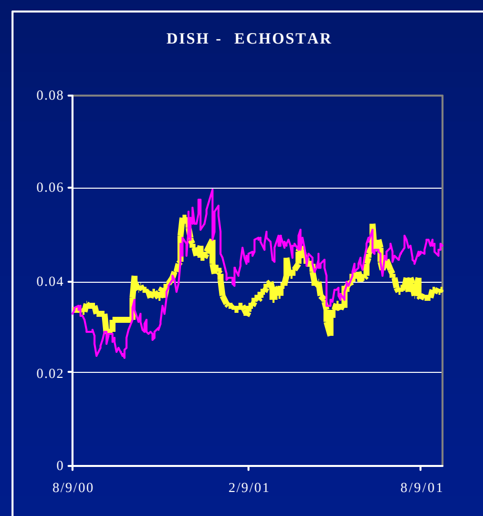
Market and model spreads (examples) (cont'd)

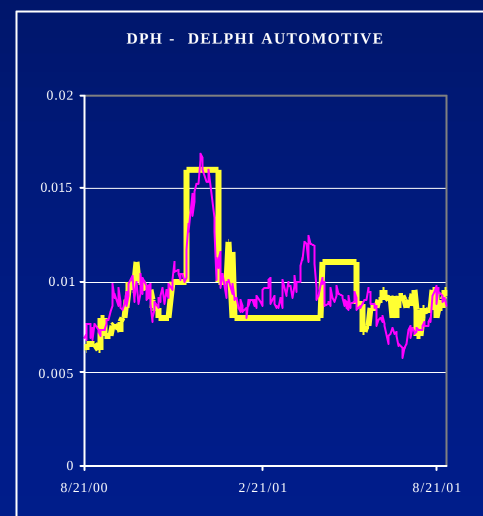






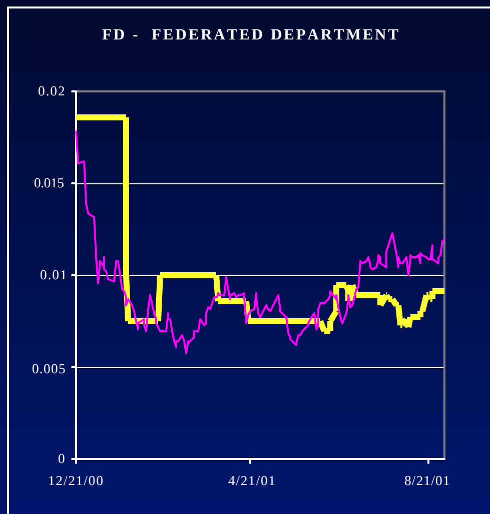


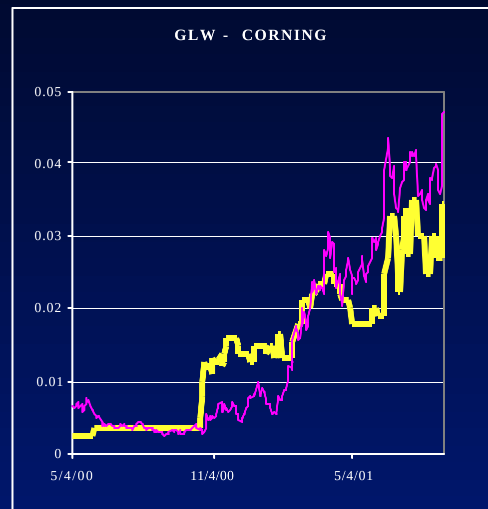


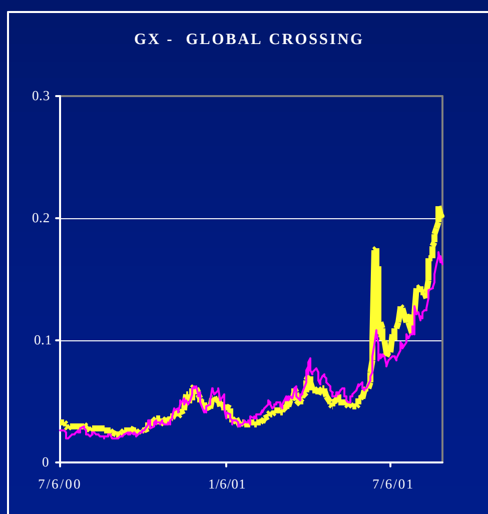


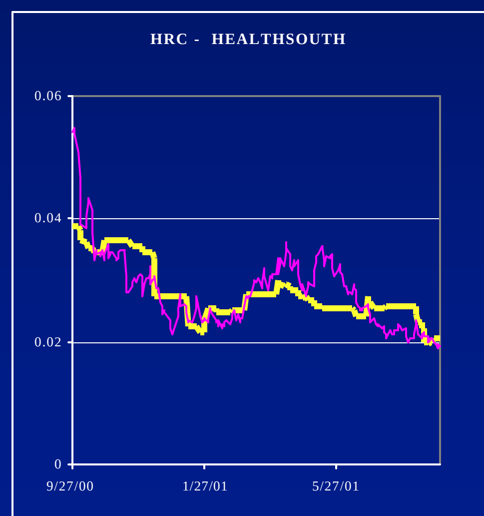
Name	ARM	CCU	CD	DAL	DISH	DPH
Correlation of monthly variation	0.40	0.25	0.29	0.15	0.76	0.60

