L2 – Production Functions II

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Identifying production functions

Identification challenge: observed input (and entry/exit) decisions are endogenous to unobserved productivity, which also affects output directly

Last time, discussed three sources of identification:

1. Experimental or quasi-experimental variation; e.g., instruments
2. Statistical restrictions on productivity; e.g., fixed effects or AR(1)
3. Economic restrictions on input choices, e.g., optimal factor demand

Today, discuss identification + estimation in three canonical papers leveraging the latter two sources of identification $\hookrightarrow$

Olley and Pakes (1996)

I. OP as an empirical framework

II. OP as an applied contribution

III. Extensions
Levinsohn and Petrin (2003)
Ackerberg Caves Fraser (2015)

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Framework

- Research question: what do differences in firm-level output and inputs over time imply about their productivity?

- Strategy: estimate production functions to recover productivity, then study how productivity evolves over time and interacts with the changing composition of firms

- Recall our problem from last time: data on

$$\{q_{it}, \ell_{it}, k_{it}, z_{it}\}$$

  for a set of firms i over times $t \geq 0$.

- Assume that

$$q_{it} = f(\ell_{it}, k_{it}, z_{it}, \omega_{it}) + \varepsilon_{it}.$$

  We want f and $\{\omega_{it}\}_{i,t}$.

- In Olley and Pakes (1996), the control z_{it} is firm age, a_{it}, and time t.

Framework

Firm state variables: $(k_{it}, \omega_{it}, a_{it})$, where a_{it} is the firm age.

Capital evolves as

$$k_{i,t+1} = (1-\delta)k_{it} + i_{it}$$

Productivity evolves as

$$\omega_{i,t+1} = g(\omega_{it}) + \xi_{i,t+1}$$

with a cumulative distribution function

$$F_\omega(\omega_{i,t+1}|\omega_{it}).$$

Key assumptions

1. ω_{it} follows an exogenous, first-order, one-dimensional Markov process

- hard to verify (ω_{it} is unobservable...)
- rules out endogenous investments that affect productivity
  - $g(\cdot)$ only depends on ω_{it}
- rules out higher-order Markov processes (in principle, can extend with additional control variables...)
- rules out higher-dimensional productivity processes
- do OLS/FE use "weaker assumptions"? **No**
  - $\hookrightarrow$ OLS assumes $\omega_{it} = 0$ for all i and t
  - $\hookrightarrow$ FE assumes $\omega_{it} = \omega_i$ for all i and t

Key assumptions

2. Input decisions

- factor prices are assumed to be common across firms
  - no unobserved differences across firms
  - may differ over **time** — index functions by t to incorporate this
- factor prices evolve according to an exogenous first-order Markov process
  - e.g., no monopsony in labor markets, et cet
- capital is **dynamic** and **fixed** input
  - dynamic: forward-looking choice
  - fixed: cannot adjust at t, only at $t-1$
- labor is a **static** and **flexible** input
  - static: current decision does not affect production/costs in future (or past) periods
  - flexible: choose at t

Key assumptions

3. Homogenous output market

4. Timing

   1. k_{it} is determined from the last period's capital and investment choice
   2. ω_{it} is observed
   3. ℓ_{it}, i_{it} chosen; exit choice χ_{it} occurs
   4. profits are realized, exit/continuation

5. Equilibrium concept

   - Optimal investment, entry, and exit according to a symmetric Markov perfect equilibrium, or MPE (Hopenhayn and Rogerson 1993, Ericson and Pakes 1995)

Equilibrium

Ericson and Pakes (1995) show that strategies in an MPE will be

$$\chi_{it} = \begin{cases} 1 & \text{if } \omega_{it} \geq \underline{\omega}_t(a_{it}, k_{it}) \\ 0 & \text{otherwise.} \end{cases}$$

and investment is given by

$$i_{it} = \iota_t(\omega_{it}, a_{it}, k_{it}),$$

with

- $\iota_t(\cdot)$ increasing in ω_{it}
- $\underline{\omega}_t(a_{it}, k_{it})$ decreasing in k_{it}

