Homework 1

1. (Textbook Section 1.4-12) Let $A_1, A_2, \cdots$ be an arbitrary infinite sequence of events, and let $B_1, B_2, \cdots$ be another infinite sequence of events defined as follows: $B_1 = A_1, B_2 = A_1^c A_2, B_3 = A_1^c A_2^c A_3, B_4 = A_1^c A_2^c A_3^c A_4, \cdots$

Prove that

$$\Pr\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \Pr(B_i) \quad \text{for } n = 1, 2, \cdots$$

And that

$$\Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Pr(B_i)$$

(Hint: for the second part, you need to prove that $\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} B_i$ by showing that every element of $\bigcup_{i=1}^{\infty} A_i$ belongs to $\bigcup_{i=1}^{\infty} B_i$ and every element of $\bigcup_{i=1}^{\infty} B_i$ belongs to $\bigcup_{i=1}^{\infty} A_i$.)

2. (Textbook section 1.6-6) Suppose that n people are seated in a random manner in a row of n theater seats. What is the probability that two particular people Michael and Michelle will be seated next to each other?

3. (Textbook section 1.6-18) Suppose that 100 students are divided into five classes, each containing 20 students, and that awards are to be given to 10 of these students. If each student is equally likely to receive an award, what is the probability that exactly two students in each class will receive awards?

4. A package contains 10 different souvenirs about Beijing, 10 different souvenirs about Shanghai, 10 different souvenirs about Xi'an, 10 different souvenirs about Seoul and 10 different souvenirs about New York. Suppose that 10 souvenirs are randomly selected from the package.

(1) What is the probability that the selection contains no souvenirs about Seoul?

(2) What is the probability that the selection contains no souvenirs about Beijing, Shanghai or Xi'an?

(3) What is the probability that the selection misses souvenirs about at least one place?