Econ 139 Lecture 19 Notes

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1 State-Price Vectors

	Asset 1	**Asset 2**	**Asset 3**	**AD 1**	**AD 2**	**RF**
State 1	3	5	4	1	0	1
State 2	1	5	3	0	1	1
Price (P)	2	4	3	q_1	q_2	

$$\text{Let } p = (2 \quad 4 \quad 3)^T$$
$$q = (q_1 \quad q_2)^T$$
$$X = \begin{pmatrix} 3 & 5 & 4 \\ 1 & 5 & 3 \end{pmatrix}$$

*Recall: $q = (XX^T)^{-1}Xp$ if market is complete

Uniqueness of State-Price Vector: If LOOP holds and the market is complete, then a state-price vector must exist and be unique.

Options can be used to complete a market:

	Stock	**C(1)**	**C(2)**	**C(3)**
State 1	1	0	0	0
State 2	2	1	0	0
State 3	3	2	1	0
State 4	4	3	2	1
P	s_0	$C_0(1)$	$C_0(2)$	$C_0(3)$

*C(1) = call with strike of 1

$$p = (s_0 \quad C_0(1) \quad C_0(2) \quad C_0(3))^T$$
$$q = (q_1 \quad q_2 \quad q_3 \quad q_4)$$
$$X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

*Recall: $p = X^T$; $q = (X^T)^{-1}p$

$$q = \begin{pmatrix} s_0 - 2C_0(1) + C_0(2) \\ C_0(1) - 2C_0(2) + C_0(3) \\ C_0(2) - 2C_0(3) \\ C_0(3) \end{pmatrix}$$

Using puts:

	Stock	**P(4)**	**P(3)**	**P(4)**
State 1	1	3	2	1
State 2	2	2	1	0
State 3	3	1	0	0
State 4	4	0	0	0
P	s_0	$P_0(4)$	$P_0(3)$	$P_0(2)$

$$q = \begin{pmatrix} P_0(2) \\ P_0(3) - 2P_0(2) \\ P_0(4) - 2P_0(3) + P_0(2) \\ s_0 - 2P_0(4) + P_0(3) \end{pmatrix}$$

Proposition: Provided there is a security/portfolio of securities that has a different payout in every state, and call and put options can be written on it, then the market can be completed.

2 Risk-Neutral Valuation (AKA Martingale Pricing)

One approach to valuation: $p(x) = \frac{E[X]}{1+r_f}$
Alternative approach: consider state-contigent payouts

New approach: Risk-Neutral Valuation/Martingale Pricing
Setup: 2 dates ($t = 1$ and $t = 0$)
Let π_θ be the probability that state θ occurs ($\pi_\theta > 0$ for all θ)
There are M fundamental securities
$X_i(0)$ is the current price of security i
$X_i(\theta, 1)$ is the payoff in state θ at $t = 1$
There is 1 risk-free asset that pays $(1 + r_f)$ in every state
No assumptions about distribution of pricing

Goal: Attempt to find risk-neutral probability measure π_θ^{RN} such that
$X_i(0) = \frac{1}{1+r_f} \sum_{\theta=1}^{N} \pi_\theta^{RN} X(\theta, 1)$ for all i

Martingale measure: In probability theory, a stochastic process ($X = x_1, x_1, ..., x_T$) is called a martingale under a probability measure P if the conditional expectation $E[X_t|I_{t-1}] = X_{t-1}$ where I is information.
- Also known as a P-martingale

1st Fundamental Theorem of Asset Pricing: There are no arbitrage opportunities among fundamental securities iif there exists a probability measure P such that every discounted price process is a P-martingale.

$E_P[\frac{\tilde{x}}{1+r_f}] = X(0)$

Finding a risk-neutral measure amounts to collecting equations $X_i(0) = \frac{1}{1+r_f}\sum_{\theta=1}^{N}\pi_\theta^{RN}X_i(\theta,1)$ and $\sum_{\theta=1}^{N}\pi_\theta^{RN} = 1$ where $\pi_\theta^{RN} > 0\forall\theta$ and solving the system of equations

π_θ and $\pi_\theta^R N$ are said to be equivilent

Ex.

	Bond	**Stock**
State 1	1.1	3
State 2	1.1	7
P	1	4

Discounted Price Process:

Bond	**Stock**
1	3/1.1
1	7/1.1

$$\pi_1^{RN} * 1 + \pi_2^{RN} * 1 = 1$$
$$\pi_1^{RN} * \frac{3}{1.1} + \pi_2^{RN} * \frac{7}{1.1} = 4$$

$$\pi_1^{RN} = 0.65,\ \pi_2^{RN} = 0.35$$