隐函数的微分法

定义

如果在某平面区域内 F(x,y) = 0 可以确定 y 为 x 的函数

(或者 x 为 y 的函数),则称 F(x,y)=0 在该区域内为**隐函数**.

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如果在某空间区域内 F(x,y,z)=0 可以确定 z 为 x,y 的函数

(或者 x 为 y, z 的函数或 y 为 x, z 的函数),

则称 F(x,y,z)=0 在该区域内为**隐函数**.

则 dF(x,y)=0,

则 d
$$F(x,y)=0$$
,

$$\frac{\partial F}{\partial x} \, \mathrm{d}x + \frac{\partial F}{\partial y} \, \mathrm{d}y = 0$$

则
$$\mathrm{d}F(x,y)=0,$$

$$\frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy = 0 \Rightarrow \frac{dy}{dx} = -\frac{\partial F}{\partial x} / \frac{\partial F}{\partial y}$$

设 F(x,y) = 0 在某平面区域确定 x 为 y 的函数,

 $\mathbb{N} \frac{\mathrm{d}x}{\mathrm{d}y} = -\frac{\partial F}{\partial y} / \frac{\partial F}{\partial x}$

$$\frac{\partial F}{\partial x}$$

则
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$$dx = -\frac{\partial F}{\partial x} / \frac{\partial F}{\partial x} dx - \frac{\partial F}{\partial x} / \frac{\partial F}{\partial y} dy$$

$$\mathrm{d}z = -\frac{\partial F}{\partial x} / \frac{\partial F}{\partial z} \, \mathrm{d}x - \frac{\partial F}{\partial y} / \frac{\partial F}{\partial z} \, \mathrm{d}y,$$

則
$$F(x,y,z) = 0$$
,

$$\frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial z} dz = 0,$$

$$\mathrm{d}z = -\frac{\partial F}{\partial x} / \frac{\partial F}{\partial z} \, \mathrm{d}x - \frac{\partial F}{\partial y} / \frac{\partial F}{\partial z} \, \mathrm{d}y,$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{\partial F}{\partial x} / \frac{\partial F}{\partial z}, \quad \frac{\partial z}{\partial u} = \frac{\partial F}{\partial u} / \frac{\partial F}{\partial z}.$$

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$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{\partial F}{\partial x} / \frac{\partial F}{\partial z}, \quad \frac{\partial z}{\partial y} = \frac{\partial F}{\partial y} / \frac{\partial F}{\partial z}.$$

同理可计算
$$\frac{\partial x}{\partial y}$$
, $\frac{\partial x}{\partial z}$ 和 $\frac{\partial y}{\partial x}$, $\frac{\partial y}{\partial z}$

例

设 $e^{-xy}-2z-e^z=0$ 确定 z 为 x,y 的函数,求 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$.

设 $e^{-xy} - 2z - e^z = 0$ 确定 z 为 x, y 的函数,求 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$.

解: $-e^{-xy} d(xy) - 2 dz + e^z dz = 0,$ 设 $e^{-xy} - 2z - e^z = 0$ 确定 z 为 x, y 的函数,求 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. 解:

$$-e^{-xy} d(xy) - 2 dz + e^z dz = 0,$$

$$dz = \frac{e^{-xy}(x dy + y dx)}{e^z - 2}$$

设
$$e^{-xy} - 2z - e^z = 0$$
 确定 z 为 x, y 的函数,求 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. 解:

用年

$$-e^{-xy} d(xy) - 2 dz + e^z dz = 0,$$

$$dz = \frac{e^{-xy}(x dy + y dx)}{e^z - 2} = \frac{xe^{-xy}}{e^z - 2} dy + \frac{ye^{-xy}}{e^z - 2} dx$$

