函数极限的运算

本段内容要点:

函数极限的四则运算法则

复合函数的极限运算

若干函数极限的例题

定理: $\lim_{x \to x_0} f(x) = A \Leftrightarrow$ $f(x) - A = x \to x_0$ 时的无穷小量.

定理:
$$\lim_{x \to x_0} f(x) = A \Leftrightarrow$$
 $f(x) - A = x \to x_0$ 时的无穷小量.

这是因为

$$f(x) = A + [f(x) - A],$$

故 $\lim_{x \to x_0} f(x) = A \Leftrightarrow f(x) = A + \alpha(x),$
其中 $\alpha(x)$ 是一个 $x \to x_0$ 时的无穷小量.

\lim_x

$$\lim_x u(x) = A, \, \lim_x v(x) = B$$
则有:

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

 $(n) = \frac{1}{x} + \frac{1}{x}$

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

(1)
$$\lim (u(x) + v(x)) = \lim u(x) + \lim v(x) = A + B$$

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

$$\frac{1}{x}$$

因为
$$\lim_{x} u(x) = A$$
,所以 $u(x) = A + \alpha(x)$;

因为
$$\lim_{x} v(x) = B$$
,所以 $u(x) = B + \beta(x)$;
其中 $\alpha(x)$, $\beta(x)$ 为该过程中的无穷小量。

$$(1) \ u(x) \pm v(x) = (A \pm B) + (\alpha(x) \pm \beta(x))$$

$$(1) \ u(x) \pm v(x) = (A \pm B) + (\alpha(x) \pm \beta(x))$$
$$\Rightarrow \lim (u(x) \pm v(x)) = A \pm B.$$

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

(2)
$$\lim (u(x) \cdot v(x)) = (\lim u(x)) \cdot (\lim v(x)) = A \cdot B$$

$$(2)$$
 $\lim_x (u(x) \cdot v(x)) = \left(\lim_x u(x)\right) \cdot \left(\lim_x v(x)\right) = A \cdot B$ 乘和的极限等于极限的乖和。

(乘积的极限等于极限的乘积)

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

(1)
$$\lim_{n \to \infty} (u(n) + u(n)) = \lim_{n \to \infty} u(n) + \lim_{n \to \infty} u(n) = A + B$$

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

$$oldsymbol{x}$$

(2)
$$\lim_x (u(x) \cdot v(x)) = \left(\lim_x u(x)\right) \cdot \left(\lim_x v(x)\right) = A \cdot B$$

$$\lim_x (u(x) \cdot v(x)) \ = \left(\lim_x u(x)
ight) \cdot \left(\lim_x v(x)
ight) = A \cdot B$$

因为
$$\lim_x u(x) = \left(\lim_x u(x)\right) \cdot \left(\lim_x v(x)\right) = A \cdot B$$

因为
$$\lim_x v(x) = B,$$
所以 $v(x) = B + eta(x);$

其中
$$\alpha(x)$$
, $\beta(x)$ 为该过程中的无穷小量.

(2)
$$u(x)v(x) = AB + (A\beta(x) + B\alpha(x) + \alpha(x)\beta(x)),$$

 $\Rightarrow \lim_{x} (u(x) \cdot v(x)) = AB.$

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

$$\lim_{x} (u(x) \perp v(x)) = \lim_{x} u(x) \perp \lim_{x} v(x) = A \perp B$$

$${\scriptstyle (2)} \lim_x (u(x) \cdot v(x)) \ = \left(\lim_x u(x)\right) \cdot \left(\lim_x v(x)\right) = A \cdot B$$

(2')
$$\lim_{x} k \cdot u(x) = k \cdot A$$
;

 $(2) \lim k \cdot u(x) = k \cdot A;$

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

 $\lim_x (u(x) \cdot v(x)) \ = \left(\lim_x u(x)
ight) \cdot \left(\lim_x v(x)
ight) = A \cdot B$

 $(2)^n \lim_x [u(x)]^n = A^n \left(or \lim_x u^n(x) = A^n
ight)$

$$\frac{1}{x}(w(w) \perp w(w)) = \lim_{x} w(w) \perp \lim_{x} v(w) = x \perp D$$

$$\frac{1}{x}$$

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

$$\lim_{x} (u(x) \perp v(x)) = \lim_{x} u(x) \perp \lim_{x} v(x) = A \perp B$$

$$\lim_x (u(x) \cdot v(x)) = \left(\lim_x u(x)\right) \cdot \left(\lim_x v(x)\right) = A \cdot B$$

(2')
$$\lim_x k \cdot u(x) = k \cdot A$$
;

