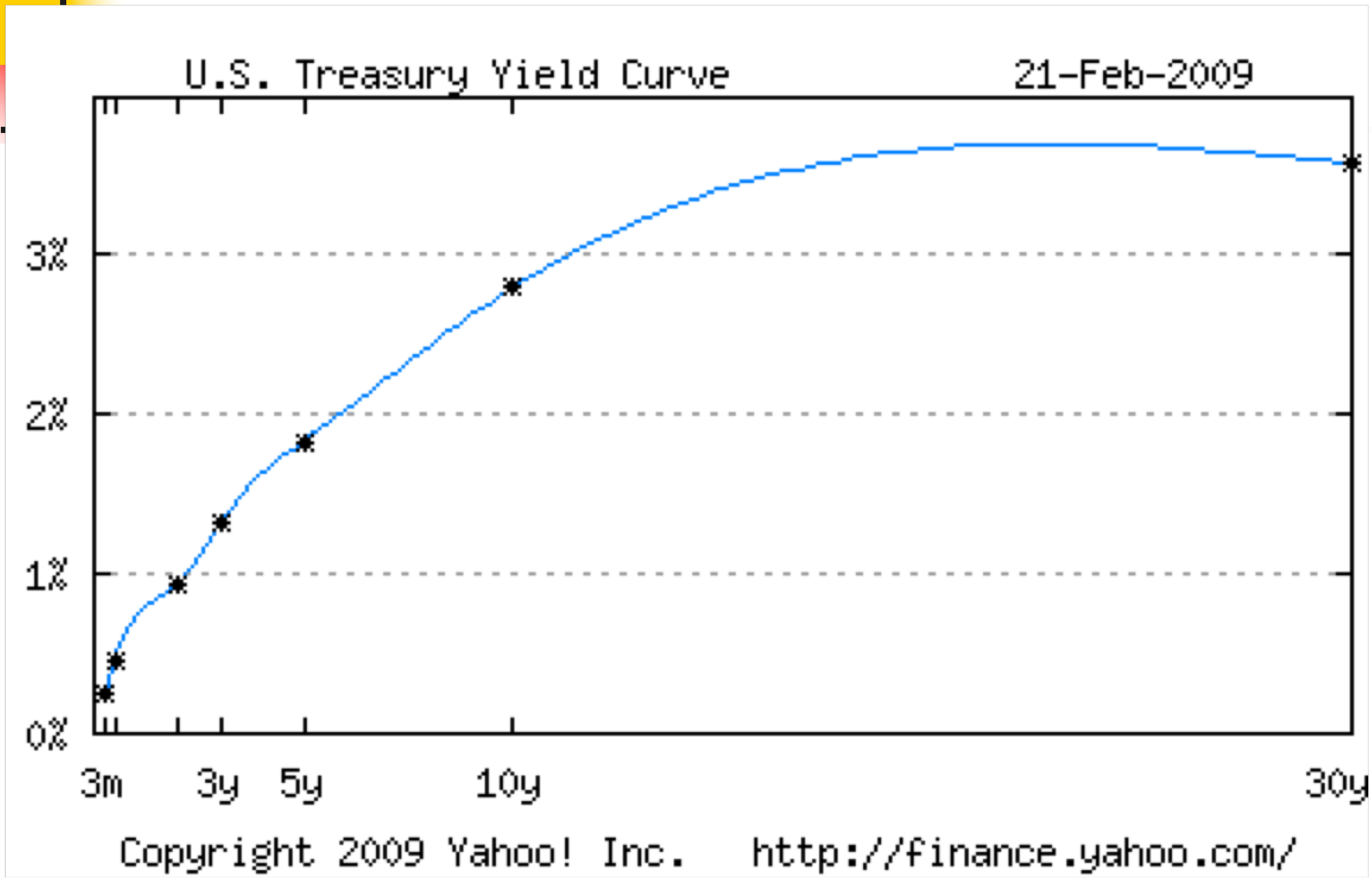
# Security Analysis and Investment

Lecture 3

# Yields and the Yield Curve


U.S. Treasury Yield Curve
21-Feb-2009
3%
2%
1%
0%
3m
3y
5y
10y
30y
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http://finance.yahoo.com/

# Spot Rates

- Definition:
  - The Spot Rate of a specific maturity is the current YTM on a zero-coupon bond of this maturity.
- Spot rates are convenient ways to quote prices:

$$P_t = \frac{M_t}{(1+s_t)^t}$$

# Spot Rates

- Spot rates can sometimes be backed out of coupon bond prices.
  - This uses a no-arbitrage style relationship to solve a system of linear equations:

$$P_1 = \frac{M_1}{1+s_1}$$

$$P_2 = \frac{C_2}{1+s_1} + \frac{M_2}{(1+s_2)^2}$$

# Using Spot Rates to price Coupon Bonds

- A coupon bond can be viewed as a series of zero coupon bonds (STRIPS)
- To find the value, each payment is discounted at the zero coupon rate
- Once the bond value is found, one can solve for the yield
- It's the reason for which similar maturity and default risk bonds sell at different yields to maturity

# Sample Bonds

| | A | B |
|---|---|---|
| Maturity | 4 years | 4 years |
| Coupon Rate | 6% | 8% |
| Par Value | 1,000 | 1,000 |
| Cash flow in 1-3 | 60 | 80 |
| Cash flow in 4 | 1,060 | 1,080 |

Assuming annual coupon payments

# Calculation of Price Using Spot Rates (Bond A)

| Period | Spot Rate | Cash Flow | PV of Cash Flow |
|---|---|---|---|
| 1 | .05 | 60 | 57.14 |
| 2 | .0575 | 60 | 53.65 |
| 3 | .063 | 60 | 49.95 |
| 4 | .067 | 1,060 | 817.80 |
| Total | | | 978.54 |

# Calculation of Price Using Spot Rates (Bond B)

| Period | Spot Rate | Cash Flow | PV of Cash Flow |
|---|---|---|---|
| 1 | .05 | 80 | 76.19 |
| 2 | .0575 | 80 | 71.54 |
| 3 | .063 | 80 | 66.60 |
| 4 | .067 | 1,080 | 833.23 |
| Total | | | 1,047.56 |

# Solving for the YTM

<u>Bond A</u>

- Bond Price = 978.54
- YTM = 6.63%

<u>Bond B</u>

- Price = 1,047.56
- YTM = 6.61%

# Discount Factors

- A discount factor is equal to the present value of $1 to be received t years in the future. (note the similarity to pure discount bond prices).
- The spot rates offer a set of discount factors for valuing coupon bonds

# Short Rate

- Spot Rate ($s_t$):
  - The YTM today for a zero-coupon bond given a specific maturity (from today to maturity date)
  - E.g.: the 2-year spot rate is 6.57%
- Short Rate ($r_t$):
  - The interest rate for a given interval (conventionally 1 year) available at different points in time
  - E.g.: the short rate today is 5%, the short rate next year will be 7.01%

# Short Rate

- Spot Rate and Short Rate:
  - The 2-year spot rate is a (geometric) average of today's short rate and next year's short rate
  - To see this, compare the total return on
    (1) buy 1-year zero; roll proceeds into 1-year zero
    with (2) buy and hold 2-year zero
    and note the two return should be equal

$$(1+s_2)^2 = (1+r_1)(1+r_2)$$

# Forward Rates

- Short Rate ($r_t$):
  - The interest rate for a given interval (1 year) available at different points in time
  - E.g.: The short rate today is 5%. The short rate next year will be 7.01%
  - But how do we know the short rate when interest rate in the future is uncertain?
- Forward Rate ($f_{t,t+1}$)
  - "the break-even" interest rate that equates the return on an n-period zero-coupon bond to that of an (n-1)-period zero-coupon bond rolled over into a 1-year bond in year n

$$(1 + s_n)^n = (1 + s_{n-1})^{n-1}(1 + f_{n-1,n})$$

# Forward Rates

- The forward rate is also the discount rate implicit in a forward contract
- In a forward contract, you agree today on a borrowing or lending rate that will prevail for some period of time at some point in the future.
  - e.g. the January 2012 one-year forward rate is 2.8%.

# Forward Rates

- Relationship between forward rates and discount rates

$$\frac{1}{(1+s_2)^2} = \frac{1}{(1+s_1)(1+f_{1,2})}$$

# Forward/Futures Contracts

- In a forward contract, you agree **today** on a delivery price for a given point in the **future.**
  - **no cash changes hands today.**
  - For riskless securities (e.g. t-bills) the forward price has an associated forward rate:

$$F_{t,T} = \frac{M_T}{(1+f_{t,T})^{T-t}}$$

# Forward Contract Example

- Suppose you enter into a forward contract on one-year t-bills.
  - The maturity of the forward contract is one year
  - Assume one-year t-bills have a face value (M) of 100.
  - One-year t-bills currently yield 5% while two-year t-bills currently yield 6%.

