Lecture 5

January 7, 2021

Firm Dynamics

- Starting point: firm size distribution
- Mobility: firm growth
- Turnover: entry and exit

Why interested in "firm dynamics"

- industrial organization: market power, industry concentration
- productivity: entrepreneurship, resource allocation, growth
- labor/urban: labor turnover, geographic agglomeration

Firm Size Distribution: Gibrat

- Gibrat's "Law" (law of proportional growth)

$$x_t - x_{t-1} = \epsilon_t x_{t-1}$$
$$x_t = x_0(1+\epsilon_1)(1+\epsilon_2)...(1+\epsilon_t)$$

- Set small time intervals

$$log(1+\epsilon) \approx \epsilon$$
$$logx_t \sim N(mt, \sigma^2 t)$$

- Limiting distribution log-normal, however, with time dependent mean/variance.
- Needs modification to maintain stationary size distribution

Firm Size Distribution: Zipf

- Power Law Distribution (Axtell 2001)
  - $Prob(S > s_i) = (\frac{s_0}{s_i})^\alpha$, $s_i > s_0, \alpha > 0$
  - $\alpha = 1$ is sometimes called "Zipf's Law"
  - Zipf's distribution has been used to approximated diverse nature/social phenomena
- Using empirical frequencies of firm size
  - rank firm size $s_0, s_1, s_2, ..., s_{n-1}$
  - construct vector $[1, \frac{n-1}{n}, \frac{n-2}{n}, ..., \frac{1}{n}]$
  - do log-log plot

Firm Size Distribution

- Firm size distribution in US, $\alpha = 1.06$

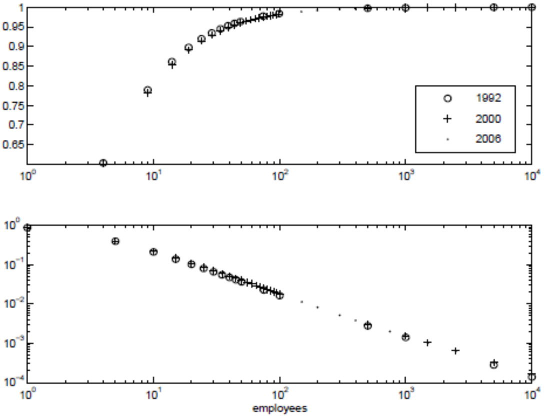


FIGURE 2 The Firm Size Distribution

Gabaix 09: Proportional Random Growth

- Let firm size $s_{t+1} = \gamma_{t+1} s_t$, growth rate γ_{t+1} IID with density $f(\gamma)$.
- We can show $G_{t+1}(x) = P(s_{t+1} > x) = P(s_t > \frac{x}{\gamma}) = \int_0^\infty G_t(\frac{x}{\gamma}) f(\gamma) d\gamma$
- Any steady state distribution has to satisfy

$$G(s) = \int_0^\infty G(\frac{s}{\gamma}) f(\gamma) d\gamma$$

- If we conjecture power law $G(s) = k/s^\zeta$, then this above equation implies $E(\gamma^\zeta) \equiv \int_0^\infty \gamma^\zeta f(\gamma) d\gamma = 1$
    - After normalizing k
- If expected growth rate is zero, i.e. $\bar{s} = E[s_{t+1}] = E(\gamma) E[s_t] = E(\gamma)\bar{s}$, then $E(\gamma) = 1$, Zipf's law.
- However, as we saw in the case of Gibrat's result, a pure random growth model doesn't have a solution for steady state, we need to add "frictions".

Nature of Frictions

- Keep the lower tail bounded
  - An added drift
  - Reflective barrier (e.g. exit)
- Process considered by Gabaix (QJE 1999)

  $$s_{t+1} = \gamma_{t+1} s_t + \epsilon_{t+1}$$

  where ϵ_{t+1} has positive mean.
  - Implies higher growth for smaller firms
  - becomes irrelevant as $s_t \to \infty$
- Result:
  - Whatever initial distribution, size distribution converges to Power Law with coefficient ζ where $E(\gamma^{\zeta}) = 1$
  - Mean growth rate of large firms is less than zero, i.e., $E[\gamma] < 1]$ so $\zeta > 1$.
  - If $E(\epsilon) = \bar{\epsilon}$ is small, $\zeta = 1 + O(\epsilon)$ so when $\bar{\epsilon} \to 0$, then $\zeta = 1$

Reflective boundary / lower bound

$$s_{t+1} = max[s_0, \gamma s_t]$$

- Axtell claims an implicit equation defining α

$$N = \frac{\alpha - 1}{\alpha}[\frac{(s_0/A)^\alpha - 1}{(s_0/A)^\alpha - (s_0/A)}]$$

- N is total (fixed) number of firms (plants), A is total number of employees, $s_0 = 1$

Kesten Process

- Let $s_t = a_t s_{t-1} + b_t$, where (a_t, b_t) are IID random variables.
- Need the following conditions
  - $E[|a|^\zeta] = 1$ and $E[|a|^\zeta max(ln(a), 0)] < \infty$ (the same as above+positive support)
  - $0 < E[|b|^\zeta] < \infty$ (b doesn't have fatter tail than PL with exponent ζ)
- Then there is steady state $s =^d as + b$, with $x^\zeta P(s > x) \to k_+$ and $x^\zeta P(s < -x) \to k_-$ with at least one of k positive.

