Answer to P.S. 2

1. (a) Natasha is risk averse. To show this, assume that she has \$10,000 and is offered a gamble of a \$1,000 gain with 50 percent probability and a \$1,000 loss with 50 percent probability. Her utility of \$10,000 is 3.162, ($u(I) = 100.5 = 3.162$). Her expected utility is:

$$EU = (0.5)(90.5) + (0.5)(110.5) = 3.158 < 3.162$$

She would avoid the gamble. If she were risk neutral, she would be indifferent between the \$10,000 and the gamble; whereas, if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by plotting the function for a few values (see Figure 1) and noting that it displays a diminishing marginal utility. (Or, note that the second derivative is negative, again implying diminishing marginal utility.)

(b) The utility of her current salary is 100.5, which is 3.162. The expected utility of the new job is

$$EU = (0.5)(50.5) + (0.5)(160.5) = 3.118$$

which is less than 3.162. Therefore, she should not take the job.

(c) Assuming that she takes the new job, Natasha would be willing to pay a risk premium equal to the difference between \$10,000 and the utility of the gamble so as to ensure that she obtains a level of utility equal to 3.162. We know the utility of the gamble is equal to 3.118. Substituting into her utility function we have, $3.118 = I^{0.5}$, and solving for I we find the income associated with the gamble to be \$9,722. Thus, Natasha would be willing to pay for insurance

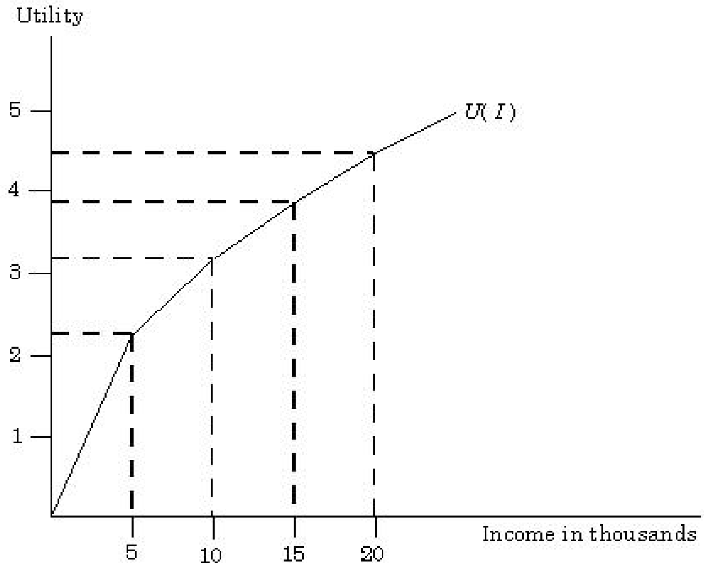

图 1: Figure 1

equal to the risk premium, \$10,000 - \$9,722 = \$278.

There is another solution. If buying insurance can ensure she obtain (0.5)16000+ (0.5)(5000) $= 10,500$, then she would willing to pay \$10,000-\$9,722=\$778 for insurance.

2. (a)

$$Min \quad rK + wL$$

s.t.

$$Q = 10K^{0.8}(L-40)^{0.2}$$

F.O.C.

$$\begin{cases} r = 8\lambda K^{0.2}(L-40)^{0.2} \\ w = 2\lambda K^{0.8}(L-40)^{-0.8} \\ Q = 10K^{0.8}(L-40)^{0.2} \end{cases}$$

Thus we have

$$\begin{cases} K = \dfrac{Q}{10}(\dfrac{4w}{r})^{0.2} \\ L = 40 + \dfrac{Q}{10}(\dfrac{r}{4w})^{0.8} \end{cases}$$

The total cost function is: $C(w,r,Q) = wL + rK = 40w + 2^{-2.6}Qr^{0.8}w^{0.2}$

(b) If wage rate is \$32 and rental rate is \$64, the total costs is $C(32,64,Q) = 1280 + 2^{3.2}Q$.
$C(32,64,\lambda Q) = 1280 + 2^{3.2}\lambda Q < 1280\lambda + 2^{3.2}\lambda Q$, if $\lambda > 1$
Hence, the technology exhibits increasing returns to scale.

