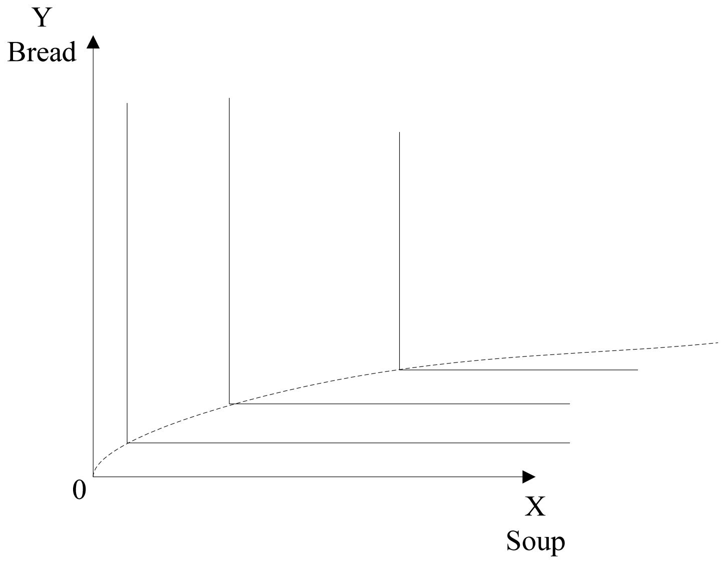
Answer to PS1

1. a)

Denote X as # pints of soup, Y as # ounces of bread.

The utility function of Ada can be denoted as $u(X, Y) = \min\{\sqrt{X}, Y\}$ or $u(X, Y) = \min\{X, Y^2\}$.

So, her indifference curve is like below figure:

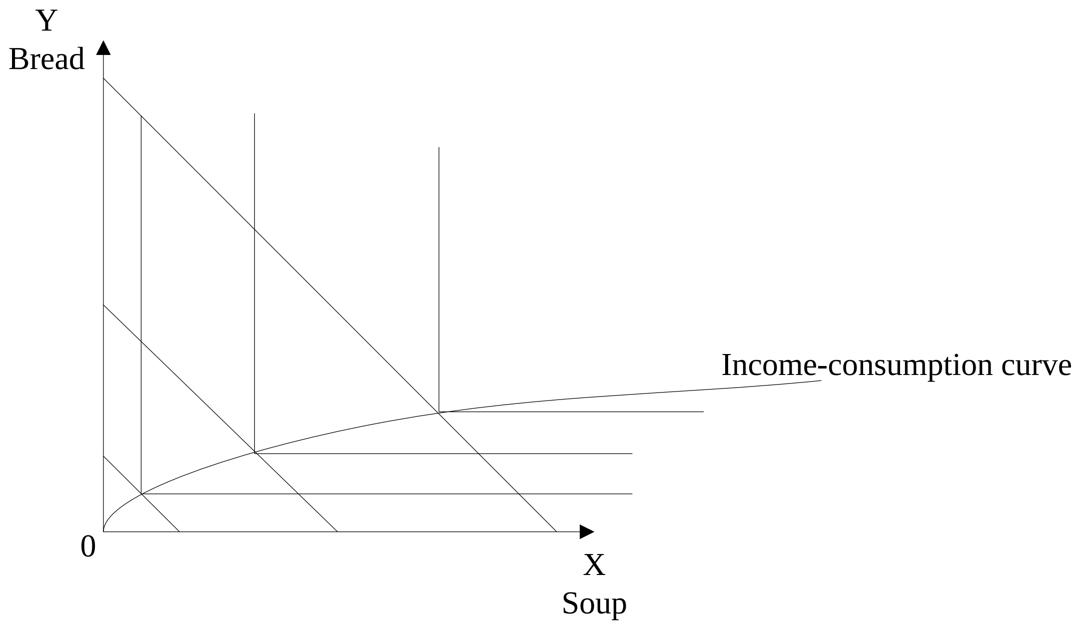


b)

