Entry and Exit in Long Run Equilibrium

- Econometrica, 1992. JEDC, 1992.
- Model turnover (entry/exit) and firm dynamics.
- Assignment of inputs to firms and aggregate productivity.
- Barriers to reallocation and productivity.

Model

- Partial equilibrium homogenous good $p = D(Q)$ strictly decreasing
- State space of firm types S, totally ordered.
- Firms are price takers: $\pi(s,p)\,,\ q(s,p)\,,\ c(s,q)$
- Assume cost decreasing in s, profits and output increasing and continuous.
- Productivity shock s follows Markov process cdf $F(s'|s)$, decreasing in s, *continuous*.
- Outside value (exit) normalized to zero.
- Cost of entry c_e and entrants draw s from cdf $G(s)$

Aggregation and equilibrium price

- Take measure of firms μ on S
- Aggregate supply

$$S(\mu, p) = \int q(s, p)\, \mu(ds)$$

- Example: μ finite suport
  $(\mu_1, \mu_2, \ldots, \mu_n)\,, \quad S(\mu, p) = \sum_i q(s_i, p)\, \mu_i$
- Remark: S is linear μ

$$S(a\mu + b\widetilde{\mu}, p) = aS(\mu, p) + bS(\widetilde{\mu}, p)$$

- Equilibrium price

$$p = D(S(\mu, p))$$

- Unique if D is strictly decreasing or S strictly increasing.

Stationary equilibrium: exit decision

- Suppose $p_t = p$ all t.
- Problem of the firm:

$$v(s,p) = \max\left(0, \pi(s,p) + \beta \int v(s',p) F(ds',s)\right)$$

- Properties of $v(\cdot)$
  1. Continuous
  2. Increasing in p
  3. Increasing in s
- Exit threshold:

$$s^* = \inf\left\{ s | \pi(s,p) + \beta \int v(s',p) F(ds',s) \geq 0 \right\}$$

$$\left[\pi(s^*,p) + \beta \int v(s',p) F(ds',s^*) \geq 0\right]$$

Entry and equilibrium price

- Expected value of an entrant

$$v^e\left(p\right) = \int v\left(s, p\right) G\left(ds\right) - c_e$$

- Free entry $v^e \leq 0$ and equal to zero if there is entry
- Unique equilibrium price such that $v^e\left(p\right) = 0$
- As in static case, price determined independently of demand.

Law of motion of measure of firms

- Timing: 1) entry; 2) shocks realized, 3) exit; 4) production
- Entrants: λ *mass:* measure λG
- Incumbents (before exit): $\mu_I(-\infty, s) = \int F(s, s_0)\,\mu(ds_0)$
- New measure of firms $(s \geq s^*)$

$$\begin{aligned} T\mu(-\infty, s) &= \lambda\left[G(s) - G(s^*)\right] + \mu_I(s^*, s) \\ &= \lambda\left[G(s) - G(s^*)\right] + \int \left[F(s, s_0) - F(s^*, s_0)\right]\mu(ds_0) \end{aligned}$$

- Invariant measure: $\mu = T\mu$

Stationary equilibrium: definition

$$\{\mu, s^*, \lambda, p\}$$

1. $v^e(p) \leq 0$ and $v^e(p)\lambda = 0$
2. s^* is optimal exit rule
3. $p = D(S(\mu, p))$
4. μ is an invariant measure

- Equilibrium with entry and exit
  - $\lambda > 0$
  - $\lambda(1 - G(s^*)) = \int F(s^*, s)\mu(ds)$
  - $v^e(p^*) = 0$
  - unique

Unique invariant measure: Discrete case

- $S = \{s_1, s_2, \dots, s_n\}$
- Suppose all firms with $s \leq s_k$ exit.
- Let $P = (p_{ij})$ be the transition matrix and $v = (\nu_i)$ probability distribution of entrants
- Let $\widetilde{v} = (0, 0, \dots, 0, \nu_{k+1}, \nu_{k+2}, \dots, \nu_n)$

$$\widetilde{P} = \begin{matrix} 0 & 0 & \dots & p_{1,k+1} & p_{1,k+2} & \dots & p_{1n} \\ 0 & 0 & \dots & p_{2,k+1} & p_{2,k+2} & \dots & p_{2n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & \dots & p_{n,k+1} & p_{n,k+2} & \dots & p_{nn} \end{matrix}$$