例

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$$e^{-xy} - 2z - e^z = 0$$
 确定 z 为 x, y 的函数,求 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. 解:

$$-e^{-xy} d(xy) - 2 dz + e^z dz = 0,$$

$$dz = \frac{e^{-xy}(x dy + y dx)}{e^z - 2} = \frac{xe^{-xy}}{e^z - 2} dy + \frac{ye^{-xy}}{e^z - 2} dx$$

$$\therefore \frac{\partial z}{\partial x} = \frac{ye^{-xy}}{e^z - 2}, \quad \frac{\partial z}{\partial y} = \frac{xe^{-xy}}{e^z - 2}.$$

(不用死记隐函数的导数公式!) 只要求全微分

例 设 F(x,y,z)=0 中,任一变量可以看作是另两个的函数,则有 $\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}=-1$ $\mathrm{d}F(x,y,z)=0, \Rightarrow F_x'\,\mathrm{d}x+F_y'\,\mathrm{d}y+F_z'\,\mathrm{d}z=0,$

$$dF(x,y,z) = 0, \Rightarrow F'_x dx + F'_y dy + F'_z dz = 0$$

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$$F_x' \qquad F'$$

$$dx = -\frac{F'_y}{F'_x} dy - \frac{F'_z}{F'_x} dz$$

$$dy = -\frac{F'_x}{F'_y} dx - \frac{F'_z}{F'_y} dz$$

$$dz = -\frac{F'_x}{F'_z} dx - \frac{F'_y}{F'_z} dy$$

$$= -\frac{F_x'}{F_y'} dx - \frac{F_z'}{F_y'} dz$$
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$$dx = -\frac{F'_y}{F'_x} dy - \frac{F'_z}{F'_z} dz \qquad \frac{\partial x}{\partial y} = -\frac{F'_y}{F'_x}, \quad \frac{\partial x}{\partial z} = -\frac{F'_z}{F'_x}$$

$$dy = -\frac{F'_x}{F'_y} dx - \frac{F'_z}{F'_y} dz \qquad \Rightarrow \frac{\partial y}{\partial x} = -\frac{F'_x}{F'_y}, \quad \frac{\partial y}{\partial z} = -\frac{F'_z}{F'_y}$$

$$dz = -\frac{F'_x}{F'_z} dx - \frac{F'_y}{F'_z} dy \qquad \frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z}, \quad \frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z}$$

$$F_y'$$
 , F_z' , ∂x , F_y' , ∂x

$$dF(x, y, z) = 0, \Rightarrow F'_x dx + F'_y dy + F'_z dz = 0,$$

$$\mathrm{d}F(x,y,z)=0, \Rightarrow F_x'\,\mathrm{d}x+F_y'\,\mathrm{d}y+F_z'\,\mathrm{d}z=0,$$
 $\mathrm{d}x=-rac{F_y'}{F_z'}\,\mathrm{d}y-rac{F_z'}{F_z'}\,\mathrm{d}z \qquad rac{\partial x}{\partial z}=-rac{F_y'}{F_z'}, \ rac{\partial x}{\partial z}=-rac{F_y'}{F_z'}$

$$= -\frac{F'_y}{F'_x} dy - \frac{F'_z}{F'_x} dz \qquad \frac{\partial x}{\partial y} = -\frac{F'_y}{F'_x}, \quad \frac{\partial x}{\partial z} = -\frac{F'_z}{F'_x}$$

$$-\frac{F'_{y}}{F'_{z}} dy - \frac{F'_{z}}{F'_{z}} dz \qquad \frac{\partial x}{\partial y} = -\frac{F'_{y}}{F'_{z}}, \quad \frac{\partial x}{\partial z} = -\frac{F'_{z}}{F'_{z}} -\frac{F'_{z}}{F'_{z}} dz \qquad \Rightarrow \frac{\partial y}{\partial z} = -\frac{F'_{z}}{F'_{z}}, \quad \frac{\partial y}{\partial z} = -\frac{F'_{z}}{F'_{z}}$$