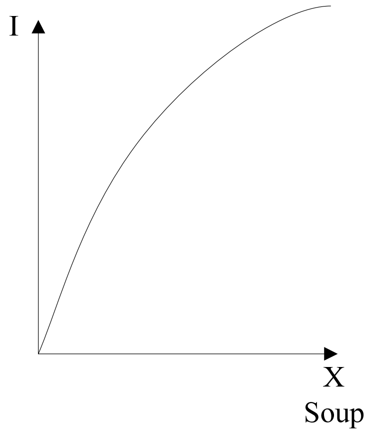
Suppose the price of soup is P_X, the price of bread is P_Y. And Ada's total income is I.

Then the budget constraint is $P_X X + P_Y Y \leq I$

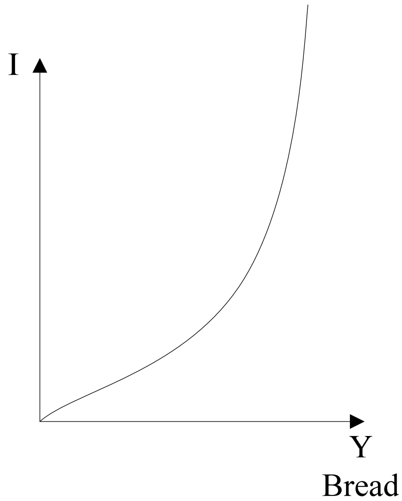
The income-consumption curve is



The Engel curve for soup is



The Engel curve for bread is



c)

Max $\min\{\sqrt{X}, Y\}$

s.t. $P_X X + P_Y Y \leq I$

From the analysis above, the optimal X and Y meet

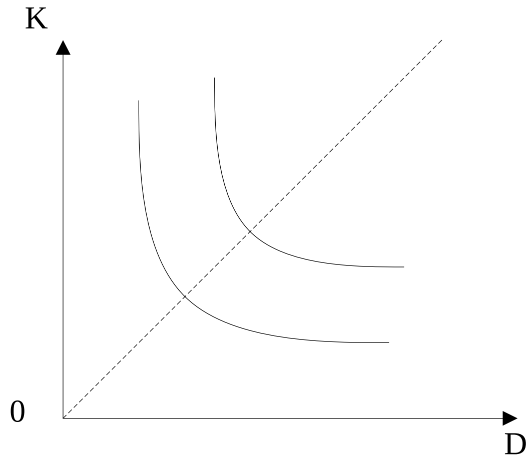
$$\begin{cases} Y = \sqrt{X} \\ P_X X + P_Y Y = I \end{cases}$$

$$\Rightarrow \begin{cases} X^* = \left(\frac{-P_Y + \sqrt{P_Y^2 + 4IP_X}}{2P_X}\right)^2 \\ Y^* = \frac{-P_Y + \sqrt{P_Y^2 + 4IP_X}}{2P_X} \end{cases}$$

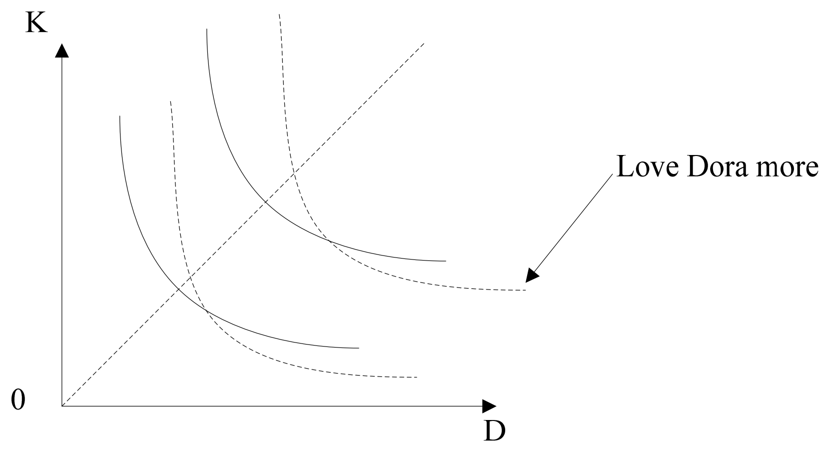
2. a)