Bottom Line

- iid growth (with frictions) leads to Pareto tails and if frictions are small to Zipf's law
- Firm growth and size distribution are intrinsically related

Empirical Facts

Does Gibrat's "Law" hold empirically?

- Ijiri and Simon incorporates entry: avoid the variance of size distribution to increase without limit, while the number of firms grow
- Edwin Mansfield worries about exit: does Gribrat's law relate to all firms (i.e. growth rate is -1 for exiting firms), or it's "conditional on survival"
- Some consensus: Gibrat's proportional growth seems to work well with larger firms, but small firms tend to have scale dependence growth.

Facts of Firm Turnover

Firm turnover: entry

- Consensus: high rate of **infant mortality**, hazard declined steadily after the first year. (Audretsch 1991).
- But the growth rates of the surviving entrants are also higher. Dunne etc (1988) found entrants account for 39 percent of end-of-5 year period firms and 16 percent market share.
- Uncertainty of success by potential entrants motivates “passive learning” in **Jovanovic 82**.

Facts of Firm Turnover

Firm turnover: exit

- Consensus: Across industries entry and exit rate tend to be positively correlated. (Geroski 1991 for UK, Dunne etc 1991 for US). This relationship reversed in early and late phases of products life cycle (Agarwal and Gort 1996).
- Explanation for exits: hazard rates decline systematically with age , size, growth rate, technology choice. (Troske 96, Doms etc 95) Manager turnover also matters (**Holmes and Schmitz 96**).
- Shakeout: the number of firms offering a product declines from its peak to long-run equilibrium (Klepper 96).

Jovanovic 1982

- Growth and failure of firms: Small firms grow faster and are more likely to fail than large firms.
- A theory of "noisy selection": Efficient firms grow and survive; inefficient firms decline and fail.
- Partial equilibrium: small industry, constant factor price, homogeneous product and time-path of the demand for product is deterministic and known.
- Costs are random: distribution is known to all, but no firm knows its "true cost". Prior belief is updated as evidence comes in.
- No aggregate uncertainty, so the path of output prices is deterministic and self-fulfilling in equilibrium.
- Firms and potential entrants make entry, production, and exit decisions.

Model

- cost of period t output q_t: $\gamma(q_t)\theta(c+\epsilon_t)$. $\gamma(.)$ is strictly convex. c is unknown firm parameter drawn from a prior distribution $g_0(c)$. ϵ_t is a random variable with zero mean.
- firms begin with an expected value of cost multiplier $\bar{\theta}_t$, then choose output levels $q(p_t/\bar{\theta}_t)$, draw values of ϵ_t, observe their profits, and update their expectations.
- exit: W is the expected present value of firm if it is employed in a different activity.

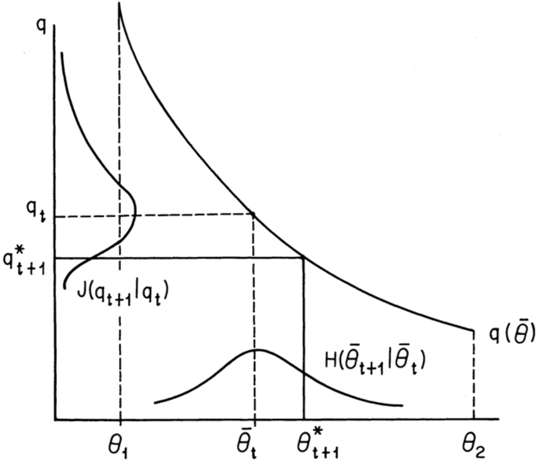
Value

- let $\eta_t = c + \epsilon_t$, then $\bar{\eta_n} = \sum_{t=1}^{n} \eta_t$ and n is sufficient for the posterior of c. Given $\theta(.)$ is strictly increasing, $\bar{\theta_t}, n$ is also a sufficient statistic.
- When the information is (θ, n) and when the price sequence is p, we have

$$V(\theta, n, t; p) = \pi(p_t, \theta) + \beta \int max[W, V(z, n+1, t+1; p)] H(dz|\theta, n)$$

- V is strictly decreasing in θ. So there exits a uniquely defined solution to equation $V(\theta, n, t; p) = W$.
- Let θ_0 be the prior mean of θ_t, then $V(\theta_0, 0, t; p) - k$ is the net value of entry at t.

Exposition of DRS on Jovanovic's Theory



Model Predictions About Failure and Growth

- The sequence of $\bar{\theta}_t$ is a Martingale.
- $q(p_t/\bar{\theta})$ is a random process which starts from $q(p_t/\theta_0)$. The shape of $J(q_{t+1}|q_t)$ depends on the curvature of $q_{t+1}(.)$ and $H(\bar{\theta_{t+1}}|\bar{\theta}_t)$.
- For each plant age, there is a failure boundary $\bar{\theta_{t+1}}*$, which could be expressed as a critical growth rate $g_t* = (q_{t+1} * - q_t)/q_t$.
- Failure: given age decline in plant size: smaller $\bar{\theta}_t$, lower prob of exceeding $\bar{\theta_{t+1}}*$. Given size decline in plant age: more precise estimate of $\bar{\theta_{t+1}}$. (Of course need $\bar{\theta}*$ not decrease too rapidly.)
- Mean and variance for nonfailing plants μ_h and σ_h^2. The mean growth rate of nonfailing plants should be an eventually decreasing function of current size, holding age fixed. (Not necessarily monotone).

