Module 9: Fixed Income Portfolio Management

Roadmap

Active Strategies

Passive Strategies

Types of Bond Strategies

- <u>Active Strategies</u>: taking active bond positions with primary objective of obtaining an <u>abnormal return</u>. They are typically speculative and include
  - Interest Rate Anticipation Strategies
  - Fundamental Valuation Strategies
- <u>Passive Strategies</u>: no change in position is necessary once bonds are selected. They include
  - Indexing
  - Cash-Flow Matching
  - Classical Immunization

Active: Interest Rate Anticipation Strategies

- Types of Interest-Rate Anticipation Strategies:
  - Rate-Anticipation Strategies
  - Strategies Based on Yield Curve Shifts

Rate-Anticipation Strategies

- Rate-Anticipation Strategies are active strategies of selecting bonds or bond portfolios with specific durations based on interest rate expectations.
  - Rate-Anticipation Swap is a rate-anticipation strategy that involves simultaneously selling and buying bonds with different durations.

Rate-Anticipation Swap

- Rate-Anticipation Swap for bond portfolio manager when interest rates are expected to <u>decrease</u> across all maturities
  - Strategy: <u>Lengthening portfolio's duration</u>: Manager could <u>sell her lower duration</u> bonds and <u>buy higher duration</u> ones.

$$\boxed{\begin{array}{l}\text{Expect } R\downarrow \text{ and } \therefore P_0^B\uparrow \\ \text{across all maturities}\end{array}} \Rightarrow \boxed{\begin{array}{l}\text{Long in high duration bonds} \\ \text{Short in low duration bonds}\end{array}}$$

  - By doing this, portfolio's value would be more sensitive to interest rate changes and as a result would subject manager to a higher return-risk position, providing greater upside gains in value if rates decrease but also greater losses in value if rates decrease.

Rate-Anticipation Swap

- Rate-Anticipation Swap for bond portfolio manager when interest rates are expected to increase across all maturities
  - Strategy: Shorten portfolio's duration: Manager could sell higher duration bonds and buy lower duration ones.

Expect $R\uparrow$ and $\therefore P_0^B\downarrow$ across all maturities $\Rightarrow$ Short in high duration bonds; Long in low duration bonds

  - Defensive Strategy: Objective is to preserve value of a bond fund.

Rate-Anticipation Swap: Cushion Bonds

- One way to shorten fund's duration is to sell high-duration bonds (possibly option-free) and then buy cushion bonds.
- A cushion bond is a callable bond with a coupon that is above current market rate, WHY?
- Cushion bond has following features:
  - High coupon yield
  - With embedded call option, its market price is lower than a comparable noncallable bond.
- Note: Interest rate swap of option-free bonds for cushion bonds provides some value preservation.

Example

- Suppose a bond manager had a fund consisting of 10-year, 10% option-free bonds valued at 113.42 with yield 8% and there were comparable 10-year, 12% coupon bonds callable at 110 with trading price close to their call price.
- If manager expected rates to <u>increase</u>, he could cushion negative price impact on fund’s value by:
  - Selling option-free bonds
  - Buying higher coupon, callable bonds – cushion bonds
- Swap of existing bonds for cushion bonds provides:
  - An immediate gain in income 113.42-110 = 3.42
  - A higher coupon income in the future: 12% instead of 10%

Example

<u>Note</u>

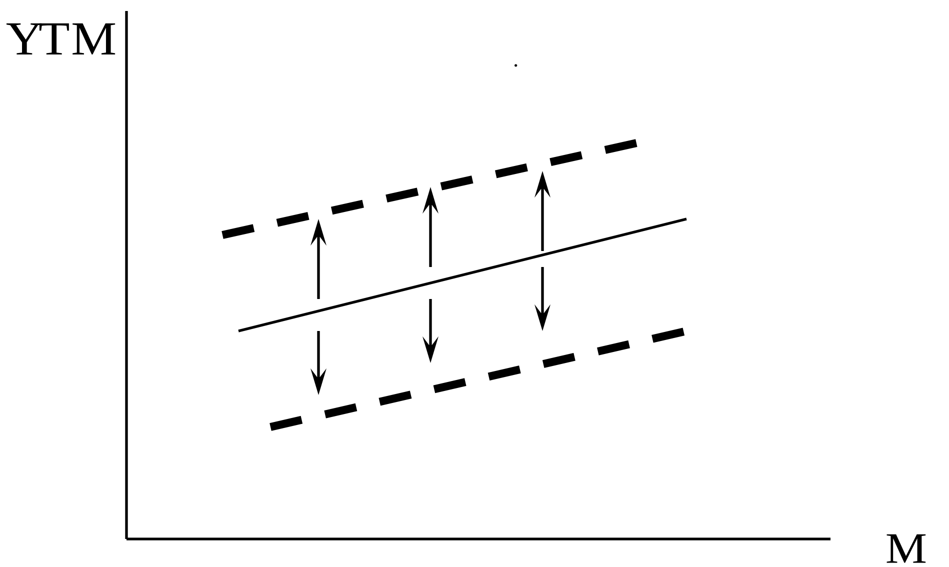
- A callable bond has a <u>lower</u> duration than a noncallable one with same maturity and coupon rate.
- 10-year cushion bond with call feature and higher coupon rate has a relatively lower duration than 10-year option-free bond.
- Thus, swap of cushion bonds for option-free bonds in this example represents a switch of longer duration bonds for shorter ones – a rate-anticipation swap.

