PHYS 2411 – Fall 2016

Homework 3 due: 09/27/2016

Problem 1 [25 pts]. (No program is required for this problem.)

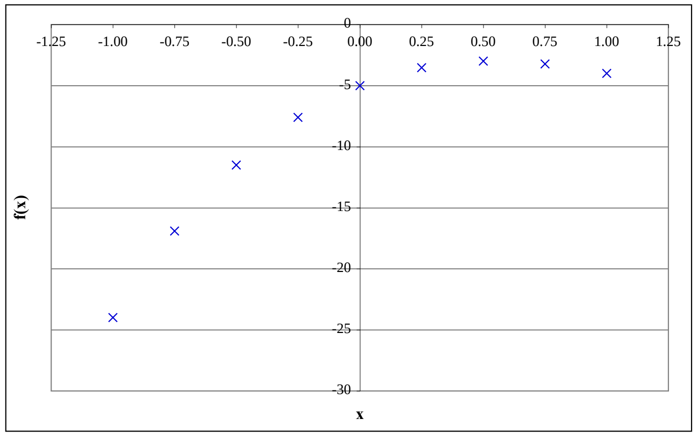
a) Show analytically that the minimal *total* error for Simpson's rule is achieved for fewer points N as compared to the trapezoid rule. Use that the relative approximation error is $\varepsilon_{\text{approx}} \approx 1/N^A$, with $A = 2$ for the trapezoid rule, and with $A = 4$ for Simpson's rule; also, the roundoff error accumulates randomly for both algorithms, that is $\varepsilon_{\text{ro}} \approx \varepsilon_{\text{m}} N^{1/2}$, where $\varepsilon_{\text{m}} \approx 10^{-15}$ is the machine precision for variables of *double data* type.

b) What are the minimal total errors for both rules? Is it possible for any of the rules to achieve an error close to machine precision?

Problem 2 [25 pts]. Use the programs you developed for **Lab 3** to numerically calculate, $\int_{-1}^{1} f(x)dx$ where an experiment has provided 9 measurements of $f(x)$ shown in the table below and the figure to the right. Use `double` data type for floating-point numbers.

a) Use Trapezoid rule or Simpson's rule: choose the method that yields the least total error.

b) Briefly comment on the use of Gaussian quadrature to integrate the tabulated function (data points) over the interval $-1 \le x \le 1$.



	Experimental data points for $f(x)$								
x	–1	–0.75	–0.50	–0.25	0.00	0.25	0.50	0.75	1.00
$f(x)$	–24.0000	–16.9063	–11.5000	–7.5938	–5.0000	–3.5313	–3.0000	–3.2188	–4.0000

Problem 3 [25 pts]. (No program is required for this problem.) The 4-th degree Legendre polynomial is given as, $P^4(x) = (3–30x^2+35x^4)/8$. Calculate the absolute approximation error of the forward difference (FD) and central difference (CD) algorithms with $h = 0.1$ for the first derivative of $P^4(x)$ as a function of x.

Problem 4 [25 pts]. Calculate the electric potential at point P at a distance d = 0.1 m on the y-axis above a uniformly charged rod with linear charge density λ=2×10^{-10} C/m and length L = 0.5 m. The potential at point P is an integral of the potential produced by infinitesimal sectors (of width dx) that make up the rod,

$$V = \int dV = \int k\frac{dq}{r} = \int_0^L \frac{k\lambda}{\sqrt{x^2+d^2}}dx = \int_0^L f(x)dx$$

with k = 9×10^9 N.m^2/C^2. Use one of the programs you created in **Lab 3** to integrates $f(x)$ in the interval $0 \le x \le L$, using <u>Simpson's rule</u> for 513 points. Give the answer in units of Volts (1V=1N.m/C). Compare your result with the exact one, $V = k\lambda \ln\frac{L+\sqrt{L^2+d^2}}{d}$, by printing to a file the numeric and exact results together with the relative error, $relErr = \left|\frac{y_{numeric} - y_{exact}}{y_{exact}}\right|$.