# Forward Contract Example

- What is the Forward Rate?
  - The forward rate solves the following equation:

$$\begin{aligned}(1+s_2)^2 &= (1+s_1)(1+f_{1,2}) \\ 1.06^2 &= (1.05)(1+f_{1,2}) \\ f_{1,2} &= 7\%\end{aligned}$$

# Forward Contracts

- This equation is the result of a no-arbitrage relationship:
  - (1) invest in 1-year zero and roll over into 1-year forward, (2) invest in 2-year zero
  - So

$$(1+s_2)^2 = (1+s_1)(1+f_{1,2})$$

  - Otherwise, short 2-year and long 1-year (if $f_{1,2}$ too high), or short 1-year and long 2-year (if $f_{1,2}$ too low)

# Example: Yield Curve -> Forward Rate

| Zero-Coupon Rates | Bond Maturity |
|---|---|
| 12% | 1 |
| 11.75% | 2 |
| 11.25% | 3 |
| 10.00% | 4 |
| 9.25% | 5 |

# Forward Rates

1yr Forward Rates

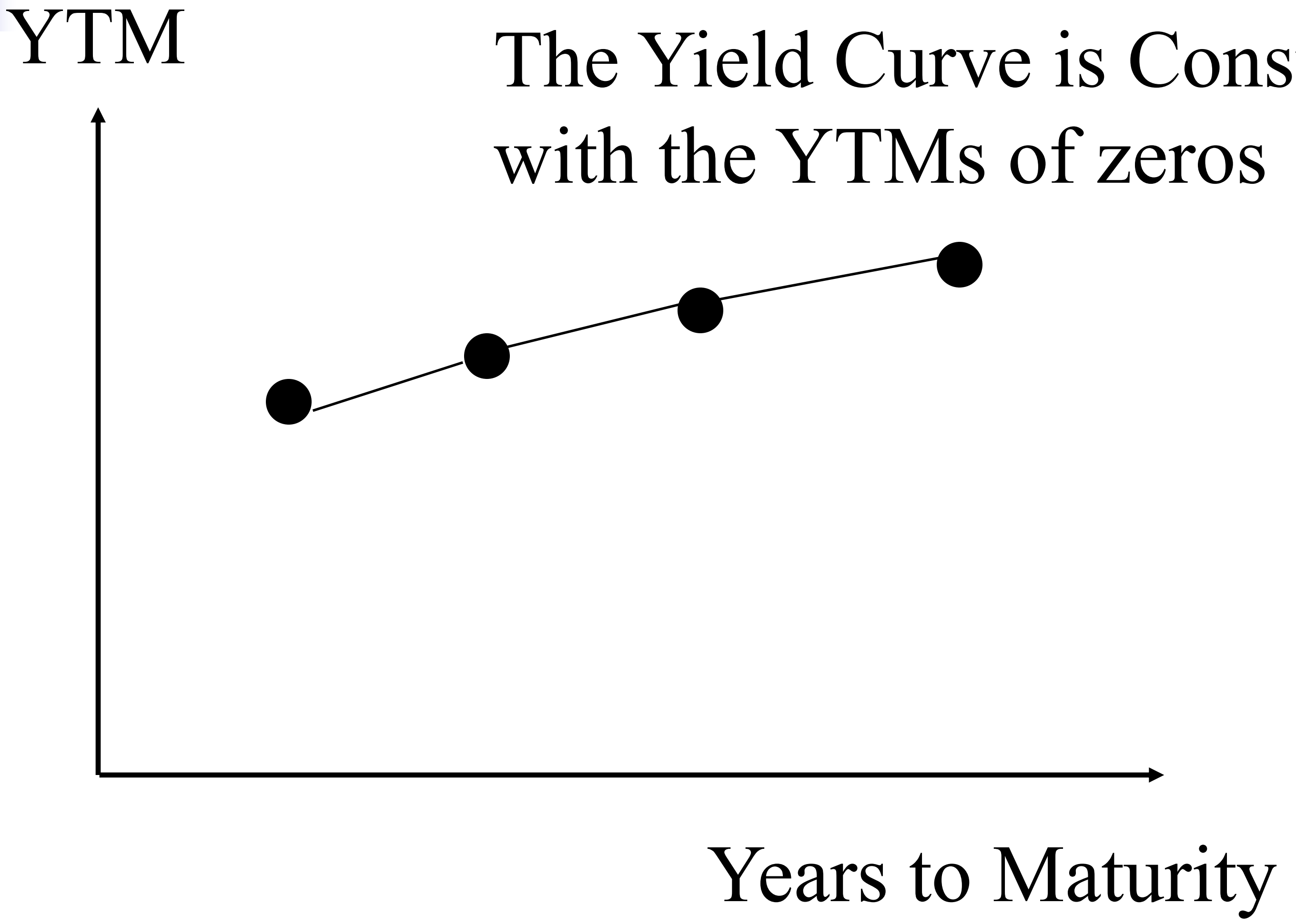
| | | |
|---|---|---|
| 1yr | $[(1.1175)^2 / 1.12] - 1$ | $=0.115006$ |
| 2yrs | $[(1.1125)^3 / (1.1175)^2] - 1$ | $=0.102567$ |
| 3yrs | $[(1.1)^4 / (1.1125)^3] - 1$ | $=0.063336$ |
| 4yrs | $[(1.0925)^5 / (1.1)^4] - 1$ | $=0.063008$ |

# Term Structure of Interest Rates

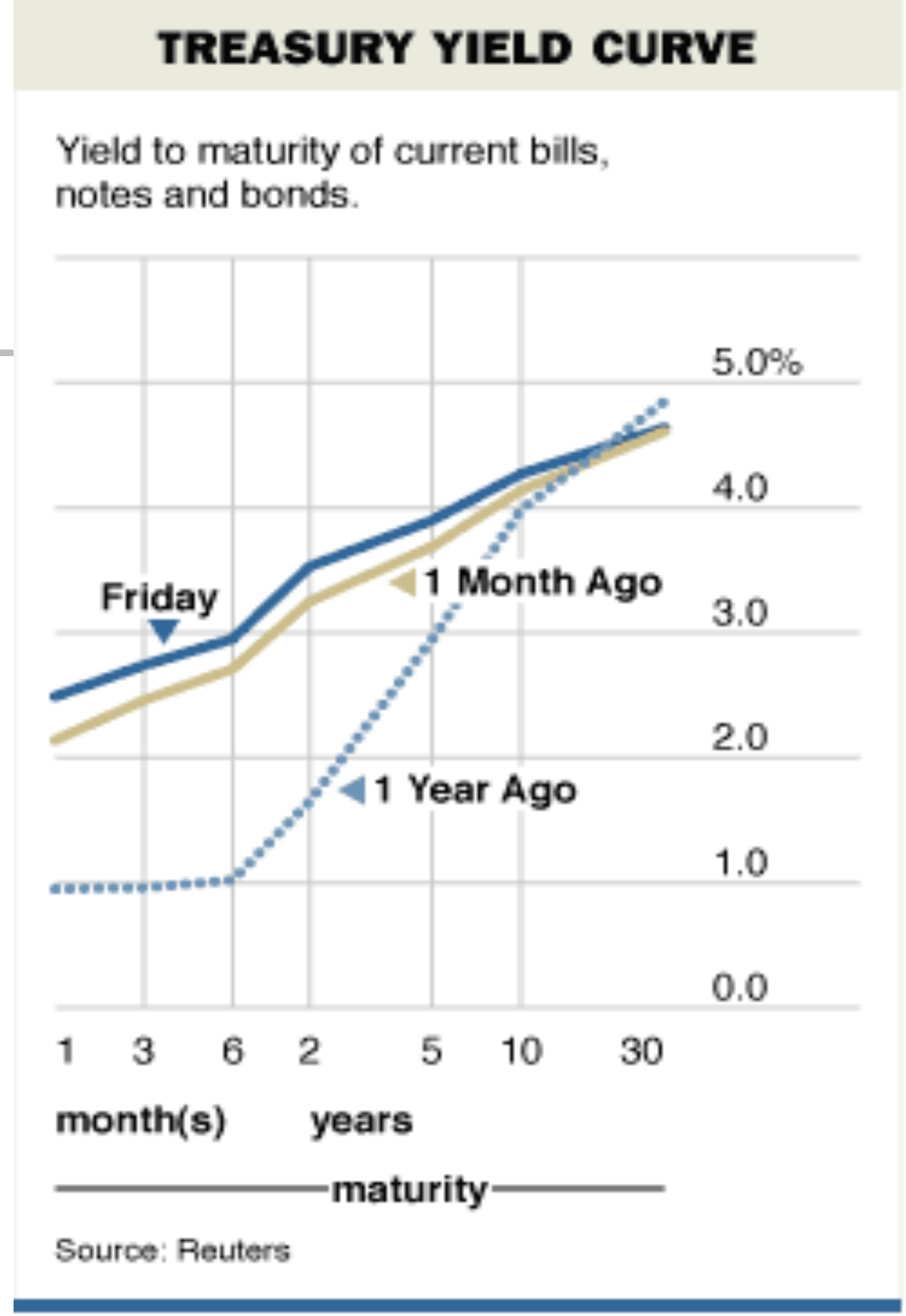
- The term structure of interest rates (yield curve) is the relationship between yields to maturity of bonds (spot rates) with different maturities

