



Chapter 12

Game Theory

Topics to be Discussed

- Gaming and Strategic Decisions
- Dominant Strategies
- The Nash Equilibrium Revisited
- Repeated Games

Topics to be Discussed

- Sequential Games
- Threats, Commitments, and Credibility
- Entry Deterrence

Gaming and Strategic Decisions

- **Game** is any situation in which *players* (the participants) make strategic decisions
 - Ex: firms competing with each other by setting prices, group of consumers bidding against each other in an auction
- Strategic decisions result in **payoffs** to the players: outcomes that generate rewards or benefits

Some Applications of Game Theory

- The study of oligopolies (industries containing only a few firms)
- The study of cartels; e.g. OPEC
- The study of externalities; e.g. using a common resource such as a fishery.
- The study of military strategies.

What is a Game?

- A game consists of
 - a set of players
 - a set of strategies for each player
 - the payoffs to each player for every possible list of strategy choices by the players.

Gaming and Strategic Decisions

- Game theory tries to determine optimal strategy for each player
- **Strategy** is a rule or plan of action for playing the game
- **Optimal strategy** for a player is one that maximizes the expected payoff
- We consider players who are rational – they think through their actions

Gaming and Strategic Decisions

- *“If I believe that my competitors are rational and act to maximize their own profits, how should I take their behavior into account when making my own profit-maximizing decisions?”*

Noncooperative vs. Cooperative Games

- Cooperative Game
 - Players negotiate binding contracts that allow them to plan joint strategies
 - Example: Buyer and seller negotiating the price of a good or service or a joint venture by two firms (i.e., Microsoft and Apple)
 - Binding contracts are possible

Noncooperative vs. Cooperative Games

- Noncooperative Game
 - Negotiation and enforcement of binding contracts between players is not possible
 - Example: Two competing firms, assuming the other's behavior, independently determine pricing and advertising strategy to gain market share
 - Binding contracts are not possible

An Example of a Two-Player Game

Player B

L

R

U

(3,9)

(1,8)

D

(0,0)

(2,1)

Player A

This is the
game's
payoff matrix.

Player A's payoff is shown first.

Player B's payoff is shown second.

Dominant Strategies

- Dominant Strategy is one that is optimal no matter what an opponent does
 - An Example
 - A and B sell competing products
 - They are deciding whether to undertake advertising campaigns

Payoff Matrix for Advertising Game

| | | <i>Firm B</i> | |
|---------------|-----------------|---------------|-----------------|
| | | Advertise | Don't Advertise |
| <i>Firm A</i> | Advertise | 10, 5 | 15, 0 |
| | Don't Advertise | 6, 8 | 10, 2 |

Payoff Matrix for Advertising Game

- Observations

- A: regardless of B, advertising is the best
- B: regardless of A, advertising is best

| | | <i>Firm B</i> | |
|---------------|-----------------|---------------|-----------------|
| | | Advertise | Don't Advertise |
| <i>Firm A</i> | Advertise | 10, 5 | 15, 0 |
| | Don't Advertise | 6, 8 | 10, 2 |

Payoff Matrix for Advertising Game

- Observations

- Dominant strategy for A and B is to advertise
- Do not worry about the other player
- Equilibrium in dominant strategy

| | | <i>Firm B</i> | |
|---------------|-----------------|---------------|-----------------|
| | | Advertise | Don't Advertise |
| <i>Firm A</i> | Advertise | 10, 5 | 15, 0 |
| | Don't Advertise | 6, 8 | 10, 2 |

Dominant Strategies

- Equilibrium in dominant strategies
 - Outcome of a game in which each firm is doing the best it can regardless of what its competitors are doing
 - Optimal strategy is determined without worrying about the actions of other players
- However, not every game has a dominant strategy for each player

Dominant Strategies

- Game Without Dominant Strategy
 - The optimal decision of a player without a dominant strategy will depend on what the other player does
 - Revising the payoff matrix, we can see a situation where no dominant strategy exists

Modified Advertising Game

| | | <i>Firm B</i> | |
|---------------|-----------------|---------------|-----------------|
| | | Advertise | Don't Advertise |
| <i>Firm A</i> | Advertise | 10, 5 | 15, 0 |
| | Don't Advertise | 6, 8 | 20, 2 |

Modified Advertising Game

- Observations

- A: No dominant strategy; depends on B's actions
- B: Dominant strategy is to advertise
- Firm A determines B's dominant strategy and makes its decision accordingly

| | | <i>Firm B</i> | |
|---------------|-----------------|---------------|-----------------|
| | | Advertise | Don't Advertise |
| <i>Firm A</i> | Advertise | 10, 5 | 15, 0 |
| | Don't Advertise | 6, 8 | 20, 2 |

The Nash Equilibrium Revisited

- A dominant strategy is stable, but in many games one or more party does not have a dominant strategy
- A more general equilibrium concept is the **Nash Equilibrium**
 - A set of strategies (or actions) such that each player is doing the best it can given the actions of its opponents

The Nash Equilibrium Revisited

- None of the players have incentive to deviate from its Nash strategy, therefore it is stable
 - In the Cournot model, each firm sets its own output assuming the other firm's outputs are fixed.
 - Cournot equilibrium is a Nash Equilibrium.

The Nash Equilibrium Revisited

- Dominant Strategy
 - “I’m doing the best I can no matter what you do. You’re doing the best you can no matter what I do.”
- Nash Equilibrium
 - “I’m doing the best I can given what you are doing. You’re doing the best you can given what I am doing.”
- Dominant strategy is a special case of Nash equilibrium

The Nash Equilibrium Revisited

- Two cereal companies face a market in which two new types of cereal can be successfully introduced, provided each type is introduced by only one firm
- Product Choice Problem
 - Market for one producer of crispy cereal
 - Market for one producer of sweet cereal
 - Each firm only has the resources to introduce one cereal
 - Noncooperative

Product Choice Problem

| | | <i>Firm 2</i> | |
|---------------|--------|---------------|--------|
| | | Crispy | Sweet |
| <i>Firm 1</i> | Crispy | -5, -5 | 10, 10 |
| | Sweet | 10, 10 | -5, -5 |

Product Choice Problem

- If Firm 1 hears Firm 2 is introducing a new sweet cereal, its best action is to make crispy
- Bottom left corner is Nash equilibrium
- What is other Nash Equilibrium?

| | | <i>Firm 2</i> | |
|---------------|--------|---------------|--------|
| | | Crispy | Sweet |
| <i>Firm 1</i> | Crispy | -5, -5 | 10, 10 |
| | Sweet | 10, 10 | -5, -5 |

Prisoners' Dilemma

| | | <i>Prisoner B</i> | |
|-------------------|---------------|-------------------|---------------|
| | | Confess | Don't Confess |
| <i>Prisoner A</i> | Confess | -1, -1 | 5, -5 |
| | Don't Confess | -5, 5 | 3, 3 |

Prisoners' Dilemma

- While there is an outcome to make these two prisoners both better off, they will simultaneously choose “Confess” which ends up in a bad outcome
- But if they could just hang tight, they would each be better off!
- The problem is that there is no way for the two prisoners to *coordinate their actions*

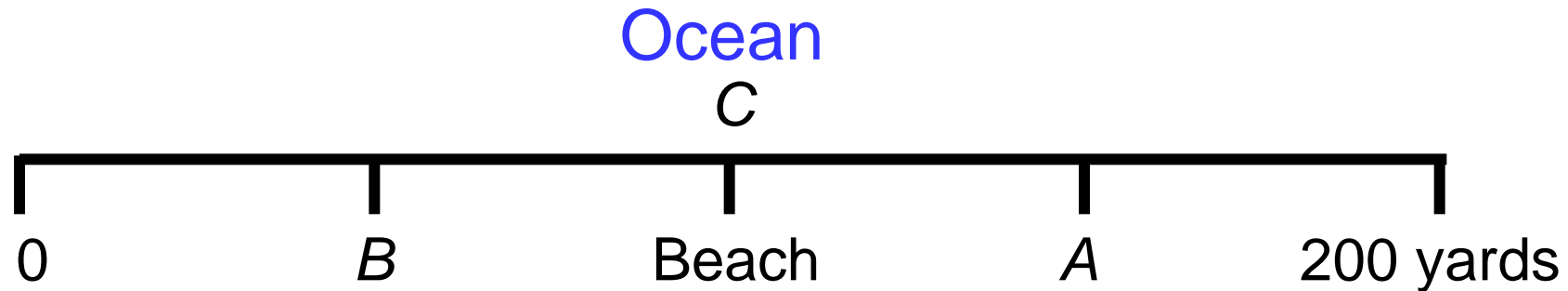
Applications of Prisoners' Dilemma

- Price War in competition
- Cheating in a cartel
- Low trust equilibrium: what if everyone behaves like Lei Feng?
- Free riding in a public goods provision
- Arm contests and control

Beach Location Game

- Scenario
 - Two competitors, Y and C, selling soft drinks
 - Beach is 200 yards long
 - Sunbathers are spread evenly along the beach
 - $\text{Price } Y = \text{Price } C$
 - Customer will buy from the closest vendor

Location Game: Hotelling Model



- Where will the competitors locate (i.e., where is the Nash equilibrium)?
- Will want to all locate in center of beach
 - Similar to groups of gas stations, car dealerships, etc.
 - The convergence of political positions in the two-party election campaign

More Applications of Game Theory

- War and Peace in the Middle East
- The balance of three kingdoms instead of two kingdoms
- The Three Musketeers and their fight: who will survive?

