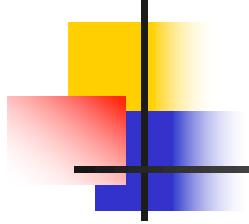




Investments

Lecture 6



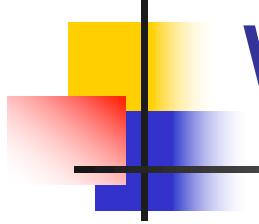
Review

$$\sigma_{11} \quad \sigma_{21} \quad \sigma_{31}$$

$$V = \sigma_{12} \quad \sigma_{22} \quad \sigma_{32}$$

$$\sigma_{13} \quad \sigma_{23} \quad \sigma_{33}$$

$$[r_1 \ r_2 \ r_3]$$



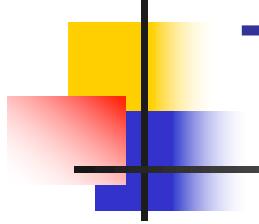
With riskfree asset

$$r_1 - rf = Z_1\sigma_{11} + Z_2\sigma_{12} + Z_3\sigma_{13}$$

$$r_2 - rf = Z_1\sigma_{21} + Z_2\sigma_{22} + Z_3\sigma_{23}$$

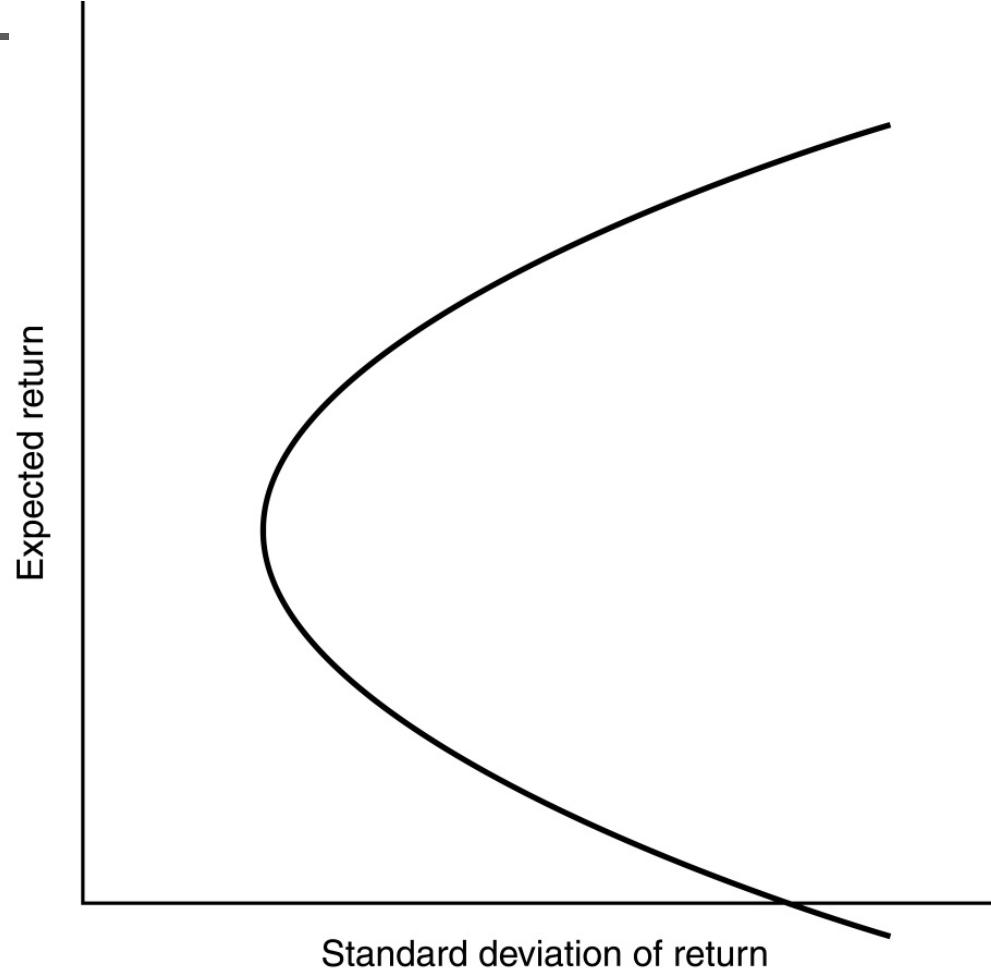
$$r_3 - rf = Z_1\sigma_{31} + Z_2\sigma_{32} + Z_3\sigma_{33}$$

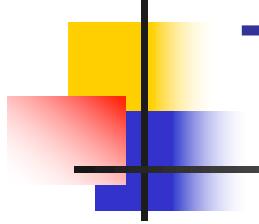
$$w_1 = \frac{Z_1}{Z_1 + Z_2 + Z_3}$$



The two-fund theorem

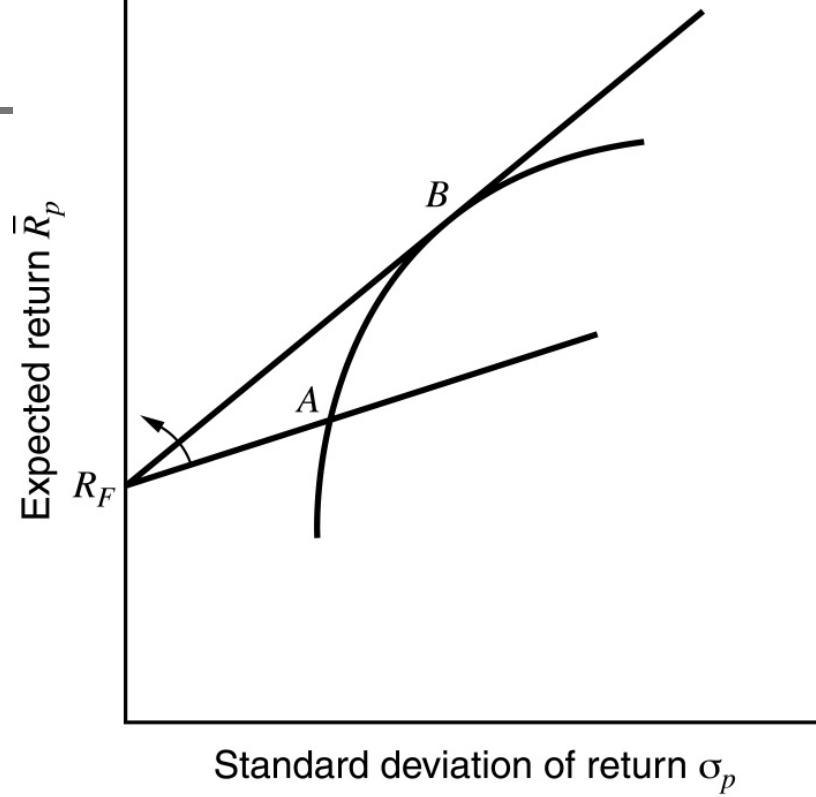
- Two efficient funds (portfolios) can be established so that any efficient portfolio can be duplicated, in terms of mean and variance, as a combination of these two. In other words, all investors seeking efficient portfolios need only invest in combinations of these two funds

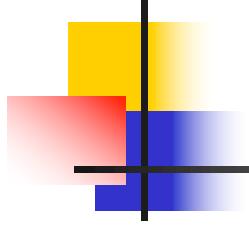




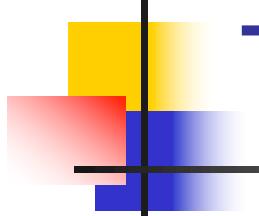
The one-fund theorem

- There is a single fund F of risky assets such that any efficient portfolio can be constructed as a combination of the fund F and the risk-free asset



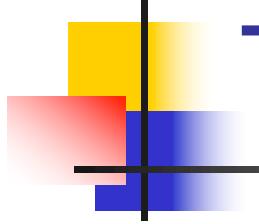


THE CAPM



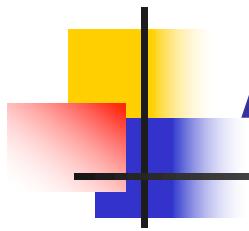
The CAPM

- Capital asset pricing model
- Sharpe, Lintner and Mossin: 1990 Nobel Prize



The CAPM

- The Capital Asset Pricing Model (CAPM) is a model that relates the expected return of a security to its systematic risk as measured by beta
 - What is the expected return of a stock?
 - Why is the expected return of gold so low?
 - Should a company undertake a risky project?

