

HW7 (due Nov. 7th)

Instructions. You *must* declare all resources that you have used on this homework (include but not limited to anyone, any book, and any webpage). Do not skip steps.

You are required to sketch the portrait BY HAND, not by computer graphing softwares.

1. Sketch the phase portrait of the following differential equation

$$\frac{dy}{dt} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} y + \begin{bmatrix} -6 \\ 24 \end{bmatrix}.$$

2. Sketch the phase portrait

$$\frac{dy}{dt} = \begin{bmatrix} 0 & 6 \\ -7 & 0 \end{bmatrix} y + \begin{bmatrix} 3 \\ 7 \end{bmatrix}.$$

3. Sketch the phase portrait

$$\begin{aligned} \frac{dy_1}{dt} &= y_1 - 2y_2 - 1 \\ \frac{dy_2}{dt} &= 2y_1 - 3y_2 - 3. \end{aligned}$$

4. (a) Find the (Floquet) multiplier of the following differential equation

$$\frac{dy}{dt} = -(\cos^2 t) y.$$

- (b) What is the asymptotic behavior of its solution, i.e. $\lim_{t \rightarrow \infty} y(t)$?

(Hints: This equation is solvable! Solve this differential equation, and find the smallest period of the coefficients in the equation.)

5. [B-N] Page 99 Problem 3. (*The proof should be around one line using the corollary listed in the question (or equivalently, Cor1 in lecture).*)
6. [B-N] Page 99 Problem 4.