Module 7: Analysis of MBS

Analysis of MBS

- Monte Carlo Simulations
- Interest Rate Risk
- Yield Analysis

Monte Carlo Simulation

Monte Carlo Simulation

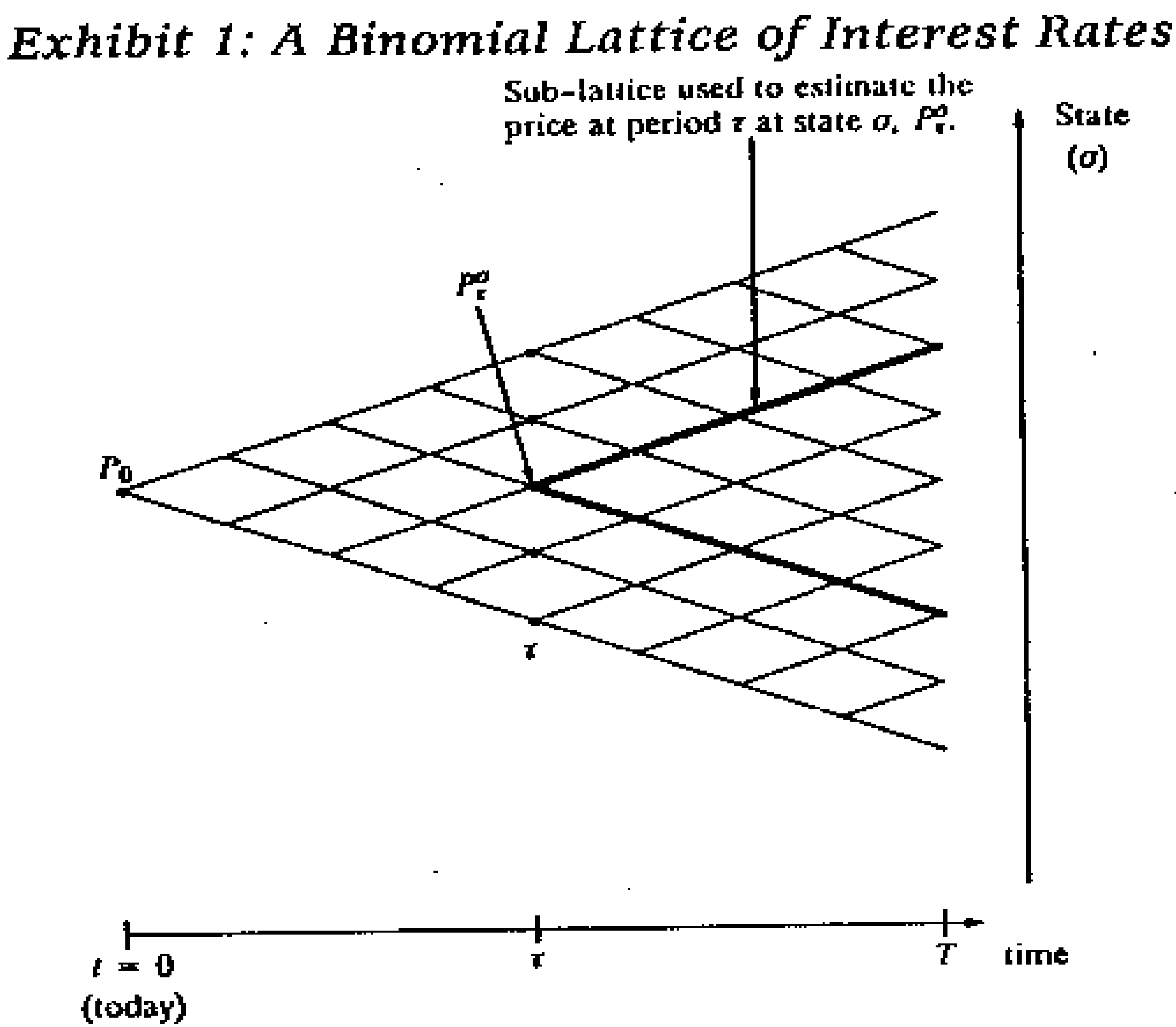
- Objective: Determine MBS’s theoretical value
- Steps:
  - Simulation of interest rates: Use a binomial interest rate tree to generate different paths for spot rates and refinancing rates.
  - Estimate cash flows of MBS or tranche for each path given a specified prepayment model based on spot rates.
  - Determine present values of each path.
  - Calculate average value – theoretical value.

Monte Carlo Simulation

- Monte Carlo simulation of term structure which is used to generate paths of risk free rates
- Generate security cash flows for each path
- Compute and average present value of discounted cash flow

$$p_{jo} = \frac{1}{|s_0|} \sum_{s=1}^{|s_0|} \sum_{t=1}^{T} \frac{C_{jt}^{s}}{\prod_{i=1}^{t} \left(1 + r_i^{s}\right)}$$

Step 1: Simulation of Interest Rate



Step 1: Simulation of Interest Rate

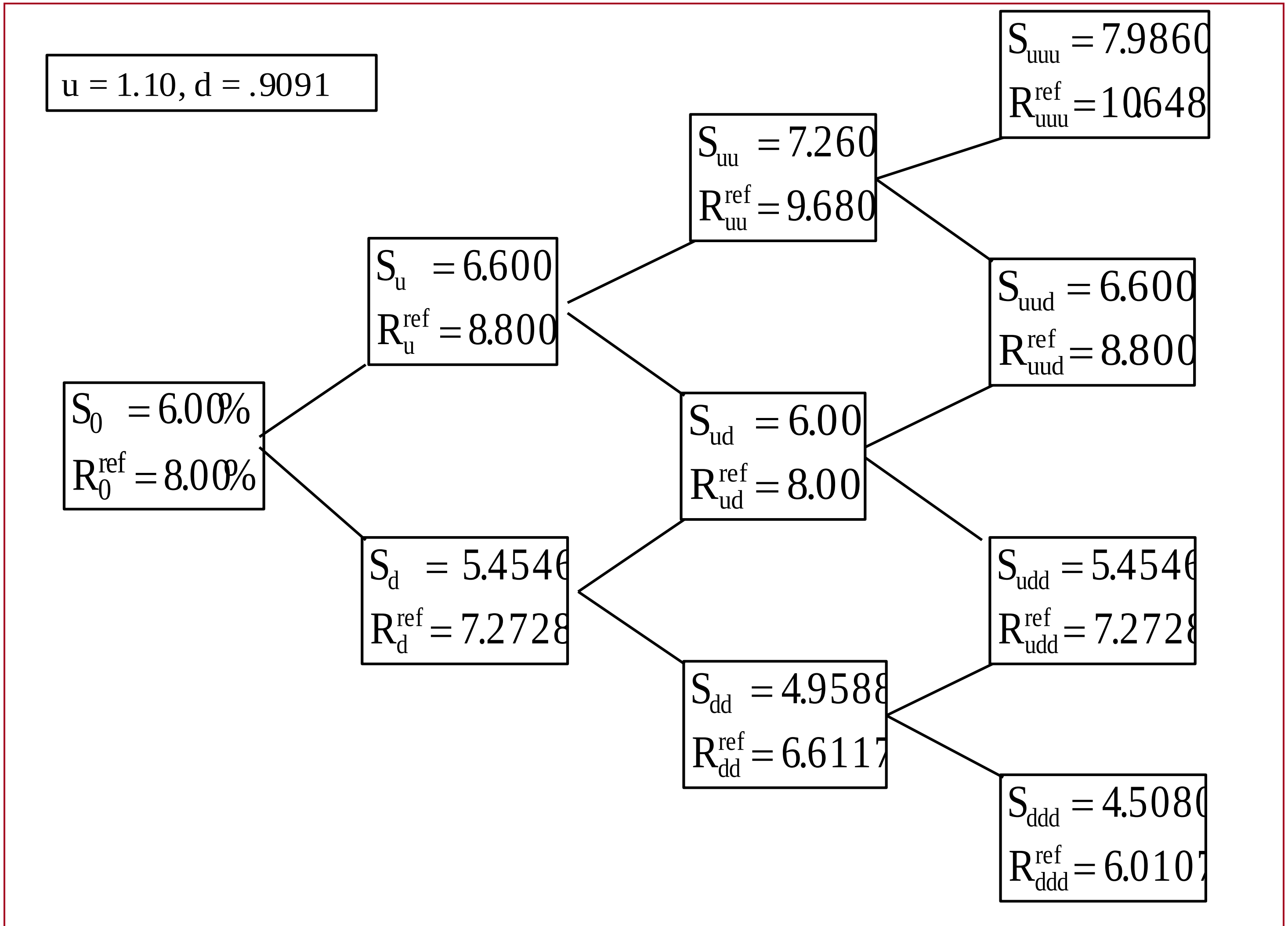
- Determination of interest rate paths from a binomial interest rate tree.
- Example:
  - Assume three-period binomial tree of one-year spot rates (S) and refinancing rates, (R^{ref}) where:

