Announcement

- Problem Set 2: Due on April 9

Reducing Risk

- Consumers are generally risk averse and therefore want to reduce risk
- Three ways consumers attempt to reduce risk are:
  1. Diversification
  2. Insurance
  3. Obtaining more information

Reducing Risk

- Diversification
  - Reducing risk by allocating resources to a variety of activities whose outcomes are not closely related
- Example:
  - Suppose a firm has a choice of selling air conditioners, heaters, or both
  - The probability of it being hot or cold is 0.5
  - How does a firm decide what to sell?

Income from Sales of Appliances

	Hot Weather	Cold Weather
Air conditioner sales	$30,000	$12,000
Heater sales	12,000	30,000

Diversification – Example

- If the firm sells only heaters or air conditioners their income will be either $12,000 or $30,000
- Their expected income would be:
  - 1/2($12,000) + 1/2($30,000) = $21,000

Diversification – Example

- If the firm divides their time evenly between appliances, their air conditioning and heating sales would be half their original values
- If it were hot, their expected income would be $15,000 from air conditioners and $6,000 from heaters, or $21,000
- If it were cold, their expected income would be $6,000 from air conditioners and $15,000 from heaters, or $21,000

Diversification – Example

- With diversification, expected income is \$21,000 with no risk
- Better off diversifying to minimize risk
- Firms can reduce risk by diversifying among a variety of activities that are not closely related

Reducing Risk – The Stock Market

- If invest all money in one stock, then take on a lot of risk
  - If that stock loses value, you lose all your investment value
- Can spread risk out by investing in many different stocks or investments
  - Ex: Mutual funds

Reducing Risk – Insurance

- Risk averse are willing to pay to avoid risk
- If the cost of insurance equals the expected loss, risk averse people will buy enough insurance to recover fully from a potential financial loss

The Decision to Insure

Insurance	Burglary (Pr = .1)	No Burglary (Pr = .9)	Expected Wealth	Standard Deviation
No	40,000	50,000	49,000	3000
Yes	49,000	49,000	49,000	0

Reducing Risk – Insurance

- For the risk averse consumer, guarantee of same income regardless of outcome has higher utility than facing the probability of risk
- Expected utility with insurance is higher than without

The Law of Large Numbers

- Insurance companies know that although single events are random and largely unpredictable, the average outcome of many similar events can be predicted
- When insurance companies sell many policies, they face relatively little risk
- Why insurance companies generally don't insure earthquake and war?
- Why nationwide compulsory health insurance?

The Law of Large Numbers: An Example

- Why pooling individual and independent random outcomes will reduce risk substantially?
- Suppose N farmers and each faces a random yearly output; they agree to pool their outputs and each gets the mean

$$X_i \sim N(\mu, \sigma^2), \quad X_i \; are \; i.i.d$$

Risk Spreading

$$\bar{X}_N = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$\bar{X}_N \to \mu$$

$$Var(\bar{X}_N) = \frac{1}{N}\sigma^2 \to 0$$

Reducing Risk – Actuarially Fair

- Insurance companies can be sure total premiums paid will equal total money paid out
- Companies set the premiums so money received will be enough to pay *expected* losses

Reducing Risk – Actuarially Fair

- Some events with very little probability of occurrence such as floods and earthquakes are no longer insured privately
  - Cannot calculate true expected values and expected losses
  - Governments have had to create insurance for these types of events
    - Ex: National Flood Insurance Program

Chapter 6
Production

Topics to be Discussed

- The Technology of Production
- Production with One Variable Input
- Isoquants
- Production with Two Variable Inputs
- Returns to Scale

Introduction

- Our study of consumer behavior was broken down into 3 steps:
  - Describing consumer preferences
  - Consumers face budget constraints
  - Consumers choose to maximize utility
- Production decisions of a firm are similar to consumer decisions
  - Can also be broken down into three steps

Production Decisions of a Firm

1. Production Technology
   - Describe how *inputs* can be transformed into *outputs*
     - Inputs: land, labor, capital and raw materials
     - Outputs: cars, desks, books, etc.
   - Firms can produce different amounts of outputs using different combinations of inputs

Production Decisions of a Firm

2. Cost Constraints
   - Firms must consider *prices* of labor, capital and other inputs
   - Firms want to minimize total production costs partly determined by input prices
   - As consumers must consider budget constraints, firms must be concerned about costs of production

Production Decisions of a Firm

3. Input Choices
   - Given input prices and production technology, the firm must choose *how much of each input* to use in producing output
   - Given prices of different inputs, the firm may choose different combinations of inputs to minimize costs
     - If labor is cheap, firm may choose to produce with more labor and less capital

Production Decisions of a Firm

- If a firm is a cost minimizer, we can also study
  - How total costs of production vary with output
  - How the firm chooses the quantity to maximize its profits
- We can represent the firm's production technology in the form of a **production function**

The Technology of Production

- Production Function:
  - Indicates the highest output (q) that a firm can produce for every specified combination of inputs
  - For simplicity, we will consider only labor (L) and capital (K)
  - *Physical vs. financial capital*
  - Shows what is technically feasible when the firm operates efficiently

The Technology of Production

- The production function for two inputs:

$$q = F(K,L)$$

  - Output (q) is a function of capital (K) and labor (L)
  - The production function is true for a given technology
    - If technology increases, more output can be produced for a given level of inputs

Production Functions

- y denotes the output level.
- The technology's production function states the ***maximum*** amount of output possible from an input bundle (assuming technical efficiency)

$$y = f(x_1, \cdots, x_n)$$

Technology Sets

- A production plan is an input bundle and an output level; $(x_1, \ldots, x_n, y)$.
- A production plan is feasible if

$$y \leq f(x_1, \cdots, x_n)$$

- The collection of all feasible production plans is the technology set.

The Technology of Production

- Short Run versus Long Run
  - It takes time for a firm to adjust production from one set of inputs to another
  - Firms must consider not only what inputs can be varied but over what period of time that can occur
  - We must distinguish between long run and short run

The Technology of Production

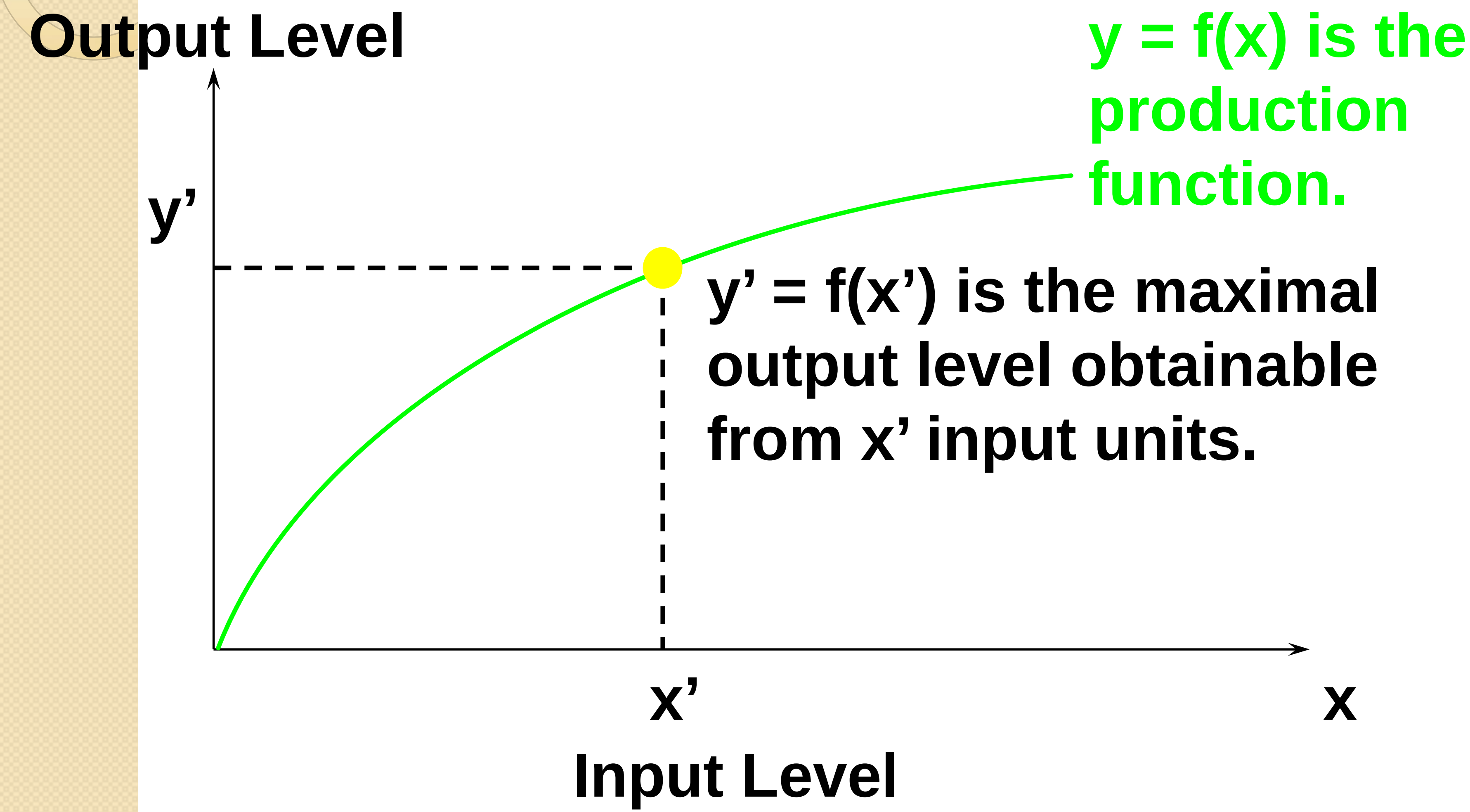
- Short Run
  - Period of time in which quantities of one or more production factors cannot be changed
  - These inputs are called fixed inputs
- Long Run
  - Amount of time needed to make all production inputs variable
- Short run and long run are not time specific

Production: One Variable Input

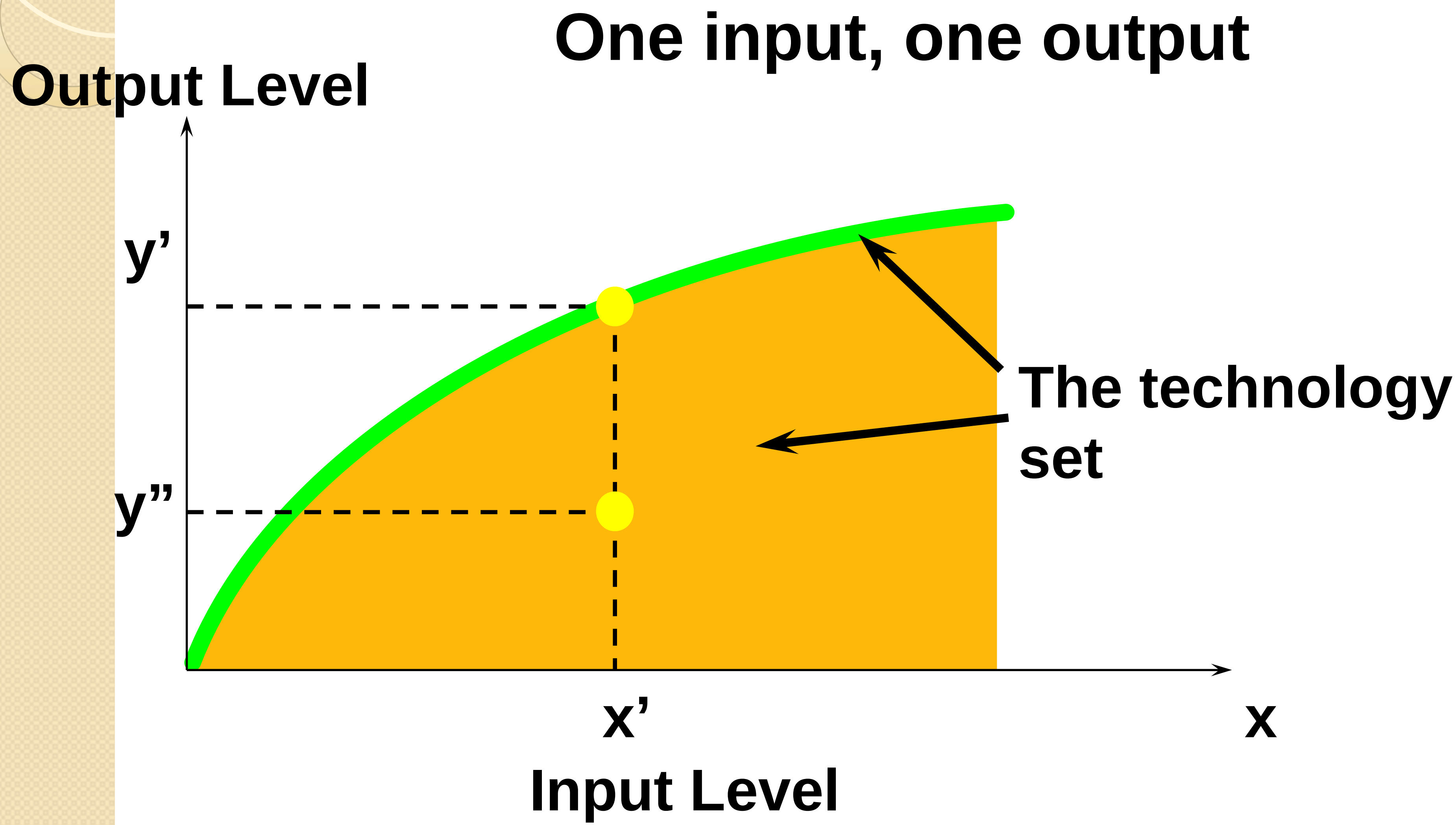
- We will begin looking at the short run when only one input can be varied
- We assume capital is fixed and labor is variable
  - Output can only be increased by increasing labor
  - Must know how output changes as the amount of labor is changed

Production Functions

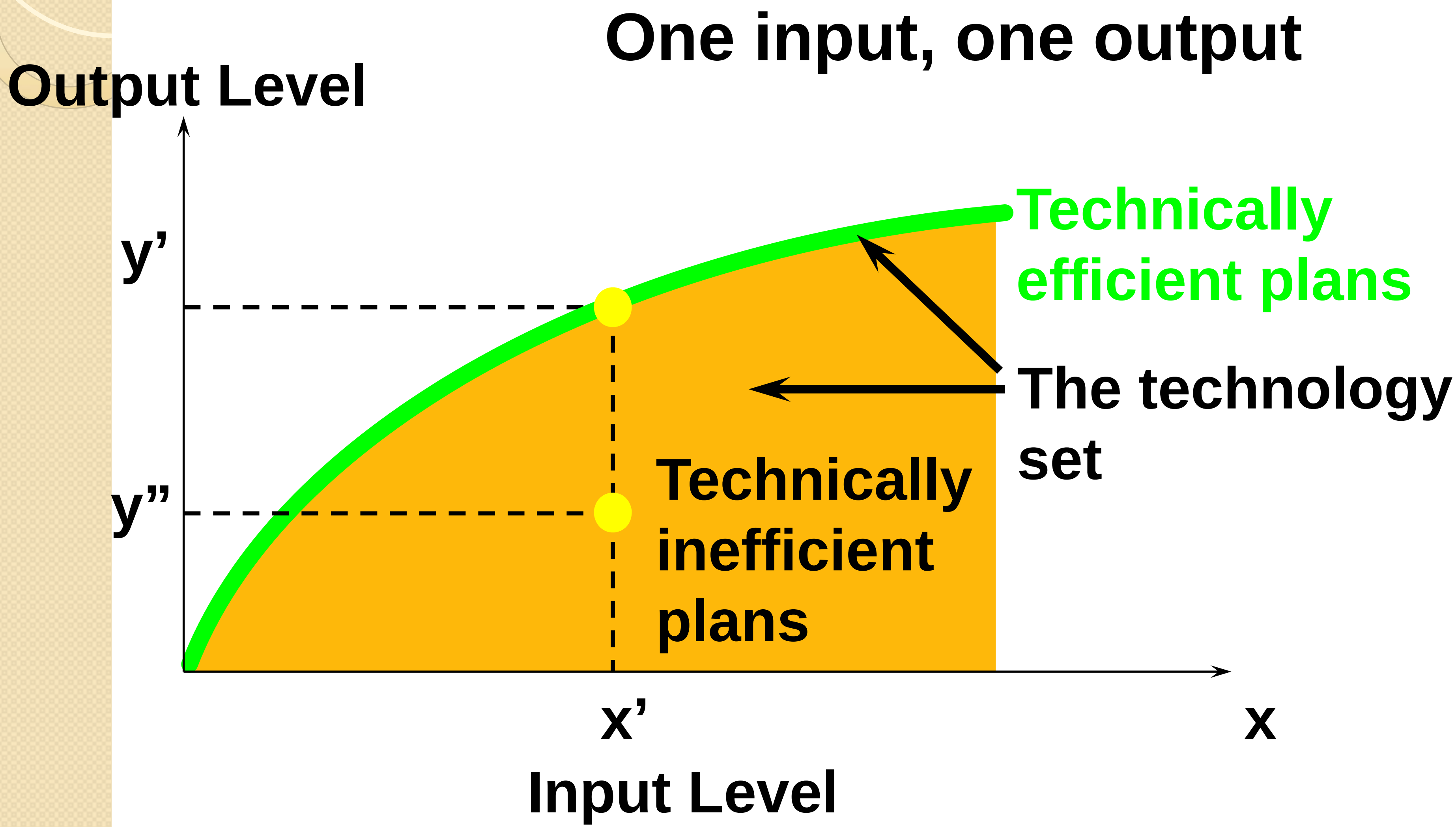
One input, one output



Production Sets



Technology Sets



Production: One Variable Input

- Firms make decisions based on the benefits and costs of production
- Sometimes useful to look at benefits and costs on an *incremental basis*
    - How much more can be produced when at incremental units of an input?
- Sometimes useful to make comparison on an *average basis*

Production: One Variable Input

- Average product of Labor - Output per unit of a particular product
- Measures the productivity of a firm's labor in terms of how much, on average, each worker can produce

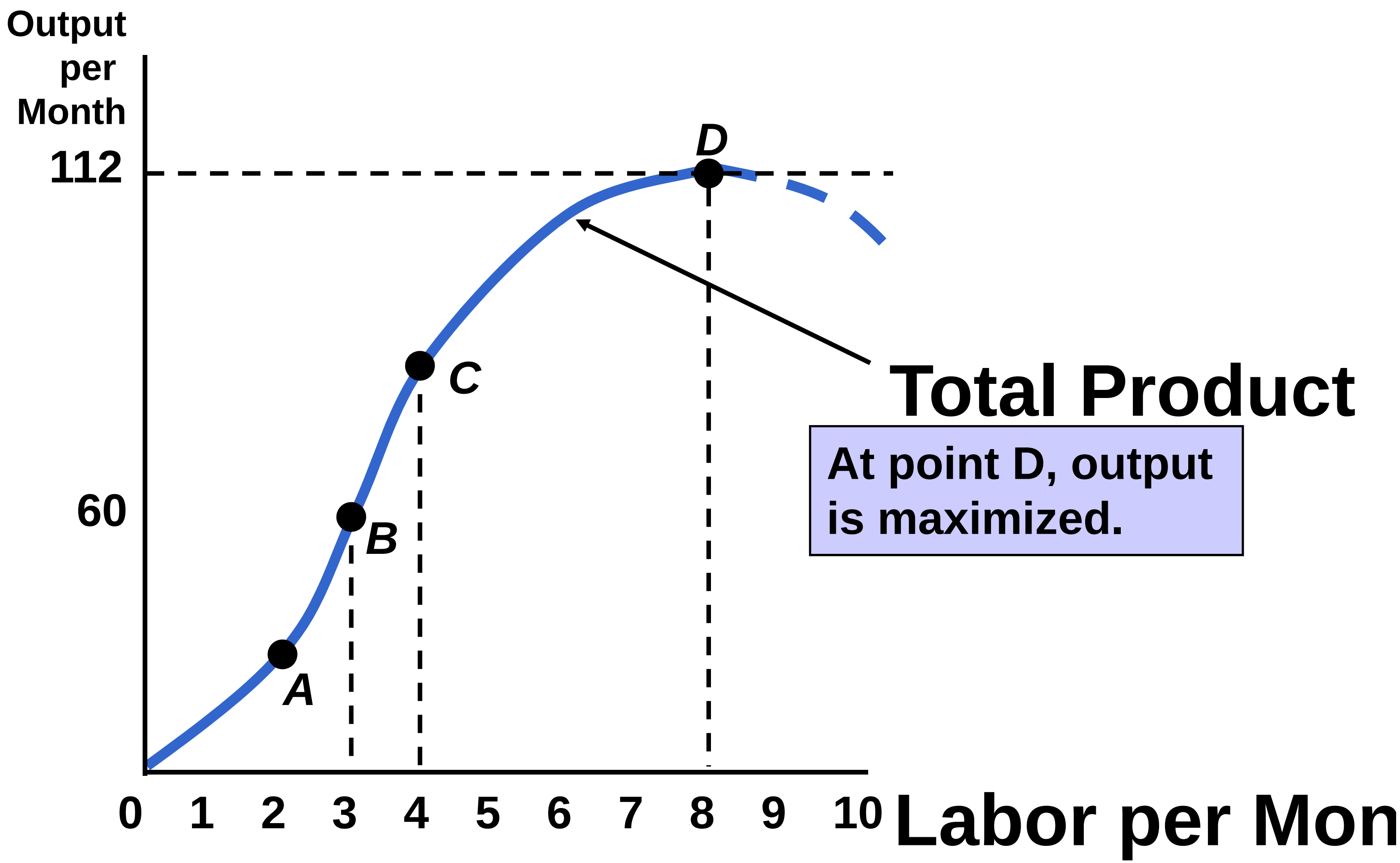
$$AP_L = \frac{Output}{Labor \quad Input} = \frac{\boldsymbol{q}}{\boldsymbol{L}}$$

Production: One Variable Input

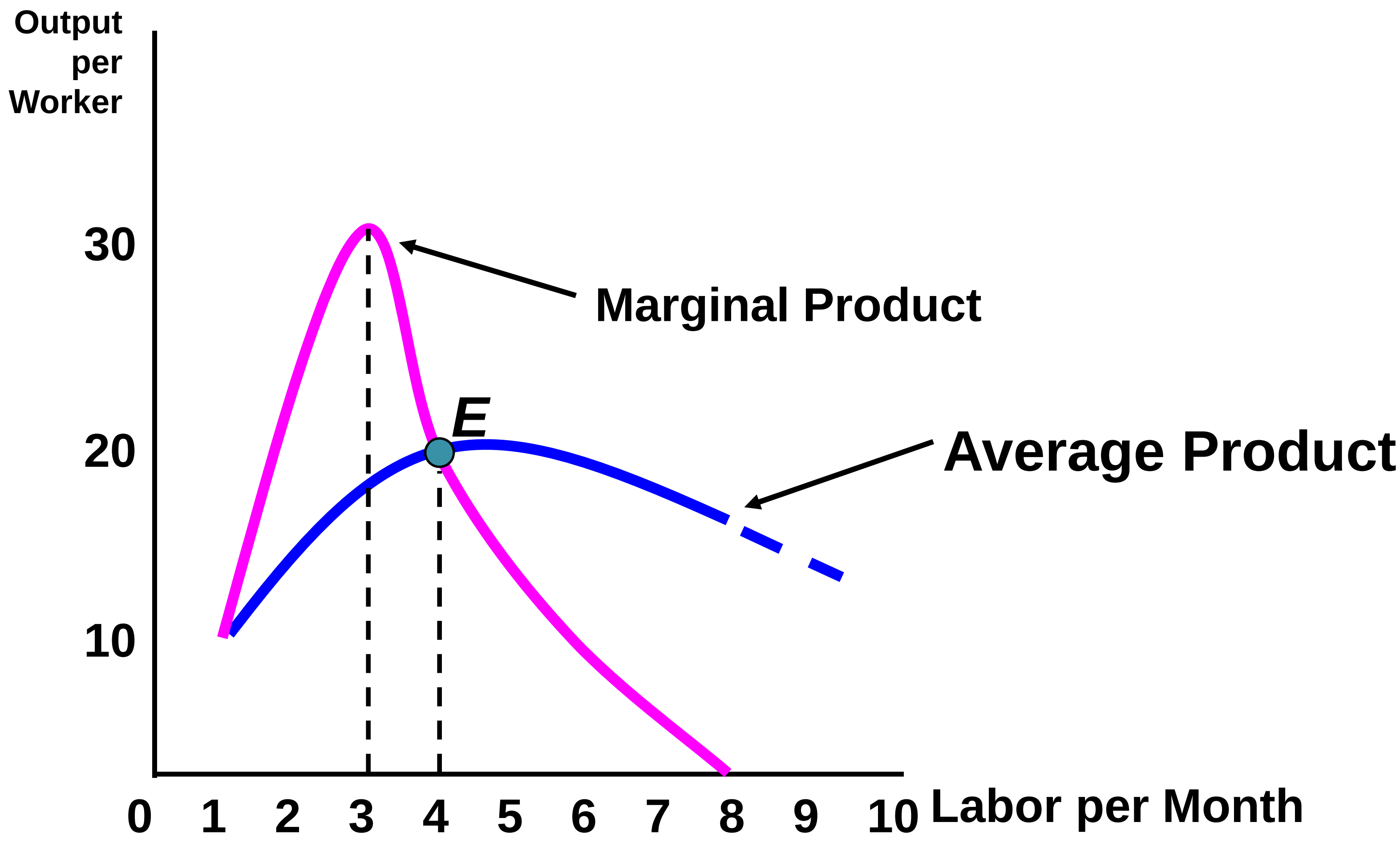
- Marginal Product of Labor – additional output produced when labor increases by one unit
- Change in output divided by the change in labor

$$\boxed{MP_L = \frac{\Delta Output}{\Delta Labor\ Input} = \frac{\Delta \boldsymbol{q}}{\Delta \boldsymbol{L}}}$$

Production: One Variable Input



Production: One Variable Input



Production: One Variable Input

- From the previous example, we can see that as we increase labor the additional output produced declines
- **Law of Diminishing Marginal Returns**: As the use of an input increases with other inputs fixed, the resulting additions to output will eventually decrease
- Note that this law occurs when ***other inputs are fixed***

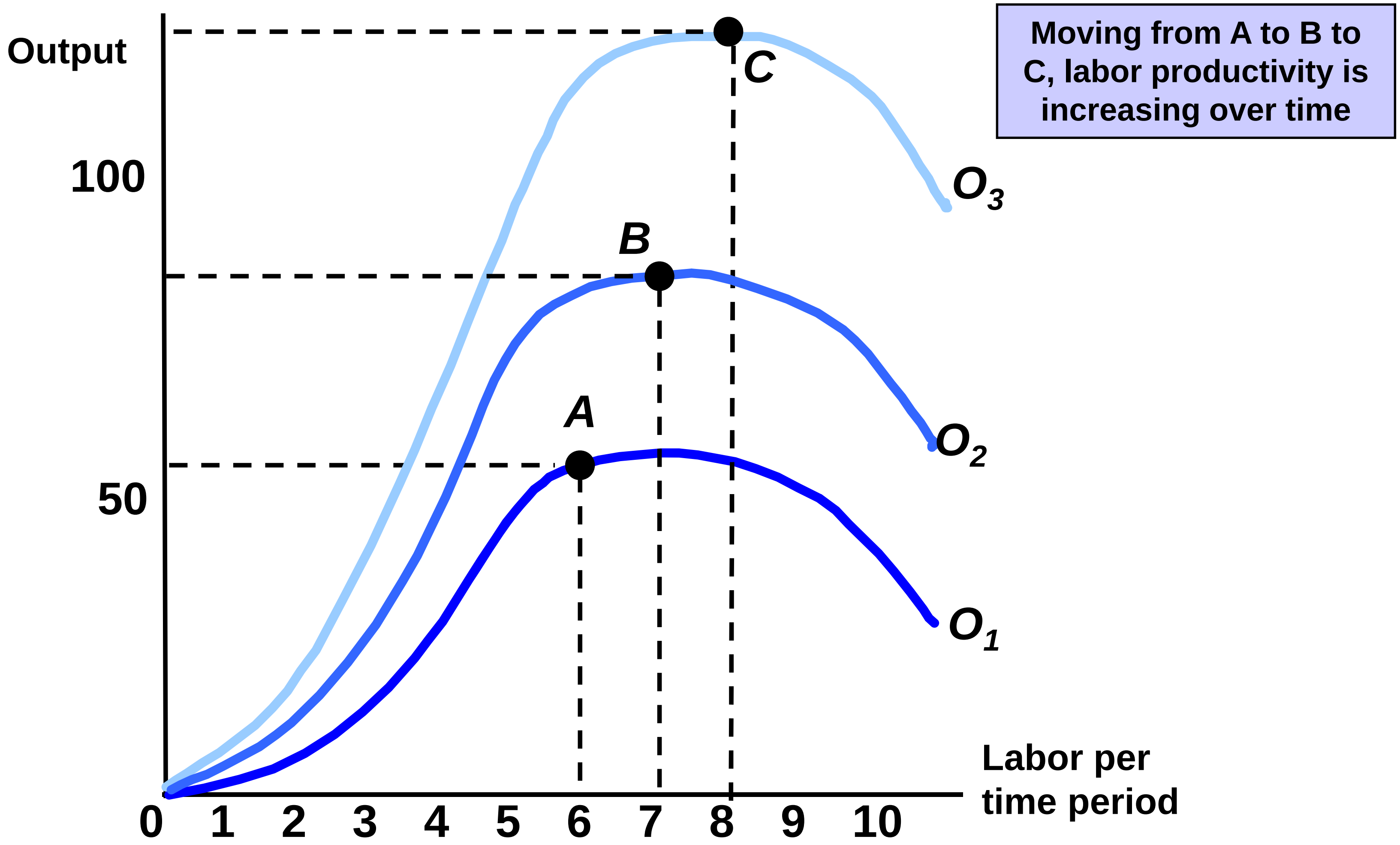
Law of Diminishing Marginal Returns

- Typically applies only for the short run when one variable input is fixed
- Can be used for long-run decisions to evaluate the trade-offs of different plant configurations
- Assumes the quality of the variable input is constant

Law of Diminishing Marginal Returns

- Assumes a constant technology
  - Changes in technology will cause shifts in the total product curve
  - More output can be produced with same inputs
  - Labor productivity can increase if there are improvements in technology, even though any given production process exhibits diminishing returns to labor

The Effect of Technological Improvement



Malthus and the Food Crisis

- Malthus predicted mass hunger and starvation as diminishing returns limited agricultural output and the population continued to grow
- Why did Malthus' prediction fail?
  - Did not take into account changes in technology
  - Although he was right about diminishing marginal returns to labor

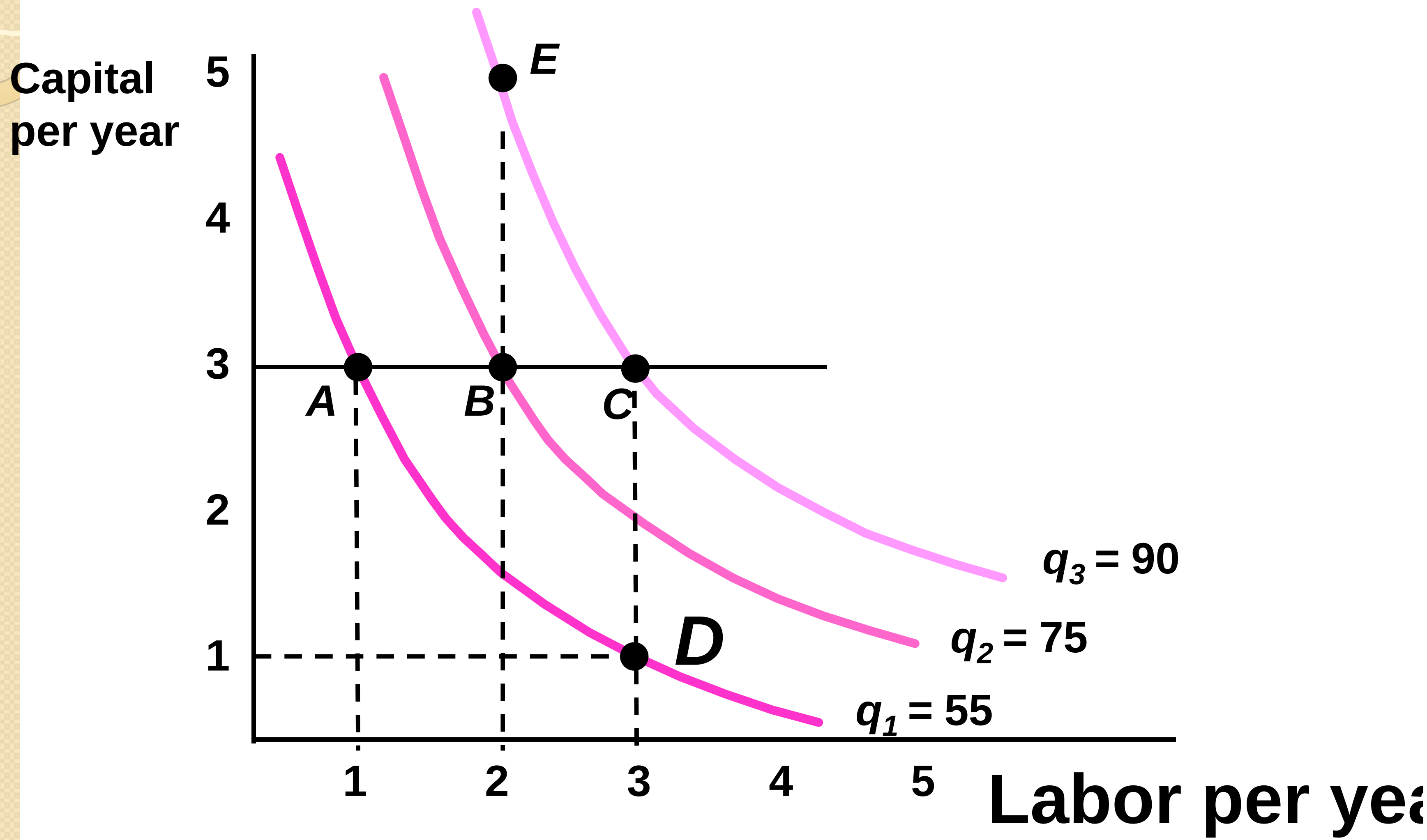
Production: Two Variable Inputs

- Firm can produce output by combining different amounts of labor and capital
- In the long run, capital and labor are both variable
- We can look at the output we can achieve with different combinations of capital and labor