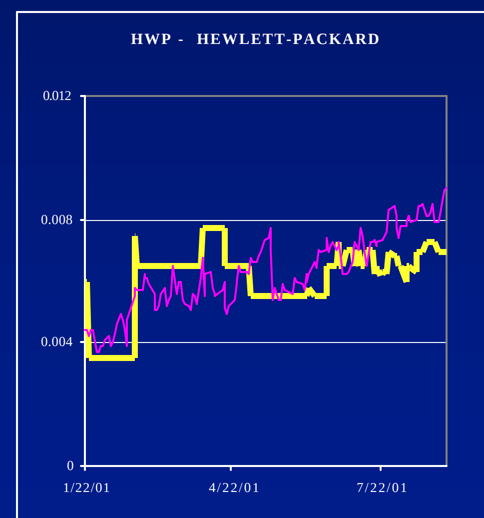
Market and model spreads (examples) (cont'd)





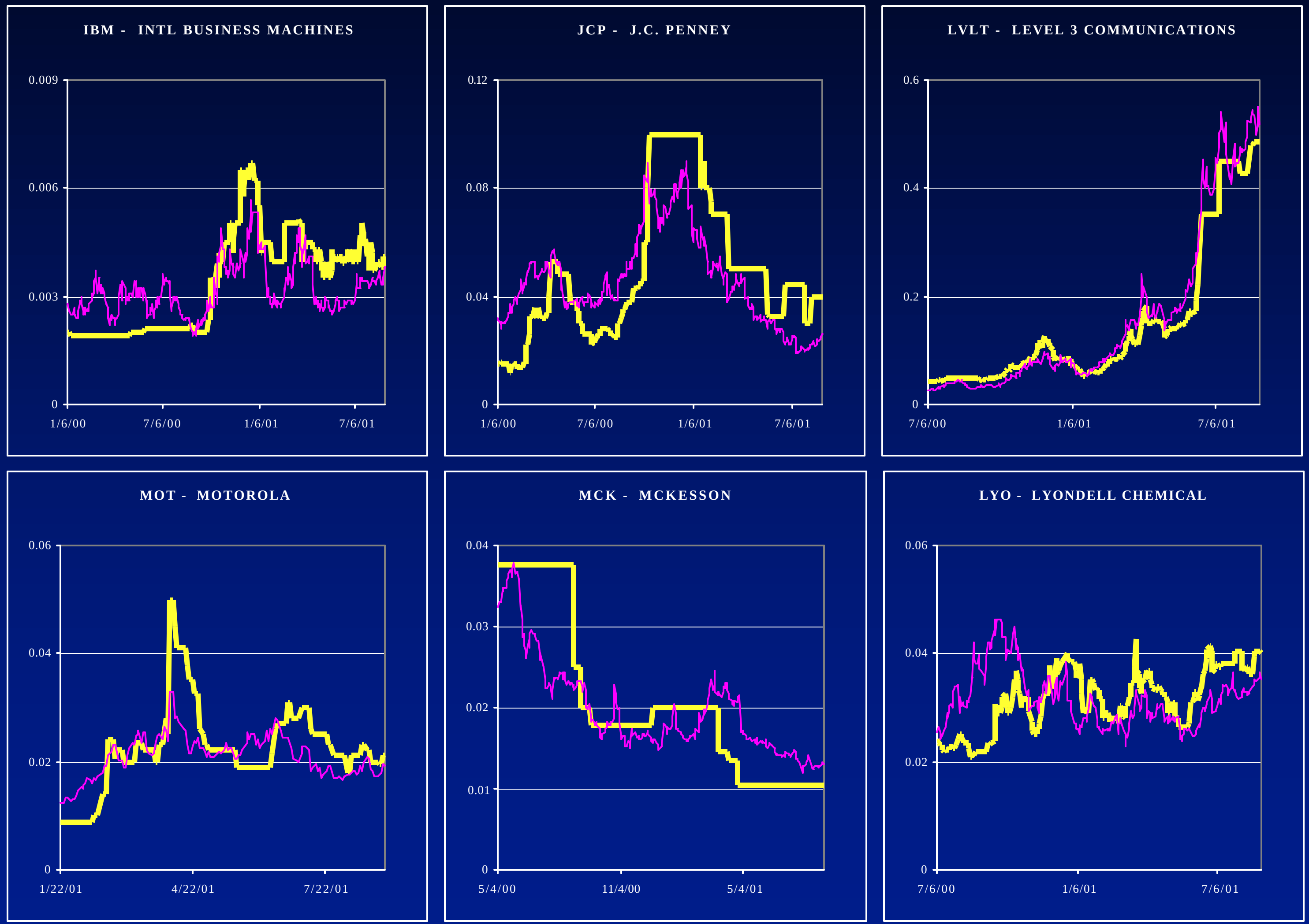






Name	FD	GLW	GM	GX	HRC	HWP
Correlation of monthly variation	0.43	0.36	0.10	0.85	0.31	0.41

