Networks Pset #1

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Question 5

Without a network, the monopolist solves the problem

$$
\begin{aligned}
&\max_p \; p \cdot q && \text{(Assume } mc = 0\text{)} \\
\theta \sim U[0,1] \implies & \; q = 1 - p \\
&\max_p \; p \cdot (1-p) \\
\frac{\partial}{\partial p} & p - p^2 = 0 && \text{(FOC)} \\
1 - 2p &= 0 \\
p &= \frac{1}{2}
\end{aligned}
$$

Now let's put this on a network. We just need to cook up a counterexample, so here's a funny network that will work. Put all $\theta < \frac{1}{2} + \delta, \delta > 0$ into singletons. Put all $\theta \geq \frac{1}{2} + \delta$ in a completely connected network. Now, as we send $\varepsilon \to 0$ we only hit the giant component (all of the $\theta \in \left[\frac{1}{2} + \delta, 1\right]$). Let's see how the monopolist's price without a network does, and if we can do better.

$$
\begin{aligned}
\pi &= p \cdot q \\
&= p \cdot \left(1 - \left[\frac{1}{2} + \delta\right]\right) \\
&= \left(\frac{1}{2}\right)^2 - \frac{\delta}{2}
\end{aligned}
$$

But the monopolist can do better! $\delta > 0 \implies \theta > p$ for all the guys that are actually buying this product. Everyone is getting some consumer surplus. We can squeeze *all* of this surplus out of the guy with $\underline{\theta} = \frac{1}{2} + \delta$, and we can squeeze *some* surplus (δ) out of everyone that's buying. Basically, increasing the price to $p' \in \left(\frac{1}{2}, \frac{1}{2} + \delta\right]$ doesn't cause anybody to stop buying the product, but does give a higher per-unit price to the monopolist.

$$
\begin{aligned}
\pi' &= p' \cdot q \\
&= \underbrace{\left(\frac{1}{2} + \delta\right)}_{\text{higher price}} \underbrace{\left(1 - \left[\frac{1}{2} + \delta\right]\right)}_{\text{same quantity}} \\
&= \left(\frac{1}{2}\right)^2 - \frac{\delta}{2} + \underbrace{\delta\left(\frac{1}{2} - \delta\right)}_{\text{additional profit}} \\
&> \pi
\end{aligned}
$$

*I worked on this problem set with Aristotle Magganas, Antonio Martner and Koki Okumura.

This holds for all $\delta \in \left(0, \frac{1}{2}\right)$.