Some motivating facts

- Size distribution of firms (measured by assets, sales or employment) is highly skewed.
- For the US follows approximately Pareto distribution with coefficient one:$(1 - F(n)) = An^{-1}$

More facts

- Firm size is persistent but the variance of innovations is quite large.
- Gross realocation of employment across firms exceeds in several orders of magnitude net reallocation.
- Variance of growth rates declines with size and age.
- Firm size increases with age.
- There is considerable degree of entry/exit into narrowly defined industries. Small and young firms have higher exit rates.
- Most firm level changes in employment correspond to idiosyncratic shocks, i.e. not explained by aggregate, geographic or industry variables.

A simplified Lucas' model

- Collection of firms $i = 1, ..., M$
- Technology $y_i = e_i n_i^{\eta}$ where $\eta < 1$
- Fixed endowment of labor N
- Competitive equilibrium $\{w, n_i\}$maximizing profits and market clearing.
- $n(e_i, w)$ labor demand and $\sum n(e_i, w) = N$
- Solves planner problem:

$$max_{n_i} \sum_i e_i n_i^{\eta}$$
$$\sum_i n_i \leq N$$

Equilibrium

- Employment:

$$\begin{aligned} n_i &= a e_i^{\frac{1}{1-\eta}} \\ a \sum e_i^{\frac{1}{1-\eta}} &= N \\ y_i &= a^\eta e_i e_i^{\frac{\eta}{1-\eta}} = a^\eta e_i^{\frac{1}{1-\eta}} \end{aligned}$$

- Remark y_i/n_i is the same for all firms!
- Solving for a and substituting:

$$\begin{aligned} y &= \sum_i y_i = a^\eta \sum e_i^{\frac{1}{1-\eta}} \\ &= \left(\sum e_i^{\frac{1}{1-\eta}} \right)^{1-\eta} N^\eta \end{aligned}$$

The Aggregate Production Function

$$
\begin{aligned}
y &= \left(\sum e_i^{\frac{1}{1-\eta}}\right)^{1-\eta} N^{\eta} \\
&= \left(E_i e_i^{\frac{1}{1-\eta}}\right)^{1-\eta} M^{1-\eta} N^{\eta}
\end{aligned}
$$

- Cobb Douglass in M, N with TFP equal to geometric average of firm shocks.
- M is like a capital stock, sometimes called "organization capital"
- Intangible?

Multiple inputs

- Results generalize to multiple inputs (Lucas 1978)
- Let $f(x)$ be homogenous of degree one and

$$y_i = e_i (f(x))^\eta$$

- aggregate endowment vector X
- Aggregate production function:

$$y = \left(E_i e_i^{\frac{1}{1-\eta}} \right)^{1-\eta} M^{1-\eta} f(X)^\eta$$

Exercise

Derive this expression.

Large number of firms

- Let $F(e)$ denote the cdf for shocks.
- Suppose M is the mass of firms (quantity of organization capital)

$$y = \left(Ee^{\frac{1}{1-\eta}}\right)^{1-\eta} M^{1-\eta} N^{\eta}$$

- Special case: $F = 1 - \left(\frac{e_{min}}{e}\right)^{\alpha}$ Pareto distribution

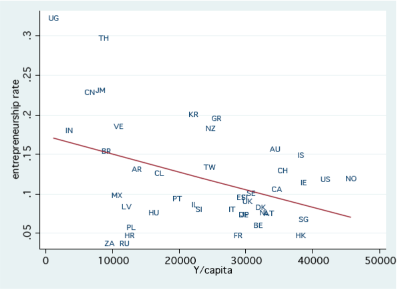
$$\left(Ee^{\frac{1}{1-\eta}}\right)^{1-\eta} = \left(\frac{e_{min}}{\alpha - \frac{1}{1-\eta}}\right)^{1-\eta} e_{min}$$

- "Tail condition": defined only when $\alpha > \frac{1}{1-\eta}$

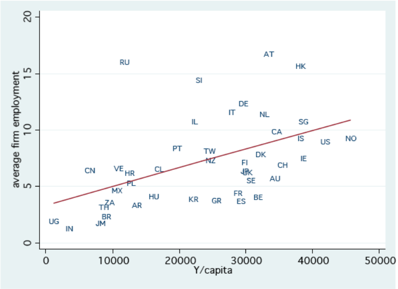
Exercise

Derive the above formula.
Show that the distribution of firm size measured in employment is Pareto. What about the distribution of output?

