Investments

Lecture 7

Factor Models

- It is difficult, sometimes even impossible, to construct the efficient frontier without making further assumptions about the **return-generating process.**
- One process we've already seen is the market model (single index model):

$$\tilde{r}_i = \alpha_{iI} + \beta_{iI}\tilde{r}_I + \tilde{\varepsilon}_{iI}$$

Factor model

- A small number of underlying basic sources of randomness: factors
- Note that the factors can be anything
  - They can be macroeconomic variables (inflation, GDP-growth, ...)
  - They can be average industry returns
  - They can be statistical factors, which do not have any economic meaning

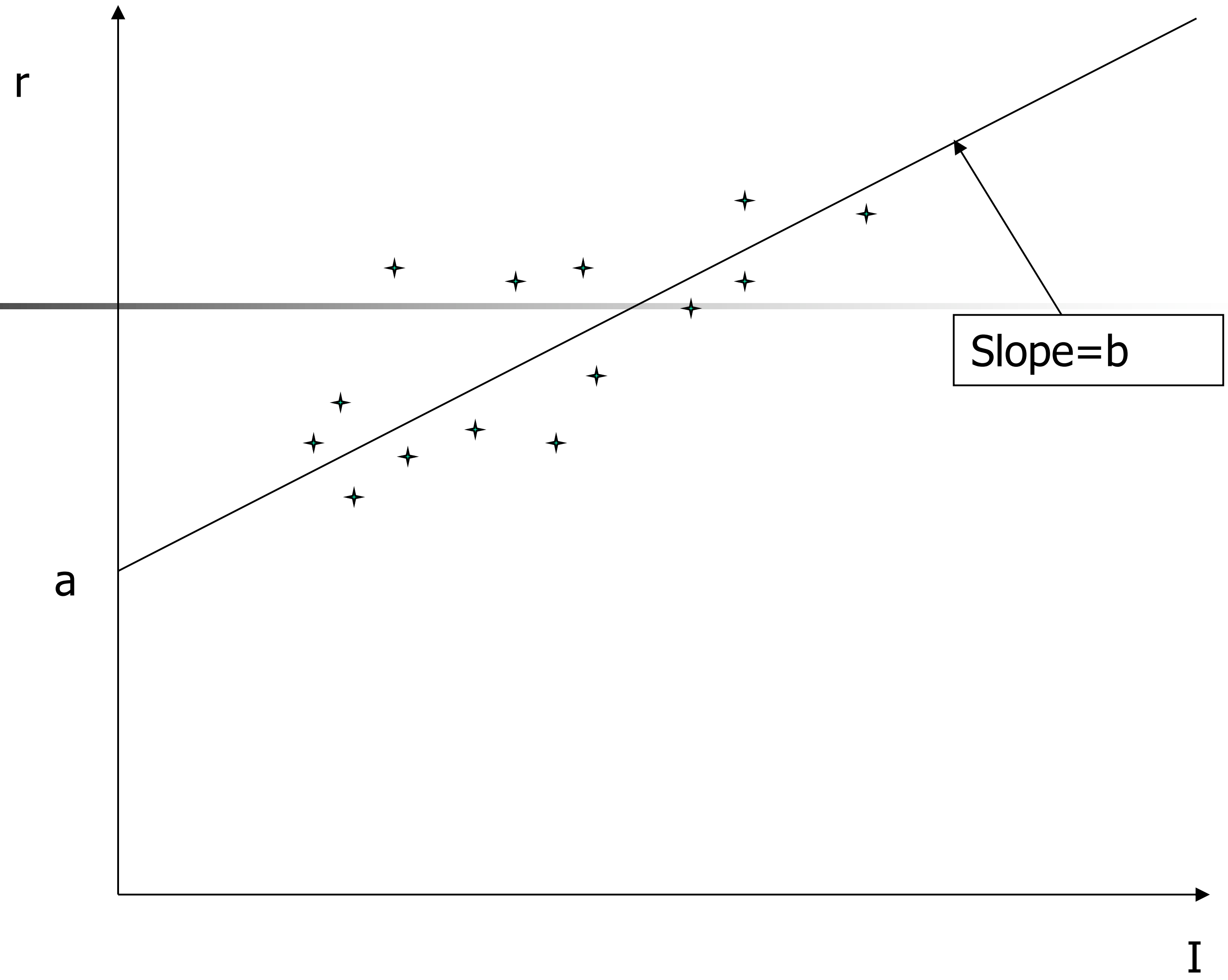
Single-factor model

$$\tilde{r}_i = a_i + b_i \tilde{I} + e_i$$

$$where$$

$$E[e_i] = 0$$

$$E[e_i(\tilde{I} - \bar{I})] = 0$$


r
a
Slope=b
I

Statistical Properties of Single-factor Model

$$\bar{r}_i = a_i + b_i \bar{I}$$

$$\sigma_i^2 = b_i^2 \sigma_I^2 + \sigma_{e_i}^2$$

$$\sigma_{ij} = b_i b_j \sigma_I^2, \qquad i \neq j$$

$$b_i = \mathrm{cov}(r_i, I) / \sigma_I^2$$

Multifactor model

$$R_i = a_i + b_{i1} I_1 + b_{i2} I_2 + \ldots + b_{in} I_n + e_i$$

$$E(e_i e_j) = 0,\ i \neq j$$

$$E[e_i(I_j - \bar{I}_j)] = 0$$

- Factors could be correlated.

$$\mathrm{cov}(r_i, I_1) = b_{i1}\sigma_{I_1}^2 + b_{i2}\sigma_{I_1 I_2}$$

$$\mathrm{cov}(r_i, I_2) = b_{i2}\sigma_{I_2}^2 + b_{i1}\sigma_{I_1 I_2}$$

Stock Return Characteristics and 2-Factor Models

- Expected Return

$$E(\tilde{r}_i) = \alpha_i + b_{i1}E(I_1) + b_{i2}E(I_2)$$

- Variance

$$\sigma_i^2 = b_{i1}^2\sigma_{I_1}^2 + b_{i2}^2\sigma_{I_2}^2 + 2b_{i1}b_{i2}\operatorname{cov}(I_1, I_2) + \sigma_{e_i}^2$$

