Two Perspectives on Asset Pricing

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Two Perspectives on Asset Pricing

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7 The Capital Asset Pricing Model

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MPT and the CAPM

The Capital Asset Pricing Model builds directly on Modern Portfolio Theory.

It was developed in the mid-1960s by William Sharpe (US, b.1934, Nobel Prize 1990), John Lintner (US, 1916-1983), and Jan Mossin (Norway, 1936-1987).

MPT and the CAPM

William Sharpe, “Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk,” *Journal of Finance* Vol.19 (September 1964): pp.425-442.

John Lintner, “The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets,” *Review of Economics and Statistics* Vol.47 (February 1965): pp.13-37.

Jan Mossin, “Equilibrium in a Capital Asset Market,” *Econometrica* Vol.34 (October 1966): pp.768-783.

MPT and the CAPM

But whereas Modern Portfolio Theory is a theory describing the demand for financial assets, the Capital Asset Pricing Model is a theory describing equilibrium in financial markets.

By making an additional assumption – namely, that supply equals demand in financial markets – the CAPM yields additional implications about the pricing of financial assets and risky cash flows.

MPT and the CAPM

Like MPT, the CAPM assumes that investors have mean-variance utility and hence that either investors have quadratic Bernoulli utility functions or that the random returns on risky assets are normally distributed.

Thus, some of the same caveats that apply to MPT also apply to the CAPM.

For example, one might hesitate before applying the CAPM to price options.

MPT and the CAPM

The traditional CAPM also assumes that there is a risk free asset as well as a potentially large collection of risky assets.

Under these circumstances, as we've seen, all investors will hold some combination of the riskless asset and the tangency portfolio: the efficient portfolio of risky assets with the highest Sharpe ratio.

MPT and the CAPM

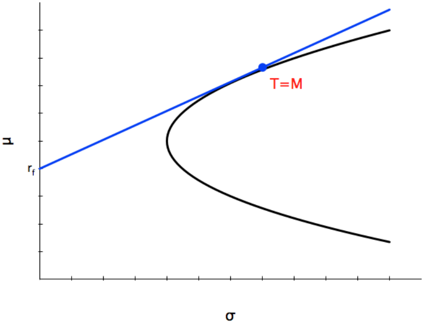
But the CAPM goes further than the MPT by imposing an equilibrium condition.

Because there is no demand for risky financial assets except to the extent that they comprise the tangency portfolio, and because, in equilibrium, the supply of financial assets must equal demand, the market portfolio consisting of all existing financial assets must coincide with the tangency portfolio.

supply = demand

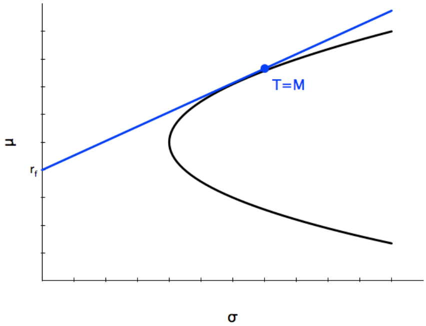
In equilibrium, that is, "everyone" must "own the market."

Deriving the CAPM



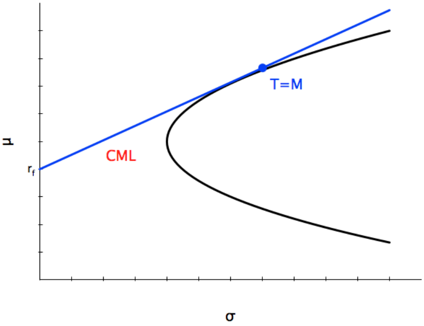
In the CAPM, equilibrium in financial markets requires the demand for risky assets – the tangency portfolio – to coincide with the supply of financial assets – the market portfolio.