$$\boxed{\begin{aligned} S_0 &= 6\% \\ u &= 1.1 \\ d &= .9091 = 1/1.1 \end{aligned}}$$

$$\boxed{\begin{aligned} R_0^{ref} &= 8\% \\ u &= 1.1 \\ d &= .9091 = 1/1.1 \end{aligned}}$$

  - With three periods, there are four possible rates after three periods (years) and eight possible paths.

Step 1: Binominal Trees



Step 1: Interest Rate Paths

Path 1	Path 2	Path 3	Path 4
6.0000% 5.4546 4.9588 4.5080	6.0000% 5.4546 4.9588 5.4546	6.0000% 5.4546 6.0000 5.4546	6.0000% 5.4546 6.0000 6.6000
Path 5	**Path 6**	**Path 7**	**Path 8**
6.0000% 6.6000 6.0000 5.4546	6.0000% 6.6000 6.0000 6.6000	6.0000% 6.6000 7.2600 6.6000	6.0000% 6.6000 7.2600 7.9860

Step 2: Estimating Cash Flow

- The second step is to estimate cash flow for each interest rate path.
- Cash flow depends on prepayment rates assumed.
- Most analysts use a prepayment model in which conditional prepayment rate (CPR) is determined by seasonality of mortgages and by a refinancing incentive that ties interest rate paths to proportion of mortgage collateral prepaid.

Step 2: Estimating Cash Flow

- To illustrate, consider a MBS formed from a mortgage pool with a par value of $1M, weighted average coupon (WAC) = 8%, and weighted-average maturity (WAM) = 10 years.
- To fit this example to three-year binomial tree, we assume that
  - Mortgages in the pool all make <u>annual</u> cash flows.
  - All have a <u>balloon payment</u> (not fully amortize) at the end of year 4.
  - Pass-through rate (PT rate) on MBS is equal to WAC.

Step 2: Estimating Cash Flow

- Mortgage pool can be viewed as a four-year asset with a principal payment made at the end of year four that is equal to original principal less amount paid down.
- As shown in next exhibit, if there were no prepayments, then the pool would generate cash flows of $149,029 each year (then sum of PV of cash flows discounted by 8% = 1M) and a balloon payment of $688,946 at the end of year 4.

Step 2: Estimating Cash Flow

- Mortgage Portfolio:
  - Par value = $1M, WAC = 8%, WAM = 10 yrs,
  - PT rate = 8%, balloon at the end of the 4th year

Year	**Balance**	**Payment**	**Interest**	**Scheduled Principal**	**Cash Flow**
1	$1,000,000	$149,029	$80,000	$69,029	$149,029
2	$930,971	$149,029	$74,478	$74,552	$149,029
3	$856,419	$149,029	$68,513	$80,516	$149,029
4	$775,903	$149,029	$62,072	$86,957	$837,975

$$Balloon = Balance(yr4) - schprin(yr4) = \$775{,}903 - \$86{,}957 = \$688{,}946$$

$$CF_4 = Balloon + p = \$688{,}946 + \$149{,}029 = \$837{,}975$$

$$CF_4 = Balance(yr4) + Interest = \$775{,}903 + \$62{,}072 = \$837{,}975$$

Step 2: Estimating Cash Flow

- Such a cash flow is, of course, unlikely given prepayment. The prepayment model assumes:
  - Annual CPR is equal to 5% if mortgage pool rate is at a par or discount (that is, if current refinancing rate is equal to WAC of 8% or greater).
  - CPR will exceed 5% if rate on mortgage pool is at a premium. CPR will increase within certain ranges as premium increases. The relationship between CPRs and the range of rates is the same in each period; that is, there is no seasoning factor.

Step 2: Estimating Cash Flow

Range: $X = WAC - R^{ref}$	CPR
$X \leq 0$	5%
$0.0\% < X \leq 0.5\%$	10%
$0.5\% < X \leq 1.0\%$	20%
$1.0\% < X \leq 1.5\%$	30%
$1.5\% < X \leq 2.0\%$	40%
$2.0\% < X \leq 2.5\%$	50%
$2.5\% < X \leq 3.0\%$	60%
$X > 3.0\%$	70%

Step 2: Estimating Cash Flow

- With this prepayment model, cash flows can be generated for eight interest rate paths.
- These cash flows are shown in Exhibit A (next two slides).

Exhibit A

Path 1	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.072728	1000000	0.08	80000	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.066117	744776	0.08	59582	59641	0.30	205540	324764	0.074546	0.077270	279846	0.5
3	0.060107	479594	0.08	38368	45089	0.40	173802	257259	0.069588	0.074703	207255	0.5
4		260703	0.08	20856				281560	0.065080	0.072289	212972	
										Value =	1010465	0.125

Path 2	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.072728	1000000	0.08	80000	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.066117	744776	0.08	59582	59641	0.30	205540	324764	0.074546	0.077270	279846	0.5
3	0.072728	479594	0.08	38368	45089	0.20	86901	170358	0.069588	0.074703	137245	0.5
4		347604	0.08	27808				375413	0.074546	0.074664	281461	
										Value =	1008945	0.125

Path 3	1.000000	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.072728	1000000	0.08	80000	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.080000	744776	0.08	59582	59641	0.05	34257	153480	0.074546	0.077270	132253	0.5
3	0.072728	650878	0.08	52070	61192	0.20	117937	231200	0.080000	0.078179	184465	0.5
4		471749	0.08	37740				509489	0.074546	0.077270	378301	
										Value =	1005411	0.125

Path 4	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.088000	1000000	0.08	80000	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.080000	884422	0.08	70754	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.072728	772918	0.08	61833	72666	0.20	140050	274550	0.080000	0.081996	216742	0.5
4		560202	0.08	44816				605018	0.074546	0.080129	444494	
										Value =	997720	0.125

Exhibit A

Path 5	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.072728	1000000	0.08	80000	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.080000	744776	0.08	59582	59641	0.05	34257	153480	0.074546	0.077270	132253	0.5
3	0.088000	650878	0.08	52070	61192	0.05	29484	142747	0.080000	0.078179	113892	0.5
4		560202	0.08	44816				605018	0.086000	0.080129	444494	
										Value =	1001031	0.125

Path 6	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.088000	1000000	0.08	80000	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.080000	884422	0.08	70754	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.088000	772918	0.08	61833	72666	0.05	35013	169512	0.080000	0.081996	133820	0.5
4		665240	0.08	53219				718459	0.086000	0.082996	522269	
										Value =	992574	0.125