Productivity and Average Size I



(a) The entrepreneurship rate

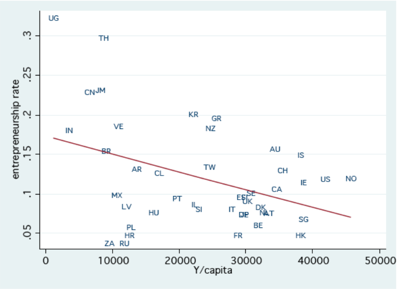


(b) Average employment

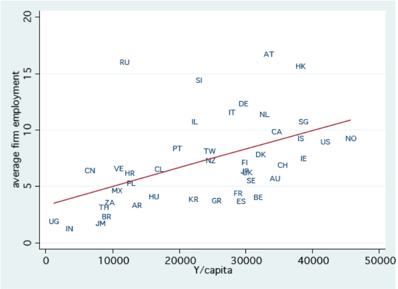
Source: Poschke, "The firm size distribution across countries and skill biased change in entrepreneurial technology.

- What would the model say about this?

Productivity and Average Size I



(a) The entrepreneurship rate



(b) Average employment

Source: Poschke, "The firm size distribution across countries and skill biased change in entrepreneurial technology.

- What would the model say about this?
- If M included as input, productivity independent of average size
- But income per capita would increase, not decrease!

Endogenizing entry

- Technology for creating firms (organization capital) takes c_e workers.
- Entrants draw e_i independently from same distribution
- Planner's problem:

$$max_{M,L} \left(Ee^{\frac{1}{1-\eta}} \right)^{1-\eta} M^{1-\eta} L^{\eta}$$
$$subject\ to:\ c_e M + L \leq N$$

- solution: $L = \eta N$ and $M = (1-\eta) N / c_e$
- Solution Independent of distribution of firm productivity
- HIgher cost of entry: less firms and larger average size of firms

Productivity with endogenous entry

- Substitute solution

$$y = \left[\left(\frac{E e^{\frac{1}{1-\eta}}}{c_e} \right)^{1-\eta} (1-\eta)^{1-\eta} \eta^{\eta} \right] N$$

- Constant returns to scale in aggregate
- Productivity negatively related to entry cost
- Higher costs of entry imply lower input per capita but higher average size of firms.

Some questions

- Repeat the analysis for entry assuming entry costs are denominated in units of output.
- Repeat the analysis for the multi-input case. What conditions do you need for the number of firms to be independent of the distribution of firms' productivity?
- Solve explicitly for the Pareto case with $\eta = 2/3$ and $\alpha = 2$ and a range of values for the cost of entry. Tabulate your results.
- Suppose firms are long-lived but with death rate δ each period and discount future flows with discount factor β. Repeat the analysis of optial entry done in the preceding slides.
- Suppose firms are long-lived but instead of facing entry costs pay a fixed cost f per period. How would the analysis change of optimal entry change?

Connection to Monopolistic Competition

- Dixit-Stiglitz (1977), Melitz (2003)
- Continuum of goods: $y = \left(\int y_i^\eta di\right)^{\frac{1}{\eta}}$
- Linear techology $y_i = e_i n_i$
- Constant markup $p_i = \frac{1}{\eta}(w/e_i)$
- $y_i \propto e_i^{\frac{1}{1-\eta}}$ and $n_i \propto e_i^{\frac{\eta}{1-\eta}}$

Aggregation in Dixit-Stiglitz

- Using labor resource constraint

$$
\begin{aligned}
y &= \left(Ee_i^{\frac{\eta}{1-\eta}}\right)^{\frac{1-\eta}{\eta}} M^{\frac{1-\eta}{\eta}} N \\
y^{\eta} &= \left(Ee_i^{\frac{\eta}{1-\eta}}\right)^{1-\eta} M^{1-\eta} N^{\eta}
\end{aligned}
$$

- With endogenous entry get same M.

