

Li-An Zhou

***Guanghua School of Management
Peking University***



INTERMEDIATE MICROECONOMICS

Introduction

- The course will cover the basic principles of microeconomics, but in a more formal and structured way than the introductory economics does
 - *Classical price theory* (consumer choice, producer theory, market equilibrium analysis)
 - *Monopoly and Oligopoly*
 - *Game theory* and information economics
 - *Market failure* (externality and public goods)
 - *Welfare economics*

Introduction

- Demonstrate the **beauty** and **rigor** of microeconomic theories
- Also apply these analytical tools to the real world and use many real world examples to show the **relevance** and **power** of economic thinking
- The principles we will cover in this course provide the foundation of almost all theories in modern economics, and also ***distinguish economists from scholars in other disciplines***

Textbooks

- The main textbook I will use for this class is *Intermediate Microeconomics* by Hal Varian
- I may also turn to the following two texts for reference
 - *Microeconomics* by Robert Pindyck and Daniel Rubinfeld
 - *Microeconomic Theory* by Walter Nicholson.

Grading Policy

- There will be **five** homework assignments graded on a check, check-plus basis. They will count for **20 percent** of the final grade
- You can drop one of them. Late problem sets will not be accepted. **No exceptions.**
- You will have a **final exam** (accounting for **80 percent**) which is in-class and closed-book

Office hours

- **My office hours: 2-4 pm, Sunday, Rm. 405, Guanghua New Building**
- My contact info:
- zhoula@gsm.pku.edu.cn, Tel: 6275-0431
- TA will also hold office hours every week, and TA sections (details to be announced later in class)



Chapter I

What is Economics?

What does Economics study?

- Lionel Robbins (1935): allocation of scarce resources
- Gary Becker (1976): any *human behavior* related to the allocation of scarce resources
 - Family and marriage
 - Crime, religion, ...
 - Law

Methodological Features of Economics

- **Rationality**
- **Stable Preferences**
- **Equilibrium**
- **Efficiency**

Methodological Features

- **Rationality:** maximizing the object function of the decision-maker
 - It's not necessarily selfishness and can be consistent with altruism
 - A kind of simplification: we care about *average* behavior of people
 - We test hypothesis, not assumptions
 - **Using rationality assumption to rule out obvious cheating traps**

Methodological Features

- **Stable preferences:** constant throughout the model
 - Preferences are **unobservable** but constraints are: **falsifiability requirement**
 - Emphasis on man-made constraints and institutional design
 - The “moral blood” of real estate developers in China
 - Why are some animals dying out?
 - The Rise and fall of Somali pirates

Methodological Features

- **Equilibrium analysis:** a tool to aggregate behaviors of individuals and predict the outcome of human interactions
 - A Puzzle of Happiness: Californian vs. Oregon
 - *Peltzman effect* and offsetting behavior
 - Reinterpretation of Marriage Law in China:
Are Mothers-in-laws hurt by this legal change?

Methodological Features

- **Efficiency criterion:** a normative notion of optimality
 - Is the economic outcome efficient?
 - Is there any room for improvement in efficiency?
 - **Economists are chiefly concerned with efficiency implications of policies or behaviors**
 - **Why should we economists care?**

Themes of Microeconomics

- Microeconomics deals with **limits or constraints**
 - Limited budgets
 - Limited time
 - Limited ability to produce
- How do we make the most of limits?
- How do we allocate scarce resources?

Themes of Microeconomics

- Workers, firms and consumers must make **trade-offs**
 - Do I work or go on vacation?
 - Do I purchase a new car or save my money?
 - Do we hire more workers or buy new machinery?
- How are these trade-offs best made?

Themes of Microeconomics

- Consumers
 - Limited incomes
 - **Consumer theory** – describes how consumers maximize their well-being, using their preferences, to make decisions about trade-offs
 - How do consumers make decisions about consumption and savings?

Themes of Microeconomics

- Workers
 - Individuals decide when and if to enter the workforce
 - Trade-offs of working now or obtaining more education/training
 - What choices do individuals make in terms of jobs or workplaces?
 - How many hours do individuals choose to work?
 - Trade-off of labor and leisure

Themes of Microeconomics

- Firms
 - What types of products do firms produce?
 - Constraints on production capacity and financial resources create needs for trade-offs
 - **Theory of the Firm** – describes how these trade-offs are best made

Themes of Microeconomics

- Prices

- *Trade-offs are often based on prices faced by consumers and producers*
- Workers make decisions based on prices for labor – wages
- Firms make decisions based on wages and prices for inputs and on prices for the goods they produce

Themes of Microeconomics

- Prices
 - How are prices determined?
 - Centrally planned economies – governments control prices
 - Market economies – prices determined by interaction of market participants
 - **Markets** – collection of buyers and sellers whose interaction determines the prices of goods

Theories and Models

- Economics is concerned with explanation of observed phenomena
 - **Theories** are used to explain observed phenomena in terms of a set of basic rules and assumptions:
 - The Theory of the Firm
 - The Theory of Consumer Behavior
 - The Theory of Government behavior

Theories and Models

- Theories are used to make predictions
 - Economic models are created from theories
 - **Models** are mathematical representations used to make quantitative predictions
 - **Parsimony and richness:** the role of a map
 - ***There is no bad math, but bad model***

Theories and Models

- Validating a Theory
 - The validity of a theory is determined by the quality of its prediction, given the assumptions
 - Theories must be tested and refined
 - Theories are invariably imperfect – but give much insight into observed phenomena and leave a lot of room for extensions and refinements

Positive & Normative Analysis

- **Positive Analysis** – statements that describe the relationship of cause and effect
 - Questions that deal with explanation and prediction
 - What will be the impact of an import quota on foreign cars?
 - What will be the impact of an increase in the gasoline excise tax?

Positive & Normative Analysis

- **Normative Analysis** – analysis examining questions of what ought to be
 - Often supplemented by value judgments
 - Should the government impose a larger gasoline tax?
 - Should the government decrease the tariffs on imported cars?

Think Like Economists

- Using baseline models as benchmarks
 - Competitive markets
 - Theories of producers and consumers
 - What makes economists different?
- Ask interesting and deep questions
 - Significant deviations from baseline models serve as a starting place for asking questions
 - Coase's famous question
 - Soft budget constraint for SOEs
 - All classical theories will receive attacks and revisions but lay foundations for further progress

The Wisdom of Economics

- Hayek's warnings to the world
 - “***The road to hell paved with good intentions***”
 - The debate about the vitality of market vs. planning economy in 1930s
 - Malthusian view on population (1798)
- How to achieve the consistency between goals and means when making policies
 - Debate on the effect of the minimum wage

Dismal Science: The Current Version

- Economists in China are not popular in the eyes of the general public
- They make arguments which evoke heated debates and often invite negative comments
- Are they really the bad people standing only for the rich and against the poor?
- The tension between economists and general public is chiefly caused by the unique logic underlying economics



Chapter 2

The Market

How Do Economists Think?

- Supply-demand analysis is a fundamental and powerful tool that can be applied to a wide range of interesting and important issues
- We will examine a model of a particular market (apartment market in a college town)
- Give you a flavor of how economists establish a theory to analyze practical issues



Economic Modeling

What causes what in economic systems?

Which variables are determined outside the model (**exogenous**)

Which variables are to be determined by the model (**endogenous**)

At what level of detail shall we model an economic phenomenon? (**simplifying assumptions**)

Modeling the Apartment Market

- Central question: *How are apartment rents determined?*
- Suppose (simplifying assumptions)
 - apartments are close or distant, but otherwise identical
 - distant apartments rents are exogenous and known
 - many potential renters and landlords, i.e., *competitive market*

A Normative Question

- Will the allocation of apartments be desirable?
- Need to know:
 - Who will rent close apartments?
 - At what price?

Two Principles in Economics

- **Optimization:** Each person tries to choose the best alternative available to him or her
- **Equilibrium:** Market price adjusts until quantity demanded equals quantity supplied.

Modeling Individual Demand

- Discrete commodity: 0 or 1 unit
 - Choose either distant or close apartment
- Close apartments are more desirable but more expensive
- Tend to choose close apartment if
 - Distant apartments are also expensive.
 - Higher income
- Decide: The maximum rent you are willing to pay for a close apartment.

Modeling Market Demand

- Suppose the most any one person is willing to pay to rent a close apartment is \$500/month. Then $p = \$500 \Rightarrow Q^D = 1$
- **Reservation price:** a person's maximum willingness to pay for something
- Suppose the price has to drop to \$490 before a 2nd person would rent. Then $p = \$490 \Rightarrow Q^D = 2$

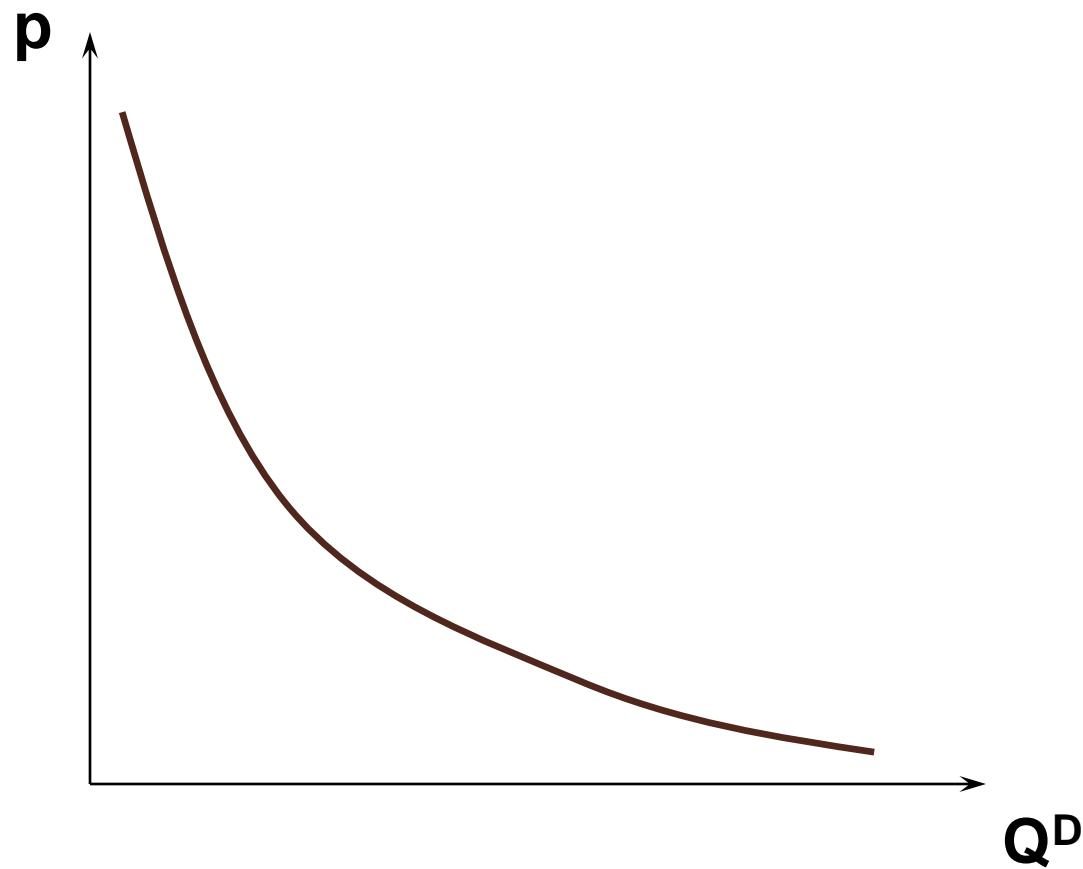
Modeling Apartment Demand

- The lower is the rental rate p , the larger is the quantity of close apartments demanded

$$p \downarrow \Rightarrow Q^D \uparrow$$

- The quantity demanded vs. price graph is the **market demand curve** for close apartments.

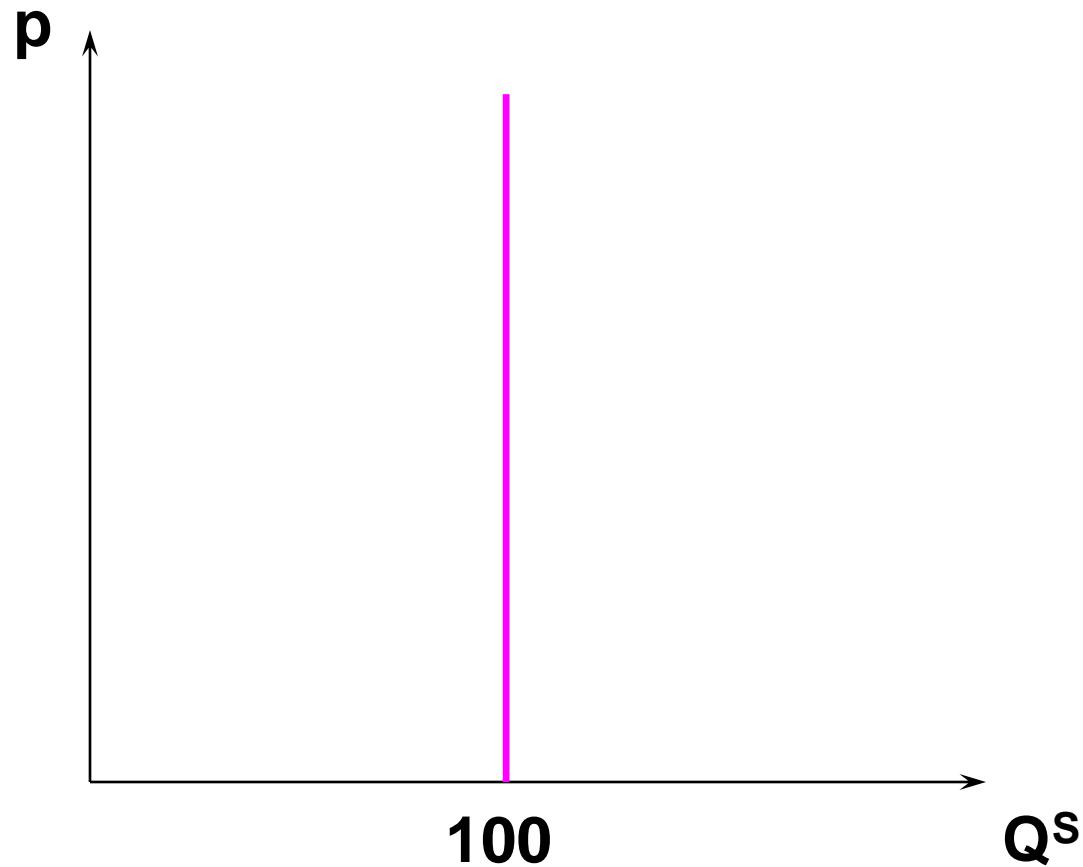
Market Demand Curve for Apartments



Modeling Apartment Supply

- **Supply:** It takes time to build more close apartments so in this short-run the quantity available is fixed (at say 100)

Market Supply Curve for Apartments



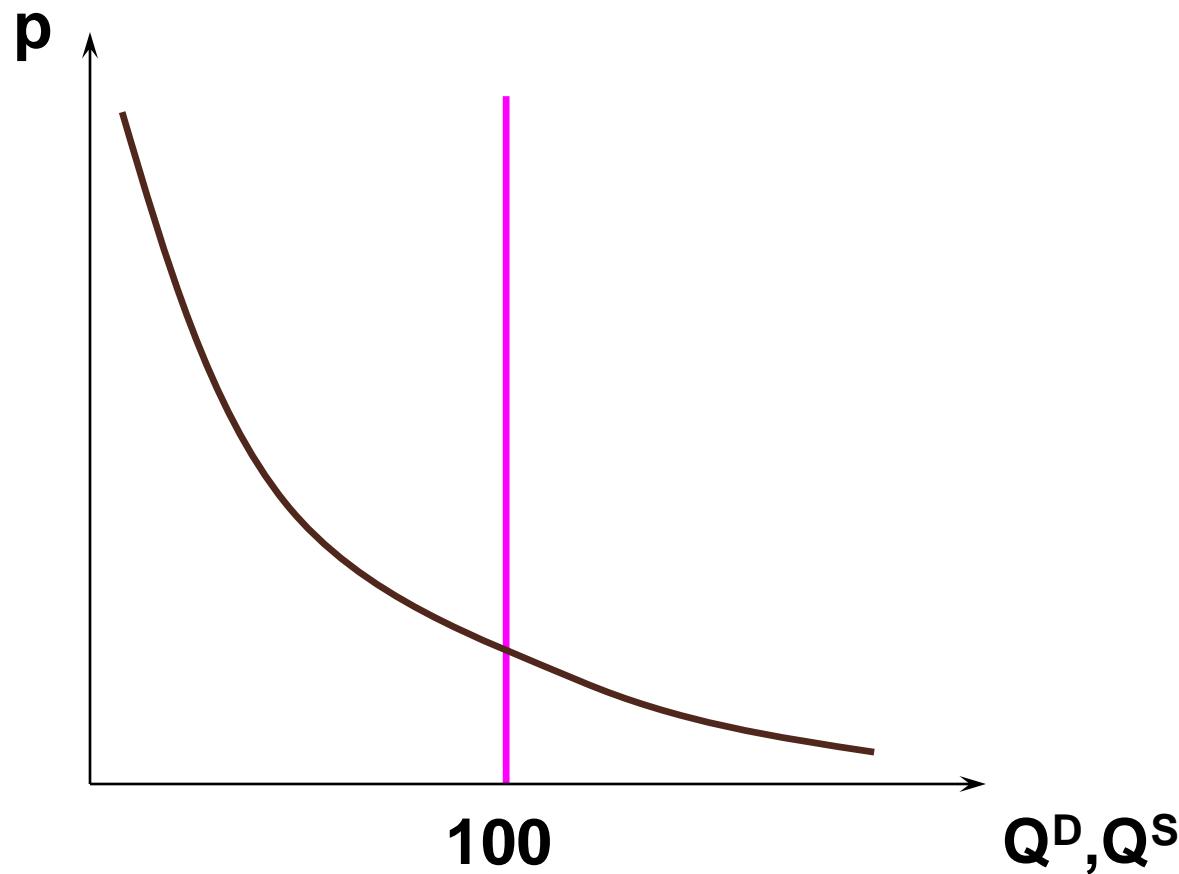
Competitive Market Equilibrium

- Assume that there are so many landlords and students in the markets that no one can influence the prevailing price in the market
 - **Price-takers**
- “low” rental price \Rightarrow quantity demanded of close apartments exceeds quantity available \Rightarrow price will rise
- “high” rental price \Rightarrow quantity demanded less than quantity available \Rightarrow price will fall.

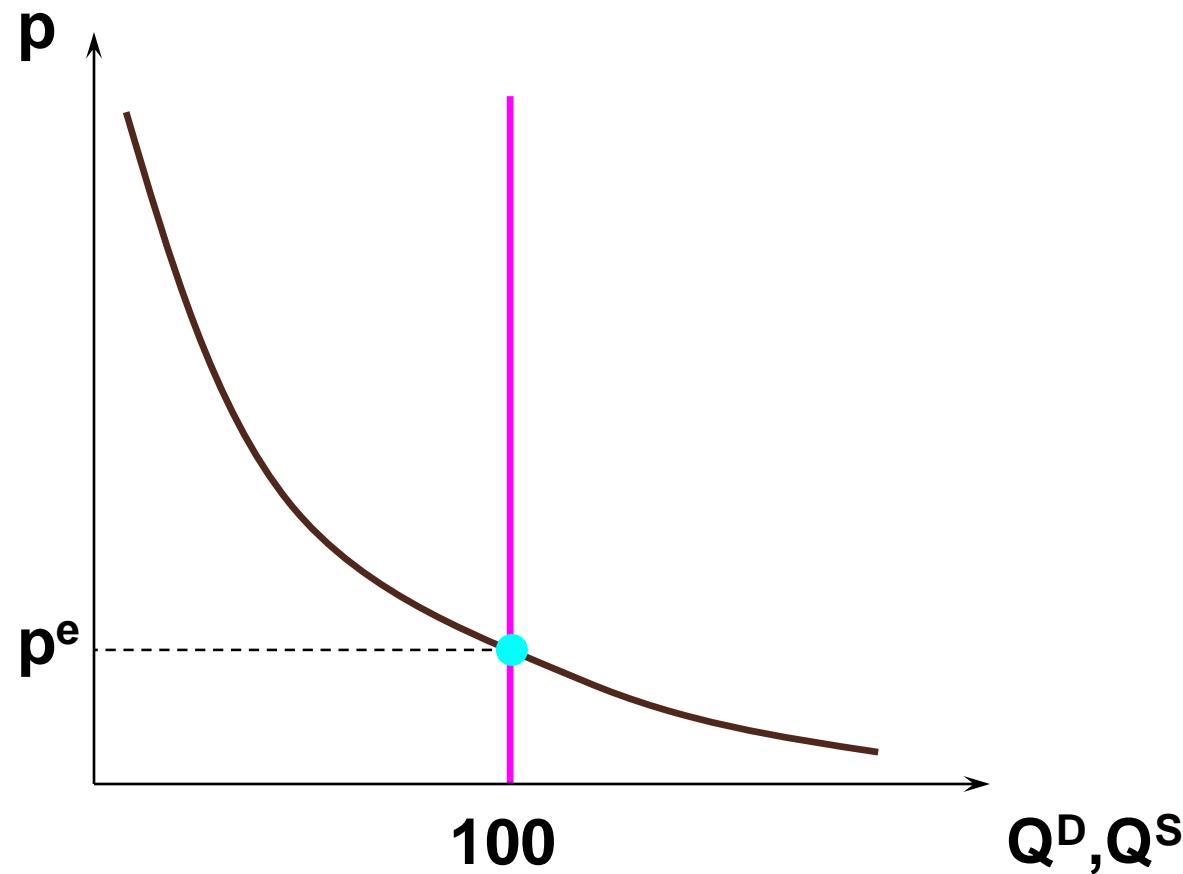
Competitive Market Equilibrium

- Quantity demanded = quantity available
⇒ price will neither rise nor fall
- so the market is at a **competitive equilibrium**
- **In the equilibrium no one is willing to change the decision he or she has made**

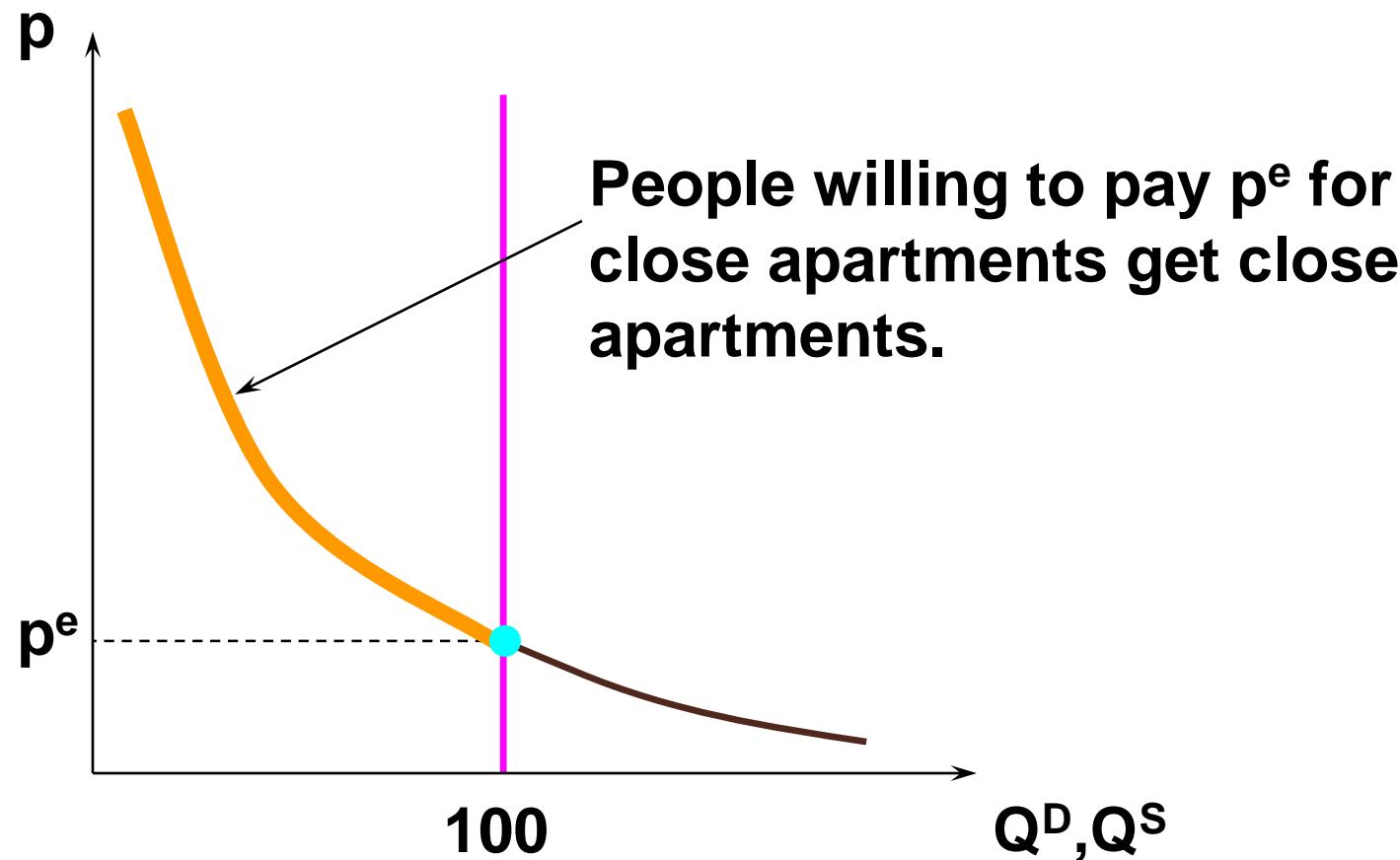
Competitive Market Equilibrium



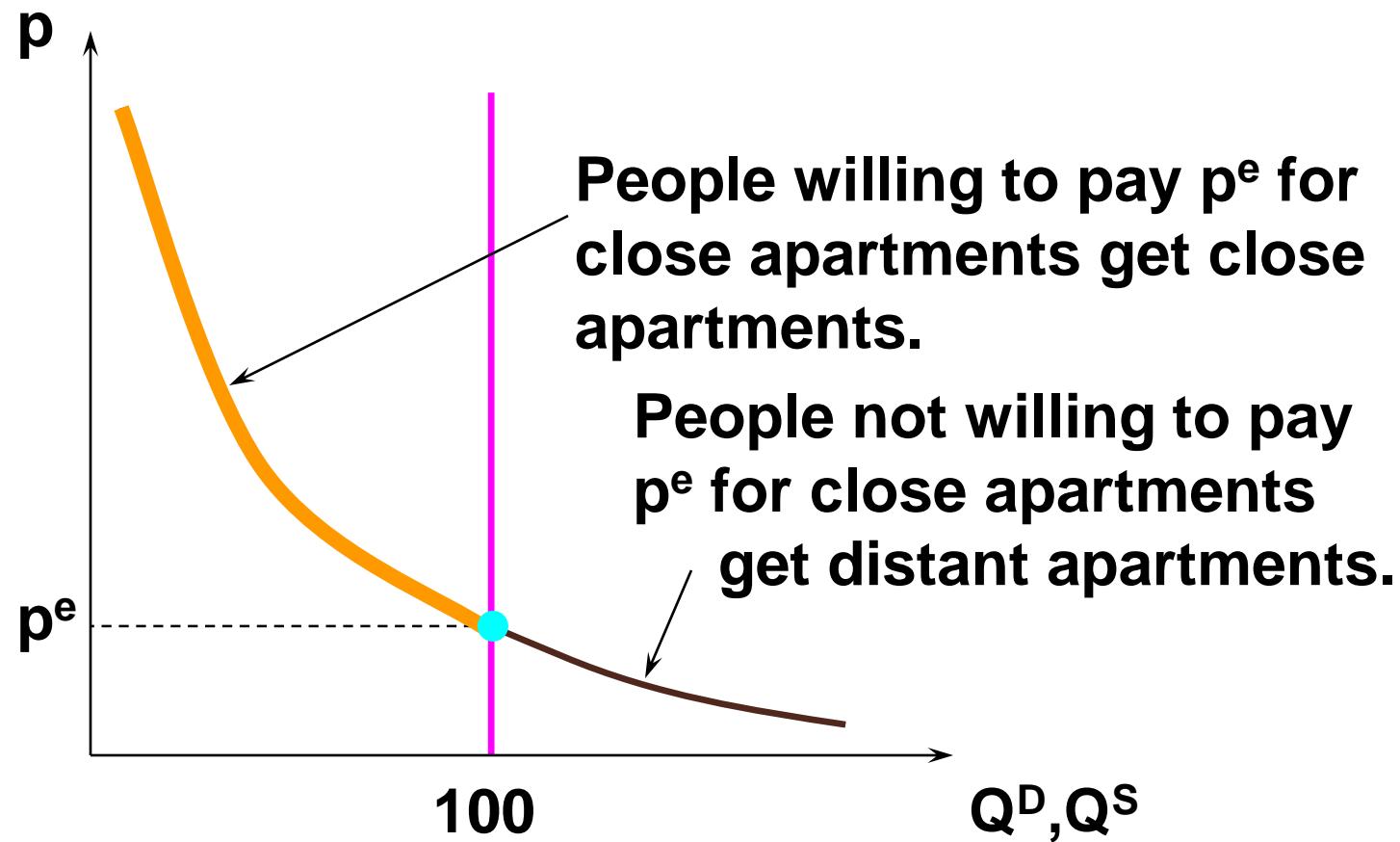
Competitive Market Equilibrium



Competitive Market Equilibrium



Competitive Market Equilibrium



Competitive Market Equilibrium

- Q: Who rents the close apartments?
A: Those most willing to pay.
- Q: Who rents the distant apartments?
A: Those least willing to pay.
- So the competitive market allocation is by “willingness-to-pay” (价高者得)

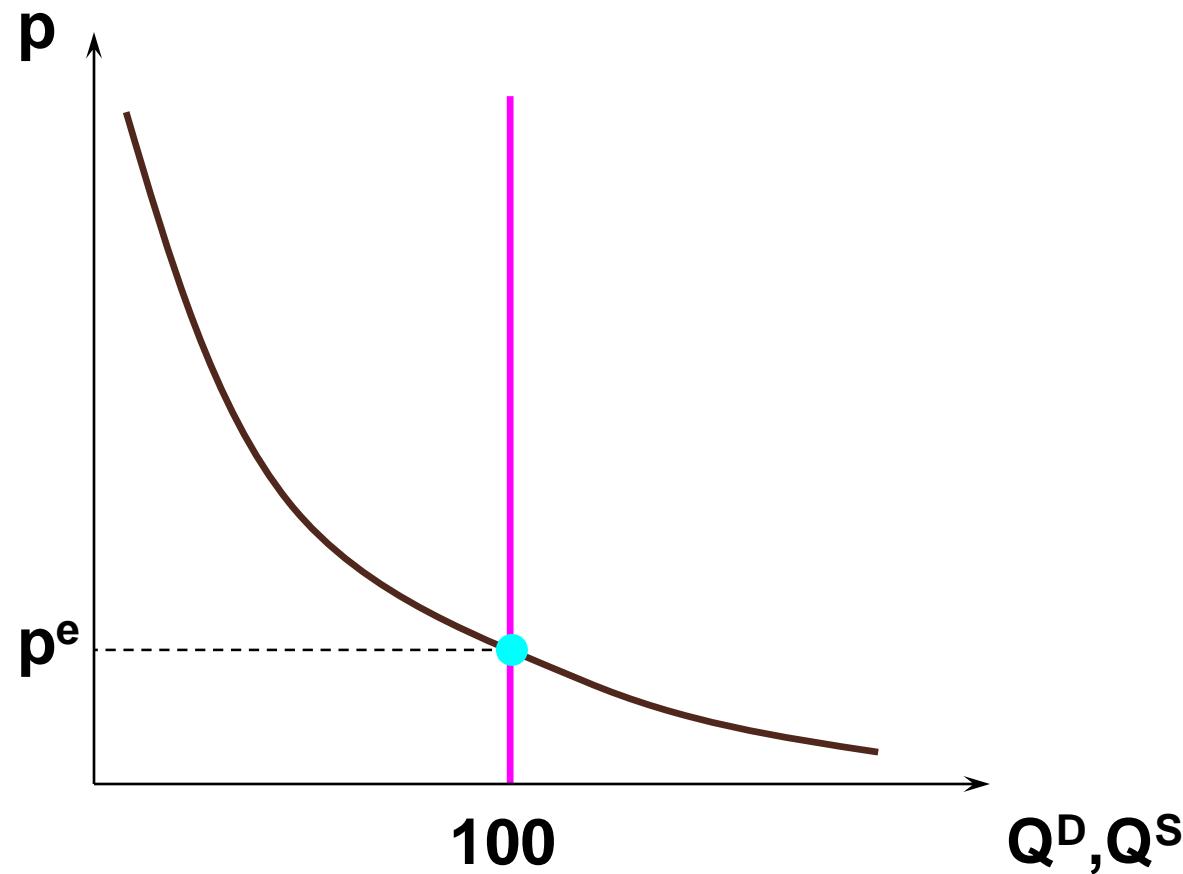
Comparative Statics

- What is exogenous in the model?
 - price of distant apartments
 - quantity of close apartments
 - incomes of potential renters
- What happens if these exogenous variables change?

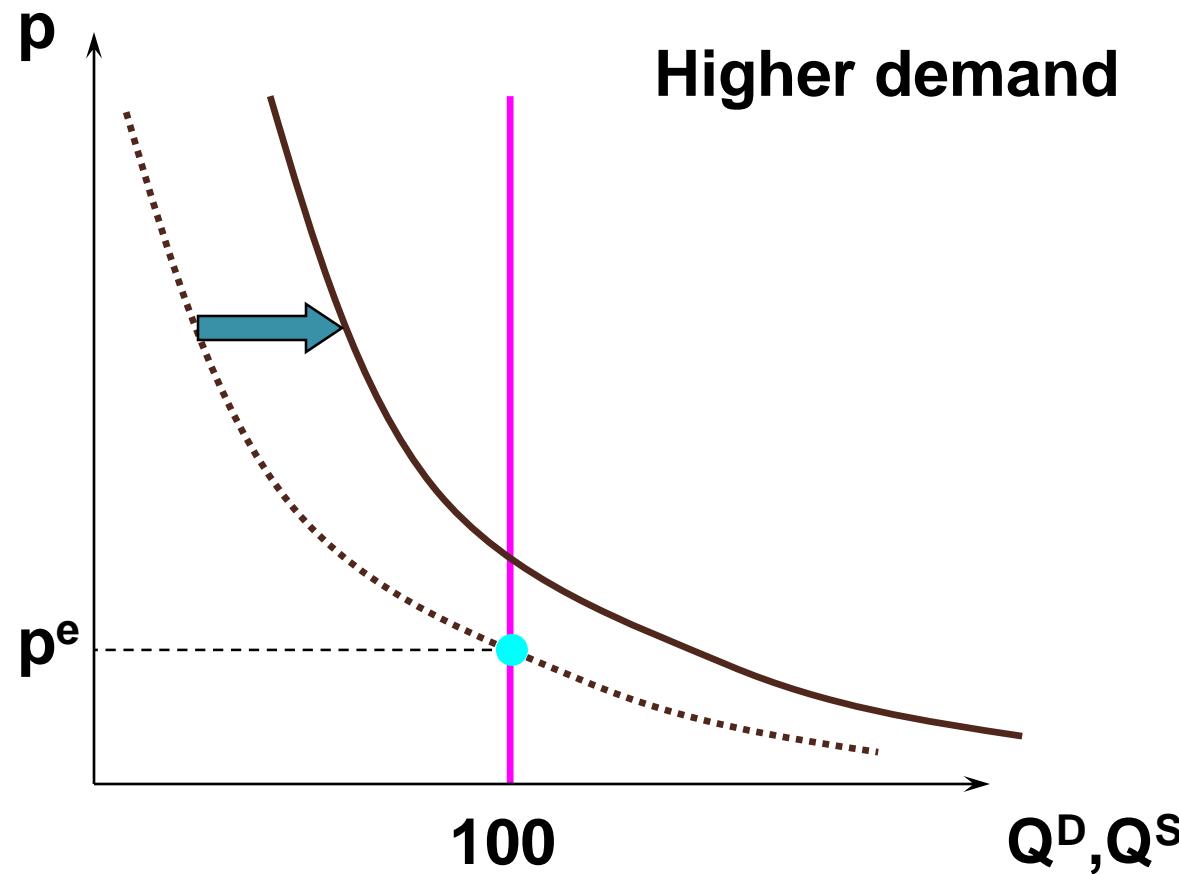
Comparative Statics

- Suppose the price of distant apartment rises
- Demand for close apartments increases (rightward shift), causing
 - a higher price for close apartments.

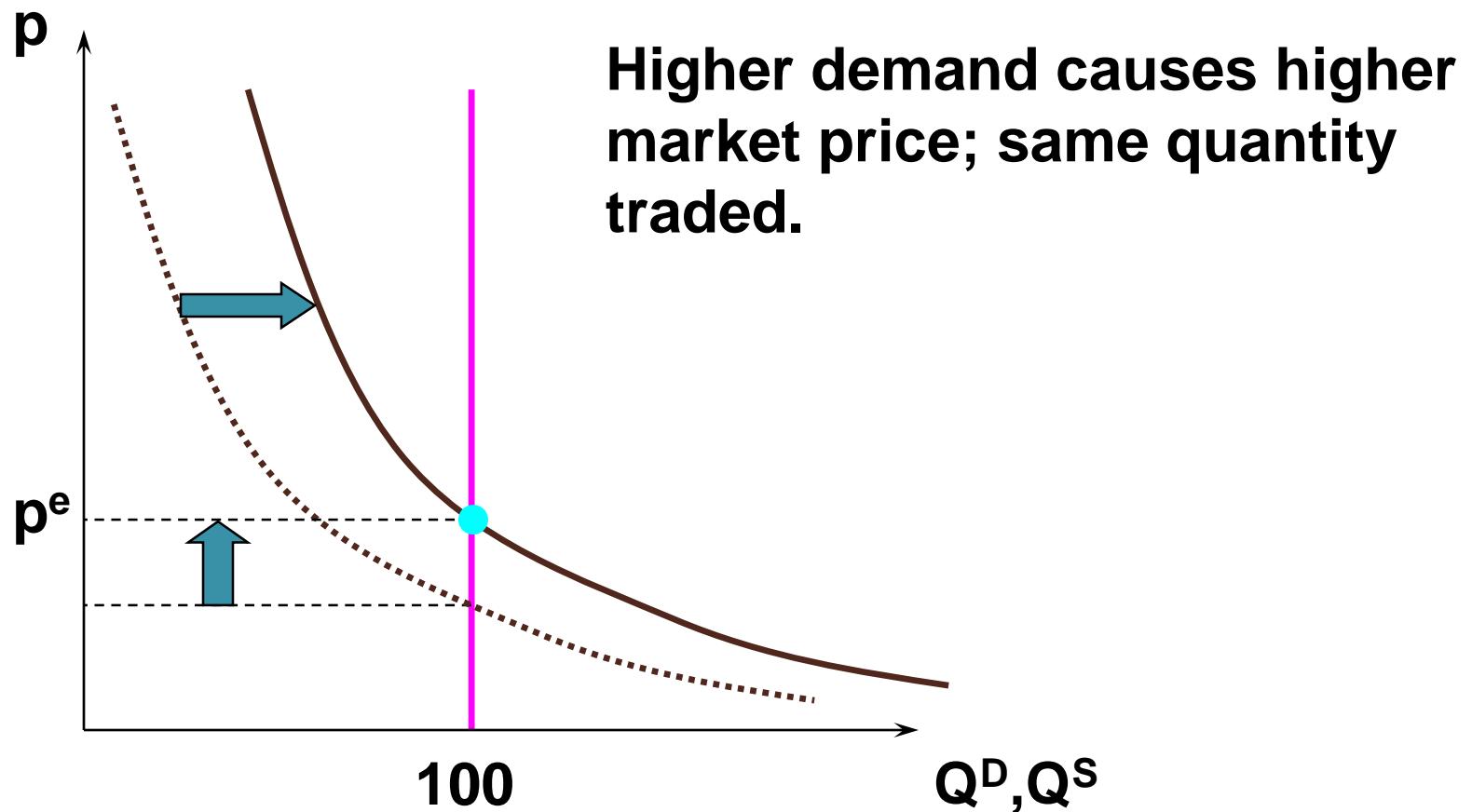
Market Equilibrium



Market Equilibrium



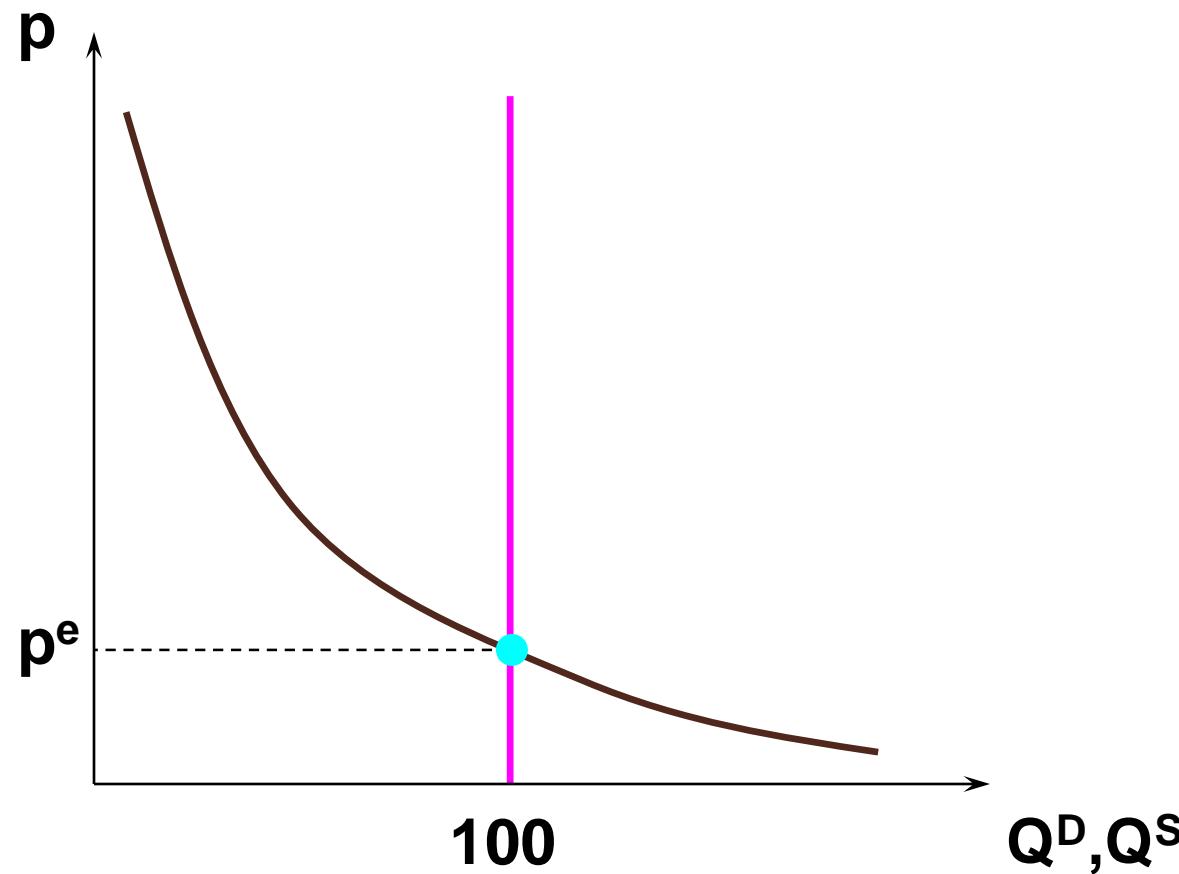
Market Equilibrium



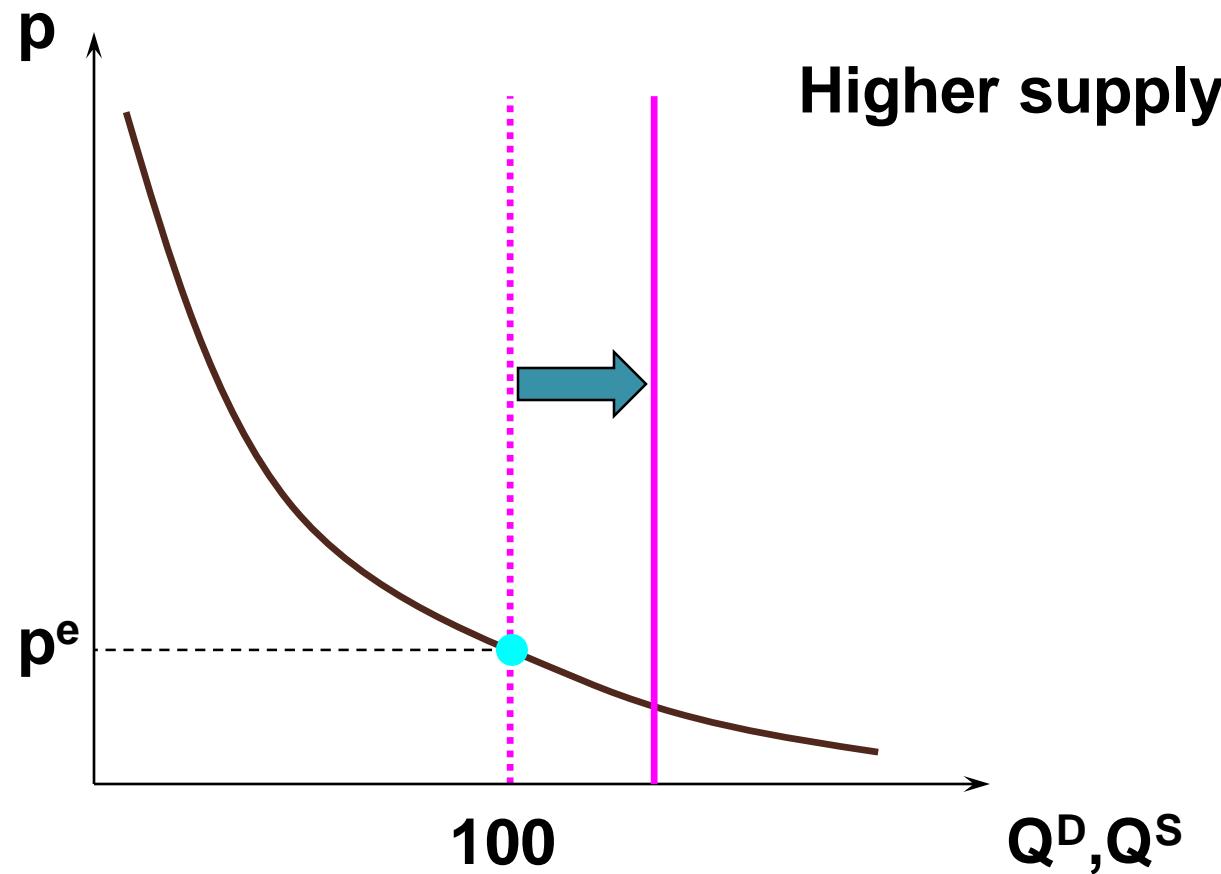
Comparative Statics

- Suppose there were more close apartments
- Supply is greater
- So the price for close apartments falls.

