Module 8: Valuing Bonds with Embedded Options

Roadmap

- Major learning outcomes:
  - Be able to calculate nominal, static (z) spread, and understand option adjusted spread (OAS).
  - Understand comparative statics of bonds with embedded options.
  - Understand how callable /putable/convertible bonds are valued using binominal tree method.

Relative Valuation Measures

- The potential benchmark interest rates that can be used in bond valuation are those in Treasury market, a specific bond sector with a given credit rating, or a specific issuer.
- Benchmark interest rates can be based on either an estimated yield curve or an estimated spot rate curve.

Relative Valuation Measures

- Relative value analysis is used to identify securities as being overpriced, underpriced, or fairly priced relative to benchmark interest rates.
- Yield spread measures are used in assessing relative value of securities. Their interpretation depends on benchmark interest rates coming from
  - Treasury securities, or
  - A specific bond sector with similar credit risk, liquidity, and maturity characteristics, or
  - Issuer’s own (other) bonds.

Traditional Yield Measures

- Nominal spread
- Static (z) spread
- Yield to worst, i.e., smallest of:
  - Yield to maturity
  - All yields to calls and puts
- These measures do not consider the effect of embedded options (reinvestment or call features).

Nominal Yield Spread

- Nominal spread is a spread relative to yield curve.

$$\text{nominal spread} = YTM_{GM} - YTM_{Tbond}$$

  - Nominal yield spread measures the compensation for additional credit risk, option risk, and liquidity risk an investor is exposed to by investing in a non-Treasury security with the same maturity.
- Nominal spread fails to consider term structure of spot rates and the fact that, for bonds with embedded options, future interest rate volatility may alter its cash flows.

Nominal Yield Spread

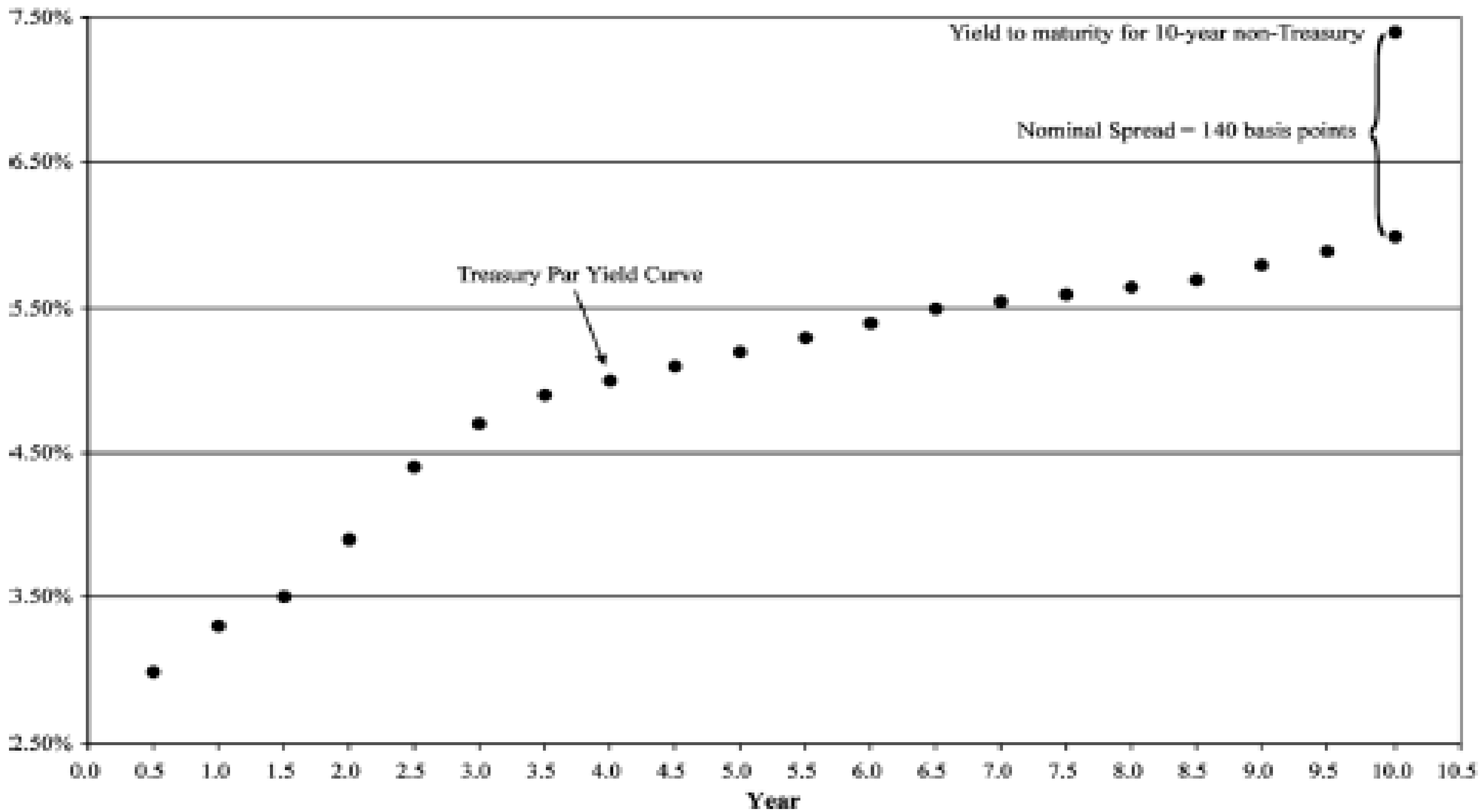


EXHIBIT 7 Illustration of the Nominal Spread

Zero-Volatility Spread

- Zero-volatility or Z- spread is a measure of spread the investor would realize over <u>entire</u> Treasury spot rate curve if bond is held to maturity.
  - It is not the spread of one point on Treasury yield curve (nominal spread), it is an average over all spot rates.
- Z-spread is also called a static spread which makes present value of cash flows from non-Treasury bond, when discounted at Treasury spot rate plus spread, equal to non-Treasury bond's price.
- Exhibits 8 and 9 show how trial and error is used to compute Z-spread

Zero-Volatility Spread

EXHIBIT 8 Determining Z-Spread for an 8% Coupon, 10-Year Non-Treasury Issue Selling at $104.19 to Yield 7.4%

Period	Years	Cash flow ($)	Spot rate (%)*	Present value ($) assuming a spread of**		
				100 bp	125 bp	146 bp
1	0.5	4.00	3.0000	3.9216	3.9168	3.9127
2	1.0	4.00	3.3000	3.8334	3.8240	3.8162
3	1.5	4.00	3.5053	3.7414	3.7277	3.7163
4	2.0	4.00	3.9164	3.6297	3.6121	3.5973
5	2.5	4.00	4.4376	3.4979	3.4767	3.4590
6	3.0	4.00	4.7520	3.3742	3.3497	3.3293
7	3.5	4.00	4.9622	3.2565	3.2290	3.2061
8	4.0	4.00	5.0650	3.1497	3.1193	3.0940
9	4.5	4.00	5.1701	3.0430	3.0100	2.9825
10	5.0	4.00	5.2772	2.9366	2.9013	2.8719
11	5.5	4.00	5.3864	2.8307	2.7933	2.7622
12	6.0	4.00	5.4976	2.7255	2.6862	2.6536
13	6.5	4.00	5.6108	2.6210	2.5801	2.5463
14	7.0	4.00	5.6643	2.5279	2.4855	2.4504
15	7.5	4.00	5.7193	2.4367	2.3929	2.3568
16	8.0	4.00	5.7755	2.3472	2.3023	2.2652
17	8.5	4.00	5.8331	2.2596	2.2137	2.1758
18	9.0	4.00	5.9584	2.1612	2.1148	2.0766
19	9.5	4.00	6.0863	2.0642	2.0174	1.9790
20	10.0	104.00	6.2169	51.1835	49.9638	48.9632
			Total	107.5416	105.7165	104.2146

Zero-Volatility Spread

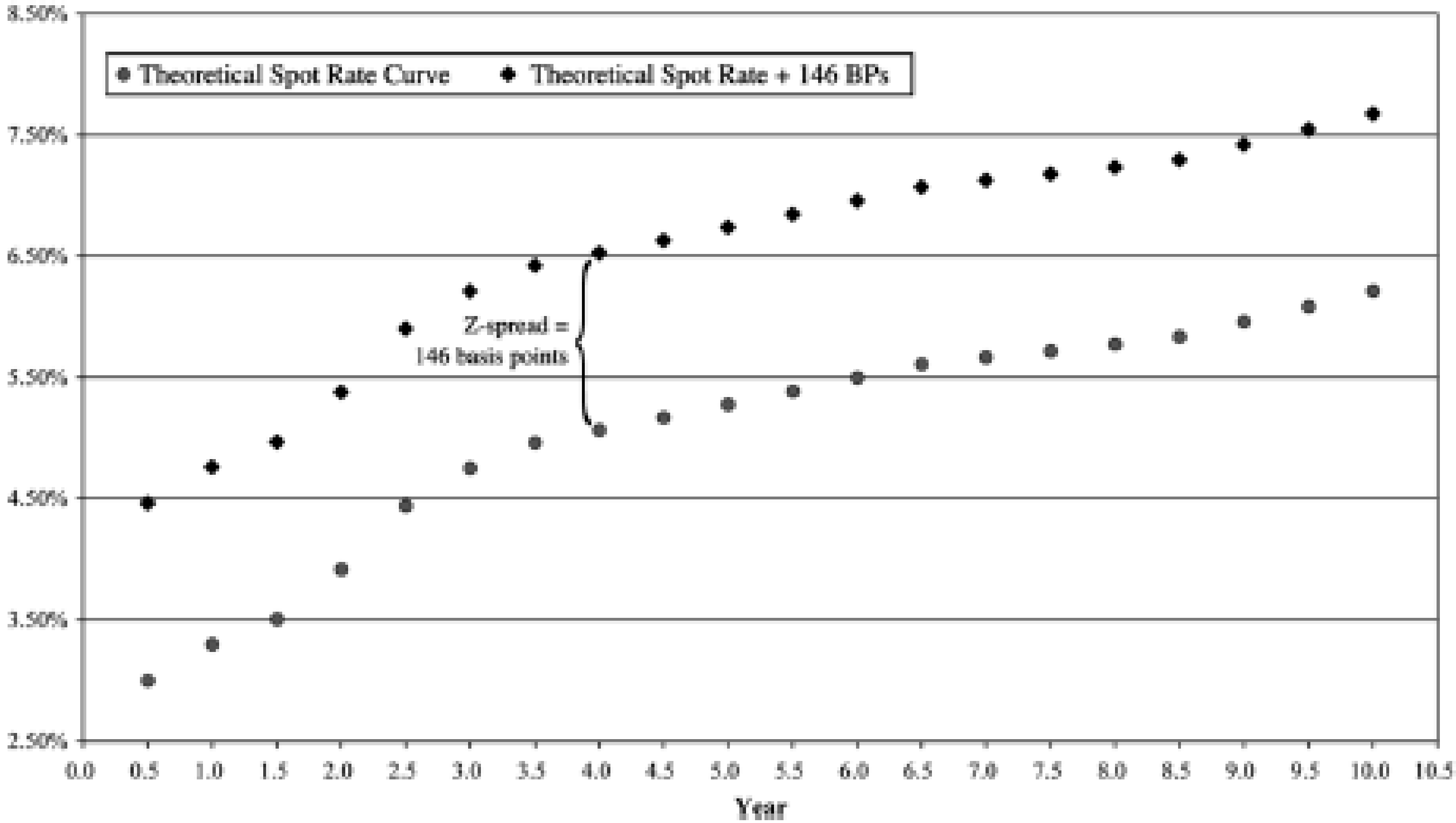


EXHIBIT 9 Illustration of the Z Spread

Zero-Volatility and Nominal Spread

- Z- and nominal spreads will not differ much for coupon-paying straight bonds. They differ if
  - Slope of term structure is steep
  - Principal is paid off before maturity (i.e. mortgage- and asset-back bonds)

Option Adjusted Spread (OAS)

- OAS was developed to take dollar difference between fair valuation and market price and convert it to a yield spread measure.
  - OAS is used to reconcile fair price (value) and market price by finding a spread that will equate the two.
  - The spread is measured in basis points.

