ECON 202A : Problem Set Fall, 2019

Lee Ohanian *

November 17, 2019

Due date: Friday, December 3 - send pdf of homework answers to Manos. Include your computer code, and comment all key lines of the code. Include all names of group members on homework. Each group turns in just one homework. If you use a program other than Matlab, then indicate that at the top of your answers.

1 Stochastic Processes and Linearized Economic Models

This problem will help you understand the connection between stochastic processes and linearized economic models.

Consider the economy in Robert Hall's 1978 Journal of Political Economy paper, "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis".

(1) In this economy, assume that $r = \delta$. Prove Hall's Corollary 3, on page 974 and show that consumption evolves as a random walk, which is an AR(1) with a unit autoregressive coefficient, no constant term, and white noise error term.

(2) Explain the economic intuition for why the stochastic process for income is irrelevant in terms of being able to forecast future consumption. Explain why the error term should be white noise.

(3) Explain the economic intuition why if $r > \delta$, then consumption evolves as a random walk with positive drift, in which there is a constant term in the regression that is positive.

(4) Obtain the quarterly real consumption data from FRED:
https://fred.stlouisfed.org/series/A794RX0Q048SBEA. Fit the following regression:

*Contact information: UCLA Bunche Hall 8391 - ohanian@econ.ucla.edu

$$\ln(c_t) = \mu + \lambda \ln(c_{t-1}) + u_t$$

Do you think that this is a reasonable statistical model of the log of consumption? (Your answer to this question may include a discussion regarding the value of the autoregressive coefficient, the R-square, and whether there is substantial correlation in the u_t residuals. Note that you should not conduct a t-test on the coefficient, as it does not have the same interpretation if $\lambda = 1$).

Next, consider the following economy.

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to:

$$\begin{aligned} z_t k_t^{\theta} + (1-\delta)k_t &= c_t + k_{t+1} \\ \ln(z_t) &= \rho \ln(z_{t-1}) + \varepsilon_t, \varepsilon \sim N(0, \sigma_\varepsilon^2) \end{aligned}$$

Assume that the time period is annual. Choose β so that the return to capital in the steady state is five percent, choose θ so that capital's share of income is 25 percent, and choose a depreciation rate such that the share of investment in the steady state is 25 percent. Choose $\rho = 0.9$ and $\sigma_\varepsilon^2 = .005$.

(6) Log-linearize this model around its deterministic steady state. (For simplicity, assume that z in the steady state is 1).

(7) Use the formula of Blanchard and Kahn to show that there is a unique stationary solution to the linearized system.

(8) Using a random number generator (Use the built-in Matlab function for this), draw 1200 values of ε to construct the z process. Choose a specific seed for the random number generator and save it, as you will also use the same seed below. Using these values of z, and assuming that k_0 is equal to its steady state value, use the linearized system to construct 1200 values values of output, consumption, and investment.

(9) Discard the first 200 observations, and then fit an AR(1) process to the log of consumption, measured as the log-deviation of consumption from the steady state value. Report the value of the AR(1) coefficient in the regression.

(10) Do the same thing as above except for choosing a value of θ so that capital's share of income is 65 percent. Adjust the depreciation rate so that the steady state share of investment remains at 25 percent. Use the same random number seed as before. Fit an AR(1) process to log consumption for this case. Which of the two cases is closer to a random walk for consumption?

What might be the economic reason for this?