(2")
$$\lim_x [u(x)]^n = A^n \left(or \, \lim_x u^n(x) = A^n
ight)$$

(3) if
$$B
eq 0$$
, then $\lim_x \dfrac{u(x)}{v(x)} = \dfrac{\lim_x u(x)}{\lim v(x)} = \dfrac{A}{B}$

(商的极限等于极限的商)

中
$$lpha(x)$$
, $eta(x)=B$,所以 $b(x)=B+eta(x)$, $=rac{A}{B}$, $a(x)$,为该过程中的无穷小量. A

(3) if B
eq 0, then $\lim_x \frac{u(x)}{v(x)} = \frac{\lim_x u(x)}{\lim_x v(x)} = \frac{A}{B}$

$$= \frac{A}{B} + (\frac{A}{B})$$

$$= \frac{A}{B} + \frac{Bc}{B}$$

$$=rac{B}{B}+rac{(\overline{B+eta(x)}-\overline{B})}{BA} = rac{A}{B}+rac{Blpha(x)-Aeta(x)}{B(B+eta(x))}$$

 $(2) \lim_{x \to a} k \cdot u(x) = k \cdot A;$

$$\lim_x u(x) = A, \lim_x v(x) = B$$
则有:

$$\lim_{x \to \infty} (u(x) + v(x)) = \lim_{x \to \infty} u(x) + \lim_{x \to \infty} v(x) = A + B$$

$$\lim_{x} (u(x) \pm v(x)) = \lim_{x} u(x) \pm \lim_{x} v(x) = A \pm B$$

$$\frac{1}{x}(x(x) - x(x)) = \lim_{x \to x} \alpha(x) + \lim_{x \to x} \alpha(x) = 21 + 2$$

 $\lim_x (u(x) \cdot v(x)) \ = \left(\lim_x u(x)
ight) \cdot \left(\lim_x v(x)
ight) = A \cdot B$

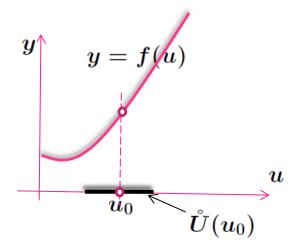
 $(2) \lim_x [u(x)]^n = A^n \left(or \lim_x u^n(x) = A^n
ight)$

(3) if B
eq 0, then $\lim_x \frac{u(x)}{v(x)} = \frac{\lim_x u(x)}{\lim v(x)} = \frac{A}{B}$

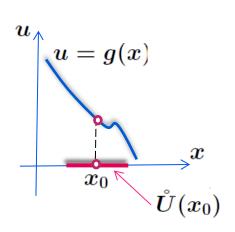
(1)
$$\lim_x (u(x) \pm v(x)) = \lim_x u(x) \pm \lim_x v(x) = A \pm B$$

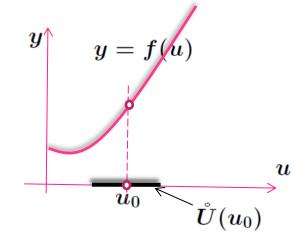
(推广:有限四则运算均成立!)

设y = f(u)在 $u \in \mathring{U}(u_0)$ 上有定义,



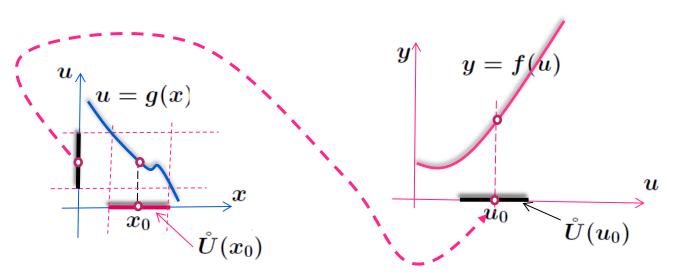
设y=f(u)在 $u\in \mathring{U}(u_0)$ 上有定义, u=g(x)在 $x\in \mathring{U}(x_0)$ 上有定义,





设y=f(u)在 $u\in \mathring{U}(u_0)$ 上有定义,u=g(x)在 $x\in \mathring{U}(x_0)$ 上有定义,

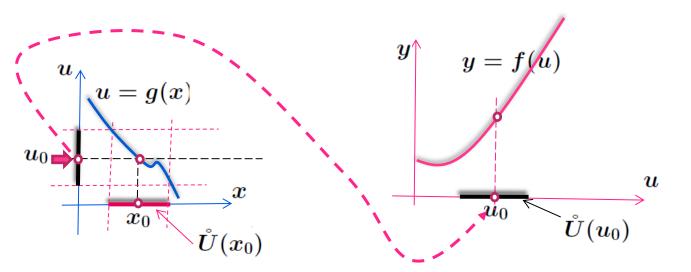
且 $x\in \mathring{U}(x_0)\Rightarrow u=g(x)\in \mathring{U}(u_0)$.



