Homework 8

October 26, 2011

1. Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a normal distribution for which both the mean and the variance are unknown. Find the M.L.E. of the 0.95 quantile of the distribution, that is, of the point θ such that $\Pr(X < \theta) = 0.95$.

2. Let $X_1, X_2, \ldots, X_n$ represent a random sample from each of the distributions having the following probability density functions: $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1, 0 < \theta < \infty$, zero elsewhere. Show that the m.l.e. $\widehat{\theta}$ of θ is consistent.

3. Suppose that a random sample is to be taken from a normal distribution for which the value of the mean θ is unknown and the standard deviation is 2.

   (a) How large a random sample must be taken in order that $E(|\overline{X}_n - \theta|^2) \leq 0.1$ for every possible value of θ?

   (b) How large a random sample must be taken in order that $E(|\overline{X}_n - \theta|) \leq 0.1$ for every possible value of θ?

(c) How large a random sample must be taken in order that $Pr(|\overline{X}_n - \theta| \leq 0.1) \geq 0.95$ for every possible value of θ?

4. (Textbook Section 7.7-10, Page 284) Suppose that a certain drug is to be administered to two different types of animals A and B. It is known that the mean response of animals of type A is the same as the mean response of animals of type B, but the common value θ of this mean is unknown and must be estimated. It is also known that the variance of the response of animals of type A if four times as large as the variance of the response of animals of type B. Let $X_1, \cdots, X_m$ denote the responses of a random sample of m animals of type A, and let $Y_1, \cdots, Y_n$ denote the responses of an independent random sample of n animals of type B. Finally, consider the estimator $\hat{\theta} = \alpha\overline{X}_m + (1-\alpha)\overline{Y}_n$.

   (a) For what values of α, m and n is $\hat{\theta}$ an unbiased estimator of θ?

   (b) For fixed values of m and n, what value of α yields an unbiased estimator with minimum variance?

5. (Textbook Section 7.4-2, Page 270) Suppose that $X_1, \cdots, X_n$ form a random sample from a normal distribution for which the mean μ and the standard deviation σ are unknown, and let $\hat{\mu}$ and $\hat{\sigma}$ denote the M.L.E.'s of μ and σ. For the sample size $n = 17$, find a value of k such that $Pr(\hat{\mu} > \mu + k\hat{\sigma}) = 0.95$.

6. (Textbook Section 7.4-3, Page 270) Suppose that the five random variables $X_1, \cdots, X_5$ are i.i.d., and each has a standard normal distribution. Determine a constant c such

that the random variable

$$\frac{c\left(X_1 + X_2\right)}{\left(X_3^2 + X_4^2 + X_5^2\right)^{1/2}}$$

will have a t distribution.