Market and model spreads (examples) (cont'd)



Name	IBM	JCP	LVLT	LYO	MCK	MOT
Correlation of monthly variation	0.56	0.41	0.85	0.55	0.17	0.67

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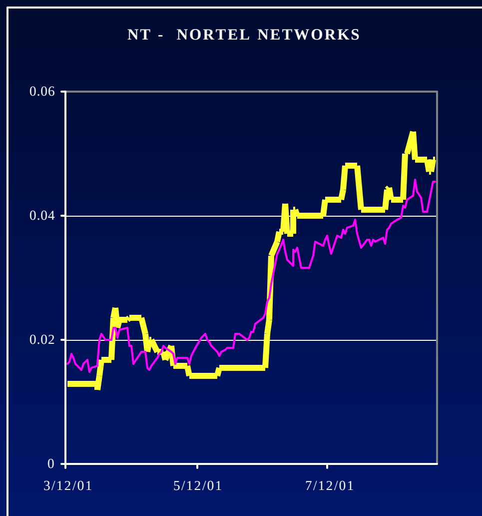
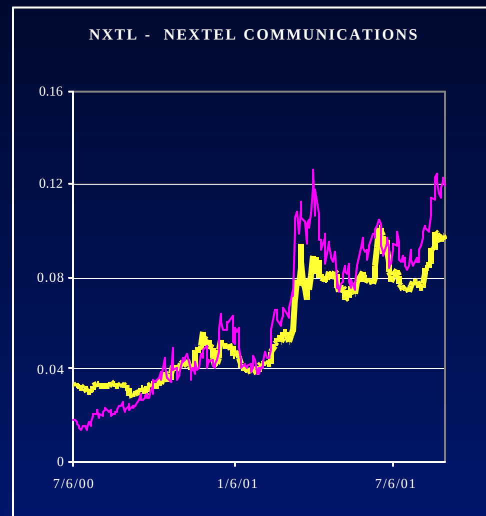
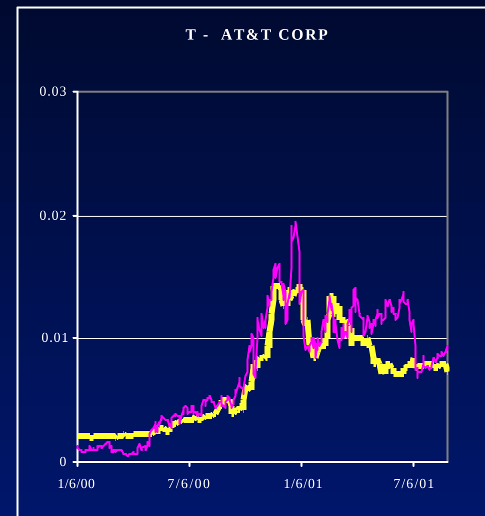
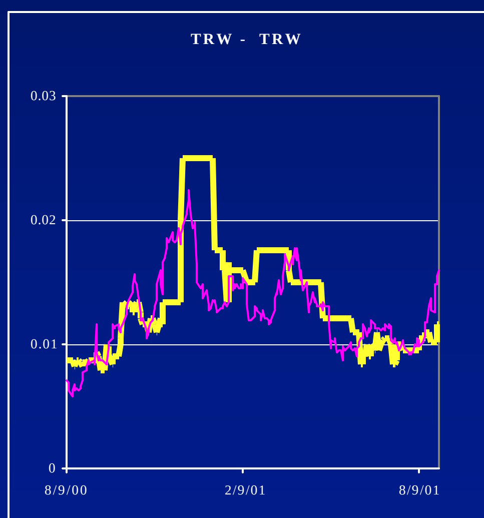
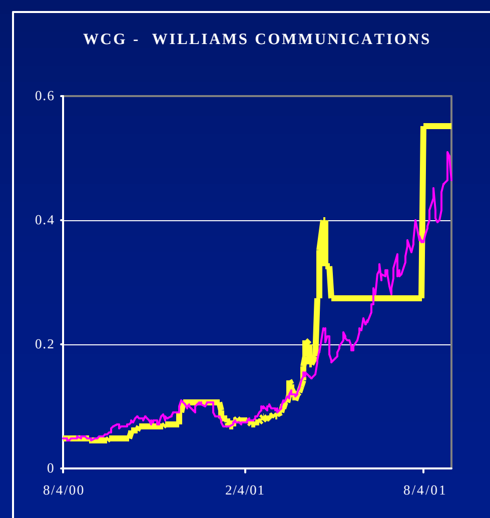
Market and model spreads (examples) (cont'd)

