偏导数

$$z=f(x,y),(x,y)\in D$$

$\forall(x_0,y_0)\in D, z=f(x,y_0)$ 是 $y=y_0$ 平面上的关于 x 的一元函数

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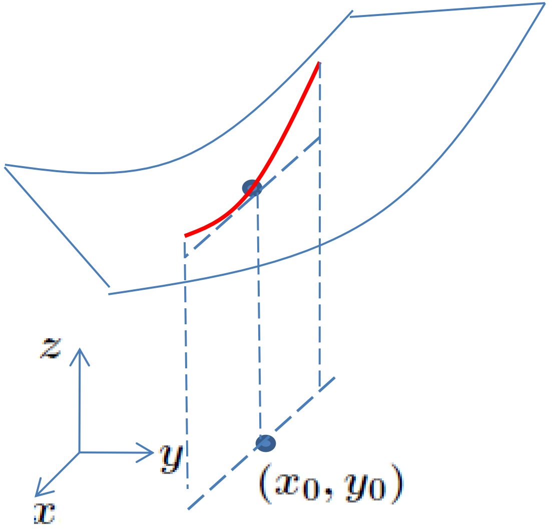

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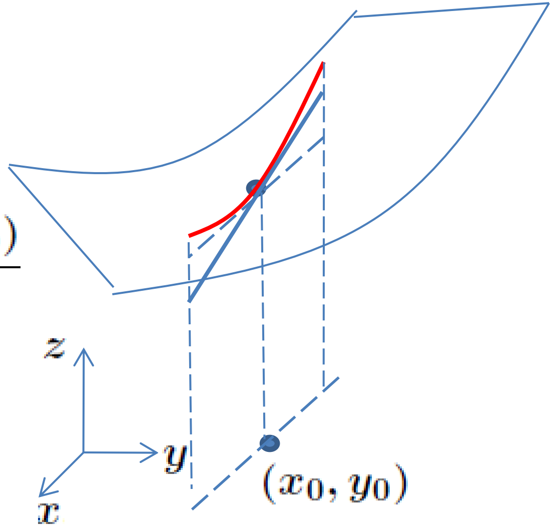

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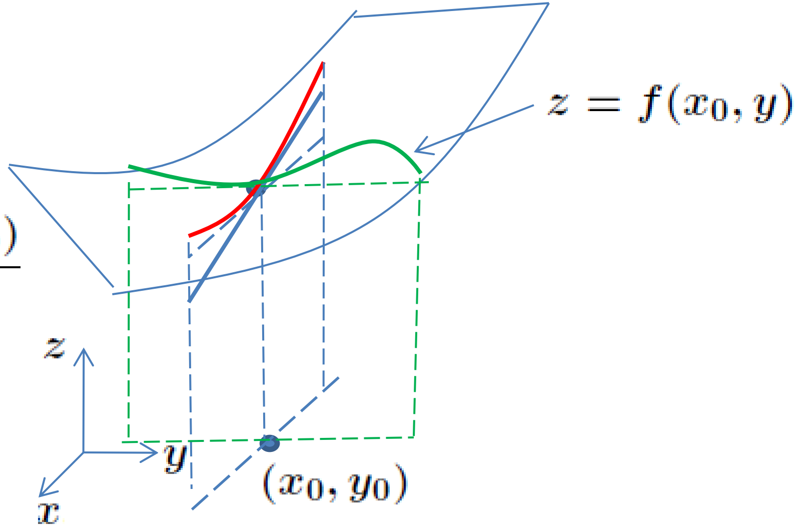

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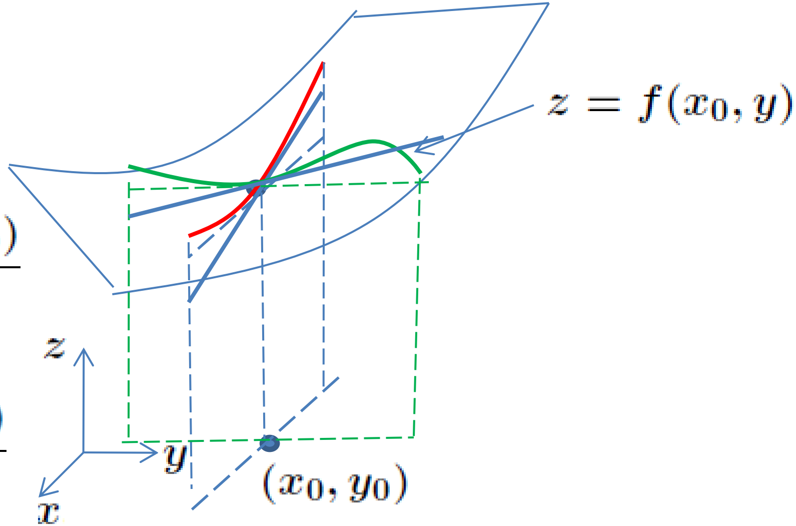