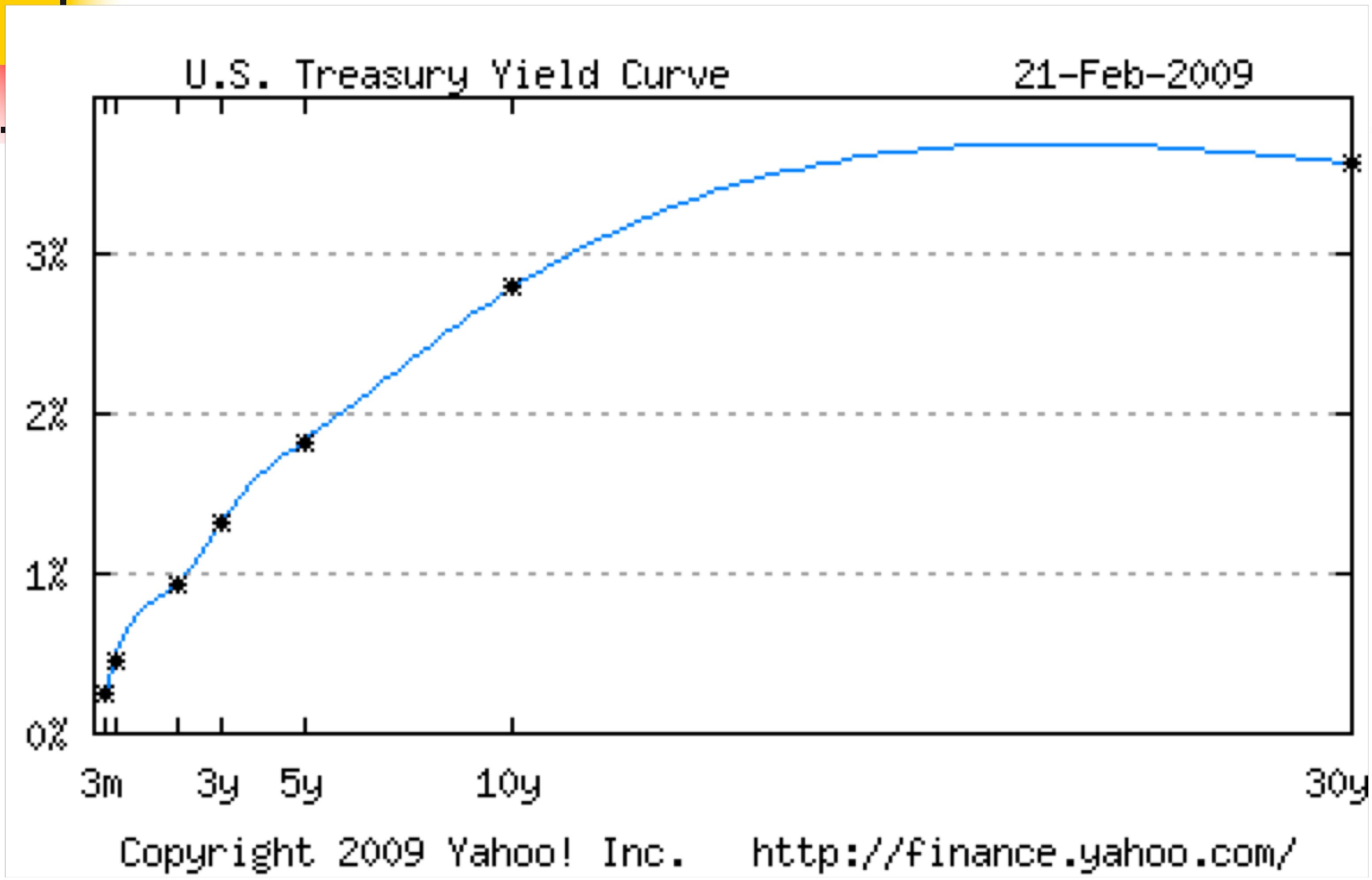
# The Yield Curve

The Yield Curve is Constructed with the YTMs of zeros



# Treasury Yield Curve for February 25, 2005




U.S. Treasury Yield Curve
21-Feb-2009
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3m
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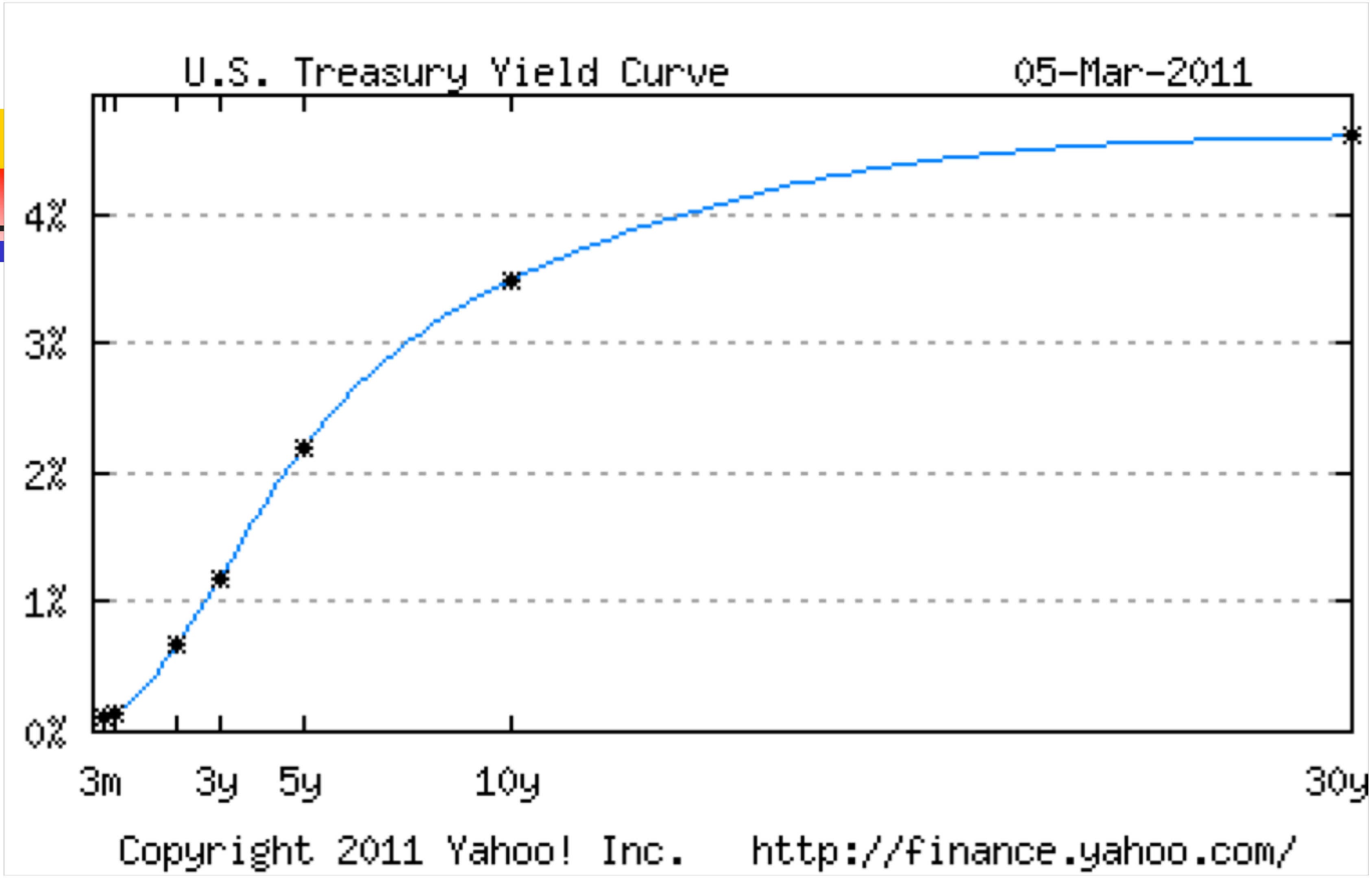
U.S. Treasury Yield Curve
05-Mar-2011
4%
3%
2%
1%
0%
3m
3y
5y
10y
30y
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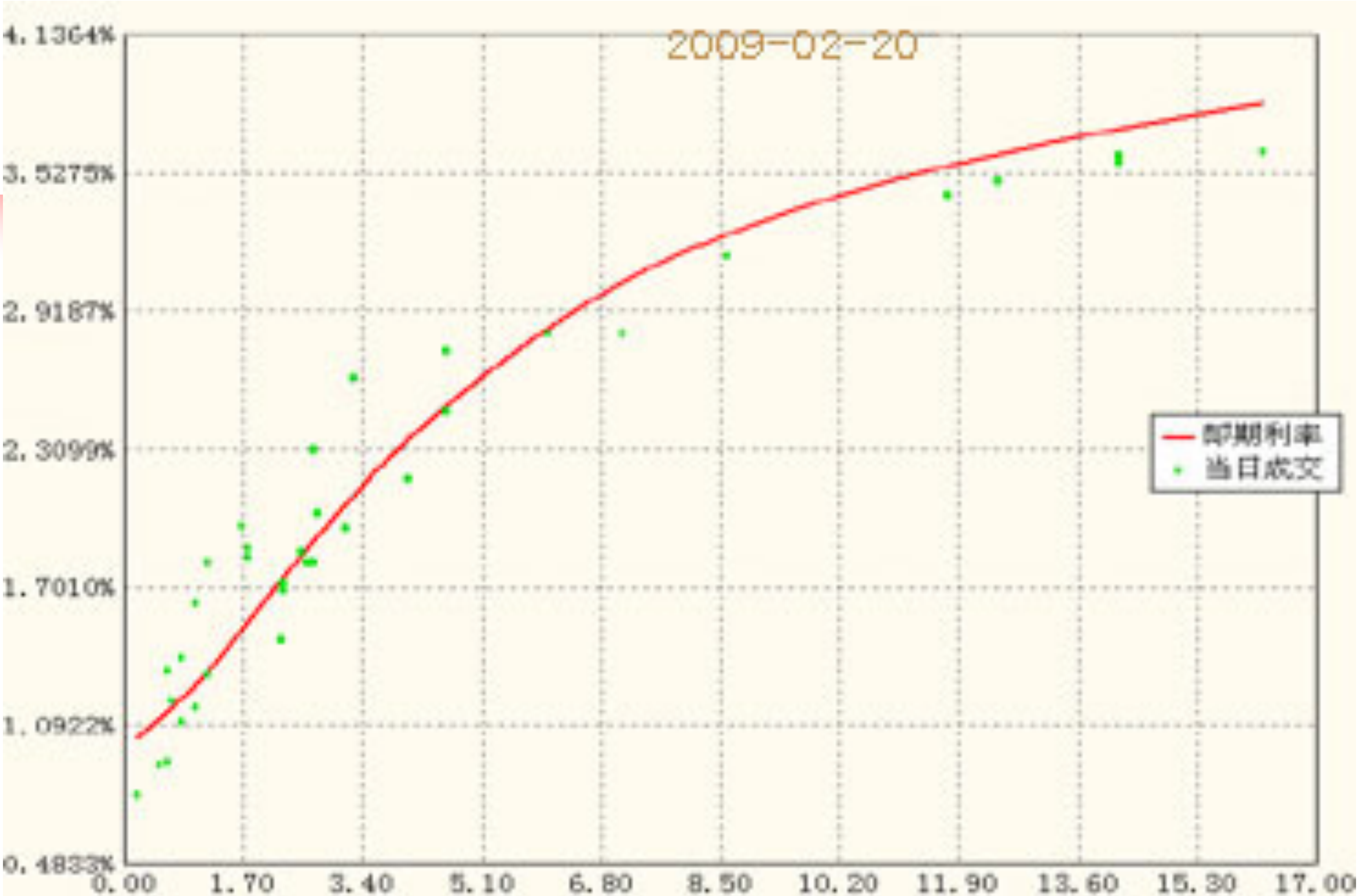