设y=f(u)在 $u\in \mathring{U}(u_0)$ 上有定义, u=g(x)在 $x\in \mathring{U}(x_0)$ 上有定义,

且
$$x \in \mathring{U}(x_0) \Rightarrow u = g(x) \in \mathring{U}(u_0)$$
.

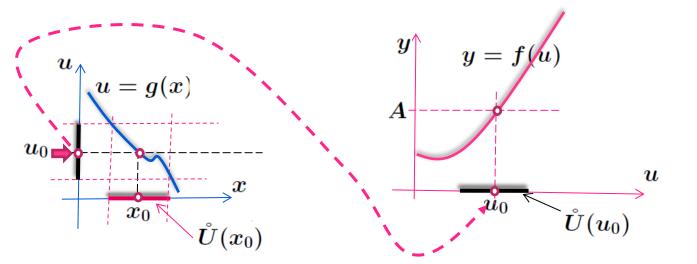
若有 $\lim_{x \to x_0} g(x) = u_0$,



设
$$y = f(u)$$
在 $u \in \mathring{U}(u_0)$ 上有定义, $u = g(x)$ 在 $x \in \mathring{U}(x_0)$ 上有定义,

且
$$x \in \mathring{U}(x_0) \Rightarrow u = g(x) \in \mathring{U}(u_0)$$
.

若有
$$\lim_{x\to x_0}g(x)=u_0, \quad \lim_{u\to u_0}f(u)=A,$$

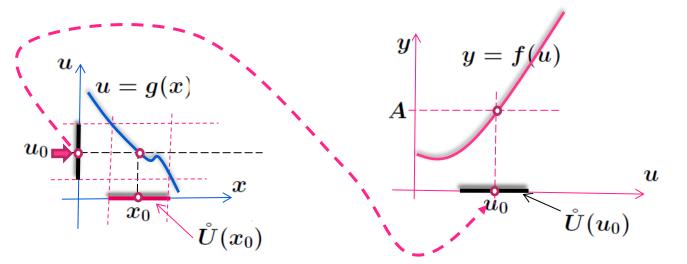


设
$$y = f(u)$$
在 $u \in \mathring{U}(u_0)$ 上有定义, $u = g(x)$ 在 $x \in \mathring{U}(x_0)$ 上有定义,

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若有
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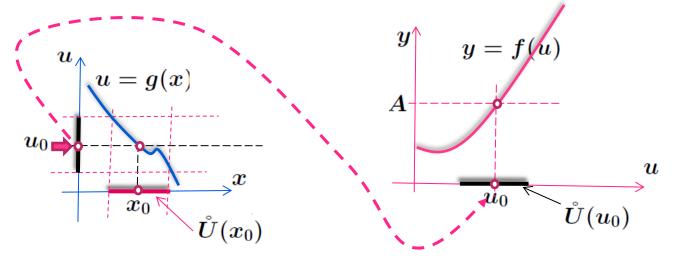
則
$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
.



证明: 往证 orall arepsilon > 0, $\exists \delta > 0$ s.t. $0 < |x - x_0| < \delta \Rightarrow |f(g(x)) - A| = |f(u) - A| < arepsilon$.

若有
$$\lim_{x \to x_0} g(x) = u_0, \quad \lim_{u \to u_0} f(u) = A,$$

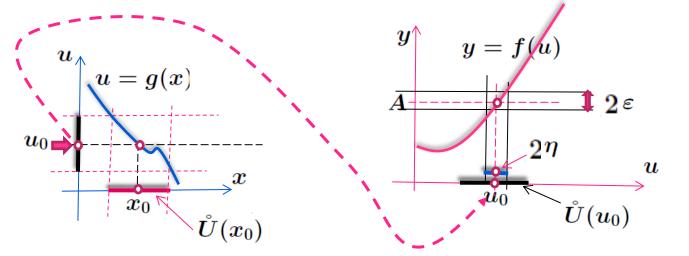
则
$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
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证明: 往证 $\forall \varepsilon > 0, \ \exists \delta > 0 \ s.t. \ 0 < |x-x_0| < \delta \ \Rightarrow |f(g(x)) - A| = |f(u) - A| < \varepsilon.$