Market Equilibrium

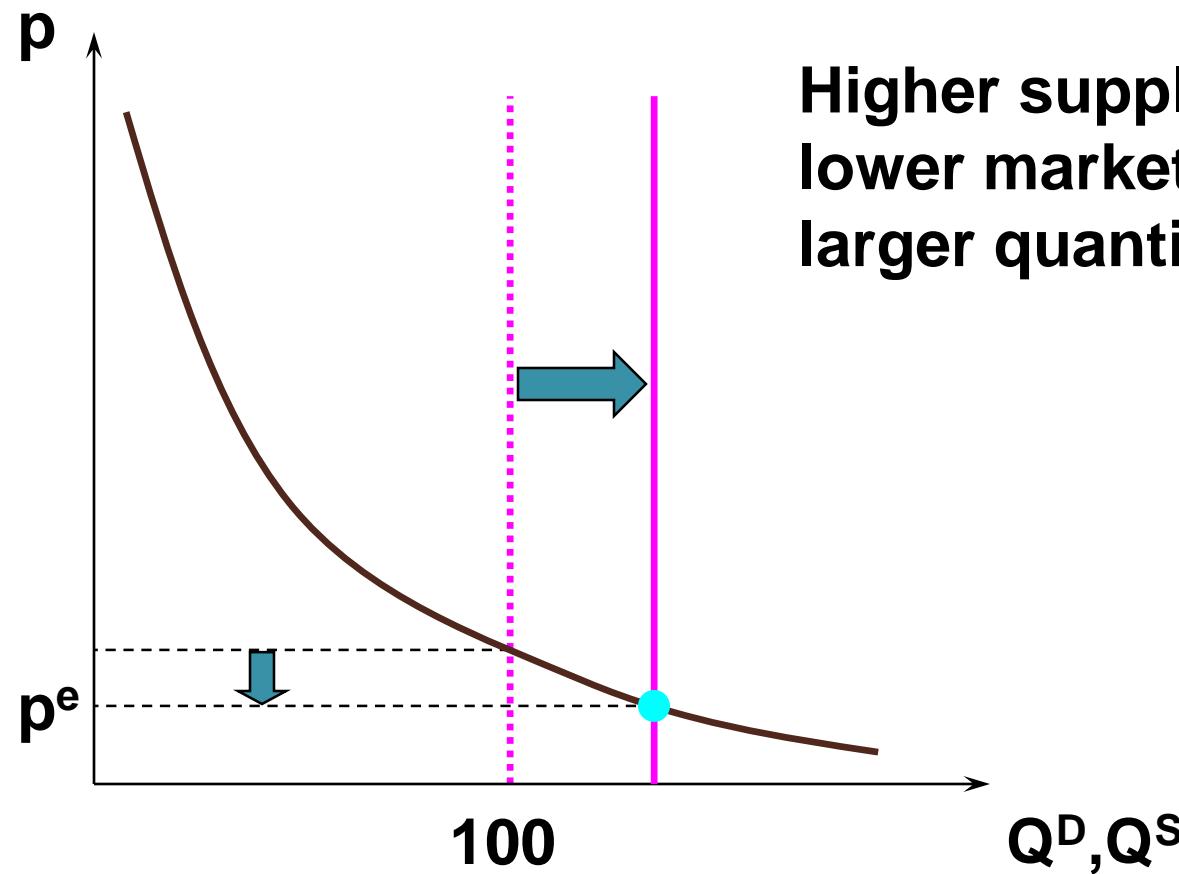


Market Equilibrium



Higher supply

Market Equilibrium

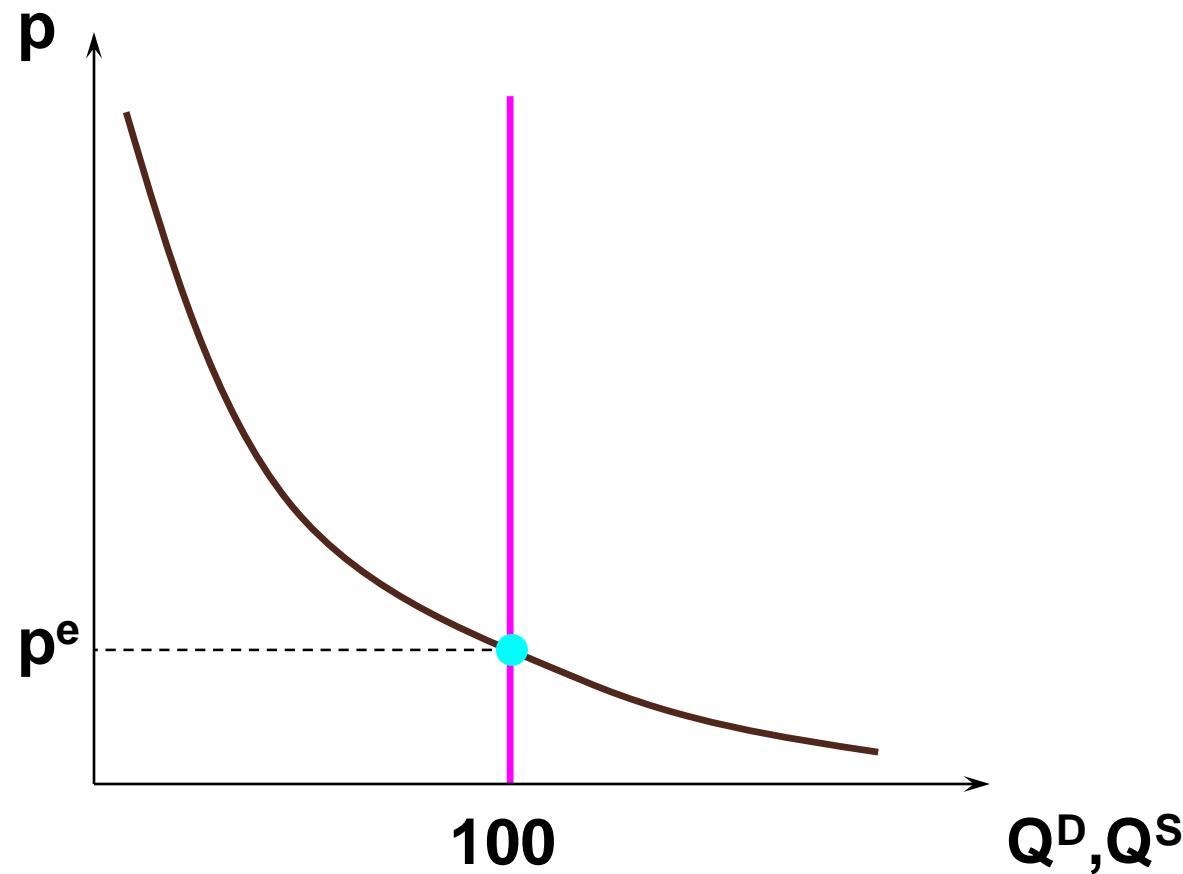


Higher supply causes a lower market price and a larger quantity traded.

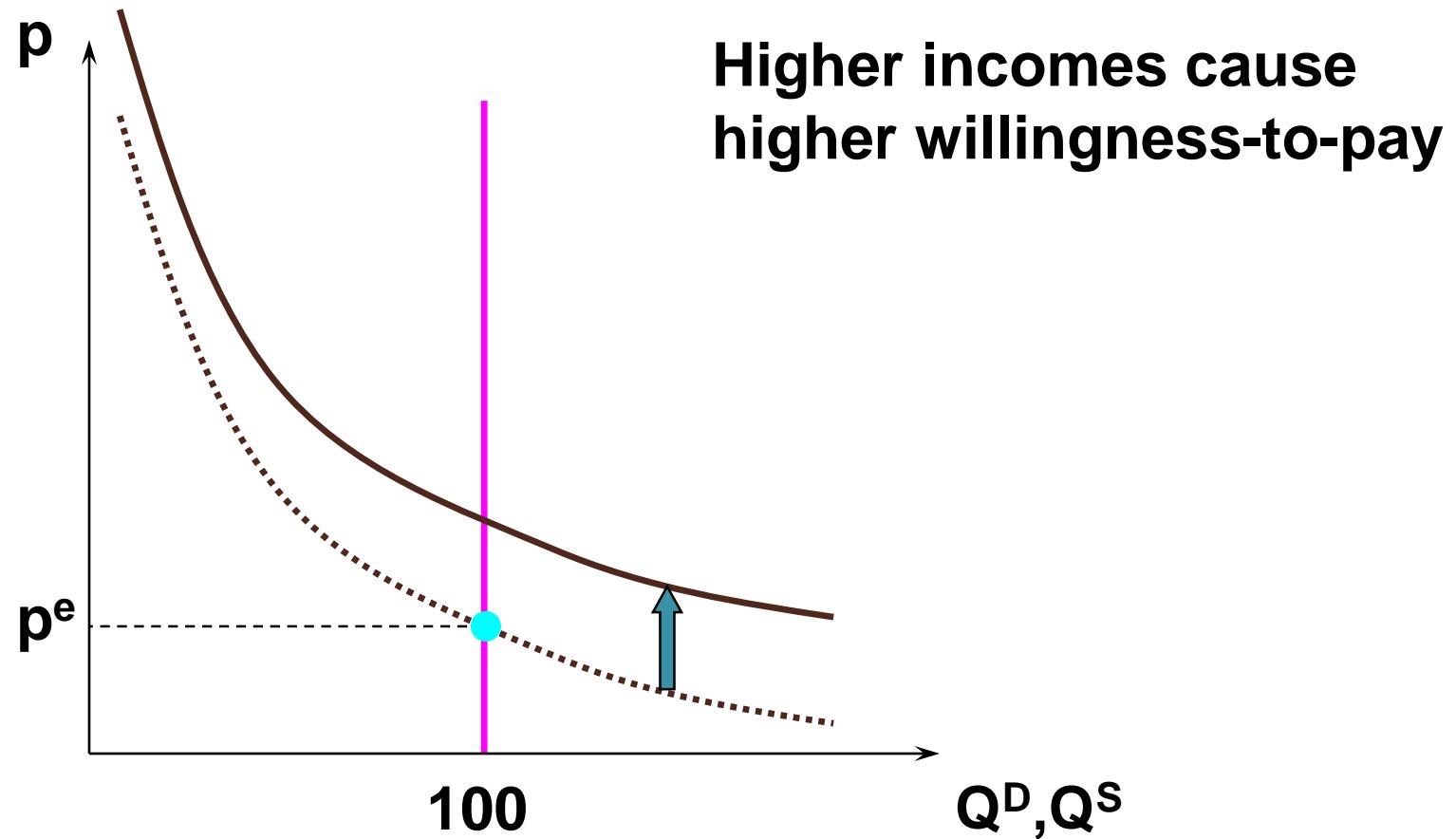
Comparative Statics

- Suppose potential renters' incomes rise, increasing their willingness-to-pay for close apartments
- Demand rises (upward shift), causing higher price for close apartments.

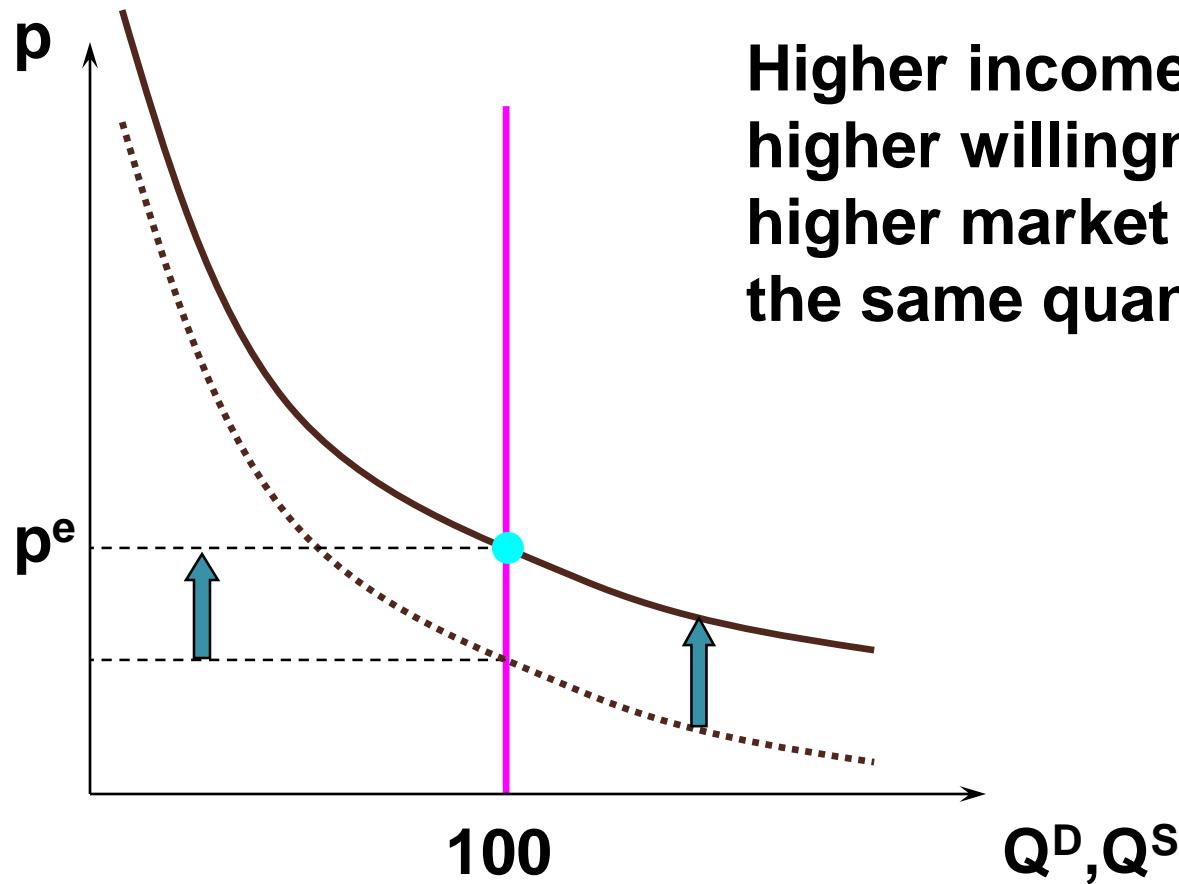
Market Equilibrium



Market Equilibrium



Market Equilibrium



**Higher incomes cause
higher willingness-to-pay,
higher market price, and
the same quantity traded.**

The Effect of an Apartment Tax

- Suppose the city council decides that there should be a tax on the apartment
- Each landlord has to pay a certain amount of tax to the city
- What will this tax policy change the apartment price?

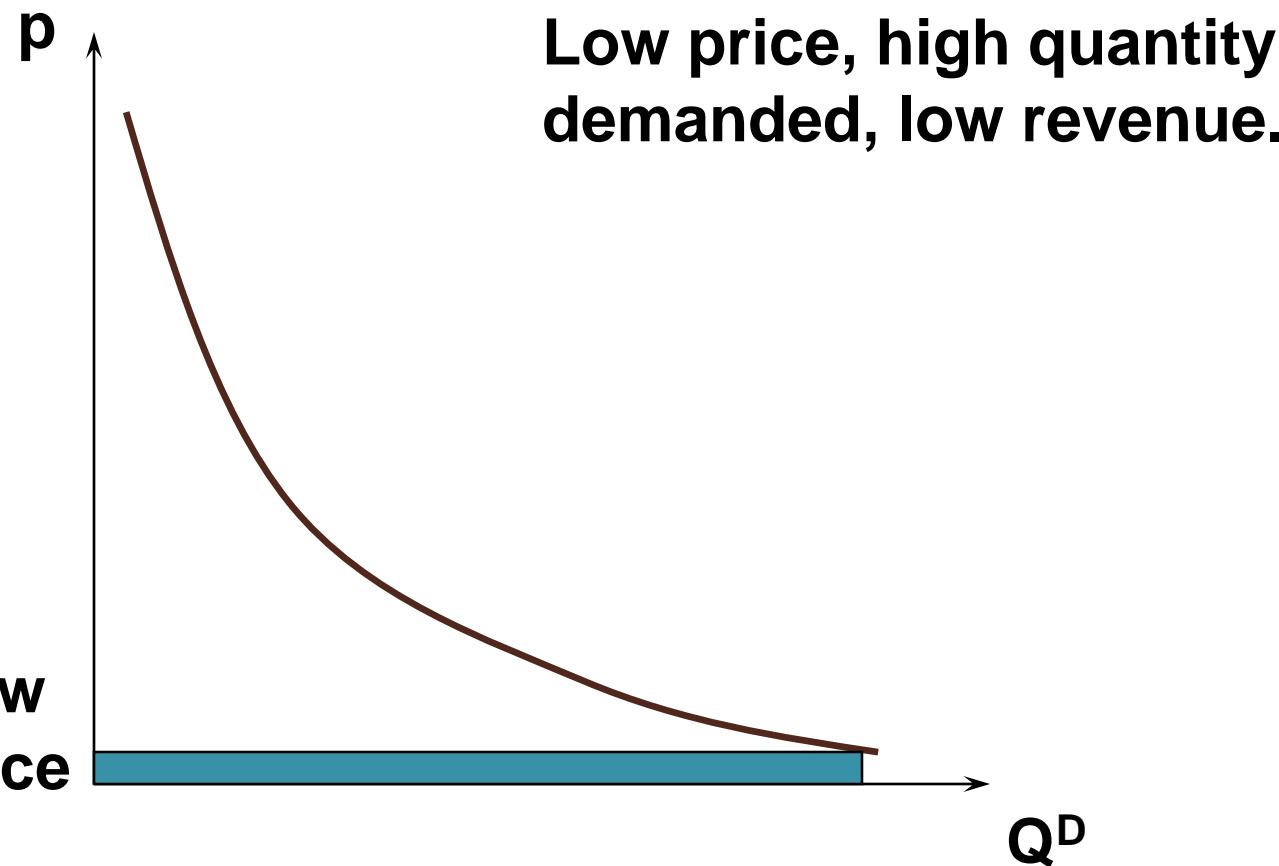
Imperfectly Competitive Markets

- Amongst many possibilities are:
 - a monopolistic landlord
 - a perfectly discriminatory monopolistic landlord
 - a competitive market subject to rent control.

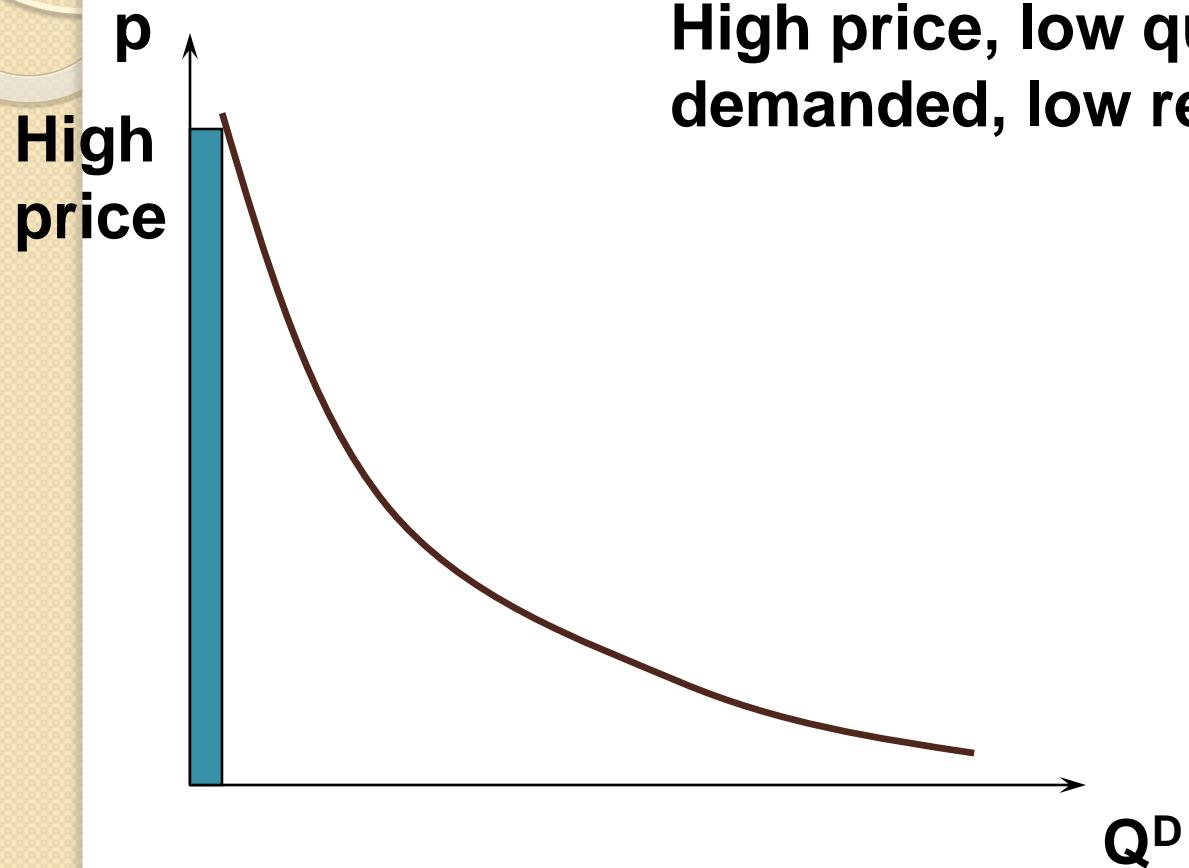
A Monopolistic Landlord

- When the landlord sets a rental price p he rents $D(p)$ apartments.
- Revenue = $pD(p)$.
- Revenue is low if $p \approx 0$
- Revenue is low if p is so high that $D(p) \approx 0$.
- An intermediate value for p maximizes revenue.

Monopolistic Market Equilibrium

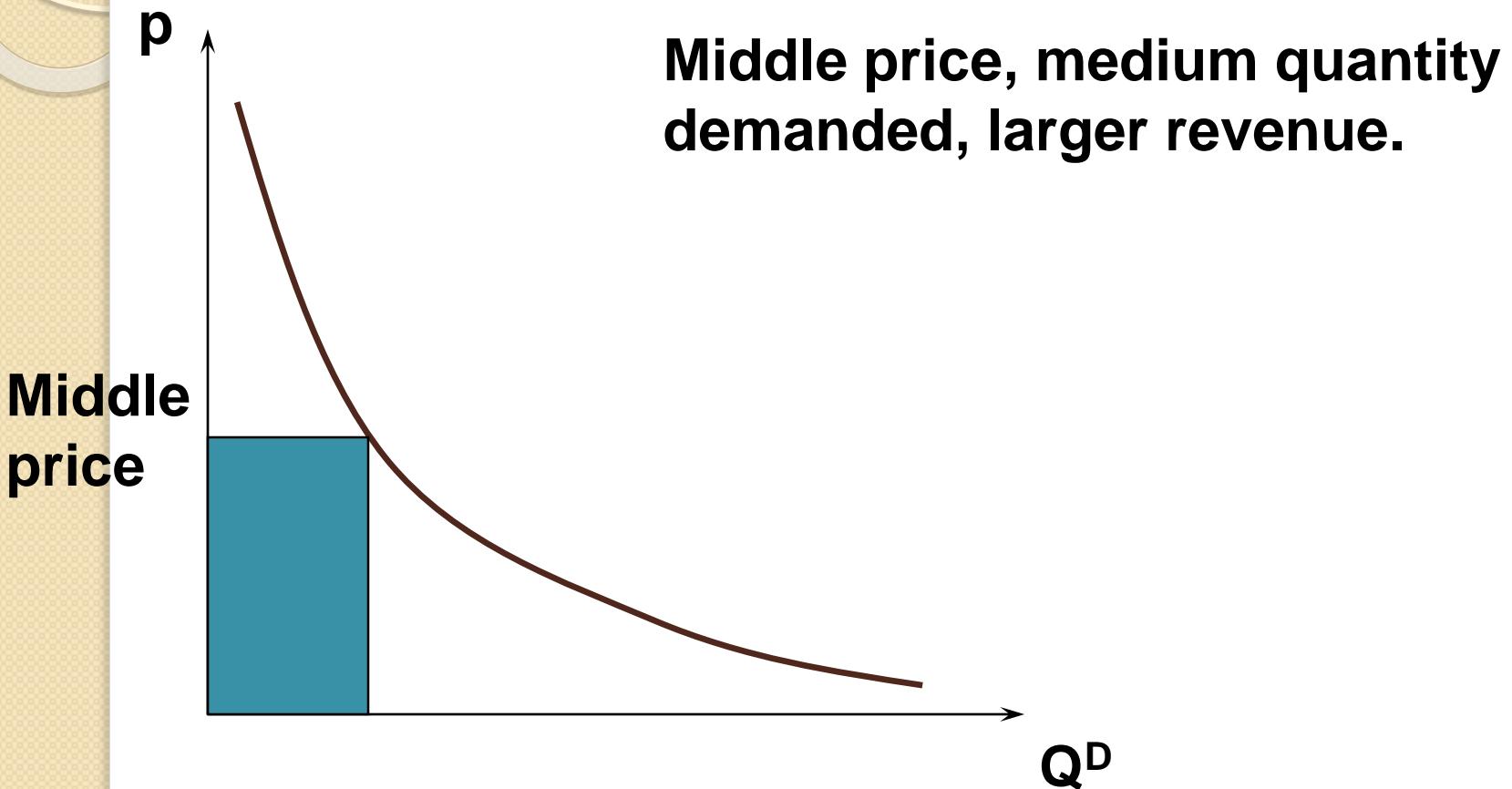


Monopolistic Market Equilibrium

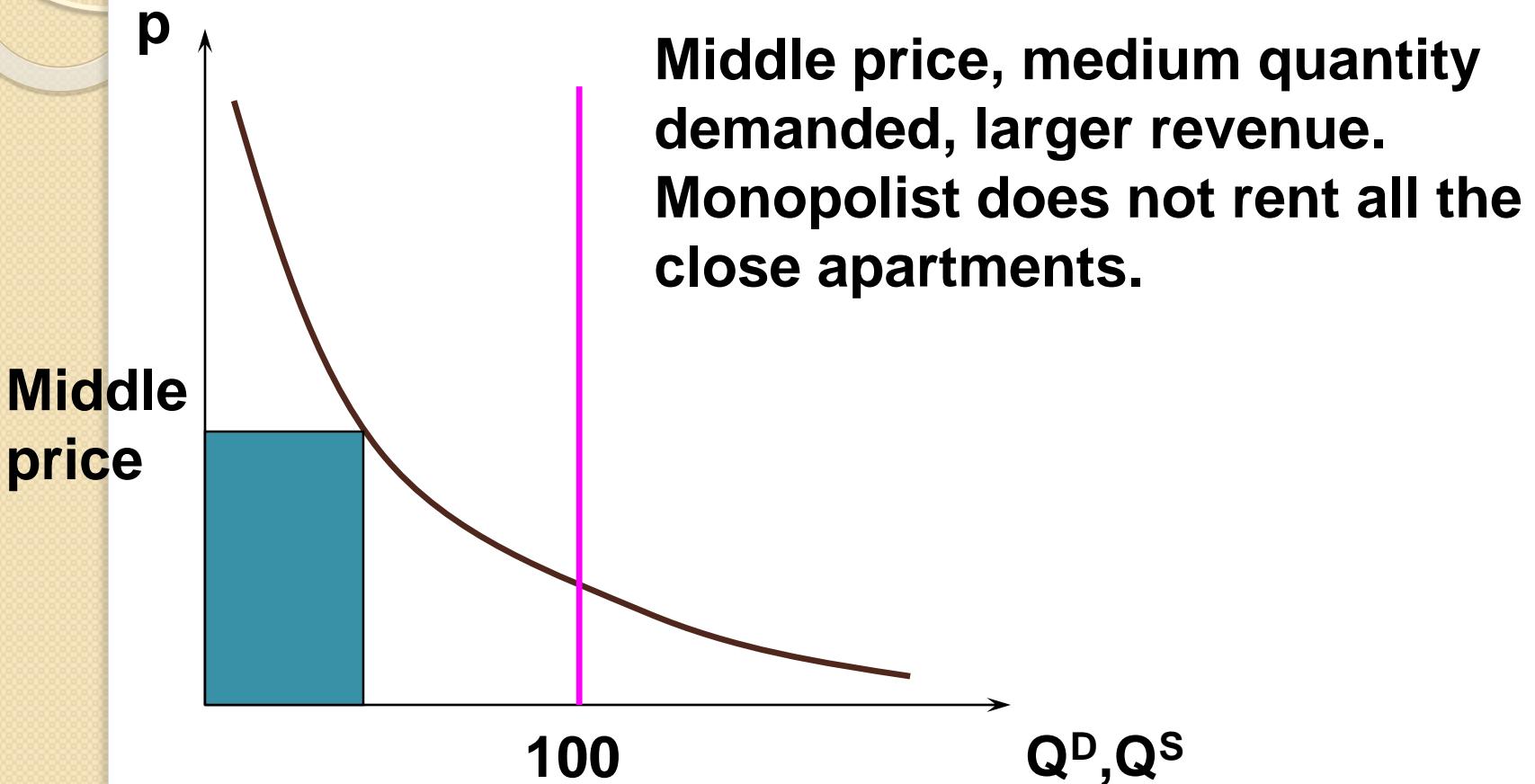


High price, low quantity demanded, low revenue.

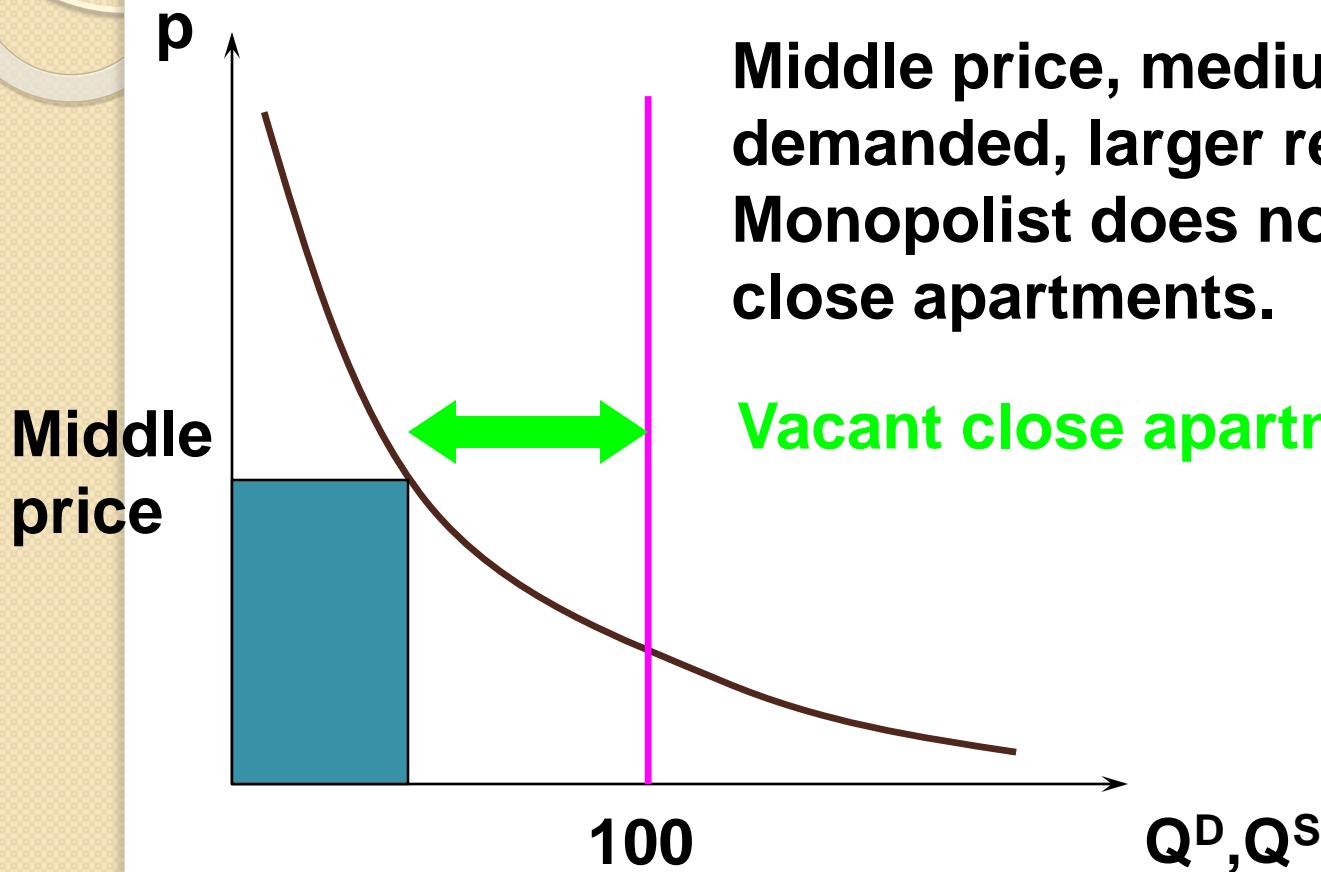
Monopolistic Market Equilibrium



Monopolistic Market Equilibrium



Monopolistic Market Equilibrium

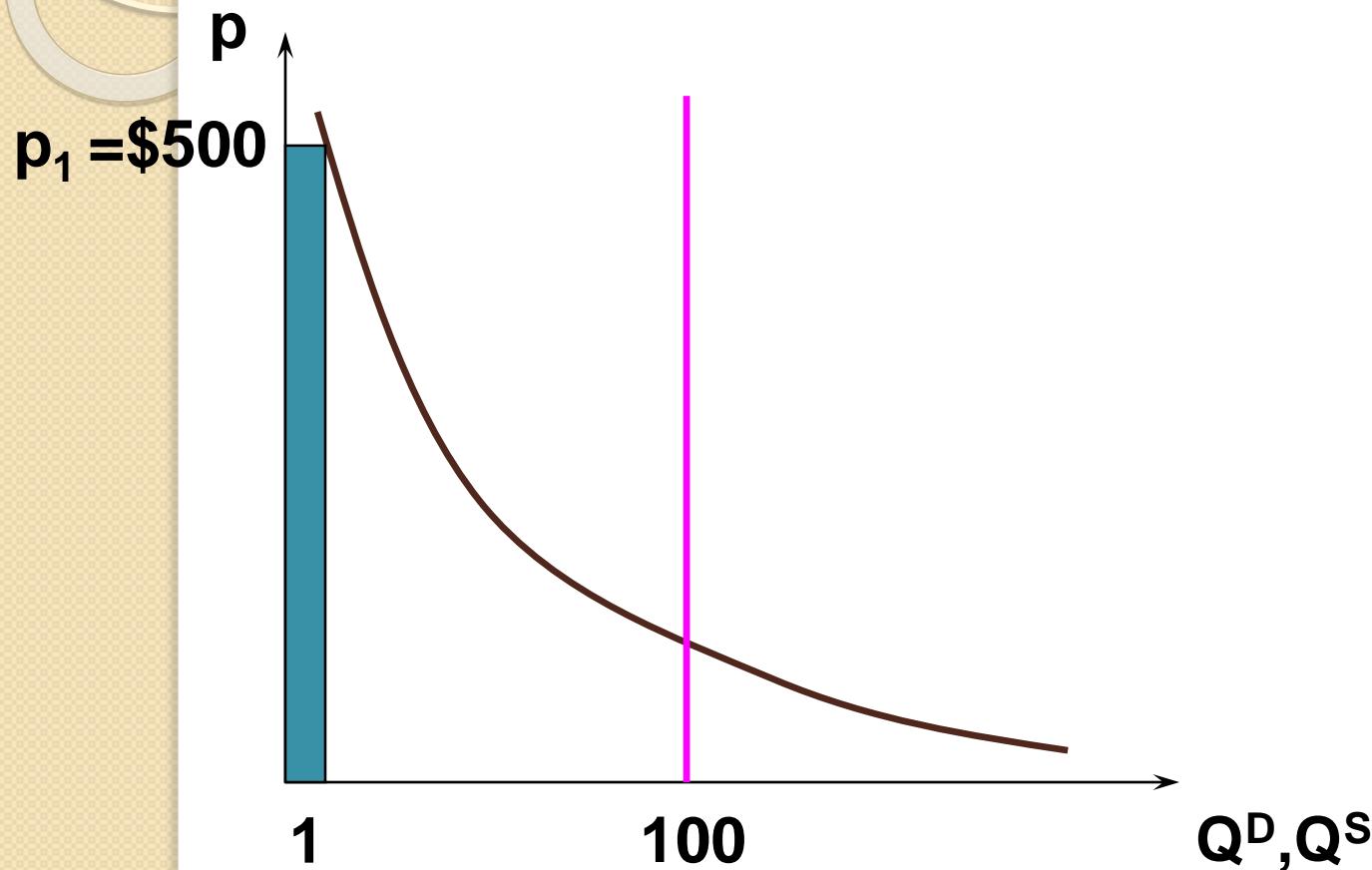


**Middle price, medium quantity demanded, larger revenue.
Monopolist does not rent all the close apartments.**

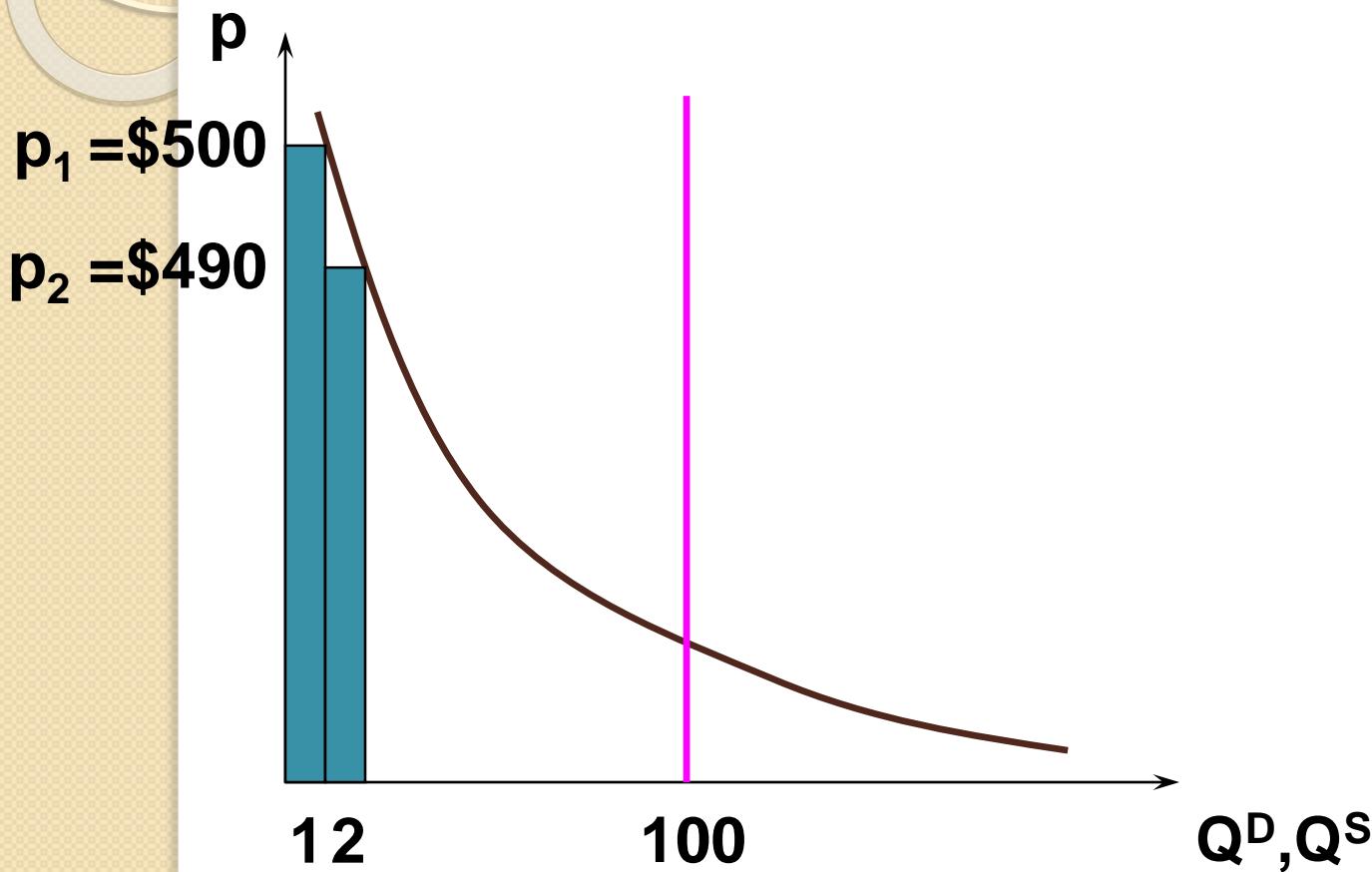
Perfectly Discriminatory Monopolistic Landlord

- Imagine the monopolist knew everyone's willingness-to-pay and he was able to charge different renters different prices
- Charge \$500 to the most willing-to-pay
- charge \$490 to the 2nd most willing-to-pay, etc.

