Homework 6

1. Suppose that X and Y have a continuous joint distribution for which the joint p.d.f. is as follows:

$$f(x,y) = \begin{cases} 12y^2, & \text{for } 0 \le y \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine $\rho(X,Y)$ (the correlation of X and Y).

2. Suppose that X, Y, and Z are three random variables such that $Var(X) = 1, Var(Y) = 4, Var(Z) = 8, Cov(X,Y) = 1, Cov(X,Z) = -1$ and $Cov(Y,Z) = 2$. Determine (a) $Var(X+Y+Z)$ and (b) $Var(3X - Y - 2Z + 1)$

3. (Textbook Section 4.7-6, Page 178) Suppose that $X_1, \cdots, X_n$ from a random sample of size n from a distribution for which the mean is 6.5 and the variance is 4. Determine how large the value of n must be in order for the following relation to be satisfied.

$$\Pr(6 \le \overline{X}_n \le 7) \ge 0.8.$$

4. Let $Z_1, Z_2, \ldots$ be a sequence of random variables; and suppose that, for $n = 1, 2, \cdots$

the distribution of Z_n is as follows:

$$\Pr(Z_n = n^2) = \frac{1}{n} \quad \text{and} \quad \Pr(Z_n = 0) = 1 - \frac{1}{n}$$

Show that

$$\lim_{n\to\infty} E(Z_n) = \infty \quad \text{but} \quad Z_n \xrightarrow{p} 0$$

5. Suppose that X has a normal distribution for which the mean is 1 and the variance is 4. Find the value of each of the following probabilities.

   1)$\Pr(X \leq 3)$ 2$\Pr(X = 1)$ 3)$\Pr(-1 < X < 0.5)$

   4)$\Pr(X \geq 0)$ 5)$\Pr(1 \leq -2X + 3 \leq 8)$

6. Suppose that $X_1, \cdots, X_n$ from a random sample of size n is to be taken from a normal distribution with mean μ and standard deviation σ. What is the minimum value of n for which

$$\Pr(|\overline{X}_n - \mu| \leq \frac{\sigma}{4}) \geq 0.99.$$