

# New Kids on the Block

*Trends in quantitative finance and their impact on investment management.*

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**T**he growing use of quantitative methods has changed finance and investment theory. Theoretical concepts such as market efficiency and market equilibrium have ceded ground to econometric methods that allow a more pragmatic investigation of asset predictability.

We look at the evolution of quantitative methods in finance and how they will change the investment management industry. We begin with a general discussion of general equilibrium theories and their limitations and shortcomings. Next we discuss one of the cornerstones in classical finance theory, mean-variance optimization, addressing some of the problems practitioners face in applying the mean-variance optimization approach to portfolio management. Following a discussion of the feedback phenomena in financial markets, we focus on asset price predictability and examples of forecastability, albeit limited. We provide an outlook on the use of data mining and machine-learning approaches for forecasting financial markets. Model risk, data snooping, and overfitting are also addressed.

We conclude by highlighting some general implications for portfolio management and provide some thoughts relative to the impact quantitative methods will have on the investment management industry.

## GENERAL EQUILIBRIUM THEORIES: LIMITATIONS AND SHORTCOMINGS

Managing financial assets depends on the ability to forecast risk and returns. This calls for a theory of financial markets. In the classical approach, market participants (or agents) do not interact directly; they are coordinated by a

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central price signal, which is the signal on which they optimize. Agents can be heterogeneous insofar as they have different utility functions, but they can be aggregated, thereby forming a “representative agent” whose optimizing behavior sets the optimal price process.

In the last 15 years, a different view of financial markets has emerged. Two interrelated aspects of this change are particularly important:

- There is now a stricter adherence to scientific and empirically validated theories.
- It has been recognized that markets are populated by agents who can make only imperfect forecasts and who influence each other *directly*.

The new perspective of interacting agents offers a natural framework to explain the presence of the feedback in financial markets that has been discovered empirically.

Modern econometric techniques have shown evidence that asset prices can be predicted, at least at some level. Asset price predictability is greater at the portfolio level than at the individual asset level. Predictions, however, remain difficult to make and are always uncertain—or we would all be rich. This explains why potential profits are often limited and are not risk-free.

Today, price processes are forecasted by factor models and other techniques based upon econometrics. For example, so-called cointegration relationships (i.e., long-term valid regressions) have been shown to exist among common stocks and bonds as well as financial and commodity futures.

Since the pioneering work of Wilfredo Pareto at the end of the 19th century, decision-making has been modeled using *utility functions*. In the second half of the 20th century, Oskar Morgenstern and John Von Neumann developed the modern theory of utility functions, now a standard model of decision-making.

Utility functions assume that choices can be ordered, and that this ordering can be expressed through a numerical utility index. Choice A is preferred to choice B if A has a higher-value utility than B.

Ordering choices is only one part of the problem. Decision-making implies that choices are identified. This in turn implies that agents can forecast the future, at least probabilistically. Economics therefore requires a model of how agents make forecasts.

Classical economics is based on the principles that 1) the agent decision-making process can be represented as the maximization of (expected) utility, and 2) that agents have a perfect knowledge of the future in the sense that the stochastic processes on which they optimize are exactly the true stochastic processes.<sup>1</sup> The instantaneous and continuous interaction among agents taking advantage of (and thereby eliminating) arbitrage opportunities in the market

is the process that will force asset prices toward equilibrium. This is the essence of *general equilibrium theories*.

If general equilibrium theories really do represent agent decision-making processes, it should be possible to validate this empirically. One way would be to measure agent utility functions and explore the real forecasting ability of agents. But this is clearly not possible; most agents cannot explicitly translate their decision-making process into a utility function. In addition, the way agents produce their forecasts is in general imperfect (often irrational) and not known in any detail.

General equilibrium theories are abstract mathematical constructs, and do not have much to do with the actual agent decision-making process and people’s cognitive behavior. They express a broad mathematical principle that is hard to refute empirically. For example, Harrison and Kreps [1979] demonstrate that *any price process where there is no arbitrage opportunity can be rationalized as a general equilibrium theory*. Lack of arbitrage opportunity is the key signal that equilibrium has been established, although often the lack of arbitrage is transitory because of the arrival of new information. This is central to Miller and Modigliani’s theory of the firm’s capital structure, and one of their greatest contributions.<sup>2</sup>

In practice, however, general equilibrium theories expressed as abstract mathematical principles do not attain any reasonable level of empirical validation. We do not have a faithful description of the utility function of the representative agent or of the exogenous variables.<sup>3</sup>

These considerations do not apply to econometrics. Because it attempts to fit generalized models that have free parameters, econometrics has a strong data-mining component. Often there is no theoretical justification behind the econometric approach. For this reason, econometrics has not been seen as mainstream economic theory but more as a set of recipes for making doubtful forecasts.

Econometrics has a strong empirical basis (in this, it resembles the physical sciences), but a relatively simple theoretical foundation. A broader application of theoretical methods typical of the physical sciences to economics and finance theory is the source of a new discipline called *econophysics*. Econophysics has brought results such as an accurate empirical analysis of the distribution of asset returns that shows significant deviations from the normal distribution. It has also reduced the importance of concepts such as market efficiency, which are deeply rooted in the general equilibrium theory. As a consequence, asset predictability has been investigated more pragmatically.

At the same time, econophysics has shown the need for a stronger theoretical base to econometrics. Perhaps its most significant contribution has been to encourage a stricter application of the scientific method to economics, with strong empirical evidence as the foundation to abstract theories—and not the other way around. This approach was

made possible by the increased availability of data, especially high-frequency data.