(c) $Q = 2000$,
The total demand of factors:

$$\begin{cases} K = \dfrac{2000}{10}(\dfrac{4\times 32}{64})^{0.2} \\ L = 40 + \dfrac{2000}{10}(\dfrac{64}{4\times 32})^{0.8} \end{cases}$$

By the assumptions of productivity:
$k^* = K/40 = 5.74$,
$l^* = L/40 = 3.87$
Or the integer solution is (6,4) or (5,6).
The marginal cost is: $MC = \frac{dC(32,64,Q)}{dQ} = 2^{3.2} = 9.19$
The average cost is: $AC = \frac{1280}{2000} + 2^{3.2} = 9.83$

3. (a) $Q(\lambda K, \lambda L) = 20(\lambda K)^{0.75}(\lambda L) = 20\lambda^{1.75}K^{0.75}L > \lambda Q(K,L)$, if $\lambda > 1$,
Thus, the technology is increasing returns to scale.

(b)

$$Min \quad 8K + 6L$$

s.t.

$$Q = 20K^{0.75}L$$

F.O.C.

$$\begin{cases} 6 = 20\lambda K^{0.75} \\ 8 = 15\lambda K^{-0.25}L \end{cases}$$

We have $\frac{K}{L} = 9/16 < 54/20$, thus this allocation is not optimal for the firm. Decreasing capital and increasing labor will decrease the cost.

4. (a) $MC_1 = 2y_1$ and $MC_2 = 8 + 2y_2$, thus

$$\begin{cases} 2y_1 = 8 + 2y_2 \\ y_1 + y_2 = 24 \end{cases}$$

We get $y_1 = 14$ and $y_2 = 10$.
In such allocation, the marginal costs in two plants are the same. If producing in one plant, the total cost will increase.

(b) For firm 1,
$max \;\; py_1 - 100 - y_1^2$
F.O.C
$y_1 = p/2$
For firm 2,
$max \;\; py_2 - 16 - 8y_2 - y_2^2$
F.O.C
$y_2 = (p-8)/2$.
So the short-run supply curve for firm 1 is $y_1 = p/2$ and for firm 2 is $y_2 = (p-8)/2$.

(c) If the price is 6, only firm 1 is active in the market. Since for firm 2, the short-run supply is $y_2 = (6-8)/2 < 0$.

5.

$$Min \quad K + 2L$$

s.t.

$$Q = KL$$

F.O.C.

$$\begin{cases} K = \sqrt{2Q} \\ L = \sqrt{Q/2} \end{cases}$$

The long run cost function is:
$C(Q) = 2\sqrt{2Q}$, $AC(Q) = 2\sqrt{2/Q}$.

6. (a) The equilibrium price satisfies that $10 - p = 4 + p$, so we have $(p, Q) = (3, 7)$

(b) By the introduction of Tax, different kinds of Tax would lead to various results
Consumption tax

$$\begin{cases} p + t_c = 10 - Q \\ p = Q - 4 \end{cases}$$

Thus, $(p_c, Q_c) = (2.5, 6.5)$. The consumer should pay $p + t = 3.5$ per good, and the firm get 2.5 per good.
Production tax

$$\begin{cases} p - t_p = Q - 4 \\ p = 10 - Q \end{cases}$$

Thus, $(p_p, Q_p) = (3.5, 6.5)$. The consumer should pay $p + t = 3.5$ per good, and the firm get 2.5 per good.
It is obvious that both of them could offer the same result.

(c) Subsidy case: $d = 1$
Consumption tax

$$\begin{cases} p + d_p = Q - 4 \\ p = 10 - Q \end{cases}$$

Thus, $(p_d, Q_d) = (2.5, 7.5)$.
The total cost of government is $7.5 \times 1000 = 7500$.