2月20日国债交易所收益率曲线图。(图片来源：北方之星数码技术有限公司)

# Measuring the term structure

- Derive spot rates from bond yields of varying maturities
- Treat each coupon as a mini-zero coupon bond (STRIPS)
- Use bonds of progressively longer maturities, starting from T-bills

# Theories of the Term Structure

- The “term structure of interest rates” is the relationship between time and spot rates.
- What determines the term structure?
  - Expectations of future interest rates.
  - Risk premia

# Term Structure Theories

- Why might interest rates change?
  - The real interest rate is expected to change (perhaps technology is expected to change; perhaps monetary policy is expected to change)
  - Inflation is expected to change

# Term Structure Theories

- Several hypotheses about term structure:
  - Unbiased expectations
  - Liquidity preference
  - Market Segmentation
  - Preferred Habitat

# Expectations Theory

- Observed long-term rate is a function of today's short-term rate and expected future short-term rates
- Long-term and short-term securities are perfect substitutes
- Forward rates that are calculated from the yield on long-term securities are market consensus expected future short-term rates

# Unbiased Expectations

- The forward rate represents the average opinion of the expected future spot rate.

$$es_{t,T} = f_{t,T}$$

- If forward rates are expected to rise, then yield curves will be upward sloping.

$$(1+s_2)^2 = (1+s_1)(1+es_{1,2})$$

# Expectations Theory

- If you believe the expectations theory, you can use it to invest in the bond market
- For example, if your forecast the future interest rate to be lower than the market expected future interest rate implied by the yield curve, you can buy long-term bonds (whose price will rise).

# Liquidity Preference Theory

- Investors may face liquidity shocks.
- If they need to sell long-maturity instruments, they will face some price risk. (if interest rates change).
- If they hold short-maturity instruments, they will not face price risk.

# Liquidity Preference Theory

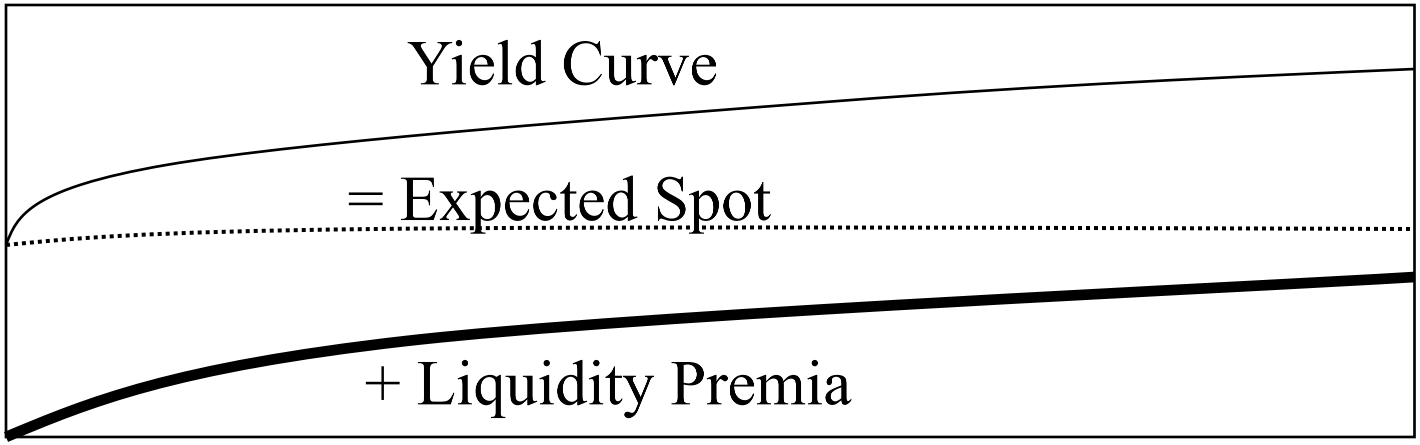
- Thus, long-maturity investments will have a risk premium.

$$f_{1,2} = es_{1,2} + L_{1,2}$$

- Remember the payoff from a forward is (P-F) and if this payment is risky, P-F must, on average, provide strictly positive profits as compensation for bearing the risk.

# Liquidity Preference Theory

- Liquidity premia make the yield curve more strongly upward sloping.
  - Intuitively, the yield curve is based on expected interest rates **plus** a liquidity premium curve:



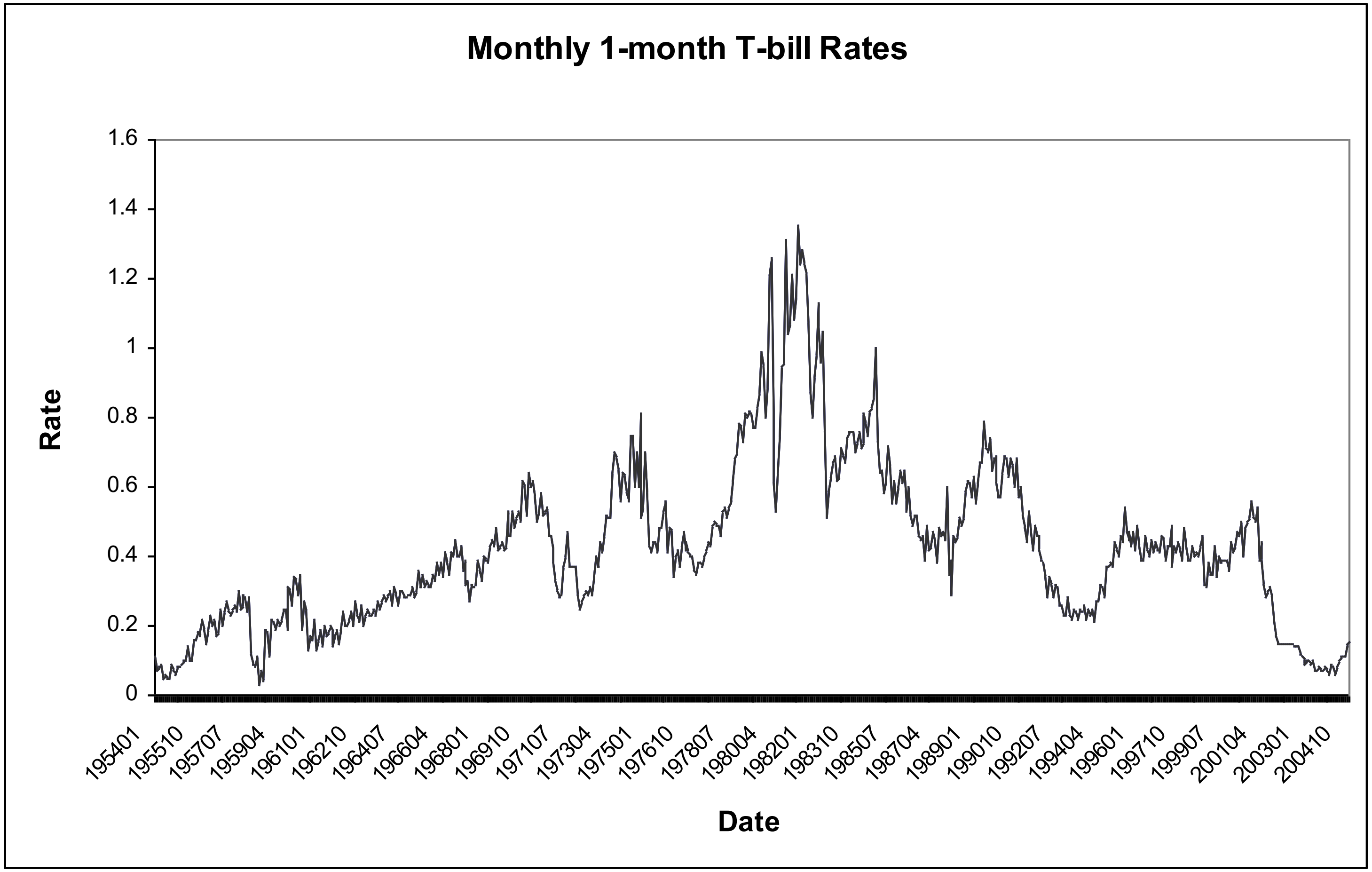
# Market Segmentation Theory

- Various investors are restricted (by law, preference, or custom) to certain securities.
- Spot rates are determined by supply and demand in their respective markets.

