CF 966

Financial Engineering and Risk Management

Tutorial Solutions: Swaps

1.

We find the PV of the cost of the two barrels based on the forward prices:

$$PV = 22e^{-0.06\times 1} + 23e^{-0.065\times 2}$$

$$= \$40.91501$$

Let x be the agreed swap price. Then

$$xe^{-0.06\times 1} + 23e^{-0.065\times 2} = 40.9150$$

$$\Rightarrow x = \$22.483/\text{barrel}.$$

2.

i) $$PV = 20e^{-0.06\times 1} + 21e^{-0.065\times 2} + 22e^{-0.07\times 3}$$

$$= 57.7880$$

Let x be the agreed swap price:

$$xe^{-0.06\times 1} + xe^{-0.065\times 2} + xe^{-0.07\times 3} = 57.788$$

$$\Rightarrow x = 21.969.$$

ii) First we calculate PV (cost two barrels) for years 2 & 3

$$21e^{-0.065\times 2} + 22e^{-0.07\times 3} = 36.273.$$

$\therefore$ Swap price x:

$$x(e^{-0.065\times 2}) + xe^{-0.07\times 3} = 36.273 \Rightarrow x = 21.480.$$

3.

- The total gain to all parties from the swap is therefore $1.4-0.5=0.9\%$ per annum.
- The bank gets 0.1% per annum of this gain.
- The swap should make each of A and B 0.4% per annum better off.
- This means that it should lead to A borrowing at LIBOR -0.3% and to B borrowing at 6.0%.

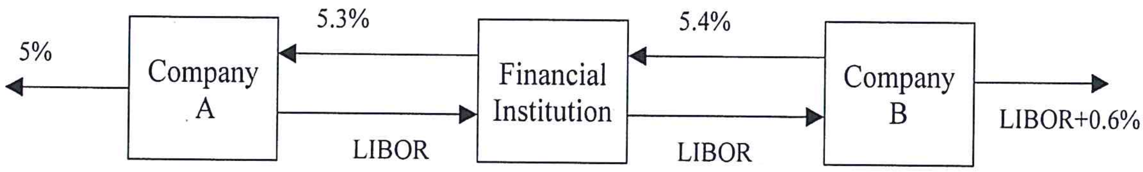

Figure 1

4.

- The total gain to all parties from the swap is therefore $1.5-0.4=1.1\%$ per annum.
- The bank requires 0.5% per annum.
- This leaves 0.3% per annum for each of X and Y.
- The swap should lead to X borrowing dollars at $9.6-0.3=9.3\%$ per annum and to Y borrowing yen at $6.5-0.3=6.2\%$ per annum.

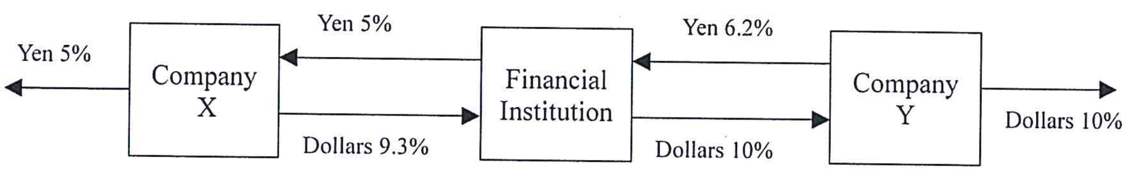

Figure 2

5. In four months \$3.5 million ($=0.5\times0.07\times\$100$ million) will be received and \$2.3 million ($=0.5\times0.046\times\$100$ million) will be paid. (We ignore day count issues.) In 10 months \$3.5 million will be received, and the LIBOR rate prevailing in four months' time will be paid. The value of the fixed-rate bond underlying the swap is

$$3.5e^{-0.05\times4/12}+103.5e^{-0.05\times10/12}=\$102.718 \text{ million}$$

The value of the floating-rate bond underlying the swap is

$$(100+2.3)e^{-0.05\times4/12}=\$100.609 \text{ million}$$

The value of the swap to the party paying floating is $\$102.718-\$100.609=\$2.109$ million. The value of the swap to the party paying fixed is $-\$2.109$ million.