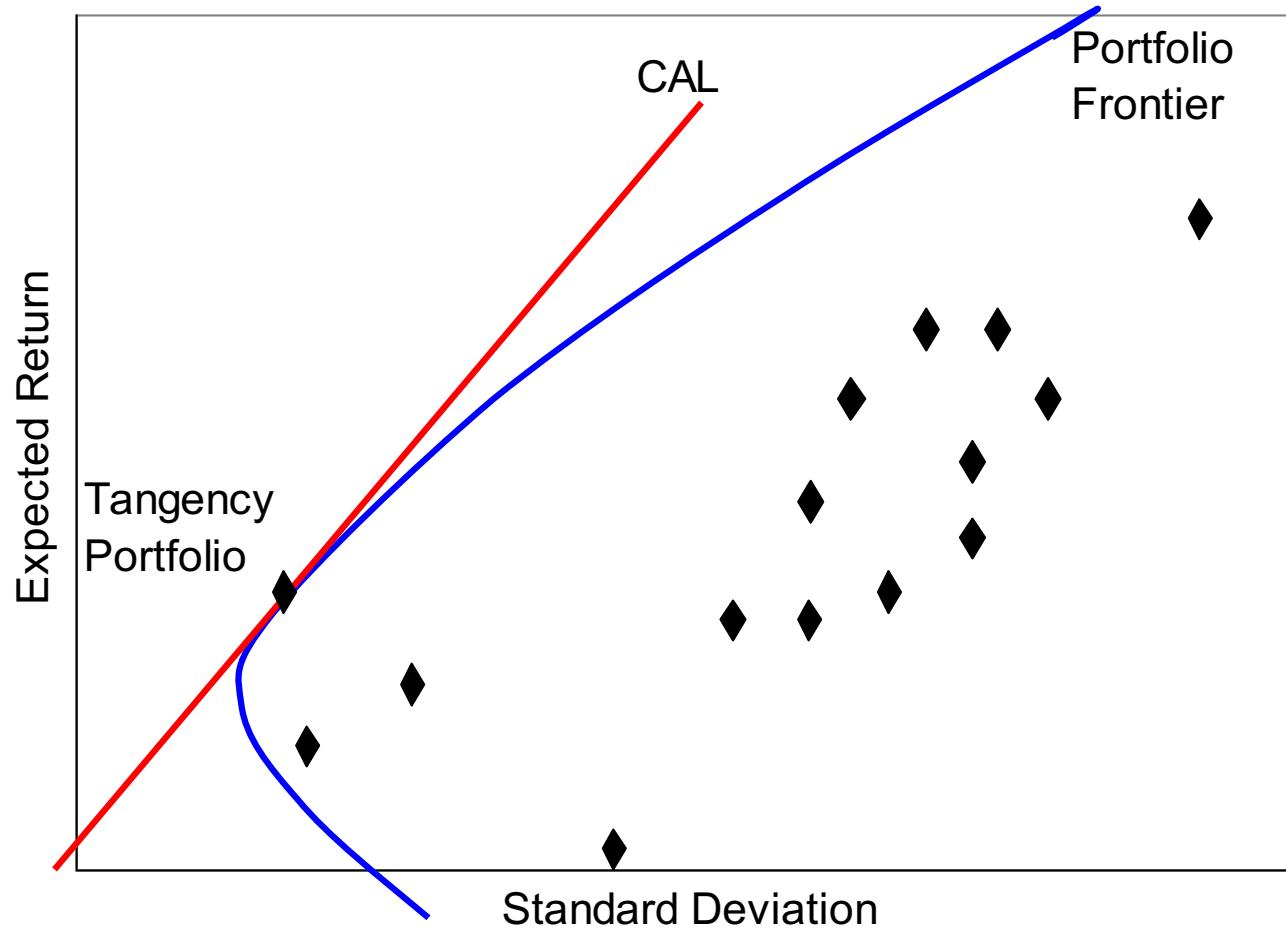


Assumptions of the CAPM

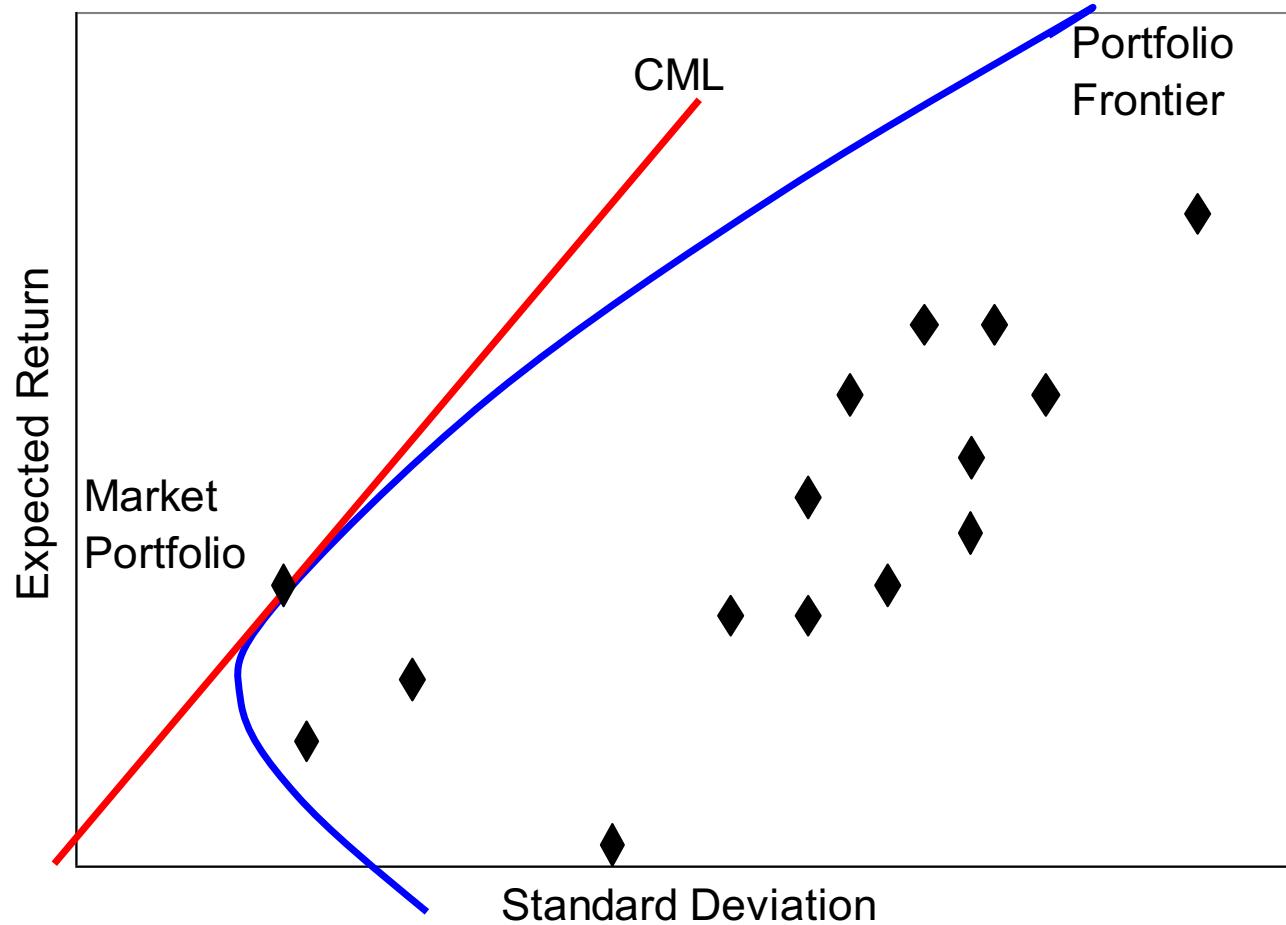
- Investors:

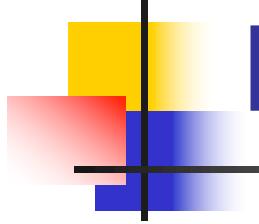
- are rational and use portfolio theory to form portfolios
- have homogenous expectations
- can buy and short-sell any asset
- cannot affect security prices
- have the same investment horizon
- pay no taxes and transaction costs

Portfolio Theory



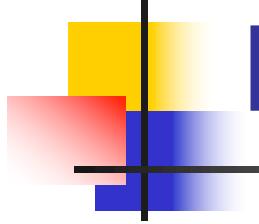
CAPM





Market Portfolio

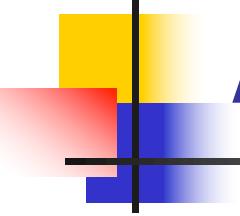
- One fund in the theorem is the ONLY fund used
- The efficient portfolio of risky assets is the market portfolio
- Equilibrium: we don't need to calculate the parameters



Market Portfolio

- All investors decide to hold the same portfolio of risky assets
- In equilibrium, this tangency portfolio is the market portfolio
 - The market portfolio includes all assets
 - The weight of each asset equals:

$$\text{Weight} = \frac{\text{Market Value of Asset}}{\text{Market Value of All Assets}}$$



A simple derivation

- * CAPM assumptions: All investors hold the market portfolio + MPT
- > if the observed market value share of Asset i is w_i , a MPT investor's optimal weight on Asset i should coincide with w_i
- * Goal: calculate $E[R_i]$ such that investors happy to hold w_i

$$\bar{R}_i - R_f = \lambda(w_1\sigma_{1i} + w_2\sigma_{2i} + \dots + w_n\sigma_{ni}) \quad (1) <- \text{MPT}$$

$$R_M = w_1R_1 + w_2R_2 + \dots + w_nR_n \quad (2) <- \text{Definition}$$

$$\text{cov}(R_M, R_i) = w_1\sigma_{1i} + w_2\sigma_{2i} + \dots + w_n\sigma_{ni} \quad (3) <- \text{Definition}$$

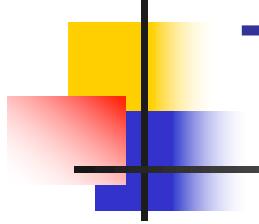
$$\bar{R}_i - R_f = \lambda \text{cov}(R_M, R_i) \quad (4) <-(1)+(3)$$

$$\bar{R}_M - R_f = \lambda \text{cov}(R_M, R_M) \quad (5) <- \text{sum up (4)}$$

$$\mathcal{O}(\epsilon)$$

$$| \phi_{\alpha} \rangle$$

$$\mathcal{O}(n^{\frac{1}{2}})$$



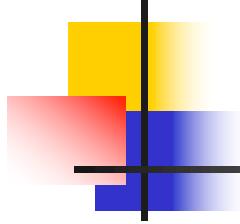
The CAPM Model

- The expected return of an asset is:

$$E(r_i) = r_F + \beta_i [E(r_M) - r_F]$$

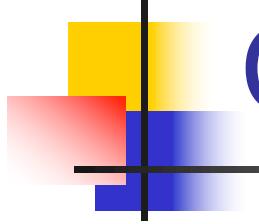
- The expected return depends linearly on the systematic risk of the asset

$$\beta_i = \frac{Cov(r_M, r_i)}{Var(r_M)}$$



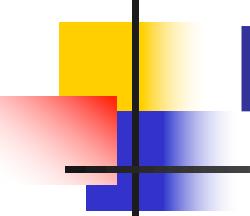
Capital Market Line

- Individuals choose any portfolio on the Capital Market Line (CML)
 - The CML connects the risk-free rate with the market portfolio
 - Risk-averse investors hold a larger portion of their assets in the risk-free asset and a smaller portion in the market portfolio
 - Risk-tolerant investors hold a smaller portion of their assets in the risk-free asset and a larger portion in the market portfolio



CAL and CML

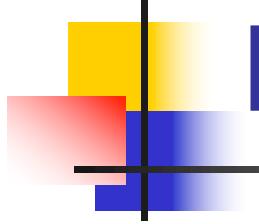
- There are infinitely many CALs
 - Given a risky asset (portfolio) A and the risk-free asset, all combinations of portfolio A and the risk-free asset lie along the CAL for portfolio A.
 - In MPT, the optimal CAL is the CAL for the tangency portfolio.
- In CAPM, CML is the CAL for the market portfolio.



Expected Returns of the Market Portfolio

- The variance of the market portfolio is the systematic risk σ_M^2
- The risk premium is the difference between the expected return and the risk-free rate
- The risk premium of the market portfolio depends on the risk-aversion of the average investor (A) and on the variance of the market:

$$E(r_M) - r_F = A \times \sigma_M^2$$



Example

- What is the expected return of the market if:
 - The risk-aversion is $A=2$
 - The standard deviation of the market is $\sigma_M=20\%$
 - The risk-free interest rate is $r_F=1\%$?

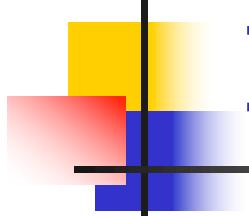
Riskiness of Individual Securities

- How much does the risk of the market portfolio change if we add an additional security?

$$Var(r_M) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j Cov(r_i, r_j)$$

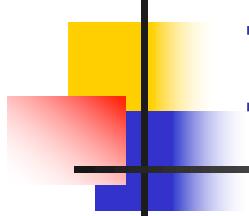
$$Var(r_M) = \sum_{i=1}^N w_i \left[\sum_{j=1}^N Cov(r_i, w_j \times r_j) \right]$$

$$Var(r_M) = \sum_{i=1}^N w_i Cov(r_i, r_M)$$



Riskiness of Individual Securities

- The risk of the market portfolio increases by the covariance of the new asset with the market
- Note that the covariance measures the contribution of an asset to the total risk of a portfolio

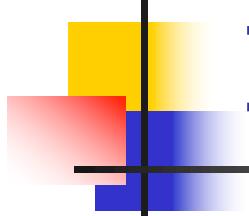


Expected Return of Individual Securities

- In equilibrium, the reward to risk ratio has to be the same for all assets:

$$\frac{E(r_M) - r_F}{Var(r_M)} = \frac{E(r_i) - r_F}{Cov(r_M, r_i)}$$

- Reshuffling terms gives the CAPM!



