2006 Financial Engineering Competition

Xinhua Development Company[1]

Glancing at his Blackberry, Brad sees he has another email from Cynthia Yang.

Brad Kaiser is a senior VP in the interest rate structuring group at Wright Derivatives. Ms Yang is the deputy treasurer of Xinhua Development Company in Shanghai.

The Xinhua Development Company is a large quasi-public developer which takes on large utility infrastructure development projects inside China. Since the late 1990s it has funded these projects by borrowing extensively from Japanese banks and insurance companies. This debt is Yen-denominated with a floating annual coupon. Details about this debt are in Exhibit 1. The completed water works, power plants, and industrial parks – which are Xinhua's primary assets – are critical in sustaining the booming Chinese economy. As a result, the fees collected from its industrial customers are now producing a steady revenue stream.

This business success has created a risk management problem for Ms Yang. The source of her problem is twofold. First, Xinhua's future receivables are in Yuan but its future interest expenses are in Yen. This exposes Xinhua to foreign exchange fluctuations in its net earnings. Ms Yang could live with this risk when Yen interest rates were at their historic lows, but Japanese rates are rising now that Japan shows signs of rebounding from its decade-long deflation.

The most direct way to eliminate the Yen/Yuan risk would be for Xinhau simply to refinance by borrowing in Yuan and then either buying back its own Yen debt or, failing that, buying other floating rate Yen-denominated bonds as a hedge. This leads to the second element in Ms Yang's problem: Borrowing in Yuan in the tightly regulated internal Chinese banking system is difficult. In addition, there is no Yuan-linked swap market in which she can borrow synthetically.

The solution Ms Yang is currently considering is to enter into a cross-currency interest rate swap in which Xinhua would receive a floating rate in Yen and pay a floating rate in US dollars. The swap would reduce her FX risk because the Yuan is officially pegged to the US dollar. Currently, the peg is around 8.02 Yuan/USD. Assuming that the peg holds in the future, Yuan receivables (from its ongoing business operations) are almost as good as USD receivables in terms of the FX risk of servicing USD debt.

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The problem with a plain vanilla swap is that the new USD coupons are likely to be a lot higher than the historical average coupon Xinhua paid on its existing Yen-denominated debt. Of course, Ms Yang understands that the rising Yen rates will also raise her Yen coupons in the future, but she is worried that the risk management merits of switching to dollars will still be lost on Xinhua's senior management given the large initial step-up in the coupon rate. Somehow the swap needs to be structured in a way that will lower the base USD rate Xinhua will pay.

Prior to contacting Brad, Ms Yang received a proposal from Rob Dudley at Wrong & Co. Mr. Dudley suggested a swap in which Xinhua would receive, in Yen, a floating rate equal to the 1-year Libor Yen rate + 125 basis points on a constant JPY 50 billion notional from Wrong & Co. In exchange, Xinhua would pay Wrong & Co., in dollars, the 1-year Libor dollar rate + a contractual base spread on a comparable USD notional[2] minus the payoffs from a series of straddles with annual caplets and floorlets on 1-year Libor dollar for the next seven years. The straddles would all be struck at-the-money-forward. The idea is simple: The more straddles Xinhua writes to Wrong & Co., the lower the USD base spread in the swap.

Ms Yang is dissatisfied with the Wrong & Co. structure because the interest rate implied volatilities priced into the embedded straddles were not much greater than the empirical volatility. This suggests that Xinhua is basically just writing fairly priced options in the Wrong proposal. "Where is the gain in that?," she asks Brad. When she expressed this objection to Mr Dudley, all she got was a lecture about market efficiency. "Which totally missed my point," she tells Brad. In her opinion, the point is not the correctness of arbitrage-free option pricing, but rather the need to identify options for which Xinhua will earn a risk premium for going short volatility. "Xinhua is willing to bear risk – in fact, we are well-positioned to bear risk – but we want to bear priced risk rather than unpriced risk when we write options." In frustration, Ms Yang contacted Brad at Wright Derivatives.

Ms Yang has specifically asked Brad to structure a cross-currency interest rate swap based on three guiding ideas:

- Xinhua will again receive, in Yen, the floating 1-year Libor Yen rate + 125 bp on a JPY 50 billion notional and will pay, in US dollars, the floating 1-year Libor dollar rate + a base spread on a comparable USD notional minus the payoffs from some new set of embedded options to reduce its USD base spread. All payments will be annual and will commence in year 2 and terminate in year 7.
- The embedded options will be structured with a better risk/reward trade-off for Xinhua than the Libor USD straddles in the Wrong & Co. structure. In particular, Ms Yang wants to write options with high prices because they include a high

[2] There is indeterminacy since both the notional and the base spread need to be chosen on the USD side of the swap. For simplicity, consider using as the "comparable USD notional" the amount that would equate values in a plain vanilla swap between Yen and USD floating payments with no spreads or options. Of course you can use something else as the USD notional if that makes more sense in your specific structure.

implicit risk premium rather than simply because they have a high fair probability of expiring in the money.

