The Network Origins of Aggregate Fluctuations
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Model

Household preferences

$$u(c_1, c_2, \ldots, c_n) = A \prod_{i=1}^{n} c_i^{\frac{1}{n}} \tag{1}$$

Sector production functions: each good produced by competitive sector; can be either consumed or used by other sectors as an input

$$x_i = z_i^{\alpha} l_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_i j} \tag{2}$$

- x_{ij}: Amount of commodity j used in the production of good i
- w_{ij}: Share of good j in total input use of firms in sector i
  - Correspond to the entries in input-output tables
- z_i: Idiosyncratic productivity shock to sector i. Independent across sectors and $\varepsilon \equiv \log(z_i) \sim F_i$.

Model

Assumption (Assumption 1)

Input shares of all sectors add up to 1: $\sum_{j=1}^{n} w_{ij} = 1 \;\; \forall \; i.$

- Can summarize the structure of intersectoral trade with the input-output matrix W, which has entries w_{ij}.
- Economy is completely specified by the tuple

$$\mathcal{E} = (\mathcal{I}, W, \{F_i\}_{i \in \mathcal{I}}), \quad \mathcal{I} \text{ is the number of sectors}$$

- Can equivalently represent the economy as a weighted directed graph on n vertices,
  - each vertex corresponds to a sector
  - A directed edge (j, i) with weight $w_{ij} > 0$ is present from vertex j to vertex i if sector j is an input supplier to sector i.

Model

Definition (Weighted Outdegree of sector i)

Share of sector i's output in the input supply of the entire economy, normalized by $1 - \alpha$,

$$d_i \equiv \sum_{j=1}^{n} w_{ji}.$$

- When all nonzero edge weights are identical, the outdegree of vertex i is proportional to the number of sectors it is a supplier for.

Competitive Equilibrium

The competitive equilibrium of the economy can be represented by value added:

$$y = \log(GDP) = \nu' \varepsilon \tag{3}$$

Definition (Influence Vector)

$$\nu = \frac{\alpha}{n}[I - (1-\alpha)W']^{-1}$$

- Aggregate output is a linear combination of log sectoral shocks with coefficients determined by the influence vector.
- Aggregate output depends on the intersectoral network of the economy through the Leontief inverse $[I - (1-\alpha)W']^{-1}$.
- Influence vector also captures how sectoral productivity shocks propagate downstream to other sectors through the input–output matrix.

Model—Influence Vector

- The influence vector can also be interpreted as a centrality measure.
- Central sectors in the network representation of the economy play a more important role in determining aggregate output.
- ν is also the sales vector of the economy in the sense that the ith element of the influence vector is equal to the equilibrium share of sales of sector i:

$$\nu_i = \frac{p_i x_i}{\sum_{j=1}^{n} p_j x_j} \tag{4}$$

Adding Network Structure

- Focus on a sequence of economics where the number of sectors increases
- Characterize how the structure of the intersectoral network affects aggregate fluctuations
- Sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$; economy n is

$$\mathcal{E} = (\mathcal{I}_n, W_n, \{F_{in}\}_{i\in\mathcal{I}_n})$$

- Since the total supply of labor is normalized to 1, increasing n (number of sectors) means disaggregating the structure of the economy

Adding Network Structure—Notation and Assumptions

- $\{y_n\}_{n\in\mathbb{N}}$ and $\{\nu_n\}_{n\in\mathbb{N}}$ are aggregate outputs and influence vectors
- w_{ij}^n and d_i^n are elements of the intersectoral matrix W_n and the degree of sector i
- $\{\varepsilon_n\}_{n\in\mathbb{N}}$ is the sequence of vectors of (log) sectoral shocks

Assumption (Assumption 2)

Given a sequence of economies $\mathcal{E}_{n\in\mathbb{N}}$, for any sector $i \in \mathcal{I}_n$ and all $n \in \mathbb{N}$,

(a) $\mathbb{E}\varepsilon_{in} = 0$

(b) $\mathrm{Var}(\varepsilon_{in}) = \sigma_{in}^2 \in (\underline{\sigma}^2, \overline{\sigma}^2)$, *where $0 < \underline{\sigma} < \overline{\sigma}$ are independent of n.*

