Networks Pset #1

Chris Ackerman*

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Question 5

Without a network, the monopolist solves the problem

$$\max_p \; p \cdot q \qquad \text{(Assume } mc = 0\text{)}$$
$$\theta \sim U[0,1] \implies q = 1 - p$$
$$\max_p \; p \cdot (1-p)$$
$$\frac{\partial}{\partial p} p - p^2 = 0 \qquad \text{(FOC)}$$
$$1 - 2p = 0$$
$$p = \frac{1}{2}$$

Now let's put this on a network. We just need to cook up a counterexample, so here's a funny network that will work. Put all $\theta < \frac{1}{2} + \delta, \delta > 0$ into singletons. Put all $\theta \geq \frac{1}{2} + \delta$ in a completely connected network. I guess the best economic "story" for this is that all of the rich guys live in a gated community and hang out together, and all the poor guys are on their own somewhere else. So not *too* crazy. Now, as we send $\varepsilon \to 0$ we only hit the giant component (all of the $\theta \in \left[\frac{1}{2} + \delta, 1\right]$). Let's see how the monopolist's price without a network does, and if we can do better.

$$\begin{aligned}\pi &= p \cdot q \\ &= p \cdot \left(1 - \left[\frac{1}{2} + \delta\right]\right) \\ &= \left(\frac{1}{2}\right)^2 - \frac{\delta}{2}\end{aligned}$$

But the monopolist can do better! $\delta > 0 \implies \theta > p$ for all the guys that are actually buying this product. Everyone is getting some consumer surplus. We can squeeze *all* of this surplus out of the guy with $\underline{\theta} = \frac{1}{2} + \delta$, and we can squeeze *some* surplus (δ) out of everyone that's buying. Basically, increasing the price to $p' \in \left(\frac{1}{2}, \frac{1}{2} + \delta\right]$ doesn't cause anybody to stop buying the product, but does give a higher per-unit price

*I worked on this problem set with Aristotle Magganas, Antonio Martner and Koki Okumura.

to the monopolist.

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\begin{aligned}
\pi' &= p' \cdot q \\
&= \underbrace{\left(\frac{1}{2} + \delta\right)}_{\text{higher price}} \underbrace{\left(1 - \left[\frac{1}{2} + \delta\right]\right)}_{\text{same quantity}} \\
&= \left(\frac{1}{2}\right)^2 - \frac{\delta}{2} + \underbrace{\delta\left(\frac{1}{2} - \delta\right)}_{\text{additional profit}} \\
&> \pi
\end{aligned}
$$

This holds for all $\delta \in \left(0, \frac{1}{2}\right)$.