Econ202A Homework #1

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1 Question 1

(a)

$$\begin{aligned} \max \sum_{t=0}^{\infty} 0.99 \log c_t \\ \text{s.t. } c_t + k_{t+1} \leq k_t^{0.36} \\ k_0 \text{ given} \end{aligned}$$

We can formulate this as a dynamic programming problem,

$$V(k) = \max_{k'} [\log(c_t) + 0.99V(k')]$$

Starting from $v_0(k) = 0$,

$$\max \log c_t \text{ s.t. } c_t \leq k_t^{0.36} - k_{t+1}$$

We know, from the functional forms for production and utility, that the solution takes the form

$$\begin{aligned} V(k) = A + \frac{0.36}{1 - 0.36 \cdot 0.99} \log(k), \\ \implies k' = 0.36 \cdot 0.99 k^{0.36} \end{aligned}$$

(b)

Starting from the Bellman Equation

$$V(k) = \max_{k'} [\log(c_t) + 0.99V(k')]$$

We can take the first order condition with respect to k' and then apply the envelope theorem to get

$$\begin{aligned} \frac{1}{c_t} = \beta \frac{1}{c_t} 0.36 k^{0.36-1} \\ \bar{k} = 0.19948 \end{aligned}$$

(c) Matlab code:

```
beta = 0.99;
delta = 0;
epsilon = 1e-6;
alpha = 0.36;
delta = 1;

ks = ((1 - beta*(1 - delta))/(alpha * beta))^(1/(alpha  - 1));

kmin = 0;
kmax = 0.2;
grid_size = 10000;

dk = (kmax-kmin)/(grid_size - 1);
kgrid = linspace(kmin, kmax, grid_size);
v = zeros(grid_size, 1);
dr = zeros(grid_size, 1);
norm = 1;

while norm > epsilon;
    for i=1:grid_size
        tmp = (kgrid(i)^alpha + (1 - delta)*kgrid(i) - kmin);
        imax = min(floor(tmp/dk) + 1, grid_size);

        c = kgrid(i)^alpha + (1 - delta)*kgrid(i) - kgrid(1:imax);
        util = log(c);
        [tv(i), dr(i)] = max(util + beta*(1:imax));
    end;
    norm = max(abs(tv - v));
    v = tv;
end;
```

This returns a steady-state capital stock value of 0.1995

(e) Since there is depreciation, I increased the maximum capital stock and used enough more grid points than recommended in the problem.

Matlab code

```
beta = 0.99;
epsilon = 1e-6;
alpha = 0.36;
delta = .02;

ks = ((1 - beta*(1 - delta))/(alpha * beta))^(1/(alpha  - 1));

kmin = 0;
kmax = 50;
grid_size = 10000;

dk = (kmax-kmin)/(grid_size - 1);
kgrid = linspace(kmin, kmax, grid_size);
v = zeros(grid_size, 1);
dr = zeros(grid_size, 1);
norm = 1;

while norm > epsilon;
    for i=1:grid_size
        tmp = (kgrid(i)^alpha + (1 - delta)*kgrid(i) - kmin);
        imax = min(floor(tmp/dk) + 1, grid_size);

        c = kgrid(i)^alpha + (1 - delta)*kgrid(i) - kgrid(1:imax);
        util = log(c);
        [tv(i), dr(i)] = max(util + beta*(1:imax));
    end;
    norm = max(abs(tv - v));
    v = tv;
end;
```

This returns a steady-state capital stock value of 48.2992

2 Question 2

(a)

$$V(x,k) = \max_{k'}[u(e^{z_t}k_t^\theta - k_{t+1}) + \beta \int_{z'} V(z',k')dG(z')]$$
$$\text{s.t. } c_t + k_{t+1} \leq e^{z_t}k_t^\theta$$

From the common functional forms that we know, this implies the law of motion

$$k' = E + F\log + Gz$$

(b) Since we have two states, we need to set up two functions

$$\begin{aligned}
v_l(k) &\equiv v(z_l,k) \\
v_h(k) &\equiv v(z_h,k) \\
Tv_l(k) &= \max_{k'}(u(z_l)f(k) - k') + \beta[qv_l(k') + (1-q)v_h(k')] \\
Tv_h(k) &= \max_{k'}(u(z_h)f(k) - k') + \beta[(1-q)v_l(k') + qv_h(k')]
\end{aligned}$$

For this two-state Markov problem, the functional form is

$$\begin{aligned}
v_0^l(k) &= E_l F\log(k) \\
v_0^h(k) &= E_h F\log(k)
\end{aligned}$$

And we split our guesses across states.

$$k' = \begin{cases} E_l F\log(k) & \text{if } z = z_l \\ E_h F\log(k) & \text{if } z = z_h \end{cases}$$

3 Question 3

(a)

$$
\begin{aligned}
V(k) = & \max_{c,i,h_1,h_2,k_1,k'} \{u(c, 1 - h_1 - h_2) + \beta V(k')\} \\
\text{s.t. } & i = f_1(k_1, h_1) \\
& c = f_2(k - k_1, h_2) \\
& k' = i + (1 - \delta)k
\end{aligned}
$$

(b) Our RCE in this case has four components and four constraints:

1. Household decision rules $c(K, k)$, $h(K, k)$, and $k'(K, k)$ along with a value function $V(K, k)$
2. Two decision rules for each firm, $k^c(K)$, $h^c(K)$, $k^i(K)$, $h^i(K)$
3. Three price functions, $p_c(K)$, $r(K)$, $w(K)$
4. A law of motion for the aggregate capital stock $K' = \hat{G}(K)$

such that

(a) Given the price functions and perceived aggregate law of motion, the household decision rules solve the household's problem.

(b) Given the price functions, $k(K)$ and $h(K)$ solve the firm's problem (for each firm)

(c) Markets clear:

$$
\begin{aligned}
h(K, K) &= h^c(K) + h^i(K) \\
K &= k^c(K) + k^i(K) \\
k'(K, K) - (1 - \delta)K &= F_1(k^i, h^i)
\end{aligned}
$$

(d) $\hat{G}(K) = k'(K, K)$