## CAN WE GET MEAN-VARIANCE OPTIMIZATION TO WORK?

Markowitz [1952] first introduced portfolio selection using a quantitative optimization procedure that balances the trade-off between risk and return. Markowitz's work laid the ground for the capital asset pricing model (CAPM). The most fundamental general equilibrium theory in modern finance, CAPM states that the expected value of the excess return of any asset is proportional to the excess return of the total investable market, where the constant of proportionality is the covariance between the asset return and the market return.

More than 50 years after Markowitz's seminal work, it appears that full risk-return optimization at the portfolio level is done at only the more quantitative firms, where processes for automated forecast generation and risk control are already installed. Today, at many firms, portfolio management remains a pure judgmental process based on qualitative, not quantitative, assessments. The first quantitative efforts at most firms appear to be focused on providing risk measures to portfolio managers. These measures offer asset managers a view of the level of risk in a particular portfolio, where risk is defined as underperformance relative to a mandate.

Although optimization technology is considered to be mature, many asset managers have had problems applying it or have avoided it altogether. One reason is that in practical applications, classical mean-variance optimization is very sensitive to the inputs (i.e., expected returns of each asset and their covariance matrix). For example, "optimal" portfolios often have extreme or non-intuitive weights for some of the individual assets. The practitioner's solution has generally been to add constraints to the original problem in order to limit extreme or non-intuitive portfolio weights. As a result, the constraints—instead of the forecasts—often determine the portfolio, making the risk-return optimization process pointless.

Practitioners applying mean-variance portfolio allocation often face additional problems, such as:

- Poor model ex post performance, coupled in many instances with the risk of maximizing error rather than minimizing it.
- Difficulty in estimating a stable covariance matrix for a large number of assets.
- Sensitivity of portfolio weights to small changes in forecasts.

From a practical point of view, one important objective is to make the portfolio allocation process more robust

to different sources of risk—including estimation risk and model risk. Two techniques appear promising: 1) Bayesian approaches, and 2) robust portfolio allocation. Both help to reduce sensitivity to estimation error and to improve model strength, resulting in better performance.

A common critique of the mean-variance optimization framework is its oversimplistic and unrealistic assumption that asset returns are normally distributed. Many return distributions in financial markets exhibit fat tails and other effects that can be taken into account only by incorporating higher moments, beyond the mean and variance. The earliest studies showing non-normality of asset returns, date to Mandelbrot [1963]. Extensions to classical mean-variance optimization incorporating higher moments are described in, for example, Athayde and Flôres [2004] and Harvey et al. [2003].<sup>4</sup>

## Bayesian Approaches

The classical approach to estimating future expected returns assumes that the "true" expected returns and covariances of returns are unknown and *fixed*. A point estimate (i.e., an estimate of the most likely return represented by a single number) is obtained using forecasting models of observed market data and proprietary data. It is difficult to make accurate estimates, and the mean-variance portfolio allocation decision is influenced by the estimation error of the forecasts. For example:

- Equally weighted portfolios often outperform mean-variance optimized portfolios (see Jobson and Korkie [1981]).
- Mean-variance optimized portfolios are not necessarily well diversified (Jorion [1985]).
- Uncertainty of returns tends to have more influence than risk in mean-variance optimization (Chopra and Ziemba [1993]).

The Bayesian approach, on the other hand, assumes that the true expected returns are unknown and *random*. Named after the English mathematician, Thomas Bayes, the Bayesian approach is based on the *subjective* interpretation of probability. A probability distribution is used to represent an investor's belief as to the probability that a specific event will actually occur. This probability distribution, called the "prior distribution," reflects an investor's knowledge about the probability before any data are observed. After more information is provided (data observed), the investor's opinions about the probability might change.

Bayes' rule is the formula for computing this new probability distribution, called the *posterior distribution*. The posterior distribution is based on knowledge of the prior probability distribution plus the new data.

A posterior distribution of expected return is derived by combining the forecast from the empirical data with a prior distribution. For example, in the Black-Litterman [1990, 1991] model, an estimate of future expected returns is based on combining market equilibrium (e.g., the CAPM equilibrium) with an investor's views. Such views are expressed as absolute or relative deviations from equilibrium together with confidence levels of the views (measured by the standard deviation of the views).

The Black-Litterman expected return is calculated as a weighted average of the market equilibrium and the investor's views. The weights depend upon 1) the volatility of each asset and its correlations with the other assets, and 2) the degree of confidence in each forecast. The resulting expected return, which is the mean of the posterior distribution, is then used as input to the classical mean-variance optimization process. Portfolio weights computed this way tend to be more intuitive and less sensitive to small changes in the original inputs, such as forecasts of market equilibrium, investor views, and the covariance matrix. The Black-Litterman model modifies the inputs to the mean-variance framework—but the risk-return optimization is the same as in Markowitz's classic approach.

The ability to incorporate exogenous insight, such as a portfolio manager's judgment, into formal models is important; such insight might be the most valuable input the model uses. The Bayesian framework allows forecasting systems to use such external information sources and subjective interventions (i.e., modification of the model due to judgment) in addition to traditional information sources such as market data and proprietary data.