Option Adjusted Spread (OAS)

- OAS depends upon valuation model employed.
  - OAS models primarily differ in how they forecast interest rate changes.
- What are the key modeling differences?
  - Interest rate volatility is a crucial assumption. The higher the interest rate volatility, the lower OAS.
  - OAS is a spread over Treasury spot rate curve or issuer's benchmark. Spot rate curve is the result of a series of assumptions that allow for changes in interest rates.

Option Adjusted Spread and Z-Spread

- Z-spread, which looks at measuring spread over a spot rate curve, has a problem in that it ignores how interest rate changes can impact cash flows – which is why it is referred to as zero-volatility
- OAS is referred to as “option adjusted” because bond’s embedded options can change cash flows and therefore value of security.

Summary of Spread Measures

Spread Measure	Benchmark	Reflects compensation for:
Nominal Spread	Treasury yield curve	Credit risk, option risk, liquidity risk
Zero Spread	Treasury spot rate curve	Credit risk, option risk, liquidity risk
Option-Adjusted Spread	Treasury spot rate curve	Credit risk, liquidity risk

Example 1

- Benchmark: Treasury market rates
- BBB corporate bond with an embedded option.
- Nominal spread between BBB bond and benchmark = 170 bps
- Nominal spread between option-free BBB's and benchmark = 145 bps
- Z-spread = 160 bps
- OAS = 125 bps
- Is the bond cheap, rich, or fair?

Example 1

- Bond has an embedded option, so OAS should be used.
- The only comparison that can be made is OAS of 125 bps to nominal spread for option-free BBB's of 145.
- Based on this comparison, bond is rich (i.e., overvalued).
  - Recall that z-spread and nominal spread are typically close, given a reasonably sloped yield curve.

Example 2

- Benchmark: AA-rated callable corporate bonds
- BBB corporate bond with an embedded option.
- Nominal spread between BBB bond and benchmark = 110 bps
- Nominal spread between benchmark and option-free AA's = 90 bps
- Z-spread = 100 bps
- OAS = 80 bps
- Is the bond cheap, rich, or fair?

Example 2

- The bond has an embedded option, so OAS should be used.
- Only comparison that can be made is OAS of 80 bps to nominal spread for option-free AA's of 90.
- Based on this comparison, bond is rich (i.e., overvalued).
  - Recall that z-spread and nominal spread are typically close, given a reasonably sloped yield curve.

Example 3

- Benchmark: Rates on other issues from specific issuer of our bond.
- BBB corporate bond with an embedded option.
- Nominal spread between BBB bond and benchmark = 30 bps
- Nominal spread between benchmark and option-free AA's = 90 bps
- Z-spread = 20 bps
- OAS = -25 bps
- Is the bond cheap, rich, or fair?

Example 3

- Bond has an embedded option, so OAS should be used.
- Only measure we need to look at is OAS.
- Sine OAS is negative, bond is rich (i.e., overvalued).

Bond Valuation Model

- A valuation model must produce arbitrage-free values; that is, producing a value for each on-the-run issue that is equal to its market price.
- There are several arbitrage-free models that can be used to value bonds with embedded options but they all follow same principle
  - they generate a tree of interest rates based on some interest rate volatility assumption
  - they require rules for determining when any of embedded options will be exercised,
  - and they employ backward induction methodology.

Bond Valuation Model

- A valuation model involves generating an interest rate tree based on

  (1) benchmark interest rates,

  (2) an assumed interest rate model, and

  (3) an assumed interest rate volatility.

Binomial Model

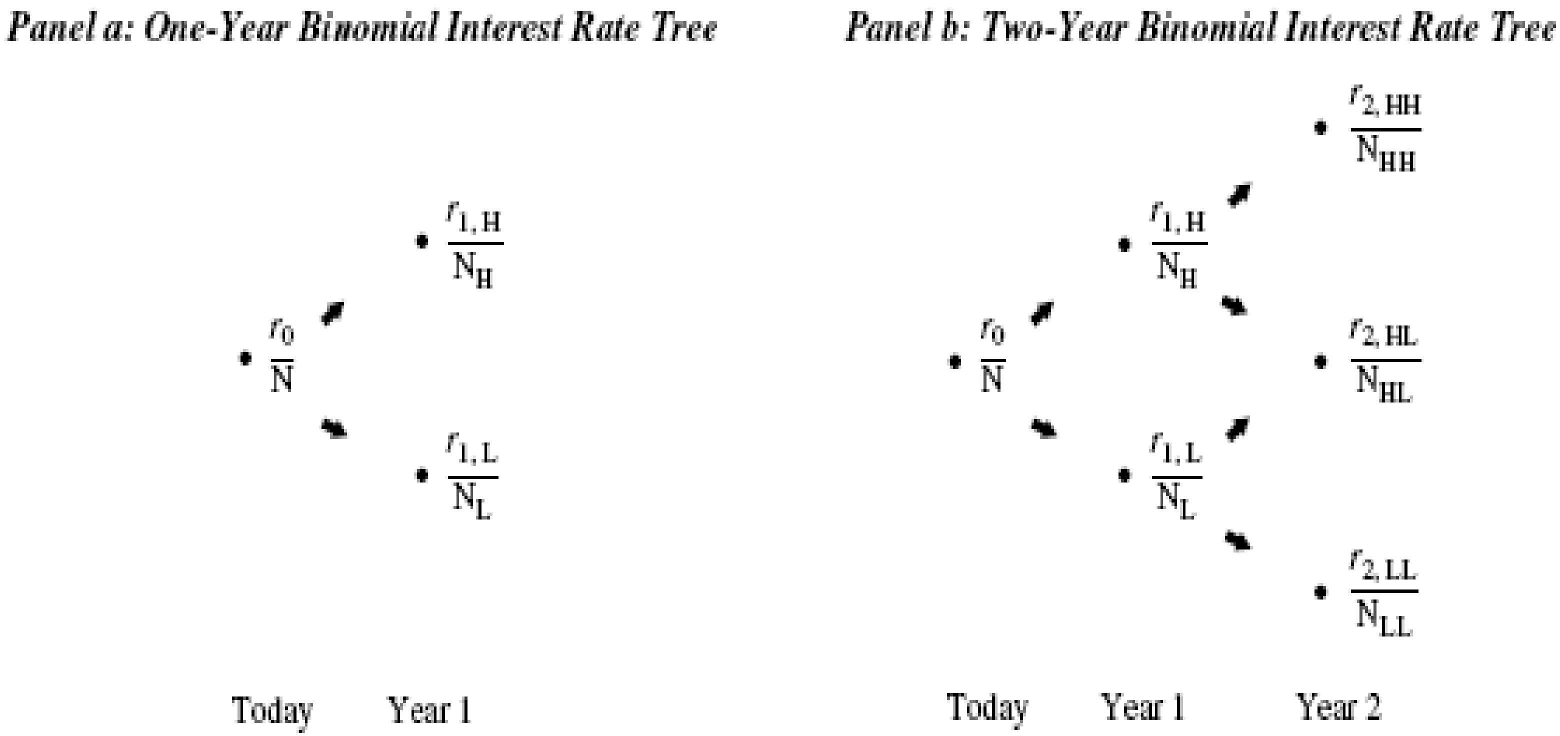


EXHIBIT 2 Binomial Interest Rate Tree

Binomial Model

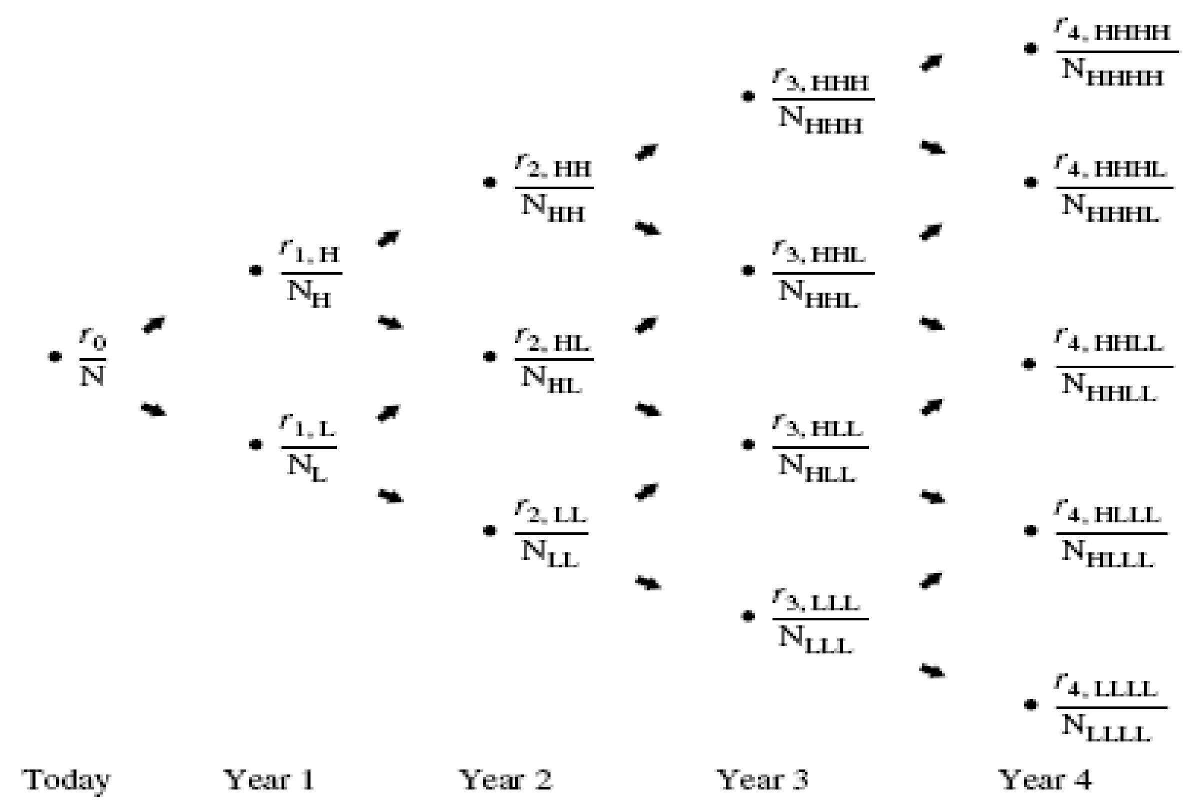


EXHIBIT 3 Four-Year Binomial Interest Rate Tree

Binomial Model

- Assumed volatility of interest rates incorporates uncertainty about future interest rates into analysis.
- Interest rate tree is constructed using a process that is similar to bootstrapping but requires an iterative procedure to determine interest rates that will produce a value for on-the-run issues equal to their market value.
- At each node of tree there are interest rates and these rates are effectively forward rates; thus, there is a set of forward rates for each year.