Dunne Roberts Samuelson 1989

The Growth and Failure of U.S. Manufacturing Plants

- Growth and failure are viewed as outcomes of a single economic process of industry or market development
- Plant size, age, ownership type, and interaction terms, are determinants of plant growth and failure
- Mean growth rate declines with size for successful plants. But when failure probability is integrated into the analysis, plant growth and size is negative for plants owned by single-plant firms but positive for plants owned by multi-plant firms: growth v.s. selection.

Measurement of Plant Employment Growth

- $g'_t = (S_{t+1} - S_t)/S_t$, $j(g'|x)$ is the distribution of potential growth rate, given a set of characteristics x.
- realized growth rate $g_t = g'_t$ if $g'_t \geq g*$ and $g_t = -1$ if $g'_t < g*$, $f(g|x)$ is the density of realized growth rates, defined over $[-1, \infty]$.
- $h(g|x)$ is the density of realized growth rates for all non-failing plants. Define $P_s(x) = \int_{-1} h(g|x)dg$.
- Now we can define mean growth rate of successful plants as: $\mu_h(x) = \frac{\int_{-1} gh(g|x)dg}{P_s(x)}$. And the mean growth rate of ALL plants is $\mu_f(x) = \mu_h(x)P_s(x) - (1 - P_s(x))$, which is always less or equal.
- The inference we can make on μ_f from μ_h depends on the relationship $P_s(x)$. Here we could observe BOTH.

A First Look at Data

TABLE I
PLANT GROWTH AND EXIT RATES

	Size (number of employees)					
Age (years)	5–19	20–49	50–99	100–249	>250	Total
a. Mean employment growth rate of successful plants						
1–5	0.606	0.299	0.187	0.132	0.067	0.446
6–10	0.338	0.136	0.066	0.011	−0.011	0.202
11–15	0.310	0.055	−0.006	−0.015	−0.018	0.153
Total	0.519	0.226	0.130	0.077	0.026	0.353
b. Plant exit rates						
1–5	0.412	0.396	0.390	0.327	0.229	0.397
6–10	0.347	0.268	0.281	0.245	0.158	0.303
11–15	0.304	0.206	0.234	0.212	0.131	0.255
Total	0.391	0.347	0.346	0.291	0.191	0.363
c. Mean employment growth rate of all plants						
1–5	−0.056	−0.216	−0.276	−0.238	−0.178	−0.129
6–10	−0.127	−0.169	−0.234	−0.236	−0.167	−0.162
11–15	−0.089	−0.163	−0.239	−0.224	−0.147	−0.141
Total	−0.074	−0.199	−0.261	−0.236	−0.170	−0.138
d. Number of plant-year observations on successful plants/failing plants						
1–5	75,959/53,325	29,938/19,649	13,758/8,794	9,472/4,601	3,281/977	132,408/87,346
6–10	27,409/14,569	15,268/5,584	7,577/2,961	5,829/1,889	2,630/494	58,713/25,947
11–15	7,773/3,400	4,675/1,216	2,198/673	1,568/421	911/137	17,125/5,847
Total	111,141/71,294	49,881/26,449	23,533/12,428	16,869/6,911	6,822/1,608	208,246/118,690

Data and Empirical Model

- The Census of Manufactures, more than $300,000$ US manufacturing plants. Five census years: 1963, 1967, 1972, 1977, 1982. Longitudinal, great for analyzing entry, exit, and growth.
- One aspect that has not been explored more by later studies: ownership.
- Put each plant in year t into a cell c based on:
  - age: 1-5, 6-10, 11-15
  - size: 5-19, 20-49, 50-99, 100-249, more than 250
  - 20 two-digit industry SIC
  - 2 ownership: single-unit, multi-unit
  - initial-size class: less than, equal to, greater than current class
- Construct growth rate $\bar{g_{ct}}$, variance of growth S_{ct}^2, and failure rate F_{ct} for each cell c.

- $Y_{ct} = \sum_{i=1}^{60} \alpha_i D_i + \sum_{k=1}^{18} \beta_k D_k^c + \epsilon_{ct}$.
- D_i each of the twenty industries in each of the three time periods.
- D_k^c include 14 dummy variables to represent 15 current size-age combinations. The rest is on initial size class for age group 2 and 3.
- Applied to SU and MU.