Deriving the CAPM



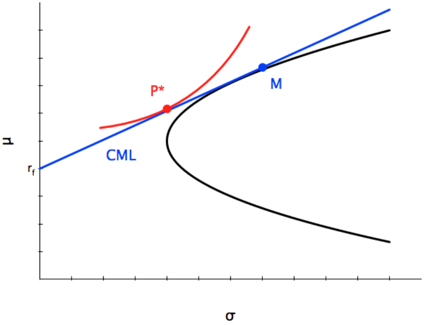
The CAPM's first implication is immediate: the market portfolio is efficient.

Deriving the CAPM



The line originating at $(0, r_f)$ and running through $(\sigma_M, E(\tilde{r}_M))$ is called the capital market line (CML).

Deriving the CAPM



Hence, it also follows that all individually optimal portfolios are located along the CML and are formed as combinations of the risk free asset and the market portfolio.

Deriving the CAPM

Recall that the trade-off between the standard deviation and expected return of any portfolio combining the riskless asset and the tangency portfolio is described by the linear relationship

$$E(\tilde{r}_P) = r_f + \left[\frac{E(\tilde{r}_T) - r_f}{\sigma_T}\right] \sigma_P.$$

Since the CAPM implies that the tangency and market portfolios coincide, the formula for the Capital Market Line is likewise

$$E(\tilde{r}_P) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \sigma_P.$$

Deriving the CAPM

And since all individually optimal portfolios are located along the CML, the equation

$$E(\tilde{r}_P) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right]\sigma_P.$$

implies that the market portfolio's Sharpe ratio

$$\frac{E(\tilde{r}_M) - r_f}{\sigma_M}$$

measures the equilibrium price of risk: the expected return that each investor gives up when he or she adjusts his or her total portfolio to reduce risk.

Deriving the CAPM

Next, let's consider an arbitrary asset – "asset j" – with random return $\tilde{r}_j$, expected return $E(\tilde{r}_j)$, and standard deviation σ_j.

MPT would take $E(\tilde{r}_j)$ and σ_j as "data" – that is, as given.

The CAPM again goes further and asks: if asset j is to be demanded by investors with mean-variance utility, what restrictions must $E(\tilde{r}_j)$ and σ_j satisfy?

Deriving the CAPM

To answer this question, consider an investor who takes the portion of his or her initial wealth that he or she allocates to risky assets and divides it further: using the fraction w to purchase asset j and the remaining fraction $1 - w$ to buy the market portfolio.

Note that since the market portfolio already includes some of asset j, choosing $w > 0$ really means that the investor "overweights" asset j in his or her own portfolio. Conversely, choosing $w < 0$ means that the investor "underweights" asset j in his or her own portfolio.

Deriving the CAPM

Based on our previous analysis, we know that this investor's portfolio of risky assets now has random return

$$\tilde{r}_P = w\tilde{r}_j + (1-w)\tilde{r}_M,$$

expected return

$$E(\tilde{r}_P) = wE(\tilde{r}_j) + (1-w)E(\tilde{r}_M),$$

and variance

$$\sigma_P^2 = w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM},$$

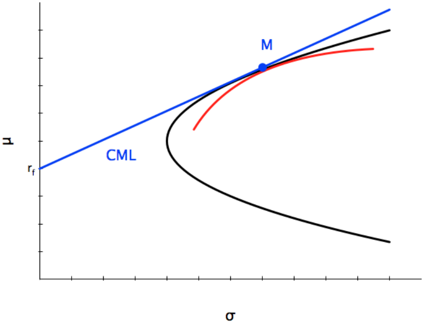
where σ_{jM} is the covariance between $\tilde{r}_j$ and $\tilde{r}_M$.

Deriving the CAPM

$$E(\tilde{r}_P) = wE(\tilde{r}_j) + (1-w)E(\tilde{r}_M),$$
$$\sigma_P^2 = w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM},$$

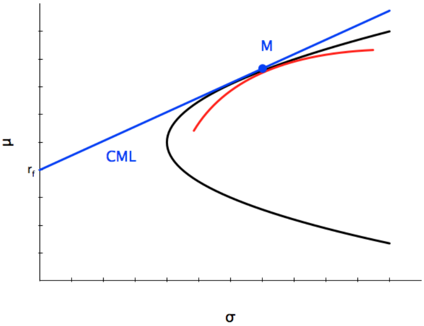
We can use these formulas to trace out how σ_P and $E(\tilde{r}_P)$ vary as w changes.

