Problem Set 5

1. *The Laplace Distribution.* Suppose that a random variable X has a continuous distribution for which the p.d.f. is as follows:

$$f(x) = \frac{1}{2\lambda} e^{-\frac{1}{\lambda}|x-\mu|}, x \in \mathbb{R}, (\lambda > 0).$$

Find the value of $E(X)$ and $Var(X)$.

2. (Textbook Section 4.2-8, Page 138) Suppose that X and Y have a continuous joint distribution for which the joint p.d.f. is as follows:

$$f(x,y) = \begin{cases} 12y^2, & \text{for } 0 \leq y \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of $E(XY)$ and $Var(XY)$.

3. Suppose that $X_1, \ldots, X_n$ are n independent random variables from a uniform distribution on the interval $[0,\ 1]$. Determine the value of $E[(X_1 + \cdots + X_n)^2]$ and $Var(X_1 + \cdots + X_n)$.

4. Suppose X is a random variable. Prove that $f(c) = E(X-c)^2$ is minimum if and only if $c = E(X)$.

5. Suppose that a certain examination is to be taken by five students independently of one another, and the number of minutes required by any particular student to complete the examination has an exponential distribution for which the mean is 80. Suppose that the examination begins at 9:00 A.M. Determine the probability that at least one of the students will complete the examination before 9:40 A.M.

   Now suppose the first student to complete the examination finishes at 9:25 A.M. Determine the probability that at least one other student will complete the examination before 10:00 A.M. Further determine the probability that no two students will complete the examination within 10 minutes of each other.