Stock Return Characteristics

- Covariance

$$\sigma_{ij} = b_{i1}b_{j1}\sigma_{I_1}^2 + b_{i2}b_{j2}\sigma_{I_2}^2$$

$$+ (b_{i1}b_{j2} + b_{i2}b_{j1})\,\mathrm{cov}(I_1, I_2)$$

Portfolio Characteristics

- Factor loadings, etc. are weighted average of component loadings:

$$\beta_{pj} = \sum_{i=1}^{N} X_i \beta_{ij}$$

Example

Calculate the expected return and standard deviation of the following portfolio:

Security	Zero Factor	Factor 1 Sensitivity	Factor 2 Sensitivity	Nonfactor Risk	Proportion
A	2%	.3	2	196	.7
B	3	.5	1.8	100	.3

Factor 1 (2) has an expected value of 15% (4%) and a standard deviation of 20% (5%). The factors are uncorrelated.

Selection of factors

- External factors: GDP, CPI, …
- Extracted factors:
  - Constructed from returns of securities
  - Industrial factors
  - Complex ways
- Firm characteristics: effective additions

Example:
3-Factor Model

- 3-Factor Model (Fama-French)
  - 1st factor: Market Premium ($r_M - r_F$)
    - The difference between the market return and the risk-free rate
  - 2nd factor: Small-Stock Premium ($r_S - r_L$)
    - The difference between the return of small and large companies, measured according to their market capitalization
  - 3nd factor: Value-Stock Premium ($r_V - r_G$)
    - The difference between the return of value and growth stocks. Value stocks are mature companies and growth stocks are companies with large growth potential

Example:
3-Factor Model

- You estimate the 3-Factor model for a mutual fund. You get the following betas:
  - Market beta: $\beta_M = 1.30$
  - Size beta: $\beta_S = 0.45$
  - Value beta: $\beta_V = -0.25$
- What investment strategy does the fund follow?

Example:
3-Factor Model

- In one month the fund has an excess return of 12% above the risk-free interest rate
- The factor returns are as follows:
  - Market premium: $r_M - r_F = 10\%$
  - Small-stock premium: $r_S - r_L = -2\%$
  - Value-stock premium: $r_V - r_G = -3\%$
- What is the abnormal return (the alpha) of the fund?

Why are Factor Models Useful?

- Imposing a factor structure makes estimating the efficient frontier possible.
  - If you can identify portfolios that have only factor risk in them, they can be used to build the efficient frontier.
  - The returns on these portfolios are called **factor mimicking returns.**

Example: Building "factor mimicing portfolios"

- Suppose three investments x, y, and z have the following returns:

$$\tilde{r}_x = 0.08 + 2\tilde{F}_1 + 3\tilde{F}_2 + \varepsilon_x$$

$$\tilde{r}_y = 0.10 + 3\tilde{F}_1 + 2\tilde{F}_2 + \varepsilon_y$$

$$\tilde{r}_z = 0.10 + 3\tilde{F}_1 + 5\tilde{F}_2 + \varepsilon_z$$

- To form the factor 1 portfolio solve the system:

$$\left.\begin{array}{rrrl} 2X_x + & 3X_y + & 3X_z = & 1 \\ 3X_x + & 2X_y + & 5X_z = & 0 \\ X_x + & X_y + & X_z = & 1 \end{array}\right\}$$

- This gives: $X_x = 2; \quad X_y = 1/3; \quad X_z = -4/3$

- Hence (what are we assuming?): $\tilde{R}_{p1} = 0.06 + \tilde{F}_1 + 0\tilde{F}_2$

and the factor premium is: $\lambda_1 = 0.06 + \overline{F}_1 - r_f$

Building the Efficient Frontier using Factor-Mimicking Portfolios

- When the efficient frontier has only factor risk, we can restrict our attention to the factor-mimicking returns only.

CAPM as a factor model

- The assumption of efficiency
- CAPM: Market Portfolio is THE mean-variance efficient portfolio (in terms of Sharpe Ratio)
- Multi-factor Model: Not necessarily

Another Way of Writing Factor Model

$$\tilde{r}_i = E(\tilde{r}_i) + b_{i,1}\tilde{f}_1 + b_{i,2}\tilde{f}_2 + \ldots + \tilde{e}_i$$

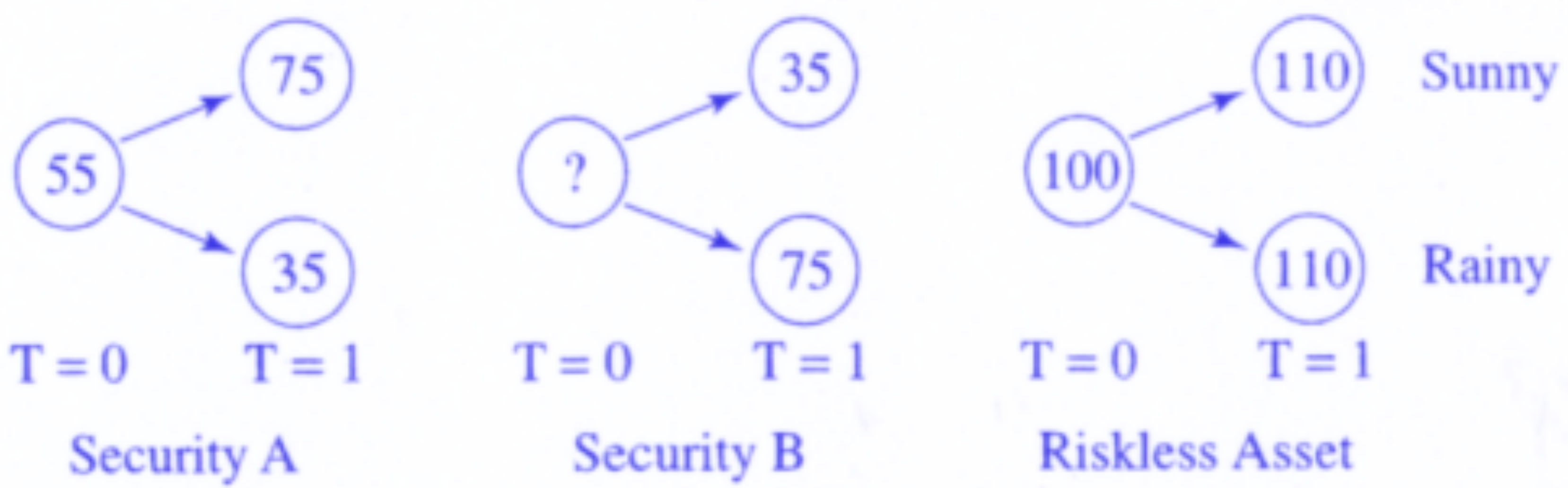
$$E(\tilde{f}_j) = 0$$

- This means that, instead of defining an *f* directly as economic growth, we would have to define it as the deviation of economic growth from what was expected.

