Econ 139 Lecture 24 Notes

Bolun Deng, Valoryn Liu, Jeffrey Ehlers, Troy Loizzo

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CAPM Anomalies

1. Value Effect: Value stocks (high book to market ratio) have empirically high returns relative to their CAPM prediction

2. Size Effect: Small stocks have high returns relative to CAPM

3. Momentum Effect: Momentum stocks (stocks that have performed well in the last year) tend to outperform their predictions from CAPM

4. Reversal Effect: Stocks that have done well over the last 2-5 years tend to under-perform CAPM prediction

Fama-French Model

Identify 3 factors:

1. Market

2. High Minus Low (HML) also called value factor: $r_{\widetilde{HML}} = r_{HighB\widetilde{ookto}Market} - r_{LowB\widetilde{ookto}Market}$

3. Small Minus Big (SMB) also called size factor: $r_{\widetilde{SMB}} = \widetilde{r_S} - \widetilde{r_B}$

Suggest replacing market model with a three factor model:

$\widetilde{r_j} = \alpha_j + \beta_{j,m}(\widetilde{r_m} - E[\widetilde{r_m}]) + \beta_{j,v}(r_{\widetilde{HML}} - E[r_{\widetilde{HML}}]) + \beta_{j,s}(r_{\widetilde{SMB}} - E[r_{\widetilde{SMB}}]) + \widetilde{\varepsilon_j}$

$\mathrm{E}[\widetilde{r_p}] = r_f + \beta_{p,m}(E[\widetilde{r_m}] - r_f) + \beta_{p,v}E[r_{\widetilde{HML}}] + \beta_{j,s}E[r_{\widetilde{SMB}}]$

Chen, Roll, and Ross (1986)

Identify 5 macroeconomic factors:

1. Industrial Production (IP)

2. Unexpected Inflation (UI)

3. Expected Inflation (EI)

4. Term Structure (TS): Difference in yield between long-term and short-term treasury bonds

5. Risk Premium (RP): Difference in yield between low and high grade bonds

CCR suggest replacing market model with:

$$\check{IP} + \beta_{jUI}\check{UI} + \beta_{jEI}\check{EI} + \beta_{jTS}\check{TS} + \beta_{jRP}\check{RP} + \widetilde{\varepsilon_j}$$

$\widetilde{r_j} = \alpha_j + \beta_{jIP}$

$\Rightarrow (\check{x} = \tilde{x} - E[\tilde{x}])$

1 Idiosyncratic Risk of a Portfolio of N Assets

Suppose each asset has weight ω_i

$$\widetilde{\varepsilon_p} = \sum_{i=1}^{N} \omega_i \widetilde{\varepsilon_i}$$

$$\text{Var}(\widetilde{\varepsilon_p}) = Var(\sum_{i=1}^{N} \omega_i \tilde{\varepsilon}_i) = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j Cov(\tilde{\varepsilon}_i \tilde{\varepsilon}_j)$$

$$\text{Var}(\widetilde{\varepsilon_p}) = \sum_{i=1}^{N} \varepsilon_i^2 Var(\tilde{\varepsilon}_i) + \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j Cov(\tilde{\varepsilon}_i \tilde{\varepsilon}_j)$$

$$\text{Var}(\widetilde{\varepsilon_p}) = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2$$

For an equal weighted portfolio ($\text{W}_i = 1/N, \forall$) :

$$\sigma_{\varepsilon p}^2 = \sum_{i=1} \frac{1}{N^2} \sigma_{\varepsilon i}^2 = 1/N^2 \sum_{i=1} \sigma_{\varepsilon i}^2 = \frac{1}{N} \sigma_{\varepsilon i}^2$$

<u>**Summary**</u>**:** 1. APT assumes that asset returns are generated by a factor model:

$$\widetilde{r_p} = E[\widetilde{r_p}] + \beta(\widetilde{r_m} - E[\widetilde{r_m}]) + \widetilde{\varepsilon_p}$$