Binomial Model

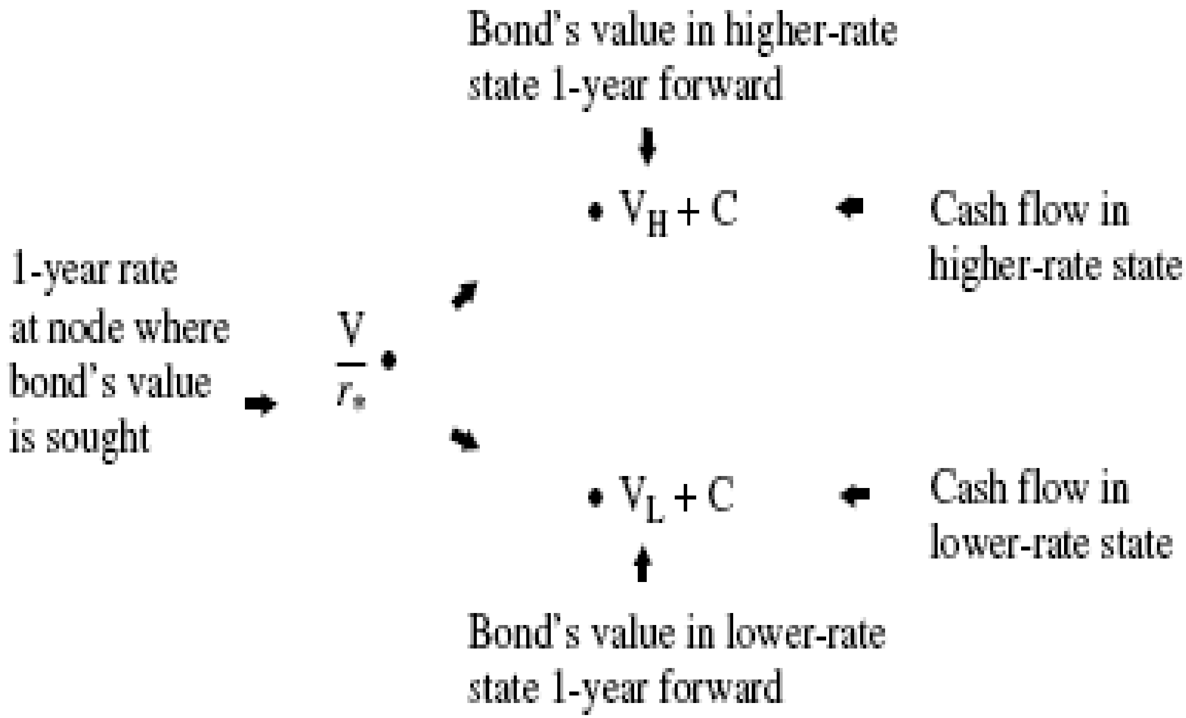


EXHIBIT 4 Calculating a Value at a Node

Binomial Model

- Using interest rate tree, arbitrage-free value of any bond can be determined.
- In valuing a callable bond using interest rate tree, cash flows at a node are modified to take into account <u>call option</u>.
- Value of embedded call option is the <u>difference</u> between value of an option-free bond and value of callable bond.

Binomial Model

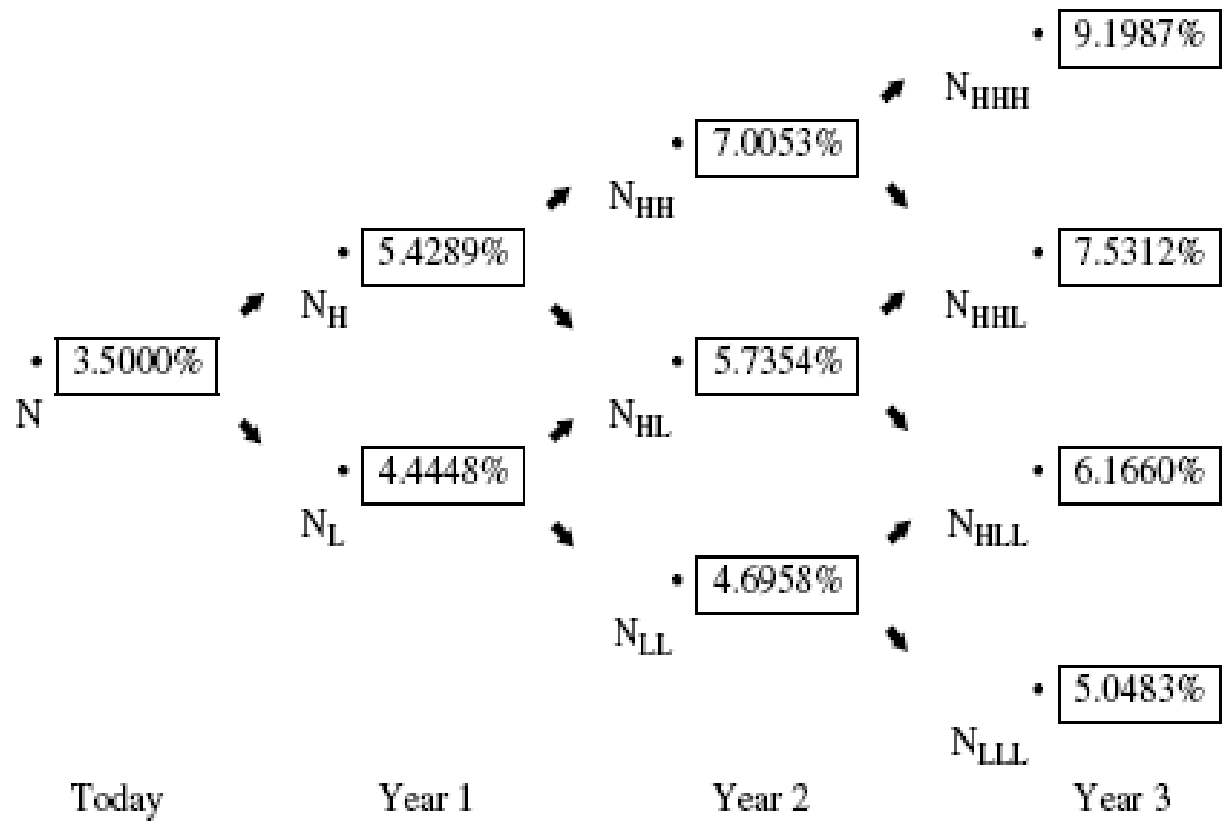


EXHIBIT 5 Binomial Interest Rate Tree for Valuing an Issuer's Bond with a Maturity Up to 4 Years (10% Volatility Assumed)

Binomial Model

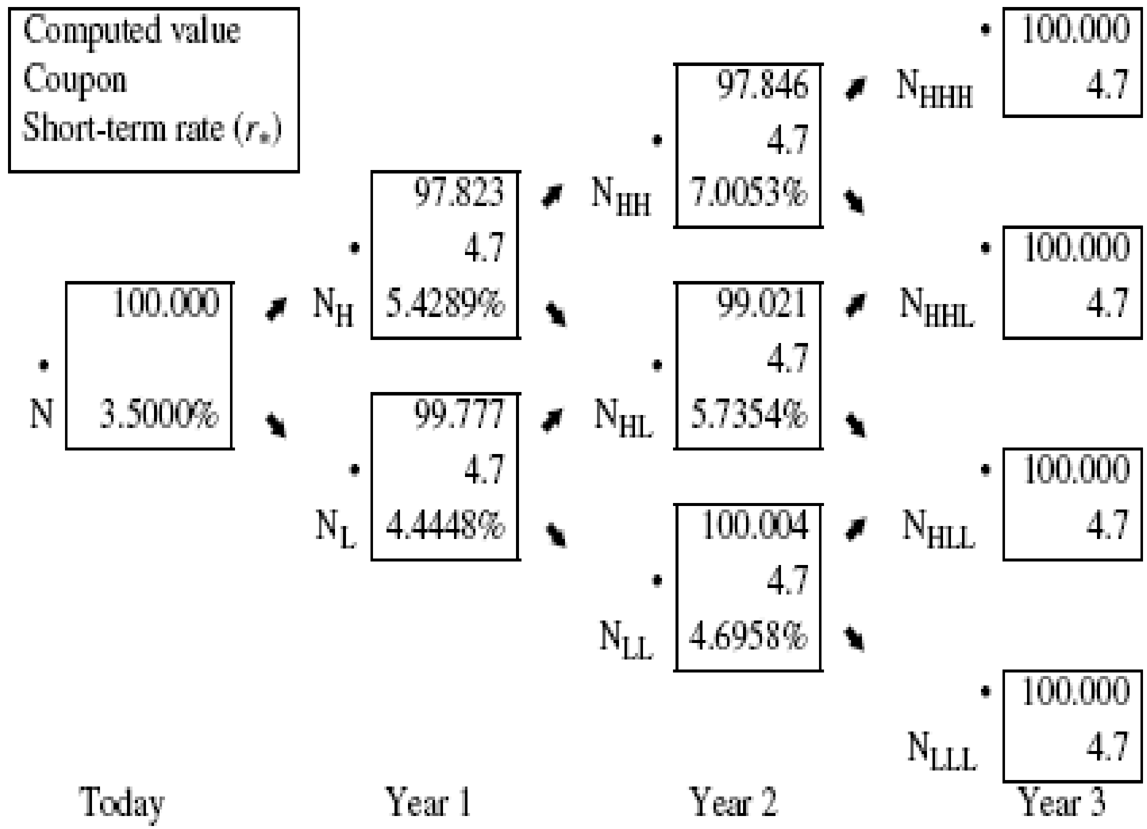


EXHIBIT 6 Demonstration that the Binomial Interest Rate Tree in Exhibit 5 Correctly Values the 3-Year 4.7% On-the-Run Issue