Aggregate Volatility

- Assumption 2(a) and independent sectoral shocks imply that we can write aggregate volatility as

$$(\operatorname{Var} y_n)^{1/2} = \sqrt{\sum_{i=1}^{n} \sigma_{in}^2 \nu_{in}^2}.$$

- For any sequence of economies satisfying Assumption 2(b),

$$(\operatorname{Var} y_n)^{1/2} = \Theta(\|\nu_n\|_2).$$

- Aggregate volatility scales with the Euclidian norm of the influence vector as the economy becomes disaggregated.
- The rate of decay of aggregate volatility may not be equal to $\sqrt{n}$ (the standard prediction from the diversification argument).
- If $\|\nu\|_2$ is bounded away from zero for all n, then aggregate volatility does not disappear as $n \to \infty$.

Asymptotic Distributions

Theorem (Theorem 1)

Consider a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ *and assume that* $\mathbb{E}\varepsilon_{in}^2 = \sigma^2$ *for all* $i \in \mathcal{I}_n$ *and all* $n \in \mathbb{N}$

(a) *If* $\{\varepsilon_{in}\}$ *are normally distributed for all* i *and all* n*, then* $\frac{1}{\|v_n\|_2} y_n \xrightarrow{d} \mathcal{N}(0, \sigma^2)$

(b) *Suppose that there exist constant* $a > 0$ *and random variable* $\bar{\varepsilon}$ *with bounded variance and cumulative distribution function* $\bar{F}$*, such that* $F_{in}(x) < \bar{F}(x)$ *for all* $x < -a$*, and* $F_{in}(x) > \bar{F}(x)$ *for all* $x > a$*. Also suppose that* $\frac{\|v_n\|_\infty}{\|v_n\|_2} \longrightarrow 0$*. Then* $\frac{1}{\|v_n\|_2} y_n \xrightarrow{d} \mathcal{N}(0, \sigma^2)$

(c) *Suppose that* $\{\varepsilon_{in}\}$ *are identically, but not normally distributed for all* $i \in \mathcal{I}_n$ *and all* n*. If* $\frac{\|v_n\|_\infty}{\|v_n\|_2} > 0$*, then the asymptotic distribution of* $\frac{1}{\|v_n\|_2} y_n$*, when it exists, is nonnormal and has finite variance* σ^2*.*

First-Order Interconnections

- Characterize the rate of decay of aggregate volatility in terms of the structural properties of the intersectoral network.
- First result: The extent of asymmetry between sectors shapes the relationship between sectoral shocks and aggregate volatility.

Definition (Coefficient of Variation)

Given an economy $\mathcal{E}_n$ with sectoral degrees $\{d_1^n, d_2^n, \ldots, d_n^n\}$, the coefficient of variation is

$$\mathrm{CV}_n \equiv \frac{1}{\bar{d}_n}\left[\frac{1}{n-1}\sum_{i=1}^{n}\left(d_i^n - \bar{d}_n\right)^2\right]^{1/2}$$

where $\bar{d}_n = \left(\sum_{i=1}^{n} d_i^n\right)/n$ is the average degree.

First-Order Interconnections

Theorem (Theorem 2)

Given a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$, aggregate volatility satisfies

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(\frac{1}{n}\sqrt{\sum_{i=1}^{n}(d_i^n)^2}\right)$$

and

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(\frac{1+\mathrm{CV}_n}{\sqrt{n}}\right).$$

- High variability in degree sequence of intersectoral network $\implies$ high variability in effect of shocks on aggregate output.
- High CV $\implies$ few sectors are responsible for most inputs.
- Low productivity $\implies$ low productivity in downstream sectors.
- Aggregate volatility decays slower than $\sqrt{n}$.

Interpreting Theorem 2

Definition (Power Law Degree Sequence)

A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ has a power law degree sequence if there exist a constant $\beta > 1$, a slowly varying function $L(\cdot)$ satisfying $\lim_{t\to\infty} L(t)t^{\delta} = \infty$ and $\lim_{t\to\infty} L(t)t^{-\delta} = 0$ for all $\delta > 0$, and a sequence of positive numbers $c_n = \Theta(1)$ such that, for all $n \in \mathbb{N}$ and all $k < d^n_{\max} = \Theta\left(n^{1/\beta}\right)$, we have

$$P_n(k) = c_n k^{-\beta} L(k)$$

where $P_n(k) \equiv \frac{1}{n}\left|\{i \in \mathcal{I}_n : d^n_i > k\}\right|$ is the empirical counter-cumulative distribution function and $d^n_{\max}$ is the maximum degree of $\mathcal{E}_n$.

