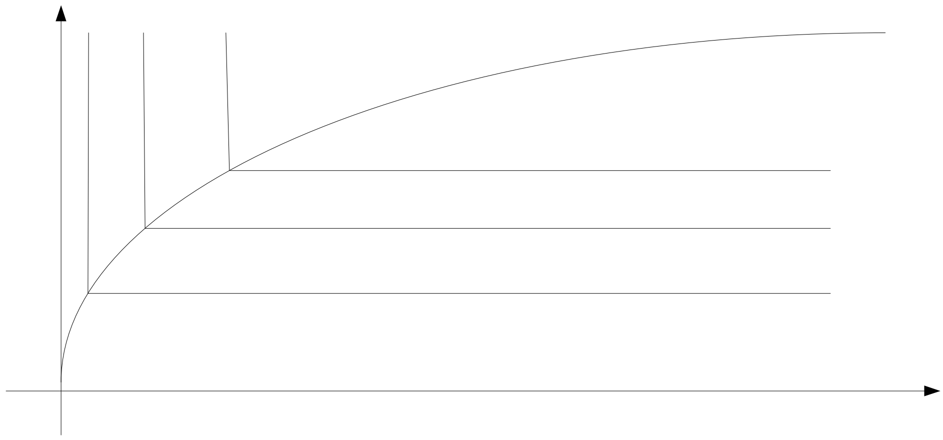
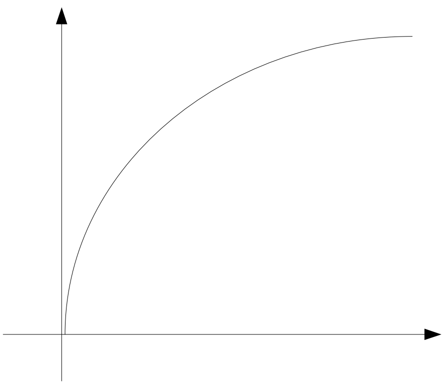
Answers to Problem Set 1

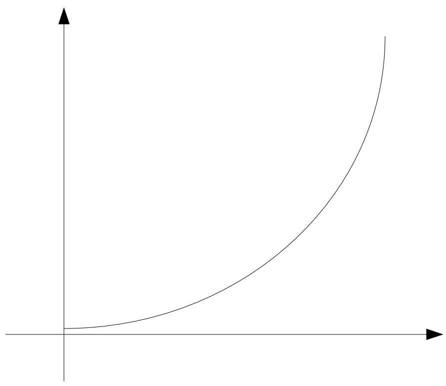
1. a) Indifference curve: $Y = \sqrt{X}$ ,

   b) Engle curve for soup (X): $X + p\cdot\sqrt{X} = m$

   Engle curve for bread (Y): $Y^2 + p\cdot Y = m$







Indiffer

c) From the analysis above:

Y

$$x = y^2$$

$$p_x x + p_y y = m$$

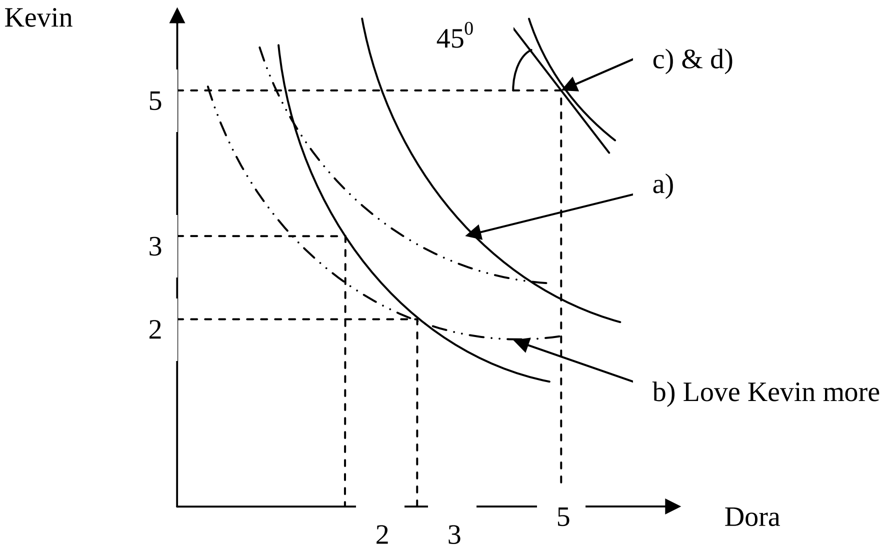
(Bread)