Yield Curve Shifts and Strategies

- Yield Curve Strategies: Some rate-anticipation strategies are based on forecasting type of yield curve shift and then implementing an appropriate strategy to profit from forecast.
- Three types of yield curve shifts occur:
  - Parallel Shifts
  - Shifts with Twists
  - Shifts with Humpedness

Yield Curve Shifts: Parallel

- <u>Parallel Shifts</u>: Rates on all maturities change by the same number of basis points.

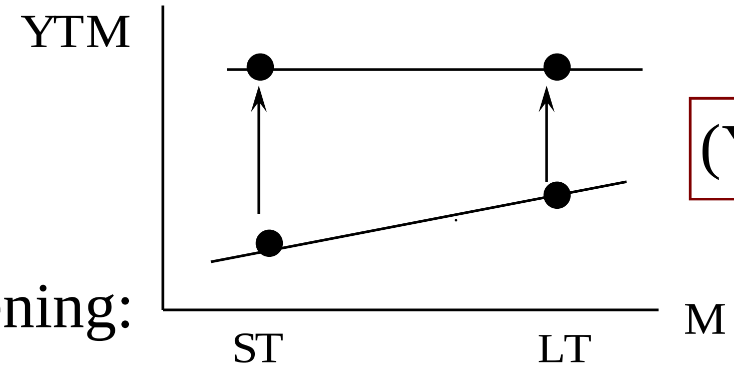


Yield Curve Shifts: Twist

- Shifts with a Twist: A <u>twist</u> is a non-parallel shift, with either a flattening or steepening of yield curve.
  - <u>Flattening</u>: spread between long-term and short-term rates decreases.
  - <u>Steepening</u>: spread between long-term and short-term rates increases.

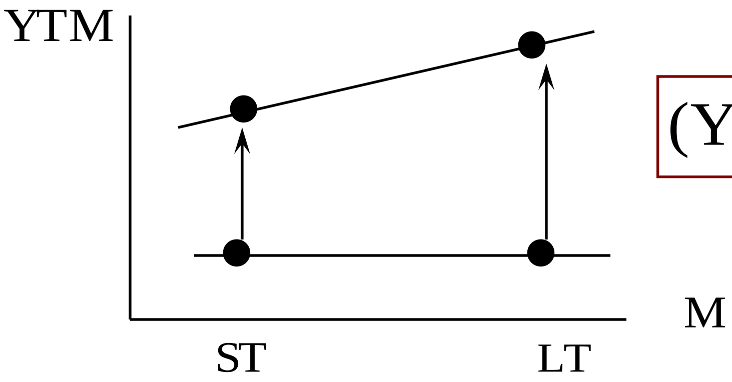
Yield Curve Shifts: Twist

- Shifts with a Twist:
  - Flattening:



$$\boxed{(YTM_{LT} - YTM_{ST}) \downarrow}$$

  - Steepening:



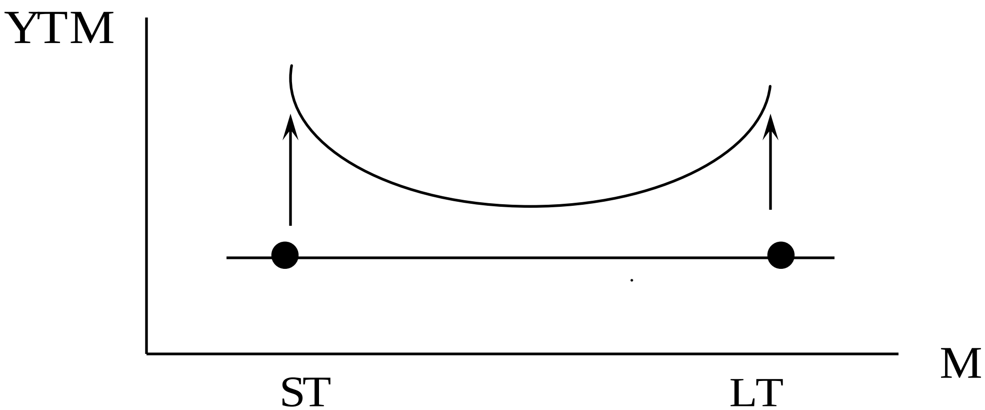
$$\boxed{(YTM_{LT} - YTM_{ST}) \uparrow}$$

Yield Curve Shifts: Humpedness

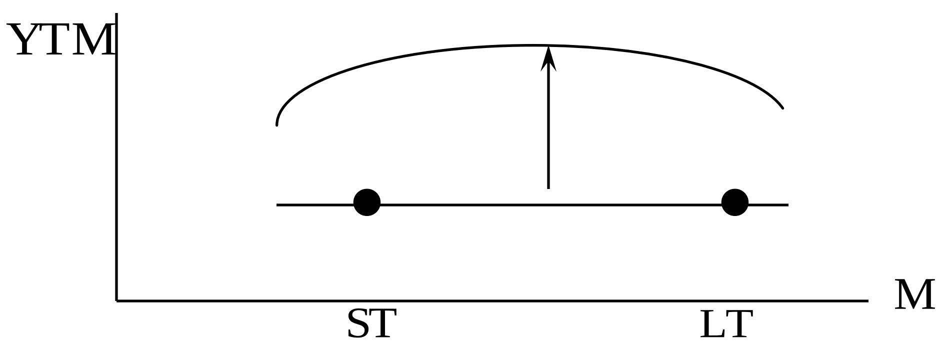
- A <u>shift with humpedness</u> is a non-parallel shift in which short-term and long-term rates change by greater magnitudes than intermediate rates.
  - Positive Butterfly*:* There is an increase in both short and long-term rates relative to intermediate rates.
  - Negative Butterfly: There is a decrease in both short and long-term rates relative to intermediate rates.