 $dx = -\frac{F'_y}{F'_x} dy - \frac{F'_z}{F'_x} dz \qquad \frac{\partial x}{\partial y} = -\frac{F'_y}{F'_x}, \quad \frac{\partial x}{\partial z} = -\frac{F'_z}{F'_x}$ $dy = -\frac{F'_x}{F'_y} dx - \frac{F'_z}{F'_y} dz \qquad \Rightarrow \frac{\partial y}{\partial x} = -\frac{F'_x}{F'_y}, \quad \frac{\partial y}{\partial z} = -\frac{F'_z}{F'_z}$ $dz = -\frac{F'_x}{F'_z} dx - \frac{F'_y}{F'_z} dy \qquad \frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z}, \quad \frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z}$

$$z = -\frac{x}{F_z'} dx - \frac{s}{F_z'} dy$$
 $\frac{\partial x}{\partial x} = -\frac{1}{F_z'}$

$$\frac{\partial z}{\partial y} \frac{\partial x}{\partial y} \frac{\partial y}{\partial y} = -1$$

$$= -1$$

例 设
$$F(x,y,z)=0$$
 中,任一变量可以看作是另两个的函数,则有 $\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}=-1$

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$$F(x,y,z)=0$$
 中,任一变量可以看作是另两个的函数,则有 $\frac{z}{\partial x}$ $\frac{z}{\partial y}$ $\frac{z}{\partial z}=-1$
 另法:
$$F(x,y,z)=0 \Rightarrow F'_x+F'_z\frac{\partial z}{\partial x}=0 \Rightarrow \frac{\partial z}{\partial x}=-\frac{F'_x}{F'_z}$$

$$F'_y+F'_z\frac{\partial z}{\partial y}=0 \Rightarrow \frac{\partial z}{\partial y}=-\frac{F'_y}{F'_z}$$

$$F(x,y,z)=0$$

$$F'_x + F'_z \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z}$$

 $F'_{x} + F'_{y} \frac{\partial y}{\partial x} = 0 \Rightarrow \frac{\partial y}{\partial x} = -\frac{F'_{x}}{F'_{y}}$ $F'_{z} + F'_{y} \frac{\partial y}{\partial z} = 0 \Rightarrow \frac{\partial y}{\partial z} = -\frac{F'_{z}}{F'_{y}}$

另 设
$$F(x,y,z)=0$$
 中,任一支重可以有行定另例 计例函数,则有 $\frac{\partial z}{\partial x}$ $\frac{\partial z}{\partial y}$ $\frac{\partial z}{\partial z}=-1$
 $F(x,y,z)=0$ \Rightarrow $F'_x+F'_z\frac{\partial z}{\partial x}=0$ \Rightarrow $\frac{\partial z}{\partial x}=-\frac{F'_x}{F'_z}$ $F'_y+F'_z\frac{\partial z}{\partial y}=0$ \Rightarrow $\frac{\partial z}{\partial y}=-\frac{F''_y}{F'_z}$

另法:
$$F(x,y,z) = 0 \Rightarrow F_x' + F_z' \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{F_x'}{F_z'}$$
$$F_y' + F_z' \frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{F_y'}{F_z'}$$

$$\exists z : F'_x + F'_z \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z}$$

 $F'_{x} + F'_{y} \frac{\partial y}{\partial x} = 0 \Rightarrow \frac{\partial y}{\partial x} = -\frac{F'_{x}}{F'_{y}}$ $F'_{z} + F'_{y} \frac{\partial y}{\partial z} = 0 \Rightarrow \frac{\partial y}{\partial z} = -\frac{F'_{z}}{F'_{z}}$

 $\Rightarrow F_x' \frac{\partial x}{\partial y} + F_y' = 0 \Rightarrow \frac{\partial x}{\partial y} = -\frac{F_y'}{F_x'}$ $F_x' \frac{\partial x}{\partial z} + F_z' = 0 \Rightarrow \frac{\partial x}{\partial z} = -\frac{F_z'}{F_z'}$

$$F_x' + F_z' rac{\partial z}{\partial x} = 0 \Rightarrow rac{\partial z}{\partial x} = -rac{F_x'}{F_z'}$$

$$F_x' + F_z' rac{\partial z}{\partial x} = 0 \Rightarrow rac{\partial z}{\partial x} = -rac{F_x'}{F_z'}$$

推广: F(x,y,z,t) = 0, 若任一变量可以看作是另三个的函数,

则
$$\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial t} \frac{\partial t}{\partial x} = 1.$$

例 设 F(x-y,y-z)=0 决定 z 是 x,y 的函数,求 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$.