Binomial Model

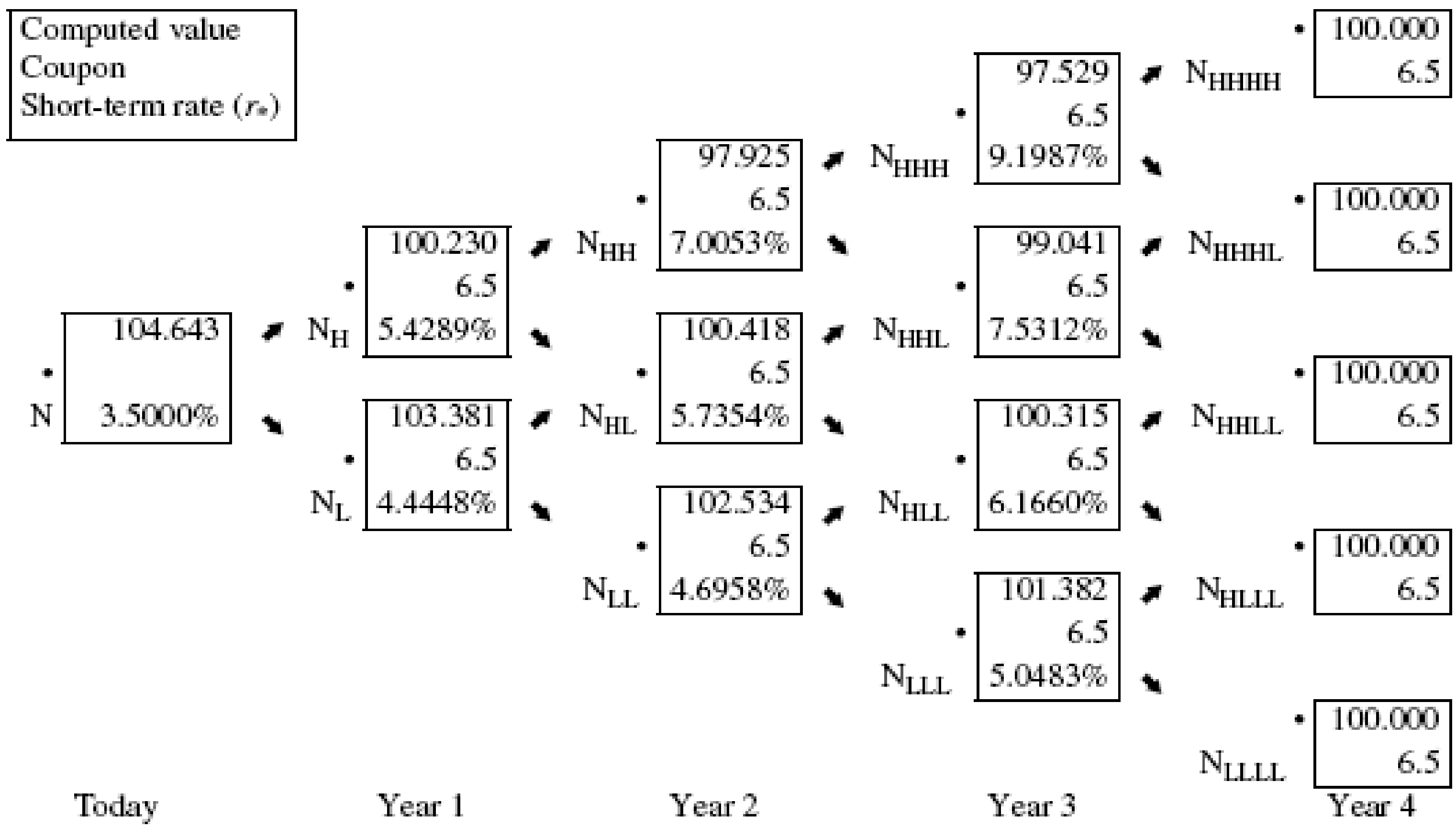


EXHIBIT 7 Valuing an Option-Free Bond with Four Years to Maturity and a Coupon Rate of 6.5% (10% Volatility Assumed)

Valuing Callable Bond

- Valuation process for callable bond is the same as for an option-free bond except
  - when call option may be exercised by issuer, bond value at a node must be changed to reflect the lesser of its values if it is not called (i.e., value obtained by applying backward induction method) and call price.

Valuing Callable Bond

- For example, a 6.5% bond with four years remaining to maturity that is callable in one year at $100.
- Exhibit 8 shows two values at each node of binomial interest rate tree. The second value is based on whether issue will be called. For simplicity, let's assume that this issuer calls issue if it exceeds call price

Valuing Callable Bond

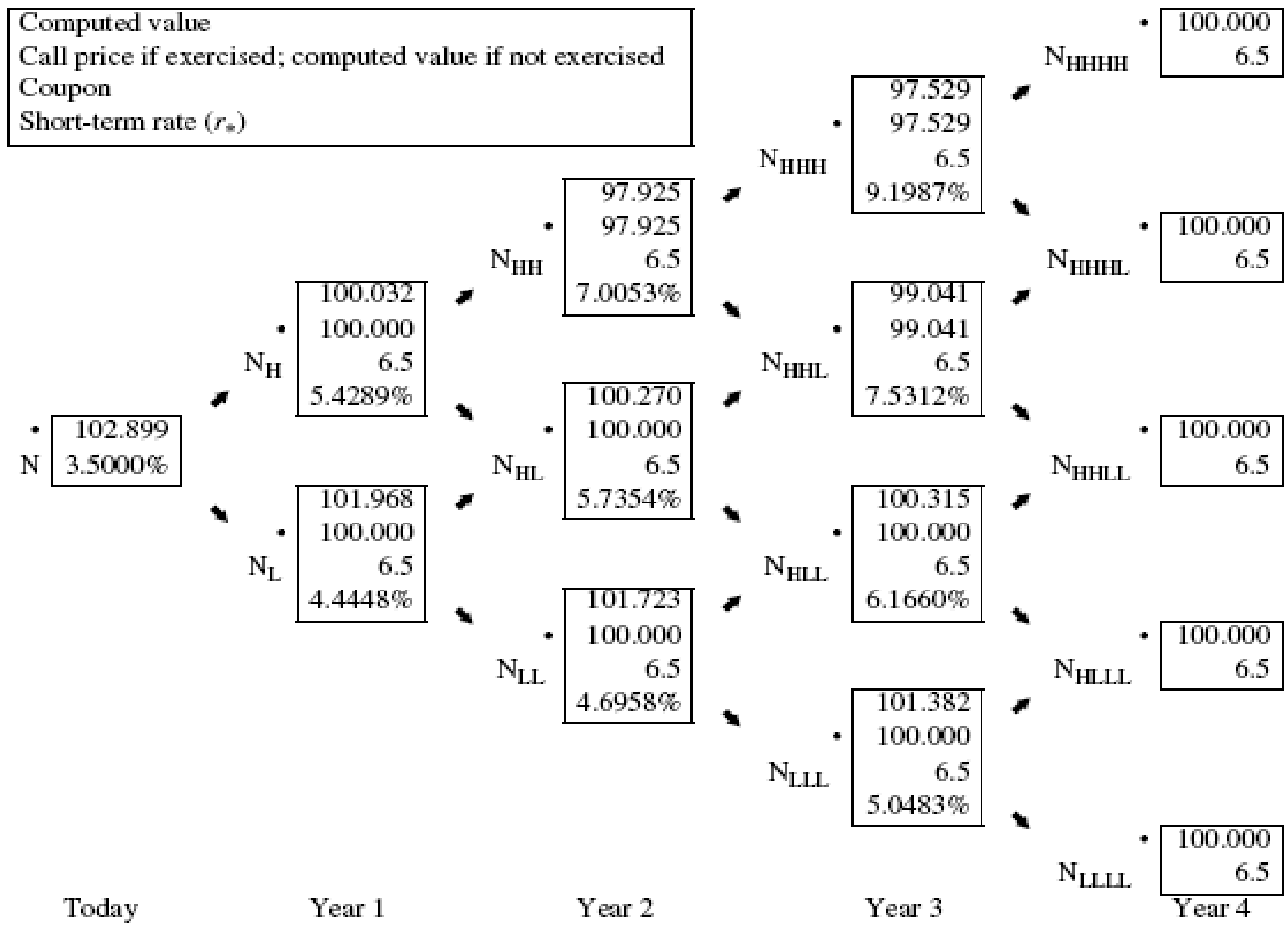
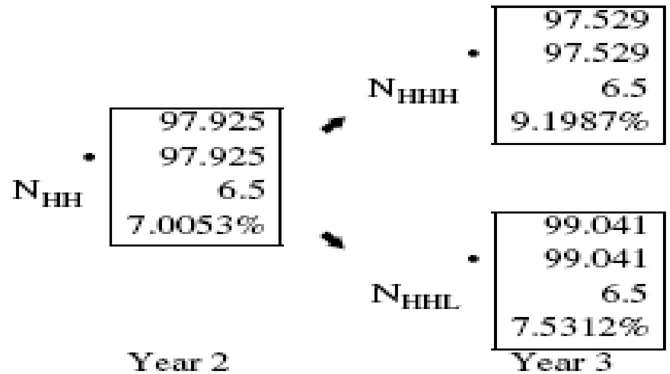


EXHIBIT 8 Valuing a Callable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and Callable in One Year at 100 (10% Volatility Assumed)

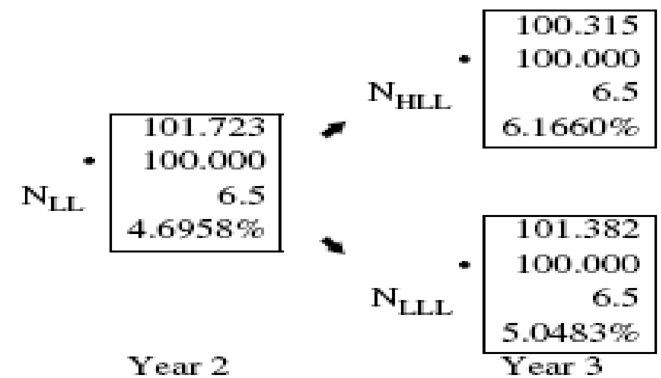
Valuing Callable Bond

- Panel a of Exhibit 9 shows nodes where issue is not called in years 2 and 3. Values are the same as valuation of an option-free bond.
- Panel b shows nodes where issue is called in years 2 and 3. Notice how methodology changes cash flows.
  - In year 3, at node NHLL backward induction method produces a value (i.e., cash flow) of 100.315. Given call rule, this issue would be called. Therefore, 100 is shown as second value at the node and it is this value that is then used in backward induction methodology.

Valuing Callable Bond



a. Nodes where call option is not exercised



b. Selected nodes where the call option is exercised

EXHIBIT 9 Highlighting Nodes in Years 2 and 3 for a Callable Bond

Valuing Callable Bond

- Suppose call price schedule is 102 in year 1, 101 in year 2, and 100 in year 3. Bond will not be called unless it exceeds call price for that year.
- Exhibit 10 shows value at each node and value of callable bond. Call price schedule results in a greater value for callable bond, $103.942 compared to $102.899 when call price is 100 in each year.

Valuing Callable Bond

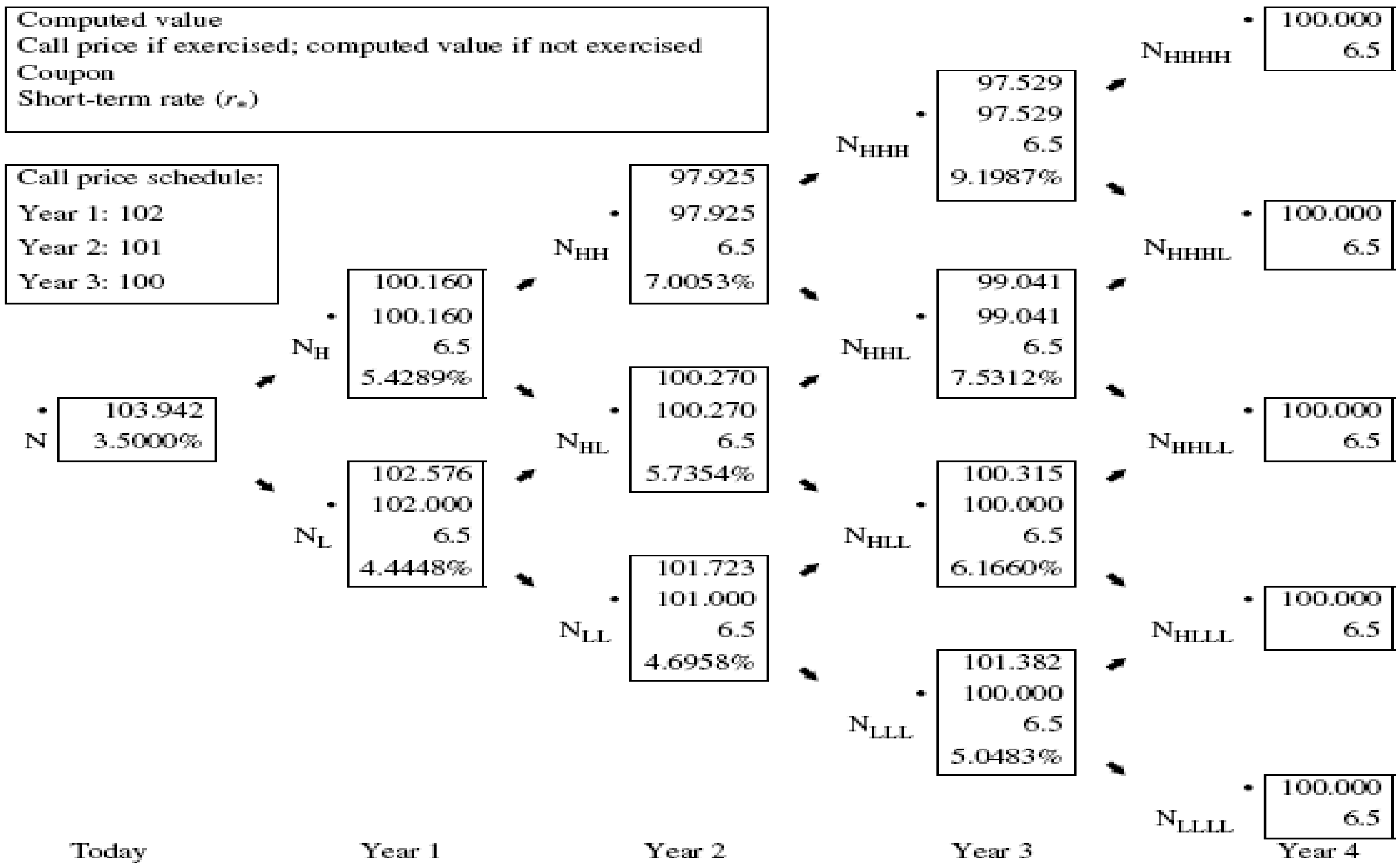


EXHIBIT 10 Valuing a Callable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and with a Call Price Schedule (10% Volatility Assumed)