Expected Returns of Individual Securities

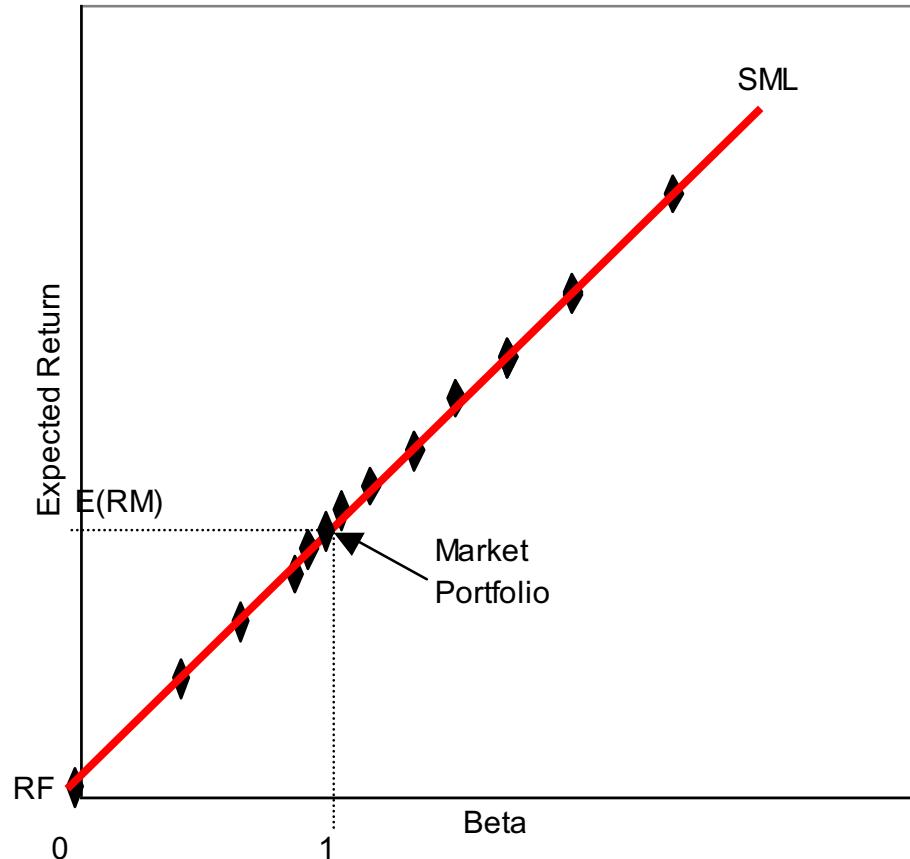
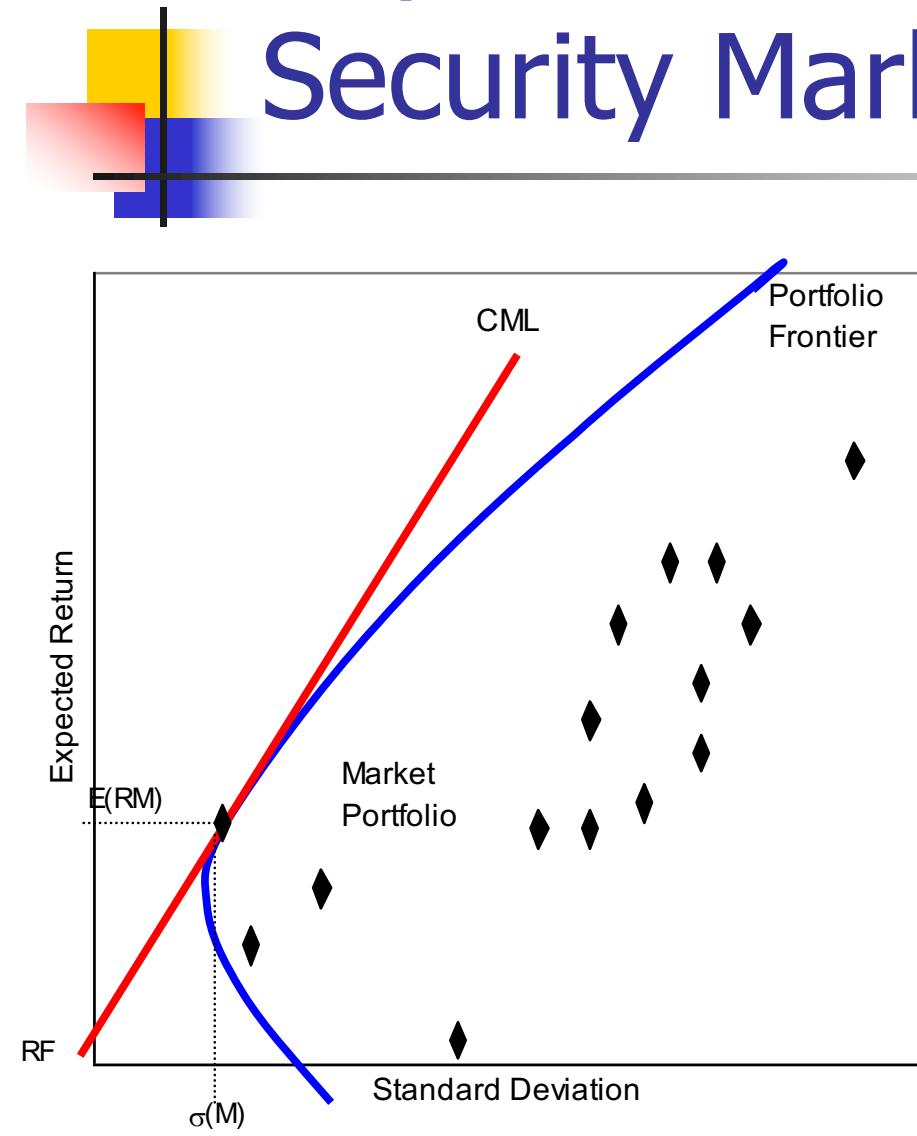
- The CAPM determines the expected return of a stock

$$E(r_i) = r_F + \beta_i [E(r_M) - r_F]$$

- The expected return depends linearly on the systematic risk of the asset

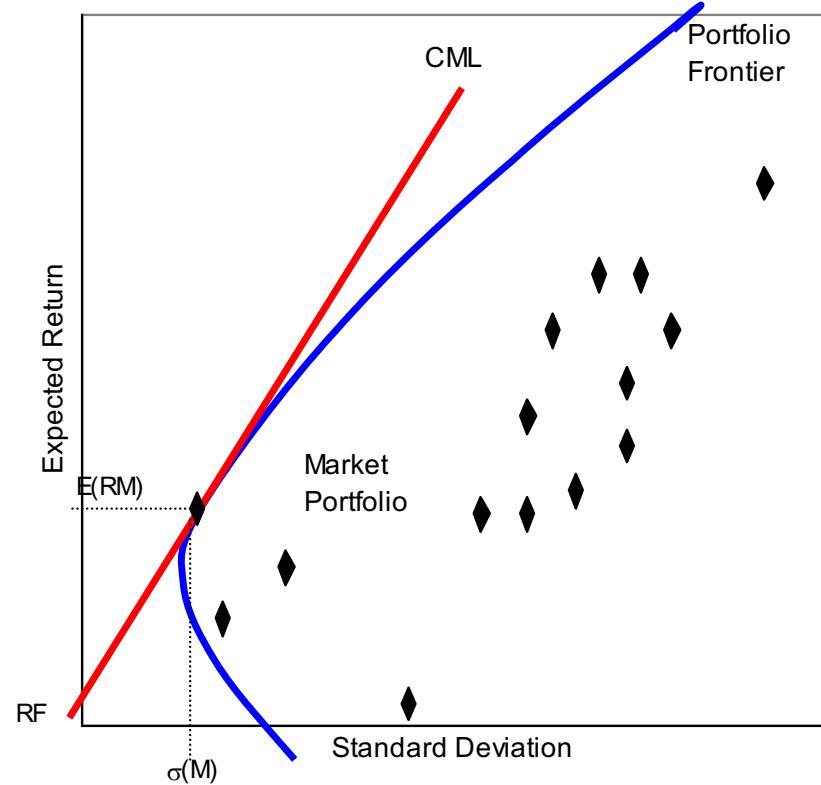
$$\beta_i = \frac{Cov(r_M, r_i)}{Var(r_M)}$$

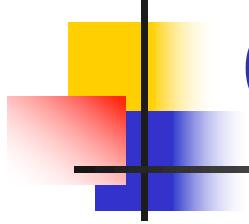
Capital Market Line and Security Market Line



Capital Market Line I

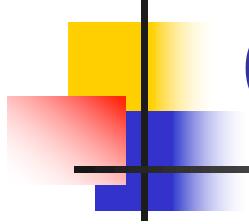
$$\begin{aligned}\bar{R}_e &= R_f + \frac{\sigma_{eM}}{\sigma_M^2} (\bar{R}_M - R_f) \\ &= R_f + \frac{\sigma_e}{\sigma_M} (\bar{R}_M - R_f) \\ &= R_f + \sigma_e \frac{\bar{R}_M - R_f}{\sigma_M}\end{aligned}$$





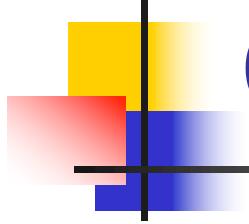
Capital Market Line II

- In equilibrium, every investor faces the same capital allocation line.
- CAL gives us the set of efficient or optimal risk-return combinations.
 - What is an efficient portfolio?



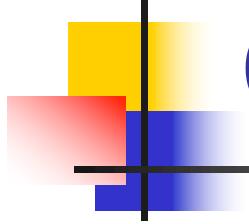
Capital Market Line III

- Note that this says that all investors should only hold combinations of the market and the risk-free asset.
 - Young and Old?
 - How does this relate to the increased popularity of index funds ?



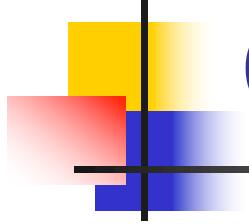
Capital Market Line IV

- Now let's ask the question of what we can say about inefficient portfolios (or individual assets). Can we say anything about their expected returns and standard deviations in equilibrium?



Capital Market Line IV

- We will see that we can, because investors will only want to hold a security in their portfolio if it provides a reasonable amount of extra reward (*i.e.*, expected return) in return for the risk (or variance) it adds to the portfolio



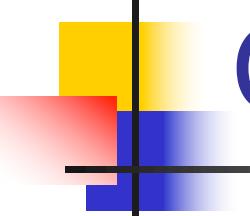
Capital Market Line V

- How much does the risk of the market portfolio change if we add an additional security?

$$Var(r_M) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j Cov(r_i, r_j)$$

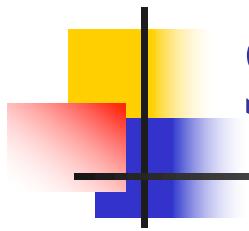
$$Var(r_M) = \sum_{i=1}^N w_i \left[\sum_{j=1}^N Cov(r_i, w_j \times r_j) \right]$$

$$Var(r_M) = \sum_{i=1}^N w_i Cov(r_i, r_M)$$



Capital Market Line VI

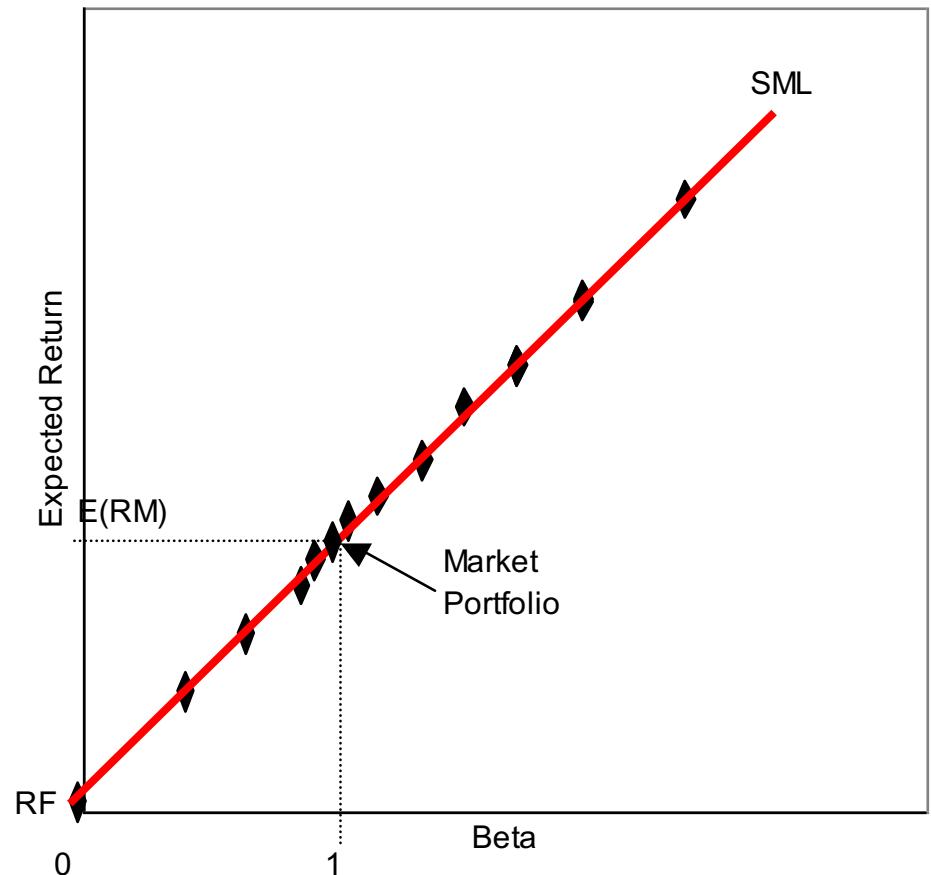
- The main result is that, for all securities, what a security adds to the risk of a portfolio will be just offset by what it adds in terms of expected return. The ratio of ***marginal return to marginal variance*** must be the same for all assets.
 - We see that what it adds in expected return is its expected excess return
 - What it adds in risk is proportional to its covariance with the portfolio.