# Preferred Habitat Theory

- Borrowers and lenders have maturity segments in which they prefer to operate.
- If prices get too high they switch to other segments (this is not true for the Market Segmentation Theory)

# Fluctuations of Interest Rates

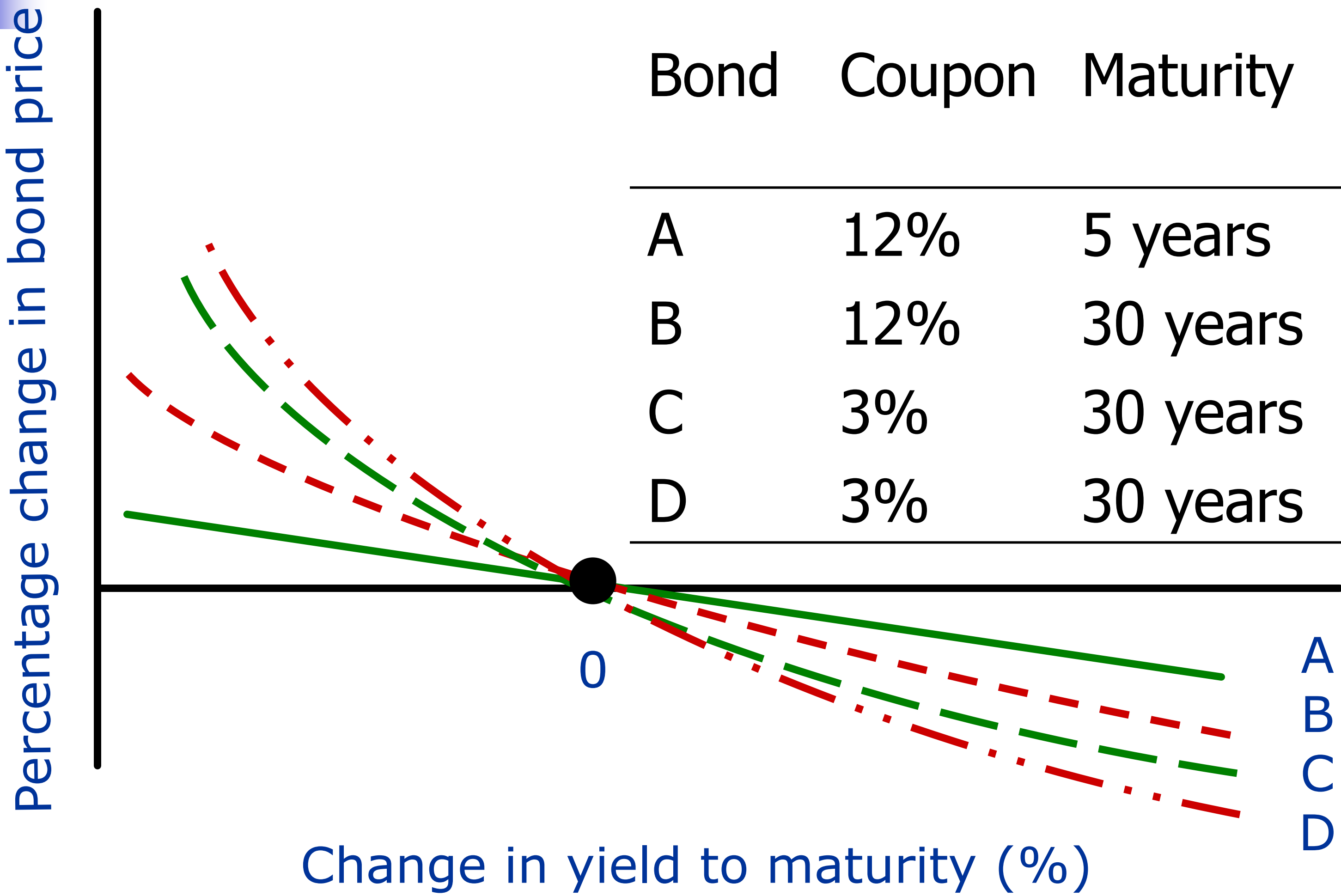


# Duration and Convexity

- You would like to buy the 9.25% Treasury bond with maturity in Feb 2018
- Currently interest rates are relatively low and you are worried that interest rates might increase in the near future
- How much would you lose if interest rates increase by 1 percentage point

# Interest Rate Sensitivity

| Bond | Coupon | Maturity | Initial YTM |
|---|---|---|---|
| A | 12% | 5 years | 10% |
| B | 12% | 30 years | 10% |
| C | 3% | 30 years | 10% |
| D | 3% | 30 years | 6% |



# Bond Pricing Relationships

- Inverse relationship between price and yield
- An increase in a bond's yield to maturity results in a smaller price decline than the gain associated with a decrease in yield
- Long-term bonds tend to be more price sensitive than short-term bonds

# Bond Pricing Relationships (cont'd)

- As maturity increases, price sensitivity increases at a decreasing rate
- Price sensitivity is inversely related to a bond's coupon rate
- Price sensitivity is inversely related to the yield to maturity at which the bond is selling

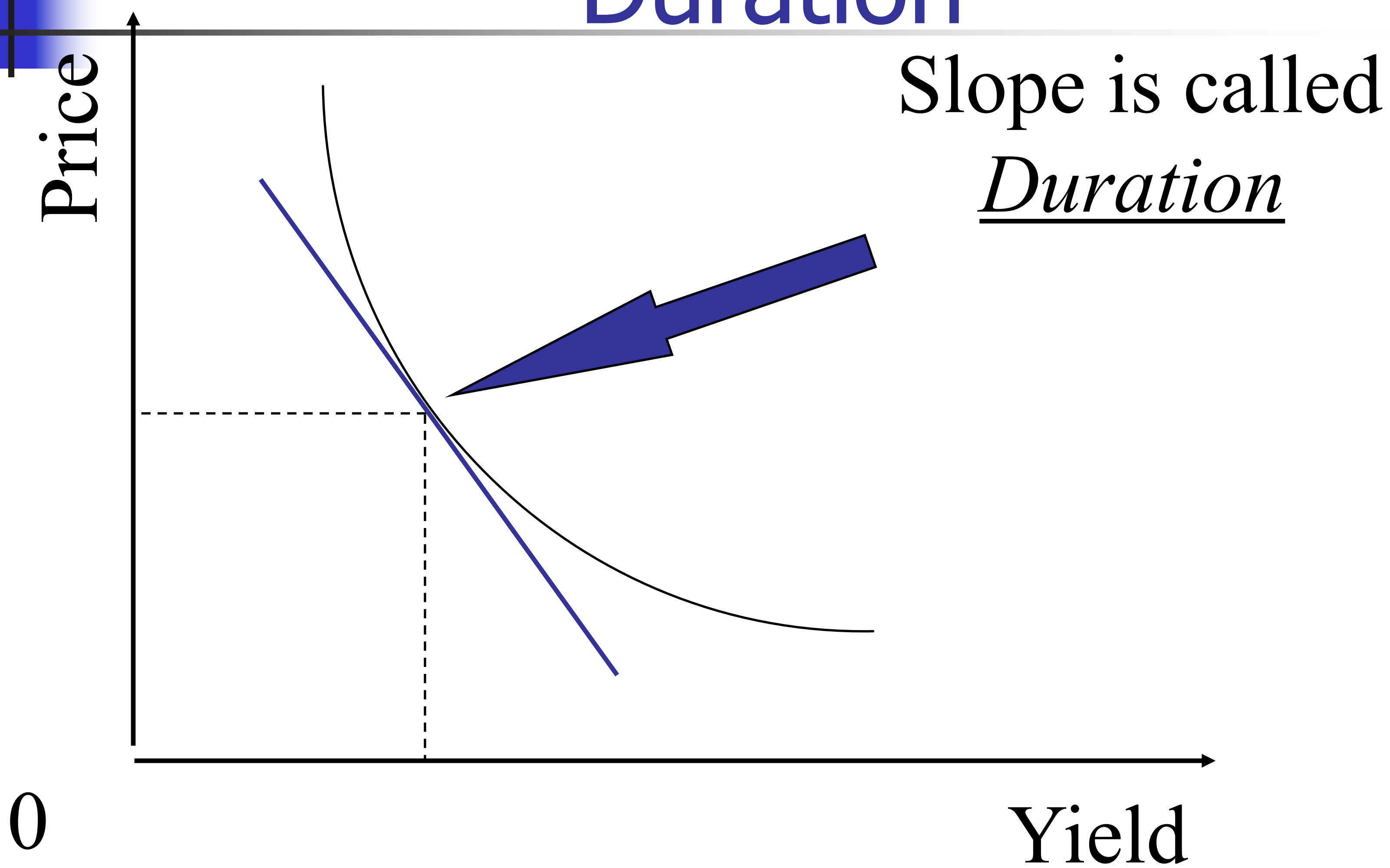
# Interest Rate Risk

- The value of the bond is given by the sum of the present values *PV* of the coupon payments *C* and the face value *FV:*

$$V_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{FV_T}{(1+y)^T} = \sum_{t=1}^{T} PV(C_t) + PV(FV_T)$$

  - How do changes in interest rates affect the values of bonds?

