Assignment 1, Spring 2021

ECON 202C

Chris Ackerman, Paige Pearcy, Luna Shen

April 11, 2021

1 Numerical exercise on Armington Model

For the following 4-country version of the Armington model, assume $\sigma = 5$, $a_{ij} = 1$ for all i, j, $L_1 = L_2 = L_3 = L_4 = 1$, $A_1 = 1$, $A_2 = A_3 = A_4 = 0.5$, and trade costs τ_{ij} correspond to the i, j element in

$$\mathcal{T} = \begin{bmatrix} 1 & 1.3 & 1.4 & 1.2 \\ 1.3 & 1 & 1.3 & 1.2 \\ 1.4 & 1.4 & 1 & 1.3 \\ 1.4 & 1.4 & 1.3 & 1 \end{bmatrix}$$

1. The equilibrium wages in countries 2, 3 and 4 are

$$0.5619, 0.5400, 0.5331.$$

2. The trade shares, λ_{ij}, are

$$\begin{bmatrix} 0.620217734098387 & 0.255577283985955 & 0.175172839223195 & 0.265589014755254 \\ 0.136161205543548 & 0.457697222493548 & 0.147736606723888 & 0.166530093153695 \\ 0.118706532240127 & 0.139709551344339 & 0.494790659680884 & 0.141776504201275 \\ 0.124914528117938 & 0.147015942176159 & 0.182299894372033 & 0.426104387889776 \end{bmatrix}$$

3. (a) The welfare in country 2 with the initial parameter values is 0.607890610207557, and with the new parameter values it is 2.21273016767756.

   (b) If country 2 is under autarky, then

$$W_2 = \lambda_{22}^{\frac{1}{1-\sigma}} a_{22}^{\frac{1}{1-\sigma}} A_2 = A_2$$

   Therefore the change in welfare is equal to the change in productivity, $A_2 - A_2'$

   (c) The increase is larger under autarky. When there is trade, the country with the productivity shock continues to consume goods produced in countries that did not experience an increase in productivity (due to their preference for diversity), so the "effective" increase in productivity is smaller.

4. Now assume that the matrix for trade costs is given by

$$\mathcal{T}' = \begin{bmatrix} 1 & 1 & 1 & 1.2 \\ 1 & 1 & 1.3 & 1.2 \\ 1 & 1.4 & 1 & 1.3 \\ 1 & 1.4 & 1.3 & 1 \end{bmatrix}$$

   The new wages for countries 2–4 are

$$0.554516371140729, 0.543085715012605, 0.549281219715067,$$

and the new trade shares are

$$\begin{bmatrix} 0.326147847659487 & 0.493393157494855 & 0.456556718492958 & 0.277286263806797 \\ 0.215594185696546 & 0.326148698455949 & 0.105668089724799 & 0.183295111953884 \\ 0.234326289727384 & 0.0922757276994401 & 0.328020689580818 & 0.144639339604683 \\ 0.223931676916583 & 0.0881824163497557 & 0.109754502201426 & 0.394779284634636 \end{bmatrix}$$

Welfare under the new trade costs is

$$1.32326361398792, 0.661631375507836, 0.660685377572939, 0.630785084384098$$

for countries 1–4. The change in wages for each country is

$$1.0000, 1.0133, 0.9942, 0.9706.$$

These values are the same regardless of whether we use "hat algebra" (the equation for changes) or directly solve the model twice, and then compare results. The advantage to using hat algebra is that it is computationally easier. This model is very small and we can solve it quickly, but for more complicated models the computation time may be an important consideration.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                             ARMINGTON MODEL

% 2021 Spring Econ 202 C, UCLA
% Chang He and Paula Beltran

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Description: In this file we have some numerical exercises on the
% Armington model from the Lecture Notes.
% We start with the basic 2-country model. Then we expand then number of
% countries to 4.
% This script uses the following functions:
% 1) findeq2country:            This function is required to find the
%                               equilibrium in the 2 country model
% 2) hatalgebra2country:        Hat algebra equations for 2-country model
% 3) modelcalculations2country: Computes additional measures in the model
%                               given wages
% 4) findeq: General code for S countries
% 5) findeqhatalgebra: Hat Algebra
% 6) modelcalculations: code to compute some statistics
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
clc;
% Filling in the parameters
global A S L Tau a_mat ssigma
A      = [1,1.25]            ;                                     % Technology
L      = [15,14]             ;                                     % Labor Supply
Tau    = [1,1.2;1.2,1]       ;                                     % Iceberg costs
a_mat  = [0.8, 0.2; 0.1, 0.9]  ;                                   % Preference shares
ssigma = 5                   ;                                     % Elasticity of substitutio
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solving the initial equilibrium
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initial value for the wage in the second country
% Note: My initial guess is the same wage as in country 1
w0          =  1                                  ;
```