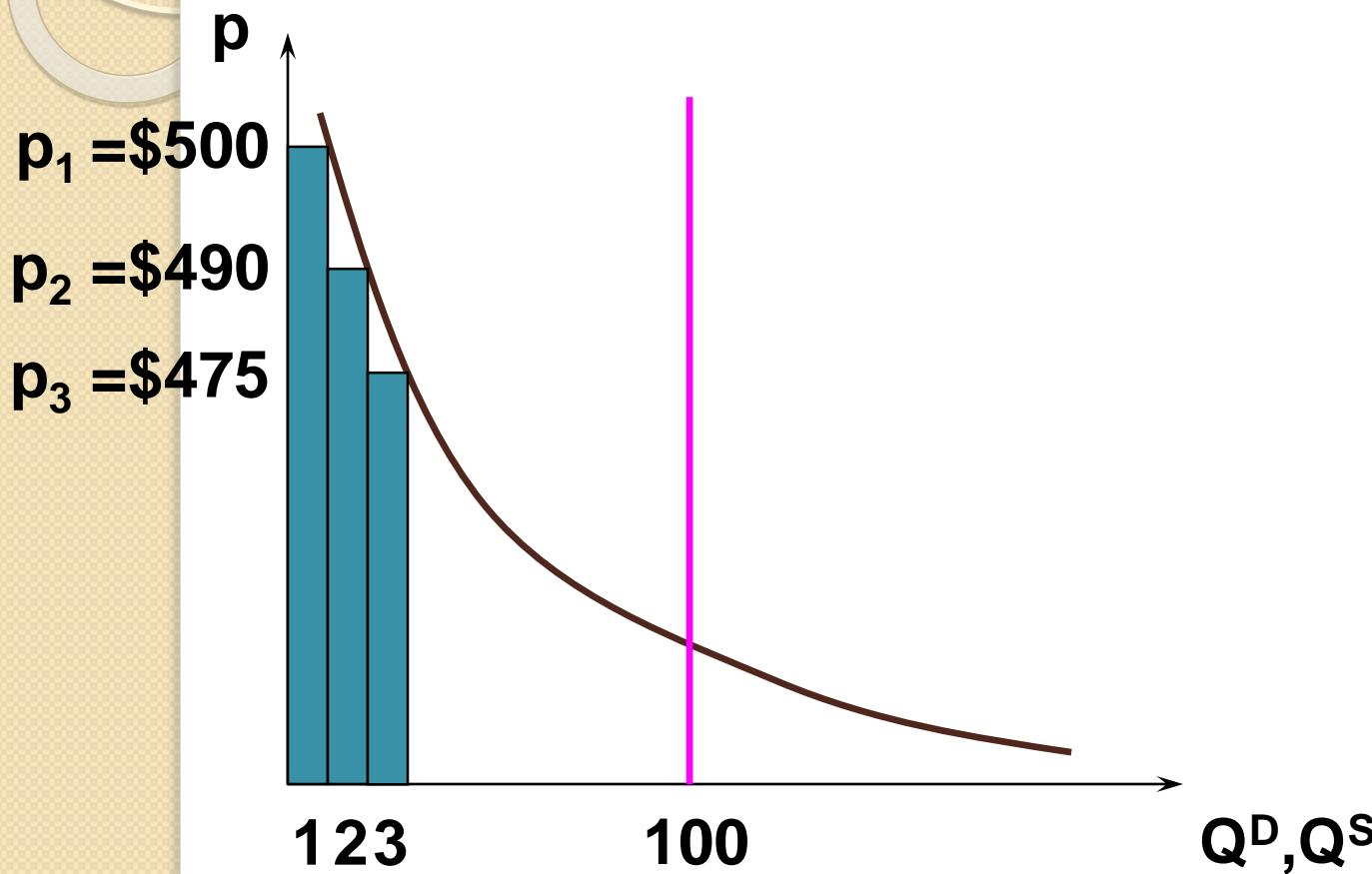
Discriminatory Monopolistic Market Equilibrium



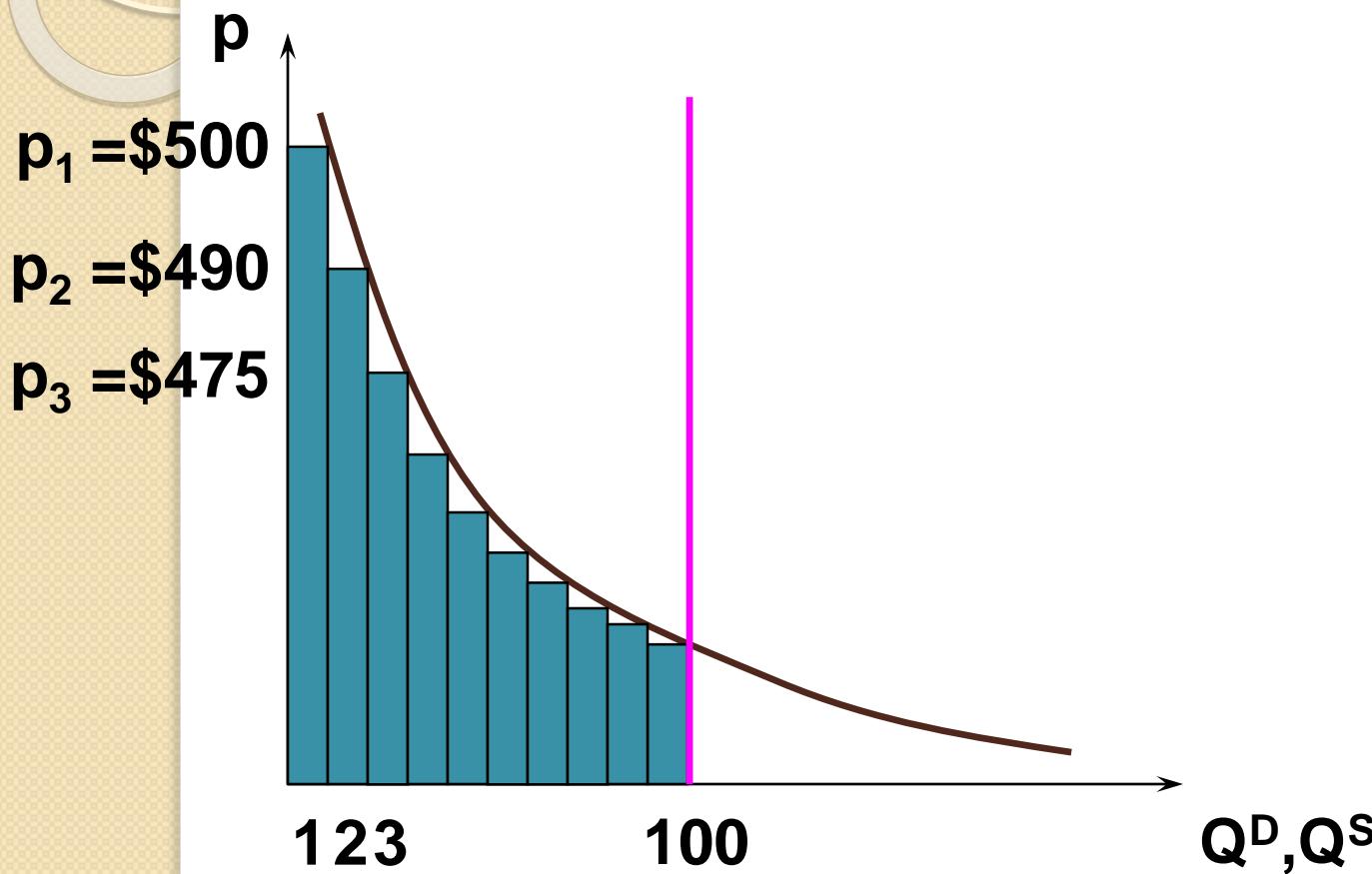
Discriminatory Monopolistic Market Equilibrium



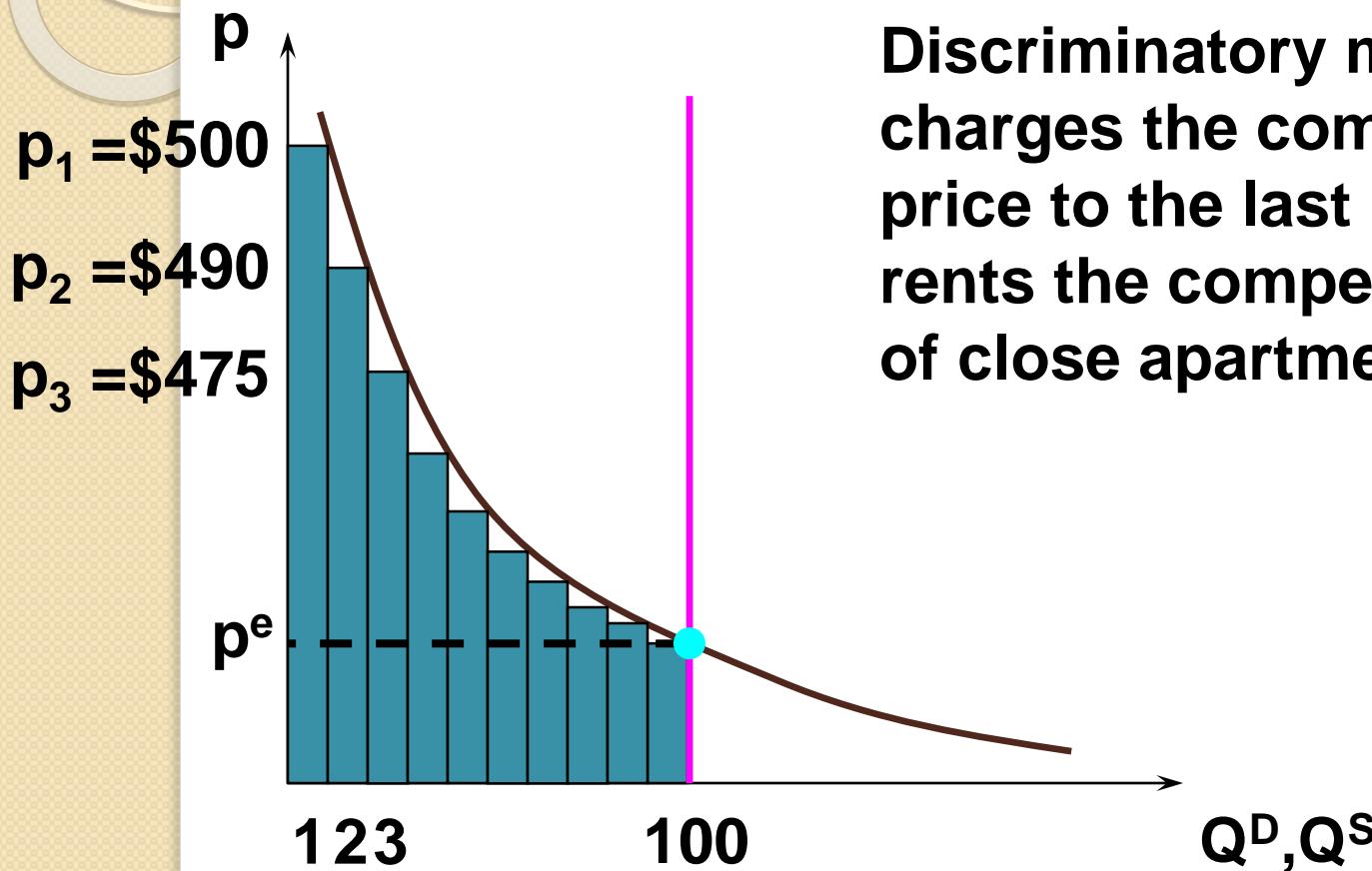
Discriminatory Monopolistic Market Equilibrium



Discriminatory Monopolistic Market Equilibrium



Discriminatory Monopolistic Market Equilibrium

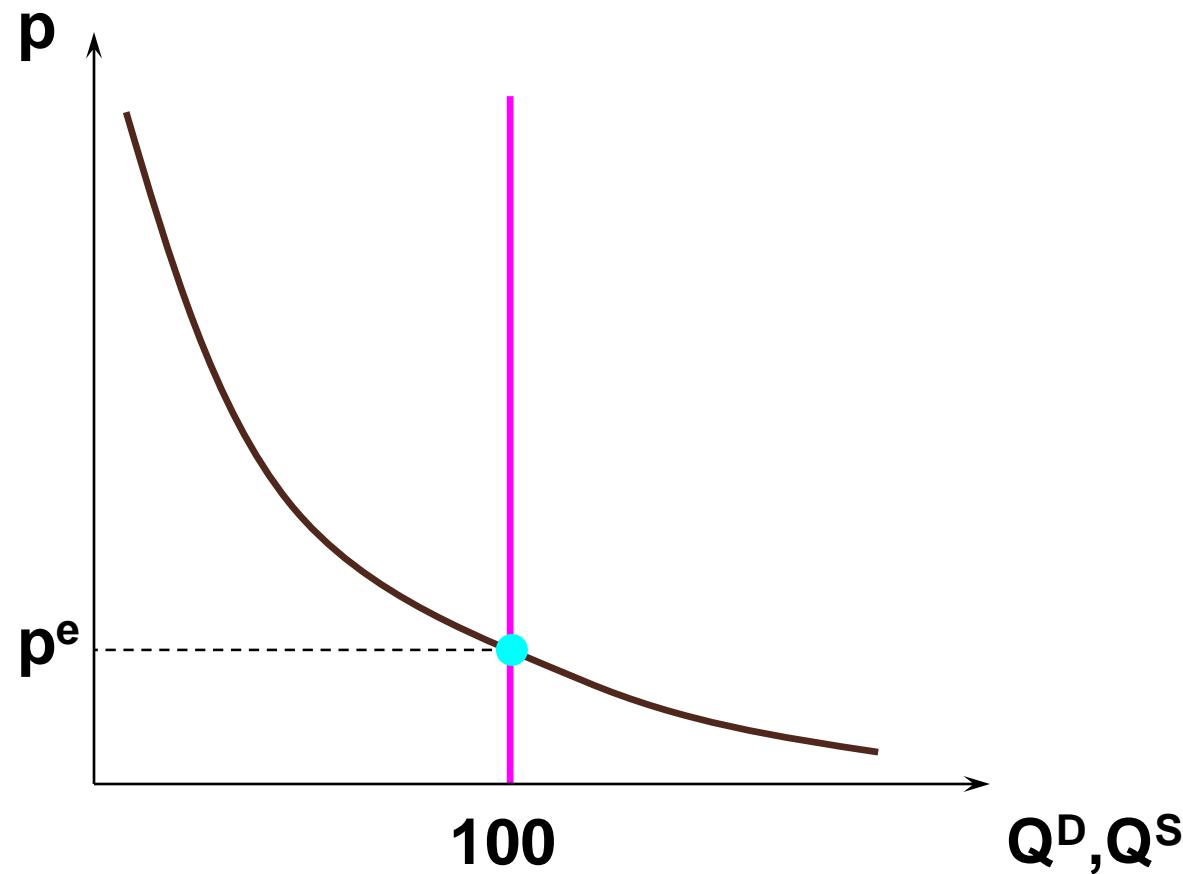


Discriminatory monopolist charges the competitive market price to the last renter, and rents the competitive quantity of close apartments.

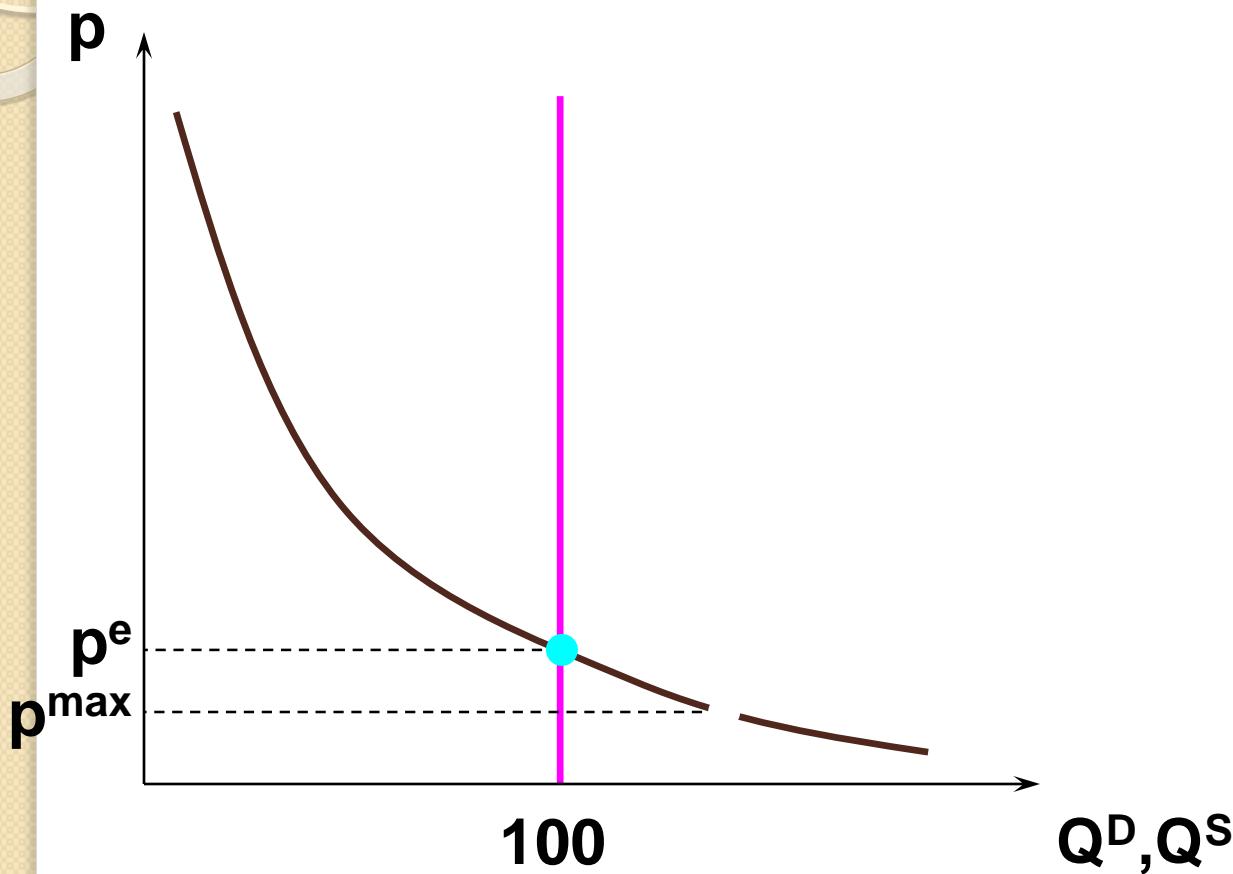
Rent Control

- Local government imposes a maximum legal price, $p^{\max} < p^e$, the competitive price.

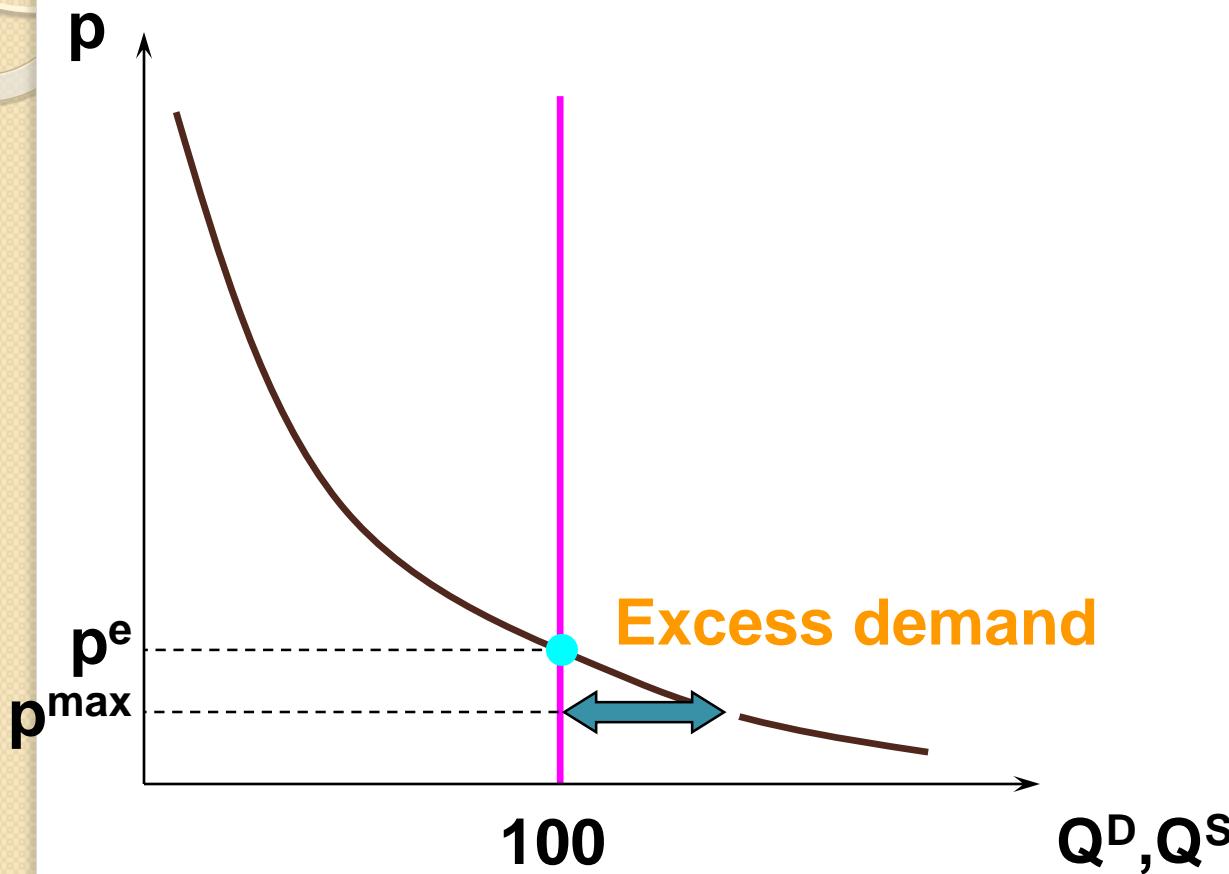
Market Equilibrium



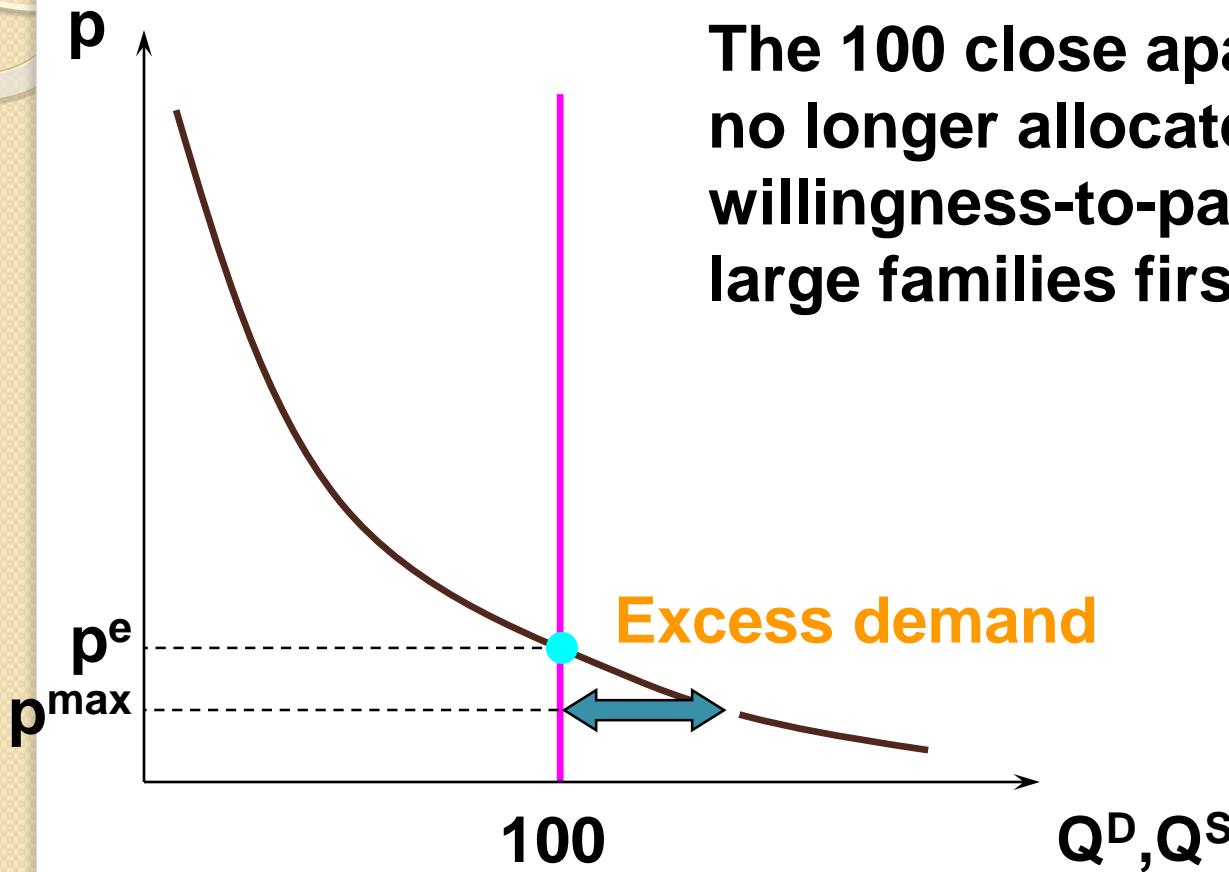
Market Equilibrium



Market Equilibrium



Market Equilibrium



Which Market Outcomes Are Desirable?

- Which is better?
 - Rent control
 - Perfect competition
 - Monopoly
 - Discriminatory monopoly

Pareto Efficiency



- Vilfredo Pareto; 1848-1923.
- A Pareto outcome allows no “wasted welfare”;
- i.e. the only way one person’s welfare can be improved is to lower another person’s welfare.

Pareto Improvement

- Given an allocation, if we can find a way to make some people better off without making anybody else worse off, we have a ***Pareto improvement***
- If an allocation allows for a Pareto improvement, it is called ***Pareto inefficient***
- If an allocation doesn't allow for any Pareto improvement, it is called ***Pareto efficient***

Pareto Efficiency

- An allocation is Pareto efficient if we can't find another allocation such that no one is worse off and at least someone is better off
- Pareto efficiency **exhausts** any Pareto improvement opportunities
- This efficiency criterion requires minimum value judgments and favors *status quo*
- Side-payment: Hicks-Kaldor compensation

Pareto Efficiency

- David has an apartment; Jack does not
- David values the apartment at \$200; Jack would pay \$400 for it
- David could sublet the apartment to Jack for \$300
- Both gain, so it was Pareto inefficient for David to have the apartment.

Criterion for Pareto Efficiency

- A Pareto inefficient outcome means there remain *unrealized mutual gains-to-trade*
- Any market outcome that achieves all possible **gains-from-trade** must be Pareto efficient
- Implications: maintain status quo and seek additional and mutual gains
 - 不动存量动增量，渐进改革，让时间解决存量问题
 - 求同存异，多做加法，少做减法

Efficiency and Equity

- Efficiency and equity may not go together, and often conflict with each other
- The side payment or Hick-Kaldor compensation may not come by easily
 - *Economists tend to use this unreal compensation to justify Pareto improvement*
- An efficient allocation often leads to unequal distribution of interests

Pareto Efficiency

- Competitive equilibrium:
 - all close apartment renters value them at the market price p^e or more
 - all others value close apartments at less than p^e
 - so no mutually beneficial trades remain
 - so the outcome is Pareto efficient.

Pareto Efficiency

- Discriminatory Monopoly:
 - assignment of apartments is the same as with the perfectly competitive market
 - so the discriminatory monopoly outcome is also Pareto efficient
 - **Note that Pareto efficient outcomes could be very unequal**

Pareto Efficiency

- Monopoly:
 - not all apartments are occupied
 - so a distant apartment renter could be assigned a close apartment and have higher welfare without lowering anybody else's welfare.
 - so the monopoly outcome is Pareto inefficient.

Pareto Efficiency

- Rent Control:
 - some close apartments are assigned to renters valuing them at below the competitive price p^e
 - some renters valuing a close apartment above p^e don't get close apartments
 - Pareto inefficient outcome



Chapter 3

Consumer Behavior

Consumer Behavior

- There are three steps involved in the study of consumer behavior

- 1. Consumer Preferences*

- To describe how and why people prefer one good to another

- 2. Budget Constraints*

- People have limited incomes

Consumer Behavior

3. Given preferences and limited incomes, what amount and type of goods will be purchased?
 - What combination of goods will consumers buy to maximize their satisfaction?

Budget Constraint

- Describe budget constraint
 - Algebra
 - Graph
- Describe changes in budget constraint
- Government programs and budget constraints
- Non-linear budget lines

Consumption Bundle

- A consumption bundle containing x_1 units of commodity 1, x_2 units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector (x_1, x_2, \dots, x_n) .

Physical Constraints

- Non-negative:

Consumption set:

$$X = \{ (x_1, \dots, x_n) \mid x_1 \geq 0, \dots, x_n \geq 0 \}$$

- A **consumption set** is the collection of all physically possible consumption bundles to the consumer
 - You only have 24 hours a day
 - Subsistence need

Budget Constraints

- Commodity prices are p_1, p_2, \dots, p_n .
- Q: When is a bundle (x_1, \dots, x_n) affordable at prices p_1, \dots, p_n ?
- A: When

$$p_1 x_1 + \dots + p_n x_n \leq m$$

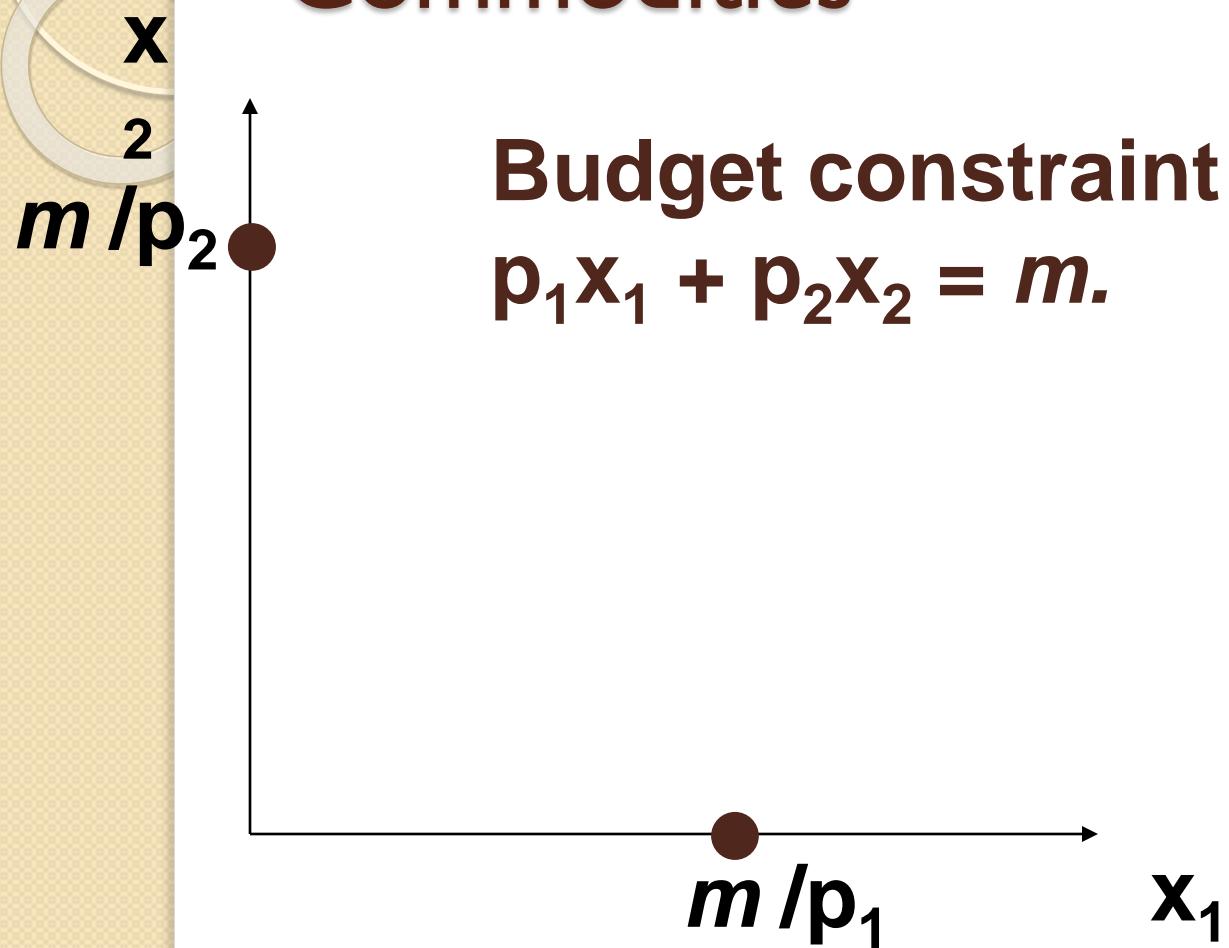
where m is the consumer's (disposable) income.

Budget Constraints

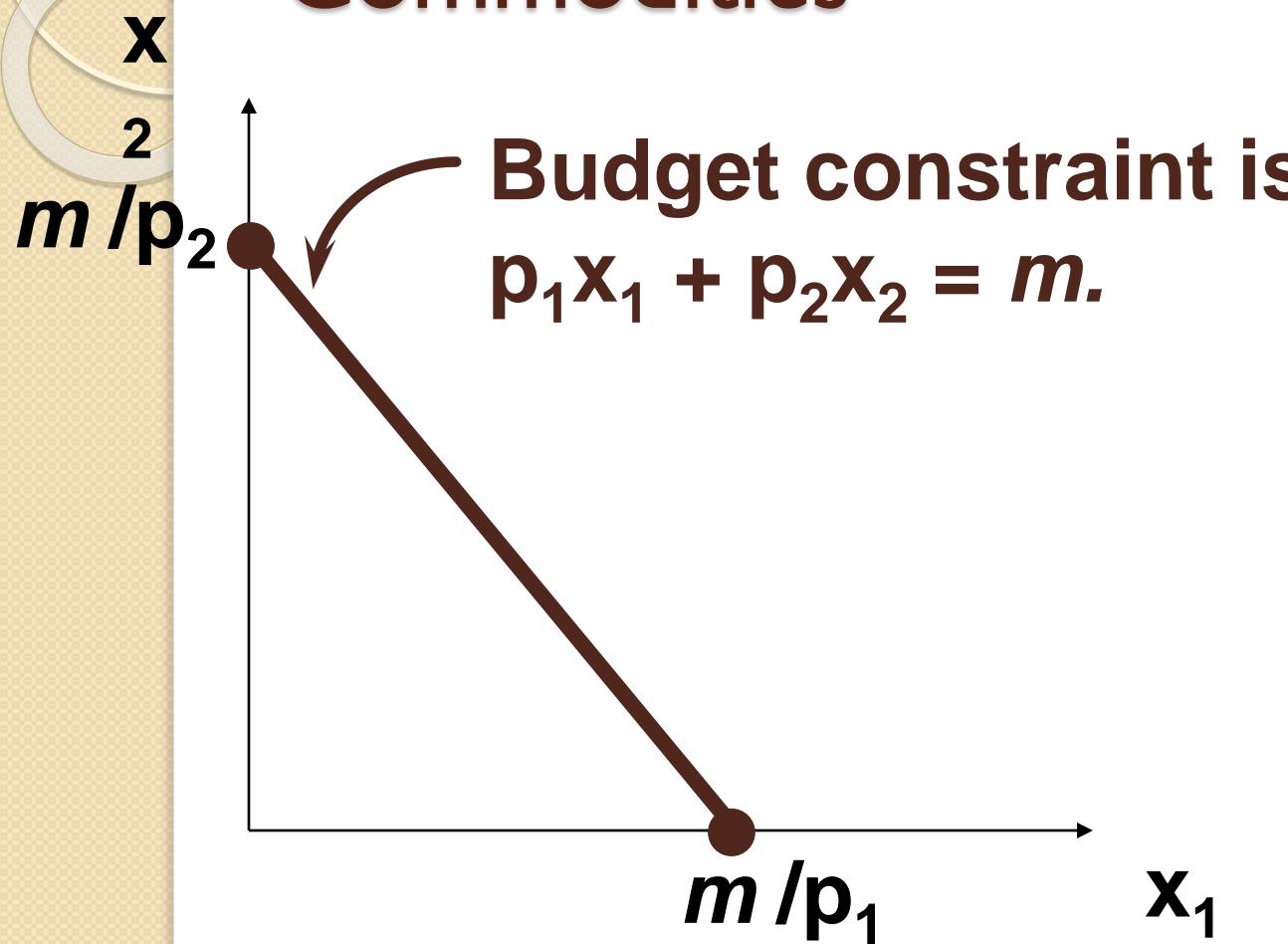
- The bundles that are only just affordable form the consumer's budget constraint.
This is the set

$$\{ (x_1, \dots, x_n) \mid x_1 \geq 0, \dots, x_n \geq 0 \text{ and } p_1 x_1 + \dots + p_n x_n = m \}.$$

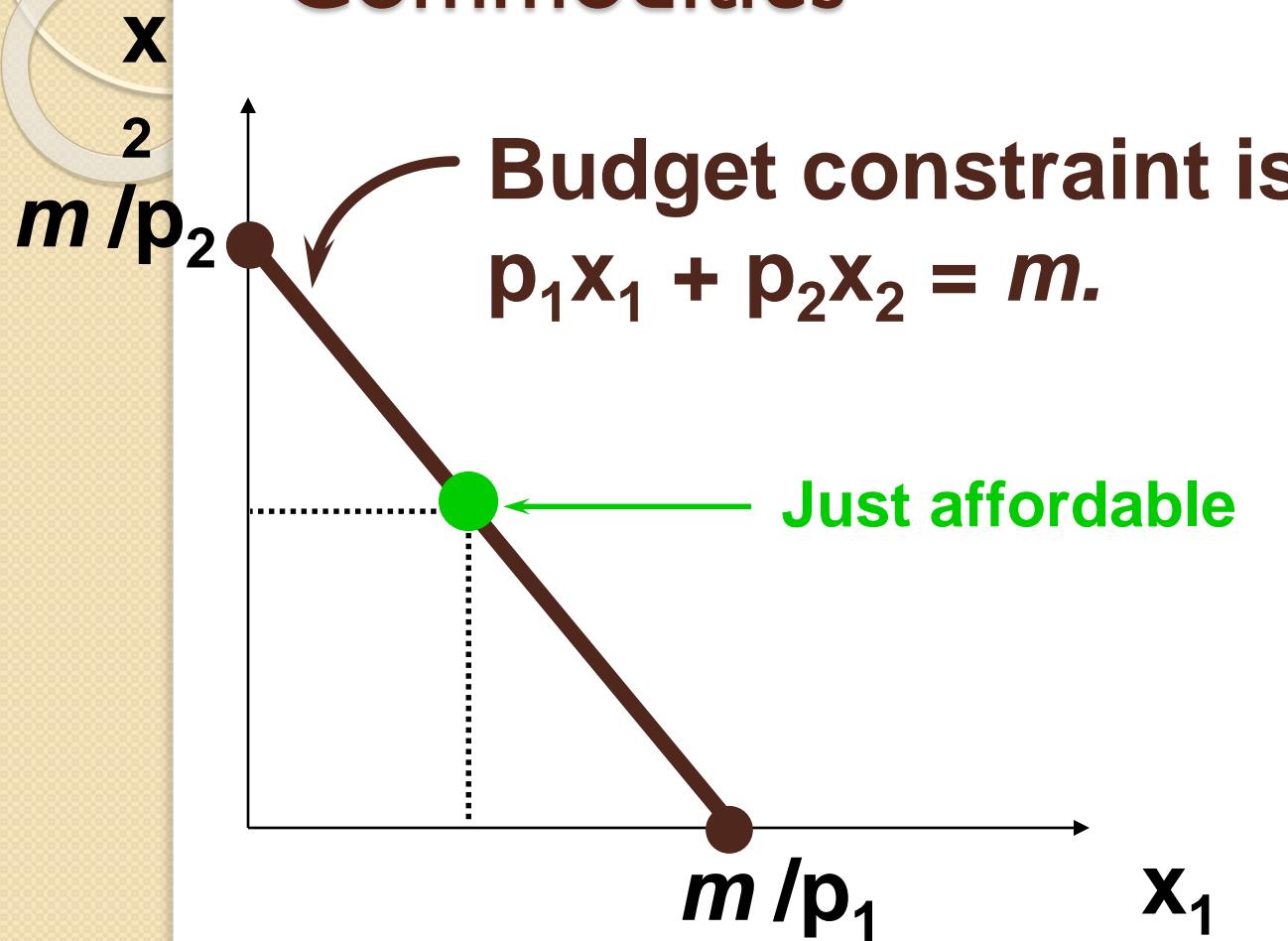
Budget Set and Constraint for Two Commodities



