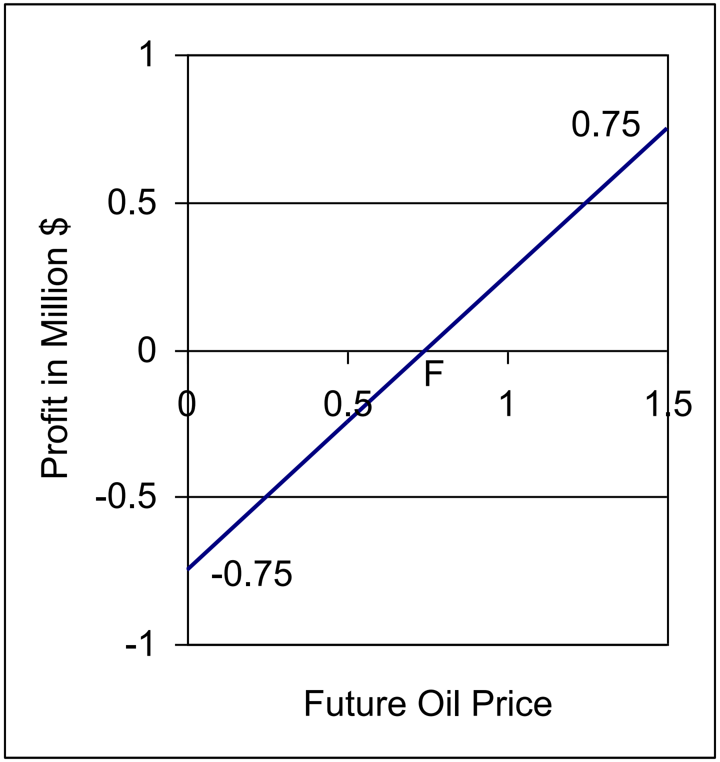
Investments

Lecture 10

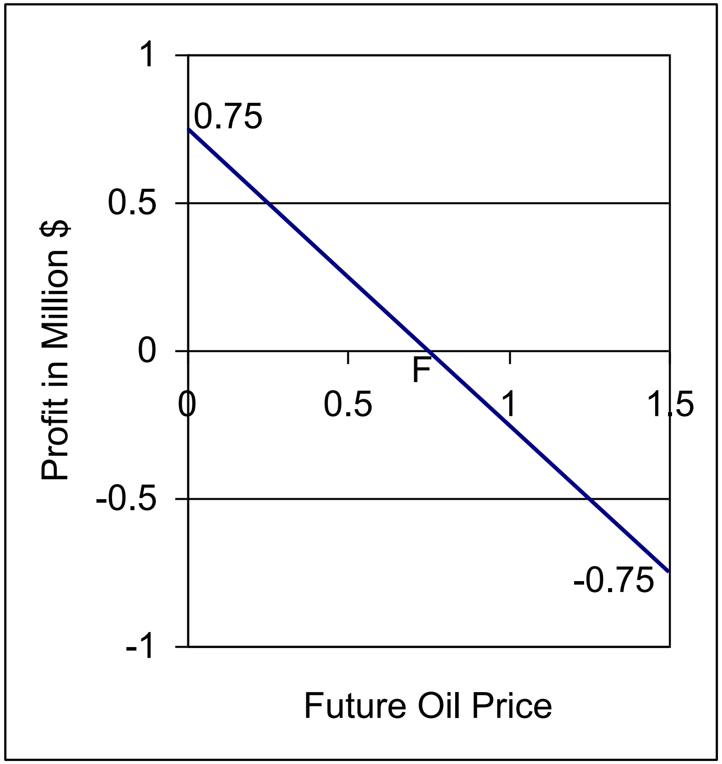
Pricing: Futures

Possible Future Profits of Future Contract

- Long Position



- Short Position



Speculation on Disney

- You believe that Disney will perform very well in the next couple of months, but you do not have any cash
- What can you do?

Speculation on Disney

- Strategy 1:
  - Buy a one year Disney forward contract
- Strategy 2:
  - Buy Disney stock today using funds borrowed for one year

Market Prices

- Current spot Disney price:
  - S_t = \$26.35
- Let's assume that the forward price for a contract expiring in one year is:
  - F = \$26.35 (F_t_T)
- 1-year interest rate:
  - r = 2%

Cash Flows of Strategy 1: Forward Contract

Action	Today	In one year
Buy Forward Contract	0	$S_T - F$ $= S_T - 26.35$

Cash Flows of Strategy 2: Synthetic Forward Contract

Action	Today	In 3 months
Buy Disney	–26.35	S_T
Borrow 26.35 at 2%	26.35	–26.48
Net Cash Flows	0	S_T –26.48

Summary

- Both strategies allow the speculator to participate in the price changes of Disney
- Payoffs after one year:
  - Strategy 1: $S_T - F_{t,T} = S_T - \$26.35$
  - Strategy 2: $S_T - S_t e^{r(T-t)} = S_T - \$26.48$
- Strategy 1 has always higher payoffs than Strategy 2 ($F < Se^{r(T-t)}$)
- Can this be an equilibrium?
- Does $Var(S_T)$ matter?

