Notes for *The Network Origins of Aggregate Fluctuations*

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1 Overview

- Paper was written shortly after the 2008 Financial Crisis
- **Leading question:** How does the organization of the input-output network in an economy affect economic volatility?
- **Leading example: One sector**
  - If there's one sector and one firm, then any shock to that firm shuts down the entire economy.
  - But if we send $n_{\text{firms}} \to \infty$ then shocks to individual firms become unimportant, and the economy is resilient to uncorrelated shocks.
- **Leading Counterexample: Automakers during 2008**
  - Some suppliers make small but important parts for all the carmakers
  - If one of these small companies collapses, there's a chance that none of the carmakers will be able to continue making cars
  - Ford asked congress to support Chrysler and GM so that their mutual suppliers wouldn't experience any disruptions
- **Research Question:** Can we write down a network model of these interactions to generalize this insight? And can we fit this model to actual input/output data from the US?
- NB we can put figures 1–3 on slides to show the 2 leading examples of symmetric networks, and then the actual 1997 network to show that it exhibits meaningful asymmetry.

2 Approach

- Consider a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ corresponding to different levels of disaggregation
- Each economy $\mathcal{E}_n$ has n sectors whose input requirements are captured by an $n \times n$ matrix W_n
- entry (i, j) captures the share of sector j's product in sector i's production technology
- jth column sum, the *degree* of sector j, is the share of j's output in the input of the entire economy
- Given a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$, investigate whether *aggregate volatility* (st. dev. of log otuput) vanishes as $n \to \infty$.
  - **Preview of results: SOMETIMES**
- Main focus: when does LLN hold and the network structure has an important effect on aggregate fluctuations
  - Aggregate output might concentrate around its mean at a rate slower than $\sqrt{n}$; sectoral shocks may have a significant role in creating aggregate shocks, even if a disaggregated economy

  - Two causes of slow rates of aggregate volatility decay:
    1. First-order interconnections: shocks to a sector that is a supplier to lots of other sectors; direct propagation
    2. Higher-order interconnections: low productivity in one sector might reduce productivity in a sequence of interconnected sectors

3 Results

Theorem 2: Provides a lower bound on asymmetry across sectors captured by variation in sectoral degrees. Higher variation in the degree of different sectors implies lower rates of decay for aggregate volatility.

Theorem 3: Tighter lower bound on second-order interconnectivity between different sectors. Two economics with identical empirical degree distributions (first-order connections) may have significantly different levels of aggregate volatility because of interactions with downstream sectors.

Theorem 4: Sectoral shocks average out at rate $\sqrt{n}$ for *balanced* networks. The nature of aggregate fluctuations resulting from sectoral shocks is not related to the sparseness of the input-output matrix, but the extent of asymmetry between sectors.

- Empirical exercise (section 4):
  - Empirical distribution of both first- and second-order degrees have Pareto tails, with second-order tail having a shape parameter of $\zeta = 1.18$
  - If this degree distribution also holds for large n, aggregate volatility in the US economy decays at rate slower than $n^{0.15}$
  - US input-output network more similar to a star network than complete network
  - In practice we might see sizable aggregate fluctuations from idiosyncratic shocks to different sectors

4 Model

- Representative household with one unit of inelastic labor; Cobb-Douglas preferences over n distinct goods:

$$u(c_1, c_2, \ldots, c_n) = A \prod_{i=1}^{n} (c_i)^{1/n}$$

- Each good is produced by a competitive sector and can be either consumed or used by other sectors as an input for production. Each sector uses C-D production with CRS; output of sector i is

$$x_i = z_i^{\alpha} \ell_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha)w_{ij}}$$

  z_i are productivity shocks across sectors, $\varepsilon_i \equiv \log(z_i) \sim F_i$. $w_{ij} \geq 0$ is the share of good j in the intermediate inputs of firms in sector i. This definition is nice because w_{ij} corresponds to the entries of input-output tables; will use this in section 4 for the calibration bit.

- Can summarize the structure of intersectoral trade with the input-output matrix W, which has entries w_{ij}.

- Economy is completely specified by the tuple

$$\mathcal{E} = (\mathcal{I}, W, \{F_i\}_{i \in \mathcal{I}}),$$

  where $\mathcal{I}$ is the number of sectors.

- Can equivalently represent the economy as a weighted directed graph on n vertices