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解:

$$\underline{F_1'}\operatorname{d}(x-y) + \underline{F_2'}\operatorname{d}(y-z) = 0,$$

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解:

$$F_1' d(x-y) + F_2' d(y-z) = 0,$$

$$F_1' dx - F_1' dy + F_2' dy - F_2' dz = 0,$$

例 设 F(x-y,y-z)=0 决定 z 是 x,y 的函数,求 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$.

解

$$F_1' d(x - y) + F_2' d(y - z) = 0,$$

$$F_1' dx - F_1' dy + F_2' dy - F_2' dz = 0,$$

$$dz = \frac{F_1'}{F_2'} dx + \frac{F_2' - F_1'}{F_2'} dy,$$

 $^{ ext{M}}$ 设 F(x-y,y-z)=0 决定 z 是 x,y 的函数,求 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$.

解

$$F'_1 d(x-y) + F'_2 d(y-z) = 0,$$

$$F_1' dx - F_1' dy + F_2' dy - F_2' dz = 0,$$

$$\mathrm{d}z = \frac{F_1'}{F_2'}\,\mathrm{d}x + \frac{F_2' - F_1'}{F_2'}\,\mathrm{d}y,$$

$$\therefore \frac{\partial z}{\partial x} = \frac{F_1'}{F_2'}, \quad \frac{\partial z}{\partial y} = \frac{F_2' - F_1'}{F_2'}.$$

(三)
$$\begin{cases} F(x,y,z) = 0 \\ G(x,y,z) = 0 \end{cases}$$
 决定
$$\begin{cases} y = y(x) \\ z = z(x) \end{cases}$$
 (两面定一线),空间曲线方程 求 $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}z}{\mathrm{d}x}$.

$$z = z(x)$$

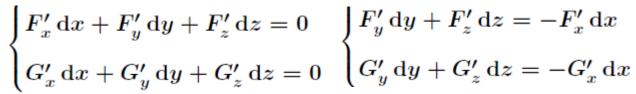
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$$\frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}z}{\mathrm{d}x}.$$

$$_{x}^{\prime}\,\mathrm{d}x$$
 -

$$\begin{cases} F'_x dx + F'_y dy + F'_z dz = 0 \\ G'_x dx + G'_y dy + G'_z dz = 0 \end{cases}$$

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$$F_x'\,\mathrm{d}x$$

$$F_x^\prime$$
 ϵ

(三)
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(两面定一线),空间曲线方程 求
$$\frac{dy}{dx}$$
, $\frac{dz}{dx}$.

$$\int_{x}^{x}\mathrm{d}x+F_{y}$$

$$-F'_x dx \quad F'_x dx$$

$$=\frac{\begin{vmatrix} -F_x \cdot \mathrm{d}x & F_z \\ -G_x' \cdot \mathrm{d}x & G_z' \end{vmatrix}}{\begin{vmatrix} F_z' & F_z' \end{vmatrix}} =$$

$$= \frac{ \left| \begin{array}{c|c} -G'_x \, \mathrm{d}x & G'_z \end{array} \right|}{ \left| \begin{array}{c|c} F' & F' \end{array} \right|}$$