TABLE II

REGRESSION COEFFICIENTS FOR PLANT FAILURE RATES: 1967, 1972, 1977 ENTRANTS
(STANDARD ERRORS IN PARENTHESES)

		Single-plant	Multiplant
Intercept[a]		0.426	0.471
Age/initial-size (IS) versus current-size (CS) category			
2	IS > CS	0.084 (0.012)*	0.104 (0.010)*
2	IS < CS	0.008 (0.010)	0.001 (0.007)
3	IS > CS	0.043 (0.020)	0.065 (0.021)
3	IS < CS	0.002 (0.016)	−0.013 (0.012)
Age/current-size category			
1	2	−0.035 (0.005)*	−0.064 (0.006)*
1	3	−0.002 (0.009)	−0.116 (0.006)*
1	4	0.055 (0.014)	−0.206 (0.007)*
1	5	−0.039 (0.056)	−0.306 (0.008)*
2	1	−0.061 (0.005)*	−0.070 (0.009)*
2	2	−0.166 (0.008)*	−0.158 (0.008)*
2	3	−0.151 (0.012)*	−0.206 (0.008)*
2	4	−0.095 (0.019)*	−0.271 (0.008)*
2	5	−0.174 (0.097)	−0.356 (0.008)*
3	1	−0.087 (0.008)*	−0.150 (0.018)*
3	2	−0.199 (0.013)*	−0.238 (0.015)*
3	3	−0.180 (0.021)*	−0.250 (0.015)*
3	4	−0.099 (0.037)	−0.315 (0.014)*
3	5	−0.193 (0.179)	−0.386 (0.014)*

Age categories: 1 = 1–5 years; 2 = 6–10 years; 3 = 11–15 years.
Size categories: 1 = 5–19 employees; 2 = 20–49 employees; 3 = 50–99 employees; 4 = 100–249 employees; 5 = ≥250 employees.
a. The reported intercept is the average value of the 60 estimated industry-time intercepts.
*Significant at the 0.05 level using Leamer's]1978] correction for sample size.

TABLE IV

REGRESSION COEFFICIENTS FOR SUCCESSFUL PLANTS: 1967, 1972, 1977 ENTRANTS (STANDARD ERRORS IN PARENTHESES)

		Mean growth rate		Variance of growth rate	
		Single-plant	Multiplant	Single-plant	Multiplant
Intercept[a]		0.492	0.709	1.650	2.446
Age–initial-size (IS) versus current-size (CS) category					
2	IS > CS	−0.038 (0.025)	0.010 (0.018)	−0.103 (0.014)*	−0.052 (0.010)*
2	IS < CS	0.061 (0.014)	0.026 (0.011)	0.080 (0.012)*	0.015 (0.008)
3	IS > CS	−0.023 (0.037)	0.009 (0.029)	−0.078 (0.020)	−0.056 (0.012)
3	IS < CS	0.045 (0.022)	0.004 (0.017)	0.057 (0.015)	0.004 (0.007)
Age–current-size category					
1	2	−0.292 (0.013)*	−0.360 (0.034)*	−1.166 (0.080)*	−1.812 (0.287)*
1	3	−0.448 (0.016)*	−0.504 (0.032)*	−1.383 (0.078)*	−1.939 (0.284)*
1	4	−0.551 (0.023)*	−0.598 (0.032)*	−1.488 (0.078)*	−2.072 (0.284)*
1	5	−0.664 (0.054)*	−0.683 (0.033)*	−1.651 (0.079)*	−2.167 (0.284)*
2	1	−0.229 (0.014)*	−0.432 (0.039)*	−1.110 (0.083)*	−2.037 (0.285)*
2	2	−0.450 (0.015)*	−0.575 (0.032)*	−1.465 (0.078)*	−2.117 (0.284)*
2	3	−0.550 (0.018)*	−0.636 (0.032)*	−1.545 (0.077)*	−2.148 (0.284)*
2	4	−0.579 (0.024)*	−0.680 (0.032)*	−1.640 (0.077)*	−2.174 (0.283)*
2	5	−0.587 (0.074)*	−0.717 (0.032)*	−1.715 (0.082)*	−2.202 (0.283)*
3	1	−0.310 (0.021)*	−0.517 (0.054)*	−1.257 (0.086)*	−2.130 (0.285)*
3	2	−0.529 (0.020)*	−0.664 (0.036)*	−1.547 (0.078)*	−2.208 (0.284)*
3	3	−0.628 (0.027)*	−0.663 (0.036)*	−1.602 (0.078)*	−2.205 (0.284)*
3	4	−0.681 (0.045)*	−0.664 (0.035)*	−1.616 (0.080)*	−2.220 (0.284)*
3	5	−0.506 (0.269)	−0.683 (0.036)*	−1.472 (0.234)*	−2.248 (0.284)*

Age categories: 1 = 1–5 years; 2 = 6–10 years; 3 = 11–15 years.
Size categories: 1 = 5–19 employees; 2 = 20–49 employees; 3 = 50–99 employees; 4 = 100–249 employees; 5 = ≥250 employees.
a. The reported intercept is the average value of the 60 estimated industry-time intercepts.
*Significant at the 0.05 level using Leamer's [1978] correction for sample size.

