Investments

Lecture 9

Futures and Options: Objectives

1. Understand uses of futures and options;
2. Understand characteristics of futures and options investments, including:
   - Risk
   - Pricing
   - Hedging
3. Understand and apply mathematical and statistical techniques required to
   - Value futures and options;
   - Hedge using futures and options.

Derivative Securities

- Financial instruments that derive their values from other traded claims are called **derivatives**.
- Typically, the value of these instruments is very closely related to the value of the underlying asset.
- As a result derivatives are useful for:
  - **Speculating** on the underlying asset, and;
  - **Hedging** the underlying asset.
- Furthermore, **arbitrage** opportunities may be possible if the underlying asset and the derivative asset are not priced consistently.

Speculators and Hedgers

- **Speculators** are individuals hope to make a profit by closing out their positions at a price that is better than the initial price. They **do not produce or use the asset** in their daily course of business.
- **Hedgers** are individuals who use derivatives to offset an otherwise risky position in the underlying asset. They **either produce or use the asset** in their daily course of business.

Example: Wheat Forwards

- In a wheat forward contract two counter-parties agree to exchange some quantity of wheat at some date in the future **at a price negotiated today**.
- A wheat farmer has **exposure** to the future **spot price** of wheat.
  - The spot price is the market price of wheat for immediate delivery.
  - The crop planted in the spring and harvested in the fall will be sold at fall spot prices. Since these spot prices are uncertain, the profits on the farmer's crop are risky.
- A risk-averse farmer can hedge this risk by selling wheat now using a forward contract.

Wheat Forwards

- Who might take the opposite side of this trade?
  - A bread producer may wish to hedge production costs.
  - A weather forecaster may speculate that the future spot price will be well above the forward price and therefore use this contract as part of a trading strategy (buy using the forward contract and sell in the future spot market).

Arbitrageurs

- Two basic types of arbitrage trades:
  - Invest nothing and make positive future profits;
  - Receive profits today without any future obligations.
- Arbitrageurs use derivative contracts to extract arbitrage profits.
  - Their actions, along with normal supply and demand forces, ensure consistent relationships among the underlying asset prices and the derivative security prices.

Wheat Forwards

- Who might be in a position to derive arbitrage profits from wheat forward contracts?
  - If you have a technology for storing wheat and the forward price is high relative to today's spot price, you may want to:
    - Borrow money to buy wheat now,
    - Sell it with the forward contract,
    - Store it until the fall,
    - Deliver the wheat and use the proceeds to pay back your lenders.
- Notice that the cost of storage and lending rates will place a bound on how high the forward price can be (a sort of no-arbitrage bound).

Futures

Futures Contracts - Definition

- A futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. (Futures = Standardized forward contracts!)
- Characteristics of futures contracts:
  - Traded on an exchange;
  - Contracts are standardized;
  - Clearing houses eliminate default risk: Some margin is required by them.

Futures - Common Examples

- Commodity futures:
  - Wheat;
  - Crude oil;
  - Gold;
  - Live cattle.
- Financial futures:
  - S&P 500 index futures;
  - T-bill futures;

Futures – Contract Specification

- Components of contract specification:
  - Asset;
  - Contract size;
  - Delivery arrangements;
    - Cash or physical delivery;
    - Place;
    - Time.
  - Price quotes;
  - …