Because portfolio managers might not be willing to give up control to a black box, the incorporation of exogenous insights into formal models through Bayesian techniques is one way to give managers better control in a quantitative framework. Forecasts are represented through probability distributions that can be modified or adjusted to incorporate other sources of information deemed relevant. The only restriction is that such additional information (i.e., the investor's "views") be combined with the model following the laws of probability. In effect, incorporating Bayesian views into a model lets us "rationalize" subjectivity within a formal, quantitative framework. "The rational investor is a Bayesian," as Markowitz noted [1987, p. 57].

Interventions can be either feed-forward (anticipatory actions) or feed-back (corrective actions) (see, for example, West and Harrison [1989]). The Bayesian framework also allows for mixing, selecting, and switching among dynamic models in a common framework. During the last decade, progress in Bayesian modeling has put these general and powerful computational techniques within reach of practitioners in the financial markets (see Carlin, Polson, and Stoffer [1992]; Carter and Kohn [1994]; and Fruehwirth-Schnatter [1994]).

Incorporating dynamic factors and Bayesian analysis into structured stochastic volatility models should allow more accurate estimates of volatility. Stochastic volatility models consider volatility as a variable term that should be forecasted. More in general, not only volatility but also the entire covariance matrix can be regarded as a set of variable terms to forecast. But, as we know, estimates of the covariance matrix are not stable but vary with time.

An early (not entirely satisfactory) attempt to deal with this problem was covariance matrix discounting. *Covariance matrix discounting* assumes that the covariance matrix changes with time. At any moment, there is a "local" covariance matrix. The covariance matrix is estimated as a weighted average of past covariance matrices. Weighting factors typically decay exponentially with time. Since its introduction in the 1980s, covariance discounting has been used as a component of applied Bayesian forecasting models in financial applications.<sup>5</sup>

Covariance matrix discounting methods do not have any real predictive power. They (simplistically) provide exponentially smoothed estimates of the local covariance structure (i.e., the covariance matrix that is supposed to hold at a given time) within the Bayesian modeling framework. They estimate change rather than forecast change. As a consequence, these models tend to work reasonably well in slow-changing volatility environments, but do poorly in fast-moving markets or when structural change occurs.

Much greater flexibility is achieved by incorporating dynamic factor models that can explicitly capture change through patterns of variation in process parameters throughout time. In other words, the covariance matrix is driven by a multifactor model. This approach has already shown significant improvement in short-term forecasting of multiple financial and economic time series, and appears to be a promising technique for the intermediate and the long term as well. Although Bayesian dynamic factor models are computationally demanding and often require time-consuming simulations, the availability of more powerful computers and recent advances in Markov chain Monte Carlo methods will contribute to a growing use of these models.

## Robust Portfolio Allocation

In the classic mean-variance optimization problem, the expected returns and the covariance matrix of returns are uncertain and have to be estimated. After the estimation of these quantities, the portfolio optimization problem is typically treated and solved as a deterministic problem with no uncertainty. However, it makes sense for the uncertainty of expected returns and risk to enter into the optimization process, thereby creating a more realistic model. Using point estimates of the expected returns and the covariance matrix of returns, and treating them as error-free in portfolio allocation,



tion, does not necessarily correspond to prudent investor behavior.

The investor would probably be more comfortable choosing a portfolio that would perform well under a number of different scenarios, thereby also attaining some protection from estimation risk and model risk. Obviously, to have some insurance in the event of less likely but more extreme cases (e.g., scenarios that are highly unlikely under the assumption that returns are normally distributed), the investor must be willing to give up some of the upside that would result under the more likely scenarios. Such an investor seeks a “robust” portfolio, i.e., a portfolio that is assured against some worst case model misspecification.

The estimation process can be improved through robust statistical techniques such as the Bayesian estimators we have discussed. Yet jointly considering estimation risk and model risk in the financial decision-making process is becoming more important.

The estimation process does not deliver a point forecast (that is, one single number) but a full distribution of expected returns. Recent approaches attempt to integrate estimation risk into the mean-variance framework by using the expected return distribution in the optimization. A simple approach is to sample from the return distribution and average the resulting portfolios (Monte Carlo approach), but as a mean-variance problem has to be solved for each draw, this is computationally intensive for larger portfolios. In addition, the averaging does not guarantee that the resulting portfolio weights will satisfy all constraints.

Introduced in the late 1990s by Ben-Tal and Nemirovski [1998, 1999] and Ghaoui and Lebret [1997], the robust optimization framework is computationally more efficient than the Monte Carlo approach. This development in optimization technology allows for the efficient solution of the robust version of the mean-variance optimization problem in about the same time as the classical mean-variance optimization problem. The technique uses the explicit distribution from the estimation process to find a robust portfolio in one single optimization. It thereby incorporates uncertainties of inputs into a deterministic framework. The classic portfolio optimization formulations such as the mean-variance portfolio selection problem, the maximum Sharpe ratio portfolio problem, and the value at risk (VaR) portfolio problem all have robust counterparts that can be solved in roughly the same amount of time as the original problem (see Goldfarb and Iyengar [2003]).

The resulting robust mean-variance portfolios are more stable to changes in inputs and tend to offer better performance than classical mean-variance portfolios. The robust optimization framework also gives additional flexibility. It can exploit the fact that there are statistically equivalent portfolios that are cheaper to trade into. This becomes very important in large-scale portfolio management with many complex

constraints such as transaction costs, turnover, and market impact.

For example, with robust optimization we can calculate the new portfolio that 1) minimizes trading costs with respect to the current holdings and 2) has an expected return that is statistically equivalent to the expected return of the classic mean-variance portfolio.