TABLE V

REGRESSION COEFFICIENTS FOR ALL PLANTS: 1967, 1972, 1977 ENTRANTS (STANDARD ERRORS IN PARENTHESES)

		Mean growth rate		Variance of growth rate	
		Single-plant	Multiplant	Single-plant	Multiplant
Intercept[a]		−0.126	−0.051	1.503	2.09
Age–initial-size (IS) versus current-size (CS) category					
2	IS > CS	−0.106 (0.022)*	−0.104 (0.017)*	−0.073 (0.024)	−0.001 (0.016)
2	IS < CS	0.043 (0.014)	0.012 (0.011)	0.094 (0.015)	0.033 (0.011)*
3	IS > CS	−0.054 (0.036)	−0.045 (0.034)	−0.016 (0.039)	−0.007 (0.026)
3	IS < CS	0.037 (0.023)	0.006 (0.019)	0.061 (0.020)	0.002 (0.016)
Age–current-size category					
1	2	−0.143 (0.010)*	−0.132 (0.020)*	−0.877 (0.048)*	−1.197 (0.155)*
1	3	−0.278 (0.013)*	−0.168 (0.019)*	−1.093 (0.047)*	−1.452 (0.151)*
1	4	−0.415 (0.017)*	−0.143 (0.019)*	−1.235 (0.047)*	−1.606 (0.150)*
1	5	−0.458 (0.056)*	−0.116 (0.021)*	−1.400 (0.054)*	−1.730 (0.150)*
2	1	−0.072 (0.011)*	−0.188 (0.025)*	−0.801 (0.053)*	−1.550 (0.153)*
2	2	−0.118 (0.013)*	−0.173 (0.020)*	−1.184 (0.047)*	−1.595 (0.150)*
2	3	−0.212 (0.017)*	−0.170 (0.020)*	−1.274 (0.047)*	−1.681 (0.150)*
2	4	−0.316 (0.025)*	−0.144 (0.020)*	−1.332 (0.049)*	−1.734 (0.150)*
2	5	−0.234 (0.100)*	−0.097 (0.021)	−1.401 (0.073)*	−1.812 (0.150)*
3	1	−0.090 (0.018)*	−0.152 (0.041)	−0.968 (0.058)*	−1.663 (0.157)*
3	2	−0.149 (0.020)*	−0.159 (0.027)*	−1.279 (0.049)*	−1.704 (0.151)*
3	3	−0.248 (0.028)*	−0.144 (0.027)*	−1.350 (0.049)*	−1.732 (0.151)*
3	4	−0.360 (0.045)*	−0.085 (0.026)	−1.353 (0.055)*	−1.774 (0.151)*
3	5	−0.156 (0.241)	−0.058 (0.027)	−1.205 (0.216)*	−1.828 (0.151)*

Age categories: 1 – 1–5 years; 2 – 6–10 years; 3 – 11–15 years.
Size categories: 1 – 5–19 employees; 2 – 20–49 employees; 3 – 50–99 employees; 4 – 100–249 employees; 5 – ≥250 employees.
[a] The reported intercept is the average value of the 60 estimated industry-time intercepts.
*Significant at the 0.05 level using Leamer's [1978] correction for sample size.

Competitive Industry Dynamics

- Introduce stationary model of industry dynamics, i.e. Hopenhayn (1992)
- Adding frictions to Hopenhayn (1992)
  - Hopenhayn and Rogerson (1993): Labor market
  - Asplund and Nocke (2006): Product market
  - Gomes (2001), Cooley and Quadrini (2001): Financing market

Hopenhayn: overview

- Workhorse model of industry dynamics
- Steady-state model: firms enter, grow and decline, and exit, but overall distribution of firms is unchanging
- Endogenous stationary distribution of firm-size etc, straightforward comparative statics
- Competitive firms, no strategic interactions
- Useful to characterize long-run industry structure

Key elements

- Continuum of firms, each measure zero, produce with DRS
- No aggregate risk: deterministic paths for producer price and factor price(s) taken as given
- But idiosyncratic risk: individual firm productivities follow a first-order Markov process
- Fixed cost to enter, fixed cost to operate each period

Model

- Time $t = 0, 1, 2, \ldots$
- Output and input prices p and w taken as given
- Output y produced with labor n given productivity a

$$y = af(n)$$

- Static profits

$$\pi(a, p, w) := \max_n \Big[paf(n) - wn - k \Big]$$

where $k > 0$ is per-period fixed cost of operating

- Let $n(a, p, w)$ denote optimal employment and let $y(a, p, w)$ denote associated output

Model

- Assumptions

  - $n(\cdot), y(\cdot), \pi(\cdot)$ are all strictly increasing in productivity a

  - productivity draws follow a first-order Markov process with distribution function $F(a' \,|\, a)$ and $F(\cdot \,|\, a)$ is strictly decreasing in a

    i.e., if $a_1 > a_2$ then $F(\cdot \,|\, a_1)$ FOSD's $F(\cdot \,|\, a_2)$

  - entrants draw initial productivity a_0 from separate distribution $G(a)$, pay sunk cost $k_e > 0$ to do so

- Timing within a period

  - incumbents decide to stay or exit, entrants decide to enter or not

  - incumbents that stay pay k, entrants pay k_e

  - *after* paying k or k_e, operating firms learn their productivity draws

Incumbent's problem

- Let $z = \{p_t, w_t\}_{t=0}^{\infty}$ denote sequence of prices a firm takes as given

- Let $v_t(a, z)$ denote the value of incumbency to a firm with current productivity draw a

- Bellman equation for an incumbent firm

$$v_t(a, z) = \pi(a, p_t, w_t) + \beta \max \left[0\,, \int v_{t+1}(a', z)\, dF(a' \,|\, a) \right]$$