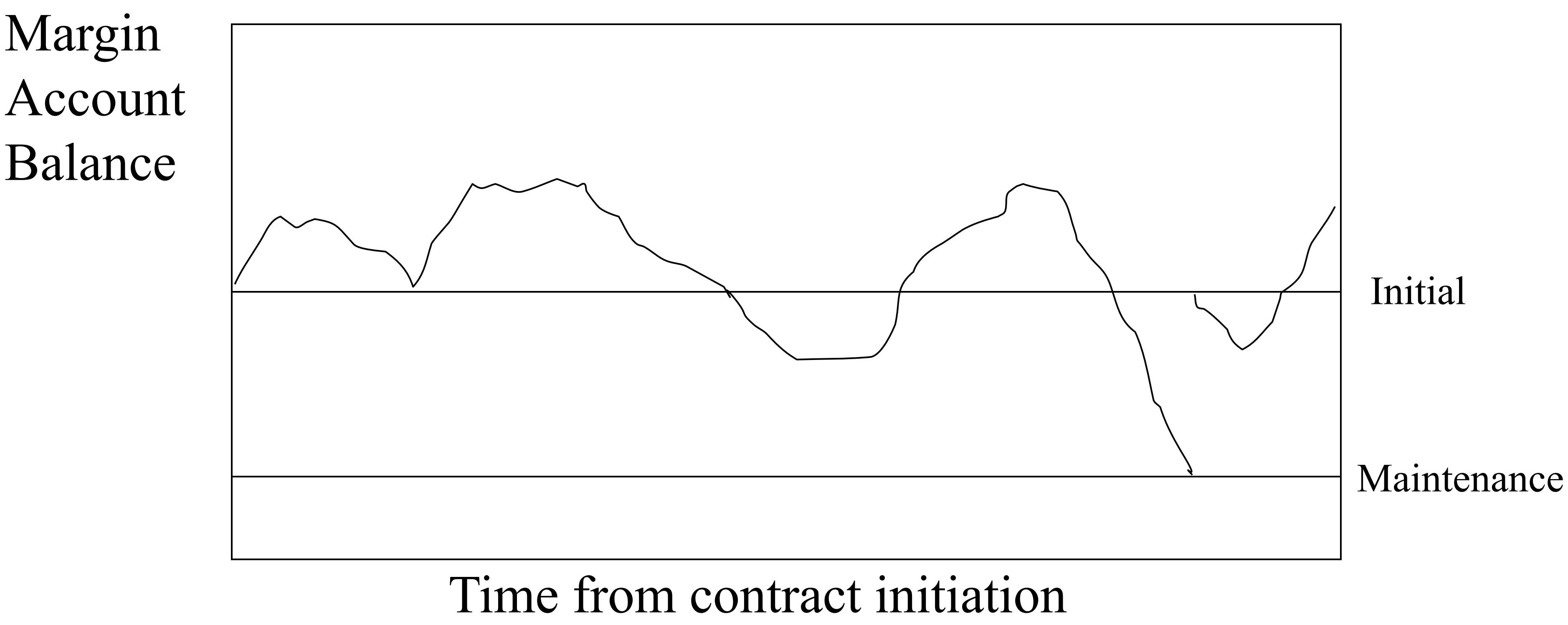
Futures Contracts - Margin

- When you enter into a futures contract, the broker typically require that you deposit funds into a margin account.
  - You may or may not earn interest on this account.
- At contract initiation you deposit an initial margin.
- This account is "marked-to-market" daily.
  - Daily profits or losses, as represented by changes in the futures price, are credited to or debited from your margin account.

Margin (cont'd)

- Maintenance margin (e.g. 10%):
- If your margin account balance falls below the **maintenance margin** you will receive a margin call, in which case you must deposit additional funds (the variation margin) to bring your balance back to the **initial margin** level.
- If you do not honor a margin call, your position is closed out.

Margin Example



Margin Example

- Futures on index
  - Two E-mini S&P 500 contracts, contract size = $50 * S&P500 (2,884 on May 8, 2019. hypothetically set to 1000 in this example.)
- Initial margin - $1500 / contract
- Maintenance margin - $1000

Day	Index Level	Daily Gain	Margin Balance	Margin Call
	1000		3000	
1-Jun	995	-500	2500	
2-Jun	1015	2000	4500	
3-Jun	1005	-1000	3500	
4-Jun	990	-1500	2000	1000
5-Jun	995	500	3500	

Important Points

- As an investor you have more than your initial margin at risk prior to maturity.
  - Previous example, suppose you always honor the margin call, you could lose up to 1000 in index terms or (1000 x $50 x 2) = $100,000
  - On an minimum investment of $2000 (maintenance margin), this is a 50 times exposure to the market

Futures Payout

- Although, in practice, the lifetime payout from a futures accrues over time and accumulates in the margin account, it is useful to think of the payout from the contract as being received at the maturity date.
- It is important to understand the relationship between this hypothetical payout and the spot price.

Futures Payoffs

- Let F be the futures price and S_T be the spot price at maturity.
- The payout from a long position in a futures contract:

$$\boxed{S_T - F}$$

- The payout from a short position in a futures contract:

$$\boxed{F - S_T}$$

Risk of a Futures

- Suppose that
  - you are long an S&P 500 E-Mini index futures with one month to maturity. The current index value is 2,884.05 and the futures price is 2889.75
  - The initial margin is $ 3000 and earns the risk-free rate of interest.
  - You make no margin payments until maturity when you settle the position.
- What is the Beta of the investment?

Beta of Futures

- The return on the investment:

$$\tilde{R} = \frac{\tilde{S}_T - F + (1 + r_f)M}{M}$$

- The Beta is

$$\beta = \frac{\operatorname{cov}\left(\dfrac{\tilde{S}_T}{M}, \tilde{r}_M\right)}{\operatorname{var}(\tilde{r}_M)} = \frac{\operatorname{cov}\left(\dfrac{\tilde{S}_T}{S_0}\dfrac{S_0}{M}, \tilde{r}_M\right)}{\operatorname{var}(\tilde{r}_M)} = \frac{S_0}{M}$$

E-Mini Beta

- In this case, the beta of the contract, with margin, is:

$$\beta = \frac{S_0}{M} = \frac{2884.05 \times 50}{3000} = 48.07$$

- Your investment is almost 50 times more risky than an investment in the market!

Options

Option Contracts

- In an option contract the **writer** grants the **buyer** the option, but not the obligation, to buy from or to sell to the writer a specific asset at a specific price (called the strike or exercise price) within a specified period of time.