- Look at the special case where the intersectoral networks have power law degree sequences.
- The first part of Theorem 2 says that aggregate volatility is higher in economies whose degree sequences have "heavier tails".

Interpreting Theorem 2

Corollary (Corollary 1)

Consider a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ *with a power law degree sequence and the corresponding shape parameter* $\beta \in (1,2)$. *Then, aggregate volatility satisfies*

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(n^{-(\beta-1)/\beta-\delta}\right)$$

where $\delta > 0$ *is arbitrary.*

- If the degree sequence of the intersectoral network exhibits heavy tails, aggregate volatility decreases at a much slower rate than predicted by the diversification argument.
- Note that so far the authors have only provided a lower bound on the rate at which aggregate volatility vanishes.
- Higher-order structural properties of the intersectoral network can still prevent output volatility from decaying at rate $\sqrt{n}$.

Second-Order Interconnections and Cascades

- First-order interconnections provide little information about how shocks to a sector affect the downstream customers of downstream customers of the affected sector, etc.

- The next theorem provides a lower bound on the decay rate of aggregate volatility in terms of second-order interconnections in the intersectoral network.

Definition (Definition 3—2nd-Order Interconnectivity Coefficient)

The second-order interconnectivity coefficient of economy $\mathcal{E}_n$ is

$$\tau_2(W_n) \equiv \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i,j} w_{ji}^n w_{ki}^n d_j^n d_k^n .$$

- Measures extent to which high degree sectors are connected to each other via common suppliers

Second-Order Interactions and Cascades

Theorem (Theorem 3)

Given a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$, aggregate volatility satisfies

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\mathrm{CV}_n}{\sqrt{n}} + \frac{\sqrt{\tau_2\left(W_n\right)}}{n}\right)$$

- Shows how second-order interactions, captured by τ_2, affect aggregate volatility.
- Even if the empirical degree distributions of two sequences of economies are identical for all n, their aggregate volatilities may exhibit considerably different behaviors.
- This is a refinement of Theorem 2; it captures the notion that there is a clustering of significant sectors because they have common suppliers.

Interpreting Theorem 3

Corollary (Corollary 2)

Suppose that $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ *is a sequence of economies whose second-order degree sequences have power law tails with shape parameter* $\zeta \in (1,2)$*(cf. Definition 2). Then, aggregate volatility satisfies*

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(n^{-(\zeta-1)/\zeta-\delta}\right)$$

for any $\delta > 0$.

- If the distributions of second-order degrees have heavy tails, aggregate volatility decreases much more slowly than predicted by diversification.
- Second-order effects may dominate first-order effects.
- If a sequence of economies has power law tails for both first- and second-order degrees, with exponents β and ζ, then the tighter bound for the decay rate of aggregate volatility is determined by $\min\{\beta, \zeta\}$.

Balanced Structures

- With limited variations in the degrees of different sectors, aggregate volatility decays at rate $\sqrt{n}$.

Definition (Definition 4—Balanced Sequence of Economics)

A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ is balanced if $\max_{i\in\mathcal{I}_n} d_i^n = \Theta(1)$.

- When the intersectoral network is balanced and the role of intermediate inputs is not too large, volatility decays at rate $\sqrt{n}$.
- Other structural properties of the network cannot contribute to aggregate volatility.

Theorem (Theorem 4)

Consider a sequence of balanced economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$. Then there exists $\bar{\alpha}\in(0,1)$ such that, for $\alpha \geq \bar{\alpha}$, (var $y_n)^{1/2} = \Theta(1/\sqrt{n})$.

Interpreting Theorem 4

- Theorem 4 is both an aggregation and an irrelevance result for balanced economies.

- As an aggregation result, it suggests observational equivalence between the one-sector economy and any balanced multi-sector economy.

- As an irrelevance result, it shows that different input-output matrices generate roughly the same volatility for balanced economies.