Homework 9

1. (Textbook Section 7.5-2, Page 276) Suppose that a random sample of eight observations is taken from a normal distribution for which both the mean μ and the variance σ^2 are unknown; and that the observed values are 3.1, 3.5 ,2.6, 3.4, 3.8, 3.0, 2.9 and 2.2. Find the confidence interval for μ with each of the following three confidence coefficients: (a)0.90, (b) 0.95, and(c) 0.99.

2. (Textbook Section 7.5-4, Page 276) Suppose that $X_1, ..., X_n$form a random sample from a normal distribution for which the mean μ is known and the variance σ^2 is known. How large a random variable sample must be taken in order that there will be a confidence interval for μ with confidence coefficient 0.95 and length less than 0.1σ?

3. (Textbook Section 8.1-1, Page 299) Let X have an exponential distribution with parameter β. Suppose that we wish to test the hypotheses

$$H_0 : \beta \geq 1,$$

$$H_1 : \beta < 1.$$

   Consider the test procedure δ that rejects H_0 if $X \geq 1$.
   (a) Determine the power function of the test.
   (b) Compute the size of the test.

4. (Textbook Section 8.1-2, Page 299) Suppose that $X_1, ..., X_n$ form a random sample from a uniform distribution on the interval $[0, \theta]$, and that the following hypotheses are to be tested:

$$H_0 : \theta \geq 2,$$

$$H_1 : \theta < 2.$$

   Let $Y_n = max(X_1, ..., X_n)$, and consider a test procedure such that the critical region contains all the outcomes for with $Y_n \leq 1.5$.
   (a) Determine the power function of the test.
   (b) Determine the size of the test.

5. (Textbook Section 8.1-3, Page 299) Suppose that the proportion p of defective items in a large population of items is unknown, and that it is desired to test the following hypotheses:

$$H_0 : p = 0.2,$$

$$H_1 : p \neq 0.2.$$

Suppose also that a random sample of 20 items is drawn from the population. Let Y denote the number of defective items in the sample, and consider a test procedure δ such that the critical region contains all the outcomes for which either $Y \geq 7$ or $Y \leq 1$.

(a) Determine the value of the power function $\pi(p|\delta)$ at the points $p = 0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1; and sketch the power function.

(b) Determine the size of the test.