Value of Call Option

- Value of a callable bond is equal to value of an option-free bond minus value of call option.

$$\text{Value of Call Option} = \text{Value of callable bond} - \text{Value of option-free bond}$$

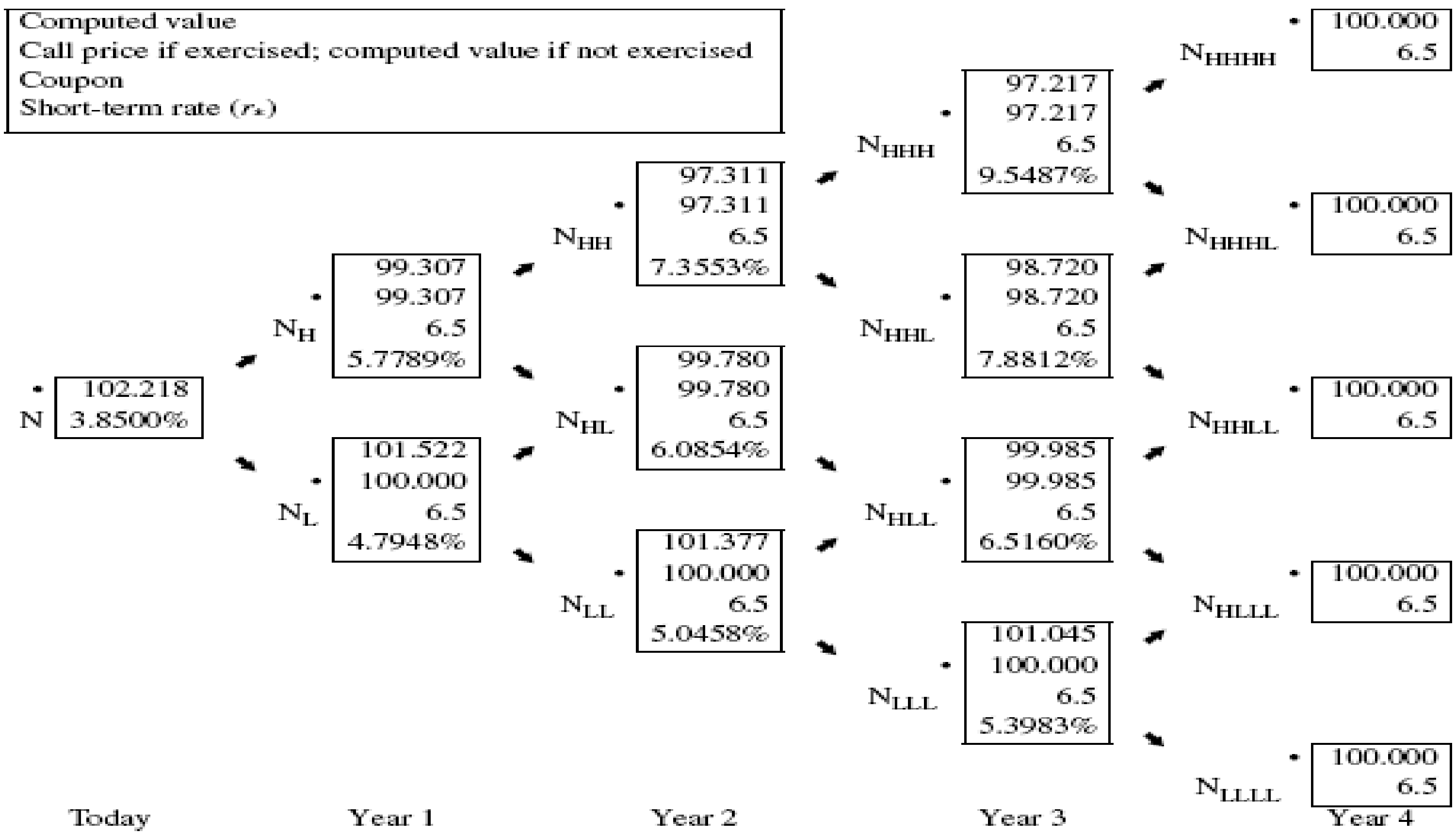
- In our illustration, value of option-free bond is \$104.643. If call price is \$100 in each year and value of callable bond is \$102.899 assuming 10% volatility for 1-year rate, value of call option is \$1.744 (= \$104.643 − \$102.899).

Option-Adjusted Spread

- OAS is constant spread that when added to all1-year rates on binomial interest rate tree that will make arbitrage-free value (i.e., value produced by binomial model) equal to market price.
- In our illustration, if market price is $102.218, OAS would be constant spread added to every rate in Exhibit 5 that will make arbitrage-free value equal to $102.218. The solution in this case would be 35 basis points. This can be verified in Exhibit 11 which shows value of this issue by adding 35 basis points to each rate.

Option-Adjusted Spread

EXHIBIT 11 Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



*Each 1-year rate is 35 basis points greater than in Exhibit 5.

Exhibit 11

- Exhibit 11 demonstrates that OAS is 35 bps.
- Thus, a positive OAS (in this case) is consistent with bond being undervalued.
    - Note: "In this case" because benchmark rates are from this specific issuer, so credit risk and liquidity risk differences are controlled for.

Exhibit 11

- Again assume that market price is 102.218.
- But now, assume that volatility forecast is 20% (rather than 10%).
- At 20% volatility, model value of bond would decrease, and option value would increase.
- But since market price didn't change, <u>OAS must decrease.</u>
- Thus, OAS estimate, and relative value analysis for bonds with embedded options, depend heavily on volatility estimate!

Exhibit 11 Problem

- Can you solve following problem?
- Calculate bond's price in Exhibit 11 if OAS is 50 bps instead of 35 bps.
- Draw binomial tree diagram, as in Exhibit 11, showing computed value and call price if exercised, and adjusted interest rates.

Exhibit 11 Problem

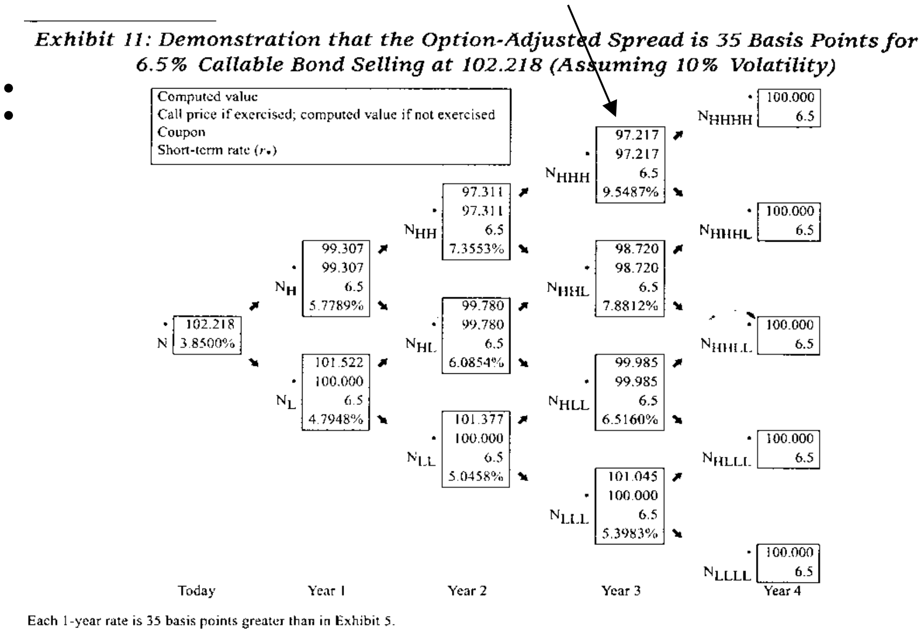
- Node N_{HHH} will be:

97.084

97.084

6.5

9.6987%

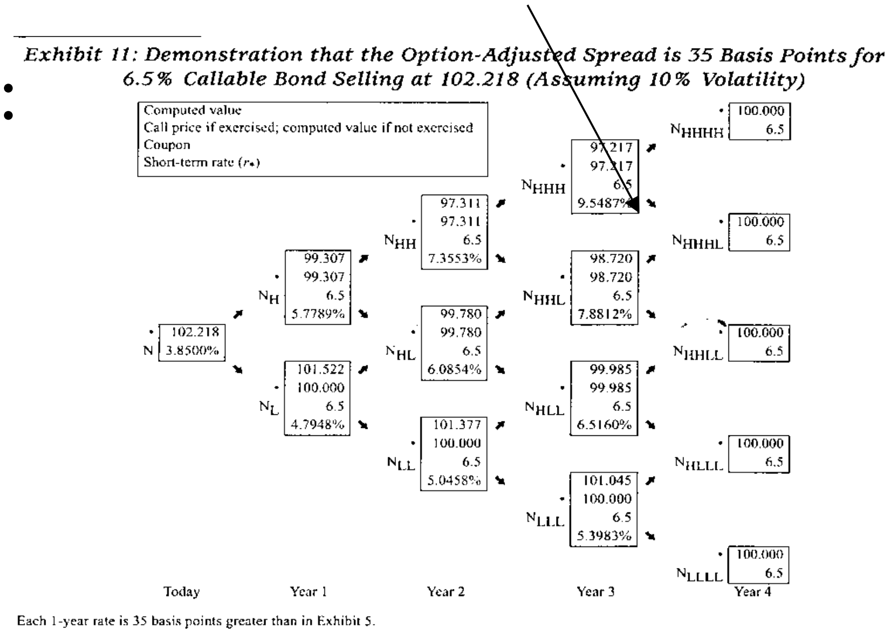


- 9.6987% = 9.5487% + (0.50%-0.35%)
- 97.084 = 106.5/(1.096987) < 100
- Will not be called.