# Duration



# Interest Rate Risk

- The change in bond values given a change in interest rates is:

$$\frac{dV}{dy} = \sum_{t=1}^{T} \frac{-t \times C_t}{(1+y)^{t+1}} + \frac{-T \times FV}{(1+y)^{T+1}} = -\frac{1}{1+y}\left[\sum_{t=1}^{T} t \times PV(C_t) + T \times PV(FV_T)\right]$$

- The percentage change in bond values given a change in interest rates is:

$$\frac{dV/dy}{V} = -\frac{1}{1+y}\left[\sum_{t=1}^{T} t\,\frac{PV(C_t)}{V} + T\,\frac{PV(FV_T)}{V}\right]$$

# Duration

- The Duration of a bond is defined as:

$$D = \sum_{t=1}^{T} t \frac{PV(C_t)}{V} + T \frac{PV(FV_T)}{V}$$

- Thus, the sensitivity of the bond price to interest rates is:

$$\frac{dV}{V} = -\frac{dy}{1+y} D$$

# Duration: Calculation

$$w_t = \frac{CF_t \big/ (1+y)^t}{\text{Price}}$$

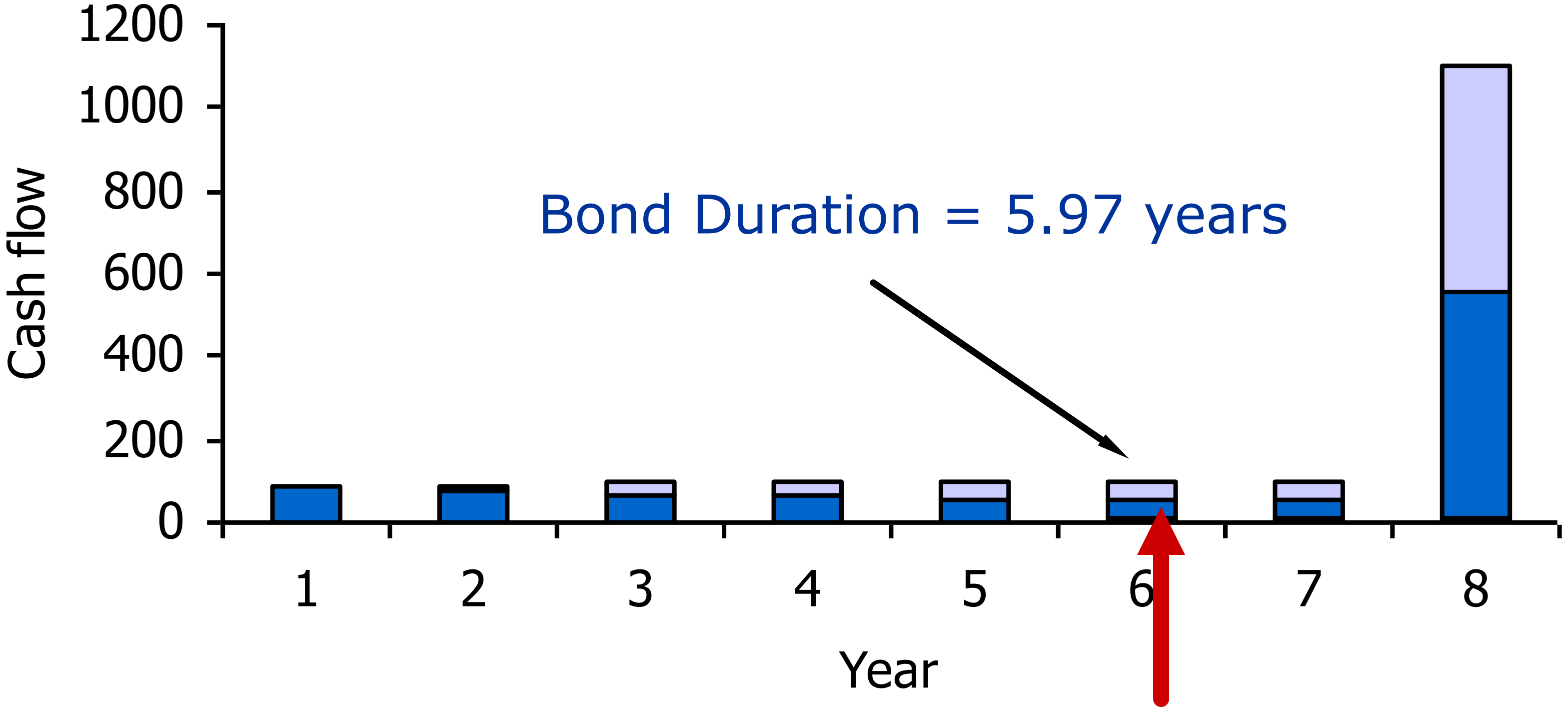
$$D = \sum_{t=1}^{T} t \times w_t$$

$CF_t$ = Cash Flow for period $t$

# Duration

- A measure of the effective maturity of a bond
- The weighted average of the times until each payment is received, with the weights proportional to the present value of the payment

# Example: 8-year, 9% annual coupon bond



# Duration

- The duration is the weighted average maturity of a bond
  - The weight of each time period depends on the relative importance of its cash flow payment
- The duration measures the sensitivity of bonds to interest rate changes
  - Bonds with long durations react more if interest rates change

# Duration: Examples

- What is the duration of a zero-coupon bond with a maturity of 10 years?
  - D=10 years

- What is the duration of the 9% Treasury Bond expiring in 11 years using a discount rate of 4%?
  - D= years (computed using Excel)

# Interest Rate Sensitivity: Example

- Suppose that the yield to maturity of the Treasury bond increases by one percentage point from 4% to 5%
  - How does that affect the value of the bond?

# Interest Rate Sensitivity: Example

- First Method: Solve for the present discounted value using the new yield to get the exact value:

$$V_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{FV_T}{(1+y)^T} = \$$$

  - The bond decreases by percent from \$ to \$

# Interest Rate Sensitivity: Example

- Second Method: Use the duration formula to get an approximate estimate of the value change:

$$\frac{\Delta V}{V} = -\frac{1}{1+y} \times D \times \Delta y = -\frac{1}{1.04} \times \_\_\_ \times 0.01 = \_\_\_\_$$

  - The bond is estimated to decrease by ___ percent

# Modified Duration (MD)

- This relation has led practitioners to define “modified duration” as

$$MD = D/(1 + y)$$

- Modified duration has the property that

$$\Delta V/V = -MD^{*}\,\Delta y$$

- Thus, modified duration can be interpreted as (minus) the return on a bond for a 1% change in interest rates

# Duration

- First, what is the duration of a 3-year zero?
- What about a 25-year zero?
- Now, what is the duration of a 2-year bond that pays a coupon of $100 this year and pays $1100 at maturity if the price = $1000?
- The YTM is the y that solves the problem:

$$1000 = \_\_\_/(1+y) + \_\_\_/(1+y)^2$$

# Duration

- Solving this problem gives YTM =

Thus, w1 =

and w2 =

- Notice that w1 + w2 =  . This is always true
- Now the duration is just  _(w1) + _(w2),

  Duration =
- Why is it that duration < maturity?

# Duration

- So which has a higher duration, a 10-year zero coupon bond or a 10-year 10% coupon bond? Why?
- Which bond will be more sensitive to interest rate risk?

# Duration - Portfolio

- The duration of a portfolio equals the weighted average of the durations of the assets in the portfolio
- What will be the duration of the equally weighted portfolio with D1 = 3.5; D2 = 2.3 and D3 = 7.1?

