Answer to Problem Set 3

1. From the point of view of social optimum, the value of any good is the utilities it can bring to consumers and the costs of it is the production costs. Hence, it is socially best to maximize the difference between consumers' utilities and the production costs. That is to say, the social optimal outcome is the one in which the sum of the consumer surplus and the producer surplus is maximized. However, in the case of monopoly, the market is commanded by the monopolistic firm. From the monopoly's point of view, it is optimal to maximize the difference between its revenue and production costs. So the firm will produce to a level such that the marginal revenue is equal to the marginal production costs. Since the demand curve is sloping downward, the marginal revenue curve is below the demand curve in the Q-P plane. Hence the marginal revenue curve intersects the marginal cost curve at a production quantity level lower than that of the intersection of the demand curve and the marginal cost curve. Therefore, the monopolistic optimal production is less than the social optimal production. If the monopolist produces one unit more, there is a consumer who will get a utility higher than the production costs of this good. So this additional unit of good should be produced in the spirit of improving social welfare. However, the profit-maximizing monopolist will not produce this unit. In sum, due to the inconsistence in the objectives, there is social costs to market power.

   The above social costs of monopoly power cannot be eliminated even if the gain to producer from monopoly power can be redistributed to consumers. That is because that once there is monopoly power, the production quantity will be less than the social optimal production. Then the social surplus—the sum of the consumer surplus and the producer surplus is less than that in the social optimal level. The difference between the two is a kind of "deadweight loss". This loss still exists even under redistribution. Hence, redistribution can't eliminate the social costs to monopoly power.

   (Note: We don't consider price discrimination here. The monopolist can only sell the goods to all the consumers at the same price.)

2. A) The profit maximization problem is as follows,

$$\max_{Q} \left(100-0.01Q\right)Q-\left(50Q+30000\right)$$

F.O.C $\quad -0.02Q^{*}+50=0$

So we get the solution.

Optimal weekly production $\quad Q^{*}=2500$,

Optimal price $\quad P^{*}=75(cents)$,

Optimal total profits per week $\quad \pi^{*}=32500(cents)$.

B) When the tax is levied on the firm, the actual revenue the firm gets for each unit of goods produced is

$$100-0.01Q-10=90-0.01Q.$$

And when the tax is levied on the consumers for each unit bought, given that the production is Q, the price paid per unit by the consumers is

$$100-0.01Q.$$

So the revenue for each product sold is still

$$90-0.01Q$$

for the firm, Hence, either the tax is levied on the consumers or the firm, the profit-maximizing problem for the firm is as follows,

$$\max_{Q}\left(90-0.01Q\right)Q-\left(50Q+30000\right)$$

By solving the first-order-condition, we get the following results,

The new level of production is $\tilde{Q}^{*}=2000$;

If the tax is levied on the consumers, the price designed by the firm is $\tilde{P}^{*}=70$ (cents);

If the tax is levied on the firm, the price designed by the firm is $\tilde{P}^{*}=80$ (cents);

In either case, the profits the firm gets are $\tilde{\pi}^{*}=10000$ (cents).

C) If the monopoly could implement the first-degree price discrimination, then price varies by consumers' willingness to pay.

MC=50, 100-0.01Q=50➔Q=5000

And the total profit should be (100-50)*5000/2-30000=95000 (cents)

D) Let CS,PS,SW denote consumer surplus, producer surplus and social welfare, then

CS(A)=(100-75)*2500/2=31250
PS(A)=62500
SW(A)=CS(A)+PS(A)=93750

CS(B)=(100-80)*2000/2=20000
PS(B)=40000
TAX(B)=10*2000=20000
SW(B)=CS(B)+PS(B)+TAX(B)=80000

SW(B)<SW(A) with the implement of tax, i.e. there is social welfare loss when charging tax.

SW(C)=PS(C)=125000
CS(C)=0

SW(C)>SW(A)>SW(B)
The social welfare increases with first-degree price discrimination as the monopoly has more information.

