Solutions to Lab 5: Revision Week 16 – Week 21

Question 1:
(See Hull – Practice Question 7.12)

Company A has a comparative advantage in the Canadian dollar fixed-rate market. Company B has a comparative advantage in the U.S. dollar floating-rate market. (This may be because of their tax positions.) However, company A wants to borrow in the U.S. dollar floating-rate market and company B wants to borrow in the Canadian dollar fixed-rate market. This gives rise to the swap opportunity.

The differential between the U.S. dollar floating rates is 0.5% per annum, and the differential between the Canadian dollar fixed rates is 1.5% per annum. The difference between the differentials is 1% per annum. The total potential gain to all parties from the swap is therefore 1% per annum, or 100 basis points. If the financial intermediary requires 50 basis points, each of A and B can be made 25 basis points better off. Thus, a swap can be designed so that it provides A with U.S. dollars at Floating + 0.25% per annum, and B with Canadian dollars at 6.25% per annum. The swap is shown in the following scheme.

Principal payments flow in the opposite direction to the arrows at the start of the life of the swap and in the same direction as the arrows at the end of the life of the swap. The financial institution would be exposed to some foreign exchange risk which could be hedged using forward contracts.

Question 2:
(See Hull – Practice Question 26.12)

The value of the option is given by the formula:

$$V_0 e^{-q_2 T} N(d_1) - U_0 e^{-q_1 T} N(d_2),$$

where

$$d_1 = \frac{\ln\left(\frac{V_0}{U_0}\right) + \left(q_1 - q_2 + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

and

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

Here we have: $V_0 = 1520$, $U_0 = 16 \cdot 100 = 1600$, $q_1 = 0$, $q_2 = 0$, $T = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.2$, $\rho_{12} = 0.7$.

We get that $\sigma = 0.1549$, $d_1 = -0.2536$, and $d_2 = -0.4086$.

Therefore the option price is

$$1520 \cdot 1 \cdot N(-0.2536) - 1600 \cdot N(-0.4086) =$$
$$= 1520 \cdot 0.3999 - 1600 \cdot 0.3414 = \$61.61$$

<u>Question 3</u>:

(See Hull – Practice Question 19.18)

a) For a call option on a non-dividend-paying stock,

$$\Delta = N(d_1)$$
$$\Gamma = \frac{N'(d_1)}{S_0 \cdot \sigma \cdot \sqrt{T}}$$
$$\Theta = -\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2 \cdot \sqrt{T}} - r \cdot K \cdot e^{-r \cdot T} \cdot N(d_2)$$

Hence, the left-hand side of the given equation is:

$$-\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2 \cdot \sqrt{T}} - r \cdot K \cdot e^{-r \cdot T} \cdot N(d_2) + r \cdot S_0 \cdot N(d_1) + \frac{1}{2} \cdot \sigma \cdot S_0 \cdot \frac{N'(d_1)}{\sqrt{T}} =$$
$$= r \cdot [S_0 \cdot N(d_1) - K \cdot e^{-r \cdot T} \cdot N(d_2)] = r \cdot \Pi$$

b) For a put option on a non-dividend-paying stock,

$$\Delta = N(d_1) - 1 = -N(-d_1)$$
$$\Gamma = \frac{N'(d_1)}{S_0 \cdot \sigma \cdot \sqrt{T}}$$
$$\Theta = -\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2 \cdot \sqrt{T}} - r \cdot K \cdot e^{-r \cdot T} \cdot N(-d_2)$$

Hence, the left-hand side of the given equation is:

$$-\frac{S_0 \cdot N'(d_1) \cdot \sigma}{2 \cdot \sqrt{T}} + r \cdot K \cdot e^{-r \cdot T} \cdot N(-d_2) - r \cdot S_0 \cdot N(-d_1) + \frac{1}{2} \cdot \sigma \cdot S_0 \cdot \frac{N'(d_1)}{\sqrt{T}} =$$
$$= r \cdot [K \cdot e^{-r \cdot T} \cdot N(-d_2) - S_0 \cdot N(-d_1)] = r \cdot \Pi$$

c) For a portfolio of options, Π, Δ, Θ and Γ are the sums of their values for the individual options in the portfolio. It follows that the given equation is true for any portfolio of European put and call options.

Question 4:

(See Hull – Practice Question 24.1)

From equation $\bar{\lambda}(T) = \frac{s(T)}{1-R}$, the average hazard rate over the three years is

$$\overline{\lambda_{1-3}} = \frac{0.0050}{1-0.3} = 0.0071$$

or 0.71% per year.

Question 5:

(See Hull – Practice Question 24.2)

From equation $\bar{\lambda}(T) = \frac{s(T)}{1-R}$, the average hazard rate over the five years is

$$\overline{\lambda_{1-5}} = \frac{0.0060}{1-0.3} = 0.0086$$

or 0.86% per year. Using the results in Question 4, the hazard rate is $\overline{\lambda_{1-3}} = 0.0071$ per year for the first three years.

By definition,

$$\overline{\lambda_{1-5}} = \frac{1}{5} \cdot \sum_{i=1}^{5} \overline{\lambda_i}$$

therefore $\overline{\lambda_1} + \overline{\lambda_2} + \overline{\lambda_3} + \overline{\lambda_4} + \overline{\lambda_5} = 5 \cdot \overline{\lambda_{1-5}}$.

Also, by definition,

$$\overline{\lambda_{1-3}} = \frac{1}{3} \cdot \sum_{i=1}^{3} \overline{\lambda_i}$$

therefore $\overline{\lambda_1} + \overline{\lambda_2} + \overline{\lambda_3} = 3 \cdot \overline{\lambda_{1-3}}$.

From the above, we have that

$$\overline{\lambda_4} + \overline{\lambda_5} = \overline{\lambda_1} + \overline{\lambda_2} + \overline{\lambda_3} + \overline{\lambda_4} + \overline{\lambda_5} - \left(\overline{\lambda_1} + \overline{\lambda_2} + \overline{\lambda_3}\right) = 5 \cdot \overline{\lambda_{1-5}} - 3 \cdot \overline{\lambda_{1-3}}$$

So, the average hazard rate in years 4 and 5 is

$$\frac{\overline{\lambda_4} + \overline{\lambda_5}}{2} = \frac{5 \cdot \overline{\lambda_{1-5}} - 3 \cdot \overline{\lambda_{1-3}}}{2} = \frac{5 \cdot 0.0086 - 3 \cdot 0.0071}{2} = 0.0109$$

or 1.09% per year.