Deriving the CAPM



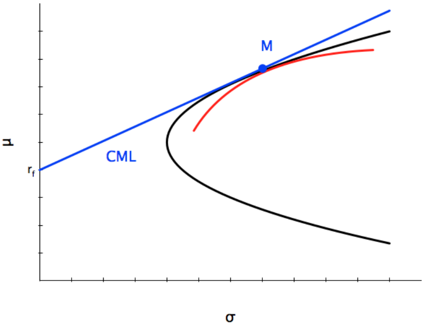
The red curve these traces out how σ_P and $E(\tilde{r}_P)$ vary as w changes, that is, as asset j gets underweighted or overweighted relative to the market portfolio.

Deriving the CAPM



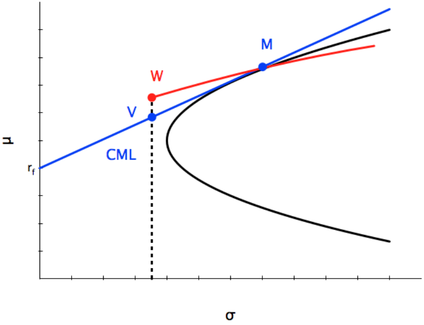
The red curve passes through M, since when $w = 0$ the new portfolio coincides with the market portfolio.

Deriving the CAPM



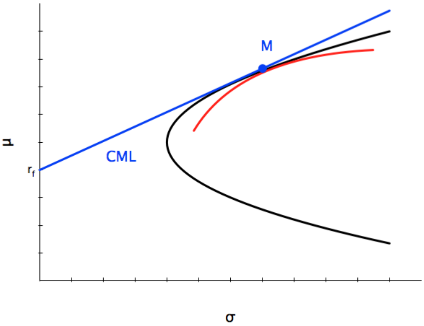
For all other values of w, however, the red curve must lie below the CML.

Deriving the CAPM



Otherwise, a portfolio along the CML would be dominated in mean-variance by the new portfolio. Financial markets would no longer be in equilibrium, since some investors would no longer be willing to hold the market portfolio.

Deriving the CAPM



Together, these observations imply that the red curve must be tangent to the CML at M.

Deriving the CAPM

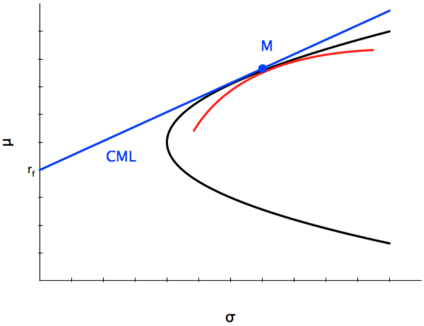
Tangent means equal in slope.

We already know that the slope of the Capital Market Line is

$$\frac{E(\tilde{r}_M) - r_f}{\sigma_M}$$

But what is the slope of the red curve?

Deriving the CAPM



Let $f(\sigma_P)$ be the function defined by $E(\tilde{r}_P) = f(\sigma_P)$ and therefore describing the red curve.

