期中考试：4月23日

金融经济学

第二讲(A)

Consumer Choice in the Time Dimension 回顾

张宇

作业：掌握重点
内容

考试题型与作业一致，有原题

Consumer Optimization: The Time Dimension

[illegible] → 时间

Irving Fisher (US, 1867-1947) was the first to recognize that the basic theory of consumer decision-making could be used to understand how to optimally allocate spending intertemporally, that is, over time, as well as how to optimally allocate spending across different goods in a static, or point-in-time, analysis.

Consumer Optimization: The Time Dimension

Following Fisher, return to the case of two goods, but reinterpret:

$$c_0 = \text{consumption today}$$
$$c_1 = \text{consumption next year}$$

Suppose that the consumer's utility function is

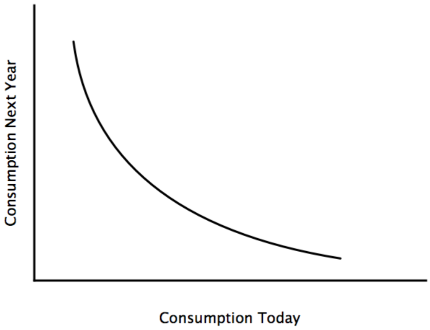
$\beta < 1$ 及时行乐

$$u(c_0) + \beta u(c_1),$$

$\beta > 1$ 先苦后甜

where β now has a more specific interpretation, as the discount factor, a measure of patience. (一般 $\beta < 1$)

Consumer Optimization: The Time Dimension



A concave utility function implies that indifference curves are convex, so that the consumer has a preference for a smoothness in consumption.

Consumer Optimization: The Time Dimension

Next, let

$$Y_0 = \text{income today}$$

→ endowment

$$Y_1 = \text{income next year}$$

$$s = \text{amount saved (or borrowed if negative) today}$$

$$r = \text{interest rate}$$

把今天的钱留到明天花

Consumer Optimization: The Time Dimension

Today, the consumer divides his or her income up into an amount to be consumed and an amount to be saved:

$$Y_0 \geq c_0 + s.$$

考试：代入数，计算

Next year, the consumer simply spends his or her income, including interest earnings if s is positive or net of interest expenses if s is negative:

$$Y_1 + (1 + r)s \geq c_1.$$

Consumer Optimization: The Time Dimension

Divide both sides of next year's budget constraint by $1 + r$ to get

$$\frac{Y_1}{1+r} + s \geq \frac{c_1}{1+r}.$$

Now combine this inequality with this year's budget constraint

$$Y_0 \geq c_0 + s.$$

to get

$$Y_0 + \frac{Y_1}{1+r} \geq c_0 + \frac{c_1}{1+r}.$$

present value

Consumer Optimization: The Time Dimension

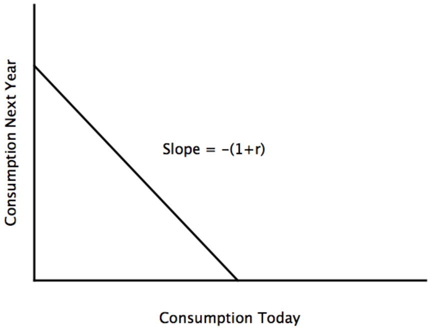
The "lifetime" budget constraint

$$Y_0 + \frac{Y_1}{1+r} \geq c_0 + \frac{c_1}{1+r}$$

says that the present value of income must be sufficient to cover the present value of consumption over the two periods. It also shows that the "price" of consumption today relative to the "price" of consumption next year is related to the interest rate via

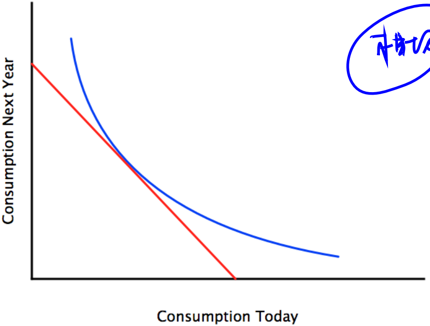
$$\frac{p_0}{p_1} = 1 + r.$$

Consumer Optimization: The Time Dimension



The slope of the intertemporal budget constraint is $-(1 + r)$.