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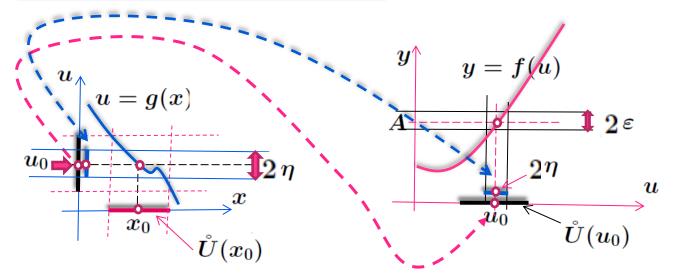
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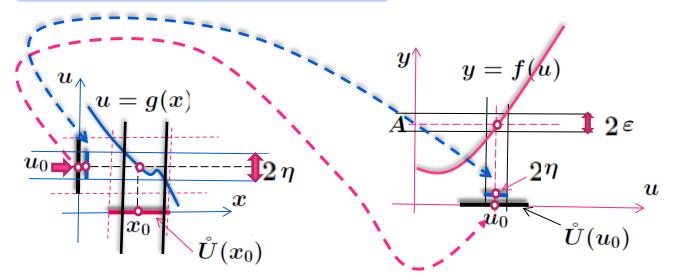
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证明: 往证 orall arepsilon > 0, $\exists \delta > 0$ s.t. $0 < |x - x_0| < \delta \Rightarrow |f(g(x)) - A| = |f(u) - A| < arepsilon$.

若有
$$\lim_{x\to x_0}g(x)=u_0,\quad \lim_{u\to u_0}f(u)=A,$$

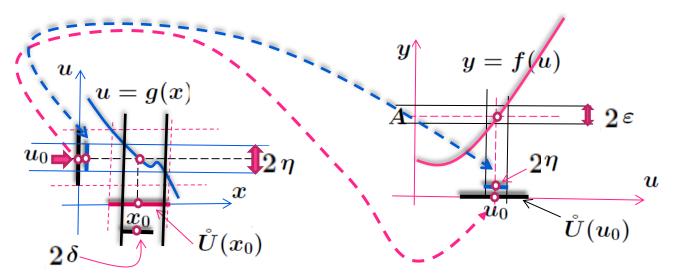
則
$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
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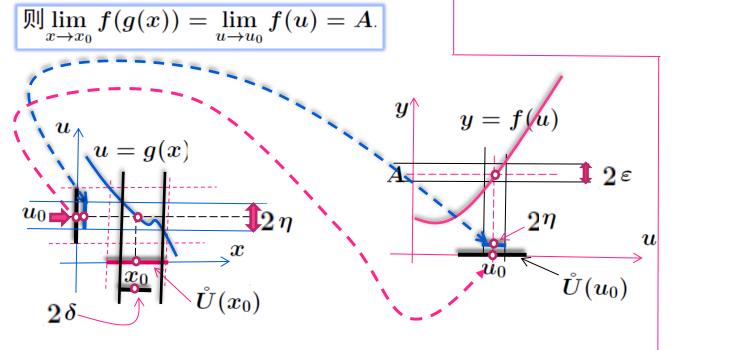
证明: 往证 orall arepsilon > 0, $\exists \delta > 0$ s.t. $0 < |x - x_0| < \delta \Rightarrow |f(g(x)) - A| = |f(u) - A| < \varepsilon$.

若有
$$\lim_{x \to x_0} g(x) = u_0, \quad \lim_{u \to u_0} f(u) = A,$$

則
$$\lim_{x o x_0} f(g(x)) = \lim_{u o u_0} f(u) = A$$
.



证明: 往证 $\forall \varepsilon > 0$, $\exists \delta > 0$ s.t. $0 < |x - x_0| < \delta \Rightarrow |f(g(x)) - A| = |f(u) - A| < \varepsilon$. 对上述的 $\varepsilon > 0$,因为 $\lim_{u \to u_0} f(u) = A$,所以 $\exists \eta > 0$ s.t. $0 < |u - u_0| < \eta \Rightarrow |f(u) - A| < \varepsilon$; 若有 $\lim_{x \to x_0} g(x) = u_0$, $\lim_{u \to u_0} f(u) = A$,



证明: 往证 $\forall \varepsilon > 0, \ \exists \delta > 0 \ s.t. \ 0 < |x - x_0| < \delta \Rightarrow |f(g(x)) - A| = |f(u) - A| < \varepsilon.$ 对上述的 $\varepsilon > 0$,因为 $\lim_{n \to \infty} f(u) = A$,所以 又由于 $\lim g(x) = u_0$, 故对于 $\eta > 0$, $\exists \eta > 0 \ s.t. \ 0 < |u - u_0| < \eta \Rightarrow |f(u) - A| < \varepsilon;$ $\exists \delta > 0 \ s.t. \ 0 < |x - x_0| < \delta$ $\Rightarrow |u-u_0|=|g(x)-u_0|<\eta$ 若有 $\lim_{x\to x_0}g(x)=u_0,\quad \lim_{u\to u_0}f(u)=A,$ 则 $\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$. u = g(x) $\mathring{U}(u_0)$

