Economics 202A
Fall 2017

Answers to Midterm Exam

Instructions: Answer all questions in a bluebook(s). Budget your time wisely. Total points for the exam equal 100.

1. Consider an optimal growth model in which the planner makes consumption and investment decisions to maximize $\sum_{t=0}^{\infty}\beta^t(\log c_t + A\log c_{t-1})$ The technology for producing output, *y*, is $y_t = k_t^{\theta}h_t^{1-\theta}$. Hours worked, h_t, is constrained to be between 0 and 1. Capital, k_t, depreciates fully each period, so the resource constraint is given by $c_t + k_{t+1} = k_t^{\theta}h_t^{1-\theta}$.

A. Write the planner's problems as a dynamic program. (10 points)

Ans: $v(k,d) = \max_{h,k'}\{\log c + A\log d + \beta v(k',d')\}$

subject to $c + k' = k^{\theta}h^{1-\theta}$

$d' = c$

$0 \le h \le 1$

B. Use value iteration and the method of undetermined coefficients to solve for the optimal law of motion for the capital stock. Carefully explain each step of your solution procedure. (20 points)

Ans: $k' = \beta\theta k^{\theta}$

C. Assume a decentralized version of this economy with markets for the consumption good, labor, capital services, and **one-period bonds** that when purchased in period *t* deliver one unit of consumption in period *t*+1. Define a *recursive competitive equilibrium* for this economy. Derive the equilibrium aggregate law of motion for capital and the pricing functions that satisfy your definition. (20 points)

Ans:
Household's Problem:
$v(K,D,k,d,b) = \max_{h,b',k'}\{\log c + A\log d + \beta v(K',D',k',d',b')\}$

subject to
$c + k' + q(K,D)b' = w(K,D)h + r(K,D)k + b$

$d' = c$

$K' = G(K,D)$ *and* $D' = F(K,D)$

Firm's Problem:
$\max_{h_f,k_f}\{k_f^{\theta}h_f^{1-\theta} - r(K,D)\,k_f - w(K,D)h_f\}$

A recursive competitive equilibrium is set of decision rules for the households ($k'(K,D,k,d,b)$, $h(K,D,k,d,b)$, $b'(K,D,k,d,b)$*), a set of decision rules for the firm (* $k_f(K,D)$ *and* $h_f(K,D)$*), a set of pricing functions (*$q(K,D)$, $w(K,D)$, $r(K,D)$*), and aggregate policy functions* $G(K,D)$ *and* $F(K,D)$ *such that*

1. *Given the functions q, w, r, F and G, the function b', k' and h solve the household's dynamic program.*
2. *Given the functions w and r, the functions* k_f *and* h_f *solve the firm's problem.*
3. *Markets clear:* $b'(K,D,K,D,0)=0$, $k_f(K,D)=K$ *and* $h_f(K,D)=h(K,D,K,D,0)$.
4. *Rational expectations:* $G(K,D)=k'(K,D,K,D,0)$ *and* $F(K,D)=K^{\theta}(h(K,D,K,D,0))^{1-\theta}-k'(K,D,K,D,0)$

According to this definition:

$G(K,D)=\beta\theta K^{\theta}$

$w(K,D)=(1-\theta)K^{\theta}$

$r(K,D)=\theta K^{\theta-1}$

$q(K,D)=\beta^{1-\theta}\theta^{-\theta}K^{\theta(1-\theta)}$

2.A $V(\hat{k}, \hat{c}_2) = \max_{h, \hat{k}', \hat{c}_2'} \left\{ \alpha \log \hat{c}_1 + (1-\alpha) \log \hat{c}_2 + A \log(1-h) + \beta \eta V(\hat{k}', \hat{c}_2') \right\}$

subject to $\hat{c}_1 + \hat{x} + \hat{i} = \hat{k}^{\theta} h^{1-\theta}$

$$\eta \gamma^{\frac{1}{1-\theta}} \hat{c}_2' = (1-\delta_D)\hat{c}_2 + \hat{x}$$

$$\eta \gamma^{\frac{1}{1-\theta}} \hat{k}' = (1-\delta)\hat{k} + \hat{i}$$

Where $\hat{c}_{1t} \equiv \dfrac{c_{1t}}{\gamma^{\frac{t}{1-\theta}}}$, $\hat{c}_{2t} = \dfrac{c_{2t}}{\gamma^{\frac{t}{1-\theta}}}$

$$\hat{x}_t = \frac{x_t}{\gamma^{\frac{t}{1-\theta}}}, \quad \hat{i}_t = \frac{i_t}{\gamma^{\frac{t}{1-\theta}}}$$