These results can also be derived by decomposing the swap into forward contracts. Consider the party paying floating. The first forward contract involves paying \$2.3 million and receiving \$3.5 million in four months. It has a value of $1.2e^{-0.05\times4/12}=\$1.180$ million. To value the second forward contract, we note that the forward interest rate is 5% per annum with continuous compounding, or 5.063% per annum with semi-annual compounding. The value of the forward contract is

$$100\times(0.07\times0.5-0.05063\times0.5)e^{-0.05\times10/12}=\$0.929 \text{ million}$$

The total value of the forward contracts is therefore $\$1.180+\$0.929=\$2.109$ million.

6. A swap rate for a particular maturity is the average of the bid and offer fixed rates that a market maker is prepared to exchange for LIBOR in a standard plain vanilla swap with that maturity. The swap rate for a particular maturity is the LIBOR/swap par yield for that maturity.

7. The spread between the interest rates offered to X and Y is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments.

- This means that the total apparent benefit to all parties from the swap is 0.8%perannum.
- 0.2% per annum will go to the bank.
- This leaves 0.3% per annum for each of X and Y. In other words, company X should be able to get a fixed-rate return of 8.3% per annum while company Y should be able to get a floating-rate return LIBOR + 0.3% per annum.
- The bank earns 0.2%, company X earns 8.3%, and company Y earns LIBOR + 0.3%.

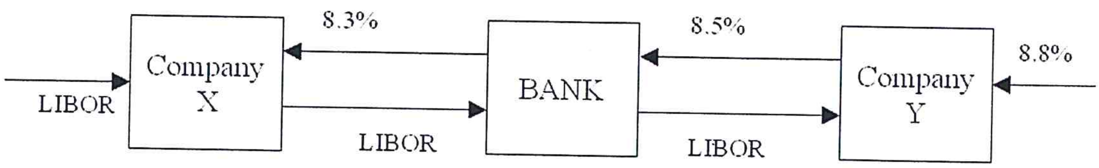

Figure 3

8. The total benefit to all parties from the swap is therefore: $80-40=40$ basis points.

It is possible to design a swap which will earn 10 basis points for the bank while making each of A and B 15 basis points better off than they would be by going directly to financial markets. One possible swap is shown in Figure 4. Company A borrows at an effective rate of

6.85% per annum in U.S. dollars. Company B borrows at an effective rate of 10.45% per annum in sterling. The bank earns a 10-basis-point spread. The way in which currency swaps such as this operate is as follows. Principal amounts in dollars and sterling that are roughly equivalent are chosen. These principal amounts flow in the opposite direction to the arrows at the time the swap is initiated.

Note that the bank is exposed to some exchange rate risk in the swap. It earns 65 basis points in U.S. dollars and pays 55 basis points in sterling. This exchange rate risk could be hedged using forward contracts.

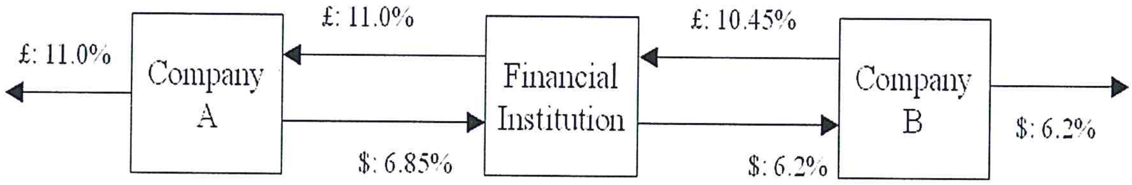

Figure 4 One Possible Swap

9. Discount rate (continuous compounded) = 11.82%.
   PV(new floating payment) = 100.94 m.
   PV(fixed) = 98.678 m.
   Swap value = $2.263 m.