Path 7	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.088000	1000000	0.08	80000	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.096000	884422	0.08	70754	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.088000	772918	0.08	61833	72666	0.05	35013	169512	0.092600	0.086188	132277	0.5
4		665240	0.08	53219				718459	0.086000	0.086141	516247	
										Value =	985008	0.125

Path 8	1	2		3	4	5	6	7	8	9	10	11
Year	R^{ref}	Balance	WAC	Interest	Sch. Prin.	CPR	Prepaid Prin.	CF	$Z_{1,t-1}$	Z_{t0}	Value	Prob.
1	0.088000	1000000	0.08	80000	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.096000	884422	0.08	70754	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.106480	772918	0.08	61833	72666	0.05	35013	169512	0.092600	0.086188	132277	0.5
4		665240	0.08	53219				718459	0.099860	0.089590	509741	
										Value =	978502	0.125
											Wt. Value	$997,457

Step 2: Estimating Cash Flow

Path 1

- Cash flows for path 1 (path with three consecutive decreases in rates) consist of
  - $335,224 in year 1 (interest = $80,000, scheduled principal = $69,029.49, and prepaid principal = $186,194.10, reflecting a CPR of .20).
  - $324,764 in year 2, with $205,540 being prepaid principal (CPR = .30).
  - $257,259 in year 3, with $173,802 being prepaid principal (CPR = .40) $251,560 in year 4.
- The year 4 cash flow with balloon payment is equal to principal balance at beginning of year and 8% interest on that balance.

Step 2: Estimating Cash Flow

- Calculations for CF for Path 1:
  - $335,224 in year 1
  - Interest = $80,000
  - scheduled principal = $69,029.49
  - prepaid principal =$186,194.10, reflecting a CPR of 0.2

$$p = \frac{\$1{,}000{,}000}{\dfrac{1-(1/(1.08))^{10}}{.08}} = \$149{,}029$$

$$\text{interest} = .08(\$1{,}000{,}000) = \$80{,}000$$

$$\textit{scheduled principal} = \$149{,}029 - \$80{,}000 = \$69{,}029$$

$$\textit{prepaid principal} = .20(\$1{,}000{,}000 - \$69{,}029) = \$186{,}194$$

$$CF_1 = \$80{,}000 + \$69{,}029 + \$186{,}194 = \$335{,}224$$

Step 2: Estimating Cash Flow

- Calculations for CF for Path 1:
  - $324,764 in year 2
  - Interest = $59,582
  - scheduled principal = $59,641
  - prepaid principal = $205,540, reflecting a CPR of .30

$$\text{Balance} = \$1{,}000{,}000 - (\$69{,}029 + \$186{,}194) = \$744{,}776$$

$$p = \frac{\$744{,}776}{\frac{1-(1/(1.08))^9}{.08}} = \$119{,}223$$

$$\text{interest} = .08(\$744{,}776) = \$59{,}582$$

$$\text{scheduled principal} = \$119{,}223 - \$59{,}582 = \$59{,}641$$

$$\text{prepaid principal} = .30(\$744{,}776 - \$59{,}641) = \$205{,}540$$

$$CF_2 = \$59{,}582 + \$59{,}641 + \$205{,}540 = \$324{,}764$$

Step 2: Estimating Cash Flow

- Calculations for CF for Path 1:
  - $257,259 in year 3:
  - Interest = $38,368
  - scheduled principal = $45,089
  - prepaid principal = $173,802, reflecting a CPR of .40

$$\text{Balance} = \$744{,}776 - (\$59{,}641 + \$205{,}540) = \$479{,}594$$

$$p = \frac{\$479{,}594}{\dfrac{1-(1/(1.08))^8}{.08}} = \$83{,}456$$

$$\text{interest} = .08(\$479{,}594) = \$38{,}368$$

$$\text{scheduled principal} = \$83{,}456 - \$38{,}368 = \$45{,}089$$

$$\text{prepaid principal} = .40(\$479{,}594 - \$45{,}089) = \$173{,}802$$

$$CF_3 = \$38{,}368 + \$45{,}089 + \$173{,}802 = \$257{,}259$$

Step 2: Estimating Cash Flow

- Calculations for CF for Path 1:
  - $281,560 in year 4:
  - The year 4 cash flow with balloon payment is equal to principal balance at beginning of the year and 8% interest on that balance.

$$\text{Balance} = \$479{,}594 - (\$45{,}089 + \$173{,}802) = \$260{,}703$$

$$\text{interest} = .08(\$260{,}703) = \$20{,}856$$

$$CF_4 = \$260{,}703 + \$20{,}856 = \$281{,}560$$

Step 2: Estimating Cash Flow

Path 8

- In contrast, cash flows for path 8 (path with three consecutive interest rate increases) are smaller in first three years and larger in year 4, reflecting low CPR of 5% in each period.

Step 3: Valuing Each Path

- Like any bond, a MBS or CMO tranche should be valued by discounting cash flows by appropriate risk-adjusted spot rates.
- For a MBS or CMO tranche, risk-adjusted spot rate, z_t, is equal to riskless spot rate, S_t, plus a risk premium.
- If underlying mortgages are insured against default, then risk premium would only reflect additional return needed to compensate investors for prepayment risk they are assuming. This premium is referred to as *option-adjusted spread (OAS).*

