多元函数极值

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对确定的 x, y, x_0, y_0 ,作一元辅助函数

$$F(t) = f(x_0 + t\Delta x, y_0 + t\Delta y), t \in [0, 1]$$

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根据一元函数Taylor公式,现有
$$F(t) = F(0) + F'(0)t + \frac{F''(0)}{2!}t^2 + \dots + \frac{F^{(n)}(0)}{n!}t^n + \frac{F^{(n+1)}(\theta)}{(n+1)!}t^{n+1}, \theta \in (0,t).$$

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 $F''(t) = \frac{\partial^2 f}{\partial x^2}(x_0 + t\Delta x, y_0 + t\Delta y)(\Delta x)^2 + 2\frac{\partial^2 f}{\partial y \partial x}(x_0 + t\Delta x, y_0 + t\Delta y)\Delta x\Delta y$

 $+\frac{\partial^2 f}{\partial y^2}(x_0+t\Delta x,y_0+t\Delta y)(\Delta y)^2=(\Delta x\frac{\partial}{\partial x}+\Delta y\frac{\partial}{\partial y})^2f(x_0+t\Delta x,y_0+t\Delta y);$

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$$F^{(n)}(t) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^n f(x_0 + t\Delta x, y_0 + t\Delta y)$$
其中, $(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^n = \sum_{k=0}^n C_n^k (\Delta x)^k (\Delta y)^{n-k} \frac{\partial^k}{\partial x^k} \frac{\partial^{n-k}}{\partial y^{n-k}}$

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$$+ \frac{\partial^{2} f}{\partial y^{2}}(x_{0} + t\Delta x, y_{0} + t\Delta y)(\Delta y)^{2} = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^{2} f(x_{0} + t\Delta x, y_{0} + t\Delta y);$$

$$\dots$$

$$F^{(n)}(t) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^{n} f(x_{0} + t\Delta x, y_{0} + t\Delta y)$$

$$\Rightarrow F^{(n)}(0) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^{n} f(x_{0}, y_{0})$$

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$$= (\Delta a)$$

根据一元函数Taylor公式,现有

$$4x\frac{\partial}{\partial x} + 2$$

 $F(t) = f(x_0 + t\Delta x, y_0 + t\Delta y), t \in [0, 1]$

 $F^{(n)}(t) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^n f(x_0 + t\Delta x, y_0 + t\Delta y)$ $\Rightarrow F^{(n)}(0) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^n f(x_0, y_0)$

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$$= (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}) f(x_0, y_0) + \frac{1}{2!} (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^2 f(x_0, y_0) + \dots$$

 $+\frac{1}{n!}(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^{n} f(x_{0}, y_{0}) + \frac{1}{(n+1)!}(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^{n+1} f(x_{0} + \theta \Delta x, y_{0} + \theta \Delta y)$

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$$f(x,y) - f(x_0, y_0) \neq F(1) - F(0)$$

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结论

若 z = f(x,y)于 (x_0,y_0) 点取到极值,且 $\frac{\partial f}{\partial x}(x_0,y_0), \frac{\partial f}{\partial y}(x_0,y_0)$ 存在,则

$$\frac{\partial f}{\partial x}(x_0, y_0) = 0, \frac{\partial f}{\partial y}(x_0, y_0) = 0.$$



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使 $\frac{\partial f}{\partial x}(x_0,y_0), \frac{\partial f}{\partial y}(x_0,y_0)$ 为零的点,称为 z=f(x,y)的驻点

极值可疑点

设
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要么全为正((A > 0)),为开口向上的抛物面,全在xoy平面上方; 要么全为负 ((A < 0)),为开口向下的抛物面,全在 xoy 平面下方;

即 $AC - B^2 > 0$ 时,函数 $z = Ax^2 + 2Bxy + Cy^2$ (除在 (0,0)点外)的值有确定的符号 与 A同号

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若 $AC-B^2<0$,则函数值 z对不同的 x,y有正有负,比如 y=0时, $z=Ax^2$ 与 A同号, $x=-\frac{B}{A}y$ 时, $z=\frac{AC-B^2}{A}y^2$ 与 A异号.所以, $AC-B^2<0$ 时,函数 $z=Ax^2+2Bxy+Cy^2$ 的值(在原点附近)不具有确定的符号.

事实上,如果学过线性代数,就知道:

$$A > 0, AC - B^2 > 0$$
时,二次型 $Ax^2 + 2Bxy + Cy^2$ 正定; $A < 0, AC - B^2 > 0$ 时,二次型 $Ax^2 + 2Bxy + Cy^2$ 负定;

 $A < 0, AC - B^2 > 0$ 时,二次型 $Ax^2 + 2Bxy + Cy^2$ 负定; $AC - B^2 < 0$ 时,二次型 $Ax^2 + 2Bxy + Cy^2$ 不定.