Production: Two Variable Inputs

- The information can be represented graphically using **isoquants**
  - Curves showing all possible combinations of inputs that yield the same output
- Curves are smooth to allow for use of fractional inputs
  - Curve 1 shows all possible combinations of labor and capital that will produce 55 units of output

Isoquant Map



Production: Two Variable Inputs

- Diminishing Returns to Labor with Isoquants
- Holding capital at 3 and increasing labor from 0 to 1 to 2 to 3
  - Output increases at a decreasing rate (0, 55, 20, 15) illustrating diminishing marginal returns from labor in the short run and long run

Production: Two Variable Inputs

- Substituting Among Inputs
  - Slope of the isoquant shows how one input can be substituted for the other and keep the level of output the same
  - The negative of the slope is the **marginal rate of technical substitution (MRTS)**
    - Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant

Production: Two Variable Inputs

- The marginal rate of technical substitution equals:

$$MRTS = -\frac{Change\ in\ Capital\ Input}{Change\ in\ Labor\ Input}$$

$$MRTS = -\Delta K / \Delta L \text{ (for a fixed level of } q\text{)}$$

MRTS and Isoquants

- We assume there is diminishing MRTS
  - Increasing labor in one unit increments from 1 to 5 results in a decreasing MRTS from 1 to 1/2
  - Productivity of any one input is limited
- Diminishing MRTS occurs because of diminishing returns and implies isoquants are convex
- There is a relationship between MRTS and marginal products of inputs

MRTS and Marginal Products

- If we are holding output constant, the net effect of increasing labor and decreasing capital must be zero
- Using changes in output from capital and labor we can see

$$\boxed{(MP_L)(dL)+(MP_K)(dK)=0}$$

MRTS and Marginal Products

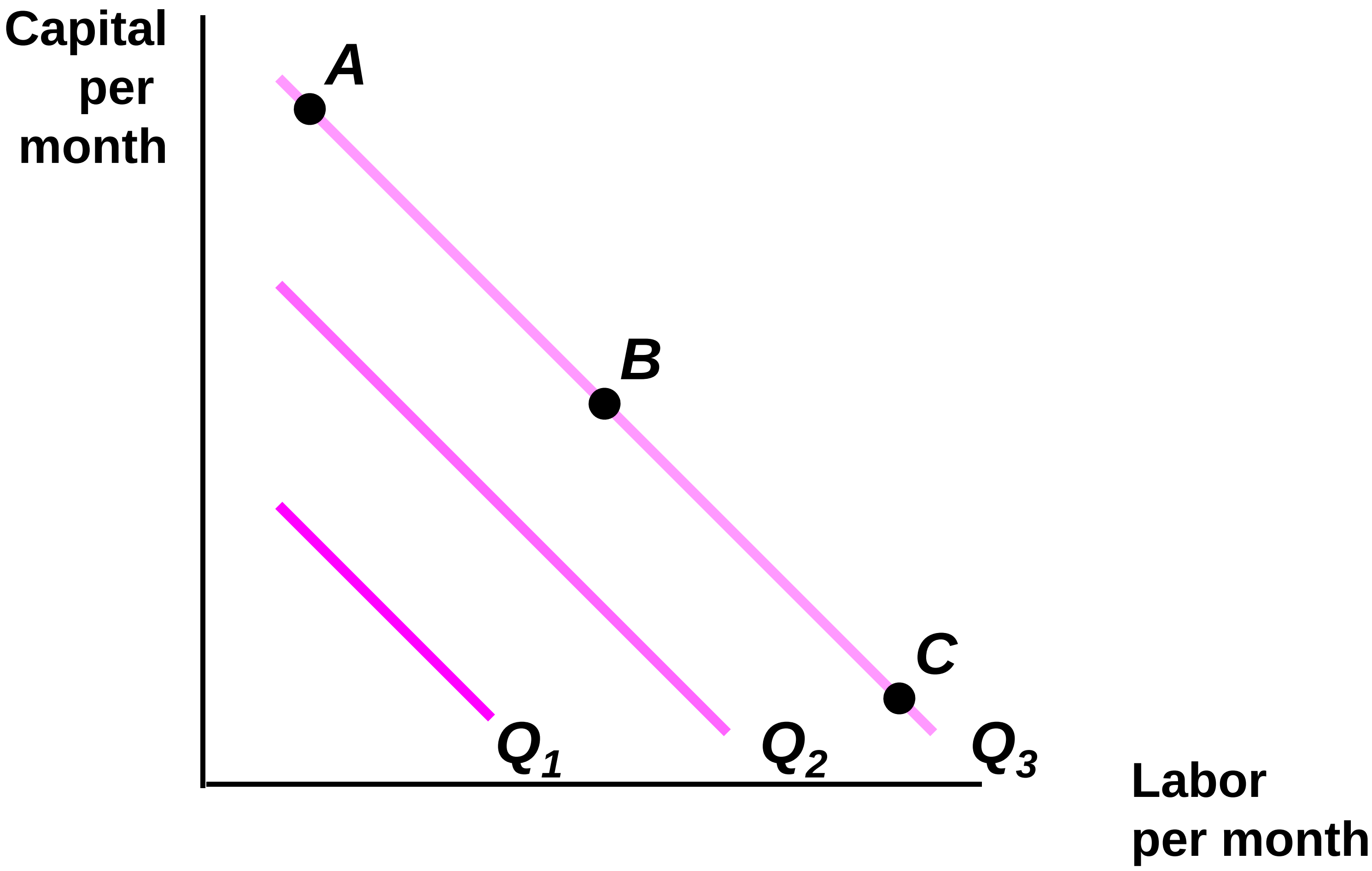
- Rearranging equation, we can see the relationship between MRTS and MPs

$$(MP_L)(dL) + (MP_K)(dK) = 0$$

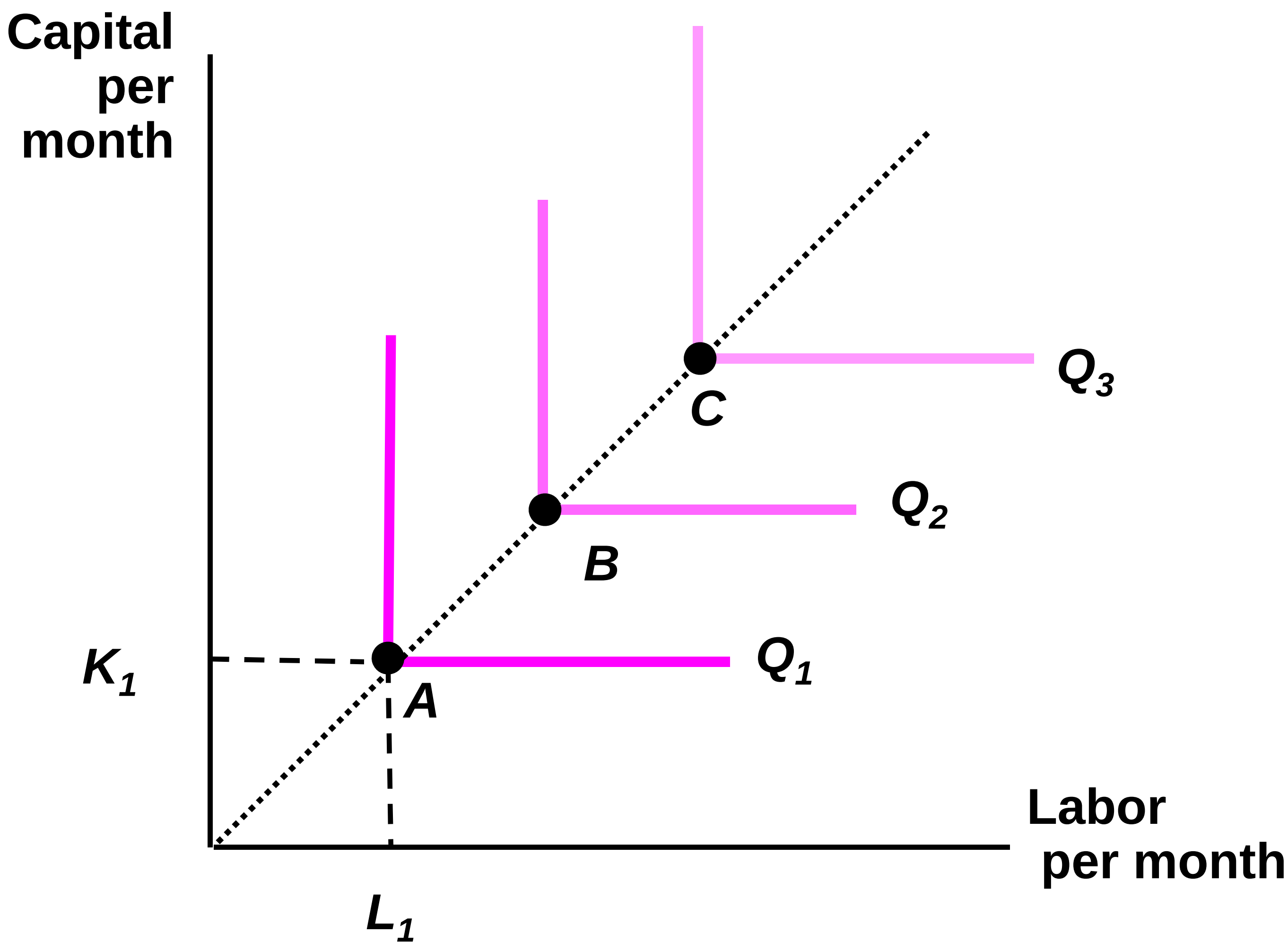
$$(MP_L)(dL) = -(MP_K)(dK)$$

$$\frac{(MP_L)}{(MP_K)} = -\frac{dK}{dL} = MRTS$$

Perfect Substitutes



Fixed-Proportions Production Function

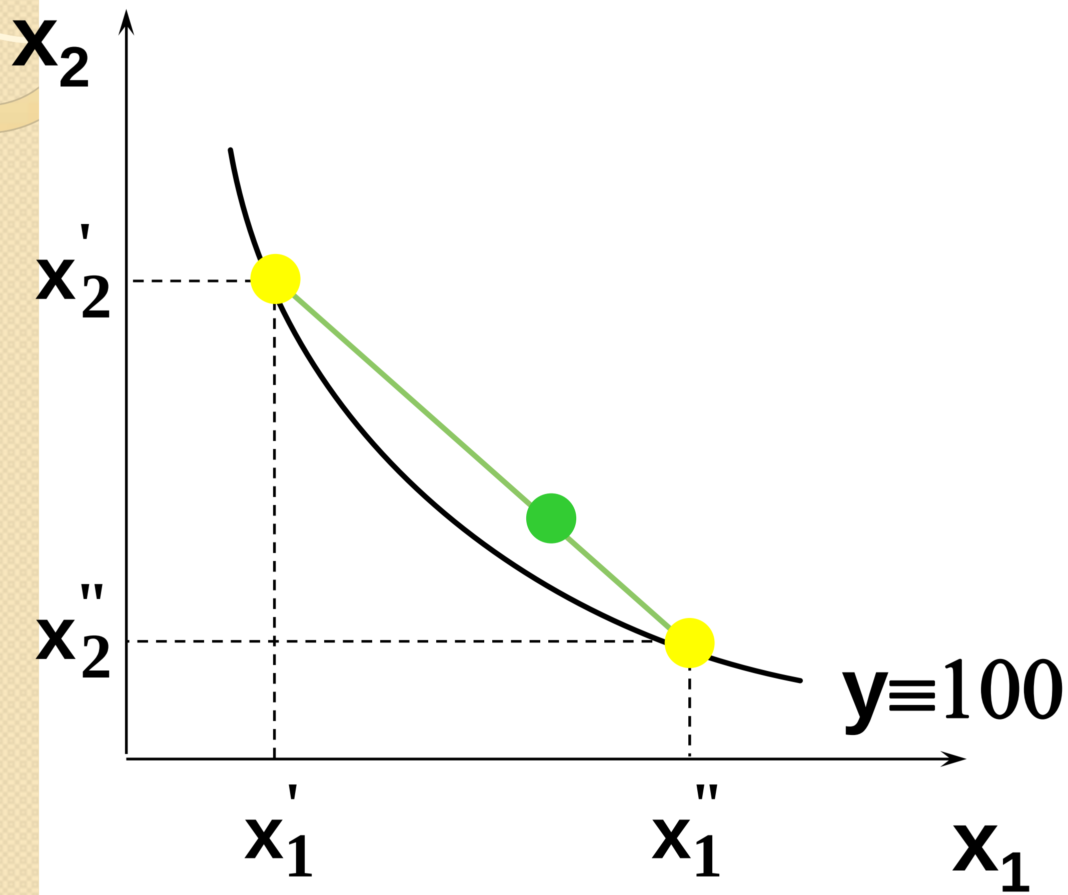


Well-behaved Technology

- **Monotonicity**: if we increase the amount of at least one of the inputs, it should be possible to produce at least as much output as we producing originally

- This is sometimes referred to as the ***property of free disposal***

Well-Behaved Technologies - Convexity



Returns to Scale

- In addition to discussing the tradeoff between inputs to keep production the same
- How does a firm decide, in the long run, the best way to increase output?
  - Can change the scale of production by increasing all inputs in proportion?
  - If double inputs, output will most likely increase but by how much?

Returns to Scale

- Rate at which output increases as inputs are increased proportionately
  - Increasing returns to scale
  - Constant returns to scale
  - Decreasing returns to scale

Constant Return to Scale

$$f(tx_1, tx_2, \ldots, tx_n) = t \bullet f(x_1, x_2, \ldots, x_n), \quad t > 0$$

Diminishing MP and CRTS

$$y = x_1^{\alpha} x_2^{1-\alpha} \quad (0 < \alpha < 1)$$

$$f(tx_1, tx_2) = (tx_1)^{\alpha} (tx_2)^{1-\alpha} = t \bullet x_1^{\alpha} x_2^{1-\alpha} = t \bullet y$$

$$\frac{\partial f}{\partial x_1} = \alpha \bullet x_1^{\alpha-1} x_2^{1-\alpha} > 0$$

$$\frac{\partial^2 f}{\partial x_1^2} = \alpha(\alpha - 1) \bullet x_1^{\alpha-2} x_2^{1-\alpha} < 0$$

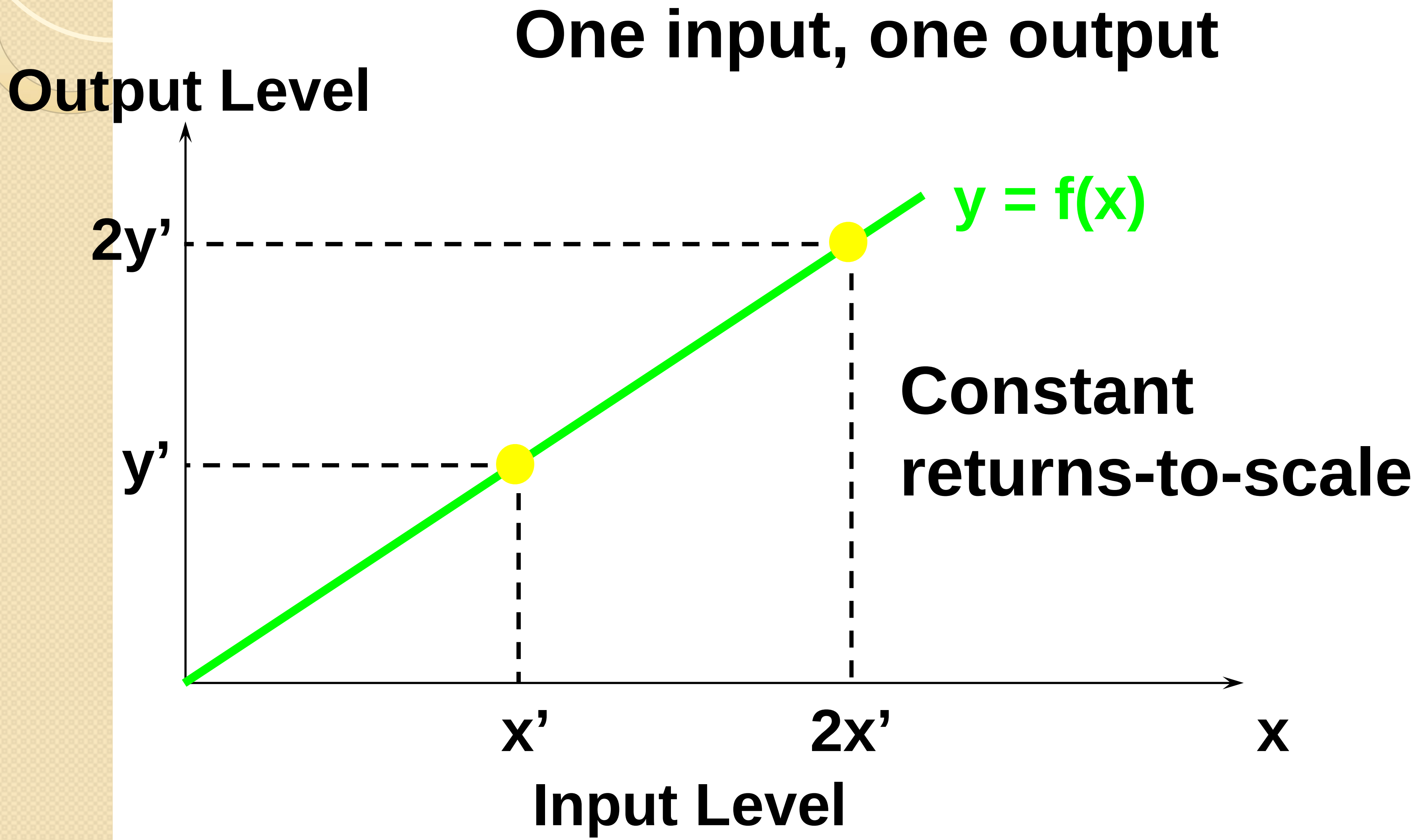
Increasing Return to Scale

$$f(tx_1, tx_2, \ldots, tx_n) > t \bullet f(x_1, x_2, \ldots, x_n),$$
$$for\ any\ t > 1$$

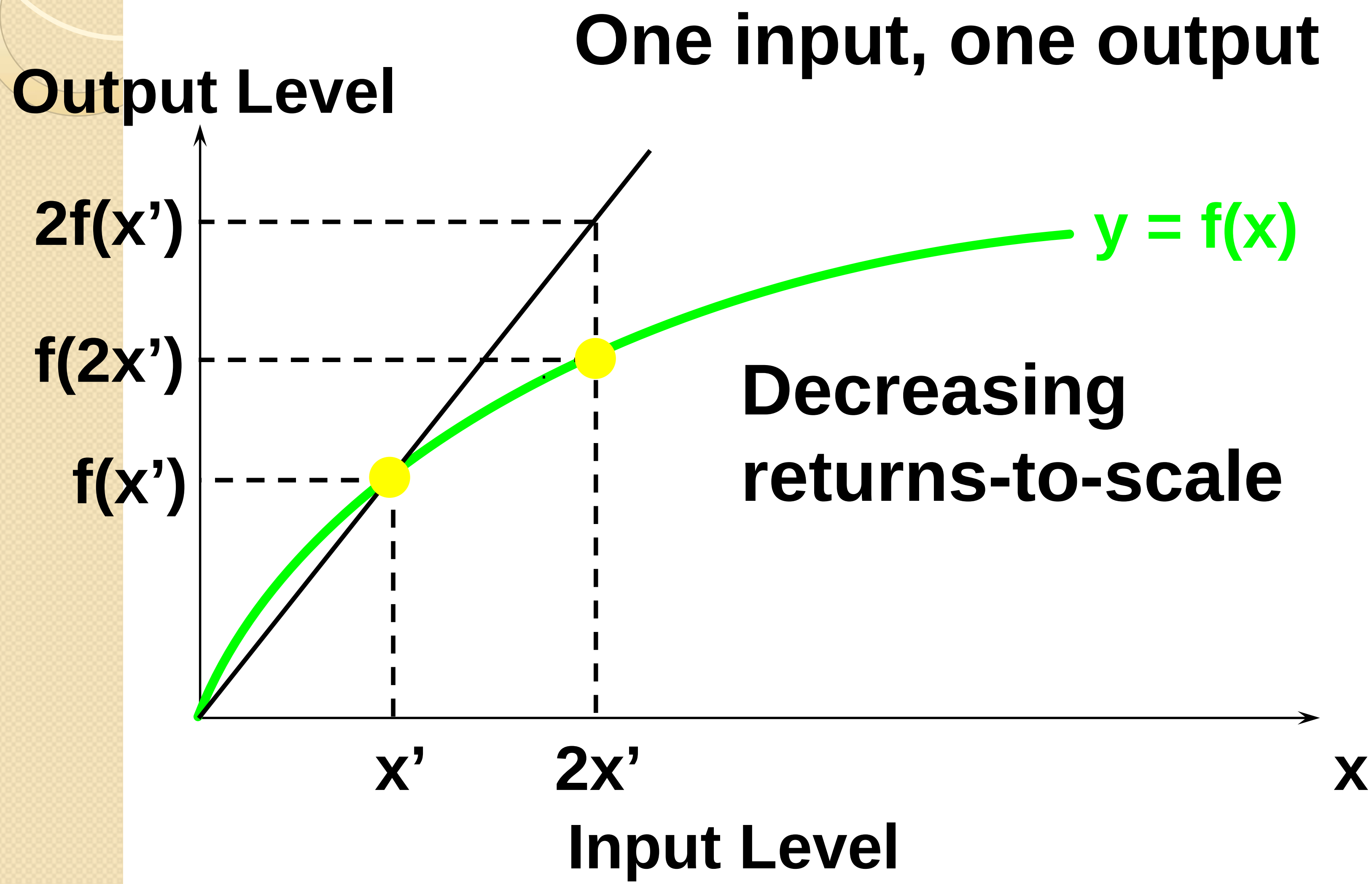
Decreasing Return to Scale

$$f(tx_1, tx_2, ..., tx_n) < t \bullet f(x_1, x_2, ..., x_n),$$
$$for\ any\ t > 1$$

Returns-to-Scale

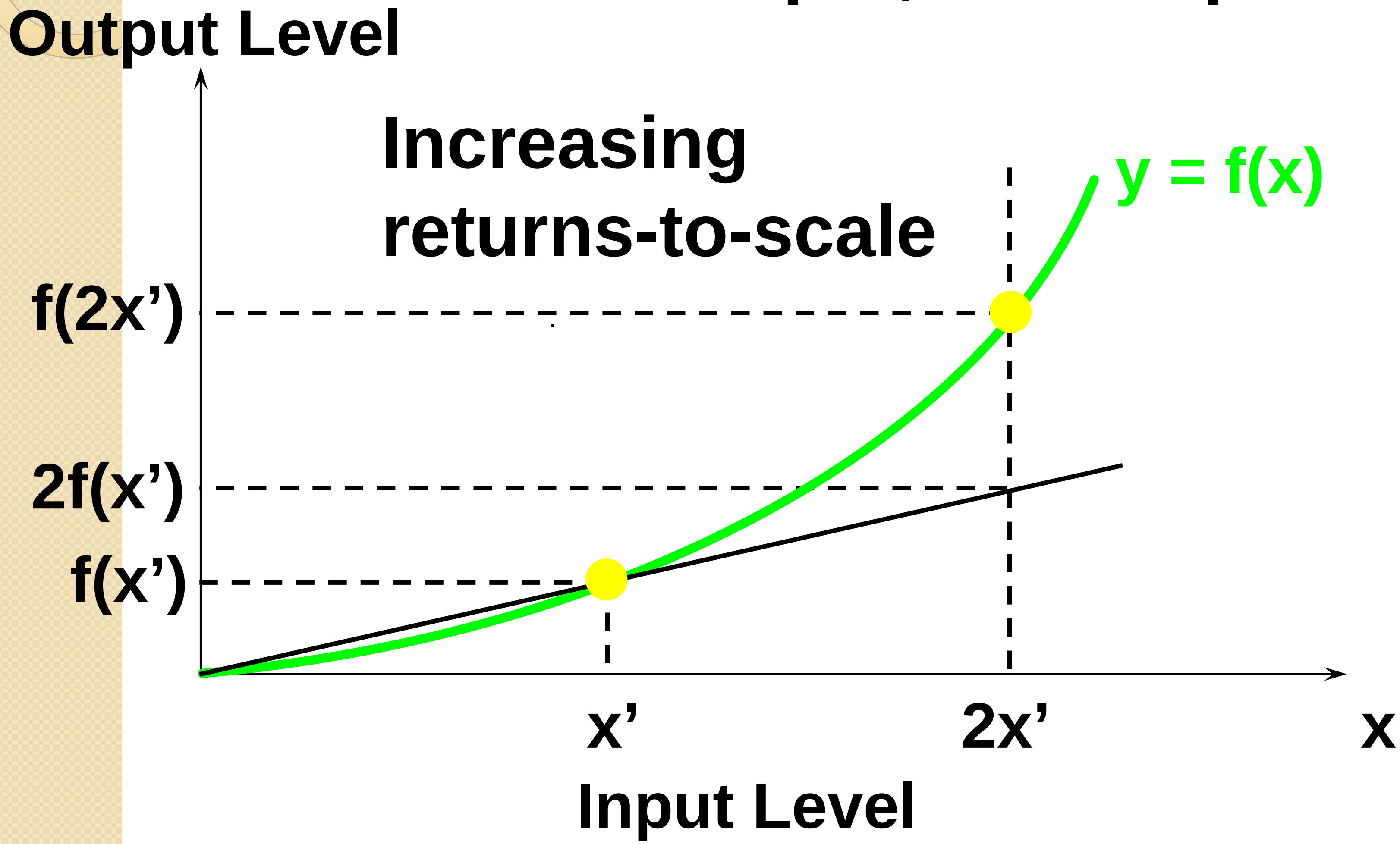


Returns-to-Scale

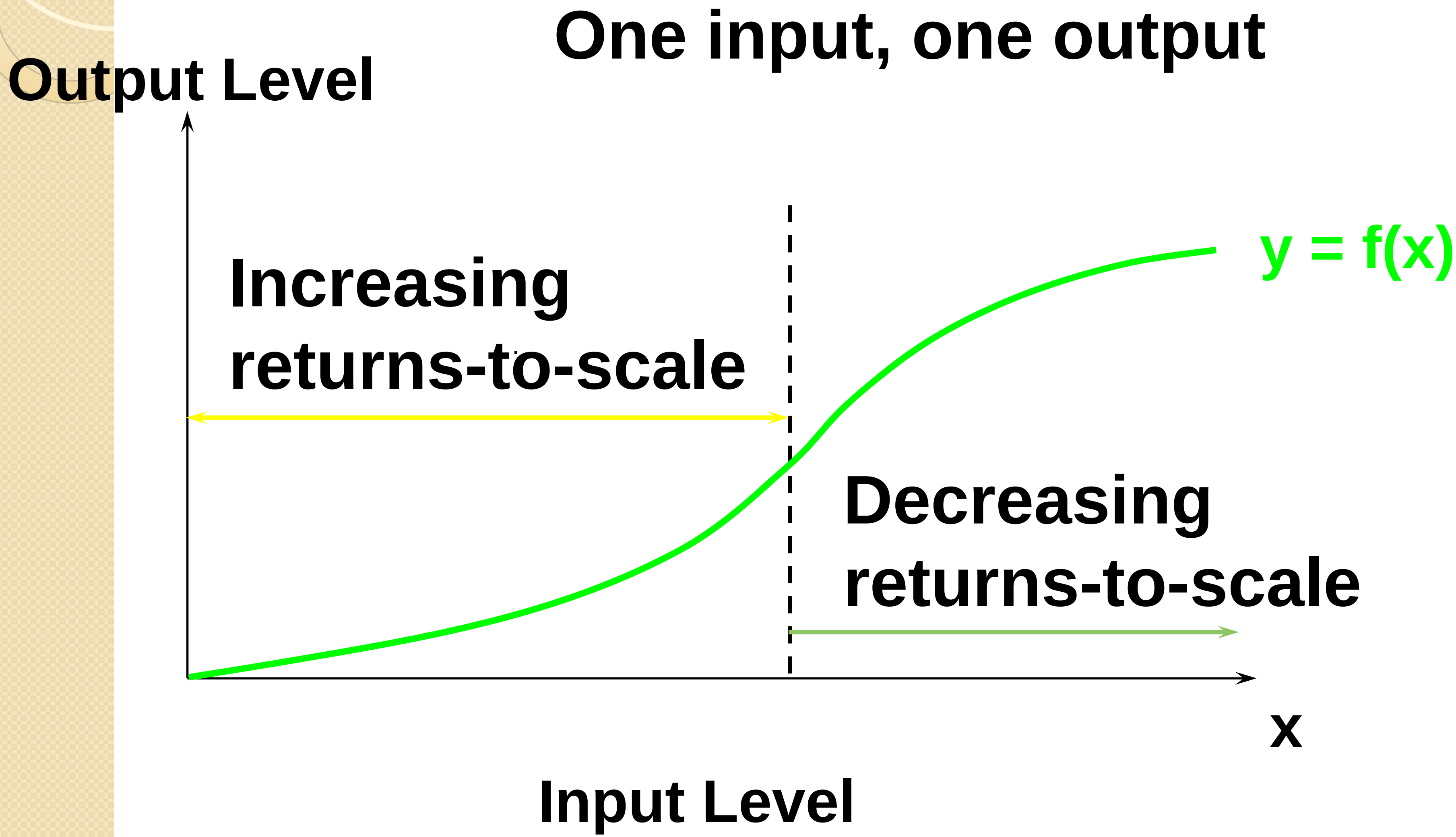


Returns-to-Scale

One input, one output



Returns-to-Scale



Examples of RTS: Cobb-Douglas Function

$$y = Ax_1^{\alpha} x_2^{\beta}$$

$$\alpha + \beta = 1 \Rightarrow CRS$$

$$\alpha + \beta > 1 \Rightarrow IRS$$

$$\alpha + \beta < 1 \Rightarrow DRS$$

Examples of Returns-to-Scale

The Cobb-Douglas production function is

$$y = x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$$

$$(kx_1)^{a_1}(kx_2)^{a_2} \cdots (kx_n)^{a_n} = k^{a_1+\cdots+a_n} y$$

The Cobb-Douglas technology's returns-to-scale is

constant ***if*** $a_1 + \ldots + a_n = 1$
increasing ***if*** $a_1 + \ldots + a_n > 1$
decreasing ***if*** $a_1 + \ldots + a_n < 1$.