Common Options

1. Call Option
   - Buyer has the right to buy the asset at a given price (the exercise price) at a given date.
   - Writer has commitment to sell the underlying asset to the holder at the exercise price if exercised.
2. Put Option
   - Buyer has the right to sell the asset at a given price (the exercise price) at a given date.
   - Writer has commitment to buy the underlying asset for the holder at the exercise price if exercised.

Terminology

1. American Option
   - Option that can be exercised **at any time prior** to expiration date.
2. European Option
   - Option that can be exercised only at expiration date.
3. In-the-money
   - stock price > exercise price
4. Out-of-the-money
   - stock price < exercise price
5. At the money
   - stock price ≈ exercise price

Notation

- S (or S_t): Current stock price at time t.
- K: exercise or strike price.
- T: time to expiration (maturity)
- C (c): value of American (European) call.
- P (p): value of American (European) put.

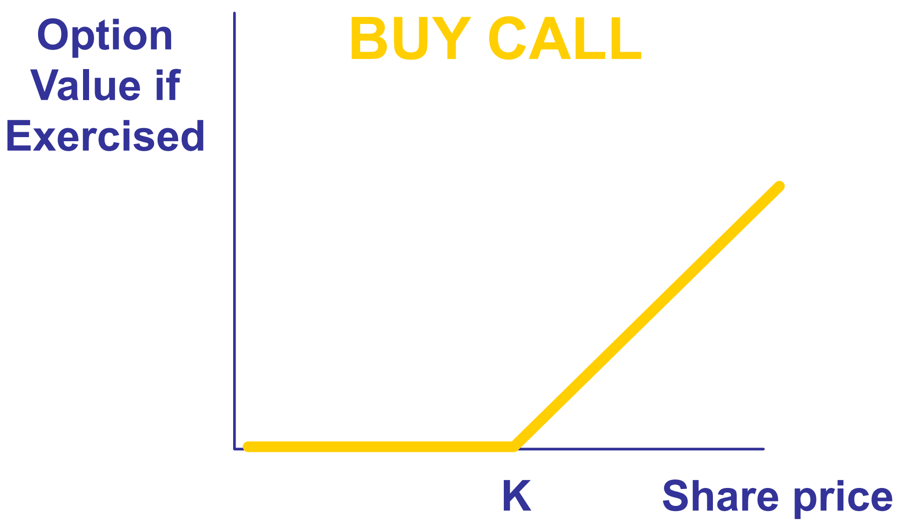
Values of options at expiration

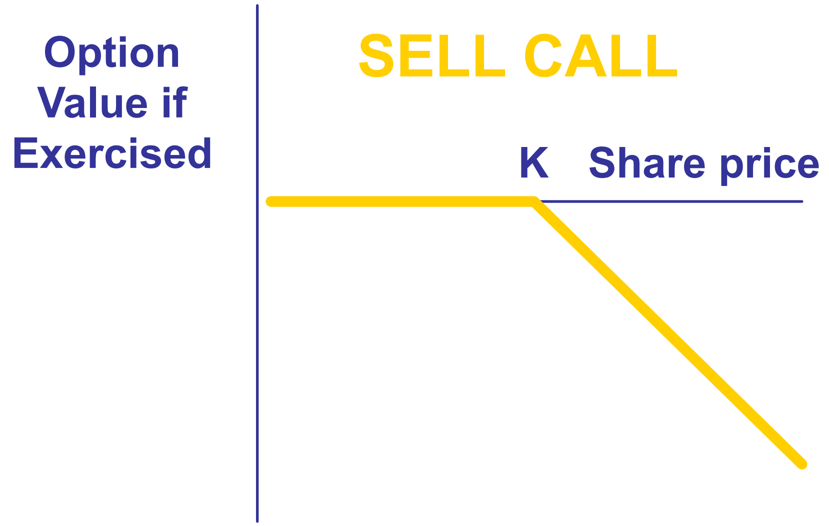
- A Payout Graph shows the cashflows resulting from a position in an option as a function of of the underlying asset’ s price.
  - eg. Call Options:
    - If $S_T < K$ then payoff to call owner = 0
    - If $S_T \geq K$ then payoff to call owner = $S_T - K$
  - Where
    - S_T = value of stock at expiration, and
    - K = Strike Price

Example: Call Option

- Call option on MOT: K =$90
- Value of option at different stock prices:

Stock Price	$80	$90	$100	$110	$120
Option Payoff	$0	$0	$10	$20	$30





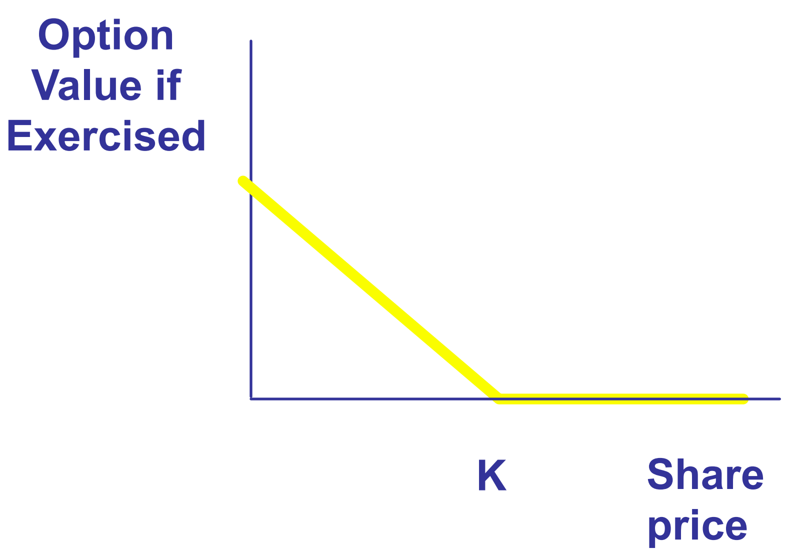
Example: Put Option

- Value of the put option at expiration:

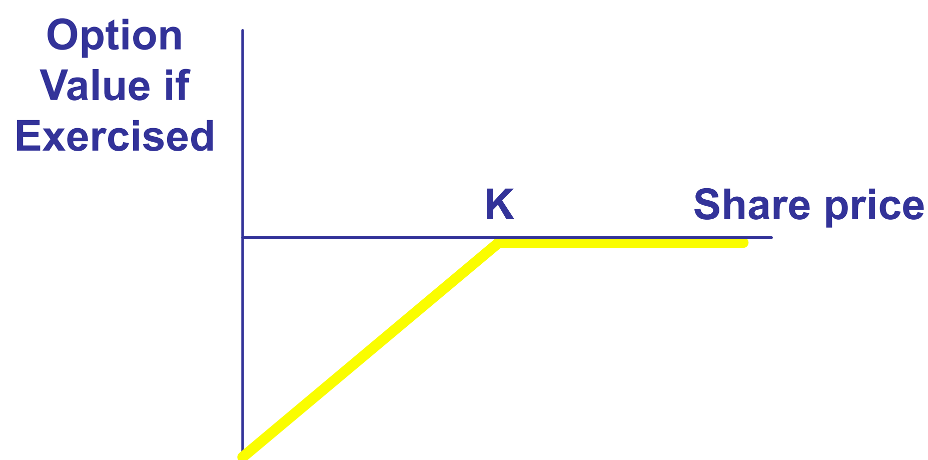
  Payoff to put owner = $K - S_T$ if $S_T < K$

  Payoff to put owner = 0 if $S_T \geq K$

BUY PUT



SELL PUT



Payoffs

- For the owner of a call option:

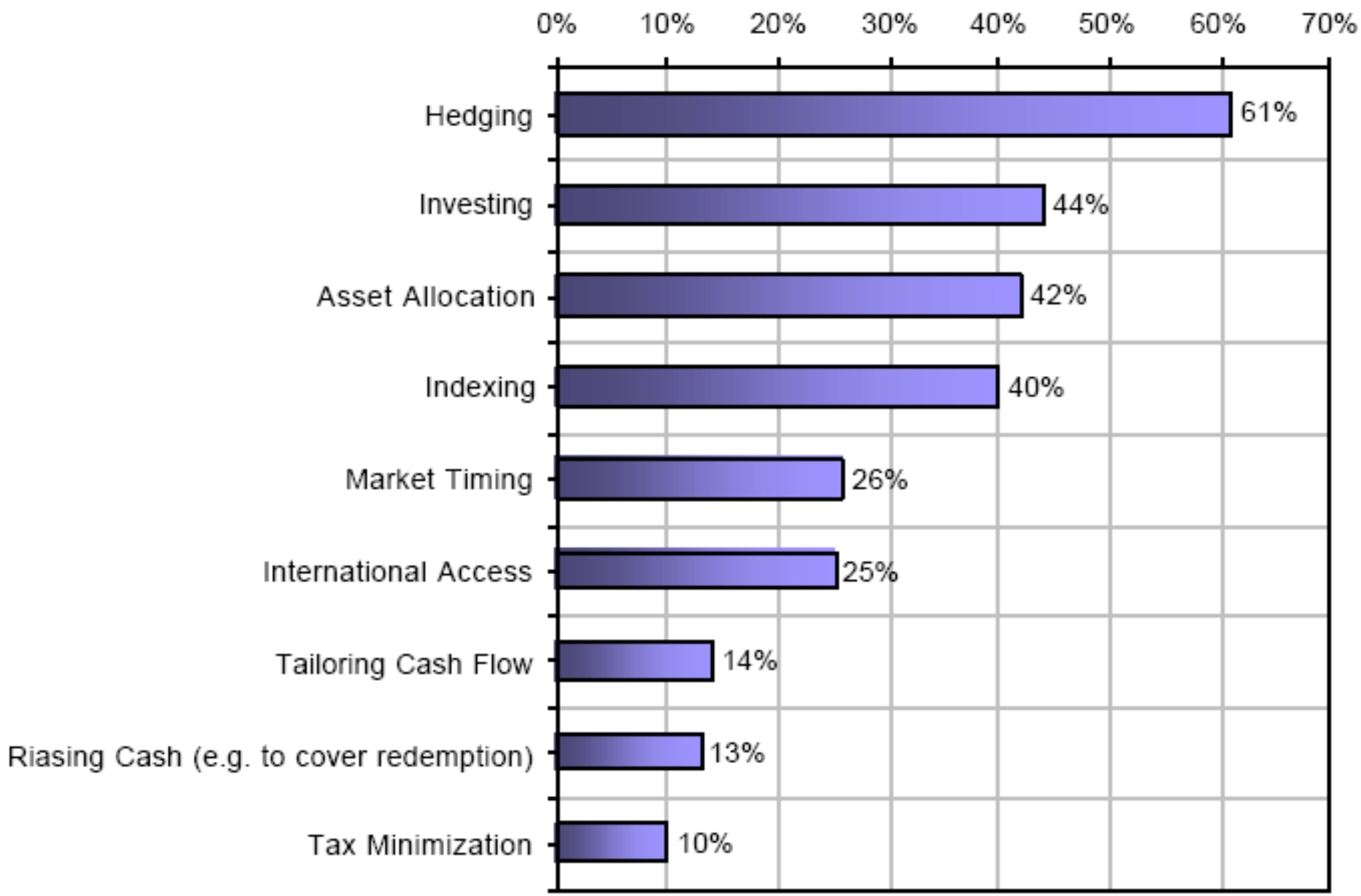
$$\max\{S_T - K, 0\} \equiv \begin{cases} S_T - K & \text{if } S_T > K \\ 0 & \text{if } S_T \le K \end{cases}$$