Deriving the CAPM

Next, define the functions $g(w)$ and $h(w)$ by

$$g(w) = wE(\tilde{r}_j) + (1-w)E(\tilde{r}_M),$$

$$h(w) = [w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2},$$

so that

$$E(\tilde{r}_P) = g(w)$$

and

$$\sigma_P = h(w).$$

Deriving the CAPM

Substitute

$$E(\tilde{r}_P) = g(w)$$

and

$$\sigma_P = h(w).$$

into

$$E(\tilde{r}_P) = f(\sigma_P)$$

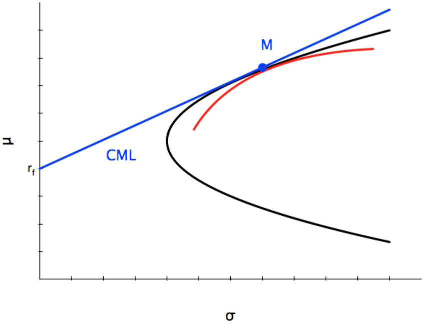
to obtain

$$g(w) = f(h(w))$$

and use the chain rule to compute

$$g'(w) = f'(h(w))h'(w) = f'(\sigma_P)h'(w)$$

Deriving the CAPM



Let $f(\sigma_P)$ be the function defined by $E(\tilde{r}_P) = f(\sigma_P)$ and therefore describing the red curve. Then $f'(\sigma_P)$ is the slope of the curve.

Deriving the CAPM

Hence, to compute $f'(\sigma_P)$, we can rearrange

$$g'(w) = f'(\sigma_P)h'(w)$$

to obtain

$$f'(\sigma_P) = \frac{g'(w)}{h'(w)}$$

and compute $g'(w)$ and $h'(w)$ from the formulas we know.

Deriving the CAPM

$$g(w) = wE(\tilde{r}_j) + (1 - w)E(\tilde{r}_M),$$

implies

$$g'(w) = E(\tilde{r}_j) - E(\tilde{r}_M)$$

Deriving the CAPM

$$h(w) = [w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2},$$

implies

$$h'(w) = \frac{1}{2}\left\{\frac{2w\sigma_j^2 - 2(1-w)\sigma_M^2 + 2(1-2w)\sigma_{jM}}{[w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2}}\right\}$$

or, a bit more simply,

$$h'(w) = \frac{w\sigma_j^2 - (1-w)\sigma_M^2 + (1-2w)\sigma_{jM}}{[w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2}}$$

Deriving the CAPM

$$f'(\sigma_P) = g'(w)/h'(w)$$

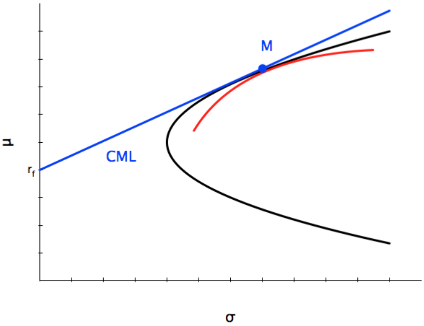
$$g'(w) = E(\tilde{r}_j) - E(\tilde{r}_M)$$

$$h'(w) = \frac{w\sigma_j^2 - (1-w)\sigma_M^2 + (1-2w)\sigma_{jM}}{[w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2}}$$

imply

$$\begin{aligned} f'(\sigma_P) &= [E(\tilde{r}_j) - E(\tilde{r}_M)] \\ &\quad \times \frac{[w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2}}{w\sigma_j^2 - (1-w)\sigma_M^2 + (1-2w)\sigma_{jM}} \end{aligned}$$

Deriving the CAPM



The red curve is tangent to the CML at M. Hence, $f'(\sigma_P)$ equals the slope of the CML when w=0.

Deriving the CAPM

When $w = 0$,

$$\begin{aligned} f'(\sigma_P) &= [E(\tilde{r}_j) - E(\tilde{r}_M)] \\ &\quad \times \frac{[w^2\sigma_j^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_{jM}]^{1/2}}{w\sigma_j^2 - (1-w)\sigma_M^2 + (1-2w)\sigma_{jM}} \end{aligned}$$

implies

$$f'(\sigma_P) = \frac{[E(\tilde{r}_j) - E(\tilde{r}_M)]\sigma_M}{\sigma_{jM} - \sigma_M^2}$$