Mixed Strategy

- Pure Strategy
 - Player makes a specific choice or takes a specific action
- Mixed Strategy
 - Player makes a random choice among two or more possible actions, based on a set of chosen probabilities

Matching Pennies

| | | <i>Player B</i> | |
|-----------------|-------|-----------------|-------|
| | | Heads | Tails |
| <i>Player A</i> | Heads | 1, -1 | -1, 1 |
| | Tails | -1, 1 | 1, -1 |

Matching Pennies

- Pure strategy: No Nash equilibrium
 - No combination of head and tails leaves both players better off
- Mixed strategy: Random choice is a Nash equilibrium

| | | <i>Player B</i> | |
|-----------------|-------|-----------------|-------|
| | | Heads | Tails |
| <i>Player A</i> | Heads | 1, -1 | -1, 1 |
| | Tails | -1, 1 | 1, -1 |

Matching Pennies

- Player A might flip coin playing heads with $\frac{1}{2}$ probability and tails with $\frac{1}{2}$ probability
- If both players follow this strategy, there is a Nash equilibrium – both players will be doing the best they can given what their opponent is doing
- Although the outcome is random, the expected payoff is 0 for each player
- Zero-sum game: excessive competition

Solving Mixed Strategies

- Indifference Condition: Pick up a mixing strategy to make your opponent to be indifferent in playing each pure strategy

Matching Pennies

| | | <i>Player B</i> | |
|-----------------|------------------|-----------------|-------|
| | | Heads | Tails |
| <i>Player A</i> | Heads p | 1, -1 | -1, 1 |
| | Tails $(1-p)$ | -1, 1 | 1, -1 |

Solving Mixed Strategies

- Suppose that player A plays Heads in the probability of p , and Tails in $(1-p)$
- For player B, the expected payoff of playing Heads and Tails is $-p + (1-p)$ and $p - 1 + p$ respectively
- Indifference condition means
- $-p + (1-p) = p - 1 + p$
- $P^* = 1/2$
- Similarly $r^* = 1/2$

Mixed Strategy

- One reason to consider mixed strategies is when there is a game that does not have any Nash equilibriums in pure strategy
- When allowing for mixed strategies, every game has a Nash equilibrium
- Mixed strategies are popular for games like poker

The Battle of the Sexes

| | | <i>Joan</i> | |
|------------|-----------|-------------|-------|
| | | Wrestling | Opera |
| <i>Jim</i> | Wrestling | 2,1 | 0,0 |
| | Opera | 0,0 | 1,2 |

The Battle of the Sexes

- Pure Strategy
 - Both watch wrestling
 - Both watch opera
- Mixed Strategy
 - Jim ($2/3$, $1/3$)
 - Joan ($1/3$, $2/3$)

| | | <i>Joan</i> | |
|------------|-----------|-------------|-------|
| | | Wrestling | Opera |
| <i>Jim</i> | Wrestling | 2,1 | 0,0 |
| | Opera | 0,0 | 1,2 |

The Battle of the Sexes

- For Jim, pick up p such that
 - $P = 2(1-p)$
 - $P^* = 2/3$
- For Joan, pick up r such that
 - $2r = 1-r$
 - $r^* = 1/3$

| | | | |
|-----|------------------|------------------|----------------|
| | | Joan | |
| | | Wrestling r | Opera $1-r$ |
| Jim | Wrestling p | 2,1 | 0,0 |
| | Opera (1-p) | 0,0 | 1,2 |

Repeated Games

- Game in which actions are taken and payoffs are received over and over again
- Oligopolistic firms play a repeated game
- With each repetition of the Prisoners' Dilemma, firms can develop reputations about their behavior and study the behavior of their competitors

Pricing Problem

| | | <i>Firm 2</i> | |
|---------------|------------|---------------|------------|
| | | Low Price | High Price |
| <i>Firm 1</i> | Low Price | 10, 10 | 100, -50 |
| | High Price | -50, 100 | 50, 50 |

Pricing Problem

- How does a firm find a strategy that would work best on average against all or almost all other strategies?
- Trigger Strategy:
 - If you don't cooperate this time, I won't cooperate with you forever
- Tit-for-tat strategy
 - A player responds in kind to an opponent's previous play, cooperating with cooperative opponents and retaliating against uncooperative ones

Trigger Strategy

- What if the game is infinitely repeated?
 - Competitors repeatedly set price every month, forever
- Think about the following strategy for firm I:
 - Play High Price in the first stage. In the t -th stage, if the outcome of all $t-1$ preceding stages has been (H, H) , then play H ; otherwise play L forever

Trigger Strategy

- Given firm 1's above strategy, what is the best response of firm 2?
- If firm 2 deviates and plays L in the first stage, then the present value of his sequence of payoff is

$$100 + 10\delta + 10\delta^2 + \dots = 100 + \frac{10\delta}{1-\delta}; (\delta = 1/1+r)$$

- But if firm 2 cooperates all the time, then

$$50 + 50\delta + \dots = \frac{50}{1-\delta}$$

- Firm 2 will cooperate if $\frac{50}{1-\delta} > 100 + \frac{10\delta}{1-\delta} \Rightarrow \delta^* > \frac{5}{9}$

Finitely Repeated Games

- What if repeated a finite number of times?
 - After the last month, there is no retaliation possible, so each will choose low price in the last period
 - But in the month before last month, knowing that will charge low price in last month, will charge low price in month before
 - Keep going and see that only rational outcome is for both firms to charge low price every month
 - This is due to the common knowledge of rationality and the failure of the punishment in the last period

Sequential Games

- Players move in turn, responding to each other's actions and reactions
 - Ex: Stackelberg model
 - Responding to a competitor's ad campaign
 - Entry decisions
 - Responding to regulatory policy

How to Solve Sequential Game

- Backward induction: if the game has several stages, go to the last stage first and then solve the game backward
- This is consistent with the principle of dynamic optimization
- Consider a game where five barons have to decide how to distribute the 100 gold coins

Game Rule of Distribution

- Each one is randomly assigned a number from 1 to 5
- The guy picking up 1 proposes the distribution scheme, and if it receives over-one-half of the support, then the scheme will be implemented. Otherwise the proposer will be thrown into the sea
- Then No.2 makes a proposal and follows the same rule as above
- The process will be repeated until the last one if there is no proposal approved by the majority rule

How to Solve this Game

- Backward induction
- If there are only 4 and 5, then 5 will veto the proposal by 4
- If 3, 4, and 5, then $(100, 0, 0)$
- If 2, 3, 4, and 5, then $(98, 0, 1, 1)$
- If 1, 2, 3, 4, and 5, then $(97, 0, 1, 2, 0)$ or $(97, 0, 1, 0, 2)$

Sequential Games

- Going back to the product choice problem
 - Two new (sweet, crispy) cereals
 - Successful only if each firm produces one cereal
 - Sweet will sell better
- What if Firm 1 sped up production and introduced new cereal first?
 - Now there is a sequential game
 - Firm 1 will think about what Firm 2 will do

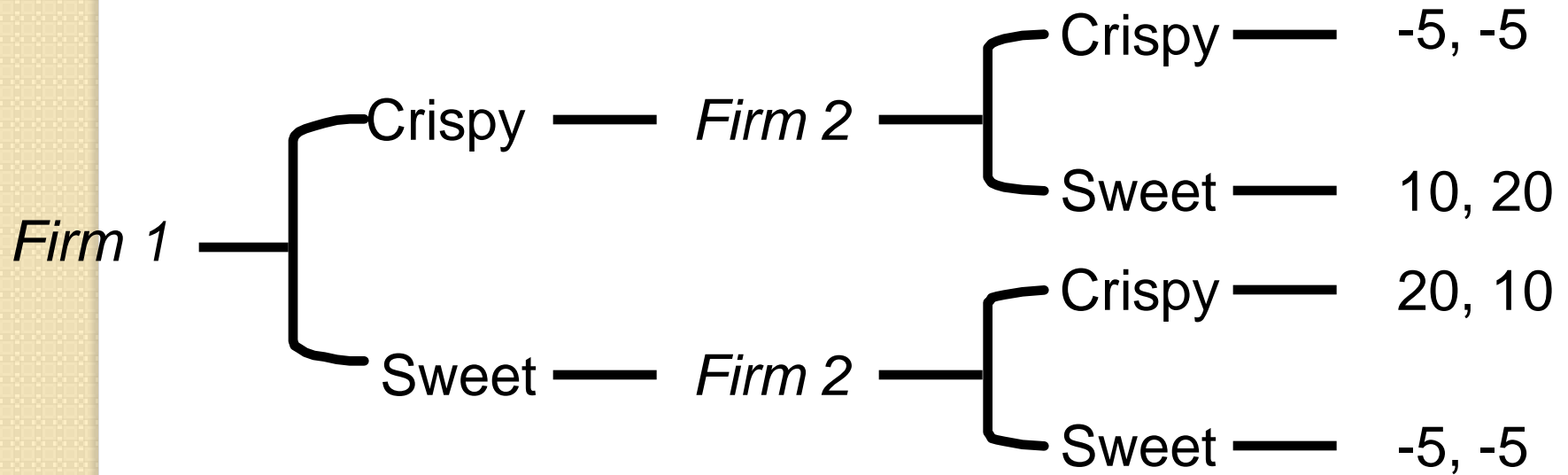
Modified Product Choice Problem

| | | <i>Firm 2</i> | |
|---------------|--------|---------------|--------|
| | | Crispy | Sweet |
| <i>Firm 1</i> | Crispy | -5, -5 | 10, 20 |
| | Sweet | 20, 10 | -5, -5 |

Extensive Form of a Game

- Extensive Form of a Game
 - Representation of possible moves in a game in the form of a decision tree
 - Allows one to work backward from the best outcome for Firm I

Product Choice Game in Extensive Form



Sequential Games

- The Advantage of Moving First
 - In this product-choice game, there is a clear advantage to moving first
 - The first firm can choose a large level of output, thereby forcing second firm to choose a small level
 - Can show the firm's mover advantage by revising the Stackelberg model and comparing to Cournot