- For the owner of a put option:

$$\max\{K - S_T, 0\} \equiv \begin{cases} K - S_T & \text{if } S_T \le K \\ 0 & \text{if } S_T > K \end{cases}$$

Derivative Strategies

Reasons Institutions, Use Equity Derivatives



Derivative Strategies

- We will examine how derivatives may be used to:
  - Eliminate risk (hedge);
  - Modify risk (partially hedge);
  - Replicate other payoffs and create synthetic payoffs.

Hedging: Determining Exposure

- Consider an individual who wishes to hedge (say using futures).

- In order to determine exactly how to hedge, we must determine the ratio of the existing risk to be hedged relative to each unit of the hedging instrument --- the **hedge ratio (for forwards and futures)**.

Hedging Example 1

- Suppose you are an index portfolio manager with $4,000,000 invested in the S&P 500 stocks. You have chosen to eliminate any risk in the portfolio by using the E-mini S&P 500 contract (remember that each contract provides a payout equal to $50 times the S&P 500 level). Determine the number of contracts you should buy or sell in order to achieve the hedge.

Step 1: Describe the Payoffs

Portfolio return:

$$4{,}000{,}000 \times \widetilde{R}_p = 4{,}000{,}000 \times \widetilde{R}_M = \frac{4{,}000{,}000 \times \widetilde{S}_T}{S_0}$$

Futures Contract Payoff: $50 \times (\widetilde{S}_T - F)$

Step 2: Determine the Hedge Ratio

- If the sensitivity is to be zero, we need the payoff to be zero, regardless of the realized return on the market. Calculate the hedge ratio:

$$\frac{4{,}000{,}000 \times \tilde{S}_T}{S_0} + h \times 50 \times (\tilde{S}_T - F) = \text{constant}$$

$$\Rightarrow \frac{4{,}000{,}000}{S_0} + h \times 50 = 0$$

$$\Rightarrow h = \frac{-4{,}000{,}000}{50 \times S_0}$$

Hedging Example 2

- Suppose you are an index portfolio manager with $4,000,000 invested in a stock portfolio. You have chosen to eliminate the **market risk** in the portfolio by using the E-mini S&P 500 contract. Using a market model regression you have determined that the portfolio beta is 1.175. How many contracts you should buy or sell in order to achieve the hedge.

$$R_P = \alpha_P + \beta_P \widetilde{R}_M + \widetilde{\varepsilon}_P$$

Partial Hedging Example

- Partial Hedging
  - You need not offset all the risk in the portfolio. In fact, by using index futures you can tune your portfolio to have any exposure you desire.
  - Example: Given the data in the previous example, describe how you can use the futures contract to change the portfolio beta to 0.5.

Let's Do it together on the Blackboard

- $4,000,000 in a portfolio with beta of 1.175, with $R_P = \alpha_P + \beta_P \widetilde{R}_M + \widetilde{\varepsilon}_P$ Reduce beta to 0.5 with the E-mini.
- Payoff of portfolio to be hedged:
- Payoff of the E-mini S&P500 future:
- Hedge ratio: h=

Creating Synthetic Payoffs

- Derivatives can be used to **replicate** payoffs.
- Example 1: Describe how you would use a futures contract that mimics the payoff to investing $50,000 in the S&P 500.