 $\begin{cases} F'_x dx + F'_y dy + F'_z dz = 0 \\ G'_x dx + G'_y dy + G'_z dz = 0 \end{cases} \begin{cases} F'_y dy + F'_z dz = -F'_x dx \\ G'_y dy + G'_z dz = -G'_x dx \end{cases}$

$$\Rightarrow dy = \frac{ \begin{vmatrix} -F'_x dx & F'_z \\ -G'_x dx & G'_z \end{vmatrix}}{ \begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \end{vmatrix}} = - \frac{ \begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \end{vmatrix}}{ \begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx,$$

$$rac{\left. egin{array}{c|c} G_z' & \left| & - \left| egin{array}{c|c} G_x' & G_z' & \left| & \ F_y' & F_z' &
ight| \end{array}
ight. \end{array}
ight] \mathrm{d}x,$$

$$\mathbf{g}_{z}^{\prime}\,\mathrm{d}z=$$

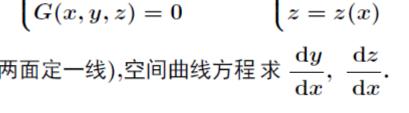
$$\mathrm{d}x,$$

$$r$$
 .

$$_{z}$$
 $\mathrm{d}z=0$

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$$\begin{cases} y = y(x) \\ z = z(x) \end{cases}$$

$$G(x,y,z)=0$$
 $z=z(x)$ (两面定一线).空间曲线方程 求 $\frac{\mathrm{d}y}{z}$.



(两面定一线),空间曲线方程 求 $\frac{dy}{dx}$, $\frac{dz}{dx}$.

$$egin{cases} F_x'\,\mathrm{d}x + F_y'\,\mathrm{d}y + F_z'\,\mathrm{d}z &= 0 \ G_x'\,\mathrm{d}x + G_y'\,\mathrm{d}y + G_z'\,\mathrm{d}z &= 0 \end{cases} egin{cases} F_y'\,\mathrm{d}y + F_z'\,\mathrm{d}z &= -F_x'\,\mathrm{d}x \ G_y'\,\mathrm{d}y + G_z'\,\mathrm{d}z &= -G_x'\,\mathrm{d}x \end{cases} \ egin{cases} -F_x'\,\mathrm{d}x + F_z'\,\mathrm{d}z &= -F_x'\,\mathrm{d}z \end{cases} \ egin{cases} F_x'\,\mathrm{d}x + F_z'\,\mathrm{d}z &= -F_x'\,\mathrm{d}z \end{cases} \ egin{cases} F_x'\,\mathrm{d}x + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z &= -F_x'\,\mathrm{d}z \end{cases} \ egin{cases} F_x'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z \end{cases} \ egin{cases} F_x'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}z \end{cases} \ egin{cases} F_x'\,\mathrm{d}z + F_z'\,\mathrm{d}z + F_z'\,\mathrm{d}$$

- $\Rightarrow dy = \frac{\begin{vmatrix} -F'_x dx & F'_z \\ -G'_x dx & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} = -\frac{\begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx, \quad dz = -\frac{\begin{vmatrix} F'_y & F'_x \\ G'_y & G'_x \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx$

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$$\begin{cases} F(x,y,z) = 0 \\ G(x,y,z) = 0 \end{cases}$$
 决定
$$\begin{cases} y = y(x) \\ z = z(x) \end{cases}$$
 引入记号:
$$\frac{D(F,G)}{D(x,z)} = \begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \end{vmatrix}, \text{则} \quad \text{Jacobi 行列式}$$

$$= y(x)$$

$$= z(x)$$

$$z'$$

 $\Rightarrow dy = \frac{\begin{vmatrix} -F'_x dx & F'_z \\ -G'_x dx & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} = -\frac{\begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx, \quad dz = -\frac{\begin{vmatrix} F'_y & F'_x \\ G'_y & G'_x \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx$

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,则 $egin{bmatrix} egin{bmatrix} egin{$

$$\frac{dy}{dx} = -\frac{D(F,G)}{D(x,z)} / \frac{D(F,G)}{D(y,z)}, \quad \frac{dz}{dx} = -\frac{D(F,G)}{D(y,x)} / \frac{D(F,G)}{D(y,z)}.$$

$$\Rightarrow dy = \frac{\begin{vmatrix} -F'_x dx & F'_z \\ -G'_x dx & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} = -\frac{\begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx, \quad dz = -\frac{\begin{vmatrix} F'_y & F'_x \\ G'_y & G'_x \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx$$