Arbitrage strategy

- Short the asset
- Invest the proceeds for a period T-t at r
- Take a long position in the forward for F
- What if $F > S\_t \, e^{r(T-t)}$?

Forward Pricing

- In equilibrium, the price of a forward contract on a financial asset with no income equals:

$$F = Se^{r(T-t)} = S(1+r)^{T-t}$$

$$F = 26.35 * 1.005 = \$26.48$$

Pricing Financial Futures: Index Futures – No Dividends

- By definition, a futures contract requires **no cash outlay at initiation**.
- At delivery, the payoff is:
  - $S_T - F$ to a long position;
  - $F - S_T$ to a short position;

Payoff Diagram

- Payoff diagram for index futures contract:

Replicating the Futures Payoff

- To replicate the payoff:

- So the Futures Price must satisfy:

Pricing Financial Futures: Index Futures – With Dividends

- Again, by definition, a futures contract requires **no cash outlay at initiation**.
- At delivery, the payoff is:
  - $S_T - F$ to a long position;
  - $F - S_T$ to a short position;
- However, if you invest in the index you would receive dividends ($D = y\ S_0$) in addition to the capital gain incorporated into S_T.

Payoff Diagram for Index Futures Contract

Payoff Diagram for Stock and Bond

Replicating the Futures Payoff

- To replicate the payoff:

- So the Futures Price must satisfy:

Summary: Index Futures Pricing

- Index option without dividends:
  - $F = S_0(1+r_f)$
- Index option with dividend yield of y%:
  - $F = S_0(1 + r_f - y)$

Example

- Assume that the SP500 currently has a value of 2000. The dividend yield on the underlying stocks in the index is expected to be 4% over the next six months. New-issue six-month Treasury bills now sell for a six-month yield of 6%
  - What is the theoretical value of a six month futures contract on the SP500.
  - Suppose the futures price was instead 2120. Describe an arbitrage opportunity that is available.

Commodity Futures

- When we consider commodity futures, the hedging strategy becomes more tricky since to implement the hedging strategy you have to hold the commodity.
  - Storage costs become important.
  - Benefits of ownership of the physical good become important. (convenience yield)

Convenience Yield

- The benefit of owning the physical commodity is often referred to as **convenience yield.**
- The effect of convenience yield on the futures price is similar to the effect of dividends on index futures prices.

Payoff from Futures and from Holding Commodity

Replicating the Futures Payoff

- To replicate the payoff:

- So the Futures Price must satisfy:

Storage Costs

- Storage costs are like negative dividends and affect the pricing relationship in the obvious way:

Example

- Consider a futures contract on mangoes that calls for the delivery of 2000 pounds of the fruit three months from now.
- The spot price of mangoes is $2 per pound.
- The three-month risk-free rate is 2%.
- It costs $.10/pound to store mangoes for three months.
- What should be the price of this futures contract?

Future/Forward Price v.s. Current Spot Price --- Summary

- Let’s summarize on the whiteboard

Contango and Backwardation: Take caution! Which contango/backwardation?

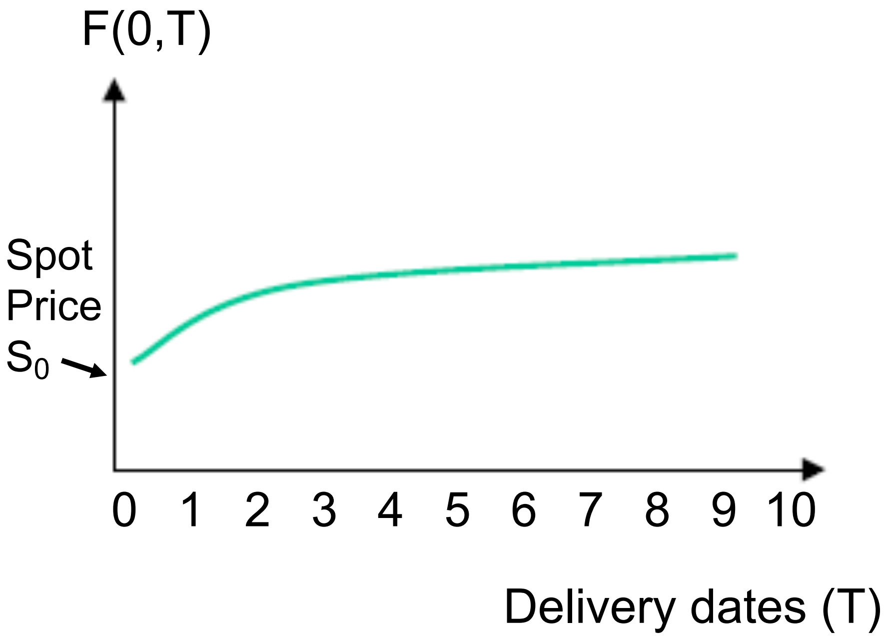
- Over delivery dates (same t):
  Do forward prices increase in time-to-maturity? Yes – Contango; No – Backwardation
  - Arbitrage (S_0): Interest rates, storage costs
- Over time (same delivery date T):
  Do forward prices converge to S_T from above or below? Above – Contango; Below – Backwardation
  - Who's hedging demand? Supplier or customer