Exhibit 11 Problem

- Node N_{HHL} will be:

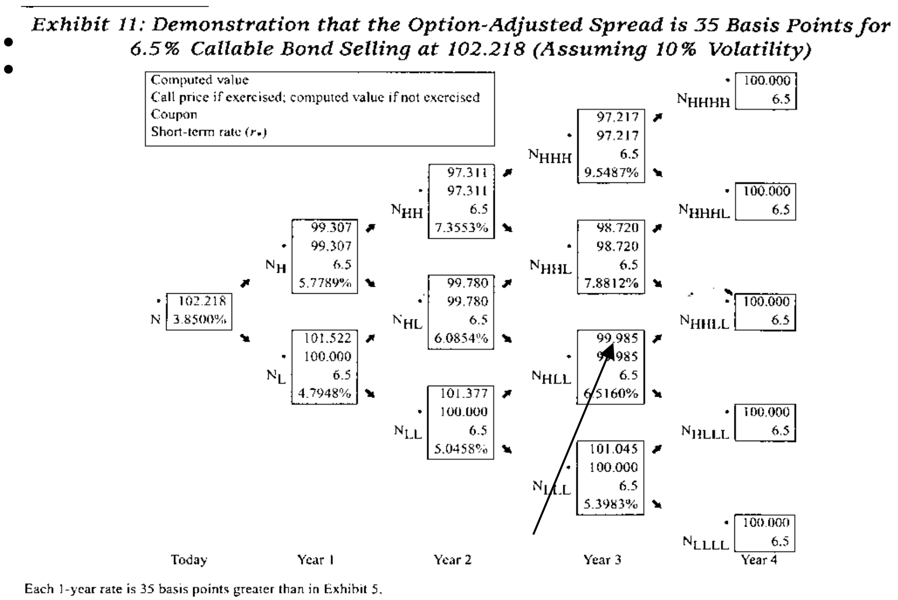
98.327
98.327
6.5
8.0312%



- $8.0312\% = 7.8812\% + (0.50\%-0.35\%)$
- $98.327 = 106.5/(1.08312) < 100$
- Will not be called.

Exhibit 11 Problem

- Node N_{HLL} will be:



99.844

99.844

6.5

5.5483%

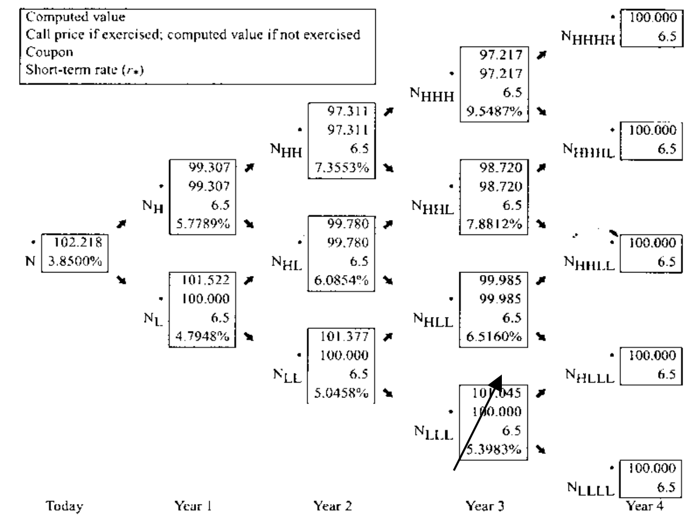
- 6.6660% = 6.5160% + (0.50%-0.35%)
- 99.844 = 106.5/(1.06666) < 100
- Will not be called.

Exhibit 11 Problem

- Node N_{LLL} will be:

100.902
100.000
6.5
5.5483%

Exhibit 11: Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



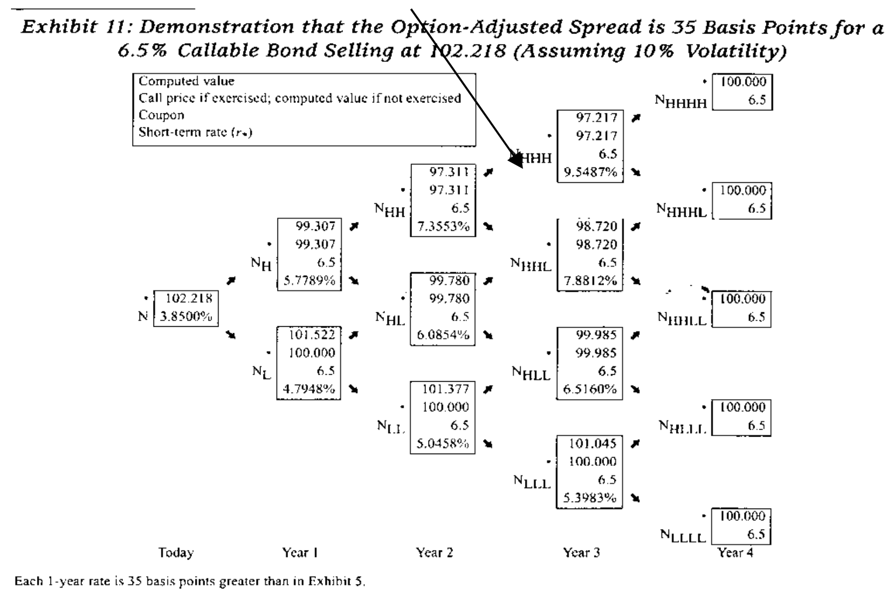
Each 1-year rate is 35 basis points greater than in Exhibit 5.

- 5.5483% = 5.3983% + (0.50%-0.35%)
- 100.902 = 106.5/(1.055483) > 100
- Will be called at 100.

Exhibit 11 Problem

- Node N_{HH} will be:

96.931
96.931
6.5
7.5053%



- 7.5053% = 7.3553% + (0.50%-0.35%)

$$96.931 = \frac{1}{2}\left[\frac{97.084 + 6.5}{1.075053} + \frac{98.327 + 6.5}{1.075053}\right] < 100$$

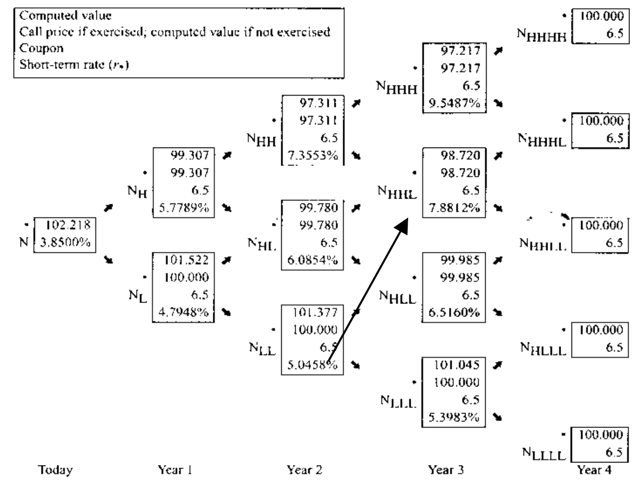
- Will not be called.

Exhibit 11 Problem

- Node N_{HL} will be:

99.388
99.388
6.5
6.2354%

Exhibit 11: Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



Each 1 year rate is 35 basis points greater than in Exhibit 5.

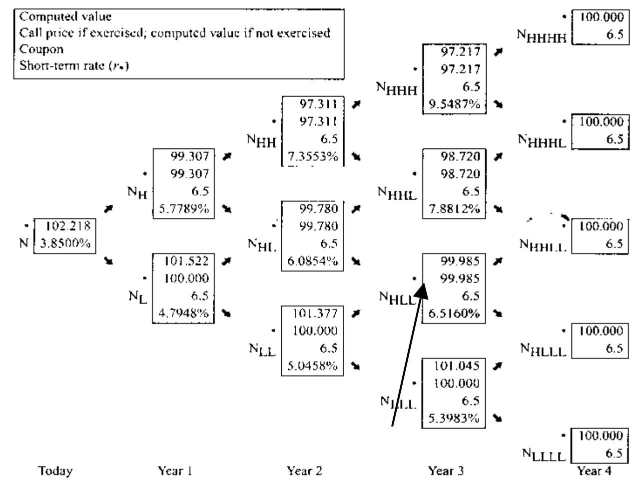
- 6.2354% = 6.0854% + (0.50%-0.35%)
- $$99.388 = \frac{1}{2}\left[\frac{98.327+6.5}{1.062354} + \frac{99.844+6.5}{1.062354}\right] < 100$$
- Will not be called.

Exhibit 11 Problem

- Node N_{LL} will be:

101.166
100.000
6.5
5.1958%

Exhibit 11: Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



Each 1 year rate is 35 basis points greater than in Exhibit 5.

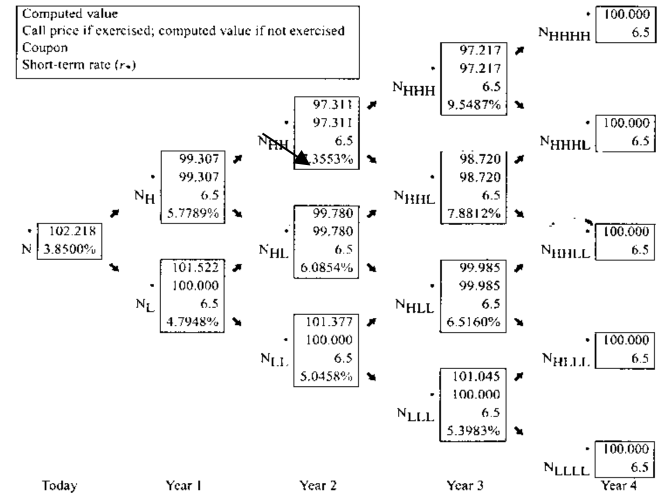
- 5.1958% = 5.0458% + (0.50%-0.35%)
- $$101.166=\frac{1}{2}\left[\frac{99.844+6.5}{1.051958}+\frac{100+6.5}{1.051958}\right]>100$$
- Will be called at 100.

Exhibit 11 Problem

- Node N_H will be:

98.874
98.874
6.5
5.9289%

Exhibit 11: Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



Each 1-year rate is 35 basis points greater than in Exhibit 5.

- 5.9289% = 5.7789% + (0.50%-0.35%)

- $$98.874 = \frac{1}{2}\left[\frac{97.084 + 6.5}{1.059289} + \frac{99.388 + 6.5}{1.059289}\right] < 100$$

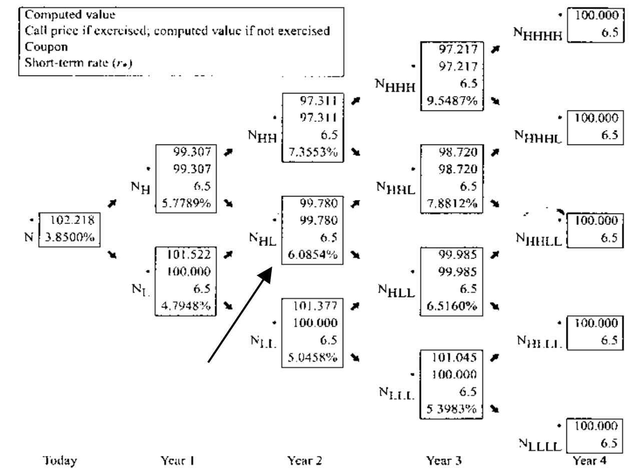
- Will not be called.

Exhibit 11 Problem

- Node N_L will be:

$$\boxed{\begin{array}{l} 101.190 \\ 100.000 \\ 6.5 \\ 4.9448\% \end{array}}$$

Exhibit 11: Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



Each 1-year rate is 35 basis points greater than in Exhibit 5.

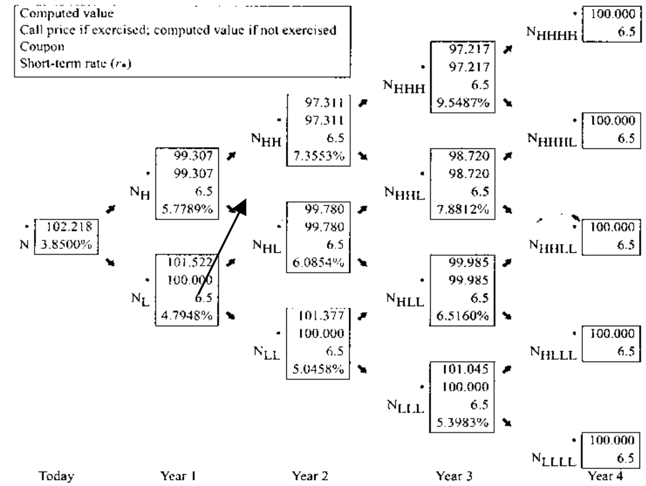
- 4.9448% = 4.7948% + (0.50%-0.35%)
- $$101.190 = \frac{1}{2}\left[\frac{99.388 + 6.5}{1.049448} + \frac{100 + 6.5}{1.049448}\right] > 100$$
- Will be called at 100.