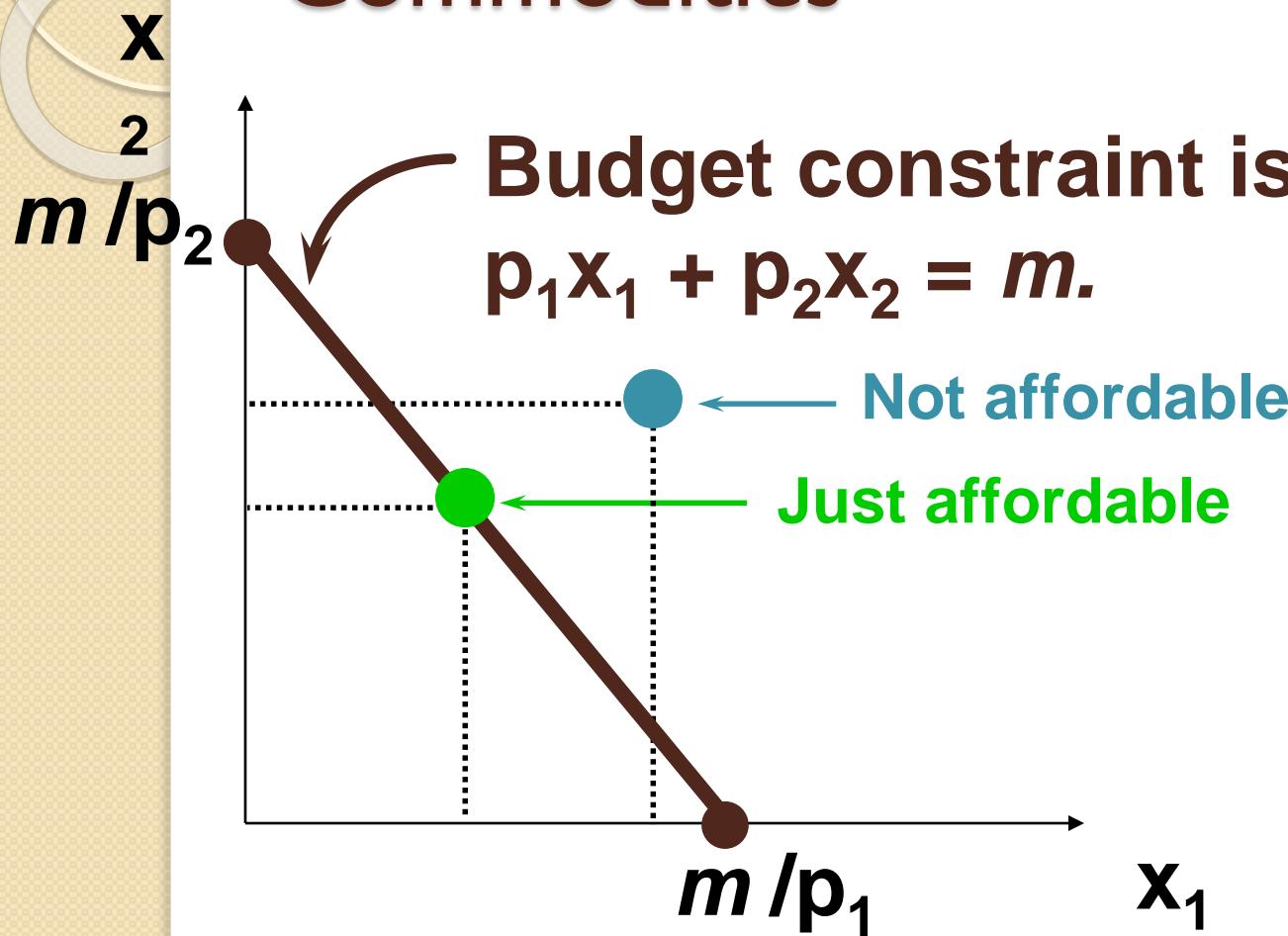
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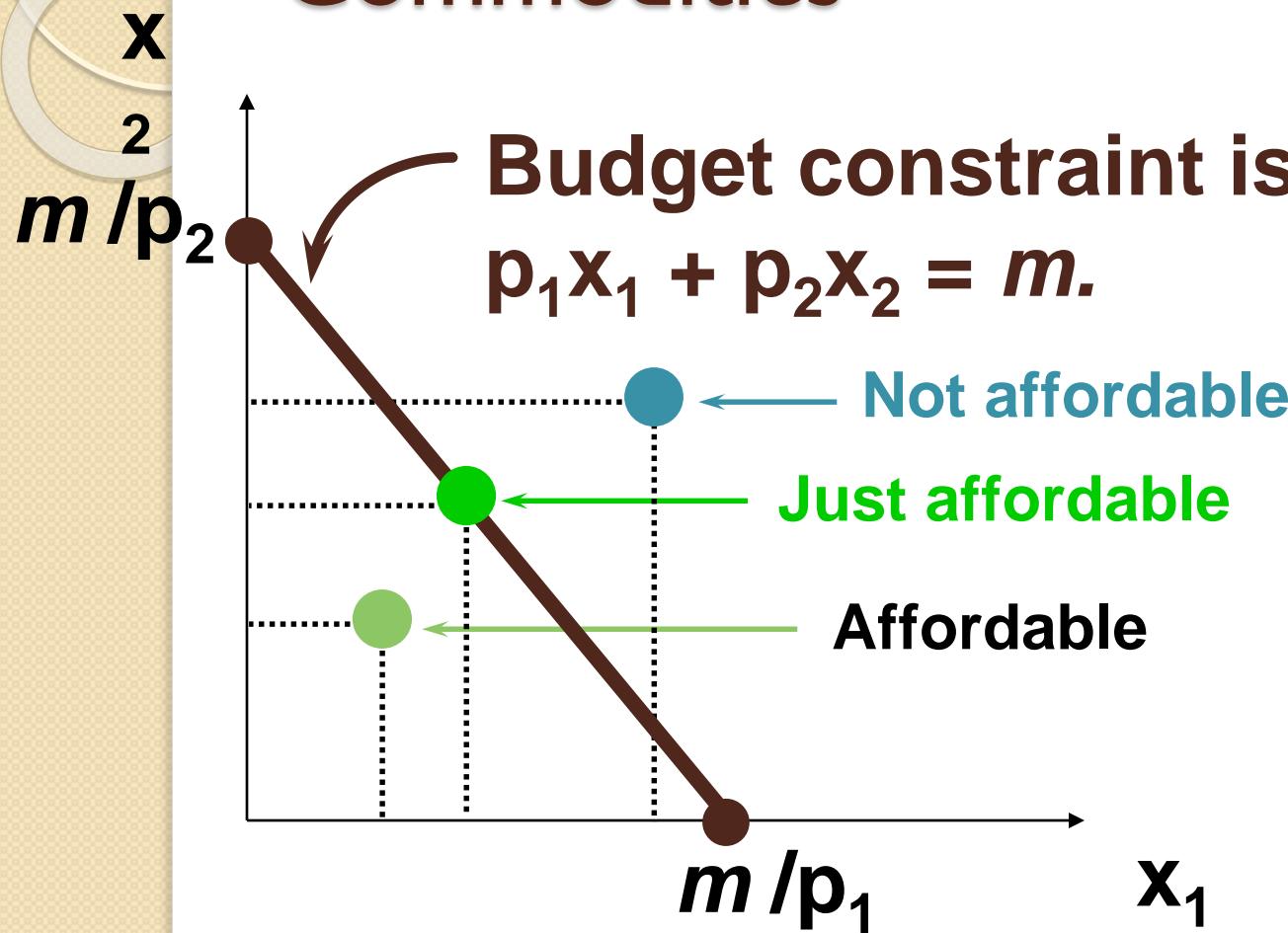
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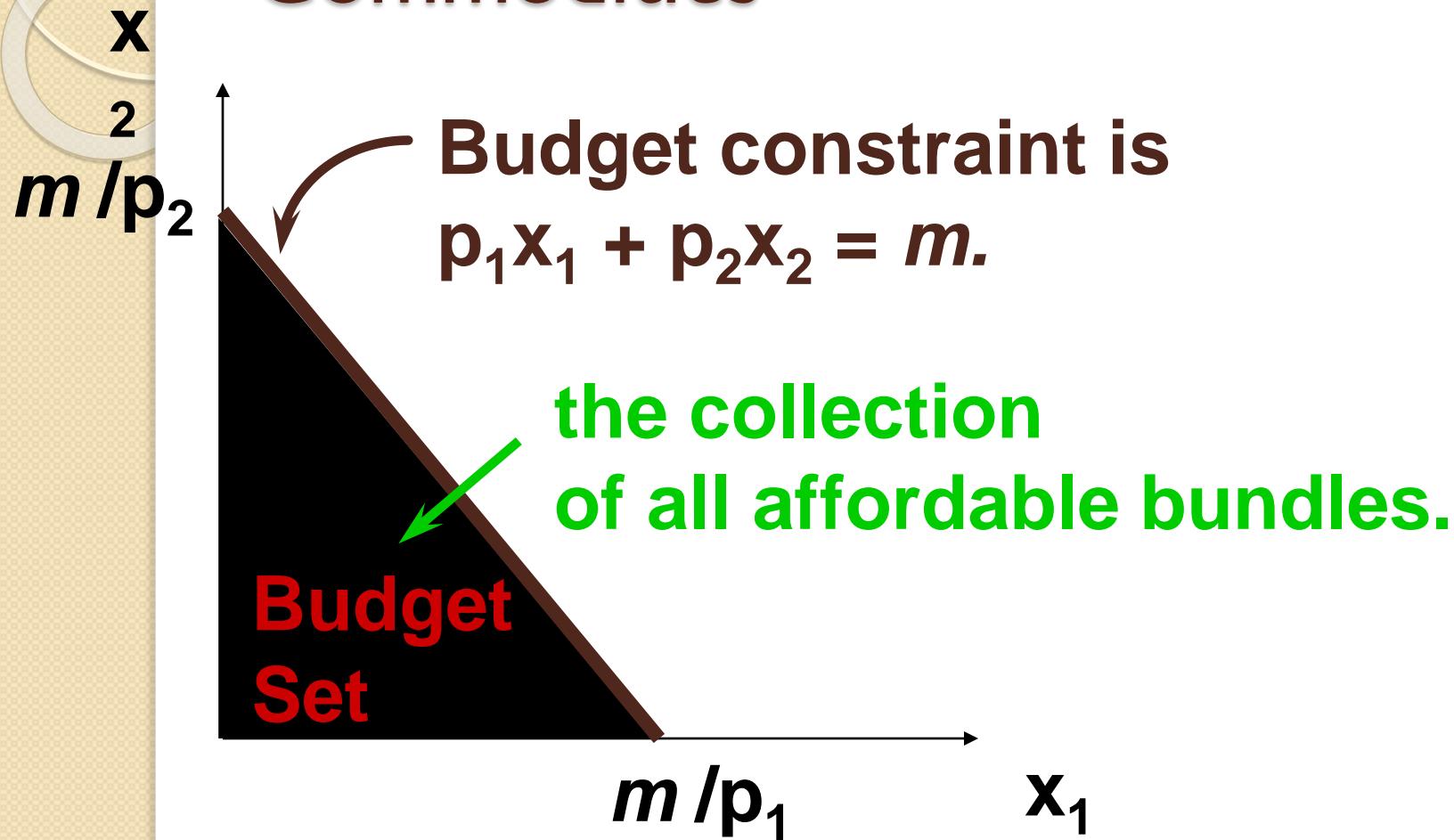
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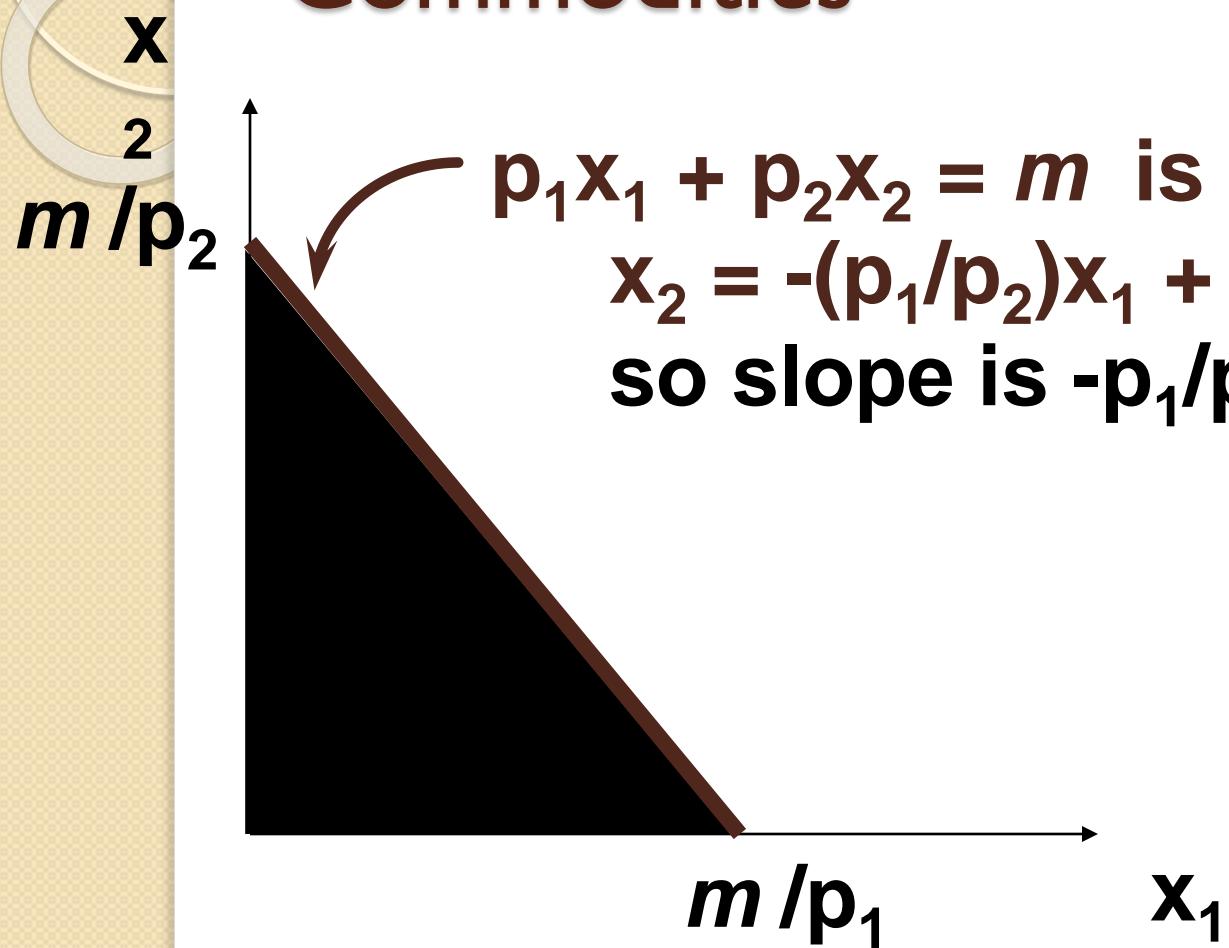
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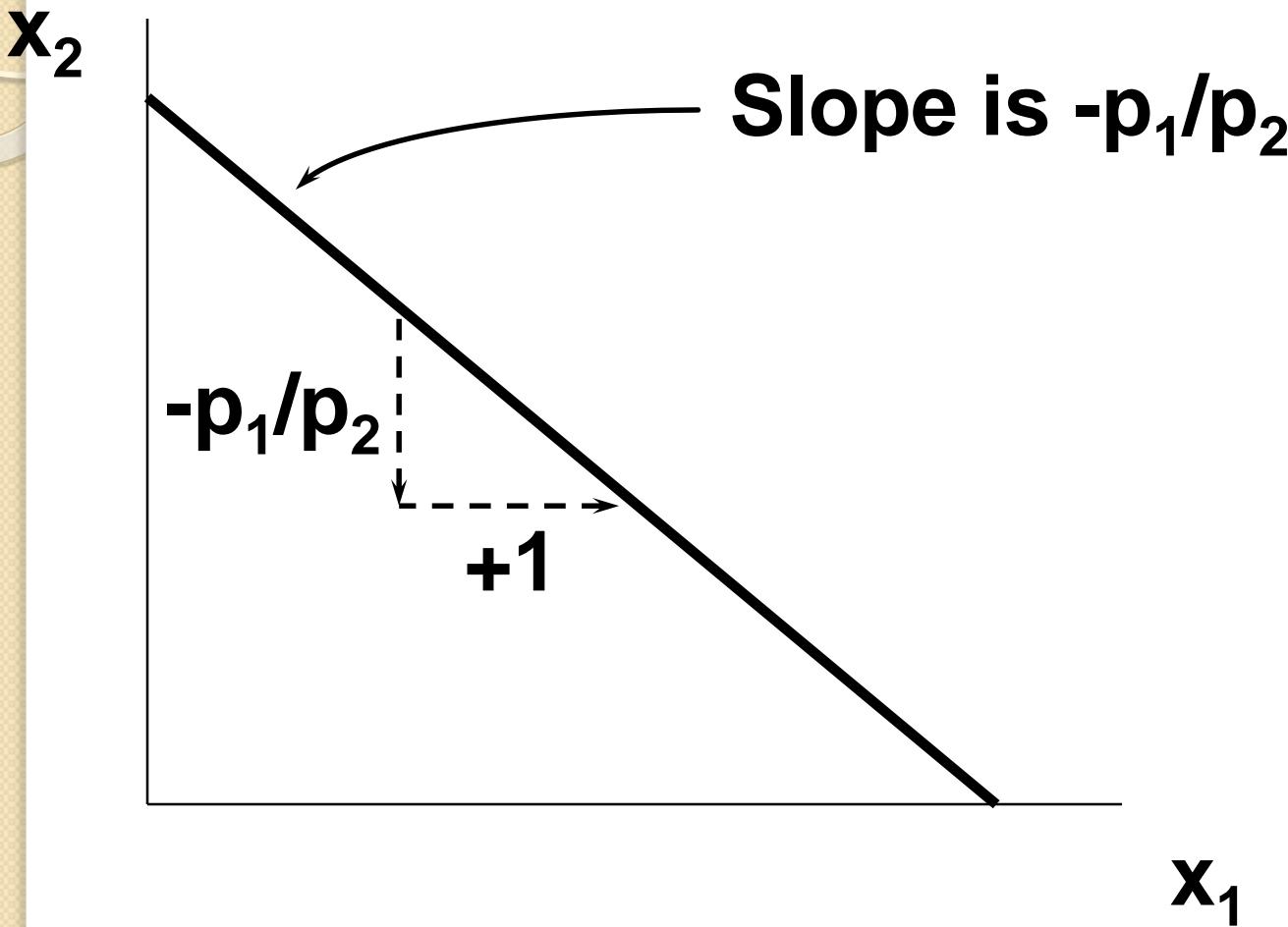
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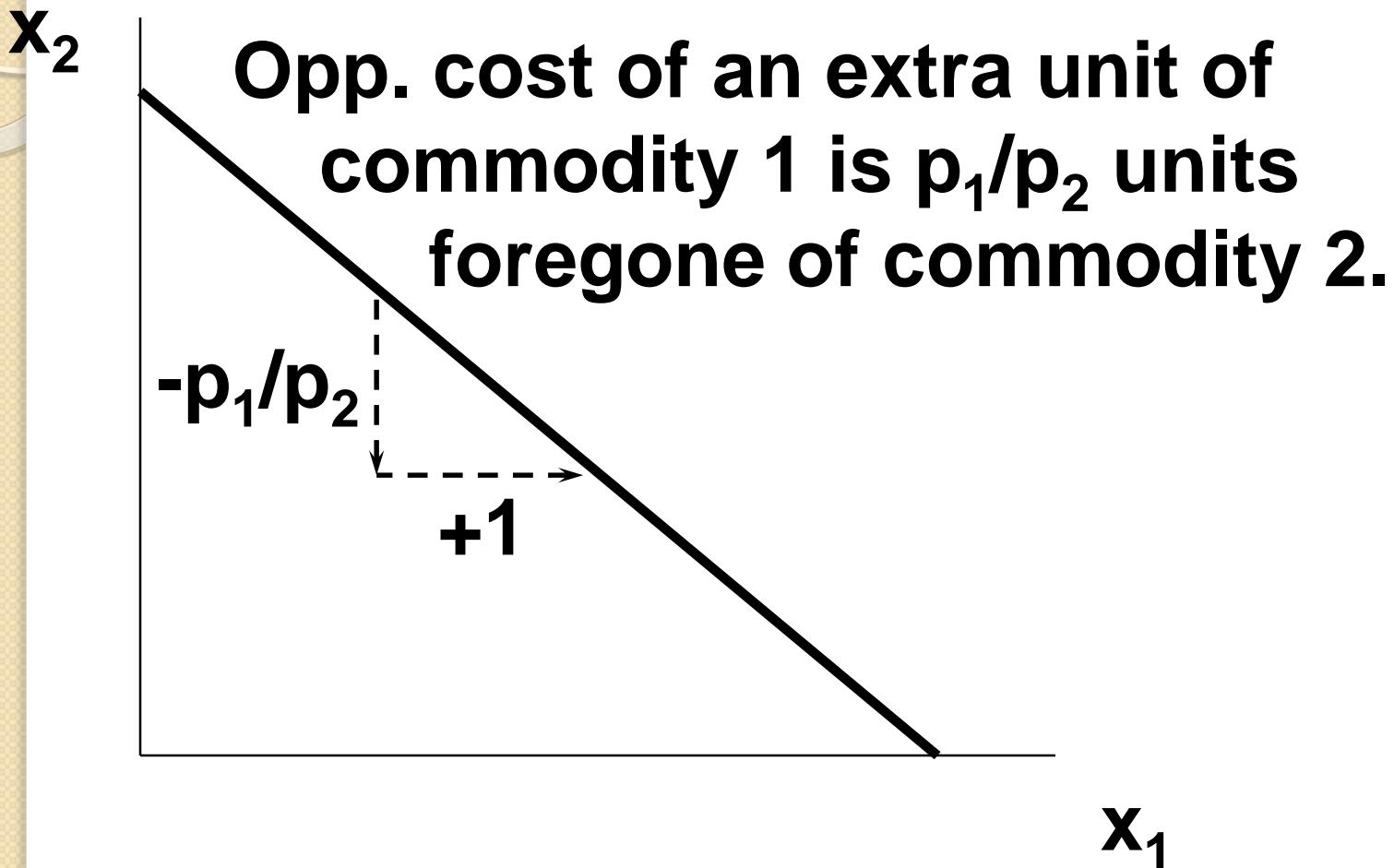
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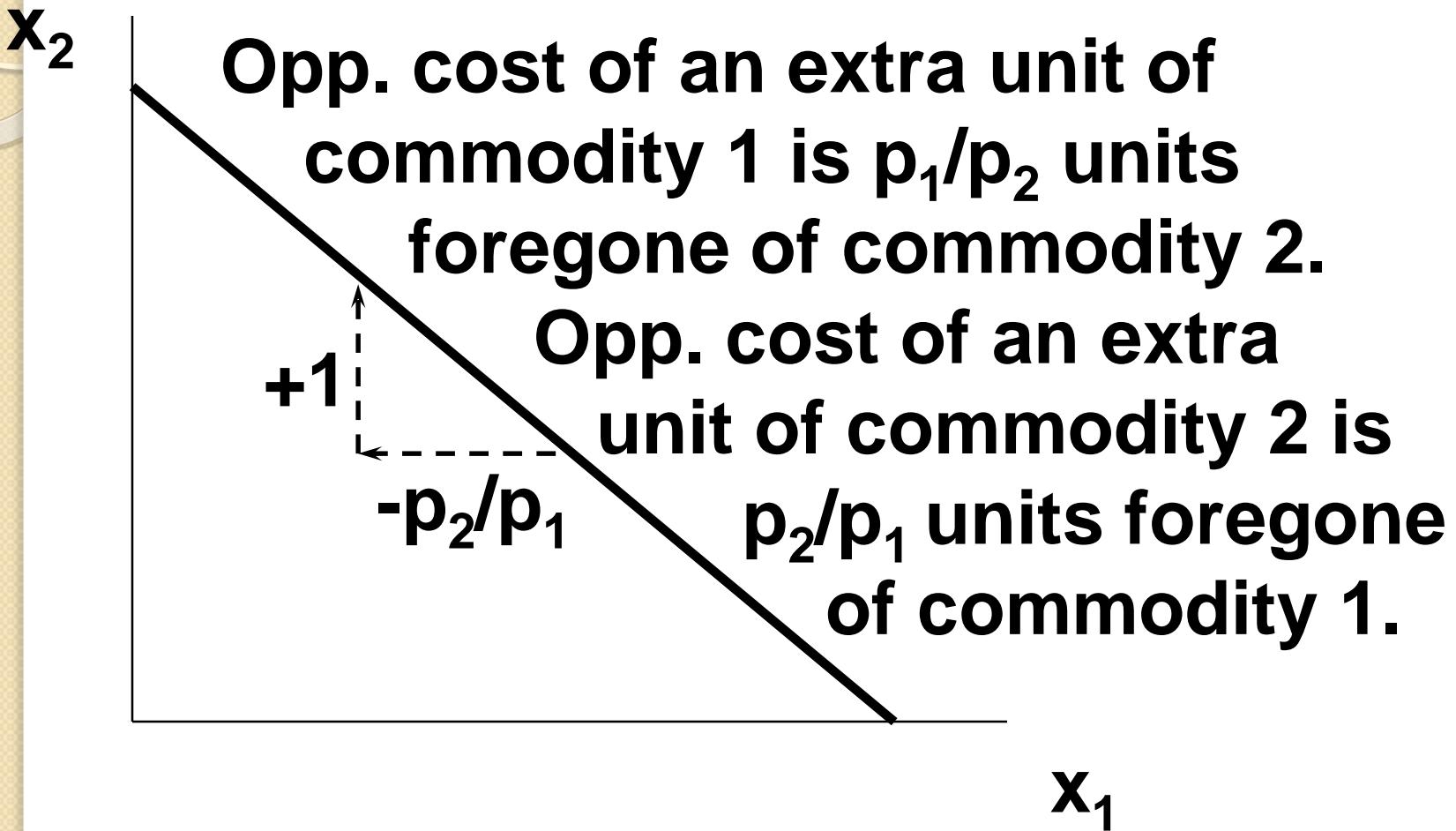
Budget Constraints



Budget Constraints



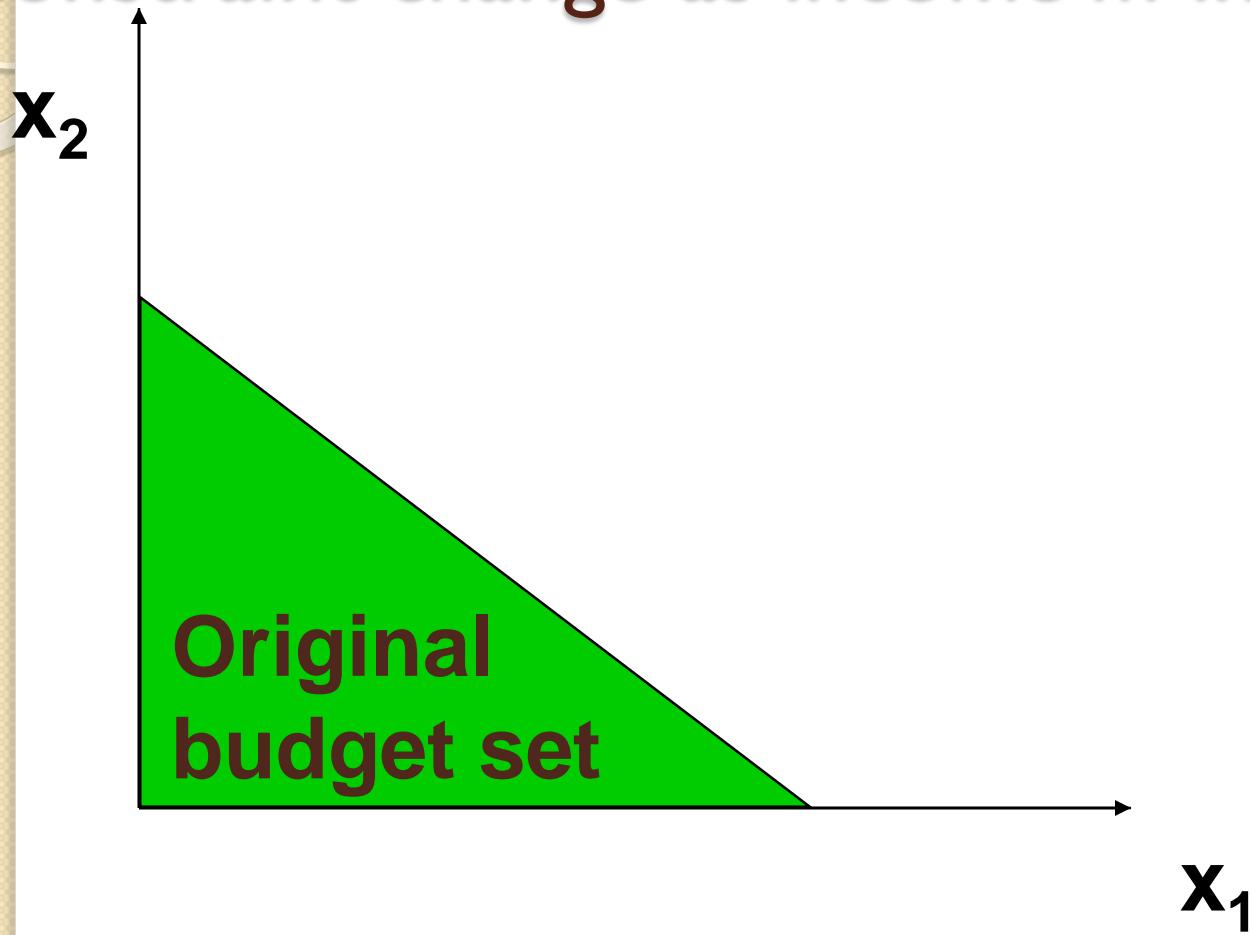
Budget Constraints



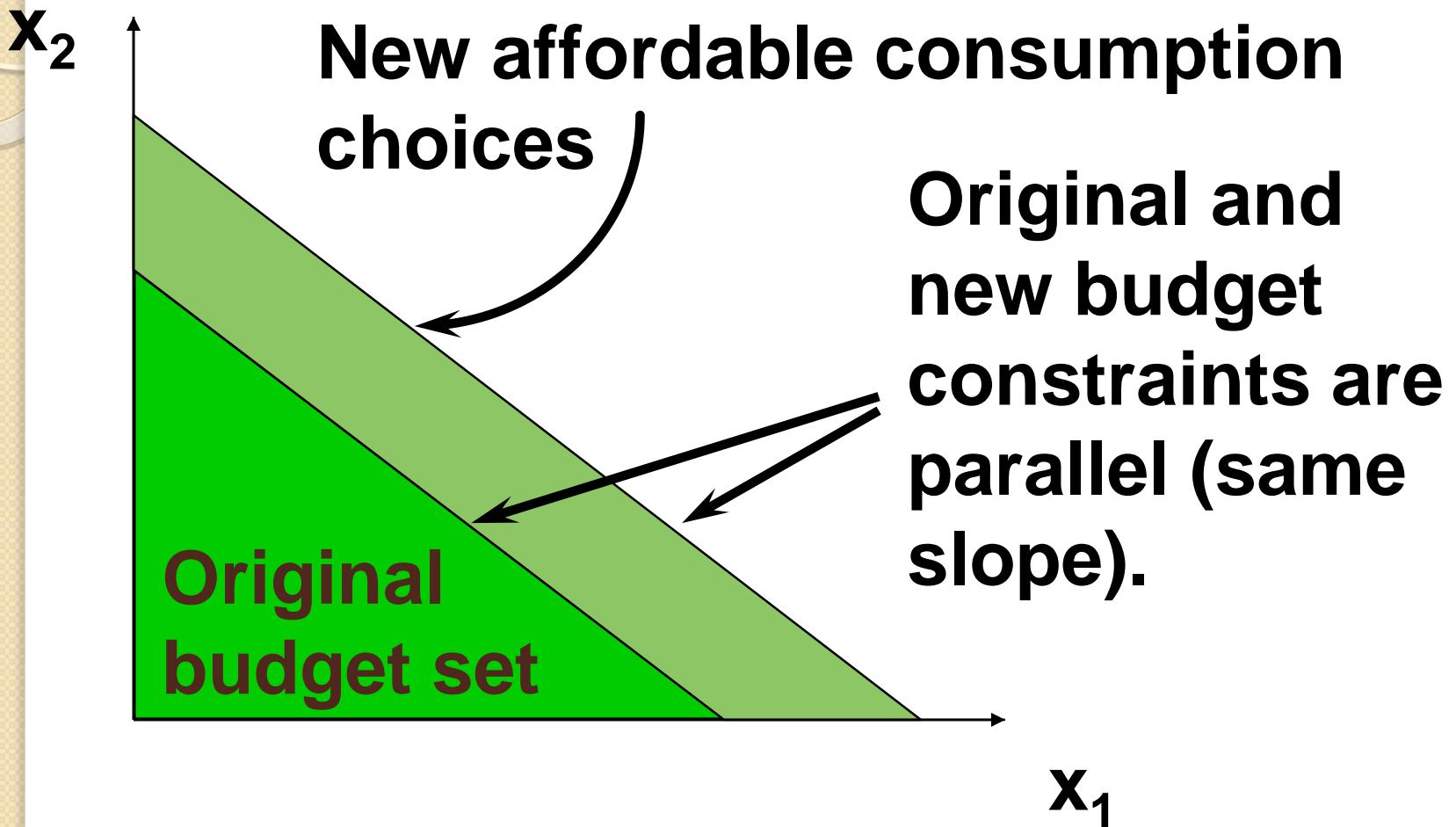
Budget Sets & Constraints; Income and Price Changes

- The budget constraint and budget set depend upon prices and income. What happens as prices or income change?

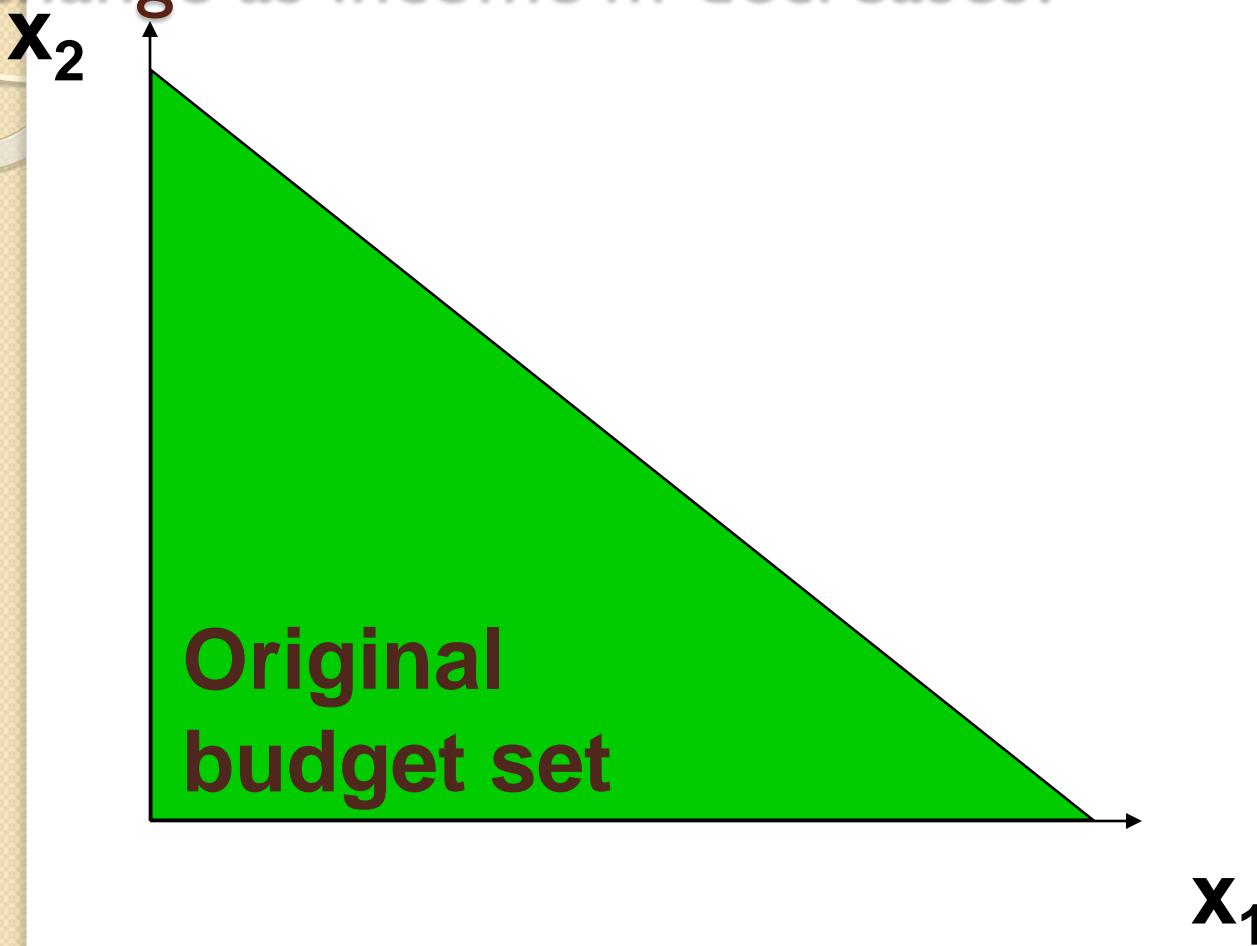
How do the budget set and budget constraint change as income m increases?



Higher income gives more choice



How do the budget set and budget constraint change as income m decreases?



Budget Constraints - Income Changes

- Increases in income m shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.
- Decreases in income m shift the constraint inward in a parallel manner, thereby shrinking the budget set and reducing choice.

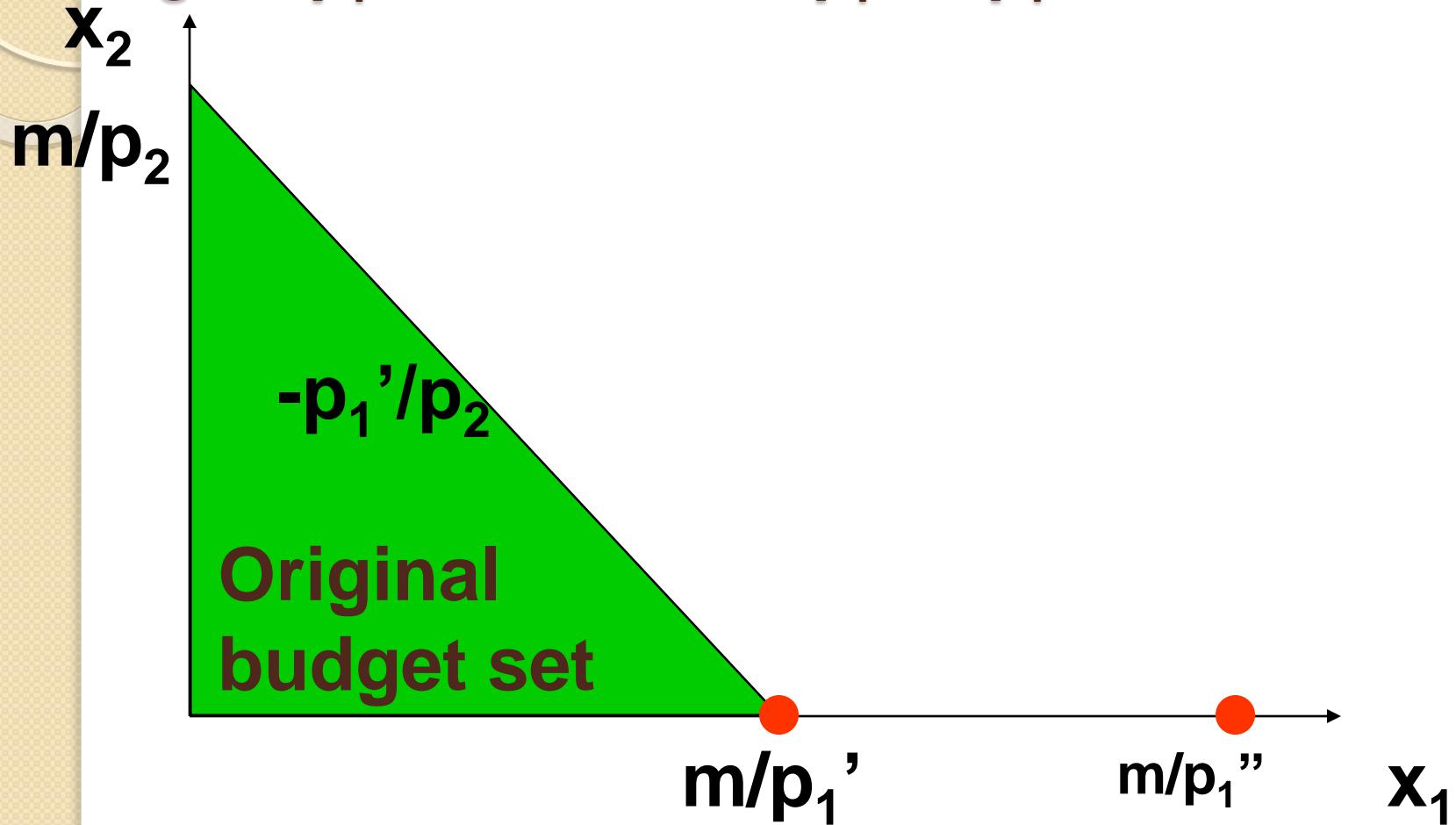
Budget Constraints - Income Changes

- No original choice is lost and new choices are added when income **increases**, so higher income cannot make a consumer worse off.
- An income **decrease** may (typically will) make the consumer worse off.

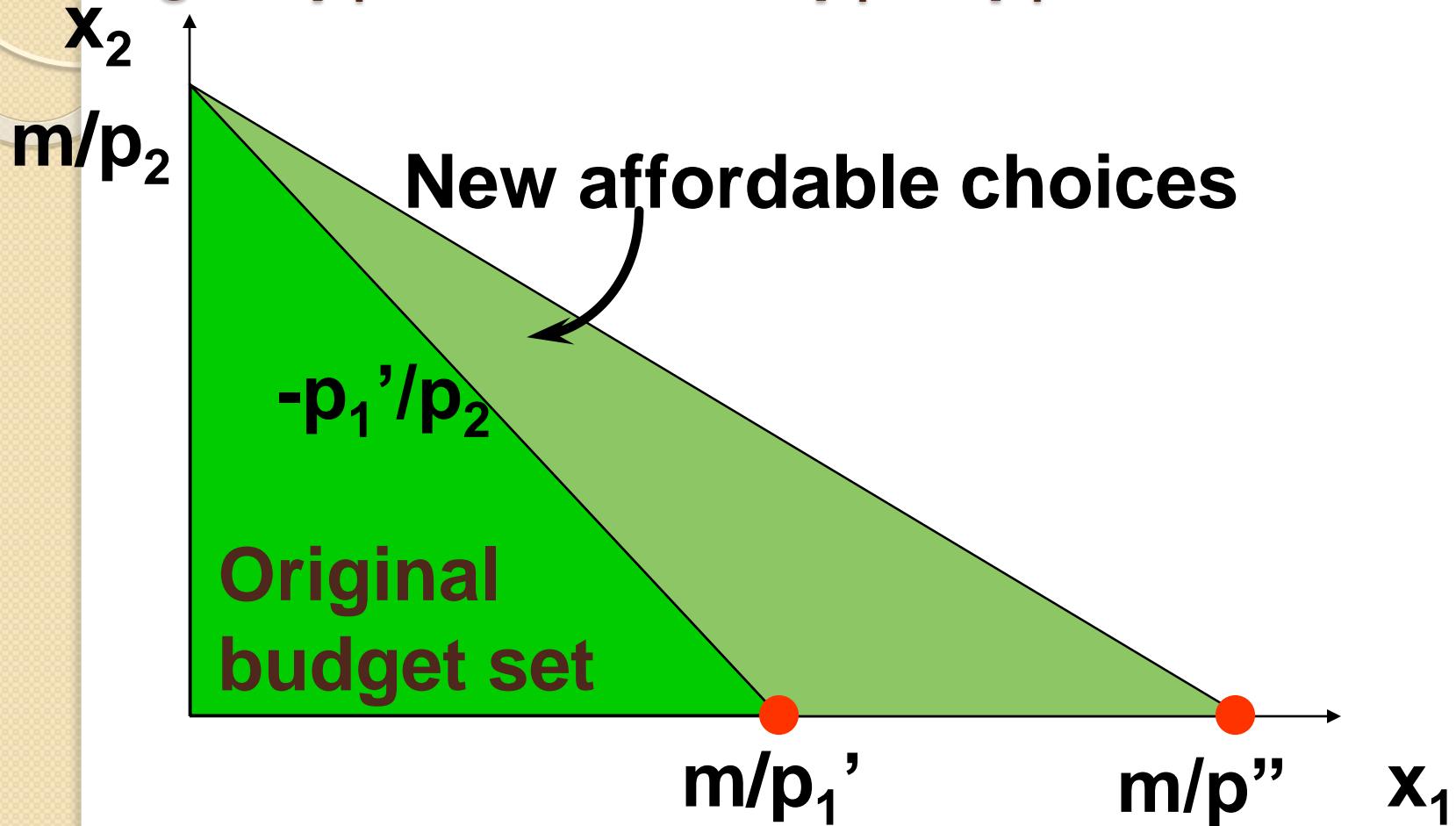
Budget Constraints - Price Changes

- What happens if just one price decreases?
- Suppose p_1 decreases.

How do the budget set and budget constraint change as p_1 decreases from p_1' to p_1'' ?