 $\mathring{U}(x_0)$

 2δ

证明: 往证 $\forall \varepsilon > 0, \ \exists \delta > 0 \ s.t. \ 0 < |x - x_0| < \delta \Rightarrow |f(g(x)) - A| = |f(u) - A| < \varepsilon.$ 又由于 $\lim g(x) = u_0$, 故对于 $\eta > 0$, 对上述的 $\varepsilon > 0$,因为 $\lim_{n \to \infty} f(u) = A$,所以 $\exists \eta > 0 \ s.t. \ 0 < |u - u_0| < \eta \Rightarrow |f(u) - A| < \varepsilon$ $\exists \delta > 0 \ s.t. \ 0 < |x - x_0| < \delta$ $\Rightarrow |u-u_0|=|g(x)-u_0|<\eta$ 若有 $\lim_{x\to x_0}g(x)=u_0,\quad \lim_{u\to u_0}f(u)=A,$ 从而 則 $\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$. $\forall \varepsilon > 0, \ \exists \delta > 0 \ s.t. \ 0 < |x - x_0| < \delta$ $\Rightarrow |f(g(x)) - A| = |f(u) - A| < \varepsilon$ y = f(u)u = g(x) $\mathring{U}(u_0)$ $\mathring{U}(x_0)$ 2δ

设
$$y = f(u)$$
在 $u \in \mathring{U}(u_0)$ 上有定义, $u = g(x)$ 在 $x \in \mathring{U}(x_0)$ 上有定义,

且
$$x \in \mathring{U}(x_0) \Rightarrow u = g(x) \in \mathring{U}(u_0)$$
.

若有
$$\lim_{x\to x_0}g(x)=u_0,\quad \lim_{u\to u_0}f(u)=A,$$

則
$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
.

此结论对所有的极限过程都成立。

设
$$y = f(u)$$
在 $u \in \mathring{U}(u_0)$ 上有定义, $u = g(x)$ 在 $x \in \mathring{U}(x_0)$ 上有定义,

且
$$x \in \mathring{U}(x_0) \Rightarrow u = g(x) \in \mathring{U}(u_0)$$
.

若有 $\lim_{x\to x_0}g(x)=u_0,\quad \lim_{u\to u_0}f(u)=A,$

則
$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
.

注意:
$$\lim_{x \to x_0} g(x) = u_0, \lim_{u \to u_0} f(u) = A$$
时,

$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
 .

不能导致

$$\lim_{x o x_0}f(g(x))=f(\lim_{x o x_0}g(x))=f(u_0)$$
 .

设
$$y = f(u)$$
在 $u \in \mathring{U}(u_0)$ 上有定义, $u = g(x)$ 在 $x \in \mathring{U}(x_0)$ 上有定义,

且
$$x \in \mathring{U}(x_0) \Rightarrow u = g(x) \in \mathring{U}(u_0)$$
.

若有
$$\lim_{x\to x_0}g(x)=u_0,\quad \lim_{u\to u_0}f(u)=A$$
,

則
$$\lim_{x o x_0}f(g(x))=\lim_{u o u_0}f(u)=A$$
.

注意:
$$\lim_{x \to x_0} g(x) = u_0$$
, $\lim_{u \to u_0} f(u) = A$ 时,

$$\lim_{x\to x_0} f(g(x)) = \lim_{u\to u_0} f(u) = A.$$

不能导致
$$\lim_{x \to x_0} f(g(x))$$
 $f(\lim_{x \to x_0} g(x)) = f(u_0)$.

这是因为
$$\lim_{n \to \infty} f(u)$$
不一定= $f(u_0)$.

例如:
$$f(u)=\left\{ egin{array}{ll} u\sin u, \ u
eq 0, \ 1, \ u=0, \end{array}
ight. \ g(x)=(x-1)^2, \ \lim_{x
ightarrow 1} g(x)=0, \ \iint f(g(x))=\left\{ egin{array}{ll} (x-1)^2\sin(x-1)^2, \ x
eq 1, \ 1, \ x=1, \end{array}
ight. \ \left. \lim_{x
ightarrow 1} f(g(x))=\lim_{x
ightarrow 1} f(u)=0
eq f(0) \end{array}
ight.$$

$$\lim_{x \to 1} f(g(x)) = \lim_{u \to 0} f(u) = 0 \neq f(0)$$
 $f(0) = f(\lim_{x \to 1} g(x)) = 1.$

注意:
$$\lim_{x \to x_0} g(x) = u_0, \lim_{u \to u_0} f(u) = A$$
时,一般只能导致 $\lim_{x \to x_0} f(g(x)) = \lim_{u \to u_0} f(u) = A$.

