Problem 1 [25 pts]. Write a program to determine the mass of several different weights. We have a weight of mass m_1, two weights of mass m_2, three weights of mass m_3, four weights of mass m_4 and five weights of mass m_5, and the following five measurements have been performed. First, all of the masses (all 15 weights) are weighed and their total mass is measured to be 3 kg. Then, at each measurement a weight of each mass is removed and the total masses are found to be, 2.03 kg (for a weight of mass m_2, two m_3 weights, three m_4 weights and four m_5 weights), 1.16 kg, 0.44 kg and 0.02 kg, respectively. The five measurements provide a system of linear equations. Use the Gaussian Elimination algorithm to find the unknown masses: clearly, the square 5×5 matrix is already in an upper triangular form, so program only the backsubstitution method of this algorithm. *Hint*: there is a very simple way to assign the elements of the square matrix (**A**) in your program—just note that $a_{i,j} = -i + j + 1$ for $j \geq i$, and zero, otherwise (i, j=1,…,5).

Problem 2 [25 pts]. (No program is required for this problem.) Can the Gaussian Elimination algorithm (with no pivoting) be used to solve the following systems of equations? Briefly explain.

1) $$\left| \begin{array}{l} 2x + y = 15 \\ 5y = 5 \end{array} \right.$$

2) $$\left| \begin{array}{l} 3z^2 - 4x = 1 \\ 2.5z^2 + \ln x = 4.12 \end{array} \right.$$

3) $$\left| \begin{array}{l} 3u^2 - 4v^2 = -3 \\ (u - v)^2 + 2v^2 + 2uv = 4 \end{array} \right.$$

4) $$\begin{pmatrix} 0 & 5 & 1 \\ 1 & 2 & -3 \\ 4 & 9 & 11 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$$

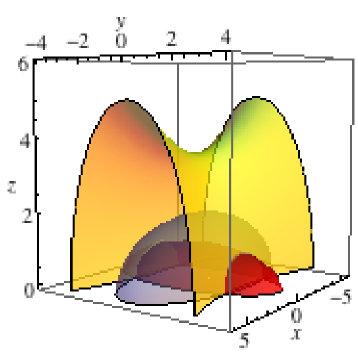
5) $$\left| \begin{array}{l} 3\cos\alpha - 2\tan^2\phi = 1 \\ 2.5\cos\alpha + 11\tan^2\phi = 8 \end{array} \right.$$

Problem 3 [25 pts]. Write a program that will calculate the point(s), where the following three surfaces intersect,

$$\begin{array}{llll} x^2 & +8y^2 & +10z^2 & = 36 \\ 2x^2 & +1y^2 & +11z^2 & = 12 \\ 5x^2 & -50y^2 & -14z^2 & = -180 \end{array},$$

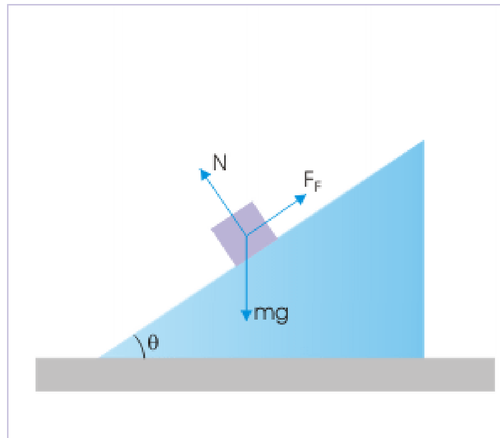
given that,

$$\mathbf{A} = \begin{pmatrix} 1 & 8 & 10 \\ 2 & 1 & 11 \\ 5 & -50 & -14 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 8 & 10 \\ 0 & -15 & -9 \\ 0 & 0 & -10 \end{pmatrix}.$$



Problem 4 [25 pts]. Write a program to determine the angle of an incline, θ, and the normal force, N, on an object of mass m=2 kg, which sits at rest on the incline. The coefficient of friction is μ=0.4. Use the two-dimensional Newton-Raphson algorithm to find the roots of the two functions,

$$f(\theta, N) = mgcos\theta - N = 0$$
$$h(\theta, N) = mgsin\theta - \mu N = 0$$



Use that, for two dimensions, the system of linear equations for the Newton-Raphson algorithm is,

$$\begin{pmatrix} f_\theta f_N \\ h_\theta h_N \end{pmatrix}\Bigg|_{(\theta_0,N_0)} \begin{pmatrix} \theta_1 - \theta_0 \\ N_1 - N_0 \end{pmatrix} = -\begin{pmatrix} f \\ h \end{pmatrix}\Bigg|_{(\theta_0,N_0)} \Leftrightarrow$$

$$\begin{pmatrix} -mgsin\theta_0 & -1 \\ mgcos\theta_0 & -\mu \end{pmatrix} \begin{pmatrix} \theta_1 - \theta_0 \\ N_1 - N_0 \end{pmatrix} = -\begin{pmatrix} f \\ h \end{pmatrix}\Bigg|_{(\theta_0,N_0)},$$

can be solved for the new guesses, θ_1 and N_1, in the following way,

$$\theta_1 = \theta_0 - \frac{f h_N - h f_N}{f_\theta h_N - h_\theta f_N}\Bigg|_{(\theta_0,N_0)}, \quad N_1 = N_0 - \frac{h f_\theta - f h_\theta}{f_\theta h_N - h_\theta f_N}\Bigg|_{(\theta_0,N_0)},$$

where θ_0 and N_0 are some initial guesses (f_N denotes the partial derivative of f with respect to N, $\partial f/\partial N$; similarly, for f_θ, h_N, and h_θ). Start with initial guesses, $\theta_0 = 0.1$ rad and $N_0 = 19.6$ N ($=mg$) and repeat while $|f(\theta,N)|$ <u>or</u> $|h(\theta,N)|$ is greater than 1×10^{-8}, <u>and</u> the number of trials is ≤ 15.

Problem 5 [25 pts bonus]. (No program is required for this problem.) The following system of equations needs to be solved for the unknowns, x and t:

$$\begin{vmatrix} f_1(x,t) = -0.03x^3 + 4\sin t - 3.97 = 0 \\ f_2(x,t) = 2.5x^3 + \sin t - 3.5 = 0 \end{vmatrix}$$

Among the three algorithms, (1) Gaussian Elimination, (2) LU Decomposition with Pivoting, and (3) Newton-Raphson algorithm, choose the <u>best</u> numerical algorithm to solve the system above. *Explain*.