- If possible, the swap should be structured so that Xinhua's maximum possible coupon on the USD leg is capped in some absolute or relative way. "My internal political insurance policy," is how Ms Yang described this feature. "If US rates go against me, I don't want my interest expense to explode."

Brad's initial thoughts about the deal: Option pricing theory summarizes the combined impact of objective event probabilities and market risk premia on derivative pricing in terms of so-called risk neutral probabilities. Thus, Brad is basically looking for events where the implied RN probabilities differ from the objective probabilities. By writing options on events where the differential between the RN and objective probabilities goes the right way, he can help Ms Yang earn a risk premium which can then be used to reduce the base spread on the dollar leg of the swap.

Brad's option pricing model will let him simulate the risk neutral probability distributions for various events. To gain insights into the objective probabilities, Bran consults with the in-house economists at Wright Derivatives about their fundamental forecasts. From these conversations, Brad determines that the "smart money" does not have strong views about discrepancies between RN and objective probabilities for future USD term structure dynamics. However, big things may be afoot for the Yen term structure along two dimensions. First, the aforementioned recent signs of a reinvigorated post-deflation Japanese economy have widened the yield spread between 3-year and 1-year Libor Yen rates. However, WD's macroeconomic models of the Japanese economy require an unusually large risk premium to reproduce an interest rate term structure even half as steep as the current curve. Specifically, yield spreads larger than 20 basis points for the 3-year and 1-year Libor Yen zero rates seem excessive to the WD macroeconomists. Second, there is always the tried-and-true strategy of trading on possible differences between implied volatilities and statistical volatility estimates. (More details about the volatility estimates are at the very end of the case.) Given all this, Brad decides to focus his search for risk premia, of both varieties, on the yield spread between the 3-year and 1-year Libor Yen zero rates.

The heavy lifting: A good option pricing model is essential when trading exotic cross-currency interest rate options. For this deal Brad will need a model of the risk neutral zero-coupon term structure dynamics for Libor Yen, a model for the risk neutral zero-coupon term structure dynamics for Libor dollar, and a model for the risk neutral USD/JPY exchange rate dynamics. Since his anticipated swap structure specifically involves options on the future "3-year minus 1-year" Yen zero yield spreads, he decides to use a two-factor interest rate model for Japan. Given that, he will use a two-factor model for the US as well. Ultimately the deal will be priced and presented in USD terms.

Modeling the USD term structure: Let F_{tT}^{USD} denote instantaneous forward rate that can be locked in at date t for a future date T and let

$$(1) \qquad dF_{tT}^{USD} = m(t, T)\, dt + s_1(t, T)\, dz_{1t}^{USD} + s_2(t, T)\, dz_{2t}^{USD}$$

denote the risk neutral dynamics for the change over the next dt in the forward rate for period T – i.e., holding the maturity date T fixed and allowing the date t on which the forward rate is set to move ahead. The factor "loadings," $s_1(t, T)$ and $s_2(t, T)$, give the sensitivities of the change in the forward rate for date T (as of date t) to two independent Brownian motion "factors" dz_{1t}^{USD} and dz_{2t}^{USD}. The HJM model then gives the no-arbitrage RN forward rate drift $m(t, T)$ corresponding to the factor loadings. Given functional forms for $s_1(t, T)$ and $s_2(t, T)$, Brad can simulate paths of different maturity zero rates over time either by evolving term structures of forward rates or, equivalently, by evolving term structures of zero-coupon bond prices.