First Mover Advantage

- Assume: Duopoly

$$P = 30 - Q$$

$$Q = \text{Total Production} = Q_1 + Q_2$$

$$MC = 0$$

Cournot :

$$Q_1 = Q_2 = 10 \text{ and } P = 10 \quad \pi = 100 / \text{Firm}$$

First Mover Advantage

- Duopoly

With Collusion

$$Q_1 = Q_2 = 7.5 \text{ and } P = 15 \quad \pi = 112.50 / \text{Firm}$$

Firm Moves First (Stackelberg)

$$Q_1 = 15 \quad Q_2 = 7.5 \text{ and } P = 7.50$$

$$\pi_1 = 112.50 \quad \pi_2 = 56.25$$

Choosing Output

| | | <i>Firm 2</i> | | |
|---------------|-----|----------------|------------|---------------|
| | | 7.5 | 10 | 15 |
| <i>Firm 1</i> | 7.5 | 112.50, 112.50 | 93.75, 125 | 56.25, 112.50 |
| | 10 | 125, 93.75 | 100, 100 | 50, 75 |
| | 15 | 112.50, 56.25 | 75, 50 | 0, 0 |

Choosing Output

- This payoff matrix illustrates various outcomes
 - Move together, both produce 10
 - If Firm 1 moves first (Q=15), best Firm 2 can do is 7.5

| | | | | |
|---------------|-----|----------------|------------|---------------|
| | | <i>Firm 2</i> | | |
| | | 7.5 | 10 | 15 |
| <i>Firm 1</i> | 7.5 | 112.50, 112.50 | 93.75, 125 | 56.25, 112.50 |
| | 10 | 125, 93.75 | 100, 100 | 50, 75 |
| | 15 | 112.50, 56.25 | 75, 50 | 0, 0 |

Threats, Commitments, and Credibility

- Strategic Moves
 - What actions can a firm take to gain advantage in the marketplace?
 - Deter entry
 - Induce competitors to reduce output, leave, raise price
 - Implicit agreements that benefit one firm

Threats, Commitments, and Credibility

- Strategic Move
 - Action that gives a player an advantage by constraining his behavior
 - Firm 1 must constrain his behavior to the extent Firm 2 is convinced that he is committed
 - Threats or commitments must be credible

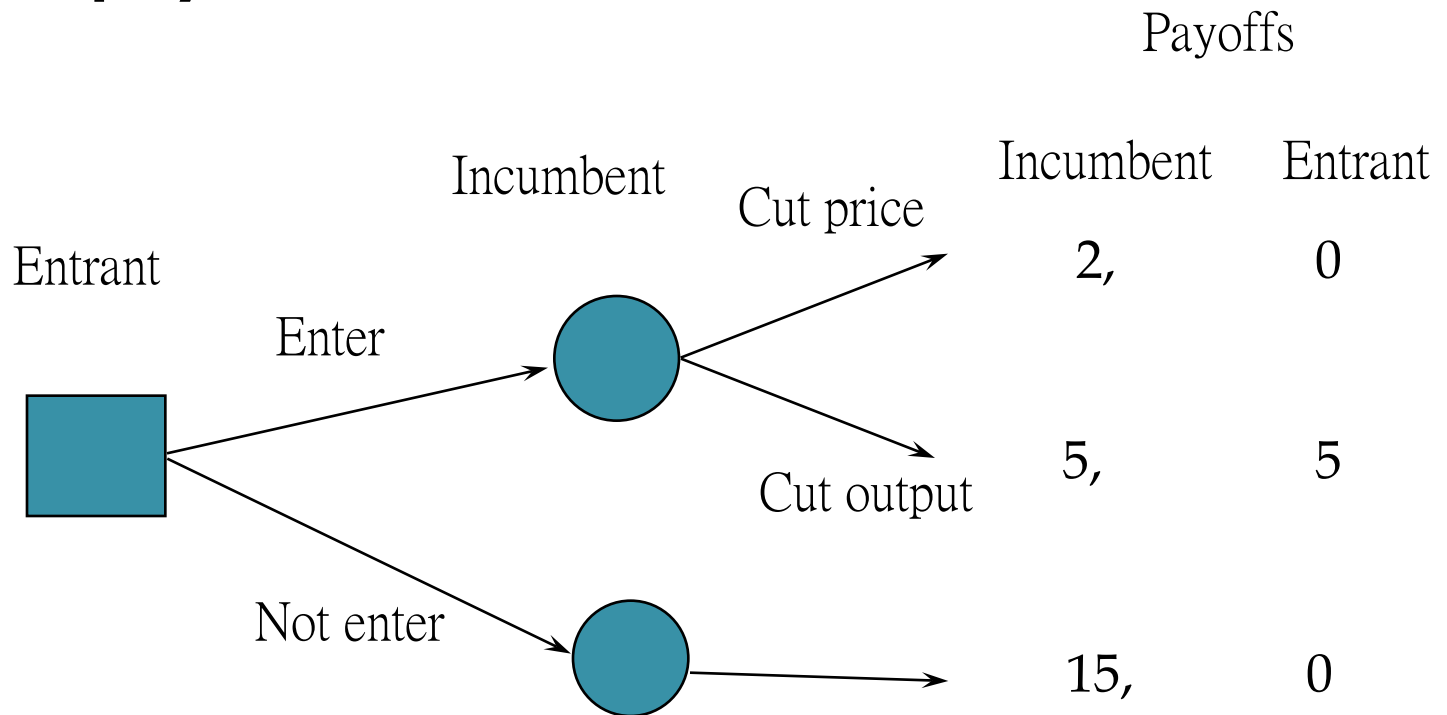
Threats, Commitments, and Credibility

- How to Make the First Move
 - Demonstrate Commitment
 - Firm 1 must do more than announce they will produce sweet cereal
 - Invest in expensive advertising campaign
 - Buy large order of sugar and send invoice to Firm 2
 - Commitment must be enough to induce Firm 2 to make the decision Firm 1 wants it to make

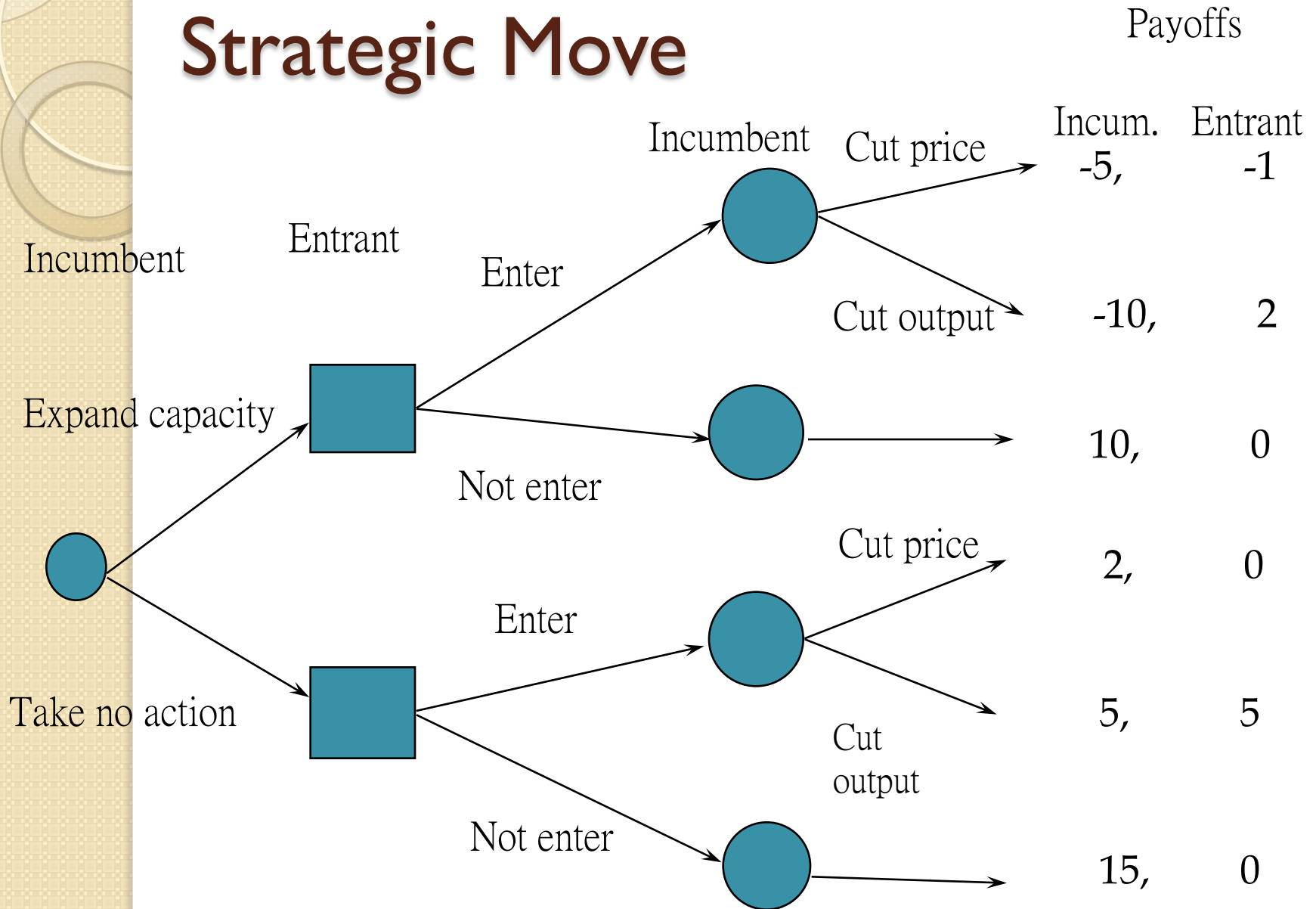
Threats, Commitments, and Credibility

- Empty Threats
 - If a firm will be worse off if it charges a low price, the threat of a low price is not credible in the eyes of the competitors
 - When firms know the payoffs of each other's actions, firms cannot make threats the other firm knows they will not follow
 - In our example, Firm 1 will always charge high price and Firm 2 knows it