  DP =

# Notes on duration

- The duration formula is derived from the bond price-yield relationship
- Calculate dP/dy and divide with P
- Approximating the bond price change using duration is equivalent to moving along the slope of the bond price-yield curve

# Uses of Duration

- Duration is used as a measure of risk for bonds, similar to the beta for stocks
- Duration is used very frequently by managers of fixed-income portfolios and also by financial institutions such as banks and insurance companies

# Rules for Duration

Rule 1 The duration of a zero-coupon bond equals its time to maturity

Rule 2 Holding maturity constant, a bond's duration is higher when the coupon rate is lower

Rule 3 Holding the coupon rate constant, a bond's duration generally increases with its time to maturity

# Rules for Duration (cont'd)

<u>Rule 4</u> Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower

<u>Rule 5</u> The duration of a level perpetuity is equal to:

$$\frac{(1 + y)}{y}$$

# Rules for Duration (cont'd)

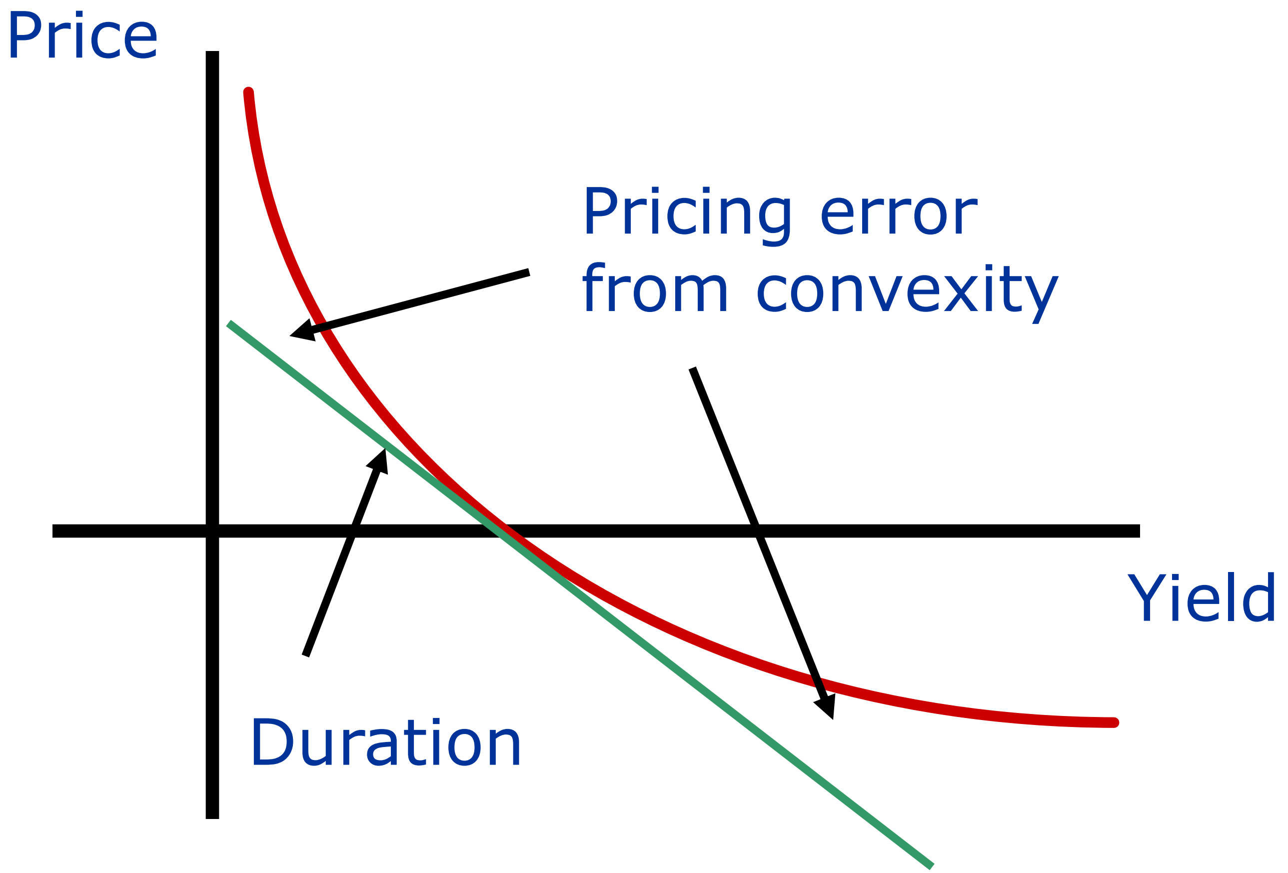
Rule 6 The duration of a level annuity is equal to:

$$\frac{1+y}{y} - \frac{T}{(1+y)^T - 1}$$

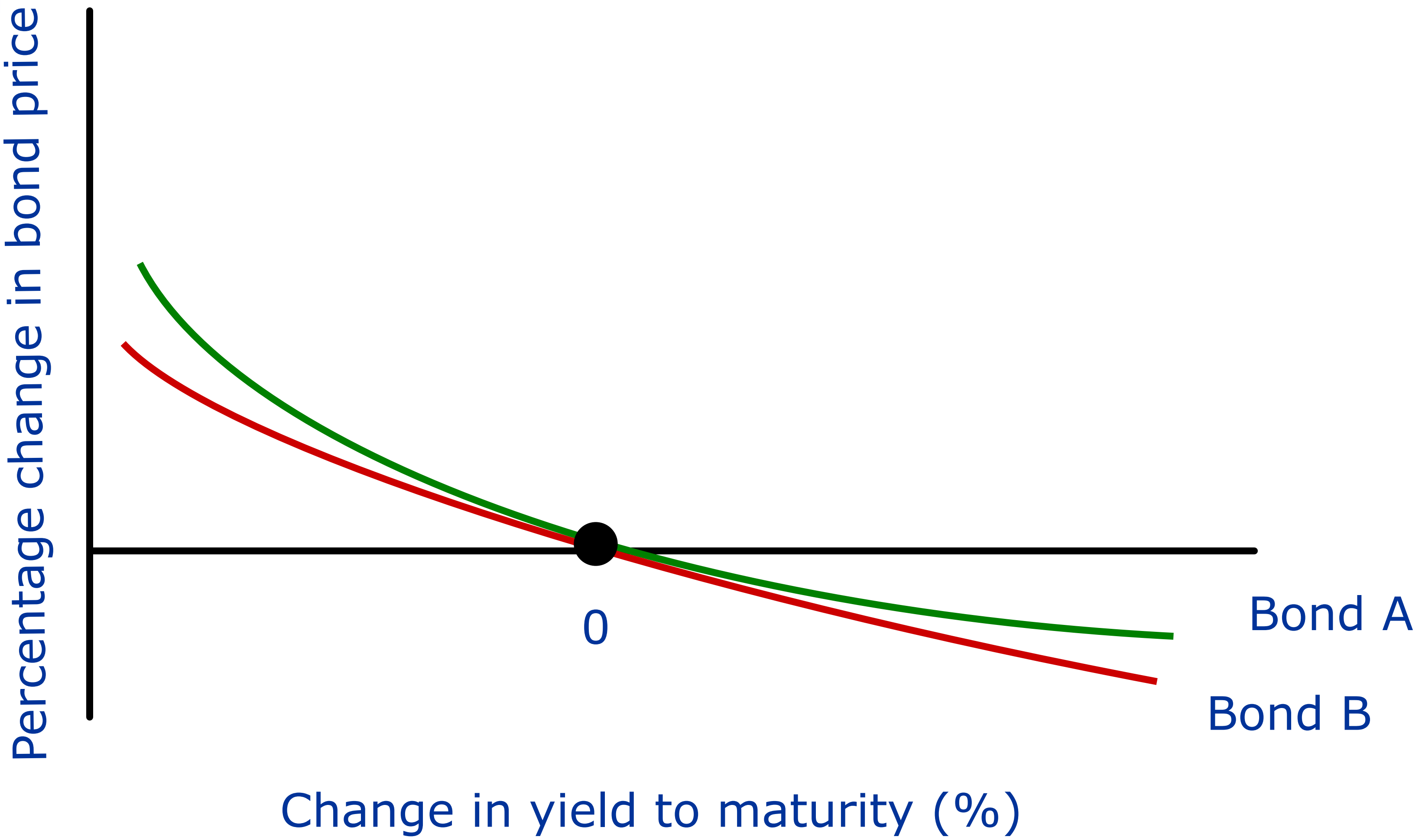
Rule 7 The duration for a coupon bond is equal to:

$$\frac{1+y}{y} - \frac{(1+y) + T(c-y)}{c[(1+y)^T - 1] + y}$$

# Duration and Convexity



# Convexity of Two Bonds



# Convexity

- The convexity enables us to get a better estimation of bond prices taking into account the curvature in addition to the slope
- The convexity is defined as:

$$C = \frac{d^2V \big/ dy^2}{V} = \frac{1}{(1+y)^2}\left[\sum_{t=1}^{T}\left(t+t^2\right)\frac{PV(C_t)}{V} + \left(T+T^2\right)\frac{PV(FV_T)}{V}\right]$$

# Correction for Convexity

$$Convexity = \frac{1}{(1+y)^2} \sum_{t=1}^{n} \left[ \frac{\frac{CF_t}{(1+y)^t}}{P} (t^2 + t) \right]$$

Correction for Convexity:

$$\frac{\Delta P}{P} = -MD * \Delta y + \frac{1}{2} \cdot Convexity \cdot (\Delta y)^2$$

# Convexity calculation

| 8% Bond | Time Sem. | Payment | PV of CF (10%) | Weight | t(t+1)x weight |
|---|---|---|---|---|---|
| | 1 | 40 | 38.095 | .0395 | .0790 |
| | 2 | 40 | 36.281 | .0376 | .2257 |
| | 3 | 40 | 34.553 | .0358 | .4299 |
| | 4 | 1040 | 855.611 | .8871 | 17.7413 |
| | | sum | 964.540 | 1.000 | 18.4759 |