How do the budget set and budget constraint change as p_1 decreases from p_1' to p_1'' ?



Budget Constraints - Price Changes

- Reducing the price of one commodity **pivots** the constraint outward. No old choice is lost and new choices are added, so reducing one price cannot make the consumer worse off
- Similarly, increasing one price **pivots** the constraint inwards, reduces choice and may (typically will) make the consumer worse off.

Uniform *Ad Valorem* Sales Taxes

- An *ad valorem sales tax* (从价营业税) levied at a rate of 5% increases all prices by 5%, from p to $(1+0.05)p = 1.05p$.
- An *ad valorem sales tax* levied at a rate of t increases all prices by tp from p to $(1+t)p$.
- A uniform sales tax is applied uniformly to all commodities.

Uniform *Ad Valorem* Sales Taxes

- A uniform sales tax levied at rate t changes the constraint from

$$P_1 X_1 + P_2 X_2 = m$$

to

$$(1+t)P_1 X_1 + (1+t)P_2 X_2 = m$$

Uniform *Ad Valorem* Sales Taxes

- A uniform sales tax levied at rate t changes the constraint from

$$P_1 X_1 + P_2 X_2 = m$$

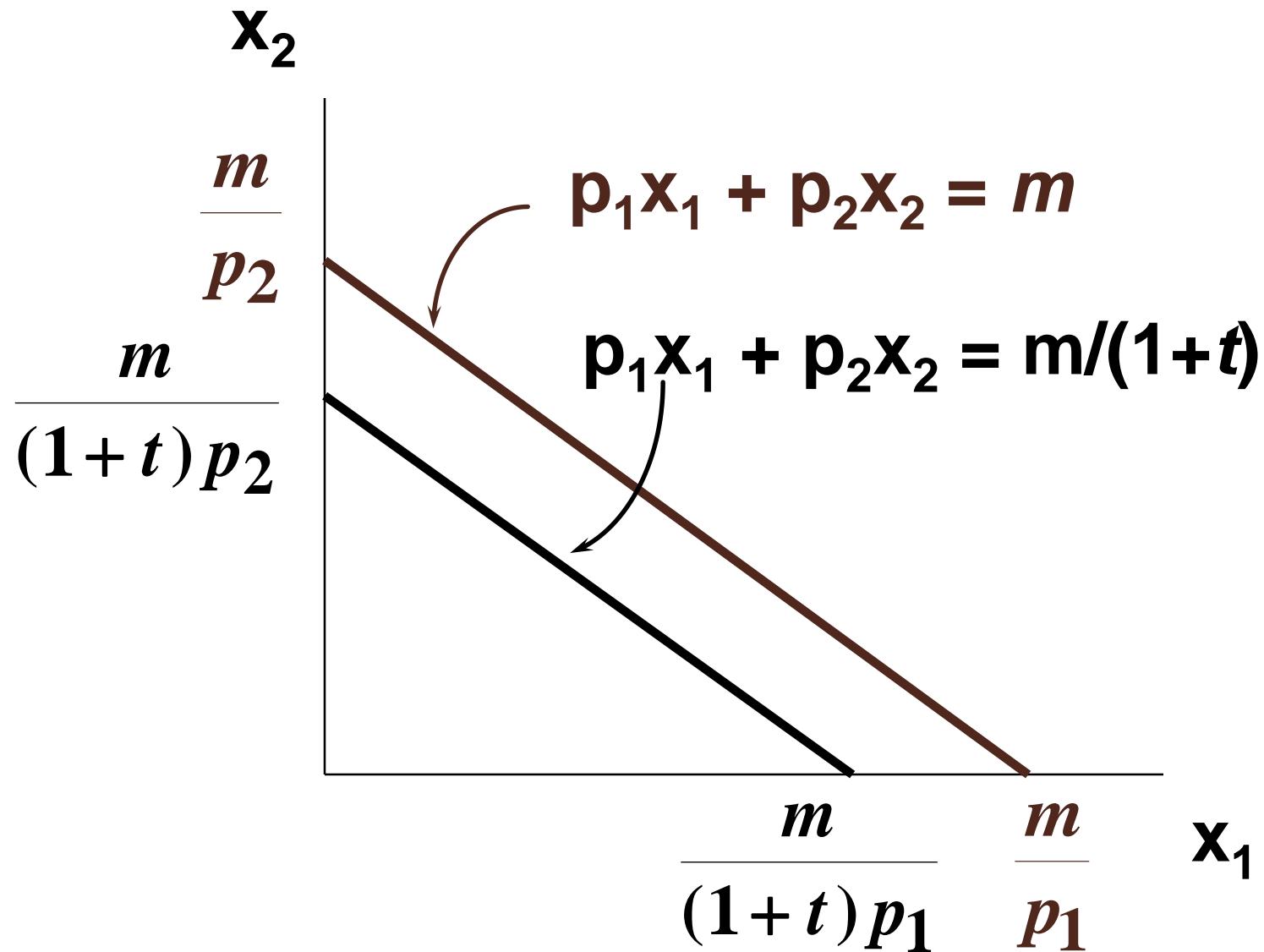
to

$$(1+t)P_1 X_1 + (1+t)P_2 X_2 = m$$

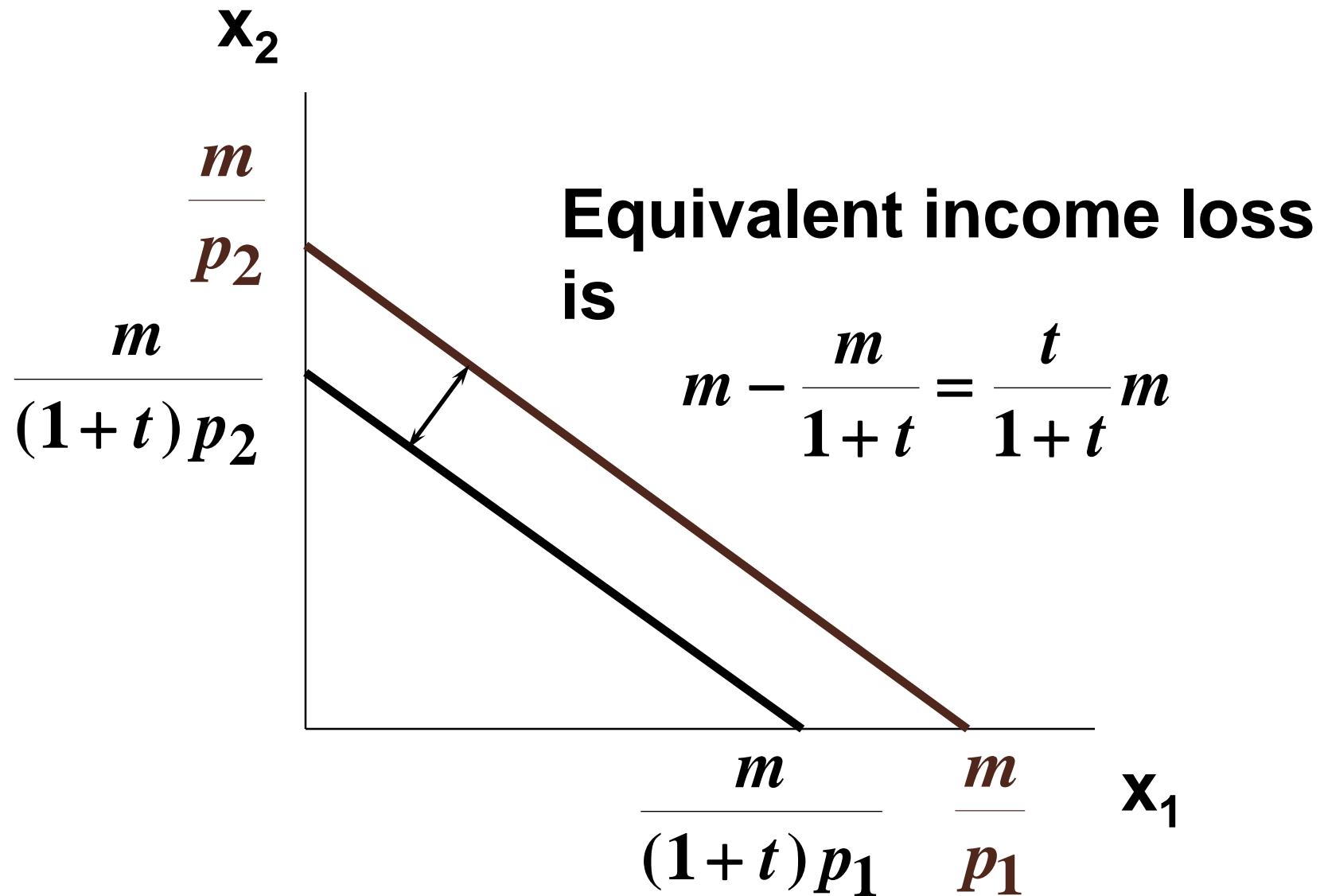
i.e.

$$P_1 X_1 + P_2 X_2 = m/(1+t).$$

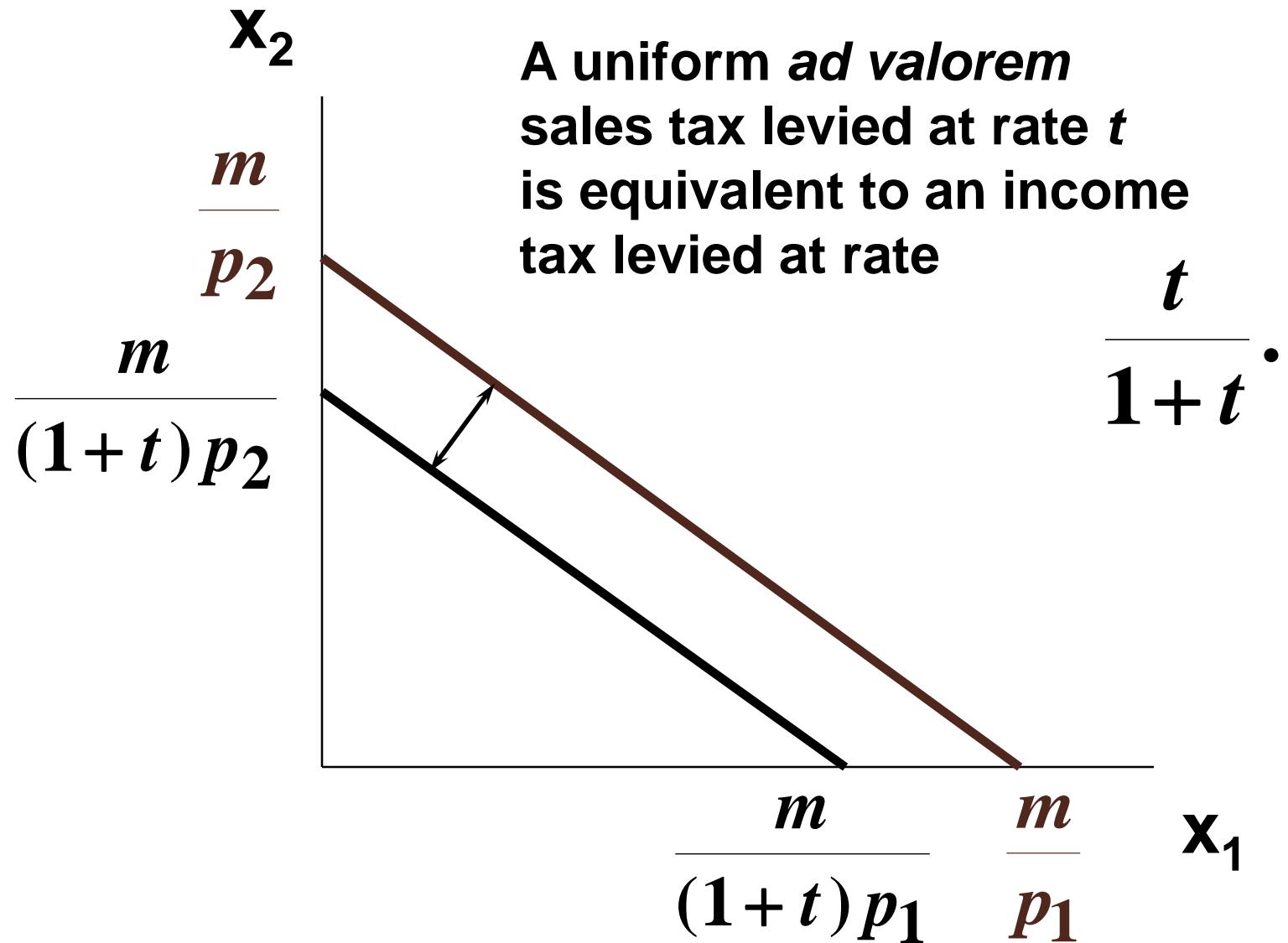
Uniform *Ad Valorem* Sales Taxes



Uniform Ad Valorem Sales Taxes



Uniform *Ad Valorem* Sales Taxes



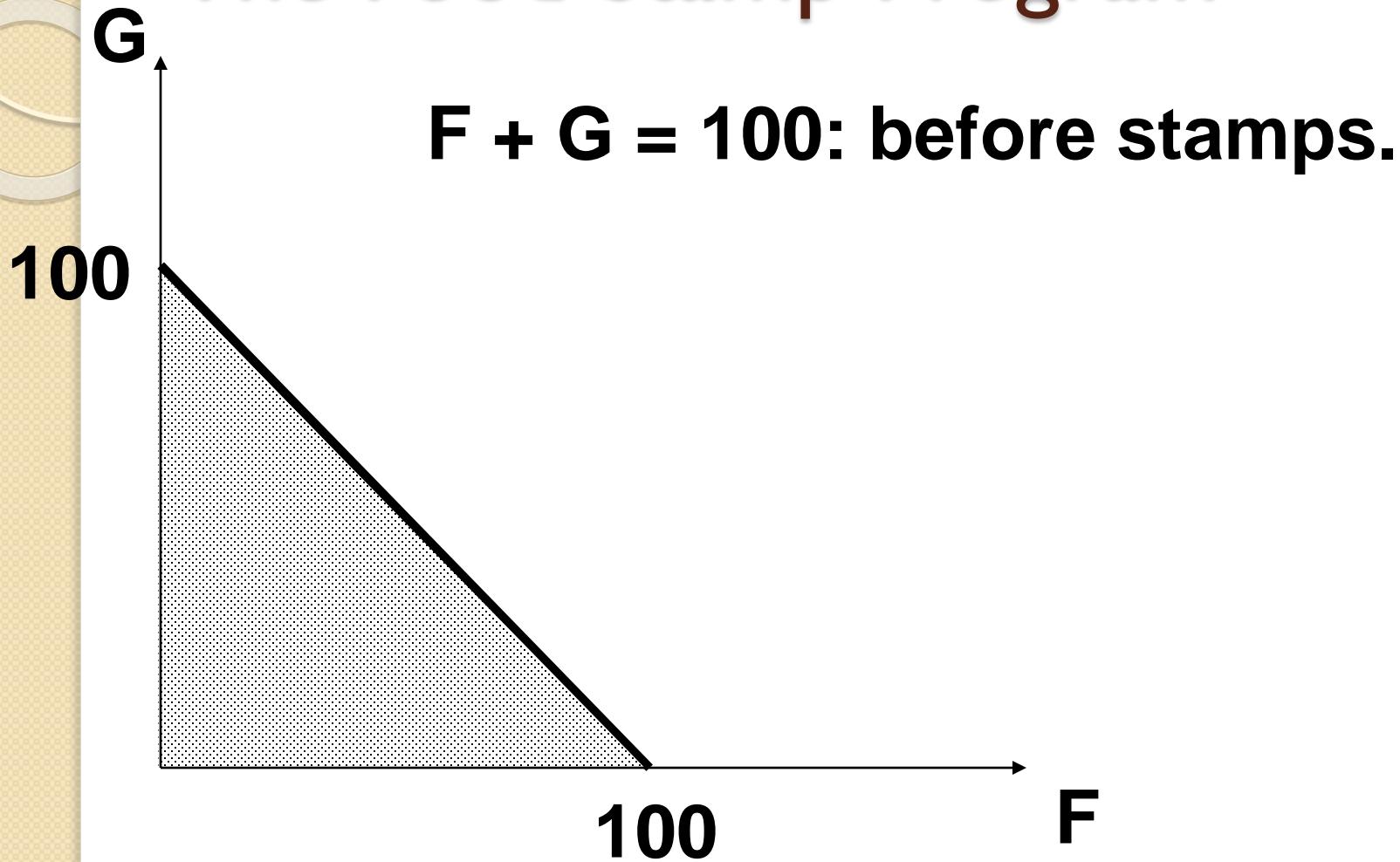
The Food Stamp Program

- Food stamps are coupons that can be legally exchanged only for food.
- How does a commodity-specific gift such as a food stamp alter a family's budget constraint?

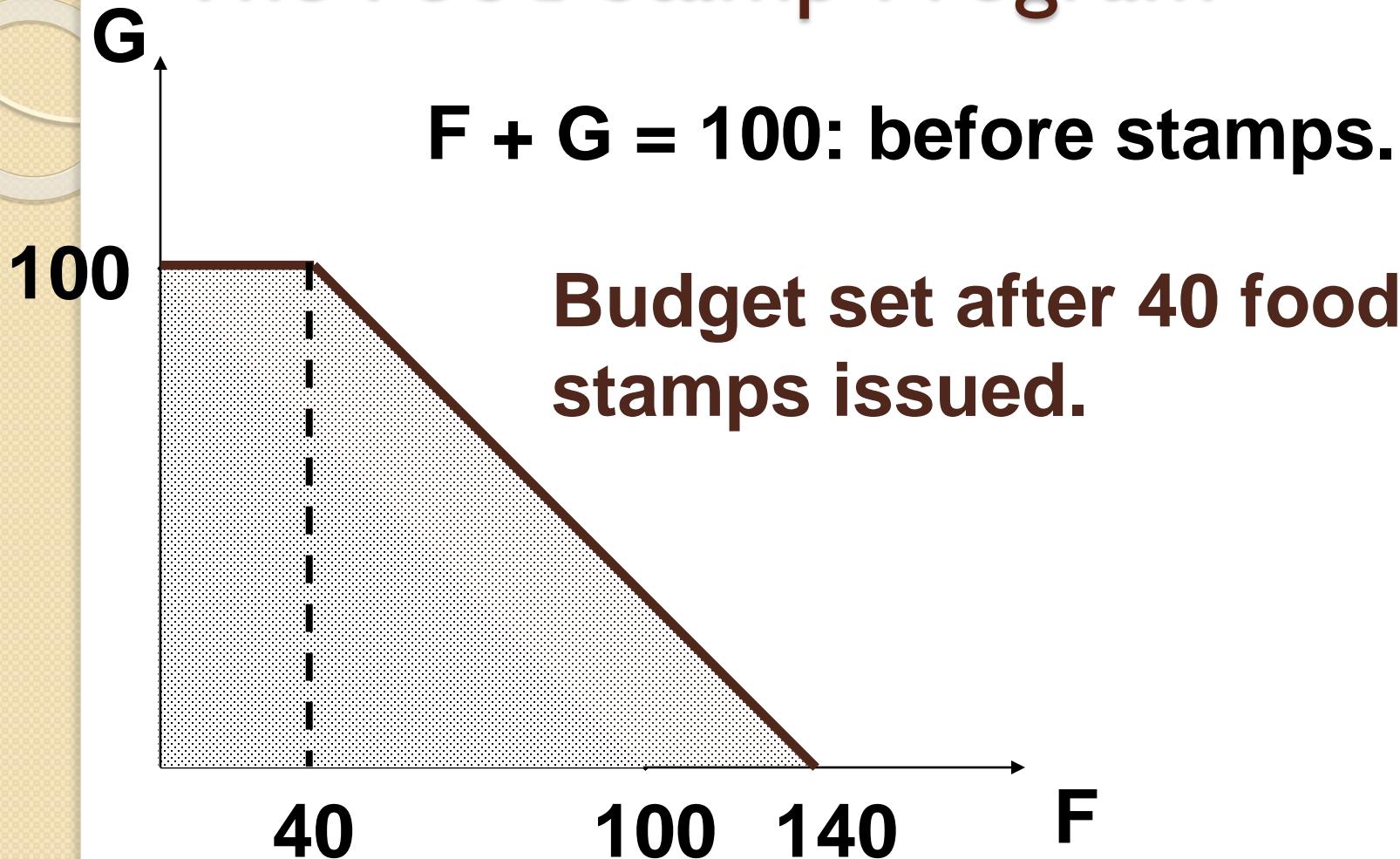
The Food Stamp Program

- Suppose $m = \$100$, $p_F = \$1$ and the price of “other goods” is $p_G = \$1$
- “Other goods” is a **composite good**
 - It simplifies the analysis to a 2-good model
- The budget constraint is then
$$F + G = 100.$$

The Food Stamp Program



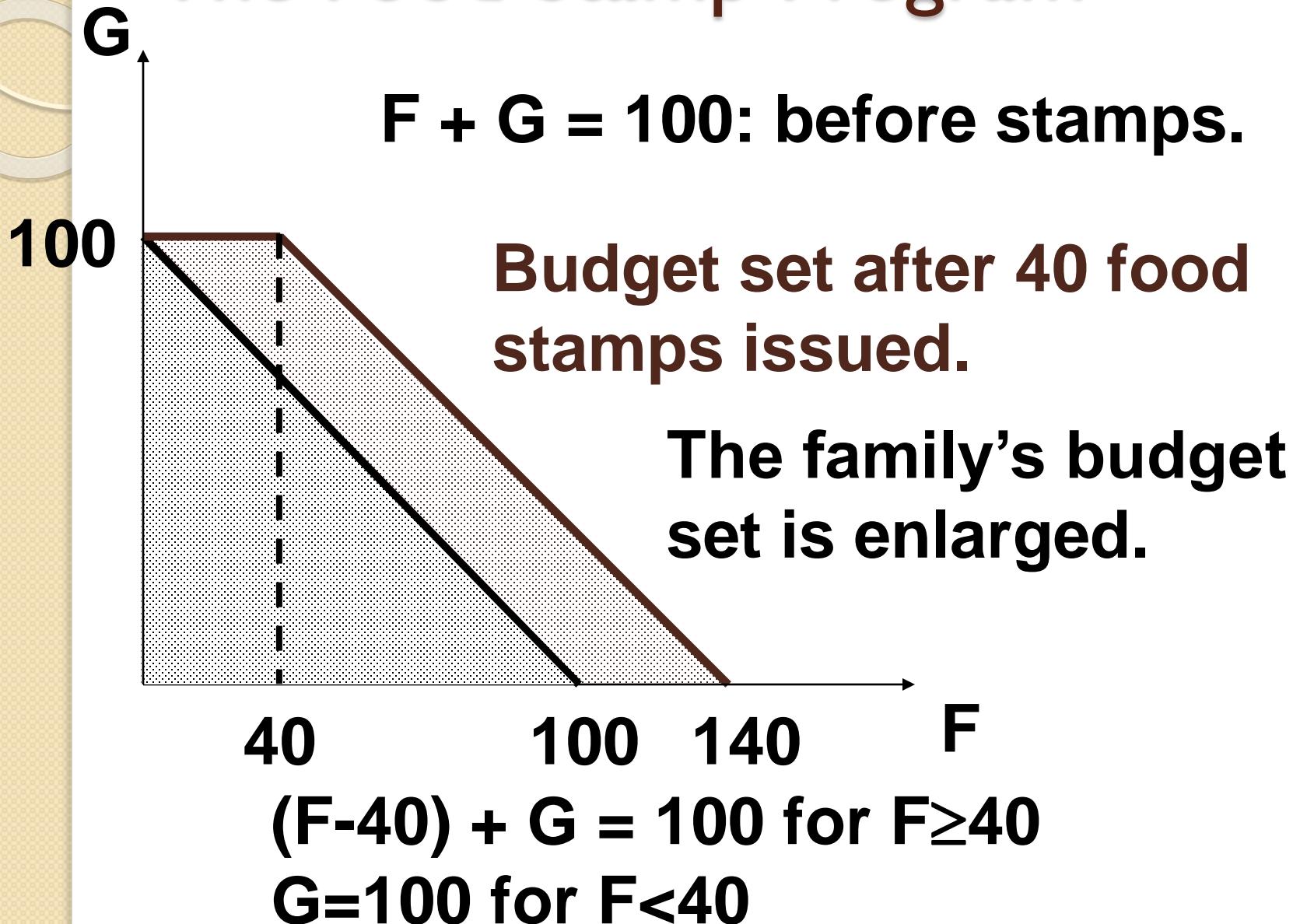
The Food Stamp Program



$$(F-40) + G = 100 \text{ for } F \geq 40$$

$$G = 100 \text{ for } F < 40$$

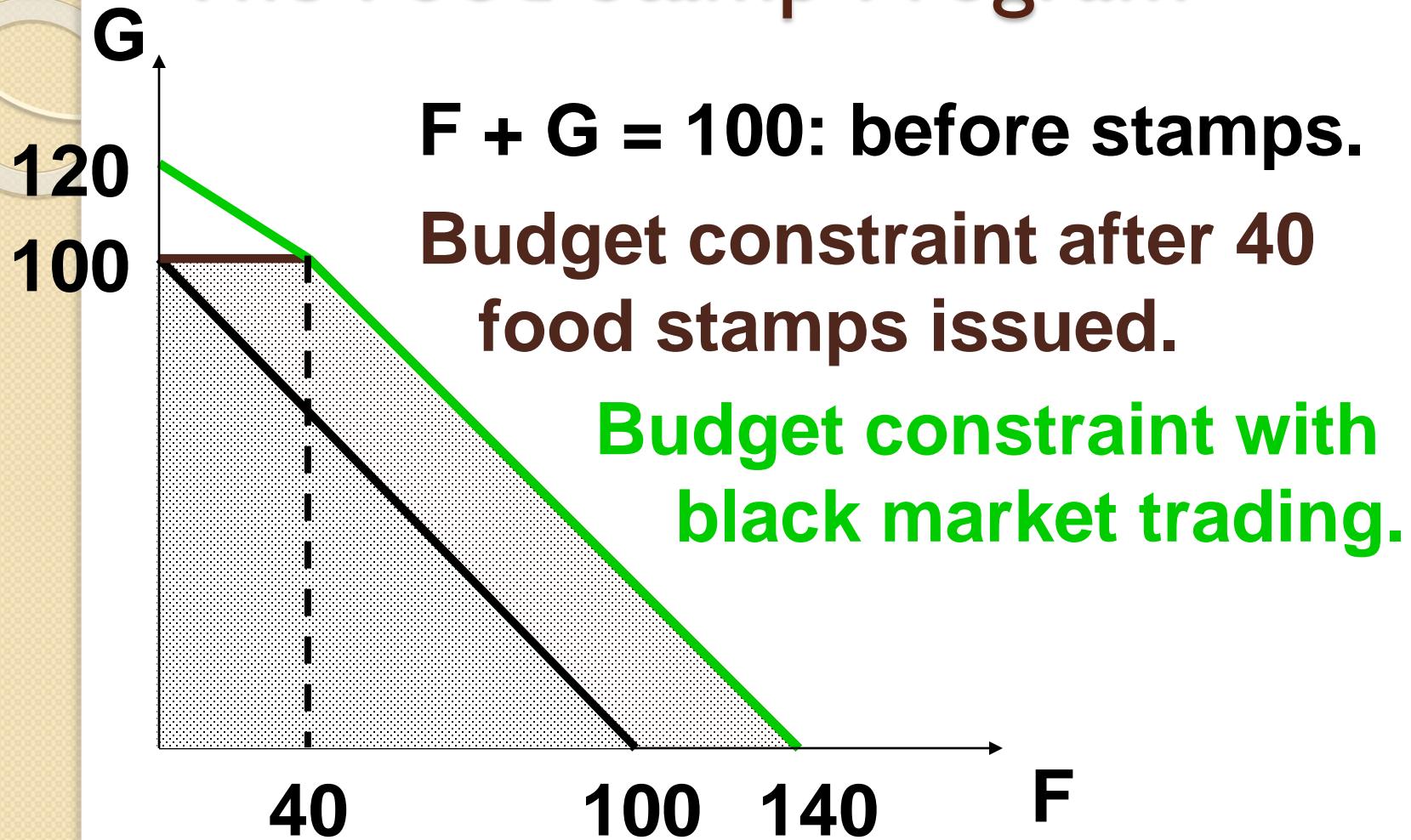
The Food Stamp Program



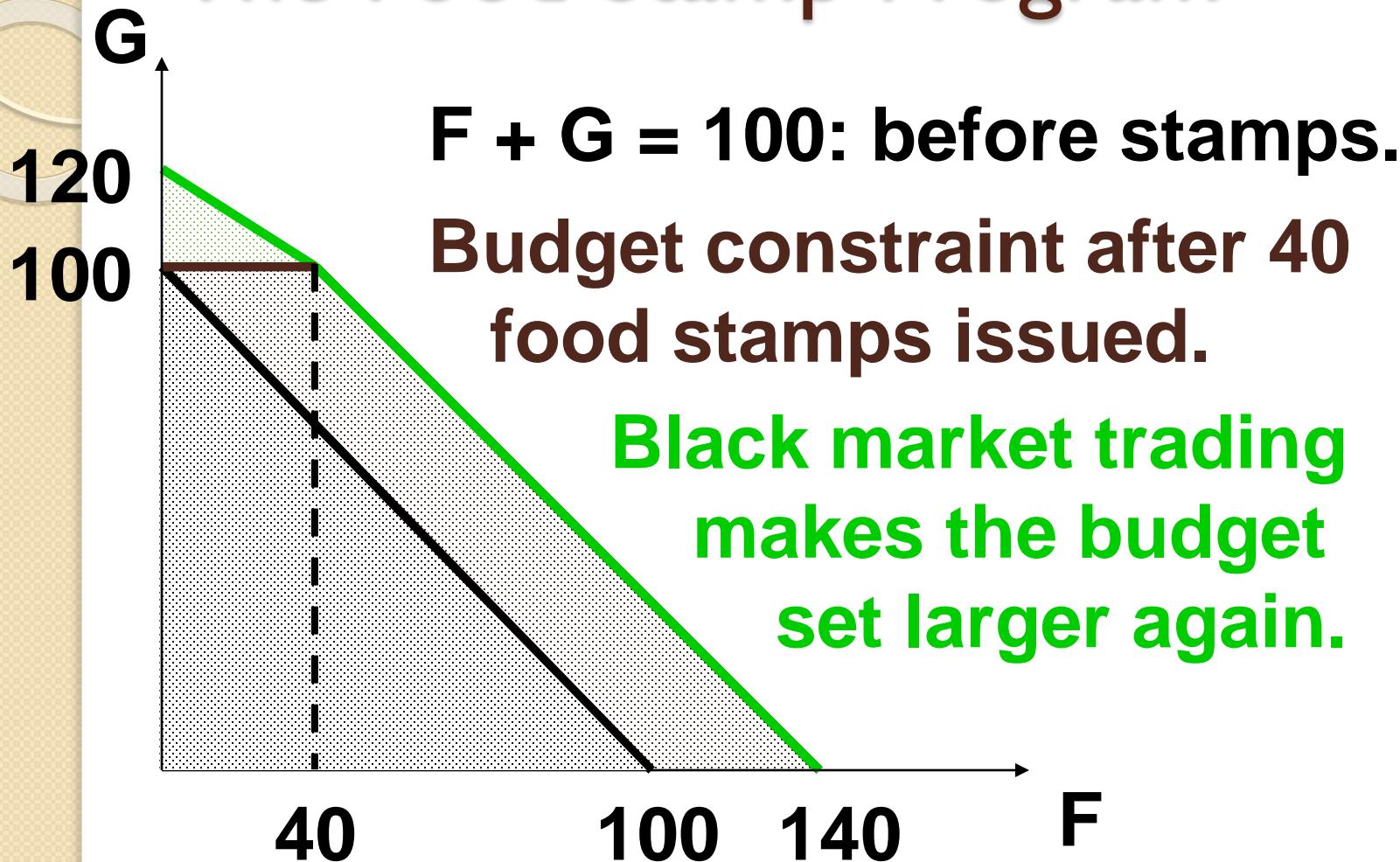
The Food Stamp Program

- What if food stamps can be traded on a black market for \$0.50 each?
- $G=100+0.5\times(40-F)$ for $F < 40$
- $(F-40)+G=100$ for $F \geq 40$

The Food Stamp Program



The Food Stamp Program



Shapes of Budget Constraints

- Q: What makes a budget constraint a straight line?
- A: A straight line has a constant slope and the constraint is

$$P_1X_1 + \dots + P_nX_n = m$$

so if prices are constants then a constraint is a straight line.

Shapes of Budget Constraints

- But what if prices are not constants?
- E.g. bulk buying discounts, or price penalties for buying “too much”.
- Then constraints will be curved.

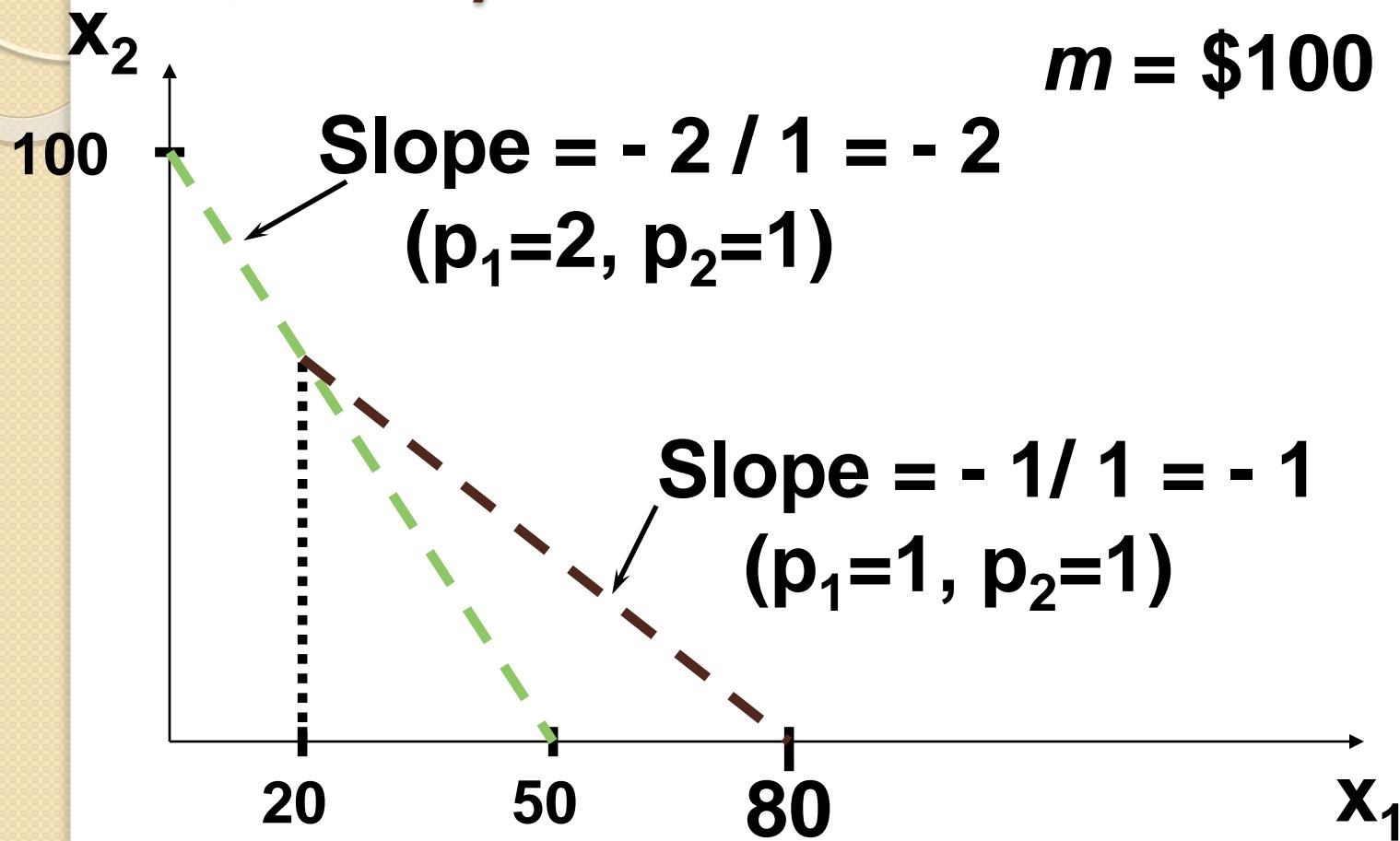
Shapes of Budget Constraints - Quantity Discounts

- Suppose p_2 is constant at \$1 but that $p_1 = \$2$ for $0 \leq x_1 \leq 20$ and $p_1 = \$1$ for $x_1 > 20$.

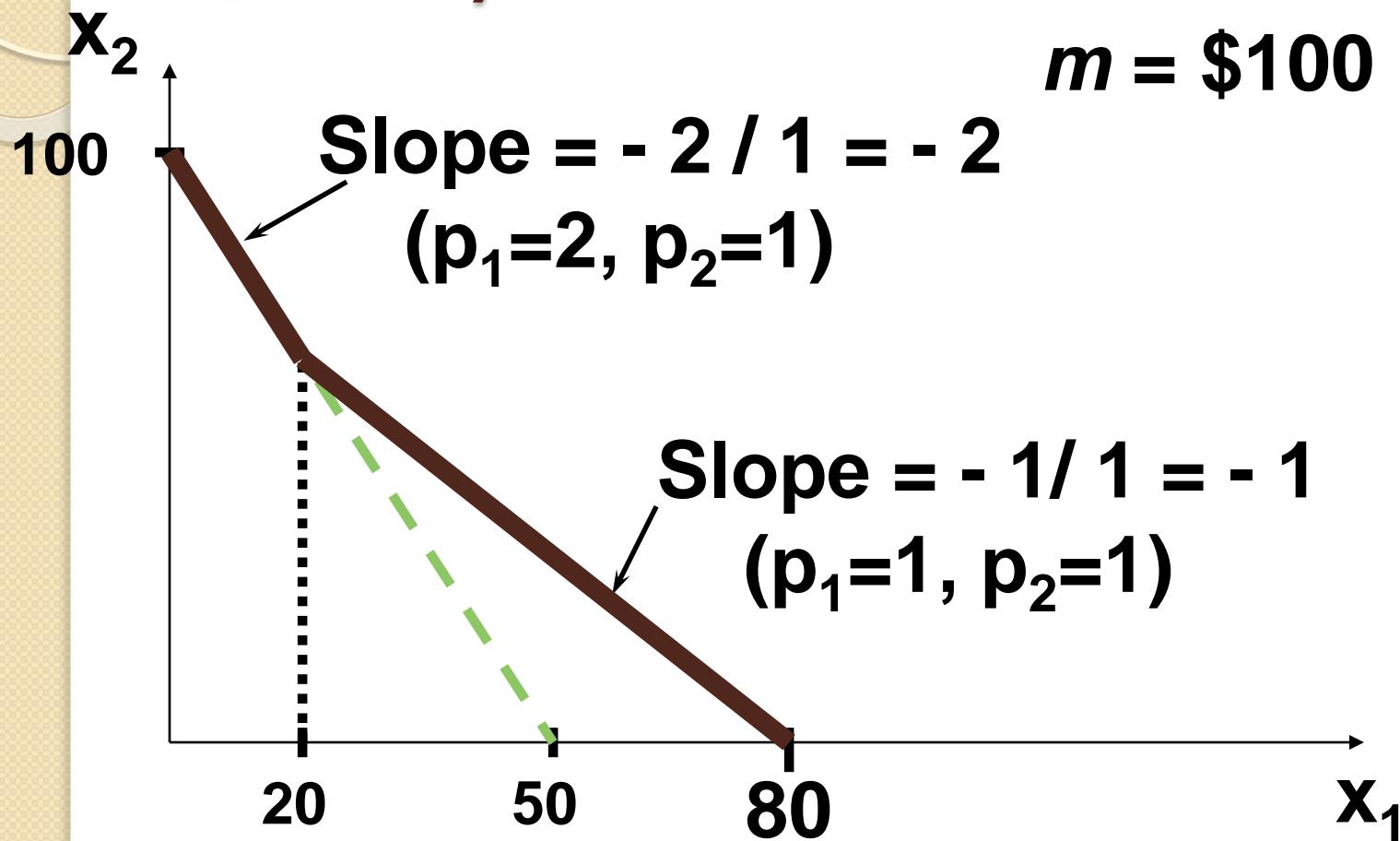
Shapes of Budget Constraints - Quantity Discounts

- Suppose p_2 is constant at \$1 but that $p_1 = \$2$ for $0 \leq x_1 \leq 20$ and $p_1 = \$1$ for $x_1 > 20$.
- Then the constraint's slope is
$$2, \text{ for } 0 \leq x_1 \leq 20$$
$$1, \text{ for } x_1 > 20$$

Shapes of Budget Constraints with a Quantity Discount



Shapes of Budget Constraints with a Quantity Discount



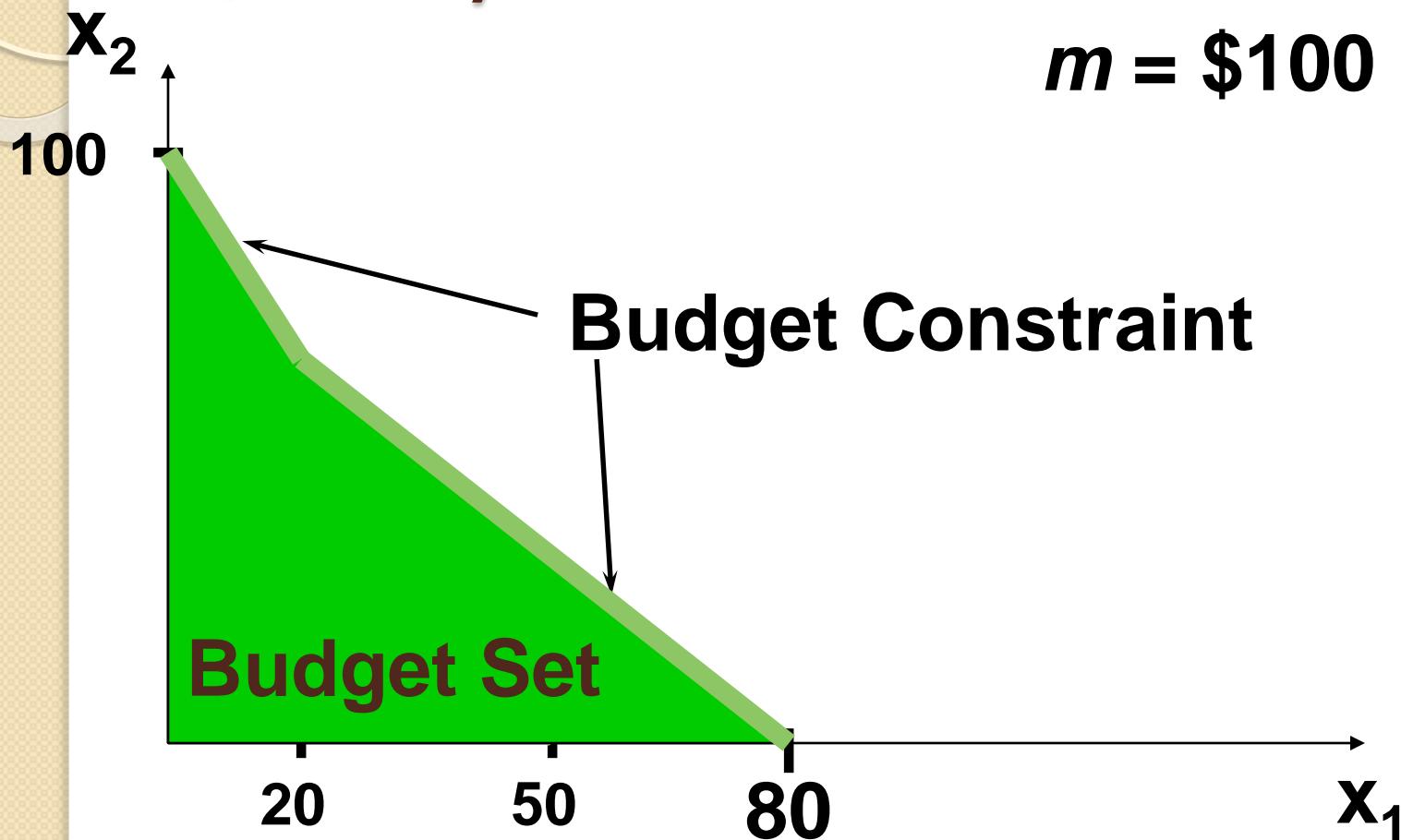
Budget Constraints with a Quantity Discount

- The constraint is

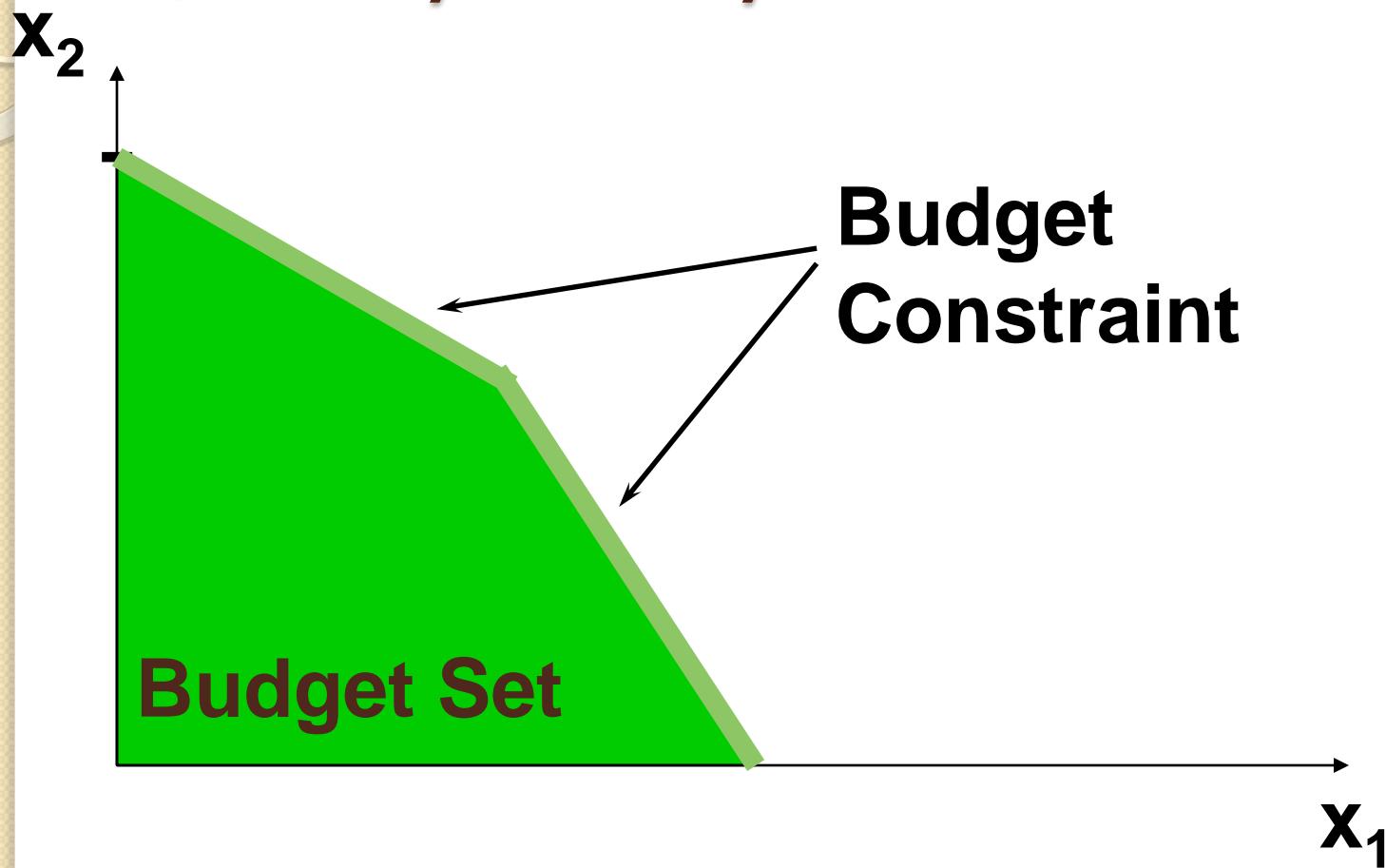
$$2x_1 + x_2 = m \text{ for } 0 \leq x_1 \leq 20$$

$$2 \times 20 + (x_1 - 20) + x_2 = m \text{ for } x_1 > 20$$

Shapes of Budget Constraints with a Quantity Discount



Shapes of Budget Constraints with a Quantity Penalty



Consumer Preferences

- Describe preferences
- Indifference curves (无差异曲线)
- Well-behaved preferences
- Marginal rate of substitution (边际替代率)

Rationality in Economics

- Behavioral Postulate:

A decision-maker always chooses its most preferred alternative from its set of available alternatives

- So to model choice we must model decision-makers' preferences.