Exhibit 11 Problem

- Node N will be:

101.863
4.0000%

Exhibit 11: Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



Each 1-year rate is 35 basis points greater than in Exhibit 5.

- 4.0000% = 3.850% + (0.50%-0.35%)
- $$101.863 = \frac{1}{2}\left[\frac{98.874+6.5}{1.04000} + \frac{100+6.5}{1.04000}\right] > 100$$
- Will be called at 100.

Effective Duration and Effective Convexity

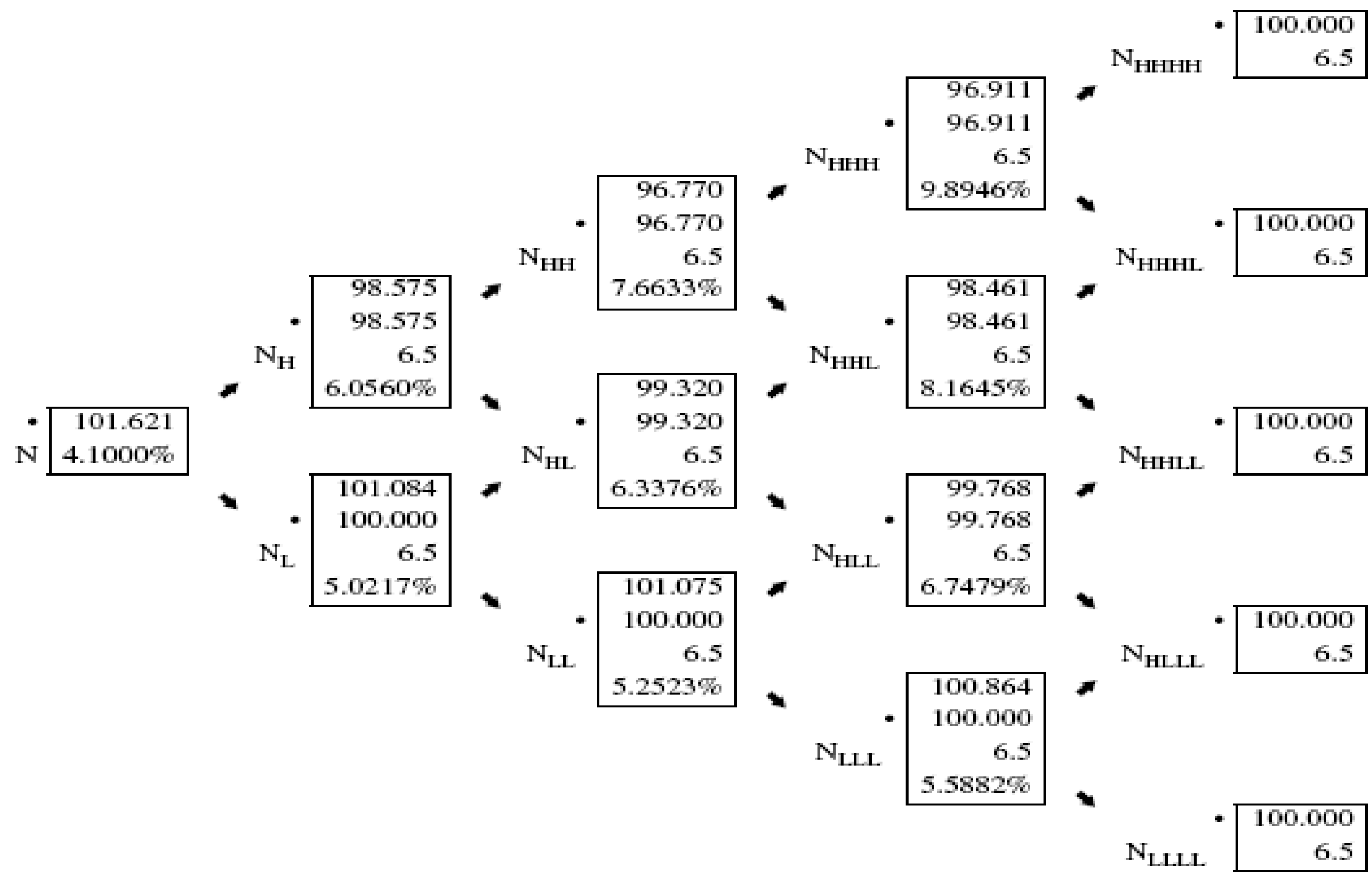
- Required values for calculating effective duration and effective convexity are found by shifting on-the-run yield curve, calculating a new binomial interest rate tree, and then determining required values by adding OAS to each short rate.
- For a bond with any embedded option, application of binomial model requires that value at each node of tree be adjusted based on whether or not option will be exercised; binomial model can be used to value bonds with multiple embedded options by determining at each node of tree whether or not one of the options will be exercised.

Effective Duration and Effective Convexity

- Procedure for calculating value of V+ is:
  - *Step 1:* Given market price of issue calculate its OAS.
  - *Step 2:* Shift on-the-run yield curve up by a small number of basis points (*y*).
  - *Step 3:* Construct a binomial interest rate tree based on new yield curve in Step 2.
  - *Step 4:* To each of 1-year rates in binomial interest rate tree, add OAS to obtain an "adjusted tree." That is, calculation of effective duration and convexity assumes that OAS will not change when interest rates change.
  - *Step 5:* Use adjusted tree found in Step 4 to determine value of the bond, which is V+.

Effective Duration and Effective Convexity

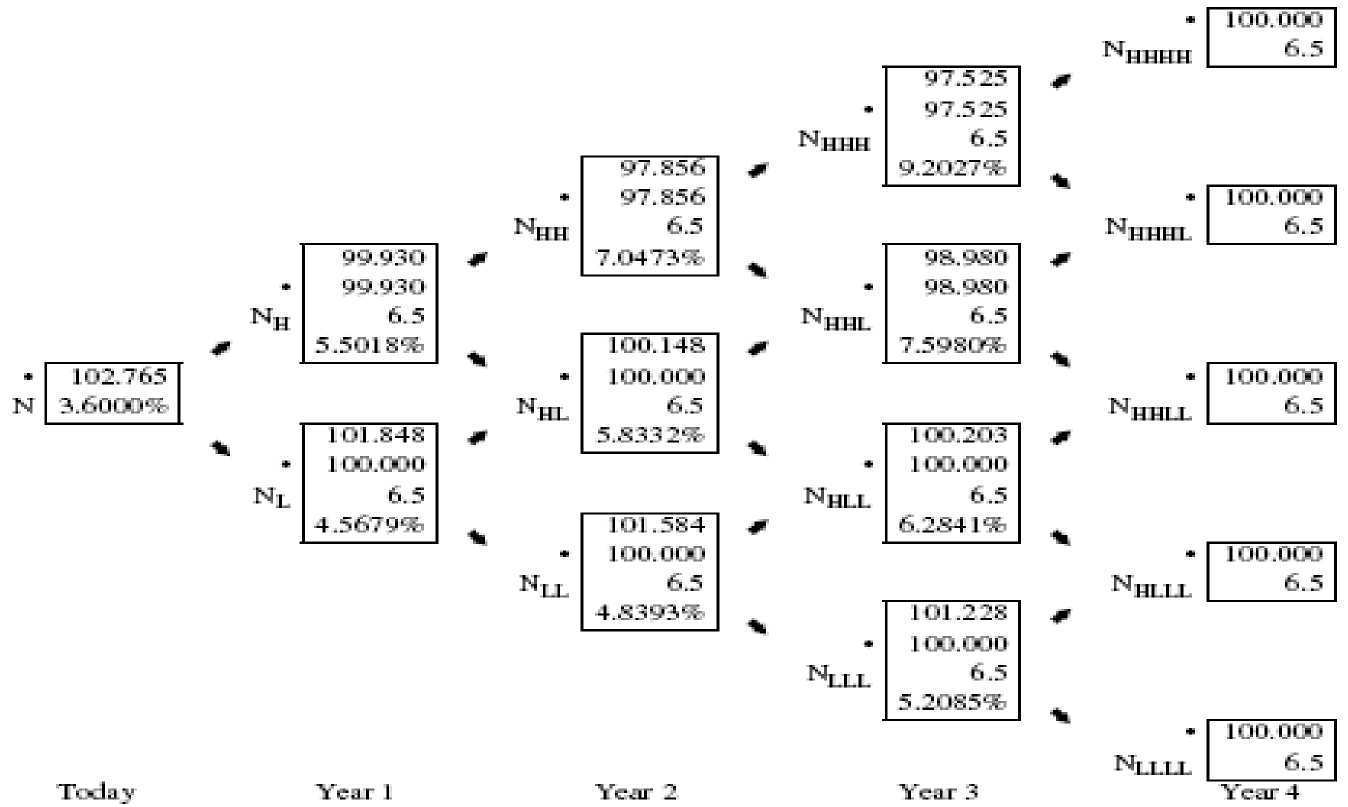
EXHIBIT 12 Determination of V_+ for Calculating Effective Duration and Convexity*



*+25 basis point shift in on-the-run yield curve.

Effective Duration and Effective Convexity

EXHIBIT 13 Determination of V_ for Calculating Effective Duration and Convexity*



*—25 basis point shift in on-the-run yield curve.

Effective Duration and Effective Convexity

The results are summarized below:

$y = 0.0025$, V+=101.621, V−=102.765, V0=102.218

Therefore,

Effective duration =
(102.765−101.621)/(2(102.218)(0.0025)) = 2.24

Effective convexity = (101.621+102.765−2(102.218))/
(2(102.218)(0.0025)^2) = −39.1321

Valuing a Putable Bond

- With a putable bond, option will be exercised if value at a node is less than price at which bondholder can put bond to issuer.
- Value of a putable bond is greater than value of an otherwise option-free bond.
- Binomial model can be used to value a single step-up callable note or a multiple step-up callable note.

Valuing Putable Bonds

- Recall that with a putable bond owner has right to force issuer to redeem bond and pay it off.
- Using binomial model, process will be same as with callable bonds, except that we assume bond will be "put" if price falls below some level.
- Let's use same interest rate tree as in Exhibit 5, and consider a 4 year 6.5% bond that is putable in one year at a price of 100.