设 z = f(x, y),在 (x_0, y_0) 点,各阶偏导数连续,则有二元函数Taylor公式:

$$f(x,y) - f(x_0,y_0) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}) f(x_0,y_0) + \frac{1}{2!} (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^2 f(x_0,y_0) + \cdots$$

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If $\frac{\partial}{\partial x} f(x_0, y_0)$, $\frac{\partial}{\partial y} f(x_0, y_0)$ 不都为零,则 $f(x_0, y_0)$ 非极值;

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 考察 $\Phi(\Delta x, \Delta y) = (\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y})^2 f(x_0, y_0)$ 在 $\forall \Delta x, \Delta y$ 变化时定号否.

定号则 $f(x_0, y_0)$ 为极值,否则 $f(x_0, y_0)$ 非极值.

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信祭
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 $\mathbb{D}\Phi = (\Delta x)^2 \frac{\partial^2 f(x_0, y_0)}{\partial x^2} + 2\Delta x \Delta y \frac{\partial^2 f(x_0, y_0)}{\partial x \partial y} + (\Delta y)^2 \frac{\partial^2 f(x_0, y_0)}{\partial y^2}$

是否关于 $\forall \Delta x, \Delta y$ 定号.

If $\frac{\partial}{\partial x} f(x_0.y_0) = \frac{\partial}{\partial y} f(x_0.y_0) = 0$

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$$\mathbb{P} \Phi = (\Delta x)^2 rac{\partial^2 f(x_0,y_0)}{\partial x_0^2} + 2\Delta x \Delta y rac{\partial x}{\partial x_0^2} + 2\Delta x \Delta y rac{\partial x}{\partial x_0^2} + 2\Delta x \Delta y rac{\partial x}{\partial x_0^2} + 2\Delta x \Delta y \frac{\partial x}{\partial x_0^$$

即
$$\Phi = (\Delta x)^2 \frac{1}{\partial x^2} + 2\Delta x \Delta y \frac{1}{\partial x \partial y}$$
是否关于 $\forall \Delta x, \Delta y$ 定号.

 $\mathbb{P}\Phi = (\Delta x)^2 \frac{\partial^2 f(x_0, y_0)}{\partial x^2} + 2\Delta x \Delta y \frac{\partial^2 f(x_0, y_0)}{\partial x \partial y} + (\Delta y)^2 \frac{\partial^2 f(x_0, y_0)}{\partial y^2}$

即考察 $A(\Delta x)^2 + 2B\Delta x\Delta y + C(\Delta y)^2$ 是否关于 $\forall \Delta x, \Delta y$ 定号.

设 z=f(x,y)z在某区域内有二阶连续偏导数,求出函数 z=f(x,y)在此区域内的驻点 (x_0,y_0) ,在驻点处求出 $A=\frac{\partial^2 f}{\partial x^2}, B=\frac{\partial^2 f}{\partial x \partial y}, A=\frac{\partial^2 f}{\partial y^2}$ 的值,判断 $AC-B^2$ 是否大于0.

令
$$A = \frac{\partial^2 f(x_0.y_0)}{\partial x^2}, B = \frac{\partial^2 f(x_0.y_0)}{\partial x \partial y}, C = \frac{\partial^2 f(x_0.y_0)}{\partial y^2},$$
即考察 $A(\Delta x)^2 + 2B\Delta x \Delta y + C(\Delta y)^2$ 是否关于 $\forall \Delta x, \Delta y$ 定号.

设 z = f(x,y)z在某区域内有二阶连续偏导数,求出函数 z = f(x,y)在此区域内的驻 点 (x_0, y_0) ,在驻点处求出 $A = \frac{\partial^2 f}{\partial x^2}$, $B = \frac{\partial^2 f}{\partial x \partial y}$, $A = \frac{\partial^2 f}{\partial y^2}$ 的值,判断 $AC - B^2$ 是否大于0.