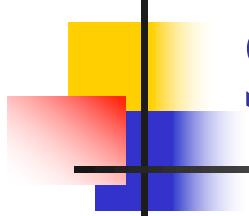


Security Market Line I

$$E(r_i) = r_F + \beta_i [E(r_M) - r_F]$$

$$\beta_i = \frac{\text{Cov}(r_M, r_i)}{\text{Var}(r_M)}$$





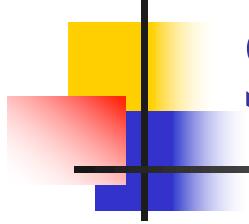
Security Market Line II

- The expected return/beta relationship comes from the evaluation of the marginal tradeoff.

$$Var(r_M) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j Cov(r_i, r_j)$$

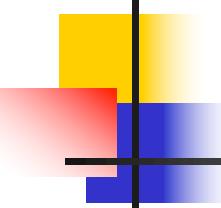
$$Var(r_M) = \sum_{i=1}^N w_i \left[\sum_{j=1}^N Cov(r_i, w_j \times r_j) \right]$$

$$Var(r_M) = \sum_{i=1}^N w_i Cov(r_i, r_M)$$



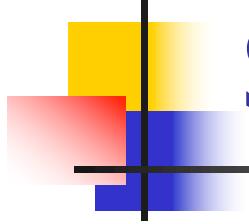
Security Market Line III

- What matters in determining the marginal increase in risk when you change the amount of a security in your portfolio is the covariance with the portfolio return. This is the intuition for why the covariance and not the variance matters



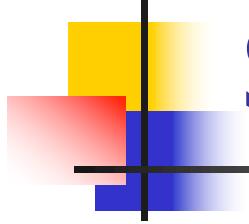
Security Market Line IV

- First, remember that we showed that, under our assumptions, all investors must hold the market portfolio.
- Second, based on our notion of equilibrium, every investor must be content with their portfolio holdings; if this were not the case then the prices of the securities would have to change.
 - This is just a supply and demand argument; if some investors want to buy IBM, and no one wants to sell, prices will have to change (move up).
 - In equilibrium, everyone must be optimally invested.
- This must mean that, in equilibrium no one can do anything to increase the Sharpe-ratio of their portfolio.



Security Market Line V

- Suppose you hold the market portfolio of risky assets. Suppose you decide to invest a small additional fraction δ_{GM} of your wealth in GM, which you finance by borrowing at the risk free rate. Let's figure out how the return and variance of your portfolio change



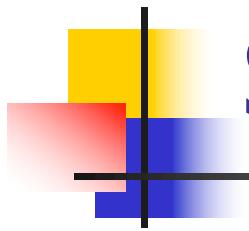
Security Market Line VI

- The return becomes:

$$\tilde{r}_c = \tilde{r}_m - \delta_{GM} r_f + \delta_{GM} \tilde{r}_{GM}$$

- So the expected return is

$$E[\tilde{r}_c] = E[\tilde{r}_m] + \delta_{GM} (E[\tilde{r}_{GM}] - r_f)$$



Security Market Line VII

- The variance is

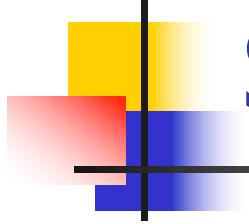
$$\sigma_c^2 = \sigma_m^2 + \delta_{GM}^2 \sigma_{GM}^2 + 2\delta_{GM} \text{cov}(\tilde{r}_m, \tilde{r}_{GM})$$

- Take the changes

$$\Delta E[\tilde{r}_c] = \delta_{GM} (E[\tilde{r}_{GM}] - r_f)$$

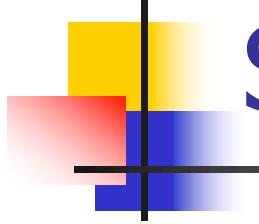
$$\Delta \sigma_c^2 = \delta_{GM}^2 \sigma_{GM}^2 + 2\delta_{GM} \text{cov}(\tilde{r}_m, \tilde{r}_{GM})$$

$$\approx 2\delta_{GM} \text{cov}(\tilde{r}_m, \tilde{r}_{GM})$$



Security Market Line VIII

- Now what if we invest δ more in GM, and invest just enough less in the IBM so that our portfolio variance stays the same. What will happen to your return?



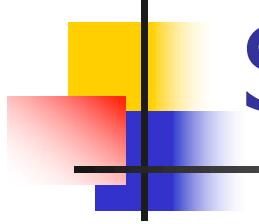
Security Market Line IX

- The change of variance is:

$$\Delta\sigma_c^2 = 2[\delta_{GM} \operatorname{cov}(\tilde{r}_m, \tilde{r}_{GM}) + \delta_{IBM} \operatorname{cov}(\tilde{r}_m, \tilde{r}_{IBM})]$$

- To make it zero, we have:

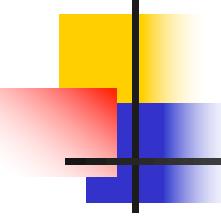
$$\delta_{IBM} = -\delta_{GM} \frac{\operatorname{cov}(\tilde{r}_m, \tilde{r}_{GM})}{\operatorname{cov}(\tilde{r}_m, \tilde{r}_{IBM})}$$



Security Market Line X

- This means the change of the expected return of the portfolio is:

$$\begin{aligned}\Delta E[\tilde{r}_c] &= \delta_{GM}(E[\tilde{r}_{GM}] - r_f) + \delta_{IBM}(E[\tilde{r}_{IBM}] - r_f) \\ &= \delta_{GM}(E[\tilde{r}_{GM}] - r_f) - \delta_{GM} \frac{\text{cov}(\tilde{r}_m, \tilde{r}_{GM})}{\text{cov}(\tilde{r}_m, \tilde{r}_{IBM})}(E[\tilde{r}_{IBM}] - r_f) \\ &= \delta_{GM}[(E[\tilde{r}_{GM}] - r_f) - \frac{\text{cov}(\tilde{r}_m, \tilde{r}_{GM})}{\text{cov}(\tilde{r}_m, \tilde{r}_{IBM})}(E[\tilde{r}_{IBM}] - r_f)]\end{aligned}$$

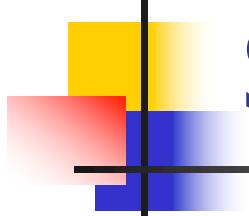


Security Market Line XI

- In equilibrium, the change must be zero

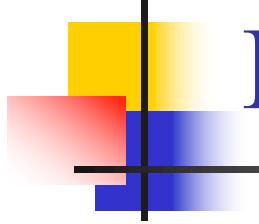
$$\frac{E(r_{GM}) - r_f}{\text{cov}(r_m, r_{GM})} = \frac{E(r_{IBM}) - r_f}{\text{cov}(r_m, r_{IBM})} = \lambda$$

- To figure out what this ratio of marginal return to marginal variance is, note that this argument holds for all assets, as well as for all portfolios. Hence we can use the market portfolio in place of IBM



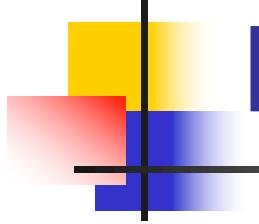
Capital Market Line and Security Market Line

- The Capital Market Line (CML) shows the combination of expected returns and *standard deviations* that are attainable
 - Individual assets are below the CML
- The Security Market Line (SML) shows the combination of expected returns and *betas*
 - Individual assets are on the SML if the CAPM is correct



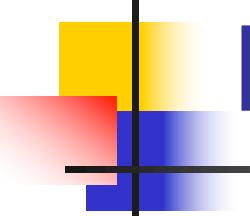
BETA

- The beta is a measure of the systematic risk of an asset
- The CAPM says that only systematic risk is priced
- Unique risk is not priced because it can be diversified away
- How do we compute betas?



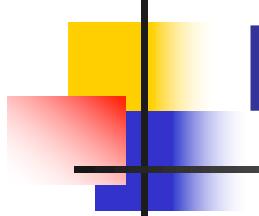
Recipe for Calculating Expected Stock Returns

- Using historical data, calculate
 - Market risk premium
 - Stock Beta
- Get the current riskless rate
- Use the CAPM to calculate expected return.



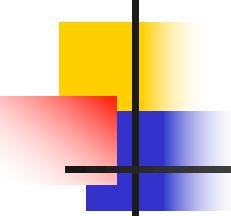
Measuring Beta

- In the Market Model, Beta is the slope from the relationship between stock returns and market returns.
- You can measure Beta using regression.
 - Get price data for a stock and an index.
 - Create returns from the prices.
 - Regress stock returns on index returns.
 - The slope of this relationship is the stock's Beta.



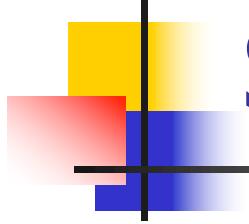
Data sources

- CSMAR
 - <http://cn.gtadata.com/Home>
- WRDS (CRSP)
 - <https://wrds-web.wharton.upenn.edu/wrds/?register=1>
- Datastream and Bloomberg terminals



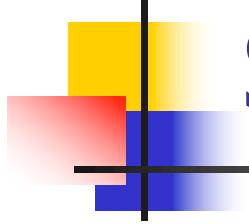
High Frequency Data

- Theoretically, by using high frequency data we can get more precise estimates of factor loadings:
 - E.g. Use daily returns to estimate betas, rather than monthly returns.
- Practically, problems arise when we use daily data:
 - E.g. A stock may not trade every day, in which case the market may move and not appear to affect the stock. If the stock does trade, the change in price may reflect information released prior to the trading day.



Solution: Sum Betas

- One potential solution to the problem is to regress daily returns on “leads and lags” of the index returns.
 - Lags will be important when the stock is less liquid than the market and information is incorporated into its price with a delay.
 - Leads will be important when the stock is more liquid than the market and the information appears to affect the market with a delay.
- The sum of the coefficients provides an estimate of the true factor loading.



Sum Betas

- Approach:
 - Collect daily returns for the stock and the index from the past year.
 - Regress daily stock returns on FIVE leads and lags of the index.
 - Add all the coefficients to produce an estimate of the factor loading.