- An *exit threshold* $a_t^*(z)$ such that firm exits if $a_t < a_t^*(z)$, solves

$$\int v_{t+1}(a', z)\, dF(a' \,|\, a^*) = 0$$

(for interior cases)

Entrant's problem

- Potential entrants are ex ante identical
- Pay $k_e > 0$ to enter, initial draw from $G(a)$ if they do
- Start producing next period
- Let $m_t \geq 0$ denote the mass of entrants, *free entry* condition

$$\beta \int v_{t+1}(a,z)\, dG(a) \leq k_e$$

with strict equality whenever $m_t > 0$

Aggregate state $\mu_t(\mathcal{A})$

- Let $\mu_t(\mathcal{A})$ be the measure of incumbents with productivity $a \in \mathcal{A}$

- $\mu_t(\mathcal{A})$ is the state variable for the aggregate economy

- $\mu_t(\mathcal{A})$ is endogenous and, in general, evolves over time

Law of motion for the state

- The measure of incumbents with productivity $a \in [0, a')$ at $t+1$ is

$$\mu_{t+1}([0, a')) = \int F(a' \,|\, a) \mathbb{1}[a \geq a_t^*] \mu_t(da) + m_{t+1} G(a'), \quad \text{all } a'$$

  (suppressing the dependence on z)

- Suppose we discretize to a grid with N elements. Then this is a linear system of the form

$$\boldsymbol{\mu}_{t+1} = \boldsymbol{\Psi}_t \boldsymbol{\mu}_t + m_{t+1} \mathbf{g}$$

  where $\boldsymbol{\Psi}$ is a $N \times N$ matrix that depends on the productivity process and exit threshold a_t^*, where $\boldsymbol{\mu}$ and $\mathbf{g}$ are $N \times 1$ vectors, and where m is a scalar

Industry demand and supply

- Industry demand curve $D(p)$, exogenous
- Industry supply curve, endogenous

$$Y = \int y(a, p_t, w_t) \mu_t(da)$$

- Market clears when

$$Y = D(p_t)$$

- Choose either p_t or w_t as numeraire. We will choose $w_t = 1$

Equilibrium: overview

- Given an initial distribution μ_0, a *perfect foresight equilibrium* consists of sequences

$$\{p_t, m_t, a_t^*, \mu_t\}_{t=0}^{\infty}$$

  such that (i) goods market clears, (ii) incumbents make optimal exit decisions, (iii) no further incentives to enter, and (iv) distribution μ_t defined recursively by law of motion above

- We will focus on a *stationary equilibrium*, constants

$$(p^*, m^*, a^*, \mu^*)$$

  that corresponds to a steady-state of the dynamical system implied by the perfect foresight equilibrium

Computing an equilibrium (sketch)

- **Step 1.** Guess output price p_0. For this price, solve the incumbent's dynamic programming problem

$$v(a, p_0) = \pi(a, p_0) + \beta \max \left[0\,, \int v(a', p_0) dF(a' \,|\, a) \right]$$

The solution of this problem also implies the optimal exit rule, i.e., the $a^*(p_0)$ that solves

$$\int v(a', p_0) dF(a' \,|\, a^*) = 0$$

- **Step 2.** Check that this price p_0 satisfies the free-entry condition

$$\beta \int v(a', p_0) dG(a') = k_e$$

For example, if the LHS is too high, then go back to Step 2 and guess a new price $p_1 < p_0$. Continue until a price p^* is found that solves the free-entry condition

Computing an equilibrium (sketch)

- **Step 3.** Guess a measure of entrants, m_0. Given this, calculate the stationary distribution μ_0. This solves the linear system

$$\mu_0([0,a')) = \int_{a \geq a^*(p^*)} F(a' \,|\, a)\mu_0(da) + m_0 G(a'), \quad \text{for all } a'$$

  Observe that the RHS depends on the price found at Step 2 via the exit threshold $a^*(p^*)$

- **Step 4.** Given this μ_0, calculate the total industry supply and check the market clearing condition

$$Y = \int y(a,p^*)\mu_0(da) = D(p^*)$$

  For example, if the LHS is too low, then go back to Step 3 and guess new entrants $m_1 > m_0$. Continue until a m^* is found that solves the market-clearing condition

Speeding up the last step

- Because of the linear law of motion for μ, the stationary distribution is linearly homogeneous in m

- In terms of the discretized system above

$$\boldsymbol{\mu} = \boldsymbol{\Psi}\boldsymbol{\mu} + m\mathbf{g} \qquad \Rightarrow \qquad \boldsymbol{\mu} = m(\mathbf{I} - \boldsymbol{\Psi})^{-1}\mathbf{g}$$

  where $\mathbf{I}$ is an identity matrix

- Two implications

  – no need to use simulations to find stationary distribution $\boldsymbol{\mu}$, just set up coefficient matrix $\boldsymbol{\Psi}$ (implied by $a^*(p^*)$) and calculate directly

  – only invert $(\mathbf{I} - \boldsymbol{\Psi})$ once, then just rescale by m

Comparative statics

- Increase in entry cost k_e
  - increases expected discounted profits
  - decreases exit threshold a^*
    - less selection, incumbents make more profits, more continue
    - increases average age of firms
  - decreases entrants m^*
  - decreases entry/exit rate $m^*/\mu^*(\mathbb{R})$
  - increases price p^*

