Solutions to Lab 3: Risk management and the Greek letters II

Question 1:

(See Hull – Practice Question 19.3)

In this case, $S_0 = K$, $r = 0.1$, $\sigma = 0.25$, and $T = 0.5$. Also,

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot T}{\sigma \cdot \sqrt{T}} = \frac{0 + \left(0.1 + \frac{0.25^2}{2}\right) \cdot 0.5}{0.25 \cdot \sqrt{0.5}} = 0.3712$$

The delta of the option is $N(d_1) = N(0.3712) = 0.64$.

Question 2:

(See lecture slides "Week 19 - Risk management and the Greek letters II")

i) $S_0 \cdot \sqrt{T} \cdot N'(d_1)$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$ and $N'(d_1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-d_1^2/2}$

ii) $S_0 \cdot \sqrt{T} \cdot N'(d_1) \cdot e^{-q\cdot T}$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$ and $N'(d_1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-d_1^2/2}$

Question 3:

(See lecture slides "Week 19 - Risk management and the Greek letters II")

i) $S_0 \cdot \sqrt{T} \cdot N'(d_1)$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$ and $N'(d_1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-d_1^2/2}$

ii) $S_0 \cdot \sqrt{T} \cdot N'(d_1) \cdot e^{-q\cdot T}$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$ and $N'(d_1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-d_1^2/2}$

Question 4:

(See lecture slides "Week 19 - Risk management and the Greek letters II")

i) $-K \cdot T \cdot e^{-r\cdot T} \cdot N(-d_2)$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$, $d_2 = d_1 - \sigma \cdot \sqrt{T}$, and $N(-d_2) = 1 - N(d_2)$

ii) Same as in (i).

Question 5:
(See Hull – Practice Question 19.14)

We have $S_0 = 0.80, K = 0.81, r = 0.08, r_f = 0.05, \sigma = 0.15, T = 0.5833.$
Therefore,

$$d_1 = \frac{\ln\left(\frac{0.80}{0.81}\right) + \left(0.08 - 0.05 + \frac{0.15^2}{2}\right) \cdot 0.5833}{0.15 \cdot \sqrt{0.5833}} = 0.1016$$

$$d_2 = d_1 - 0.15 \cdot \sqrt{0.5833} = -0.0130,$$

which gives

$$N(d_1) = 0.5405, \qquad N(d_2) = 0.4948.$$

i) The delta of one call option is $e^{-r_f \cdot T} \cdot N(d_1) = e^{-0.05 \cdot 0.5833} \cdot 0.5405 = 0.5250.$

ii) Also,

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} = \frac{1}{\sqrt{2\pi}} \cdot e^{-0.0007} = 0.3969.$$

We have that the gamma of one call option is

$$\frac{N'(d_1) \cdot e^{-r_f \cdot T}}{S_0 \cdot \sigma \cdot \sqrt{T}} = \frac{0.3969 \cdot e^{-0.05 \cdot 0.5833}}{0.80 \cdot 0.15 \cdot \sqrt{0.5833}} = 4.206.$$

iii) The vega of one call option is

$$S_0 \cdot \sqrt{T} \cdot N'(d_1) \cdot e^{-r_f \cdot T} = 0.80 \cdot \sqrt{0.5833} \cdot 0.3969 \cdot 0.9713 = 0.2355$$

iv) The theta of one call option is

$$-\frac{S_0 \cdot N'(d_1) \cdot \sigma \cdot e^{-r_f \cdot T}}{2\sqrt{T}} + r_f \cdot S_0 \cdot N(d_1) \cdot e^{-r_f \cdot T} - r \cdot K \cdot e^{-r \cdot T} \cdot N(d_2) =$$
$$= -\frac{0.80 \cdot 0.3969 \cdot 0.15 \cdot e^{-0.05 \cdot 0.5833}}{2 \cdot \sqrt{0.5833}} + 0.05 \cdot 0.8 \cdot 0.5405 \cdot e^{-0.05 \cdot 0.5833} -$$
$$-0.08 \cdot 0.81 \cdot e^{-0.08 \cdot 0.5833} \cdot 0.4948 =$$
$$= -0.0399.$$

v) The rho of one call option is

$$K \cdot T \cdot e^{-r \cdot T} \cdot N(d_2) = 0.81 \cdot 0.5833 \cdot 0.9544 \cdot 0.4948 = 0.2231$$

vi) Delta can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the value of an option to buy one yen increases by 0.525 times that amount.
Gamma can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the delta increases by 4.206 times that amount.

Vega can be interpreted as meaning that, when the volatility (measured in decimal form) increases by a small amount, the option's value increases by 0.2355 times that amount. When volatility increases by 1% ($= \ 0.01$), the option price increases by 0.002355.
Theta can be interpreted as meaning that, when a small amount of time (measured in years) passes, the option's value decreases by 0.0399 times that amount. In particular, when one calendar day passes, it decreases by $0.0399/365 = 0.000109$.
Finally, rho can be interpreted as meaning that, when the interest rate (measured in decimal form) increases by a small amount, the option's value increases by 0.2231 times that amount. When the interest rate increases by 1% ($= \ 0.01$), the options value increases by 0.002231.