Empty Threat



Strategic Move





Chapter 13

General Equilibrium

Topics to be Discussed

- General Equilibrium Analysis
- Efficiency in Exchange
- Equity and Efficiency
- Efficiency in Production

Topics to be Discussed

- The Gains from Free Trade
- An Overview: The Efficiency of Competitive Markets
- Why Markets Fail

General Equilibrium Analysis

- Up to this point, we have been focused on partial equilibrium analysis
 - Activity in one market has little or no effect on other markets
- Market interrelationships can be important
 - Complements and substitutes
 - Increase in firms' input demand can cause market price of the input and product to rise

General Equilibrium Analysis

- To study how markets interrelate, we can use **general equilibrium analysis**
 - Simultaneous determination of the prices and quantities in all relevant markets, taking into account feedback effects
- The **feedback effect** is the price or quantity adjustment in one market caused by price and quantity adjustments in related markets

Efficiency in Exchange

- We showed before that competitive markets are efficient because consumer and producer surpluses are maximized
- We can study this in more detail by examining an exchange economy
 - Market in which two or more consumers trade two goods among themselves
 - Same for two countries

Efficiency in Exchange

- An efficient allocation of goods is one where no one can be made better off without making someone else worse off
 - Pareto efficiency
- Voluntary trade between two parties is mutually beneficial and increases economic efficiency

Gain from Trade

- Assumptions
 - Two consumers (countries)
 - Two goods
 - Both people know each other's preferences
 - Exchanging goods involves zero transaction costs
 - James and Karen have a total of 10 units of food and 6 units of clothing

Gain from Trade

| Individual | Initial Endowment | Trade | Final Allocation |
|------------|-------------------|----------|------------------|
| James | 7F, 1C | -1F, +1C | 6F, 2C |
| Karen | 3F, 5C | +1F, -1C | 4F, 4C |

- To determine if they are better off, we need to know the preferences for food and clothing

Gain from Trade

- James' MRS of food for clothing is only $\frac{1}{2}$
 - He will give up $\frac{1}{2}$ unit of clothing for 1 unit of food
- Karen has a lot of clothing and little food
 - MRS of food for clothing is 3
 - To get 1 unit of food, she will give up 3 units of clothing

Gain from Trade

- There is room for trade
 - James values clothing more than Karen
 - Karen values food more than James
 - Karen is willing to give up 3 units of clothing to get 1 unit of food, but James is willing to take only $\frac{1}{2}$ unit of clothing for 1 unit of food
- Actual terms of trade are determined through bargaining
 - Trade for 1 unit of food will fall between $\frac{1}{2}$ and 3 units of clothing

Gain from Trade

- Suppose Karen offers James 1 unit of clothing for 1 unit of food
 - James will have more clothing, which he values more than food
 - Karen will have more food, which she values more
- Whenever two consumers' MRSs are different, there is room for mutually beneficial trade
 - Allocation of resources is inefficient

Gain from Trade

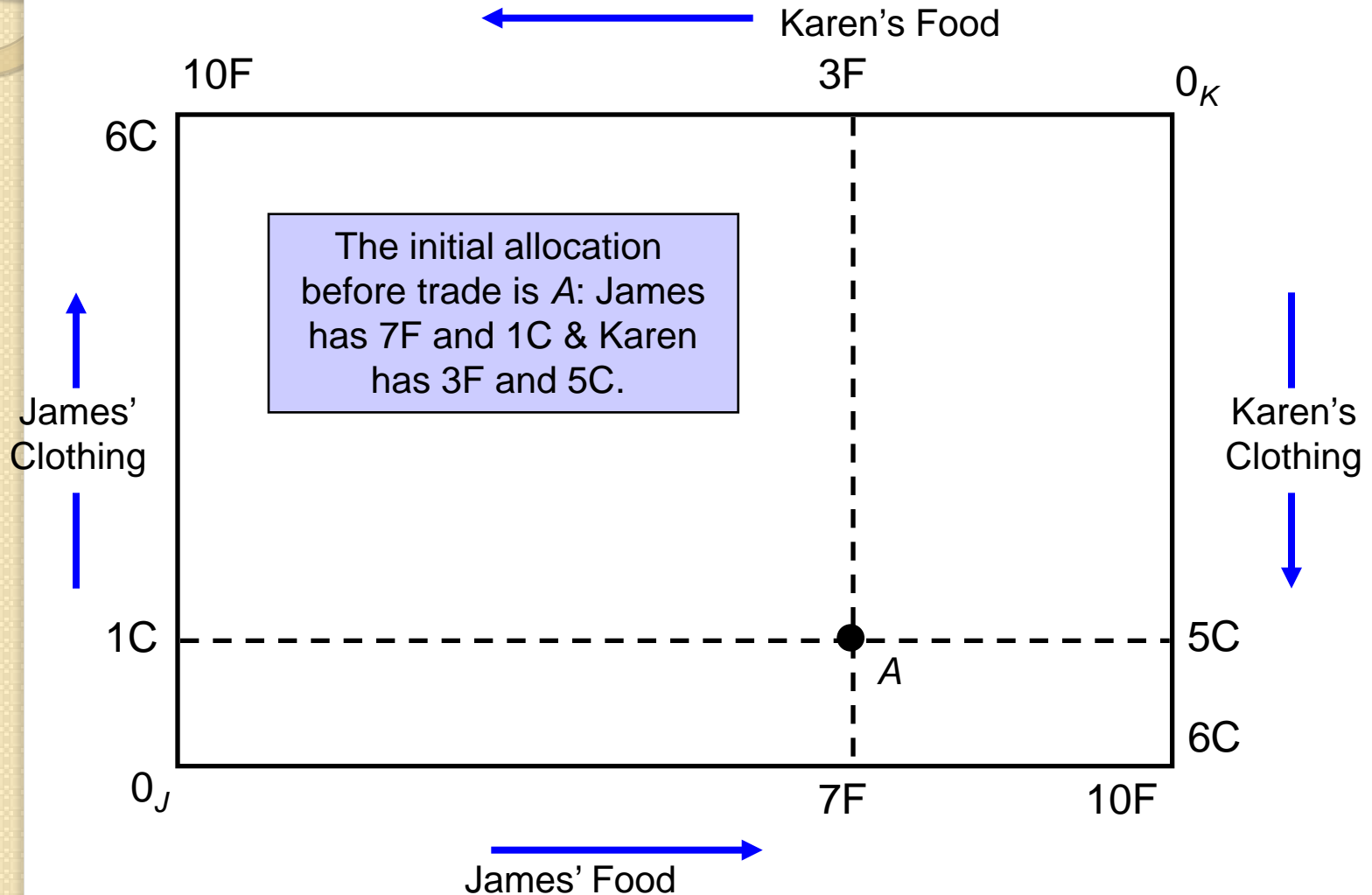
- From this analysis we obtain an important result:

An allocation of goods is efficient only if the goods are distributed so that the marginal rate of substitution between any pair of goods is the same for all consumers

