

**Problem Set 2**  
(Due Date: Nov. 17)

1. Jane owns a house that is worth \$100,000. She cares only about her wealth, which consists entirely of the house. In any given year, there is a 20% chance that the house will burn down. If it does, its scrap value will be \$22,500. Jane's utility function of income is  $u(W) = W^{1/2}$ .
  - a) Is Jane risk averse, risk neutral or a risk lover? Explain.
  - b) What is the expected monetary value of Jane's uncertainty?
  - c) What is the maximum that Jane is willing to pay to completely insure her house against being destroyed by the fire?
  - d) Say that Homer is the president of an insurance company. He is risk neutral and has an utility function of the following type  $u(W) = W$ , where W is his wealth. What is the lowest price at which he is willing to provide a fair insurance contract (insurance premium equals expected loss) to completely insure Jane's risk?
  
2. Suppose that the process of producing lightweight parkas by Polly's Parkas is described by the function  $Q = 10K^{0.8}(L-40)^{0.2}$ , where Q is the number of parkas produced, K is the number of machine hours, and L is the number of person-hours of labors.
  - 1) Derive the cost-minimizing demands for K and L as a function of Q, wage rates (w), and rental rates on machines (r). Use these to derive the total cost function.
  - 2) This process requires skilled workers, who earn \$32 per hour. The rental rate is \$64 per hour. At these factor prices, what are total costs as a function of Q? Does this technology exhibit decreasing, constant, or increasing returns to scale?
  - 3) Polly's Parkas plans to produce 2000 parkas per week. At the factor prices given above, how many workers should they hire (at 40 hours per week) and how many machines should they rent (at 40 machine-hours per week)? What are the marginal and average costs at this level of production?
  
3. 某个企业的生产函数如下：
 
$$Q = \prod_{i=1}^m z_i^{\alpha_i} = z_1^{\alpha_1} z_2^{\alpha_2} \dots z_m^{\alpha_m}$$
 其中 Q 为产量， $z_i$  是第 i 个投入品， $\alpha_i > 0, i=1,2,\dots,m$ 。
  - 1) 请求出该企业的长期成本函数（假设  $z_i$  的价格为  $w_i$ ）。
  - 2) 企业平均成本的变化与企业规模报酬的性质是什么关系？请予以说明。
  
4. A manufacturer has two workshops. The production function of one workshop is  $x_1^{1/2} x_2^{1/2}$ , and the production function of other is  $x_1^{1/4} x_2^{3/4}$ . Prices of  $x_1$  and  $x_2$  are 2 and 5

respectively.

- a) What is the cost function of two workshops respectively?
  - b) What is the cost function of the manufacturer?
5. A school teaches students how to drive construction machinery. The number of students the school can educate per week is given by  $q = 10(\min(k, l))^\gamma$ , where  $k$  is the number of machines the firm rents per week, and  $l$  is the number of instructors hired each week, and  $\gamma$  is a parameter indicating the returns to scale in this production function.
- a) Explain why development of a profit-maximizing model here requires  $0 < \gamma < 1$ .
  - b) Supposing  $\gamma = 0.5$ , calculate the firm's total-cost and profit functions.
  - c) If the price of machine is  $v = 1000$  and the wage of instructor is  $w = 500$ , and tuition paid by the student is  $P = 600$ , how many students will the school serve and what are its profits?
  - d) If the price students are willing to pay rises to  $P = 900$ , how much will profit change?
  - e) Graph the school's supply curve for students slots, and how that the increase in profits calculated in part (d) can be shown on that graph.
6. An enterprising entrepreneur purchases two firms to produce widgets. Each firm produces identical products, and each has a production function given by

$$q_i = \sqrt{k_i l_i}, i = 1, 2.$$

The firms differ, however, in the amount of capital equipment each has. In particular, firm 1 has  $k_1 = 25$  whereas firm 2 has  $k_2 = 100$ . Rental rates for  $k$  and  $l$  are given by  $w = v = \$1$ .

- a. If the entrepreneur wishes to minimize short-run total costs of widget production, how should output be allocated between the two firms?
- b. Given that output is optimally allocated between the two firms, calculate the short-run total, average, and marginal cost curves. What is the marginal cost of the 100th widget? The 200th widget?
- c. How should the entrepreneur allocate widget production between the two firms in the long run? Calculate the long-run total, average, and marginal cost curves for widget production.
- d. How would your answer to part (c) change if both firms exhibited diminishing returns to scale?