Chapter 7

The Cost of Production

Topics to be Discussed

- Cost Minimization
- Costs in the Short Run and Long Run
- Measuring Cost: Which Costs Matter?
- Long-Run Versus Short-Run Cost Curves

Cost Minimizing Input Choice

- How does a firm select inputs to produce a given output at minimum cost?
- Assumptions
  - Two Inputs: Labor (L) and capital (K)
  - Price of labor: wage rate (w)
  - The price of capital (r)
    - r = depreciation rate + interest rate
    - Or rental rate if not purchasing
    - These are equal in a competitive capital market

The Isocost Line

- The Isocost Line
  - A line showing all combinations of L & K that can be purchased for the same cost
  - Total cost of production is sum of firm's labor cost, wL, and its capital cost, rK:

$$C = wL + rK$$

  - For each different level of cost, the equation shows another isocost line

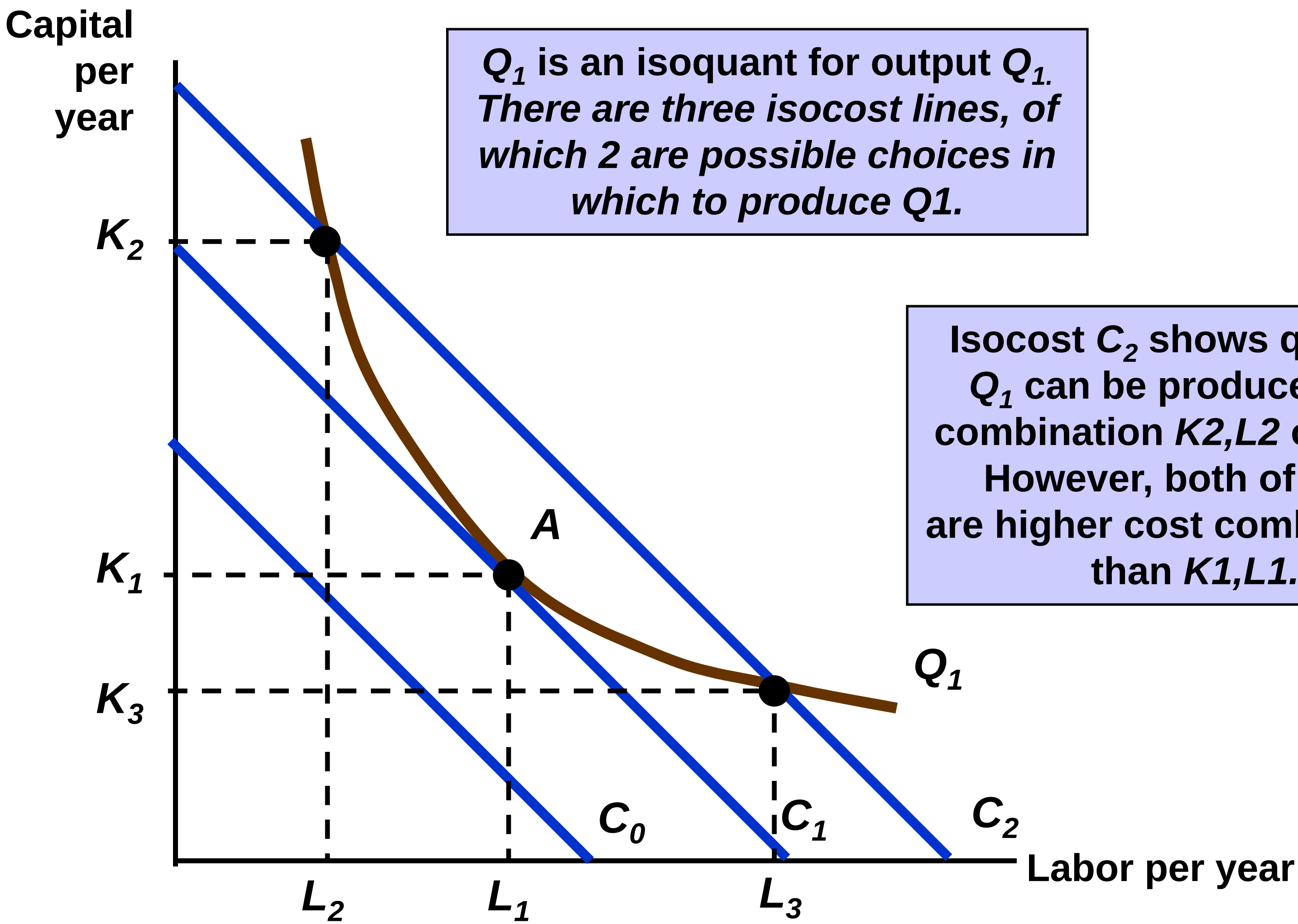
The Isocost Line

- Rewriting C as an equation for a straight line:
  - K = C/r - (w/r)L
  - Slope of the isocost: $dK/dL = -(w/r)$
    - -(w/r) is the ratio of the wage rate to rental cost of capital.
    - This shows the rate at which capital can be substituted for labor with no change in cost

Choosing Inputs

- We will address how to minimize cost for a given level of output by combining isocosts with isoquants
- We choose the output we wish to produce and then determine how to do that at minimum cost
  - Isoquant is the quantity we wish to produce
  - Isocost is the combination of K and L that gives a set cost
  - ***We assume that firms face no liquidity constraints***

Producing a Given Output at Minimum Cost



Q_1 is an isoquant for output Q_1. There are three isocost lines, of which 2 are possible choices in which to produce Q1.

Isocost C_2 shows quantity Q_1 can be produced with combination *K2,L2* or *K3,L3*. However, both of these are higher cost combinations than *K1,L1*.

Input Substitution When an Input Price Change

- If the price of labor changes, then the slope of the isocost line changes, -(w/r)
- It now takes a new quantity of labor and capital to produce the output
- If price of labor increases relative to price of capital, and capital is substituted for labor

First-Order Condition

- How does the isocost line relate to the firm's production process?

$$\text{MRTS} = -\, dK/dL = -\, \text{MP}_\text{L}/\text{MP}_\text{K}$$

$$\text{Slope of isocost line} = dK/dL = w/r$$

$$MP_L/MP_K = w/r \ \text{ when firm minimizes cost}$$

Cost in the Long Run

- The minimum cost combination can then be written as:

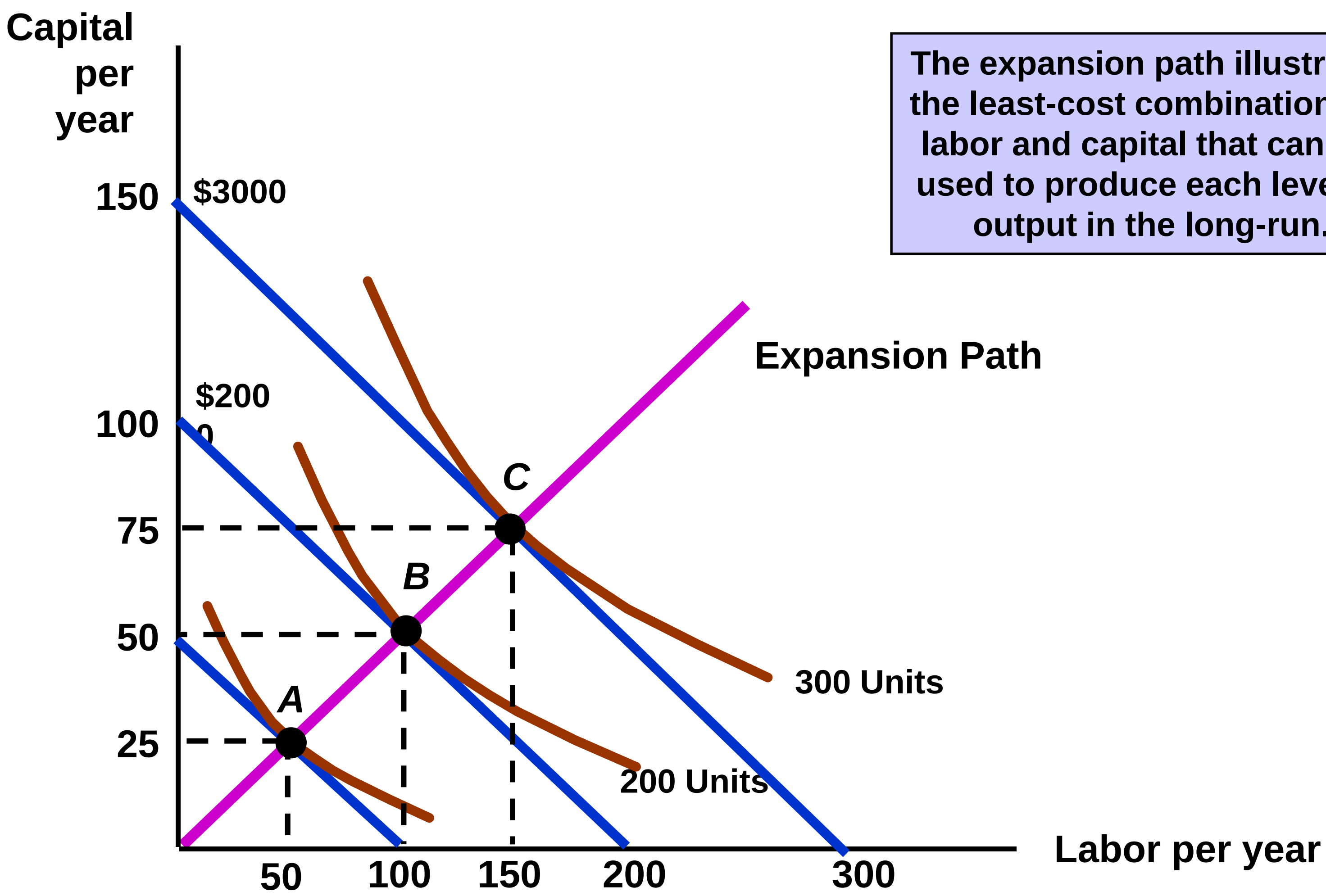
$$\mathrm{MP_L}/w = \mathrm{MP_K}/r$$

  - Minimum cost for a given output will occur when each dollar of input added to the production process will add an equivalent amount of output.

Cost in the Long Run

- Cost minimization with Varying Output Levels
  - For each level of output, there is an isocost curve showing minimum cost for that output level
  - A firm's **expansion path** shows the minimum cost combinations of labor and capital at each level of output
  - Slope equals $\Delta K/\Delta L$

A Firm's Expansion Path



The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

Expansion Path and Long Run Costs

- Firm's expansion path has same information as long-run total cost curve
- To move from expansion path to LR cost curve
  - Find tangency with isoquant and isocost
  - Determine min cost of producing the output level selected
  - Graph output-cost combination

From Cost Minimization to Cost Functions

$$Min \;\; w_1 x_1 + w_2 x_2 \quad s.t. \; f(x_1, x_2) = \bar{y}$$

$$L(x_1, x_2, \lambda) = w_1 x_1 + w_2 x_2 + \lambda[\bar{y} - f(x_1, x_2)]$$

$$\Rightarrow x_i^*(w_1, w_2, y) \; (i = 1{,}2),$$

$$conditional \;\; factor \; deamnd$$

$$\Rightarrow C[x_1^*(w_1, w_2, y), x_2^*(w_1, w_2, y)] = C(w_1, w_2, y)$$

Return to Scale and the Cost Functions

- Constant Returns to Scale
  - If input is doubled, output will double
  - AC cost is constant at all levels of output
- Denote c(w1, w2, 1) as the unit cost function
- The minimal cost to produce y units of output, c(w1, w2, y) = c(w1, w2, 1)y
- C(w1, w2, y)/y = AC = c(w1, w2, 1)

Return to Scale and the Cost Functions

2. Increasing Returns to Scale
   - If input is doubled, output will more than double
   - AC decreases at all levels of output
3. Decreasing Returns to Scale
   - If input is doubled, output will less than double
   - AC increases at all levels of output

Long Run Versus Short Run Costs

- In the short run, some costs are fixed
- In the long run, firm can change anything including plant size
  - Can produce at a lower average cost in long run than in short run
  - Capital and labor are both flexible
- We can show this by holding capital fixed in the short run and flexible in long run

Short and Long Run Cost Functions

- The cost function c(w1, w2, y) measures the ***minimal*** costs of producing y units of output given the factors prices w1 and w2
- The distinction between short run and long-run cost functions arises from the production functions in different time horizons (short-run vs. long run) used to solve the cost minimization problem

Relationship between LR and SR Costs

$$Min \;\; w_1x_1 + w_2x_2 \;\; s.t. \;\; f(x_1, \bar{x}_2) = y$$

$$\Rightarrow C_s(y, \bar{x}_2) = w_1 x_1^S (w_1, w_2, \bar{x}_2, y) + w_2 \, \bar{x}_2$$

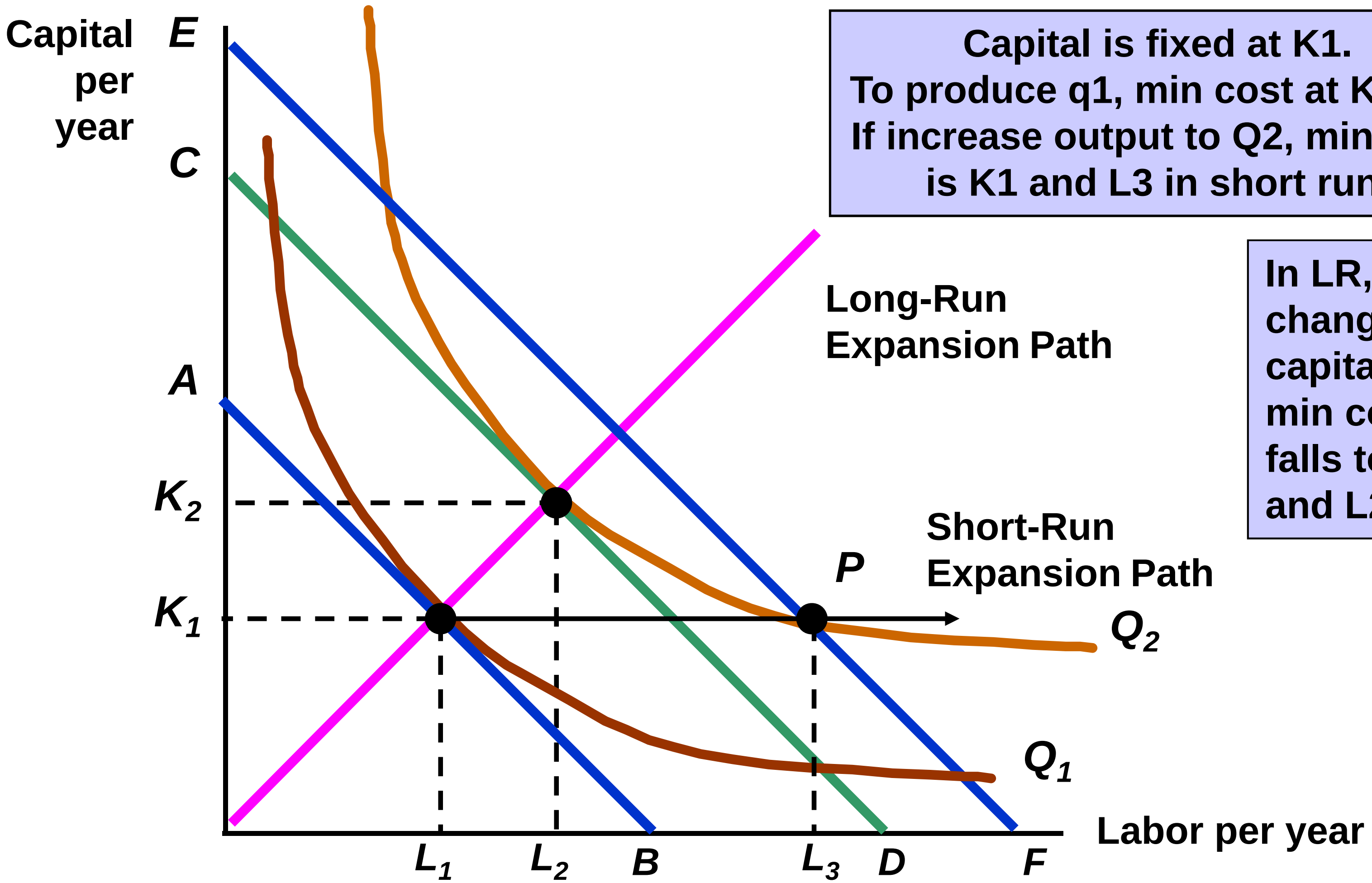
$$Min \;\; w_1x_1 + w_2x_2 \;\; s.t. \;\; f(x_1, x_2) = y$$

$$\Rightarrow C \;\; (y) = w_1 x_1 (w_1, w_2, y) + w_2 x_2 (w_1, w_2, y)$$

$$\Rightarrow C(y) \leq C_s(y, \bar{x}_2)$$

$$\Rightarrow C(y) = C_s(y, x_2(y))$$

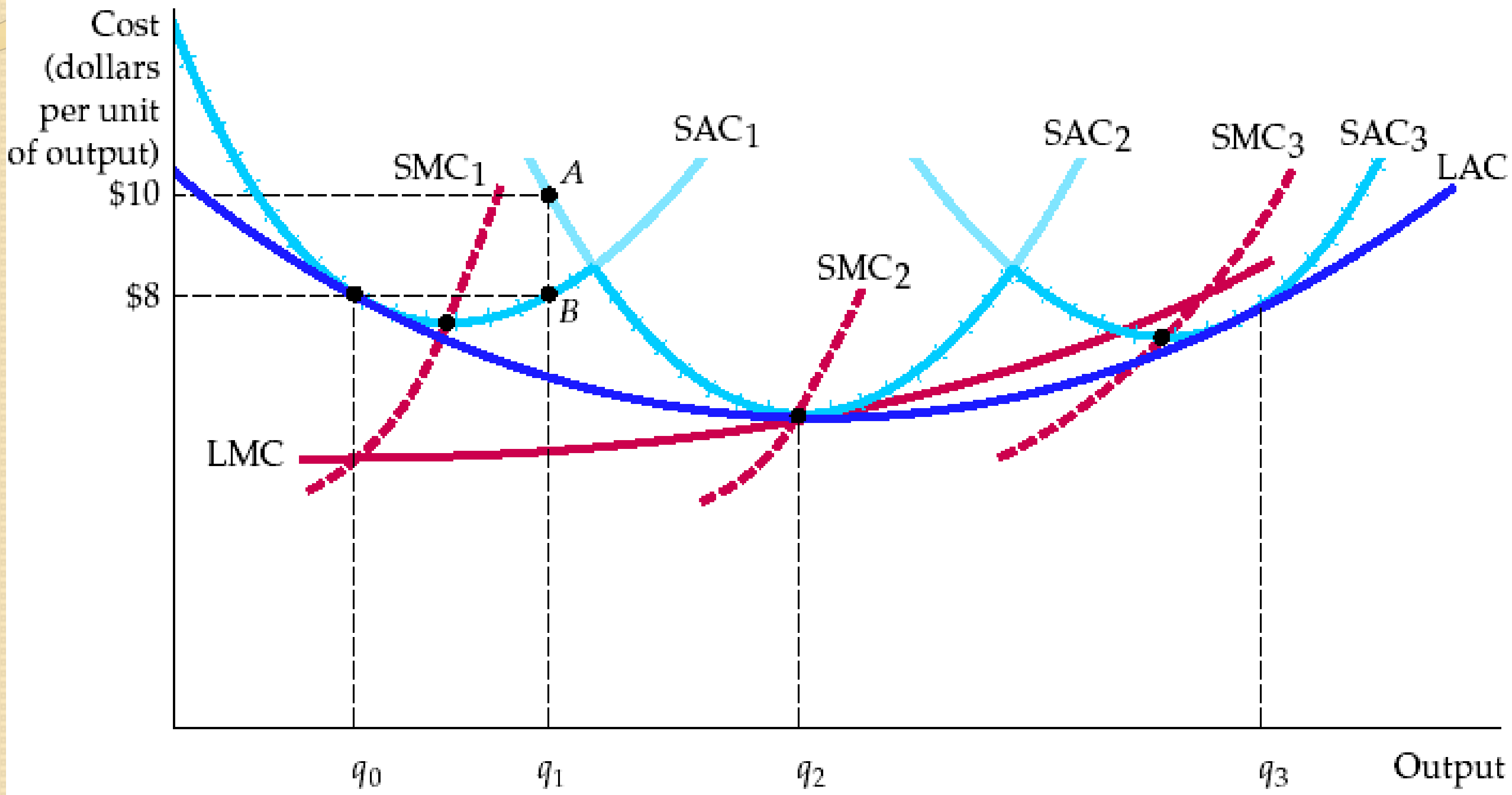
The Inflexibility of Short Run Production



Capital is fixed at K1.
To produce q1, min cost at K1,L1.
If increase output to Q2, min cost is K1 and L3 in short run.

In LR, can change capital and min costs falls to K2 and L2.

Long Run Cost as Envelope Curve



Envelope Theorem

$$C(y) = C_s(y, k(y))$$

$$\frac{dC(y)}{dy} = \frac{\partial C_s(y^*, k^*)}{\partial y} + \frac{\partial C_s(y^*, k^*)}{\partial k}\frac{dk(y^*)}{dy} = \left.\frac{\partial C_s}{\partial y}\right|_{k=k(y)}$$

$$\because \frac{\partial C_s(y^*, k^*)}{\partial k} = 0$$

Note that k* is the optimal choice of k at the output level y*

Measuring Cost: Which Costs Matter?

- For a firm to minimize costs, we must clarify what is meant by *costs* and how to measure them
  - It is clear that if a firm has to rent equipment or buildings, the rent they pay is a cost
  - What if a firm owns its own equipment or building?
    - How are costs calculated here?

Measuring Cost: Which Costs Matter?

- Accountants tend to take a *retrospective* view of firms' costs, whereas economists tend to take a *forward-looking* view
- Accounting Cost
  - Actual expenses plus depreciation charges for capital equipment
- Economic Cost
  - Cost to a firm of utilizing economic resources in production, including opportunity cost

Measuring Cost: Which Costs Matter?

- Economic costs distinguish between costs the firm can control and those it cannot
  - Concept of opportunity cost plays an important role
- **Opportunity cost**
  - Cost associated with opportunities that are foregone when a firm's resources are not put to their highest-value use

Opportunity Cost

- An Example
  - A firm owns its own building and pays no rent for office space
  - Does this mean the cost of office space is zero?
  - The building could have been rented instead
  - Foregone rent is the opportunity cost of using the building for production and should be included in the economic costs of doing business

Measuring Cost: Which Costs Matter?

- Although opportunity costs are hidden and should be taken into account, sunk costs should not
- **Sunk Cost**
  - Expenditure that has been made and cannot be recovered
  - Should not influence a firm's *future* economic decisions

Sunk Cost

- Firm buys a piece of equipment that cannot be converted to another use
- Expenditure on the equipment is a sunk cost
  - Has no alternative use so cost cannot be recovered – opportunity cost is zero
  - Decision to buy the equipment might have been good or bad, but now does not matter

Fixed and Variable Costs

- Some costs vary with output, while some remain the same no matter the amount of output
- Total cost can be divided into:

1. Fixed Cost
    - Does not vary with the level of output
2. Variable Cost
    - Cost that varies as output varies

Fixed and Variable Costs

- Total output is a function of variable inputs and fixed inputs
- Therefore, the total cost of production equals the fixed cost (the cost of the fixed inputs) plus the variable cost (the cost of the variable inputs), or…

$$\boxed{TC = FC + VC}$$

Fixed and Variable Costs

- Which costs are variable and which are fixed depends on the time horizon
- Short time horizon – most costs are fixed
- Long time horizon – many costs become variable
- In determining how changes in production will affect costs, must consider if fixed or variable costs are affected.

Marginal and Average Cost

- In completing a discussion of costs, must also distinguish between
  - Average Cost
  - Marginal Cost
- After definition of costs is complete, one can consider the analysis between short-run and long-run costs

Measuring Costs

- Marginal Cost (MC):
  - The cost of expanding output by one unit
  - Fixed costs have no impact on marginal cost, so it can be written as:

$$\boxed{MC = \frac{dVC}{dq} = \frac{dTC}{dq}}$$

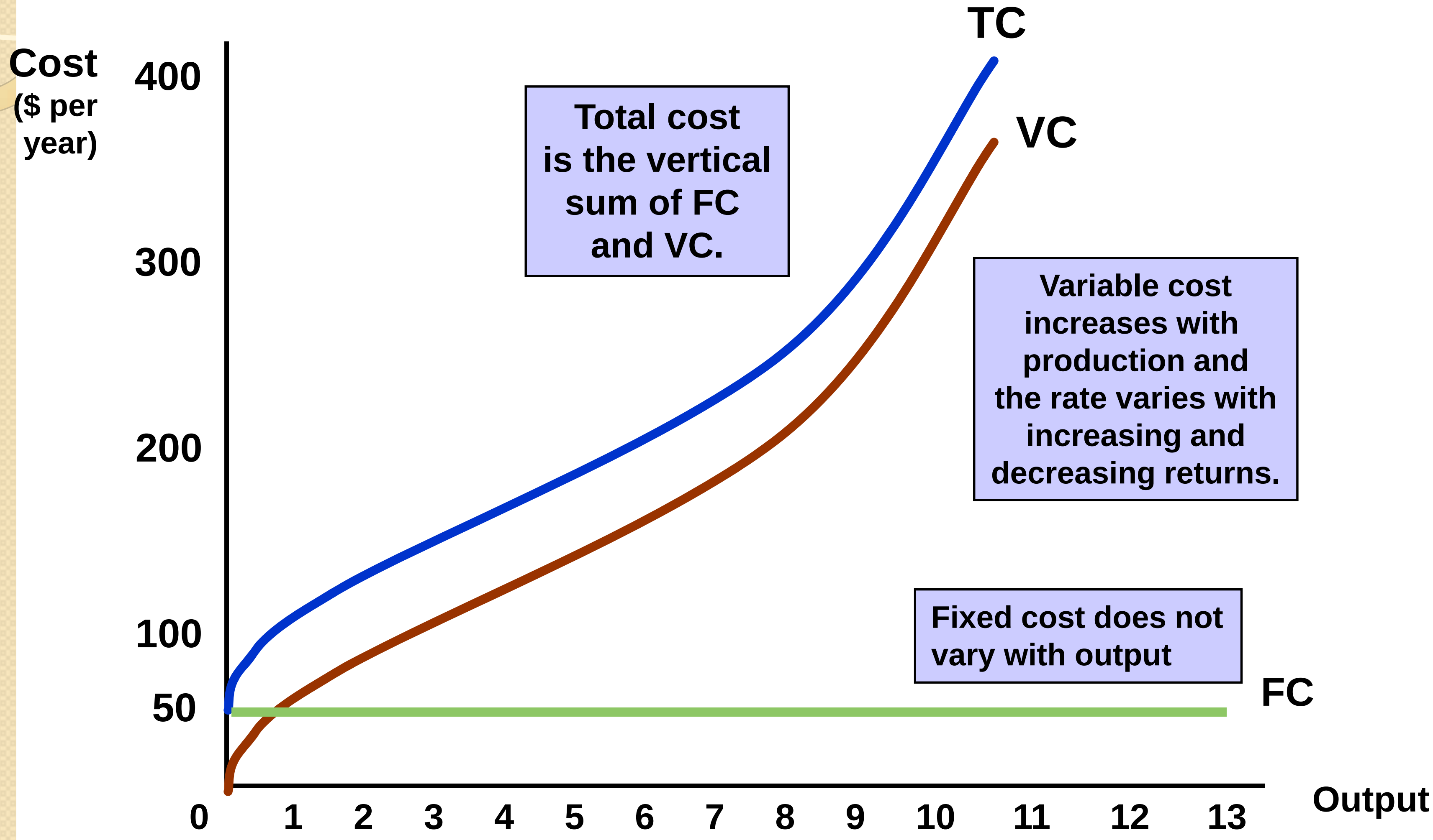
Measuring Costs

- Average Total Cost (ATC)
  - Cost per unit of output
  - Also equals average fixed cost (AFC) plus average variable cost (AVC)

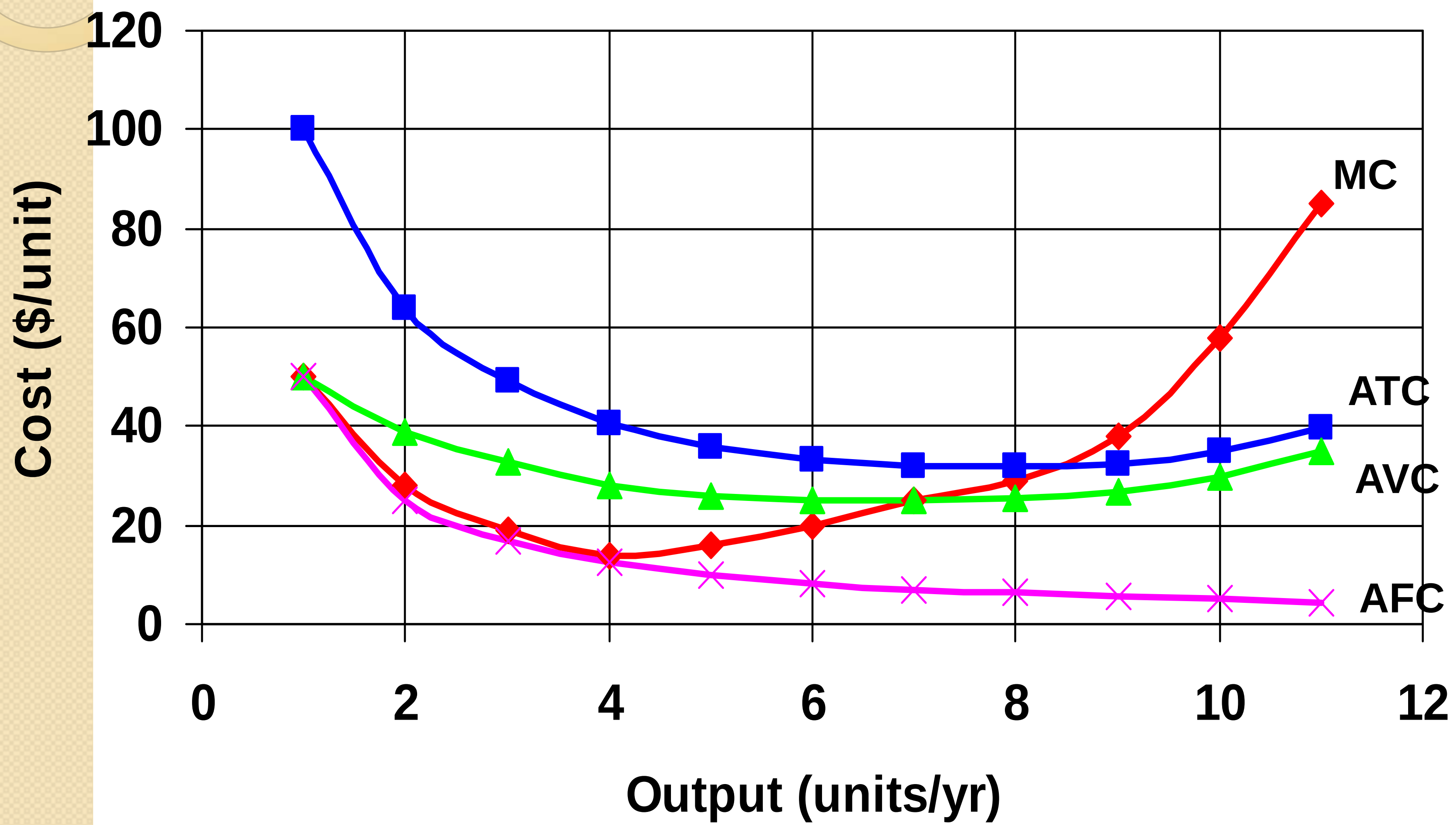
$$ATC = \frac{TC}{q} = AFC + AVC$$

$$ATC = \frac{TC}{q} = \frac{TFC}{q} + \frac{TVC}{q}$$

Cost Curves for a Firm



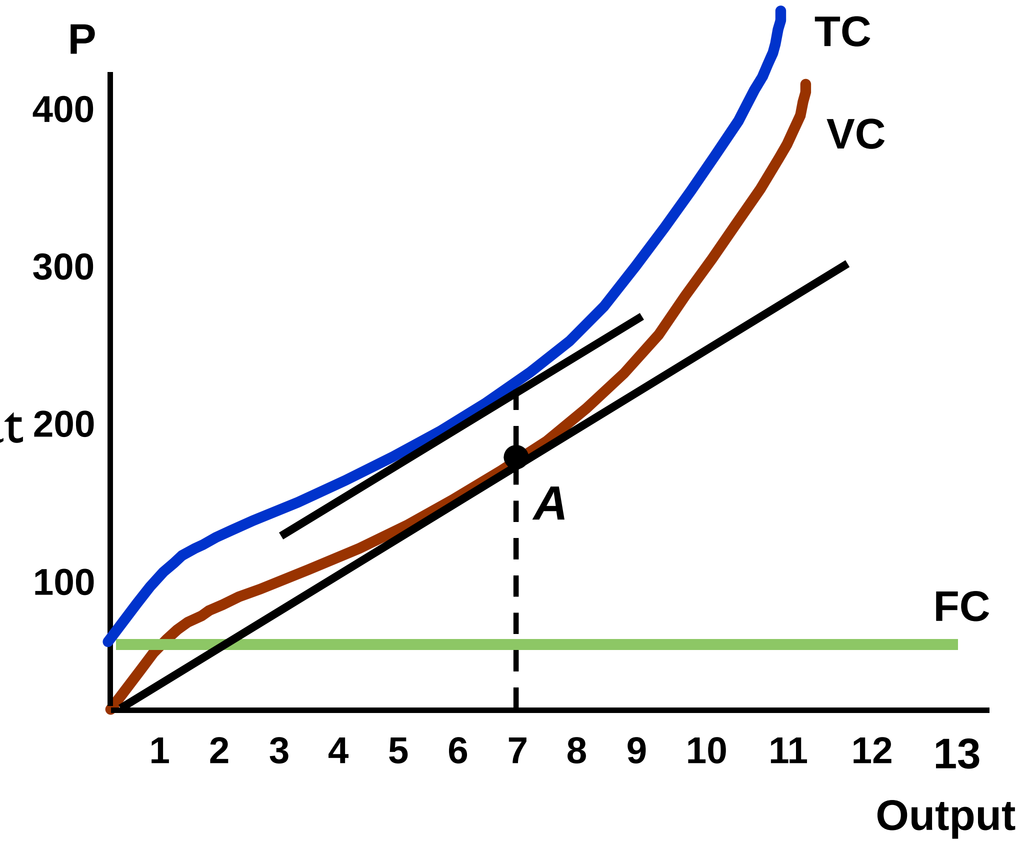
Cost Curves



Cost ($/unit)
120
100
80
60
40
20
0
0
2
4
6
8
10
12
Output (units/yr)
MC
ATC
AVC
AFC

Cost Curves for a Firm

- The line drawn from the origin to the variable cost curve:
  - Its slope equals AVC
  - The slope of a point on VC or TC equals MC
  - Therefore, MC = AVC at 7 units of output (point A)



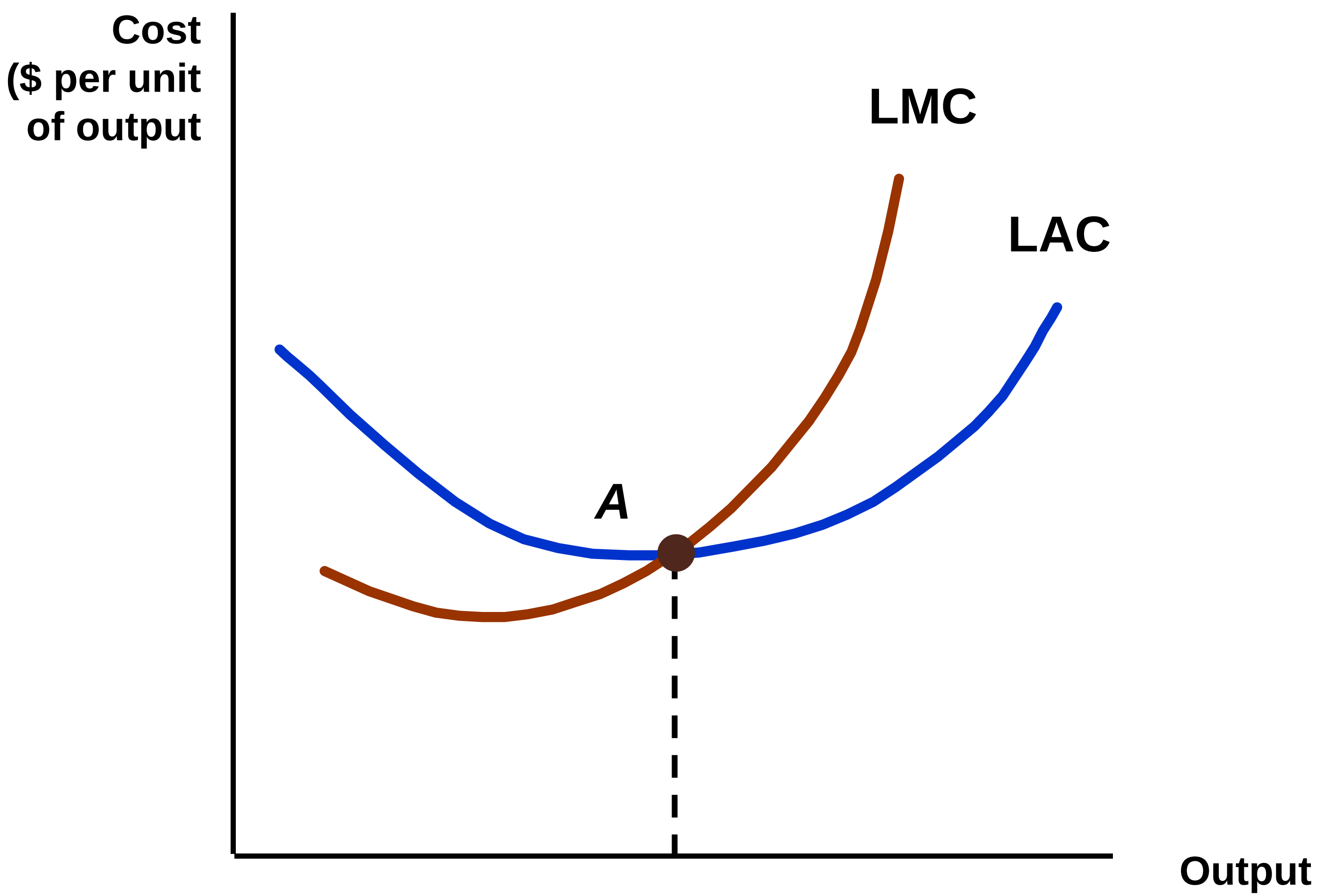
Long Run Versus Short Run Cost Curves

- In the long run:
  - Firms experience increasing and decreasing returns to scale and therefore long-run average cost is "U" shaped.
  - Source of U-shape is due to returns to scale instead of decreasing returns to scale like the short-run curve
  - Long-run marginal cost curve measures the change in long-run total costs as output is increased by 1 unit

Long Run Versus Short Run Cost Curves

- Long-run marginal cost leads long-run average cost:
  - If LMC < LAC, LAC will fall
  - If LMC > LAC, LAC will rise
  - Therefore, LMC = LAC at the minimum of LAC
- In special case where LAC is constant, LAC and LMC are equal

Long Run Average and Marginal Cost



Long Run Costs

- As output increases, firm's AC of producing is likely to decline to a point
  1. On a larger scale, workers can better specialize
  2. Scale can provide flexibility – managers can organize production more effectively
  3. Firm may be able to get inputs at lower cost if can get quantity discounts. Lower prices might lead to different input mix.