Preference Relations

- Comparing two different consumption bundles, x and y :
 - **strict preference** (严格偏好): x is more preferred than y
 - **Indifference** (无差异): x is exactly as preferred as y
 - **weak preference** (弱偏好): x is as at least as preferred as is y

Assumptions about Preference Relations

- **Completeness** (完备性): For any two bundles x and y it is always possible to make the comparison between x and y
- **Reflexivity** (反身性): Any bundle x is always at least as preferred as itself

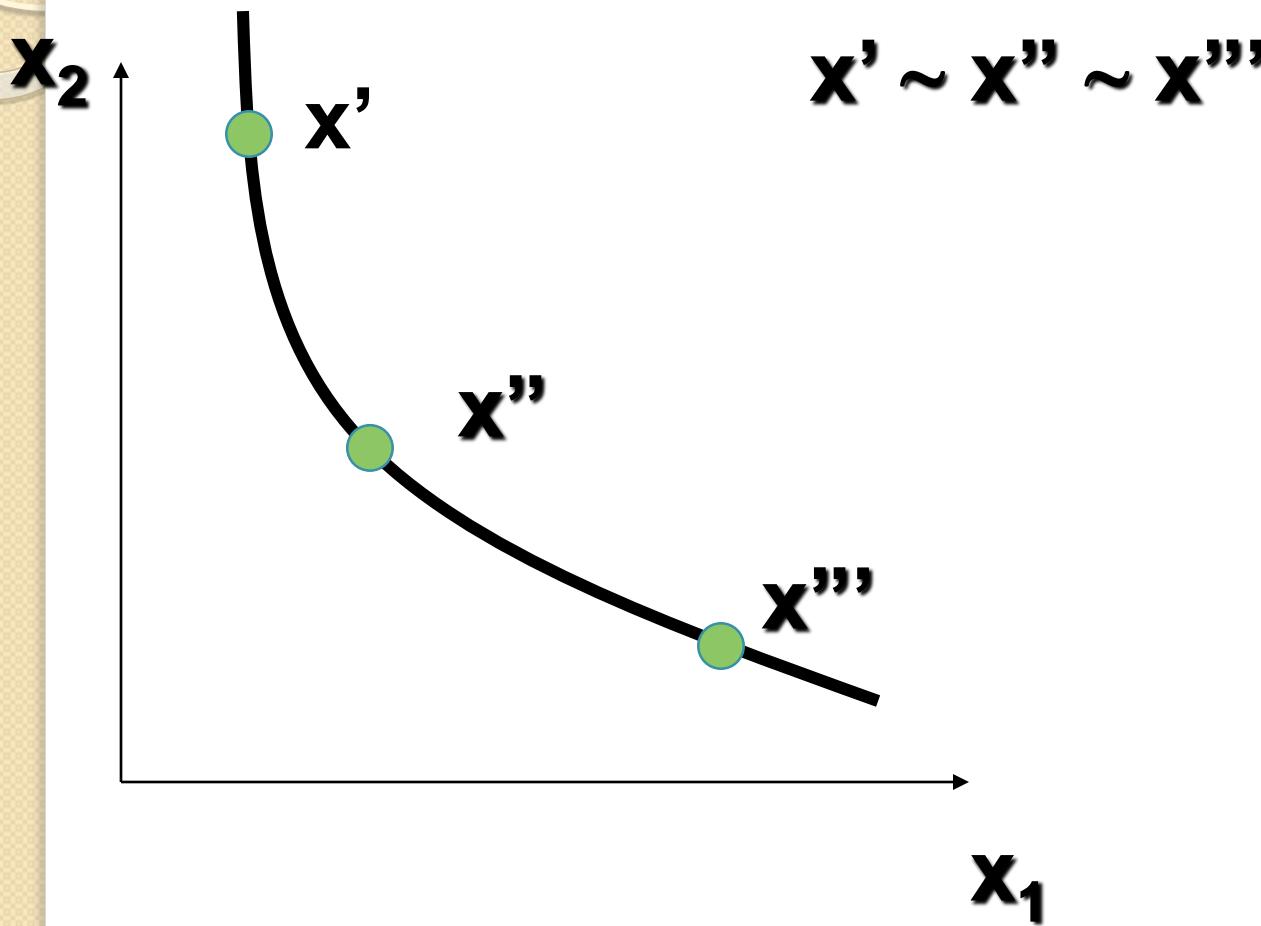
Assumptions about Preference Relations

- **Transitivity** (传递性): If x is at least as preferred as y , and y is at least as preferred as z , then x is at least as preferred as z .

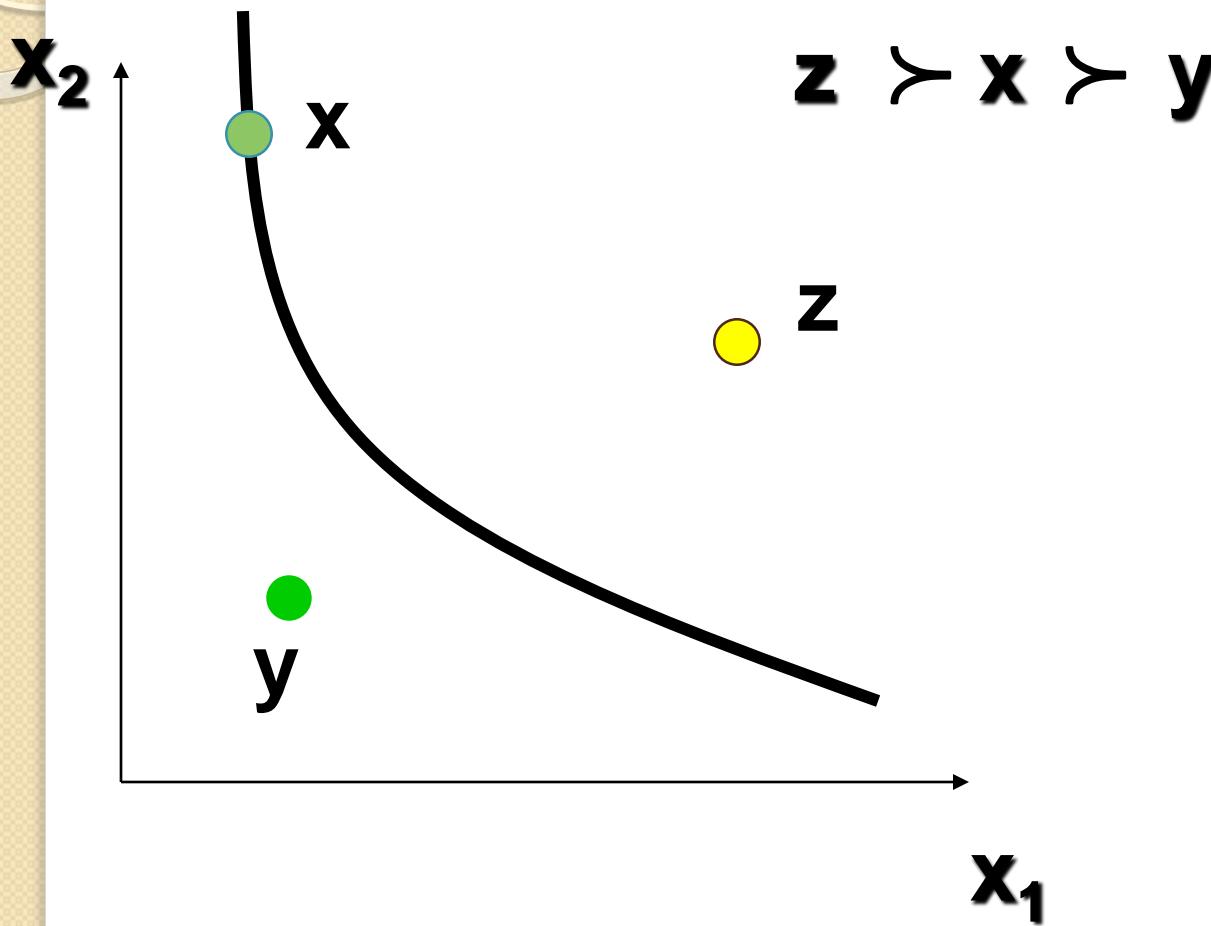
Indifference Curves

- Take a reference bundle x' . The set of all bundles equally preferred to x' is the indifference curve containing x' ; the set of all bundles $y \sim x'$.

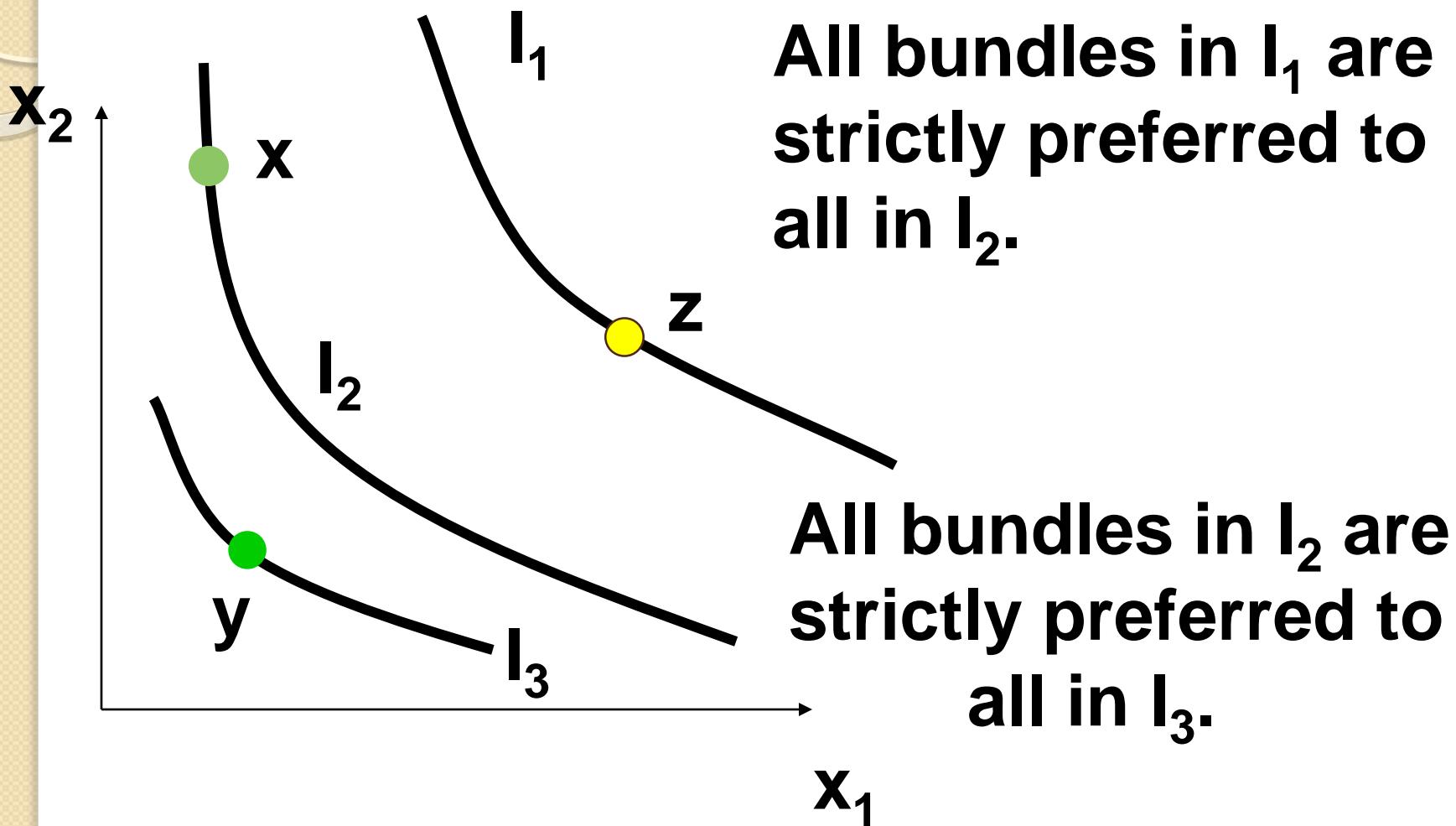
Indifference Curves



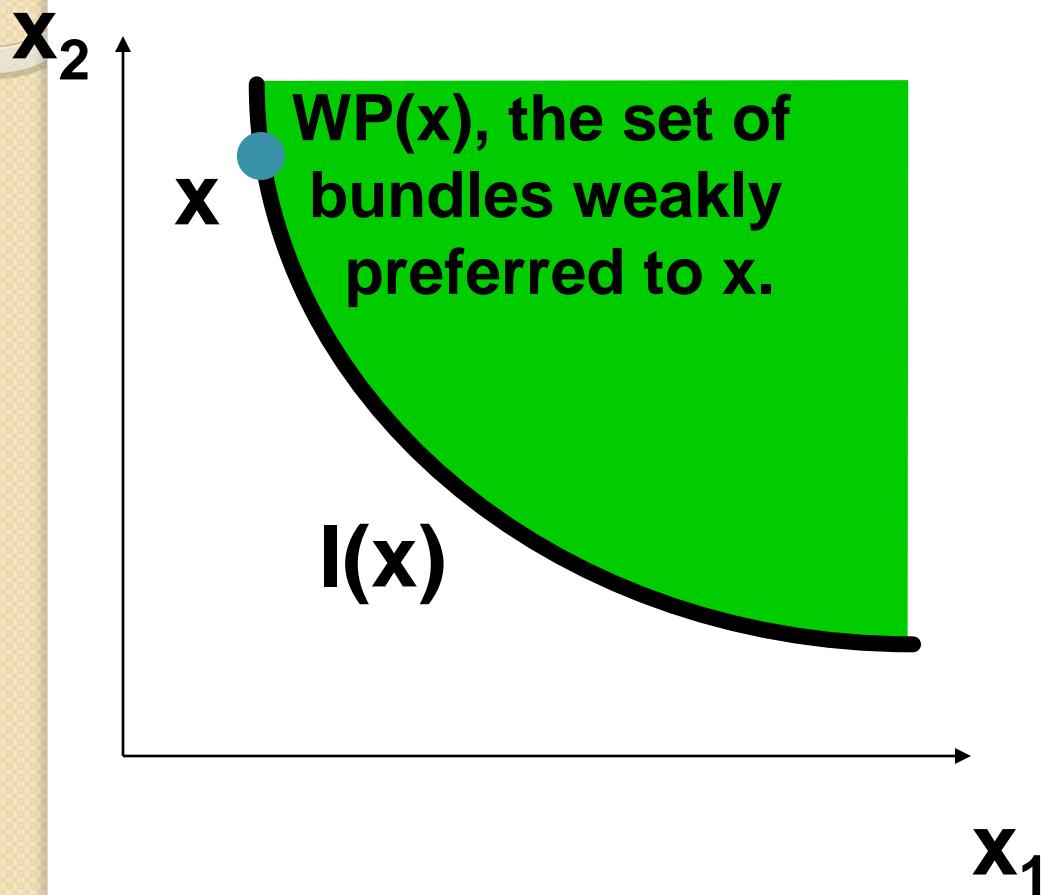
Indifference Curves



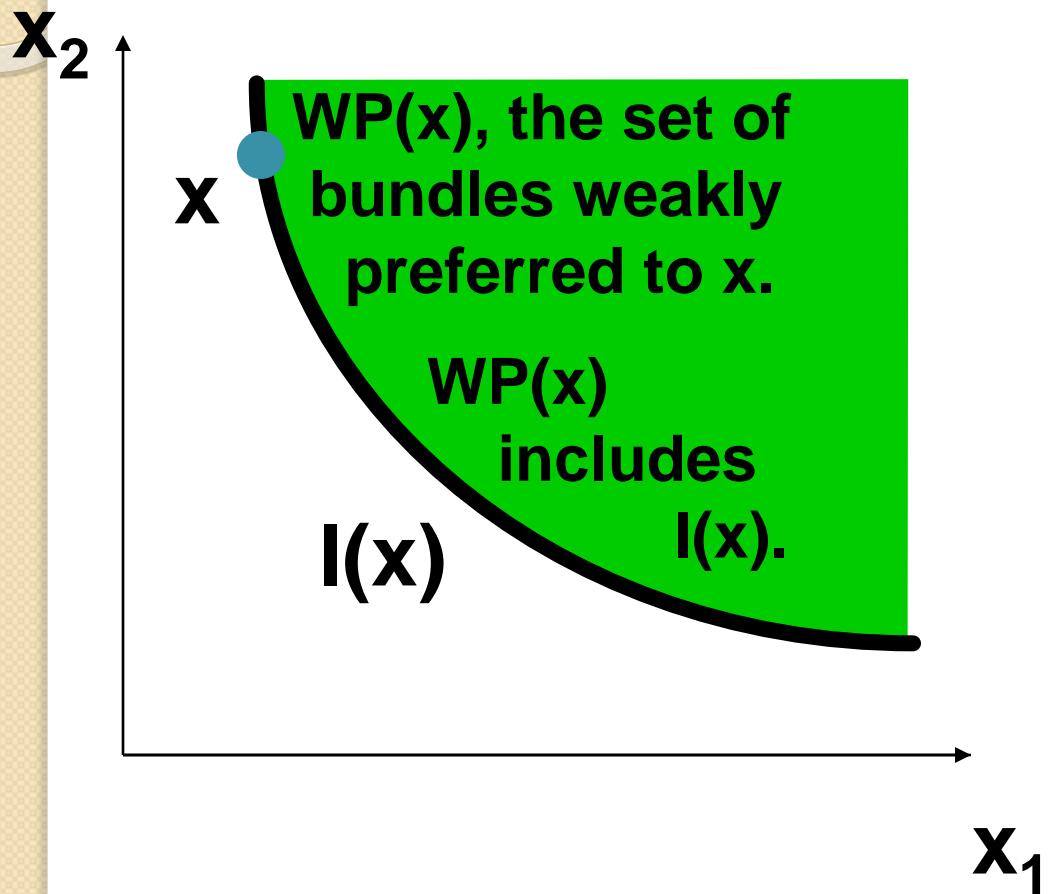
Indifference Curves



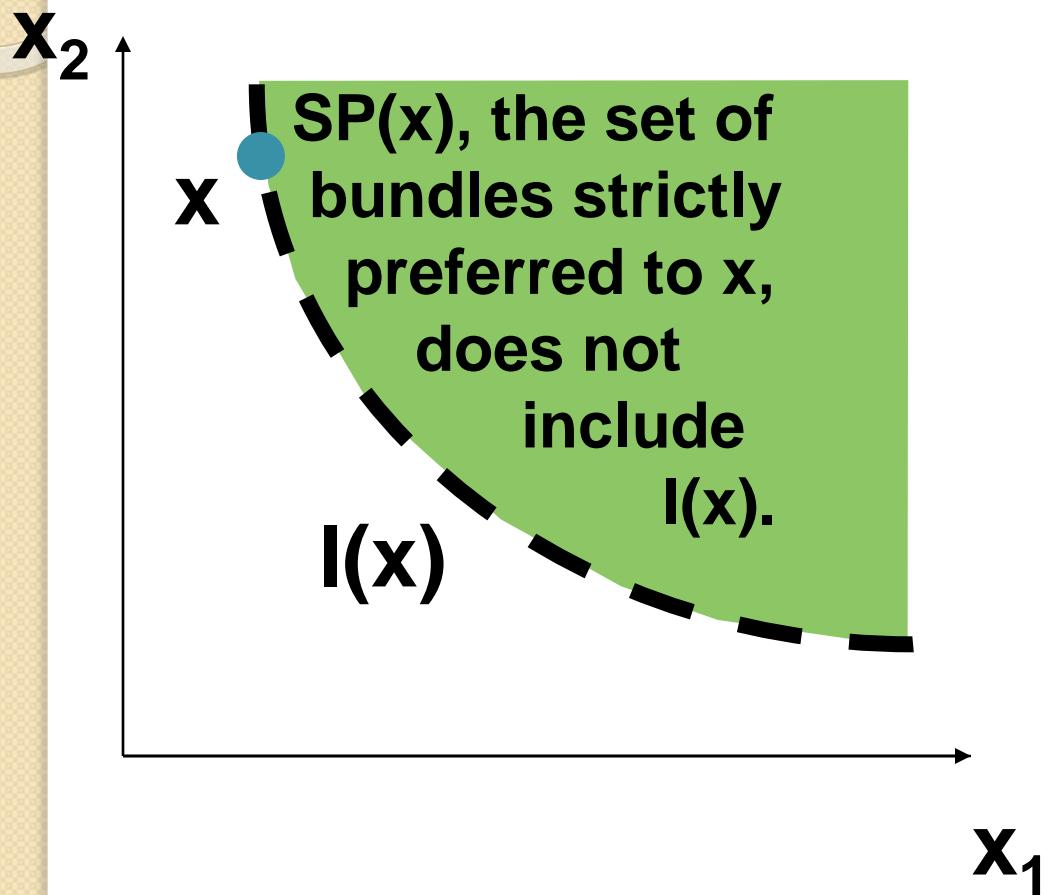
Weakly Preferred Set (弱偏好集)



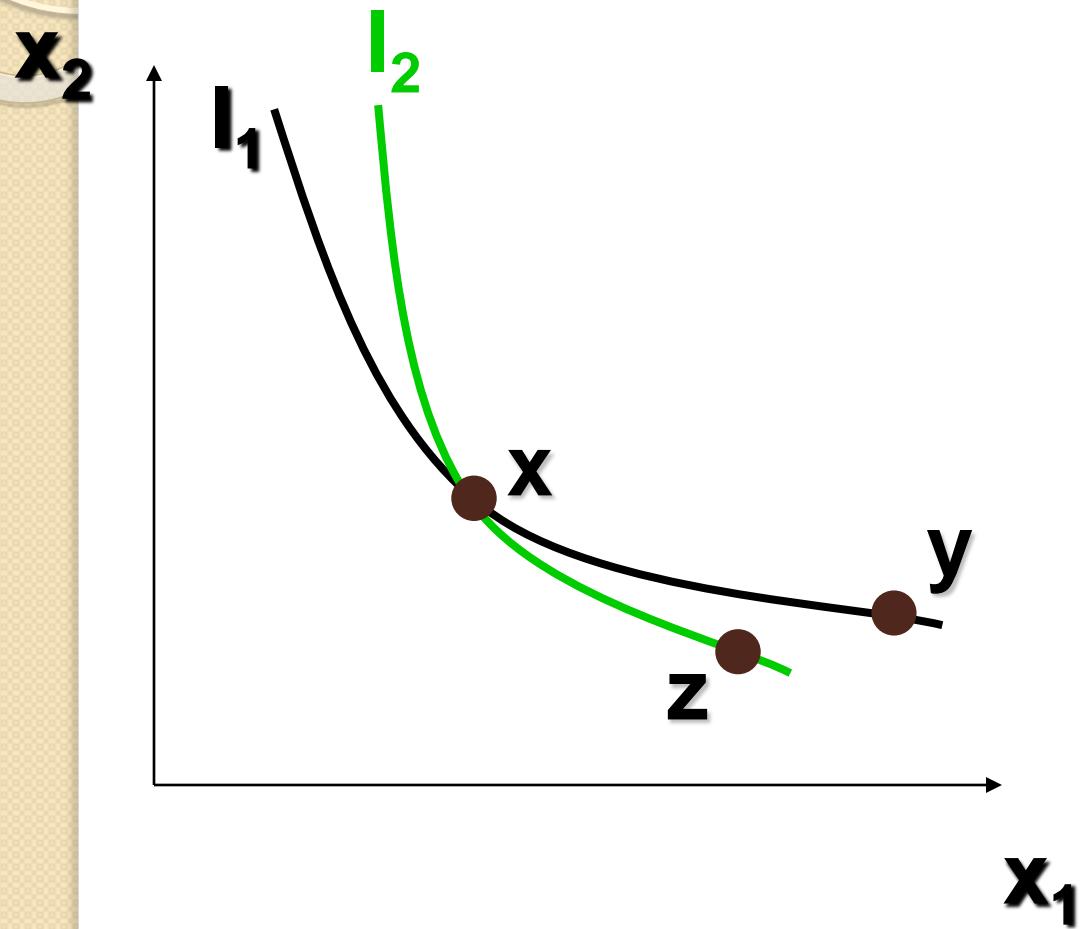
Weakly Preferred Set (弱偏好集)



Strictly Preferred Set (严格偏好集)



Indifference Curves Cannot Intersect

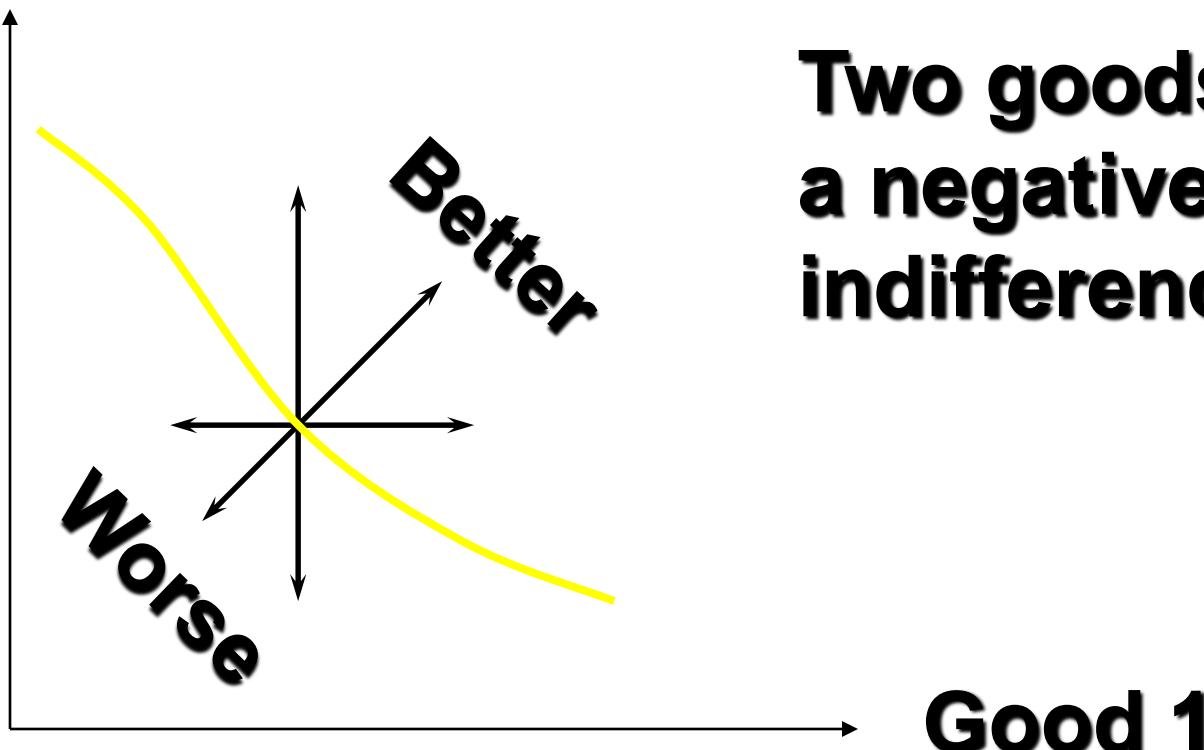


Slopes of Indifference Curves

- When more of a commodity is always preferred, the commodity is a **good**.
- If every commodity is a good then indifference curves are negatively sloped.

Slopes of Indifference Curves

Good 2



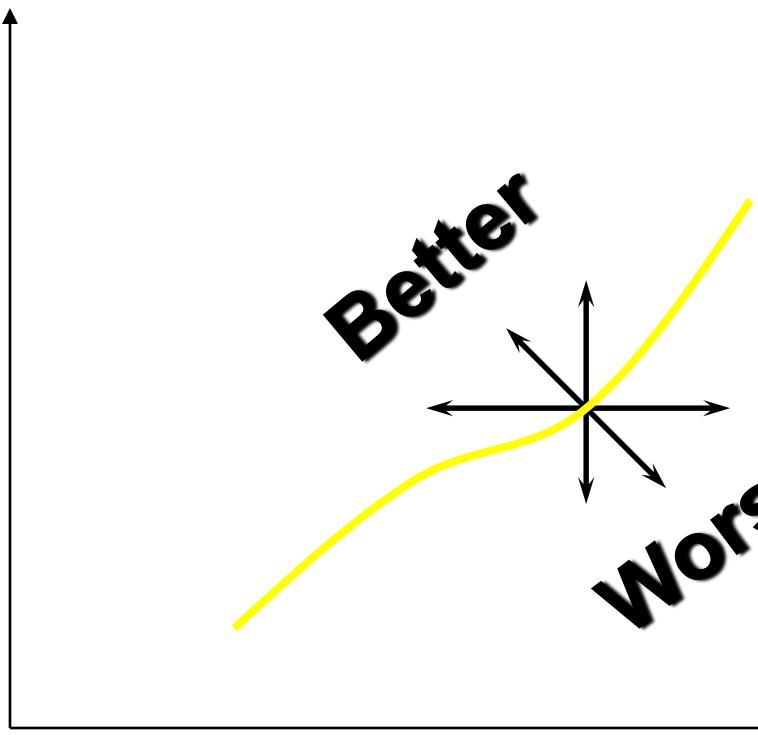
**Two goods →
a negatively sloped
indifference curve.**

Slopes of Indifference Curves

- If less of a commodity is always preferred then the commodity is a **bad**.

Slopes of Indifference Curves

Good 2

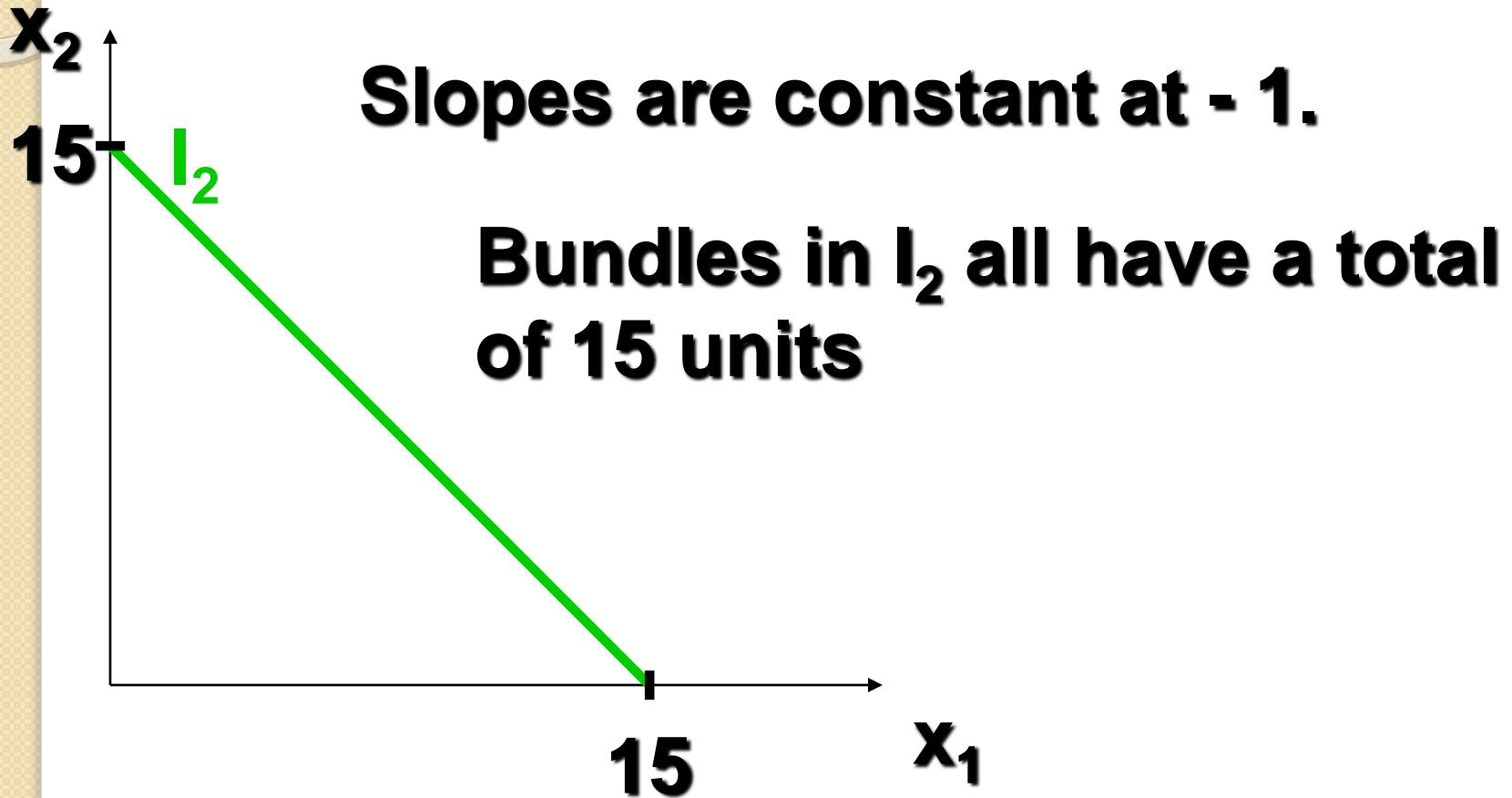


One good and one bad → a
positively sloped indifference curve.

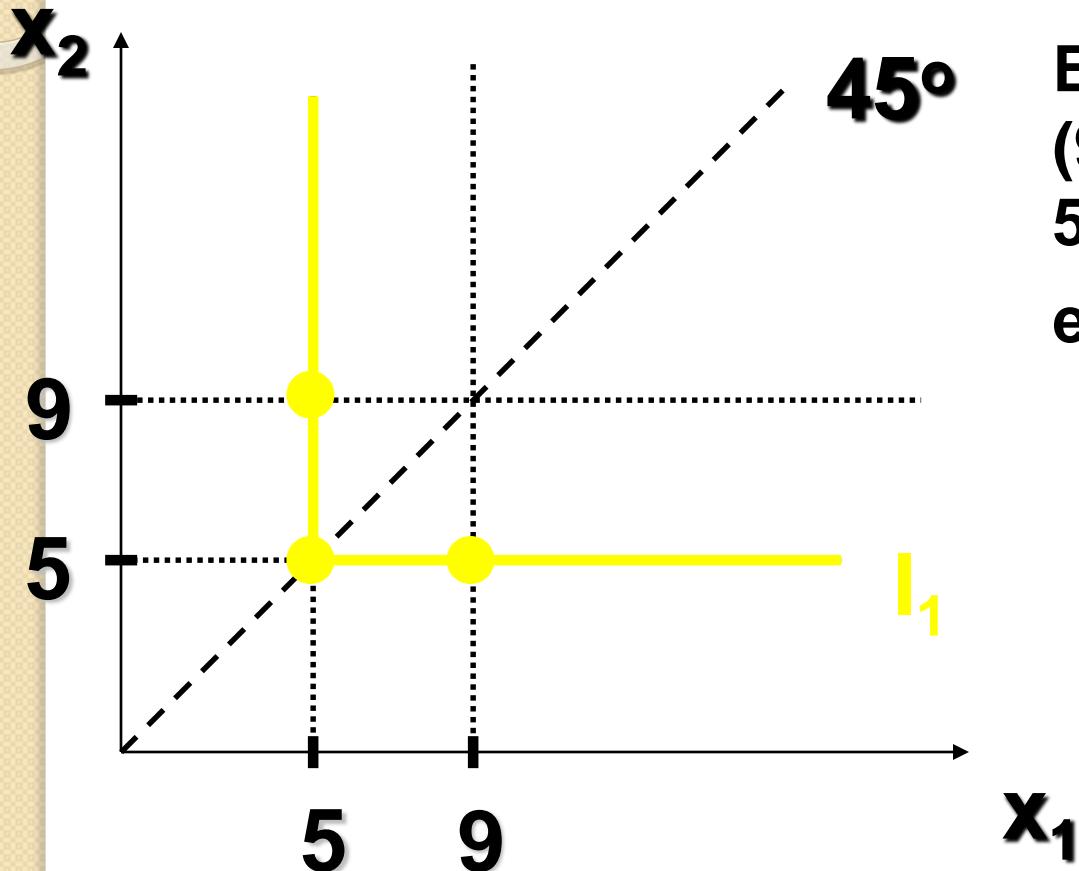
Examples

- Perfect substitutes (完全替代)
- Perfect complements (完全互补)
- Satiation (餍足)

Extreme Cases of Indifference Curves: Perfect Substitutes

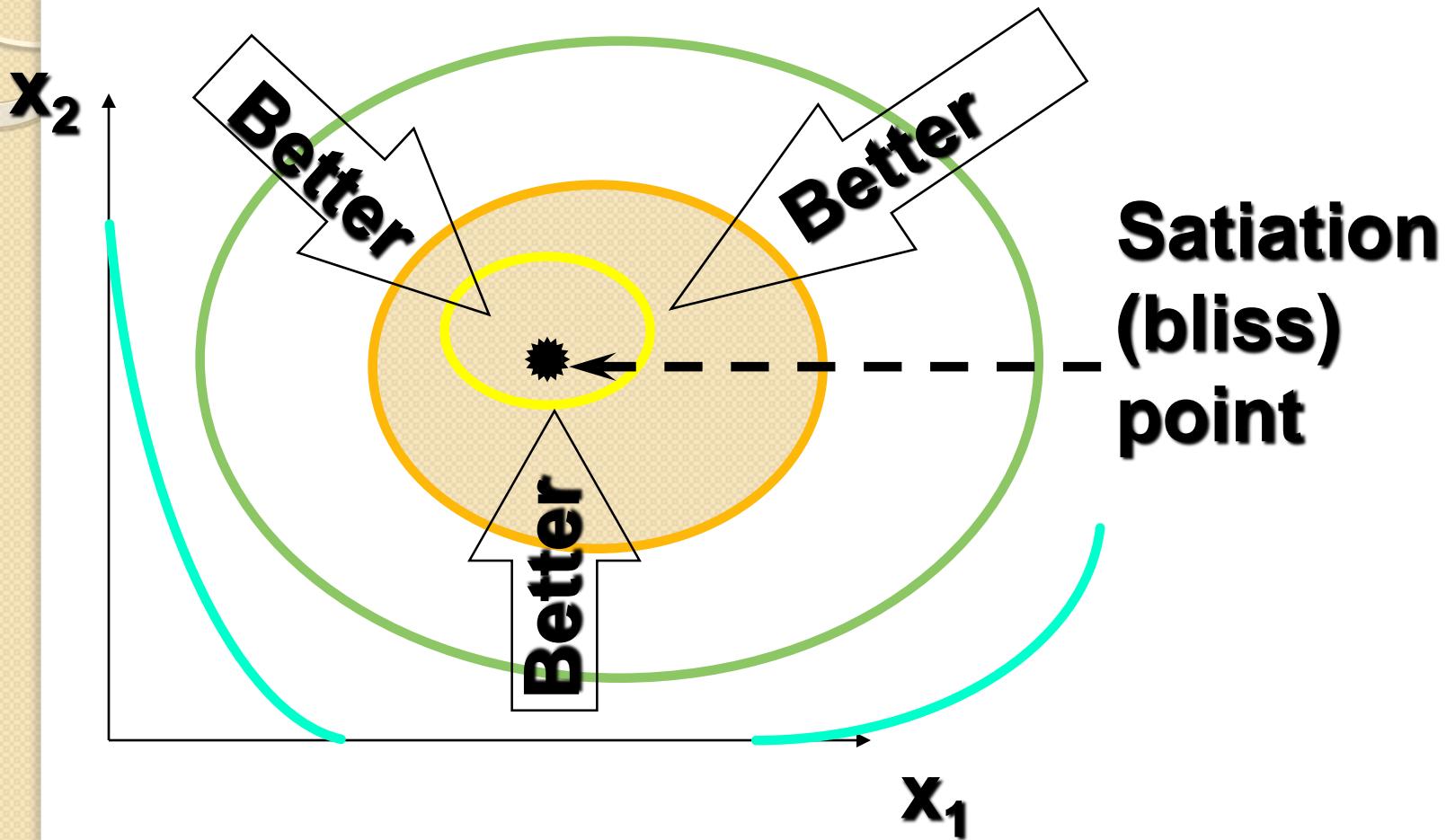


Extreme Cases of Indifference Curves: Perfect Complements



Each of $(5,5)$, $(5,9)$ and $(9,5)$ contains 5 pairs so each is equally preferred.

