Integration of a double exponential between r_{in} and r_0 and between 0 and z_0

$$I = 4\pi \int_{r_{in}}^{r_0} \int_{0}^{z_0} A_0 \, e^{-\frac{r}{\lambda_s}} e^{-\frac{z}{z_s}} \, r \, dr \, dz$$

$$I_r = \int_{r_{in}}^{r_0} e^{-\frac{r}{\lambda_s}} \, r \, dr$$

$$f(x) = r$$
$$g'(x) = e^{-\frac{r}{\lambda_s}}$$

$$I_r = -\lambda_s e^{-\frac{r}{\lambda_s}} r \Big|_{r_{in}}^{r_0} - \int_{r_{in}}^{r_0} (-\lambda_s) e^{-\frac{r}{\lambda_s}} \, dr =$$

$$= -\lambda_s \left(e^{-\frac{r_0}{\lambda_s}} r_0 - e^{-\frac{r_{in}}{\lambda_s}} r_{in} \right) + \lambda_s (-\lambda_s) e^{-\frac{r}{\lambda_s}} \Big|_{r_{in}}^{r_0} =$$

$$= -\lambda_s r_0 e^{-\frac{r_0}{\lambda_s}} + \lambda_s r_{in} e^{-\frac{r_{in}}{\lambda_s}} - \lambda_s^2 e^{-\frac{r_0}{\lambda_s}} + \lambda_s^2 e^{-\frac{r_{in}}{\lambda_s}}$$

$$I_r = \lambda_s^2 \left(e^{-\frac{r_{in}}{\lambda_s}} - e^{-\frac{r_0}{\lambda_s}} + \frac{r_{in}}{\lambda_s} e^{-\frac{r_{in}}{\lambda_s}} - \frac{r_0}{\lambda_s} e^{-\frac{r_0}{\lambda_s}} \right)$$

$$I_r = 4\pi z_s \lambda_s^2 A_0 \left(1 - e^{-\frac{z_0}{z_s}}\right)\left(e^{-\frac{r_{in}}{\lambda_s}} - e^{-\frac{r_0}{\lambda_s}} + \frac{r_{in}}{\lambda_s} e^{-\frac{r_{in}}{\lambda_s}} - \frac{r_0}{\lambda_s} e^{-\frac{r_0}{\lambda_s}} \right)$$

Integration of a distribution which is exponential in vertical direction and constant in radial direction

$$I = 4\pi \int_0^{r_{in}} \int_0^{z_0} const \cdot e^{-\frac{z}{z_s}} \, r \, dr \, dz = 4\pi \cdot \frac{r^2}{2} \Big|_0^{r_{in}} const \cdot z_s \cdot \left(1 - e^{-\frac{z_0}{z_s}}\right) = 2\pi r_{in}^2 z_s \left(1 - e^{-\frac{z_0}{z_s}}\right) const.$$

$$\underline{\underline{I = 2\pi r_{in}^2 z_s \left(1 - e^{-\frac{z_0}{z_s}}\right) const}}$$

For the case that the constant is the value of an exponential function in radial direction at $r = r_{in}$

$$\Rightarrow I = 2\pi r_{in}^2 z_s \left(1 - e^{-\frac{z_0}{z_s}}\right) \cdot A_0 \, e^{-\frac{r_{in}}{h_s}}$$

where A_0 would be the central emissivity of a double exponential