3. A) We solve the following profit-maximizing problem of the firm.

$$\max_{Q_1,Q_2}\left(700-5Q_1-5Q_2\right)(Q_1+Q_2)-10Q_1^2-20Q_2^2$$

Then we get the subsequent first-order conditions.

$$700-10(Q_1^*+Q_2^*)-20Q_1^*=0,$$

$$700-10(Q_1^*+Q_2^*)-40Q_2^*=0.$$

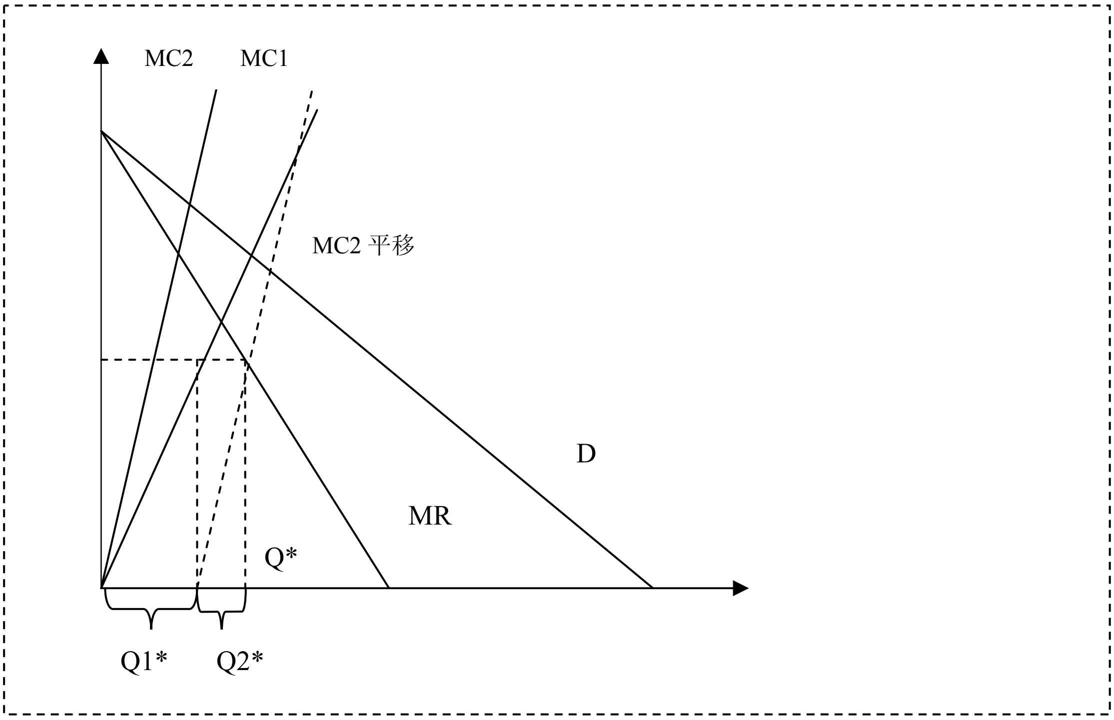
So we can get the optimal production levels at each factory as

$$\begin{cases} Q_1^* = 20, \\ Q_2^* = 10. \end{cases}$$

The optimal total production is $Q^* = 30$.

And the optimal market price is $P^* = 550$.

This result can be demonstrated by the following graph.



B) Let the cost function of the first factory by denoted by

$$\tilde{C}_1\left(Q_1\right) = aQ_1^2 \text{, where } \ a > 10 .$$

So the profit-maximizing problem of the firm changes into the following one.

$$\max_{Q_1, Q_2} \left(700 - 5Q_1 - 5Q_2\right)(Q_1 + Q_2) - aQ_1^2 - 20Q_2^2 .$$

Then we have the first-order conditions,

$$700 - 10(\tilde{Q}_1^* + \tilde{Q}_2^*) - 2a\tilde{Q}_1^* = 0 ,$$

$$700 - 10(\tilde{Q}_1^* + \tilde{Q}_2^*) - 40\tilde{Q}_2^* = 0 .$$

By simple calculations, we get that

$$\begin{cases} \tilde{Q}_1^* = \dfrac{280}{4+a}, \\ \tilde{Q}_2^* = \dfrac{14a}{4+a}. \end{cases} .$$

The optimal total production is now $\tilde{Q}^* = \dfrac{280 + 14a}{4+a}$.

