Integration of a distribution which is exponential in vertical direction and linearly varying with radius.

$$I = 4\pi \int_0^{r_{in}} \int_0^{z_0} const \cdot r \cdot e^{-\frac{z}{z_s}} \, r \, dr \, dz =$$

$$= 4\pi \, const \, \frac{r^3}{3} \Big|_0^{r_{in}} \cdot z_s \left(1 - e^{-\frac{z_0}{z_s}}\right) = \frac{4\pi}{3} r_{in}^3 \cdot z_s \cdot const \cdot \left(1 - e^{-\frac{z_0}{z_s}}\right)$$

For the case that the linear function is equal to the value of the exponential function at radius r_{in}:

$$const = A_0 \, r_{in}^{-1} \, e^{-\frac{r_{in}}{h_s}}$$

where A_0 would be the central emissivity of the double exponential

$$I = \frac{4\pi}{3} r_{in}^{\cancel{3}\,2} \cdot z_s \cdot A_0 \cdot e^{-\frac{r_{in}}{h_s}} \cdot \cancel{r_{in}} \left(1 - e^{-\frac{z_0}{z_s}}\right)$$

$$I = \frac{4\pi}{3} A_0 \, r_{in}^2 \cdot z_s \, e^{-\frac{r_{in}}{h_s}} \left(1 - e^{-\frac{z_0}{z_s}}\right)$$