Step 3: Valuing Each Path

- If we assume no default risk, then risk-adjusted spot rate can be defined as

$$\boxed{z_t = S_t + k_t}$$

where: k = OAS

Step 3: Valuing Each Path

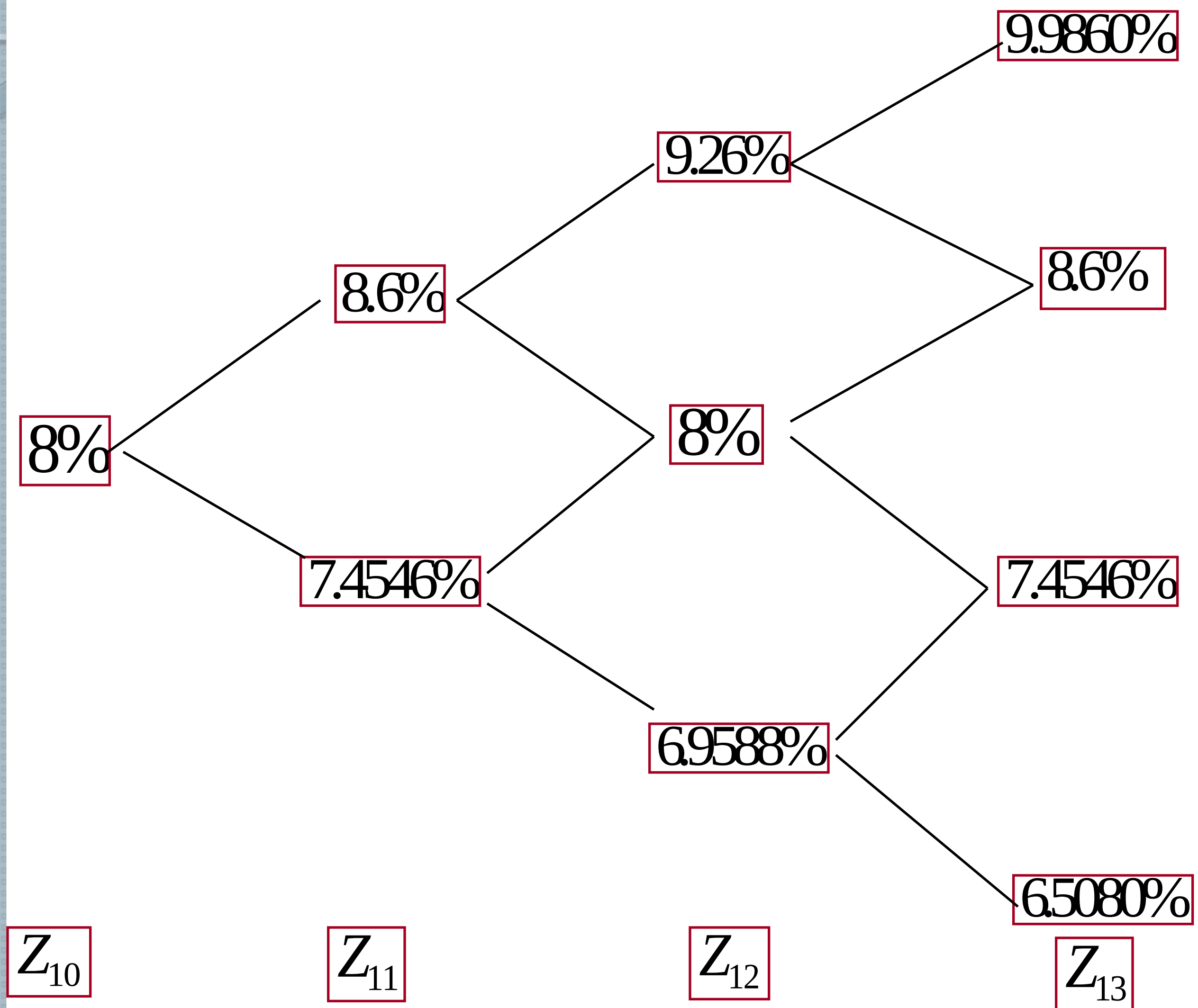
- Value of each path can be defined as

$$V_i^{Path} = \sum_{M=1}^{T} \frac{CF_M}{(1+z_M)^M} = \frac{CF_1}{1+z_1} + \frac{CF_2}{(1+z_2)^2} + \frac{CF_3}{(1+z_3)^3} + \cdots + \frac{CF_T}{(1+z_T)^T}$$

where:
i = i-th path
z_M = spot rate on bond with M-year maturity
T = maturity of MBS

For this example, assume option-adjusted spread (k) is 2% greater than one-year, risk-free spot rates.

Step 3: Valuing Each Path



Step 3: Valuing Each Path

- From these current and future one-year spot rates, current 1-year, 2-year, 3-year, and 4-year equilibrium spot rates can be obtained for each path by using geometric mean:

$$\boxed{z_M = \left((1+z_{10})(1+z_{11})\cdots(1+z_{1,M-1})\right)^{1/M} - 1}$$

Step 3: Valuing Each Path

- The set of spot rates z_1, z_2, z_3, and z_4 needed to discount cash flows for path 1 (three consecutive decreases) would be:

$$z_1 = .08$$

$$z_2 = \left((1+z_{10})(1+z_{11})\right)^{1/2} - 1$$

$$= \left((1.08)(1.074546)\right)^{1/2} - 1 = .07727$$

$$z_3 = \left((1+z_{10})(1+z_{11})(1+z_{12})\right)^{1/3} - 1$$

$$= \left((1.08)(1.074546)(1.069588)\right)^{1/3} - 1 = .074703$$

$$z_4 = \left((1+z_{10})(1+z_{11})(1+z_{12})(1+z_{13})\right)^{1/4} - 1$$

$$= \left((1.08)(1.074546)(1.069588)(1.06508)\right)^{1/4} - 1 = .07228$$

Step 3: Valuing Each Path

- Using these rates, value of MBS following path 1 is $1,010,465:

$$V_1^{Path} = \frac{\$335{,}224}{1.08} + \frac{\$324{,}764}{(1.077217)^2} + \frac{\$257{,}259}{(1.074703)^3} + \frac{\$281{,}560}{(1.072289)^4} = \$1{,}010{,}465$$

- Spot rates and values of each of eight paths are shown in columns 9 and 10 of Exhibit A.

Step 4: Theoretical Path

- Theoretical value of MBS is defined as average of values of all interest rate paths:

$$\overline{V} = \frac{1}{N}\sum_{i=1}^{N} V_i^{path}$$

- In this example, theoretical value of MBS issue is $997,457 or 99.7457% of its par value (see bottom of Exhibit A).

Step 4: Theoretical Path

- Theoretical value along with standard deviation of path values are useful measures in evaluating a MBS or CMO tranche relative to other securities.
- A MBS's theoretical value can also be compared to its actual price to determine if MBS is over or underpriced.
- E.g., if theoretical value is 98% of par and actual price is at 96%, then mortgage security is underpriced, '$2 cheap', and if it is priced at par, then it is considered overpriced, '$2 rich.'

Option-Adjusted Spread

- Instead of determining theoretical value of MBS or tranche given a path of spot rates and option-adjusted spreads, analysts can use a Monte Carlo simulation to estimate mortgage security's rate of return given its market price.
- Since security's rate of return is equal to a riskless spot rate plus OAS (assuming no default risk), analysts use simulation to estimate OAS.
- From simulation, OAS satisfies that theoretical value of MBS equals its market price.

Option-Adjusted Spread

- This spread can be found by iteratively solving for k that satisfies following equation:

$$\begin{aligned} &MarketPrice \\ &= \frac{1}{N}\left(\left[\sum_{M=1}^{T}\frac{CF_{(1)M}}{(1+S_{(1)M}+k)^{M}}\right]+\left[\sum_{M=1}^{T}\frac{CF_{(2)M}}{(1+S_{(2)M}+k)^{M}}\right]+\cdots+\left[\sum_{M=1}^{T}\frac{CF_{(N)M}}{(1+S_{(N)M}+k)^{M}}\right]\right)\end{aligned}$$

where: N = number of paths

Interest Rate Risk

Interest Rate Risk

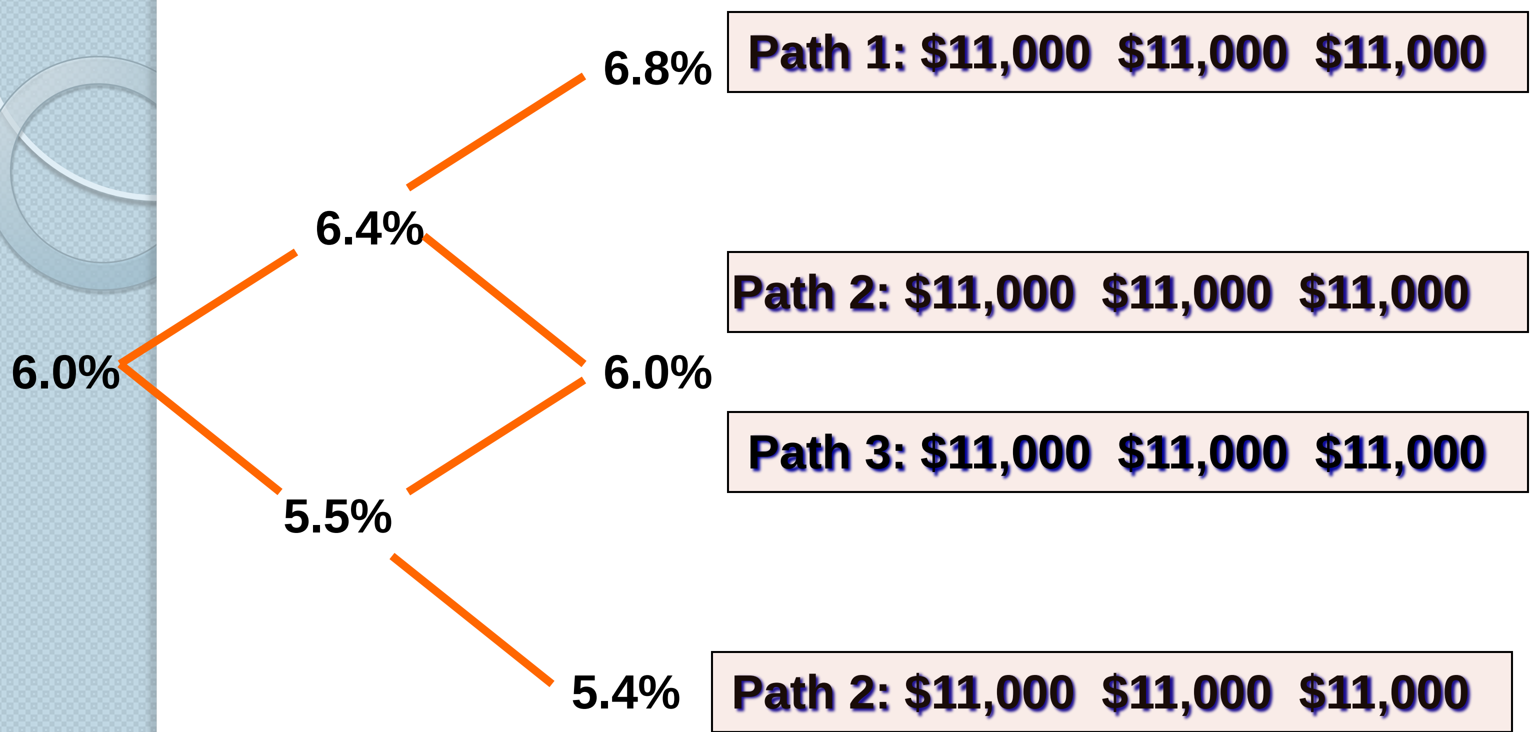
Recall:

We can define Modified Duration as this measure:

$$D_{\text{mod}} = -\frac{1}{P}\frac{dP}{dy} = \frac{1}{P\cdot\left(1+\frac{y}{2}\right)}\cdot\sum_{k=1}^{n} t_k \cdot PV\left(C_k\right)$$

- When the prepayments of an asset are variable, how do we asses interest rate risk?

Cash Flows



At sufficiently high interest rates, bond will never prepay. Therefore, we can treat this bond like a non-contingent bond.

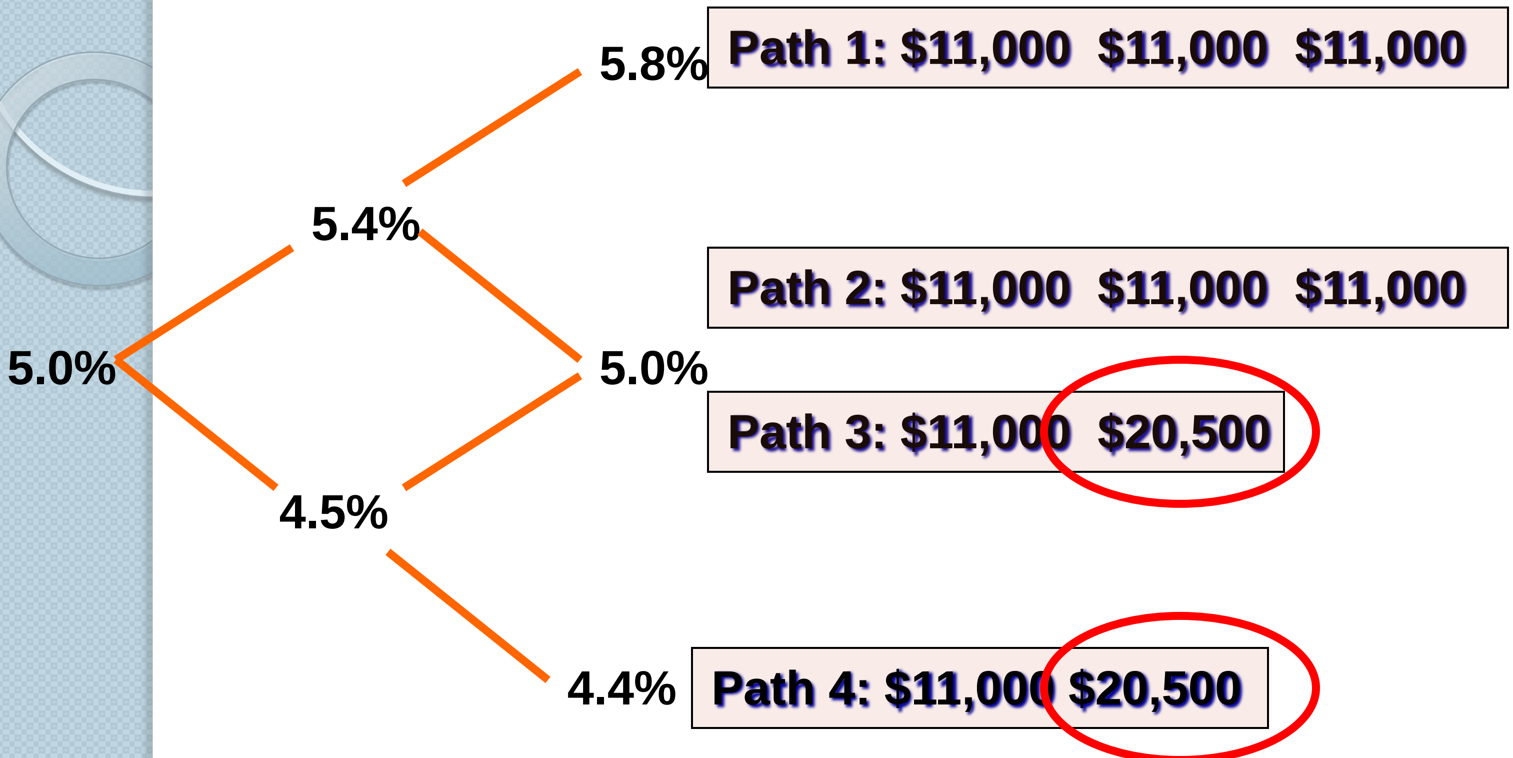
Any bond with equal monthly payments has a Macaulay duration equal to median payment date

$$P(Y=6\%) = \underbrace{\frac{\$11{,}000}{(1.06)}}_{\$10{,}377} + \underbrace{\frac{\$11{,}000}{(1.06)^2}}_{\$9{,}790} + \underbrace{\frac{\$11{,}000}{(1.06)^3}}_{\$9{,}235} = \$29{,}402$$

$$\left(\frac{\$10{,}377}{\$29{,}402}\right)-1 + \left(\frac{\$9{,}790}{\$29{,}402}\right)-2 + \left(\frac{\$9{,}235}{\$29{,}402}\right)-3 = 2$$

Macaulay Duration = -2 → Modified Duration = $\frac{-2}{1.06}$ = -1.89

Cash Flows



However, value of this bond will be very sensitive at interest rates near 4.5% (prepayment “trigger”)

Effective Duration

- We can define modified duration in terms of cash flows, yield and time to cash flows.
  - But thinking this way gives us no insight in how to extend this notion to bonds with embedded options (since they don't have fixed cash flows).
- We can think modified duration in terms of price sensitivity which allows us to extend notion of modified duration to these bonds.
  - If we have a pricing function, we can estimate derivative, and compute price sensitivity.
  - This measure is referred to as *Effective Duration* or *Option Adjusted Duration*.

Effective Duration & Convexity

- There is a duration measure that is more appropriate for bonds with embedded options. Duration and convexity can be used with a binomial tree to measure duration of a MBS.

$$\text{Duration} = \frac{P_{-} - P_{+}}{2(P_0)\Delta y}; \quad \text{Convexity} = \frac{P_{+} - P_{-} - 2(P_0)}{(P_0)(\Delta y)^2}$$

P_ = *price if yield is decreased by x basis points*
P+ = *price if yield is increased by x basis points*
$\boldsymbol{P_0}$ = *initial price* (per $100 of par value)
$\Delta \boldsymbol{y}$ (or ***dy***) = *change in rate* (x basis points in decimal form)

Effective Duration & Convexity

- When approximate duration formula is applied to a bond with an embedded option, new prices at higher and lower yield levels should reflect value from valuation model.
- Differences between modified duration and effective duration are summarized next slide.
- Standard convexity measure may be inappropriate for a bond with embedded options because it does not consider the effect of a change in interest rates on bond's cash flow.