The demand function is:

$$y(p_x, p_y, m) = \frac{1}{2p_x}(-p_y + \sqrt{p_y^2 + 4mp_y})$$

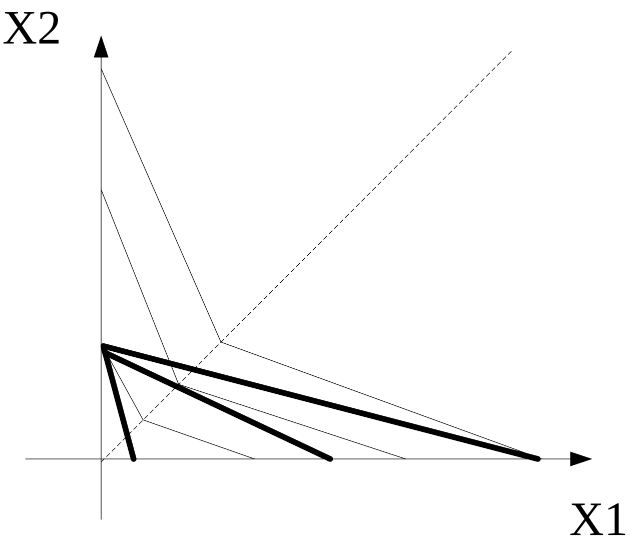
$$x(p_x, p_y, m) = y(p_x, p_y, m)^2$$

2.




3. $U(X_1, X_2) = \min(2X_1 + X_2, 2X_2 + X_1) = \begin{cases} 2X_1 + X_2 & \text{if } X_1 \le X_2 \\ 2X_2 + X_1 & \text{if } X_1 > X_2 \end{cases}$

The indifference curve is as the following:




The demand function of $X_2$ is symmetric. So, let's fix p2 and take $X_1$ as example.
The demand function of $X_1$ is

Answers to Problem Set 1

$$x_1(p_1, p_2, I) = \begin{cases} 0 & \text{if } \frac{p_1}{p_2} > 2 \\ [0, \frac{I}{p_1 + p_2}] & \text{if } \frac{p_1}{p_2} = 2 \\ \frac{I}{p_1 + p_2} & \text{if } \frac{p_1}{p_2} \in (\frac{1}{2}, 2) \\ [\frac{I}{p_1 + p_2}, \frac{I}{p_1}] & \text{if } \frac{p_1}{p_2} = \frac{1}{2} \\ \frac{I}{p_1} & \text{if } \frac{p_1}{p_2} < \frac{1}{2} \end{cases}$$

For simplicity concern, let $A = (2p_2, +\infty), B = (\frac{1}{2}p_2, 2p_2), C = [0, \frac{1}{2}p_2)$, we de-composite the total effect (TE) into Hicksian substitution effect (SE) and Income effect (IE).
Suppose the price changes from p1 to p1', with the consumption bundle from x to x'. Hicksian substitution effect denote an intermediate bundle x", where x" is the optimal consumption bundle under the price (p1', p2) and brings the consumer the same utility as the original bundle x.

TE = x1' – x1

SE = x1" – x1

IE = x1' – x1"

In addition, if either p1 or p1' is equal to 2p2 or 0.5p2, then the demand is not well defined, so there is no slutsky de-composition. So we ignore those cases.
Next, we discuss the value of x1, x1', x1"

Case I: From A to A.

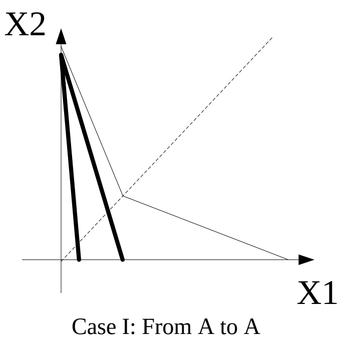
x1 = x1' = x1" = 0

Case II: From B to B.

