Homework 10

1. (Textbook Section 8.1-4, Page 300) Suppose that $X_1, ..., X_n$ form a random sample from a normal distribution for which the mean μ is unknown and the variance is 1. Suppose also that μ_0 is a certain specified number, and that the following hypotheses are to be tested:

$$H_0 : \mu = \mu_0;$$

$$H_1 : \mu \neq \mu_0.$$

Finally, suppose that the sample size n is 25, and consider a test procedure such that H_0 is to be accepted if $|\overline{X_n} - \mu_0| < c$. Determine the value of c such that the size of the test will be 0.05.

2. (Textbook Section 8.1-15, Page 302) Let X have a uniform distribution on the interval $[0, \theta]$, and suppose that we wish to test the hypotheses

$$H_0 : \theta \leq 1;$$

$$H_1 : \theta > 1.$$

We shall consider test procedures of the form "reject H_0 if $X \geq c$". For each possible value x of X, find the p-value if $X = x$ is observed.

3. (Textbook Section 8.2-1, Page 312) Use the data in Example 7.5.1, comprising a sample of $n = 10$ lactic acid measurements in cheese. Assume, as we did there, that the lactic acid measurements are a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. Suppose that we wish to test the following hypotheses:

$$H_0 : \mu \leq 1.2,$$

$$H_1 : \mu > 1.2.$$

(a) Perform the level $\alpha_0 = 0.05$ test of these hypotheses.
(b) Compute the p-value.

4. (Textbook Section 8.2-3, Page 313) The manufacturer of a certain type of automobile claims that under typical urban driving conditions the automobile will travel on average at least 20 miles per gallon of gasoline. The owner of this type of automobile notes the mileages that she has obtained in her own driving when she fills he automobile's tank with gasoline on nine different occasions. She finds that the results, in miles per gallon, are as follows: 15.6, 18.6, 18.3, 20.1, 21.5, 18.4, 19.1, 20.4, and 19.0. Test the manufacturer's claims by carrying out a test at the level of significance $\alpha_0 = 0.05$. List carefully the assumptions you must make.

5. The table below records the ALb content (mg/L) in urine samples of 10 breast cancer patients before and after chemotherapy. Please test whether chemotherapy has effects on ALb content.

Patient ID	1	2	3	4	5	6	7	8	9	10
ALb content before chemotherapy	3.3	11.7	9.4	6.8	2.0	3.1	5.3	3.7	21.8	17.6
ALb content after chemotherapy	33.0	30.8	8.8	11.4	42.6	5.8	1.6	19.0	22.4	30.2