Meanwhile, we know that the slope of the CML is

$$\frac{E(\tilde{r}_M) - r_f}{\sigma_M}$$

Deriving the CAPM

The tangency of the red curve with the CML at M therefore requires

$$\frac{[E(\tilde{r}_j) - E(\tilde{r}_M)]\sigma_M}{\sigma_{jM} - \sigma_M^2} = \frac{E(\tilde{r}_M) - r_f}{\sigma_M}$$

$$E(\tilde{r}_j) - E(\tilde{r}_M) = \frac{[E(\tilde{r}_M) - r_f][\sigma_{jM} - \sigma_M^2]}{\sigma_M^2}$$

$$E(\tilde{r}_j) - E(\tilde{r}_M) = \left(\frac{\sigma_{jM}}{\sigma_M^2}\right)[E(\tilde{r}_M) - r_f] - [E(\tilde{r}_M) - r_f]$$

$$E(\tilde{r}_j) = r_f + \left(\frac{\sigma_{jM}}{\sigma_M^2}\right)[E(\tilde{r}_M) - r_f]$$

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left(\frac{\sigma_{jM}}{\sigma_M^2}\right)[E(\tilde{r}_M) - r_f]$$

Let

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2}$$

so that this key equation of the CAPM can be written as

$$E(\tilde{r}_j) = r_f + \beta_j[E(\tilde{r}_M) - r_f]$$

where β_j, the "beta" for asset j, depends on the covariance between the returns on asset j and the market portfolio.

A simple derivation

* CAPM assumptions: All investors hold the market portfolio + MPT
-> if the observed market value share of Asset i is w_i, a MPT investor’s optimal weight on Asset i should coincide with w_i
* Goal: calculate E[Ri] such that investors happy to hold w_i

$$\overline{R}_i - R_f = \lambda(w_1\sigma_{1i} + w_2\sigma_{2i} + \ldots + w_n\sigma_{ni}) \qquad \text{(1)<-MPT}$$

$$\overline{R}_M = w_1 R_1 + w_2 R_2 + \ldots + w_n R_n \qquad \text{(2)<-Definition}$$

$$\operatorname{cov}(R_M, R_i) = w_1\sigma_{1i} + w_2\sigma_{2i} + \ldots + w_n\sigma_{ni} \qquad \text{(3)<-Definition}$$

$$\overline{R}_i - R_f = \lambda \operatorname{cov}(R_M, R_i) \qquad \text{(4)<-(1)+(3)}$$

$$\overline{R}_M - R_f = \lambda \operatorname{cov}(R_M, R_M) \qquad \text{(5)<-sum up (4)}$$

$$\bar{R}_i = R_f + \frac{\sigma_{iM}}{\sigma_M^2}(\bar{R}_M - R_f)$$

$$\bar{R}_e = R_f + \frac{\sigma_{eM}}{\sigma_M^2}(\bar{R}_M - R_f)$$

$$= R_f + \frac{\sigma_e}{\sigma_M}(\bar{R}_M - R_f)$$

$$= R_f + \sigma_e \frac{\bar{R}_M - R_f}{\sigma_M}$$

Deriving the CAPM

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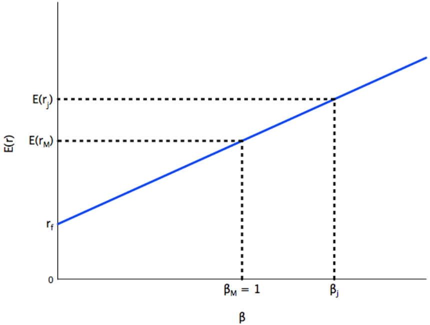
$$E(\tilde{r}_j) = r_f + \beta_j[E(\tilde{r}_M) - r_f]$$

This equation summarizes a very strong restriction.

It implies that if we rank individual stocks or portfolios of stocks according to their betas, their expected returns should all lie along a single security market line with slope $E(\tilde{r}_M) - r_f$.

Deriving the CAPM



According to the CAPM, all assets and portfolios of assets lie along a single security market line. Those with higher betas have higher expected returns.

Deriving the CAPM

There are several complementary ways of interpreting this result.

All bring us back to the theme of diversification emphasized by MPT.