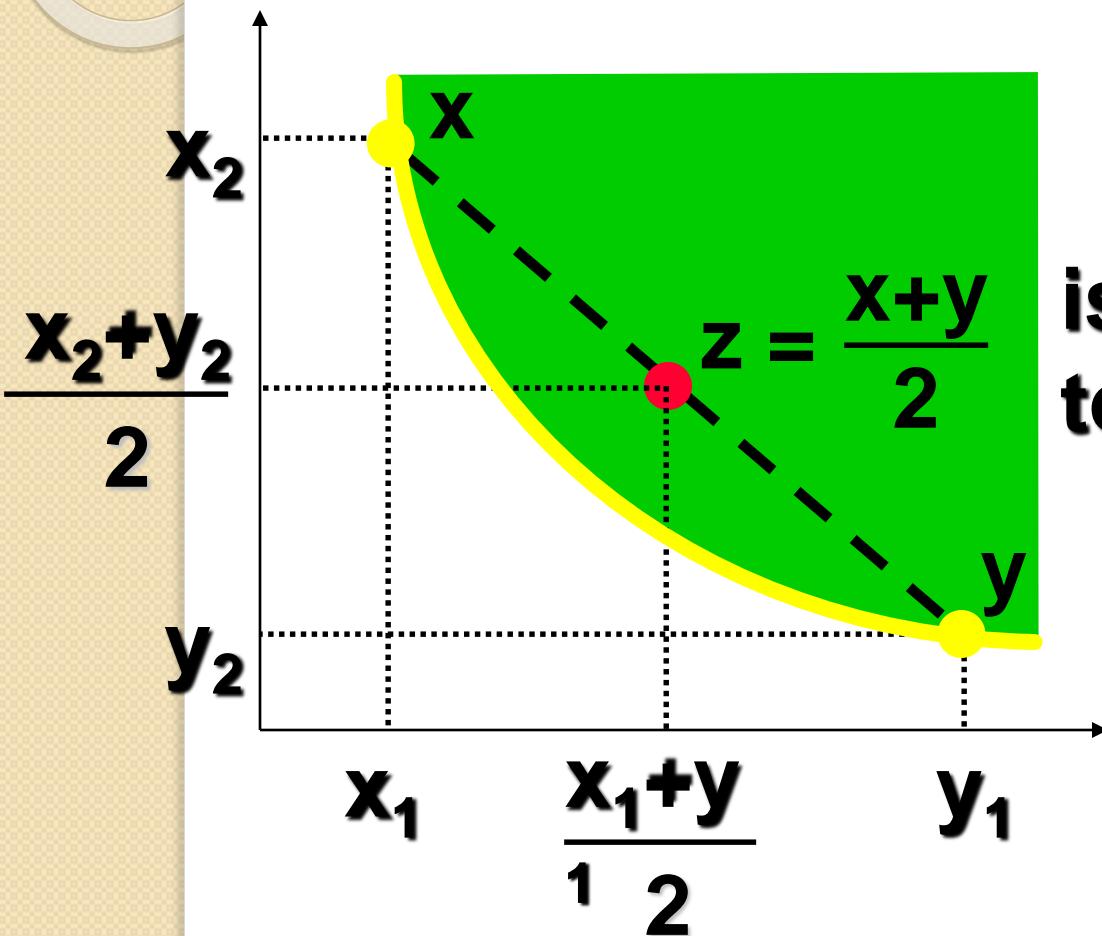
Indifference Curves Exhibiting Satiation



Well-Behaved Preferences

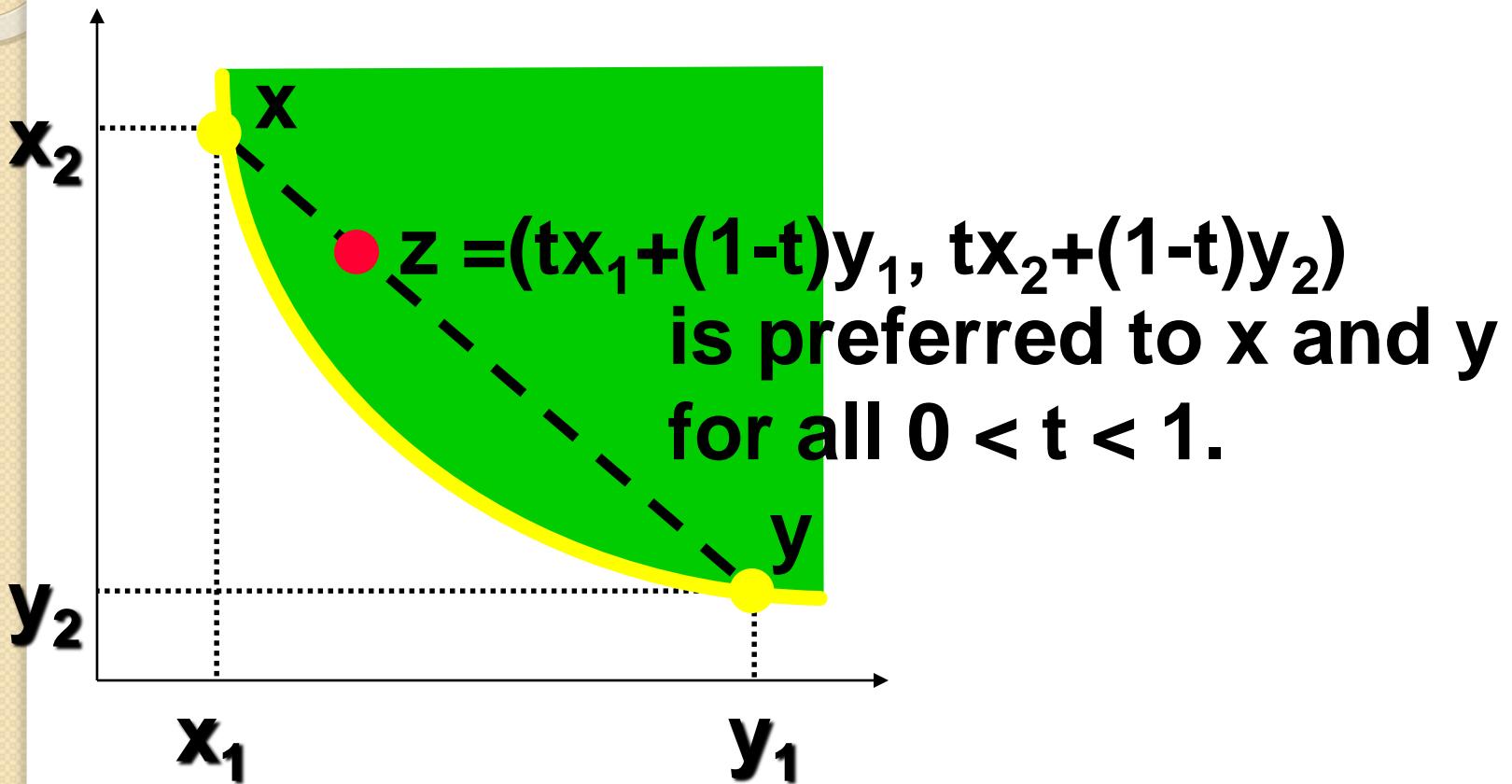
- **Convexity:** Mixtures of bundles are (at least weakly) preferred to the bundles themselves.
- E.g., the 50-50 mixture of the bundles x and y is
$$z = (0.5)x + (0.5)y.$$
 z is at least as preferred as x or y .

Well-Behaved Preferences -- Convexity.

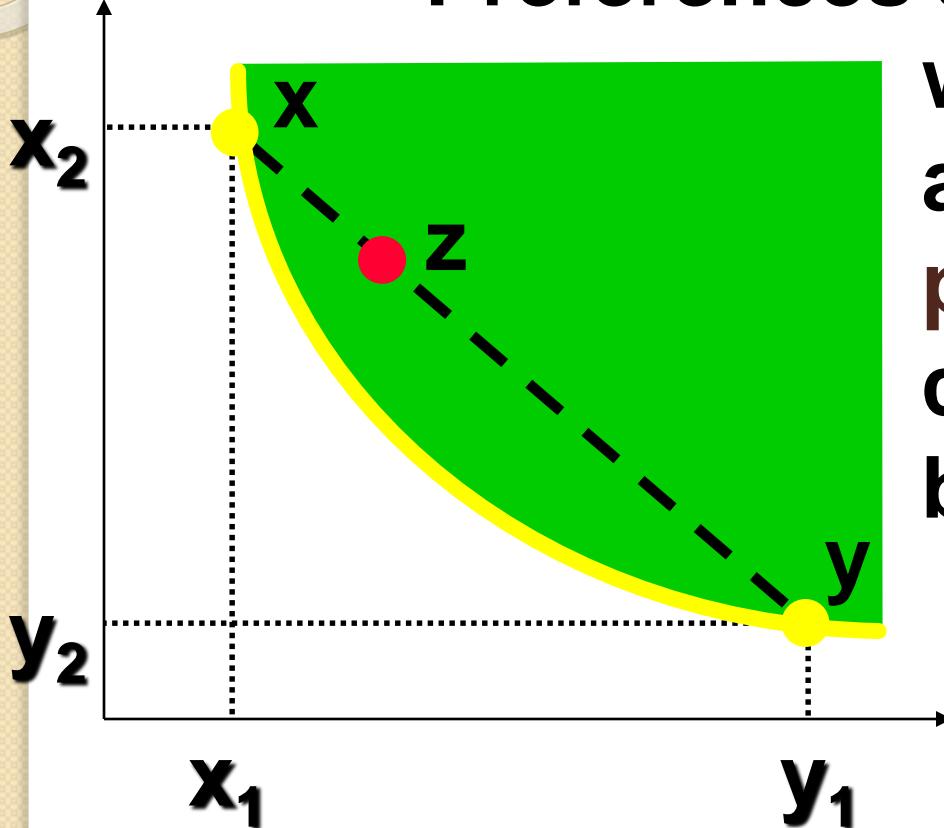


**is strictly preferred
to both x and y .**

Well-Behaved Preferences -- Convexity.

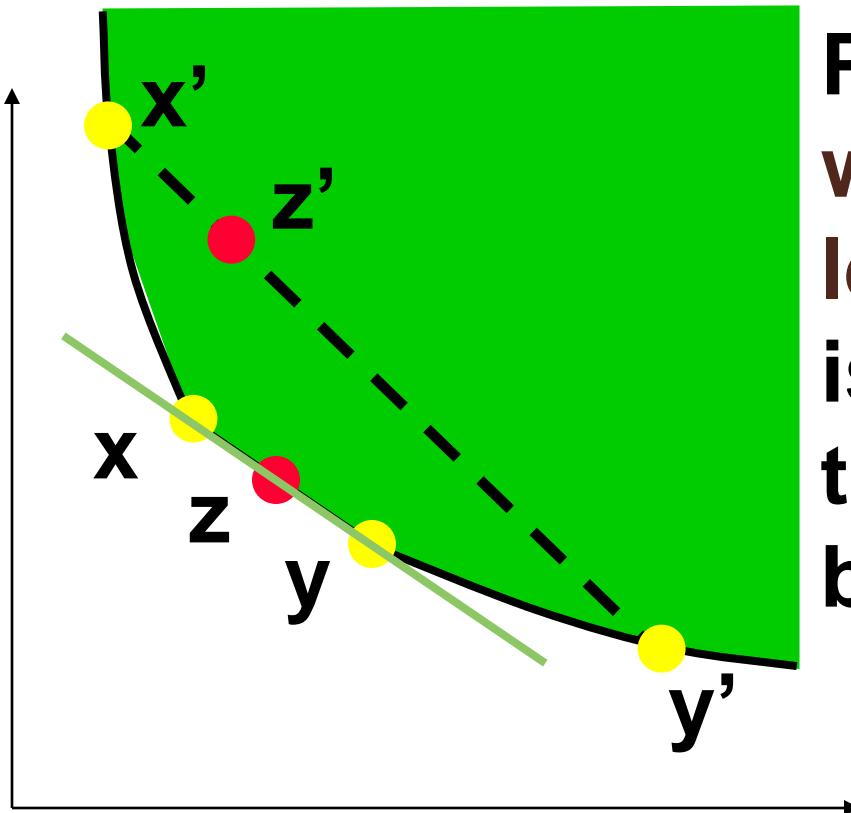


Well-Behaved Preferences -- Convexity.



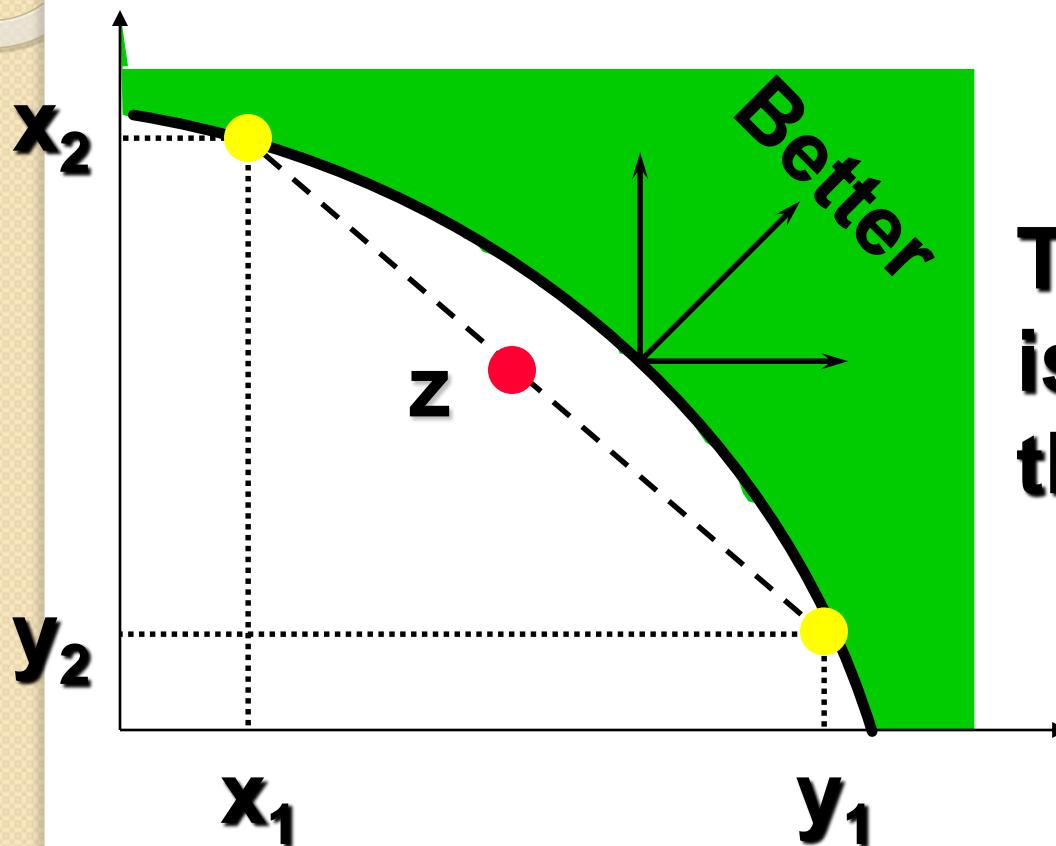
Preferences are strictly convex when all mixtures z are strictly preferred to their component bundles x and y .

Well-Behaved Preferences -- Weak Convexity.



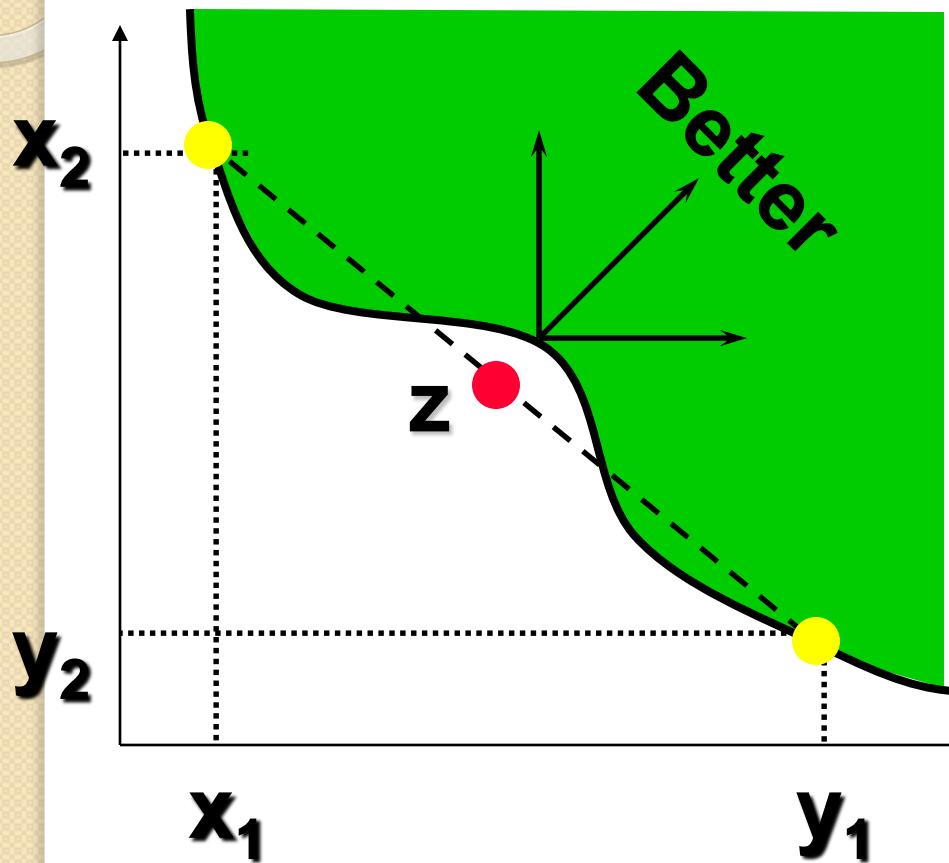
Preferences are weakly convex if at least one mixture z is equally preferred to a component bundle.

Non-Convex Preferences



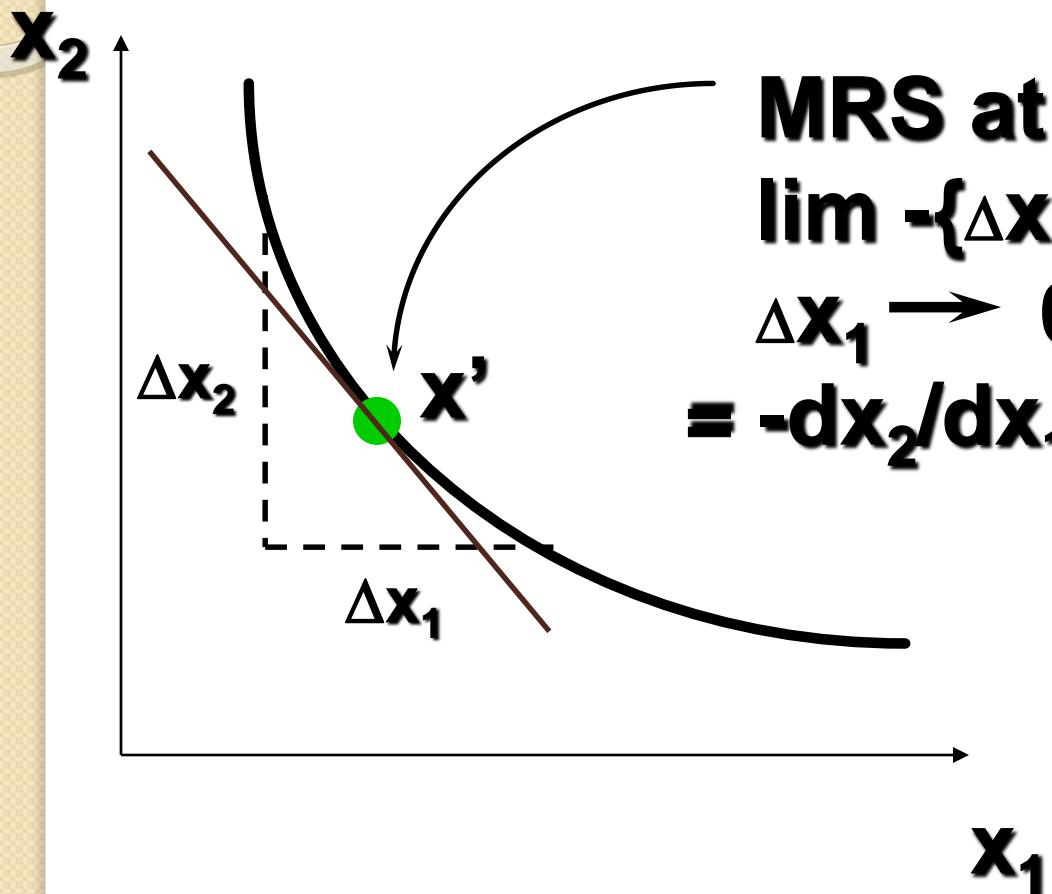
The mixture z is less preferred than x or y .

More Non-Convex Preferences



The mixture z is less preferred than x or y .

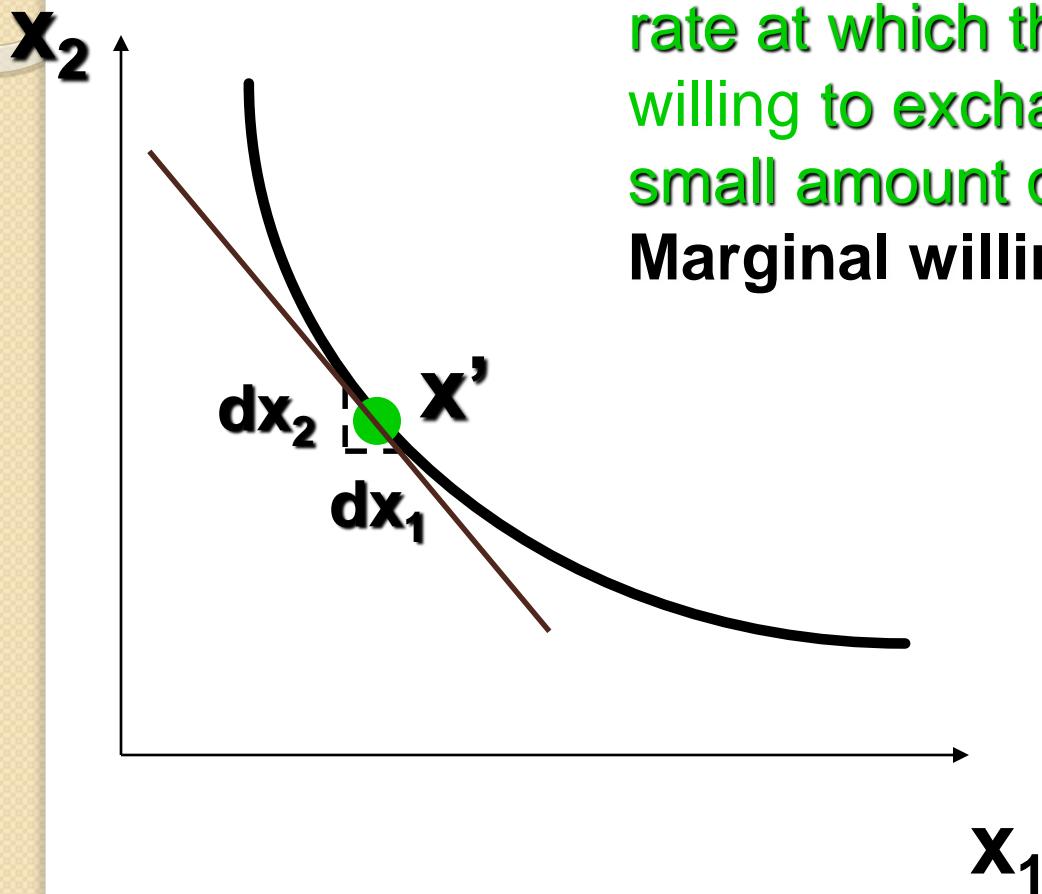
Marginal Rate of Substitution



MRS at x' is
$$\lim_{\Delta x_1 \rightarrow 0} -\{\Delta x_2 / \Delta x_1\}$$

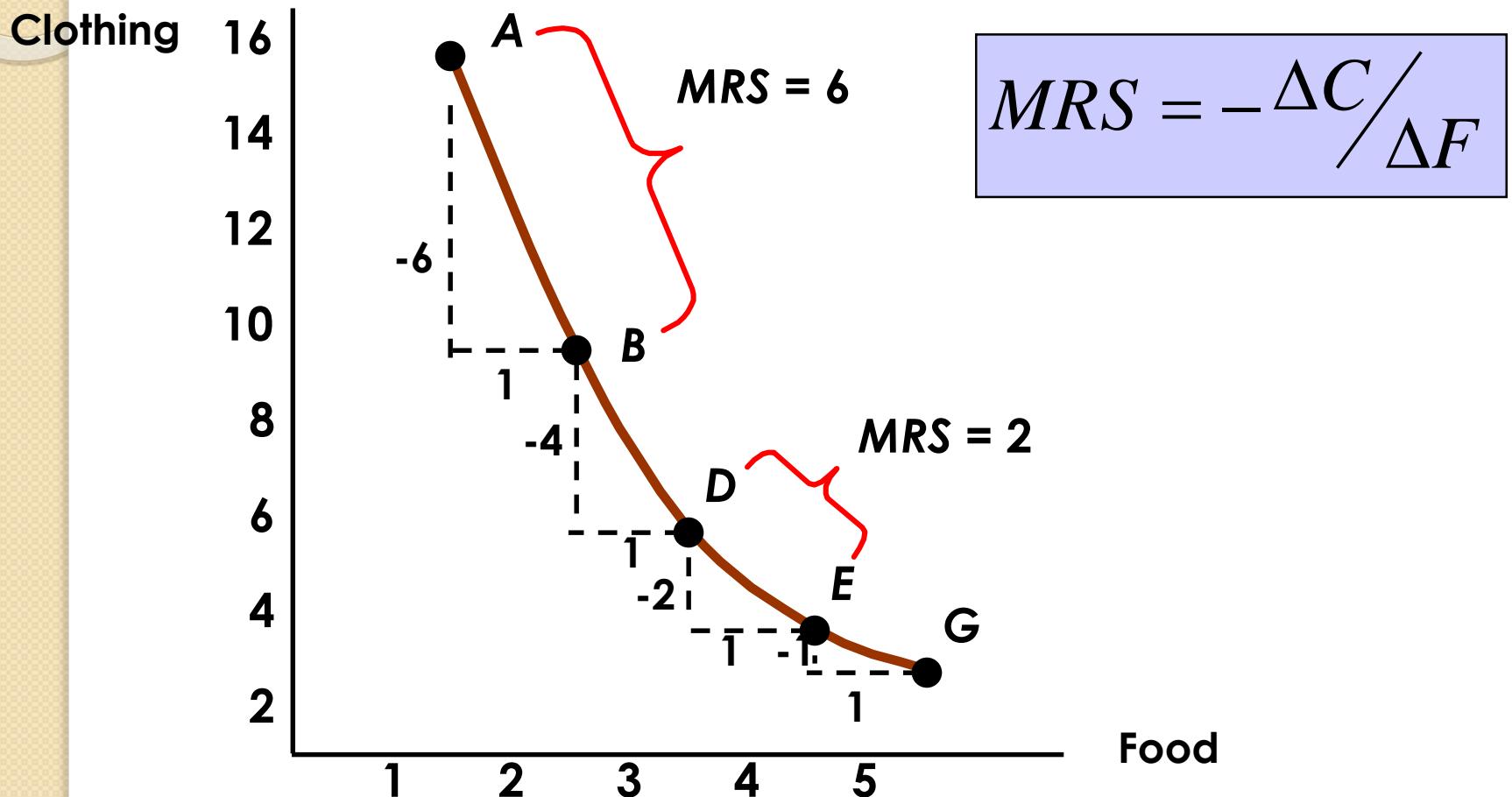
$$= -dx_2/dx_1 \text{ at } x'$$

Marginal Rate of Substitution

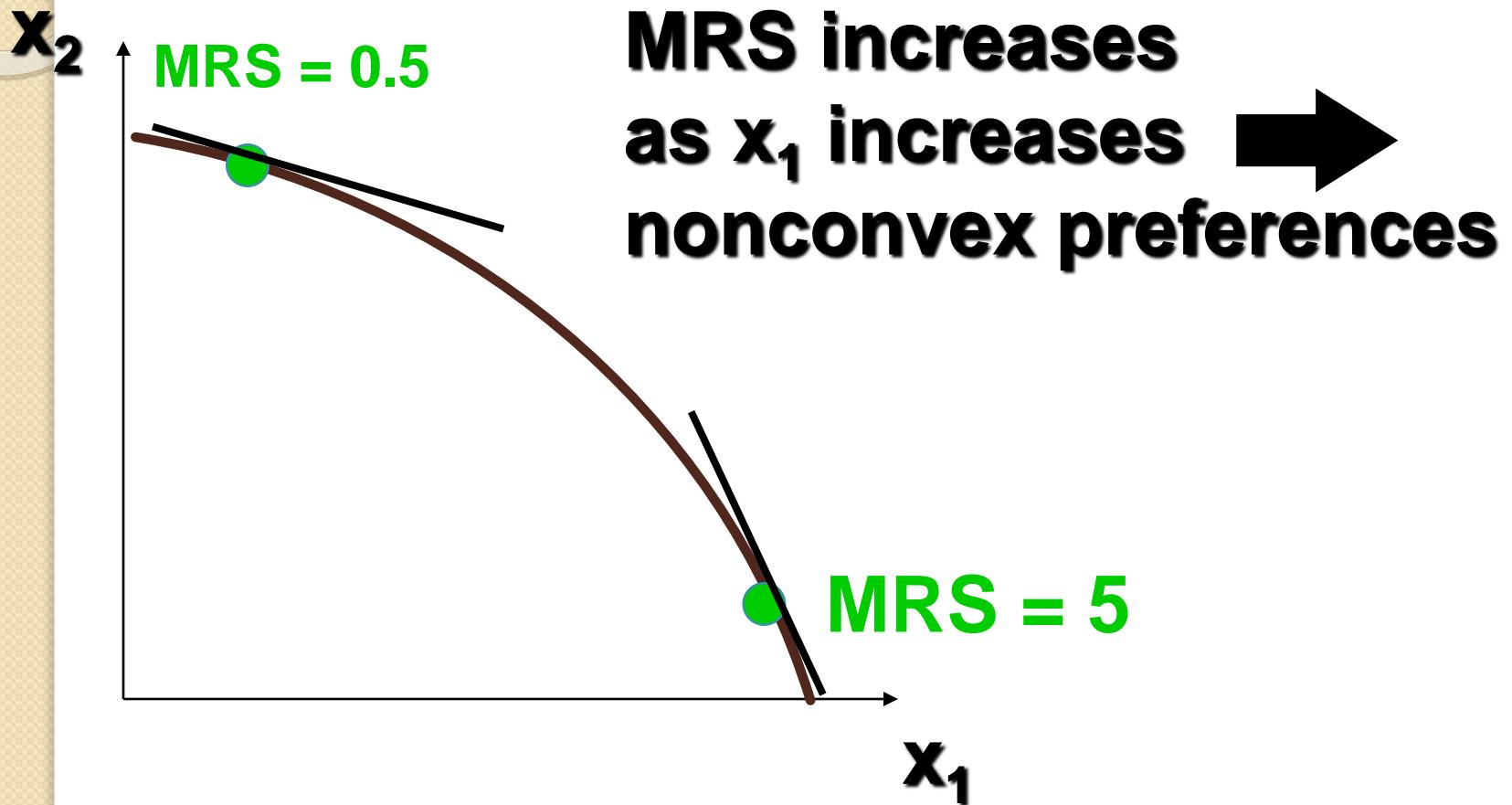


$dx_2 = MRS * dx_1$ so, at x' , MRS is the rate at which the consumer is only just willing to exchange commodity 2 for a small amount of commodity 1.
Marginal willingness to pay.

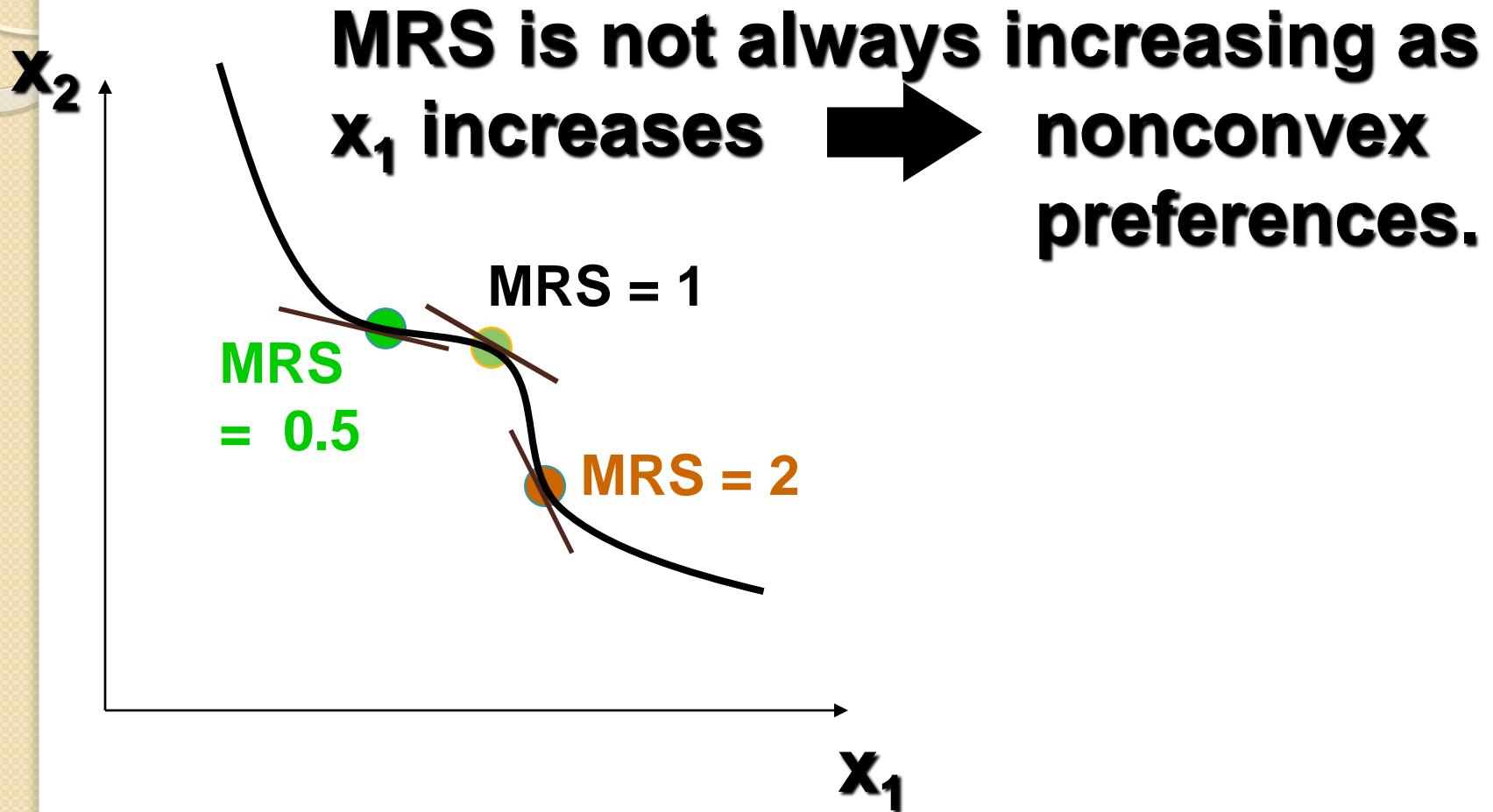
Marginal Rate of Substitution



MRS & Ind. Curve Properties



MRS & Ind. Curve Properties



MRS and Utility Function

$$U(x_1, x_2) = \bar{U}$$

$$d\bar{U} = dU(x_1, x_2) = \frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0$$

$$MRS = -\frac{dx_2}{dx_1} \Big|_{u=\bar{u}} = \frac{\partial U}{\partial x_1} / \frac{\partial U}{\partial x_2}$$

Utility Function and Preferences

- A consumer's preferences can be represented by a utility function
- The “perfect substitutes” preference can be represented by $U(x, y) = x + y$
- The “perfect complements” preferences:
 $U(x, y) = \min \{x, y\}$

Is the utility function representation unique for a given preference?

Utility Function and Preferences

- The utility representation is not unique
- A *positive monotonic transformation* of a utility function represents the same preference as the original utility function
- As long as the utility functions give the same *ordering* of the consumption bundles, we say they represent the same preference
- The utility is *ordinal*, rather than cardinal

Positive monotonic transformation

$$V(x_1, x_2) = f[u(x_1, x_2)], \quad f'(u) > 0$$

$$MRS = \frac{\partial v / \partial x_1}{\partial v / \partial x_2} = \frac{f'(u)}{f'(u)} \frac{\partial u / \partial x_1}{\partial u / \partial x_2} = \frac{\partial u / \partial x_1}{\partial u / \partial x_2}$$

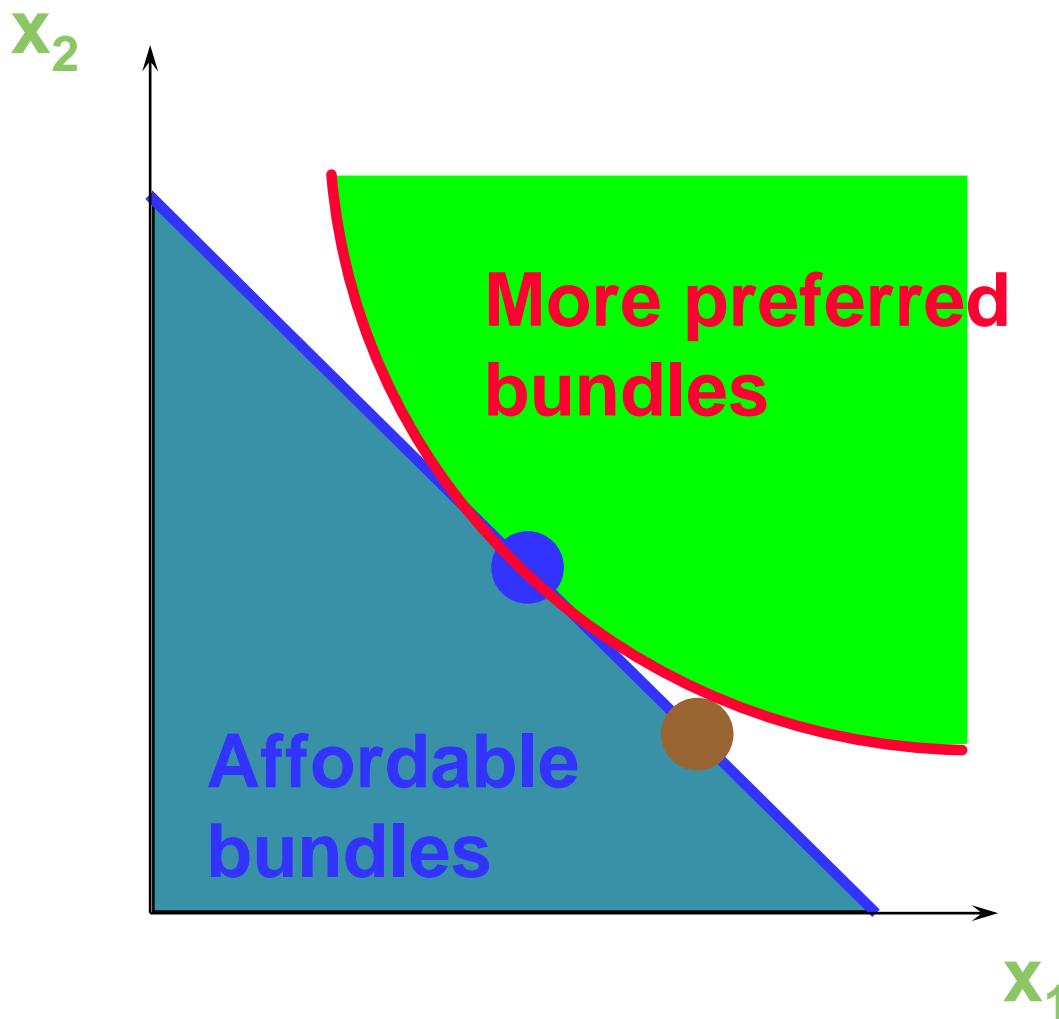
Consumer Choice

- Rational constrained choice
- Computing ordinary demands
 - Interior solution (内在解)
 - Corner solution (角点解)

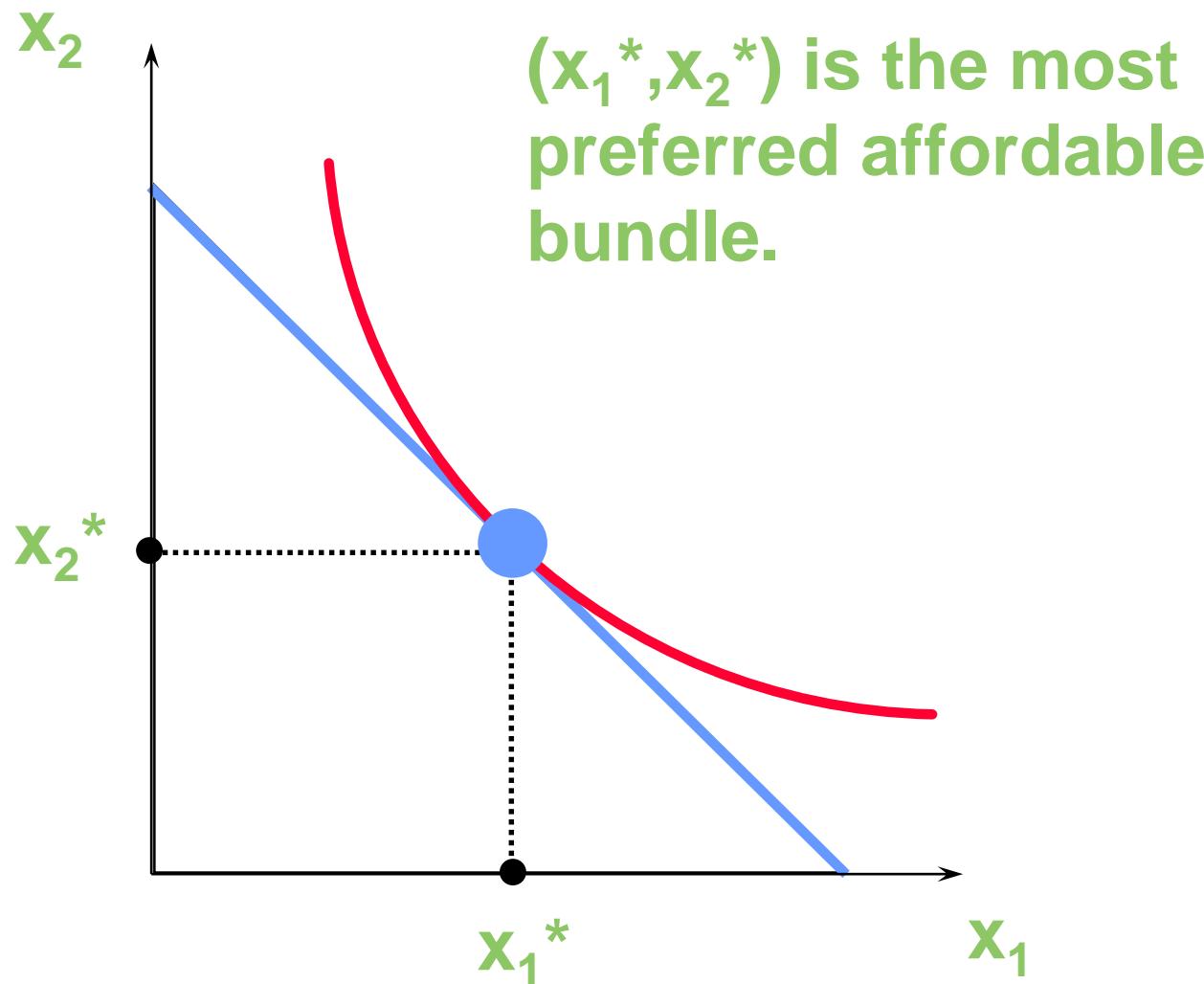
Economic Rationality

- The principal behavioral postulate is that a decision-maker chooses its most preferred alternative from those available to it.
- The available choices constitute the choice set.
- How is the most preferred bundle in the choice set located?