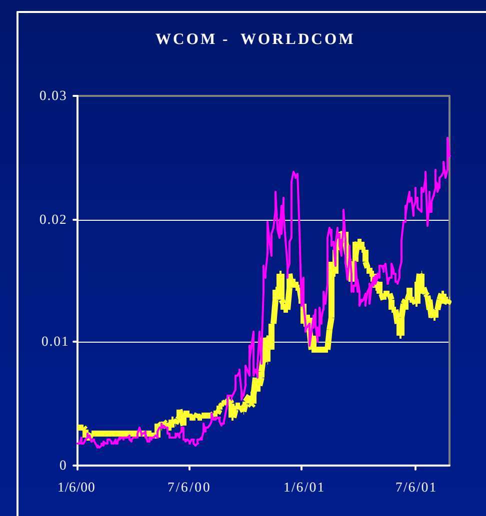






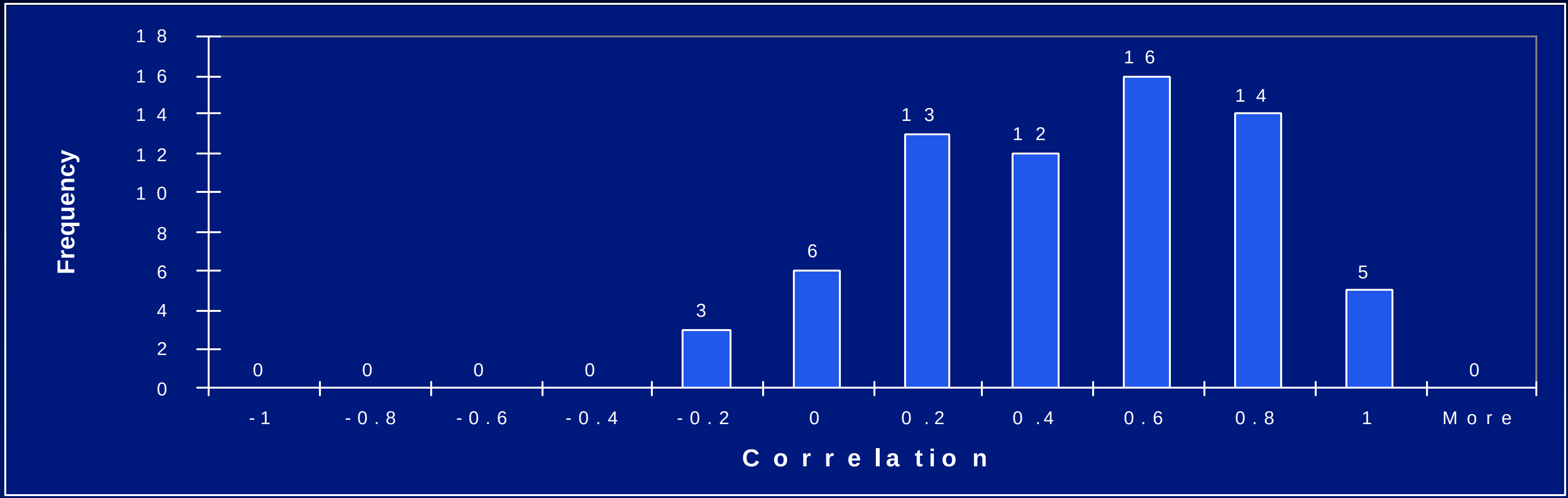






Name	NT	NXTL	T	TRW	WCG	WCOM
Correlation of monthly variation	0.76	0.75	0.40	0.33	0.61	0.48

Distribution of correlation between monthly percentage change of market and model spreads (subset of 69 most liquid names)



	# of names	Average correlation	Stdev
AA	4	0.10	0.29
A	15	0.28	0.33
BBB	25	0.32	0.25
BB	18	0.50	0.29
B	6	0.72	0.08
CCC	1	0.46	NA

	# of names	Average correlation	Stdev
Basic materials	4	0.07	0.33
Communications	23	0.51	0.25
Consumer, cyclical	18	0.29	0.26
Consumer, non-cyclical	7	0.16	0.18
Energy	1	0.68	NA
Industrial	7	0.28	0.45
Technology	6	0.6	0.22
Utilities	3	0.33	0.23

- Although the precision of the model is independent from rating, the correlation of the change in spreads is better for High Yield names