Both take us a step further, by emphasizing as well the idea of aggregate risk, which cannot be "diversified away," and idiosyncratic risk, which can be diversified away.

Deriving the CAPM

The first interpretation goes directly back to the MPT: a stock with low and especially negative σ_{jM} will be most useful for diversification.

But then all investors will want to hold that stock. In equilibrium, therefore, the stock's price will be high and, given future cash flows, its expected return will be low.

Therefore, stocks with low or negative betas will have low expected returns. Investors hold these stocks, despite their low expected returns, because of they are useful for diversification.

Deriving the CAPM

Conversely, a stock with high, positive σ_{jM} will not be very useful for diversification.

In equilibrium, therefore, the stock will sell for a low price.

Therefore, stocks with high betas will have high expected returns. The high expected return is needed to compensate investors, because the stock is not very useful for diversification.

Deriving the CAPM

The second interpretation uses the CAPM equation in its original form

$$E(\tilde{r}_j) = r_f + \left(\frac{\sigma_{jM}}{\sigma_M^2}\right)[E(\tilde{r}_M) - r_f]$$

together with the definition of correlation, which implies

$$\rho_{jM} = \frac{\sigma_{jM}}{\sigma_j \sigma_M}$$

to re-express the CAPM relationship as

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \rho_{jM}\sigma_j$$

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \rho_{jM}\sigma_j$$

The term inside brackets is the equilibrium price of risk.

And since the correlation lies between -1 and 1, the term $\rho_{jM}\sigma_j$, satisfying

$$\rho_{jM}\sigma_j \leq \sigma_j,$$

represents the "portion" of the total risk σ_j in asset j that is correlated with the market return.

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \rho_{jM}\sigma_j$$

The idiosyncratic risk in asset j, that is, the portion that is uncorrelated with the market return, can be diversified away by holding the market portfolio.

Since this risk can be freely shed through diversification, it is not "priced."

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \rho_{jM}\sigma_j$$

Hence, according to the CAPM, risk in asset j is priced only to the extent that it takes the form of aggregate risk that, because it is correlated with the market portfolio, cannot be diversified away.

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \rho_{jM}\sigma_j$$

Thus, according to the CAPM:

1. Only assets with random returns that are positively correlated with the market return earn expected returns above the risk free rate. They must, in order to induce investors to take on more aggregate risk.

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M}\right] \rho_{jM}\sigma_j$$

Thus, according to the CAPM:

2. Assets with returns that are uncorrelated with the market return have expected returns equal to the risk free rate, since their risk can be completely diversified away.

Deriving the CAPM

$$E(\tilde{r}_j) = r_f + \left[\frac{E(\tilde{r}_M) - r_f}{\sigma_M} \right] \rho_{jM} \sigma_j$$

Thus, according to the CAPM:

3. Assets with negative betas – that is, with random returns that are negatively correlated with the market return – have expected returns below the risk free rate! For these assets, $E(\tilde{r}_j) - r_f < 0$ is like an "insurance premium" that investors will pay in order to insulate themselves from aggregate risk.

Deriving the CAPM

The third interpretation is based on a statistical regression of the random return $\tilde{r}_j$ on asset j on a constant and the market return $\tilde{r}_M$:

$$\tilde{r}_j = \alpha + \beta_j \tilde{r}_M + \varepsilon_j$$

可能 ε_i 与 ε_j 相关

This regression breaks the variance of $\tilde{r}_j$ down into two "orthogonal" (uncorrelated) components:

1. The component $\beta_j \tilde{r}_M$ that is systematically related to variation in the market return.
2. The component ε_j that is not.

Do you remember the formula for β_j, the slope coefficient in a linear regression?

Deriving the CAPM

Consider a statistical regression of the random return $\tilde{r}_j$ on asset j on a constant and the market return $\tilde{r}_M$:

$$\tilde{r}_j = \alpha + \beta_j \tilde{r}_M + \varepsilon_j$$

Do you remember the formula for β_j, the slope coefficient in a linear regression? It is

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2}$$

the same "beta" as in the CAPM!