Modified Duration Versus Effective Duration

Duration

Interpretation: generic description of sensitivity of a bond's price (as a percent of initial price) to a parallel shift in yield curve

Modified Duration

Duration measure in which yield changes do not change expected cash flow

Effective Duration

Duration measure in which yield changes may change expected cash flow

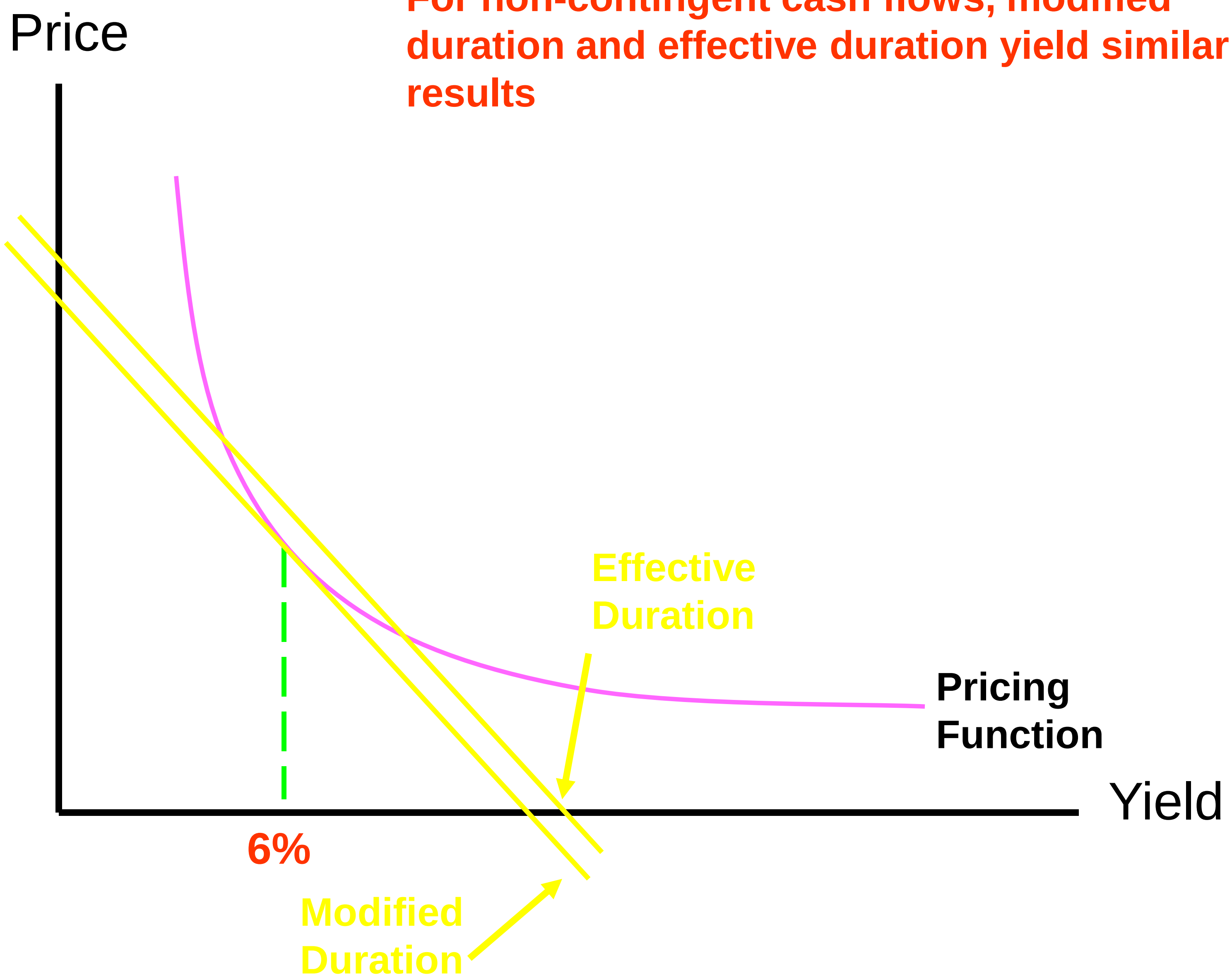
Effective Duration and Convexity

- Steps for using binomial tree to estimate duration and convexity:
  - Take yield curve estimated with bootstrapping and value MBS, P_0, using calibration approach.
  - Let yield curve estimated with bootstrapping decrease by a small amount and then estimate price of MBS using calibration approach -- P_-.
  - Let yield curve estimated with bootstrapping increase by a small amount and then estimate price of MBS using calibration approach -- P_+.
  - Calculate effective duration and convexity.

$$\boxed{D = \frac{P_- - P_+}{2(P_0)\Delta y};\ \text{Convexity} = \frac{P_+ - P_- - 2(P_0)}{(P_0)(\Delta y)^2}}$$