And the optimal market price is $\tilde{P}^* = \dfrac{1400 + 630a}{4+a}$.

By taking partial derivatives with respect to the parameter a, it is derived that

$$\frac{\partial \tilde{Q}_1^*}{\partial a} = -\frac{280}{(4+a)^2} < 0,$$

$$\frac{\partial \tilde{Q}_2^*}{\partial a} = \frac{56}{(4+a)^2} > 0,$$

$$\frac{\partial \tilde{Q}^*}{\partial a} = -\frac{224}{(4+a)^2} < 0,$$

$$\frac{\partial \tilde{P}^*}{\partial a} = \frac{1120}{(4+a)^2} > 0.$$

Therefore, given the labor costs in factory 2, increasing the costs of factory 1 will reduce the optimal production of factory 1, increase the optimal production of factory 2, reduce the optimal total production and increase the profit-maximizing price.

Other solution to B)

Some students assume $C_1(Q_1) = 10Q_1^2 + \phi(Q_1), \phi'(\cdot) > 0$, which is also right.

With the picture above, as labor costs increase in Factory 1, MC1 increases. And MC1 shifts upwards, this change may induce MC shifts upwards. Then we get an increase with Q2 and P, and decrease with Q1 and Q.

4. 1) If the monopolist can maintain the separation between the two markets, it can design different prices in the two markets. So there are two optimization problems.

$$\max_{P_1} (P_1 - 5)(55 - P_1)$$

and

$$\max_{P_2} (P_2 - 5)(70 - 2P_2).$$

Hence, we can get the following first-order conditions,

$$60 - 2P_1 = 0,$$

$$80 - 4P_2 = 0.$$

Then the optimal output levels and prices in the two markets are

$$Q_1^* = 25, \quad P_1^* = 30;$$

$$Q_2^* = 30, \quad P_2^* = 20,$$

and the total profits are $\pi^* = 1075$.

2) In this case, the demand functions in the two markets change into the subsequent discontinuous forms,

$$Q_1 = \begin{cases} 55 - P_1 + 70 - 2(P_1 + 5), if P_1 < P_2 - 5; \\ 55 - P_1, if \left|P_1 - P_2\right| \leq 5 \\ 0, if P_1 > P_2 + 5. \end{cases}$$

$$Q_2 = \begin{cases} 55 - (P_2 + 5) + 70 - 2P_2, if P_2 < P_1 - 5; \\ 70 - 2P_2, if \left|P_1 - P_2\right| \leq 5 \\ 0, if P_2 > P_1 + 5. \end{cases}.$$

So the monopolist will consider three cases when designing the optimal pricing systems.

A) $\left|P_1 - P_2\right| \leq 5$.

In this case, the profits maximization problem for the firm is

$$\max_{P_1, P_2} P_1(55 - P_1) + P_2\left(70 - 2P_2\right) - 5(125 - P_1 - 2P_2)$$

$$\text{s.t} \quad P_1 - P_2 \leq 5,$$

$$P_1 - P_2 \geq -5.$$

Because by the results in 1), at the optimum, at least one of the constraints must be binding (To be binding is to take equality). Meanwhile, the two constraints cannot be binding simultaneously. Hence, at the optimum, we must have only one of the constraints binding. There are two cases,

A1) $P_1 - P_2 = 5$.

Substitute it into the objective function and take derivatives with respect to P_1, we get that

$$P_1 = 80/3, \; P_2 = 65/3, \; \pi = 9525/9.$$

A2) $P_1 - P_2 = -5$.

Repeat the above steps, we get that

$$P_1 = 20, \quad P_2 = 25, \quad \pi = 925 < 9525/9.$$

So the first case is superior. That is to say,

$$P_1^A = 80/3, \; P_2^A = 65/3, \quad \pi^A = 9525/9.$$

B) $P_1 - P_2 < -5$.