不能导致 $\lim_{x o x_0} f(g(x))$ 人 $f(\lim_{x o x_0} g(x)) = f(u_0)$.

这是因为
$$\lim_{u\to u_0} f(u)$$
不一定= $f(u_0)$.

推论: $\lim_x u(x) = A > 0 \quad \Rightarrow \lim_x \ln u(x) = \lim_{u \to A} \ln u = \ln A$

推论: $\lim_{x} u(x) = A > 0 \quad \Rightarrow \lim_{x} \ln u(x) = \lim_{u \to A} \ln u = \ln A$

$$\lim_{x} u(x) = A > 0 \quad \Rightarrow \lim_{x} \ln u(x) = \lim_{u \to A} \ln u = \ln A$$

推论:
$$\lim_x u(x) = A \qquad \Rightarrow \lim_x e^{u(x)} = \lim_{u \to A} e^u = e^A$$

推论: $\lim_{x} u(x) = A > 0 \quad \Rightarrow \lim_{x} \ln u(x) = \lim_{u \to A} \ln u = \ln A$

$$x$$
 x $u{ o}A$

推论:
$$\lim_x u(x) = A \qquad \Rightarrow \lim_x e^{u(x)} = \lim_{u \to A} e^u = e^A$$

推论: 若
$$\lim_{x \to a} f(x) = A$$
, 则 $\lim_{x \to a} \sin f(x) = \sin A$

若
$$\lim_{x o x_0}f(x)=A$$
,则 $\lim_{x o x_0}\sin f(x)=\sin A$

推论: $\lim_{x} u(x) = A > 0 \quad \Rightarrow \lim_{x} \ln u(x) = \lim_{u \to A} \ln u = \ln A$

$$x$$
 $u{ o}A$

推论:
$$\lim u(x) = A \qquad \Rightarrow \lim e^{u(x)} - \lim e^{u} - e^{A}$$

$$\lim_x u(x) = A \qquad \Rightarrow \lim_x e^{u(x)} = \lim_{u \to A} e^u = e^A$$

$$\lim_{x} u(x) = A \qquad \Rightarrow \lim_{x} e \quad \Rightarrow \lim_{u \to A} e = e$$

推论: 若
$$\lim_{x \to a} f(x) = A$$
, 则 $\lim_{x \to a} \sin f(x) = \sin A$

若
$$\lim_{x o x_0}f(x)=A$$
,则 $\lim_{x o x_0}\sin f(x)=\sin A$

$$(|\sin f(x) - \sin A| \leqslant |f(x) - A|).$$

推论: $\lim_x u(x) = A > 0 \quad \Rightarrow \lim_x \ln u(x) = \lim_{u \to A} \ln u = \ln A$

$$x$$
 $u \rightarrow A$

推论:
$$\lim_x u(x) = A$$
 $\Rightarrow \lim_x e^{u(x)} = \lim_{u \to A} e^u = e^A$

$$\lim_{x} a(x) = A \qquad \Rightarrow \lim_{x} e^{-x} = \lim_{u \to A} e^{-u} = e$$

若
$$\lim_{x o x_0}f(x)=A$$
,则 $\lim_{x o x_0}\sin f(x)=\sin A$

推论:
$$\lim_x u(x) = A \geqslant 0 \quad \Rightarrow \lim_x u^{\alpha}(x) = \lim_{u \to A} u^{\alpha} = A^{\alpha}$$

推论:

 $\lim_{x} u(x) = A > 0$, $\lim_{x} v(x) = B$ $\Rightarrow \lim_{x} u(x)^{v(x)} = A^{B}$.

推论:

損敗に
$$\lim_x u(x) = A > 0, \ \lim_x v(x) = B \quad \Rightarrow \lim_x u(x)^{v(x)} = A^B$$
.

事实上
$$\lim_x u(x)^{v(x)} = \lim_x e^{v(x) \ln u(x)}$$

推论: $\lim_x u(x) = A > 0, \ \lim_x v(x) = B \quad \Rightarrow \lim_x u(x)^{v(x)} = A^B.$

$$\lim_{x} u(x) = A > 0, \lim_{x} v(x) = B \longrightarrow \lim_{x} u(x) = A.$$

事实上 $\lim_x u(x)^{v(x)} = \lim_x e^{v(x) \ln u(x)}$

$$\overline{\mathbb{I}}\lim_x u(x) = A > 0$$

$$\Rightarrow \lim_x \ln u(x) = \lim_{u \to A} \ln u = \ln A$$

所以 $\lim e^{v(x)\ln u(x)} = e^{B\ln A} = A^B.$ 推论:

$$\lim_x u(x) = A > 0, \ \lim_x v(x) = B \quad \Rightarrow \lim_x u(x)^{v(x)} = A^B.$$

注意, 只要 $A \neq 1$, 上述公式对 $B = +\infty$, $B = -\infty$ 的情况也成立. 推论:

$$\lim_{x} u(x) = A > 0$$
, $\lim_{x} v(x) = B$ $\Rightarrow \lim_{x} u(x)^{v(x)} = A^{B}$.

