Solutions to Lab 4: Credit Risk

Question 1:

Face value of bond: $F = \$100$
Annual coupon rate: 4%
Semi-annual coupon rate: $4\%/2 = 2\%$
Coupon payments: $C = 2\% \cdot 100 = \$2$
Periods of semi-annual coupon payments: $T = 4 \cdot 2 = 8$ (4 years × 2 payments per year)
Yield to maturity: $r = 5\%/2 = 2.5\%$ (semi-annual compounding)
Risk-free yield: 3% per year, which translates to $r_f = 1.5\%$ semi-annual rate

The risk-free value in year 4 is: $100 + 2 = \$102$
The risk-free value in year 3 is: $2 + 2 \cdot e^{-0.03\cdot 0.5} + 102 \cdot e^{-0.03\cdot 1} = \$102.96$
The risk-free value in year 2 is: $2 + 2 \cdot e^{-0.03\cdot 0.5} + 2 \cdot e^{-0.03\cdot 1} + 2 \cdot e^{-0.03\cdot 1.5} + 102 \cdot e^{-0.03\cdot 2} = \$103.88$
The risk-free value in year 1 is: $2 + 2 \cdot e^{-0.03\cdot 0.5} + 2 \cdot e^{-0.03\cdot 1} + 2 \cdot e^{-0.03\cdot 1.5} + 2 \cdot e^{-0.03\cdot 2} + 2 \cdot e^{-0.03\cdot 2.5} + 102 \cdot e^{-0.03\cdot 3} = \$104.78$

The loss computed by considering the default is then found by subtracting the recovery amount from the risk-free value.

The discount factor for year t is computed by

$$e^{-r_f \cdot t} = e^{-0.03\cdot t}$$

Then the present value of expected loss is found by multiplying the loss (given default) with the discount factor and the default probability.

The total present value of expected loss is the sum of losses from each year.

Time (years)	Default probability	Recovery amount ($)	Risk-free value ($)	Loss given default ($)	Discount factor	PV of expected loss
1.0	Q	30	104.78	74.78	0.9704	$72.57 \cdot Q$
2.0	Q	30	103.88	73.88	0.9418	$69.58 \cdot Q$
3.0	Q	30	102.96	72.96	0.9139	$66.68 \cdot Q$
4.0	Q	30	102.00	72.00	0.8869	$63.86 \cdot Q$
						Total: $272.69 \cdot Q$

The present value of semi-annual coupon payments is:

$$PV_c = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^T} = C \cdot \left(\frac{1 - \left(\frac{1}{1+r}\right)^{T+1}}{1 - \frac{1}{1+r}} - 1 \right)$$

Multiplying numerator and denominator of the fraction by $1 + r$ we get

$$PV_c = \frac{1 - (1+r)^{-T}}{r} = \frac{1 - (1.025)^{-8}}{0.025} = \$14.34$$

The present value of the face value is:

$$PV_F = \frac{F}{(1+r)^T} = \frac{100}{1.025^8} = \$82.07$$

So the total market value of the bond is $14.34 + 82.07 = \$96.41$

The risk-free value of the bond is similarly obtained by discounting the promised cash flows at 3% annual rate, which translates to $r_f = 1.5\%$ semi-annual rate.

$$PV_c^{rf} = \frac{1-\left(1+r_f\right)^{-T}}{r} = \frac{1-(1.015)^{-8}}{0.015} = \$14.97$$

Its present face value is similarly

$$PV_F^{rf} = \frac{F}{\left(1+r_f\right)^T} = \frac{100}{1.015^8} = \$88.77$$

So the total risk-free value of the bond is $14.97 + 88.77 = \$103.74$

The total loss from defaults should therefore be equated to $103.74 - 96.41 = \$7.33$. The value of Q implied by the bond price is therefore given by $272.69 \cdot Q = 7.33$, that is, $Q = 0.0269$. The implied probability of default is 2.69% per year.

Question 2:

For the derivation of the following table, see solution of Question 1 (the numbers might be slightly different in the magnitude of ± 0.2 due to approximations).

Time (years)	Default probability	Recovery amount ($)	Risk-free value ($)	Loss given default ($)	Discount factor	PV of expected loss
0.5	Q	40	106.73	66.73	0.9753	$65.08 \cdot Q$
1.5	Q	40	105.97	65.97	0.9277	$61.20 \cdot Q$
2.5	Q	40	105.17	65.17	0.8825	$57.52 \cdot Q$
3.5	Q	40	104.34	64.34	0.8395	$54.01 \cdot Q$
4.5	Q	40	103.46	63.46	0.7985	$50.67 \cdot Q$
						Total: $\mathbf{288.48 \cdot Q}$

The risk-free value of the bond is $\$104.09$, while the market price of the corporate bond is $\$95.34$. Therefore the total loss from defaults is $\$8.75$.
Then, $288.48 \cdot Q = 8.75$, or equivalently, $0.03033 = 3.033\%$