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$$\frac{D(F,G)}{D(x,z)} = \begin{vmatrix} F'_x & F'_z \\ G' & G' \end{vmatrix}, 则 \quad \text{Jacobi 行列式}$$

$$\frac{dy}{dx} = -\frac{D(F,G)}{D(x,z)} / \frac{D(F,G)}{D(y,z)}, \quad \frac{dz}{dx} = -\frac{D(F,G)}{D(y,x)} / \frac{D(F,G)}{D(y,z)}.$$

$$\Rightarrow dy = \frac{\begin{vmatrix} -F'_x dx & F'_z \\ -G'_x dx & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} = -\frac{\begin{vmatrix} F'_x & F'_z \\ G'_x & G'_z \\ G'_y & G'_z \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx, \quad dz = -\frac{\begin{vmatrix} F'_y & F'_x \\ G'_y & G'_x \end{vmatrix}}{\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}} dx$$

根据实际情况可以灵活应用,不必记公式!

例
$$\left\{ \begin{array}{l} x^2+y^2+z^2=a^2 \\ x+y+z=0 \end{array} \right. \left(- \uparrow \right) , \\ \bar{x} \, \frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}z}{\mathrm{d}x}. \end{array} \right.$$

例 $\begin{cases} x^2 + y^2 + z^2 = a^2 \\ x + y + z = 0 \end{cases} (- \uparrow 大 \otimes),$ $\frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}z}{\mathrm{d}x}.$ $\begin{cases} x \, \mathrm{d}x + y \, \mathrm{d}y + z \, \mathrm{d}z = 0 \\ \mathrm{d}x + \mathrm{d}y + \mathrm{d}z = 0 \end{cases}$

例
$$\begin{cases} x^2 + y^2 + z^2 = a^2 \\ x + y + z = 0 \end{cases} (- \uparrow 大 \otimes), \stackrel{d}{x} \frac{dy}{dx}, \frac{dz}{dx}.$$

$$\begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ dx + dy + dz = 0 \end{cases} \Rightarrow \begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ z \, dx + z \, dy + z \, dz = 0 \end{cases}$$

$$\begin{cases} x \, \mathrm{d}x + \\ \mathrm{d}x + \end{cases}$$

 $\begin{cases} x^2 + y^2 + z^2 = a^2 \\ x + y + z = 0 \end{cases} (- \uparrow + \downarrow \exists), \stackrel{\text{d}y}{=}, \frac{\text{d}z}{\text{d}x}.$ $\begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ dx + dy + dz = 0 \end{cases} \Rightarrow \begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ z \, dx + z \, dy + z \, dz = 0 \end{cases}$ $\Rightarrow (x - z) \, dx + (y - z) \, dy = 0 \Rightarrow \frac{dy}{dx} = \frac{x - z}{z - y}.$

$$\Rightarrow (x-z)\,\mathrm{d}x +$$

 $\begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ y \, dx + y \, dy + y \, dz = 0 \end{cases}$

$$(x) dx +$$

$$(y -$$











$$x \, \mathbf{d}$$

$$\begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ dx + dy + dz = 0 \end{cases} \Rightarrow \begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ z \, dx + z \, dy + z \, dz = 0 \end{cases}$$
$$\Rightarrow (x - z) \, dx + (y - z) \, dy = 0 \Rightarrow \frac{dy}{dx} = \frac{x - z}{z - y}.$$

$$\begin{cases} x \, dx + y \, dy + z \, dz = 0 \\ y \, dx + y \, dy + y \, dz = 0 \end{cases}$$

 $\Rightarrow (x - y) dx + (z - y) dz = 0 \Rightarrow \frac{dz}{dx} = \frac{x - y}{y - z}.$