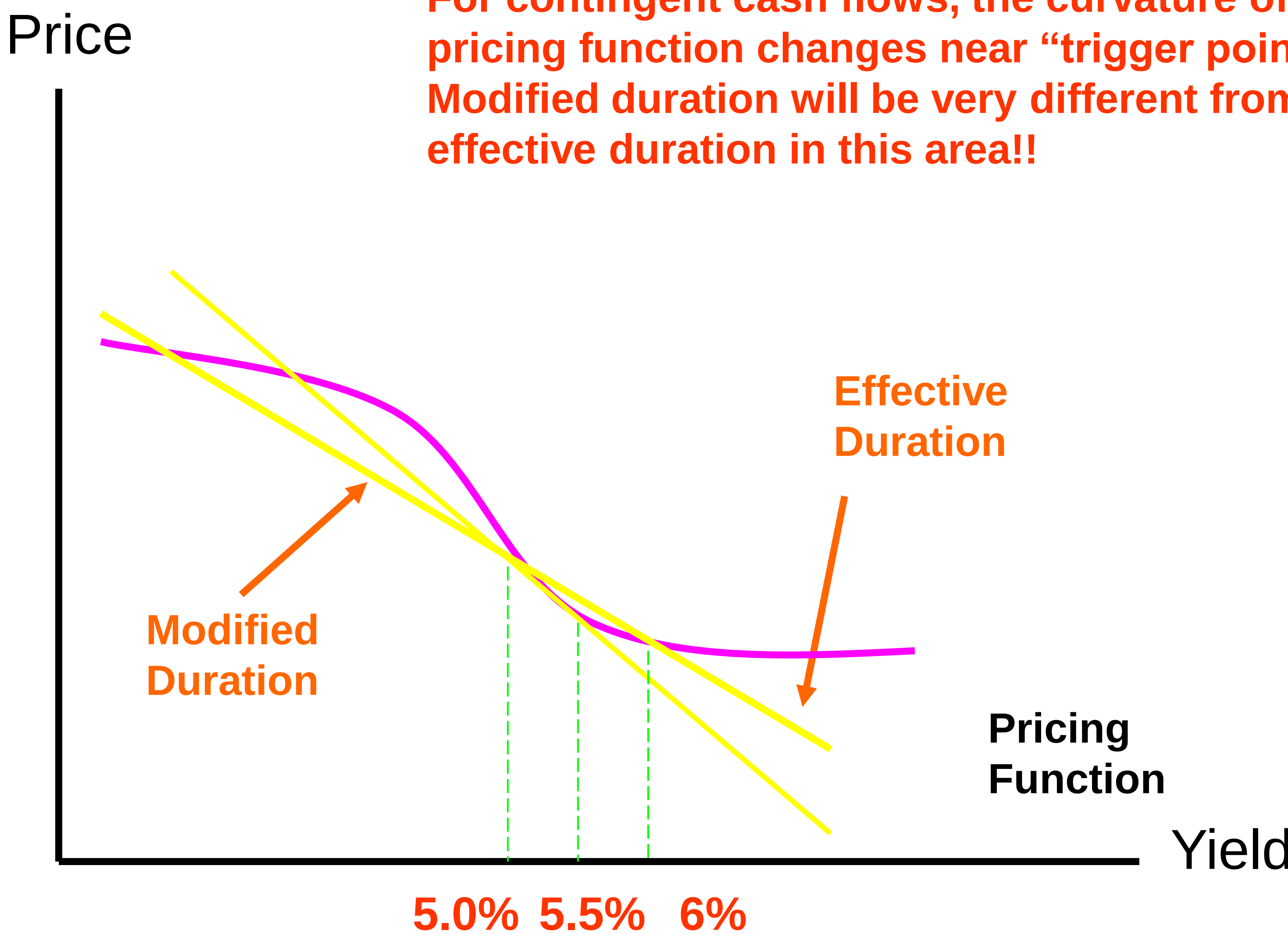
Effective Duration and Convexity

For non-contingent cash flows, modified duration and effective duration yield similar results



Effective Duration and Convexity

For contingent cash flows, the curvature of pricing function changes near "trigger point". Modified duration will be very different from effective duration in this area!!



Other Duration Measures

- Cash flow duration
- Coupon curve duration
- Empirical duration

Cash Flow Duration

- Cash flow duration is computed assuming that, if interest rates change, prepayment rate will change when computing new value. The typical procedure is:
  - Shock the rate, assume a new fixed set of cash flows consistent with new rate, and price bond by discounting new cash flows at new rates.
  - It allows cash flows to vary with rates, but ignores the fact that under each scenario interest rates can change again. It is not a dynamic but a static approach.

Cash Flow Duration

- It is better than modified duration in that it recognizes that cash flows change when interest rates change, whereas, modified duration holds cash flows fixed.
- It is worse than effective duration in that it does not capture the value of prepayment option, the way that effective duration does. Effective duration accounts for optionality inherent in MBS cash flows.

Coupon Curve Duration

- Coupon curve duration is based on relationship between coupon rates and prices for similar MBSs. The typical procedure is:
  - Shocked prices which come from other MBS with different coupons.
  - For example, when rates rise 1%, new price of the 9% coupon MBS will equal old price of the 8% coupon MBS. With two such observations, apply standard mathematical formula to find the sensitivity to interest rates.

Example

- Suppose that coupon curve of prices for a MBS is as follows:

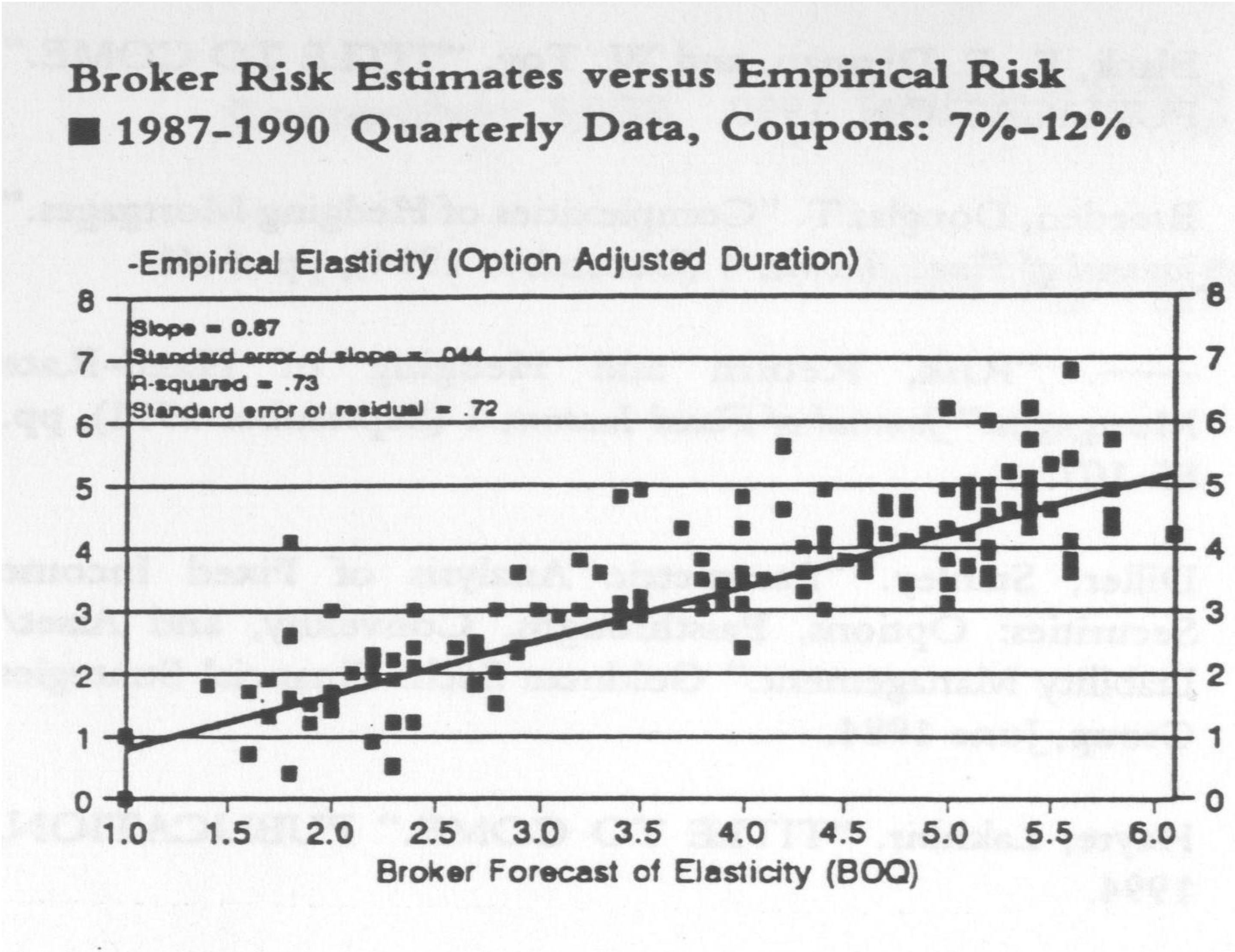
Coupon	6%	7%	8%	9%	10%
Price	96	98	100	102	105

  - If yield *declines* by 1%, price of 7% coupon MBS will *increase* to price of 8% coupon MBS (V_-): 100.
  - If yield *increases* by 1%, price of 7% coupon MBS will *decline* to price of 6% coupon MBS (V_+): 96.
  - V_0 = 98, and Δy = 0.01.
  - Coupon rate duration = (100-96)/(2 x 98 x 0.01) = 2.04

