2、

a. The consumer price index uses the consumption bundle in year 1 to figure out how much weight to put on the price of a given good:

$$
\begin{aligned}
\text{CPI}^2 &= \frac{(\$2 \times 10) + (\$1 \times 0)}{(\$1 \times 10) + (\$2 \times 0)} \\
&= \frac{(P^2_{red} \times Q^1_{red}) + (P^2_{green} \times Q^1_{green})}{(P^1_{red} \times Q^1_{red}) + (P^1_{green} \times Q^1_{green})} \\
&= 2.
\end{aligned}
$$

According to the CPI, prices have doubled.

b. Nominal spending is the total value of output produced in each year. In year 1 and year 2, Abby buys 10 apples for $1 each, so her nominal spending remains constant at $10. For example,

$$
\begin{aligned}
\text{Nominal Spending}_2 &= (P^2_{red} \times Q^2_{red}) + (P^2_{green} \times Q^2_{green}) \\
&= (\$2 \times 0) + (\$1 \times 10) \\
&= \$10.
\end{aligned}
$$

c. Real spending is the total value of output produced in each year valued at the prices prevailing in year 1. In year 1, the base year, her real spending equals her nominal spending of $10. In year 2, she consumes 10 green apples that are each valued at their year 1 price of $2, so her real spending is $20. That is,

$$
\begin{aligned}
\text{Real Spending}_2 &= (P^1_{red} \times Q^2_{red}) + (P^1_{green} \times Q^2_{green}) \\
&= (\$1 \times 0) + (\$2 \times 10) \\
&= \$20.
\end{aligned}
$$

Hence, Abby's real spending rises from $10 to $20.

d. The implicit price deflator is calculated by dividing Abby's nominal spending in year 2 by her real spending that year:

$$
\begin{aligned}
\text{Implicit Price Deflator}_2 &= \frac{\text{Nominal Spending}_2}{\text{Real Spending}_2} \\
&= \frac{\$10}{\$20} \\
&= 0.5.
\end{aligned}
$$

Thus, the implicit price deflator suggests that prices have fallen by half. The reason for this is that the deflator estimates how much Abby values her apples using prices prevailing in year 1. From this perspective green apples appear very valuable. In year 2, when Abby consumes 10 green apples, it appears that her consumption has increased because the deflator values green apples more highly than red apples. The only way she could still be spending $10 on a higher consumption bundle is if the price of the good she was consuming fell.

e. If Abby thinks of red apples and green apples as perfect substitutes, then the cost of living in this economy has not changed-in either year it costs $10 to consume 10 apples. According to the CPI, however, the cost of living has doubled. This is because the CPI only takes into account the fact that the red apple price has doubled; the CPI ignores the fall in the price of green apples because they were not in the consumption bundle in year 1. In contrast to the CPI, the implicit price deflator estimates the cost of living has been cut in half. Thus, the CPI, a Laspeyres index, overstates the increase in the cost of living and the deflator, a Paasche index, understates it.

3

a. A Cobb-Douglas production function has the form Y =A$K^{\alpha}L^{1-\alpha}$ The text showed that the marginal products for the Cobb-Douglas production function are:

$$MPL = (1-\alpha)Y/L.$$
$$MPK = \alpha Y/K.$$

Competitive profit-maximizing firms hire labor until its marginal product equals the real wage, and hire capital until its marginal product equals the real rental rate. Using these facts and the above marginal products for the Cobb-Douglas production function, we find:

$$W/P = MPL = (1-\alpha)Y/L.$$
$$R/P = MPK = \alpha Y/K.$$

Rewriting this:

$$(W/P)L = MPL \times L = (1-\alpha)Y.$$
$$(R/P)K = MPK \times K = \alpha Y.$$

Note that the terms (W/P)L and (R/P)K are the wage bill and total return to capital, respectively. Given that the value of a= 0. 3, then the above formulas indicate that labor receives 70 percent of total output (or income), which is (1 - 0. 3), and capital receives 30 percent of total output (or income).

b. To determine what happens to total output when the labor force increases by 10 percent, consider the formula for the Cobb-Douglas production function:

$$Y = AK^{\alpha}L^{1-\alpha}.$$

Let Y1 equal the initial value of output and Y2 equal final output. We know that a= 0.3. We also know that labor L increases by 10 percent:

$$Y_1 = AK^{0.3}L^{0.7}.$$
$$Y_2 = AK^{0.3}(1.1L)^{0.7}.$$

Note that we multiplied L by 1. 1 to reflect the 10-percent increase in the labor force.

