Solutions to Lab 4: Credit Risk

Question 1:
(See Hull – Practice Question 24.12)

Face value of bond: $F = \$100$
Annual coupon rate: 4%
Semi-annual coupon rate: $4\%/2 = 2\%$
Coupon payments: $C = 2\% \cdot 100 = \$2$
Years: 4
Yield to maturity: $r = 5\%$ per annum
Risk-free yield: $r_f = 3\%$ per annum

The risk-free value in year 4 is: $100 + 2 = \$102$
The risk-free value in year 3 is: $2 + 2 \cdot e^{-0.03 \cdot 0.5} + 102 \cdot e^{-0.03 \cdot 1} = \$102.96$
The risk-free value in year 2 is: $2 + 2 \cdot e^{-0.03 \cdot 0.5} + 2 \cdot e^{-0.03 \cdot 1} + 2 \cdot e^{-0.03 \cdot 1.5} + 102 \cdot e^{-0.03 \cdot 2} = \$103.88$
The risk-free value in year 1 is: $2 + 2 \cdot e^{-0.03 \cdot 0.5} + 2 \cdot e^{-0.03 \cdot 1} + 2 \cdot e^{-0.03 \cdot 1.5} + 2 \cdot e^{-0.03 \cdot 2} + 2 \cdot e^{-0.03 \cdot 2.5} + 102 \cdot e^{-0.03 \cdot 3} = \$104.78$

The loss computed by considering the default is then found by subtracting the recovery amount from the risk-free value.

The discount factor for year t is computed by

$$e^{-r_f \cdot t} = e^{-0.03 \cdot t}$$

Then the present value of expected loss is found by multiplying the loss (given default) with the discount factor and the default probability.

The total present value of expected loss is the sum of losses from each year.

Time (years)	Default probability	Recovery amount ($)	Risk-free value ($)	Loss given default ($)	Discount factor	PV of expected loss
1.0	Q	30	104.78	74.78	0.9704	$72.57 \cdot Q$
2.0	Q	30	103.88	73.88	0.9418	$69.58 \cdot Q$
3.0	Q	30	102.96	72.96	0.9139	$66.68 \cdot Q$
4.0	Q	30	102.00	72.00	0.8869	$63.86 \cdot Q$
						Total: $272.69 \cdot Q$

So the market (present) value of the bond is

$$PV = 2 \cdot e^{-0.05 \cdot 0.5} + 2 \cdot e^{-0.05 \cdot 1} + 2 \cdot e^{-0.05 \cdot 1.5} + 2 \cdot e^{-0.05 \cdot 2} + 2 \cdot e^{-0.05 \cdot 2.5} + 2 \cdot e^{-0.05 \cdot 3} + \\ +2 \cdot e^{-0.05 \cdot 3.5} + 102 \cdot e^{-0.05 \cdot 4} = \$96.19$$

The risk-free (present) value of the bond is similarly obtained by discounting the promised cash flows at 3% annual rate.

$$PV_{rf} = 2 \cdot e^{-0.03 \cdot 0.5} + 2 \cdot e^{-0.03 \cdot 1} + 2 \cdot e^{-0.03 \cdot 1.5} + 2 \cdot e^{-0.03 \cdot 2} + 2 \cdot e^{-0.03 \cdot 2.5} + 2 \cdot e^{-0.03 \cdot 3} + \\ +2 \cdot e^{-0.03 \cdot 3.5} + 102 \cdot e^{-0.03 \cdot 4} = \$103.66$$

The total loss from defaults should therefore be equated to $103.66 - 96.19 = \$7.47$. The value of Q implied by the bond price is therefore given by $272.69 \cdot Q = 7.47$, that is, $Q = 0.0274$. The implied probability of default is 2.74% per year.