Deriving the CAPM

Consider a statistical regression: 常用：

$$\tilde{r}_j = \alpha + \beta_j \tilde{r}_M + \varepsilon_j \text{ with } \beta_j = \sigma_{jM}/\sigma_M^2$$

the same “beta” as in the CAPM!

But this is not an accident: to the contrary, it restates the conclusion that, according to the CAPM, risk in an individual asset is priced – and thereby reflected in a higher expected return – only to the extent that it is correlated with the market return.

Valuing Risky Cash Flows

We can also use the CAPM to value risky cash flows.

Let $\tilde{C}_{t+1}$ denote a random payoff to be received at time $t+1$ ("one period from now") and let P_t^C denote its price at time t ("today.")

If $\tilde{C}_{t+1}$ was known in advance, that is, if the payoff were riskless, we could find its value by discounting it at the risk free rate:

$$P_t^C = \frac{\tilde{C}_{t+1}}{1+r_f}$$

Valuing Risky Cash Flows

But when $\tilde{C}_{t+1}$ is truly random, we need to find its expected value $E(\tilde{C}_{t+1})$ and then "penalize" it for its riskiness either by discounting at a higher rate

$$P_t^C = \frac{E(\tilde{C}_{t+1})}{1 + r_f + \psi}$$

or by reducing its value more directly

$$P_t^C = \frac{E(\tilde{C}_{t+1}) - \Psi}{1 + r_f}$$

Valuing Risky Cash Flows

$$P_t^C = \frac{E(\tilde{C}_{t+1})}{1 + r_f + \psi}$$

$$P_t^C = \frac{E(\tilde{C}_{t+1}) - \Psi}{1 + r_f}$$

The CAPM can help us identify the appropriate risk premium ψ or Ψ.

Our previous analysis suggests that, broadly speaking, the risk premium implied by the CAPM will somehow depend on the extent to which the random payoff $\tilde{C}_{t+1}$ is correlated with the return on the market portfolio.

Valuing Risky Cash Flows

To apply the CAPM to this valuation problem, we can start by observing that with price P_t^C today and random payoff $\tilde{C}_{t+1}$ one period from now, the return on this asset or investment project is defined by

$$1 + \tilde{r}_C = \frac{\tilde{C}_{t+1}}{P_t^C}$$

or

$$\tilde{r}_C = \frac{\tilde{C}_{t+1} - P_t^C}{P_t^C}$$

where the notation $\tilde{r}_C$ emphasizes that this return, like the future cash flow itself, is risky.

Valuing Risky Cash Flows

Now the CAPM implies that the expected return $E(\tilde{r}_C)$ must satisfy

$$E(\tilde{r}_C) = r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

where the project's beta depends on the covariance of its return with the market return:

$$\beta_C = \frac{\sigma_{CM}}{\sigma_M^2}$$

This is what takes skill: with an existing asset, one can use data on the past correlation between its return and the market return to estimate beta. With a totally new project that is just being planned, a combination of experience, creativity, and hard work is often needed to choose the right value for β_C.

Valuing Risky Cash Flows

But once a value for β_C is determined, we can use

$$E(\tilde{r}_C) = r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

together with the definition of the return itself

$$\tilde{r}_C = \frac{\tilde{C}_{t+1}}{P_t^C} - 1$$

to write

$$E\left(\frac{\tilde{C}_{t+1}}{P_t^C} - 1\right) = r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

Valuing Risky Cash Flows

$$E\left(\frac{\tilde{C}_{t+1}}{P_t^C} - 1\right) = r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

implies

$$\left(\frac{1}{P_t^C}\right) E(\tilde{C}_{t+1}) = 1 + r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

$$P_t^C = \frac{E(\tilde{C}_{t+1})}{1 + r_f + \beta_C[E(\tilde{r}_M) - r_f]}$$

Valuing Risky Cash Flows

Hence, through

$$P_t^C = \frac{E(\tilde{C}_{t+1})}{1 + r_f + \beta_C[E(\tilde{r}_M) - r_f]}$$

the CAPM implies a risk premium of

$$\psi = \beta_C[E(\tilde{r}_M) - r_f]$$

which, as expected, depends critically on the covariance between the return on the risky project and the return on the market portfolio.