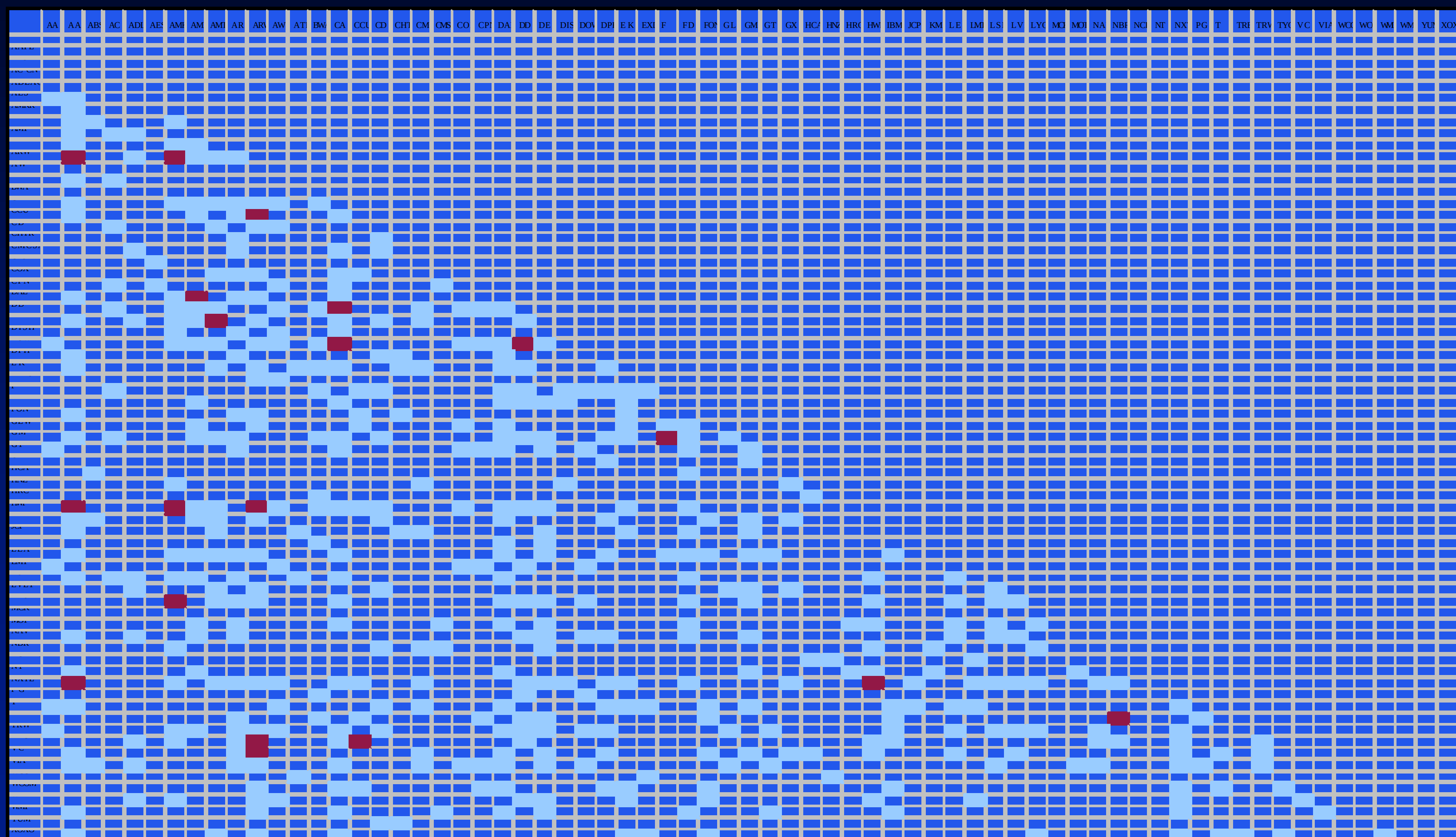
Analysis of pairwise correlation of the basis between market and model spreads

$$CDS_{Closed-form\ formula} = CDS_{Market} + white\ noise$$

Low correlation of the noise between two names

- 69 names with most frequent updates of market spreads
- Correlation between the daily moves of the basis between any pair of names
- Average correlation is 0.15 overall (69 names, March 2001–August 2001)
- Previous analysis on High Yield names displayed an average correlation of 0.05 (15 names, March 2000–August 2000)

The residual noise between the E2C model and market spread is diversifiable like a white noise



Scale

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Risk measurement

- Credit spread risk of a portfolio is traditionally measured by its sensitivity to credit spreads and expressed in terms of PVBP – Present value of a Basis Point by names, industries, maturity buckets, etc . . .
- At the same time, equity market data is more dynamic and real time stock prices are widely available

$\rightarrow$ It's natural to try to combine the information of the two markets for a better risk management framework