Long Run Costs

- At some point, AC will begin to increase
  1. Factory space and machinery may make it more difficult for workers to do their jobs efficiently
  2. Managing a larger firm may become more complex and inefficient as the number of tasks increase
  3. Bulk discounts can no longer be utilized. Limited availability of inputs may cause price to rise.

Economies and Diseconomies of Scale

- Economies of Scale
  - Increase in output is greater than the increase in inputs
- Diseconomies of Scale
  - Increase in output is less than the increase in inputs
- U-shaped LAC shows economies of scale for relatively low output levels and diseconomies of scale for higher levels

Production with Two Outputs – Economies of Scope

- Many firms produce more than one product and those products are closely linked
- Examples:
  - Chicken farm--poultry and eggs
  - Automobile company--cars and trucks
  - University--teaching and research

Production with Two Outputs – Economies of Scope

- Advantages

1. Both use capital and labor
2. The firms share management resources
3. Both use the same labor skills and types of machinery

Production with Two Outputs – Economies of Scope

- The **degree of economies of scope (SC)** can be measured by percentage of cost saved producing two or more products jointly:

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}$$

  - $C(q_1)$ is the cost of producing q_1
  - $C(q_2)$ is the cost of producing q_2
  - $C(q_1,q_2)$ is the joint cost of producing both products

Chapter 8

Profit Maximization and Supply Behavior

Profit Maximization

- A basic assumption for firm's rational behavior
- Do firms really maximize profits?
- The agency problem for CEOs
- The presence of not-for-profit organizations (hospitals and universities)
- Alchian's survival test theory: a strong argument for profit maximization as an approximation

Economic Profit

- A firm uses inputs $j = 1\ldots,m$ to make products $i = 1,\ldots n$.
- Output levels are $y_1,\ldots,y_n$.
- Input levels are $x_1,\ldots,x_m$.
- Product prices are $p_1,\ldots,p_n$.
- Input prices are $w_1,\ldots,w_{m.}$

Perfectly Competitive Markets

- Price Taking
  - The individual firm sells a very small share of the total market output and, therefore, cannot influence market price
  - Each firm takes market price as given – price taker
  - The individual consumer buys too small a share of industry output to have any impact on market price

The Competitive Firm

- The competitive firm takes all output prices $p_1,\ldots,p_n$ and all input prices $w_1,\ldots,w_m$ as given constants.

Economic Profit

- The economic profit generated by the production plan $(x_1,\ldots,x_m,y_1,\ldots,y_n)$ is

$$\Pi = p_1 y_1 + \cdots + p_n y_n - w_1 x_1 - \cdots w_m x_m.$$

Economic Profit

- How do we value a firm?
- Suppose the firm's stream of periodic economic profits is $\Pi_0, \Pi_1, \Pi_2, \ldots$ and r is the rate of interest.
- Then the present-value of the firm's economic profit stream is

$$\mathbf{PV} = \Pi_0 + \frac{\Pi_1}{1+r} + \frac{\Pi_2}{(1+r)^2} + \cdots$$

Marginal Revenue, Marginal Cost, and Profit Maximization

- We can study profit maximizing output for any firm, whether perfectly competitive or not
  - Profit (π) = Total Revenue - Total Cost
  - If q is output of the firm, then total revenue is price of the good times quantity
  - Total Revenue (R) = Pq

Marginal Revenue, Marginal Cost, and Profit Maximization

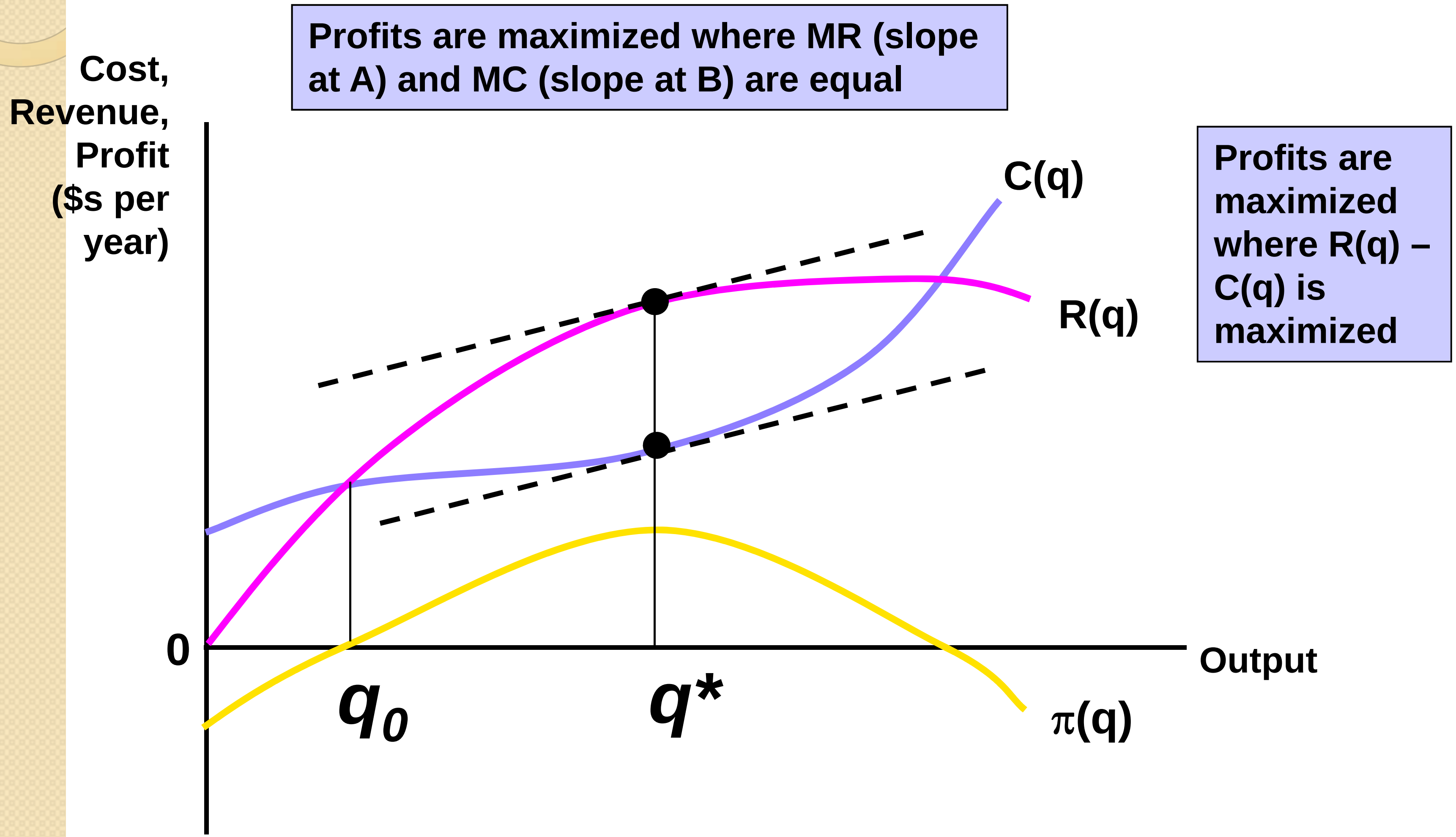
- Costs of production depends on output
  - Total Cost (C) = C(q)
- Profit for the firm, π, is difference between revenue and costs

$$\boxed{\pi(q) = R(q) - C(q)}$$

Marginal Revenue, Marginal Cost, and Profit Maximization

- Firm selects output to maximize the difference between revenue and cost
- We can graph the total revenue and total cost curves to show maximizing profits for the firm
- Distance between revenues and costs show profits

Profit Maximization – Short Run



Marginal Revenue, Marginal Cost, and Profit Maximization

- Profit is maximized at the point at which an additional increment to output leaves profit unchanged

$$\pi(q) = R(q) - C(q)$$

$$\frac{d\pi}{dq} = \frac{dR}{dq} - \frac{dC}{dq} = 0$$

$$= MR - MC = 0$$

$$MR = MC$$

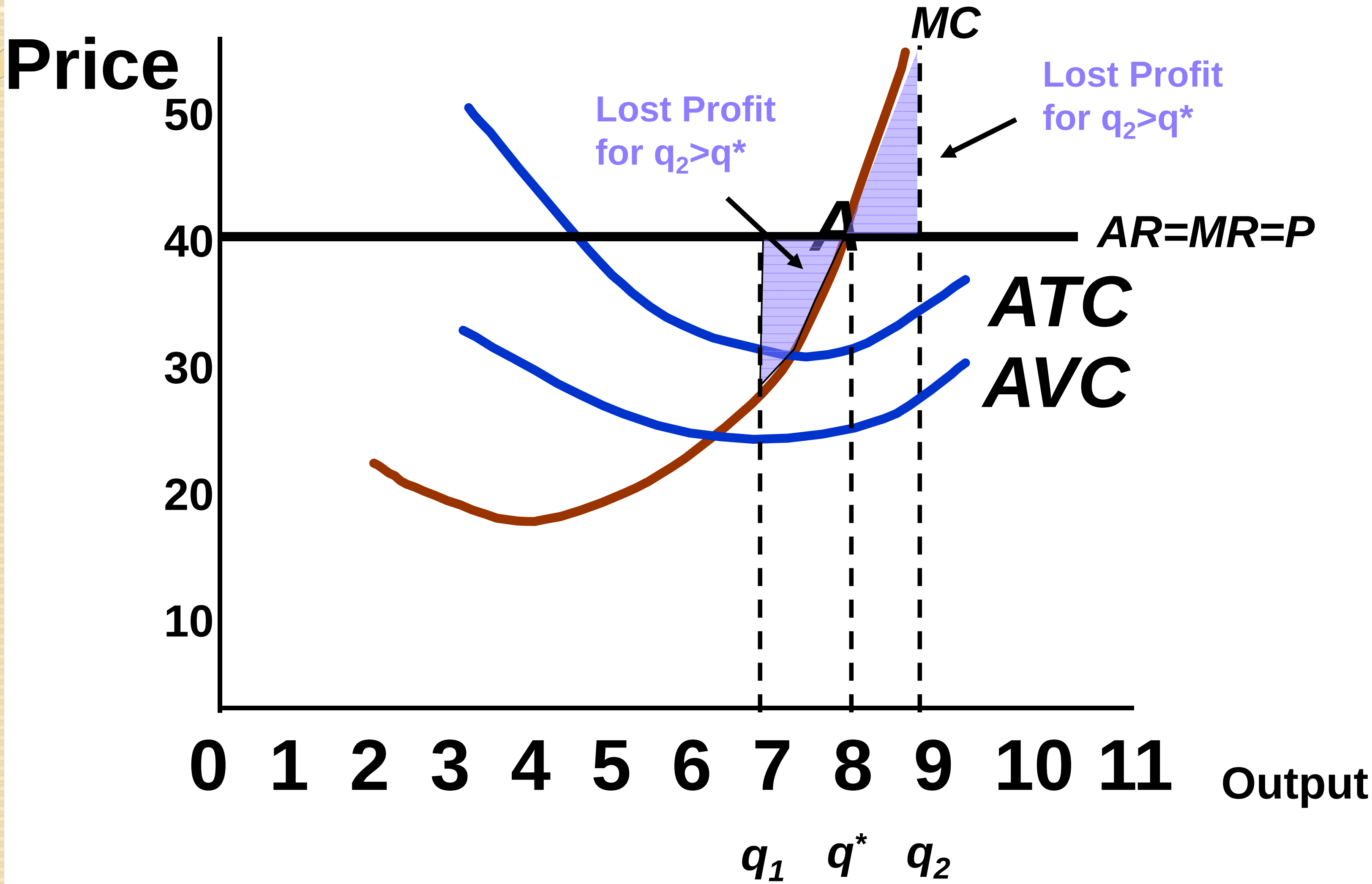
Choosing Output: Short Run

- We will combine revenue and costs with demand to determine profit maximizing output decisions
- In the short run, capital is fixed and firm must choose levels of variable inputs to maximize profits
- We can look at the graph of MR, MC, ATC and AVC to determine profits

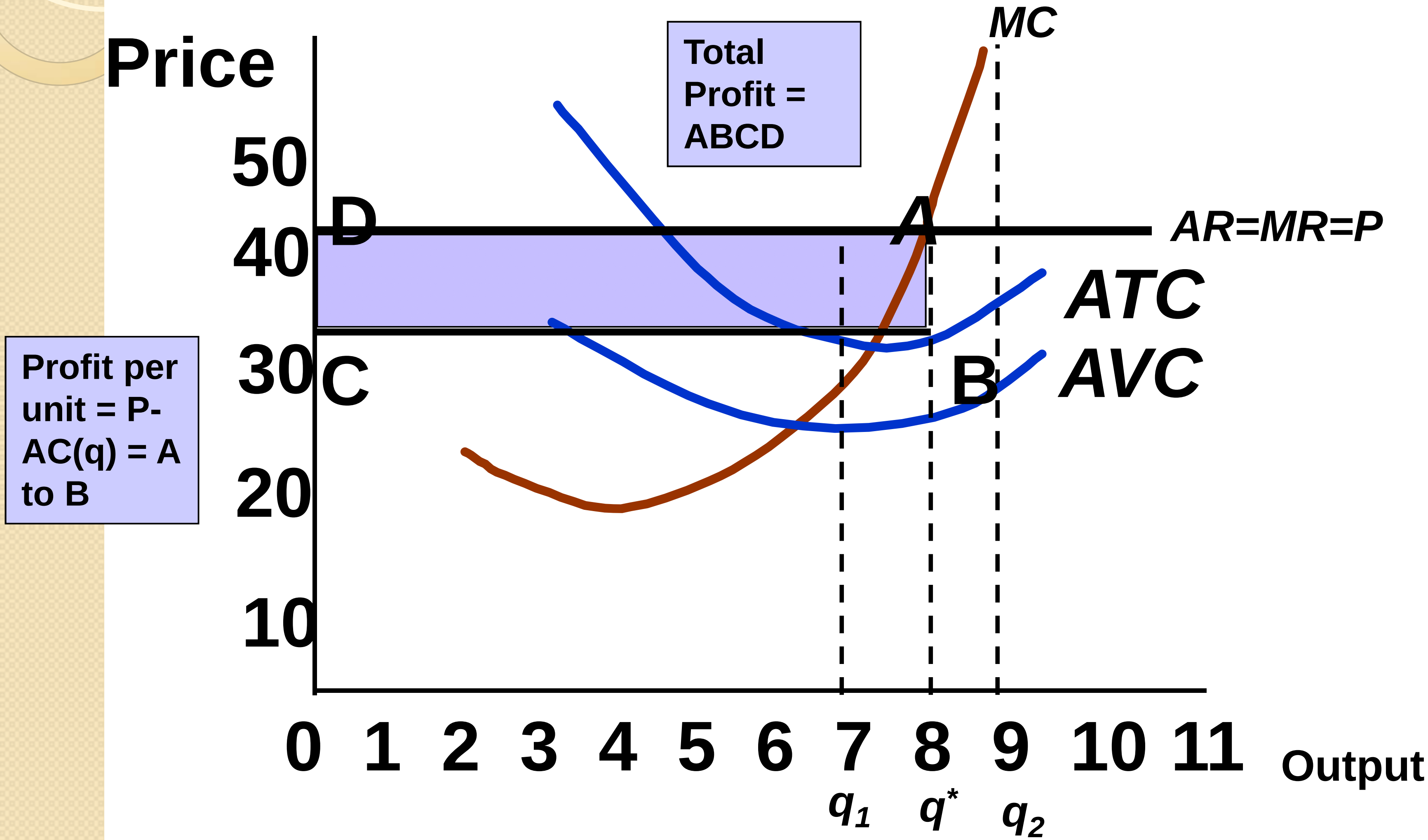
Choosing Output: Short Run

- The point where MR = MC, the profit maximizing output is chosen
  - MR = MC at quantity, q^*, of 8
  - At a quantity less than 8, MR > MC, so more profit can be gained by increasing output
  - At a quantity greater than 8, MC > MR, increasing output will decrease profits

A Competitive Firm



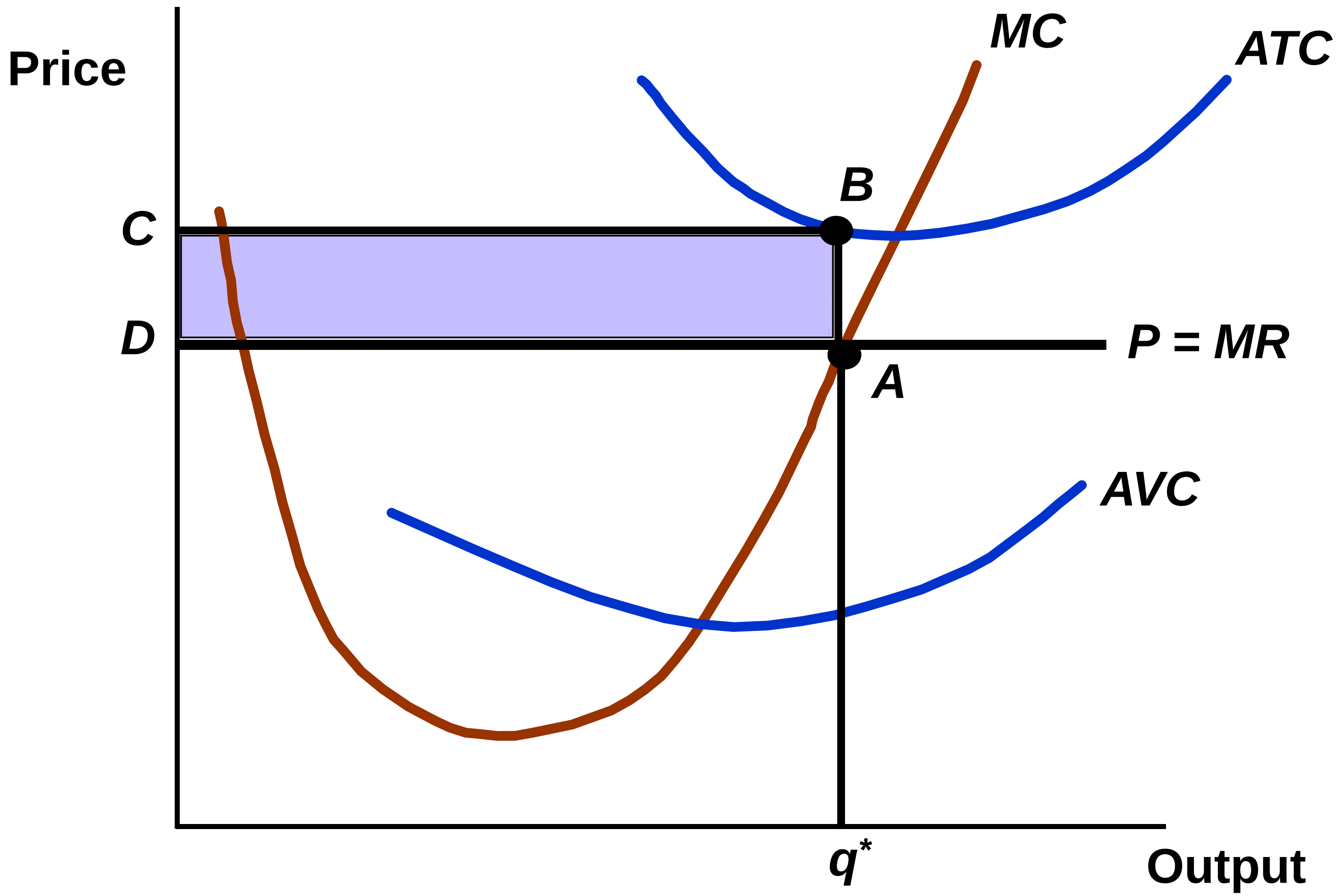
A Competitive Firm – Positive Profits



The Competitive Firm

- A firm does not have to make profits
- It is possible a firm will incur losses if the P < AC for the profit maximizing quantity
  - Still measured by profit per unit times quantity
  - Profit per unit is negative (P – AC < 0)

A Competitive Firm – Losses



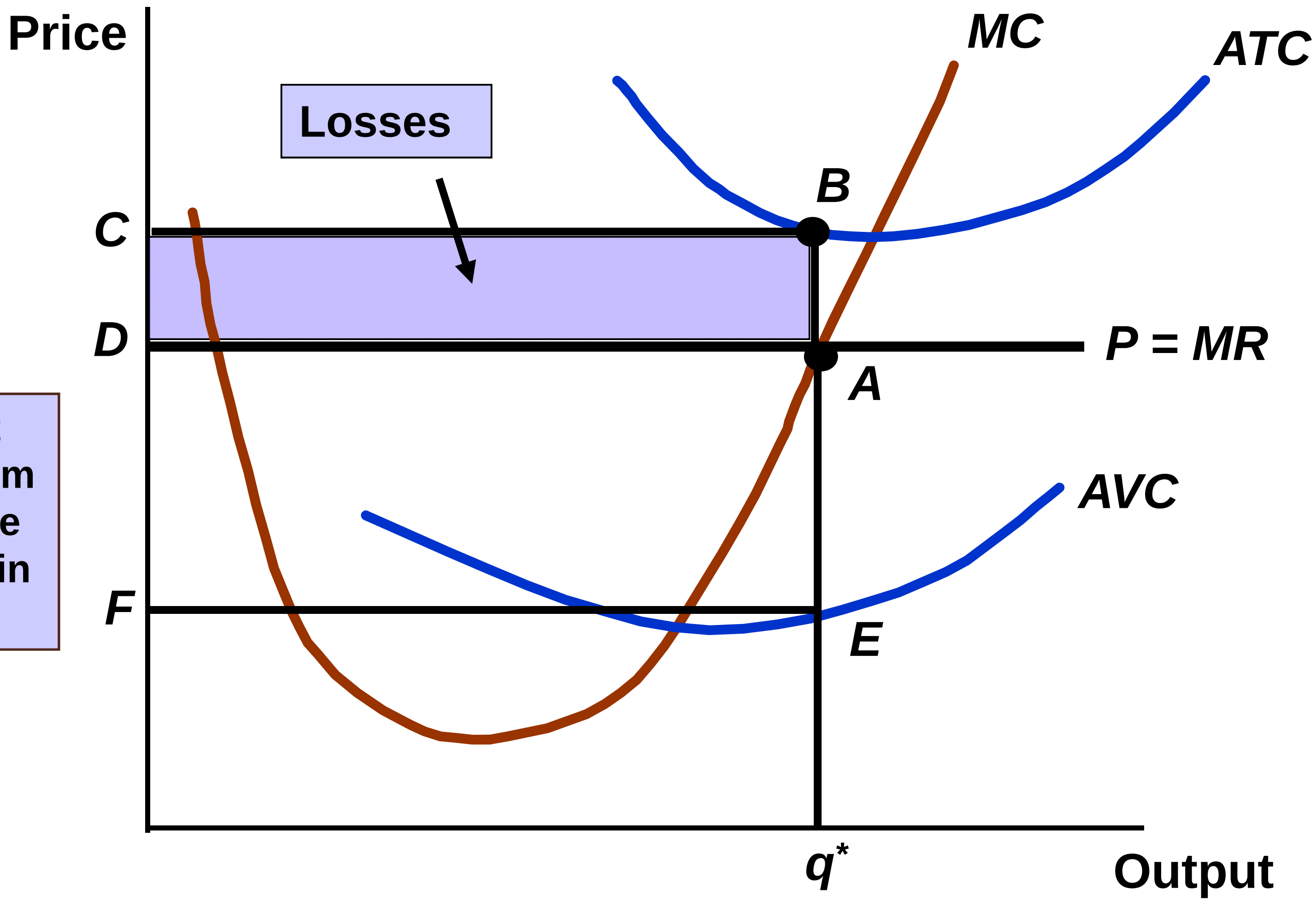
Short Run Production

- Why would a firm produce at a loss?
  - Might think price will increase in near future
  - Shutting down and starting up could be costly
- Firm has two choices in short run
  - Continue producing
  - Shut down temporarily
  - Will compare profitability of both choices

Short Run Production

- When should the firm shut down?
  - If $AVC < P < ATC$, the firm should continue producing in the short run
    - Can cover all of its variable costs and some of its fixed costs
  - If $AVC > P < ATC$, the firm should shut down
    - Cannot cover its variable costs or any of its fixed costs

A Competitive Firm – Losses

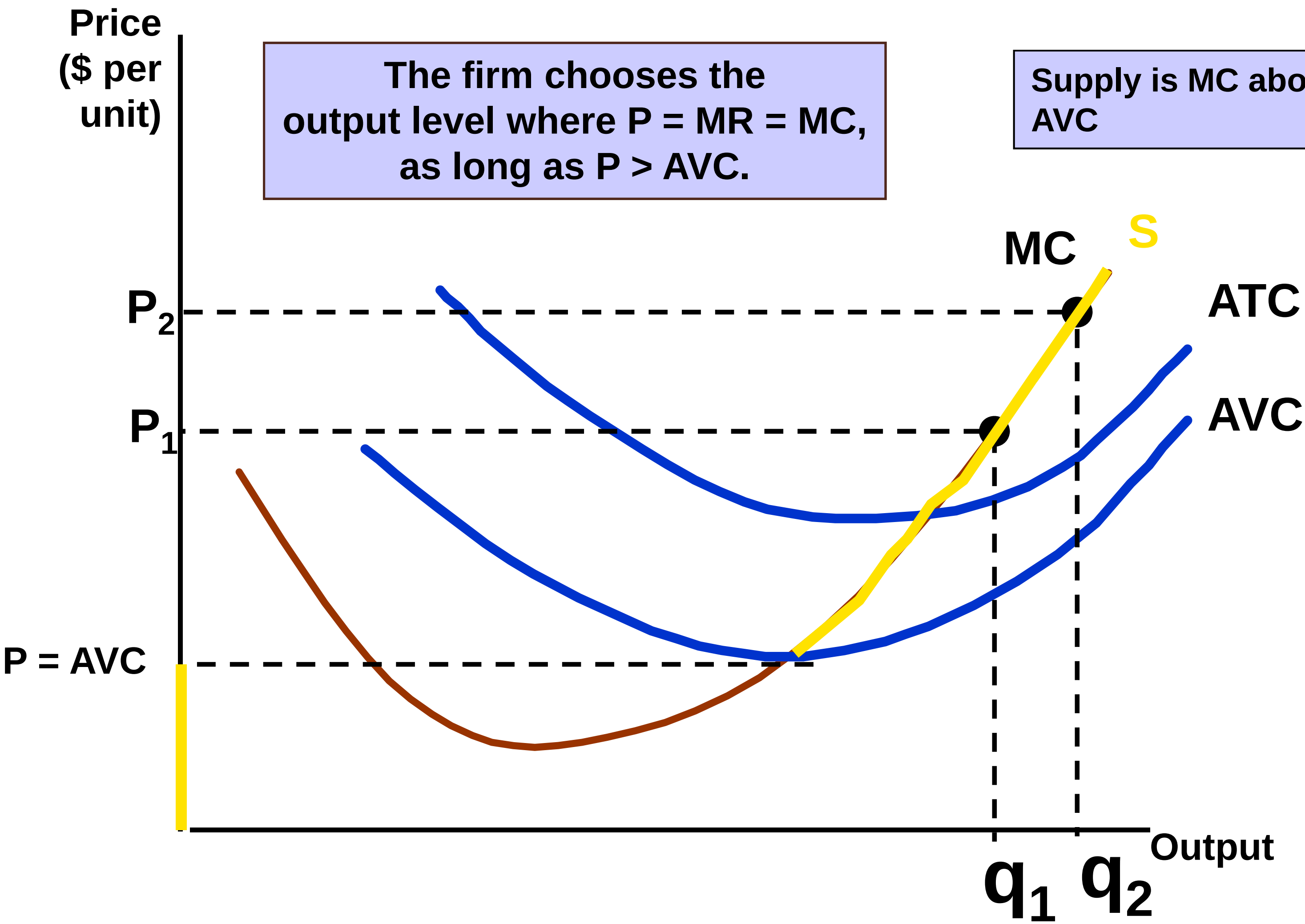


P < ATC but
➢AVC so firm will continue to produce in short run

Competitive Firm – Short Run Supply

- Supply curve tells how much output will be produced at different prices
- Competitive firms determine quantity to produce where P = MC
  - Firm shuts down when P < AVC
- Competitive firms' supply curve is portion of the marginal cost curve above the AVC curve

A Competitive Firm's Short-Run Supply Curve



A Competitive Firm's Short-Run Supply Curve

- Supply is upward sloping due to diminishing returns
- Higher price compensates the firm for the higher cost of additional output and increases total profit because it applies to all units