Example

- Describe how you can use a futures contract and investments in t-bills to create a well diversified portfolio with a beta of 0.5.
- S: S&P 500 index ($). B: value of t-bills.
- N: no. of contracts. M: margin / contract.

$$R_P = \frac{B\left(1+r_f\right) + N\times 50(\tilde{S}_T - F)}{B + N\times M}$$

$$R_P = \frac{B\left(1+r_f\right) - N\times 50\times F}{B+N\times M} + \frac{N\times 50\times S_0}{B+N\times M}\frac{\tilde{S}_T}{S_0}$$

Options Strategies

- Basic option payoffs can be added together to create compound payoffs that have almost arbitrary characteristics. We'll examine:
  - Put-Call parity;
  - Covered strategies;
  - Spreads.

Put-Call Parity

- Suppose you simultaneously buy a call and write a put, both having a strike price K and maturity T.
  - Summary of payoffs from this position at T:

	$S_T < K$	$S_T > K$
Payoff from call purchased	0	$S_T - K$
Payoff from put written	$-(K - S_T)$	0
Total	$S_T - K$	$S_T - K$

  - So the total payoffs to these positions will be $S_T - K$.

An Alternative

- Borrow $\frac{K}{(1+r_f)^T}$ today and repay K at maturity
- Buy 1 share of stock
- Your payoffs from these positions at time T:
  - Payoff = Asset - Liability = $S_T - K$
- The **two positions** give you **identical payoffs.**
- By **no arbitrage**, the costs of establishing these positions must be identical

The Put-Call Parity Relationship

1. Cost to establish option positions:
   a. Purchase call option for C_0
   b. Sell put option for P_0
   c. Total cost of establishing position: C_0-P_0
2. Cost to establish levered equity position:
   a. Cost of stock: S_0
   b. Borrowed funds: $\dfrac{K}{(1+r_f)^T}$
   c. Total cost of establishing levered stock position:

$$S_0 - \frac{K}{(1+r_f)^T}$$

3. Costs of establishing identical payoff positions must be identical:

$$C_0 - P_0 = S_0 - \frac{K}{(1+r_f)^T}$$

Two Ways of Checking Put-Call Parity

$$\mathrm{C}_0 - \mathrm{P}_0 = \mathrm{S}_0 - \frac{\mathrm{K}}{(1+r_f)^T}$$

$$\mathrm{C}_0 + \frac{\mathrm{K}}{(1+r_f)^T} = P_0 + S_0$$

If the Put-Call Parity does not hold, then arbitrage opportunities will arise.

Example: Suppose that the security price is $31, the exercise price is $30, the risk-free rate is 10% per annum, the price of a 3-month European call option is $3, and the price of a 3-month European put option is $2.25.

- Do we have mispricing?
- If yes, could we make arbitrage profits?

Value of Portfolio A:

$$c + Ke^{-r(T-t)} = 3 + 30e^{-0.1x0.25} = \$32.26$$

Value of Portfolio B:

$$p + S = 2.25 + 31 = \$33.25$$

Portfolio B is overpriced relative to portfolio A.

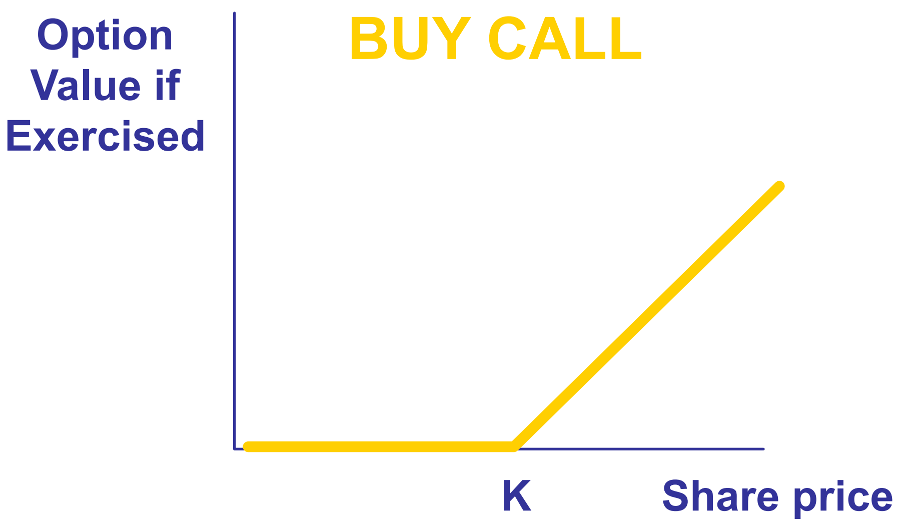
What is the arbitrage strategy?