Comparative statics

- Ambiguous implications for firm-size distribution

  – *price effect*, higher k_e increases price p^*

    hence incumbents increase output $y(a, p^*)$ and employment $n(a, p^*)$

  – *selection effect*, higher k_e reduces productivity threshold a^*

    hence more incumbent firms are relatively-low productivity firms

Static version for intuition

- Once-and-for-all productivity draw $a \sim G(a)$, once-and-for-all endogenous exit

- Static profit maximization problem

$$\pi(a, p) := \max_n \left[pan^{\alpha} - n - k \right], \qquad (w = 1 \text{ is numeraire})$$

- Implies employment, output

$$n(a, p) = (\alpha pa)^{\frac{1}{1-\alpha}}, \qquad y(a, p) = an(a, p)^{\alpha}$$

and profits

$$\pi(a, p) = (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} (pa)^{\frac{1}{1-\alpha}} - k$$

Exit and entry conditions

- Exit threshold a^* satisfies

$$\pi(a^*, p) = 0$$

such that firms immediately exit for all $a < a^*$

- Value of a firm given once-and-for-all choices

$$v(a, p) = \max\left[0\,,\,\sum_{t=0}^{\infty}\beta^t\pi(a, p)\right] = \max\left[0, \frac{\pi(a, p)}{1-\beta}\right]$$

- Free entry condition

$$k_e = \beta\int v(a, p^*)\,dG(a) = \beta\int_{a^*}^{\infty}\frac{\pi(a, p^*)}{1-\beta}\,dG(a)$$

Two conditions in two unknowns a^*, p^*

Implications for selection

- Substituting the profit function into the free entry condition

$$(1-\beta)k_e = \beta\int_{a^*}^{\infty}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(p^* a)^{\frac{1}{1-\alpha}} - k\right]dG(a)$$

- Using the exit condition for a^* to eliminate p^* gives

$$(1-\beta)\frac{k_e}{k} = \beta\int_{a^*}^{\infty}\left[\left(\frac{a}{a^*}\right)^{\frac{1}{1-\alpha}} - 1\right]dG(a)$$

- Increase in k_e (or decrease in k) reduces cutoff a^* and increases p^*

$\Rightarrow$ Larger entry barriers *weaken the selection effect and allow more unproductive firms to operate*

- Let's now turn to an application of these ideas

Aw, Chung, Roberts (2003)

- Comparison of Taiwanese and Korean manufacturers
- Background: at aggregate level, Taiwan and Korea similar export-oriented economies, but interesting micro-level differences
- Goal:
  - interpret relationships between market concentration, producer turnover and productivity through the lens of a Hopenhayn model
  - compare these relationships between Taiwan and Korea

Overview

- Korean industries characterized by
  - more market concentration
  - more cross-sectional productivity dispersion
  - less producer turnover
  - more low-productivity producers operating
  - more output attributable to high-productivity producers
  - larger prod. differentials between surviving and failing producers

- Suggestive of Hopenhayn model where entry sunk costs k_e are higher in Korea than Taiwan. Remember
  - higher k_e discourages entry
  - higher k_e protects incumbents

  $\Rightarrow$ reduces productivity threshold a^*, weakens selection effect on existing producers, more incumbent firms are low-productivity firms

Large firms account for more output in Korea

Distribution of Gross Output by Employment Categories

	Taiwan (1986)	South Korea (1988)
Textiles		
5–99 workers	0.235	0.193
100–499 workers	0.400	0.223
500+ workers	0.365	0.553
Apparel		
5–99 workers	0.340	0.285
100–499 workers	0.401	0.383
500+ workers	0.259	0.332
Chemicals		
5–99 workers	0.430	0.140
100–499 workers	0.233	0.361
500+ workers	0.337	0.499
Plastics		
5–99 workers	0.324	0.352
100–499 workers	0.342	0.263
500+ workers	0.334	0.385
Fabricated metals		
5–99 workers	0.640	0.330
100–499 workers	0.272	0.352
500+ workers	0.088	0.317
Electrical machinery/electronics		
5–99 workers	0.184	0.105
100–499 workers	0.262	0.168
500+ workers	0.554	0.727
Transport equipment		
5–99 workers	0.225	0.070
100–499 workers	0.237	0.114
500+ workers	0.538	0.816

Productivity dispersion is higher in Korea

Cross-sectional Productivity Dispersion (average values over three census years)

Industry	Taiwan (1981–91)	South Korea (1983–93)
Textiles		
Standard Deviation	0.266	0.380
Range (10^{th}–90^{th} Percentile)	0.635	0.893
Mean *TFP* Growth	0.159	0.095
Apparel		
Standard Deviation	0.273	0.376
Range (10^{th}–90^{th} Percentile)	0.643	0.910
Mean *TFP* Growth	0.039	0.217
Chemicals		
Standard Deviation	0.193	0.013
Range (10^{th}–90^{th} Percentile)	0.255	0.400
Mean *TFP* Growth	0.576	0.936
Plastics		
Standard Deviation	0.237	0.314
Range (10^{th}–90^{th} Percentile)	0.553	0.713
Mean *TFP* Growth	0.119	0.120
Fabricated Metals		
Standard Deviation	0.244	0.330
Range (10^{th}–90^{th} Percentile)	0.575	0.791
Mean *TFP* Growth	0.052	0.083
Electrical Machinery/Electronics		
Standard Deviation	0.236	0.326
Range (10^{th}–90^{th} Percentile)	0.546	0.756
Mean *TFP* Growth	0.173	0.061
Transport Equipment		
Standard Deviation	0.240	0.310
Range (10^{th}–90^{th} Percentile)	0.550	0.722
Mean *TFP* Growth	−0.020	0.115