Yield Curve Shifts: Humpedness

– Positive Butterfly: ST and LT rates change more than intermediate:



– Negative Butterfly: Intermediate rates change more than ST and LT:



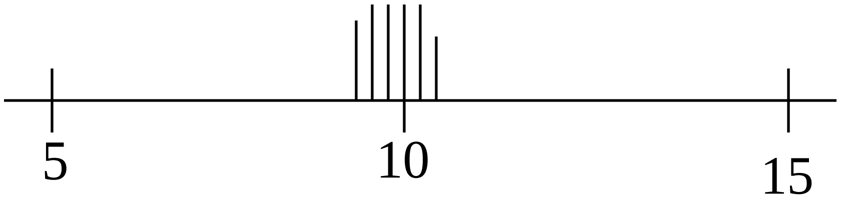
Yield Curve Shift Strategies

- Yield Curve Strategies
  - Bullet strategy is formed by constructing a portfolio concentrated in one maturity area.
  - Barbell strategy is formed with investments concentrated in both short-term and long-term bonds.
  - Ladder strategy is formed with equally allocated investments in each maturity group.

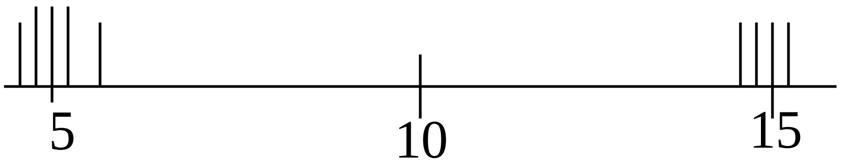
Yield Curve Strategies

Yield Curve Strategies:

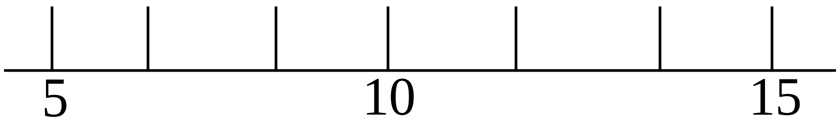
- Bullet Strategy:



- Barbell Strategy:



- Ladder Strategy:



Yield Curve Shift Strategies

- Strategies Based on Expectations
  - ➢ Bullet strategy could be formed based on an expectation of a downward shift in yield curve with a twist such that long-term rates were expected to decrease more than short-term. If investors expected a downward parallel shift in yield curve, a bullet strategy with longer duration bonds would yield greater returns than an investment strategy in intermediate or short-term bonds if expectation turns out to be correct.
  - ➢ Barbell strategy could be profitable for an investor who is forecasting an upward negative butterfly yield curve shift.

Yield Curve Strategies: Total Return Analysis

- Correct yield curve strategy depends on forecast.
- One approach to use in identifying appropriate strategy is Total Return Analysis, which involves determining possible returns from different yield curve strategies given different yield curve shifts.

Example

- Consider three bonds:
    - Bond A: 5-year, 6% bond selling at par, duration = 4.46.
    - Bond B: 11-year, 6% bond selling at par, duration = 8.36.
    - Bond C: 20-year, 6% bond selling at par, duration =12.16
- Assume yield curve is currently flat at 6%.
- Consider two strategies:
  - Barbell: Invest 50% in A and 50% in C.
  - Bullet: 100% in Bond B.

Example

- Consider two types of yield curve shifts one year later:
  - Parallel shifts ranging between -200 BP and + 200 BP.
  - Yield curve shifts characterized by a flattening where for each change in Bond B (intermediate bond), Bond A increases 25 BP more and Bond C decreases by 25 BP less:

    $\Delta y^A = \Delta y^B + 25BP$ and $\Delta y^C = \Delta y^B - 25BP$

Example: Parallel Shifts

Yield Curve Change in BP	Value A	Value B	Value C	Return A	Return B	Return C	Return Barbell	Return Bullet	Difference
200	93.38	86.58	80.79	-0.62	-7.42	-13.21	-6.92	-7.42	0.50
150	94.98	89.70	85.06	0.98	-4.30	-8.94	-3.98	-4.30	0.31
100	96.61	92.98	89.66	2.61	-1.02	-4.34	-0.86	-1.02	0.16
50	98.29	96.41	94.63	4.29	2.41	0.63	2.46	2.41	0.05
25	99.14	98.18	97.26	5.14	4.18	3.26	4.20	4.18	0.02
0	100.00	100.00	100.00	6.00	6.00	6.00	6.00	6.00	0.00
-25	100.87	101.86	102.84	6.87	7.86	8.84	7.86	7.86	0.00
-50	101.75	103.77	105.80	7.75	9.77	11.80	9.78	9.77	0.01
-100	103.55	107.72	112.09	9.55	13.72	18.09	13.82	13.72	0.09
-150	105.38	111.87	118.89	11.38	17.87	24.89	18.14	17.87	0.27
-200	107.26	116.22	126.27	13.26	22.22	32.27	22.76	22.22	0.54

Bond Return = (Value-100) + 6
Bullet Return = .5(Bond Return for A) + .5(Bond Return for C)
Barbell Return = Bond Return for B

Note: Bullet portfolio has a duration of 8.31 (= (.5)(4.46) + (.5)(12.16)).
This is approximately the same as the duration of Bond B.

Example: Parallel Shifts

- Observations:
  - For different parallel shifts in yield curve, there is not much difference in returns on bullet portfolio and barbell. This is due to both having the same duration.
  - If one were expecting a significant downward shift in yield curve, Bond C with largest duration would give you greatest gains.
  - If one were expecting a significant upward shift in yield curve, Bond A with lowest duration would give you minimum loss.
  - Comment: Returns are consistent with duration as a measure of a bond’s price sensitivity to interest rate changes.