From sensitivities to equity equivalents

Credit spread risk management system:

$$PVBP = \frac{\partial Portfolio}{\partial Spread}$$

per name and bucket

Portfolio equity equivalents – sensitivities of portfolio to one percent change in equity price

$$= PVBP \cdot S \frac{\partial Spread}{\partial S}$$

Benchmark spread and equity sensitivities

$$DV01 = \frac{\partial P}{\partial Spread}$$

$$\$Delta = S \frac{\partial P}{\partial S}$$

Spread curve behavior – movement of spread curve per one percent change in equity price

$$S \frac{\partial Spread}{\partial S} = \frac{\$Delta}{DV01}$$

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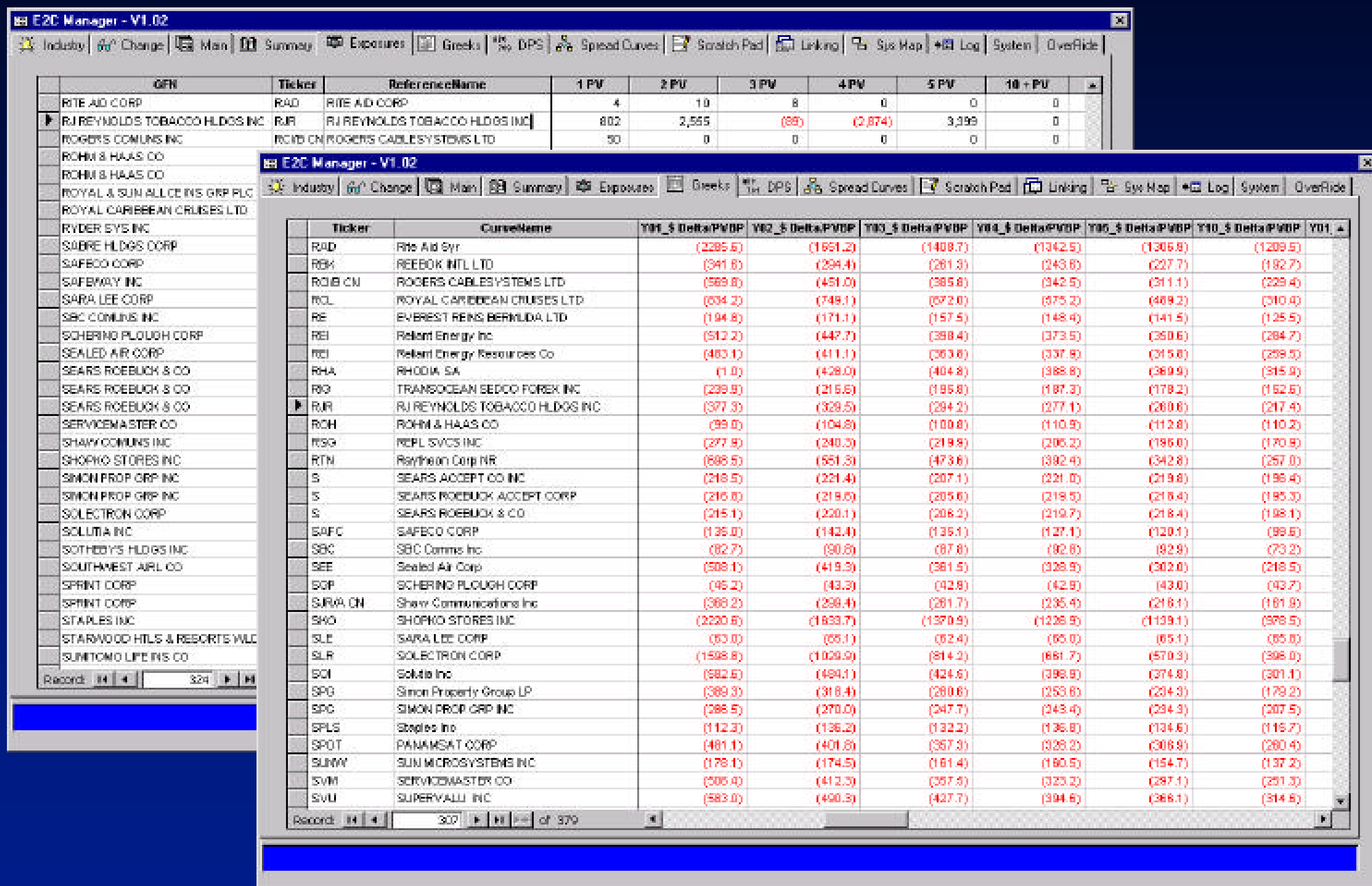
From sensitivities to equity equivalents (cont'd)



