

## Problem Set 4

(Due Date: Dec. 4)

- Two players play the Rock-paper-scissors (剪刀石头布) game. Assume that the winning utility is 1, the even utility is 0, and the losing utility is -1.
  - Draw the payoff matrix for this game.
  - Find the Nash equilibrium (pure strategy and mixed strategy).
  - Calculate the expected payoff of both players.
- Two firms (A and B) are considering bringing out competing brands of a healthy cigarette. Payoffs to the companies when they compete simultaneously are shown in the table:

		B	
		Produce	Not Produce
A	Produce	3,3	5,4
	Not Produce	4,5	2,2

- Identify the Nash equilibrium of this game.
  - Assume that A is the first mover in the market so the game becomes a sequential game. Please describe the game both in terms of the normal form (the usual payoff matrix) in extensive form.
  - Identify both Nash equilibrium and subgame perfect equilibrium (using backward induction) in this sequential game.
  - Based on the results you get in c), discuss the relation between Nash equilibrium and subgame perfect equilibrium.
- 假设一对夫妻都喜欢干净的房间胜于脏乱的房间，但是都不喜欢打扫。他们的收益矩阵如下

		妻子	
		打扫	不打扫
丈夫	打扫	5, 5	2, 6
	不打扫	6, 2	3, 3

- 博弈双方是否存在占优策略(dominant strategy)? 如果是，请指出，并给出原因。这个博弈的纳什均衡(Nash equilibrium)是什么？请给出原因。
- 假设我们给上述博弈之后再加上一个阶段使之变成一个动态博弈。在这个阶段中，丈夫和妻子可以同时选择是否吵架。无论谁先开始吵，吵架对双方都会造成 2 单位的效用损失。现在妻子威胁丈夫说如果他不主动打扫就一定会和他吵架。假设双方都是理性的，请问这个威胁可信么?这个动态博弈的纳什均衡是什么？
- 回到（1）的静态博弈。假设丈夫和妻子被某种他们不能控制的情绪支配着，导致他们如果看到对方没有打扫房间没法不吵架。同样我们假定，无论谁先开始吵，吵架对双方都会造成 2 单位的效用损失。请给出这个博弈的新的收益矩阵。并找出其纯策略纳什均衡(pure strategy Nash equilibrium)。
- 假设丈夫和妻子不是以吵架的方式解决问题，而是通过采取扳机策略(trigger strategy)威胁对方，如果对方不打扫卫生，那么以后就永远不会打扫。假设二人时间偏好上的贴现

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因子为  $\delta$ ，请问当贴现因子满足什么条件的时候这个策略会使得两人都打扫卫生（假设无限期重复博弈）。

4. A monopolist can produce at a constant average and marginal cost  $AC = MC = \$5$ . It faces a market demand curve given by  $Q = 50 - P$ .
- Calculate the profit-maximizing price and quantity for this monopolist. Also calculate his profit.
  - Suppose a second firm with the same production technology enters the market and they two determine the output level simultaneously. What is the equilibrium price and quantity of each firm? Also calculate the profit for each firm.
  - Suppose there are  $N$  firms with the identical technology above. Calculate the equilibrium price, quantity and profit for each firm.
  - Return to the two firms case. Now suppose the first firm is the Stackelberg leader. Find the equilibrium quantity, price and profit for each firm.
  - Suppose there are three firms. Firm 1 is the Stackelberg leader who decides quantity first, firm 2 and firm 3 decides quantity simultaneously after the decision of firm 1. Find out the equilibrium quantity, price and profit for each firm.

5. Two firms compete by choosing price. Since their products are slightly differentiated, their demand functions are

$$Q_1 = 20 - P_1 + P_2; \quad Q_2 = 20 - P_2 + P_1$$

where  $P_1$  and  $P_2$  are the prices charged by each firm, respectively, and  $Q_1$  and  $Q_2$  are the resulting demands. Marginal costs are zero.

- Suppose the two firms set their prices at the *same time*. Find the resulting Nash equilibrium prices.
- Suppose Firm 1 sets its price *first*, and then Firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be? Does the first mover have a first-mover advantage?