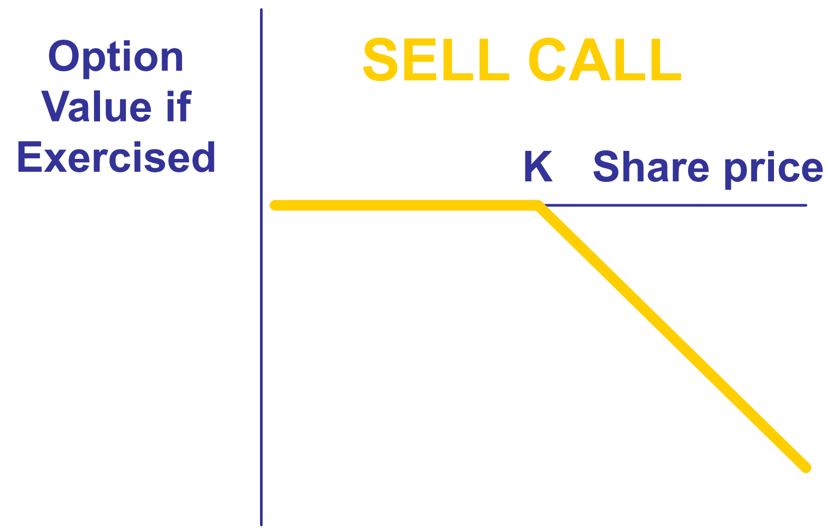
Buy securities in Portfolio A and short the securities in Portfolio B.

Covered Strategies: Review

- Recall: Call Options

$$\max\{S_T - K, 0\} \equiv \begin{cases} S_T - K & \text{if } S_T > K \\ 0 & \text{if } S_T \le K \end{cases}$$



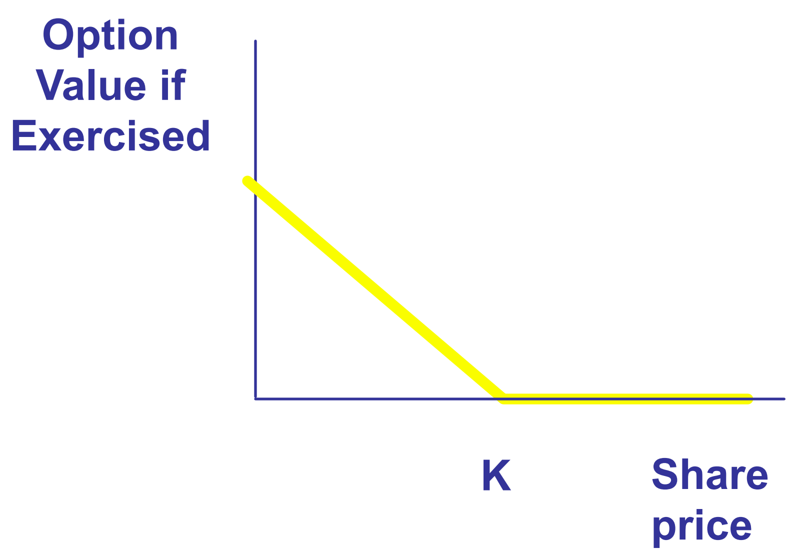


Covered Strategies: Review

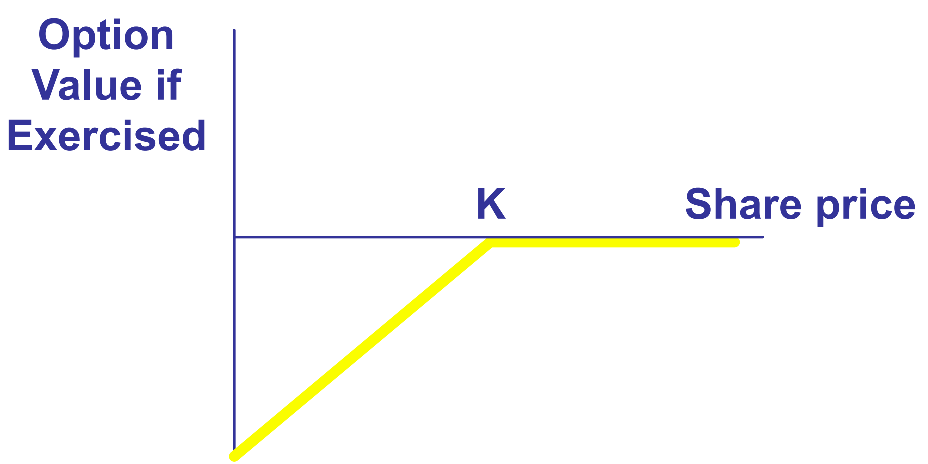
Recall: Put Options

$$\max\{K - S_T, 0\} \equiv \begin{cases} K - S_T & \text{if } S_T \le K \\ 0 & \text{if } S_T > K \end{cases}$$