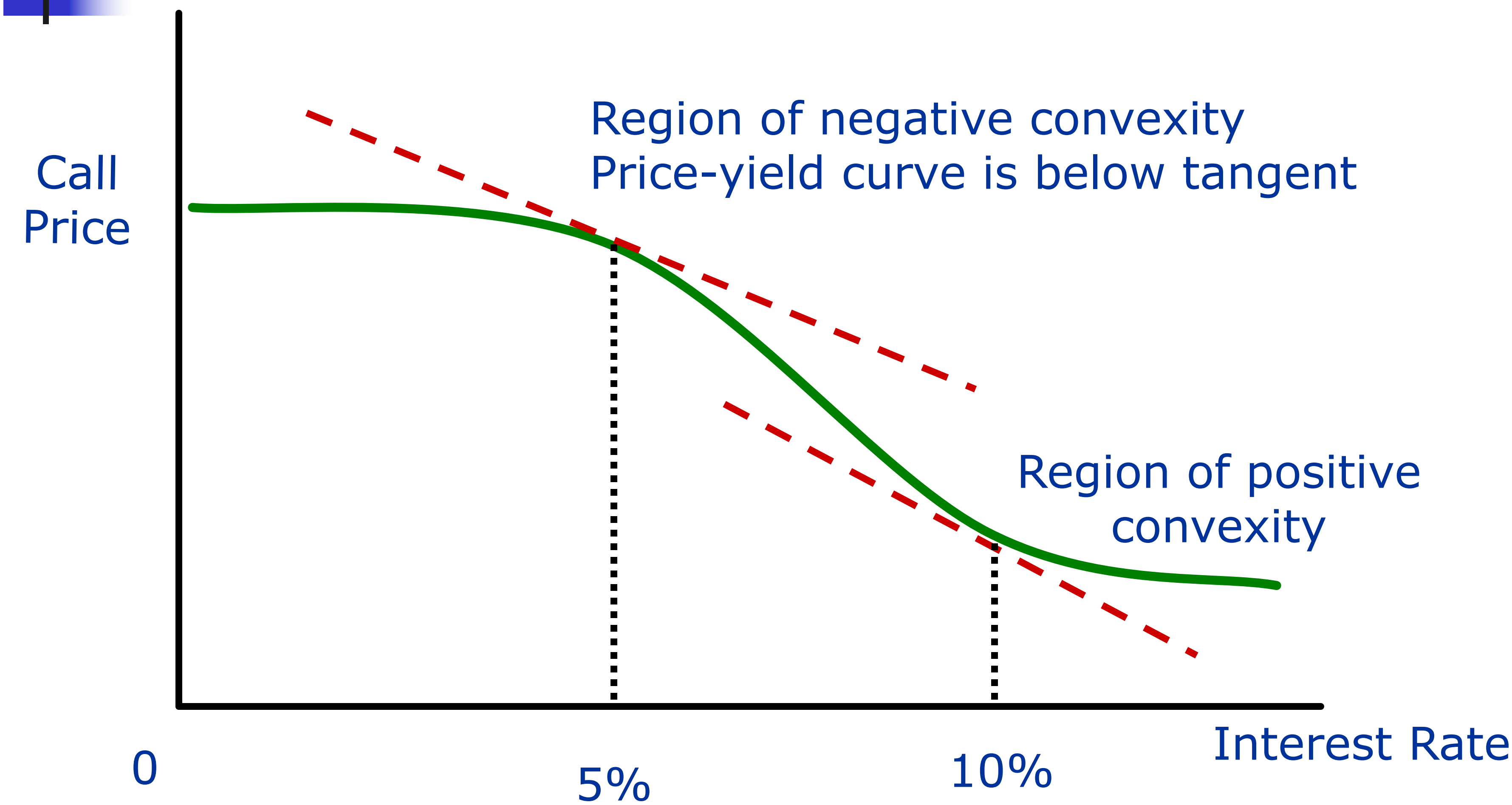
# Convexity calculation (cont.)

- Convexity is computed like duration, as a weighted average of the terms $(t^2+t)$ (rather than t) divided by $(1+y)^2$
- Thus, in the above example, it is equal to

$$18.4759/1.05^2 = 16.7582$$

in semester terms.

# Duration and Convexity of Callable Bonds



# Convexity and Duration

- The percentage change in the value of a bond can be estimated using both duration and convexity:

$$\frac{\Delta V}{V} = -\frac{1}{1+y} \times D \times \Delta y + \frac{1}{2} \times C \times (\Delta y)^2$$

  - How much does the bond change in value if both duration and convexity are included?

# Convexity and Duration: Example

$$\frac{\Delta V}{V} = -\frac{1}{1+y} \times D \times \Delta y + \frac{1}{2} \times C \times (\Delta y)^2 =$$

# Managing Fixed Income Securities: Basic Strategies

- Active strategy
  - Trade on interest rate predictions
  - Trade on market inefficiencies
- Passive strategy
  - Control risk
  - Balance risk and return

# Managing interest rate risk

- Bond price risk
- Coupon reinvestment rate risk
- Matching maturities to needs
- The concept of duration
- Duration-based strategies
- Controlling interest rate risk with derivatives

# Overview

- Passive Bond Management
  - Passive managers take prices as set and seek to control the risk of their fixed-income portfolios
- Active Bond Management
  - Active managers try to create value by forecasting interest rates and by identifying mispriced securities

# Passive Bond Management

- Immunization
  - A strategy to shield net worth from interest rate movements
  - Example:
    - Asset & Liability Management for Banks

# Example:
# Asset & Liability Management

- Banks have a natural mismatch between the maturities of assets and liabilities
  - Assets: Long-term commercial and consumer loans or mortgages
  - Liabilities: Short-term deposits
- What is the main risk?

# Example: Asset & Liability Management

- Assets:
  - Present value: $50M
  - Average Duration: 10 years
- Liabilities:
  - Present value: $45M
  - Average Duration: 1 year
- Equity:
  - Present value: $5M
  - Average Duration: ? years

# Example: Asset & Liability Management

- Suppose that interest rates increase from 4% to 6%. How does that affect the equity of the bank?
  - Value of Assets:

$$\frac{\Delta V_A}{V_A} = -\frac{1}{1+y} \times D_A \times \Delta y = -\frac{1}{1.04} \times 10 \times 0.02 = -0.1923$$

  - Value of Liabilities:

$$\frac{\Delta V_L}{V_L} = -\frac{1}{1+y} \times D_L \times \Delta y = -\frac{1}{1.04} \times 1 \times 0.02 = -0.0192$$

# Example: Asset & Liability Management

- The value of the assets will decrease by about 19.23% from $50M to $40.385M
- The value of the liabilities will decrease by about 1.92% from $45M to $44.135M
- Thus, the value of equity will decrease by about 175% from $5M to –$3.75M!

# Example: Asset & Liability Management

- What is the duration of the equity of the bank?

  $D_E =$

- What can the bank do to manage interest rate risk?

# Example: Asset & Liability Management

- Risk Management Techniques
  - Reduce the duration of the assets
  - Increase the duration of the liabilities
  - Increase the equity by raising capital
  - Use derivative instruments, such as interest rate swaps

# Immunization

- Managers use duration to immunize their portfolios from interest rate risk
- One way to immunize a portfolio is to set the duration (and magnitude) of its liabilities to the duration of its assets
- Banks immunize their portfolios this way
- Banks naturally have high duration assets (loans) & low duration liabilities (deposits)

# Immunization

- Another way to immunize is to set the duration of your portfolio to the maturity of a large future payment you will make
- Pension funds and insurance companies immunize their liabilities this way
- When you immunize, you need to update your immunized position as time goes by and the variables determining D change

# Question

- Suppose there is a one-year zero coupon bond with price 952.4. There is a two-year bond with coupon rate 5% and its price is 982.1. A three-year bond with coupon rate 10% has a price of 1057.5

- One-year, two-year, three year spot rates?
- One-year forward rate for one year from today, two year from today
- Two-year forward rate for one year from today
- A three-year bond with coupon rate 5% is priced at 953. Any problem?

# Review

$$\frac{1}{(1+s_2)^2} = \frac{1}{(1+s_1)(1+f_{1,2})}$$

$$\frac{\Delta V}{V} = -\frac{1}{1+y} \times D \times \Delta y + \frac{1}{2} \times C \times (\Delta y)^2$$

# Immunization

- Example:  Suppose an insurance company must make payments to a customer of $10 million in 1 year and $4 million in 5 years
- Suppose the yield curve is flat at 10%
- If the company wants to fully fund and immunize its obligation with 1 zero, what should it buy?  What will the zero cost?

# Immunization

- (Example) Duration of payments:
- value =
- weight of 10 =
- weight of 4 =
- duration =
- So we should buy zeros with a maturity of ____ years

# Immunization

- (Example) So we will buy zeros of maturity ___, but how many should we buy?
- The market value of our zeros must be set equal to the market value of the obligation
- So buy _______ worth of zeros
- This works out to ___________ of face value in zeros
- Maybe they should buy 1 and 4 year zeros