Problem Set 4
(Due Date: Dec. 12)

1. Two players play the Rock-paper-scissors (剪刀石头布) game. Assume that the winning utility is 1, the even utility is 0, and the losing utility is -1.
   a) Draw the payoff matrix for this game.
   b) Find the Nash equilibrium (pure strategy or mixed strategy).
   c) Calculate the expected payoff of both players.

2. Two firms (A and B) are considering bringing out competing brands of a healthy cigarette. Payoffs to the companies when they compete simultaneously are shown in the table:

		B	
		Produce	Not Produce
A	Produce	3,3	5,4
	Not Produce	4,5	2,2

   a) Identify the Nash equilibrium of this game.
   b) Assume that A is the first mover in the market so the game becomes a sequential game. Please describe the game both in terms of the normal form (the usual payoff matrix) in extensive form.
   c) Identify both Nash equilibrium and subgame perfect equilibrium (using backward induction) in this sequential game.
   d) Based on the results you get in c), discuss the relation between Nash equilibrium and subgame perfect equilibrium.

3. A monopolist can produce at a constant average and marginal cost $AC = MC = \$5$. It faces a market demand curve given by $Q = 50 - P$.
   a) Calculate the profit-maximizing price and quantity for this monopolist. Also calculate his profit.
   b) Suppose a second firm with the same production technology enters the market and they two determine the output level simultaneously. What is the equilibrium price and quantity of each firm? Also calculate the profit for each firm.
   c) Suppose there are N firms with the identical technology above. Calculate the equilibrium price, quantity and profit for each firm.
   d) Return to the two firms case. Now suppose the first firm is the Stackelberg leader. Find the equilibrium quantity, price and profit for each firm.
   e) Suppose there are three firms. Firm 1 is the Stackelberg leader who decides quantity first, firm 2 and firm 3 decides quantity simultaneously after the decision of firm 1. Find out the equilibrium quantity, price and profit for each firm.

4. Suppose firms A and B each operate under conditions of constant average and marginal cost, but that $MC_A = 10$, $MC_B = 8$. The demand for the firms' output is given by

$$Q_D = 500 - 20P$$

a) If the firms practice Bertrand competition, what will be the market price under a Nash Equilibrium?
b) What will the profits be for each firm?
c) Will this equilibrium be Pareto efficient?
d) What if $MC_A = 15$?

5. Two firms compete by choosing price. Since their products are slightly differentiated, their demand functions are

$$Q_1 = 20 - P_1 + P_2; \quad Q_2 = 20 - P_2 + P_1$$

where P$_1$ and P$_2$ are the prices charged by each firm, respectively, and Q$_1$ and Q$_2$ are the resulting demands. Marginal costs are zero.

1) Suppose the two firms set their prices at the *same time*. Find the resulting Nash equilibrium prices.
2) Suppose Firm 1 sets its price *first*, and then Firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be? Does the first mover have a first-mover advantage?

6. 有两个企业生产相同的产品,在一个市场上竞争。企业 1 的成本函数为 $C(x_1) = x_1^2$,企业 2 的成本函数为 $C(x_2) = 8x_2$。市场需求曲线为 $P(x) = 90 - x_1 - x_2$,两个企业的产量之和必须低于 90,否则价格为零。

1) 假定两个企业选择产量进行竞争,求出古尔诺均衡的产量、价格和利润。
2) 如果两个企业合并,合并之后利润最大化的产量、价格和利润为多少?
3) 两个企业合并之后,两个企业变成了两个独立的生产车间,但企业试图在两个车间进行产量的最优配置,使得成本最小化。请求出合并企业层面的成本函数 C(x)。