# Answers to Problem Set 4

Question 1.
(a)

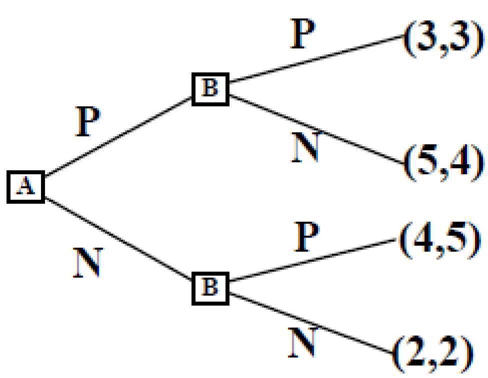
| | | play 2 | | |
|---|---|---|---|---|
| | | 1 剪刀 | 2 石头 | 3 布 |
| | 1 剪刀 | 0,0 | -1,1 | 1,-1 |
| play 1 | 2 石头 | 1,-1 | 0,0 | -1,1 |
| | 3 布 | -1,1 | 1,-1 | 0,0 |

方格内第一个数字是play 1效用，第二个数字是player 2 效用

(b)
There is no pure strategy equilibrium.
For mixed strategy equilibrium, suppose player 1 take (x,y,1-x-y), by equivalence principle, $x*0 – y + (1-x-y) = x + y*0 – (1-x-y) = -x +y + 0*(1-x-y)$ ，
Which means $x = y = 1/3$.
So the mixed strategy equilibrium is ((1/3,1/3,1/3),(1/3,1/3,1/3))

(c) The expected payoff for player 1 :
1/9*（0+1-1+0+1-1+0+1-1）=0
The expected payoff for player 2:
1/9*（0+1-1+0+1-1+0+1-1）=0

Question 2.
(a) pure strategy Nash Equilibrium: (N, P) and (P, N).
For mixed strategy equilibrium, suppose (x,1-x) is taken by A. By equivalence principle, we have $x = 3/4$. So the mixed strategy equilibrium is ((3/4,1/4), (3/4,1/4)).
(b) Extensive form：
Suppose A moves first, the game tree is as follows.



The perfect Nash Equilibrium here is (P,N), with payoff (5,4).
So both players have first-move advantage.

Normal form ：

| | | B | | | |
|---|---|---|---|---|---|
| | | P,P | N,P | P,N | N,N |
| A | P | 3,3 | 5,4 | 3,3 | 5,4 |
| | N | 4,5 | 4,5 | 2,2 | 2,2 |

（Note: P denotes Produce; N denotes Not Produce.）

Pure strategy Nash equilibrium: (P,(N,P) )和(N,(P,P))
Sub-game perfect equilibrium: (P,(N,P))

(c) Sub-game perfect equilibrium must be Nash equilibrium in the corresponding normal form game. However, not all Nash equilibrium in the normal form game can be sub-game perfect equilibrium (Because some may suffer from issues like Non-credible threat, they are equilibrium in the Game, but they are not equilibrium in all sub-games).

Question 3.

(a)In monopoly case,

$$\max_p (50-p)(p-5)$$

with first order condition

$$p=\frac{55}{2}, q=\frac{45}{2}, \pi=\frac{2025}{4}=506.25$$

b) In Cournot case, for firm 2:

$$\max_{q_2}(50-q_1-q_2-5)q_2$$

with first order condition

$$q_2=\frac{1}{2}(45-q_1)$$

Similarly,

$$q_1=\frac{1}{2}(45-q_2)$$

So

$$q_1=q_2=15, p=20, \pi_1=\pi_2=225=\frac{2025}{9}$$

c) Suppose there are N firms, for firm i

$$\max_{q_i}(50-\Sigma q_j-5)q_i$$

with first order condition

$$q_i=\frac{1}{2}(45-\Sigma_{j\neq i}q_j)$$

By symmetry, $q_1=q_2=.=q_N=\frac{1}{N}Q$, so

$$q_i=\frac{45}{N+1}, p=50-\frac{N}{N+1}45, \pi_i=\frac{2025}{(N+1)^2}$$

d) In the Stackerberg case, the best response for firm 2 is similar as above,

$$q_2=\frac{1}{2}(45-q_1)$$

For firm 1:

$$\max_{q_1}(50-q_1-\frac{1}{2}(45-q_1)-5)q_1$$

By first order condition,

$$q_1=\frac{45}{2}, q_2=\frac{45}{4}, p=\frac{65}{4}, \pi_1=\frac{2025}{8}, \pi_2=\frac{2025}{16}$$

e) For firm 2

$$\max_{q_2}(50-q_1-q_2-q_3-5)q_2$$

With first order condition

$$q_2=\frac{1}{2}(45-q_1-q_3)$$

Similarly,

$$q_3=\frac{1}{2}(45-q_1-q_2)$$

So

$$q_2 = q_3 = 15 - \frac{1}{3}q_1$$

For firm 1,

$$\max_{q_1} (50 - q_1 - 2(15 - \frac{1}{3}q_1) - 5)q_1$$

With first order condition,

$$q_1 = \frac{45}{2}, q_2 = q_3 = \frac{15}{2}, p = \frac{25}{2}, \pi_1 = \frac{2025}{12} = 168.75, \pi_2 = \pi_3 = \frac{2025}{36} = 56.25$$

Question 4.

a) The Bertrand price is 10.

b) $\pi_A = 0, \pi_B = 2 \times 300 = 600$

c) The equilibrium is not Pareto efficient. If we allow B to monopolize, the profit for A is still 0, i.e. A is no worse off, but the profit for B becomes

$$\max_p (p-8)(500-20p)$$

So $p = \frac{33}{2}, \pi = 5 \times 17^2 > 600$

d) If $MC_A$=15, the outcome is unchanged, since monopoly price is larger than 15.

Question 5

a) For firm 1

$$\max_{p_1} p_1(20 - p_1 + p_2)$$

With first order condition

$$p_1 = 10 + \frac{1}{2}p_2$$

Similarly,

$$p_2 = 10 + \frac{1}{2}p_1$$

So $p_1 = p_2 = 20$.

b) For firm 2,

$$p_2 = 10 + \frac{1}{2}p_1$$

For firm 1,

$$\max_{p_1} p_1(20 - p_1 + (10 + \frac{1}{2}p_1))$$

By first order condition,

$$p_1 = 30, p_2 = 25, Q_1 = 15, Q_2 = 25, \pi_1 = 450, \pi_2 = 625$$

So first mover has no advantage.

Question 6

(1) According to the approach in Cournot Competition:

$$\pi_1 = p*q - c*q = (90 - x_1 - x_2)*x_1 - x_1^2$$

$$\pi_2 = p*q - c*q = (90 - x_1 - x_2)*x_2 - 8x_2$$

According to F.O.C, we have:

$$4x_1 + x_2 = 90$$

$$x_1 + 2x_2 = 82$$

Thus, $x_1$ =14, $x_2$ =34, P=42, $\pi_1 = 392, \pi_2 = 1156$

(2) If the two firms merge, the optimization problem is:

Max $\pi = (90 - x_1 - x_2)(x_1 + x_2) - x_1^2 - 8x_2$

According to F.O.C, we have:

$$4x_1 + 2x_2 = 90$$

$$2x_1 + 2x_2 = 82$$

Thus, $x_1$ =4, $x_2$ =37 P=49 $\pi = 1697$

(3) 两个车间合并之后，两个企业变成了独立的生产车间，但企业试图在两个企业之间进行最优的产量配置，使得成本最小化，请求出成本函数。

首先算出两个企业的边际成本函数，

MC1=$2x_1$, MC2=8,

所以，当总产量 x 小于 4 时，全部在第一个车间生产，成本最低

当总产量 x 大于 4 时，前 4 个单位在第一个车间生产，之后的 x-4 个单位全部放在第二个车间生产产量最低，故成本函数为：

当 x 小于等于 4 时 ，C(x)= $x^2$

当 x 大于 4 时 ，C(x)= $8x - 16$