These are two key equations we can take to the data. ✓

Endogeneity concerns

1. Simultaneity bias in labor ℓ_{it}.
   - in contrast, k_{it} is decided prior to the realization of ω_{it}
   - **however**, could still be correlated with ω_{it} through $\omega_{i,t-1}$
2. Selection bias:
   - all else equal, firms with
     - higher draws of ω_{it}
     - more capital k_{it}

     will be less likely to exit.
   - this means that $\mathbb{E}[\omega_{i,t+1}|\omega_{it}, \chi_{it} = 1] \neq \mathbb{E}[\omega_{i,t+1}|\omega_{it}]$
   - also implies correlation of k_{it} with ω_{it} through $\omega_{i,t-1}$

Estimation (1/3)

First, control for ω_{it} with investment:

$$\omega_{it} = \iota_t^{-1}(a_{it}, k_{it}; i_{it}).$$

In practice, first step is to estimate

$$q_{it} = \beta_0 + \beta_\ell \ell_{it} + \underbrace{\beta_a a_{it} + \beta_k k_{it} + \iota_t^{-1}(a_{it}, k_{it}; i_{it})}_{\phi_t(i_{it}, a_{it}, k_{it})} + \varepsilon_{it}$$

where $\phi_t(\cdot)$ is nonparametric (e.g., with a flexible polynomial expansion)

Remark. OP estimate β_ℓ in this step, alongside ϕ_t.

- the model implies there is no bias in β_ℓ
- but what generates variation in ℓ_{it} after controlling for ω_{it}? We will return to this issue soon.

Estimation (2/3)

Second, control for nonrandom exit. The selection problem is that

$$\mathbb{E}[\omega_{i,t+1}|\omega_{it}, \chi_{i,t+1} = 1]$$

in the data does not equal the "true" $\mathbb{E}[\omega_{i,t+1}|\omega_{it}]$. OP write this as a function of ω_{it} and a nonparametric choice probability,

$$\begin{aligned}
\mathbb{P}(\chi_{i,t+1} = 1|\underline{\omega}_{t+1}(\cdot), \omega_{it}, a_{it}, k_{it}) &= 1 - F(\underline{\omega}_{t+1}(\cdot)|\omega_{it}) \\
&\equiv P_t(i_{it}, a_{it}, k_{it}) \\
&= P_{it}
\end{aligned}$$

so that $\mathbb{E}[\omega_{i,t+1}|\omega_{it}, \chi_{t+1} = 1] = g(P_{it}, \underbrace{\phi_{it} - \beta_k k_{it} - \beta_a a_{it}}_{\omega_{it}})$.

In practice, can estimate this exit probability $\hat{P}_t(\cdot)$ using kernel estimation techniques or probit in polynomial of state variables.

Estimation (3/3)

So far, we have g, P, ϕ, and β_ℓ.

In the third step, OP obtain β_k using moments

$$q_{i,t+1} - \beta_\ell \ell_{i,t+1} = \beta_0 + \beta_a a_{i,t+1} + \beta_k k_{i,t+1} + g(P_{it}, \phi_{it} - \beta_k k_{it} - \beta_a a_{it}) + \xi_{i,t+1} + \varepsilon_{i,t+1}$$

which can be used to identify β_k via

$$\mathbb{E}[k_{i,t+1}\xi_{i,t+1}] = 0$$

because of the timing of the capital decision.

Telecommunications

Institutional background:

1. technological change
   - mechanical $\mapsto$ electronic tech for signal switching, transmission
2. regulatory change
   - FCC rulings starting with Carterphone in 1968
   - divestiture of the monopolist AT&T by January 1984
   - implications for AT&T / Western Electric vertical contract

Data: plant-level panel from the U.S. Census, 1963–1987

- producers of any "customer premise and network telecommunications equipment" (p. 1266)

Data

TABLE I

CHARACTERISTICS OF THE DATA

Year	Plants	Firms	Shipments (billions 1982 $)	Employment
1963	133	104	5.865	136899
1967	164	131	8.179	162402
1972	302	240	11.173	192248
1977	405	333	13.468	192259
1982	473	375	20.319	222058
1987	584	481	22.413	184178

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Divestiture, 1982–1984

TABLE II

BELL COMPANY EQUIPMENT PROCUREMENT
(PERCENT PURCHASED FROM WESTERN ELECTRIC)

1982	1983	1984	1985	1986[E]
92.0	80.0	71.8	64.2	57.6

[E] Estimated for 1986.
Source: NTIA (1988, p. 336, and discussion pp. 335–337).

