**Week 1 HW (due Sept. 10th)**

**Instructions.** You *must* declare all resources that you have used on this homework (include but not limited to anyone, any book, and any webpage).

1. Read Example 1 on Page 11 of [B-N] and complete Problem 1-3 on Page 11 of [B-N]

   Consider the differential equation
   $$y' = y^2.$$

   Here $n = 1$, the region $D$ in the whole $(t, y)$ space, $f(t, y) = y^2$ is defined everywhere, and
   $$\phi(t) = \frac{\eta}{1 - \eta(t - t_0)}$$

   is a solution on an interval $I$ containing $t_0$ for which $\phi(t_0) = \eta$.

   a) Construct the above solution $\phi$ by the method of separation of variables.

   b) What is the interval of validity of the above equation $\phi$ if $\eta < 0$?

   c) Discuss the case $\eta = 0$.

2. [B-N] Page 12 Problem 8(a))

   $$y' = ty^3, \quad y(0) = 1$$

   Verify that the following function
   $$\phi(t) = \left(1 - t^2\right)^{-1/2}$$

   is a solution of the given differential equation satisfiying the given initial condition. Determine the interval of validity.

3. Determine the order of the equation, and whether the equation is autonomous and linear. If it is linear, rewrite the equation into a system of equations in the form of $\frac{d\vec{x}}{dt} = A(t)\vec{x} + g(t)$.

   (a) $\dfrac{d^3y}{dx^3} + \left(y^5 + xy + 1\right)\dfrac{dy}{dx} = 1$

   (b) $x^2\dfrac{d^4x}{dt^4} + \left(\dfrac{dx}{dt}\right)^5 = \cos x$

   (c) $t^3 x'''(t) + x''(t) - 16x(t) = 0$

4. (Revised from [B-N] Page 13 problem 11)

   Solve
   $$y' = \frac{y^2 - 1}{2},$$
   and sketch the solution for different initial values.

5. Solve equation
   $$\frac{dy}{dt} + \frac{2y}{t} = 2t\sqrt{y}.$$

6. Find the general solutions of

   a) $y'' + 12y' + 36y = 0$

   b) $y'' - 3y = 0$

   c) $y'' + y' + y = 0$