Homework 7

1. Suppose that X and Y have a continuous joint distribution for which the joint p.d.f.is as follows:

$$f(x,y) = \begin{cases} x+y, & \text{for } 0 \le x \le 1, \text{ and } 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

   (1) Find $E(Y|X)$ and $Var(Y|X)$.

   (1) Find $Var[E(Y|X)]$ and $E[Var(Y|X)]$.

2. Suppose that on a certain examination in advanced mathematics, students from university A achieve scores that are normally distributed with a mean of 625 and a variance of 100, and students from university B achieves scores which are normally distributed with a mean of 600 and a variance of 150. If two students from university A and three students from university B take this examination, what is the probability that the average of the scores of the two students from university A will be greater than the average of the scores of the three students form university B? Hint: Determine the distribution of the difference between the two averages.

3. Suppose that a random variable X has a normal distribution, and for every x, the conditional distribution of another random variable Y given $X = x$ is a normal distribution with mean $ax + b$ and variance σ^2 , where a, b and σ^2 are constants. Prove that the joint distribution of X and Y is a bivariate normal distribution.

4. Let $X_1, X_2, \ldots, X_n$ represent a random sample from each of the distributions having the following probability density functions:

   (a) $f(x;\theta) = \theta^x e^{-\theta}/x!, x = 0, 1, 2, \ldots, 0 \le \theta < \infty$,zero elsewhere, where $f(0; 0) = 1$.

   (b) $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1, 0 < \theta < \infty$, zero elsewhere.

   (c) $f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}, -\infty < x < \infty, -\infty < \theta < \infty$.

   In each case find the m.l.e. $\widehat{\theta}$ of θ.

5. The *Pareto distribution* is frequently used as a model in study of incomes and has the distribution function

$$F(x; \theta_1, \theta_2) = 1 - (\theta_1/x)^{\theta_2}, \theta_1 \le x,$$

   zero elsewhere, where $\theta_1 > 0$ and $\theta_2 > 0$

   If $X_1, X_2, \ldots, X_n$ is a random sample from this distribution, find the maximum likelihood estimators of θ_1 and θ_2.