Forward Curves (over T, same t)

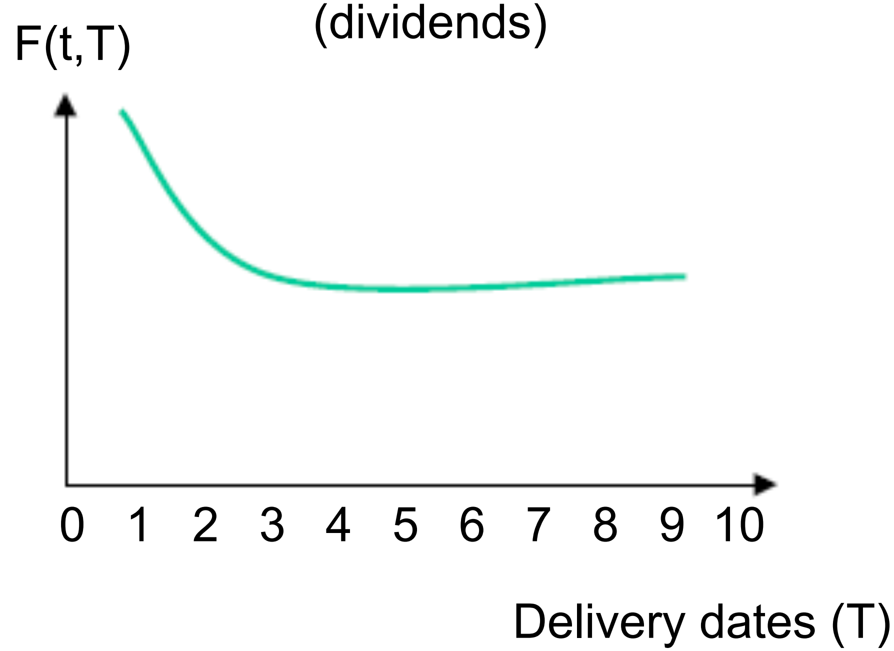
- The relationship between the delivery dates (or equivalently the time-to-maturity) of futures contracts and the futures price is called the forward curve.
- Depending on the underlying asset, these curves may be upward sloping (in **contango**) or downward sloping (in **backwardation**)

Forward Curves (over T, same t)

Contango



Backwardation
(dividends)



Forward Curves (over T, same t)

- Index futures:
  - Dividends

Forward Curves (over T, same t)

- Commodity futures:
  - Storage costs
  - Convenience yields

Future price over time (same T, over t)

- Assume r=0 for simpicity
- Normal backwardation (Keynes)
  - Farmers sell wheat forward at F_0 and are the natural hedgers. Speculator will only buy wheat forward if $E[P_T]-F_0>0$
- Contango
  - Grain processors buy wheat forward at F_0 and are the natural hedgers. Speculator will only sell wheat forward if $F_0-E[P_T]>0$
- Expectation hypothesis: $F_0-E[P_T]=0$

Future price over time (same T, over t)