$$\hat{k}_t = \frac{k_t}{\gamma^{\frac{t}{1-\theta}}}$$

B. $\dfrac{\cancel{\alpha} \cancel{\eta} \gamma^{\frac{1}{1-\theta}}}{\cancel{\bar{c}_1}} = \beta \left[\dfrac{\theta \bar{k}^{\theta-1} \bar{h}^{1-\theta} + 1 - \delta}{\cancel{\bar{c}_1}} \right]$

$$\frac{\alpha \gamma^{\frac{1}{1-\theta}}}{\bar{c}_1} = \beta \left[\frac{(1-\alpha)}{\bar{c}_2} + \frac{\alpha(1-\delta_D)}{\bar{c}_1} \right]$$

$$\frac{\alpha(1-\theta)\bar{k}^{\theta}\bar{h}^{-\theta}}{\bar{c}_1} = \frac{A}{1-\bar{h}}$$

$$\bar{c}_1 + \bar{c}_2[\beta\gamma^{\frac{1}{1-\theta}} - 1 + \delta_D] + \bar{k}[\beta\gamma^{\frac{1}{1-\theta}} - 1 + \delta].$$

$$= \bar{k}^{\theta}\,\bar{h}^{1-\theta}$$

These 4 equation can be solved to get $\bar{c}_1, \bar{c}_2, \bar{k}, \bar{h}$

Hence, balanced growth path is

$$c_{1t} = \gamma^{\frac{t}{1-\theta}}\bar{c}_1, \quad c_{2t} = \gamma^{\frac{t}{1-\theta}}\bar{c}_2$$

$$i_t = \gamma^{\frac{t}{1-\theta}}[\beta\gamma^{\frac{1}{1-\theta}} - 1 + \delta]\bar{k}$$

$$x_t = \gamma^{\frac{t}{1-\theta}}[\beta\gamma^{\frac{1}{1-\theta}} - 1 + \delta_D]\bar{c}_2$$

$$k_t = \gamma^{\frac{t}{1-\theta}}\bar{k}$$

$$h_t = \bar{h}$$

C. i. No need to impute a service flow from consumer durables since these are explicitly modeled and provide utility.

No need to provide detail on components of NIPA. Just need to describe time series that need to be obtained from NIPA.

Model	NIPA
c_{1t}	Consumption of services + nondurables + gov. consumption
c_{2t}	stock of consumer durables
x_t	purchases of durable consumption goods
i_t	Gross private domestic investment + gov. investment + net exports

Model	Nipa
k_t	given (capital used in production)
y_t	RGNP

ii. Steady state conditions can be rewritten

(1) $\dfrac{\gamma^{\frac{1}{1-\theta}}}{\beta} - 1 + \delta = \theta \dfrac{\bar{y}}{\bar{k}}$

(2) $\alpha\left[\dfrac{\gamma^{\frac{1}{1-\theta}}}{\beta} - 1 + \delta_D\right] = (1-\alpha)\dfrac{\bar{c}_1}{\bar{c}_2}$

(3) $\dfrac{\alpha(1-\theta)\,\bar{y}}{\bar{c}_1\,\bar{h}} = \dfrac{A}{1-\bar{h}}$

(4) $\dfrac{\bar{x}}{\bar{c}_2} = [\gamma\gamma^{\frac{1}{1-\theta}} - 1 + \delta_D]$

(5) $\dfrac{\bar{i}}{\bar{k}} = [\gamma\gamma^{\frac{1}{1-\theta}} - 1 + \delta]$

~~from (1)~~ we can get

$$\theta = \text{avg}\left\{\frac{\text{capital income}}{\text{RGNP}}\right\}$$

Where GNP=capital income+labor income. Details on how this income accounting is done is not required.

$$\gamma^{\frac{1}{1-\theta}} = \text{avg}\left\{\frac{RGNP_{t+1}}{RGNP_t}\right\}$$

$\Rightarrow$ can solve for γ

$$\frac{\bar{x}}{\bar{c}_2} = \text{avg}\left(\frac{x_t}{c_{2t}}\right)$$

$\Rightarrow$ can get δ_D from (4)

$$\frac{\bar{i}}{\bar{k}} = \text{avg}\left\{\frac{i_t}{k_t}\right) \Rightarrow \text{solve for } \delta \text{ from (5)}$$

$$\frac{\bar{k}}{\bar{y}} = \text{avg}\left(\frac{k_t}{y_t}\right) \Rightarrow \text{solve for } \beta \text{ from (1)}$$

$$\frac{\bar{c}_1}{\bar{c}_2} = \text{avg}\left(\frac{c_{1t}}{c_{2t}}\right) \Rightarrow \text{solve for } \alpha \text{ from (2)}$$

$$\frac{\bar{c}_1}{\bar{y}} = \text{avg}\left(\frac{c_{1t}}{y_t}\right) \Rightarrow \text{Solve for } A \text{ from (3)}$$