Econ202A Assignment 1

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1 Problem 1

Consider the following problem solved be a representative agent:

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t, \quad 0 < \beta < 1$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq k_t^\theta, \quad 0 < \theta < 1, \\ k_0 &\text{ given.} \end{aligned}$$

(a)

$$U'(f(k_t) - k_{t+1}) = \beta U'(f(k_{t+1}) - k_{t+2}) f'(k_{t+1}) \qquad \text{(Euler Equation)}$$

$$\frac{1}{k_t^\theta - k_{t+1}} = \beta \frac{\theta k_{t+1}^{\theta-1}}{k_{t+1}^\theta - k_{t+2}} \tag{1}$$

$$\frac{1}{k_{T-1}^\theta - k_T} = \beta \frac{\theta k_T^{\theta-1}}{k_T^\theta} \tag{2}$$

$$k_T = \frac{\theta\beta}{1+\theta\beta} k_{T-1}^\theta \tag{3}$$

$$k_{t+1} = \frac{\theta\beta(1-(\theta\beta)^{T-t})}{1-(\theta\beta)^{T-t+1}} k_t^\theta \tag{4}$$

$$k_{t+1} = \theta\beta k_t^\theta \tag{5}$$

(b)

$$\overline{k} = \theta\beta \overline{k}^\theta \tag{6}$$

$$\overline{k}^{1-\theta} = \theta\beta \tag{7}$$

$$\overline{k} = (\theta\beta)^{\frac{1}{1-\theta}} \tag{8}$$

(c)

$$u'(f(k) - k') = \beta v'(k') \qquad \text{(FOC from Bellman)}$$

$$U'(f(k_t) - k_{t+1}) = \beta U'(f(k_{t+1}) - k_{t+2}) f'(k_{t+1}) \qquad \text{(Euler Equation)}$$

$$V'(S) = R_k(k) + \beta V'(k') B_S(S) \qquad \text{(Envelope Condition Read up on this)}$$

$$\frac{1}{k^\theta - k'} = \frac{\theta\beta}{1-\theta\beta} \frac{1}{k'} \tag{9}$$

$$k' = \theta\beta k^\theta \tag{10}$$

2 Problem 2

3 Problem 3 (due to turn in)