Answer to Problem Set 5

1. Solution:

1) Consumer i' s marginal rate of substitution is as following：

$$\mathrm{MRS_i} = -\frac{dx_2^i}{dx_1^i} = \frac{MU_1^i}{MU_2^i} = \left({\alpha_1^i}/{x_1^i}\right)\Big/\left({\alpha_2^i}/{x_2^i}\right) = \frac{\alpha_1^i x_2^i}{\alpha_2^i x_1^i}$$

2) By the definition, we can have:

$$\mathrm{MES_i} = \frac{x_1^i dx_2^i}{x_2^i dx_1^i} = \frac{x_1^i}{x_2^i} \times (-\mathrm{MRS_i}) = -\frac{\alpha_1^i}{\alpha_2^i}$$

which is a constant.

3) To get the demand function of the consumer, we should solve the consumer's maximization problem:

$$\max_{x_1^i, x_2^i} \alpha_1^i \ln x_1^i + \alpha_2^i \ln x_2^i$$

$$\text{s.t. } x_1^i + p x_2^i = w_1^i + p w_2^i$$

$$L = \alpha_1^i \ln x_1^i + \alpha_2^i \ln x_2^i + \lambda(w_1^i + p w_2^i - x_1^i - p x_2^i)$$

$$\text{F.O.C} \quad \begin{cases} \dfrac{\partial L}{\partial x_1^i} = {\alpha_1^i}/{x_1^i} - \lambda = 0 \\ \dfrac{\partial L}{\partial x_2^i} = {\alpha_2^i}/{x_2^i} - \lambda p = 0 \\ \dfrac{\partial L}{\partial \lambda} = w_1^i + p w_2^i - x_1^i - p x_2^i = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x_1^i = \dfrac{\alpha_1^i}{\alpha_1^i + \alpha_2^i}(w_1^i + pw_2^i) \\ x_2^i = \dfrac{\alpha_2^i}{\alpha_1^i + \alpha_2^i} \times \dfrac{(w_1^i + pw_2^i)}{p} \end{cases}$$

Thus the demand function of consumer is as following:

$$\begin{cases} x_1^i = \dfrac{1}{1 - 1/MES_i}(w_1^i + pw_2^i) \\ x_2^i = \dfrac{1}{-MES_i + 1} \times \dfrac{(w_1^i + pw_2^i)}{p} \end{cases}$$

4) In the Walras equilibrium, we have

$$\begin{cases} x_1^A + x_1^B = w_1^A + w_1^B \\ x_2^A + x_2^B = w_2^A + w_2^B \end{cases}$$

which means

$$\begin{cases} \dfrac{\alpha_1^A}{\alpha_1^A + \alpha_2^A}\left(w_1^A + pw_2^A\right) + \dfrac{\alpha_1^B}{\alpha_1^B + \alpha_2^B}\left(w_1^B + pw_2^B\right) = w_1^A + w_1^B \\ \dfrac{\alpha_2^A}{\alpha_1^A + \alpha_2^A} \times \dfrac{(w_1^A + pw_2^A)}{p} + \dfrac{\alpha_2^B}{\alpha_1^B + \alpha_2^B} \times \dfrac{(w_1^B + pw_2^B)}{p} = w_2^A + w_2^B \end{cases}$$

$$\Rightarrow p = \frac{\dfrac{\alpha_2^A}{\alpha_1^A + \alpha_2^A} w_1^A + \dfrac{\alpha_2^B}{\alpha_1^B + \alpha_2^B} w_1^B}{\dfrac{\alpha_1^A}{\alpha_1^A + \alpha_2^A} w_2^A + \dfrac{\alpha_1^B}{\alpha_1^B + \alpha_2^B} w_2^B}$$

5) The social planer’s problem is:

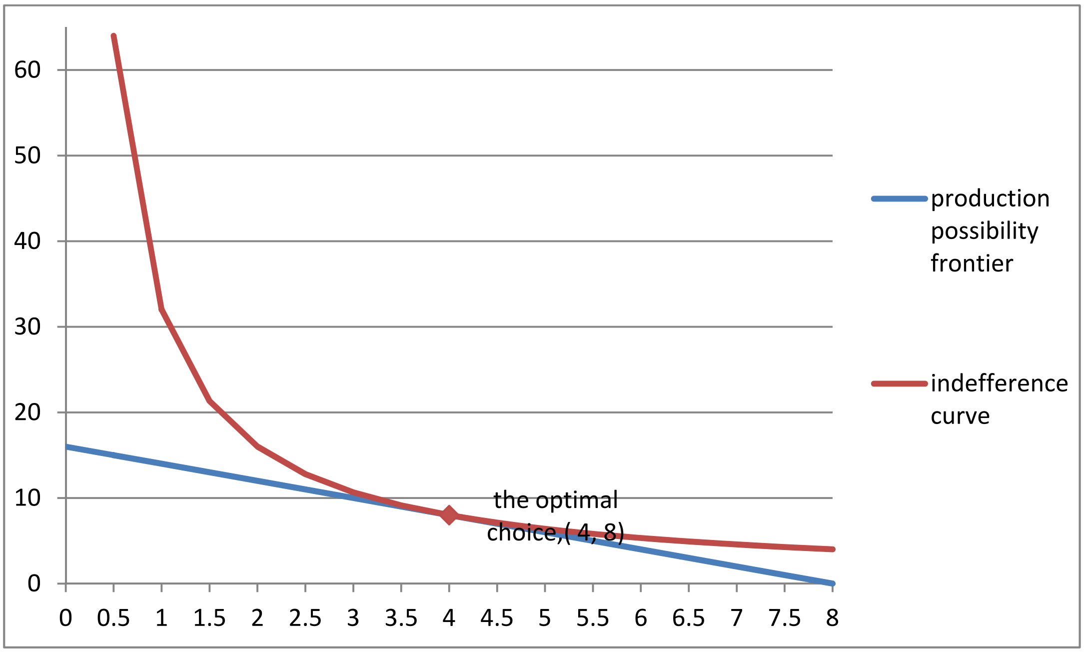
$$\max_{x_1^A, x_1^B, x_2^A, x_2^B} \alpha_1^A \ln x_1^A + \alpha_2^A \ln x_2^A + \alpha_1^B \ln x_1^B + \alpha_2^B \ln x_2^B$$

$$\text{s.t. } \begin{cases} x_1^A + x_1^B = w_1^A + w_1^B \\ x_2^A + x_2^B = w_2^A + w_2^B \end{cases}$$

$$\begin{cases} x_1^A = \dfrac{\alpha_1^A}{\alpha_1^A + \alpha_1^B}(w_1^A + w_1^B) \\ x_1^B = \dfrac{\alpha_1^B}{\alpha_1^A + \alpha_1^B}(w_1^A + w_1^B) \\ x_2^A = \dfrac{\alpha_2^A}{\alpha_2^A + \alpha_2^B}(w_2^A + w_2^B) \\ x_2^B = \dfrac{\alpha_2^B}{\alpha_2^A + \alpha_2^B}(w_2^A + w_2^B) \end{cases}$$

2. Solution:

1) Since Robinson can catch 1 fish per hour or collect 2 coconuts per hour, with the constraint that he would like to spend 8 hours per day to work, his production possibility frontier will be $F + C/2 = 8 \Rightarrow C = 16 - 2F$. Also we can know his indefference curve will be $C = \overline{U}/F$, where $\overline{U}$ is the utility level. Then the graft will be as following: (The horizontal axis is F and the vertical axis is C)



2) If Robinson is the social planer, his problem is

$$\max_{F,C} FC$$

$$s.t. F + C/2 = 8$$

$$\begin{cases} F = 4 \\ C = 8 \end{cases}$$

3) When Robinson acts as a labor supplier, he decides to work 8 hours per day. So we must have the supply of the labor in the labor market $L^s = 8$ and the worker's income $I = 8p_l$, where p_l is the price of labor per hour.

When Robinson acts as a producer, we assume there are two firms in this market. One produces fish and one produces coconut. For the fish firm:

$$\max_{F^s L_f^d} p_f F^s - p_l L_f^d$$

$$s.t.\ \ F^s = L_f^d$$

$$\Rightarrow \max_{L_f^d}(p_f - p_l)L_f^d$$

where p_f is the market price of fish. F^s is the number of fish the producer produces. L_f^d is hours of labor per day the producer employs to produce fish. As we can see, if $p_f - p_l \geq 0$, fish firm would like to employs labor as mush as possible, and the supply of fish is $F^s = L_f^d$. Its profit is$\pi_f = (p_f - p_l)L_f^d$.

