Solutions to Lab 1: Swaps

Question 1:
(See lecture slides "Week 17 – Swaps and risk management I")

Year 1 present value: $PV_1 = 20 \cdot e^{-0.06 \cdot 1} = \$20.72$ per barrel
Year 2 present value: $PV_2 = 23 \cdot e^{-0.065 \cdot 2} = \$20.20$ per barrel
Present value: $PV_{12} = 20.72 + 20.20 = \$40.92$ per two barrels

Let price/barrel = x for each year of 2-year swap
$x \cdot e^{-0.06 \cdot 1} + x \cdot e^{-0.065 \cdot 2} = 40.92 \implies x = \$22.483$ per barrel

Question 2:
(See lecture slides "Week 17 – Swaps and risk management I")

Year 1 present value: $PV_1 = 20 \cdot e^{-0.06 \cdot 1} = \$18.84$ per barrel
Year 2 present value: $PV_2 = 21 \cdot e^{-0.065 \cdot 2} = \$18.44$ per barrel
Year 3 present value: $PV_3 = 22 \cdot e^{-0.07 \cdot 3} = \$17.83$ per barrel

i) Present value: $PV_{123} = 18.84 + 18.44 + 17.83 = \$55.11$ per three barrels
Let price/barrel = x for each year of 3-year swap
$x \cdot e^{-0.06 \cdot 1} + x \cdot e^{-0.065 \cdot 2} + x \cdot e^{-0.07 \cdot 3} = 55.11 \implies x = \$20.951$ per barrel

ii) First we calculate the cost of two barrels for years 2 and 3.
Present value: $PV_{23} = 18.44 + 17.83 = \$36.27$ per two barrels
Let price/barrel = x for each year of the 2-year swap starting after one year
$x \cdot e^{-0.065 \cdot 2} + x \cdot e^{-0.07 \cdot 3} = 36.27 \implies x = \$21.478$ per barrel

Question 3:
(See Hull - Chapter 7.4)

The total gain to all parties from the swap is
$(6.4 - 5) - \big((LIBOR + 0.6) - (LIBOR + 0.1)\big) = 1.4 - 0.5 = 0.9\%$ per annum.
The bank (financial institution) gets 0.1% per annum of this gain.
The swap should make each of A and B gain of $\frac{0.9-0.1}{2} = 0.4\%$ per annum.
This means that it should lead to A borrowing at $LIBOR - 0.3\%$ and to B borrowing at 6%.
The following scheme represents the cash flows:

Question 4:
(See Hull - Chapter 7.7)

The total gain to all parties from the swap is
$(6.5-5)-(10-9.6)=1.5-0.4=1.1\%$ per annum.
The bank (financial institution) requires 50bp, that is, 0.5% per annum of this gain.
The swap should make each of X and Y gain of $\frac{1.1-0.5}{2}=0.3\%$ per annum.
The swap should lead to X borrowing dollars at $9.6-0.3=9.3\%$ per annum and to Y borrowing yen at $6.5-0.3=6.2\%$ per annum.

The following scheme represents the cash flows:

Question 5:
(See Hull – Practice Question 7.2)

Consider the party paying floating. The first exchange involves paying \$1.2 million and receiving \$2.0 million in four months. It has a value of $0.8e^{-0.027\cdot 4/12}=\$0.7928$ million.
To value the second forward contract, we note that the forward interest rate is 3% per annum with semiannual compounding. The value of the forward contract is
$100\cdot(0.04\cdot 0.5-0.03\cdot 0.5)\cdot e^{-0.027\cdot 10/12}=\$0.4889$ million.
The total value of the forward contracts is therefore $\$0.7928+\$0.4889=\$1.2817$. This is the value of the swap to the party paying floating. For the party paying fixed, the value is $-\$1.2817$.

Question 6:
(See Hull – Practice Question 7.9)

The spread between the interest rates offered to X and Y is 0.8% per annum on fixed rate investments and 0.0% per annum on floating rate investments. This means that the total apparent benefit to all parties from the swap is 0.8% per annum. Of this, 0.2% per annum will go to the bank. This leaves 0.3% per annum for each of X and Y. In other words, company X should be able to get a fixed-rate return of 8.3% per annum while company Y should be able to get a floating-rate return LIBOR + 0.3% per annum.

The required swap is shown in the following scheme. The bank earns 0.2%, company X earns 8.3%, and company Y earns LIBOR $+ 0.3\%$.