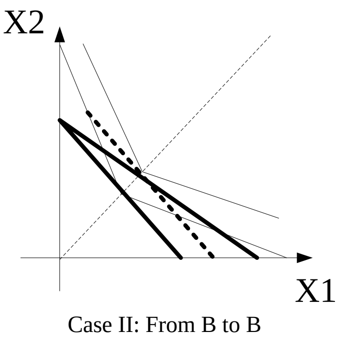
x1 = I / (p1 + p2) = x1", x1' = I / (p1' + p2)

Case II: From C to C

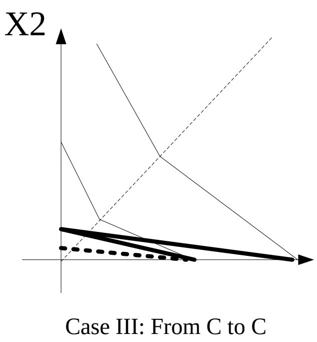
x1 = I / p1 = x1", x1' = I / p1'




Case I: From A to A




Case II: From B to B




Case III: From C to C

Case IV: From A to B

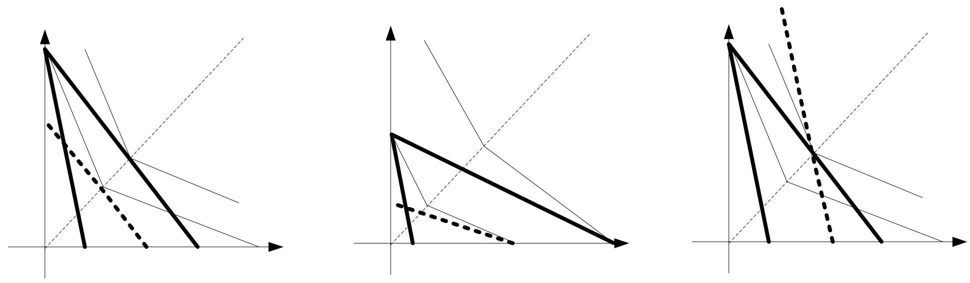
x1 = 0, x1' = I / (p1' + p2), x1" = I / (3p2)

Case V: From A to C

x1 = 0, x1' = I / p2, x1" = I / p1'

Case VI: From B to A

x1 = I / (p1 + p2) = x1", x1' = 0



Case VII: From B to C

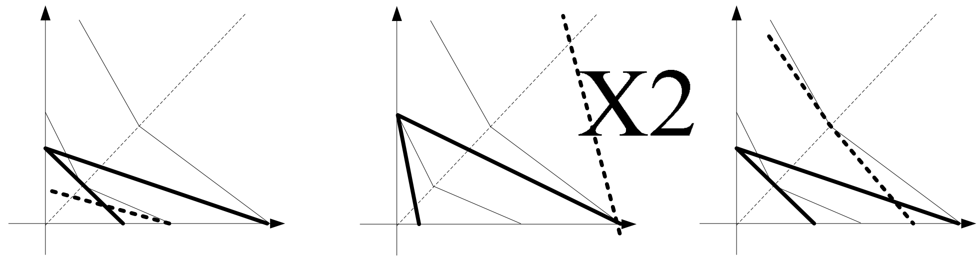
x1 = I / (p1 + p2), x1' = I / p1', x1" = 3I / (p1 + p2)

Case VIII: From C to A

x1 = I / p1, x1' = 0 = x1"

Case IX: From C to B

x1 = I / p1, x1' = I / (p1 + p2'), x1" = I / 3p1




4.

$$\max\ \log U(x,\ y) = 2\log x + 3\log y$$
$$s.t.\ p_x x + p_y y = I$$

So

$$x(p_x, p_y, I) = \frac{2I}{5p_x};\ y(p_x, p_y, I) = \frac{3I}{5p_y}$$

b) A: x(1,8,30)=12; y(1,8,30)=2.25; A=(12, 2.25)

C: x(1,2,30)=12, y(1,2,30)=9; C=(12, 9)

c)

If we use Slutsky de-composition:

If as the price of y drops to 2, and the income changes to keep the purchasing power, then the income would satisfy: I'=12*1+2*2.25=16.5, this gives x(1,2,16.5)=6.6, y(1,2,16.5)=4.95, so B = (6.6, 4.95) Thus, the slusky substitution effect is 4.96-2.25=2.71 as price goes down; the income effect is 9-4.96=4.04 as price goes down.