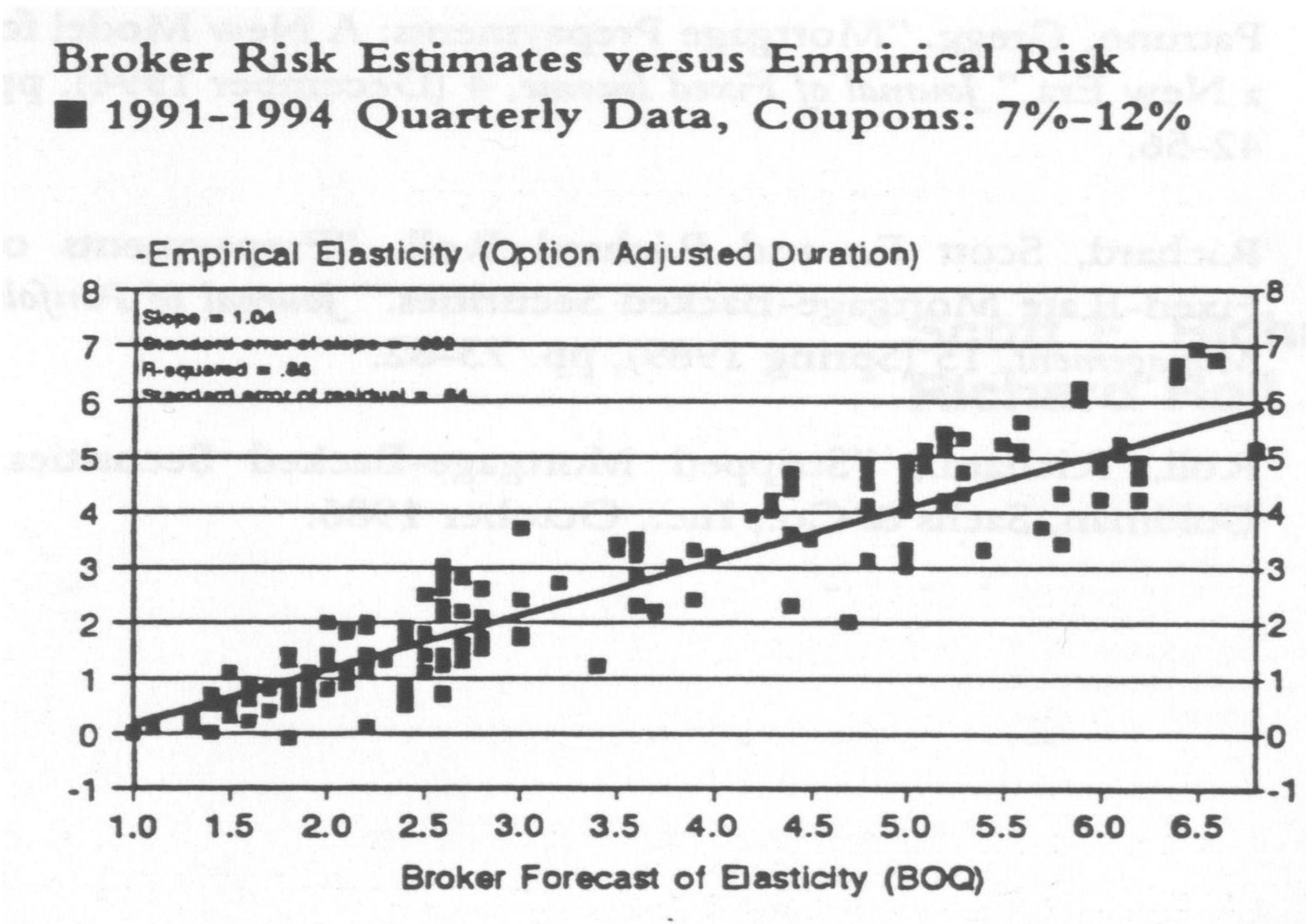
Empirical Duration

- Empirical duration is observed sensitivity of bond's price to actual interest rate changes. It is determined using regression analysis with historical yields and prices. There are three problems with this duration.
  - No observed price series for thinly-traded bonds.
  - Non-linearities or other aberrant option price reactions are possible.
  - There may have been changes in volatility that accompanied changes in interest rates.

Empirical Duration



Empirical Duration



Yield Analysis

Yield Analysis

- Like all securities, MBS can be evaluated in terms of their characteristics. However, it is more complex because of the difficulty in estimating cash flows due to prepayment.
- One approached used to evaluate MBS and CMO tranches is yield analysis. *Yield analysis* involves calculating yields on MBS or CMO tranches given *different prices and prepayment speed assumptions* or alternatively calculating values on MBS or CMO tranches given *different rates and speeds*.

Example

- For example, suppose an institutional investor is interested in buying a MBS issue that has a par value of $100M, WAC = 8, WAM = 355 months, and a PT rate of 7.5%.
- The value, as well as average life, maturity, duration, and other characteristics of this security would depend on the rate the investor requires on MBS and prepayment speed he estimates.

Example

- If investor's required return on MBS is 9% and his estimate of PSA speed is 150, then he would value MBS issue at $93,702,142. At that rate and speed, MBS would have an average life of 9.18 years.
- Whether this purchase represents a good investment *depends*, in part, on rates for other securities with similar maturities, durations, and risk, and in part, on how good prepayment rate assumption is.

Yield Analysis

- In general, the decision on whether or not to invest in a particular MBS or tranche depends on the price investor can command.
- For example, based on an expectation of a 100% PSA, our investor might conclude that a yield of 9% on MBS would make it a good investment.
- In this case, investor would be willing to offer no more than $92,732,145 for MBS issue, and average life would be 11.51 years.

Yield Analysis

- One common approach used in conducting a yield analysis is to generate a matrix of different yields by varying prices and prepayment speeds.
- Next exhibit shows different values of MBS given different required rates and prepayment speeds.
- Using this matrix, investor could determine, for a given price and assumed speed, estimated yield, *or* determine, for a given speed and yield, price. Using this approach, an investor can also evaluate for each price average yield and standard deviation over a range of PSA speeds.

Yield and Vector Analysis

- Mortgage Portfolio = \$100M, WAC = 8%, WAM = 355 Months, PT Rate = 7.5.

Rate/PSA	50	100	150
	Value	Value	Value
7%	\$106,039,631	\$105,043,489	\$104,309,207
8%	\$98,251,269	\$98,526,830	\$98,732,083
9%	\$91,442,890	\$92,732,145	\$93,702,142
10%	\$85,457,483	\$87,554,145	\$89,146,871
Average Life	14.95	11.51	9.18
	Vector	*Vector*	*Vector*
	Month Range: PSA	*Month Range: PSA*	*Month Range: PSA*
	1-50: 200	1-50: 200	1-50: 200
	51-150: 250	51-150: 300	51-150: 150
	151-250: 150	151-250: 350	151-250: 100
	251-355: 200	251-355: 400	251-355: 50
Rate	Value	Value	Value
7%	\$103,729,227	\$103,473,139	\$104,229,758
8%	\$98,893,974	\$98,964,637	\$98,756,370
9%	\$94,465,328	\$94,794,856	93,826,053
10%	\$90,395,704	\$90,929,474	89,,364,229

Yield Analysis

- One of limitations of above yield analysis is the assumption that PSA speed used to estimate yield is constant during the life of MBS.
- In fact, such an analysis is sometimes referred to as ***static yield analysis***.
- In practice, prepayment speeds change over the life of a MBS as interest rates change in the market.

Vector Analysis

- A more dynamic yield analysis, known as ***vector analysis***, can be used.
- In applying vector analysis, PSA speeds are assumed to change over time.
- In the above case, a matrix of values for different rates can be obtained for different PSA vectors formed by dividing total period into a number of periods with different PSA speeds assumed for each period.
- A vector analysis example is also shown at the bottom of last exhibit.

Web Sites

MBS Price Information:

Wall Street Journal

- Go to http://online.wsj.com/public/us, “Market,” “Bonds, Rates, & Credit Markets,” and “Mortgage-Backed Securities, CMO.”

Investinginbonds.com

- **MBS Price Index:** Merrill Lynch Mortgage-Backed Securities (MBS) Index is a statistical composite tracking overall performance of MBS market over time. Index includes U.S. dollar-denominated 30-year, 15-year and balloon pass-through mortgage securities.
- Go to http://investinginbonds.com/; click “MBS/ABS Market At-A-Glance.”