注意, 只要
$$A \neq 1$$
,
上述公式对 $B = +\infty$, $B = -\infty$ 的情况也成立.

$$\lim_{n \to \infty} (1 + \frac{1}{n})^n = e, \ \lim_{n \to \infty} (1 + \frac{1}{n})^{-n} = e^{-1}$$

例: $\lim_{x o 1}rac{x^2-1}{2x^2-x-1}$

例:
$$\lim_{x o 1}rac{x^2-1}{2x^2-x-1}$$

 $= \lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)(2x+1)}$

例:
$$\lim_{x o 1}rac{x^2-1}{2x^2-x-1}$$

 $= \lim_{x \to 1} \frac{x+1}{2x+1}$

$$= \lim_{x \to 1} 2x^2 - x - 1$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)(2x+1)}$$

$$-x-1$$

$$-x-1$$

$$\overline{r-1}$$

$$\frac{1}{x-1}$$

$$\frac{1}{-1}$$

例:
$$\lim_{x o 1}rac{x^2-1}{2x^2-x-1}$$

$$x \to 1 \ 2x^2 - x - 1$$

$$x \rightarrow 1$$
 $2x^2 - x - 1$
$$(x - 1)(x$$

$$x \rightarrow 1$$
 $2x^2 - x - 1$
$$(x - 1)(x + 1)$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{(x-1)(2x+1)}$$

$$(x-1)(x+1)$$

 $= \lim_{x \to 1} \frac{x+1}{2x+1}$

 $= \frac{\lim_{x \to 1} (x+1)}{\lim_{x \to 1} (2x+1)} = \frac{2}{3}.$

$$x \rightarrow 1$$
 $2x^2 - x - 1$
$$(x - 1)(x + 1)$$

$$\overline{x-1}$$

例:
$$\lim_{x \to -1} \left(\frac{1}{x+1} - \frac{3}{x^3+1} \right)$$

 $= \lim_{x \to -1} \frac{x^2 - x + 1 - 3}{(x+1)(x^2 - x + 1)}$

例:
$$\lim_{x o -1}\left(rac{1}{x+1}-rac{3}{x^3+1}
ight)$$

例:
$$\lim \left(\frac{1}{3} - \frac{3}{3}\right)$$

 $= \lim_{x \to -1} \frac{x^2 - x + 1 - 3}{(x+1)(x^2 - x + 1)}$

 $= \lim_{x \to -1} \frac{x^2 - x - 2}{(x+1)(x^2 - x + 1)}$

例:
$$\lim_{x \to a} \left(\frac{1}{x^2 + a^2} - \frac{3}{x^2 + a^2} \right)$$

例:
$$\lim_{x o -1} \left(rac{1}{x+1} - rac{3}{x^3+1}
ight)$$

例:
$$\lim_{x \to 1} \left(\frac{1}{x+1} - \frac{3}{x^3+1} \right)$$

例:
$$\lim_{x o -1}\left(rac{1}{x+1}-rac{3}{x^3+1}
ight)$$

 $= \lim_{x \to -1} \frac{x^2 - x + 1 - 3}{(x+1)(x^2 - x + 1)}$

 $= \lim_{x \to -1} \frac{x^2 - x - 2}{(x+1)(x^2 - x + 1)}$

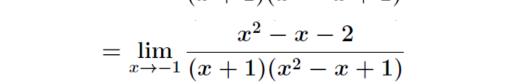
 $= \lim_{x \to -1} \frac{(x+1)(x-2)}{(x+1)(x^2-x+1)}$