Example: Yield Curve Shifts Characterized by a Flattening

Yield Change for B in BP	Value A	Value B	Value C	Return A	Return B	Return C	Return Barbell	Return Bullet	Difference
200	92.59	86.58	82.89	-1.41	-7.42	-11.11	-6.26	-7.42	1.16
150	94.17	89.70	87.32	0.17	-4.30	-6.68	-3.25	-4.30	1.04
100	95.79	92.98	92.10	1.79	-1.02	-1.90	-0.05	-1.02	0.97
50	97.45	96.41	97.26	3.45	2.41	3.26	3.35	2.41	0.95
25	98.29	98.18	100.00	4.29	4.18	6.00	5.14	4.18	0.96
0	99.14	100.00	102.84	5.14	6.00	8.84	6.99	6.00	0.99
-25	100.00	101.86	105.80	6.00	7.86	11.80	8.90	7.86	1.04
-50	100.87	103.77	108.88	6.87	9.77	14.88	10.88	9.77	1.11
-100	102.64	107.72	115.42	8.64	13.72	21.42	15.03	13.72	1.31
-150	104.46	111.87	122.50	10.46	17.87	28.50	19.48	17.87	1.61
-200	106.32	116.22	130.19	12.32	22.22	36.19	24.25	22.22	2.03

$\Delta y^A = \Delta y^B + 25BP$ and $\Delta y^C = \Delta y^B - 25BP$

Example: Yield Curve Shifts Characterized by a Flattening

- Observation: In contrast to parallel shifts, there are differences between barbell and bullet portfolios when yield curve shift has a twist, even though they have the same durations.

Fundamental Valuation Strategies

- The objective of fundamental bond analysis is the same as that of fundamental stock analysis.
- It involves determining a bond’s intrinsic value and then comparing that value with bond’s market price.
- Active management of a bond portfolio using a fundamental strategy involves buying bonds that are determined to be underpriced and selling or avoiding those determined to be overpriced.

Fundamental Valuation Strategies

- A bond fundamentalist often tries to determine a bond's intrinsic value by estimating required rate for discounting bond's cash flows.
- This rate, R, depends on current level of interest rates as measured by risk-free rate, R_f, and bond's risk premiums: default risk premium (DRP), liquidity premium (LP), and option-adjusted spread (OAS):

$$\boxed{R = R_f + DRP + LP + OAS}$$

Fundamental Valuation Strategies

- Fundamentalists use various models to estimate various spreads. These include:
  - Regressions
  - Multiple discriminant analysis
  - Option pricing models

Yield Pickup Swaps

- A variation of fundamental bond strategies is a <u>yield pickup swap</u>. In a yield pickup swap, investors or arbitrageurs try to find bonds that are identical, but for some reason are temporarily mispriced, trading at different yields.
- Strategy: When two identical bonds trade at different yields, abnormal return can be realized by going long in underpriced (higher yield) bond and short in overpriced (lower yield) bond, then closing positions once prices of two bonds converge.

Yield Pickup Swaps

- The strategy underlying a yield pickup swap can be extended from comparing different bonds to comparing a bond with a portfolio of bonds constructed to have the same features.

Example

- Suppose a portfolio consisting of an AAA rating, 10-year, 10% coupon bond and an A rating, 5-year, 5% coupon bond is constructed such that it has same cash flows and features as an AA rating, 7.5-year, 7.5% coupon bond. If the AA quality bond and portfolio do not provide same yield, then an arbitrageur or speculator could form a yield pickup swap by taking opposite positions in portfolio and bond.
- A fundamentalist could also use this methodology for identifying underpriced bonds: buying all AA quality, 7.5-year, 7.5% coupon bonds with yields exceeding portfolio formed with those features.

Other Swaps: Callable/Noncallable Swap

- During periods of high interest rates, spread between yields on callable and noncallable bonds is greater than during periods of relatively low interest rates.
- Accordingly, if investors expect rates to decrease in the future, causing spread between callable and noncallable bonds to narrow, they could capitalize by forming a callable/noncallable bond swap: short in callable bond and long in noncallable one.
- To effectively apply this bond swap requires investors to not only forecast interest rate changes, but to also forecast changes in spread.

Passive Strategies

- Passive Strategies: Strategies that once they are formed do not require active management or changes.
- Objectives of passive management strategies include:
  - A <u>buy-and-hold</u> investing in bonds with specific maturities, coupons, and ratings until holding bonds to maturity.
  - Forming portfolios with returns that <u>mirror returns on a bond index</u>.
  - Constructing portfolios that ensure there are sufficient funds to meet future liabilities.

Passive Strategies

- We look at following passive strategies:
  - Bond Indexing
  - Cash-flow Matching
  - Classical Immunization

Bond Indexing

- Bond Indexing is constructing a bond portfolio whose returns over time replicate returns of a bond index.
- Indexing is often used by investment fund managers who believe that actively managed bond strategies do not outperform bond market indices.

Bond Indexing

First step in constructing a bond index fund is to select appropriate index. Bond indices can be

- General:
  - Shearson-Lehman Aggregate Index
  - Merrill-Lynch Composite Index
- Specialized:
  - Salomon Smith Barney's Global Government Bond Index.
- Customized:
  - Some investment companies offer their own customized index specifically designed to meet certain investment objectives.