Case IV: From A to

Therefore, the substitution effect if negative, the total and income effect are both negative.

Since the consumption of y increases as the income increases, y is not inferior; and the total effect is 9-2.25 as price goes down, y is not giffen. Y is a normal good.

If we use Hicksian de-composition:

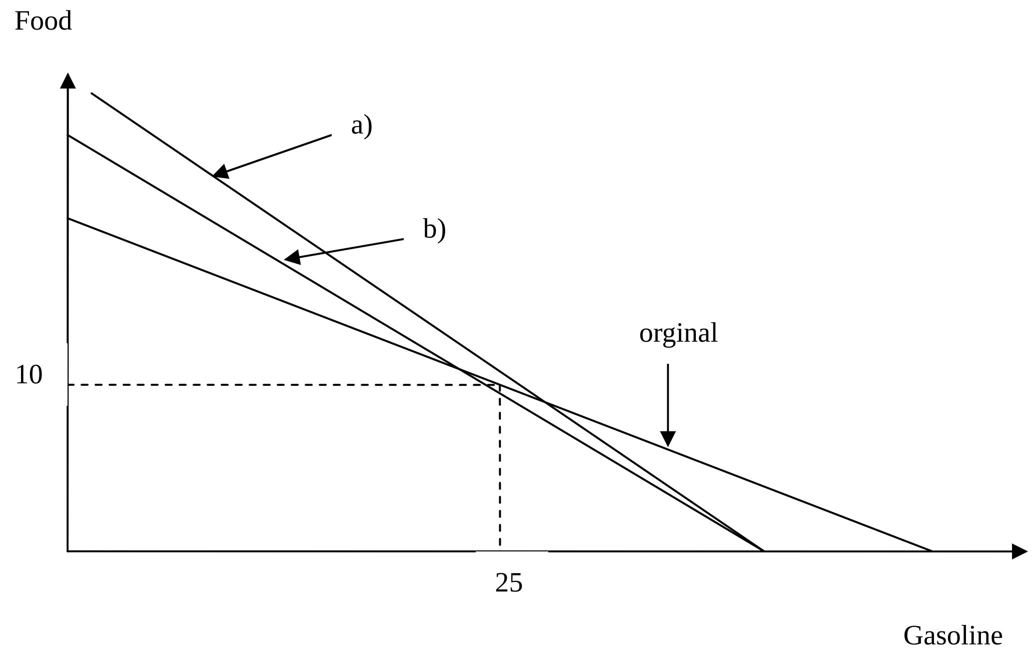
Suppose the intermediate bundle B=(x,y)

$$U(A) = U(B)$$

$$x/y = 2py' / 3px = 4 / 3$$

So B = (5.22, 3.91)

5.




The slopes of the original, a) and b) budget lines are 2/5, 5/7, 5/8 respectively. When prices change to case a), the original optimal choice is affordable, may or may not be chosen. Therefore, Ashley is better off in case a).

(a). Since 5 * 25 + 7 * 10 = 195 < 200, so the initial bundle is affordable in the new environment, not chosen. So the consumer is better off.

(b)

(i) 5 * 25 + 8* 10 = 205 > 200, the original bundle is not affordable. So it is ambiguous whether the consumer is better off or worse off.

(ii) Food = 11, Gasoline = (200 – 8 * 11) / 5 = 22.4

In the original environment, 5 * 11 + 2 * 22.4<100, the new bundle is affordable in the original price, but not chosen, so the consumer is worse off.

(iii)Food = 15, Gasoline = 16,    5 * 15 + 2 * 16 >100, so it is ambiguous whether the consumer is better off or worse off.