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Economics 202C
Problem Set 1
UCLA

Due date: Friday, December 11, 12pm before the TA session- send pdf of homework answers to Paula. Include your computer code, and comment all key lines of the code. Include all names of group members on homework. Each group turns in just one homework. If you use a program other than Matlab, then indicate that at the top of your answers.

I. Stochastic Processes and Linearized Economic Models

This problem will help you understand the connection between stochastic processes and linearized economic models.

Consider the economy in Hall's 1978 paper, "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis", Journal of Political Economy.

(1) In this economy, assume that $r = \delta$. Prove Hall's Corollary 1 and 2, and 4. In addition, how would you go about estimating the implied regression in Corrolary 4?

(2) Explain the economic intuition for why the stochastic process for income is irrelevant in terms of being able to forecast future consumption.

(3) Explain the economic intuition why if $r < \delta$, then consumption evolves as a random walk with positive drift, in which there is a constant term in the regression that is negative.

(4) Obtain quarterly real consumption (in chained dollars) from the U.S. national income and product accounts from 1950 through 2019. Fit the following regression:

$$\ln(c_t) = \mu + \lambda \ln(c_{t-1}) + u_t$$

(5) Do you think that this is a reasonable statistical model of the log of consumption? (Your answer to this question may include a discussion regarding the value of the autoregressive coefficient, the R-square, and whether there is autocorrelation in the u_t residuals.)

Next, consider the following economy.

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to:

$$\begin{aligned} z_t A_t^{1-\theta} k_t^{\theta} + (1-\delta)k_t &= c_t + k_{t+1}, A_t = (1+\gamma)^t, t = 0, 1, ... \\ \ln(z_t) &= \rho \ln(z_{t-1}) + \varepsilon_t, \varepsilon \tilde{} N(0, \sigma_{\varepsilon}^2) \end{aligned}$$

Assume that the time period is annual. Construct a detrended version of this economy and show the first order conditions. Choose β so that the return to capital in the steady state of the detrended economy is five percent, choose θ so that capital's share of income is 30 percent, and choose a depreciation rate such that the share of investment to GDP in the steady state is 20 percent. Choose $\rho = 0.95$, $\sigma_{\varepsilon}^2 = .002$ and $\gamma = 0.02$.

(6) Log-linearize this model around its deterministic steady state. (For simplicity, assume that z in the steady state is 1).

(7) Use the formula of Blanchard and Kahn to show that there is a unique stationary solution to the linearized system.

(8) Using a random number generator (Matlab has a built-in function for this), draw 1100 values of ε to construct the z process. Using these values of z, and assuming that k_0 is equal to its steady state value, use the linearized system to construct 1100 values values of output, consumption, and investment.

(9) Discard the first 100 observations, and then fit an AR(1) process to the log of consumption, measured as the log-deviation of consumption from the steady state value. Report the value of the AR(1) coefficient in the regression, and evaluate whether there is autocorrelation in the residuals.

(10) Compare the regression coefficient in (9) and your assessment of the autocorrelation in the residuals, to your answers in (4) and (5). Does the RBC model provide a good approximation to consumption dynamics? What does it tell us about using consumption data to try to discriminate between the Hall model and the RBC model?