Homework Assignment #4

Due: In class, two weeks after distribution

§1 One-Period Binomial Lattice Model

Suppose there is a risky asset S and a risk-free asset B. On the second day, the risky asset will be worth $S_1 = uS_0$ with probability p and $S_1 = dS_0$ with probability $1-p$. Let r be the constant risk-free rate.

1. Prove that the economy (S, B) admits no arbitrage if and only if $0 < d < 1 + r < u$.

2. Let P_0 be the "no-arbitrage" (fair) price of a put option with strike K; and let C_0 be the "no-arbitrage" (fair) price of a call option with strike K. Prove the put-call parity:

$$C_0 - P_0 = S_0 - \frac{K}{1+r}.$$

3. Suppose you observe that the market trading price of a call option with strike K is less than the "no-arbitrage" price C_0. If you believe the model, how do you arbitrage from this?

§2 Multi-Period Binomial Lattice Model

Consider a two-period binomial lattice model with $S_0 = 4,\ u = 2,\ d = \frac{1}{2}$. Suppose that the real-world probability for the stock to go up at each period is $p = \frac{1}{3}$. For simplicity, we assume the risk-free rate to be zero.

1. Find the no-arbitrage price of a call option with strike 6.

2. Find the corresponding Delta-hedging strategy, i.e. the number of stock shares in the replicating portfolio.

3. We may note that the initial no-arbitrage price of this option is irrelevant to p. However, a Goldman Sachs analyst said, if p were higher, this option would be more favorable. Do you agree? Why? Can you give some explanation?