Solutions to Lab 2: Risk management and the Greek letters I

Question 1:

(See lecture slides "Week 19 - Risk management and the Greek letters II")

i) $\Delta = \frac{\partial f}{\partial S}$, $\Gamma = \frac{\partial^2 f}{\partial S^2}$, $\mathcal{V} = \frac{\partial f}{\partial \sigma}$ (Remember to define the symbols f, S, σ)

ii) If the portfolio is delta-hedged and has a high value of Γ then it will require more frequent rebalancing or larger trades than one with a low value of Γ. Keeping gamma close to zero minimises the need for rebalancing.
If $\mathcal{V}$ has a low value then the portfolio is relatively insensitive to changes in the value of volatility. A low value of $\mathcal{V}$ is sought.

Question 2:

(See lecture slides "Week 18 - Risk management and the Greek letters I" , and "Week 19 - Risk management and the Greek letters II")

i) $N(d_1)$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$

ii) $e^{-q\cdot T}\cdot N(d_1)$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r-q+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$

Question 3:

(See lecture slides "Week 19 - Risk management and the Greek letters II")

i) $\frac{N'(d_1)}{S_0\cdot\sigma\cdot\sqrt{T}}$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$ and $N'(d_1) = \frac{1}{\sqrt{2\pi}}\cdot e^{-d_1^2/2}$

ii) $\frac{N'(d_1)\cdot e^{-q\cdot T}}{S_0\cdot\sigma\cdot\sqrt{T}}$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r-q+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$ and $N'(d_1) = \frac{1}{\sqrt{2\pi}}\cdot e^{-d_1^2/2}$

Question 4:

(See lecture slides "Week 19 - Risk management and the Greek letters II")

i) $-\frac{S_0\cdot N'(d_1)\cdot\sigma}{2\sqrt{T}} - r\cdot K\cdot e^{-r\cdot T}\cdot N(d_2)$, where $d_1 = \frac{ln\left(\frac{S_0}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}}$, $d_2 = d_1 - \sigma\cdot\sqrt{T}$,
and $N'(d_1) = \frac{1}{\sqrt{2\pi}}\cdot e^{-d_1^2/2}$

ii) $-\frac{S_0 \cdot N'(d_1) \cdot \sigma \cdot e^{-q \cdot T}}{2\sqrt{T}} + q \cdot S_0 \cdot N(d_1) \cdot e^{-q \cdot T} - r \cdot K \cdot e^{-r \cdot T} \cdot N(d_2)$,

where $d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right) \cdot T}{\sigma \cdot \sqrt{T}}$, $d_2 = d_1 - \sigma \cdot \sqrt{T}$, and $N'(d_1) = \frac{1}{\sqrt{2\pi}} \cdot e^{-d_1^2/2}$

Question 5:

(See Hull – Practice Question 19.10 based on Chapter 19.12)

The delta of a European futures call option is usually defined as the rate of change of the option price with respect to the futures price (not the spot price). It is

$$e^{-rT} N(d_1).$$

Here we have $F_0 = 8, K = 8, r = q = 0.12, \sigma = 0.18, T = 0.6667.$

Therefore,

$$d_1 = \frac{\ln\left(\frac{8}{8}\right) + \left(0.12 - 0.12 + \frac{0.18^2}{2}\right) \cdot 0.6667}{0.18 \cdot \sqrt{0.6667}} = 0.0735$$

Then we have $N(d_1) = 0.5293$

and the delta of the option is

$$e^{-0.12 \cdot 0.6667} \cdot 0.5293 = 0.4886$$

So, the delta of the short position in 1,000 futures options is

$$-1000 \cdot 0.4886 = -488.6$$