Homework Assignment #9 and #10
Due: In class, two weeks after distribution

§1 Option Pricing under Bachelier's Arithmetic Brownian Motion Model

Louis Jean-Baptiste Alphonse Bachelier (March 11, 1870 – April 28, 1946) was a French mathematician at the turn of the 20th century. He is credited with being the first person to model the stochastic process now called Brownian motion, which was part of his PhD thesis The Theory of Speculation, (published 1900). His thesis, which discussed the use of Brownian motion to evaluate stock options, is historically the first paper to use advanced mathematics in the study of finance. Thus, Bachelier is considered a pioneer in the study of financial engineering and stochastic processes.

In this exercise, we appreciate his work. Suppose the underlying stock price follows the arithmetic Brownian Motion, i.e. a Brownian motion with drift

$$dS(t) = \mu dt + \sigma dW(t), S(0) = S_0.$$

We note that this model is not realistic but may serve as an example to set up the foundation of the modern option pricing theory.

- Derive the PDE and its terminal condition for pricing a put option with payoff $(K - x)^+$, where K is the strike;
- What is the strategy for hedging a short position of such a put option? Please find the "Greeks" explicitly.
- Apply the Feynman-Kac theorem to express the initial option price as a "risk-neutral" expectation of the discounted payoff.
- Based on the previous result, derive an explicit formula for pricing this call option.
- (Bonus question) Can you solve this PDE using analytical methods directly? Note that your result should be the same as that from the second question.

§2 Shreve Vol II. Exercise. 4.12 (i) and (ii)

§3 Shreve Vol II. Exercise. 4.11

§4 Show that the implied volatility calculated from call and put options are the same.