Bond Market Indexes

Index	No. of Issues	Maturity	Size	Subindexes
U.S. Investment Grades Bond				
Lehman Brothers Aggregate	5000	Over 1 year	Over $100M	Government, corporate,
Merrill Lynch Composite	5000	Over 1 year	Over $50M	Government/corporate mortgage-backed, asset-backed
Salomon Smith Barney Composite	5000	Over 1 year	Over $50M	Government, corporate, government/corporate mortgage-backed, Bond Investment Grades, Treasury/Agency, corporate, mortgages
U.S. High Yield bond				
First Boston	423	All maturities	Over $75M	Composite and by ratings
Lehman Brother	624	Over 1 year	Over $100M	Composite and by ratings
Merrill Lynch	735	Over 1 year	Over $25M	Composite and by ratings
Salomon Smith Barney	300	Over 7 years	Over $50M	Composite and by ratings

Bond Market Indexes

Index	Number of Issues	Maturity	Size	Subindexes
Global Government Bond				
Lehman Brothers	800	Over 1 year	Over $200M	Composite and 13 countries in local currency and U.S.$
Merrill Lynch	9735	Over 1 year	Over $100M	Composite and 9 countries in local currency and U.S.$
J.P. Morgan	445	Over 1 year	Over $200M	Composite and 11 countries in local currency and U.S.$
Salomon Smith Barney	525	Over 1 year	Over $250M	Composite and 14 countries in local currency and U.S.$

Bond Indexing

- Second step in constructing a bond index fund is to determine how to replicate index's performance.
- One approach is to purchase all of bonds comprising index in the same proportion that they appear in the index. This is known as pure bond indexing or full-replication approach.
  - This approach would result in a perfect correlation between bond fund and index.
  - However, with some indices consisting of as many as 5000 bonds, transaction costs involved in acquiring all of bonds is very high.

Bond Indexing

- An alternative to selecting all bonds is to use <u>only a sample</u>.
  - By using a smaller size portfolio, transaction costs incurred in constructing index fund would be smaller.
  - However with fewer bonds, there may be less than perfect positive correlation between index and index fund.
  - Difference between returns on index and index fund are referred to as <u>tracking errors</u>.
- When a sample approach is used, index fund can be set up using an optimization approach to determine allocation of each bond in the fund such that it <u>minimizes tracking error</u>.

Bond Indexing: Cell Matching

- Another approach is to use a cell matching strategy.
- A cell matching strategy involves decomposing index into cells, with each cell defining a different mix of features of index (duration, credit rating, sector, etc.).
- Example: Suppose we decompose a bond index into
  - 2 durations ($D > 5$, $D < 5$)
  - 2 sectors (Corporate, Municipal)
  - 2 quality ratings (AA, A)

Bond Indexing: Cell Matching

- With these feature, eight cells can be formed:

C1 = D < 5, AAA, Corp
C2 = D < 5, AAA, Muni
C3 = D < 5, AA, Corp
C4 = D < 5, AA, Muni
C5 = D > 5, AAA, Corp
C6 = D > 5, AAA, Muni
C7 = D > 5, AA, Corp
C8 = D > 5, AA, Muni

- Index fund is constructed by selecting bonds to match each cell and then allocating funds to each type of bond based on each cell’s allocation.

Bond Indexing: Cell Matching

- One cell matching approach is to base the cell identification on just <u>two features</u> such as durations and sectors or durations and ratings.
  - <u>Duration/sector index</u> is formed by matching amounts of index's durations that make up each of sectors. This requires estimating duration for each sector comprising the index and determining each sector's percentage of value to the index.
  - <u>Duration/rating index</u> is formed by determining percentages of value and average durations of each rating group making up the index.

Duration/Sector and Duration/Rating

Sector	Percentage of Value	Duration
Treasury	20%	4.50
Federal Agency	10%	3.25
Municipals	15%	5.25
Corporate Industry	15%	6.00
Corporate Utility	10%	6.25
Corporate Foreign	10%	5.55
Sovereign	10%	5.75
Asset-Backed	10%	6.25
	100%	Weighted Average = 5.29
Rating	**Percentage of Value**	**Duration**
AAA	60%	5.25
AA	15%	5.35
A	10%	5.25
BBB	5%	5.65
BB	5%	5.25
B	5%	5.30
	100%	Weighted Average = 5.29

Bond Indexing: Enhanced Bond Indexing

- A variation of straight indexing is enhanced bond indexing. It allows for minor deviations in order to try attain a return better than the index.
  - Usually deviations are in ratings or sectors, and not in durations, and they are based on some active management strategy.
- Example: A fund indexed primarily to Merrill-Lynch composite but with more weight given to lower crediting bonds based on an expectation of an improving economy would be an enhanced index fund combining indexing and sector rotation.

Cash Flow Matching

- A cash flow matching strategy involves constructing a bond portfolio with cash flows that match outlays of liabilities. It is also referred to as a dedicated portfolio strategy.
- One method that can be used for cash flow matching is to start with final liability for time T and work backwards.

Cash Flow Matching: Method

1. For last period, one would select a bond with a principal (F_T) and coupon (C_T) that matches the amount of that final liability (L_T):

$$L_T = F_T + C_T$$
$$L_T = F_T(1 + C^{RO})$$
$$\text{where}: C^{R0} = C_T / F_T$$

- To meet this liability, one could buy $L_T /(1+ C^{R0})$ of par value of bonds maturing in T periods.

Cash Flow Matching: Method

2. To match liability in period T-1, one would need to select bonds with a principal of F_{T-1} and coupon C_{T-1} (or coupon rate of $C^{R1} = C_{T-1}/F_{T-1}$) that is equal to projected liability in period T-1 (L_{T-1}) less coupon amount of C_T from T-period bonds selected:

$$\boxed{\begin{aligned} L_{T-1} - C_T &= F_{T-1} + C_{T-1} \\ L_{T-1} - C_T &= F_{T-1}(1 + C^{R1}) \end{aligned}}$$

- To meet this liability, one could buy $(L_{T-1}-C_T)/(1+C^{R1})$ of par value of bonds maturing in T-1 periods.

