Class Note 1

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1. Assets

    (a) Real assets: Lands, Buildings, Machines, Knowledge that's used to produce goods and services.

    (b) Financial assets: Claims to the income generated by real assets. Stocks = Ownership

    Example:

    - Issue stock.
    - Use proceeds to build factory which generates revenue.
    - Pay workers and other expenses, left with earnings.
    - In order to expand, issue *bonds* (Corporate bonds).
      Bonds offer a fixed payment of interest (except for floating rate bond).
      Bonds = Debt (Obligations)
      Transfer wealth across time, potential states of nature.

2. Intertemporal Choice

    - Assume people live for two periods.
      $t = 0$: Young.
      $t = 1$: Older.
    - 2 goods: c_0, c_1
    - prices: $p_0 = p_1 = 1$
    - Credit Market available.
      Save or borrow at interest rate r.
    - Income: M_0, M_1
    - Budget Constraint:
      Income at $t = 1$?

$$M_1 + (M_0 - p_0 c_0)(1 + r) = p_1 c_1$$
$$M_1 + (M_0 - c_0)(1 + r) = c_1$$
$$c_0 + \frac{c_1}{1 + r} = M_0 + \frac{M_1}{1 + r}$$

    - Utility Function:

$$U(c_0, c_1) = u(c_0) + \frac{1}{1 + \delta} u(c_1)$$

where $u^{'} > 0$.

Maximize utility (budget constraint)

$$\max_{c_0, c_1} U(c_0) + \frac{1}{1+\delta}u(c_1)$$

s.t.

$$c_0 + \frac{c_1}{1+r} = M_0 + \frac{M_1}{1+r}$$

$$\mathcal{L}(c_0, c_1, r) = u(c_0) + \frac{1}{1+\delta}u(c_1) - \lambda(c_0 + \frac{c_1}{1+r} - M_0 - \frac{M_1}{1+r})$$

<u>FOC</u>:

(a)

$$\frac{\partial \mathcal{L}}{\partial c_0} = u'({c_0}^*) - \lambda = 0$$

(b)

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{1+\delta}u'({c_1}^*) - \frac{\lambda}{1+r} = 0$$

(c)

$$\frac{\partial \mathcal{L}}{\partial \lambda} = {c_0}^* + \frac{{c_1}^*}{1+r} - M_0 - \frac{M_1}{1+r} = 0$$

From (a), (b)

$$\Rightarrow \frac{u'({c_0}^*)}{u'({c_1}^*)} = \frac{1+r}{1+\delta}$$

<u>Cases</u>:

(a) <u>Case 1</u>: $r = \delta$

$$\Rightarrow u'({c_0}^*) = u'({c_1}^*) \Rightarrow {c_0}^* = {c_1}^*$$

Let $c^* = {c_0}^* = {c_1}^*$,

$$ {c_0}^* + \frac{{c_1}^*}{1+r} = M_0 + \frac{M_1}{1+r} \Rightarrow c^* = M_0(\frac{1+r}{2+r}) + M_1(\frac{1}{2+r})$$

(b) <u>Case 2</u>: $M_1 = 0$ (retirement)

$$c^* = M_0(\frac{1+r}{2+r}) < M_0 \Rightarrow Saving$$

Assume strictly concave utility function: $(u'' < 0)$
recall:

$$f(\alpha x + (1-\alpha)y) > \alpha f(x) + (1-\alpha)f(y), \forall \alpha \in (0,1)$$

(c) <u>Case 3</u>: $r > \delta$

$$u'({c_0}^*) = \frac{1+r}{1+\delta}u'({c_1}^*) > u'({c_1}^*) \Rightarrow {c_0}^* < {c_1}^*$$

(d) <u>Case 4</u>: $r < \delta$

$$u'({c_0}^*) = \frac{1+r}{1+\delta}u'({c_1}^*) < u'({c_1}^*) \Rightarrow {c_0}^* > {c_1}^*$$

Suppose:

$$u(c) = \begin{cases} \frac{c_A}{A} & \text{if } A \neq 1, A < 1 \\ \log(c) & \text{if } A = 0 \end{cases}$$

For simplicity, assume $\delta = 0$.

$$u'({c_0}^*) = (1+r)u'({c_1}^*) \Rightarrow {c_0}^{A-1} = (1+r){c_1}^{A-1} \Rightarrow \frac{{c_1}^*}{{c_0}^*} = (1+r)^{\frac{1}{1-A}}$$

So called "Smooth Consumption".

$$r = 5\%, A = 0 \Rightarrow c_1 = (1+r)c_0 : c_1 \text{ is } 5\% \text{ higher.}$$

What if:

$$A = -3 \Rightarrow c_1 = (1+r)^{1/4}c_0 : c_1 \text{ is } 1.2\% \text{ higher. Averse to fluctuation.}$$