# 无穷小量的比较及应用

本段内容：

定义同阶无穷小量、高阶无穷小量和等价无穷小量

等价无穷小在极限运算中的使用

同一极限过程中的无穷小量的加法和乘法运算

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如果$\lim\limits_{x} \dfrac{\alpha}{\beta^k} = A \neq 0$，则称$\alpha$是$\beta$的$k$阶无穷小.

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$$\lim_{x\to 0}\frac{\arcsin^2 x}{x^2}=\lim_{x\to 0}\frac{x^2}{x^2}=1.$$

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$$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \lim_{x \to 0} \frac{x - x}{x^3} = 0$$

事实上, $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3} = \dfrac{1}{6}$.

例: $x \to 0$时, $x^3$是$x$的3阶无穷小.

无穷小量的运算

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$$\Rightarrow \boxed{\alpha + \beta = o(x^2)},\ \boxed{\alpha \cdot \beta = o(x^5)}$$

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$$\lim_{x\to 0} \frac{\alpha \cdot \beta}{x^5} = \lim_{x\to 0} \frac{\alpha}{x^2} \frac{\beta}{x^3}$$

$$= \lim_{x\to 0} \frac{o(x^2)}{x^2} \lim_{x\to 0} \frac{o(x^3)}{x^3} = 0.$$

本段要点：

同阶无穷小量、高阶无穷小量和等价无穷小量的概念

等价无穷小在极限的乘除法运算中可以代换，在加减法中通常不能代换

同一极限过程中的无穷小量的加法结果为较低阶，乘法结果为阶数之和