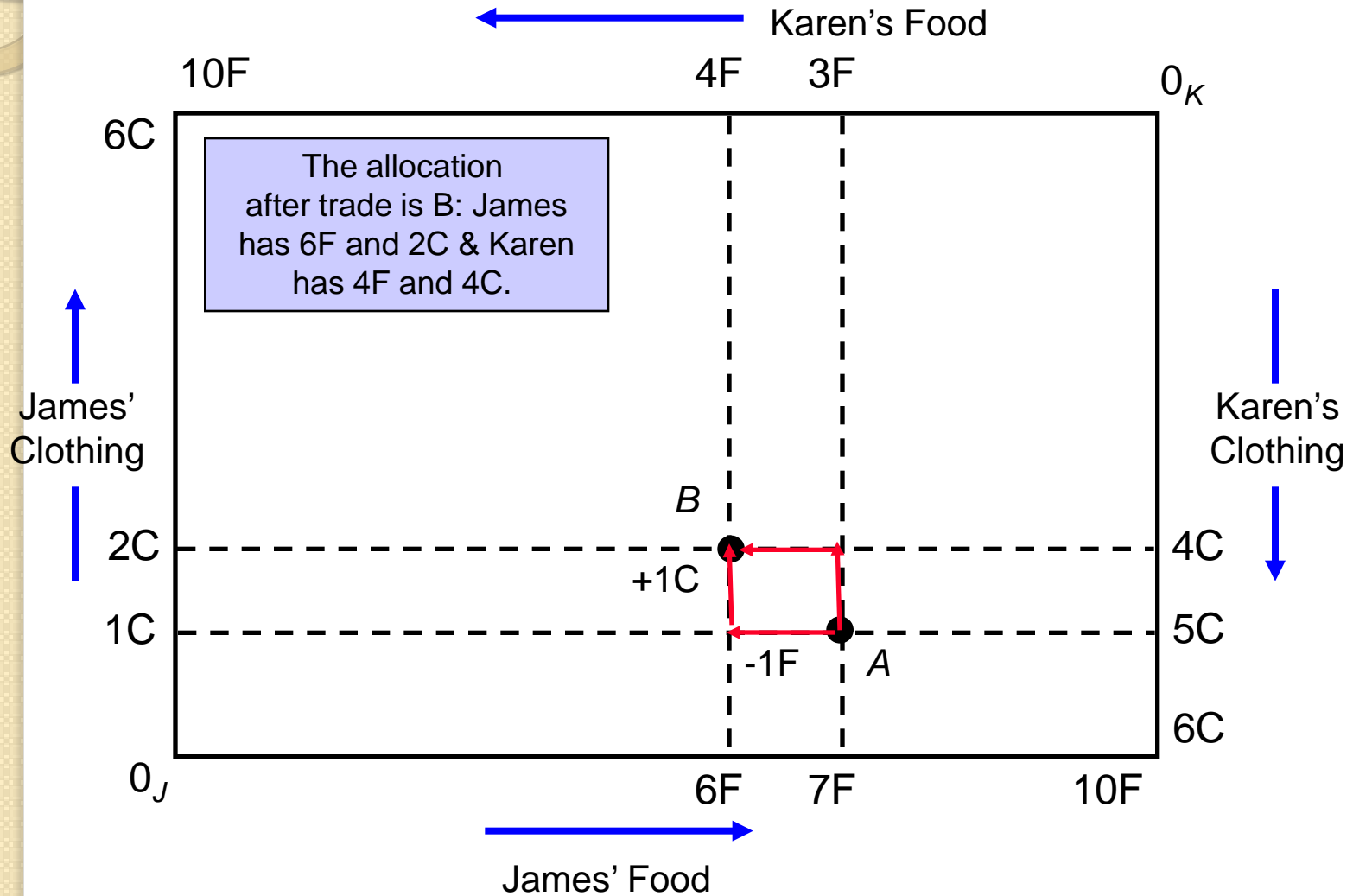
The Edgeworth Box Diagram

- A diagram showing all possible allocations of either two goods between two people or of two inputs between two production processes is called an **Edgeworth Box**
- Let's consider an exchange economy