Arbitrage Pricing Theory (APT)

- Arbitrage: **<u>the law of one price</u>**
- Less strict assumptions:
  - Utility function: not necessary
  - More than mean and variance
  - Still homogeneous belief
  - Return generating process
  - Infinite number of securities

Example


55
75
35
T = 0
T = 1
Security A
?
35
75
T = 0
T = 1
Security B
100
110
110
Sunny
Rainy
T = 0
T = 1
Riskless Asset

Arbitrage

- An arbitrage is a trading strategy that generates only positive cash flows
  - No initial investment is required upfront
  - Strictly positive cash flows occur either today or sometimes in the future
  - Under no conditions can you lose any money
  - Arbitrages follow from mispricings of assets

Arbitrage

- Pricing restrictions in the APT come from the Absence of Arbitrage.
- Absence of Arbitrage in Financial markets means that no security exists which has a price $<= 0$ and a payoff >0.
    - Also, no security can be created which has this property.

Arbitrage (cont.)

- This rule also implies that:
  - Two securities that always have the same payoff must have the same price.
  - No security exists which has a zero price and a strictly positive payoff.
- In an efficiently functioning financial market arbitrage opportunities cannot exist (for very long).

Arbitrage (cont.)

- Unlike for equilibrium rules such as the CAPM, arbitrage rules require only that there be one intelligent investor in the economy.
  - This is why derivatives security pricing models do a better job of predicting prices than equilibrium based models.
  - we will see how to apply the same concept to pricing portfolios of assets.
- If arbitrage rules are violated, then unlimited risk-free profits are possible.

APT

$$\tilde{R}_i = a_i + b_{i1}\tilde{I}_1 + b_{i2}\tilde{I}_2 + \ldots + b_{in}\tilde{I}_n + e_i$$

$$E(e_i e_j) = 0,\ i \neq j$$

$$E[e_i(I_j - \bar{I}_j)] = 0$$

Independence assumptions!

Trading Strategy

- Suppose asset returns are given by:

$$r_i - r_F = \alpha_i + \beta_i \left[r_M - r_F \right] + \varepsilon_i$$

- You have the following assets:
  - Asset S: $E(r_S)=15\%$ $\beta_S=1$ $\sigma_S=40\%$
  - Asset D: $E(r_D)=5\%$ $\beta_D=1$ $\sigma_D=40\%$
  - Market: $E(r_M)=10\%$ $\beta_M=1$ $\sigma_M=20\%$
  - T-Bills: $r_F=2\%$ $\beta_F=0$ $\sigma_F=0\%$
- What could you do?

Trading Strategy

- Buy S and short-sell D
  - If we ignore margin requirements, then you do not need to put any money down. You just use the proceeds from the short-sale of D to buy S
  - What is the expected return and the variance of your portfolio?

Trading Strategy

- Buy S and short-sell D
  - The return of this portfolio is:

$$
\begin{aligned}
\tilde{r}_P &= \tilde{r}_S - \tilde{r}_D \\
&= \alpha_S - \alpha_D + [\beta_S - \beta_D][\tilde{r}_M - r_F] + \tilde{\varepsilon}_S - \tilde{\varepsilon}_D \\
&= \alpha_S - \alpha_D + \tilde{\varepsilon}_S - \tilde{\varepsilon}_D \\
&= 0.1 + \tilde{\varepsilon}_S - \tilde{\varepsilon}_D
\end{aligned}
$$

Example of Trading Strategy

- The expected return is:

$$E(r_P) = \alpha_S - \alpha_D = 10\%$$

- The variance and standard deviation are:

$$Var(r_P) = Var(\varepsilon_S - \varepsilon_D) = 2Var(\varepsilon) = 0.24$$

$$\sigma_P = \sqrt{Var(r_P)} = \sqrt{0.24} = 49\%$$

  - Note that the variance of the firm-specific risk is given by the variance of the individual stocks minus the systematic variance:

$$Var(\varepsilon_i) = \sigma_i^2 - \beta^2 \sigma_M^2 = 0.4^2 - 1^2 \times 0.2^2 = 0.12$$

Example of Trading Strategy

- Note that this strategy is very risky with an expected return of 10% and a standard deviation of 49%
- What happens to the risk if we include additional stocks?
  - Suppose we have two stocks of type S and two stocks of type D
  - Buy half of each S-stock and short-sell half of each D-stock

Example of Trading Strategy

- This portfolio has the following returns:

$$r_P = 0.5\left(r_{S,1} + r_{S,2}\right) - 0.5\left(r_{D,1} + r_{D,2}\right)$$

$$= \alpha_S - \alpha_D + 0.5\left(\varepsilon_{S,1} + \varepsilon_{S,2} - \varepsilon_{D,1} - \varepsilon_{D,2}\right)$$

$$E\left(r_P\right) = \alpha_S - \alpha_D = 10\%$$

$$Var\left(r_P\right) = Var\left(0.5\left(\varepsilon_{S,1} + \varepsilon_{S,2} - \varepsilon_{D,1} - \varepsilon_{D,2}\right)\right)$$

$$= 0.25 \times Var\left(\varepsilon_{S,1} + \varepsilon_{S,2} - \varepsilon_{D,1} - \varepsilon_{D,2}\right)$$

$$= 0.25 \times 4 \times Var\left(\varepsilon\right) = Var\left(\varepsilon\right) = 0.12$$

$$\sigma_P = \sqrt{Var\left(r_P\right)} = \sqrt{0.12} = 35\%$$

Trading Strategy

- By introducing additional assets we can decrease the firm-specific risk through diversification
- By short-selling asset D and by buying asset S we completely eliminate systematic risk
- Thus, the total risk of this trading strategy approaches zero if the number of available stocks increases