The loading functions $s_1(t, T)$ and $s_2(t, T)$ control the extent to which different maturity forward rates react differently to factors dz_{1t}^{USD} and dz_{2t}^{USD} and, thus, they control how the shape of the term structure of interest rates – and, in particular, the 1-year and 3-year zero rates and the corresponding spread between them – will change over time. Choosing a functional form for the factor loadings and calibrating them is a key step in Brad's option valuation. WD Research suggests using forward rate factor loadings to model US interest rates of the form:

(2) $\quad s_1(t, T) = a(t)\, h_1(T\text{-}t) = a(\mathrm{t})\, \phi$

(3) $\quad s_2(t, T) = a(t)\ \ h_2(T\text{-}t) = a(\mathrm{t})\, (\exp[-2*(T\text{-}t\text{-}1)] - 0.5)$

where $a(t)$ can take different values in different years t (but is the same for factors 1 and 2 within any given year t) and where $h_1(T\text{-}t) = \phi$ and $h_2(\mathrm{T\text{-}t}) = \exp[-2*(T\text{-}t\text{-}1)] - 0.5$ determine the relative mix of the two shocks on different forward rates.[3] Given this recommendation, Brad needs to calibrate the constants $a(t_1)$ through $a(t_6)$ for the next six[4] years t_1 through t_6 and also calibrate the constant ϕ. To calibrate his US interest rate dynamics, Brad has the current Libor dollar term structure in Exhibit 3, the market prices for Libor dollar caplets in Exhibit 4, and some statistical correlation estimates in Exhibit 2.

Aside: The specification of interest rate dynamics in terms of forward rates in (1) is simply for clarity about the definition of the forward rate factor loadings. Associated with these RN forward rate dynamics are corresponding RN dynamics for bond prices. Either representation can then be discretized to simulate term structures over time.

Modeling the Yen term structure: WD Research recommends using the same functional forms for the Yen forward rate factor loadings

(4) $\quad k_1(t, T) = a(t)\, h_1(T\text{-}t) = b(\mathrm{t})\, \phi_{\mathrm{Yen}}$

(5) $\quad k_2(t, T) = a(t)\ \ h_2(T\text{-}t) = b(\mathrm{t})\, (\exp[-2*(T\text{-}t\text{-}1)] - 0.5)$

[3] Note that the function $\exp[-2*(T\text{-}t\text{-}1)] - 0.5$ is between +0.5 and -0.5 for $T\text{-}t \geq 1$ where $T\text{-}t = 1$ year is the shortest maturity forward rate given an annualized time step.

[4] The final payment in year 7 will depend on interest rates and spreads set in year 6.

on two different random factors, dz_{1t}^{JPY} and dz_{2t}^{JPY}. The calibration of the Yen b's and ϕ_{Yen} will, however, differ from the USD calibration. Exhibits 2 through 4 also provide market data and historical statistical information for the JPY calibrations.

Modeling the USD/JPY spot exchange rate: Once he has the risk neutral spot interest rate dynamics, Brad will need to simulate the corresponding RN spot USD/JPY exchange rate dynamics. The current Yen/USD exchange rate is 117 Yen per USD (which will need to be converted into USD/Yen). Brad estimates that the market implied (proportional) volatility for USD/JPY is 11.75 percent which is well within the normal range around the current statistical volatility estimate. "No risk premium there," thinks Brad.

Other correlations: All of the random variables in the case are potentially correlated. Since you already have plenty to do, you can make some simplifying assumptions about these other correlations in your numerical analysis. In particular, assume that the interest rate and FX innovations and also the cross-currency interest rate factors are all uncorrelated. Be forewarned, however: Qualitative questions about these correlations are fair game in the Q&A.

The WD volatility forecasts (revisited): Given the many potential numerical pitfalls in this case, you should assume that when you talked to the WD economists, their objective Yen yield spread volatility forecast happened to be exactly <u>one half</u> of the average of your six calibrated RN Yen yield spread volatilities over the life of the swap. In addition, assume that the objective forecast for the Yen 1-year zero rate volatility was only slightly less than its calibrated RN counterparts. Remember that the volatilities we are talking about here are for the <u>levels</u> of the yield spreads at each date.

In the real world, you would have specific numbers for the macroeconomic forecasts of the Yen yield spread volatility which you would compare with your calculations of the RN Yen yield spread volatilities. Again, the point here is to insure that the "optimal" structure for Ms Yang will be largely independent of any differences in parametric calibration and/or numerical mistakes you might make.

Comments about modeling and numerical issues:

- More ambitious numerical implementations are always preferred to less ambitious implementations, but don't go overboard numerically! Your priority should be on having a valuation that is logically sound and on understanding and being able to explain the implications of any numerical or modeling shortcuts. For example, discretizations with annual time steps are fine.
- You can use any two-factor term structure model of your choice given the data in the case.
- If additional background information is needed for your sales pitch, feel free to make up things as long as they are plausible and consistent with the spirit of the

case. Your numerical analysis should, however, stay within the parameters of the case.

Preliminary technical questions: The general Q&A will begin with a few questions to help the judges assess the technical accuracy of your analysis independent of the particular structure you are proposing to Ms Yang.