Valuing a Putable Bond

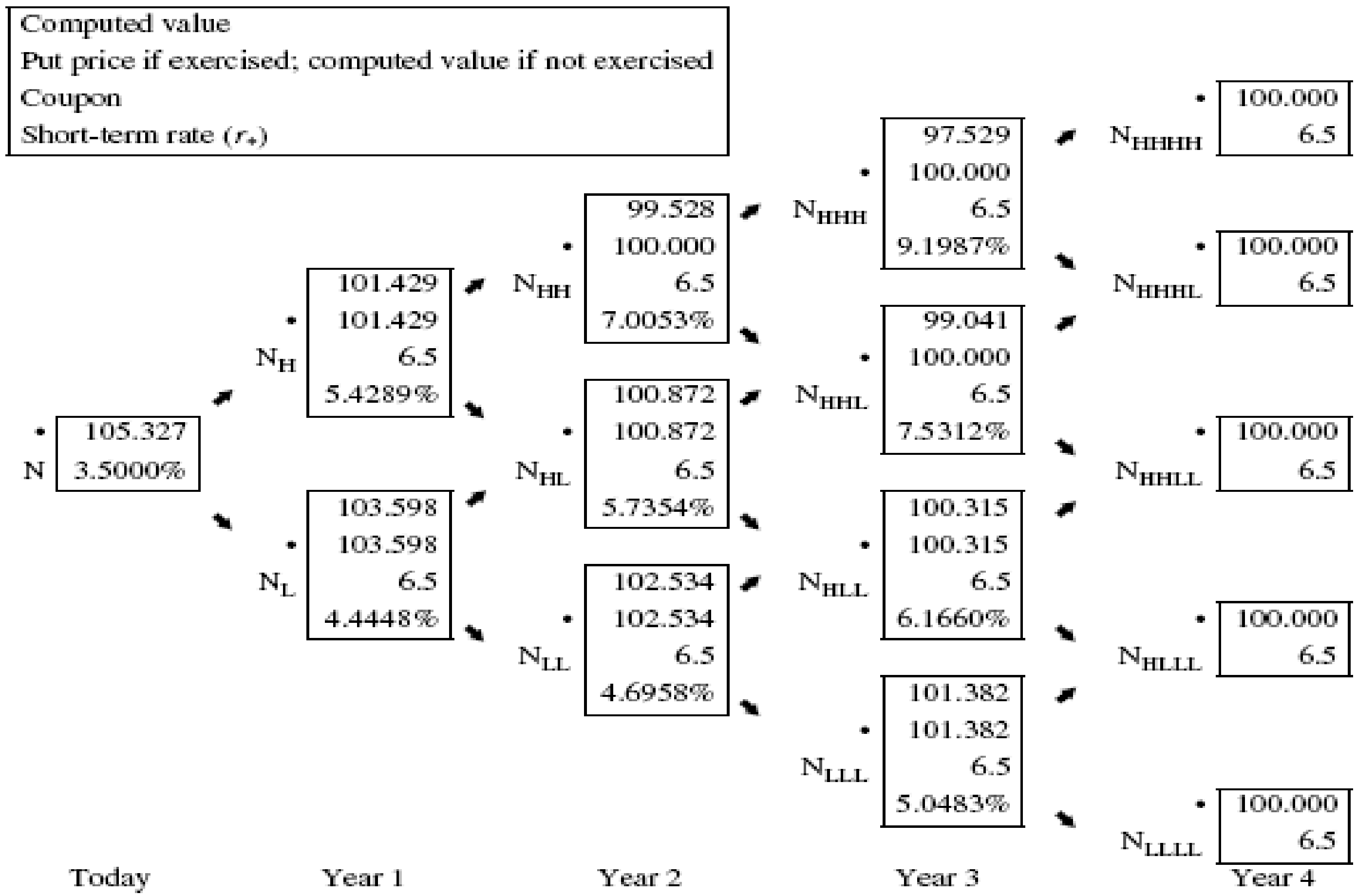
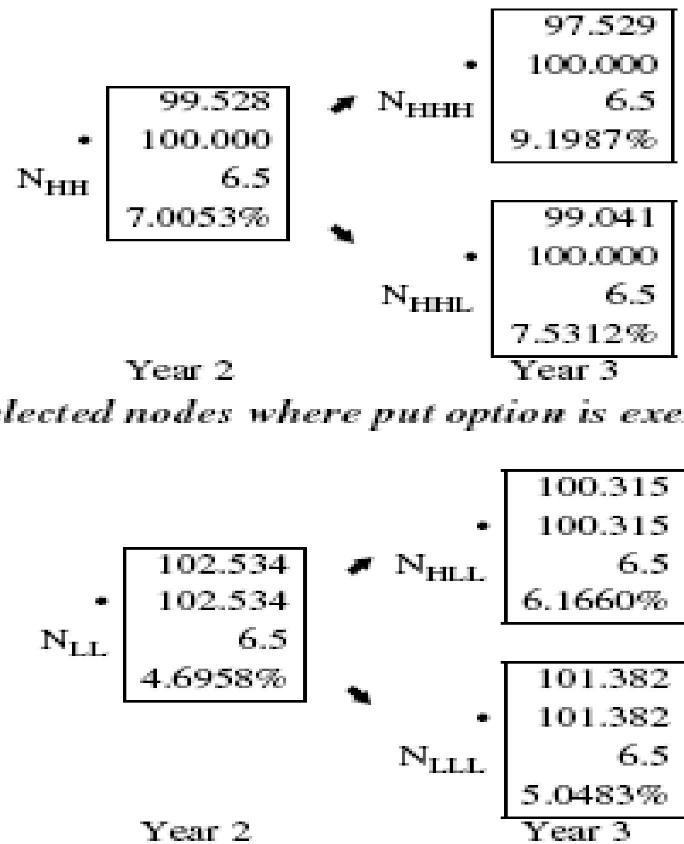


EXHIBIT 14 Valuing a Putable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and Putable in One Year at 100 (10% Volatility Assumed)

Valuing a Putable Bond



(a) Selected nodes where put option is exercised

(b) Nodes where put option is not exercised

EXHIBIT 15 Highlighting Nodes in Years 2 and 3 for a Putable Bond

Valuing a Putable Bond

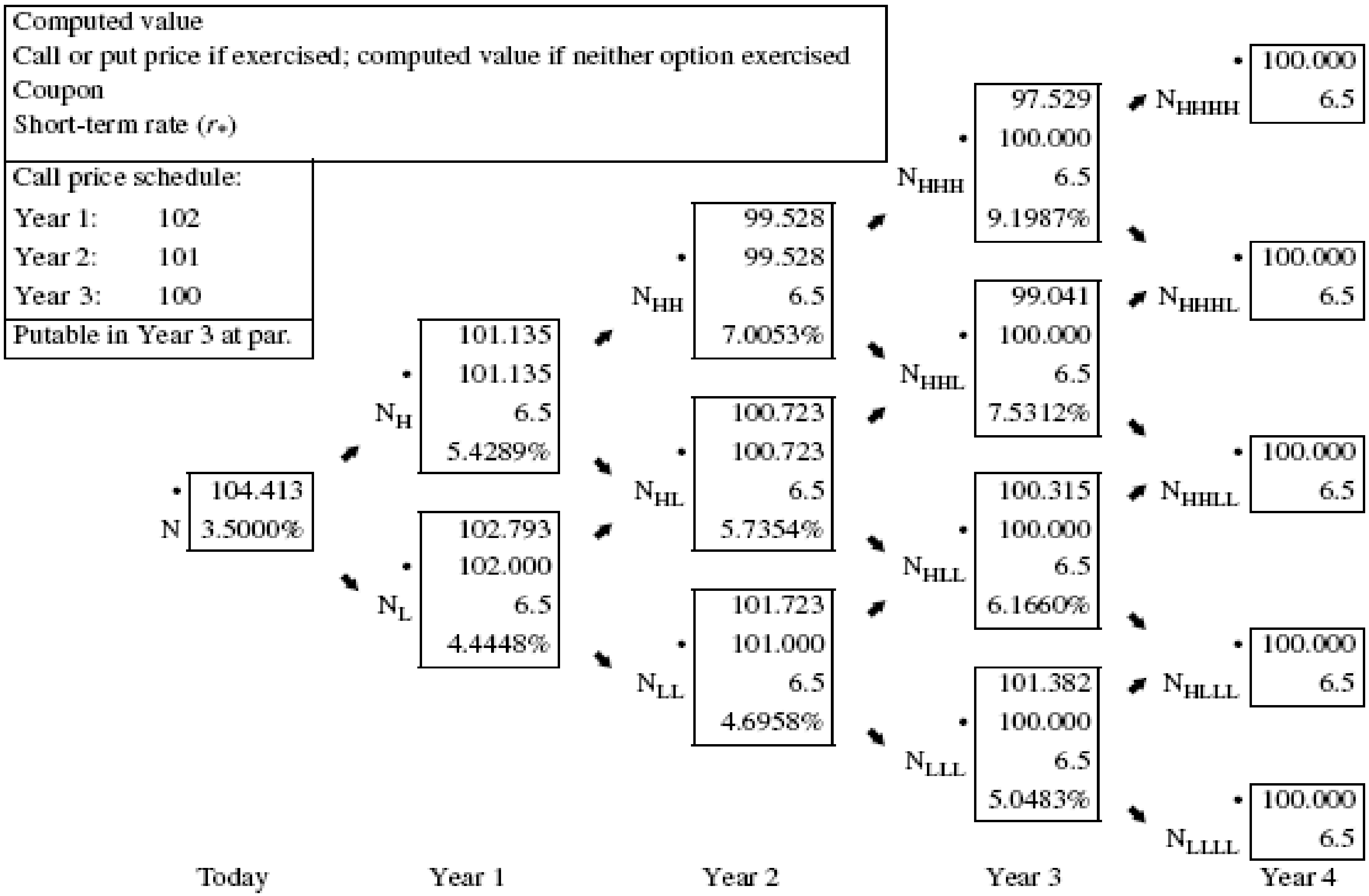


EXHIBIT 16 Valuing a Putable/Callable Issue (10% Volatility Assumed)

Exhibit 15

- From Node N_{HH}:

$$99.528 = \frac{1}{2}\left[\frac{100 + 6.5}{1.070053} + \frac{100 + 6.5}{1.070053}\right]$$

Value of Put Option

- We know from earlier coverage that this bond, if it is option-free, is priced at $104.643.
- Exhibit 14 shows putable bond is priced at $105.327.
- Since:

$$\text{Value of Put Option} = \text{Value of putable bond} - \text{Value of option-free bond}$$

option value is $105.327 - $104.643 = $0.684

Valuing a Floating-Rate Note

- To value a floating-rate note that has a cap, coupon at each node of tree is adjusted by determining whether or not cap is reached at a node; if rate at a node does exceed cap, rate at the node is capped rate rather than rate determined by floater’s coupon formula.
- For a floating-rate note, binomial method must be adjusted to account for the fact that a floater pays in arrears; that is, coupon payment is determined in a period but not paid until next period.

Convertible Securities

- Convertible and exchangeable securities can be converted into shares of common stock.
- Conversion ratio is the number of common stock shares for which a convertible security may be converted.
- Almost all convertible securities are callable and some are putable.
- Conversion value is value of convertible bond if it is immediately converted into common stock.

Convertible Securities

- Market conversion price is price that an investor effectively pays for common stock if convertible security is purchased and then converted into common stock.
- Premium paid for common stock is measured by market conversion premium per share and market conversion premium ratio.
- Straight value of a convertible security is its value if there was no conversion feature.
- Minimum value of a convertible security is the greater of conversion value and straight value.

Convertible Securities

- A fixed income equivalent refers to situation where straight value is considerably higher than conversion value so that security will trade much like a straight security.
- A common stock equivalent refers to situation where conversion value is considerably higher than straight value so that convertible security trades as if it were an equity instrument.
- A hybrid equivalent refers to situation where convertible security trades with characteristics of both a fixed income security and a common stock instrument.

Convertible Securities

- While downside risk of a convertible security usually is estimated by calculating premium over straight value, limitation of this measure is that straight value changes as interest rates change.
- An advantage of buying convertible rather than common stock is the reduction in downside risk.
- Disadvantage of a convertible relative to straight purchase of common stock is upside potential give-up because a premium per share must be paid.

Convertible Securities

- An option-based valuation model is a more appropriate approach to value convertible securities than traditional approach because it can handle multiple embedded options.
- There are various option-based valuation models: one-factor and multiple-factor models.
- The most common convertible bond valuation model is one-factor model in which one factor is stock price movement.