若 $AC - B^2 < 0$,则该驻点非极值点;若 $AC - B^2 > 0$,则该驻点为极值点,此时

令
$$A = \frac{\partial^2 f(x_0.y_0)}{\partial x^2}, B = \frac{\partial^2 f(x_0.y_0)}{\partial x \partial y}, C = \frac{\partial^2 f(x_0.y_0)}{\partial y^2},$$
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点
$$(x_0,y_0)$$
,在驻点处求出 $A=\frac{\partial^2 f}{\partial x^2}$, $B=\frac{\partial^2 f}{\partial x \partial y}$, $A=\frac{\partial^2 f}{\partial y^2}$ 的值,判断 $AC-B^2$ 是否大于0. 若 $AC-B^2<0$,则该驻点非极值点;若 $AC-B^2>0$,则该驻点为极值点,此时

$$(x_0,y_0)$$
,在驻点处求出 $A=\frac{\sigma J}{\partial x^2}$, $B=\frac{\sigma J}{\partial x\partial y}$, $A=\frac{\sigma J}{\partial y^2}$ 的值,判断 $AC-B^2$ 是否大于0. $AC-B^2<0$,则该驻点非极值点;若 $AC-B^2>0$,则该驻点为极值点,此时
$$\begin{cases} A>0\Rightarrow f(x_0,y_0) 是极小值 \\ A<0\Rightarrow f(x_0,y_0) 是极大值 \end{cases}$$

令 $A = \frac{\partial^2 f(x_0.y_0)}{\partial x^2}, B = \frac{\partial^2 f(x_0.y_0)}{\partial x \partial y}, C = \frac{\partial^2 f(x_0.y_0)}{\partial y^2},$ 即考察 $A(\Delta x)^2 + 2B\Delta x \Delta y + C(\Delta y)^2$ 是否关于 $\forall \Delta x, \Delta y$ 定号.

设 z = f(x,y)z在某区域内有二阶连续偏导数,求出函数 z = f(x,y)在此区域内的驻

点
$$(x_0,y_0)$$
,在驻点处求出 $A=\frac{\partial^2 f}{\partial x^2}$, $B=\frac{\partial^2 f}{\partial x \partial y}$, $A=\frac{\partial^2 f}{\partial y^2}$ 的值,判断 $AC-B^2$ 是否大于0. 若 $AC-B^2<0$,则该驻点非极值点;若 $AC-B^2>0$,则该驻点为极值点,此时

若
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若
$$AC - B^2 = 0$$
,情况待定.

即考察 $A(\Delta x)^2 + 2B\Delta x\Delta y + C(\Delta y)^2$ 是否关于 $\forall \Delta x, \Delta y$ 定号.

设 z = f(x,y)z在某区域内有二阶连续偏导数,求出函数 z = f(x,y)在此区域内的驻 点 (x_0, y_0) ,在驻点处求出 $A = \frac{\partial^2 f}{\partial x^2}$, $B = \frac{\partial^2 f}{\partial x \partial y}$, $A = \frac{\partial^2 f}{\partial y^2}$ 的值,判断 $AC - B^2$ 是否大于0.

点
$$(x_0, y_0)$$
,在驻点处求出 $A = \frac{\partial^2 f}{\partial x^2}$, $B = \frac{\partial^2 f}{\partial x \partial y}$, $A = \frac{\partial^2 f}{\partial y^2}$ 的值,判断 $AC - B^2$ 是否大于0. 若 $AC - B^2 < 0$,则该驻点非极值点;若 $AC - B^2 > 0$,则该驻点为极值点,此时

$$AC - B^{\circ} < 0$$
,则该驻总非极值总;名 $AC - B^{\circ} > 0$,则该驻总为极值总,此时 $A > 0 \Rightarrow f(x_0, y_0)$ 是极小值 $A < 0 \Rightarrow f(x_0, y_0)$ 是极大值

若 $AC - B^2 = 0$,情况待定.

如
$$z = x^4 + y^4 - x^2 - 2xy - y^2$$
有三个驻点 $(-1, -1), (0, 0), (1, 1)$ 在 $(0, 0)$ 处 $A = -2, B = -2, C = -2, AC - B^2 = 0$,此时无法去决定 $(0, 0)$ 是否是极值点.

在
$$(0,0)$$
处 $A=-2$, $B=-2$, $C=-2$, $AC-B^2=0$,此时无法去决定 $(0,0)$ 是否是极值点。
但在 $y=x$ 上, $z=x^4-4x^2=x^2(x^2-4)<0$ 0 $(0<|x|<1)$,在 $y=-x$ 上, $z=2x^4>$

 $0(0 < |x| \ll 1)$,所以 (0,0) 不是极值点.

即考察 $A(\Delta x)^2 + 2B\Delta x\Delta y + C(\Delta y)^2$ 是否关于 $\forall \Delta x, \Delta y$ 定号.

例 $\bar{x}z = 3xy - x^3 - y^3$ 的极值.

解:
$$\begin{cases} \frac{\partial z}{\partial x} = 3y - 3x^2 \\ \frac{\partial z}{\partial x} = 3x - 3y^2 \end{cases}, \Leftrightarrow \begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial x} = 0 \end{cases}, \mathbb{N} \begin{cases} 3y - 3x^2 = 0 \\ 3x - 3y^2 = 0 \end{cases}, \begin{cases} y = x^2 \\ x = y^2 \end{cases}$$

例 $xz = 3xu - x^3 - y^3$ 的极值.