Cash Flow Matching: Method

3. To match liability in period T-2, one would need to select bonds with a principal of $F_{T\text{-}2}$ and coupon $C_{T\text{-}2}$ (or coupon rate of $C^{R2} = C_{T\text{-}2}/ F_{T\text{-}2}$) that is equal to projected liability in period T-2 ($L_{T\text{-}2}$) less coupon amounts of C_T and $C_{T\text{-}1}$ from T-period and T-1-period bonds selected:

$$\boxed{\begin{aligned} L_{T-2} - C_T - C_{T-1} &= F_{T-2} + C_{T+2} \\ L_{T-2} - C_T - C_{T-1} &= F_{T-2}(1 + C^{R2}) \end{aligned}}$$

- To meet this liability, one could buy
  $(L_{T\text{-}2} - C_T - C_{T\text{-}1})/(1+ C^{R2})$ of par value of bonds maturing in T-2 periods.

Cash Flow Matching: Example

- Example: A simple cash-flow matching case is presented in the following exhibits.
- It shows matching of liabilities of \$4M, \$3M, and \$1M in years 3, 2, and 1 with 3-year, 2-year, and 1-year bonds each paying 5% annual coupons and selling at par.

Year	1	2	3
Liability	\$1M	\$3M	\$4M

Cash Flow Matching: Example

Bonds	**Coupon Rate**	**Par**	**Yield**	**Market Value**	**Liability**	**Year**
3-Year	5%	100	5%	100	$4M	3
2-year	5%	100	5%	100	$3M	2
1-year	5%	100	5%	100	$1M	1

Cash Flow Matching: Example

Cash-Flow Matching Strategy:

- \$4M liability at the end of year 3 is matched by buying \$3,809,524 worth of three-year bonds:

  $$\$3,809,524 = \$4,000,000/1.05$$

- \$3M liability at the end of year 2 is matched by buying \$2,675,737 of 2-year bonds:

  $$\$2,675,737 = (\$3,000,000 - (.05)(\$3,809,524))/1.05.$$

- \$1M liability at the end of year 1 is matched by buying \$643,559 of 1-year bonds:

  $$\$643,559 = (\$1,000,000 - (.05)(\$3,809,524) - (.05)(\$2,675,737))/1.05$$

Cash Flow Matching: Example

1	2	3	4	5	6
Year	**Total Bond Values**	**Coupon Income**	**Maturing Principal**	**Liability**	**Ending Balance (3) + (4) – (5)**
1	$7,128,820	$356,441	$643,559	$1,000,000	0
2	$6,485,261	$324,263	$2,675,737	$3,000,000	0
3	$3,809,524	$190,476	$3,809,524	$4,000,000	0

Cash Flow Matching: Features

- With cash-flow matching the basic goal is to construct a portfolio that will provide a stream of payments from coupons, sinking funds, and maturing principals that will match liability payments.
- A dedicated portfolio strategy is subject to some minor market risk given that some cash flows may need to be reinvested forward.

Cash Flow Matching: Features

- It also can be subject to <u>default risk</u> if lower crediting bonds are purchased.
- The biggest risk with cash-flow matching strategies is that bonds selected to match forecasted liabilities may be <u>called</u>, forcing investment manager to purchase new bonds yielding lower rates.

Classical Immunization

- Immunization is a strategy of minimizing market risk by selecting a bond or bond portfolio with a duration equal to horizon date.
- For liability management cases, liability payment date is liability's duration, D_L.
- Immunization can be described as a duration-matching strategy of equating duration of bond or asset to duration of liability.

Classical Immunization

- When a bond's duration is equal to liability's duration, direct interest-on-interest effect and inverse price effect exactly offset each other.
- As a result, rate from investment (ARR) or value of investment at horizon or liability date does not change because of an interest rate change.

Classical Immunization: History

- The foundation for bond immunization strategies comes from a 1952 article by F.M. Redington:
- He argued that a bond investment position could be immunized against interest rate changes by matching durations of the bond and the liability.
- Redington’s immunization strategy is referred to as classical immunization.

Classical Immunization: Example

- A fund has a single liability of $1,352 due in 3.5 years, D_L = 3.5 years, and current investment funds of $968.30.
- Current yield curve is flat at 10%.
- Immunization Strategy: Buy bond with Macaulay's duration of 3.5 years.
  - Buy 4-year, 9% annual coupon at YTM of 10% for P_0 = $968.30. This Bond has D = 3.5.
  - This bond has both a duration of 3.5 years and is worth $968.50, given a yield curve at 10%.

Classical Immunization: Example

- If fund buys this bond, then any parallel shift in yield curve in very near future would have price and interest rate effects that exactly offset each other.
- As a result, cash flow or ending wealth at year 3.5, referred to as <u>accumulation value or target value</u>, would be exactly $1,352.