- What are your calibrated values of the US *a*s and ϕ and the Yen *b*s and ϕ_{yen}?
- What is your term structure of volatility for the 3-year US zero rates? In particular, what is the standard deviation of your distribution of simulated 3-year Libor dollar rates for year 1, year 2, …, and year 6 (i.e., the levels of the rates, not the changes in rates)?
- What is the corresponding term structure of volatility for the 3-year Libor Yen rates? For the Yen yield spread?

Some other considerations: Here are some additional issues to consider in this deal.

- There is no liquid market for Yuan-linked derivatives, but the prices of comparatively illiquid, cash-settled "non-deliverable forwards" on the Yuan suggest the FX market thinks the Yuan is overvalued at current peg. If the Chinese government were to allow the Yuan to float more freely against the US dollar, how would that affect the qualitative logic behind this deal?
- What risk exposures does Wright Derivatives have in this deal? How can it hedge them?

Competition Format: The format is the same as in previous years.

- You should give Ms Yang (and the judges) a short written "term sheet" outlining the key terms of your proposal (e.g., pricing, payoff rule) at the start of your presentation.
- Each presentation will begin with a 10-15 minute formal sales pitch addressed to the client. Ms Yang, while knowledgeable about derivatives, is primarily interested in how your proposal will help solve her business problem. Questions from the judges during the sales pitch will be from the client's perspective.
- A single spokesperson can represent the team or more than one presenter can be involved as you see fit. Most presentations are in PowerPoint, but you may use other media if you choose.
- Following the sales pitch, the judges will have 25-30 minutes for general Q&A about the details of the modeling and technical implementation and about the profitability of the deal and risk management issues at Wright Derivatives. In the Q&A, the judges are no longer restricted to the role of a client.
- The judges may ask questions about alternative parameter values. Bring your spreadsheet (or whatever numerical package you use) to the presentation so you can plug them in your model and discuss them.

Judging criteria: Teams will be evaluated on business intuition, marketing quality, structuring creativity, and technical proficiency. Some specific considerations are:

- *Salesmanship.* The sales pitch should focus on the "forest" (the business drivers behind your proposal) rather than the "trees" (modeling details).
- *Pricing and structure.* Completing the analysis (i.e., structuring and pricing your proposal) is clearly a necessary first step. You should also be able to explain the pros and cons of your modeling choices.
- *Clarity.* When working with technically complicated financial products, clarity and intuition are vital.

Good luck!

Exhibit 1
Key terms for outstanding yen-denominated debt of the Xinhua Development Company

Currency:	Japanese yen
Principal:	JPY 50 billion
Remaining time to maturity now:	7 years
Coupon:	Floating, payable annually
Coupon due in year t:	125 b.p. + 1 year Libor Yen rate at end of year t-1
Prepayment:	Not allowed
Sinking fund:	No

Exhibit 2
Statistical correlation estimates

Correlation of changes in the 1-year and 3-year Libor USD rates:	0.81
Correlation of changes in the 1-year and 3-year Libor Yen rates:	0.72

Note: Perfect positive correlation = 1.

Exhibit 3
Current market term structures for 1-year forward rates (1 percent = 1.0)

Years forward	Libor USD	Libor Yen
(spot rate) 0	5.20	0.50
1	5.25	0.80
2	5.13	1.51
3	5.22	1.92
4	5.30	2.19
5	5.35	2.38
6	5.38	2.54
7	5.42	2.64
8	5.47	2.71
9	5.50	2.77

Exhibit 4
Market implied volatilities from Black model (1 percent = 0.01)

Time (years)	Libor Dollar	Libor Yen
1	0.172	0.192
2	0.164	0.165
3	0.155	0.146
4	0.146	0.132
5	0.140	0.122
6	0.136	0.111
7	0.131	0.109

Notes:

- The caplet time convention is best explained by an example: The Black ISD of 0.155 for the "3-year" US caplet is for a caplet with a payoff contingent on the 1-year rate set in 3 years (i.e., for the period between year 3 and year 4). The actual payoff occurs in arrears one year later in year 4. See Hull for more detail.
- All quotes are for caplets struck at-the-money forward. For example, the strike for the "3-year" US caplet is today's 2-year ahead 1-year forward rate (i.e., 5.22 percent from Exhibit 3 which is the rate you can lock in today for the period between year 3 and 4).
- Caplet prices for Libor Dollar cap are quoted in USD. Caplet prices for Libor Yen are in Yen.
- Black model assumes lognormal interest rate. They are relative volatilities wrt the spot interest rate level rather than absolute volatilities. This lognormality is, however, just a quotation convention and is not relevant for the volatility specification in your HJM model.