Although we are unaware of any portfolio manager currently using the robust optimization framework, results reported by Ceria [2003] indicate that the technique might help in achieving inherent robustness in portfolio optimization models and in minimizing trading costs.

## NEW TERRITORY

The question as to whether general equilibrium theories are appropriate representations of economic systems cannot be answered empirically. When general equilibrium theories are implemented in simplified versions, they lead to theoretical predictions that are at odds with empirical data. Perhaps the easiest way to see this is through CAPM, the most fundamental general equilibrium theory.

CAPM implies that there is a spectrum of expected returns. As returns cumulate exponentially (through compounding), this implies that in the long run price processes are exponentially divergent. But exponentially diverging price processes are practically impossible and inconsistent with empirical findings. In other words, CAPM does not correctly describe the long-term behavior of the financial markets as we observe them.

Feedbacks that prevent prices from diverging indefinitely are naturally introduced in the context of interacting agents. To understand how interacting agents generate feedback, imagine that the market is segmented into two major sectors and that there is mutual interaction between the sectors. A classic example is a predator-prey model. The predator population feeds on the prey and grows when there is abundance of prey. But a growing population of predators needs more prey. At some point, there is an inversion, and the predator population begins to falter.

Predator-prey models are non-linear models with highly complex chaotic behavior. In a similar way, by considering the exchange of limited resources, the theory of multiple interacting agents offers a natural framework for understanding and modeling feedback in financial markets (Aoki [1998, 2004]).<sup>6</sup>

Direct interaction between agents might also help explain the presence of non-normal distributions or fat tails in many financial variables such as asset returns and risk factors. Multiple interacting agent systems are subject to contagion and propagation phenomena that might—and very often do—produce fat tails. This is a possible explanation of how markets crash or are subject to large movements, and

is indeed a very different view from that in the CAPM and mean-variance optimal worlds.

A view of financial markets that is gaining ground is one of a multitude of interacting agents that form a complex system characterized by a high level of uncertainty. Uncertainty is embedded in the probabilistic structure of models and is therefore model-related. This theoretical approach has given a new twist to the econometrics of financial markets.

Econometrics is no longer simple data mining, i.e., the analysis of data to find relationships and patterns with no previous knowledge or a thorough understanding of the meaning of the underlying data. Today, econometrics supplies the empirical foundation of economics. This is a significant change. Science is highly stratified; we can build complex theories on the foundation of simpler theories.<sup>7</sup>

To lay the scientific foundation for a better understanding of financial markets, we begin with the collection of econometric data. When modeled, these data give us statistical facts of an empirical nature that provide the building blocks of future theoretical development.

The view that economic agents are heterogeneous, make mistakes, and mutually interact gives more freedom to economic theory (Mantegna and Stanley [1999] and Aoki [2004]). Yet this approach, although scientifically rigorous, cannot eliminate the basic fact that financial models are statistical models based on historical data and are therefore subject to a fundamental uncertainty; models are educated guesses, not foresight.

## TRENDS AND MEAN REVERSION: ARE MARKETS FORECASTABLE?

The statistical methodologies used in asset management are ultimately related to making forecasts of risk and return. According to classical theory, equity markets are not forecastable, in the sense that future returns do not depend upon past returns. As we have noted, this view is more theoretical than empirical. It is based on the efficient markets hypothesis (EMH) that present prices perfectly reflect all available information. Any new information would affect present prices instantaneously. In the words of Samuelson [1965], "properly anticipated prices fluctuate randomly."

To test the EMH would require an understanding of just what restrictions it imposes on probabilistic models of returns. From the theoretical point of view, the EMH is embodied in the martingale theory of dynamic price processes. In a martingale, the expected value of a process at future dates is its current value. If after appropriate discounting (by taking into account the time value of money and risk), all price processes behave as martingales, the best forecast of future prices is present prices. In other words, prices might have different distributions, but the condi-

tional mean, after appropriate discounting, is the present price.

Now, the martingale theory of price processes states that prices are martingales *after discounting by a factor that takes into account risk*. If one allows general time-varying discount factors, testing the EMH is problematic. Therefore, in its most general form, the theory of price processes as martingales (just like the general equilibrium theories) is empirically weak. In principle, it is satisfied by any price process that precludes arbitrage opportunities.<sup>8</sup>

We find here with the martingale principle the problems already encountered with respect to general equilibrium theories. In its most general form, the theory is almost impossible to refute.

If one makes the assumption that the discount factor is constant, the martingale approach is typically embodied in theories such as CAPM or arbitrage pricing theory (APT). According to the CAPM and the APT, returns are assumed to be multivariate stationary processes.<sup>9</sup>

The mean of the returns is related to the level of risk. Risk is the beta in CAPM or the betas with respect to the factors of APT. The presence of common factors implies that there are limitations to the benefits of diversification insofar as only idiosyncratic risk can be diversified. As there is risk due to common factors, portfolio risk cannot be arbitrarily reduced by diversification. This means, in turn, that there are long-range correlations driven by the common factors. Under this assumption, all portfolios are risky, and the difference between returns simply reflects the price of risk.

This is, in the essence, the classical view and the basic principle underlying the classical mean-variance optimization framework. In this simple form, the martingale hypothesis is not a satisfactory long-term model of returns. In fact, although return processes are mutually correlated so that risk does not disappear in aggregate, with the compounding of returns, different average returns translate into exponentially diverging prices. Therefore, there must be feedback from prices to returns.