例:
$$\lim_{n \to \infty} \left(\frac{1}{n+1} - \frac{3}{n^3+1} \right)$$

例:
$$\lim_{x \to -1} \left(\frac{1}{x+1} - \frac{3}{x^3+1} \right)$$

$$= \lim_{x \to -1} \frac{x^2 - x + 1 - 3}{(x+1)(x^2 - x + 1)}$$

$$x^2 - x - 2$$



$$x \to -1 \ (x+1)(x^2 - x + 1)$$

$$= \lim_{x \to -1} \frac{(x+1)(x-2)}{(x+1)(x^2 - x + 1)}$$

$$= \lim_{x \to -1} \frac{(x+1)(x-2)}{(x+1)(x^2 - x + 1)}$$

$$= \lim_{x \to -1} \frac{x-2}{x^2 - x + 1}$$

$$\lim_{x \to -1} \frac{(x+1)(x-2)}{(x+1)(x^2 - x + 1)}$$

$$= \lim_{x \to -1} \frac{(x+1)(x^2 - x + 1)}{(x+1)(x^2 - x + 1)}$$

$$= \lim_{x \to -1} \frac{x-2}{x^2 - x + 1}$$

$$x o -1 (x+1)(x^2 - x + 1)$$

$$= \lim_{x o -1} \frac{x-2}{x^2 - x + 1}$$

$$= \lim_{x o -1} (x^2 - x + 1) = \frac{-3}{3} = -1.$$

例: $\lim_{x o +\infty} x (\sqrt{x^2+1}-x)$

例: $\lim_{x \to +\infty} x(\sqrt{x^2+1}-x)$

$$= \lim_{x \to +\infty} \frac{x}{\sqrt{x^2 + 1} + x}$$

 $= \lim_{x \to +\infty} \frac{x}{\sqrt{x^2 + 1} + x}$

 $=\lim_{x\to +\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}$

例:
$$\lim_{x o +\infty} x(\sqrt{x^2+1}-x)$$

例:
$$\lim x(\sqrt{x^2+1}-x)$$

$$\langle F_{\parallel} \rangle = 1$$
: $\sim \sim \left(\sqrt{\sim^2 + 1} \right)$

例: $\lim_{x \to +\infty} x(\sqrt{x^2+1}-x)$

 $=\lim_{x\to +\infty}\frac{x}{\sqrt{x^2+1}+x}$

 $= \lim_{x \to +\infty} \frac{1}{\sqrt{1 + \frac{1}{x^2} + 1}}$

例:
$$\lim_{x o\infty}\left(rac{x^2+1}{x+1}-x
ight)$$

$$(x^2+1)$$

例:
$$\lim \left(\frac{x^2+1}{x^2}-x\right)$$

 $= \lim_{x \to \infty} \frac{x^2 + 1 - x^2 - x}{x + 1}$ $= \lim_{x \to \infty} \frac{1 - x}{x + 1}$

 $= \lim_{x \to \infty} \frac{1/x - 1}{1 + 1/x} = \frac{-1}{1} = -1.$

例:
$$\lim_{x o\infty}\left(rac{x^2+1}{x+1}-x
ight)$$

例:
$$\lim_{x \to 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$$

•

例:
$$\lim_{x o 8}rac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$$

$$\overline{z} - 2$$

$$\overline{x}-2$$

$$\frac{2x-}{\overline{c}-2}$$

$$\frac{2x-x}{x-2}$$

$$\frac{x-5}{-2}$$

$$\frac{\overline{c}-5}{2}$$

$$\frac{5-5}{2}$$

$$\frac{5-5}{2}$$



$$\frac{-5}{2}$$





 $= \lim_{x \to 8} \frac{(\sqrt{9+2x}-5)(\sqrt{9+2x}+5)}{\sqrt{9+2x}+5} \frac{\sqrt[3]{x^2}+\sqrt[3]{x}+4}{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+\sqrt[3]{x}+4)}$



例:
$$\lim_{x \to 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$$

例:
$$\lim_{x \to 8} \frac{\sqrt{3} + 2x}{\sqrt[3]{x} - 2}$$

$$= \lim_{x \to 8} \frac{(\sqrt{9 + 2x} - 5)(\sqrt{9 + 2x} + 5)}{\sqrt{9 + 2x} + 5} \frac{\sqrt[3]{x^2} + \sqrt[3]{x} + 4}{(\sqrt[3]{x} - 2)(\sqrt[3]{x^2} + \sqrt[3]{x} + 4)}$$

$$\frac{2\omega}{-2}$$

 $= \lim_{x \to 8} \frac{2x - 16}{\sqrt{9 + 2x} + 5} \frac{\sqrt[3]{x^2} + \sqrt[3]{x} + 4}{x - 8}$

 $= \lim_{x \to 8} \frac{2(\sqrt[3]{x^2} + \sqrt[3]{x} + 4)}{\sqrt{9 + 2x} + 5}$ $= \frac{20}{10} = 2.$

本段知识要点:

和差积商的极限等于极限的和差积商

复合函数的极限等于极限的复合

不能直接使用极限运算规则的函数要进行变形

分式求极限的关键是约分掉无穷小量或无穷大量