```
% Using fsolve to find the equilibrium wage
w1          =  fsolve( @(x)findeq2country(x),w0) ;
% Some useful calculations
[model_init]=  modelcalculations2country(w1)     ;
% Saving the initial iceberg costs
Tau0        =  Tau                               ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Counterfactual change in Iceberg trade costs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tau(1,2)    =  1.6                               ;
Tau(2,1)    =  Tau(1,2)                          ;
% Finding the solution using the equilibrium conditions
w2          =  fsolve( @(x)findeq2country(x),w1) ;
[model_end] =  modelcalculations2country(w2)     ;


% Finding the changes using hat algebra
global Khat Lhat lambda0 Yshare0
Khat        = (Tau./Tau0).^(1-ssigma)              ;
Lhat        = ones(size(L))                       ;
lambda0     = model_init.lambda                   ;
Yshare0     = model_init.Yshare                   ;
x0          = ones(5,1)                           ;
xvec        = fsolve( @(x)hatalgebra2country(x),x0)   ;
w_hat(1,1)  = 1                                   ;
w_hat(2,1)  = xvec(1)                             ;
lambda_hat  = xvec(2:end)                         ;
lambda_hat  = reshape(lambda_hat,2,2)             ;



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 4 Country model (Homework 1, Q1)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% PARAMETERS
A      =  [1, .5, .5, .5];
S      =  size(A,2);
L      =  [1, 1, 1, 1];
Tau    =  [1 ,       1.3,       1.4,       1.2 ;...
          1.3,  1  ,    1.3,       1.2  ;...
          1.4,       1.4,       1  ,       1.3  ;...
          1.4,       1.4,       1.3,       1]   ;
a_mat  =  [1 ,       1,       1,       1; ...
           1,       1,       1,       1; ...
           1,       1,       1 ,       1; ...
           1,       1,       1,       1];
ssigma = 5;
% Finding Initial Equilibrium
winit=[1,1,1];
w0=fsolve(@(x)findeq(x),winit);
model_init=modelcalculations(w0);
% Finding Final equilibrium
Tau = [1, 1, 1, 1.2;...
```