Classical Immunization: Example

DURATION-MATCHING

Ending Values at 3.5 Years Given Different Interest Rates for 4- Year, 9% Annual Coupon Bond with Duration of 3.5

Time (yr)	9%	10%	11%
1	$\$\ 90(1.09)^{2.5} = \$111.64$	$\$\ 90(1.10)^{2.5} = \$114.21$	$\$\ 90(1.11)^{2.5} = \$116.83$
2	$90(1.09)^{1.5} = \$102.42$	$90(1.10)^{1.5} = \$103.83$	$90(1.11)^{1.5} = \$105.25$
3	$90(1.09)^{.5} = \$\ 93.96$	$90(1.10)^{.5} = \$\ 94.39$	$90(1.11)^{.5} = \$\ 94.82$
3.5	$1090/(1.09)^{.5} = \underline{\$1044.03}$	$1090/(1.10)^{.5} = \underline{\$1039.27}$	$1090/(1.11)^{.5} = \underline{\$1034.58}$
Target Value	\$1352	\$1352	\$1352

Classical Immunization

- Note that in addition to matching duration, immunization also requires that initial investment or current market value of assets purchased to be equal or greater than present value of liability using current YTM as a discount factor.
- In this example, present value of $1,352 liability is $968.50 (= $1,352/(1.10)$^{3.5}$), which equals current value of bond and implies a 10% rate of return.

Classical Immunization

- Redington's duration-matching strategy works by having offsetting price and reinvestment effects.
- In contrast, a <u>maturity-matching strategy</u> where a bond is selected with a maturity equal to horizon date has no price effect and therefore no way to offset reinvestment effect.
- This can be seen in next exhibit where unlike duration-matched bond, a 10% annual coupon bond with a maturity of 3.5 years has different ending values given different interest rates.

Classical Immunization: Example

MATURITY-MATCHING

Ending Values at 3.5 Years Given Different Interest Rates for 10% Annual Coupon Bond with Maturity of 3.5 Years

Time (yr)	9%	10%	11%
1 2 3 3.5	$ $100(1.09)^{2.5}$ = $124.04 $100(1.09)^{1.5}$ = $113.80 $100(1.09)^{.5}$ = $104.40 1050 = $1050 $1392	$ $100(1.10)^{2.5}$ = $126.91 $100(1.10)^{1.5}$ = $115.37 $100(1.10)^{.5}$ = $ 104.88 1050 = $1050 $1397	$ $100(1.11)^{2.5}$ = $129.81 $100(1.11)^{1.5}$ = $116.95 $100(1.11)^{.5}$ = $ 105.36 1050 = $1050 $1402

Immunization and Rebalancing

- Fisher and Weil (1971) compared duration-matched immunization with maturity-matched ones under a number of interest rate scenarios. They found: Duration-matched positions were closer to their initial YTM than maturity-matched strategies, but that they were not absent of market risk. They offered two reasons for presence of market risk with classical immunization.
  - Shifts in yield curves were not parallel.
  - Immunization only works when duration of assets and liabilities are match <u>at all times</u>.
- To achieve immunization, they argued that <u>duration of bond must be equal to remaining time in horizon period</u>.

Immunization and Rebalancing

- Durations of assets and liabilities change with both time and yield changes:

  (1) Duration declines more slowly than terms to maturity.

    - In our earlier example, our 4-year, 9% bond with a Maculay duration of 3.5 years when rates were 10%, one year later would have duration of 2.77 years with no change in rates.

  (2) Duration inversely changes with interest rate changes.

Immunization and Rebalancing

- Thus, a bond and liability that currently have same durations will not necessarily be equal as time passes and rates change.
- Immunized positions require active management, called <u>rebalancing</u>, to ensure that duration of bond position is always equal to remaining time to horizon.
- Rebalancing Strategies when $D_B \neq D_L$
  - Sell bond and buy new one
  - Add a bond to change D_p
  - Reinvest cash flows differently
  - Use futures or options.

Bond Immunization: Focus Strategy

- For a single liability, immunization can be attained with a focus strategy or a barbell strategy.
- In a <u>focus strategy</u>, a bond is selected with a duration that matches duration of liability or a bullet approach is applied where a portfolio of bonds are selected with all bonds close to desired duration.
- Example: If duration of liability is 4 years, one could select a bond with a 4-year duration or form a portfolio of bonds with durations of 4 and 5 years.

Bond Immunization: Barbell Strategy

- In a barbell strategy, duration of liability is matched with a bond portfolio with durations more at extremes.
- Example: For a duration liability of 4 years, an investor might invest half of his funds in a bond with a two-year duration and half in a bond with a six-year duration.
- Note: The problem with barbell strategy is that it may not immunize position if shift in the yield curve is not parallel.

Bond Immunization: Immunizing Multiple-Period Liabilities

- For multiple-period liabilities, bond immunization strategies can be done by either:
  - Matching duration of each liability with appropriate bond or bullet bond portfolio.
  - Constructing a portfolio with a duration equal to weighted average of durations of liabilities (D^P_L).
- <u>Example</u>: If a fund had multiple liabilities of $1M each in years 4, 5, and 6, it could either:
  - invest in three bonds, each with respective durations of 4 years, 5 years, and 6 years, or
  - it could invest in a bond portfolio with duration equal to 5 years:

$$\boxed{D_L^P = \left(\frac{\$1M}{\$3M}\right)*4 + \left(\frac{\$1M}{\$3M}\right)*5 + \left(\frac{\$1M}{\$3M}\right)*6 = 5yrs}$$

Bond Immunization: Immunizing Multiple-Period Liabilities

- The portfolio approach is relatively simple to construct, as well as to manage.
- Bierwag, Kaufman, and Tuevs (1983) found that matching portfolio's duration of assets with duration of liabilities does not always immunize positions.
- Thus, for multiple-period liabilities, the best approach is generally considered to be one of immunizing each liability.
- As with single liabilities, this also requires rebalancing each immunized position.

Combination Matching

- An alternative to frequent rebalancing is a combination matching strategy:
- Combination Matching:
  - Use cash flow matching strategy for early liabilities
  - Immunization for longer-term liabilities.

