Game Theory

Problem Set 2

1. Consider the following coordination game. There are two players $i = 1, 2$, who can choose actions $a_i \in \{0, 1\}$. Let θ be a parameter drawn from a uniform distribution on $\{0, 1, 2, ..., 10\}$ before the game is played. Payoffs are given by:

$$u_i(a_i, a_j, \theta) = \begin{cases} a_j + \frac{1}{10}\theta - c & \text{if } a_i = 1 \\ 0 & \text{if } a_i = 0 \end{cases}$$

where c is known with $\frac{3}{2} < c < 2$.

   a. Describe the Nash equilibria of this game given that the realization of θ is commonly known.

   b. Now suppose that instead of observing θ, players' information is given by the following partitions:

$$\begin{aligned} \text{Player 1} &: \quad (\{0\}, \{1,2\}, \{3,4\}, ..., \{9,10\}) \\ \text{Player 2} &: \quad (\{0,1\}, \{2,3\}, ..., \{8,9\}, \{10\}) \end{aligned}$$

   That is, if $\theta = 3$, then player 1 learns that $\theta \in \{3, 4\}$ for sure (he then assigns equal probability to both of these values due to the uniform prior), while player 2 learns that $\theta \in \{2, 3\}$ (and similarly assigns equal probability to each of these values). Show that in this game, neither player will ever play $a = 1$.

   c. Connect this model to the global game model discussed in class (no need to write anything).

2. (Only for PhDs!!!) Consider the following global game with continuum of players. Each player chooses an action in $\{0, 1\}$. Let $u(a, l, \theta)$ be the utility function of each player, where $a \in \{0, 1\}$ is his action, $l \in [0, 1]$ is the proportion of players that choose action 1 and θ is the state of the world. Now, each player observes only $x = \theta + \varepsilon_i$, where θ is drown from an improper uniform prior over the real line, and ε_i are distributed independently across players with some cdf F (with pdf f). Let $\pi(l, \theta) = u(1, l, \theta) - u(0, l, \theta)$.

Below you can take the following results for granted: conditional on θ, x has a distribution function $F_{x|\Theta}(x|\theta)$ equal to $F_{\varepsilon}(x-\theta)$ and a density function $f_{x|\Theta}(x|\theta)$ equal to $f_{\varepsilon}(x-\theta)$. Also conditional on observing x, θ has a distribution function $f_{\Theta|x}(\theta|x)$ equal to $f_{\varepsilon}(x-\theta)$.

a. Let $\Psi(l, x, k)$ be the probability that a player assigns to a proportion less than l observing signal greater than k, if he himself observed signal x. Show that

$$\Psi(l, x, x) = l,$$

independently of F.

b. Let $\pi^*(x, k)$ be the expected gain to choosing action 1 if the player got signal x and knows others choose 1 if their signal is greater than k. Show that the result in a. implies that

$$\pi^*(x, x) = \int_{l=0}^{1} \pi(l, x)\, dl.$$

In the solution you will see how to apply the results from exercise 4 to the following model of currency crises. The current value of a currency is e. If the currency is attacked and not defended by the central bank, the value of the currency drops to $\zeta(\theta) < e$. The CB will not defend the currency and the devaluation occurs if a proportion $a(\theta)$ of speculators attacks, where a is increasing (this is a reduced model of costs of defending that depend on θ). The cost of attacking to the speculators is t. Thus, if $a = 1$ stands for "not attack" we have:

$$\begin{aligned} u(1, l, \theta) &= 0, \\ u(0, l, \theta) &= \varepsilon - \zeta(\theta) - t \text{ if } l < 1 - a(\theta), \\ u(0, l, \theta) &= -t \text{ if } l > 1 - a(\theta). \end{aligned}$$

Basically, we will be able to write down an equation that implicitly defines a θ^* such that there is an equilibrium with the devaluation occuring if the realized θ is below θ^*.