

HW6 (due Oct. 31th) Happy Halloween in advance!

Instructions. You *must* declare all resources that you have used on this homework (include but not limited to anyone, any book, and any webpage). Do not skip steps.

You do NOT need to re-prove the theorems discussed in class. The idea of this HW is to understand and apply them!

1. Find the fundamental matrix of

$$\frac{dy}{dt} = Ay,$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -2 & 2 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}.$$

2. [B-N] In the Lemma of the estimate of $|e^{tA}|$ (aka, [B-N] Page 80 Thm 2.10), explain why $\rho = \max_{j=1, \dots, k} \operatorname{Re} \lambda$ does not work. (Hints: Recall the remark of this lemma in class!)
3. [B-N] Page 81 Problem 2.
4. [B-N] Page 83 Problem 5. (Hints: Estimate the right hand side of the equation using the Theorem for the estimate of non-homogeneous equation.)

5. Consider

$$\frac{dy}{dt} = Ay + g(t),$$

where

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -\pi & 0 & 0 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 4 & -1 \end{bmatrix}.$$

- (a) If $g(t) = 0$, show that

$$\lim_{t \rightarrow \infty} y(t) = 0.$$

- (b) Show that the above limit still holds if $|g(t)| = e^{-t}$.