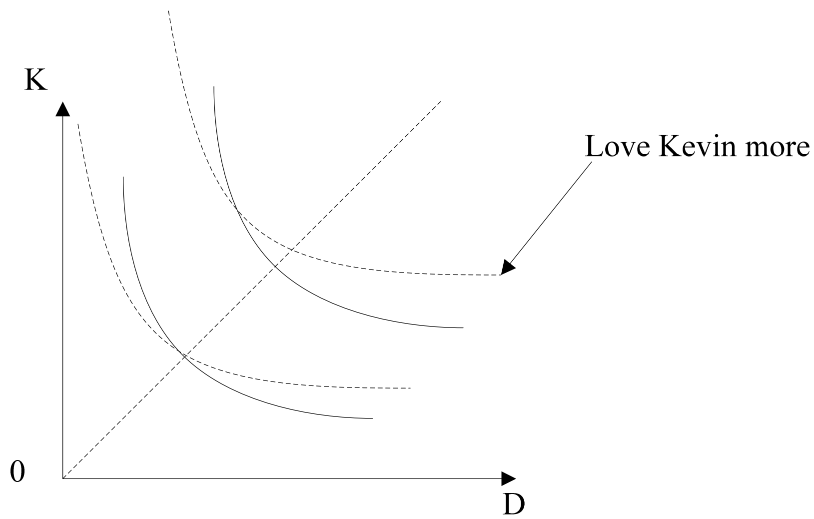
Suppose K is # yummies Kevin consumes and D is # yummies Dora consumes.

Gary loves both children equally, which means Gary's indifference curves are symmetric about the line K=D.

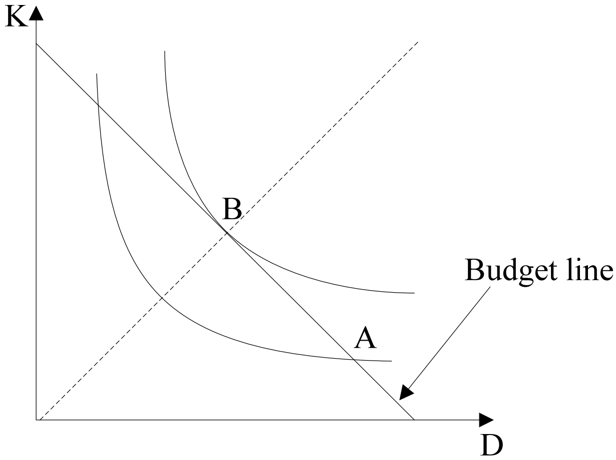


b) Suppose Gary loved Dora more than Kevin. He is willing to lose more yummies for Kevin to substitute 1 unit yummy for Dora.





c) Budget constraint: $\mathrm{K} + \mathrm{D} = 10$



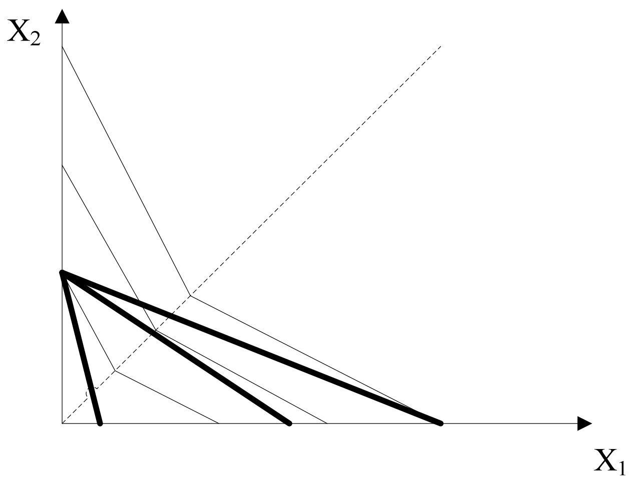
Point A is the original distribution. Point B is the optimal distribution.

d) Only the original distribution changes. The optimal distribution reaches still at Point B.