Derivation of the Supply Curve

$$Max\ \pi(q) = pq - C(q)$$

$$s.t.\ \pi \geq -F \Rightarrow p \geq AVC$$

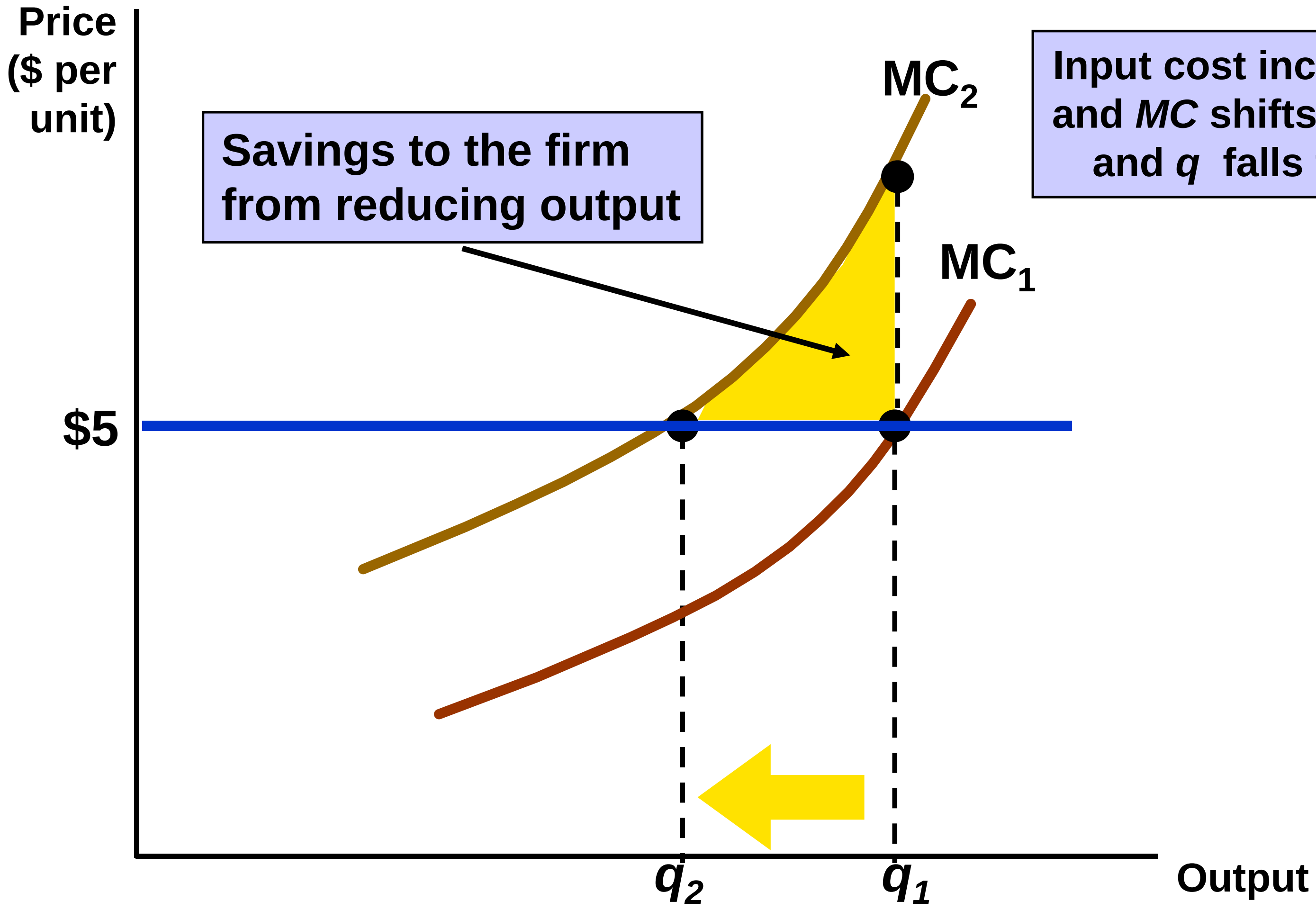
$$d\pi / dp = p - C'(q) = 0 \Rightarrow p = C'(q)$$

$$dp / dq = C''(q) > 0$$

A Competitive Firm's Short-Run Supply Curve

- Over time, prices of product and inputs can change
- How does the firm's output change in response to a change in the price of an input?
  - We can show an increase in marginal costs and the change in the firm's output decisions

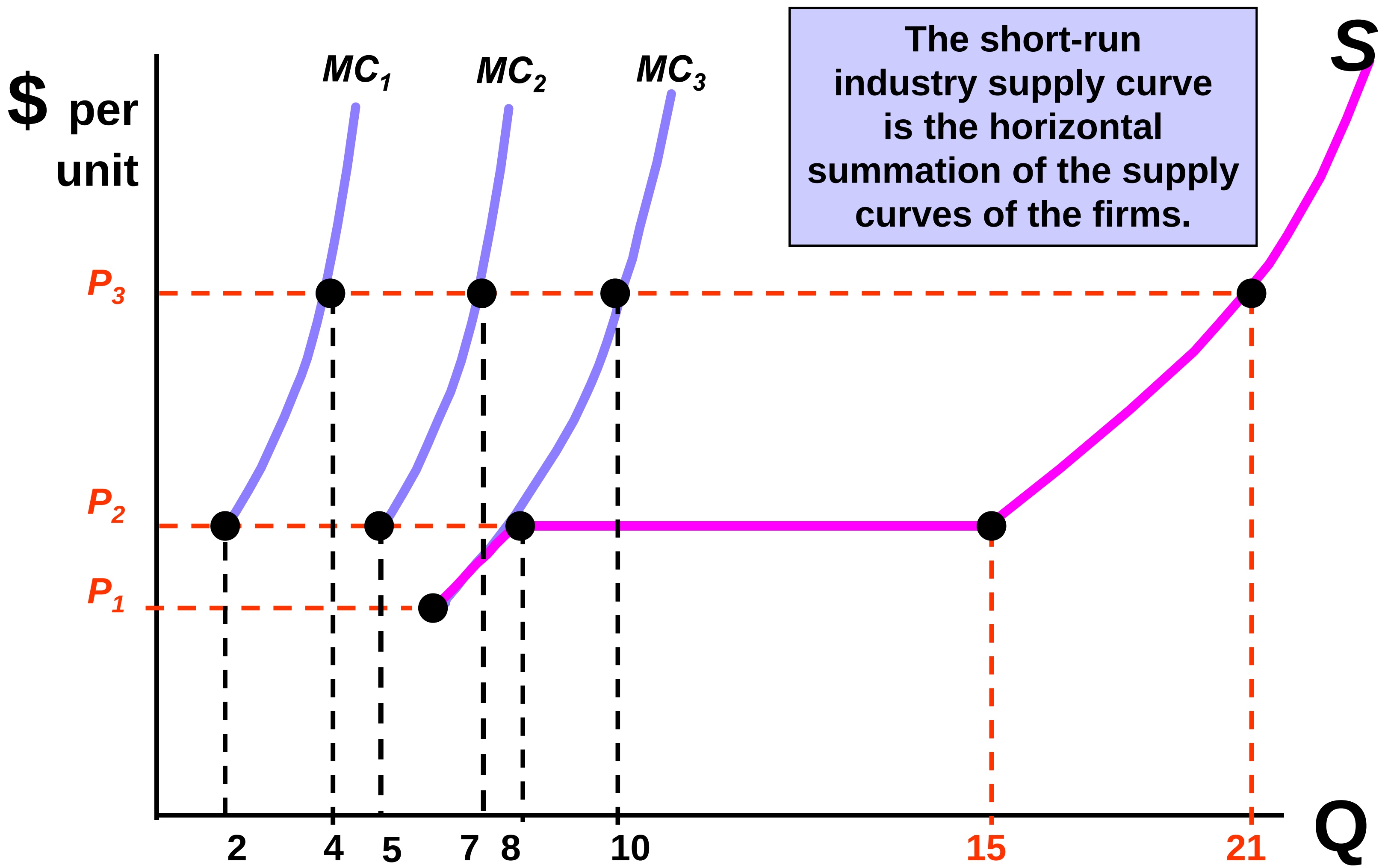
The Response of a Firm to a Change in Input Price



Short-Run Market Supply Curve

- Shows the amount of product the whole market will produce at given prices
- Is the sum of all the individual producers in the market
- We can show graphically how we can sum the supply curves of individual producers

Industry Supply in the Short Run



Elasticity of Market Supply

- Elasticity of Market Supply
  - Measures the sensitivity of industry output to market price
  - The percentage change in quantity supplied, Q, in response to 1-percent change in price

$$E_s = (\Delta Q / Q)/(\Delta P / P)$$

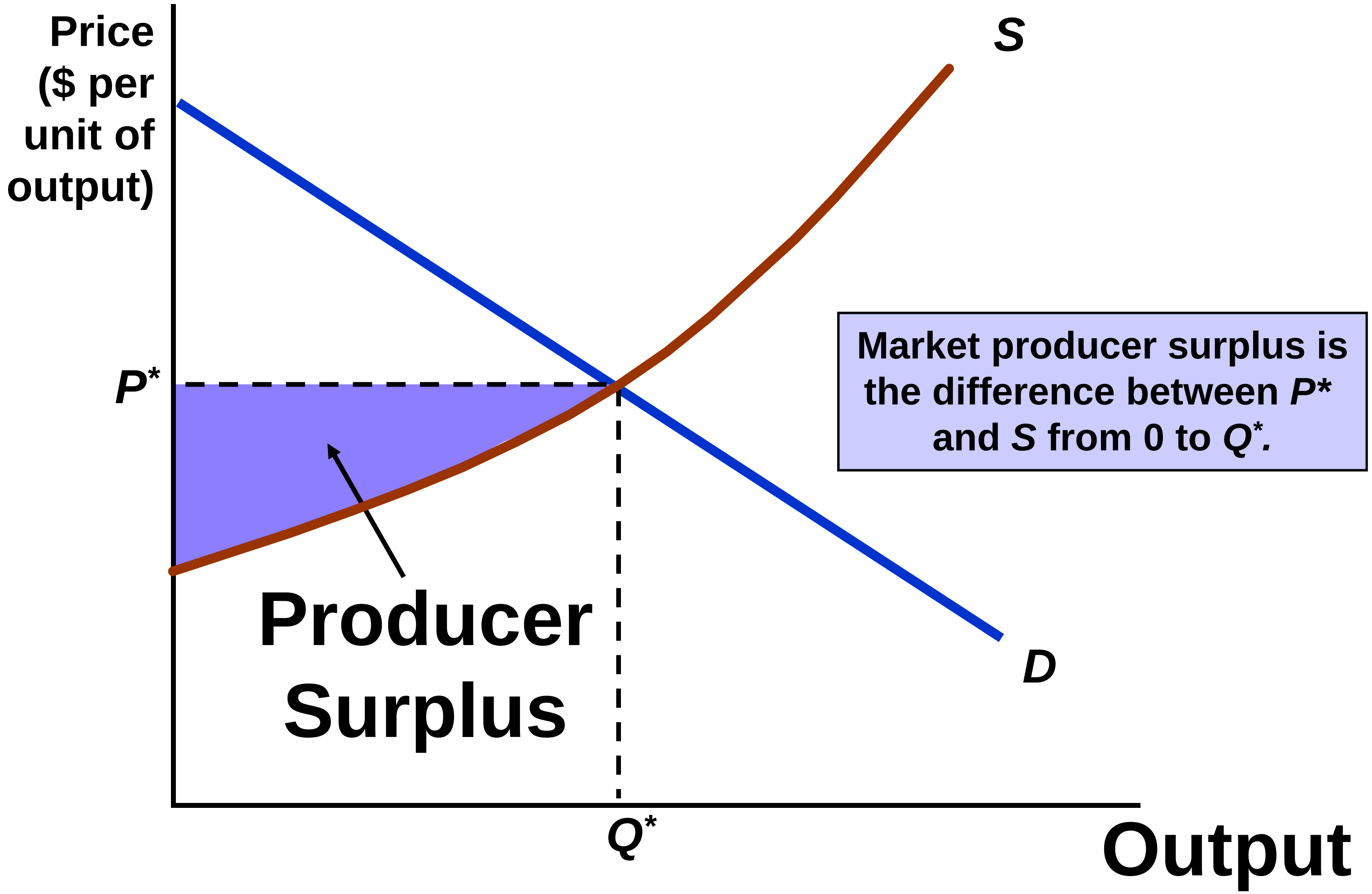
Elasticity of Market Supply

- When MC increases rapidly in response to increases in output, elasticity is low
- When MC increases slowly, supply is relatively elastic
- **Perfectly inelastic** short-run supply arises when the industry's plant and equipment are so fully utilized that new plants must be built to achieve greater output
- **Perfectly elastic** short-run supply arises when marginal costs are constant

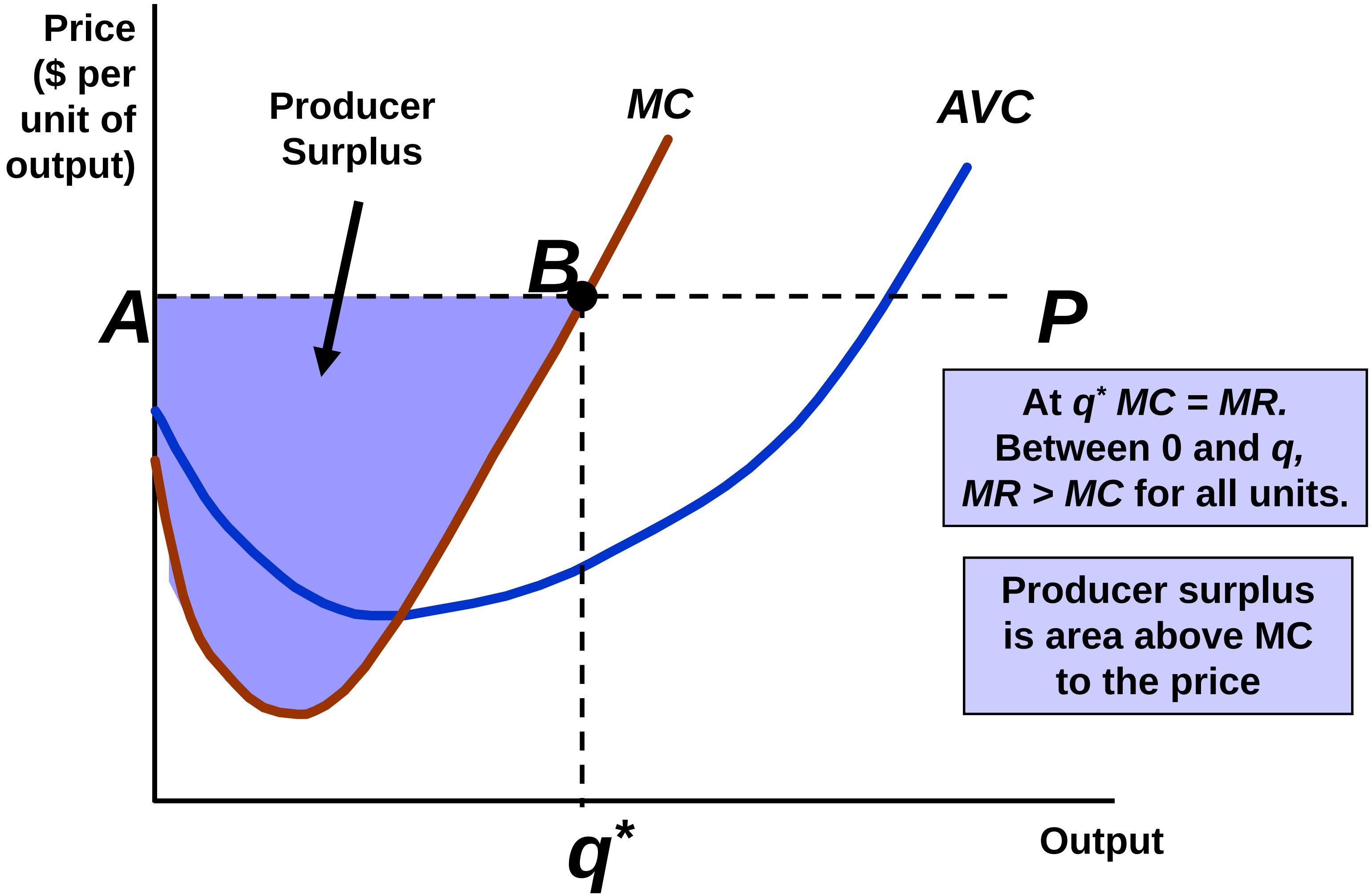
Producer Surplus in the Short Run

- Price is greater than MC on all but the last unit of output
- Therefore, surplus is earned on all but the last unit
- The **producer surplus** is the sum over all units produced of the difference between the market price of the good and the marginal cost of production
- Area above supply curve to the market price

Producer Surplus for a Market



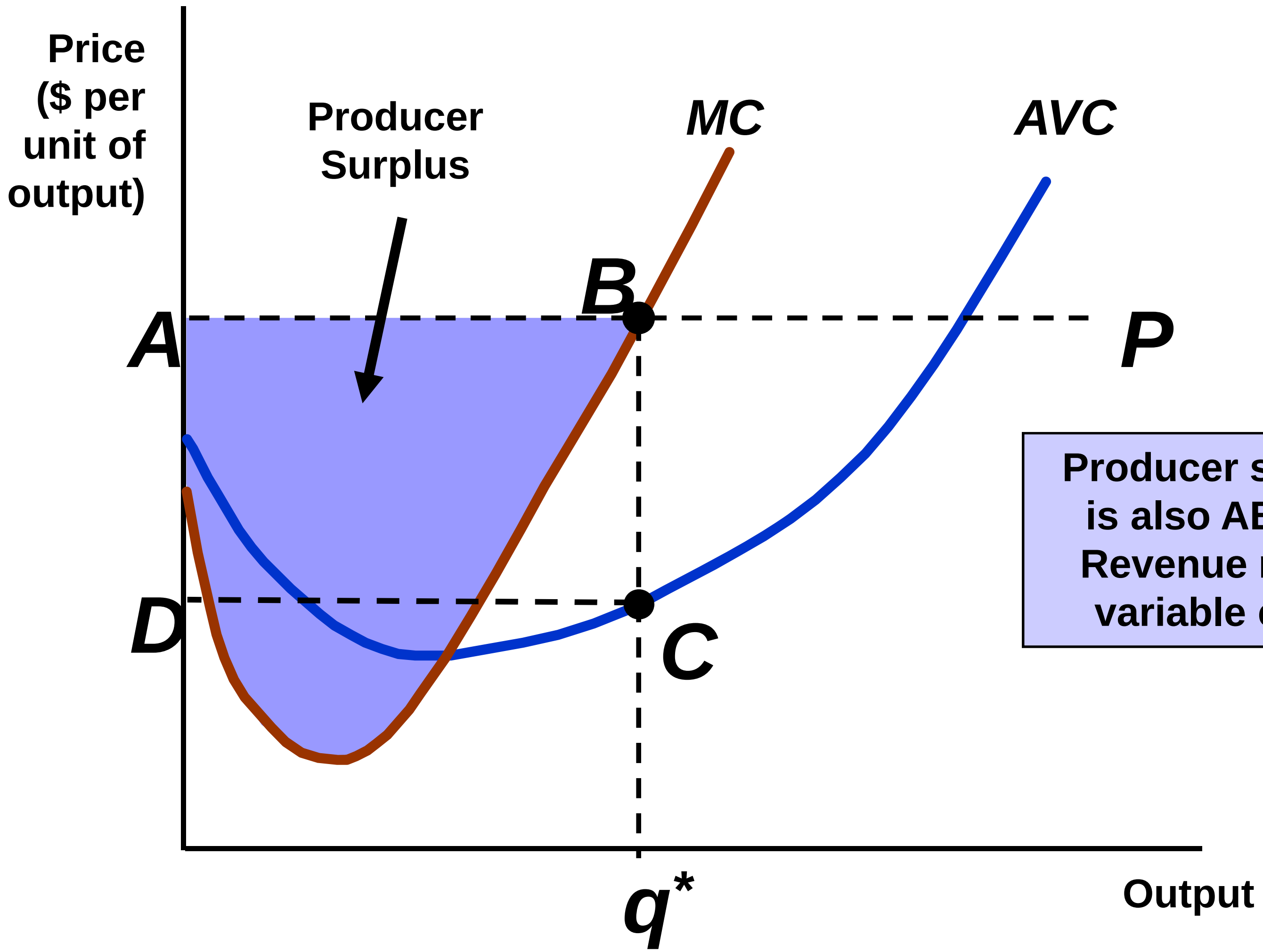
Producer Surplus for a Firm



The Short-Run Market Supply Curve

- Sum of MC from 0 to q*, it is the sum of the total variable cost of producing q*
- Producer Surplus can be defined as the difference between the firm's revenue and its total variable cost
- We can show this graphically by the rectangle ABCD
  - Revenue (0ABq*) minus variable cost (0DCq*)

Producer Surplus for a Firm



Producer surplus is also ABCD = Revenue minus variable costs

Producer Surplus Versus Profit

- Profit is revenue minus total cost (not just variable cost)
- When fixed cost is positive, producer surplus is greater than profit

$$\text{Producer Surplus} = PS = R - VC$$

$$\text{Profit} = \pi = R - VC - FC$$

Producer Surplus Versus Profit

- Costs of production determine magnitude of producer surplus
  - Higher cost firms have less producer surplus
  - Lower cost firms have more producer surplus
  - Adding up surplus for all producers in the market given total market producer surplus
  - Area below market price and above supply curve

Choosing Output in the Long Run

- In short run, one or more inputs are fixed
  - Depending on the time, it may limit the flexibility of the firm
- ***In the long run, a firm can alter all its inputs, including the size of the plant***
- We assume free entry and free exit
  - No legal restrictions or extra costs

Economic Profit

- Suppose the firm is in a short-run circumstance in which $x_2 \equiv \tilde{x}_2$.
- Its short-run production function is

$$y = f(x_1, \tilde{x}_2).$$

Profit Max: Another Look

- Suppose the firm is in a short-run circumstance in which $x_2 \equiv \tilde{x}_2$.
- Its short-run production function is
$$y = f(x_1, \tilde{x}_2).$$
- The firm's fixed cost is $FC = w_2\tilde{x}_2$ and its profit function is
$$\Pi = py - w_1x_1 - w_2\tilde{x}_2.$$

Short-Run Iso-Profit Lines

- A $\$\Pi$ iso-profit line contains all the production plans that provide a profit level $\$\Pi$.
- A $\$\Pi$ iso-profit line's equation is

$$\Pi \equiv py - w_1 x_1 - w_2 \tilde{x}_2 .$$

Short-Run Iso-Profit Lines

- A $\$\Pi$ iso-profit line contains all the production plans that yield a profit level of $\$\Pi$.
- The equation of a $\$\Pi$ iso-profit line is

$$\Pi \equiv py - w_1x_1 - w_2\tilde{x}_2.$$

- I.e.

$$y = \frac{w_1}{p}x_1 + \frac{\Pi + w_2\tilde{x}_2}{p}.$$

Short-Run Iso-Profit Lines

$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}$$

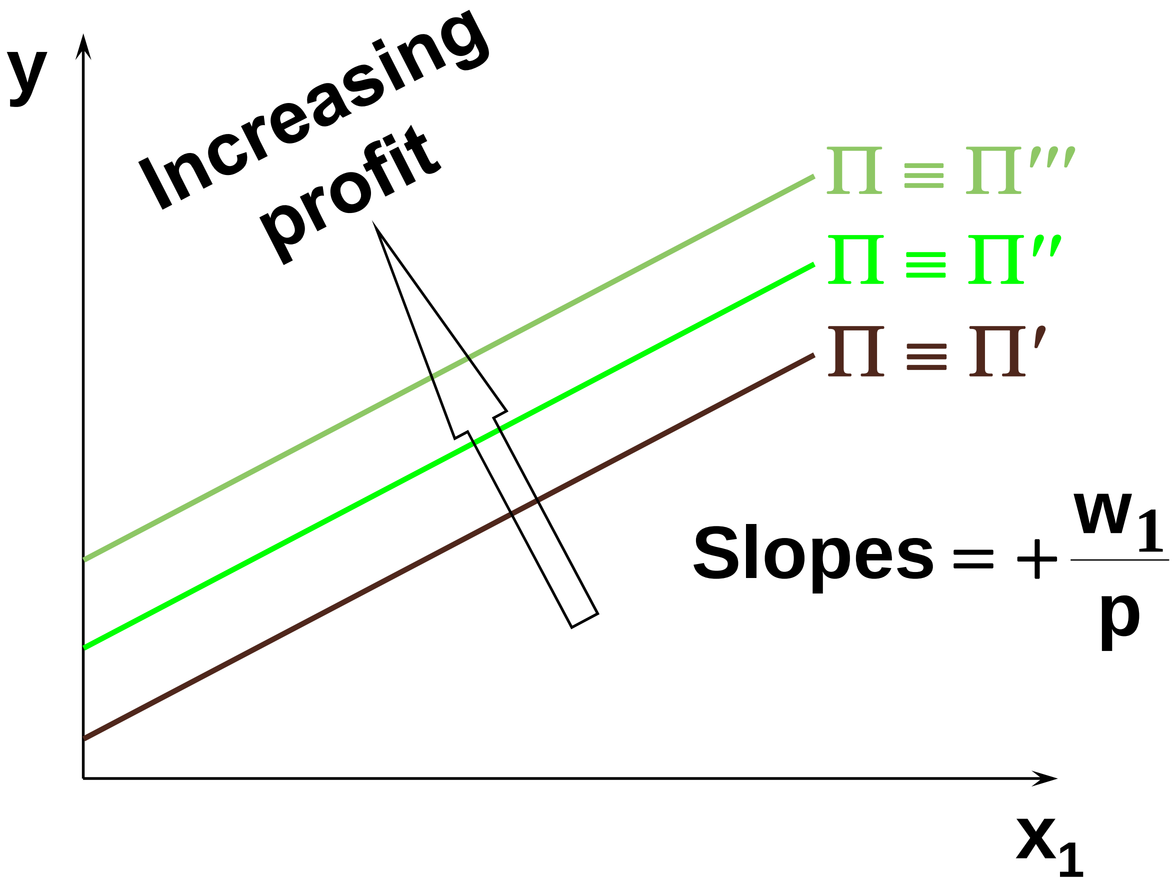
has a slope of

$$+\frac{w_1}{p}$$

and a vertical intercept of

$$\frac{\Pi + w_2 \tilde{x}_2}{p}.$$

Short-Run Iso-Profit Lines



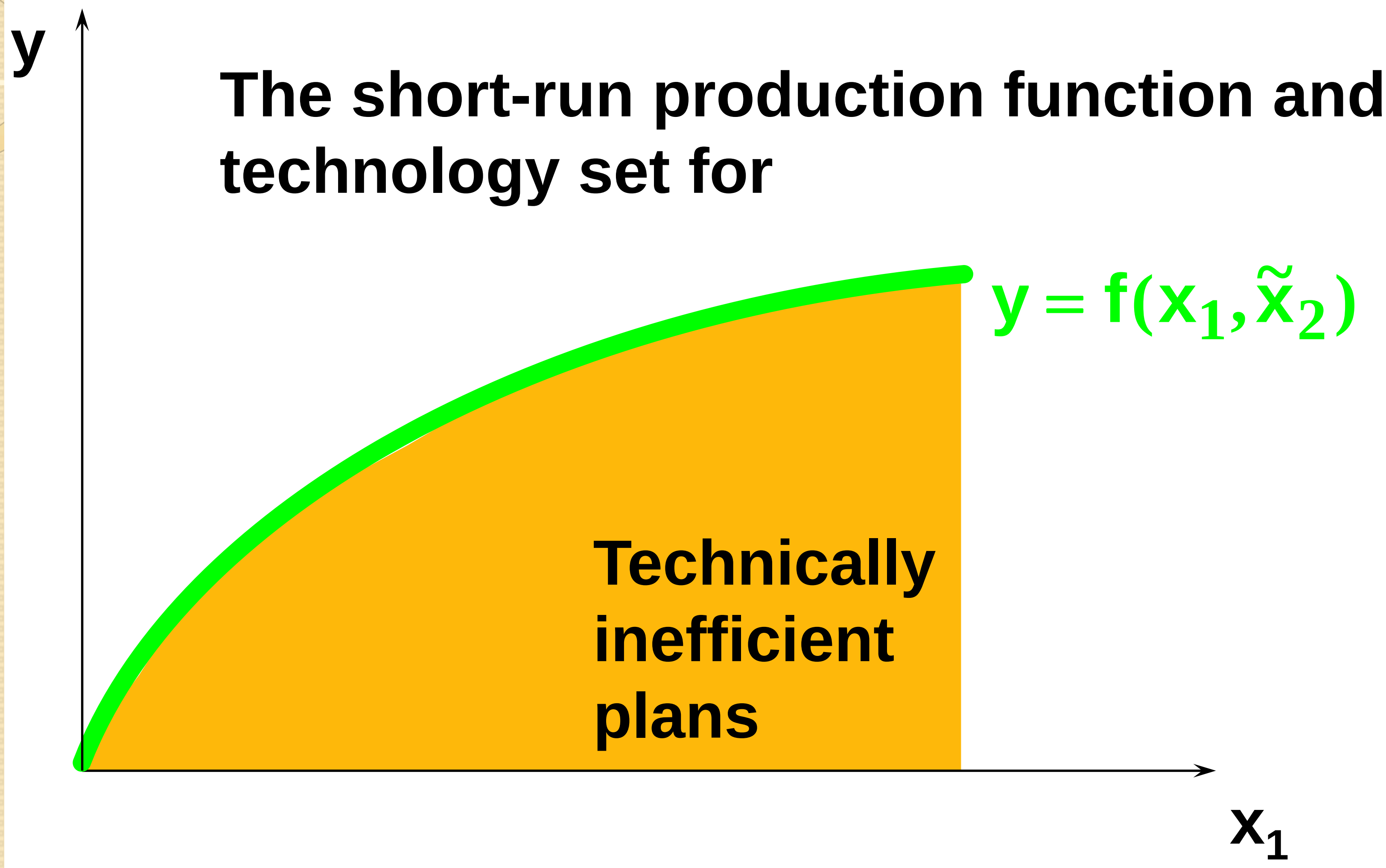
Short-Run Profit-Maximization

- The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans.
- Q: What is this constraint?

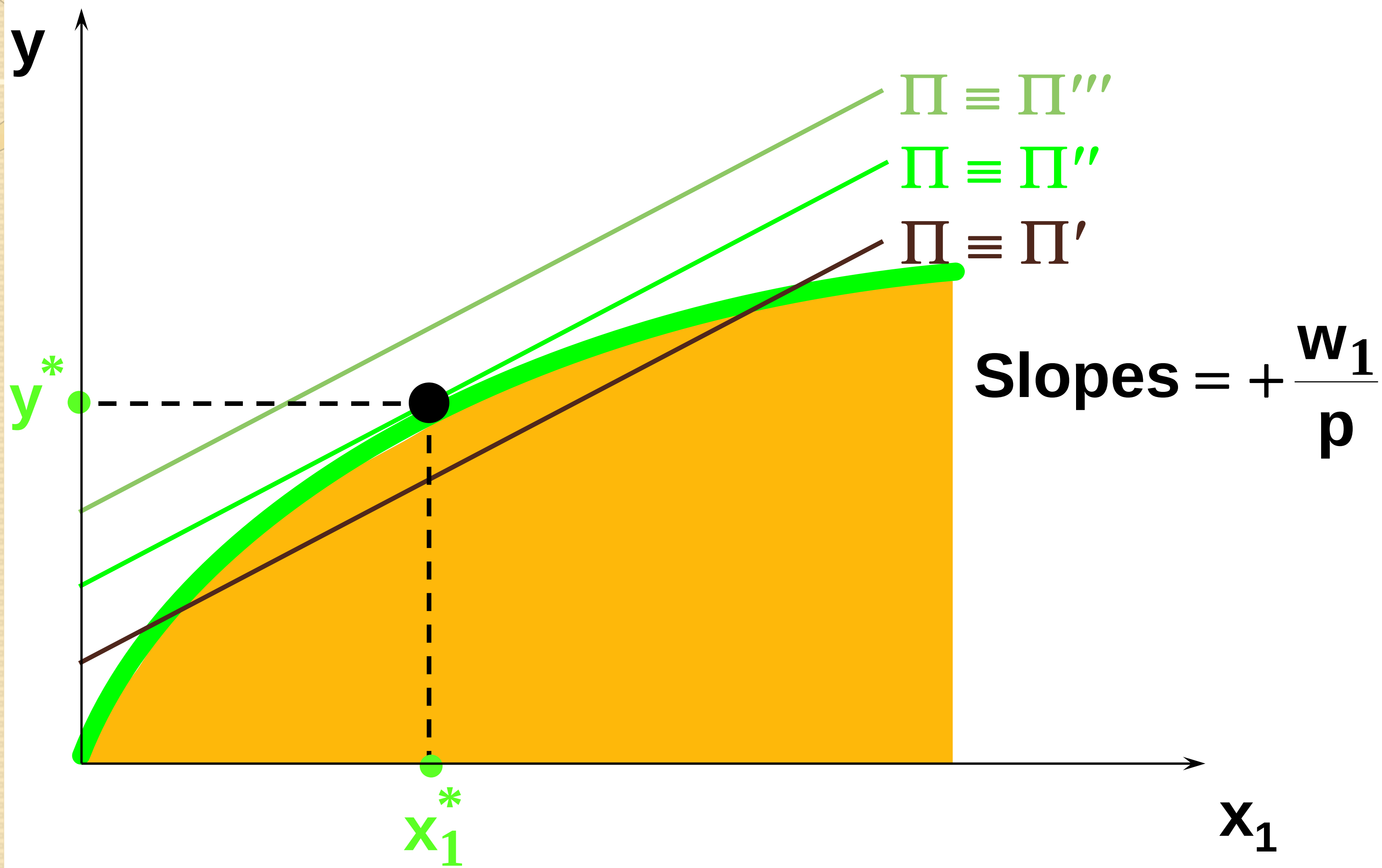
Short-Run Profit-Maximization

- The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans.
- Q: What is this constraint?
- A: The production function.