To calculate the percentage change in output, divide Y2 by Y1:

$$\frac{Y_2}{Y_1} = \frac{AK^{0.3}(1.1L)^{0.7}}{AK^{0.3}L^{0.7}}$$
$$= (1.1)^{0.7}$$
$$= 1.069.$$

That is, output increases by 6. 9 percent.

To determine how the increase in the labor force affects the rental price of capital, consider the formula for the real rental price of capital R/P:

$$R/P = MPK = \alpha AK^{\alpha-1}L^{1-\alpha}.$$

We know that a= 0. 3. We also know that labor (L) increases by 10 percent. Let (R/P)1 equal the initial value of the rental price of capital, and (R/P)2 equal the final rental price of capital after the labor force increases by 10 percent. To find (RI/P)2, multiply L by 1. 1 to reflect the 10-percent increase in the labor force:

$$(R/P)_1 = 0.3AK^{-0.7}L^{0.7}.$$
$$(R/P)_2 = 0.3AK^{-0.7}(1.1L)^{0.7}.$$

The rental price increases by the ratio

$$\frac{(R/P)_2}{(R/P)_1} = \frac{0.3AK^{-0.7}(1.1L)^{0.7}}{0.3AK^{-0.7}L^{0.7}}$$
$$= (1.1)^{0.7}$$
$$= 1.069.$$

So the rental price increases by 6. 9 percent.

To determine how the increase in the labor force affects the real wage, consider the formula for the real wage W/P:

$$W/P = MPL = (1-\alpha)AK^{\alpha}L^{-\alpha}.$$

We know that a = 0. 3. We also know that labor (L) increases by 10 percent. Let (W/P)1 equal the initial value of the real wage and (W/P)2 equal the final value of the real wage. To find (W/P)2, multiply L by 1. 1 to reflect the 10-percent increase in the labor force:

$$(W/P)_1 = (1-0.3)AK^{0.3}L^{-0.3}.$$
$$(W/P)_2 = (1-0.3)AK^{0.3}(1.1L)^{-0.3}.$$

To calculate the percentage change in the real wage, divide (W/P)2 by (W/P)1:

$$\frac{(W/P)_2}{(W/P)_1} = \frac{(1-0.3)AK^{0.3}(1.1L)^{-0.3}}{(1-0.3)AK^{0.3}L^{-0.3}}$$
$$= (1.1)^{-0.3}$$
$$= 0.972.$$

That is, the real wage falls by 2.8 percent.

C. We can use the same logic as in part (b) to set

$$Y_1 = AK^{0.3}L^{0.7}.$$
$$Y_2 = A(1.1K)^{0.3}L^{0.7}.$$

Therefore, we have:

$$\frac{Y_2}{Y_1} = \frac{A(1.1K)^{0.3}L^{0.7}}{AK^{0.3}L^{0.7}}$$
$$= (1.1)^{0.3}$$
$$= 1.029.$$

This equation shows that output increases by about 3 percent. Notice that a< 0.5 means that proportional increases to capital will increase output by less than the same proportional increase to labor.

Again using the same logic as in part (b) for the change in the real rental price of capital:

$$\frac{(R/P)_2}{(R/P)_1} = \frac{0.3A(1.1K)^{-0.7}L^{0.7}}{0.3AK^{-0.7}L^{0.7}}$$
$$= (1.1)^{-0.7}$$
$$= 0.935.$$

The real rental price of capital falls by 6.5 percent because there are diminishing returns to capital; that is, when capital increases, its marginal product falls.