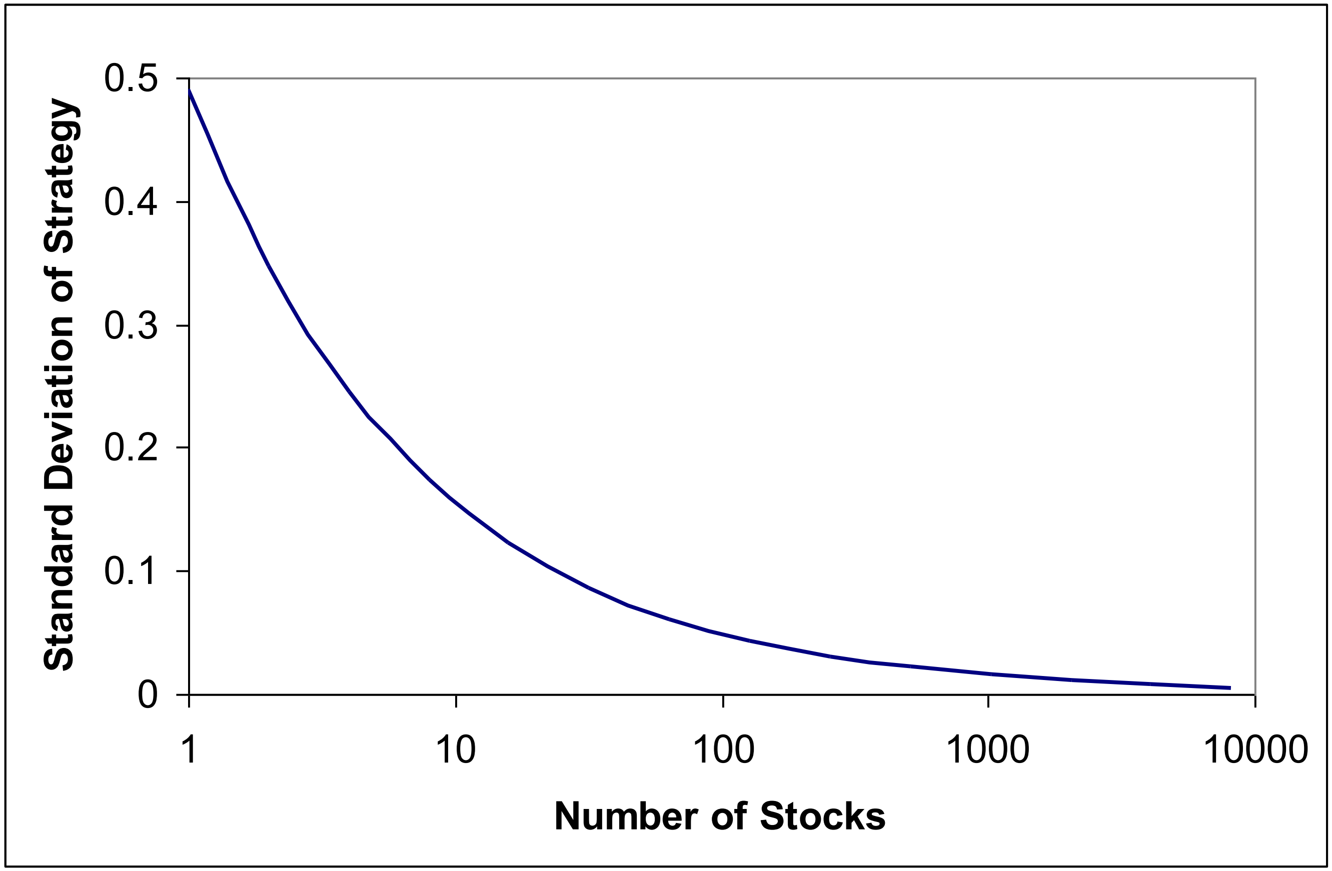
Example of Trading Strategy

- You can show that the variance of the portfolio with N assets is:

$$Var(r_P) = \frac{2}{N} Var(\varepsilon)$$

- Thus, as the number of assets increases, the risk of our portfolio will go towards zero

Standard Deviation of Trading Strategy



Trading Strategy

- If we have 10,000 stocks of each type, then the standard deviation of the portfolio decreases to just 0.5%
- Now, this strategy looks very attractive and many investors will undertake this trading strategy
- What happens to the prices of the stocks and their alphas?

Arbitrage Pricing

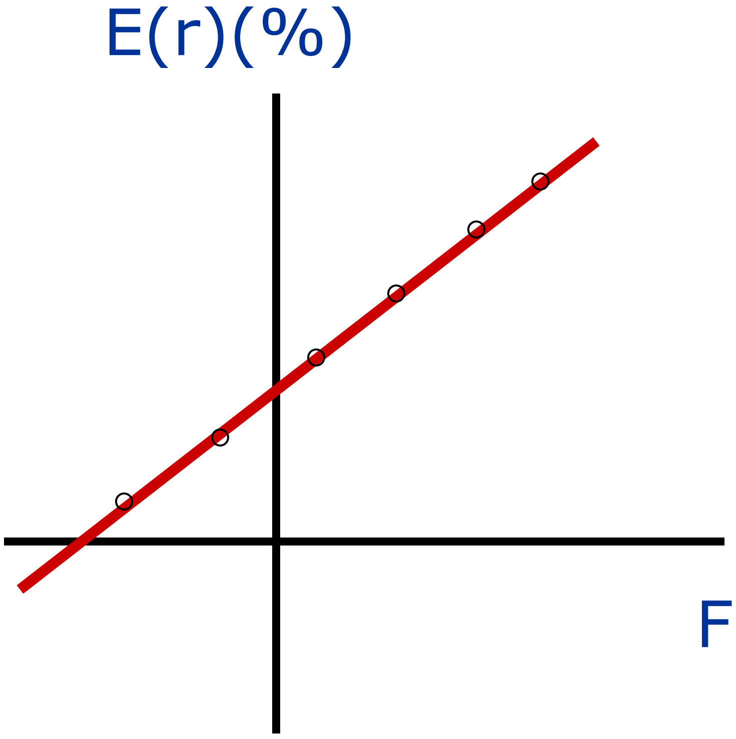
- In equilibrium, such attractive trading strategies are not possible
- What happens to the stock prices and the alphas if many investors follow such trading strategies?