Similarly, for the coconut firm:

$$\max_{C^s L_c^d} p_c C^s - p_l L_c^d$$

$$\text{s.t. } C^s = 2L_c^d$$

$$\Rightarrow \max_{L_c^d}(2p_c - p_l)L_c^d$$

if $2p_c - p_l \geq 0$, this coconut firm would like to employs labor as mush as possible, and the supply of fish is $C^s = 2L_C^d$. Its profit is$\pi_c = (2p_c - p_l)L_c^d$.

In the equilibrium, we must have $\pi_f = \pi_c = 0$. If not, the firm with a positive profit has incentive to increase the wage of labor to make more profit.

In the labor market, we must have $L_f^d + L_c^d = L^s = 8$

Thus we must have

$$p_f = 2p_c = p_l \tag{1}$$

When Robinson acts as a consumer,

$$\max_{F^d, C^d} F^d C^d$$

$$\text{s.t. } p_f F^d + p_c C^d \le I + \pi_f + \pi_c = 16p_c + (p_f - 2p_c)L_f^d$$

$$\Rightarrow \begin{cases} F^d = \frac{1}{2} \times \frac{16p_c + (p_f - 2p_c)L_f^d}{p_f} \\ C^d = \frac{1}{2} \times \frac{16p_c + (p_f - 2p_c)L_f^d}{p_c} \end{cases}$$

In the good market, we must have $F^d = F^s$ and $C^d = C^s$. Therefore,

$$16p_c = (p_f + 2p_c)L_f^d \tag{2}$$

Combining equation (1) and (2), we have $L_f^d = 4$.

In general equilibrium,

$$\begin{cases} F^d = F^s = 4 \\ C^d = C^s = 8 \\ L^d = L^s = 8 \\ p_l = p_f = 2p_c \end{cases}$$

3. Solution:

1) Suppose B be the original value when A=0. Then at the level of emissions abatement A, the social value is $V = B + \int_0^A (500 - 20x)dx - \int_0^A (200 + 5x)dx$. Then at the socially efficient level of emissions abatement, we must have $500 - 20A = 200 + 5A \Rightarrow A^* = 12$, i.e. the socially efficient level of emissions abatement is 12.

2) $MB = 500 - 20A^* = 260$

$MC = 200 + 5A^* = 260$

3) If you abate one million more tons, $\Delta V = \int_{12}^{13}(500 - 20x)dx - \int_{12}^{13}(200 + 5x)dx = -12.5$.

If one million fewer, $\Delta V = \int_{12}^{11}(500 - 20x)dx - \int_{12}^{11}(200 + 5x)dx = -12.5$.

4) Suppose that it is socially efficient to abate until total benefits equal total costs when $A = A_1$, but marginal benefits does not equal to marginal costs. Then $MB(A_1) > MC(A_1)$ or $MB(A_1) < MC(A_1)$.

If $MB(A_1) > MC(A_1)$, we increase A_1 by a unit and will gain more benefit than cost, which means our net social benefits will increase. Thus it contrasts the definition of A_1, which is the socially efficient level.

If $MB(A_1) < MC(A_1)$, we decrease A_1 by a unit and will loss less benefit than cost, which means our net social benefits will increase. Thus it also contrasts the definition of A_1, which is the socially efficient level.

Therefore A_1 cannot be the socially efficient level. And at the socially efficient level, we must have MB=MC.

4. Solution:

a) Denote N' is the number of wells exist in the competitive market now. Then, we consider operating a new well. The revenue of a new well brings is $10 \times (500 - N' - 1)$ and the cost is 1000. If the new well is worth to operate, we must have $10 \times (500 - \mathrm{N}' - 1) \geq 1000$. As N' increases, the revenue a new well brings decreases. Thus a new well will not be operated when$10 \times (500 - \mathrm{N}') = 1000 \Rightarrow \mathrm{N}' = 400$. Therefore, in this perfectly competitive case, the equilibrium number of wells is 400 and the equilibrium output is $\mathrm{Q} = 40000$.