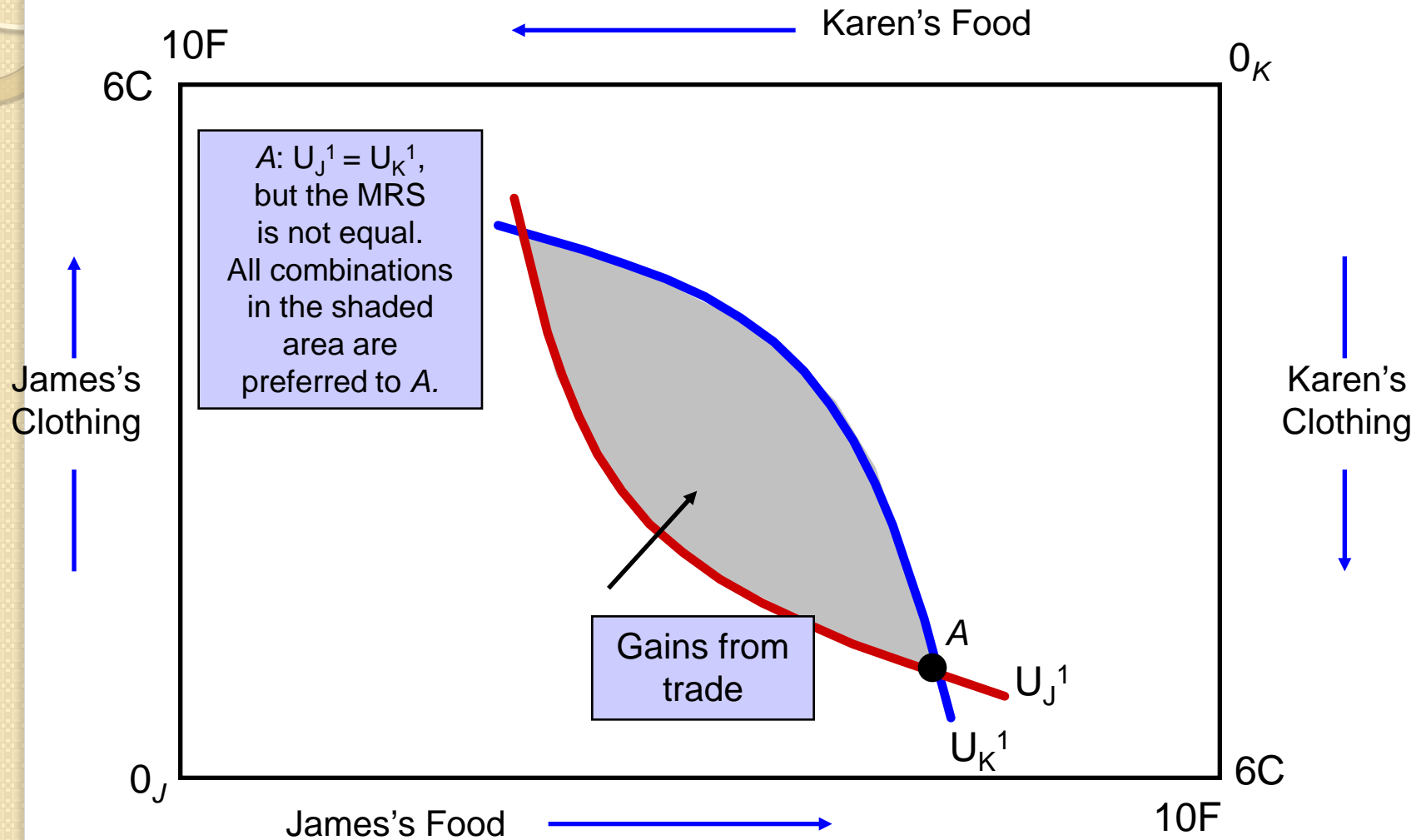
The Edgeworth Box



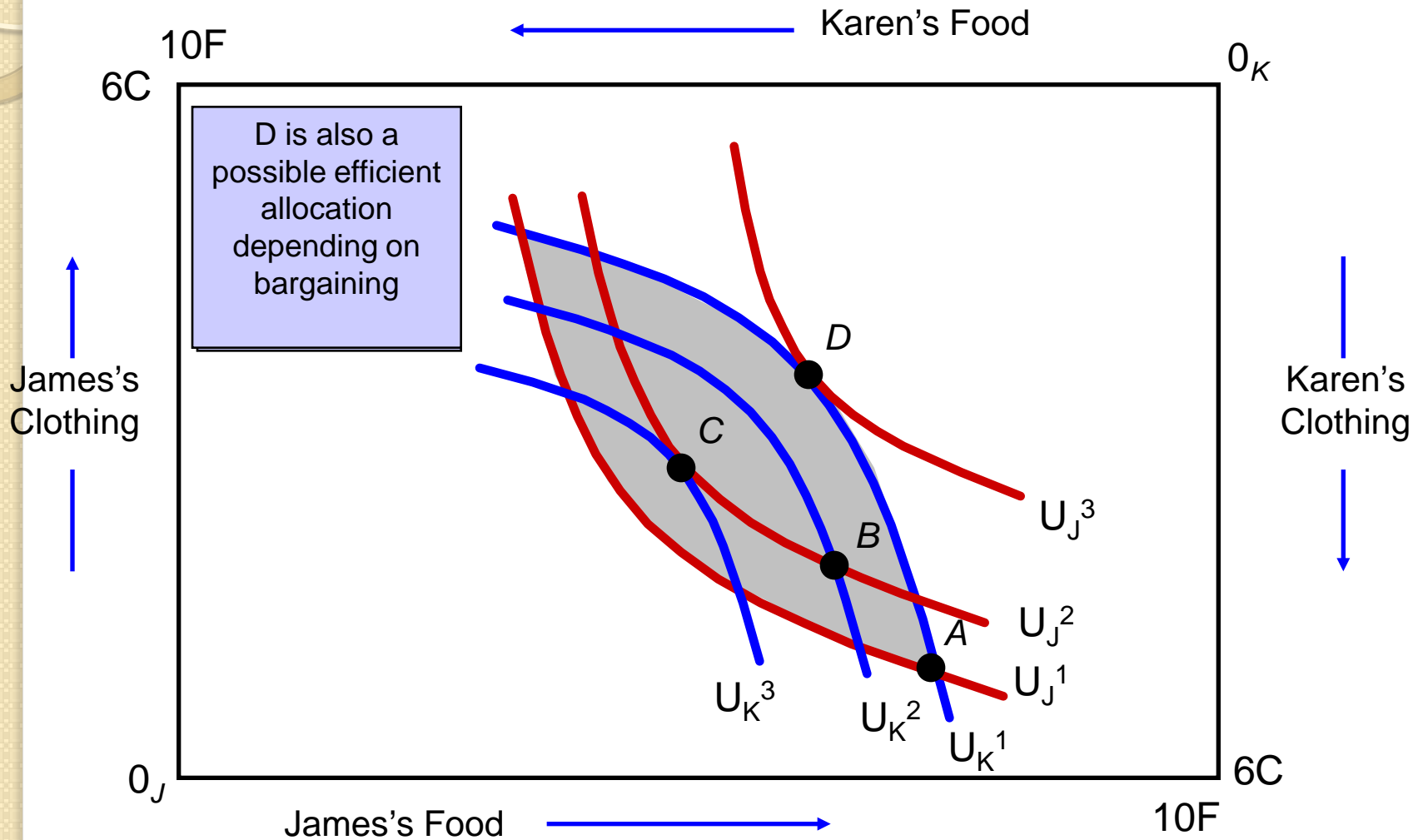
Exchange in an Edgeworth Box



Gains from Trade



Efficiency in Exchange

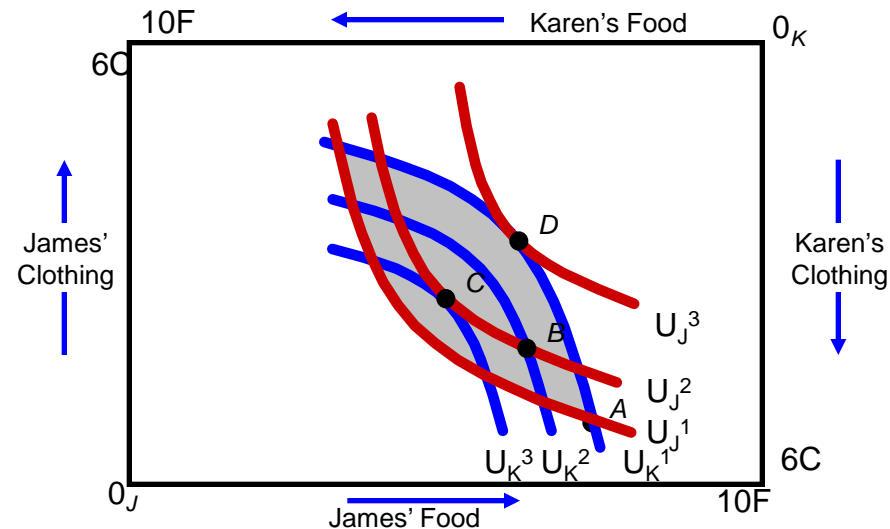


Efficient Allocations

- How do these parties reach an efficient allocation?
 - When there is no more room for trade
 - When their MRSs are equal
 - They will keep trading, reaching higher indifference curves, until they can no longer do so and still make each better off
 - This is when indifference curves are tangent – they have the same slope and same MRS

Efficiency in Exchange

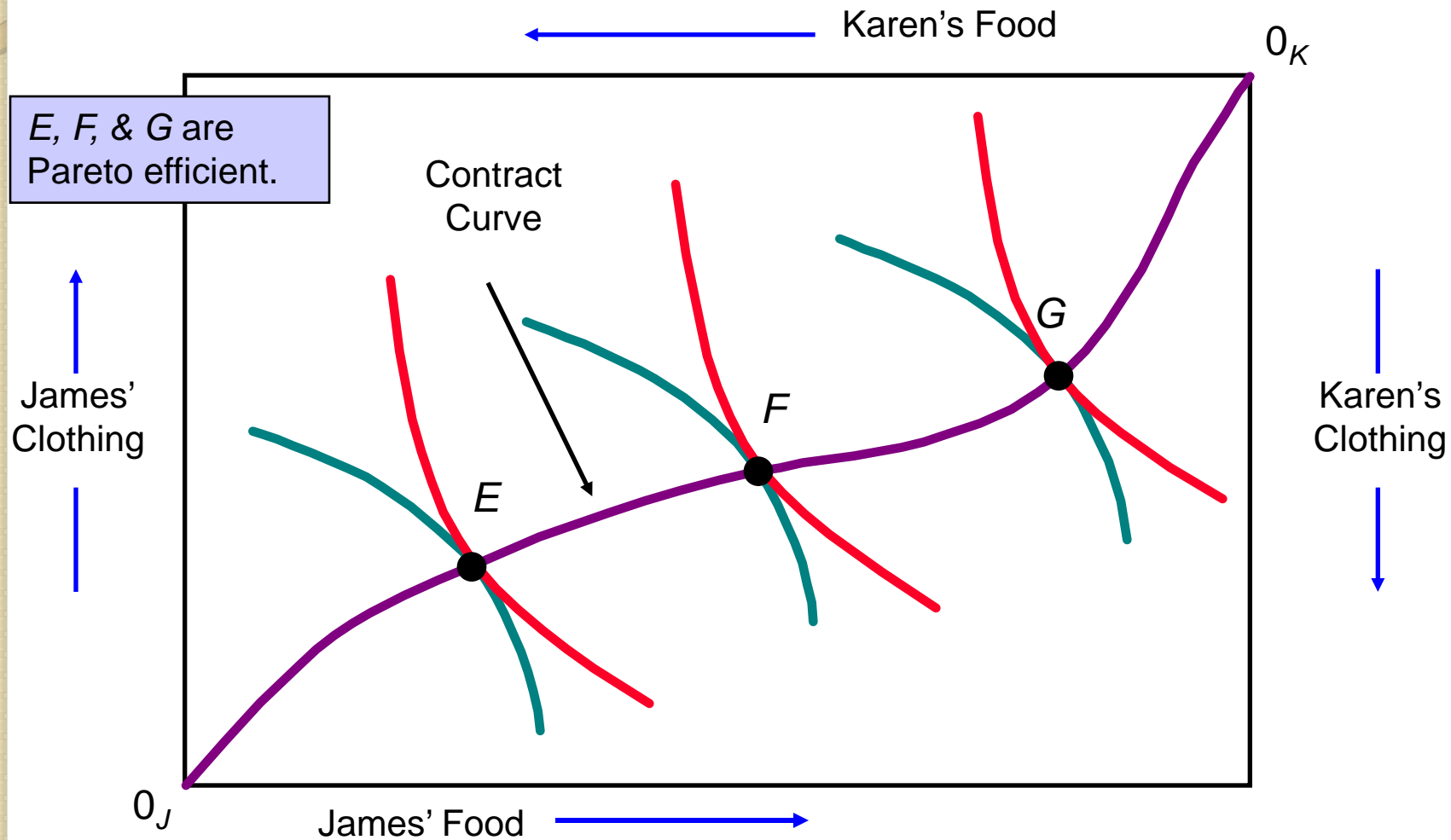
- Any move outside the shaded area will make one person worse off (closer to their origin)
- B is a mutually beneficial trade--higher indifference curve for each person
- Trade may be beneficial but not efficient
- MRS is equal when indifference curves are tangent and the allocation is efficient



The Contract Curve

- To find all possible efficient allocations of food and clothing between Karen and James, we would look for all points of tangency between each of their indifference curves
- The **contract curve** shows all the efficient allocations of goods between two consumers, or of two inputs between two production functions

The Contract Curve



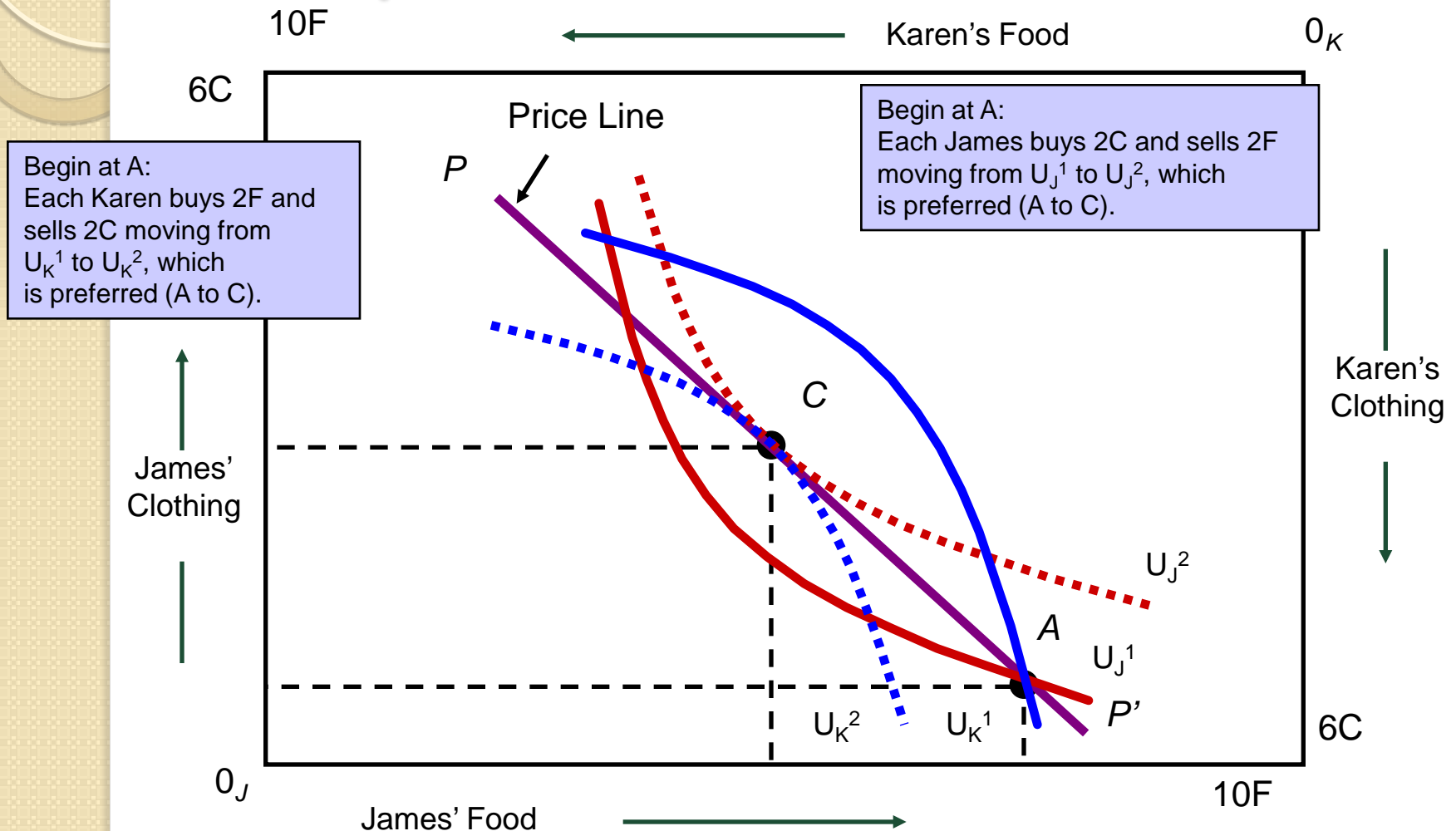
Contract Curve

- All points of tangency between the indifference curves are efficient
 - MRS of individuals is the same
 - No more room for trade
- The contract curve shows all allocations that are Pareto efficient
 - Pareto efficient allocation occurs when further trade will make someone worse off
 - *Pareto set itself does not depend on the initial endowment*