Finally, the change in the real wage is:

$$\frac{(W/P)_2}{(W/P)_1} = \frac{0.7A(1.1K)^{0.3}L^{-0.3}}{0.7AK^{0.3}L^{-0.3}}$$

$$= 1.029.$$

Hence, real wages increase by 2.9 percent because the added capital increases the marginal productivity of the existing workers. (Notice that the wage and output have both increased by the same amount, leaving the labor share unchanged-a feature of Cobb-Douglas technologies.)

d. Using the same formula, we find that the change in output is:

$$\frac{Y_2}{Y_1} = \frac{(1.1A)K^{0.3}L^{0.7}}{AK^{0.3}L^{0.7}}$$

$$= 1.1.$$

This equation shows that output increases by 10 percent. Similarly, the rental price of capital and the real wage also increase by 10 percent:

$$\frac{(R/P)_2}{(R/P)_1} = \frac{0.3(1.1A)K^{-0.7}L^{0.7}}{0.3AK^{-0.7}L^{0.7}}$$

$$= 1.1.$$

$$\frac{(W/P)_2}{(W/P)_1} = \frac{0.7(1.1A)K^{0.3}L^{-0.3}}{0.7AK^{0.3}L^{-0.3}}$$

$$= 1.1.$$

4.

a. The marginal product of labor MPL is found by differentiating the production function with respect to labor:

$$MPL = \frac{dY}{dL}$$

$$= \frac{1}{3}K^{1/3}H^{1/3}L^{-2/3}.$$

An increase in human capital will increase the marginal product of labor because more human capital makes all the existing labor more productive.

b. The marginal product of human capital MPH is found by differentiating the production function with respect to human capital:

$$\begin{aligned} MPH &= \frac{dY}{dH} \\ &= \frac{1}{3} K^{1/3} L^{1/3} H^{-2/3}. \end{aligned}$$

An increase in human capital will decrease the marginal product of human capital because there are diminishing returns.

c. The labor share of output is the proportion of output that goes to labor. The total amount of output that goes to labor is the real wage (which, under perfect competition, equals the marginal product of labor) times the quantity of labor. This quantity is divided by the total amount of output to compute the labor share:

$$\begin{aligned} \text{Labor Share} &= \frac{(\frac{1}{3} K^{1/3} H^{1/3} L^{-2/3}) L}{K^{1/3} H^{1/3} L^{1/3}} \\ &= \frac{1}{3}. \end{aligned}$$

We can use the same logic to find the human capital share:

$$\begin{aligned} \text{Human Capital Share} &= \frac{(\frac{1}{3} K^{1/3} L^{1/3} H^{-2/3}) H}{K^{1/3} H^{1/3} L^{1/3}} \\ &= \frac{1}{3}, \end{aligned}$$

so labor gets one-third of the output, and human capital gets one-third of the output. Since workers own their human capital , it will appear that labor gets two-thirds of output.

d. The ratio of the skilled wage to the unskilled wage is:

$$\begin{aligned} \frac{W_{\text{skilled}}}{W_{\text{unskilled}}} &= \frac{MPL + MPH}{MPL} \\ &= \frac{\frac{1}{3} K^{1/3} L^{-2/3} H^{1/3} + \frac{1}{3} K^{1/3} L^{1/3} H^{-2/3}}{\frac{1}{3} K^{1/3} L^{-2/3} H^{1/3}} \\ &= 1 + \frac{L}{H}. \end{aligned}$$

Notice that the ratio is always greater than 1 because skilled workers get paid more than unskilled workers. Also, when H increases this ratio falls because the diminishing returns to human capital lower its return, while at the same time increasing the marginal product of unskilled workers.

e. If more college scholarships increase H, then it does lead to a more egalitarian society. The policy lowers the returns to education, decreasing the gap between the wages of more and less educated workers. More importantly, the policy even raises the absolute wage of unskilled workers because their marginal product rises when the number of skilled workers rises.