Question 2:

Face value of bond: $F = \$100$
Annual coupon rate: 6%
Semi-annual coupon rate: $6\%/2 = 3\%$
Coupon payments: $C = 3\% \cdot 100 = \$3$
Years: 5
Yield to maturity: $r = 7\%$ per annum
Risk-free yield: $r_f = 5\%$ per annum

The risk-free value in year 4.5 is: $3 + 103 \cdot e^{-0.05 \cdot 0.5} = \$103.46$
The risk-free value in year 3.5 is: $3 + 3 \cdot e^{-0.05 \cdot 0.5} + 3 \cdot e^{-0.05 \cdot 1} + 103 \cdot e^{-0.05 \cdot 1.5} = \$104.34$
The risk-free value in year 2.5 is: $3 + 3 \cdot e^{-0.05 \cdot 0.5} + 3 \cdot e^{-0.05 \cdot 1} + 3 \cdot e^{-0.05 \cdot 1.5} + 3 \cdot e^{-0.05 \cdot 2} + 103 \cdot e^{-0.05 \cdot 2.5} = \$105.17$
The risk-free value in year 1.5 is: $3 + 3 \cdot e^{-0.05 \cdot 0.5} + 3 \cdot e^{-0.05 \cdot 1} + 3 \cdot e^{-0.05 \cdot 1.5} + 3 \cdot e^{-0.05 \cdot 2} + 3 \cdot e^{-0.05 \cdot 2.5} + 3 \cdot e^{-0.05 \cdot 3} + 103 \cdot e^{-0.05 \cdot 3.5} = \$105.97$
The risk-free value in year 0.5 is: $3 + 3 \cdot e^{-0.05 \cdot 0.5} + 3 \cdot e^{-0.05 \cdot 1} + 3 \cdot e^{-0.05 \cdot 1.5} + 3 \cdot e^{-0.05 \cdot 2} + 3 \cdot e^{-0.05 \cdot 2.5} + 3 \cdot e^{-0.05 \cdot 3} + 3 \cdot e^{-0.05 \cdot 3.5} + 3 \cdot e^{-0.05 \cdot 4} + 103 \cdot e^{-0.05 \cdot 4.5} = \$106.73$

The loss computed by considering the default is then found by subtracting the recovery amount from the risk-free value.

The discount factor for year t is computed by

$$e^{-r_f \cdot t} = e^{-0.05 \cdot t}$$

Then the present value of expected loss is found by multiplying the loss (given default) with the discount factor and the default probability.

The total present value of expected loss is the sum of losses from each year.

Time (years)	Default probability	Recovery amount ($)	Risk-free value ($)	Loss given default ($)	Discount factor	PV of expected loss
0.5	Q	40	106.73	66.73	0.9753	$65.08 \cdot Q$
1.5	Q	40	105.97	65.97	0.9277	$61.20 \cdot Q$
2.5	Q	40	105.17	65.17	0.8825	$57.52 \cdot Q$
3.5	Q	40	104.34	64.34	0.8395	$54.01 \cdot Q$
4.5	Q	40	103.46	63.46	0.7985	$50.67 \cdot Q$
						Total: $288.48 \cdot Q$

So the market (present) value of the bond is

$$PV = 3 \cdot e^{-0.07 \cdot 0.5} + 3 \cdot e^{-0.07 \cdot 1} + 3 \cdot e^{-0.07 \cdot 1.5} + 3 \cdot e^{-0.07 \cdot 2} + 3 \cdot e^{-0.07 \cdot 2.5} + 3 \cdot e^{-0.07 \cdot 3} + \\ +3 \cdot e^{-0.07 \cdot 3.5} + 3 \cdot e^{-0.07 \cdot 4} + 3 \cdot e^{-0.07 \cdot 4.5} + 103 \cdot e^{-0.07 \cdot 5} = \$95.34$$

The risk-free (present) value of the bond is similarly obtained by discounting the promised cash flows at 5% annual rate.

$$PV_{rf} = 3 \cdot e^{-0.05 \cdot 0.5} + 3 \cdot e^{-0.05 \cdot 1} + 3 \cdot e^{-0.05 \cdot 1.5} + 3 \cdot e^{-0.05 \cdot 2} + 3 \cdot e^{-0.05 \cdot 2.5} + 3 \cdot e^{-0.05 \cdot 3} + \\ +3 \cdot e^{-0.05 \cdot 3.5} + 3 \cdot e^{-0.05 \cdot 4} + 3 \cdot e^{-0.05 \cdot 4.5} + 103 \cdot e^{-0.05 \cdot 5} = \$104.09$$

Therefore the total loss from defaults is $104.09 - 95.34 = \$8.75$.
Then, $288.48 \cdot Q = 8.75$, or equivalently, $Q = 0.03033$. So, the default probability is 3.033% per year.