```
    1, 1, 1.3, 1.2;...
    1, 1.4, 1, 1.3;...
    1, 1.4, 1.3, 1];
w0=fsolve(@(x)findeq(x),w0);
model_final=modelcalculations(w0);
% Hat algebra
% Some initial steps
global lambda0 Yshare0 Khat Lhat
L0=model_init.L;
L1=L;
A0=model_init.A;
A1=[1, .5, .5, .5];
Tau0=model_init.iceberg;
Tau1=Tau;
a_mat0=model_init.a_mat;
a_mat1=a_mat;
K0=model_init.K;
K1=(a_mat1.*(Tau1.^(1-ssigma))).*((ones(S,1)*A1.^(ssigma-1))');
Yshare0=model_init.Yshare;
lambda0=model_init.lambda;
Khat=K1./K0;
Lhat=L1./L0;
% Initial values
x0=[model_init.wages(1:end-1)*0+1,reshape(lambda0,1,S*S)*0+1];
% Finding the solution
xsol=fsolve(@(x) findeqhatalgebra(x),x0');
lambda_hat=xsol(S:end);
lambda_hat=reshape(lambda_hat,S,S);
w_hat=[1;xsol(1:S-1)];
% check that hat algebra gives the same solution as direct calculation
model_final.wages./model_init.wages
```

2 Intermediate Inputs

1.

$$
\begin{aligned}
\max_{L_i, M_i} \; & p_i A_i L_i^{\alpha} M_i^{1-\alpha} - w_i L_i - P_i M_i \\
p_i \alpha A_i L_i^{1-\alpha} &= w_i \qquad (L_i) \\
p_i (1-\alpha) A_i L_i^{\alpha} M_i^{-\alpha} &= P_i \qquad (M_i) \\
\implies \frac{\alpha}{1-\alpha} \frac{M_i}{L_i} &= \frac{W_i}{P_i} \\
\implies M_i &= \frac{1-\alpha}{\alpha} L_i \frac{w_i}{p_i} \\
p_i &\propto A_i L_i^{\alpha-1} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} L_i^{1-\alpha} \left(\frac{w_i}{p_i}\right)^{1-\alpha} \\
&= W_i \\
&\propto A_i \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \left(\frac{w_i}{p_i}\right)^{1-\alpha} \\
\implies p_i &= \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{w_i^{\alpha} P_i^{1-\alpha}}{A_i} \\
&= K \frac{w_i^{\alpha} P_i^{1-\alpha}}{A_i}
\end{aligned}
$$

2. We can use the first order conditions from the previous part.

$$
\begin{aligned}
\frac{\alpha}{1-\alpha} \frac{M_i}{L_i} &= \frac{w_i}{P_i} \\
\frac{\alpha}{1-\alpha} &= \frac{w_i L_i}{P_i M_i} \\
w_i L_i + P_i M_i &= Y_i - p_i Q_i \\
\implies w_i L_i &= \alpha p_i Q_i \\
P_i M_i &= (1-\alpha) p_i Q_i
\end{aligned}
$$

3.

$$
\begin{aligned}
P_i (C_i + M_i) &= w_i L_i + P_i M_i \\
* &= p_i Q_i \\
w_i L_i &= \alpha p_i Q_i \\
\implies p_i Q_i &= \frac{1}{\alpha} w_i L_i \\
P_i (C_i + M_i) &= \frac{1}{\alpha} w_i L_i
\end{aligned}
$$

4. (a)

$$max_{q_{ij}} \quad \left(\sum_{i\in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\text{s.t.} \quad \sum_i p_{ij}q_{ij} \leq p_j Q_j$$

$$\frac{\sigma}{\sigma-1}\left(\sum_{i\in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{-1}{\sigma}} = \lambda p_{ij} \tag{FOC}$$

$$\begin{aligned}
Y_j &= \frac{X_{ij}}{w_j L_j} \\
&= \frac{a_{ij} Y_j p_i^{\sigma-1} p_{ij}^{1-\sigma}}{w_j L_j} \\
&= \frac{a_{ij}\frac{1}{\alpha} w_j L_j p_j^{\sigma-1}(\tau_{ij}p_{ij})^{1-\sigma}}{w_j L_j} \\
&= a_{ij}\tau_{ij}^{1-\sigma} p_i^{1-\sigma}\frac{1}{\alpha}p_j^{\sigma-1} \\
&= a_{ij}\tau_{ij}^{1-\sigma}\left(k\frac{w_i^{\alpha}p_i^{1-\alpha}}{A_i}\right)^{1-\sigma}\frac{1}{\alpha}p_j^{\sigma-1}
\end{aligned}$$

(b)

$$\begin{aligned}
\lambda_{ii} &= a_{ii}\tau_{ii}^{1-\sigma}\left(k\frac{w_i^{\alpha}p_i^{1-\alpha}}{A_i}\right)^{1-\sigma}\frac{1}{\alpha}p_i^{\sigma-1} \\
&= a_{ii}\left(\frac{kw_i^{\alpha}}{A_i}\right)^{1-\sigma}\frac{1}{\alpha}p_i^{(1-\sigma)+(\sigma-1)-\alpha(1-\sigma)} \\
&= \frac{a_i i}{\alpha}\left(\frac{k}{A_i}\right)^{1-\sigma}\left(\frac{w_i}{P_i}\right)^{\alpha(1-\sigma)}
\end{aligned}$$

5.

$$\begin{aligned}
W_i &= \frac{U_i}{L_i} \\
&= \frac{w_i L_i}{P_i L_i} \\
&= \left(\lambda_{ii}\frac{\alpha}{\alpha_{ii}}\right)^{\frac{1}{\alpha(1-\sigma)}}\left(\frac{A_i}{k}\right)^{\frac{1}{\alpha}} \\