Consumer Optimization: The Time Dimension



At the optimum, the intertemporal marginal rate of substitution equals the slope of the intertemporal budget constraint.

Consumer Optimization: The Time Dimension

We now know the answer ahead of time: if we take an algebraic approach to solve the consumer's problem, we will find that the IMRS equals the slope of the intertemporal budget constraint:

$$\frac{u'(c_0)}{\beta u'(c_1)} = \overset{\text{MRS}}{1 + r}.$$

But let's use calculus to derive the same result.

也可以用 Lagrange method

Consumer Optimization: The Time Dimension

The problem is to choose c_0 and c_1 to maximize utility

$$u(c_0) + \beta u(c_1)$$

subject to the budget constraint

$$Y_0 + \frac{Y_1}{1+r} \geq c_0 + \frac{c_1}{1+r}.$$

The Lagrangian is

$$L = u(c_0) + \beta u(c_1) + \lambda \left(Y_0 + \frac{Y_1}{1+r} - c_0 - \frac{c_1}{1+r} \right).$$

Consumer Optimization: The Time Dimension

Lagrangian

$$L = u(c_0) + \beta u(c_1) + \lambda \left(Y_0 + \frac{Y_1}{1+r} - c_0 - \frac{c_1}{1+r} \right).$$

Lagrangian multiplier

The first-order conditions

$$u'(c_0^*) - \lambda^* = 0$$

$$\beta u'(c_1^*) - \lambda^* \left(\frac{1}{1+r} \right) = 0.$$

lead directly to the graphical result

$$\frac{u'(c_0^*)}{\beta u'(c_1^*)} = 1 + r.$$

Consumer Optimization: The Time Dimension

At first glance, Fisher's model seems unrealistic, especially in its assumption that the consumer can borrow at the same interest rate r that he or she receives on his or her savings.

A reinterpretation of saving and borrowing in this framework, however, can make it more applicable, at least for some consumers.

Investment Strategies and Cash Flows

Investment Strategy	Cash Flow at $t = 0$	Cash Flow at $t = 1$
Saving	-1	$+(1+r)$
Buying a bond (long position in bonds)	-1	$+(1+r)$

$\approx$ equivalent

Investment Strategies and Cash Flows

Investment Strategy	Cash Flow at $t = 0$	Cash Flow at $t = 1$
Borrowing	$+1$	$-(1 + r)$
Issuing a bond	$+1$	$-(1 + r)$
Short selling a bond (short position in bonds)	$+1$	$-(1 + r)$
Selling a bond (out of inventory)	$+1$	$-(1 + r)$

Investment Strategies and Cash Flows

Investment Strategy	Cash Flow at $t = 0$	Cash Flow at $t = 1$
Buying a stock (long position in stocks)	$-P_0^s$	$+P_1^s$
Short selling a stock (short position in stocks)	$+P_0^s$	$-P_1^s$
Selling a stock (out of inventory)	$+P_0^s$	$-P_1^s$

不考虑风险

Consumer Optimization: The Time Dimension

Someone who already owns bonds can "borrow" by selling a bond out of inventory. In fact, theories like Fisher's work better when applied to consumers who already own stocks and bonds.

Greg Mankiw and Stephen Zeldes, "The Consumption of Stockholders and Nonstockholders," *Journal of Finance*, 1991.

Annette Vissing-Jorgensen, "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 2002.

Consumer Optimization: The Time Dimension

On completely different topics:

Arvind Krishnamurthy and Annette Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, 2011.

Anna Cieslak, Adair Morse, and Annette Vissing-Jorgensen, "Stock Returns over the FOMC Cycle," Unpublished Manuscript, June 2016.

Copyright