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Entry and exit

TABLE III

ENTRANTS ACTIVE IN 1987

	Number	Share of Number Active in 1987 (%)	Share of 1987 Shipments (%)	Share of 1987 Employment (%)
Plants: New since 1972	463	79.0	32.8	36.0
Firms: New since 1972	419	87.0	30.0	41.4
Plants: New since 1982	306	52.0	12.0	13.5
Firms: New since 1982	299	60.1	19.4	27.5

TABLE IV

INCUMBENTS EXITING BY 1987

	Number	Share of Number Active in Base Year (%)	Share of Shipments in Base Year (%)	Share of Employment in Base Year (%)
Plants active in 1972 but not in 1987	181	60.0	40.2	39.0
Firms active in 1972 but not in 1987	169	70.0	13.8	12.1
Plants active in 1982 but not in 1987	195	41.2	26.0	24.1
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Production function estimates I

TABLE VI

ALTERNATIVE ESTIMATES OF PRODUCTION FUNCTION PARAMETERS[a]
(STANDARD ERRORS IN PARENTHESES)

Sample:	Balanced Panel		Full Sample[c, d]						
								Nonparametric F_ω	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimation Procedure	Total	Within	Total	Within	OLS	Only P	Only h	Series	Kernel
Labor	.851	.728	.693	.629	.628			.608	
	(.039)	(.049)	(.019)	(.026)	(.020)			(.027)	
Capital	.173	.067	.304	.150	.219	.355	.339	.342	.355
	(.034)	(.049)	(.018)	(.026)	(.018)	(.02)	(.03)	(.035)	(.058)
Age	.002	−.006	−.0046	−.008	−.001	−.003	.000	−.001	.010
	(.003)	(.016)	(.0026)	(.017)	(.002)	(.002)	(.004)	(.004)	(.013)
Time	.024	.042	.016	.026	.012	.034	.011	.044	.020
	(.006)	(.017)	(.004)	(.017)	(.004)	(.005)	(.01)	(.019)	(.046)
Investment	—	—	—	—	.13	—	—	—	—
					(.01)				
Other Variables	—	—	—	—	—	Powers of P	Powers of h	Full Polynomial in P and h	Kernel in P and h
# Obs.[b]	896	896	2592	2592	2592	1758	1758	1758	1758

Balanced v. full sample

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Control for selection

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Control for past productivity

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Preferred estimate

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Production function estimates II

TABLE VIII

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(STANDARD ERRORS IN PARENTHESES)

	Labor	Capital	Age	Time	# Obs.
1974–1978					
OLS	.78	.27	−.00	.03	832
	(.03)	(.03)	(.003)	(.01)	
3-Step Procedure[a]	.71	.29	.01	.10	578[b]
	(.05)	(.05)	(.03)	(.19)	
1982–1987					
OLS	.62	.33	−.01	−.02	1212
	(.03)	(.03)	(.002)	(.010)	
3-Step Procedure	.55	.40	.01	−.01	729
	(.05)	(.13)	(.02)	(.11)	
Switch Makers[c]					
OLS	.79	.25	−.01	.03	562
	(.05)	(.05)	(.003)	(.01)	
3-Step Procedure	.66	.31	.01	.01	387
	(.07)	(.16)	(.04)	(.16)	

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Productivity analysis (1/4)

TABLE IX

INDUSTRY PRODUCTIVITY GROWTH RATES[a]

Time Period	(1) Full Sample	(2) Balanced Panel
1974–1975	−.279	−.174
1975–1977	.020	−.015
1978–1980	.146	.102
1981–1983	−.087	−.038
1984–1987	.041	.069
1974–1987	.008	.020
1975–1987	.032	.036
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Productivity analysis (2/4)

TABLE X

VARIABLE COST EFFICIENCY[a]
(MINIMUM COST OF PRODUCTION DIVIDED BY ACTUAL COST OF PRODUCTION)

Years	Total	Interfirm	Intrafirm
1974–1977	.77	.84	.91
1978–1980	.69	.76	.91
1981–1983	.65	.72	.91
1984–1987	.72	.80	.89

[a] See text for details.