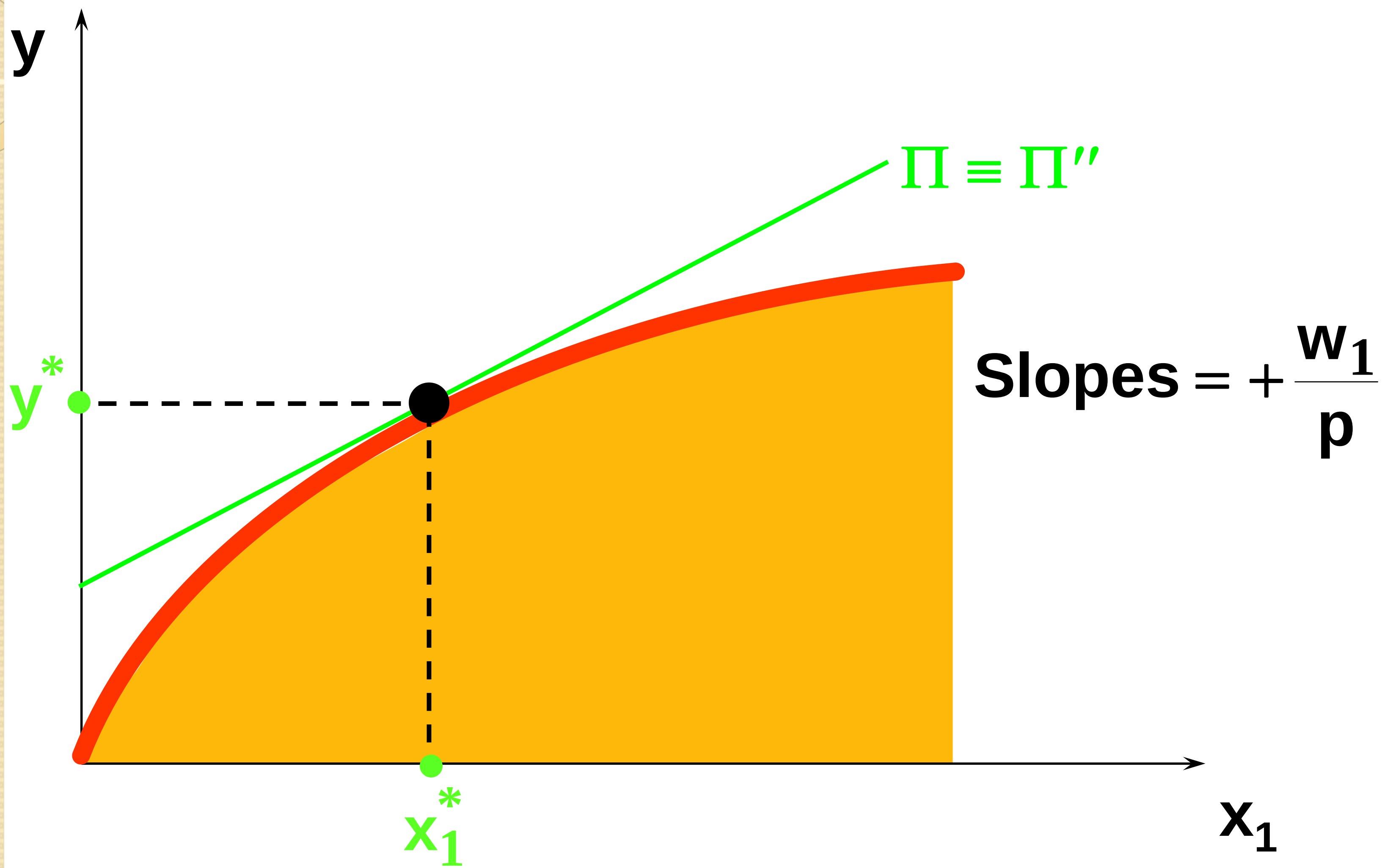
Short-Run Profit-Maximization



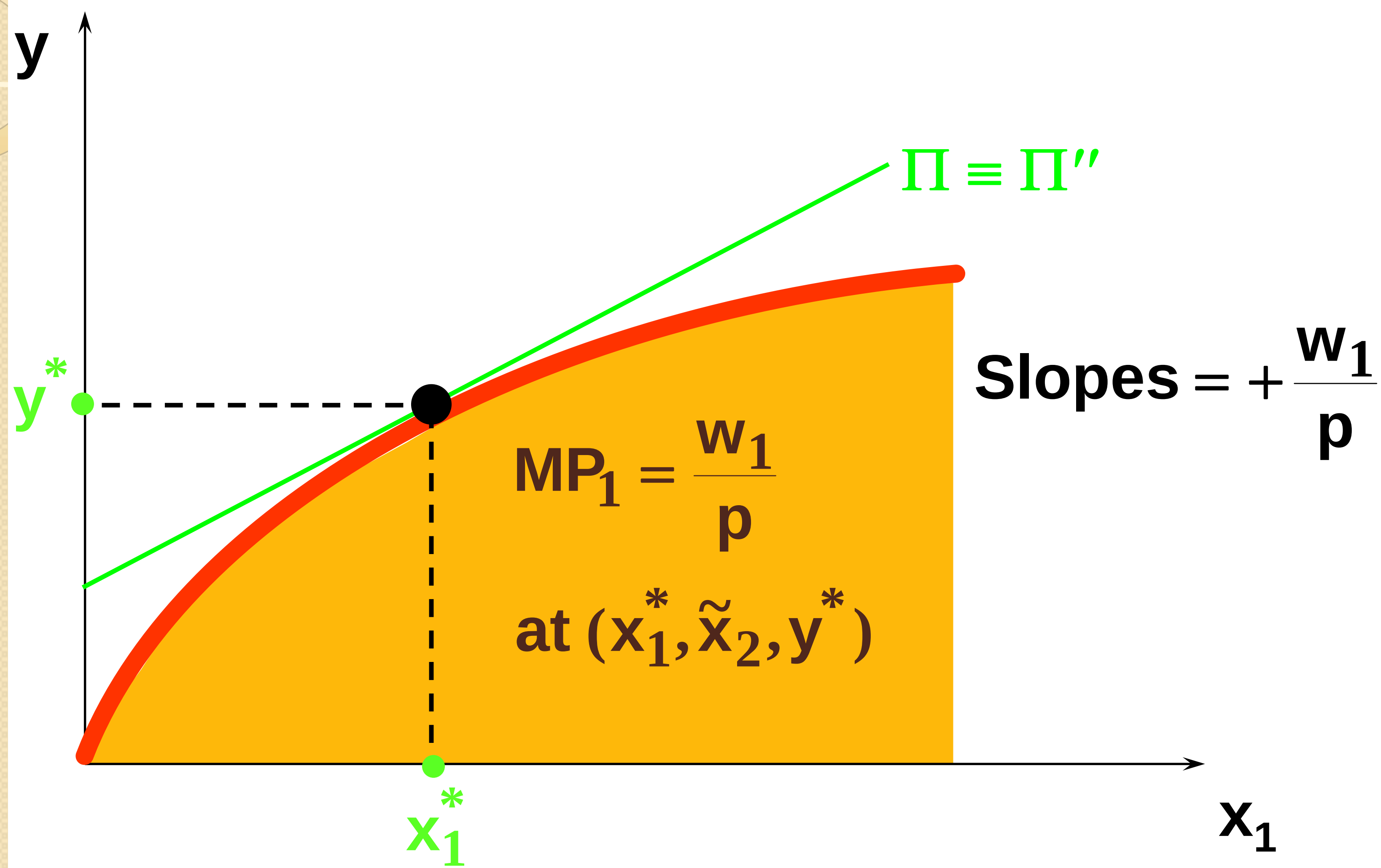
Short-Run Profit-Maximization



Short-Run Profit-Maximization



Short-Run Profit-Maximization



Short-Run Profit-Maximization

$$MP_1 = \frac{w_1}{p} \quad \Leftrightarrow \quad p \times MP_1 = w_1$$

$p \times MP_1$ is the **marginal revenue product of input 1**, the rate at which revenue increases with the amount used of input 1.

If $p \times MP_1 > w_1$ then profit increases with x_1.

If $p \times MP_1 < w_1$ then profit decreases with x_1.

Comparative Statics of Short-Run Profit-Maximization

The equation of a short-run iso-profit line is

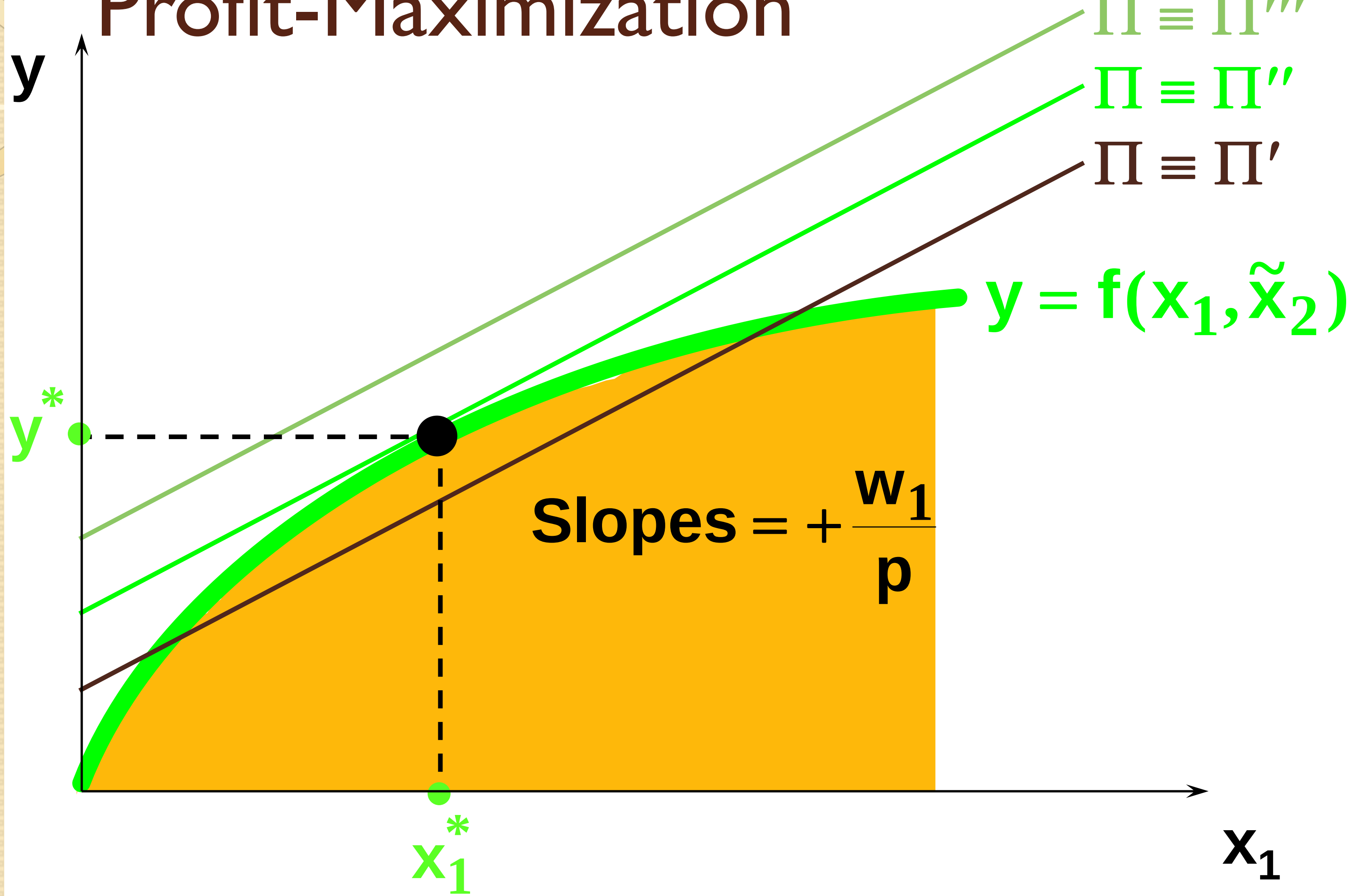
$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}$$

so an increase in w_1 causes

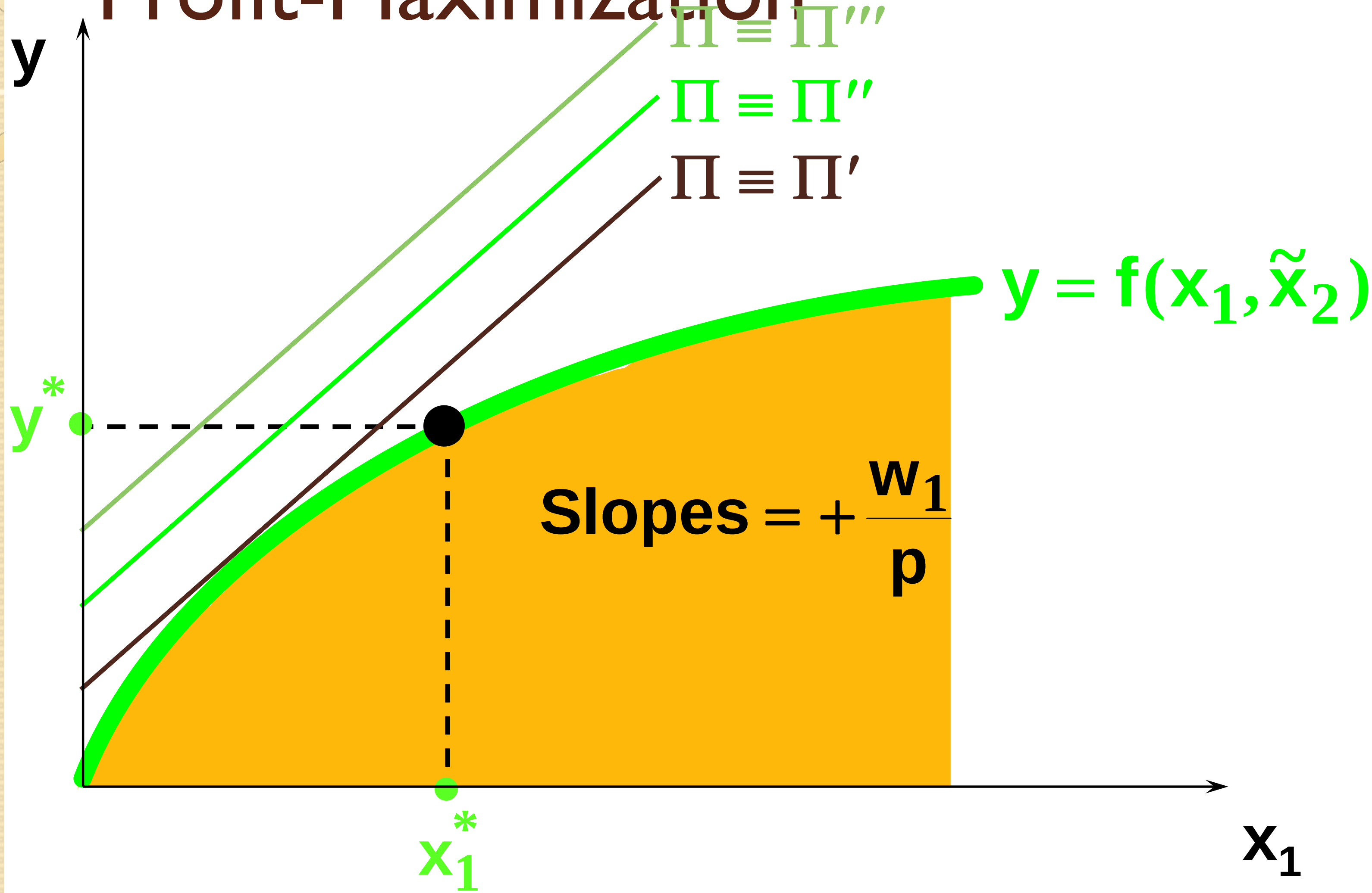
-- an increase in the slope, and

-- no change to the vertical intercept.

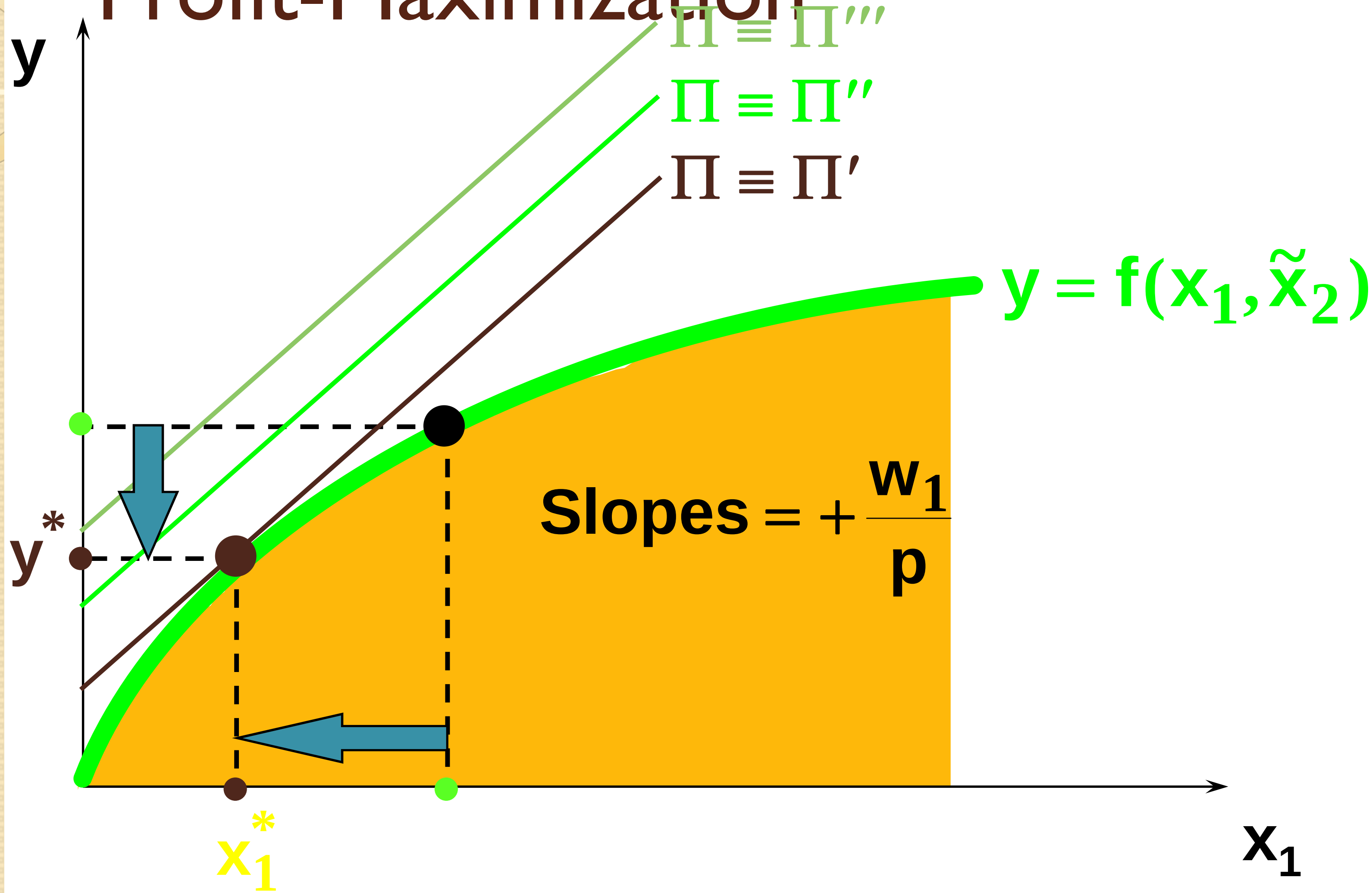
Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization

- An increase in w_1, the price of the firm's variable input, causes
  - a decrease in the firm's output level (the firm's supply curve shifts inward), and
  - a decrease in the level of the firm's variable input (the firm's demand curve for its variable input slopes downward).

Long-Run Profit-Maximization

- Now allow the firm to vary both input levels.
- Since no input level is fixed, there are no fixed costs.

Long-Run Profit-Maximization

- Both x_1 and x_2 are variable.
- Think of the firm as choosing the production plan that maximizes profits for a given value of x_2, and then varying x_2 to find the largest possible profit level.

Long-Run Profit-Maximization

The equation of a long-run iso-profit line is

$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 x_2}{p}$$

so an increase in x_2 causes

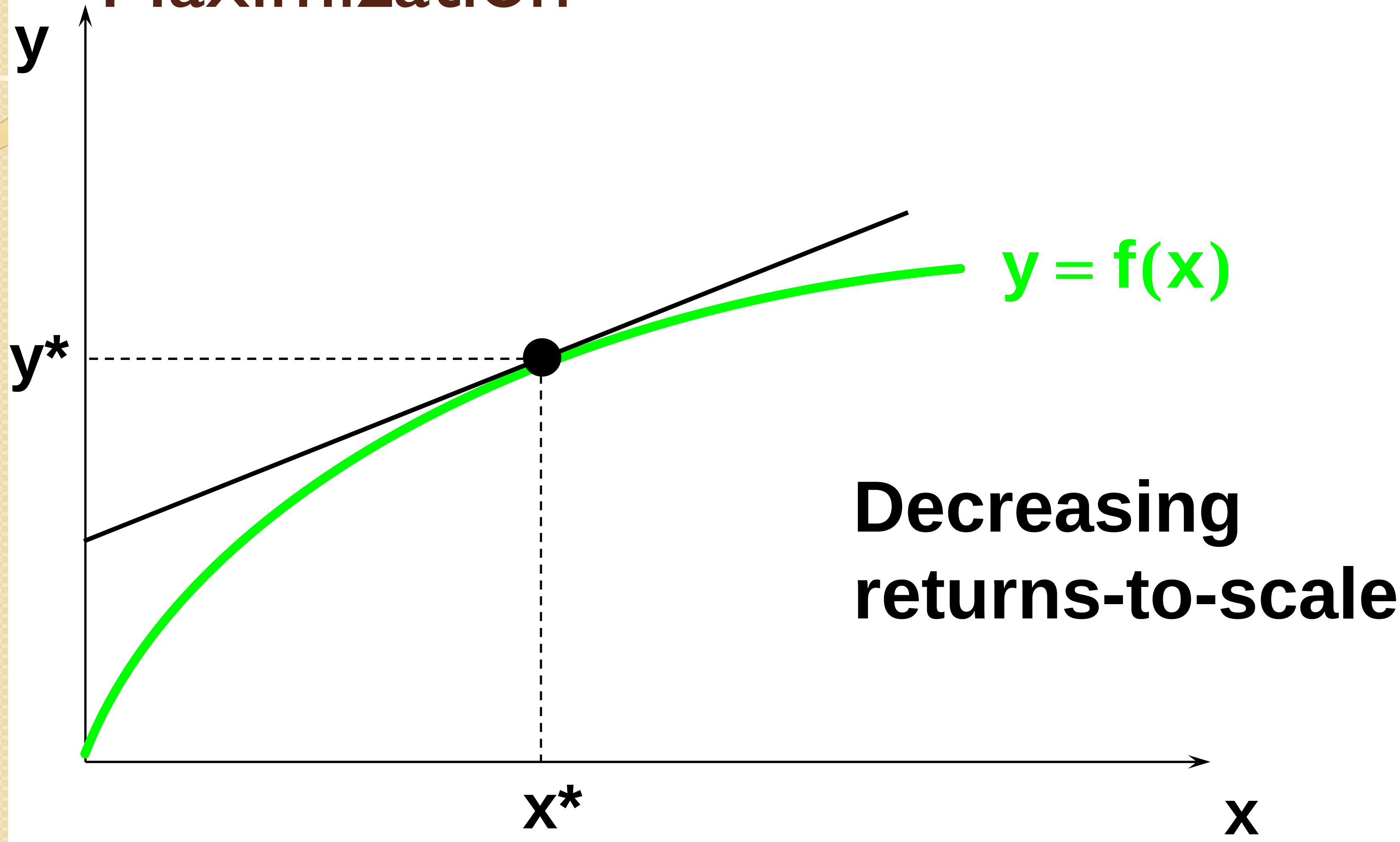
-- no change to the slope, and

-- an increase in the vertical intercept.

Returns-to-Scale and Profit-Maximization

- If a competitive firm's technology exhibits decreasing returns-to-scale then the firm has a single long-run profit-maximizing production plan.

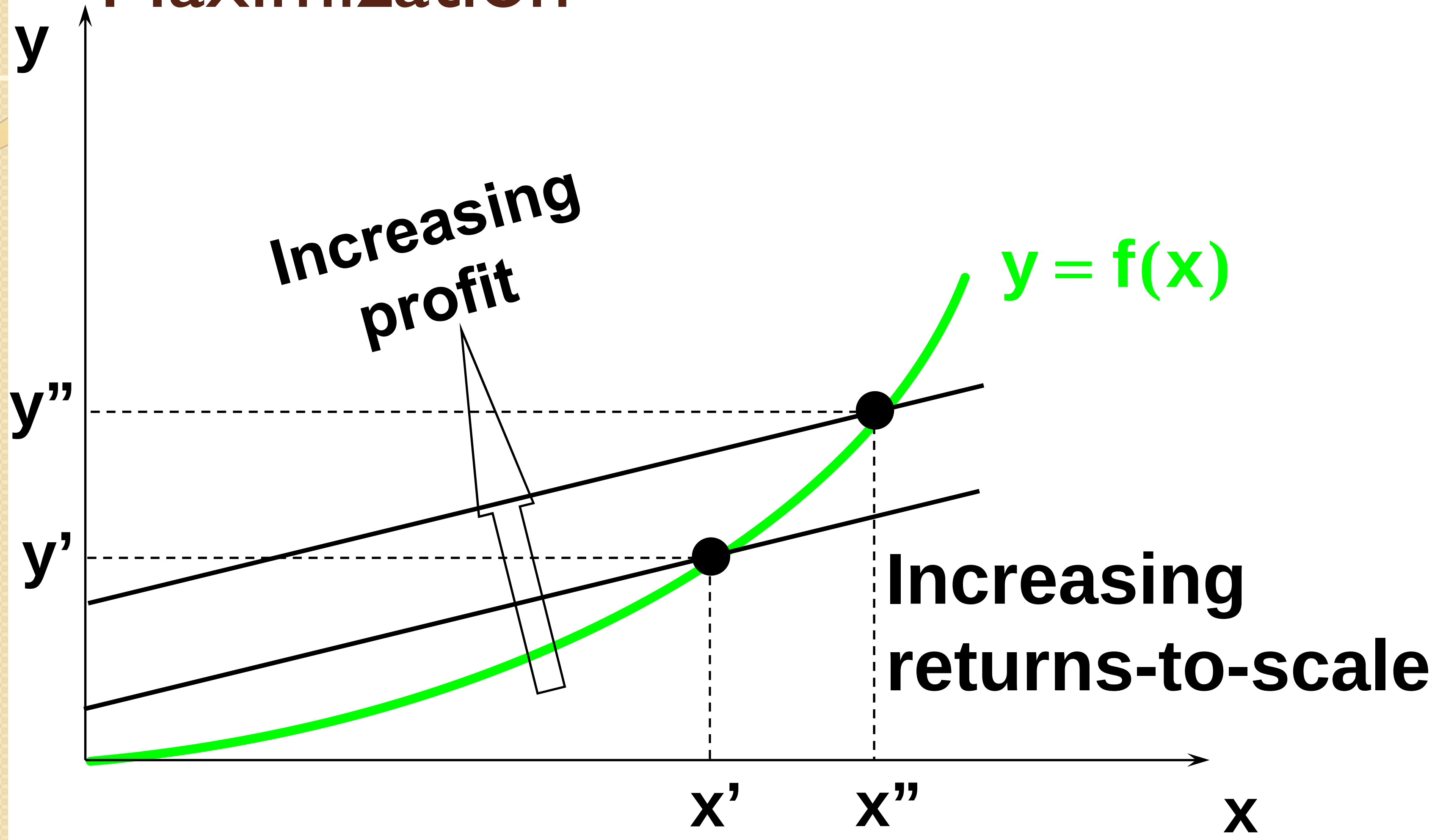
Returns-to Scale and Profit-Maximization



Returns-to-Scale and Profit-Maximization

- If a competitive firm's technology exhibits increasing returns-to-scale then the firm does not have a profit-maximizing plan.

Returns-to Scale and Profit-Maximization



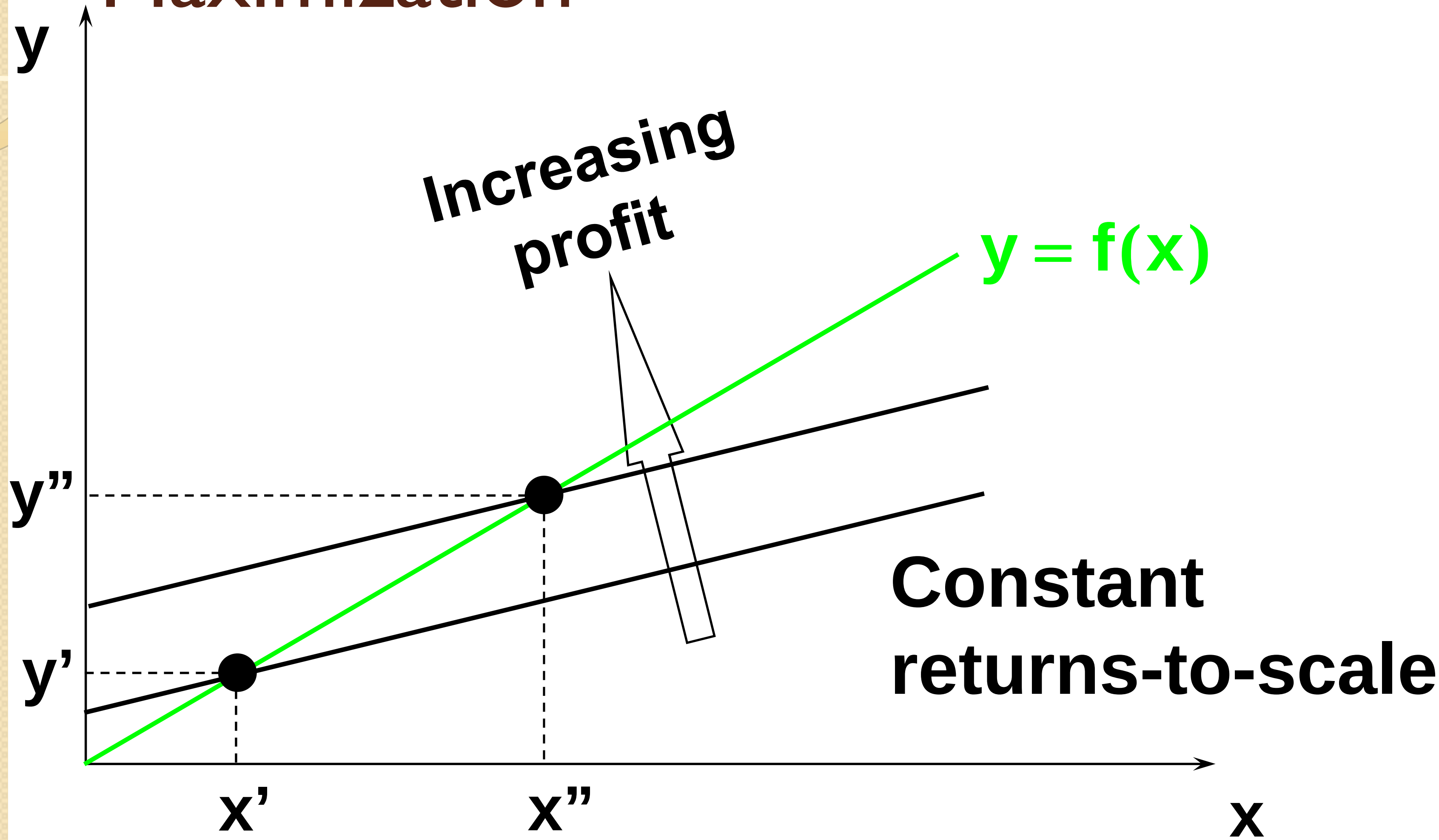
Returns-to-Scale and Profit-Maximization

- So an increasing returns-to-scale technology is inconsistent with firms being perfectly competitive.

Returns-to-Scale and Profit-Maximization

- What if the competitive firm's technology exhibits constant returns-to-scale?