2. Assume N large enough that idiosyncratic risk vanishes, which implies : $\widetilde{r_p} = E[\widetilde{r_p}] + \beta(\widetilde{r_m} - E[\widetilde{r_m}])$for my well diversified portfolio

<u>**Proposition**</u> : Absence of arbitrage requires that portfolios with the same Beta have the same expected returns

<u>Proof</u>: Consider two well diversified portfolios

$$\widetilde{r_p^1} = E[\widetilde{r_p^1}] + \beta_p(\widetilde{r_m} - E[\widetilde{r_m}])$$

$\widetilde{r_p^2} = E[\widetilde{r_p^2}] + \triangle + \beta_p(\widetilde{r_m} - E[\widetilde{r_m}])$

Suppose $\triangle > 0$, consider taking a long position of x in port 2 and a short position of -x in port 1

Payoff: x$(1 + \widetilde{r_p^2}) - x(1 + \widetilde{r_p^1})$

xE$[\widetilde{r_p^1}] + x\triangle + x\beta_p(\widetilde{r_m} - E[\widetilde{r_m}]) - xE[\widetilde{r_p^1}] - x\beta_p(\widetilde{r_m} - E[\widetilde{r_m}])$

Suppose ($\triangle < 0$), short port 2 for -x and go long port 1 for x

Payoff: (-x$\triangle > 0$) absence of arbitrage implies $\triangle = 0$

Proposition: Absence of arbitrage requires expected returns on all well-diversified portfolios to satisfy

E$[\widetilde{r_p}] = r_f + \beta_p(E[\widetilde{r_m}] - r_f)$

Proof: First consider portfolio with $\beta_p = 0$ so that $\widetilde{r_p^1} = E[\widetilde{r_p}] + 0 \times (\widetilde{r_m} - E[\widetilde{r_m}])$

$\Rightarrow \widetilde{r_p^1} = E[\widetilde{r_p}] = r_f$

Next, consider portfolio with $\beta_p = 0$ so that $\widetilde{r_p^2} = E[\widetilde{r_p^2}] + 1 \times (\widetilde{r_m} - E[\widetilde{r_m}]) = \widetilde{r_m}$

By construction, portfolios 1 and 2 satisfy: E$[\widetilde{r_p}] = r_f + \beta_p(E[\widetilde{r_m}] - r_f)$

Consider a third well-diversified portfolio with $\beta_p \neq 0, \beta_p \neq 1$ such that: $E[\widetilde{r_p^3}] = r_f + \beta_p(E[\widetilde{r_m}] - r_f) + \triangle$

So far: $\widetilde{r_p^1} = r_f, \widetilde{r_p^2} = E[\widetilde{r_m]} + (\widetilde{r_m} - E[\widetilde{r_m]}),$
$\widetilde{r_p^3} = r_f + \beta_p(E[r_m] - r_f) + \widetilde{\triangle + \beta_p}(\widetilde{r_m} - E[\widetilde{r_p}])$

Construct a fourth portfolio from portfolios 1 and 2 by allocating (1 - β_p)to portfolio 1 and$\beta_p$to portfolio 2:

$\widetilde{r_p^4} = (1 - \beta_p)\widetilde{r_p^1} + \beta_p\widetilde{r_p^2}$

$\widetilde{r_p^4} = (1 - \beta_p r_f + \beta_p E[\widetilde{r_m}] + \beta_p(\widetilde{r_m} - E[\widetilde{r_m}])$

$\widetilde{r_p^4} = r_f + \beta_p(E[\widetilde{r_m}] - r_f) + \beta_p(\widetilde{r_m} - E[\widetilde{r_m}])$

$\Rightarrow \widetilde{r_p} = r_f + \beta_p(E[\widetilde{r_m}] - r_f) + \beta_p(\widetilde{r_m} - E[\widetilde{r_m}])$first prop:$\triangle = 0$

$\Rightarrow E[\widetilde{r_p}] = r_f + \beta_p(E[\widetilde{r_m}] - r_f)$