

**Problem Set 5**

1. George has an endowment of 50 nuts (n) and 80 apples (a). Bob has an endowment of 200 nuts (n) and 120 apples (a). Assume both have the same utility function  $U = n \cdot a$ , so their  $MRS = n / a$ .
  - a) Is there room for beneficial exchange?
  - b) What is the equilibrium price ratio,  $\frac{P_a}{P_n}$ ?
  - c) How many nuts and apples will George and Bob consume in the competitive equilibrium?
  - d) Draw the contract curve.

2. 考虑一个两人的交换经济，消费者  $i$  的效用函数为  $u^i(x_1^i, x_2^i) = \alpha_1^i \log x_1^i + \alpha_2^i \log x_2^i$ ，其中  $\alpha_j^i > 0$ ,  $i = A, B$ 。消费者  $i$  关于商品  $j$  的初始禀赋为  $w_j^i$ 。我们不妨假设商品 1 的价格为 1，商品 2 的价格为  $p$ 。

- 1) 求出消费者  $i$  的边际替代率 (marginal rate of substitution, MRS)。
  - 2) 定义消费者  $i$  的边际替代弹性 (marginal elasticity of substitution, MES) 为  $\frac{x_1^i dx_2^i}{x_2^i dx_1^i}$ 。证明  $MES_i$  为一个常数。
  - 3) 求出消费者  $i$  对商品  $j$  的需求函数，并把它表示为 MES 的函数。
  - 4) 求出该交换经济中的瓦尔拉斯均衡的价格。
  - 5) 假定有一个社会计划者试图实现两个消费者效用之和的最大化，前提是最后形成的配置必须是可行的 (feasible)。请求出满足社会计划者目标和约束的最优配置。
3. Assume that scientific studies provide you with the following information concerning the costs and benefits of sulfur dioxide emissions:

Benefits of abating (reducing) emissions	$MB = 500 - 20A$
Cost of abating emissions:	$MC = 200 + 5A$

where  $A$  is the quantity abated in millions of tons and the benefits and costs are given in dollars per ton.

- 1) What is the socially efficient level of emissions abatement?
  - 2) What are the marginal benefit and marginal cost of abatement at the socially efficient level of abatement?
  - 3) What happens to net social benefits (benefits minus costs) if you abate one million more tons than the efficient level? One million fewer?
  - 4) Why is it socially efficient to set marginal benefits equal to marginal costs rather than abating until total benefits equal total costs?
4. Suppose that the oil industry in Utopia is perfectly competitive and that all firms draw oil from a single (and practically inexhaustible) pool. Assume that each competitor believes that he or she can sell all the oil he or she can produce at a stable world price of \$10 per barrel and that the cost of operating a well for one year is \$1,000.  
Total output per year ( $Q$ ) of the oil field is a function of the number of wells ( $N$ ) operating in the fields. In particular

$$Q = 500N - N^2$$

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And the amount of oil produced by each well ( $q$ ) is given by

$$q = 500 - N$$

- a) Describe the equilibrium output and the equilibrium number of wells in this perfectly competitive case. Is there a divergence between private and social marginal cost in the industry?
  - b) Suppose now that the government nationalizes the oil field. How many oil wells should it operate? What will total output be? What will the output per well be?
  - c) As an alternative to nationalization, the Utopian government is considering an annual license fee per well to discourage overdrilling. How large should this license fee be if it is to prompt the industry to drill the optimal number of wells?
5. Suppose that the inverse demand curve for paper is  $p = 200 - Q$ , the private marginal cost (unregulated competitive market supply) is  $MC^p = 80 + Q$ , and the marginal harm from gunk is  $MC^g = Q$ .
- a) What is the unregulated competitive equilibrium?
  - b) What is the social optimum? What specific tax (per unit of output or gunk) results in the social optimum?
  - c) What is the unregulated monopoly equilibrium?
  - d) How would you optimally regulate the monopoly? What is the resulting equilibrium?
6. There are three groups in a community. Their demand curves for public television in hours of programming,  $T$ , are given respectively by

$$W_1 = \$150 - T, W_2 = \$200 - 2T, \text{ and } W_3 = \$250 - T.$$

Suppose public television is a pure public good that can be produced at a constant marginal cost of \$200 per hour.

- a) What is the efficient number of hours of public television?
  - b) How much public television would a competitive private market provide?
7. 一个企业的成本函数是:  $C = 4q^2$ , 其中  $q$  是企业的产量。企业能够以单位价格 64 元销售任意数量的产品。但是, 企业的生产需要排出污水, 给邻近居民带来损失, 总的损失  $D$  依赖于企业的产量:  $D = 4q + q^2$ 。
- 1) 假定企业具有污染的产权。它会生产多少单位产品? 如果居民与企业进行协商, 那么, 双方协商的产量是多少? 居民最少要向企业付多少钱, 才能使得企业生产这个产量? 最多需要付多少钱? 假定双方协商的成本为零。
  - 2) 假定居民具有不受污染的产权。企业生产多少单位产品? 如果企业与居民进行协商, 那么, 双方协商的产量是多少? 企业最少要向居民付多少钱, 才能使得企业生产这个产量? 最多需要付多少钱? 假定双方协商的成本为零。
  - 3) 假定企业具有污染的产权。如果政府想通过税收来控制外部性, 它应该是多少? 政府的税收收入是多少?