Question 1.

a)

Smith \ Jones	1	2	3
1	3,-3	-1,1	-1,1
2	-1,1	3,-3	-1,1
3	-1,1	-1,1	3,-3

b) There is no pure strategy equilibrium.
For mixed strategy equilibrium, suppose Smith take (x,y,1-x-y), by equivalence principle,

$$3x - y - (1-x-y) = -x + 3y - (1-x-y) = -x - y + 3(1-x-y)$$

Which means

$$x = y = 1/3.$$

So the mixed strategy equilibrium is ((1/3,1/3,1/3),(1/3,1/3,1/3))

c) The expected payoff for Smith:

$$1/3 * 3 + 1/3 * (-1) + 1/3 * (-1) = 1/3.$$

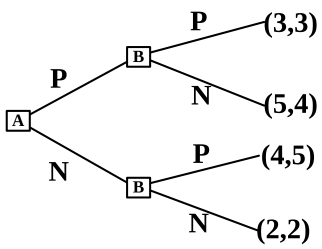
The expected payoff for Jones:

$$1/3 * (-3) + 1/3 * 1 + 1/3 * 1 = -1/3$$

Question 2.

a) There are two pure strategy Nash Equilibrium: (N, P) and (P, N).
For mixed strategy equilibrium, suppose (x,1-x) is taken by A. By equivalence principle, we have x = 3/4. So the mixed strategy equilibrium is ((3/4,1/4), (3/4,1/4)).

b) Suppose A moves first, the game tree is as follows.



The perfect Nash Equilibrium here is (P,N), with payoff (5,4).
So both players have first-move advantage.

c) There is some ambiguous in this problem. DON'T count!
If B can commit on his behavior ex-ante, i.e., he can commit that he will produce and A believe his commitment, then he does not need to pay A to stay out of the market.
If B cannot commit, then they will mix ex-ante, with expected payoff 1/2 * 3 + 1/2 * 4 = 3.5. If B can monopolize, he will get 5. Since 5 > 3.5, he will bribe A not to come into the market.

Question 3.

a) In monopoly case,

$$\max_p (50 - p)(p - 5)$$

with first order condition

$$p = \frac{55}{2}, q = \frac{45}{2}, \pi = \frac{2025}{4} = 506.25$$

b) In Cournot case, for firm 2:

$$\max_{q_2} (50 - q_1 - q_2 - 5) q_2$$

with first order condition

$$q_2 = \frac{1}{2}(45 - q_1)$$

Similarly,

$$q_1 = \frac{1}{2}(45 - q_2)$$

So

$$q_1 = q_2 = 15, p = 20, \pi_1 = \pi_2 = 225 = \frac{2025}{9}$$

c) Suppose there are N firms, for firm i

$$\max_{q_i} (50 - \Sigma q_j - 5) q_i$$

with first order condition

$$q_i = \frac{1}{2}(45 - \Sigma_{j \neq i} q_j)$$

By symmetry, $q_1 = q_2 = . = q_N = \frac{1}{N} Q$, so

$$q_i = \frac{45}{N+1}, p = 50 - \frac{N}{N+1} 45, \pi_i = \frac{2025}{(N+1)^2}$$

d) In the Stackerberg case, the best response for firm 2 is similar as above,

$$q_2 = \frac{1}{2}(45 - q_1)$$

For firm 1:

$$\max_{q_1} (50 - q_1 - \frac{1}{2}(45 - q_1) - 5) q_1$$

By first order condition,

$$q_1 = \frac{45}{2}, q_2 = \frac{45}{4}, p = \frac{65}{4}, \pi_1 = \frac{2025}{8}, \pi_2 = \frac{2025}{16}$$

e) For firm 2

$$\max_{q_2} (50 - q_1 - q_2 - q_3 - 5) q_2$$

With first order condition

$$q_2 = \frac{1}{2}(45 - q_1 - q_3)$$

Similarly,

$$q_3 = \frac{1}{2}(45 - q_1 - q_2)$$

So

$$q_2 = q_3 = 15 - \frac{1}{3}q_1$$

For firm 1,

$$\max_{q_1} (50 - q_1 - 2(15 - \frac{1}{3}q_1) - 5)q_1$$

With first order condition,

$$q_1 = \frac{45}{2}, q_2 = q_3 = \frac{15}{2}, p = \frac{25}{2}, \pi_1 = \frac{2025}{12} = 168.75, \pi_2 = \pi_3 = \frac{2025}{36} = 56.25$$

Question 4.

a) The Bertrand price is 10.

b) $\pi_A = 0, \pi_B = 2 \times 300 = 600$

c) The equilibrium is not Pareto efficient. If we allow B to monopolize, the profit for A is still 0, i.e. A is no worse off, but the profit for B becomes

$$\max_p (p - 8)(500 - 20p)$$

So $p = \frac{33}{2}, \pi = 5 \times 17^2 > 600$

d) If MC_A =15, the outcome is unchanged, since monopoly price is larger than 15.

Question 5

a) For firm 1

$$\max_{p_1} \; p_1(20 - p_1 + p_2)$$

With first order condition

$$p_1 = 10 + \frac{1}{2}p_2$$

Similarly,

$$p_2 = 10 + \frac{1}{2}p_1$$

So $p_1 = p_2 = 20$.

b) For firm 2,

$$p_2 = 10 + \frac{1}{2}p_1$$

For firm 1,

$$\max_{p_1} \; p_1(20 - p_1 + (10 + \frac{1}{2}p_1))$$

By first order condition,

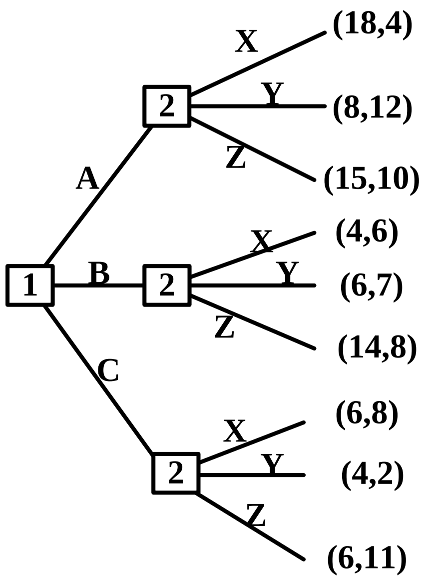
$$p_1 = 30, p_2 = 25, Q_1 = 15, Q_2 = 25, \pi_1 = 450, \pi_2 = 625$$

So first mover has no advantage.

Question 6.

a) The pure strategy Nash Equilibrium is (A,Y).
There is no mixed strategy Nash Equilibrium, because player1 has strictly dominant strategy A. For any strategy played by Player2, Player1's best response is always playing A.

b) Using back ward induction, the Nash Equilibrium is (B,Z).



c) The first round is redundant, and the game is equivalent to a two stage game, where player 2 chooses first, player1 follows.
By backward induction (game tree is omitted here), the PNE is (A,Y)