Exercise

Analyze entry in this model when entry costs are denominated in goods, with and without increasing returns. If you prefer you can use the Pareto distribution in your analysis.

Matching size distribution

$$
\begin{aligned}
y &= \left(Ee_i^{\frac{\eta}{1-\eta}}\right)^{\frac{1-\eta}{\eta}} M^{\frac{1-\eta}{\eta}} N \\
y^\eta &= \left(Ee_i^{\frac{\eta}{1-\eta}}\right)^{1-\eta} M^{1-\eta} N^\eta
\end{aligned}
$$

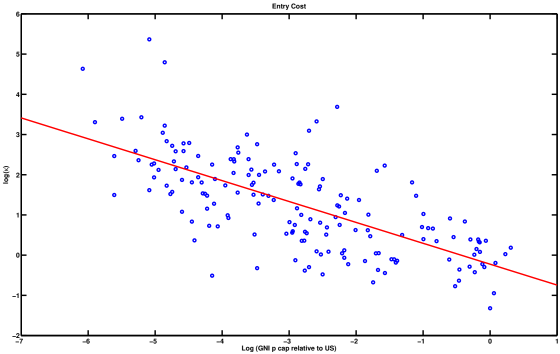
- $n_i \propto e_i^{\eta/(1-\eta)}$ while in previous case proportional to $e_i^{1/(1-\eta)}$
- Use same distribution Gas before, but need to transform $\tilde{e} = e^{1/\eta}$
- Get same TFP term!

General or Partial Equilibrium: Aggregation

- Partial equilibrium:
  - Aggregate demand $D(p)$
  - Cost function $c(e,q)$, supply function $s(e,p)$
  - Total supply $S(p) = MEs(e,p)$
- Correspondence
  - set $w = 1$
  - $c(e,q) = f^{-1}(q/e)$
  - Entry cost c_e
  - Solve for unique p^* that makes expected profits $= c_e$
  - Find M so that $ME_e s(e,p^*) = D(p^*)$
  - $p^* = 1/w^*$

Costs of entry and TFP

- Calculate regulatory costs of creating business measured in units of annual labor κ
- Lowest US $\kappa = 0.3$, highest Liberia $\kappa = 616.8$, 29 countries with $\kappa < 1$ and 31 with $\kappa > 10$.



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- Regression line $d \ln y / d \ln \kappa = -2$ suggests very large effect

Effects of entry costs in the model

- From our previous derivations:

$$\begin{aligned} y &= \left[\left(\frac{E e^{\frac{1}{1-\eta}}}{c_e + \kappa} \right)^{1-\eta} (1-\eta)^{1-\eta} \eta^{\eta} \right] N \\ d \ln y / d \ln (c_e + \kappa) &= -(1-\eta) \end{aligned}$$

- Effects of distortions to entry costs depend on the degree of decreasing returns
- Calibration of η :
  - In basic model, return to intangibles. Literature uses 0.85
  - In Dixit-Stiglitz, $\eta = 1/(1+\text{markup}) = 1/1.25 = 0.8$
  - $d \ln y / d \ln \kappa = -0.15$ or -0.2

Cost of entry and underdevelopment

- Barseghyan (2008): SD in $ln\kappa$ 1.61
  - would give SD in GNP per capita 0.3
  - Using instrumental variables, finds elasticity between 25 and 30%
- Back of the envelope (data Moscoso-Boedo and Mukoyama)
  - baseline $c_e = 36$. Compare $\kappa = 10$ and $\kappa = 100$ to $\kappa = 0$ (US)
  - $\kappa = 10 \rightarrow TFP = 0.9(0.95)$, $\kappa = 100 \rightarrow TFP = 0.6(0.76)$ (numbers in brackets using elasticity of -0.2 above)
  - Differences between average entry cost of lower income countries (GNI = 0.2 US) gives 21% TFP gap.
- Sizeable but far from observed differences in TFP.