Example from Ibbotson Beta Book

Beta Book Example

Ticker
PG

Company
PROCTER & GAMBLE CO

CAPM: Ordinary Least Squares

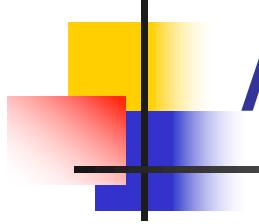
Levered					Unlevered	
OLS Beta	t-Stat	R-Sqr	Pr. Grp Beta	Adj. Beta	OLS Beta	Adj Beta
0.49	1.96	0.06	0.77	0.50	0.44	0.45

CAPM: Sum Beta (Including Lag)

Levered					Unlevered	
SUM Beta	t-Stat	R-Sqr	Pr. Grp Beta	Adj Beta	SUM Beta	Adj Beta
0.73	2.84	0.07	0.82	0.73	0.65	0.65

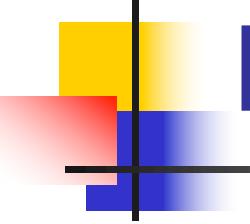
Fama-French Three-Factor Model

FF Beta	FF t-Stat	SMB Prem	SMB t-Stat	HML Prem	HML t-Stat	FF R-Sqr
0.53	1.92	-0.38	-1.09	0.72	2.52	0.09



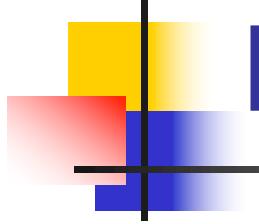
Assignment

- Estimate the standard betas of the two listed firms.
- Estimate the sum betas with lag 2
- Estimate the sum betas with lead 2
- Estimate the sum betas with lag2 and lead2.



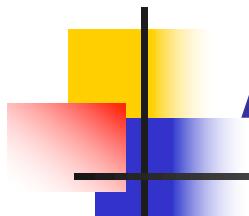
Example

- The risk-free rate is 5% while the market portfolio's expected return is 12%. If the standard deviation of the market portfolio is 13%, what is the equilibrium expected return of a security that has a covariance with the market of 186?



Example

- What is the expected return of MSFT stock and gold?
 - The betas are $\beta_{MSFT}=1.36$ and $\beta_{Gold}=-0.05$
 - The expected return of the market is $E(r_M)=10\%$
 - The risk-free interest rate is $r_F=1\%$

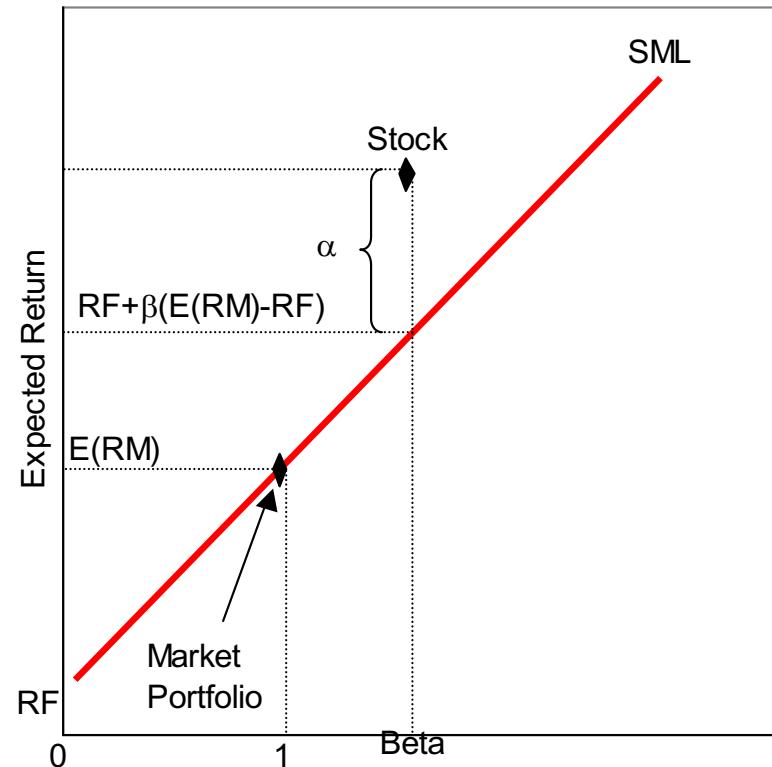


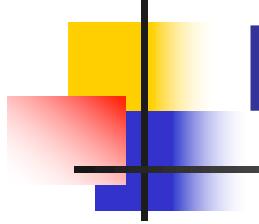
Applications of the CAPM (1)

- Investment management industry
 - Security analysts compare their individual return expectations with the *fair* expected return from the CAPM model
 - Jensen's index (alpha)
 - A stock's alpha denotes the abnormal expected return of a security in excess of the expected return predicted by the CAPM model
 - Stocks with high alphas are under-valued and should be acquired in a portfolio

- The alpha of a security is defined as the excess performance above the CAPM-return

$$\alpha = E(r_s) - [r_F + \beta(E(r_M) - r_F)]$$

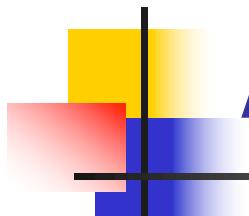




Beta of a Portfolio

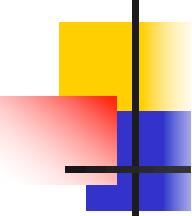
- The beta of a portfolio is simply the weighted average of the betas of the assets
- The beta of a portfolio of assets A and B with weights w_A and w_B is:

$$\begin{aligned}\beta_P &= \frac{Cov(r_M, w_A r_A + w_B r_B)}{Var(r_M)} \\ &= w_A \frac{Cov(r_M, r_A)}{Var(r_M)} + w_B \frac{Cov(r_M, r_B)}{Var(r_M)} = w_A \beta_A + w_B \beta_B\end{aligned}$$



An example: ABC fund

- The ABC mutual fund has the 10-year record of rates of return shown in the table. We would like to evaluate this fund's performance in terms of mean-variance portfolio theory and the CAPM. Is it a good fund that we could recommend?



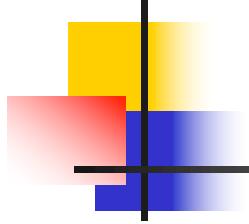
ABC Fund Performance

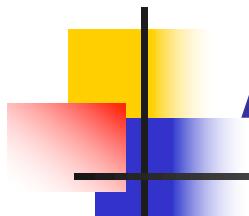
Year	Rate of return percentages		
	ABC	S&P	T-bills
1	14	12	7
2	10	7	7.5
3	19	20	7.7
4	-8	-2	7.5
5	23	12	8.5
6	28	23	8
7	20	17	7.3
8	14	20	7
9	-9	-5	7.5
10	19	16	8

- Step 1: calculate mean, std of ABC
- Step 2: means, std. of T-bill SP500 and covariance

ABC Fund Performance

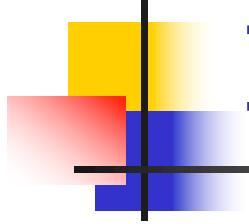
Year	Rate of return percentages		
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1	14	12	7
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5	23	12	8.5
6	28	23	8
7	20	17	7.3
8	14	20	7
9	-9	-5	7.5
10	19	16	8
Average	13	12	7.6
Standard deviation	12.4	9.4	.5
Geometric mean	12.3	11.6	7.6
Cov(ABC, S&P)	0107		

- 
- Step 3: Beta
 - Step 4: Jensen index (or alpha)
 - (Optional: Sharpe ratio, Treynor measure)

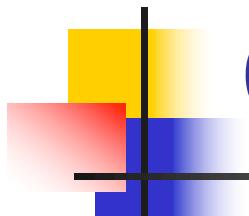


Applications of the CAPM (2)

- Capital Budgeting Decisions
 - The CAPM provides the return a project needs to yield to be acceptable to investors

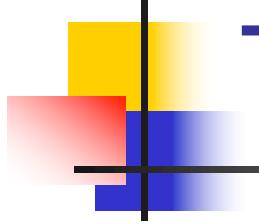


Index Models



CAPM and Index Models

- The CAPM relies on the theoretical market portfolio, which includes all assets (real estate, human capital, etc.)
 - An index model uses actual portfolios, such as the S&P 500 or the Wilshire 5000 Index to represent systematic factors
- The CAPM predicts relationships among expected returns, which are not observed
 - An index model uses historically realized returns



The CAPM and Index Models

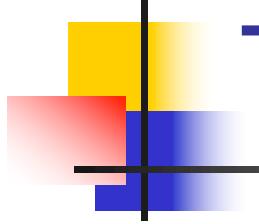
- The CAPM states:

$$E(r_i) - r_F = \beta_i [E(r_M) - r_F]$$

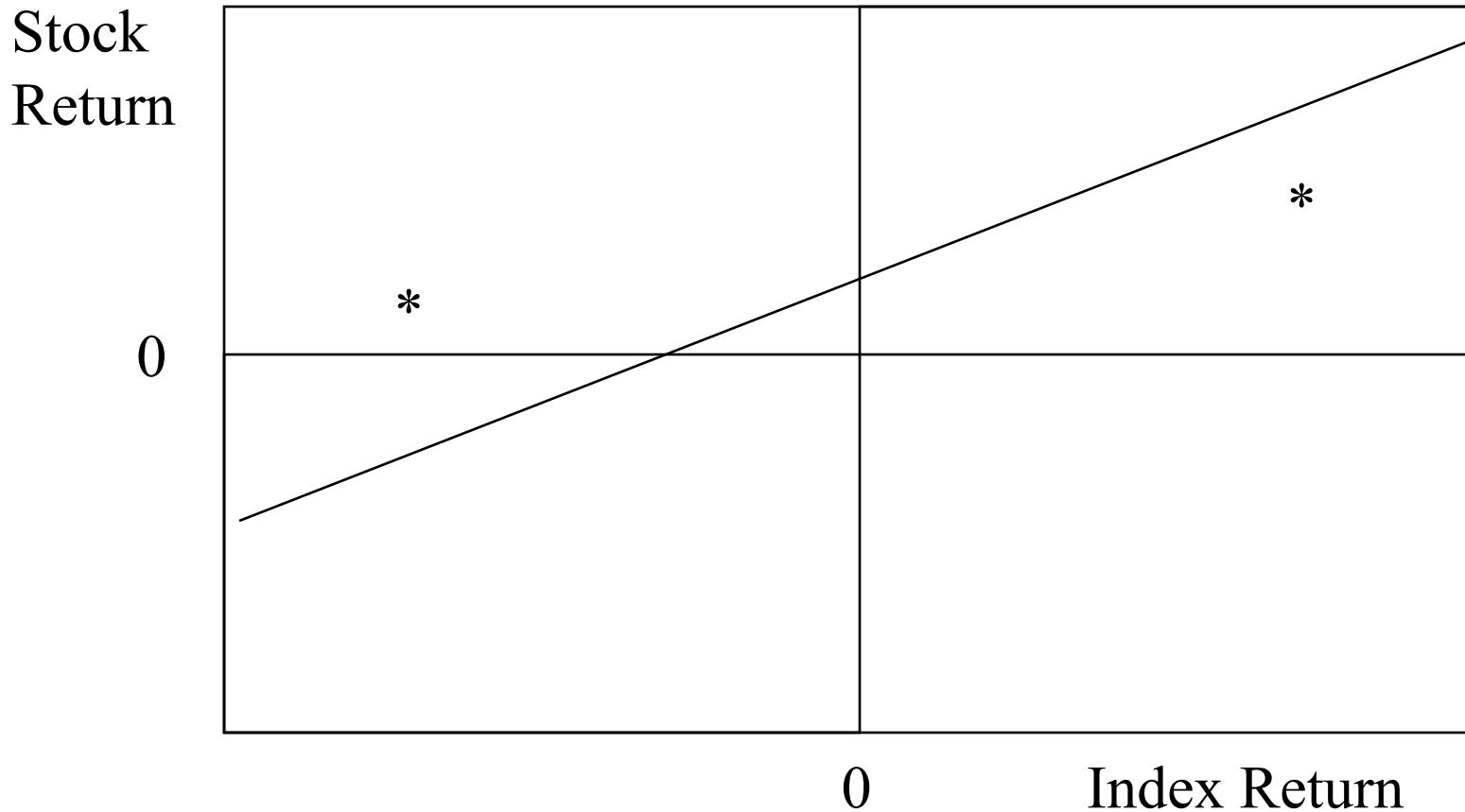
- The Single Index Model (Market Model) states:

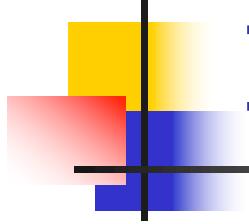
$$r_i - r_F = \alpha_i + \beta_i [r_M - r_F] + \varepsilon_i$$

- α : Abnormal return of security
- ε : Firm-specific unique risk



The Market Model





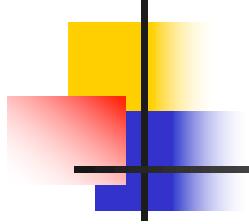
Implications of Market Model

- Slope measures covariance of stock and index returns:

$$\beta_{iI} = \frac{\text{cov}(\tilde{r}_i, \tilde{r}_I)}{\text{var}(\tilde{r}_I)}$$

- Stock risk is market risk plus unique risk:

$$\sigma_{iI} = \beta_{iI}^2 \text{var}(\tilde{r}_I) + \text{var}(\tilde{\varepsilon}_{iI})$$

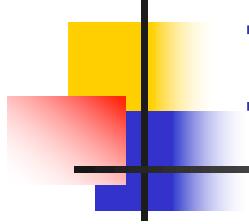


Systematic and Firm-Specific Risk

- The total risk of a stock has two parts:

$$\begin{aligned}Var(r_i - r_F) &= Var(\alpha_i + \beta_i[r_M - r_F] + \varepsilon_i) \\&= Var(\beta_i[r_M - r_F]) + Var(\varepsilon_i) \\&= \beta_i^2 Var(r_M) + Var(\varepsilon_i)\end{aligned}$$

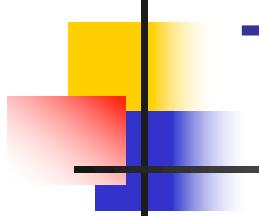
- The first part $\beta_i^2 Var(r_M)$ is the systematic risk of the stock
- The second part $Var(\varepsilon_i)$ is the firm-specific risk of the stock



Implications of Market Model

- Portfolio Beta is the weighted average of the individual stock Betas:

$$\beta_{pI} = \sum_{i=1}^N X_i \beta_{iI}$$

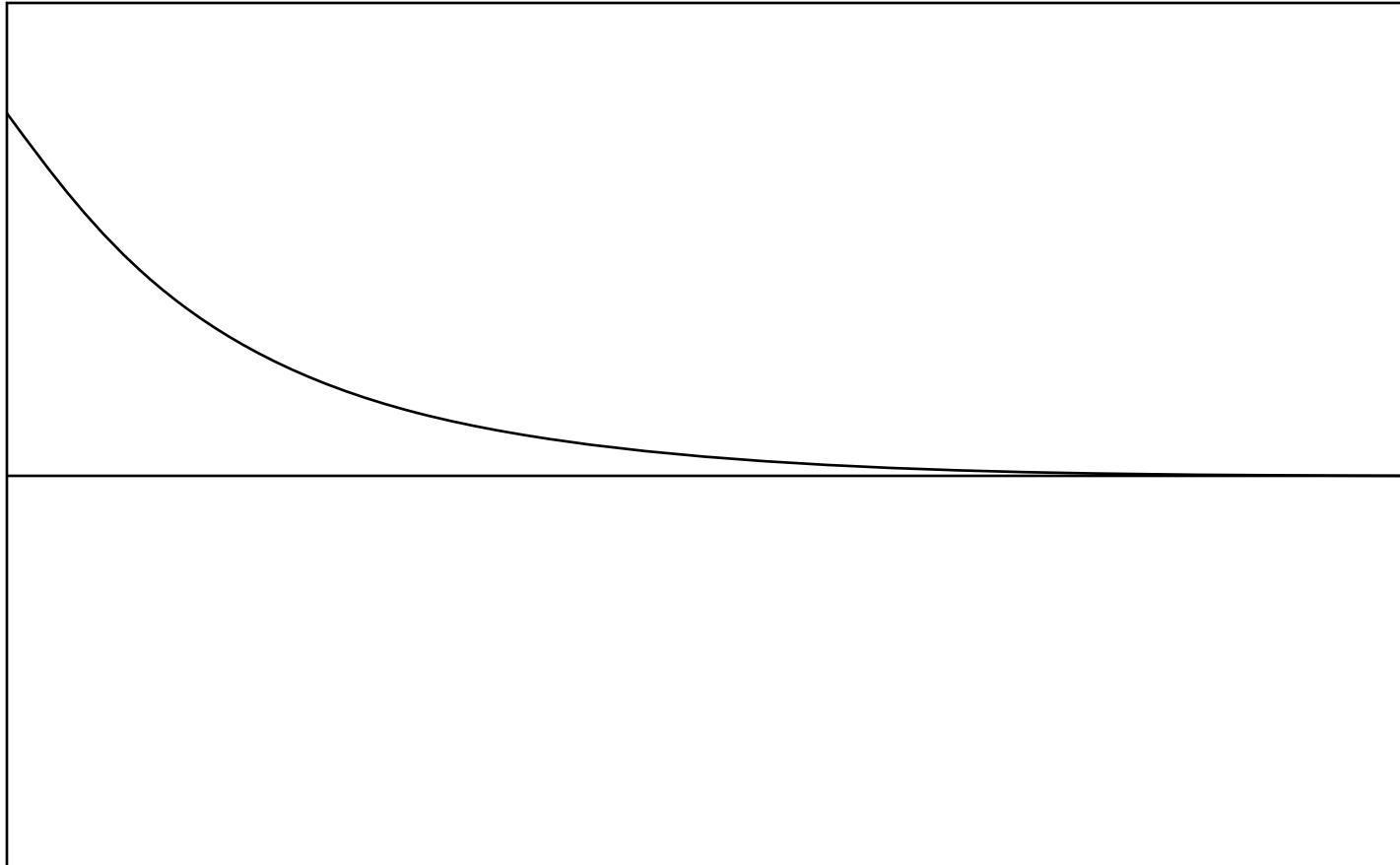


The Market Model

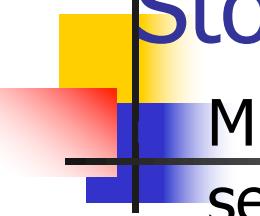
- The unique risk of a well-diversified portfolio is zero:

$$\tilde{r}_p = \alpha_{pI} + \beta_{pI} \tilde{r}_I$$

Decomposition of Stock Variance



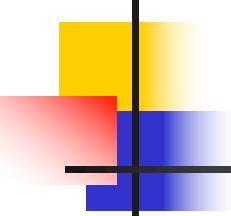
Example: The Market Model and Stock Variance



Mr. Powers owns a portfolio composed of three securities with the following characteristics.

Security	Beta	Std. Dev of Random Error	Proportion
A	1.20	5%	.3
B	1.05	8	.5
C	0.90	2	.2

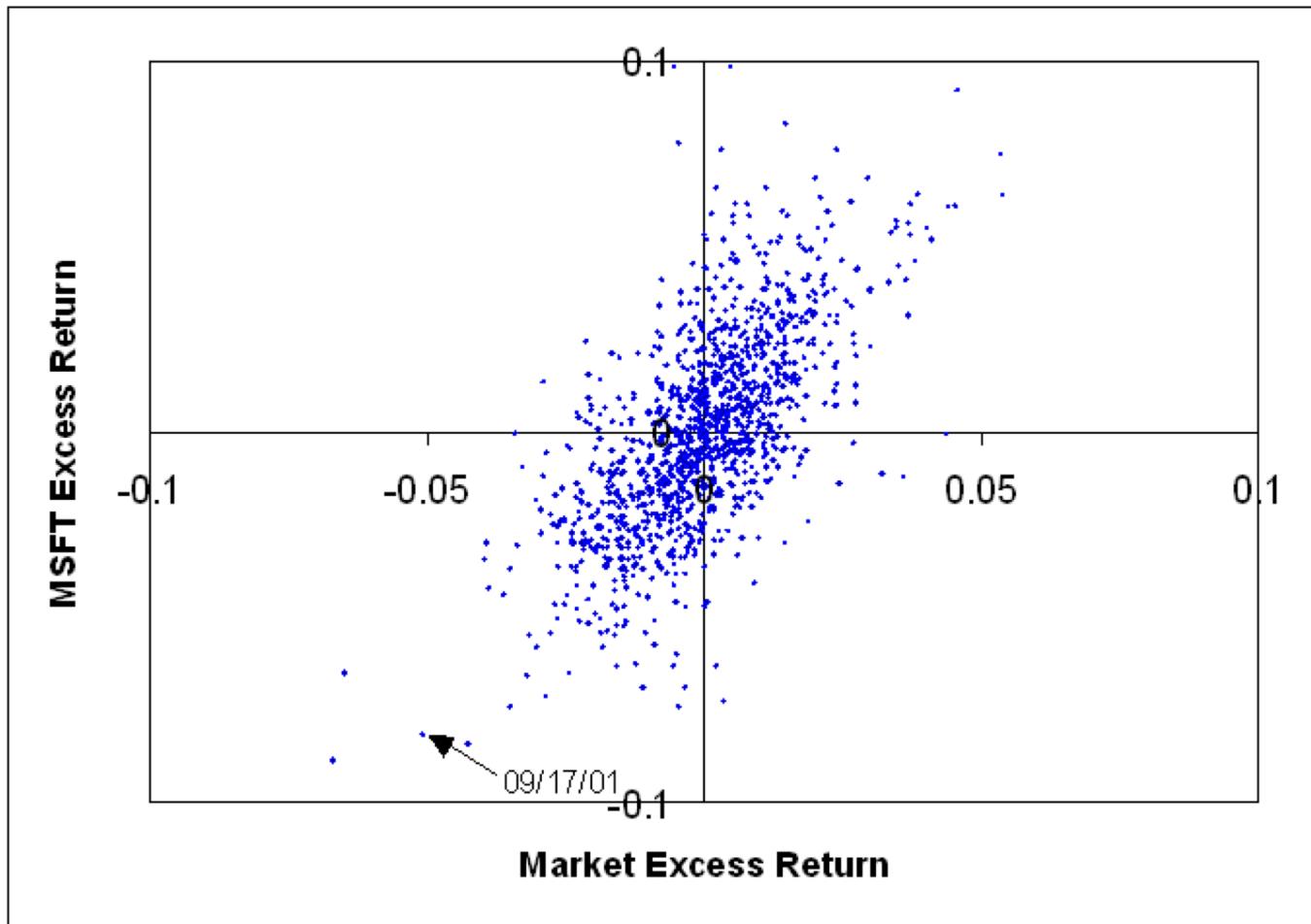
- If the standard deviation of the market index is 18%, what is the total risk of Mr. Power's portfolio?