Immunization: Surplus Management

- The major users of immunization strategies are pensions, insurance companies, and commercial banks and thrifts.
  - Pensions and life insurance companies use multiple-period immunization to determine investments that will match a schedule of forecasted payouts.
  - Insurance companies, banks and thrifts, and other financial corporations also use immunization concepts for <u>surplus management</u>.

Immunization: Surplus Management

- Surplus management refers to managing surplus value of assets over liabilities.
- This surplus can be measured as economic surplus, defined as difference between market value of assets and present value of liabilities:

$$\boxed{Economic\ \ Surplus = V_A - V_L}$$

- Example: A pension with a bond portfolio currently valued at $200M and liabilities with a present value of $180M would have an economic surplus of $20M.

Immunization: Surplus Management

- An economic surplus can change if interest rates change.
- Direction and extent of the change depends on surplus's duration gap, which is the difference in duration of assets and duration of liabilities.
  - If duration of bond portfolio > duration of liabilities, then economic surplus will vary inversely to interest rates.
  - If duration of bond portfolio < duration of liabilities, then surplus value will vary positively with interest rates.
  - If duration of bond portfolio = duration of liabilities, then surplus will be invariant to rate changes – an immunized position.

Immunization and Surplus Management

- Duration Gap and Economic Surplus and Rate Relation:

$$\boxed{\text{Economic Surplus} = V_A - V_L}$$

$$\boxed{D_A > D_L \Rightarrow \frac{\Delta(\text{Ec Sur})}{\Delta r} < 0}$$

$$\boxed{D_A < D_L \Rightarrow \frac{\Delta(\text{Ec Sur})}{\Delta r} > 0}$$

$$\boxed{D_A = D_L \Rightarrow \frac{\Delta(\text{Ec Sur})}{\Delta r} = 0} \Rightarrow \boxed{\text{immunized}}$$

Bond Immunization: Surplus Management

Example:

$$D_A = 7,\ D_L = 5,\ V_A = \$200M,\ V_L = \$180M,$$
$$\text{Economic Surplus} = \$20M$$

$$\text{Assume } \Delta r = -1\%:$$
$$V_A \uparrow \text{ by } 7\% \Rightarrow V_A = \$200M(1.07) = \$214M$$
$$V_L \uparrow \text{ by } 5\% \Rightarrow V_L = \$180M(1.05) = \$189M$$
$$\text{Econmic Surplus} \uparrow \text{ by } \$5M \text{ to } \$25M$$
$$(= \$214M - \$189M = \$25M)$$

Immunization: Duration Gap Analysis by Banks

- Duration gap analysis is used by banks and other deposit institutions to determine changes in market value of institution's net worth to changes in interest rates.
- With gap analysis, a bank's asset sensitivity and liability sensitivity to interest rate changes is found by estimating Macaulay's duration for assets and liabilities and then using formula for modified duration to determine percentage change in value to a percentage change in interest rates.

$$\%\Delta P = -(\text{Macaulay's Duration})\ (\Delta R/(1+R)$$

Immunization: Duration Gap Analysis by Banks

- Example: Consider a bank with following balance sheet:
  - Assets and liabilities each equal to $150M.
  - Weighted Macaulay duration of 2.88 years on its assets.
  - Weighted duration of 1.467 on its liabilities.
  - Interest rate level of 10%.

Immunization:
Duration Gap Analysis by Banks

Assets	Amount in millions of $	Macaulay Duration	Weighted Duration	Liabilities	Amount in millions of $	Macaulay Duration	Weighted Duration
Reserves	10	0.0	0.000	Demand Deposits	15	1.0	0.100
Short-Term Securities	15	0.5	0.050	Nonnegotiable Deposits	15	0.5	0.050
Intermediate Securities	20	1.5	0.200	Certificates of Deposit	35	0.5	0.117
Long-Term Securities	20	5.0	0.667	Fed Funds	5	0.0	0.000
Variable-Rate Mortgages	10	0.5	0.033	Short-Term Borrowing	40	0.5	0.133
Fixed-Rate Mortgages	25	6.0	1.000	Intermediate-Term Borrowing	40	4.0	1.067
Short-Term Loans	20	1.0	0.133		150		1.467
Intermediate Loans	30	4.0	0.800				
	150		2.88				

Immunization: Duration Gap Analysis by Banks

- Bank's positive duration gap of 1.413 suggests an inverse relation between changes in rates and net worth.
  - If interest rate were to increase from 10% to 11%, bank's asset value would decrease by 2.62% and its liabilities by 1.33%, resulting in a decrease in bank's net worth of \$1.93M:

$$\%\Delta P = -(\text{Macaulay's Duration})\,(\Delta R/(1+R))$$

$$\text{Assets: } \%\Delta P = -(2.88)\,(.01/1.10) = -.0262$$

$$\text{Liabilities: } \%\Delta P = -(1.467)\,(.01/1.10) = -.0133$$

$$\text{Change in Net Worth} = (-.0262)(\$150M) - (-.0133)(\$150M) = -\$1.93M$$

  - If rates were to decrease from 10% to 9%, then bank's net worth would increase by \$1.93M.

Immunization: Duration Gap Analysis by Banks

- With a positive duration gap an increase in rates would result in a loss in bank’s capital and a decrease in rates would cause bank’s capital to increase.
- If bank’s duration gap had been negative, then a direct relation would exist between bank’s net worth and interest rates.
- If gap were zero, then its net worth would be invariant to interest rate changes.
- As a tool, duration gap analysis helps bank’s management ascertain the degree of exposure that its net worth has to interest rate changes.