Homework 11

1. (Textbook Section 8.3-1, Page 322) Suppose that a certain drug A was administered to eight patients selected at random, and after a fixed time period, the concentration of the drug in certain body cells of each patient was measured in appropriate units. Suppose that these concentrations for the eight patients were found to be as follows:

$$1.23, 1.42, 1.41, 1.62, 1.55, 1.51, 1.60, and 1.76.$$

Suppose also that a second drug B was administered to six different patients selected at random, and when the concentration of drug B was measured in a similar way for these six patients, the results were as follows:

$$1.76, 1.41, 1.87, 1.49, 1.67, and 1.81.$$

Assuming that all the observations have a normal distribution with a common unknown variance, test the following hypotheses at the level of significance 0.10: The null hypotheses is that the mean concentration of drug A among all patients is at least as large as the mean concentration of drug B. The alternative hypotheses is that the mean concentration of drug B is larger than that of drug A.

2. (Textbook Section 8.4-7 Page 331) Consider two different normal distributions for which both the means μ_1 and μ_2 and the variances σ_1^2 and σ_2^2 are unknown, and suppose that it is desired to test the following hypotheses:

$$H_0 : \sigma_1^2 \leq \sigma_2^2,$$

$$H_1 : \sigma_1^2 > \sigma_2^2.$$

Suppose further that a random sample consisting of 16 observations for the first normal distribution yields the values $\sum_{i=1}^{16} X_i = 84$ and $\sum_{i=1}^{16} X_i^2 = 563$, and an independent random sample consisting of 10 observations from the second normal distribution yields the value $\sum_{i=1}^{10} Y_i = 18$ and $\sum_{i=1}^{10} Y_i^2 = 72$.
(a) What are the M.L.E.'s of σ_1^2 and σ_2^2?
(b) If an F test is carried out at the level of significance 0.05, is the hypothesis H_0 accepted or rejected?

3. (Textbook Section 9.1-4 Page 340) According to a simple genetic principle, if both the mother and the father of a child have genotype Aa, then there is probability 1/4 that the child will have genotype AA, the probability 1/2 that she will have genotype Aa,

and probability 1/4 that she will have genotype aa. In a random sample of 24 children having both parents with genotype Aa, it is found that 10 have genotype AA, 10 have genotype Aa, and four have genotype aa. Investigate whether the simple genetic principle is correct by carrying out a χ^2 test of goodness-of-fit.

4. (Textbook Section 9.2-2 Page 347) At the fifth hockey game of the season at a certain arena, 200 people were selected at a random and asked how many of the precious four games they had attended. The results are given in Table 9.5. Test the hypothesis that these 200 observed values can be regarded as a random sample from a binomial distribution; that is, there exits a number $\theta(0 < \theta < 1)$such that the probabilities are as follows:

$$p_0 = (1-\theta)^4, p_1 = 4\theta(1-\theta)^3, p_2 = 6\theta(1-\theta)^2,$$
$$p_3 = 4\theta^3(1-\theta), p_4 = \theta^4.$$

Number of games previously attended	Number of people
0	33
1	67
2	66
3	15
4	19

5. Let the result of a random experiment be classified as one of the mutually exclusive and exhaustive ways A_1, A_2, A_3 and also as one of the mutually exclusive and exhaustive ways B_1, B_2, B_3, B_4. Two hundred independent trials of the experiment result in the following data:

	B_1	B_2	B_3	B_4
A_1	10	21	15	6
A_2	11	27	21	13
A_3	6	19	27	24

Test, at the 0.05 significance level, the hypothesis of independence of the A attribute and the B attribute, namely $H_0 : Pr(A_iB_j) = Pr(A_i)Pr(B_j), i = 1,2,3$ and $j = 1,2,3,4$, against the alternative of dependence.