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Productivity analysis (3/4)

TABLE XI

DECOMPOSITION OF PRODUCTIVITY[a]
(EQUATION (16))

Year	p_t	$\bar{p}_t$	$\Sigma_i\, \Delta s_{it}\, \Delta p_{it}$	$\rho(p_t, k_t)$
1974	1.00	0.90	0.01	−0.07
1975	0.72	0.66	0.06	−0.11
1976	0.77	0.69	0.07	−0.12
1977	0.75	0.72	0.03	−0.09
1978	0.92	0.80	0.12	−0.05
1979	0.95	0.84	0.12	−0.05
1980	1.12	0.84	0.28	−0.02
1981	1.11	0.76	0.35	0.02
1982	1.08	0.77	0.31	−0.01
1983	0.84	0.76	0.08	−0.07
1984	0.90	0.83	0.07	−0.09
1985	0.99	0.72	0.26	0.02
1986	0.92	0.72	0.20	0.03
1987	0.97	0.66	0.32	0.10

Productivity analysis (3/4)

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1974	1.00	0.90	0.01	−0.07
1975	0.72	0.66	0.06	−0.11
1976	0.77	0.69	0.07	−0.12
1977	0.75	0.72	0.03	−0.09
1978	0.92	0.80	0.12	−0.05
1979	0.95	0.84	0.12	−0.05
1980	1.12	0.84	0.28	−0.02
1981	1.11	0.76	0.35	0.02
1982	1.08	0.77	0.31	−0.01
1983	0.84	0.76	0.08	−0.07
1984	0.90	0.83	0.07	−0.09
1985	0.99	0.72	0.26	0.02
1986	0.92	0.72	0.20	0.03
1987	0.97	0.66	0.32	0.10

Productivity analysis (3/4)

TABLE XI

DECOMPOSITION OF PRODUCTIVITY[a]
(EQUATION (16))

Year	p_t	$\bar{p}_t$	$\Sigma_i \, \Delta s_{it} \, \Delta p_{it}$	$\rho(p_t, k_t)$
1974	1.00	0.90	0.01	−0.07
1975	0.72	0.66	0.06	−0.11
1976	0.77	0.69	0.07	−0.12
1977	0.75	0.72	0.03	−0.09
1978	0.92	0.80	0.12	−0.05
1979	0.95	0.84	0.12	−0.05
1980	1.12	0.84	0.28	−0.02
1981	1.11	0.76	0.35	0.02
1982	1.08	0.77	0.31	−0.01
1983	0.84	0.76	0.08	−0.07
1984	0.90	0.83	0.07	−0.09
1985	0.99	0.72	0.26	0.02
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Productivity analysis (4/4)

TABLE XII

PROBIT MODELS OF EXIT PROBABILITIES[a]
(STANDARD ERRORS IN PARENTHESES)

	1	2	3
Intercept	−1.39 (.11)	−0.69 (.25)	−0.63 (.25)
Productivity	−0.16 (.06)	−0.15 (.06)	−0.16 (.06)
Age		0.00 (.01)	−0.00 (.01)
Capital		−0.09 (.03)	−0.10 (.03)
D2			−0.37 (.20)
D3			0.10 (.14)
D4			0.47 (.12)
# Obs.	2098	2098	2098
Log Likelihood	−392.2	−387.1	−372.1

[a] The dummy variables are defined as follows: Base period is 1974−1977; D2 = 1 for years 1978 to 1980, 0 otherwise; D3 = 1 for years 1981 to 1983, 0 otherwise; D4 = 1 for years 1984 to 1987, 0 otherwise.