In this case, the optimal problem for the monopolist is as follows,

$$\max_{P_1} \left(P_1 - 5\right)(115 - 3P_1)$$

Then we get that

$$P_1^B = 65/3,\ P_2^B > 80/3,\ \pi^B = 2500/3 < 9525/9.$$

C) $P_1 - P_2 > 5$.

By symmetry, the optimal problem for the monopolist is as follows,

$$\max_{P_2} \left(P_2 - 5\right)(120 - 3P_2)$$

And the optimal solution is

$$P_1^C > 55/2,\ P_2^C = 45/2,\ \pi^C = 1225/4 < 9525/9.$$

In sum, if it only costs the demanders \$5 to transport goods between the two markets, the optimal prices and production in the two markets are given by

$$\tilde{P}_1^* = 80/3,\ \tilde{Q}_1^* = 85/3;$$

$$\tilde{P}_2^* = 65/3,\ \tilde{Q}_2^* = 80/3.$$

The monopolist's new profits are $\tilde{\pi}^* = 9525/9$.

Another solution:

Consider the No arbitrage Condition : |P1 − P2| = 5, and we just need to compare π_1 (whenP1 − P2 = 5) with π_2 (whenP1 − P2 = −5).

3) In this case, $P_1 = P_2 = P$ and the profit-maximizing problem for the monopolist is as follows.

$$\max_{P} \left(P - 5\right)(125 - 3P)$$

The corresponding first-order condition is

$$125 - 3P - 3(P - 5) = 0$$

So at the optimum,

$$\hat{P}_1^* = \hat{P}_2^* = 70/3,\ \hat{Q}_1^* = 95/3,\ \tilde{Q}_2^* = 70/3,\ \hat{\pi}^* = 3025/3.$$

4) In this case, the marginal prices in the two markets should be equal to the common marginal production costs. That is to say,

$$P_1^{'} = P_2^{'} = 5.$$

Then the demand quantities in the two markets are

$$Q_1^{'} = 50, \quad Q_2^{'} = 60.$$

So the lump-sum entry fees in the two markets are

$$E_1 = \int_0^{50} (55 - x)dx - 5\times 50 = 1250,$$

$$E_2 = \int_0^{60} (70 - 2x)dx - 5\times 60 = 900.$$

That is to say, the monopolist can set both of the marginal prices in the two markets to be equal to the marginal production costs \$5. He also sets the lump-sum entry fees in the first market to be \$1250 and that of the second market to be \$300.

5. A) let Pi be the separate price of goods i, and P be the bundling price. Let Xi be the total profit of firm, X be the total profit. (i=1,2)

(i) Selling separately

When P1<30 and P1>40, X<0

As $P_1 \in (30, 40]$, X>=0 and X is increasing with P1. Thus, we only need to compare the profit for the cases P1=30 and 40.

As X1(P1=30)=0 < X1(P1=40)=10, then the optimal price for 1 is 40 and profit for goods 1 is 10.

Similarly, we only need to compare the profit for the cases P2=40, 60 and 80.

As X2(P2=60)=60 > X2(P2=80)=50 >X2(P2=40)=30, then the optimal price is 60 and profit for goods 2 is 60.

Total Profit is X=X1+X2=10+60=70.

(ii) Bundling

The cost is 60 under such condition, and the optimal price should be among 60(=20+40), 90(=30+60) and 120(=40+80).

As X(P=90)=60 = X(P=120)=60 > X(P=60)=0, thus the optimal price could be 90 or 120, and total profit is 60.

Comparing the two cases above, we could see selling separately is a better choice for the firm.

B) with a similar logic as above, we could get the following results:

(i) Selling separately

X1(P1=80)=100 > X1(P1=100)=70 > X1(P1=40)=30 > X1(P1=25)=0

Similarly, $P2^*=80$ and $X2^*=100$

Thus, the optimal prices would be 80 and 80, and the total profit is $X=100+100=200$

(ii) Bundling

In this case, $P^*=120$ and $X^*=240$

Comparing the two cases above, we could see bundling is a better choice for the firm.

From (A) and (B), we could see that when consumers have heterogeneous demands and such demands for different parts of the bundle product are inversely correlated (Case B is an example), bundling might be a better choice for the firm to get more profit.