Returns-to Scale and Profit-Maximization



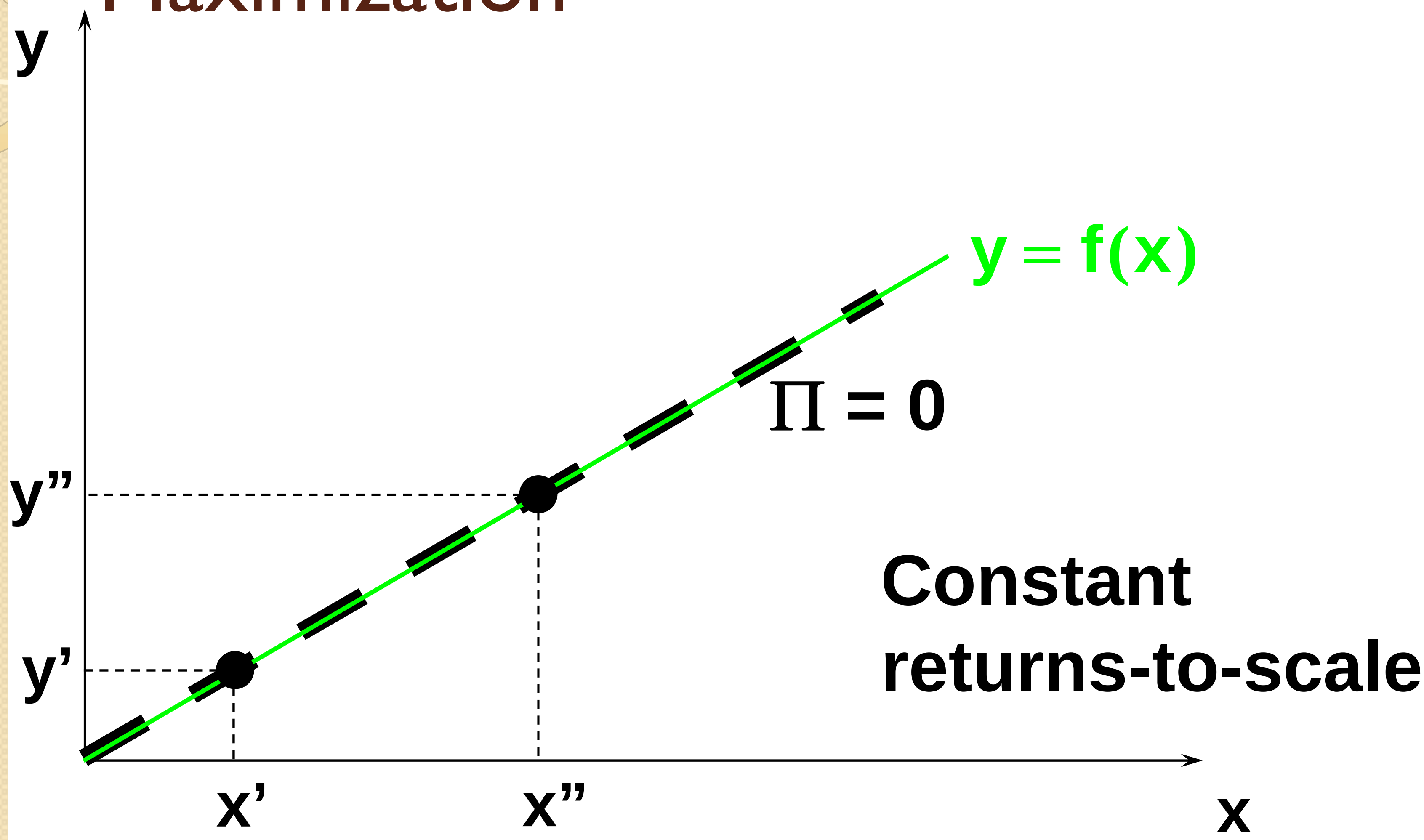
Returns-to Scale and Profit-Maximization

- So if any production plan earns a positive profit, the firm can double up all inputs to produce twice the original output and earn twice the original profit.

Returns-to Scale and Profit-Maximization

- Therefore, when a firm's technology exhibits constant returns-to-scale, earning a positive economic profit is inconsistent with firms being perfectly competitive.
- Hence constant returns-to-scale requires that competitive firms earn economic profits of zero.

Returns-to Scale and Profit-Maximization



Long-Run Competitive Equilibrium

- For long run equilibrium, firms must have no desire to enter or leave the industry
- We can relate economic profit to the incentive to enter and exit the market
- Need to relate accounting profit to economic profit

Long-Run Competitive Equilibrium

- Accounting profit
  - Difference between firm's revenues and direct costs
- Economic profit
  - Difference between firm's revenues and direct and indirect costs
  - Takes into account opportunity costs

Long-Run Competitive Equilibrium

- Firm uses labor (L) and capital (K) with purchased capital
- Accounting Profit and Economic Profit
  - Accounting profit: π = R - wL
  - Economic profit: π = R = wL - rK
    - wl = labor cost
    - rk = opportunity cost of capital

Long-Run Competitive Equilibrium

- Zero-Profit
  - A firm is earning a normal return on its investment
  - Doing as well as it could by investing its money elsewhere
  - Normal return is firm's opportunity cost of using money to buy capital instead of investing elsewhere
  - Competitive market long run equilibrium

Long-Run Competitive Equilibrium

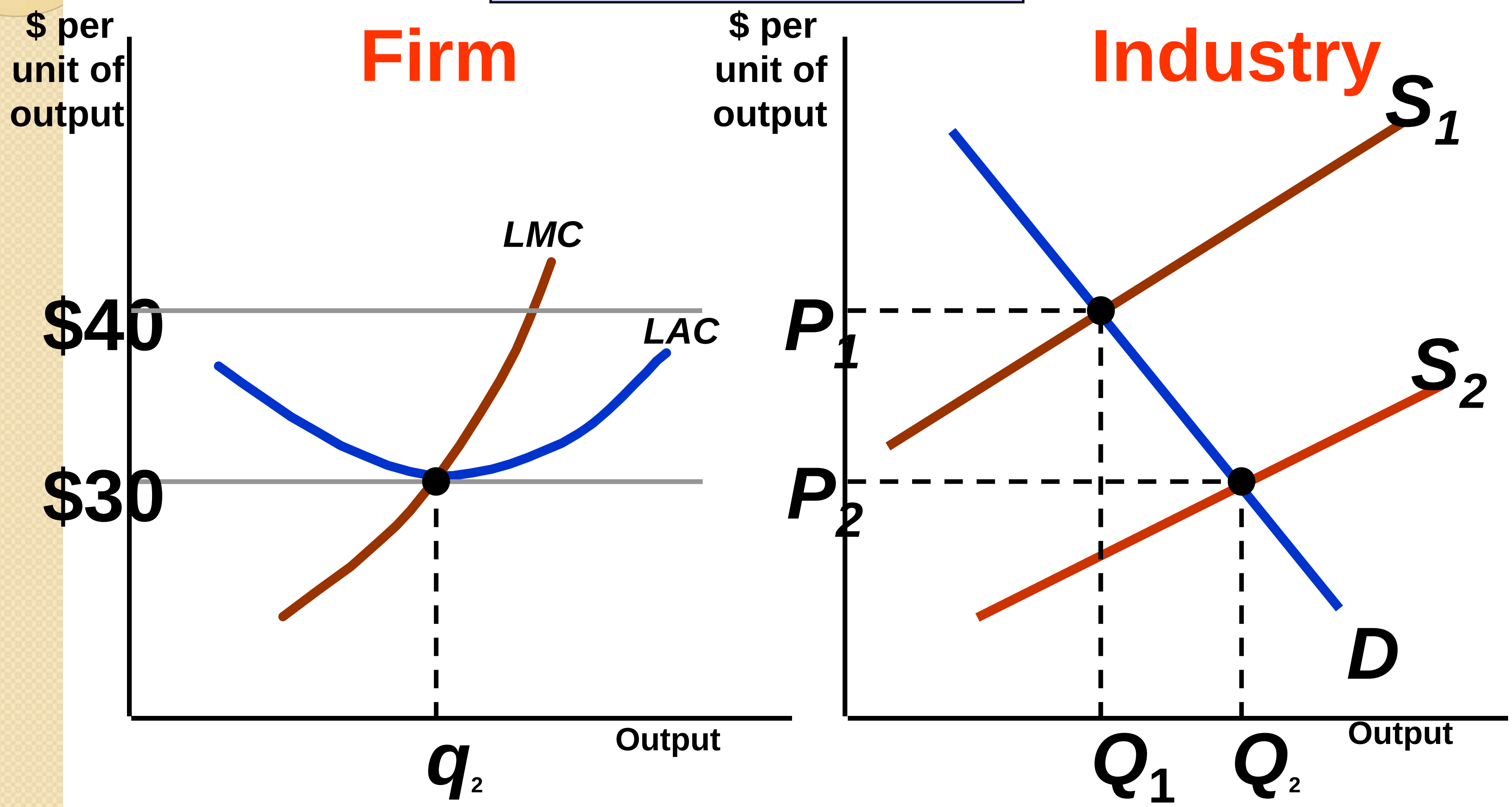
- Zero Economic Profits
  - If R > wL + rk, economic profits are positive
  - If R = wL + rk, zero economic profits, but the firm is earning a normal rate of return, indicating the industry is competitive
  - If R < wl + rk, consider going out of business

Long-Run Competitive Equilibrium

- Entry and Exit
  - The long-run response to short-run profits is to increase output and profits
  - Profits will attract other producers
  - More producers increase industry supply, which lowers the market price
  - This continues until there are no more profits to be gained in the market – zero economic profits

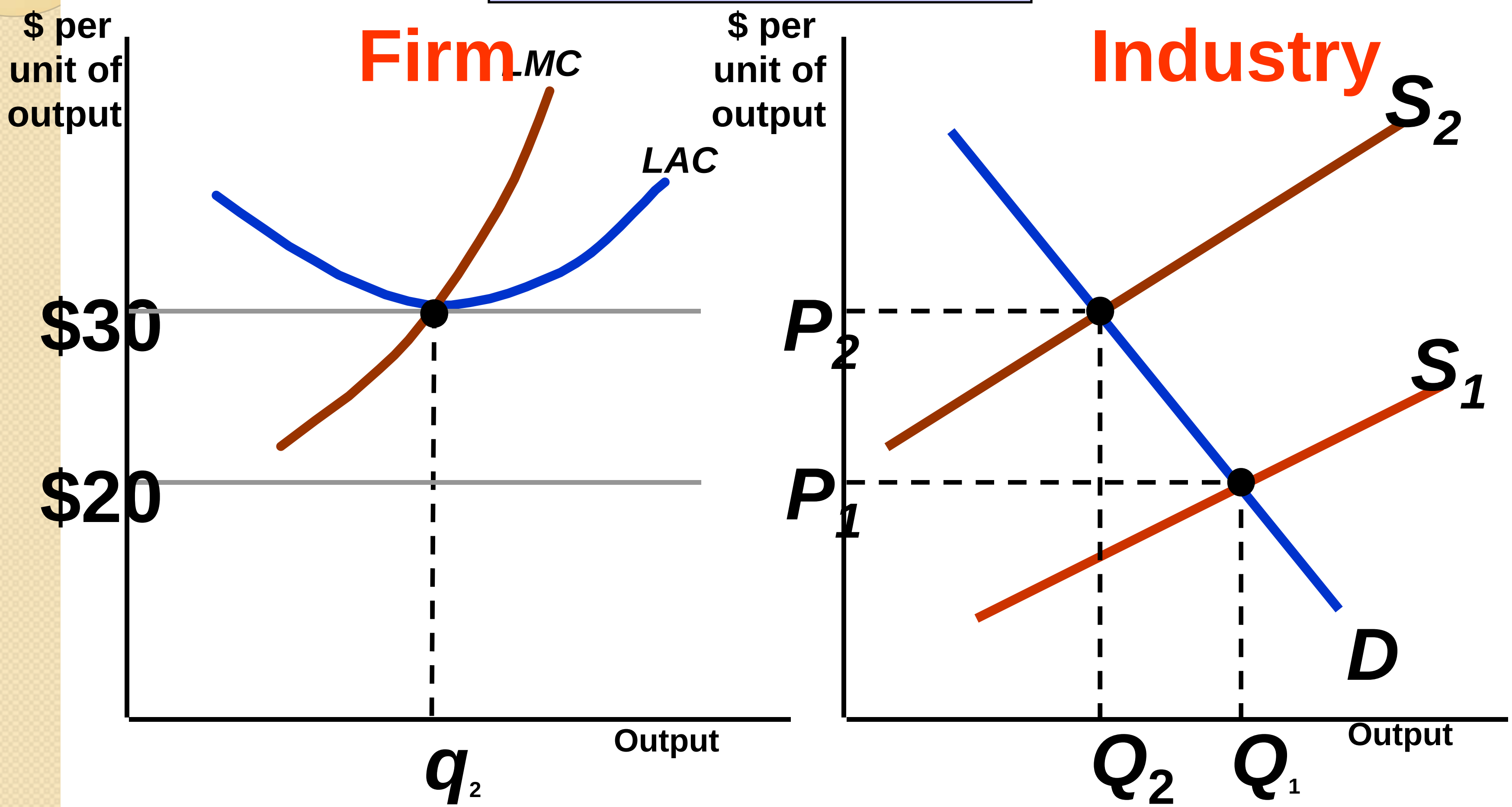
Long-Run Competitive Equilibrium – Profits

- Profit attracts firms
- Supply increases until profit = 0



Long-Run Competitive Equilibrium – Losses

- Losses cause firms to leave
- Supply decreases until profit = 0



Long-Run Competitive Equilibrium

1. All firms in industry are maximizing profits
   - MR = MC
2. No firm has incentive to enter or exit industry
   - Earning zero economic profits
3. Market is in equilibrium
   - $Q_D = Q_S$

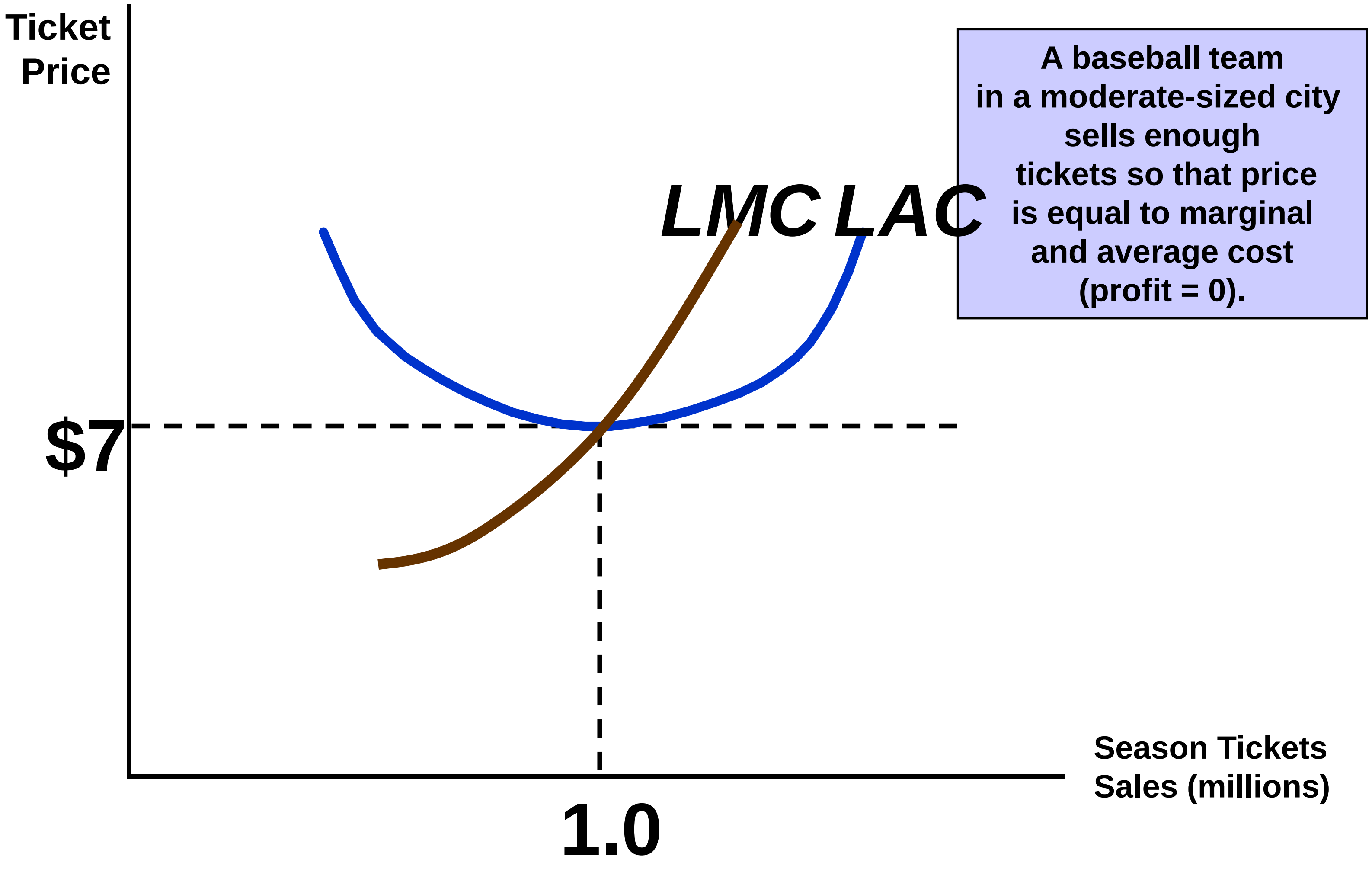
Choosing Output in the Long Run

- Economic Rent
  - The difference between what firms are willing to pay for an input less the minimum amount necessary to obtain it
  - When some have accounting profits that are larger than others, they still earn zero economic profits because of the willingness of other firms to use the factors of production that are in limited supply

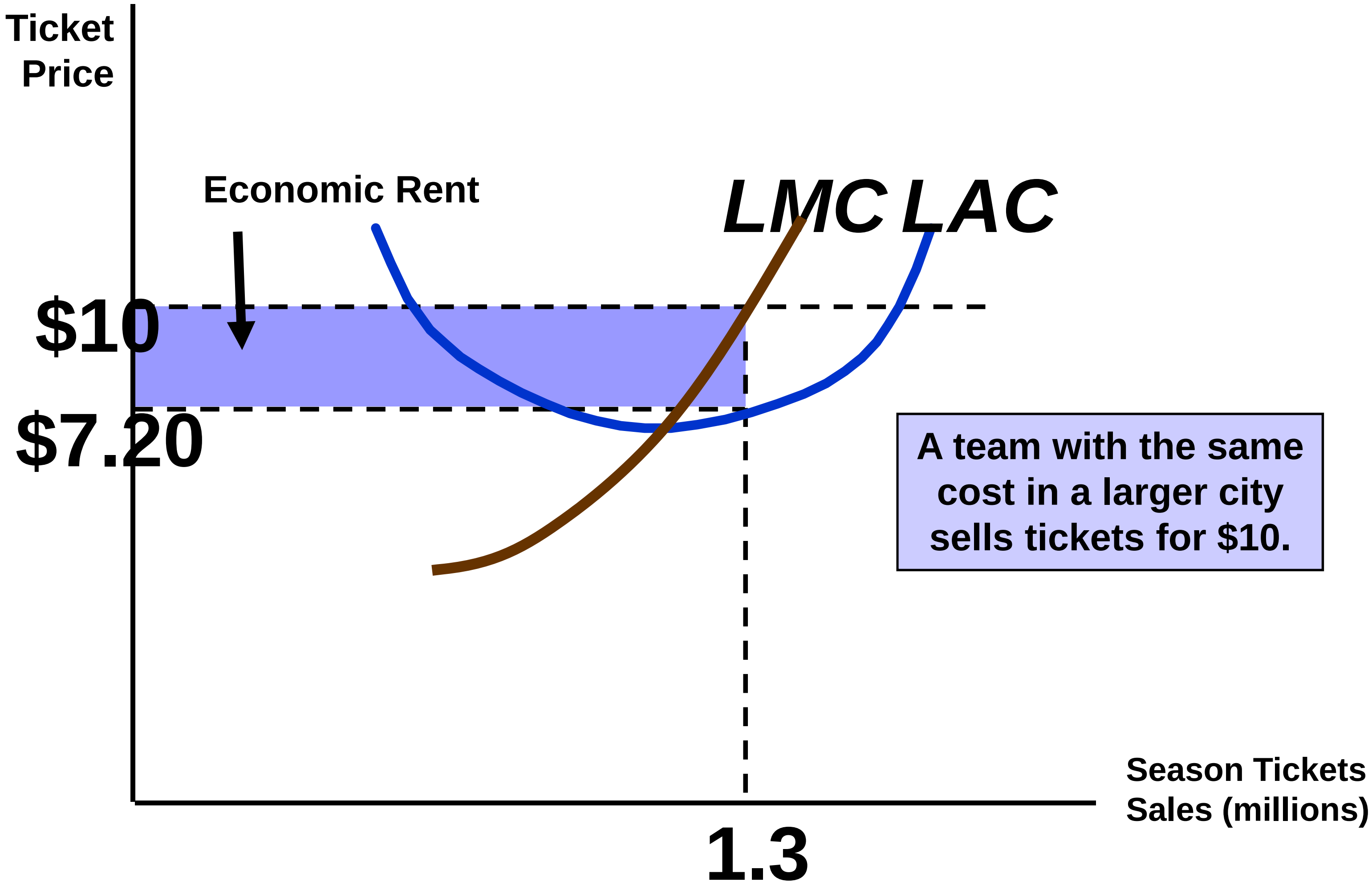
Choosing Output in the Long Run

- An Example
  - Two firms A & B that both own their land
  - A is located on a river which lowers A's shipping cost by $10,000 compared to B
  - The demand for A's river location will increase the price of A's land to $10,000 = economic rent
  - Although economic rent has increased, economic profit has become zero

Firms Earn Zero Profit in Long-Run Equilibrium



Firms Earn Zero Profit in Long-Run Equilibrium



Firms Earn Zero Profit in Long-Run Equilibrium

- With a fixed input such as a unique location, the difference between the cost of production (LAC = 7) and price ($10) is the value or opportunity cost of the input (location) and represents the economic rent from the input

Firms Earn Zero Profit in Long-Run Equilibrium

- If the opportunity cost of the input (rent) is not taken into consideration, it may appear that economic profits exist in the long run

The Industry's Long-Run Supply Curve

- The shape of the long-run supply curve depends on the extent to which changes in industry output affect the prices the firms must pay for inputs

The Industry's Long-Run Supply Curve

- Assume
  - All firms have access to the available production technology
  - Output is increased by using more inputs, not by invention
  - The market for inputs does not change with expansions and contractions of the industry

The Industry's Long-Run Supply Curve

- To analyze long-run industry supply, will need to distinguish between three different types of industries
  1. Constant-Cost
  2. Increasing-Cost
  3. Decreasing-Cost

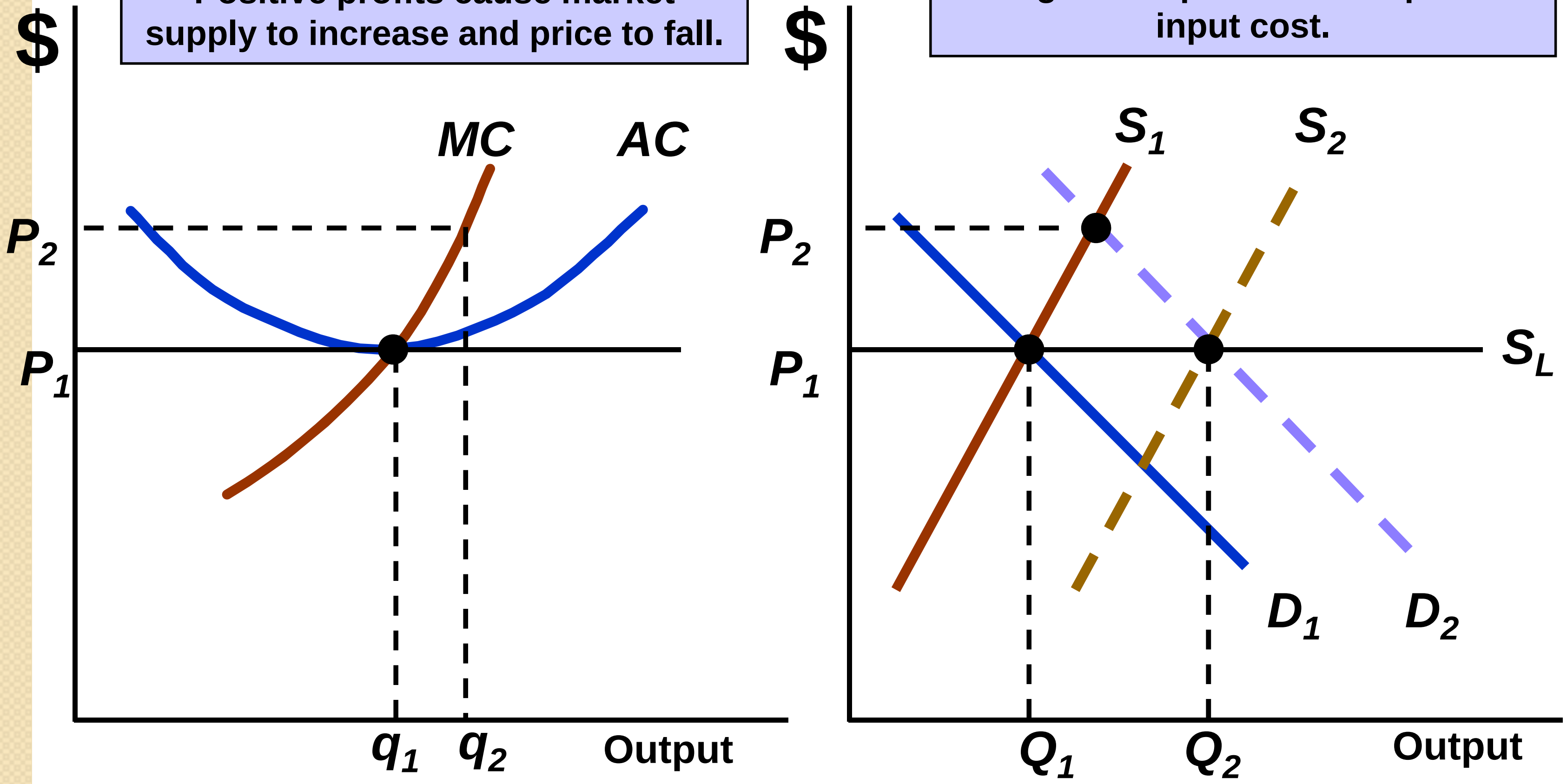
Constant-Cost Industry

- Industry whose long-run supply curve is horizontal
- Assume a firm is initially in equilibrium
  - Demand increases, causing price to increase
  - Individual firms increase supply
  - Causes firms to earn positive profits in short run
  - Supply increases, causing market price to decrease
  - Long run equilibrium – zero economic profits

Constant-Cost Industry

Increase in demand increases market price and firm output. Positive profits cause market supply to increase and price to fall.

Q_1 increases to Q_2. Long-run supply = S_L = LRAC. Change in output has no impact on input cost.



Long-Run Supply in a Constant-Cost Industry

- Price of inputs does not change
  - Firms' cost curves do not change
- In a constant-cost industry, long-run supply is a horizontal line at a price that is equal to the minimum average cost of production

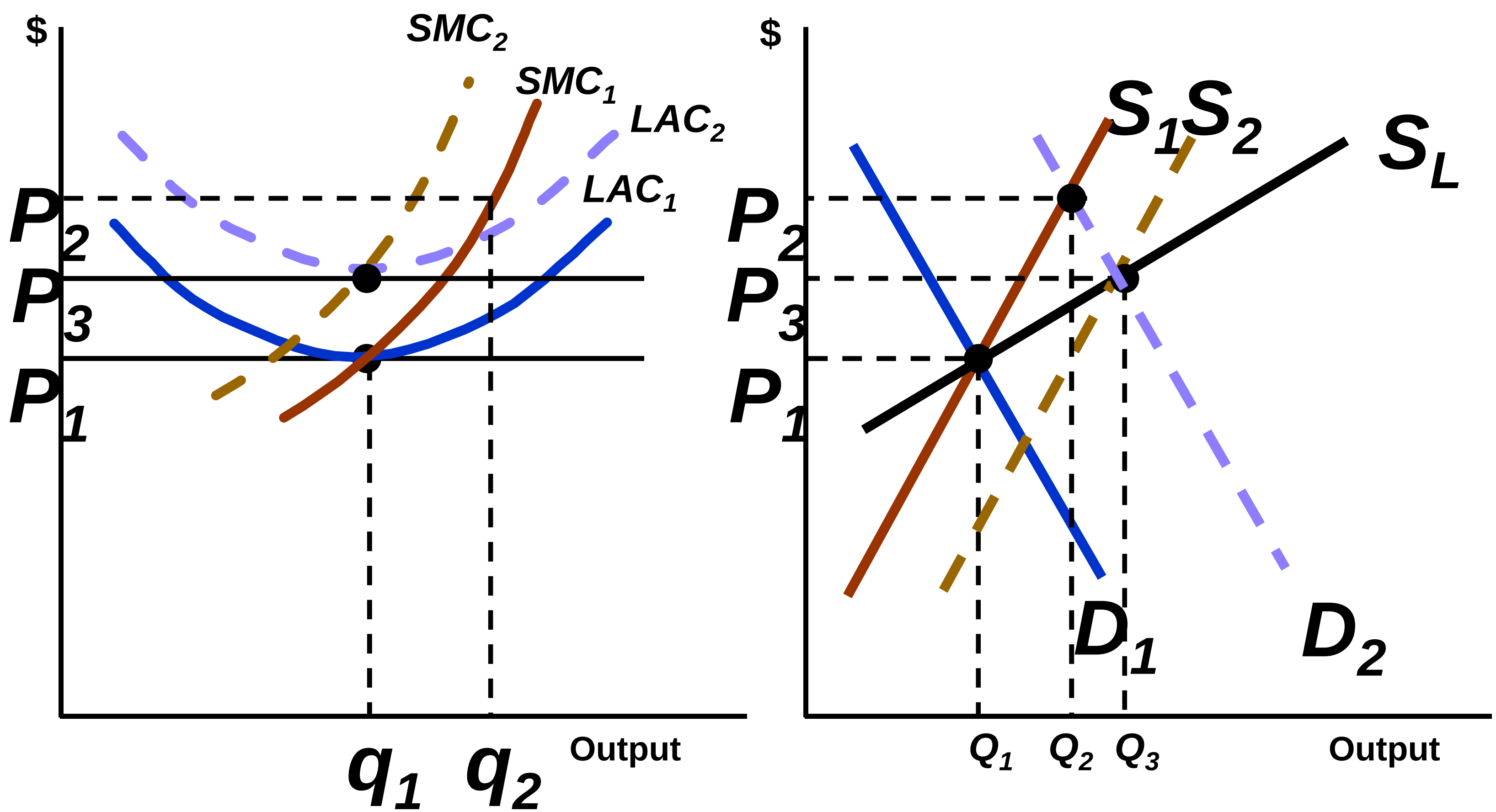
Increasing-Cost Industry

- Prices of some or all inputs rises as production is expanded when demand of inputs increases
- When demand increases, causing prices to increase and production to increase
  - Firms enter the market increasing demand for inputs
  - Costs increase, causing an upward shift in supply curves
  - Market supply increases but not as much

Long-Run Supply in an Increasing-Cost Industry

Due to the increase in input prices, long-run equilibrium occurs at a higher price.