Productivity analysis (4/4)

TABLE XII

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(STANDARD ERRORS IN PARENTHESES)

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LP

Investment data may not exist, or adjustment costs may imply many zeros, but the OP inversion requires continuous and nonzero investment.

The second crucial paper in this literature is Levinsohn and Petrin (2003).

LP point out that you can use intermediate inputs or "materials" (e.g., electricity) as a control instead, assuming

$$m_{it} = \chi_t(\omega_{it}, k_{it})$$

is monotone in ω_{it} for all k_{it}. Requires

- common input, output prices across firms
- no error in input demand function

LP, continued

A few remarks.

Rmk 1. Monotonicity somewhat testable

- LP plot χ_t and show that it is indeed increasing in ω (Figure 1)

Rmk 2. How to define m_{it}?

- in practice, often observe different kinds of materials
- this gives an overidentification test with various kinds of materials (Table 4)

Rmk 3. If F takes a parametric form, then χ_t may also take a known form.

- may imply a more efficient estimator than a fully nonparametric control (e.g., Doraszelski and Jaumandreu 2018)

Rmk 4. Where were materials in F before?

- added-value: usually "subtracted from output before estimation"
- see Gandhi Navarro Rivers (2016) for lively discussion

ACF

Ackerberg Caves Fraser 2015 ECTA.

Modern approach is to not identify β_ℓ in the first stage.

Problem: what shifts ℓ_{it} after we control for ω_{it}? ("multicollinearity problem")

ACF spent 9+ years trying to come up with cases where OP is consistent for β_ℓ.

They only find three:

- iid optimization error in ℓ_{it}, but not investment i_{it}
- iid shocks to the price of labor or output after i_{it} is chosen, but prior to ℓ_{it}
- labor is non-dynamic and chosen at $t - b$ as a function of $\omega_{i,t-b}$, whereas i_{it} is chosen at t

See paper for discussion.

ACF, continued

Propose using a **conditional** control. I.e., add ℓ_{it} to the investment equation:

$$\omega_{it} = \iota_t^{-1}(k_{it}, a_{it}, i_{it}, \ell_{it})$$

or whatever you are using as the proxy variable (e.g., materials).

Unlike LP/OP, allows for

- exogenous, serially correlated, unobserved firm-specific shocks to the price of labor
- firm-specific unobserved adjustment costs to the labor input
- labor to have dynamic effects (e.g., hiring or firing costs)

ACF, continued

No longer get β_ℓ in the first step. Instead, ACF recover ϕ and g, then identify all of f in the second stage, with

$$q_{i,t+1} = f(\ell_{i,t+1}, a_{i,t+1}, k_{i,t+1}) + g(P_{it}, \phi_{it} - f(\ell_{it}, k_{it}, a_{it})) + \xi_{i,t+1} + \varepsilon_{i,t+1}$$

using moments

- $\mathbb{E}[k_{i,t+1}\xi_{i,t+1}] = 0$ (because of timing)
- $\mathbb{E}[\ell_{it}\xi_{i,t+1}] = 0$ (lagged labor)

NB:

- now must also include ℓ_{it} as an argument in ϕ ("conditional control")
- now must also find an instrument for labor.

See ACF (Appendix A) for a two-step algorithm, or Wooldridge 2009 (Economics Letters) for a joint GMM estimator.

Some other extensions

- endogenous productivity
  - Doraszelski and Jaumandreu 2013
- unobserved factor-augmenting technical change
  - Doraszelski and Jaumandreu 2018
- disentangling markups from productivity
  - de Loecker 2011
  - de Loecker and Warzynski 2012
  - Bond Hashemi Kaplan Zoch 2021
- multiproduct firms
  - de Loecker Goldberg Khandelwal Pavcnik 2016; Orr 2018
- mergers and acquisitions
  - Braguinsky Ohyama Okazaki Syverson 2015

Next time

We will discuss factor misallocation.

Please read:

1. Syverson (2004, JPE)
2. Asker, Collard-Wexler, and De Loecker (2019, AER)