CF 966

Financial Engineering and Risk Management

Tutorial Questions: The Greeks

1. (i) Define delta, gamma and Vega for an individual derivative.
   (ii) Explain how gamma and Vega can be used in the risk management of a portfolio that is delta hedged.

2. Derive the delta of a European call option, where
   (a) The underlying stock pays no dividend,
   (b) The underlying stock pays a continuous dividend q.

3. Derive the gamma of a European call option, where
   (a) The underlying stock pays no dividend,
   (b) The underlying stock pays a continuous dividend q.

4. Using questions 2 and 3, deduce the theta of a European call option, where
   (a) The underlying stock pays no dividend,
   (b) The underlying stock pays a continuous dividend q.

5. An investment bank has written a number, x, of European call options on a non- dividend paying stock with strike price 200p, current stock price 180p, time to expiry of six months and an assumed continuously-compounded interest rate of 3% p.a. The bank is delta-hedging the option position assuming the Black-Scholes framework holds and currently holds 250,000 shares of the stock and is short £413,057 in cash.

   (i) By using the hedging position and the Black-Scholes formula for the value of the option, derive two equations satisfied by x and σ, the bank' s assumed volatility.
   (ii) Estimate the implied volatility interpolation (Hint. Calculate the ratio $N(d_1):N(d_2).$)
   (iii) Deduce the value of x.

6. Assume the Black-Scholes model applies. An ATM European call option on a stock has an exercise date one year away and a strike of $118.57. The option is priced at $10. The continuously compounded risk-free rate is 1% pa.

    (i)   Estimate the implied volatility to within 1% per annum.

    (ii)   Calculate the corresponding hedging portfolio in share and cash for 1000 options on the share.

    (iii)   Calculate the option's Vega.

    (iv)   Price a European put option on the same stock with the same expiry date and a strike of $110.

    (v)   The hedging portfolio of the call option has the same value, the same delta and the same Vega as the option. The delta of the put option is -0.29975 and its Vega is 39.435. Determine the hedging portfolio of the call option in terms of shares, cash and the put option.