Adjustment path and Jovanovic's model

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The evolution of the industry

- What happens out of steady state?
- In general, unique equilibrium.
- Under some conditions easy to characterize.
  - Positive entry in every period
  - constant price
  - could occur if demand changes are not too large and enough exit

Constant price equilibrium

- Take a path of entry: $\{\lambda_t\}_{t=0}^{T}$
- p^* be the stationary equilibrim price
- s^* the stationary equilibrium exit point
- $\hat{\mu}_j$ the measure of age j firms for a unit mass of entry
- This implies:

$$
\begin{aligned}
\mu_T &= \sum_{j=0}^{T} \lambda_{T-j}\hat{\mu}_j \\
\mu_T &= \lambda_T \tilde{\nu} + \tilde{P}\mu_{T-1}
\end{aligned}
$$

Fill-in-the-blanks equilibrium

1. Pick λ_0 so that $D(S(\lambda_0 \hat{\nu}, p^*)) = p^*$
2. For any t, pick λ_t so that $D\left(S\left(\lambda_t \hat{\nu} + \tilde{P}\mu_{t-1}, p^*\right)\right) = p^*$

- Example:

$$
\begin{aligned}
s &= \{0,1,2\}, \; q(s,p) = 2sp \\
\nu &= \left(\delta, \frac{1-\delta}{2}, \frac{1-\delta}{2}\right) \\
s^* &= \{0\}, p^* = 1/2 \\
& \quad \text{redraw with prob } (1-\alpha) \\
D(Q) &= 1 - Q
\end{aligned}
$$

- Natural rate of decrease in total output: $\delta(1-\alpha)$
- Provided demand does not fall more than that, positive entry.
- Sufficient condition: total output of cohorts decreasing and demand not decreasing.

The Jovanovic model

- Selection and the Evolution of Industry (ECTRA 82)
- Firms don't know how good they are.
- Get noisy observations over time.
- Choose output and optimal exit time.

Non-stationary exit rule

- Uncertainty on the firm's productivity $\Rightarrow$ option value for staying
- Example 1: Suppose firms draw their shock but only see it after producing.
- Example 2: Problem # 22
- Example 3:
  - Firm draw $\theta \geq 0$
  - Then pay fixed cost ϕ and choose output
  - Then draw x from $\exp(\theta)$ and that x persists forever.
- Firm's cost $xc(q)$ where c is strictly convex

Analysis of example 3

- Period 2

$$
\begin{aligned}
v(x,p) &= \max\left\{0, \frac{\pi(x,p)-\phi}{1-\beta}\right\} \\
\pi(x,p) &= \max_q pq - xc(q)
\end{aligned}
$$

- $\pi(x^*, p) = \phi$
- Exit *iff* $x \geq x^*$
- Remark: profit function convex in x

Analysis of example 3

- Period 1, firm with observed θ
- Choose output to maximize expected profit

$$\pi\left(\frac{1}{\theta}, q\right) = \max_q pq - \frac{c(q)}{\theta}$$

  - Value function

$$V(\theta, p) = \max\left\{0, \pi\left(\frac{1}{\theta}, p\right) - \phi + \beta\theta \int \exp(-\theta x)\, v(x, p)\, dx\right\}$$
$$= \max\left\{0, \pi\left(\frac{1}{\theta}, p\right) - \phi + \frac{\beta}{1-\beta}\theta \int^{x^*} \exp(-\theta x)\left(\pi(x, p) - \phi\right) dx\right\}$$

- $\theta^* = \max\{\theta | V(\theta, p) = 0\}$
- Exit if $\theta \leq \theta^*$

Option value

- $E(x|\theta^*) = 1/\theta^* > x^*$
- Proof:

$$
\begin{aligned}
0 &= \pi\left(\frac{1}{\theta^*}, p\right) - \phi + \frac{\beta}{1-\beta}\theta \int^{x^*} \exp(-\theta x)\left(\pi(x,p) - \phi\right) dx \\
&> \pi\left(\frac{1}{\theta^*}, p\right) - \phi + \frac{\beta}{1-\beta}\theta \int \exp(-\theta x)\left(\pi(x,p) - \phi\right) dx \\
&= \pi\left(\frac{1}{\theta^*}, p\right) - \phi + \frac{\beta}{1-\beta} E\left[\pi(x,p) - \phi|\theta\right] \\
&\geq \frac{\pi\left(\frac{1}{\theta^*}, p\right) - \phi}{1-\beta}
\end{aligned}
$$

Equilibrium

- Suppose some cost of entry c_e and initial distribution $G(d\theta)$
- Guess: constant price equilibrium

$$\begin{aligned} v^e(p) &= \int v(\theta, p)\, G(d\theta) - c_e \\ v^e(p^*) &= 0 \end{aligned}$$

- Solve for output of *representative* cohort

$$\begin{aligned} q_0 &= \int_{\theta^*} q\left(\frac{1}{\theta}, p^*\right) G(d\theta) \\ q_1 &= q_2 = q_n = \int_{\theta^*} \theta \int^{x^*} \exp(-\theta x)\, q(x, p^*)\, dx G(d\theta) \end{aligned}$$