Rational Constrained Choice



Rational Constrained Choice



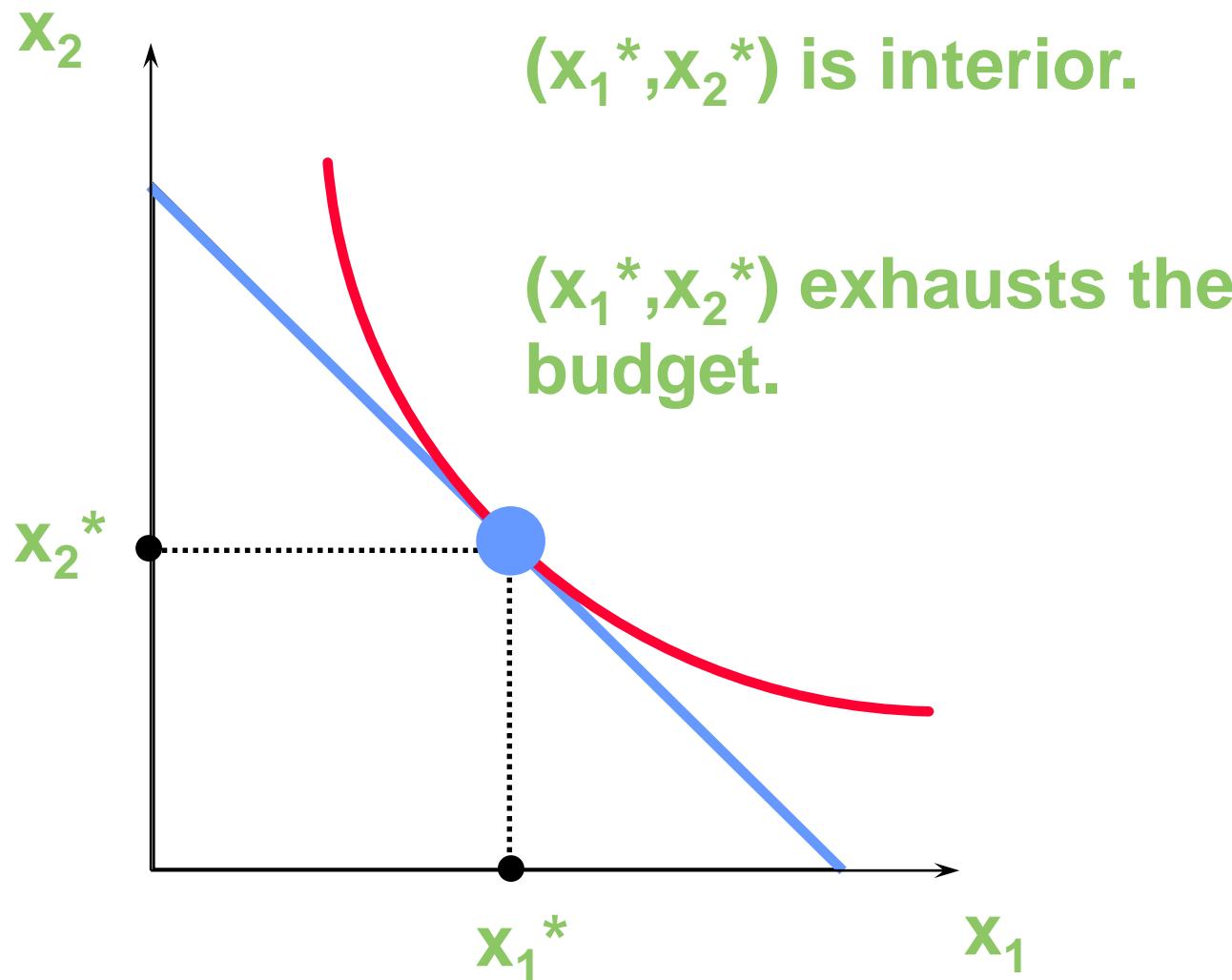
Rational Constrained Choice

- The most preferred affordable bundle is called the consumer's **ORDINARY DEMAND** (or **DEMAND**) at the given prices and budget.
- Ordinary demands will be denoted by $x_1^*(P_1, P_2, m)$ and $x_2^*(P_1, P_2, m)$.

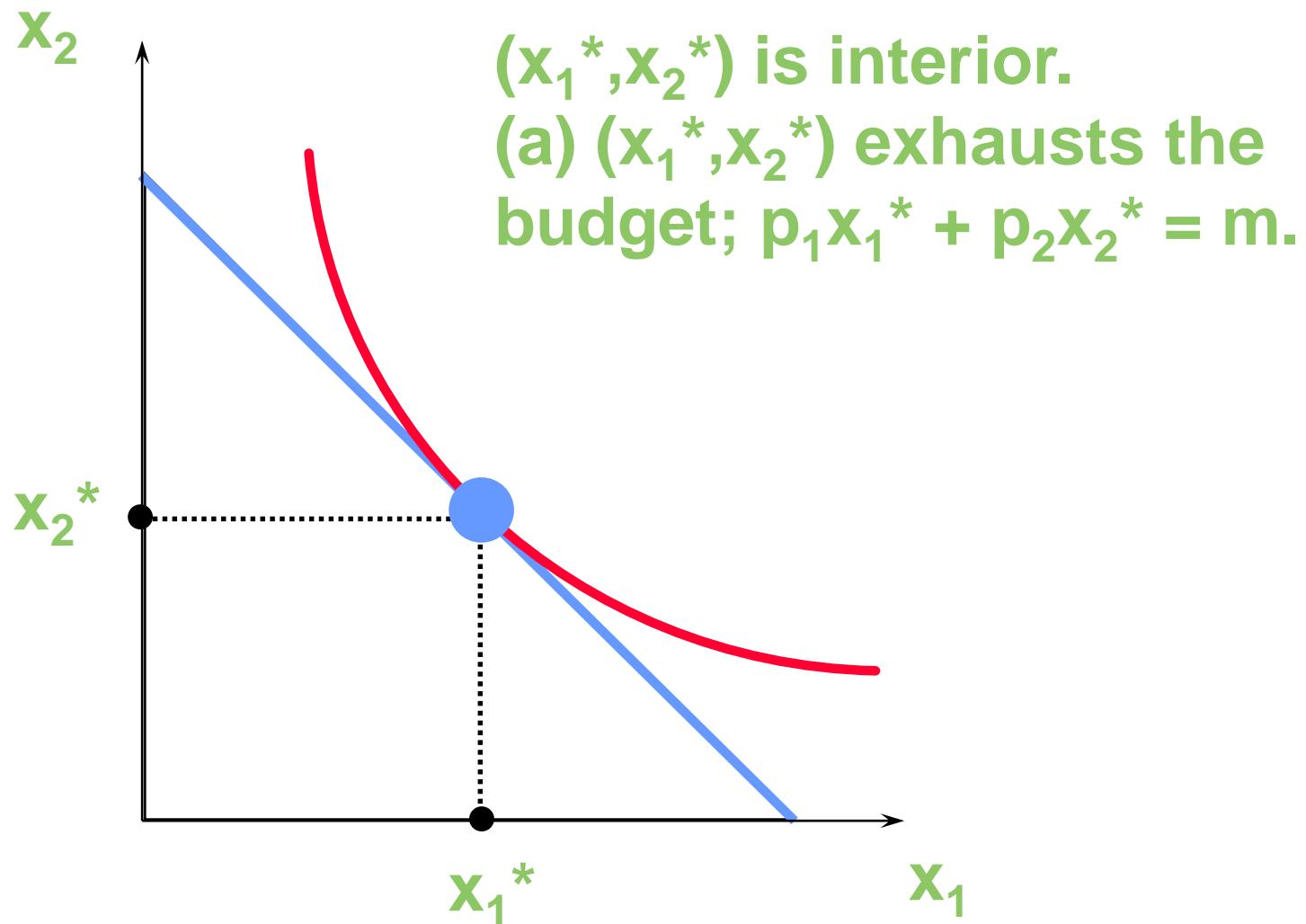
Rational Constrained Choice

- When $x_1^* > 0$ and $x_2^* > 0$ the demanded bundle is **INTERIOR**.
- If buying (x_1^*, x_2^*) costs \$m then the budget is exhausted.

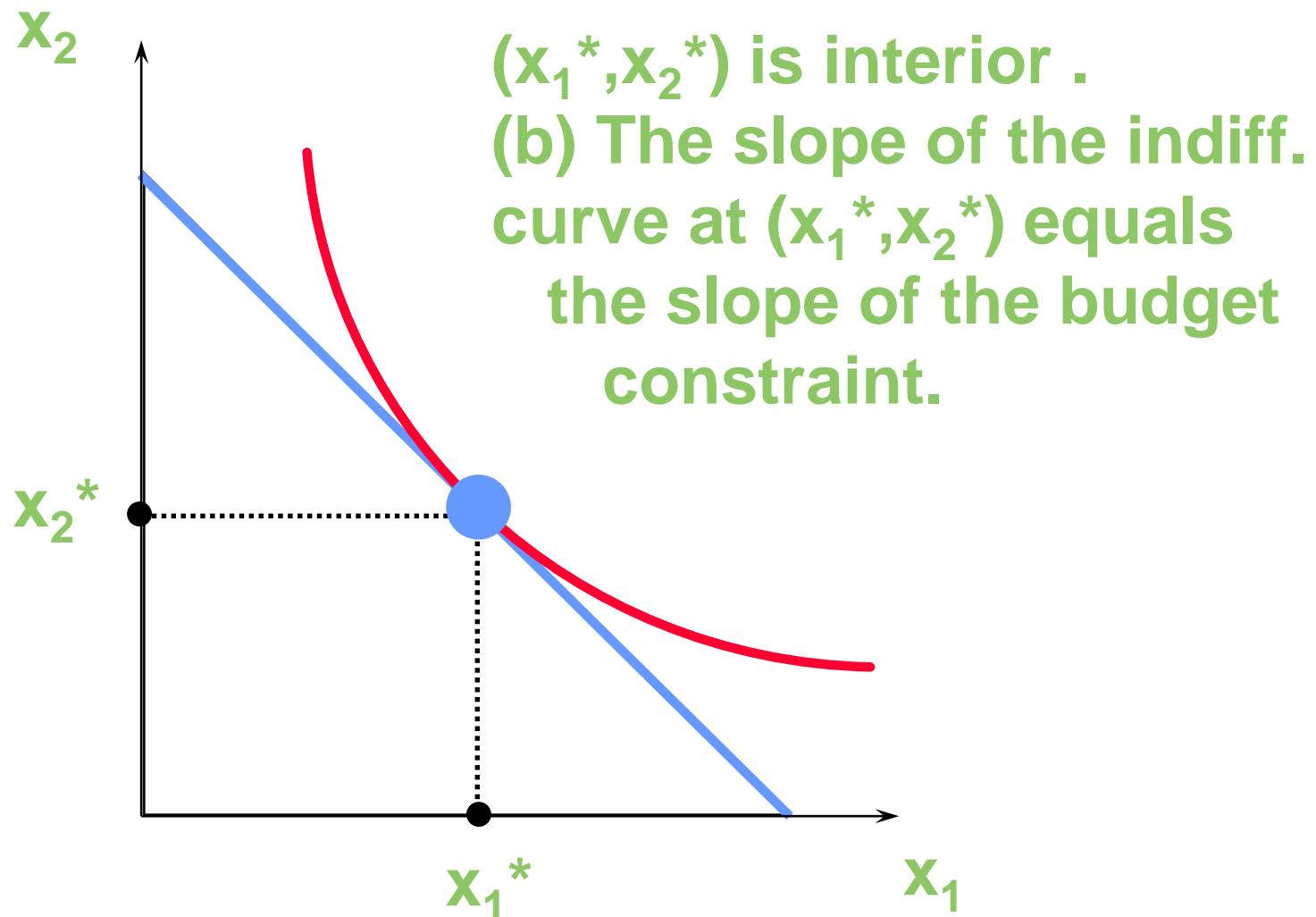
Rational Constrained Choice



Rational Constrained Choice



Rational Constrained Choice



Rational Constrained Choice

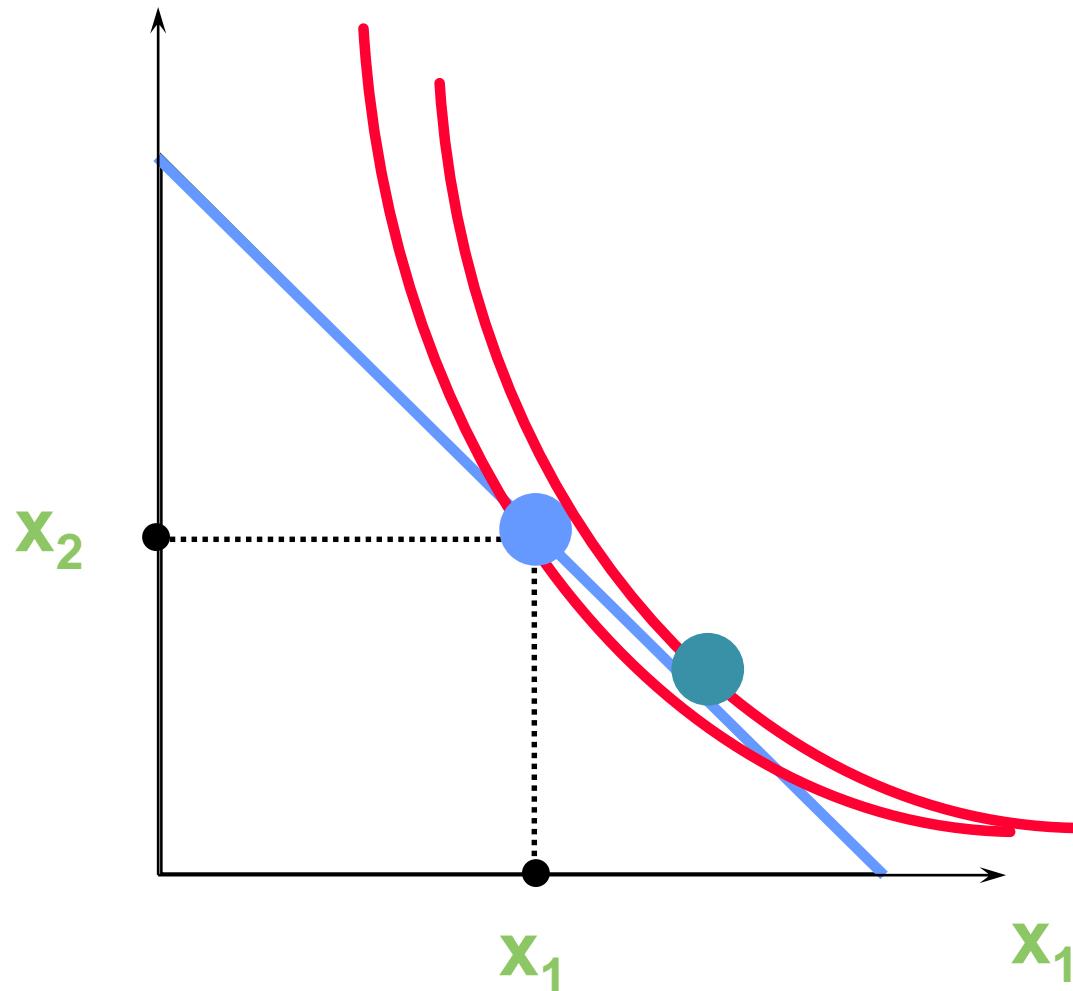
- (x_1^*, x_2^*) satisfies **two conditions**:
- (a) the budget is exhausted;

$$P_1 x_1^* + P_2 x_2^* = m$$

- (b) tangency: the slope of the budget constraint, $-P_1/P_2$, and the slope of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*) .

Meaning of the Tangency Condition

- Consumer's marginal willingness to pay equals the market exchange rate.
- Suppose at a consumption bundle (x_1, x_2) ,
$$MRS = 2, P_1/P_2 = 1$$
 - The consumer is willing to give up 2 unit of x_2 to exchange for an additional unit of x_1
 - The market allows her to give up only 1 unit of x_2 to obtain an additional x_1
- (x_1, x_2) is not optimal choice
- She can be better off increasing her consumption of x_1 .



Computing Ordinary Demands

- Solve for 2 simultaneous equations.
 - Tangency
 - Budget constraint
- The conditions may be obtained by using the Lagrangian multiplier method, i.e., constrained optimization in calculus.

Computing Ordinary Demands

- How can this information be used to locate (x_1^*, x_2^*) for given P_1, P_2 and m ?

Computing Ordinary Demands - a Cobb-Douglas Example.

- Suppose that the consumer has Cobb-Douglas preferences.

$$U(x_1, x_2) = x_1^\alpha x_2^\beta$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- At (x_1^*, x_2^*) , $MRS = p_1/p_2$ so the tangency condition ($MRS = p_1/p_2$) is
-

$$MRS = \frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}$$

$$x_2 = \frac{\beta p_1}{\alpha p_2} x_1 \quad (1)$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- (x_1^*, x_2^*) also exhausts the budget so

$$p_1 x_1 + p_2 x_2 = m \quad (2)$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- The solution to the simultaneous equations (1) and (2) is:

$$x_1^* = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1}$$

$$x_2^* = \frac{\beta}{\alpha + \beta} \frac{m}{p_2}$$

Lagrange Multipliers

$$\text{Max } U(x_1, x_2) \quad \text{s.t. } p_1x_1 + p_2x_2 = m$$

$$L = U(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = m - p_1x_1 - p_2x_2 = 0$$

Equal Marginal Principle

$$\lambda = \frac{\partial U / \partial x_1}{p_1} = \frac{\partial U / \partial x_2}{p_2}$$

In the case of $U(x_1, x_2, \dots, x_n)$,

$$\lambda = \frac{\partial U / \partial x_1}{p_1} = \frac{\partial U / \partial x_2}{p_2} = \dots = \frac{\partial U / \partial x_n}{p_n}$$

Understanding lamda

$$\frac{dU}{dm} = \frac{\partial U}{\partial x_1} \frac{dx_1}{dm} + \frac{\partial U}{\partial x_2} \frac{dx_2}{dm}$$

Since $dm = p_1 dx_1 + p_2 dx_2$, $\lambda = \frac{\partial U}{\partial x_1} / p_1 = \frac{\partial U}{\partial x_2} / p_2$

$$\frac{dU}{dm} = \lambda p_1 \frac{dx_1}{dm} + \lambda p_2 \frac{dx_2}{dm} = \lambda (p_1 dx_1 + p_2 dx_2) / dm$$

$$\Rightarrow \frac{dU}{dm} = \lambda$$

How to Allocate Time Efficiently?

$$\text{Max } U = s_1 + \dots + s_n = \sum_{i=1}^n s_i$$

$$\text{s.t. (1)} \quad s_i = f_i(t_i), \quad f'_i(t_i) > 0, \quad f''_i(t_i) < 0$$

$$(2) \quad \sum_{i=1}^n t_i \leq T$$

$$\Rightarrow \partial f_1(t_1)/\partial t_1 = \partial f_2(t_2)/\partial t_2 = \dots = \partial f_n(t_n)/\partial t_n = \lambda$$

λ : shadow price of time

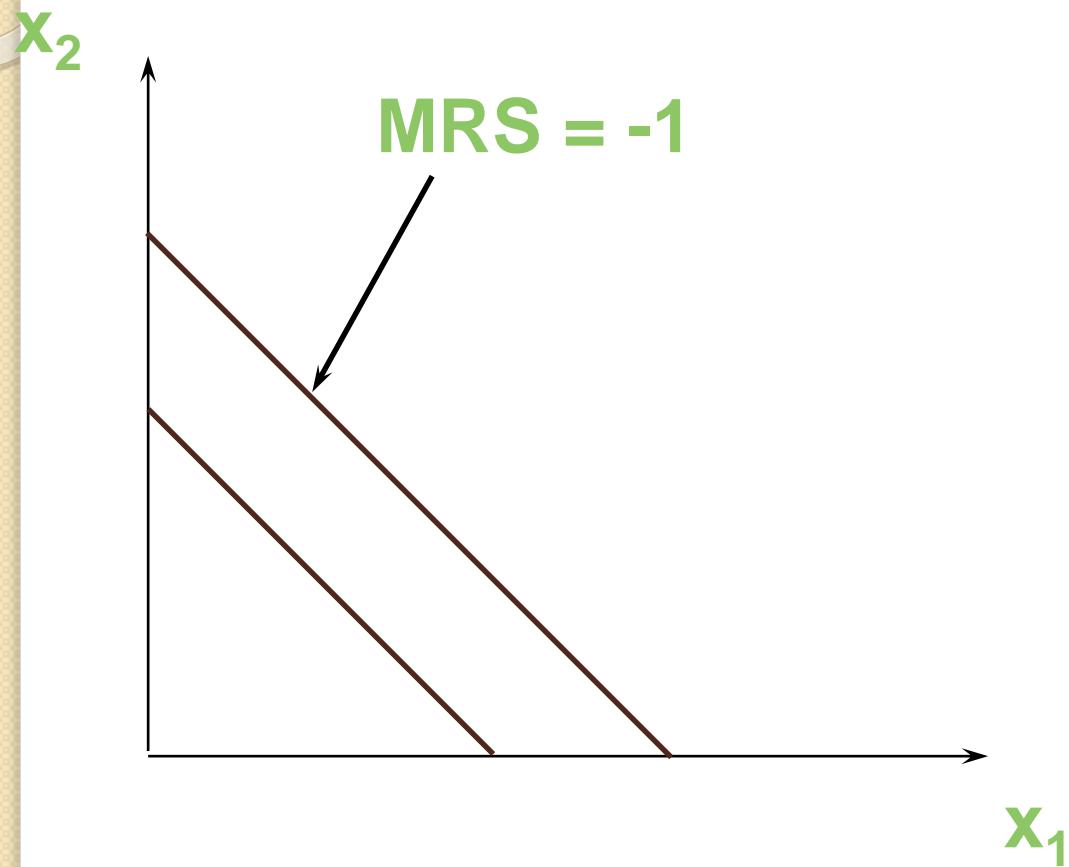
Rational Constrained Choice: Summary

- When $x_1^* > 0$ and $x_2^* > 0$ and (x_1^*, x_2^*) exhausts the budget, and indifference curves have no ‘kinks’, the ordinary demands are obtained by solving:
 - (a) $P_1 x_1^* + P_2 x_2^* = y$
 - (b) the slopes of the budget constraint, $-P_1/P_2$, and of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*) .

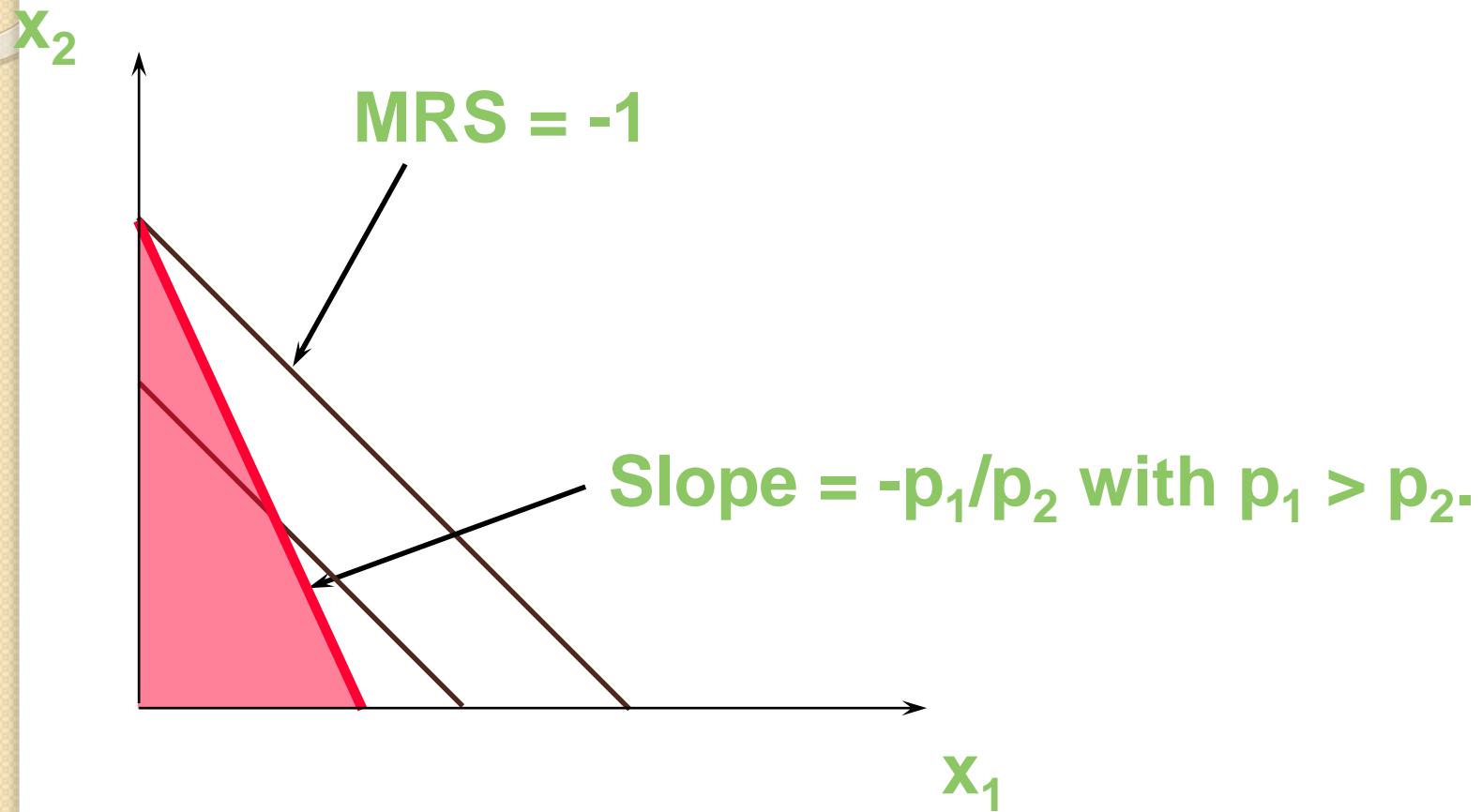
Rational Constrained Choice

- But what if $x_1^* = 0$?
- Or if $x_2^* = 0$?
- If either $x_1^* = 0$ or $x_2^* = 0$ then the ordinary demand (x_1^*, x_2^*) is at a corner solution (角点解) to the problem of maximizing utility subject to a budget constraint.

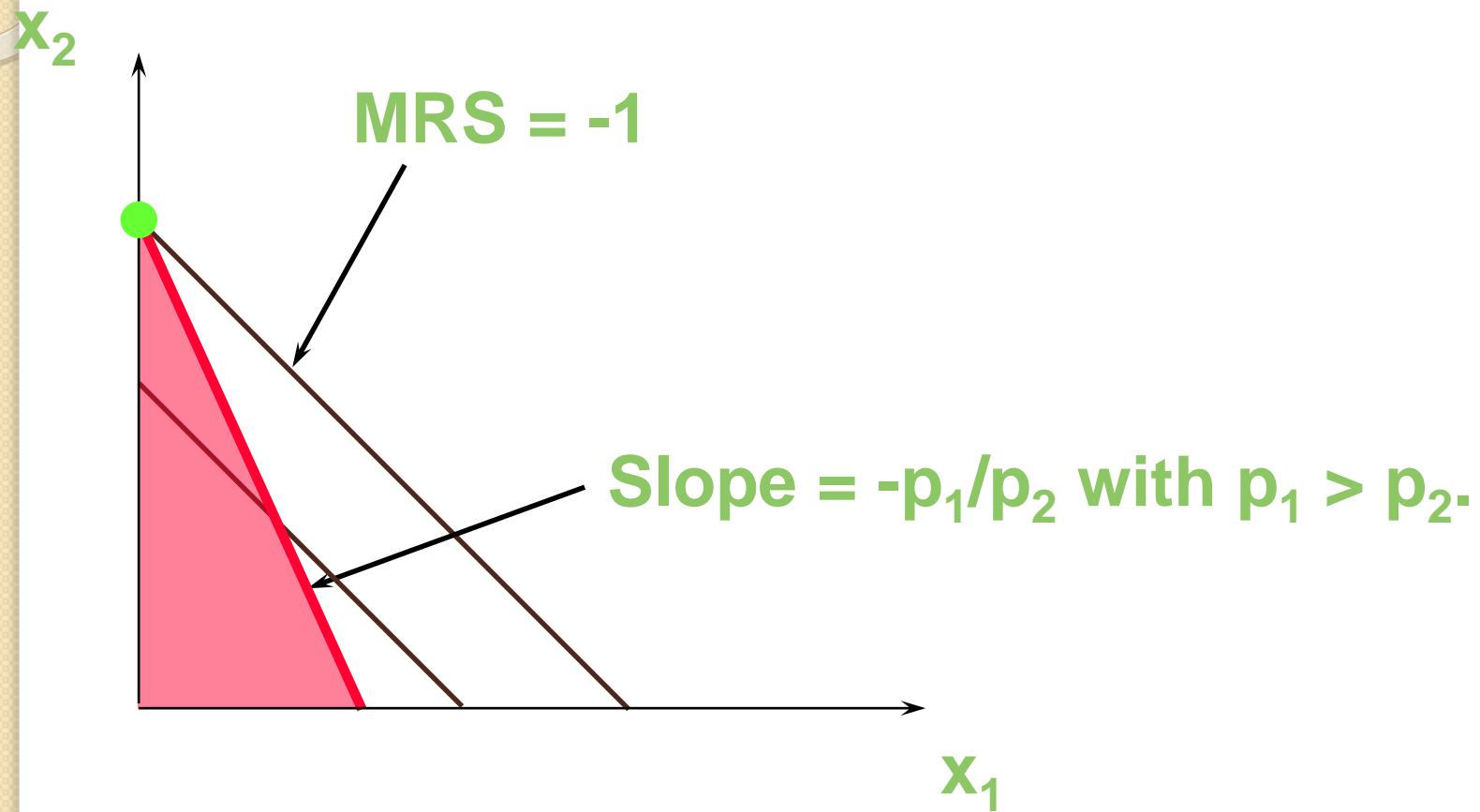
Examples of Corner Solutions -- the Perfect Substitutes Case



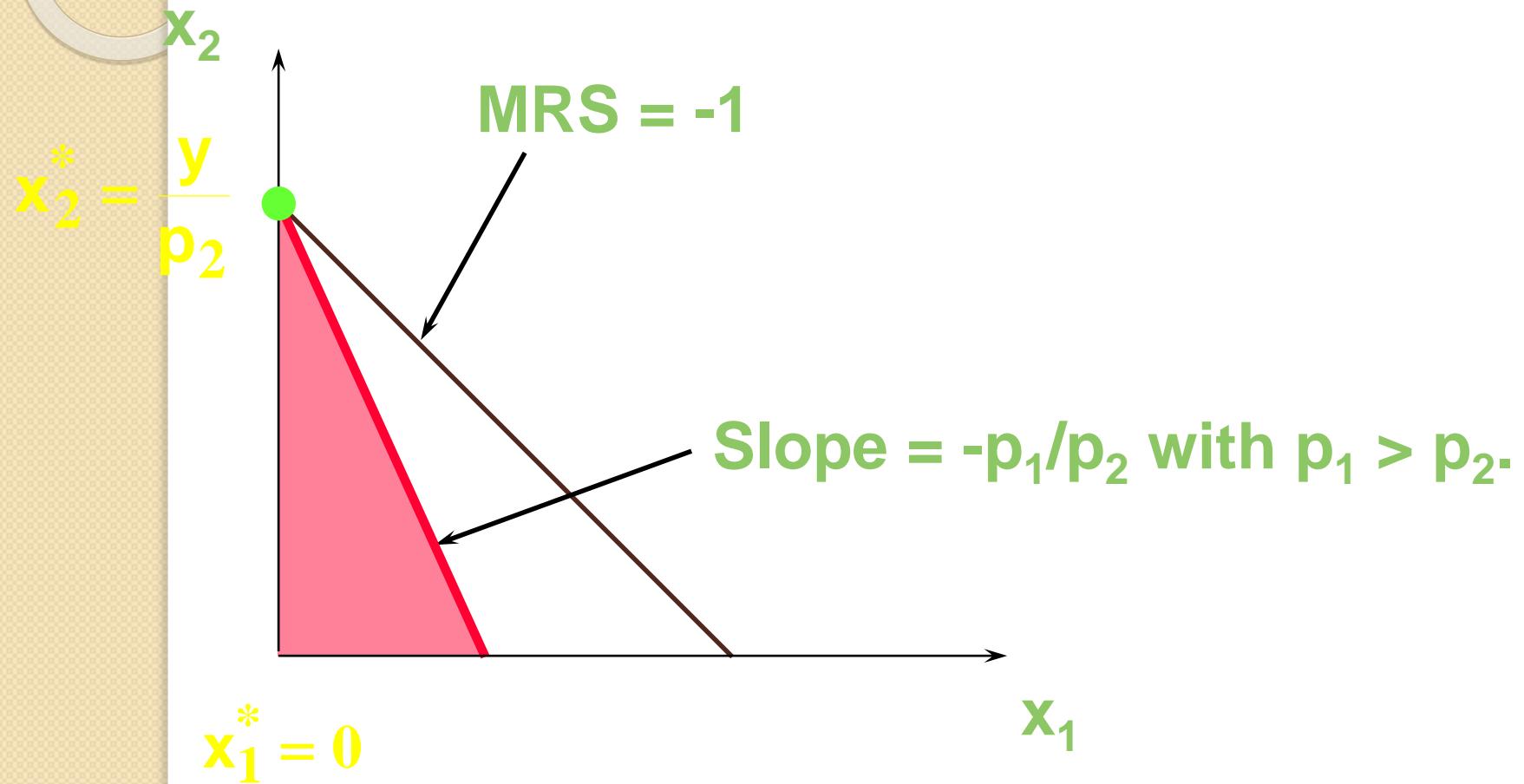
Examples of Corner Solutions -- the Perfect Substitutes Case



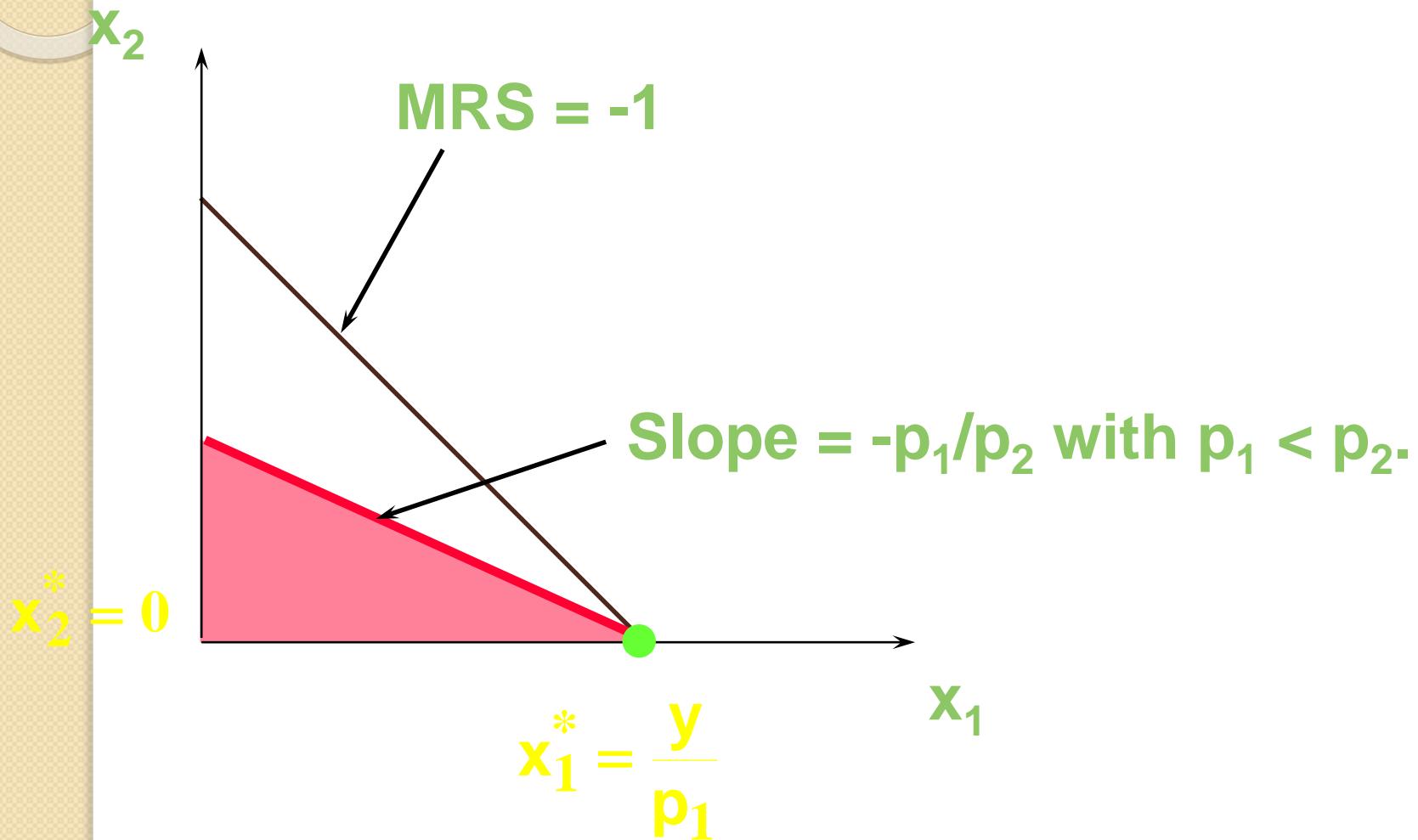
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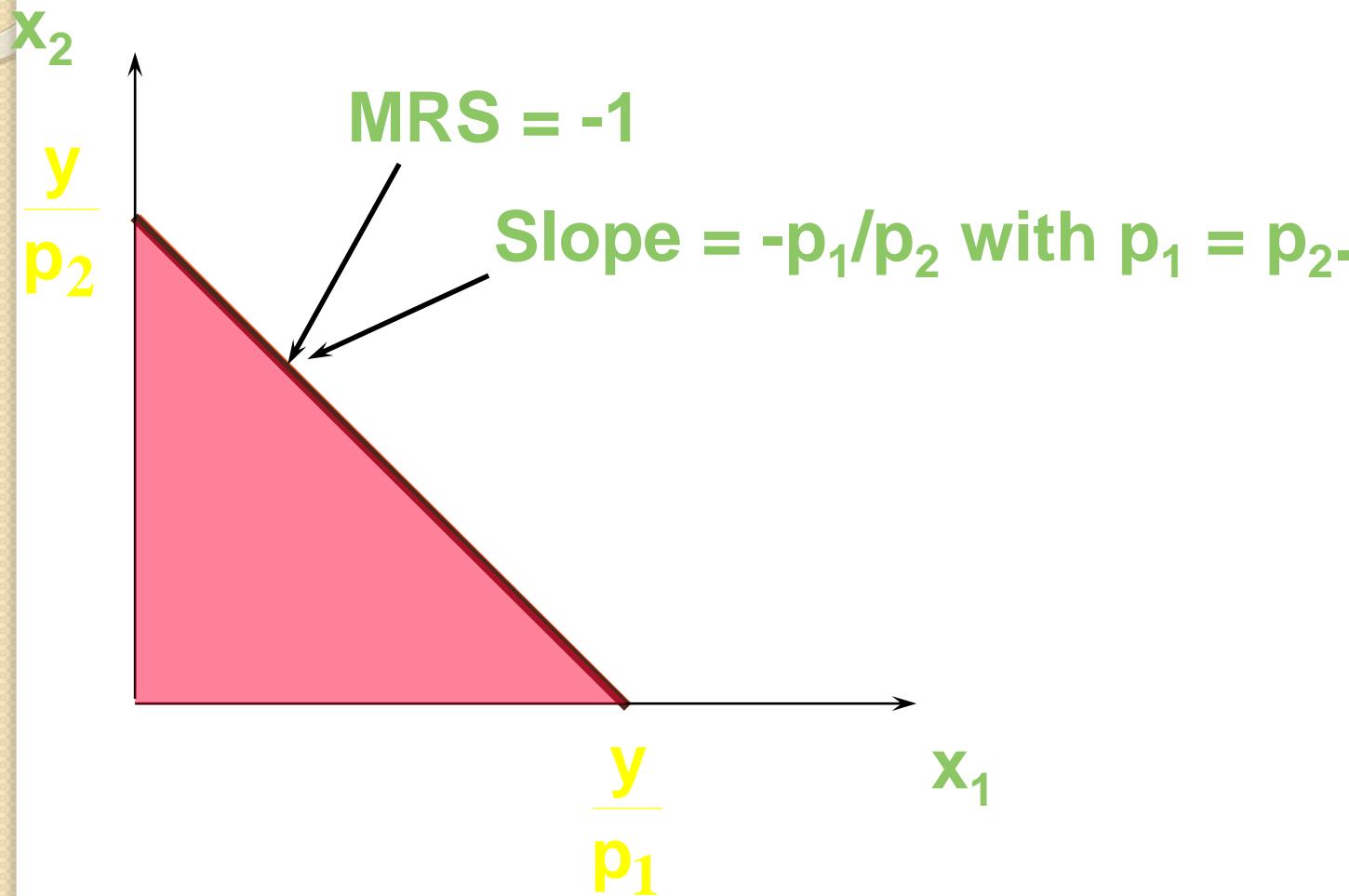
Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Is Tangency Condition Sufficient?

- Tangency condition is sufficient and necessary if
 - (1) Preferences are convex
 - (2) Solutions are interior