The novelty in financial econometrics is that these feedbacks have been discovered *empirically*. In fact, over the past 15 years, a number of stylized empirical facts have been determined. These stylized facts show that there is feedback from prices to returns. While these facts do not violate the martingale theory in its most general form, they do violate the martingale approach in restricted forms.<sup>10</sup>

Among the stylized facts, perhaps the best known is volatility clustering. Periods of high volatility are followed by periods of low volatility. The volatility clustering is embodied in autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH) models and in stochastic volatility models.<sup>11</sup>

Volatility clustering disappears if different models of

volatility are used. For example, it does not occur if one considers realized volatility an observable quantity (see Andersen, et al. [2001]).<sup>12</sup> ARCH/GARCH models depend on the sampling frequency, i.e., they are not invariant under time aggregation. This means that the results of the model will change if, for example, we use daily data as opposed to weekly data or vice versa.

Other important facts related to volatility clustering have been discovered. If returns are related to risk, in the sense that higher returns are correlated with higher risk, it seems reasonable to argue that the autoregressive, forecastable nature of volatility implies the autoregressive nature of returns. The GARCH-M (GARCH in mean) model proposed by Engle, Lilien, and Robins [1987] effectively models volatility and expected returns simultaneously. In the GARCH-M model, both returns and volatility are driven by the same autoregressive process.

Empirical evidence for the GARCH-M model is not very strong. There is no conclusive *simple* evidence that positive shocks to volatility are correlated with positive shocks to returns. For example, French, Schwert, and Stambaugh [1987] report weak evidence of positive correlation between periods of high volatility and periods of high expected returns. According to some empirical studies, such as Glosten, Jagannathan, and Runkle [1993], the contrary is true. French, Schwert, and Stambaugh [1987] report that positive shocks to volatility are correlated with negative shocks to returns.

Several extensions of GARCH and stochastic volatility models have been proposed with an entirely time-dependent variance-covariance matrix. In the case of large portfolios of stocks, the specification of models is critical. In fact, estimates are difficult to obtain, given the large number of parameters needed if one wants to estimate the entire variance-covariance matrix. A direct GARCH or stochastic volatility approach is therefore not practical. Different approaches have been proposed, either as simplified extensions of basic GARCH models or through autoregressive factors that drive a stochastic volatility model. Both covariances and expected returns can be driven by common factors. Factors are in general hidden. This means that they are not observable quantities but rather the result of mathematical manipulations.

The situation regarding the forecastability of returns and their link to covariances is complex. Two facts are reasonably well established. First, Lo and MacKinlay [1988] show that the null hypothesis that stock prices follow a random walk can be rejected. Second, while the returns of individual stocks do not seem to be autocorrelated, portfolio returns are indeed significantly autocorrelated (Campbell, Lo, and MacKinlay [1996]).<sup>13</sup> Predictability is more evident at the portfolio level than at the level of the individual stock. The presence of significant cross-autocorrelations gives rise to this

effect. Strong lead-lag relationships between the prices of large firms and of small firms appear to be one of the dominant factors.

A different type of empirical evidence of cross-autocorrelations is evident in the profit allowed by trading strategies based on momentum and reversal effects. Jegadeesh and Titman [1993] have shown that momentum effects can be detected in large portfolios of U.S. equities; Rouwenhorst [1998] confirms this finding for European equities. Contrarian strategies based on reversals are also profitable. Although individual stocks do not exhibit significant autocorrelations, significant momentum and reversal effects are found in larger portfolios. Lo and MacKinlay [1990] and Lewellen [2002] demonstrate that momentum and reversal effects are not due to significant positive or negative autocorrelation of individual stocks but to cross-autocorrelation effects and other cross-sectional phenomena.

This literature also shows that momentum and reversal effects coexist at different time horizons. These findings suggest that equity price processes should be modeled with dynamic models that capture cross-autocorrelations, but with some sort of dimensionality reduction.<sup>14</sup>

The presence of portfolio forecastability, even if single assets are unforecastable, is one of the key features of cointegration. Two or more processes are said to be *cointegrated* if there are long-term stable regression relationships between them, even if the processes themselves are individually integrated (i.e., random shocks never decay). For example, two random walks are individually unforecastable but there might still be a stable long-term regression relationship between them. This means that there are linear combinations of the processes that are autocorrelated and thus forecastable.<sup>15</sup>

The presence of cointegrating relationships is associated with common trends or common factors. A set of cointegrated price processes can be decomposed as a set of regressions over a smaller number of common factors. Cointegration and the presence of common factors are equivalent concepts. This equivalence can be expressed more formally in terms of state-space models. A state-space model is a dynamic model that represents a set of processes as regressions over a set of possibly hidden factors or state-variables (see Focardi and Fabozzi [2004] for a review of these concepts).

Over the last 15 years, econometric analysis has shown that asset prices show some level of predictability. While these findings are not contrary to the most general formulation of the martingale theory, they are contrary to models such as the CAPM or the APT, which are based on constant trends. A certain level of asset predictability arises naturally in the context of systems populated by interacting agents who make imperfect forecasts. In the literature, such agents are said to be *boundedly rational agents*.