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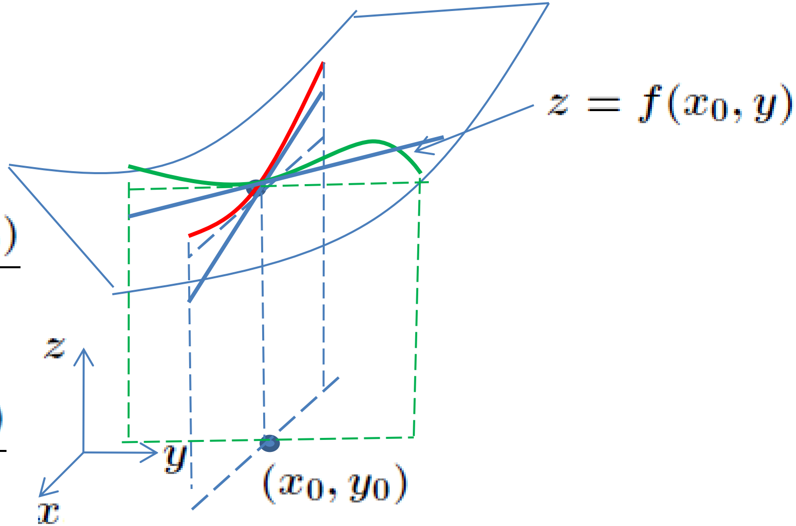

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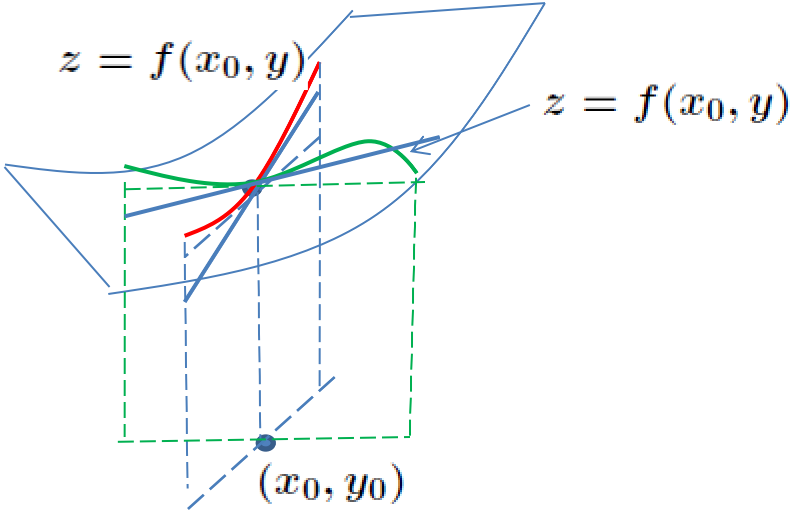

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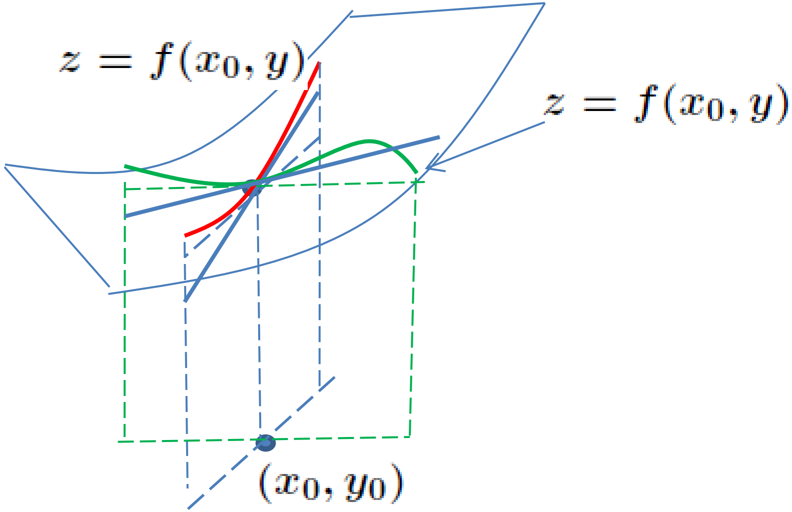

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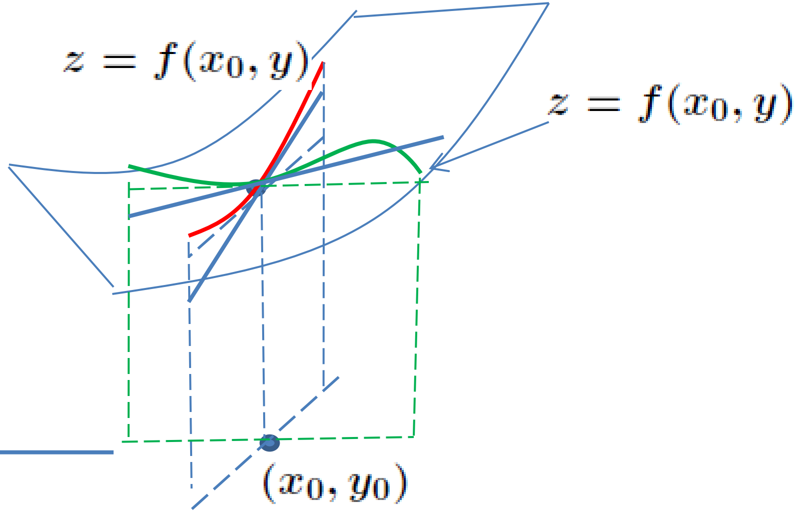