Endogenous distribution of firm productivity

- Lucas original model has $F(e)$ distribution of managerial skills in population.
- Equilibrium determines threshold for entrepreneurship e^* by indifference condition.
- Similar setting: endogenous entry of firms, but with a fixed distribution of *given* productivities $F(e)$
- Lucas model special case where cost of entry are denominated in labor and equal to one.

Endogenous productivity and the Pareto case

- Letting threshold be e_0

$$Ee^{\frac{1}{1-\eta}} = \left(\frac{\alpha}{\alpha - \frac{1}{1-\eta}}\right)^{1-\eta} e_0$$

- Number of firms (per capita): $m = 1 - F(e_0) = e_0^{-\alpha}$ so $e_0 = m^{-1/\alpha}$
  - This is called selection effect

Aggregate production function (per capita):

$$y = A(\alpha, \eta)\, m^{1-\eta-1/\alpha} n^{\eta}$$

where m and n are number of firms and productive workers per capita.

- Still constant returns in the population (assuming entrepreneurs also replicate)

Productivity, number of firms and entry

$$y = A(\alpha, \eta)\, m^{-1/\alpha} m^{1-\eta} n^{\eta}$$

- Productivity decreases with m and so does average size!
- We saw there are more entrepreneurs in less developed economies
- Quantitative exploration:
  - Average firms in India 1/3 of US, so m is 3 times larger
  - $1/\alpha$ bounded by $1-\eta$. Take $\eta = 3/4$ (conservative) so upper bound for $1/\alpha = 1/4$.
  - Effect on TFP is $3^{-.25}$ approximately -25%.
- However, if n is unchanged output per capita would increase with m!

Entry, average size and productivity

- Why more firms in India?
- Consider equilibrium entry:

$$\max_{M,L} A(\alpha, \eta) M^{1-\eta-1/\alpha} L^{\eta}$$
$$\text{subject to : } c_e M + L \leq N$$

- Take costs of entry $c_e = 1$(Lucas' model):

$$M = \frac{1-\eta-1/\alpha}{1-1/\alpha}, L = \frac{\eta}{1-1/\alpha}$$

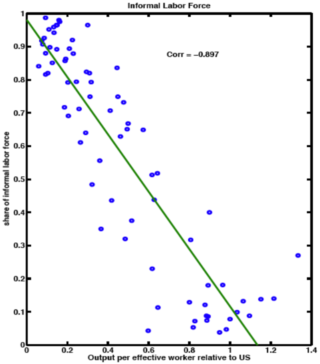
- M increases with α (less productive economy) and average size of firms decreases

Back of the envelope calculation

- Take $\eta = 3/4$
- $\alpha = 5$ gives average firm size of 16 and $\alpha = 10$ average size 6.
- Firm size Pareto distributions coefficient 1.25 and 2.5 (reasonable)
- TFP ratio is 1.65
- Large but still small compared to gap of 5 in the data.
- Alternative reading: range of average firm size across countries easily accomodated by ranges of TFP.

Informality and Firm Size

- Lots of informal (non complying) firms in less developing countries
- Relationship to development:



Source: Derasmo and Moscoso-Boedo 2012

Implications for Aggregate Productivity

- High costs of entry will lead to avoidance and informality
- Possible costs: risks of closure, limited access to markets and inputs and financial resources.
- Derasmo and Moscoso-Boedo (JME 2012) model several distortions: costs of entry, financial constraints, etc.
- Firms pay some cost and draw productivity
  - If above some threshold enter in formal market paying cost of entry
  - If below operate as informal, in which case their productivity is some given baseline
- Find all these factors explain 25% TFP gap between low income countries (between 2% and 8% US GNI) and US (36% of actual gap)
- Differences in cost of entry major force (80%). Informal sector option doesn't do much.