## DATA MINING: ARE WE GETTING THERE?

The scientific and quantitative approach to the financial markets has evolved tremendously. New models and techniques have been developed and applied to financial data. It should be no surprise that machine-learning methods that have been successful in applications such as fraud detection, credit scoring analysis, and customer behavior are now applied to “mining” the markets.<sup>16</sup>

Words of caution are in order. As data mining (and any other econometric technique) relies upon historical data, it can produce reasonable forecasts only if the future behaves similarly to the past. Sometimes forecasts are guesses at best.

Data mining and more recent machine-learning methodologies provide a range of general techniques for the classification, prediction, and optimization of structured and unstructured data. Neural networks, classification and decision trees, *k*-nearest neighbor methods, and support vector machines are some of the more common classification and prediction techniques used in machine learning today. For combinatorial optimization, genetic algorithms and reinforced learning are now fairly widespread.<sup>17</sup>

We discuss two applications for machine learning in finance. The first explores optimal decision trees to describe conditional relationships. The second applies support vector machines to analyze text.

Conditional relationships can efficiently be described in a hierarchical fashion like a decision tree. A decision tree is a simple set of if-then-else rules, making it intuitive and easy to analyze. More important relationships are considered first, and relationships of less significance are considered farther out in the branches of the tree. Decision trees address a drawback of the classical regression framework which estimates the linear trend or relationship in the underlying data. The linear framework inherent in classical regression makes it difficult to incorporate interactions or to describe conditional relationships among the variables.

Sorensen, Mezrich, and Miller [1998] apply a classification and regression technique (CART) to construct optimal decision trees to model the relative performance of the S&P 500 with respect to cash. The model assigns different probabilities to three market states: outperform, underperform, and neutral.

The CART framework allows for non-linear behavior and interactions as well as conditional relationships among the underlying variables. In this case, the explanatory variables used are capital market data such as steepness of the yield curve, credit spread, equity risk premium, and dividend yield premium (S&P 500 dividend yield minus the long bond yield).

The CART algorithm produces a hierarchy of dependent variables that results in the minimum amount of prediction errors. The final result is a decision tree of non-linear

if-then-else rules where each rule is conditioned on previous rules. The authors report that when the forecast probability for outperformance exceeds 70%, the model is correct almost 70% of the time.

Support vector machines (SVM) have been applied to the classification of “unstructured” data such as text (Joachims [1998]). While standard econometric techniques make use of only quantifiable information, financial markets are clearly affected by non-measurable information such as political, economic, and financial news. Exploiting textual information in addition to numeric time series data could improve the quality of forecasts.

A considerable amount of valuable information is available in electronic format, much of it through the Internet. Textual data provide qualifying and explanatory information beyond what can typically be found in standard numeric market data, adding another layer of information to be used in econometric forecasting. Specifically, text reports not only the effect (e.g., the S&P 500 rallied) but also the cause (e.g., the Fed reduced the short-term interest rate). While there have been some attempts to analyze textual information, especially in combination with the analysis of market data, this has not been extensively explored. Combining textual information with time series data in forecasting models remains a challenge.

Support vector machines are computational linguistic methods. An SVM can be trained to, for example, recognize messages that cause upward or downward movements in a stock's price. In this case, the SVM works by dividing news into buy and sell triggers. Schematically, the text in each message is split into “feature vectors” by the SVM, where each feature corresponds to a word in the text to be classified, together with an attribute that describes the importance of the word. The SVM algorithm then finds an optimal classification over the spanned feature space of words.

While initial studies show we cannot yet successfully predict stock returns by “mining” news (such as information from message boards and newspapers), studies have shown that Internet message boards provide relevant financial information that goes beyond what can be found in the financial and business press (see Wysocki [1999]).<sup>18</sup>

Message boards reflect the state of public information and adapt rapidly to change. They appear to have an effect on trading volumes and to be influenced by volatility (Antweiler and Frank [2004]). Despite its high noise level, the automatic analysis of events such as corporate announcements and economic or political news should be a fruitful avenue for future research.<sup>19</sup>

Despite the flexibility and generality of machine-learning techniques, there remain challenges ahead. Financial practitioners are accustomed to choosing models and model parameters using statistical procedures of inference, confidence intervals, and significance tests. These procedures are



often not straightforward for some of the popular machine-learning techniques. In particular, there are no standardized procedures for the selection of model-specific parameters such as the number of nodes in a neural network, the number of generations in a genetic algorithm, or the number of feature vectors in a support vector machine. In addition, the notion of degrees of freedom for machine-learning and other non-linear models is not well defined, making it hard to interpret and compare different models through standard measures of statistical fit.

From a financial practitioner's point of view, better statistical procedures for the specification and comparison of machine-learning techniques are essential. This is not to say that machine learning does not have a place in financial analysis. Its generality and broad applicability are some of its strengths, and machine-learning techniques can often be applied when traditional approaches are too limited or just not available.

## MODEL RISK, DATA SNOOPING, AND OVERFITTING

No discussion of empirical modeling is complete without a consideration of model risk, overfitting, and data snooping biases. A model as a simplified representation of reality can be either descriptive or predictive in nature—or both. Financial models are generally predictive, to forecast unknown or future values on the basis of current or known values using a specified equation or set of rules.

The predictive or forecasting power of a model is limited by the appropriateness of the inputs and assumptions. Therefore, the model will fail if we use it beyond its limitations. We need to challenge our models and identify sources of model risk to understand these limitations, to avoid incurring losses when we use them in the real world.