Valuing Risky Cash Flows

Alternatively,

$$E(\tilde{r}_C) = r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

and

$$\tilde{r}_C = \frac{\tilde{C}_{t+1}}{P_t^C} - 1$$

imply

$$E\left(\frac{\tilde{C}_{t+1}}{P_t^C} - 1\right) = r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

and hence

$$\left(\frac{1}{P_t^C}\right) E(\tilde{C}_{t+1}) = 1 + r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

Valuing Risky Cash Flows

$$\left(\frac{1}{P_t^C}\right) E(\tilde{C}_{t+1}) = 1 + r_f + \beta_C[E(\tilde{r}_M) - r_f]$$

can be rewritten as

$$P_t^C = \frac{E(\tilde{C}_{t+1}) - P_t^C \beta_C[E(\tilde{r}_M) - r_f]}{1 + r_f}$$

indicating that the CAPM also implies

$$\Psi = P_t^C \beta_C[E(\tilde{r}_M) - r_f]$$

which, again as expected, depends critically on the covariance between the return on the risky project and the return on the market portfolio.

Strengths and Shortcomings of the CAPM

An enormous literature is devoted to empirically testing the CAPM's implications.

Although results are mixed, studies have shown that when individual portfolios are ranked according to their betas, expected returns tend to line up as suggested by the theory.

Strengths and Shortcomings of the CAPM

A famous article that presents results along these lines is by Eugene Fama (Nobel Prize 2013) and James MacBeth, "Risk, Return, and Equilibrium," *Journal of Political Economy* Vol.81 (May-June 1973), pp.607-636.

Early work on the MPT, the CAPM, and econometric tests of the efficient markets hypothesis and the CAPM is discussed extensively in Eugene Fama's 1976 textbook, *Foundations of Finance*.

Strengths and Shortcomings of the CAPM

More recent evidence against the CAPM's implications is presented by Eugene Fama and Kenneth French, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics* Vol.33 (February 1993): pp.3-56.

This paper shows that equity shares in small firms and in firms with high book (accounting) to market value have expected returns that differ strongly from what is predicted by the CAPM alone.

Quite a bit of recent research has been directed towards understanding the source of these "anomalies."

Strengths and Shortcomings of the CAPM

Despite some empirical shortcomings, however, the CAPM quite usefully deepens our understanding of the gains from diversification.

Related, the CAPM alerts us to the important distinction between idiosyncratic risk, which can be diversified away, and aggregate risk, which cannot.

Strengths and Shortcomings of the CAPM

Like MPT, the CAPM must rely on one of the two strong assumptions – either quadratic utility or normally-distributed returns – that justify mean-variance utility.

And while the CAPM is an equilibrium theory of asset pricing, it stops short of linking asset returns to underlying economic fundamentals.

假设市场已经包含了所有的信息，，
这时候CAPM才是有效的

These last two points motivate our interest in other asset pricing theories, which are less restrictive in their assumptions and/or draw closer connections between asset prices and the economy as a whole.

Strengths and Shortcomings of the CAPM

These last two points motivate our interest in other asset-pricing theories, which are less restrictive in their assumptions and/or draw closer connections between asset prices and the economy as a whole.

→ 容纳更多因子，不需要 rely on MPT

Arbitrage Pricing Theory, to which we will turn our attention next, yields many of the same implications as the CAPM, but requires less restrictive assumptions about preferences and the distribution of asset returns.

Strengths and Shortcomings of the CAPM

These last two points motivate our interest in other asset-pricing theories, which are less restrictive in their assumptions and/or draw closer connections between asset prices and the economy as a whole.

equilibrium

The equilibrium version of Arrow-Debreu theory draws links between asset prices and the economy that are only implicit in the CAPM.