

Example 7.1

$u' = -\sin(t)$; $u(0)=1$; Find $u(2)$

| h | exact | explicitEuler | implicitEuler | Iterated RK2 | Iterated RK4 | Adams34 pc |
|--------|-----------|---------------|---------------|--------------|--------------|------------|
| 0.1 | -0.416147 | -0.369502 | -0.460431 | -0.416737 | -0.416147 | -0.41615 |
| 0.01 | -0.416147 | -0.411589 | -0.420682 | -0.416153 | -0.416147 | -0.416147 |
| 0.001 | -0.416147 | -0.415692 | -0.416601 | -0.416147 | -0.416147 | -0.416147 |
| 0.0001 | -0.416147 | -0.416101 | -0.416192 | -0.416147 | -0.416147 | -0.416147 |

Example 7.2

$u' = -10*(u-\cos(t))-\sin(t)$; $u(0)=1$; Find $u(2)$

| h | exact | explicitEuler | implicitEuler | Iterated RK2 | Iterated RK4 | Adams34 pc |
|--------|-----------|---------------|---------------|--------------|--------------|------------|
| 0.1 | -0.416147 | -0.41792 | 1 | -0.415621 | -0.416095 | -0.416148 |
| 0.01 | -0.416147 | -0.416309 | -0.415987 | -0.416143 | -0.416147 | -0.416147 |
| 0.001 | -0.416147 | -0.416163 | -0.416131 | -0.416147 | -0.416147 | -0.416147 |
| 0.0001 | -0.416147 | -0.416148 | -0.416145 | -0.416147 | -0.416147 | -0.416147 |

Example 7.3

$u' = -2100*(u-\cos(t))-\sin(t)$; $u(0)=1$; Find $u(2)$

| step size | exact | explicitEuler | implicitEuler | Iterated RK2 | Iterated RK4 | Adams34 pc |
|-----------|-----------|---------------|---------------|--------------|--------------|------------|
| 0.01 | -0.416147 | -3.83E+254 | nan | nan | nan | nan |
| 0.001 | -0.416147 | -1.45E+76 | nan | -1.33E+81 | -4.16E-01 | -4.17E+283 |
| 0.0001 | -0.416147 | -4.16E-01 | -0.416147 | -0.416147 | -0.416147 | -0.416147 |
| 0.000001 | -0.416147 | -0.416147 | -0.416147 | -0.416147 | -0.416147 | -0.416147 |

At a step size of 10^{-2} , every method failed to converge. With a step size of 10^{-3} , RK4 converged to a moderate approximation of the solution. With a step size of 10^{-4} , all methods converge to the same solution.

Comments: Implicit and explicit euler methods appear to converge at similar rates, but from opposite directions. The predictor corrector method converges faster than RK4 for the first two problems, producing an accurate result with a step size as large as 10^{-1} , but blows up to a very large number on the third problem when step size is small. Iterated RK4 does make more function calls, but the time difference is too small to be measured for the given problems.