Arbitrage Pricing

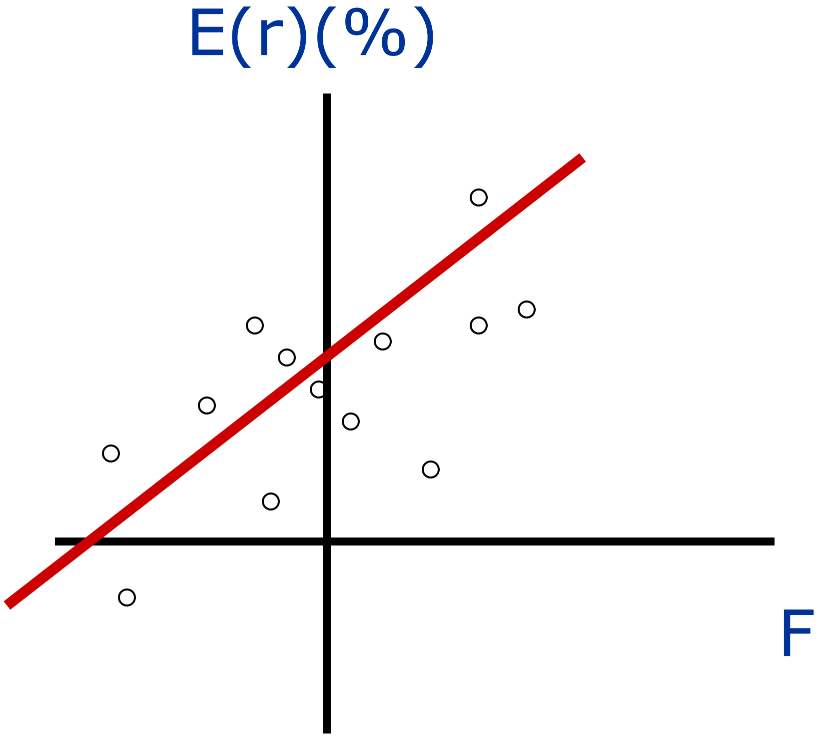
- The alpha of a large number of assets can not deviate from zero in equilibrium
- This implies that almost all assets have zero alphas:

$$r_i - r_F = \beta_i [r_M - r_F] + \varepsilon_i$$

Portfolio & Individual Security Comparison

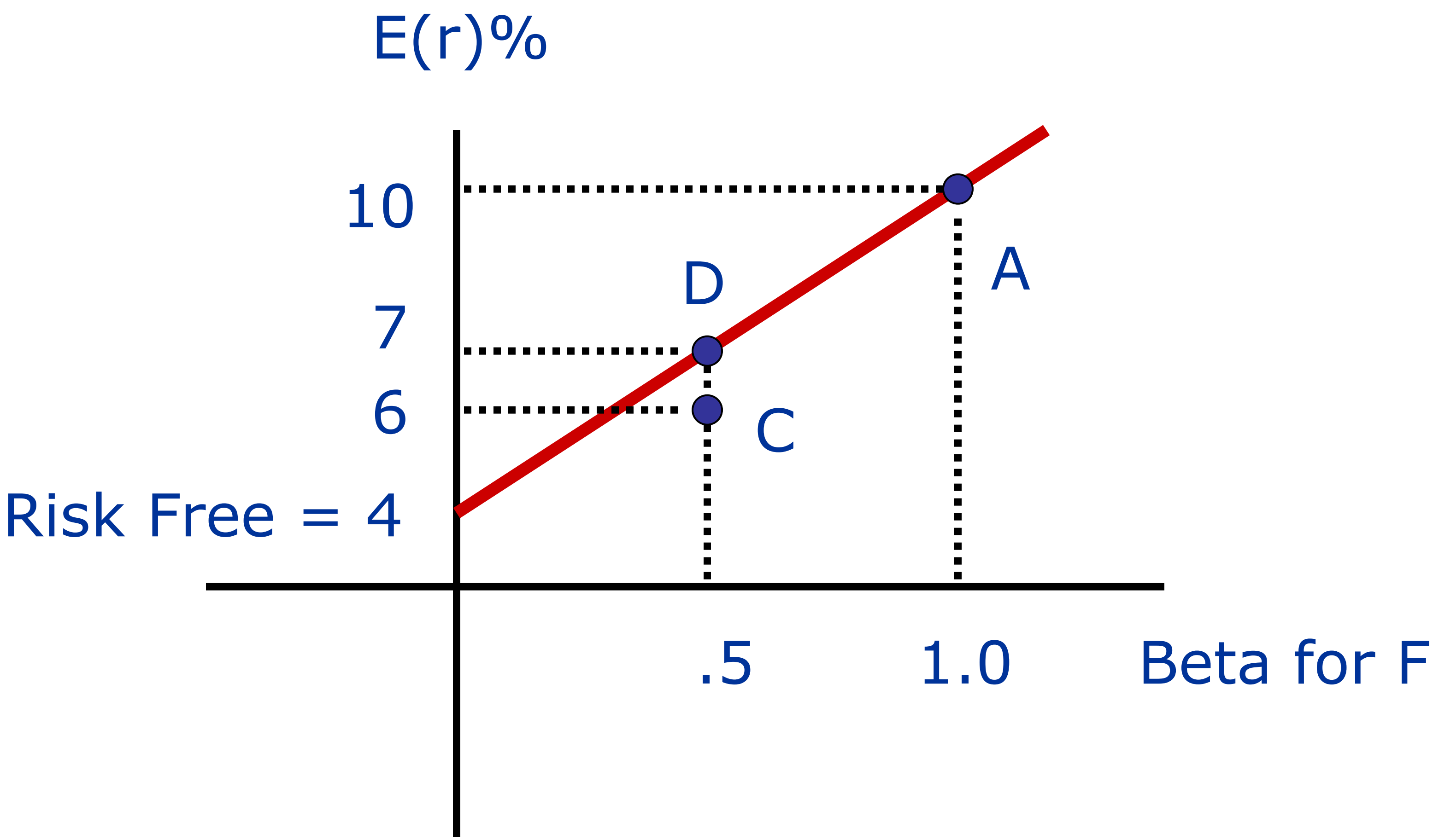


Portfolio



Individual Security

Disequilibrium Example



Disequilibrium Example

- Short Portfolio C
- Use funds to construct an equivalent risk higher return Portfolio D
  - D is comprised of A & Risk-Free Asset
- Arbitrage profit of 1%

The Arbitrage Pricing Theory

- The CAPM is a one factor model:

$$r_i - r_F = \beta_i \left[r_M - r_F \right] + \varepsilon_i$$

- The APT is a multi-factor model:

$$r_i - r_F = \beta_i^1 \left[r^1 - r_F \right] + \cdots + \beta_i^N \left[r^N - r_F \right] + \varepsilon_i$$

The Arbitrage Pricing Theory

- Note that the factors can be anything
  - They can be macroeconomic variables (inflation, GDP-growth, …)
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Example: 3-Factor Model

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Chen, Roll and Ross (1986)

- Monthly and annual unanticipated growth in industrial production (YP, MP)
- Changes in expected inflation, as measured by the change in r_{TBill} (DEI).
- Unexpected inflation (UI)
- Unanticipated changes in risk premiums, as measured by $r_{Baa} - r_{AAA}$ (UPR). This is often called the "Default Spread"
- Unanticipated changes in the slope of the term structure, as measured by $r_{T\text{-}Bond}-r_{T\text{-}Bill}$. (UTS). This is often called the "Term Spread"

Example: one-factor model

- Do these expected returns and factor sensitivities represent an equilibrium?

i	$E[r_i]$	b_i
Stock 1	15%	0.9
Stock 2	21	3.0
Stock 3	12	1.8

Arbitrage portfolio

$$X_1 + X_2 + X_3 = 0$$

$$b_1 X_1 + b_2 X_2 + b_3 X_3 = 0$$

$$\bar{r}_1 X_1 + \bar{r}_2 X_2 + \bar{r}_3 X_3 > 0$$

- Let’s figure out the arbitrage portfolio together!

APT more formally

$$\tilde{R}_i = a_i + b_{i1}\tilde{I}_1 + b_{i2}\tilde{I}_2 + \ldots + b_{in}\tilde{I}_n + e_i$$

$$E(e_i e_j) = 0,\ i \neq j$$

$$E[e_i(I_j - \bar{I}_j)] = 0$$

Independence assumptions! We can create well-diversified portfolios.

No arbitrage between them -> expected returns

Generally in equilibrium we have

$$\bar{r}_i = \lambda_0 + b_{i1}\lambda_1 + b_{i2}\lambda_2 + \ldots + b_{in}\lambda_n$$