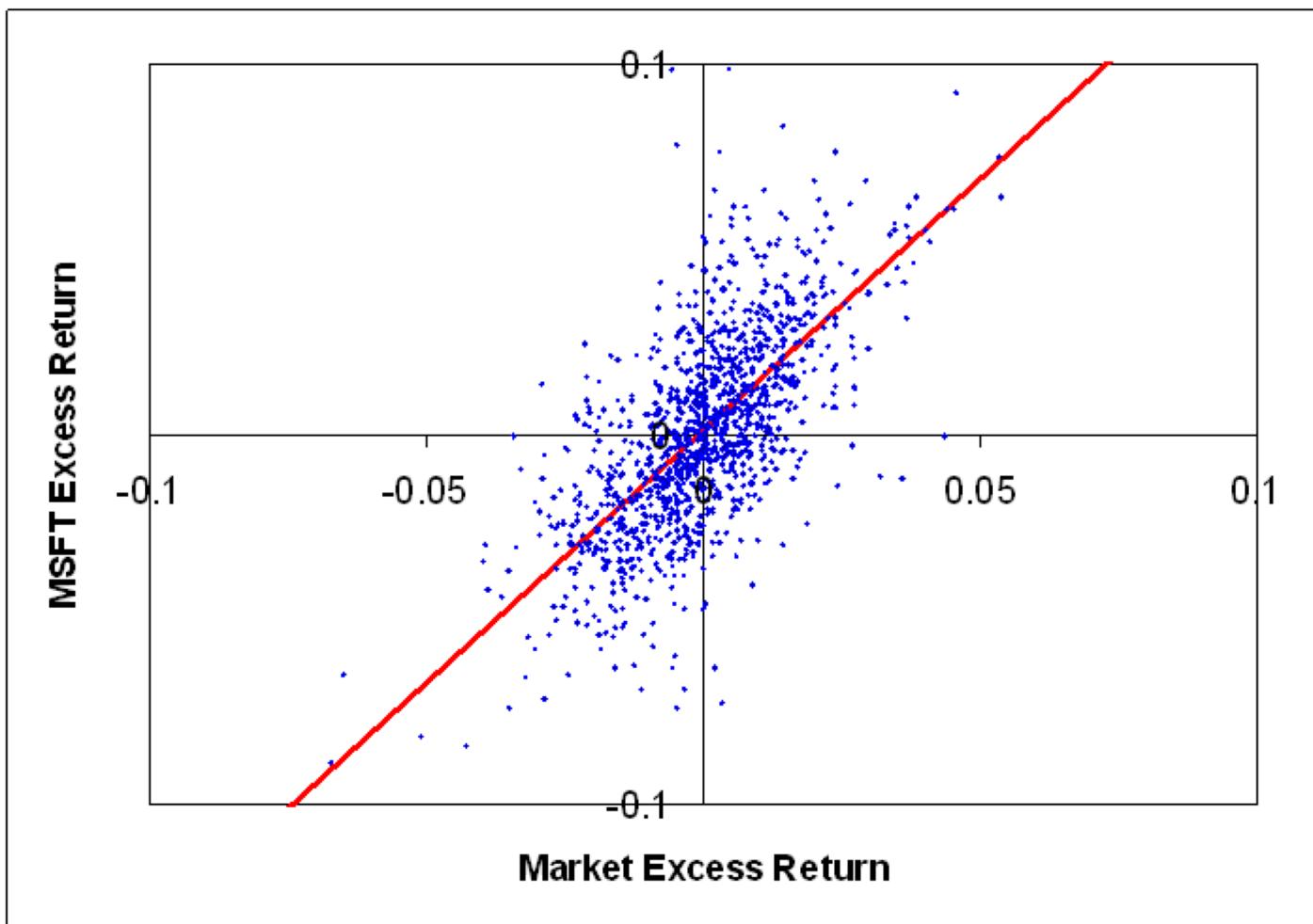
Estimating the Index Model

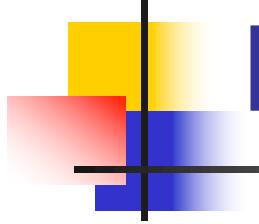
- Estimate the index model for Microsoft
 - Use daily return data on different stocks over the period between 1998-2004
 - Use the Wilshire 5000 Index fund as the relevant market index
 - Use the 3-Month Treasury bill rate as the risk-free interest rate
 - Estimate the index model using a linear regression

Excess Returns of MSFT and Stock Market



Regression Line for MSFT





Linear Regression

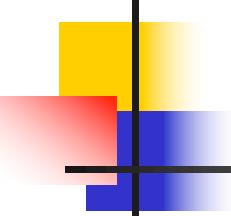
- The MSFT regression line equals:

$$r_{MSFT} - r_F = \alpha_{MSFT} + \beta_{MSFT} \times [r_M - r_F] + \varepsilon_{MSFT}$$

$$r_{MSFT} - r_F = 0.08 + 1.36 \times [r_M - r_F] + \varepsilon_{MSFT}$$

$$R^2 = 0.44$$

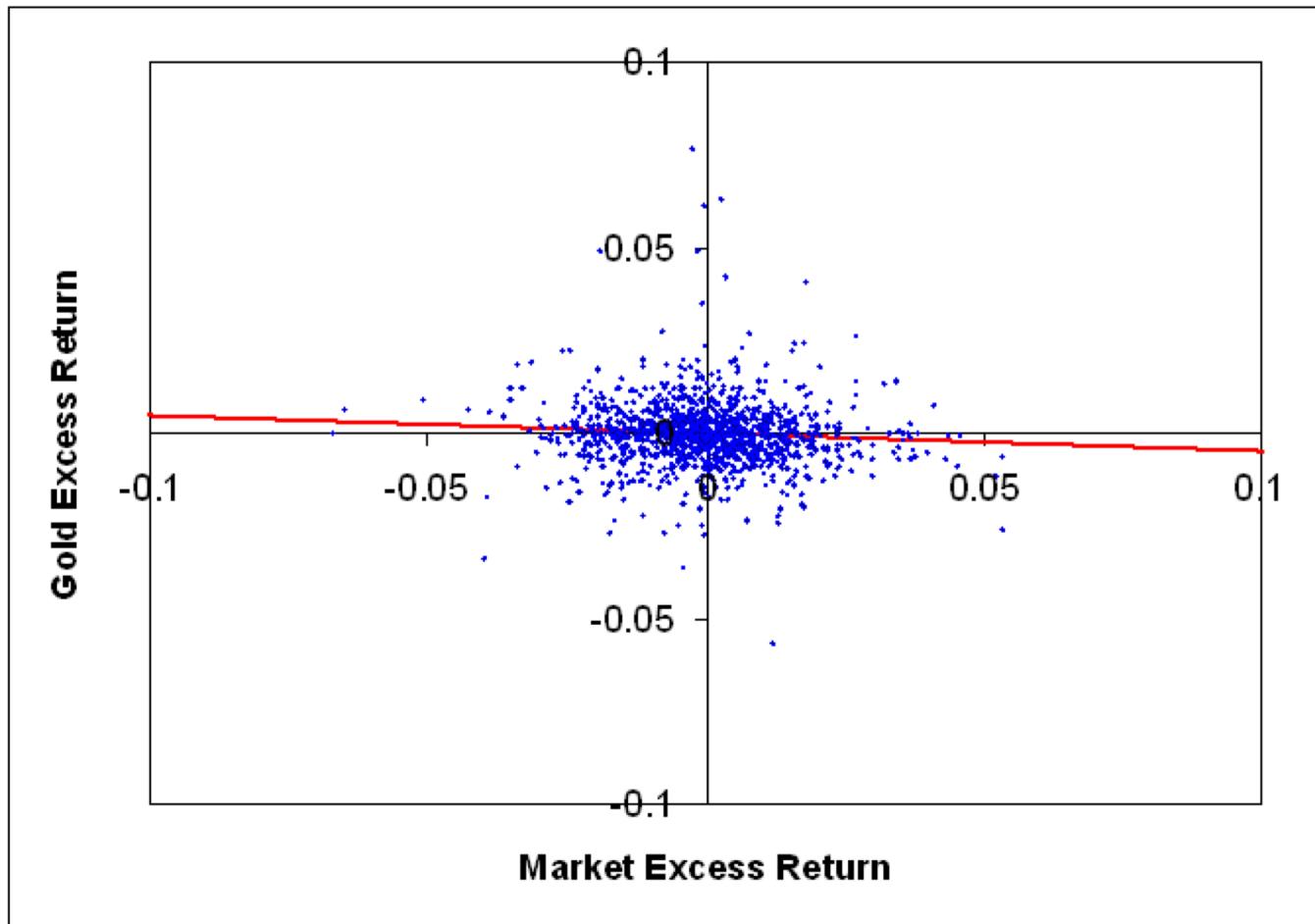
- What does all this mean?



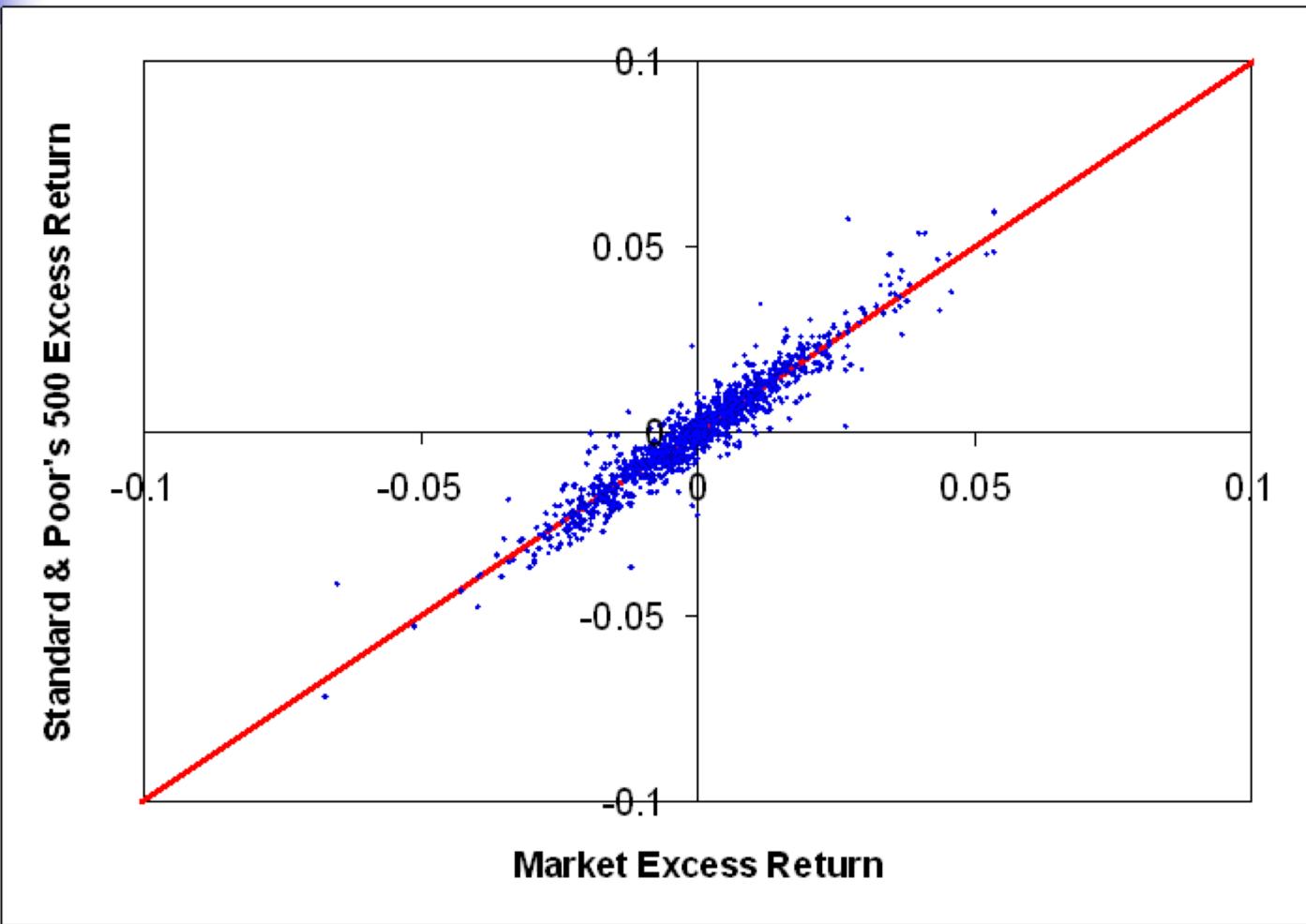
Interpretation of Linear Regressions

- The slope is the beta of MSFT
 - If the excess return of the market increases by 1%, then the excess return of MSFT increases by 1.36%
 - Beta is defined as $\beta = \text{Cov}(r_{\text{MSFT}} - r_F, r_M - r_F) / \text{Var}(r_M - r_F)$
- The intercept is the alpha of MSFT
 - MSFT generated an abnormal return of 0.08% per day relative to the CAPM
- The R² is a measure of the fit of the linear regression
 - 44% of the variation in MSFT is systematic and is explained by variations in the total market
 - 56% of the variation in MSFT is firm-specific variation

Regression Line for Gold

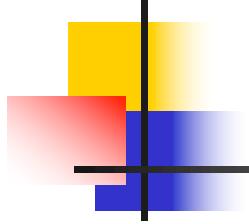


Regression Line for Standard & Poor's 500 Index



Regression Results for Selected Assets

Asset	Mean Daily Excess Return	Daily Standard Deviation	Abnormal Return per day: α	Market Risk: β	R ²
MSFT	0.06%	2.75%	0.08%	1.36	0.44
GE	0.01%	2.24%	0.03%	1.22	0.53
Ford	-0.03%	2.63%	-0.02%	0.93	0.22
Lockheed	0.03%	2.42%	0.03%	0.33	0.03
Yahoo	0.17%	5.33%	0.21%	2.25	0.32
Gold	0.00%	0.85%	0.00%	-0.05	0.01
Nasdaq 100	0.02%	2.82%	0.05%	1.83	0.75
Dow Jones	0.00%	1.29%	0.01%	0.87	0.81
SPY	-0.01%	1.40%	0.00%	1.00	0.95
MSCI EAFE	-0.02%	1.10%	-0.02%	0.41	0.25

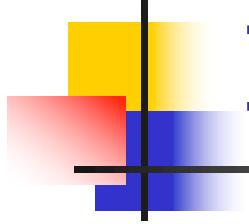


"In physics, it takes three laws to explain 99% of the data; in finance, it takes more than 99 laws to explain about 3%."