Long Run Supply is upward Sloping



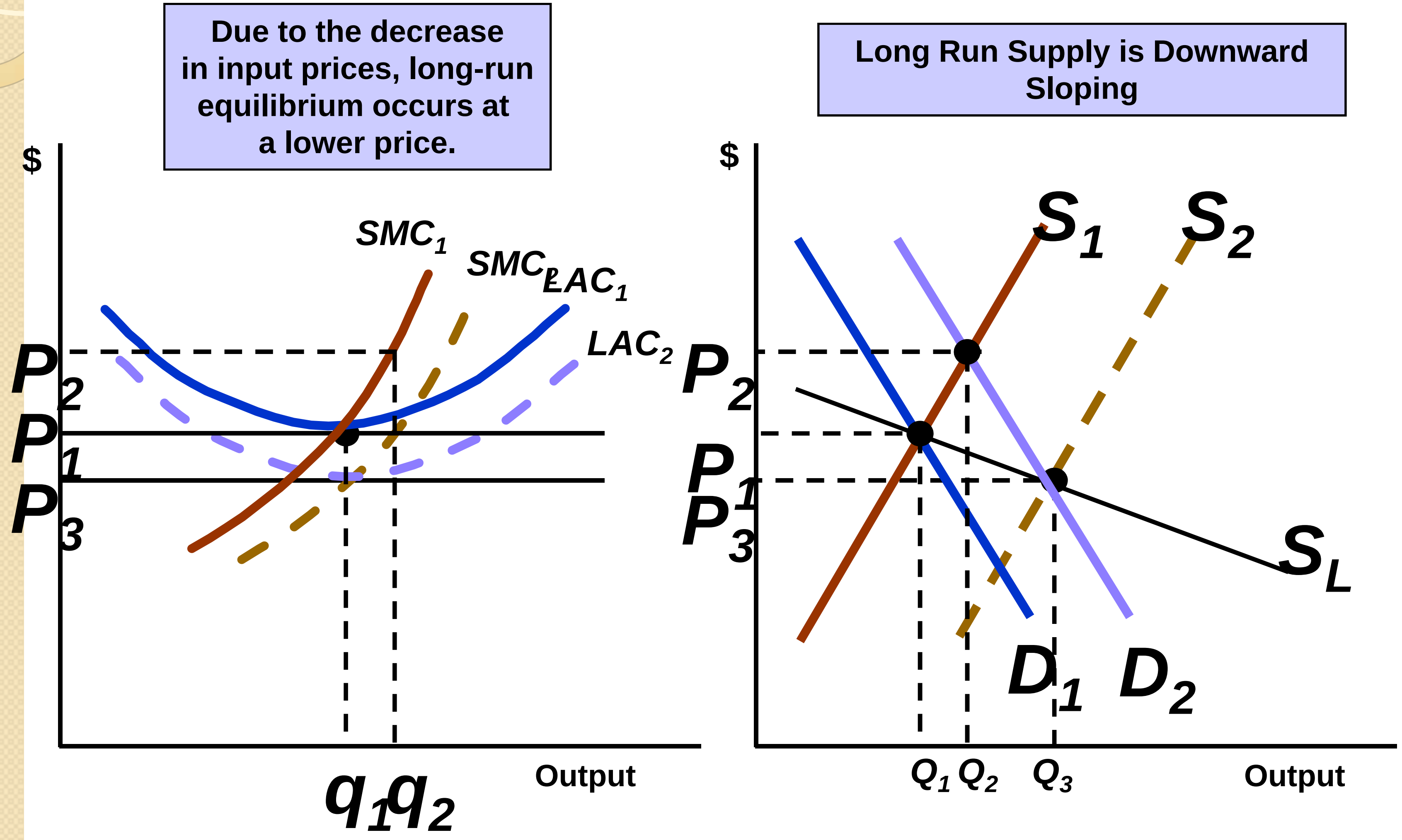
Long-Run Supply in an Increasing-Cost Industry

- In an increasing-cost industry, long-run supply curve is upward sloping
- More output is produced, but only at the higher price needed to compete for the increased input costs

Decreasing-Cost Industry

- Industry whose long-run supply curve is downward sloping
- Increase in demand causes production to increase
  - Increase in size allows firm to take advantage of size to get inputs cheaper
  - Increased production may lead to better efficiencies or quantity discounts
  - Costs shift down and market price falls

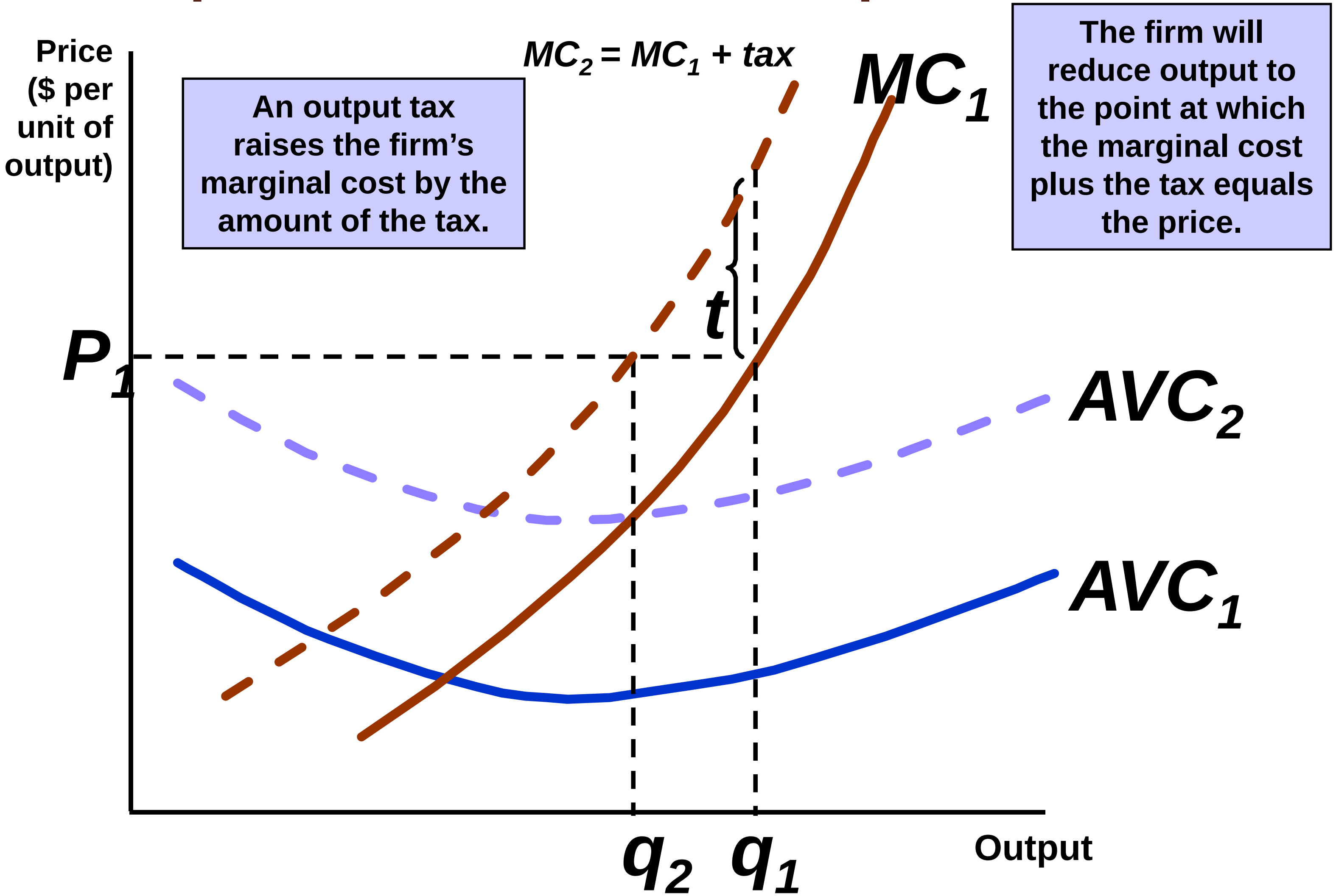
Long-Run Supply in a Decreasing-Cost Industry



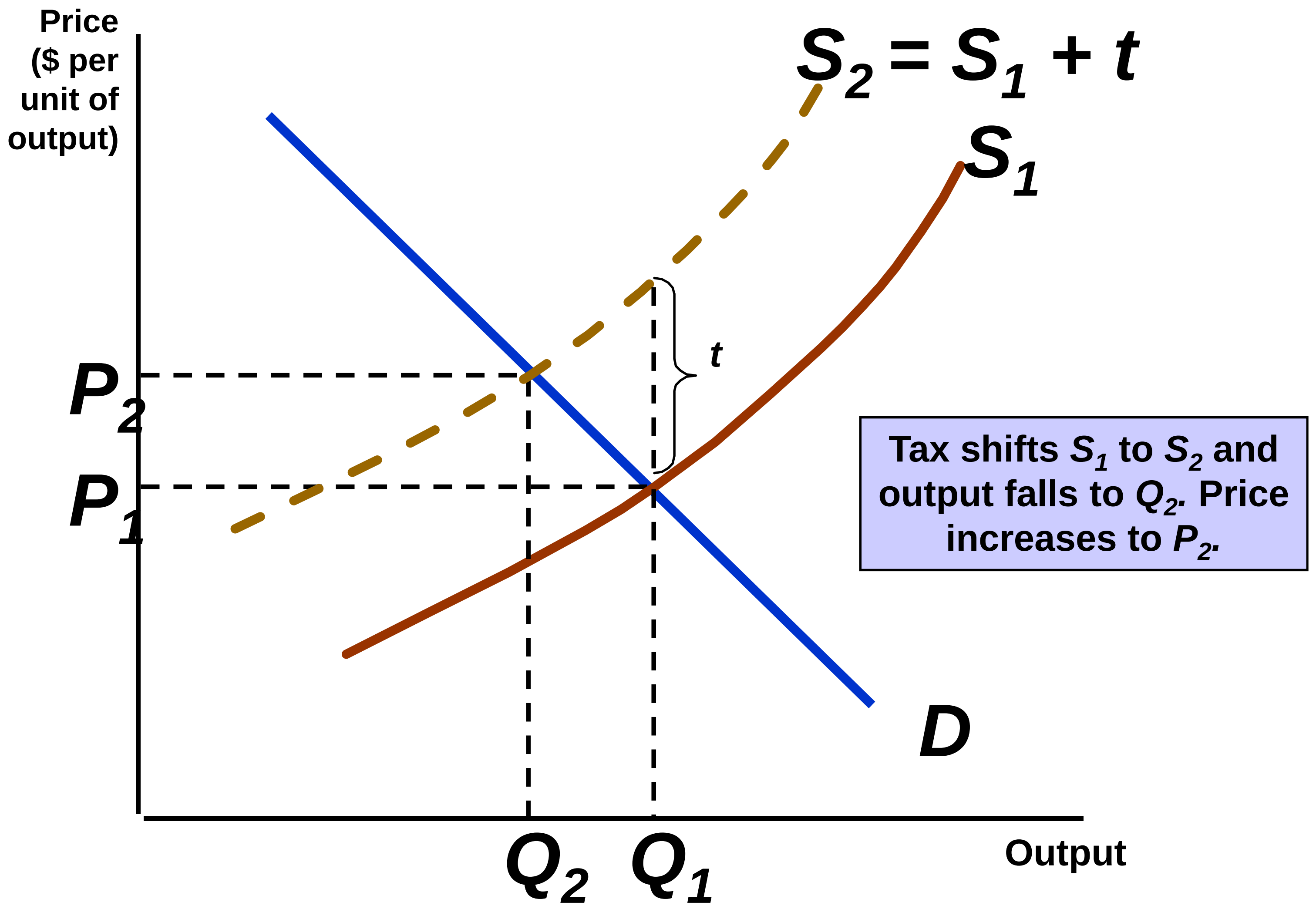
The Industry's Long-Run Supply Curve

- The Effects of a Tax
  - In an earlier chapter we studied how firms respond to taxes on an input
  - Now, we will consider how a firm responds to a tax on its output

Effect of an Output Tax on a Competitive Firm's Output



Effect of an Output Tax on Industry Output



Long-Run Elasticity of Supply

1. Constant-cost industry
   - Long-run supply is horizontal
   - Small increase in price will induce an extremely large output increase
   - Long-run supply elasticity is infinitely large
   - Inputs would be readily available

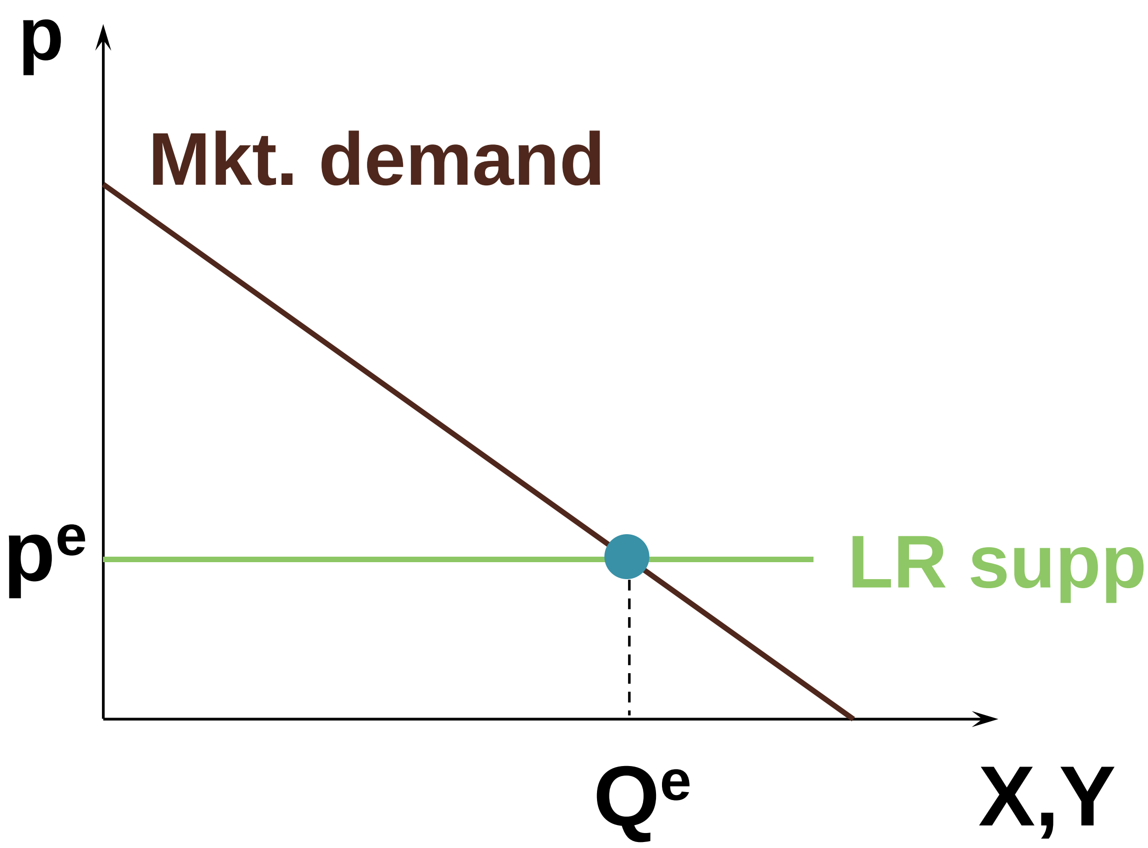
Long-Run Elasticity of Supply

2. Increasing-cost industry
   - Long-run supply is upward-sloping and elasticity is positive
   - The slope (elasticity) will depend on the rate of increase in input cost
   - Long-run elasticity will generally be greater than short-run elasticity of supply

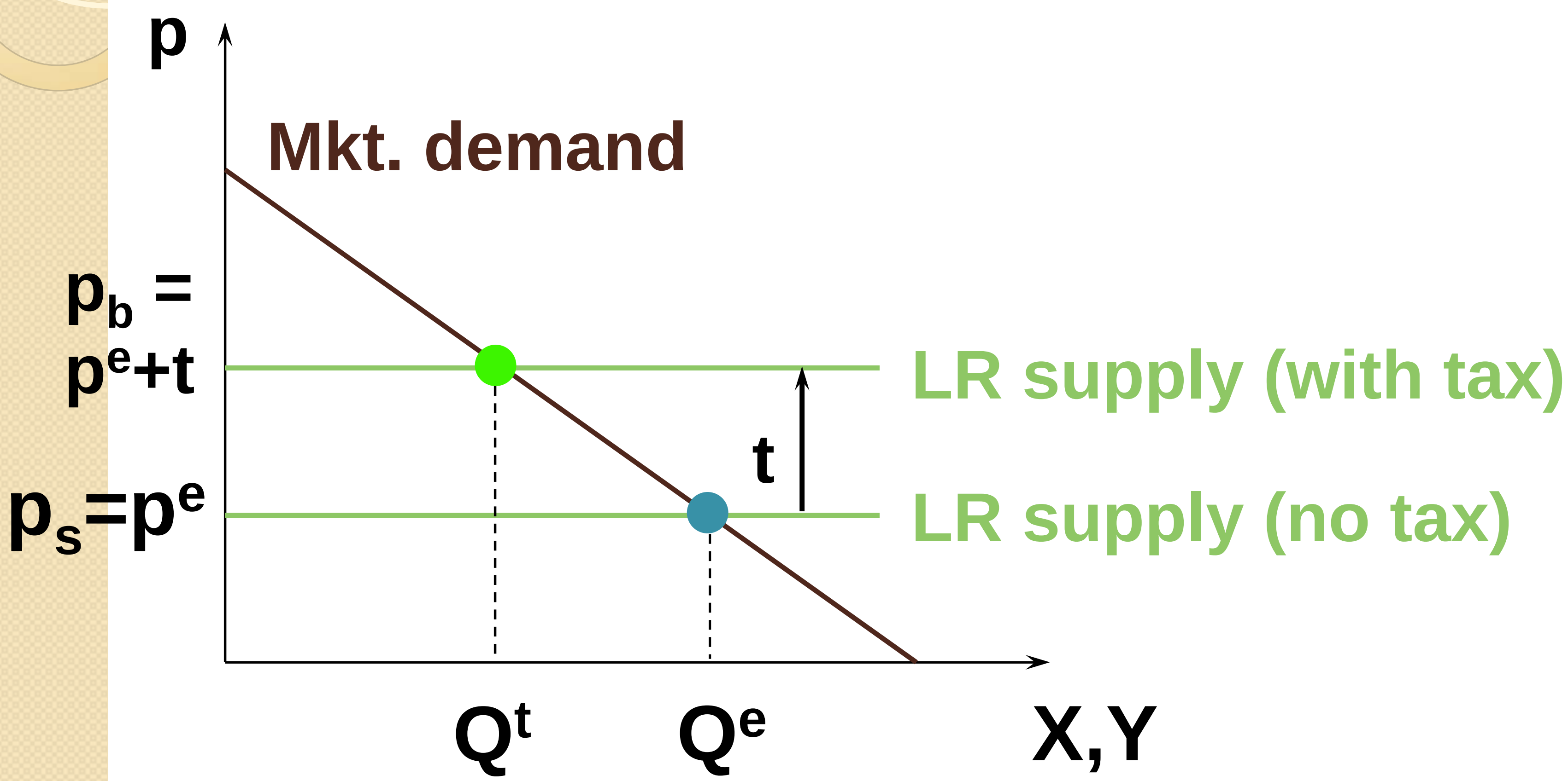
Long-Run Implications for Taxation

- In a short-run equilibrium, the burden of a sales or an excise tax is typically shared by both buyers and sellers, tax incidence of the tax depending upon the own-price elasticities of demand and supply.
- Q: Is this true in a long-run market equilibrium?

Long-Run Implications for Taxation

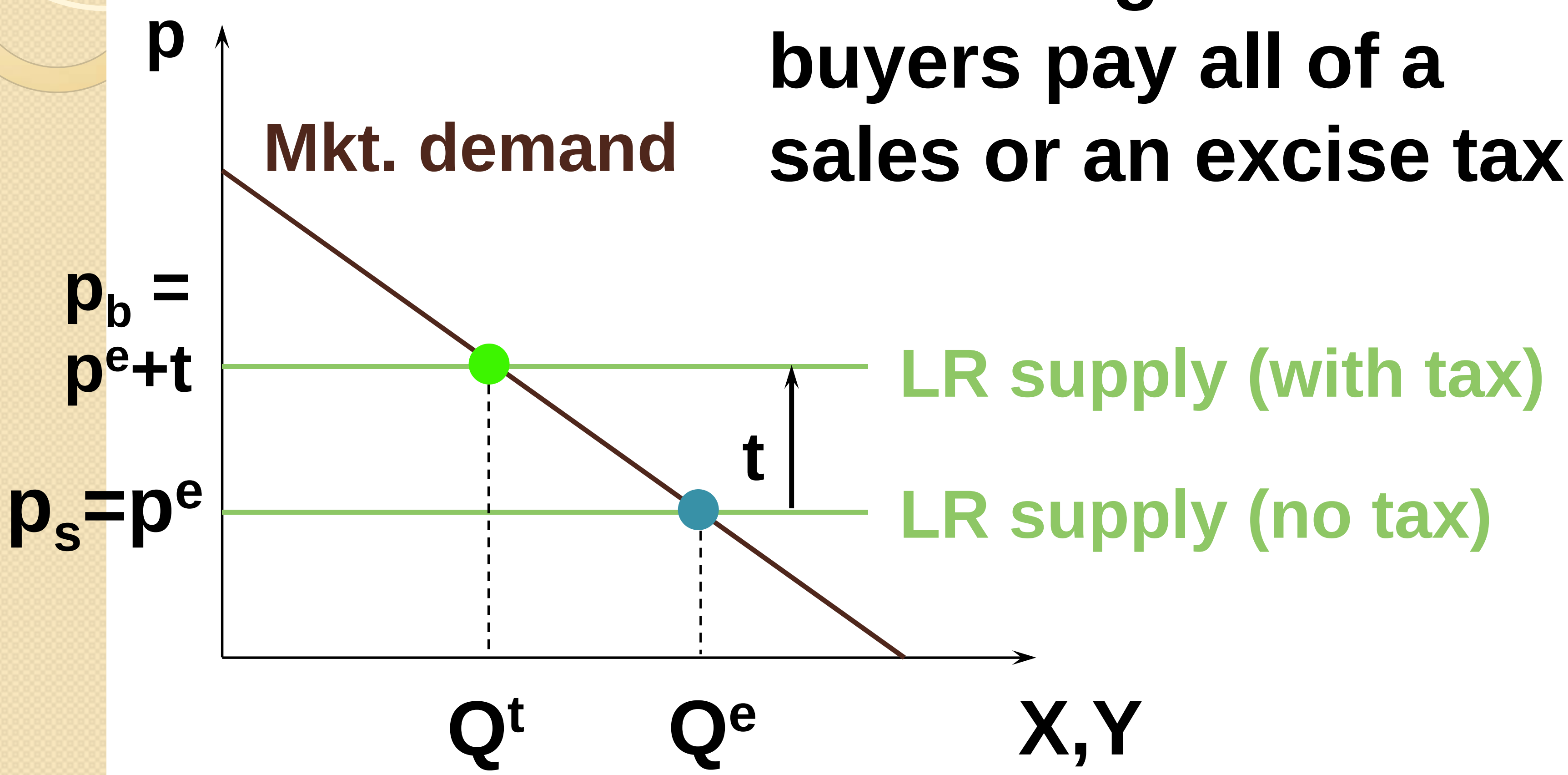


Long-Run Implications for Taxation



Long-Run Implications for Taxation

In the long-run the buyers pay all of a sales or an excise tax.



Fixed Inputs and Economic Rent

- What if there is a barriers to entry or exit?
- E.g., the taxi-cab industry has a barrier to entry even though there are lots of cabs competing with each other.
- Liquor licensing is a barrier to entry into a competitive industry.

Fixed Inputs and Economic Rent

- Q: When there is a barrier to entry, will not the firms already in the industry make positive economic profits?

Fixed Inputs and Economic Rent

- Q: When there is a barrier to entry, will not the firms already in the industry make positive economic profits?
- A: No. Each firm in the industry makes a zero economic profit. Why?

Fixed Inputs and Economic Rent

- An input (e.g. an operating license) that is fixed in the long-run causes a long-run fixed cost, F.
- Long-run total cost, $c(y) = F + c_v(y)$.
- And long-run average total cost,
$$AC(y) = AFC(y) + AVC(y).$$
- In the long-run equilibrium, what will be the value of F?

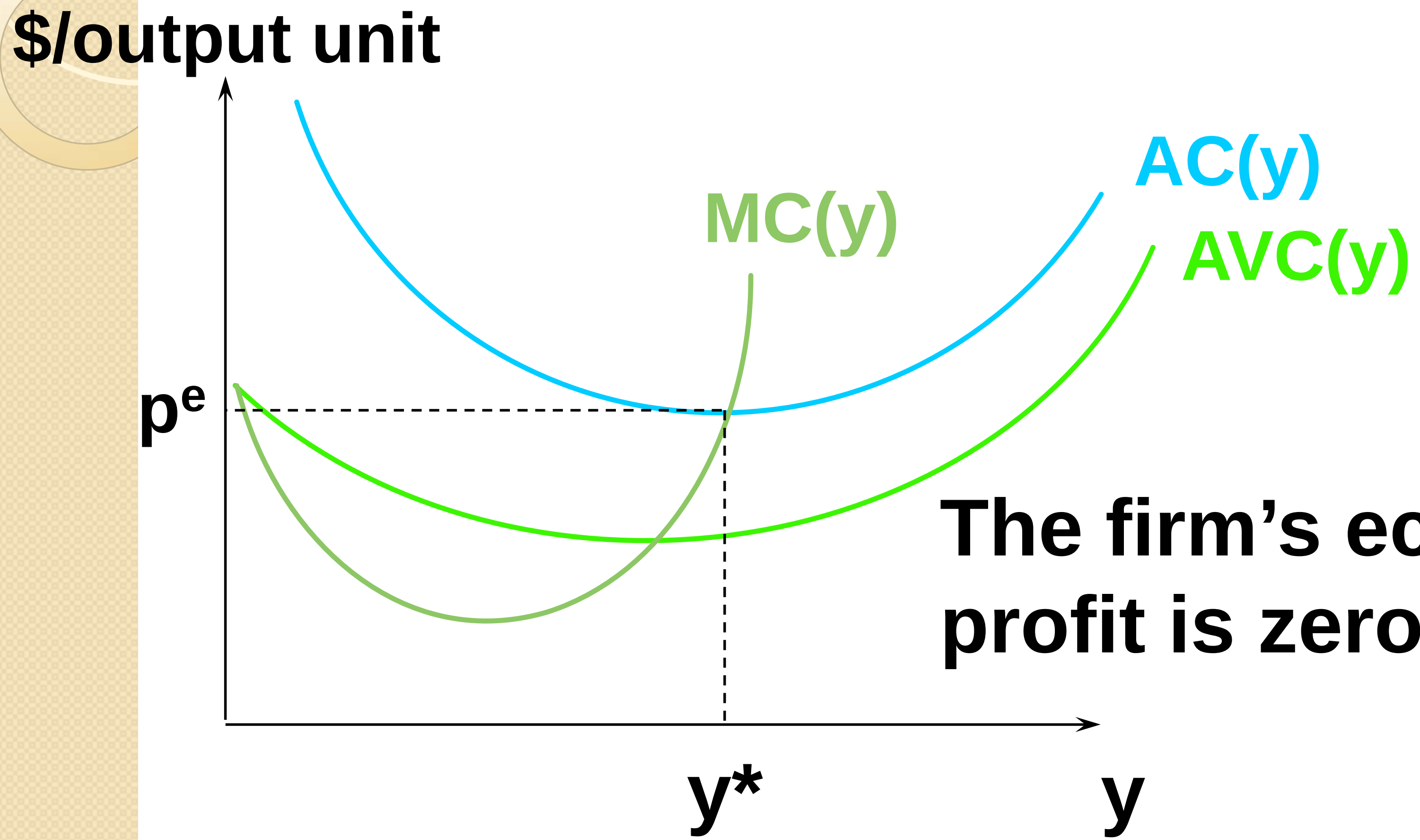
Fixed Inputs and Economic Rent

- Think of a firm that needs an operating license -- the license is a fixed input that is rented but not owned by the firm.
- If the firm makes a positive economic profit then another firm can offer the license owner a higher price for it. In this way, all firms' economic profits are competed away, to zero.

Fixed Inputs and Economic Rent

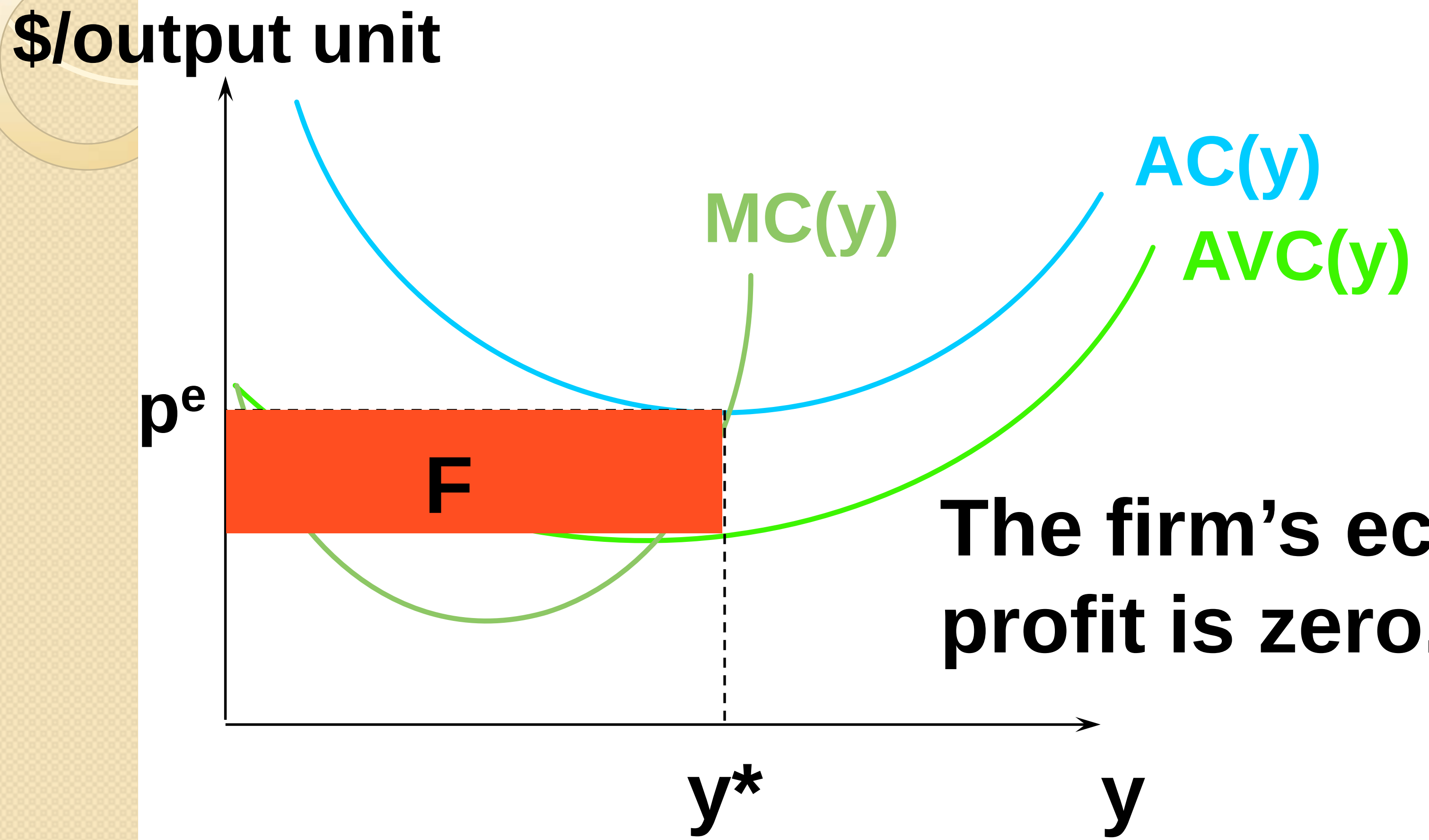
- So in the long-run equilibrium, each firm makes a zero economic profit and each firm's fixed cost is its payment for its operating license.

Fixed Inputs and Economic Rent



The firm's economic profit is zero.

Fixed Inputs and Economic Rent



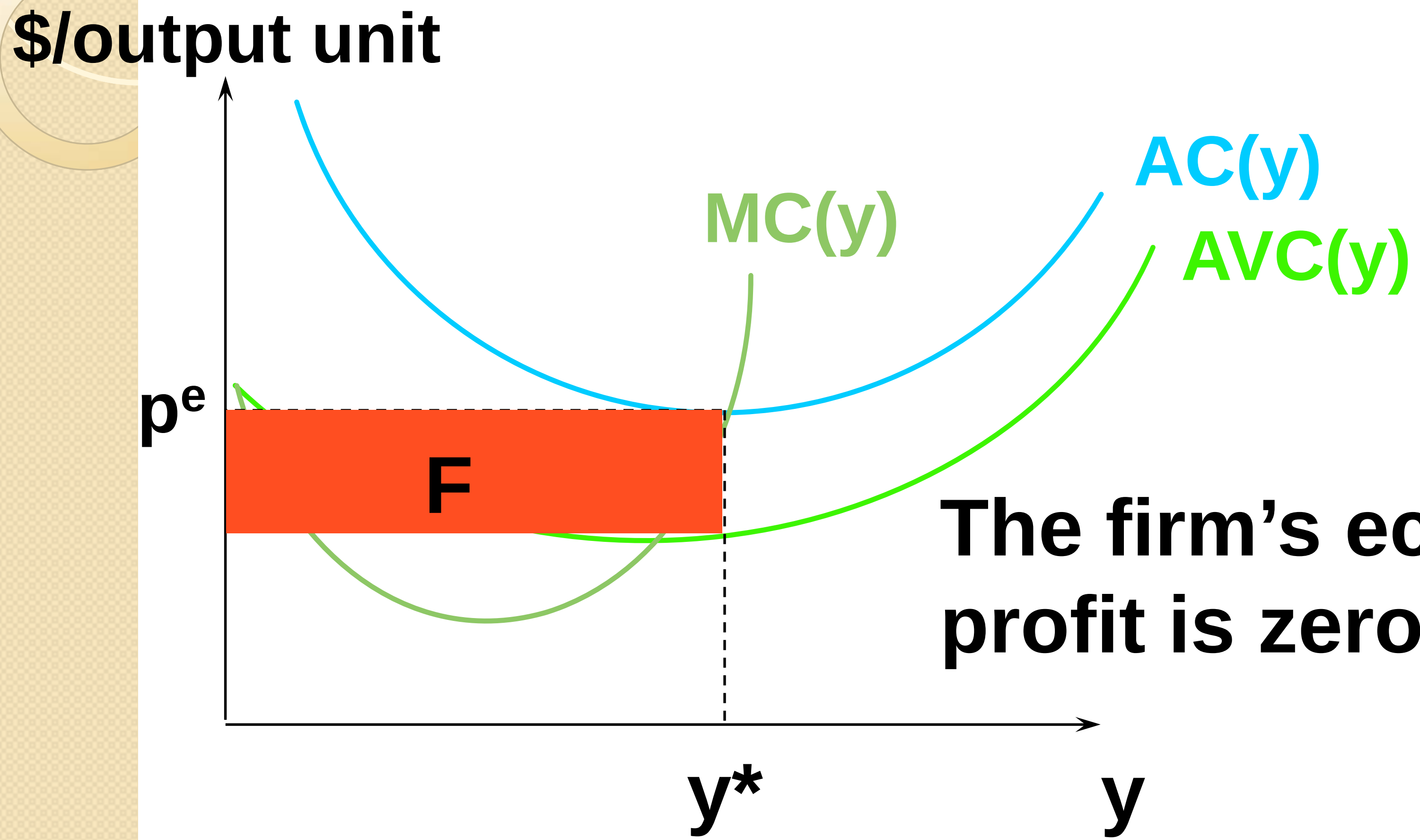
The firm's economic profit is zero.

F is the payment to the owner of the fixed input (the license).

Fixed Inputs and Economic Rent

- Economic rent is the payment for an input that is in excess of the minimum payment required to have that input supplied.
- Each license essentially costs zero to supply, so the long-run economic rent paid to the license owner is the firm's long-run fixed cost.

Fixed Inputs and Economic Rent



The firm's economic profit is zero.

F is the payment to the owner of the fixed input (the license); F = economic rent.