GFN	Ticker	ReferenceName	1 PV	2 PV	3 PV	4 PV	5 PV	10 + PV
RITE AID CORP	RAD	RITE AID CORP	4	10	8	0	0	0
RJ REYNOLDS TOBACCO HLDGS INC	RJR	RJ REYNOLDS TOBACCO HLDGS INC	802	2,565	(89)	(2,874)	3,399	0
ROGERS COMUNS INC	RCI/B CN	ROGERS CABLESYSTEMS LTD	50	0	0	0	0	0
ROHM & HAAS CO								
ROHM & HAAS CO								
ROYAL & SUN ALLCE INS GRP PLC								
ROYAL CARIBBEAN CRUISES LTD								
RYDER SYS INC								
SABRE HLDGS CORP								
SAFECO CORP								
SAFEWAY INC								
SARA LEE CORP								
SBC COMUNS INC								
SCHERING PLOUGH CORP								
SEALED AIR CORP								
SEARS ROEBUCK & CO								
SEARS ROEBUCK & CO								
SEARS ROEBUCK & CO								
SERVICEMASTER CO								
SHAW COMUNS INC								
SHOPKO STORES INC								
SIMON PROP GRP INC								
SIMON PROP GRP INC								
SOLECTRON CORP								
SOLUTIA INC								
SOTHEBY'S HLDGS INC								
SOUTHWEST AIRL CO								
SPRINT CORP								
SPRINT CORP								
STAPLES INC								
STARWOOD HTLS & RESORTS WLD								
SUMITOMO LIFE INS CO								

Record: 324

Ticker	CurveName	Y01_$ Delta/PVBP	Y02_$ Delta/PVBP	Y03_$ Delta/PVBP	Y04_$ Delta/PVBP	Y05_$ Delta/PVBP	Y10_$ Delta/PVBP	Y01_
RAD	Rite Aid Syr	(2285.5)	(1661.2)	(1408.7)	(1342.5)	(1306.8)	(1209.5)	
RBK	REEBOK INTL LTD	(341.6)	(294.4)	(261.3)	(243.6)	(227.7)	(192.7)	
RCI/B CN	ROGERS CABLESYSTEMS LTD	(569.8)	(451.0)	(385.8)	(342.5)	(311.1)	(229.4)	
RCL	ROYAL CARIBBEAN CRUISES LTD	(634.2)	(749.1)	(672.0)	(675.2)	(469.2)	(310.4)	
RE	EVEREST REINS BERMUDA LTD	(194.8)	(171.1)	(157.5)	(148.4)	(141.5)	(125.5)	
REI	Reliant Energy Inc	(512.2)	(447.7)	(398.4)	(373.5)	(350.6)	(284.7)	
REI	Reliant Energy Resources Co	(463.1)	(411.1)	(363.6)	(337.9)	(315.8)	(259.5)	
RHA	RHODIA SA	(1.0)	(428.0)	(404.8)	(388.8)	(369.9)	(315.9)	
RIG	TRANSOCEAN SEDCO FOREX INC	(239.9)	(215.6)	(195.8)	(187.3)	(178.2)	(152.6)	
RJR	RJ REYNOLDS TOBACCO HLDGS INC	(377.3)	(329.5)	(294.2)	(277.1)	(260.6)	(217.4)	
ROH	ROHM & HAAS CO	(99.0)	(104.8)	(100.8)	(110.9)	(112.8)	(110.2)	
RSG	REPL SVCS INC	(277.9)	(240.3)	(219.9)	(206.2)	(196.0)	(170.9)	
RTN	Raytheon Corp NR	(686.5)	(551.3)	(473.6)	(392.4)	(342.8)	(257.0)	
S	SEARS ACCEPT CO INC	(218.5)	(221.4)	(207.1)	(221.0)	(219.8)	(198.4)	
S	SEARS ROEBUCK ACCEPT CORP	(216.8)	(219.6)	(205.6)	(219.5)	(218.4)	(195.3)	
S	SEARS ROEBUCK & CO	(215.1)	(220.1)	(206.2)	(219.7)	(218.4)	(198.1)	
SAFC	SAFECO CORP	(135.0)	(142.4)	(135.1)	(127.1)	(120.1)	(99.6)	
SBC	SBC Comms Inc	(82.7)	(90.6)	(87.8)	(92.8)	(92.9)	(73.2)	
SEE	Sealed Air Corp	(508.1)	(419.3)	(361.5)	(328.9)	(302.0)	(218.5)	
SGP	SCHERING PLOUGH CORP	(45.2)	(43.3)	(42.8)	(42.9)	(43.0)	(43.7)	
SJR/A CN	Shaw Communications Inc	(368.2)	(298.4)	(261.7)	(235.4)	(216.1)	(161.9)	
SKO	SHOPKO STORES INC	(2220.6)	(1633.7)	(1370.9)	(1226.9)	(1139.1)	(978.5)	
SLE	SARA LEE CORP	(63.0)	(65.1)	(62.4)	(65.0)	(65.1)	(65.6)	
SLR	SOLECTRON CORP	(1598.8)	(1029.9)	(814.2)	(661.7)	(570.3)	(396.0)	
SOI	Solutia Inc	(582.5)	(484.1)	(424.6)	(398.9)	(374.8)	(301.1)	
SPG	Simon Property Group LP	(389.3)	(318.4)	(280.6)	(253.6)	(234.3)	(179.2)	
SPG	SIMON PROP GRP INC	(286.5)	(270.0)	(247.7)	(243.4)	(234.3)	(207.5)	
SPLS	Staples Inc	(112.3)	(136.2)	(132.2)	(136.8)	(134.6)	(116.7)	
SPOT	PANAMSAT CORP	(481.1)	(401.8)	(357.3)	(328.2)	(308.9)	(260.4)	
SUNW	SUN MICROSYSTEMS INC	(178.1)	(174.5)	(161.4)	(160.5)	(154.7)	(137.2)	
SVM	SERVICEMASTER CO	(506.4)	(412.3)	(357.5)	(323.2)	(297.1)	(251.3)	
SVU	SUPERVALU INC	(583.0)	(490.3)	(427.7)	(394.6)	(366.1)	(314.6)	