Most often, model risk in financial applications occurs as a result of incorrect assumptions, model identification or specification errors, and inappropriate estimation procedures—or in models used without satisfactory out-of-sample testing. The best way to manage model risk is to review models on a regular basis, thoroughly understand their weaknesses and limitations, and prohibit use beyond the purpose for which the models were originally designed.

Some financial models are very sensitive to small changes in inputs, resulting in big changes in outputs, such as the mean-variance portfolio optimization problem. Because this type of model instability might present itself only under special circumstances, such a problem is very difficult to detect and often only found the hard way, and sometimes only after big losses. To deal with these issues efficiently, future models will need to be designed with an inherent robustness to different sources of risk—a real challenge for researchers and practitioners alike.

A model is overfitted when it captures the underlying structure or the dynamics in the data as well as random noise. Use of too many model parameters that restrict the degrees of freedom relative to the size of the sample data is a common cause for overfitting. Often, this results in good in-sample fit but poor out-of-sample behavior. By systematically searching the parameter space, even an incorrect or a misspecified model can be made to fit the available data.

Obviously, this apparent fit is due to chance, and therefore does not have any descriptive or predictive power. In many cases, it is very hard to distinguish spurious phenomena that are a result of overfitting or data mining from a valid empirical finding and a truly good model.

The Super Bowl theory is probably one of the better-known and longer-lasting spurious market predictions. Its remarkable record and lack of any rational connection contribute to its popularity. According to the theory, the U.S. stock market will end the year up over the previous year if a National Football Conference team wins the Super Bowl. A less obvious but well-documented result of data mining is the calendar effect—systematic changes in the price of an asset on a specific day of the week or week of the year.

How can we avoid the dangers of model overfitting and data snooping? Clearly, the answer depends on the models or techniques being used. We present some general rules of thumb on how to account for these biases or circumvent them altogether.

A good starting point for limiting data snooping is to be prudent and disciplined in the modeling process. It is particularly important to decide upon the framework, defining how the model will be specified before beginning to analyze the actual data. Model hypotheses that make financial or economic sense should be formulated in the initial stage of model building rather than trying to find a good fit for the data and then creating an *ex post* explanation or story for the model.<sup>20</sup>

Even if one makes serious attempts to avoid storytelling, some forms of data mining often sneak into a model. One must pay careful attention to the number of dependent variables in a regression model, or the number of factors and components in a stochastic model, in order to avoid overfitting, particularly in a rich class of models with great flexibility. For example, when a genetic algorithm is used in “meta” searches over different models to find one that best describes the data, the result can be severe data snooping.

Forecasting models should not be calibrated or employed simply to find out what best fits the data or what has worked best in the past. There is no doubt that with today's computational power and enough data, we can find spurious relationships. The Super Bowl theory is a classic example.

It is a good idea to preserve some of the available data from the full sample for out-of sample tests, and not make use of it for model development purposes. True out-of-sam-

ple studies of model performance should be conducted after the model has been fully developed. Although good out-of-sample performance increases confidence in a model, historical backtests tend to overstate expected returns. One reason is that many historical backtests are not corrected for survivorship bias.<sup>21</sup> We are more comfortable with a model that also works cross-sectionally and produces similar results in different countries, but we always have to keep the disclaimer in mind when we evaluate any backtest: "Past performance is not necessarily an indication of future performance."

In some special cases, the impact of systematic parameter searches can be computed explicitly by correcting the statistical distribution used for inference (Lo and MacKinlay [1990]). In more general settings, sampling, bootstrapping, and randomization techniques can be used to evaluate whether a given model has predictive power over a benchmark model after taking into consideration the effects of data snooping (see White [2000]).<sup>22</sup>

Finally, all forecasting models should be monitored and compared on a regular basis. Deteriorating results from a model or variable should be investigated and understood.

Well-designed and thoroughly tested models should not have to be changed often, but structural shifts and major alterations in underlying market conditions due to changes in exogenous factors such as economic policy, political climate, and shocks caused by unforeseeable events sometimes trigger model revisions or recalibrations.

## IMPLICATIONS FOR PORTFOLIO MANAGEMENT

Just as automation and mechanization were the cornerstones of the Industrial Revolution at the turn of the 19th century, modern finance theory, quantitative models, and econometric techniques provide the foundation that has revolutionized the investment management industry over the past 15 years. Future success for participants in the industry will depend on their ability not only to provide excess returns in a risk-controlled fashion to investors, but also to incorporate financial innovation and process automation into their frameworks.

Quantitative models and techniques are going to play an increasingly important role in the industry. They will affect all steps in the investment management process, specifically:

- Defining the policy statement.
- Setting the investment objectives.
- Selecting investment strategies.
- Implementing the investment plan and constructing the portfolio.
- Monitoring, measuring, and evaluating investment performance.

Today, quantitative techniques are used mainly in the last two steps, although a growing number of practitioners are beginning to embrace quantitative methods throughout the entire investment management process.

Perhaps one of the most significant benefits, just as during the Industrial Revolution, is the power of automation. Automation enforces a systematic investment approach and a structured and unified framework. Completely automated risk models and marking-to-market processes provide not only a powerful tool for analyzing and tracking portfolio performance in real-time, but also a foundation for complete process and system backtests. It is important to be able to evaluate how changes in the chain of decisions influence the whole process to allow us to more fully understand, compare, and calibrate investment strategies, underlying investment objectives and policies.