$$\lambda_i : factor\ i\ risk\ premium$$

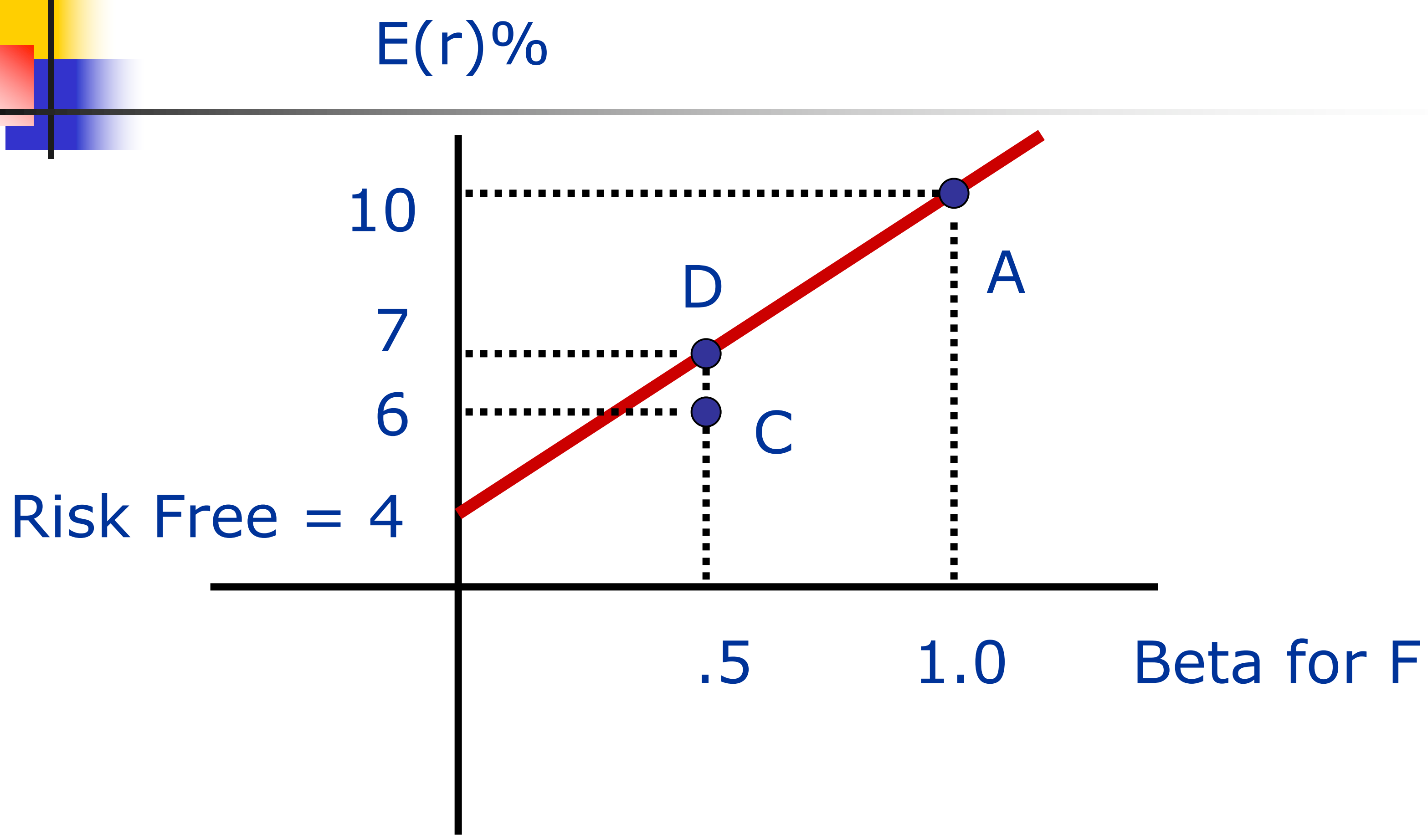
In equilibrium

- Suppose

$$\lambda_0 = 8\%$$

$$\lambda_1 = 4\%$$

- What are the equilibrium prices of 3 stocks?


E(r)%
10
D
A
7
6
C
Risk Free = 4
.5
1.0
Beta for F

Recall Chen, Roll and Ross (1986)

$$\tilde{r}_i = E(\tilde{r}_i) + b_{i,1}\tilde{f}_1 + b_{i,2}\tilde{f}_2 + \ldots + \tilde{e}_i$$

$$E(\tilde{f}_j) = 0$$

- This means that, instead of defining an *f* directly as economic growth, we would have to define it as the deviation of economic growth from what was expected.

A Simple Example

	IBM
Boom Payoff (Pr=0.5)	140
Bust Payoff (Pr=0.5)	100
$E(CF_1)$	120
Time 0 Price	100
Discount Rate	20%

- Remember that Expected Return is precisely equivalent to the Discount Rate that investors are applying to the expected cash flows.

	IBM	DELL
Boom Payoff (Pr=0.5)	140	160
Bust Payoff (Pr=0.5)	100	80
$E(CF_1)$	120	120
Time 0 Price	100	?
Discount Rate	20%	?

- The price investors will pay for DELL will be less than $100, even though DELL's expected cash flows are the same as IBM's.
- The discount rate investors will apply to DELL's cash flows will be higher than the 20% applied to IBM's cash flows.
- The expected return investors will require from DELL will be higher than the IBM's expected return of 20%.
- How do we calculate DELL's discount rate?

How this relates to APT

- Calculating the business cycle factor f_{BC} in the two states. The indicator is one (at the end of the next year) if the economy is in an expansion, and zero if the economy is in a recession.
- Assuming there is a 50%/50% chance that we will be in an expansion/recession, the expected value of the indicator is 0.5.
- This means that the business-cycle factor has a value of 0.5 = 1-0.5 if the economy booms, and -0.5 = 0-0.5 if the economy goes bust.

	IBM	DELL
Boom Payoff (Pr=0.5)	140	160
Bust Payoff (Pr=0.5)	100	80
$E(CF_1)$	120	120
Time 0 Price	\$100	\$90
Discount Rate	20%	33.33%

- For simplicity, let's assume that investors are only willing to buy up all of DELL's shares if the price of DELL is \$90, or, equivalently, that the discount rate that they will apply to DELL is 33.33%

Factor Loading

- The way of doing this is just running a time-series regression of the returns of IBM on the factor.
- Here, since there are only two things that can happen at each point in time (a boom or a bust), estimating the coefficients is the same as finding the coefficients that fit the equations in the boom, and in the bust.

$$
\begin{aligned}
0.40 &= E(r_{\text{IBM}}) + b_{\text{IBM,BC}} \cdot 0.5 \\
0.00 &= E(r_{\text{IBM}}) + b_{\text{IBM,BC}} \cdot -0.5
\end{aligned}
$$

Factor Loading Results

- Solving these gives $E(r_{IBM}) = 0.20$ (which we already knew) and $b_{IBM,BC} = 0.4$.
- Since DELL's returns in the boom and bust are 77.78% and -11.11%, respectively, similar calculations for DELL gives $E(r_{DELL}) = 0.3333$ and $b_{DELL,BC} = 0.8889$.

Factor Risk Premium

- To go from factor loadings to discount factors, we have to evaluate the APT Pricing Equation.

$$\bar{r}_i = \lambda_0 + b_{i1}\lambda_1 + b_{i2}\lambda_2 + \ldots + b_{in}\lambda_n$$

$$\lambda_i : factor\ i\ risk\ \ premium$$

- The factor risk premium is a measure of how much more investors discount a stock as a result of having one extra unit of risk relating to the BC factor.
- Let's calculate the BC factor risk premium together!
- Now you can apply this risk premium to other stocks.

Factor Forecasts

- How do we forecast stock returns?
- Estimate factor betas (loadings), then forecast factor returns
- The simplest approach to forecasting factor is to calculate a history of factor returns and take their average.
  - A factor relationship is more stable than a stock relationship
- Factor forecasts are difficult beyond historical averages

Stock	Industry	Growth	Bond β	Size	ROE	Beta
American Express	Financial services	0.17	−0.05	0.19	−0.28	1.16
AT&T	Telephones	−0.16	0.74	1.47	−0.59	0.84
Chevron	Energy reserves and production	−0.53	−0.24	0.83	−0.72	0.70
Coca-Cola	Food and beverage	−0.02	0.30	1.41	1.48	1.06
Disney	Entertainment	0.13	−0.86	0.71	0.42	1.13
Dow Chemical	Chemical	−0.64	−0.92	0.48	0.22	1.13
DuPont	Chemical	−0.10	−0.74	1.05	−0.41	0.93
Eastman Kodak	Leisure	−0.19	−0.30	0.39	−0.55	0.94
Exxon	Energy reserves and production	−0.67	0.03	1.67	−0.27	0.71
General Electric	Heavy electrical	−0.24	0.13	1.56	0.15	1.10
General Motors	Motor vehicles	2.74	−1.80	0.73	−1.24	1.25
IBM	Computer hardware	0.51	−0.62	1.16	−0.62	1.11
International Paper	Forest products and paper	−0.23	−1.08	0.01	−0.49	1.08
Johnson & Johnson	Medical products	−0.12	0.68	1.06	0.78	1.07
McDonalds	Restaurant	−0.16	0.28	0.55	0.24	1.06
Merck	Drugs	−0.04	0.46	1.37	2.28	1.10
3M	Chemical	−0.22	−0.69	0.78	0.20	0.91
Philip Morris	Tobacco	−0.01	0.30	1.60	1.22	1.02
Procter & Gamble	Home products	−0.32	0.80	1.12	0.41	1.05
Sears	Department stores	−0.34	−1.29	0.45	−0.69	1.10