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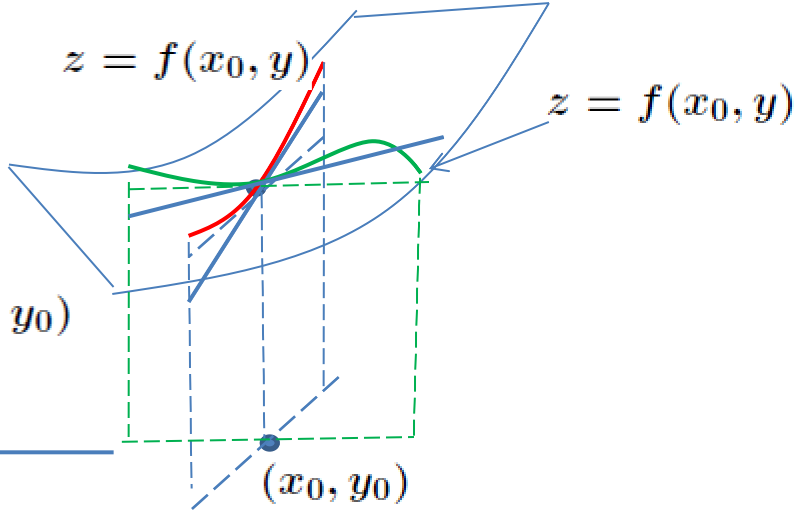

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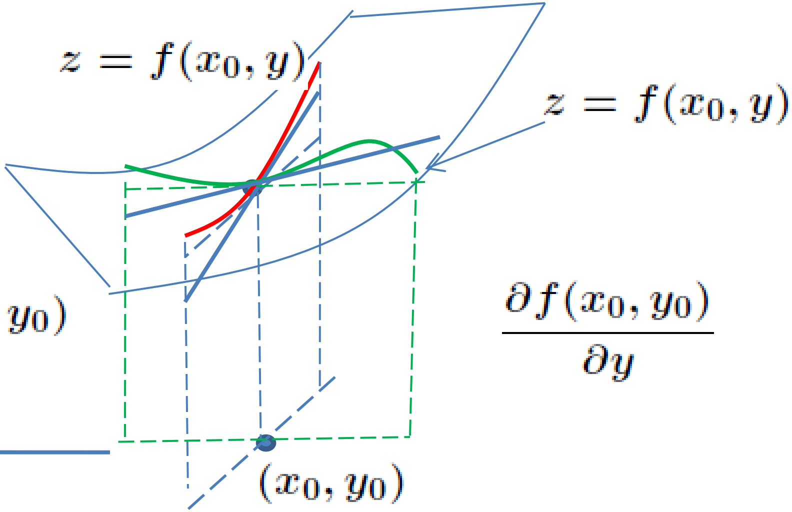

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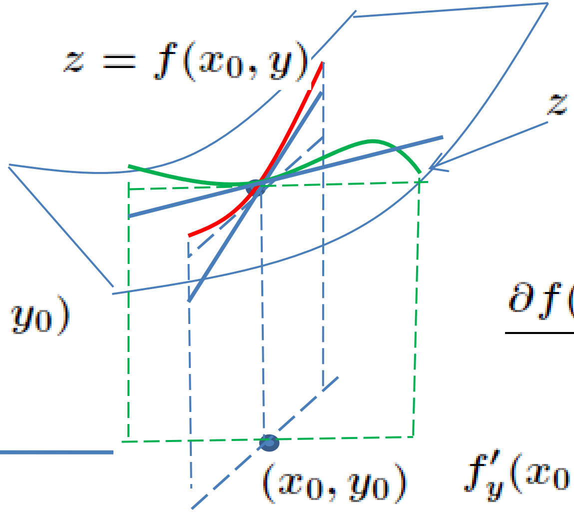

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在一般点 (x, y) 处同样定义

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

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例

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$$\frac{\partial z}{\partial x}|_{(1,1)} = 3, \quad \frac{\partial z}{\partial y}|_{(0,1)} = 2 .$$

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$$u=\sqrt{x^2+y^2+z^2}$$（三元函数）

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注' $\frac{\partial}{\partial x}$ 和 $\frac{\partial}{\partial y}$ 分别是一个整体运算符号，

不像一元微商 $\frac{dy}{dx}$ 中的 d 是一个独立的运算符号（微分运算）

有时也写成 ∂_x，∂_y 等.

对于一个二元隐函数 $F(x,y)=0$ 而言，

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$$\beta=-\lim_{\Delta P\to 0}\frac{1}{V}\cdot\frac{V(P+\Delta P,T)-V(P,T)}{\Delta P}=-\frac{1}{V}\cdot\frac{\partial V(P,T)}{\partial P}$$

在恒温下，单位体积的物体在单位压强增加量下导致的体积改变量（减小量）