Andrew Lo

Country	Source	Sample Start	Market	Monthly	Monthly	Beta With
			Capitalization Millions US\$	Arithmetic Mean Return	Standard Deviation	World Market Last Five Years
		March 1995	Annualized	Annualized	Monthly	
Argentina	IFC	October 1979	\$17,939	42.0%	93.1%	1.80
Australia	MSCI	October 1979	\$120,148	15.1%	26.6%	1.27
Austria	MSCI	October 1979	\$18,222	13.8%	25.2%	0.62
Belgium	MSCI	October 1979	\$53,215	17.6%	21.2%	0.53
Brazil	IFC	October 1979	\$81,800	33.1%	64.0%	0.68
Canada	MSCI	October 1979	\$178,656	9.7%	19.6%	0.75
Chile	IFC	October 1979	\$45,500	24.9%	32.0%	0.45
China	IFC	April 1993	\$28,407	6.3%	86.6%	0.35
Colombia	IFC	October 1985	\$10,620	39.7%	31.4%	-0.19
Denmark	MSCI	October 1979	\$36,785	15.4%	20.1%	0.70
Finland	MSCI	April 1988	\$25,685	8.1%	26.8%	1.66
France	MSCI	October 1979	\$299,756	15.0%	23.4%	1.06
Germany	MSCI	October 1979	\$311,842	14.1%	22.4%	0.67
Greece	IFC	October 1979	\$9,799	7.9%	37.9%	0.62
Hong Kong	MSCI	October 1979	\$141,414	24.2%	33.9%	1.36
Hungary	IFC	April 1993	\$759	-3.5%	41.5%	1.68
India	IFC	October 1979	\$71,904	17.4%	29.5%	-0.09
Indonesia	IFC	October 1990	\$21,841	3.3%	30.8%	0.92
Ireland	MSCI	April 1988	\$13,187	13.7%	22.6%	1.17
Italy	MSCI	October 1979	\$97,527	15.2%	27.7%	0.64
Japan	MSCI	October 1979	\$2,098,944	17.8%	25.1%	1.87

Jordan	IFC	October 1979	\$3,093	9.2%	17.5%	0.07
Malaysia	IFC	October 1985	\$138,774	16.6%	27.4%	0.73
Mexico	IFC	October 1979	\$47,962	20.8%	46.5%	0.92
Netherlands	MSCI	October 1979	\$182,581	18.6%	18.0%	0.97
New Zealand	MSCI	April 1988	\$18,906	9.2%	25.4%	1.61
Nigeria	IFC	October 1985	\$654	10.5%	54.6%	0.86
Norway	MSCI	October 1979	\$19,010	14.1%	27.9%	1.37
Pakistan	IFC	October 1985	\$7,758	19.1%	24.3%	-0.28
Peru	IFC	April 1993	\$5,266	30.2%	40.2%	1.36
Philippines	IFC	October 1985	\$30,043	42.9%	37.4%	0.68
Poland	IFC	April 1993	\$1,713	97.3%	96.3%	3.90
Portugal	IFC	October 1986	\$11,662	32.6%	44.7%	1.03
Singapore	MSCI	October 1979	\$55,835	16.1%	25.7%	0.95
South Africa	IFC	April 1993	\$140,934	43.8%	26.7%	0.80
South Korea	IFC	October 1979	\$123,159	16.0%	30.6%	0.26
Spain	MSCI	October 1979	\$73,334	15.3%	24.1%	1.42
Sri Lanka	IFC	April 1993	\$1,648	18.9%	34.1%	0.02
Sweden	MSCI	October 1979	\$76,728	21.6%	24.0%	1.66
Switzerland	MSCI	October 1979	\$240,254	14.2%	19.2%	0.77
Taiwan	IFC	October 1985	\$147,472	35.8%	51.3%	1.11
Thailand	IFC	October 1979	\$90,882	22.1%	27.1%	0.32
Turkey	IFC	October 1987	\$19,541	45.1%	73.1%	0.08
United Kingdom	MSCI	October 1979	\$779,121	16.4%	21.5%	1.14
United States	MSCI	October 1979	\$3,030,838	14.8%	15.0%	0.43
Venezuela	IFC	October 1985	\$2,992	21.7%	46.1%	0.47
Zimbabwe	IFC	October 1979	\$1,179	13.3%	35.3%	0.69

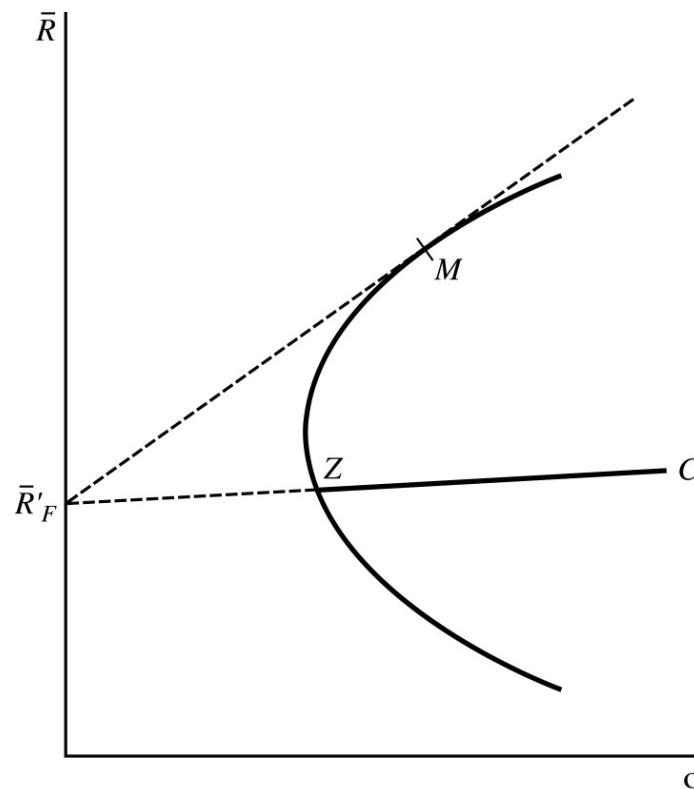


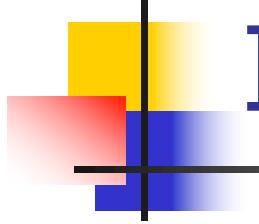
Is the CAPM correct?

- Let's go back to the assumptions:
- Investors:
 - are not always rational and not all investors use portfolio theory to form portfolios
 - have heterogeneous expectations
 - face borrowing and short-selling constraints and cannot invest freely in all assets
 - can affect security prices
 - have different investment horizons
 - and pay taxes and transaction costs

Zero-beta CAPM (CAPM with borrowing constraints)

$$E(R_i) = R_Z + \beta_i [E(R_M) - R_Z]$$



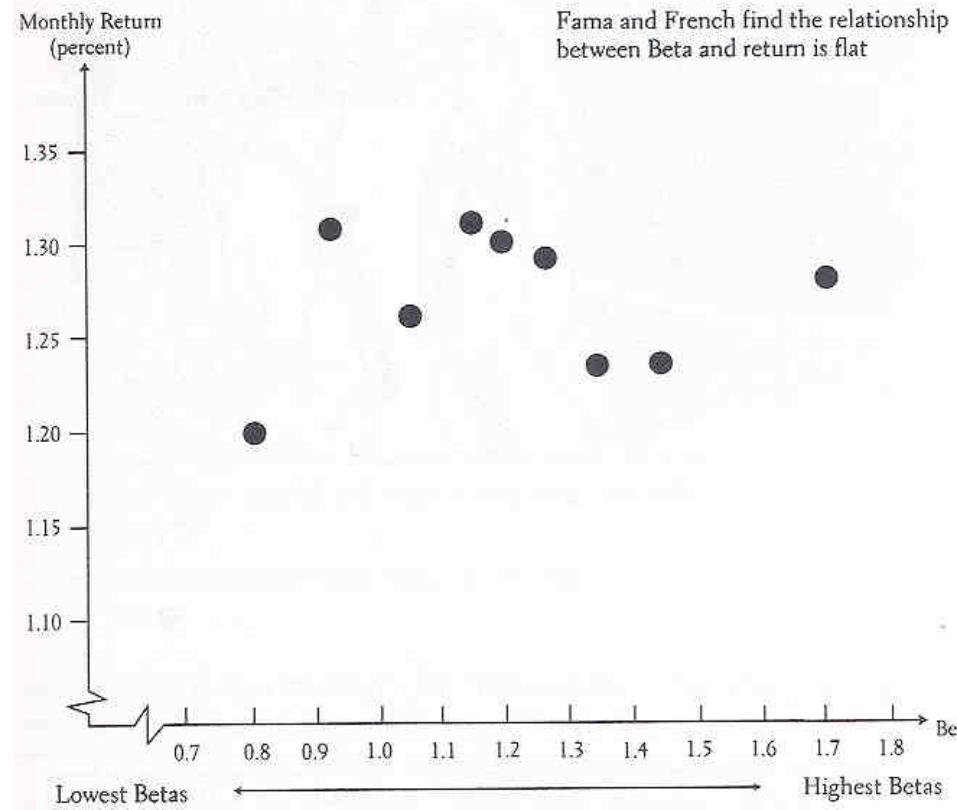


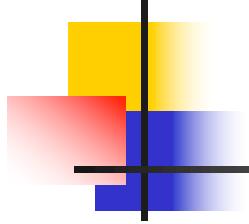
Is Beta Dead?

- Finance researchers found that the relationship between beta and the average return is relatively flat
 - Stocks with high betas did not perform significantly better than stocks with low betas

Is Beta Dead?

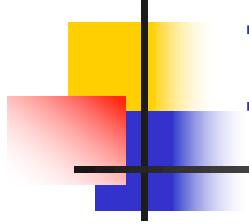
Average Monthly Return vs. Beta: 1963–90
(Fama and French Study)





"In physics, it takes three laws to explain 99% of the data; in finance, it takes more than 99 laws to explain about 3%."

Andrew Lo



Is Beta Dead?

- We should not bury beta too soon
 - The tests use a stock index as an approximation of the market index, which should also include bonds, commodities, real estate, human capital...
 - Betas do a relatively good job at predicting relative volatility in the market
 - The CAPM can be a very powerful tool to determine under-pricing of securities