解:
$$\begin{cases} \frac{\partial z}{\partial x} = 3y - 3x^2 \\ \frac{\partial z}{\partial y} = 3x - 3y^2 \end{cases}, \diamondsuit \begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases}, \bowtie \begin{cases} 3y - 3x^2 = 0 \\ 3x - 3y^2 = 0 \end{cases}, \begin{cases} y = x^2 \\ x = y^2 \end{cases}$$

得驻点 (x,y) = (0,0), (x,y) = (1,1)

$$(y) = (0,0), (x,y) = (1,1)$$

例 $xz = 3xu - x^3 - y^3$ 的极值.

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$$\begin{cases} \frac{\partial z}{\partial y} = 3x - 3y^2 \end{cases} \quad \begin{cases} \frac{\partial z}{\partial y} = 0 \end{cases} \quad 3x - 3y^2 = 0 \end{cases} \quad \begin{cases} x = y^2 \end{cases}$$
 得驻点 $(x, y) = (0, 0), (x, y) = (1, 1)$

$$\frac{\partial^2 z}{\partial z} = -6x, \frac{\partial^2 z}{\partial z} = 3, \frac{\partial^2 z}{\partial z} = -6y^2$$

例 $x = 3xy - x^3 - y^3$ 的极值.

得驻点 (x,y) = (0,0), (x,y) = (1,1)

解:
$$\begin{cases} \frac{\partial z}{\partial x} = 3y - 3x^2 \\ \frac{\partial z}{\partial y} = 3x - 3y^2 \end{cases}, \Leftrightarrow \begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases}, \mathbb{N} \begin{cases} 3y - 3x^2 = 0 \\ 3x - 3y^2 = 0 \end{cases}, \begin{cases} y = x^2 \\ x = y^2 \end{cases}$$

∴ 于 (0,0)点, $A = 0, B = 3, C = 0, AC - B^2 = -9 < 0, ∴ <math>(0,0)$ 非极值点.

例 $x = 3xu - x^3 - u^3$ 的极值.

解:
$$\begin{cases} \frac{\partial z}{\partial x} = 3y - 3x^2 \\ \frac{\partial z}{\partial y} = 3x - 3y^2 \end{cases}, \Leftrightarrow \begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases}, \text{ If } \begin{cases} 3y - 3x^2 = 0 \\ 3x - 3y^2 = 0 \end{cases}, \begin{cases} y = x^2 \\ x = y^2 \end{cases}$$

得驻点 (x,y)=(0,0),(x,y)=(1,1)

又
$$\frac{\partial^2 z}{\partial x^2} = -6x$$
, $\frac{\partial^2 z}{\partial x \partial y} = 3$, $\frac{\partial^2 z}{\partial y^2} = -6y^2$
 $\therefore \mp (0,0)$ 点, $A = 0$, $B = 3$, $C = 0$, $AC - B^2 = -9 < 0$, $\therefore (0,0)$ 非极值点.

二于 (1,1)点, A=-6, B=3, C=-6, $AC-B^2=27>0$, AC=10, AC=

∴ 于
$$(1,1)$$
点, $A = -6$, $B = 3$, $C = -6$, $AC - B^2 = 27 > 0$, ∴ $(1,1)$ 为极值点.

例 $\bar{x}z = 3xy - x^3 - y^3$ 的极值.

解:
$$\begin{cases} \frac{\partial z}{\partial x} = 3y - 3x^2 \\ \frac{\partial z}{\partial y} = 3x - 3y^2 \end{cases}, \Leftrightarrow \begin{cases} \frac{\partial z}{\partial x} = 0 \\ \frac{\partial z}{\partial y} = 0 \end{cases}, \text{ If } \begin{cases} 3y - 3x^2 = 0 \\ 3x - 3y^2 = 0 \end{cases}, \begin{cases} y = x^2 \\ x = y^2 \end{cases}$$

$$\therefore$$
于 $(0,0)$ 点, $A=0,B=3,C=0,AC-B^2=-9<0,$ \therefore $(0,0)$ 非极值点.

∴ 于
$$(1,1)$$
点, $A = -6$, $B = 3$, $C = -6$, $AC - B^2 = 27 > 0$, ∴ $(1,1)$ 为极值点.

$$\therefore A = -6 < 0 \therefore (1,1)$$
为极大值点,极大值为 $f(1,1) = 3 - 1 - 1 = 1$.