Problem Set 4

1. (Textbook Section 3.5-7, Page 91) Suppose that the joint p.d.f. of X and Y is as follows:
$$f(x,y) = \begin{cases} 2xe^{-y}, & \text{for } 0 \le x \le 1, \text{ and } 0 \le y \le \infty \\ 0, & \text{otherwise.} \end{cases}$$
Are X and Y independent?

2. Suppose that the joint p.d.f. of X and Y is as follows:
$$f(x,y) = \begin{cases} c(x+y^2), & \text{for } 0 \le x \le 1, \text{ and } 0 \le y \le 0 \\ 0, & \text{otherwise.} \end{cases}$$
Determine (a) the conditional p.d.f. of X for every given value of Y, and (b) $Pr\left(X < \frac{1}{2} | Y = \frac{1}{2}\right)$.

3. (Textbook Section 3.7-7, Page 108) Suppose that the p.d.f. of a random variable X is as follows:
$$f(x) = \begin{cases} \frac{1}{n!}x^n e^{-x}, & \text{for } x > 0 \\ 0, & \text{otherwise.} \end{cases}$$
Suppose also that for any given value $X = x(x > 0)$, the n random variables $Y_1, \ldots, Y_n$ are i.i.d. and the conditional p.d.f. g of each of them is as follows:
$$g(y|x) = \begin{cases} \frac{1}{x}, & \text{for } 0 < y < x \\ 0, & \text{otherwise.} \end{cases}$$
Determine (a) the marginal joint p.d.f. of $Y_1, \ldots, Y_n$, and (b) the conditional p.d.f. of X for any given values of $Y_1, \ldots, Y_n$.

4. Suppose that the p.d.f. of a random variable X is as follows:
$$f(x) = \begin{cases} \frac{1}{2}x, & \text{for } 0 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$
Determine the p.d.f. of $Y = 3X + 2$.

5. (Textbook Section 3.9-7, Page 126) Suppose that X_1 and X_2 are i.i.d. random variables and that the p.d.f. of each of them as follows:

$$f(x) = \begin{cases} e^{-x}, & \text{for } x > 0 \\ 0, & \text{for } x \leq 0. \end{cases}$$

Find the p.d.f. of $Y = X_1 - X_2$.

6. Let W denote the range of a random sample of n observations from a uniform distribution on the interval [0,1], Determine the value of $Pr(W > 0.9)$.