Firm dynamics: stylized facts

- Small firms grow faster (conditional on survival)
- Size of firms very persistent (close to random walk)
- Large firms have lower variance of growth rates
- Size distribution of firms stochastically increasing in age
- Exit rates decline with age

Firm dynamics - motivation

- Lots of evidence that firms' size is not constant
  - Five year AR1 of firm ln employment US manufacturing, persistence 0.92 and large variance of innovation.
- Firm size distribution stochastically increases with age.
  - average entrant 35% size of average incumbent

Firm dynamics - simple model

- entrants draw independently initial shocks from same distribution
- firm productivity evolves according to MP $F\left(e_{t+1}|e_t\right)$
- Repeated application generates probability distributions $\tilde{\mu}_s$ for firms of age s.
- exogenous death/exit rate $1-\delta$.

$$
\begin{aligned}
M_t &= \delta^t m_0 + \delta^{t-1} m_1 + \ldots + \delta m_{t-1} + m_t \\
\mu_t &= M_t^{-1}\left(m_t \tilde{\mu}_0 + \delta m_{t-1} \tilde{\mu}_1 + \ldots + \delta^t m_0 \tilde{\mu}_t\right) \\
y_t &= \left(\int e^{\frac{1}{1-\eta}} d\mu_t(e)\right)^{1-\eta} M_t^{1-\eta} L_t^{\eta}
\end{aligned}
$$

Competitive equilibrium

Given sequence of wages $w = \{w_t\}_{t=0}^{\infty}$

$$\begin{aligned} v_t(e; w) &= max_n en^{\eta} - w_t n + \beta\delta E v_{t+1}(e'; w|e) \\ v_t^e &= E_0 v_t(e; w) - w_t c_e \end{aligned}$$

Definition

A competitive equilibrium is a sequence $\{m_t, n_t(e), v_t\}$ and wages $\{w_t\}$ that satisfy the following conditions:

- Employment decisions are optimal given wages
- value functions are as defined above
- $v_t^e \leq 0$ and $m_t v_t^e = 0$
- $m_t c_e + \int n_t(e)\, \mu_t(de) = N$.

Planners problem

- Objective:

$$max_{m_t,L_t} \quad \sum_{t=0}^{\infty}\beta^t \left(\int e^{\frac{1}{1-\eta}} d\mu_t\left(e\right)\right)^{1-\eta} M_t^{1-\eta} L_t^{\eta}$$

$$subject\ to: \quad M_t = \delta^t m_0 + \delta^{t-1} m_1 + \ldots + m_t$$

$$L + c_e m_t = N$$

- Unique solution:
  - Objective strictly concave
- Constraints linear

Stationary equilibrium

- Analogous to steady state (or balanced growth path)
- Entry flow $m_t = m$ for all t.
- This implies $M = \frac{m}{1-\delta}$ and $\mu = (1-\delta)\sum_{s=0}^{\infty} \delta^s \tilde{\mu}_s$
- Value function:

$$v(e) = max_n e n^\eta - wn + \beta\delta \int v(e') F(de'e)$$

- Resource constraint: $\int n(e,w)\, d\mu(e) + mc_e = N$
- Stationary equilibrium is unique.
- Steady state productivity proportional to: $\left(\frac{Ee^{\frac{1}{1-\eta}}}{c_e}\right)^{1-\eta}$

Age-increasing size distribution

Assumption

(FOSD) $F(e', e)$ *decreasing in* e.

- Sequence $\tilde{\mu}_s$ obtained recursively as $\tilde{\mu}_{s+1}([0, e']) = \int F([0, e'], e)\, d\tilde{\mu}_s(e)$

Assumption

$F \circ G \succ G$ *(F increases G):* $\int F([0, e'], e)\, dG(e) < G([0, e'])$

- Persistence and $F \circ G \succ G$ implies $\tilde{\mu}_s$ is increasing sequence (in FOSD)

Endogenous exit and selection

- Firm exit endogenous
- Need a reason for exiting: fixed costs, opportunity costs
- Assume fixed cost f denominated in units of labor

$$v(e;w) = max\left\{0, \pi(e,w) + \beta \int v(e';w) F(de',e)\right\}$$

- Decision rules $n(e,w)$ and exit set $E(w)$.

Proposition

$(i)v(e;w)$ strictly decreasing in w if nonzero. (ii)Under (FOSD) $v(e;w)$ strictly increasing in e if nonzero and exit set is threshold $e(w)$.