3. Suppose the price of X_1 is P_1. The price of X_2 is P_2. Income is m.

$u = \min\{2X_1 + X_2, 2X_2 + X_1\}\backslash$

$$\Rightarrow \quad u = \begin{cases} 2X_1 + X_2, \text{if } X_1 \le X_2 \\ 2X_2 + X_1, \text{if } X_2 < X_1 \end{cases}$$



If $\frac{P_1}{P_2} > 2,\ X_1 = 0, X_2 = \frac{m}{P_2};$

If $\frac{P_1}{P_2} = 2, X_1 = \alpha\frac{m}{P_1+P_2}, X_2 = \alpha\frac{m}{P_1+P_2} + (1-\alpha)\frac{m}{P_2}$, where $\alpha \in [0,1]$;

If $\frac{1}{2} < \frac{P_1}{P_2} < 2, X_1 = X_2 = \frac{m}{P_1+P_2};$

If $\frac{P_1}{P_2} = \frac{1}{2}, X_1 = \beta\frac{m}{P_1+P_2} + (1-\beta)\frac{m}{P_1}, X_2 = \beta\frac{m}{P_1+P_2}$, where $\beta \in [0,1]$;

If $0 < \frac{P_1}{P_2} < \frac{1}{2},\ X_1 = \frac{m}{P_1}, X_2 = 0.$

4. a)

max log $U(x, y)$=2log x+3log y

s.t. $p_x x+p_y y=I$

So, the demand function for x and y are $\begin{cases} x^* = \frac{2I}{5p_x} \\ y^* = \frac{3I}{5p_y} \end{cases}$.

b)

x(1, 8, 30)=12, y(1, 8, 30)=2.25

Bundle A is (12, 2.25)

x(1, 2, 30)=12, y(1, 2, 30)=9

Bundle C is (12, 9)

c)

If we use Slutsky de-composition:

If as the price of y drops to 2, and the income changes to keep the purchasing power, then the income would satisfy: I'=12*1+2*2.25=16.5, this gives x(1,2,16.5)=6.6, y(1,2,16.5)=4.95, so B = (6.6, 4.95) Thus, the slusky substitution effect is 4.95-2.25=2.7 as price goes down; the income effect is 9-4.95=4.05 as price goes down.

The substitution effect is negative. The income effect is positive. And the total effect is negative.

Since the consumption of y increases as the income increases, y is not inferior; and the total effect is (9-2.25) as price goes down, y is not giffen. So, Y is a nomal good.

If we use Hicksian de-composition,

Suppose the intermediate bundle B=(x, y)

U(A)=U(B)

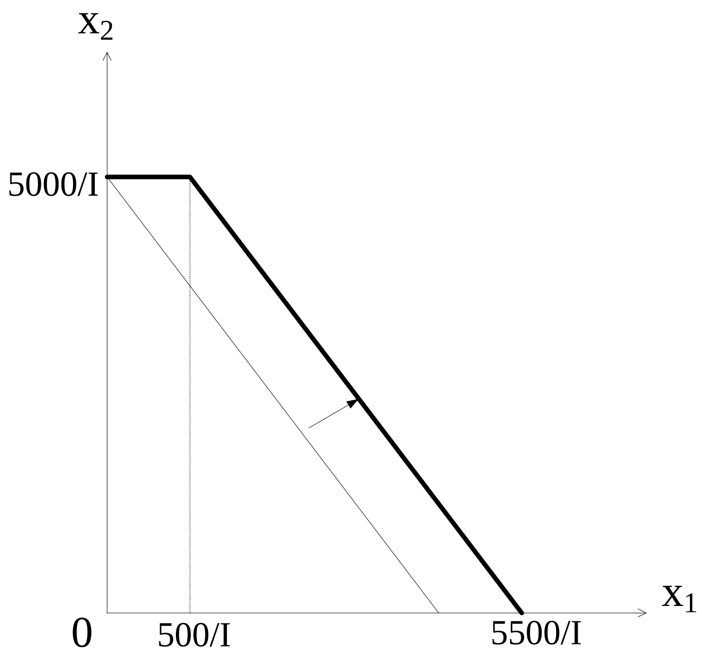
$$\frac{x}{y} = \frac{2p_y'}{3p_x} = \frac{2\times 2}{4} = \frac{4}{3}$$

So, buddle B=(5.22, 3.92).