A divergence between private and social marginal cost exists in the industry. Because the private marginal cost of operating a new well is 1000, while the social marginal cost is $10\mathrm{N}' + 1000$ (A new well will decrease the amount of oil produced by each well, which have already existed in the market and whose number is N', by 1 barrel.)

b) If the government nationalizes the oil field,

$$\max_{\mathrm{N}} 10(500\mathrm{N} - \mathrm{N}^2) - 1000\mathrm{N}$$

$$\begin{cases} \mathrm{N}^* = 200 \\ \mathrm{Q} = 60000 \\ \mathrm{q} = 300 \end{cases}$$

c) Suppose the license fee be t. The private marginal cost of a new is t+1000 and the marginal value is $10 \times (500 - \mathrm{N}' - 1)$. If $10 \times (500 - \mathrm{N}' - 1) > \mathrm{t} + 1000$, the new well will be operated; if $10 \times (500 - \mathrm{N}' - 1) < \mathrm{t} + 1000$, the new well will not be operated. Then in the equilibrium $10 \times (500 - \mathrm{N}') = \mathrm{t} + 1000$. If the fee is to prompt the industry to drill the optimal number of wells, $\mathrm{N}' = \mathrm{N}^* =$

$200 \Rightarrow \mathrm{t} = 2000$.

5. Solution:

a) We can know that for T hours of programming, the total price per hour of the three groups will pay is $\mathrm{W_1 + W_2 + W_3 = 600 - 4T}$. In the efficient number of hours of public television, $600 - 4\mathrm{T} = 200 \Rightarrow \mathrm{T}^* = 100$.

b) However in a competitive private market, group 1 will consume $\mathrm{T_1} = 0$ since $\mathrm{W_1} = 150 - \mathrm{T} < 200, \forall T \geq 0$; group 2 will consume $\mathrm{T_2} = 0$ since $\mathrm{W_2} = 200 - 2\mathrm{T} \leq 200, \forall \mathrm{T} \geq 0$; group 3 will consume $\mathrm{T_3} = 50$. Then in a competitive private market, 50 hours of public television will be provided.

6. Solution:

1) If the firm has the right to pollute, it concerns its own profit.

$$\max_{\mathrm{q_1}} 64\mathrm{q_1} - 4\mathrm{q_1^2} \Rightarrow \mathrm{q_1^*} = 8$$

And its profit is $\pi_1 = 256$. Residents' cost is $\mathrm{D_1} = 96$

If residents can negotiate with the firm, they would like to consider all the effects of the pollution and maximize the social welfare.

$$\max_{\mathrm{q}} 64\mathrm{q} - 4\mathrm{q}^2 - (4\mathrm{q} + \mathrm{q}^2) \Rightarrow \mathrm{q}^* = 6$$

Thus the firm's profit is $\pi = 240$ and residents' cost is $\mathrm{D} = 60$.

Therefore residents will pay the firm at least 16 ($\pi_1 - \pi = 256 - 240 = 16$) to make the firm produce $q^* = 6$. And the pay is at most 36 ($D_1 - D = 96 - 60 = 36$).

2) If residents have the right of no pollution, they would like:

$$\min_{q_2} 4q_2 + q_2^2 \quad \Rightarrow q_2^* = 0$$

Since the firm has no right about the pollution and there is no negotiation, it will be forced to produce at the level residents want $q_2^* = 0$. Thus the firm's profit is $\pi_2 = 0$ and residents' cost is $D_2 = 0$.

However, if there is negotiation between the firm and residents, they would like to maximize their benefits.

$$\max_q 64q - 4q^2 - (4q + q^2) \Rightarrow q^* = 6$$

Thus the firm's profit is $\pi = 240$ and residents' cost is $D = 60$.

Therefore the firm will pay residents at least 60 ($D - D_2 = 60 - 0 = 60$) to let the firm produce $q^* = 6$. And the pay is at most 240 ($\pi - \pi_2 = 240 - 0 = 240$).

3) When the firm has the right of pollution and government would like to charge a tax on the firm's production. Assume t be the tax government charges per unit.

For the firm:

$$\max_q 64q - 4q^2 - tq \Rightarrow q = 8 - t/8$$

If the government can use this tax to achieve the social efficiency, we must have $q = q^* = 6 \quad \Rightarrow t = 16$.

So the tax government charges per unit is 16 and the revenue of government is $T = tq = 96$.