- Value of entry: $v^e(w) = \int v(e;w)\, dG(e) - c_e$

Example

- Every period with probability δ maintain same e and probability $1-\delta$ draw again from G
- Profits $\pi(e,w)$ and there is a fixed cost f
- Cost of entry c_e

$$\begin{aligned} v(e,w) &= \pi(e,w) - f + \beta\delta v(e,w) + \beta(1-\delta)\int v(e,w)\,dG \\ &= \frac{\pi(e,w) - f + \beta(1-\delta)c_e}{1-\beta\delta} \end{aligned}$$

- Exit if $\pi(e^*(w), w) - f + \beta(1-\delta)c_e = 0$
- Equilibrium w obtained from

$$\int_{e^*(w)} [\pi(e,w) - f + \beta(1-\delta)c_e]\,dG(e) = c_e$$

Measure of Firms

- State variable: measure $\mu_t(de)$
  - M_t is the total mass
  - $\tilde{\mu}_t = \mu_t / M_t$ probability distribution over firm productivity
- Entrants measure $m_t \nu(de)$
  - m_t is mass of entrants
  - ν is probably distribution of shocks with cdf G
- Law of motion:

$$\mu_t \xrightarrow{F(e'|e)} \mu^I_{t+1} - \xrightarrow{\mu^I_{t+1}(e^*)} + m_t \left(G\left(e\right) - G\left(e^*\right)\right) \rightarrow \mu_{t+1}$$

Unique invariant measure: Discrete case

- $E = \{e_1, e_2, ..., e_n\}$
- Suppose all firms with $e \leq e_k$ exit.
- Let $P = (p_{ij})$ be the transition matrix and $v = (\nu_i)$ probability distribution of entrants
- Let $\widetilde{v} = (0, 0, ..., 0, \nu_{k+1}, \nu_{k+2}, ..., \nu_n)$

$$\widetilde{P} = \begin{matrix} 0 & 0 & \dots & p_{1,k+1} & p_{1,k+2} & \cdots & p_{1n} \\ 0 & 0 & \dots & p_{2,k+1} & p_{2,k+2} & \cdots & p_{2n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & \dots & p_{n,k+1} & p_{n,k+2} & \cdots & p_{nn} \end{matrix}$$

- Then

$$T\mu = \widetilde{P}'\mu + m\nu$$

Unique invariant measure discrete case ...

$$T\mu = \widetilde{P}'\mu + m\widetilde{\nu}$$

- Fixed point:

$$\mu = \widetilde{P}'\mu + m\widetilde{\nu}$$
$$\mu = \left(I - \widetilde{P}'\right)^{-1} m\nu$$

- If exists, linear in m
- Exists if $\left(I - \widetilde{P}'\right)$ is invertible
- In that case

$$\left(I - \widetilde{P}'\right)^{-1}\nu = \sum_{n=0}^{\infty}\left(\widetilde{P}'\right)^{n}\nu$$

- $\left(\tilde{P}'\right)^{n} v = \alpha_n\tilde{\mu}_n$

Unique invariant measure

- Invariant measure as a weighted sum of measure of different cohorts.
- Let α_n be the probability of surviving up to n periods

$$\mu = m \sum_{n=0}^{\infty} \alpha_n \tilde{\mu}_n$$

- Necessary and Sufficient conditoin for existence :

$$\sum_{n=0}^{\infty} \alpha_n < \infty$$

- Integrating by parts: finite expected lifetime

$$\sum_{n=0}^{\infty} \alpha_n = \sum_{n=0}^{\infty} n \left(\alpha_{n+1} - \alpha_n\right) < \infty$$

- Finite expected lifetime

Invariant measure: continuous case

- Entrants: m *mass:* measure mG
- Incumbents (before exit): $\mu_I(-\infty, e) = \int F(e, e_0)\,\mu(de_0)$
- New measure of firms ($e \geq e^*$)

$$\begin{aligned} T\mu(-\infty, e) &= m[G(e) - G(e^*)] + \mu_I(e^*, e) \\ &= m[G(e) - G(e^*)] + \int [F(e, e_0) - F(e^*, e_0)]\,\mu(de_0) \end{aligned}$$