It is telling that many portfolio managers consider portfolio implementation and construction with mean-variance optimization difficult to use and not worth maintaining. When we eliminate many of the problems associated with classical portfolio optimization, advances such as Bayesian techniques and robust optimization provide significant improvement over the classical approaches. Modern econometric techniques such as cointegration will play a major role, but we also expect ultimately to see an increased use of modeling techniques such as data mining and machine learning.

The quantitative approach is not without its risks. It introduces a new source of risk—model risk—and an inescapable dependence on historical data as its raw material. We must be cautious in how we use our models, make sure that we understand their weaknesses and limitations, and prevent applications beyond what models were originally designed for. With more parameters in the models and more sophisticated econometric techniques, we run the risk of overfitting our models. Distinguishing spurious phenomena as a result of overfitting or data mining can be a difficult task. Future models must be designed with an inherent robustness to different sources of risk.

As an industry, we are not yet there. As we address some of the challenges that need to be met to get there, it should go without saying that, to date, the development of a quantitative investment management framework is both time-consuming and resource-intensive. At the larger firms, quantitative investment management is a niche that offers a competitive advantage. It is a source of new products and can provide increased economies of scale to the asset management business as a whole. The future has yet to show its full impact.

## ENDNOTES

<sup>1</sup>Stochastic processes are random sequences of events that are governed by the laws of probability. In finance, sequences of events evolve in time so these processes are sometimes referred to as time series. Examples of stochastic processes in finance include stock returns, interest rates, and exchange rates.

<sup>2</sup>The idea that economics is a model without any realistic interpretation is a key tenet of positive economics as put forward by Milton Friedman.

<sup>3</sup>This situation can be likened to superstring theories in physics today.

<sup>4</sup>For recent empirical evidence on the distribution of asset returns and portfolio selection when distributions are non-normal, see Rachev and Mitnik [2000] and Rachev [2001].

<sup>5</sup>A recent discussion of covariance matrix discounting is provided in Aguilar and West [1998].

<sup>6</sup>A non-linear model exhibits chaotic behavior if very small perturbations lead to unbounded changes of the paths.

<sup>7</sup>In physics, this statement is a matter of principle. For example, quantum mechanics could not even be expressed without classical mechanics.

<sup>8</sup>This is not strictly true. There are technical conditions that must be satisfied in order to establish the equivalence of the martingale principle and of absence of arbitrage.

<sup>9</sup>The stationarity of a process or of a time series means that the joint probability distribution for the variables does not change over time.

<sup>10</sup>*Stylized facts* are statistical findings related to a particular model. Stylized facts are model-dependent; different models produce different stylized facts.

<sup>11</sup>An autoregressive process is a statistical process that relates the behavior of a time series variable to past values (i.e., lagged values) of that variable plus an unpredictable shock. In econometrics, heteroscedasticity indicates that the variance of the error term of the regression is not constant but is conditional on the regressor.

<sup>12</sup>One can, however, recover the usual ARCH/GARCH behavior from realized volatility models.

<sup>13</sup>Autocorrelation is the correlation between a time series variable and a lagged value of that same variable.

<sup>14</sup>Dimensionality reduction is a technique used to identify the most relevant parameters; parameters of a secondary order are regarded as noise. Dimensionality reduction techniques replace the original model with a simplified model that retains only the most relevant parameters.

<sup>15</sup>Cointegration is not limited to simple long-run regression, but also includes long-run stationary dynamic processes. In the latter case, cointegration is called dynamic cointegration.

<sup>16</sup>Machine learning is an area of artificial intelligence that develops computer algorithms that “learn” by analyzing data, rather than relying upon human insight. For a review of machine-learning techniques see Hastie, Tibshirani, and Friedman [2001].

<sup>17</sup>Structured data are data with a regular repeated structure that can be easily stored in a database table, while unstructured data are everything else that has no regular structure such as text documents, e-mails, web pages, or graphics. Neural networks are inspired by the way the human nervous system and brain process information, using a large number of highly interconnected pro-

cessing elements (neurons) working together. Neural networks, just like people, learn by example (training). Support vector machines are non-linear learning methods that can perform classification and regression tasks. A combinatorial optimization problem is an optimization problem in which the space of possible solutions is discrete instead of continuous. Genetic algorithms are evolutionary algorithms that generate new individuals (possible solutions) by combining or mutating smaller building blocks (“chromosomes”) according to a predefined set of rules.

<sup>18</sup>Cutler, Poterba, and Summers [1989] show that less than half of the variation in aggregate stock prices is explained by macroeconomic news, and that market moves coincident with major world events are often small. Because they find that large market moves occur on days without identifiable major information releases, they argue that moves in stock prices must reflect something other than news about fundamental values.

<sup>19</sup>There are many applications for text mining in finance, such as the analysis and classification of news, research reports, annual reports, and other textual information. Techniques vary, from linguistics to data mining. See Focardi and Jonas [2002].

<sup>20</sup>When Peter Bernstein interviewed Fischer Black for *Capital Ideas*, he learned that Black always expressed a new idea in words first, before he started writing down an equation. If he could not put it in words, he discarded the equation.

<sup>21</sup>Survivorship bias is a bias that appears in a sample of price processes chosen at the end of a period. For example, the present composition of any major index such as the S&P 500 includes companies that are viable today but were not viable in the past. This knowledge could be used to reveal spurious mean-reversion effects.

<sup>22</sup>Sampling methods define the strategy to be followed in sampling a data set. Bootstrap methods create additional samples from a given sample through computer simulation.

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