Denote by $\mathcal{F}_0 = \{\phi, \Omega\}$. We can verify

$$\mathbb{E}(X|\mathcal{F}_0) = \mathbb{E}X$$

directly by using our understanding of conditional expectations.

Indeed, we construct a random variable $1_\Omega : \Omega \to R$ satisfying

$$1_\Omega(\omega) = 1 \text{ for all } \omega \in \Omega.$$

It is easy to know that

$$\sigma(1_\Omega) = \mathcal{F}_0 = \{\phi, \Omega\}.$$

Thus, we have

$$\mathbb{E}(X|\mathcal{F}_0) = \mathbb{E}(X|\sigma(1_\Omega)) = \mathbb{E}(X|1_\Omega).$$

In what follows, let us show that

$$\mathbb{E}(X|1_\Omega = y) = \mathbb{E}X, \tag{1}$$

which implies that

$$\mathbb{E}(X|1_\Omega) = \mathbb{E}X.$$

To show (1), it is sufficient to verify that X and 1_Ω are independent. We show that for any arbitrary two sets A and B,

$$\mathbb{P}(X \in A, 1_\Omega \in B) = \mathbb{P}(X \in A)\mathbb{P}(1_\Omega \in B). \tag{2}$$

If $1 \notin B$, we have $\{1_\Omega \in B\} = \phi$. Thus, we have

$$\begin{aligned}
\mathbb{P}(X &\in A, 1_\Omega \in B) = \mathbb{P}(X \in A, \phi) = 0, \\
\mathbb{P}(X &\in A)\mathbb{P}(1_\Omega \in B) = \mathbb{P}(X \in A)\mathbb{P}(\phi) = 0.
\end{aligned}$$

If $1 \in B$, we have $\{1_\Omega \in B\} = \Omega$. Thus, we have

$$\begin{aligned}
\mathbb{P}(X &\in A, 1_\Omega \in B) = \mathbb{P}(X \in A, \Omega) = \mathbb{P}(X \in A), \\
\mathbb{P}(X &\in A)\mathbb{P}(1_\Omega \in B) = \mathbb{P}(X \in A)\mathbb{P}(\Omega) = \mathbb{P}(X \in A).
\end{aligned}$$