- Invariant measure: $\mu = T\mu$

Stationary equilibrium: definition

$$\{\mu, e^*, m, w\}$$

$v^e(w) \leq 0$ and $v^e(w)\, m = 0$

e^* is optimal exit rule

$N = \int (f + n(e, w))\, d\mu + mc_e$

μ is an invariant measure

- Equilibrium with entry and exit
  - $m > 0$
  - $m(1 - G(e^*)) = \int F(e^*, e)\, \mu(ds)$
  - $v^e(w^*) = 0$
  - unique

Rate of turnover

- Rate of turnover (entry/exit)

$$\frac{m}{\mu(E)} = \frac{m}{\lambda \sum_{n=0}^{\infty} \alpha_n \tilde{\mu}_n}$$
$$= 1/E(n)$$

- $E(n)$ decreases with e^* (turnover increases)
- e^* increases with w
- Higher cost of entry c_e, decrease w, decreases turnover

Turnover and Sunk Costs

- Indirect measures of sunk costs
  - Average size of firms
  - Number of firms
- Cross industry regression. Dependent: Rate of Entry

Variable	Estimate	t
Intercept	-3.10	-24-1
Log Avg Size	-0.07	-4-0
Log Num Firms	0.14	12.6

Selection and Productivity

- Productivity determined by stochastic process (G, F) and exit threshold

Formulas for homogeneous case

$$
\begin{aligned}
y &= \left(Ee^{\frac{1}{1-\eta}}\right)^{1-\eta} M^{1-\eta} L^{\eta} \\
L &= N - mc_e - Mf \\
M &= m \sum_n \alpha_n (e^*)
\end{aligned}
$$

$$
Ee^{\frac{1}{1-\eta}} = \frac{\int_{e^*} e^{\frac{1}{1-\eta}} d\mu}{M}
$$

- Selection effect: $Ee^{\frac{1}{1-\eta}}$ increases with threshold e^*so decreases with c_e.
- Other effects (possibly M decreases), but total productivity must decrease in c_e.

Selection and productivity: analysis

- No scale effects: Increasing N does not change e^* and just increases proportionally m and M.

Exercise

Aggregate productivity shocks neutral

- Changes wage proportionally (hint $\pi(e, w)$ homogeneous of degree one.)
- Does not change employment, exit or entry decisions.
- Just scales up total output

- Aggregate productivity shock that is complementary to e
  - Increases relative size of larger (higher e) firms
  - Increases exit threshold e^* (selection effect)

Identifying stochastic process

- Hopenhayn and Rogerson (JPE 1993)
- Production function: $f(e, n) = en^{\alpha}$
- Let p denote output price (labor as numeraire)
- $\ln e_{t+1} = \rho \ln e_t + \varepsilon_{t+1}$, where $\varepsilon_t \sim N\left(\bar{\varepsilon}, \sigma_{\varepsilon}^2\right)$
- First order conditions for employment:$\ln \alpha p + \ln e_t = (1-\alpha) \ln n_t$ implies:

$$
\begin{aligned}
\ln n_{t+1} &= (1-\alpha)^{-1} \ln e_{t+1} + \ln \alpha p \\
&= (1-\alpha)^{-1} \left(\rho \ln e_t + \varepsilon_{t+1}\right) + \ln \alpha p \\
&= (1-\alpha)^{-1} \left\{(1-\alpha) \rho \ln n_t + \rho \ln \alpha p + \varepsilon_{t+1}\right\} + \ln \alpha p \\
&= A + \rho \ln n_t + (1-\alpha)^{-1} \varepsilon_{t+1}
\end{aligned}
$$

- ρ and σ identified from AR1 parameters for ln firm size

More calibration

- The initial distribution determined by distribution of entrants sizes
- more parameters to determine: c_f, c_e and the mean $\bar{\varepsilon}$.
- Data to use: rate of turnover, mean size, age distribution.