Problem Set 2
Econ 139, Fall 2019
Due in class on Th October 10. No late Problem Sets accepted, sorry!

General rules for problem sets: show your work, write down the steps that you use to get a solution (no credit for right solutions without explanation), write legibly. If you cannot solve a problem fully, write down a partial solution. We give partial credit for partial solutions that are correct. Do not forget to write your name on the problem set!

Problem 1. Preference relations and simple lotteries. Prove the following statements:

1. If $\succcurlyeq$ is rational, then $\sim$ is transitive, that is, $x \sim y$ and $y \sim z$ implies $x \sim z$.

2. If $\succcurlyeq$ is rational, then $\succcurlyeq$ has the reflexive property, that is, $x \succcurlyeq x$ for all x.

For the remainder of the problem, assume the rational preference relation $\succcurlyeq$ over the space of simple lotteries $\mathcal{L}$ satisfies the independence axiom.

3. For all $\alpha \in (0, 1)$ and L_{xy}, L_{vz}, $L_{st} \in \mathcal{L}$ we have
$$L_{xy} \succ L_{vz} \iff \alpha L_{xy} + (1 - \alpha) L_{st} \succ \alpha L_{vz} + (1 - \alpha) L_{st}.$$

4. For all $\alpha \in (0, 1)$ and L_{xy}, L_{vz}, $L_{st} \in \mathcal{L}$ we have
$$L_{xy} \sim L_{vz} \iff \alpha L_{xy} + (1 - \alpha) L_{st} \sim \alpha L_{vz} + (1 - \alpha) L_{st}.$$

5. For all $\alpha \in (0, 1)$ and L_{xy}, L_{vz}, L_{st}, $L_{pq} \in \mathcal{L}$, if $L_{xy} \succ L_{st}$ and $L_{vz} \succ L_{pq}$, then
$$\alpha L_{xy} + (1 - \alpha) L_{vz} \succ \alpha L_{st} + (1 - \alpha) L_{pq}.$$

   [Hint: you may assume that $\succ$ is transitive. The proof of this is similar to the proof for indifference.]

Problem 2. Risk Aversion. Consider the following utility functions over wealth, W:

(i) $u(W) = -\frac{1}{W}$

(ii) $u(W) = \ln(W)$

(iii) $u(W) = -W^{-\gamma}$

(iv) $u(W) = -e^{-\gamma W}$

(v) $u(W) = \frac{W^{\gamma}}{\gamma}$

(vi) $u(W) = \alpha W - \beta W^2$, where $\alpha, \beta > 0$

For each utility function:

1. Check that $u'(W) > 0$ and $u''(W) < 0$. Where applicable, what restrictions on the parameter γ (or parameters α, β) are required to ensure that $u'(W) > 0$ and $u''(W) < 0$? For utility function (vi), what is the range of wealth for which we have $u'(W) > 0$ and $u''(W) < 0$?

2. Compute the absolute and relative risk-aversion coefficients.

3. Where relevant, what is the effect of the parameter γ?

4. Classify the functions as increasing, decreasing, or constant risk-aversion utility functions (both absolute and relative).

Problem 3. Certainty Equivalent and Risk Premium. Suppose an individual with zero initial wealth and utility function $u(W) = \sqrt{W}$ is confronted with the gamble $(16, 4, 0.5)$ (i.e., it pays off 16 with probability 0.5 and 4 with probability 0.5).

1. What is the certainty equivalent for this gamble?
2. Suppose there is an insurance policy that pays off -6, if the gamble pays off 16, and 6 if the gamble pays off 4. What is the maximum that the individual should be willing to pay for this policy?
3. What is the minimum required increase in the probability of the high-payoff state so that the individual will not be willing to pay any premium for such an insurance policy?
4. Now suppose the individual is faced with the gamble $(36, 16, 0.5)$. In this case, assume the insurance policy pays off -10, if the gamble pays off 36, and 10 if the gamble pays off 16. Repeat points 1-3 for this new gamble. Is the required increase in probability smaller, larger, or the same as for the first gamble? Why?

Problem 4. Insurance. An agent with a logarithmic utility function of wealth tries to maximize his expected utility. The agent faces a situation in which he will incur a loss of L with probability p. The agent has the possibility to insure against this loss. The insurance premium depends on the extent of the coverage. The amount covered is denoted by α and the price of the insurance per unit of coverage is q (hence the amount the agent spends on the insurance will be αq).

1. Calculate the amount of coverage α^* demanded by the agent as a function of the agent's initial wealth level W_0, the loss L, the probability p and the price of the insurance q.
2. What is the expected gain (expected revenue - expected cost) for an insurance company offering such a contract?
3. If there is perfect competition in the insurance market (i.e., zero expected gain), what price q will the insurance company set? [Hint: you may assume $\alpha^* > 0$.]
4. What amount of insurance α^* will the agent buy at the price calculated under point 3. What is the influence of the form of the utility function?
5. Now suppose that the insurance company is able to charge a premium $q > p$. Let $W_0 = 10000$ and $p = 0.01$. For $q \in \{0.0105, 0.0110, 0.0115, 0.0120, 0.0125\}$:
    (a) Calculate the minimum loss L for which the agent would be willing to buy insurance (i.e., the minimum L that ensures $\alpha^* \geq 0$).
    (b) Assume $L = 5000$, calculate the optimal amount of insurance α^* the agent will purchase. What is the associated deductible $(L - \alpha^*)$ in each case? Explain.

Problem 5. Expected Utility and Stochastic Dominance. An individual with a utility function $U(c) = -e^{-Ac}$, where $A = (1/30)\ln(4)$, and an initial wealth of \$50, must choose a portfolio of two assets. Each asset has a price of \$50. The first asset is riskless and pays off \$50 next period in each of the two possible states. The risky asset pays off z_s in state $s = 1, 2$. Suppose also that the individual cares only about next period consumption (denoted by c_1 or c_2 depending on the state). The probability of state 1 is denoted by π.

1. In each scenario, the individual splits his wealth equally between the two assets. Fill in the following table:

Scenario	(z_1, z_2)	π	(c_1, c_2)	$\mathbb{E}(c)$	$\mathbf{V}ar(c)$
1	(20,80)	1/5	(35,65)		
2	(38,98)	1/2			
3	(30,90)	1/3			

2. In each scenario, the individual splits his wealth equally between the two assets. How would the individual rank these three scenarios under

   (a) mean-variance analysis,

   (b) first order stochastic dominance,

   (c) second order stochastic dominance,

   (d) expected utility?

   Explain.

3. Show that in each scenario the individual's optimal decision is to invest an equal amount in each of the two assets.