- Then

$$T\mu = \widetilde{P}\mu + \lambda\widetilde{\nu}$$

Unique invariant measure discrete case ...

$$T\mu = \widetilde{P}\mu + \lambda\widetilde{\nu}$$

- Fixed point:

$$\mu = \widetilde{P}\mu + \lambda\widetilde{\nu}$$

$$\mu = \left(I - \widetilde{P}\right)^{-1}\lambda\nu$$

- If exists, linear in λ
- Exists if $\left(I - \widetilde{P}\right)$ is invertible
- In that case

$$\left(I - \widetilde{P}\right)^{-1}\nu = \sum_{t=0}^{\infty}\left(\widetilde{P}\right)^{t}\nu$$

Unique invariant measure: general case

- An invariant measure is a weighted sum of measure of different cohorts.
- Let $\bar{\mu}_n$ be the probability distribution on S of age n cohort
- Let α_n be the probability of surviving up to n periods
- An invariant distribution with positive entry must satisfy

$$\mu = \lambda \sum_{n=0}^{\infty} \alpha_n \bar{\mu}_n$$

- Necessary and Sufficient conditoin for existence :

$$\sum_{n=0}^{\infty} \alpha_n < \infty$$

- Integrating by parts:

$$\sum_{n=0}^{\infty} \alpha_n = \sum_{n=0}^{\infty} n \left(\alpha_{n+1} - \alpha_n \right) < \infty$$

- Finite expected lifetime

Properties

- Size and Age
  - In the data size distribution stochastically incerases with age
  - In model, depends on properties of F and G
  - Sufficient condtitions:

$$\frac{F(s_2, s)}{1 - F(s_1, s)} \text{ decreasing in } s \text{ for all } (s, s_1)$$

$$\bar{\mu}_1 \succeq \bar{\mu}_0$$

- Firm growth: depends on properties of F

Rate of turnover (entry/exit)

$$\frac{\lambda}{\mu(S)} = \frac{\lambda}{\lambda \sum_{n=0}^{\infty} \alpha_n \bar{\mu}_n} = 1/E(n)$$

Rate of turnover

- $E(n)$ decreases with s^* (turnover increases)
- s^* decreases with p
- Higher cost of entry c_e, increase p, decreases turnover
- No turnover under above conditions if $G(s) \leq \inf_{s_0} F(s, s_0)$
- Many papers force some turnover (e.g. binomial probability of *death*)

Turnover and Sunk Costs

- Indirect measures of sunk costs
  - Average size of firms
  - Number of firms
- Cross industry regression. Dependent: Rate of Entry

Variable	Estimate	t
Intercept	-3.10	-24-1
Log Avg Size	-0.07	-4-0
Log Num Firms	0.14	12.6

Identifying stochastic process

- Hopenhayn and Rogerson (JPE 1993)
- Production function: $f(s,n) = sn^{\alpha}$
- Let p denote output price (labor as numeraire)
- $\ln s_{t+1} = \rho \ln s_t + \varepsilon_{t+1}$, where $\varepsilon_t \tilde{} N\left(\bar{\varepsilon}, \sigma_{\varepsilon}^2\right)$
- First order conditions for employment:

$$\ln \alpha p + \ln s_t = (1-\alpha) \ln n_t$$

- Implies:

$$\begin{aligned}
\ln n_{t+1} &= (1-\alpha)^{-1} \ln s_{t+1} + \ln \alpha p \\
&= (1-\alpha)^{-1} \left(\rho \ln s_t + \varepsilon_{t+1}\right) + \ln \alpha p \\
&= (1-\alpha)^{-1} \left\{(1-\alpha)\rho \ln n_t + \rho \ln \alpha p + \varepsilon_{t+1}\right\} + \ln \alpha p \\
&= A + \rho \ln n_t + (1-\alpha)^{-1} \varepsilon_{t+1}
\end{aligned}$$

- ρ and σ identified from AR1 parameters for ln firm size

more calibration

- The initial distribution determined by distribution of entrants sizes
- more parameters to determine: c_f, c_e and the mean $\bar{\varepsilon}$.
- Data to use: rate of turnover, mean size, age distribution.