Lecture 5: Consumer Choice and Preferences

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1. Put-Call Parity cont.
2. Consumer Choice: Preferences & Utility
3. Evaluating Risky Investments

Put-Call Parity continued

Claim:

Long call Price + long zero-coupon bond price = long stock price + long put price

C_0 = Call option price (A long call is an option to buy the stock at time t, at price K. A short call is an obligation to sell the stock at time t with price equal to K, unless the person who holds the corresponding long call declines to exercise their option)
X = K = Face value of zero-coupon bond (Face value has to equal strike of call and put)
S_0 = Stock price
P_0 = Put option price (A long put is an option to sell the stock at time t with price equal to K, a short put is an obligation to buy the stock at time t with price equal to K)
T = time
r_f = risk free interest rate

$$C_0 + K\frac{1}{(1+r_f)^T} = S_0 + P_0$$

or

$$C_0 + Ke^{-r_t * T} = S_0 + P_0$$

Call payoff: $max(S_t - K, 0)$
Put payoff: $max(K - S_t, 0)$

Long call payoff:
Notice the payoff can either be stock value at time t minus strike (K) or zero. One is not obligated to exercise a long call (You are obligated when you have a short call), so if the stock is below K you will let your call expire. When one buys a long call, they bet the price of the stock will go up.

Long Put payoff:
Notice the payoff can either be K minus stock at time t or zero. Once again you are not obligated to exercise your long put, so if the stock price is above K you will let your put expire. When one buys a put they bet the stock price will go down

Consider two portfolios
1) Buy a call and a ZCB with Face Value X at maturity T.
2) Buy a Stock and a Put Option

Payoff of a call: max(S_t - X, 0) where S_t is the stock at time T.
Payoff of a put: max(X - S_t, 0) where S_t is the stock at time T.

	Portfolio 1	Portfolio 2
Payoff	max(S_t - X,0) + X	max(X - S_t,0) + ST
Price	C_0 + (K/(1+$r_f)^t$)	S_0 + P_0

Portfolio 1 payoff
Case 1 $S_t > K$

$$Payoff = S_t - K + K = S_t$$

Case 2 $S_t \le K$

Payoff=K

Portfolio 2
Case 1 $S_t > K$

Payoff=S_t

Case 2 $S_t \le K$

$$K - S_t + S_t = K$$

Consumer Choice: Preferences & Utility

Main idea:
People choose the best things they can afford

Consumption Bundle (x_1, x_2):

- The set of goods a consumer can consume
- Given two consumption bundles (x_1, x_2) and (y_1, y_2), consumers can decide:
    1. If one is strictly better than the other
    2. If they are indifferent between the two

Notation for Preference Relations:

1. $\succ$: strict preference
    a. $(x_1, x_2) \succ (y_1, y_2) \rightarrow$ bundle (x_1, x_2) is strictly preferred over bundle (y_1, y_2)
2. ~ : indifference
    a. $(x_1, x_2) \sim (y_1, y_2) \rightarrow$ the consumer is indifferent between both bundles
3. $\succsim$: weak preference
    a. $(x_1, x_2) \succsim (y_1, y_2) \rightarrow$ bundle (x_1, x_2) is strictly preferred over bundle (y_1, y_2) or the consumer is indifferent between the two

Given a consumption set $X \subseteq R_+^n$, a preference relation $\succsim$ is rational if:

1. It is complete for all $x, y \in X$
    a. Either $x \succsim y$ or $y \succsim x$ or both
2. Is is transitive for all $x, y, z \in X$
    a. If $x \succsim y$ and $y \succsim z$ then $x \succsim z$
    b. Some examples where this doesn't hold
        i. Ex. may prefer an orange to an apple, and an apple to a banana, but can prefer a banana over an orange (whereas transitivity would say the person would prefer the orange to the banana)

Preference relation $\succsim$ over x is continuous if it is preserved under limits for any sequence of pairs $\{x^n, y^n\}_{n=1}^{\infty}$ such $x^n \succsim y^n$ for all n, and $x = \lim_{n\to\infty} x^n$, $y = \lim_{n\to\infty} y^n$, then $x \succsim y$.

- Another characteristic of continuity:

  $\succsim$ over x is continuous if for all $x, y, z \in X$ such $x \succsim y \succsim z$ then there exists an $\alpha \in [0, 1]$ such that $y \sim \alpha x + (1 - \alpha)z$

Lexicographic preferences:

- Assume $x, y \in R_+^n$, $x \succsim y$ if
    1. $x_1 > y_1$ or
    2. $x_1 = y_1$ and $x_1 \geq y_1$

- Consider a sequence of bundles
  $x^n = (\frac{1}{n}, 0) \rightarrow$ i.e. $x^1 = (1, 0)$, $x^2 = (\frac{1}{2}, 0)$, … so $\lim_{n\to\infty} x^n = (0,0)$
  $y^n = (0, 1), \forall\, n$ so $\lim_{n\to\infty} y^n = (0,1)$
  For every n, $x^n \succsim y^n$, but $\lim_{n\to\infty} y^n \succsim \lim_{n\to\infty} x^n$
  → Lexicographic preferences are complete and transitive, but not continuous

<u>Evaluating Risky Investments</u>

	Asset 1	Asset 2	Asset 3	Probability
State 1	1050	500	1050	0.5
State 2	1200	1600	1600	0.5
Price	1000	1000	1000	

Notice any rational investor would pick asset 3, because it is state-by-state dominant

table of returns below

	Asset 1	Asset 2	Asset 3	Probability
State 1	5%	-50%	5%	0.5
State 2	20%	60%	60%	0.5

$E[r_1]=12.5\%$, $E[r_2]=5\%$, $E[r_3]=32.5\%$
$\sigma_1 = 7.5\%$, $\sigma_2 = 55\%$, $\sigma_3 = 27.5\%$

Notice that the expected return of asset one is greater than asset two.

Furthermore, the variance of asset one is lower than asset two. This implies asset one mean-variance dominates asset two. Now notice that the expected return of asset three is greater than asset one. Asset three also has a larger variance, so someone who is risk averse will pick asset one; however, as we saw earlier asset three is state by state dominant. The reason someone might pick asset one is because the information in the first table is not always available.

Typically, one tries to maximize expected return and minimize variance.

William Sharpe, Nobel Prize 1990

- Suggested using Sharpe ratio $S = \frac{E[r_i]}{\sigma_i}$

- Measures expected return per unit of risk. Key assumption is that expected return and risk are weighted equally.