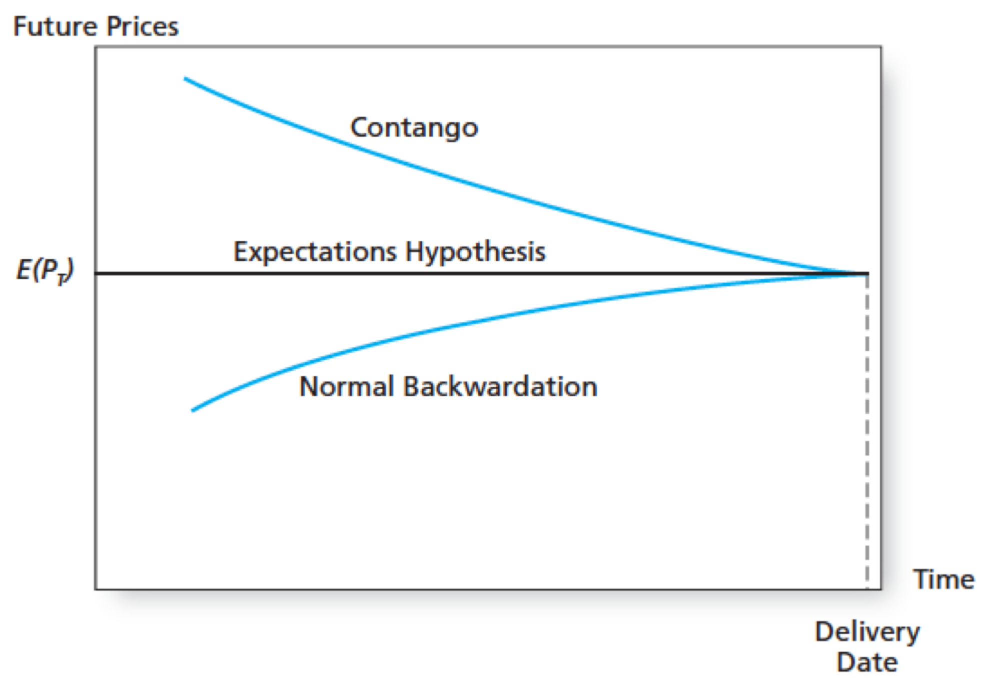


Figure 22.7 Futures price over time, in the special case that the expected spot price remains unchanged

Future price and expected spot price: From Keynes to MPT

- Normal backwardation and contango are related to risk premia:
  - future price are different from expected spot price because the speculators demand a risk premium for bearing price risk
- MPT: modern view of risk premia, future price, and expected spot price
  - Systematic risk is priced

Future price and expected spot price: From Keynes to MPT

As an example of the use of modern portfolio theory to determine the equilibrium futures price, consider once again a stock paying no dividends. If $E(P_T)$ denotes today's expectation of the time T price of the stock, and k denotes the required rate of return on the stock, then the price of the stock today must equal the present value of its expected future payoff as follows:

$$P_0 = \frac{E(P_T)}{(1+k)^T} \tag{22.4}$$

We also know from the spot-futures parity relationship that

$$P_0 = \frac{F_0}{(1+r_f)^T} \tag{22.5}$$

Therefore, the right-hand sides of Equations 22.4 and 22.5 must be equal. Equating these terms allows us to solve for F_0:

$$F_0 = E(P_T)\left(\frac{1+r_f}{1+k}\right)^T \tag{22.6}$$

You can see immediately from Equation 22.6 that F_0 will be less than the expectation of P_T whenever k is greater than r_f, which will be the case for any positive-beta asset. This means that the long side of the contract will make an expected profit [F_0 will be lower than $E(P_T)$] when the commodity exhibits positive systematic risk (k is greater than r_f).

Basis Risk

The **basis** is the difference between the futures price and the spot price.

The **basis** at initiation is $S_t e^{r(T-t)} - S_t$. The **basis** at delivery date is 0. The **basis** can fluctuate at interim dates.

Suppose you use futures to hedge (for example, you are contracted to deliver oil in 1 year and you buy future), and you hold the contract and the future until the delivery date, fluctuations in the basis does not affect your payoffs. **However**, if the contract are liquidated early, the hedger bears **basis risk**.

Speculators can also **speculate on basis risk.**

Speculating on Basis

a short position to deliver oil in the future. If the asset and futures contract are held until maturity, the hedger bears no risk. Risk is eliminated because the futures price and spot price at contract maturity must be equal: Gains and losses on the futures and the commodity position will exactly cancel. However, if the contract and asset are to be liquidated early, before contract maturity, the hedger bears **basis risk,** because the futures price and spot price need not move in perfect lockstep at all times before the delivery date. In this case, gains and losses on the contract and the asset may not exactly offset each other.

Some speculators try to profit from movements in the basis. Rather than betting on the direction of the futures or spot prices per se, they bet on the changes in the difference between the two. A long spot–short futures position will profit when the basis narrows.

Example 22.6 Speculating on the Basis

Consider an investor holding 100 ounces of gold, who is short one gold-futures contract. Suppose that gold today sells for \$991 an ounce, and the futures price for June delivery is \$996 an ounce. Therefore, the basis is currently \$5. Tomorrow, the spot price might increase to \$995, while the futures price increases to \$999, so the basis narrows to \$4.

The investor's gains and losses are as follows:

Gain on holdings of gold (per ounce): $\$995 - \$991 = \$4$

Loss on gold futures position (per ounce): $\$999 - \$996 = \$3$

The net gain is the decrease in the basis, or \$1 per ounce.