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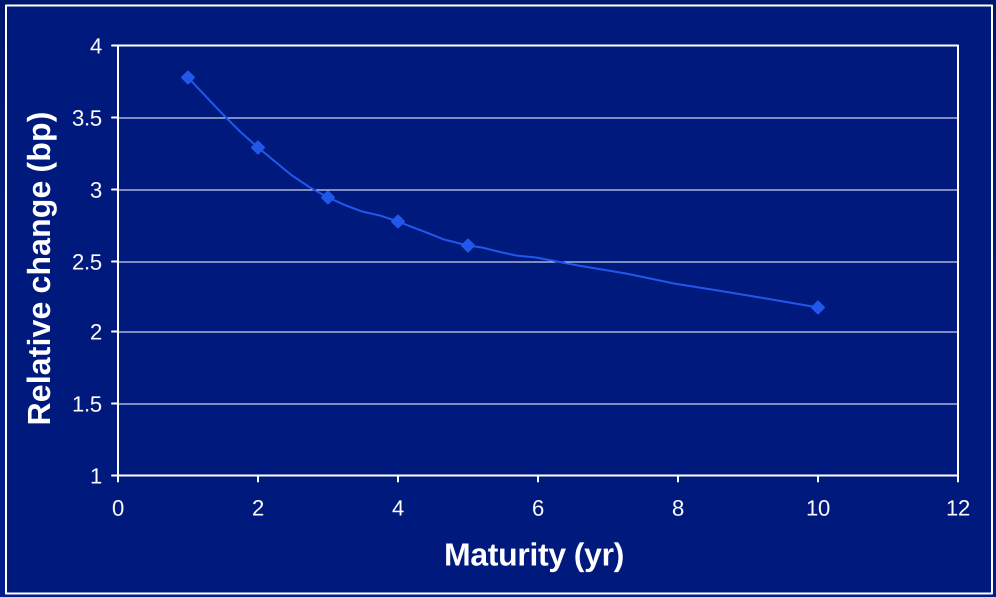
Example: RJR. Exposure in 5-year = 3,399 ($/bp), and the 5-year spread will move by 2.6 bp for 1% change in stock price, therefore, the equity equivalent position for the 5-year exposure is given by $3,399 * 2.6 * 100 = $883,740 worth of RJR shares

Implications for the behavior of credit curves

- The model implicitly provides the factor shape – the relative move of different maturity buckets of the spread curve

- Example: RJR. For every 3.3bp move in two-year spread, the five-year spread should move 2.6 bp



Agenda

- The model
- Empirical study
- Risk measurement
- **Conclusion**

Advantages of the closed-form formula rely in its simple expression . . .

- No default probability (Lardy – Finkelstein – Khuong-Huu – Yang)

$$B(T) = N\left(\frac{\ln(d)}{A_T} - \frac{A_T}{2}\right) - d \cdot N\left(-\frac{\ln(d)}{A_T} - \frac{A_T}{2}\right)$$

is expressed as a function of market observable parameters

$$A_T^{\ 2} = \left(s_S^{\ *}\frac{S^*}{S^* + \overline{L}D}\right)^2 T + l^2$$

$$d = \frac{S_0 + \overline{L}D}{\overline{L}D}\exp(l^2)$$

Notes: S_0: stock price at time T=0
S^*: reference stock price
s_S^*: reference stock volatility
D: debt per share
$\overline{L}$: global recovery
λ: percentage standard deviation of the default barrier
N: cumulative standard normal distribution function

. . . as well as in its robustness

- The formula is a simple expression of observable parameters
- Robustness and approximations are validated by boundary conditions
- It is a general result as it approximates any sophisticated model with the same boundary conditions
- Very well supported by historical back testing charts across industries and ratings, and other empirical research, including comparisons with major available credit analysis tools
- Basis (i.e., difference) between model spread and market spread
  - Unbiased
  - Precision independent of industry and rating
  - It behaves as white noise and is diversifiable

The closed-form formula has a wide range of applications

- Discovery tool to determine the fair price of credit risk
- Arbitrage of equity and credit price anomalies
- Risk management of credit instruments using equity or equity options
- Portfolio management

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- 1st - Synthetic CDO's
- 1st – Basket Default Swaps
- 1st – Exotic Credit Derivatives
- 2nd – Credit-linked Notes
- 3rd – Vanilla Default Swaps – Investment Grade – Emerging Markets and Asia

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