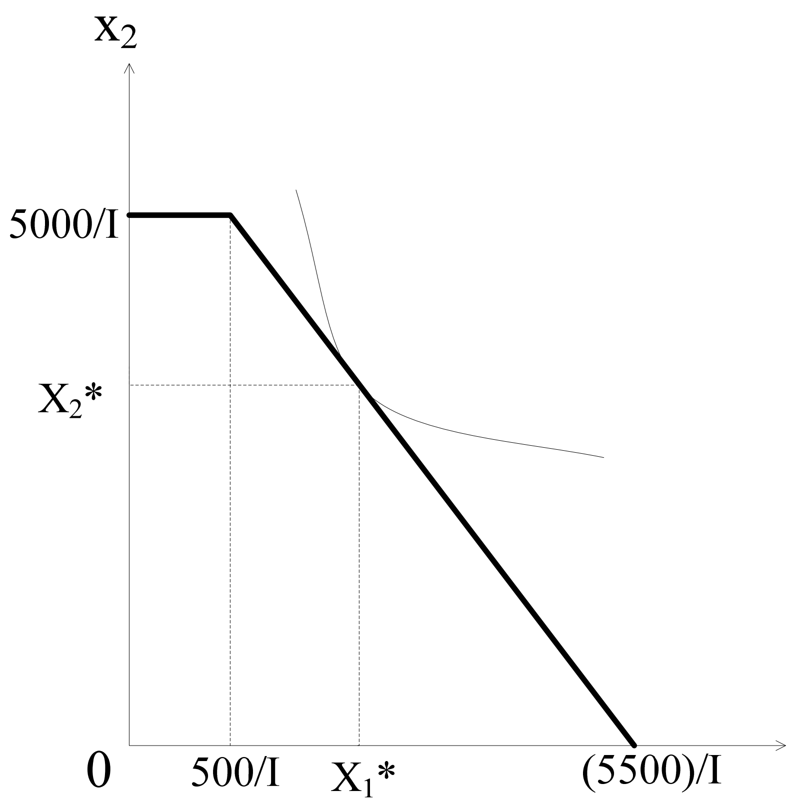
5. a)

The new budget line is

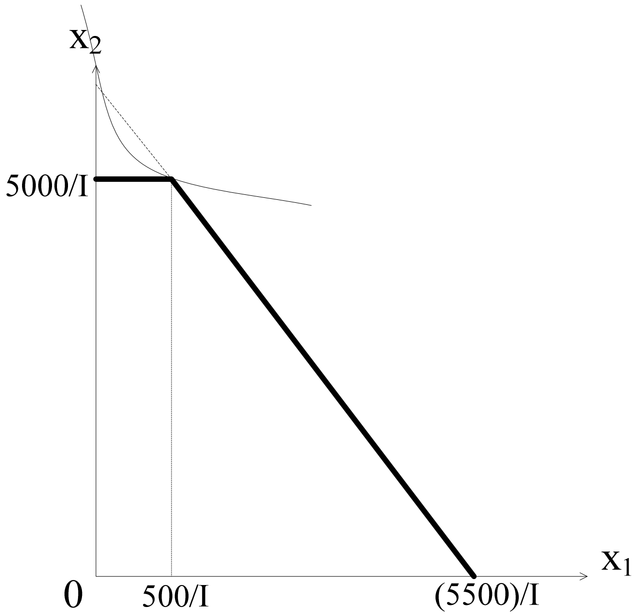
$$\begin{cases} x_2 = \frac{5000}{I} \text{ if } 0 \le x_1 < \frac{500}{I} \\ x_1 + x_2 = \frac{5500}{I} \text{ if } \frac{500}{I} \le x_1 \le \frac{5500}{I} \end{cases}$$