Less turnover in Korea

Industry Output Volatility Rates

	Taiwan		South Korea	
Industry	1981–6	1986–91	1983–8	1988–93
Textiles	0.847	0.829	0.335	0.656
Apparel	1.00	0.881	0.750	1.10
Chemicals	0.540	0.655	0.265	0.158
Plastics	0.719	0.886	0.618	0.558
Fabricated Metals	0.788	0.918	0.786	0.858
Electrical Machinery/Electronics	0.602	0.640	0.370	0.454
Transport Equipment	0.451	0.510	0.432	0.194

Volatility is defined as (entry rate + exit rate) − |entry rate − exit rate| = 2 Min(entry rate, exit rate). Entry and exit rates are measured using the value of output accounted for by the entering and exiting firms. See Dunne and Roberts (1991) or Roberts (1996) for details.

Exiting firms have low productivity

	Taiwan				
				F-statistic:	
	$\alpha_1^x - \alpha_1^S$	$\alpha_2^x - \alpha_2^S$	$\beta_2^x - \beta_2^S$	No exit differential (a)	Equal exit differential (b)
Textiles	−0.078** (0.013)	0.028 (0.021)	−0.028* (0.013)	13.41**	9.93**
Apparel	−0.072** (0.018)	0.007 (0.031)	−0.010 (0.018)	5.52**	4.06*
Chemicals	−0.074** (0.012)	−0.041* (0.018)	−0.054** (0.011)	22.90**	1.52
Plastics	−0.039** (0.009)	0.015 (0.014)	−0.011 (0.008)	6.76**	5.41**
Fabricated metals	−0.034** (0.010)	−0.026 (0.014)	−0.026** (0.007)	9.55**	0.22
Electric/electronics	−0.070** (0.010)	−0.049** (0.016)	−0.020* (0.009)	20.73**	7.07**
Transport equipment	−0.055** (0.015)	−0.041 (0.024)	−0.023 (0.013)	6.09**	1.2

*significant at the 0.05 level.
**significant at the 0.01 level.

Mean productivity difference between exiting and surviving firms

Exiting firms have low productivity

South Korea				
			F-statistic:	
$\alpha_1^x - \alpha_1^S$	$\alpha_2^x - \alpha_2^S$	$\beta_2^x - \beta_2^S$	No exit differential (a)	Equal exit differential (b)
−0.051** (0.017)	−0.115** (0.025)	0.036 (0.020)	11.12**	12.17**
−0.119** (0.020)	−0.028 (0.037)	−0.035 (0.021)	13.55**	5.13**
−0.263** (0.023)	−0.136** (0.039)	−0.046 (0.027)	47.37**	18.81**
−0.126** (0.017)	−0.028 (0.026)	−0.003 (0.014)	18.29**	15.42**
−0.129** (0.017)	−0.085** (0.027)	−0.010 (0.017)	23.63**	12.78**
−0.131** (0.018)	−0.007 (0.030)	−0.028 (0.016)	19.04**	11.35**
−0.130** (0.026)	0.015 (0.041)	0.004 (0.021)	8.30**	8.97**

Lower productivity of exiting firms especially pronounced in Korea

Korean firms more likely to enter lower tail

Cumulative Distribution of Current Productivity Conditional on Past Productivity
$F(\phi_{t+1} = 0|\phi_t)$

	Taiwan Quartile for ϕ_t				Korea Quartile for ϕ_t			
	1st	2nd	3rd	4th	1st	2nd	3rd	4th
Textiles	0.638	0.508	0.421	0.333	0.876	0.809	0.627	0.353
Apparel	0.660	0.539	0.463	0.311	0.859	0.700	0.541	0.471
Chemicals	0.654	0.512	0.420	0.268	0.836	0.689	0.551	0.340
Plastics	0.607	0.498	0.383	0.357	0.784	0.685	0.565	0.491
Fabricated metals	0.548	0.483	0.412	0.334	0.846	0.686	0.574	0.412
Electrical machinery	0.602	0.541	0.461	0.458	0.791	0.700	0.611	0.470
Transportation equipment	0.632	0.516	0.408	0.394	0.789	0.685	0.560	0.497

In Korea, more likely to move down but not more likely to exit

Summary/bigger picture

- Productivity dispersion higher and turnover lower in Korea, consistent with weaker selection effects (greater barriers to entry)

- What accounts for these weaker selection effects? If larger barriers to entry, do these reflect policy settings, or features of market structure (e.g., credit market frictions, supplier networks), or both?

- More generally, what accounts for differences in selection effects across industries and countries? Are these micro-level differences an important determinant of cross-country differences in aggregate productivity and income?