Mathematical Methods in Finance Final Exam

Dec 15, 2016

1. $W(t)$ is a standard Brownian Motion. Calculate:

(1) $\mathbb{P}(W(2) \geq 1, W(5) < W(2) + 3)$.

(2) $\mathbb{E}\big(W(5) \big| \mathcal{F}(2)\big)$.

(3) $\mathbb{E}\big(W(5) \big| W(2)\big)$.

(4) $\mathbb{E}\big(W(2) \big| \mathcal{F}(5)\big)$.

(5) $\mathbb{E}\big(W(2) \big| W(5)\big)$.

2. A 3-term binomial lattice pricing model for a put option strike at 6. $S_0 = 6$, $u = 2$, $d = 1/2$, and $r = 1/4$.

(1) Calculate V_0.

(2) How to Delta-hedge the put option?

3. Stock price S_k follows

$$S_{k+1} = S_k X_{k+1}$$

where X_k are i.i.d. Bernoulli distributed with p probability to u, and $(1-p)$ probability to $1/u$.

(1) Prove

$$M_k = \left(\prod_{l=1}^{k} X_l\right)^{\frac{\log(1-p) - \log p}{\log u}}$$

is a martingale.

(2) Find the probability that stock price hits $S_0 u^n$ before $S_0 u^m$, where $n < 0 < m$.

4. Let $R(t)$ be a stochastic process with

$$\mathrm{d}R(t) = \kappa\big(\theta(t) - R(t)\big)\mathrm{d}t + \sigma(t)\mathrm{d}W(t)$$

(1) Solve the model explicitly

(2) What is the distribution of $R(t)$? Find the expectation and variance of it.

5. An "Asset-or-Nothing" option with payoff $S(T)\mathbb{I}_{S(T)\geq K}$. Assume stock price follows Black-Scholes-Merton Model.

(1) Find the PDE of $v(t, S(t))$ with no-arbitrage pricing.

(2) Find the PDE of $v(t, S(t))$ with Feymann-Kac Equation.

(3) Calculate $v(0, S(0))$.

Bonus Question. Let $X(t)$ be a stochastic process with

$$\mathrm{d}X(t) = \mu\big(X(t)\big)\mathrm{d}t + \sigma\big(X(t)\big)\mathrm{d}W(t), X(0) = X_0$$

Let T be the time when $X(t)$ leaves the interval (A, B). Thus,

$$T = \inf\{t > 0; X(t) \notin (A, B)\}$$

Calculate:

(1) $\mathbb{P}(X(T) = A)$.

(2) $\mathbb{E}T$.