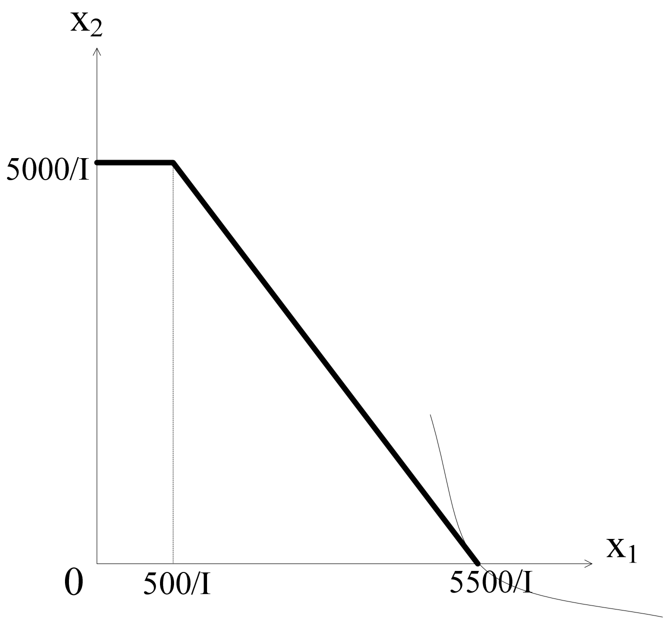
Situation I:



Situation II



Situation III:



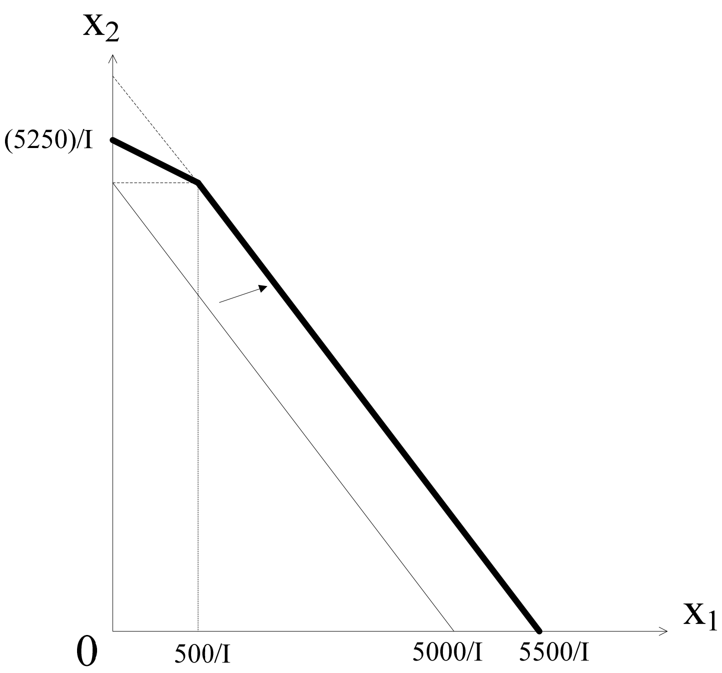
So, the optimal demand for moon cake is $[\frac{500}{I}, \frac{5500}{I}]$

b)