Market Trade

- What would happen if the two persons are in a competitive market?
- Both are price-takers
- Suppose we have a third party who acts as an “auctioneer” for the two agents
- At any prices announced by the auctioneer, if there is excess demand or supply, then she will change the prices such that demand equals demand for any good

Consumer Equilibrium in a Competitive Market



Walrasian Equilibrium

- A market **equilibrium** or Walrasian **equilibrium** is a set of prices at which the quantity demanded equals the quantity supplied in every market
 - Also called **competitive equilibrium**
- In the market equilibrium, all consumers are facing the same price, so all consumers will have the same marginal rate of substitution between any two goods

Walrasian Equilibrium

- At the equilibrium prices p^* , we should have $X_A^i(p_1^*, p_2^*) + X_B^i(p_1^*, p_2^*) = W_A^i + W_B^i$ ($i = 1, 2$)
- This means that $(X_A^i - W_A^i) + (X_B^i - W_B^i) = 0$
- Let us denote the excess demand for good i by agent j by $e_j^i = X_j^i(p_1, p_2) - W_j^i$ ($j = A, B$)
- Then, we get $z_i(p_1, p_2) = e_A^i(p_1, p_2) + e_B^i(p_1, p_2)$
- $Z(p_1, p_2)$ represents the aggregate excess demand for good i

Walrasian Equilibrium

- The prices (p_1^*, p_2^*) are equilibrium prices if

$$z_1(p_1^*, p_2^*) = 0, z_2(p_1^*, p_2^*) = 0$$

- The equilibrium prices will clear all markets simultaneously
- In order to understand the nature of these conditions, we need to know Walras' Law

The Walras' Law

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$$

At the optimal bundles $(X_A^1(p^), X_A^2(p^*))$*

$$p_1 X_A^1(p^*) + p_2 X_A^2(p^*) = p_1 W_A^1 + p_2 W_A^2 \Rightarrow p_1 e_A^1 + p_2 e_A^2 = 0$$

$$p_1 X_B^1(p^*) + p_2 X_B^2(p^*) = p_1 W_B^1 + p_2 W_B^2 \Rightarrow p_1 e_B^1 + p_2 e_B^2 = 0$$

$$\Rightarrow p_1 (e_A^1 + e_B^1) + p_2 (e_A^2 + e_B^2) = 0$$

$$\Rightarrow p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$$

More generally, $\sum_{k=1}^K p_k z_k(p) = 0$

Implications

- If $(K-1)$ markets clear, then the last market will also clear
- There are only $(K-1)$ independent prices
- Note that p^* and tp^* are both equilibrium prices, and by setting $t = 1/p_1$, then there are only $(K-1)$ relative prices
- In general we call good 1 as *numeraire* good

Equilibrium and Efficiency

- As shown before, we can see that the allocation in a competitive equilibrium is economically efficient
 - The efficient point must occur where the two indifference curves are tangent
 - If not, one of the consumers can increase their utility and be better off

A Proof of Efficiency in Market Equilibrium

- This can be proven by way of counter argument
- Suppose a market equilibrium is NOT efficient, then it implies that there is another feasible allocation (y_A, y_B)

such that

$$y_A^i + y_B^i = W_A^i + W_B^i \quad (i = 1, 2) \text{ and}$$

$$(y_A^1, y_A^2) \succ_A (X_A^1, X_A^2)$$

$$(y_B^1, y_B^2) \succ_B (X_B^1, X_B^2)$$

A Proof of Efficiency in Market Equilibrium

- If (y_A) is better than the bundle (x_A) A is choosing, then it must cost more than A can afford, similarly for B
- That is, $p_1 y_j^1 + p_2 y_j^2 > p_1 W_j^1 + p_2 W_j^2 \quad (j = A, B)$
- Adding these two equations, we get

$$p_1(y_A^1 + y_B^1) + p_2(y_A^2 + y_B^2) > p_1(W_A^1 + W_B^1) + p_2(W_A^2 + W_B^2)$$

- This contradicts with the feasibility condition with equality

First Theorem of Welfare Economics

- *Market equilibria are Pareto efficient*
- If everyone trades in a competitive marketplace, all mutually beneficial trades will be completed and the resulting equilibrium allocation of resources will be economically efficient
 - Welfare economics involves the normative evaluation of markets and economic policy

Equilibrium and Efficiency

- The first welfare theorem is the best illustration of Adam Smith's ***invisible hand***
 - Economy will automatically allocate all resources efficiently without need for regulatory control
 - Supports argument for less government intervention and more highly competitive markets
- Markets use ***minimum*** information to function well (compare this with the centralized planning)

Underlying Assumptions

- For the first theorem to hold, there are several implicit assumptions:
 - No externality or public goods
 - No monopoly power in setting prices
 - No asymmetric information: markets can be missing due to the asymmetric information

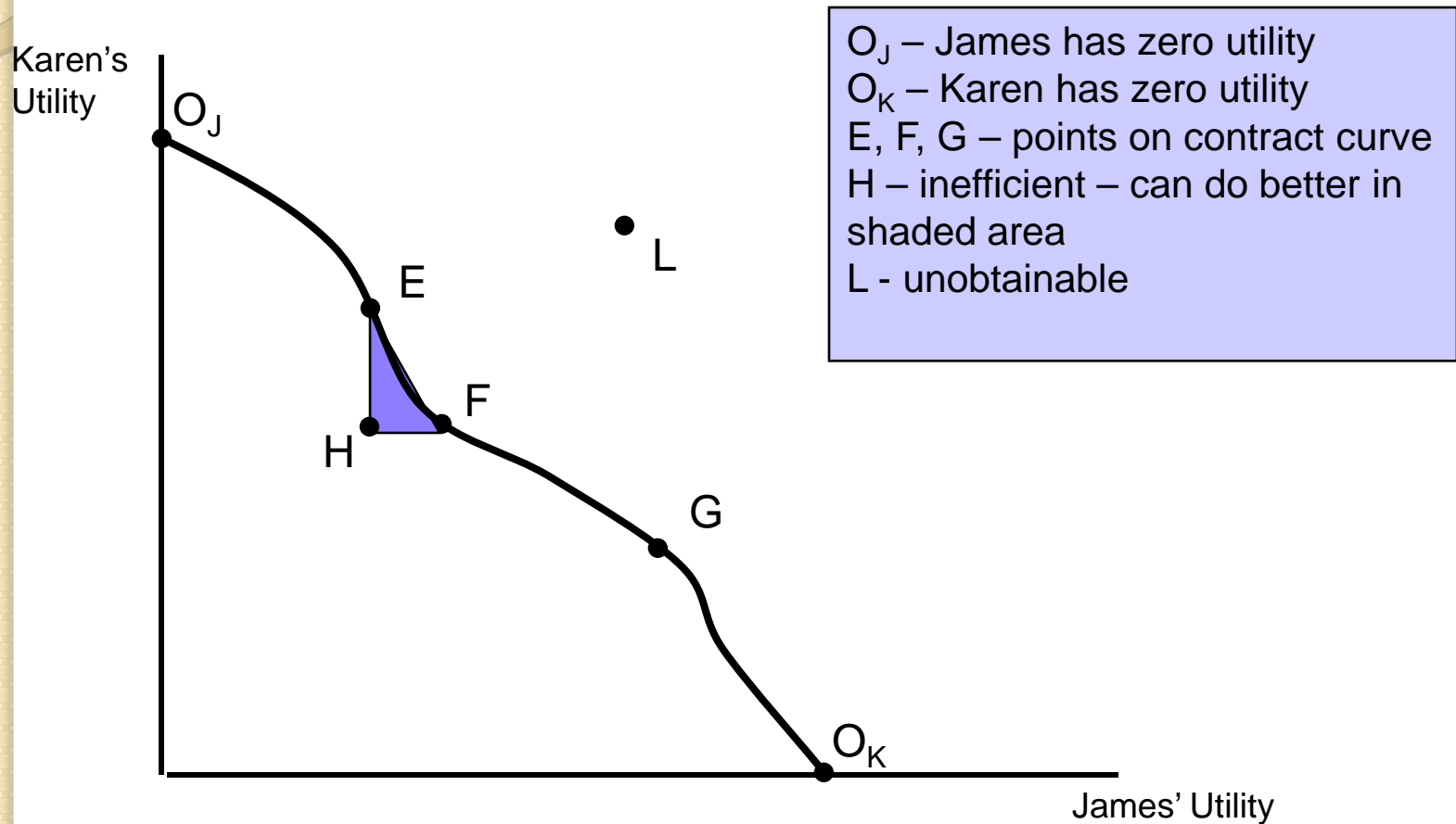
Equity and Efficiency

- Although there are many efficient allocations, some may be more fair than others
- The difficult question is, what is the most equitable allocation?
- We can show that there is no reason to believe that efficient allocation from competitive markets will give an equitable allocation

The Utility Possibilities Frontier

- From the Edgeworth Box, we showed a two person exchange
- The **utility possibilities frontier** represents all allocations that are efficient in terms of the utility levels of the two individuals
 - Shows the levels of satisfaction that are achieved when the two individuals have reached the contract curve

The Utility Possibilities Frontier



The Utility Possibilities Frontier

- From previous example, one can see that an inefficient allocation might be more equitable than an efficient one
- But how do we define an equitable allocation?
 - It depends on what we believe equity to entail
 - Requires interpersonal comparisons of utility