Factor forecasts: An example

- The factor forecasts are 2% for growth, 2.5% for bond beta, -1.5% for size and 0% for ROE.
- 8% for the chemical industry, and 6% for all the other industries.
- CAPM forecast is 6% market excess return
- E.g. IBM:
  - Factor model: 0.06+0.51* 0.02+(-0.61*0.025)+1.16*(-0.015) -0.62*0=3.7%

Stock	Industry	APT	CAPM	APT-CAPM
American Express	Finance services	5.93%	6.96%	−1.03%
AT&T	Telephones	5.33%	5.04%	0.29%
Chevron	Energy reserves and production	3.10%	4.20%	−1.11%
Coca-Cola	Food and beverage	4.60%	6.36%	−1.77%
Disney	Entertainment	3.05%	6.78%	−3.74%
Dow Chemical	Chemical	3.70%	6.78%	−3.08%
DuPont	Chemical	4.38%	5.58%	−1.21%
Eastman Kodak	Leisure	4.29%	5.64%	−1.36%
Exxon	Energy reserves and production	2.23%	4.26%	−2.03%
General Electric	Heavy electrical	3.51%	6.60%	−3.10%
General Motors	Motor vehicles	5.89%	7.50%	−1.62%
IBM	Computer hardware	3.73%	6.66%	−2.93%
International Paper	Forest products and paper	2.83%	6.48%	−3.66%
Johnson & Johnson	Medical products	5.87%	6.42%	−0.55%
McDonalds	Restaurant	5.56%	6.36%	−0.81%
Merck	Drugs	5.02%	6.60%	−1.59%
3M	Chemical	4.67%	5.46%	−0.80%
Philip Morris	Tobacco	4.33%	6.12%	−1.79%
Procter & Gamble	Home products	5.68%	6.30%	−0.62%
Sears	Department stores	1.42%	6.60%	−5.18%

Two issues

- First, define a qualified model
- Second, find the correct set of factor forecasts

A qualified model

- Go far… but how far is far enough?

- Any factor model that is good at explaining the risk of a diversified portfolio (in terms of e.g. adjusted R2) should be (nearly) qualified as an APT model.

Many factors have been considered

1. Risk factors

- Market beta (trailing 60-month regression of monthly excess returns)
- APT betas (trailing 60-month regressions on T bill returns, percentage changes in industrial production, the rate of inflation, the difference in the returns to long- and short-term government bonds, and the difference in the returns to corporate and government bonds)
- Volatility of total return (trailing 60 months)
- Residual variance (non-market-related risk over trailing 60 months)
- Earnings risk (standard error of year over year earnings per share around time trend)
- Debt to equity (most recently available book value of total debt to book value of common equity)
- Debt to equity trend (five-year trailing time trend in debt to equity)
- Times interest earned (net operating income to total interest charges)
- Times interest earned trend (five-year quarterly time trend in year over year times interest earned)
- Yield variability (five-year trailing volatility in earnings, dividend, and cash flow yield)

2. *Liquidity factors*

- Market capitalization (current market price times the most recently available number of shares outstanding)
- Market price per share
- Trading volume/market capitalization (trailing 12-month average monthly trading volume to market capitalization)
- Trading volume trend (five-year time trend in monthly trading volume)

3. *Factors indicating price level*

- Earnings to price (most recently available four quarters, earnings to current market price)
- Earnings to price trend (five-year monthly time trend in earnings to price)
- Book to price (most recently available book value to current market price)
- Book to price trend (five-year monthly time trend in book to price)
- Dividend to price (most recently available four quarters, dividend to current market price)
- Dividend to price trend (five-year monthly time trend in dividend to price)
- Cash flow to price (most recently available ratio of earnings plus depreciation per share to current market price)
- Cash flow to price trend (five-year monthly time trend in cash flow to price)
- Sales to price (most recently available four-quarters, total sales per share to current market price)
- Sales to price trend (five-year monthly time trend in sales to price)

4. Factors indicating growth potential

- Profit margin (net operating income to total sales)
- Profit margin trend (trailing five-year quarterly time trend in year over year profit margin)
- Capital turnover (total sales to total assets)
- Capital turnover trend (trailing five-year quarterly time trend in year over year capital turnover)
- Return on assets (net operating income to total assets)
- Return on assets trend (trailing five-year quarterly time trend in year over year return on assets)
- Return on equity (net income to total book value of total equity capital)
- Return on equity trend (trailing five-year quarterly time trend in year over year return on equity)
- Earnings growth (trailing five-year quarterly time trend in year over year earnings per share divided by the trailing five-year average earnings per share)

5. Technical factors

- Excess return (relative to the S&P 500) in previous one month
- Excess return (relative to the S&P 500) in previous two months
- Excess return (relative to the S&P 500) in previous three months
- Excess return (relative to the S&P 500) in previous six months
- Excess return (relative to the S&P 500) in previous 12 months
- Excess return (relative to the S&P 500) in previous 24 months
- Excess return (relative to the S&P 500) in previous 60 months

6. Sector variables

- Zero/one dummy variables reflecting firm's principal line of business (durables, nondurables, utilities, energy, construction, business equipment, manufacturing, transportation, financial, and business services)

What should we do?

- Principal components analysis: dimensionality reduction
- Lasso: model selection
- Maximum likelihood and Bayesian econometrics: model uncertainty