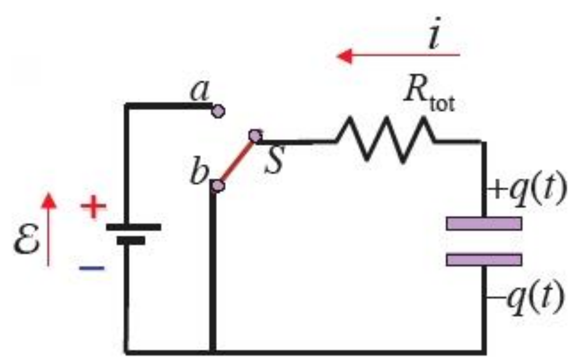
Problem 1 [25 pts]. Write a program that will calculate the electric field from the electric potential of a positively charged *conducting* sphere of radius R = 0.8 m and charge Q = 0.5 nC = 0.5×10^{-9}C. Use that the electric field is, $E=-dV/dr$, and for the sphere, the electric potential is given as ($k=9\times10^9$ N.m^2/C^2), $V=kQ/r=4.5/r$ [V], $r>R$ (outside) and $V=kQ/R=5.625$ V, $r\le R$ (inside). Calculate E using the forward differentiation with step h = 0.02 m at 21 points equally spaced in the interval of r from 0.001 m to 4 m. Plot the electric potential together with the numeric and theoretical electric fields ($E_{\text{th}}=4.5/r^2$ [V/m], outside, and zero inside) vs. r. Does the numeric result agree with the exact one?

Problem 2 [25 pts]. (No program is required for this problem.) A high-speed camera (1000 frames/sec and hence, 0.001 s between each frame) is used to calculate the speed of an accelerating object. The distance traveled by the object, as determined at each frame, is recorded in the table below. We want to calculate the speed of the object (v = d$x(t)$/dt) at time t_0 = 0.003 s using the smallest possible step h.

a) What is the minimum step h that could be used with the FD algorithm?
b) What is the minimum step h that could be used with the ED algorithm?
c) Calculate the speed of the object at time t_0 = 0.003 s using the ED algorithm with the step determined in b).

t, s	0.000	0.001	0.002	0.003	0.004	0.005	0.006
x, m	0.00	0.02	0.08	0.18	0.32	0.50	0.72

Problem 3 [25 pts]. In a RC circuit, a fully charged capacitor C starts discharging after the switch S is closed. The current coming out of the capacitor must equal the current going through the resistor, $C\dfrac{dV}{dt}+\dfrac{V}{R}=0$, and hence the voltage V across the capacitor over time is the solution of the ODE, $\dfrac{dV}{dt}=-\dfrac{V}{RC}$. Write a program using Euler's rule that will calculate the voltage V across the capacitor over time, 0s ≤ t ≤ 4 s, if $V(t_0=0)=V_0=$ 5V. The RC circuit time constant is $\tau = RC$ = 2s. Use `Double` data type for floating point numbers and step h = 0.1s (note that the h step determines the number of points in the interval 0s ≤ t ≤ 4 s used to solve the ODE). Make a plot of the calculated voltage and the exact solution, $V=V_0e^{-t/\tau}$, as a function of time. Briefly comment on the accuracy of Euler's algorithm.



Problem 4 (optional) [25 pts bonus]. Write a program that will calculate the first derivative of the 4-th degree Legendre polynomial, $P^4(x)$ = $(3-30x^2+35x^4)/8$, using three differentiation algorithms, forward difference (FD), central difference (CD), and extrapolated difference (ED). Use `Double` data type for floating-point numbers and h = 0.1. The program output should contain, x, $P^4(x)$, and the absolute errors for FD, CD, and ED [that is, numeric derivatives compared to the exact derivative $(-60x+140x^3)/8$] for 21 points of x equally spaced in the interval, –1 to 1. Make three plots for each of the absolute errors (FD, CD, and ED) as a function of x. Which differentiation method is the most accurate? What error is observed for the ED method?