Social Welfare Functions

- Weights are often applied to individual's utility to determine what is socially desirable
 - How these weights are applied comes from the social welfare functions
- The **utilitarian function** weights everyone's utility to maximize utility for the whole society

Social Welfare Functions

- Each social welfare function is associated with a particular view of equity
- Some views of equity do not assign weights and cannot be represented by a welfare function
 - Competitive market process is equitable because it rewards those who are most able and work hardest
 - Believes competitive equilibrium would be most equitable

Social Welfare Functions

- The Rawlsian view is that individuals don't know what their endowment will be
- Rawls argues that if you don't know your own fate, you will opt for the system in which the least well-off person is treated reasonably well
- *The most equitable allocation maximizes the utility of the least well-off person in society*

Social Welfare Functions

- An egalitarian view believes that goods should be equally shared by all individuals in society
- Could have situation where more productive people are rewarded, thereby producing more goods and then having more to reallocate to all of society

Four Views of Equity

| | |
|-------------------|--|
| Egalitarian | All members of society receive equal amount of goods |
| Rawlsian | Maximize the utility of the least-well-off person |
| Utilitarian | Maximize the total utility of all members of society |
| Market - Oriented | The market outcome is the most equitable |

Equity and Perfect Competition

- A competitive equilibrium can occur at any point on the contract curve depending on the initial allocation
- Since not all competitive equilibria are equitable, we rely on the government to help reach equity by redistributing income
 - Taxes
 - Public services

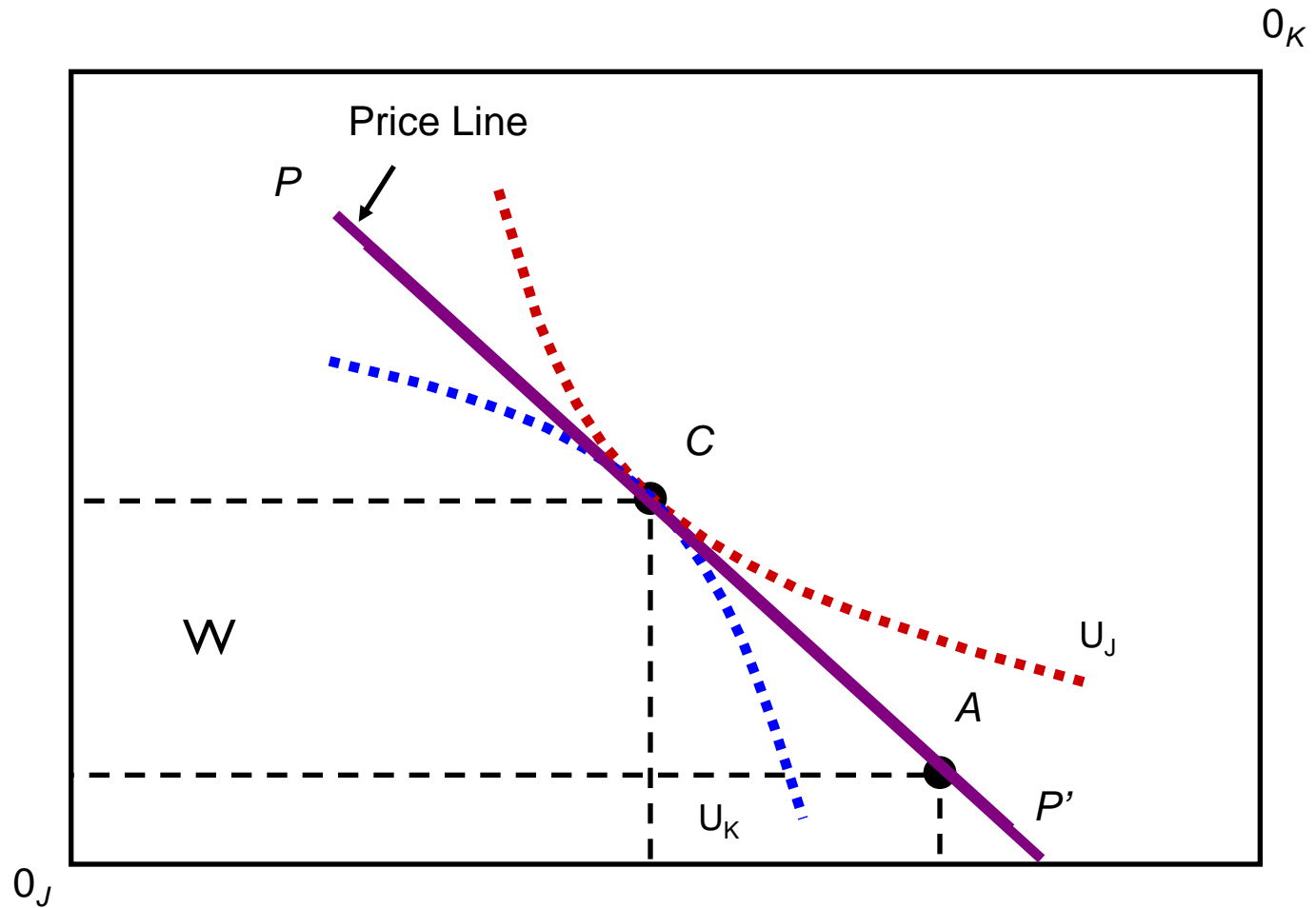
Efficiency and Equilibrium

- Must a society that wants to be more equitable necessarily operate in an inefficient world?

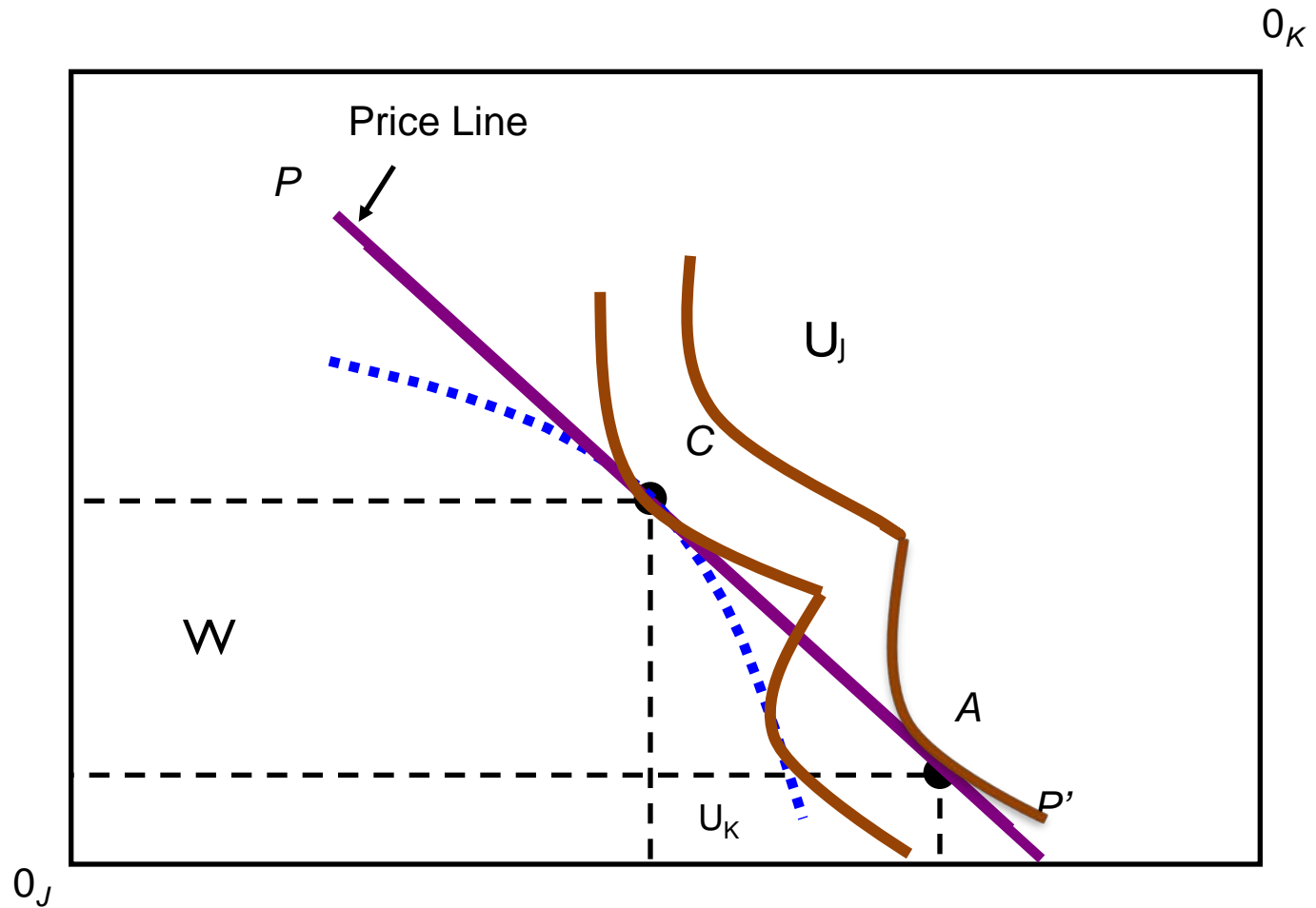
Second Theorem of Welfare Economics

If individual preferences are convex, then every efficient allocation (every point on the contract curve) is a competitive equilibrium for some initial allocation of goods

Efficiency and Equilibrium



Why Are Convex Preferences?



Implications of the Second Theorem

- The problem of distribution and efficiency can be separated
- Prices play two roles in the market system: *allocative vs. distributive*
- The allocative role of prices is to reflect relative scarcity while the distributive role is to determine the values of the endowments
- Let prices allocate resources and government redistribute wealth or income using (lump-sum) subsidy or taxes
- Don't mix up these two roles