- Fill the blanks equilibrium if $q_0 > q_1$ with aggregate output Q^*

$$\begin{aligned} \lambda_0 q_0 &= Q^* \\ \lambda_n q_0 &= Q^* - q_1 \sum_{j=1}^{n} \lambda_{n-j} \end{aligned}$$

Conditions for existence of this equilibrium

- Sufficient and necessary condition: $q_1 \leq q_0$

$$\begin{aligned}\lambda_n q_0 &= Q^* - q_1 \sum_{j=1}^{n} \lambda_{n-j} \\ &= \lambda_{n-1} q_0 + q_1 \sum_{j=2}^{n} \lambda_{n-j} - q_1 \sum_{j=1}^{n} \lambda_{n-j} \\ &= \lambda_{n-1} q_0 - \lambda_{n-1} q_1\end{aligned}$$

- Sufficient condition:

$$q\left(\frac{1}{\theta}, p^*\right) = q\left(E_\theta x, p^*\right) \geq E_\theta q\left(x, p^*\right)$$

- Holds if q is concave in x.
- Example $c(q) = \frac{1}{\alpha} q^\alpha$

$$\begin{aligned} x q^{\alpha-1} &= p \\ q &= x^{-\frac{1}{\alpha-1}} p^{\frac{1}{\alpha-1}} \end{aligned}$$

- Concave if $\alpha > 2$

Properties

- Survival:

$$\begin{aligned} \text{First period} \quad &: \quad S_1 = 1 - G(\theta^*) \\ \text{Second period} \quad &: \quad S_2 = \int_{\theta^*} (1 - \exp(-\theta x^*))\, G(d\theta) \end{aligned}$$

- Average size:

$$\begin{aligned} \text{First period} \quad &: \quad \frac{q_0}{S_1} \\ \text{Second period} \quad &: \quad \frac{q_1}{S_2} \end{aligned}$$

- Not obvious comparison
- Variability of growth rates declines with age

Cash flow dependence

- Data shows cash flow dependence on growth
- Frequently interpreted as evidence of borrowing constraints
- Cash flow period zero: $pq\left(\frac{1}{\theta}, p^*\right) - xc\left(q\left(\frac{1}{\theta}, p^*\right)\right)$
- Expected cash flow:$pq\left(\frac{1}{\theta}, p^*\right) - \frac{1}{\theta}c\left(q\left(\frac{1}{\theta}, p^*\right)\right)$
- Expand *iff* cash flow $>$ expected cash flow $\left(x < \frac{1}{\theta}\right)$

The learning process in Jovanovic

- $x_t = \xi(\eta_t)$, strictly increasing and continuous on $[\alpha_1, \infty]$, $\alpha_1 > 0$
- $\eta_t = \theta + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma^2\right)$ *iid*.
- $\theta \sim N\left(\bar{\theta}, \sigma_\theta^2\right)$
- Take sample path $\eta_1, \eta_2, ..., \eta_n$ and $\bar{\eta}_n = \frac{1}{n}\sum_i \eta_i$
- $P\left(\theta | \eta_1, \eta_2, ..., \eta_n\right)$ is

$$N\left(\frac{\theta}{\sigma_\theta^2} + \frac{n\bar{\eta}_n}{\sigma^2}\right) / \left(\frac{1}{\sigma_\theta^2} + \frac{n}{\sigma^2}\right), \left(\left(\frac{1}{\sigma_\theta^2} + \frac{n}{\sigma^2}\right)\right)^{-1}$$

- $P\left(\eta | \bar{\eta}_n, n\right)$ is also normal with same mean an additional variance σ^2

Sufficient statistic and DP problem

- $x^* = \int \xi(\eta) P(d\eta | \bar{\eta}_n, n)$
- $\pi(x^*, p)$ is profit function
- For fixed n, x^* is strictly increasing in $\bar{\eta}_n$
- So x^*, n are sufficient statististics for dist of next period's x^*.
- Outside option W (same as fixed cost $\phi = (1-\beta) W$)

$$V(x, n; p) = \max \left\{ W, \pi(x, p) + \beta \int V(z, n+1; p) P(dz | x, n) \right\}$$

Properties of the equilibrium

- Sequence x_n^* is a martingale: $E\left(x_{n+1}^* | x_n^*\right) = x_n^*$
- Exit given by boundary $x\left(n\right)$.
- Firms that survive are in average larger than firms that fail.
- Gini coefficient of firm size rises (though not necessarily monotonic)
- Average profits of firms rise over time (selection)
- HIgher Gini coefficient is associated with increase in profits of large firms but not of small ones.
- Total output of a cohort falls if $q\left(x,p\right)$ is concave.
- $\pi_t - E_t \pi_t = -c\left(q_t\right)\left(x_t - x_t^*\right)$
  - Conditional on firm size, higher profits lead to growth.