Answer to Problem Set 2

Ex1.

Answer:

a) Risk averse

As $u'(w) = 0.5w^{-0.5} > 0$
$u''(w) = -0.25w^{-1.5} < 0$

b) Expected monetary value=0.2*22500+0.8*100000=84500

c) $(100000 - x)^{0.5} = 0.2 * 22500^{0.5} + 0.8 * 100000^{0.5}$

X=19921.07

d) Y=0.2*(22500-100000)+0.8*0=15500

Ex2:

Answer:

e) $\underset{K,L}{Min} \quad rK + wL$

s.t. $Q = 10K^{0.8}(L-40)^{0.2}$

F.O.C.

$$\begin{cases} r = 8\lambda K^{-0.2}(L-40)^{0.2} \\ w = 2\lambda K^{0.8}(L-40)^{-0.8} \\ Q = 10K^{0.8}(L-40)^{0.2} \end{cases} \Rightarrow \begin{cases} K = \dfrac{Q}{10}(\dfrac{4w}{r})^{0.2} \\ L = \dfrac{Q}{10}(\dfrac{r}{4w})^{0.8} + 40 \end{cases}$$

The total cost function is:

$$C(w,r,Q) = wL + rK = 40w + 2^{-2.6}Qr^{0.8}w^{0.2}$$

f) $C(32, 64, Q) = 1280 + 2^{3.2}Q$

We could prove $Q(\lambda K, \lambda L) > \lambda Q(K, L)$

Hence, the technology exhibits increasing returns to scale.

g) Q=2000,

The total demand of factors:

$$\begin{cases} K = \dfrac{2000}{10}(\dfrac{4\times 32}{64})^{0.2} \\ L = \dfrac{2000}{10}(\dfrac{64}{4\times 32})^{0.8} + 40 \end{cases}$$

By the assumptions of productivity:

$$k^* = K/40 \approx 6$$
$$l^* = L = 40 \approx 4$$

The Marginal cost is:

$$MC = \frac{dC(32,64,Q)}{dQ} = 2^{3.2}$$

The average cost is

$$AC = \frac{1280}{Q} + 2^{3.2}$$

Ex3:

Answer:

a) MC1=2y1, MC2=8+2y2

MC1=MC2 and y1+y2=24

Then y1=14 and y2=10

If producing in each plant, the total cost will increase as the marginal cost is increasing.

b) For firm1,

$$\underset{y_1\geq 0}{Max}\, py_1\text{-}100\text{-}y_1^2 \;\Rightarrow\; y_1 = \frac{1}{2}p(p \geq 0)$$

For firm 2,

$$\underset{y_2\geq 0}{Max}\, py_2\text{-}16\text{-}8y_2\text{-}y_2^2 \;\Rightarrow\; y_2 = \begin{cases} \frac{1}{2}(p-8) & if\;\; p \geq 8 \\ 0 & if\;\; 0 \leq p \leq 8 \end{cases}$$

c) When the price is 6, y2<0 and only firm 1 is active.

Ex4:

Answer:

a) L=Q/10

TC=10+2L=10+Q/5

AC=TC/Q=10/Q+1/5

AVC=1/5

AFC=10/Q

MC=1/5

b)

$$\underset{K,L}{Min}\ K+2L$$

s.t. Q=KL

F.O.C.

$$\begin{cases} K^* = \sqrt{2Q} \\ L^* = \sqrt{\dfrac{Q}{2}} \end{cases}$$

the long run cost function is:

$$C(Q) = 2\sqrt{2Q}\ ,\quad AC(Q) = 2\sqrt{2/Q}$$

Ex5:

Answer:

a) The equilibrium price satisfies that

10-p=4+p

$(p^*, Q\ ) = (3,7)$

b) By the introduction of Tax, different kinds of Tax would lead to various results.

t=1

Consumption tax

$$\begin{cases} p + t_c = 10 - Q \\ p = Q - 4 \end{cases} \Rightarrow \quad \begin{cases} p_c = 2.5 \\ Q_c = 6.5 \end{cases}$$

the consumer should pay p+t=3.5 per good, and the firm get p=2.5 per good.

Production tax

$$\begin{cases} p - t_p = Q - 4 \\ p = 10 - Q \end{cases} \Rightarrow \quad \begin{cases} p_p = 3.5 \\ Q_p = 6.5 \end{cases}$$

the consumer should pay p =3.5 per good, and the firm get p-t=2.5 per good.

6.5*1000=6500

It is obvious that both of them could offer the same result.

c) Subsidy case d=1

$$\begin{cases} p + d_p = Q - 4 \\ p = 10 - Q \end{cases} = \quad \begin{cases} p_d = 2.5 \\ Q_d = 7.5 \end{cases}$$

The total cost of gov. is

$C(Q,d)=dQ=7.5*1000=7500$