BUY PUT



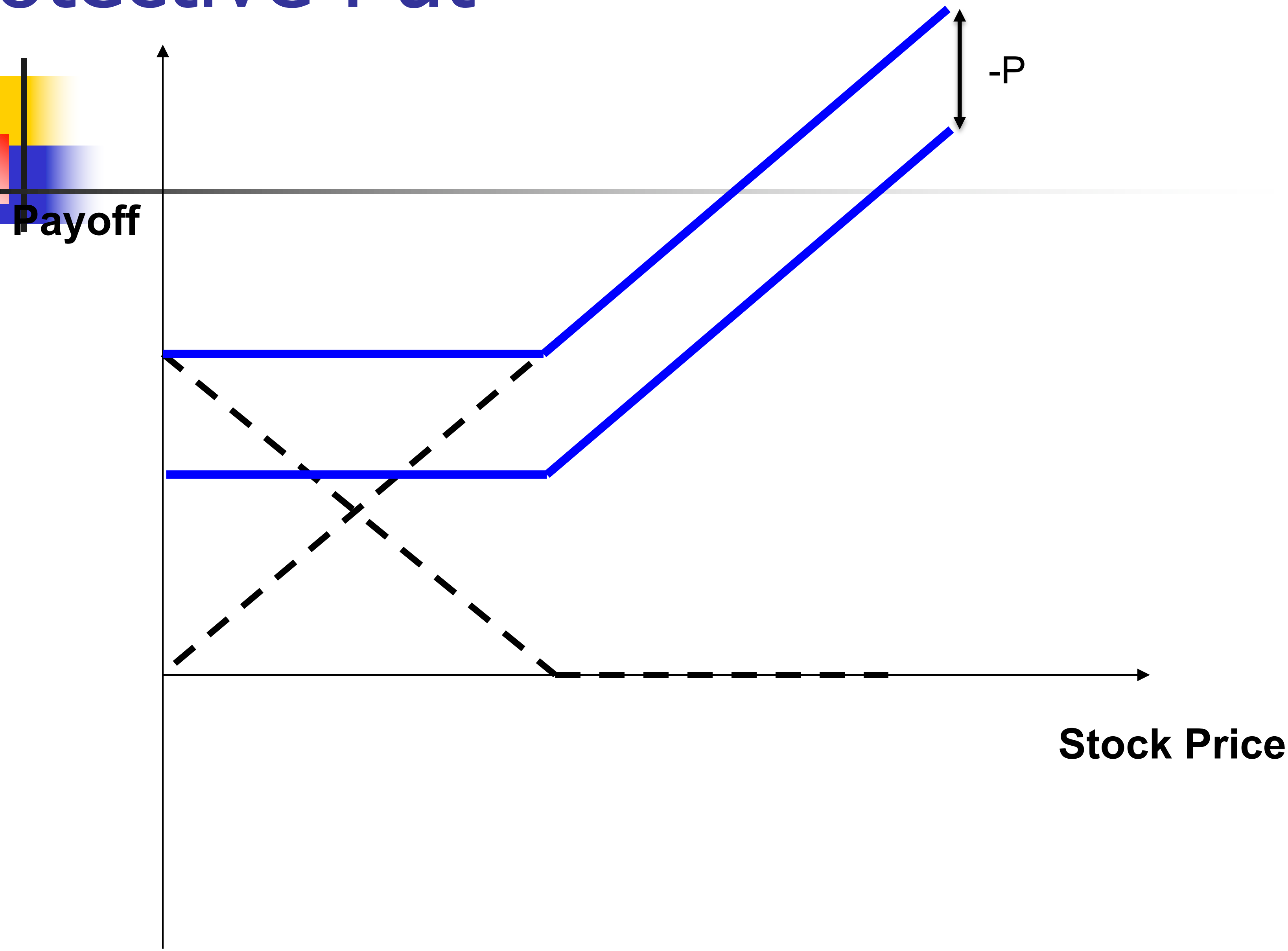
SELL PUT



Covered Strategies

- Protective Put
  - Buying a put on a long position in the underlying asset.
  - Acts like insurance by guaranteeing a floor on the total value of the portfolio.
  - Eliminate downside risk by paying up at the beginning

Protective Put



Covered Call

- Write call option on a long position in the underlying asset.
- Generates immediate income in the form of the option premium.
- Collect income at the beginning at a cost of eliminating upside opportunities

Covered Call

Straddle

- Bets on volatility
- Buy a call and put with the same strike price and expiration date.

Straddle: Graph

- Payoff comes from movement of the underlying asset in either direction.

Spreads

- Exposure to stock price movements in an interval (eliminate upside and downside).
- Bull call spread:
  - Alternative to collar (long stock, put and write call)
  - Buy a call with a low strike price and sell a call with a higher strike price.
  - Can also be achieved with put options. (buy at low strike, sell at high strike)
- Bear call spread:
  - Mirror image payoff to Bull Spread

Bull Spread