Problem Set 2

1. Each time a shopper purchases a tube of toothpaste, he choose either brand A or brand B. Suppose that for each purchase after the first, the probability is $\frac{1}{3}$ that he will choose the same brand that he chose on his preceding purchase and the probability is $\frac{2}{3}$ that he will switch brands. If he is equally likely to choose either brand A or brand B on his first purchase, what is the probability that both his first and second purchases will be brand A and both his third and fourth purchases will be brand B?

2. In the World Series of baseball, two teams A and B play a sequence of games against each other, and the first team that wins a total of four games becomes the winner of the World series. If the probability that team A will win any particular game against team B is 1/3, what is the probability that team A will win the World series?

3. Suppose that $A_1, A_2, \ldots, A_k$ are a sequence of k independent events. Let $B_1, B_2, \ldots, B_k$ be another sequence of k events such that for each value of $j (j = 1, 2, \ldots, k)$, either $B_j = A_j$ or $B_j = A_j^C$. Prove that $B_1, B_2, \ldots, B_k$ are also independent events. Hint: Use an induction argument based on the number of events B_j for which $B_j = A_j^C$.

4. Suppose that when a machine is adjusted properly, 50 percent of the items produced by it are of high quality and the other 50 percent are of medium quality. Suppose, however, that the machine is improperly adjusted during 10 percent of the time and that, under these conditions 25 percent of the items produced by it are of high quality and 75 percent are of medium quality.

1) suppose that five items produced by the machine at a certain time are selected at random and inspected. If four of these items are of high quality and one item is of medium quality. What is the probability that the machine was adjusted properly at that time?

2) Suppose that one additional item, which was produced by the machine at the same time as the other five items, is selected and found to be of medium quality. What is the new posterior probability that the machine was adjusted properly?