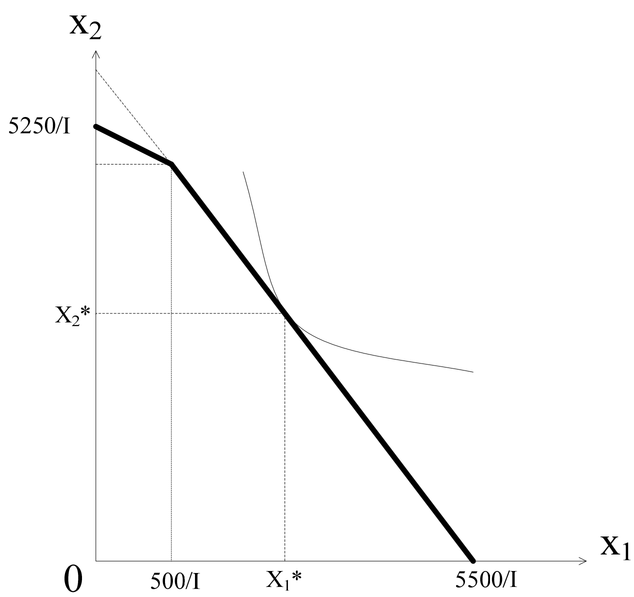
The budget line becomes:

$$\begin{cases} 0.5Ix_1 + Ix_2 = 5000 + 250 \ \text{ if } 0 \le x_1 < \frac{500}{I} \\ Ix_1 + Ix_2 = 5000 + 500 \text{ if } \frac{500}{I} \le x_1 \le \frac{5500}{I} \end{cases}$$

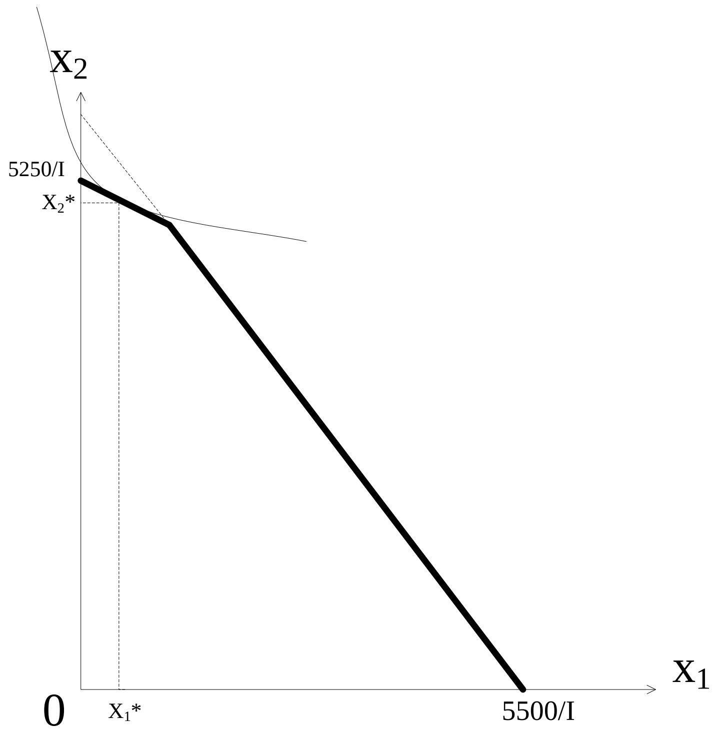
Budget line:



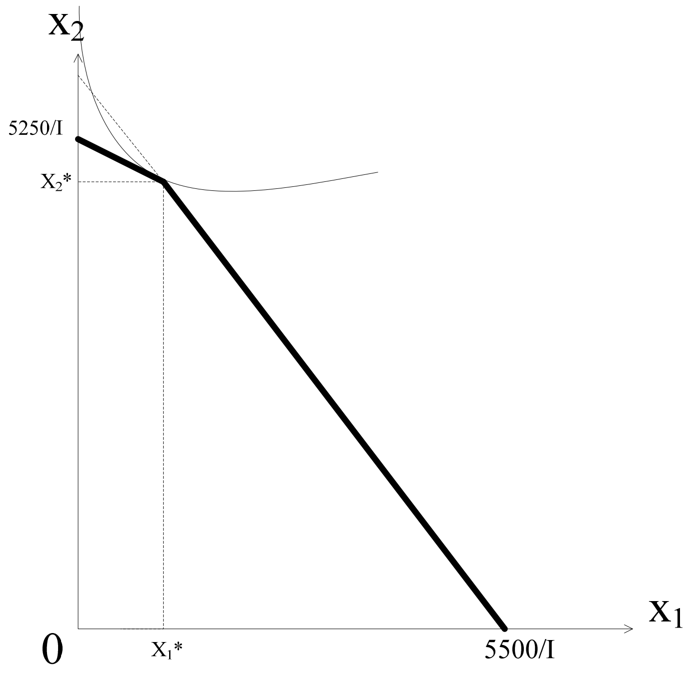
Situation I:



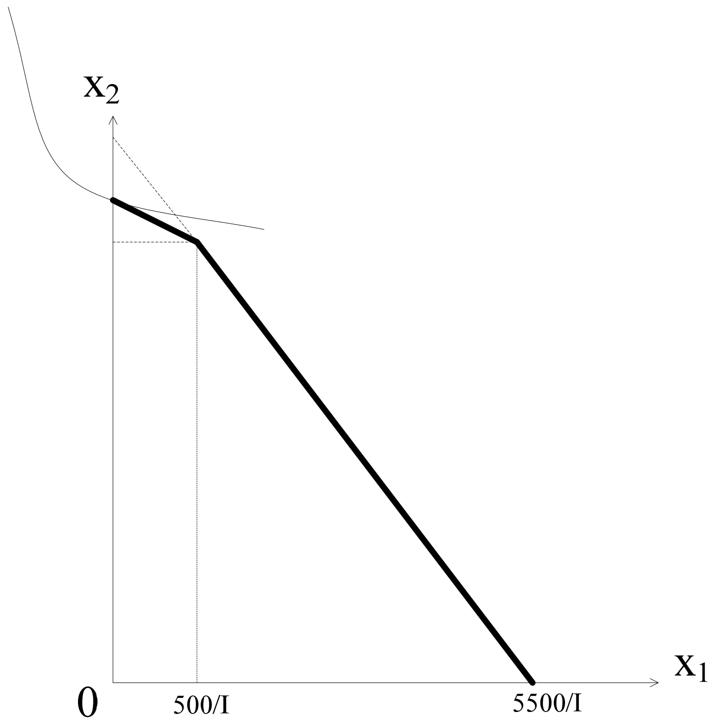
Situation II:



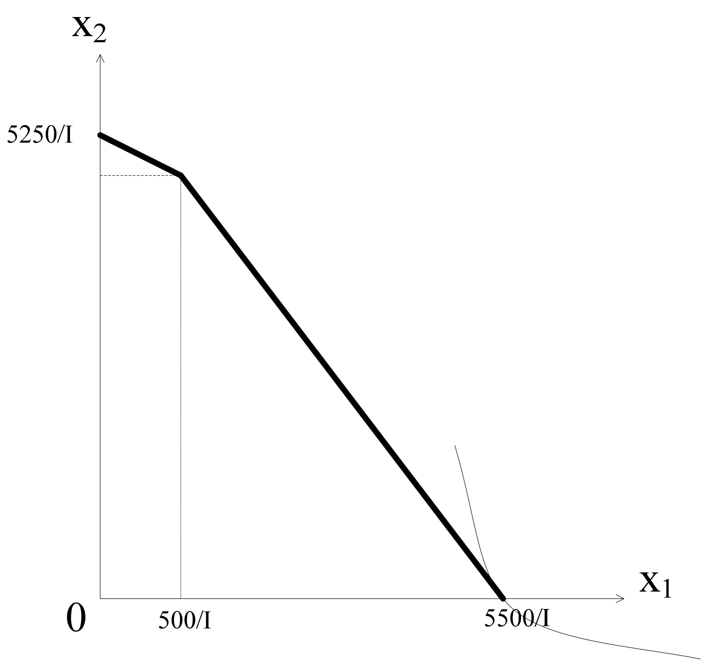
Situation III:



Situation IV:



Situation V:



So, the optimal demand for moon cake is [0, 5500/I]