\implies \frac{W_i}{W_i^A} &= \frac{\left(\lambda_i i\frac{\alpha}{a_i i}\right)^{\frac{1}{\alpha(1-\sigma)}}\left(\frac{A_i}{k}\right)^{\frac{1}{\alpha}}}{\left(1\cdot\frac{\alpha}{a_i i}\right)^{\frac{1}{\alpha(1-\sigma)}}\left(\frac{A_i}{k}\right)^{\frac{1}{\alpha}}} \\
&= \lambda_i i^{\frac{1}{\alpha(1-\sigma)}}
\end{aligned}$$

As α increases, $\frac{1}{\alpha(1-\sigma)}$ decreases, so $\frac{W_i}{W_i^A}$ decreases and the gains from trade decrease. When α is closer to 1 intermediate goods are less helpful for production, so there is a smaller loss from transitioning to autarky because the lost intermediate goods have a smaller impact on welfare.

3 Tariffs

1. From the logic in the notes, we can start by setting the price in country i equal to marginal cost,
$$p_i = \frac{w_i}{A_i}.$$
When a person in country j wants to buy this good, they have to pay the price for the good, plus the per-unit tariff $\tau_{ij} - 1$, so the price of good i in country j is
$$p_{ij} = \tau_{ij}\frac{w_i}{A_i}.$$

2.
$$\begin{aligned}
p_{ii} &= \frac{p_{ij}}{\tau_{ij}} \\
p_{ij} &= \tau_{ij}\frac{w_i}{A_i} \\
\sum_i p_{ij}q_{ij} &= w_jL_j + R_j \\
&= w_jL_j + \sum_i(\tau_{ij}-1)p_{ii}q_{ij} \\
&= w_jL_j + \sum_i(\tau_{ij}-1)\frac{p_{ij}}{\tau_{ij}}q_{ij} \\
\sum_i p_{ij}q_{ij} &= w_jL_j + \sum_i p_{ij}q_{ij} - \sum_i \frac{p_{ij}q_{ij}}{\tau_{ij}} \\
\sum_i \frac{p_{ij}q_{ij}}{\tau_{ij}} &= w_jL_j
\end{aligned}$$

3. Using our expressions for $P_j = \left(\sum_j \left(\frac{\tau_{ij}w_j}{A_j}\right)^{1-\sigma}\right)$ and $p_{ij} = \frac{w_i\tau_{ij}}{A_i}$, we can rewrite
$$X_{ij} = p_{ij}q_{ij} = a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma} Y_jP_j^{\sigma-1} = \frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}Y_j$$
Therefore, the bilateral expenditure share is given by the expression
$$\lambda_{ij} = \frac{X_{ij}}{Y_j} = \frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}$$

4. Using the labor market clearing condition, $Y_i = w_iL_i + R_i$, we can rewrite
$$X_{ij} = \frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}Y_j = \frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}(w_jL_j + R_j)$$
Then, using the goods market clearing condition, $Y_i = \sum_{j\in S} X_{ij}$, we let
$$Y_i = \sum_{j\in S} X_{ij} = \sum_{j\in S}\frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}(w_jL_j + R_j)$$
$$\Rightarrow w_iL_i + R_i = \sum_{j\in S}\frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}(w_jL_j + R_j)$$

Using our expression for $\lambda_{ij} = \frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}}$, we see that the labor market clearing condition can be written as

$$w_iL_i + R_i = \sum_{j\in S} \lambda_{ij}(wjL_j + R_j)$$

5. Define $r_i = \frac{R_i}{E_i}$ where E_i is total expenditure. The total expenditure of country i is given by $E_i = Y_i$, where $R_i = r_iY_i$. So we see that $Y_i = w_iL_i + R_i = \frac{w_iL_i}{1-r_i}$. Notice that

$$\begin{aligned} r_i = \frac{R_i}{E_i} &= \frac{R_i}{Y_i} \\ \Rightarrow \frac{w_iL_i}{Y_i} &= 1 - r_i \\ \Rightarrow \frac{R_i}{w_iL_i} &= \frac{r_i}{1-r_i} \end{aligned}$$

Thus we can derive an expression for welfare,

$$W_i = \frac{Y_i}{L_iP_i} = \frac{w_iL_i}{(1-r_i)L_iP_i} = \frac{w_i}{(1-r_i)P_i}$$

Additionally, given our expression for λ and the assumption that $\tau_{ii} = 1$, we can rewrite

$$\begin{aligned} \lambda_{ii} &= a_{ii}\tau_{ii}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma} P_j^{\sigma-1} \\ \Rightarrow \frac{w_i}{P_i} &= \left(\frac{\lambda_{ii}A_i^{1-\sigma}}{a_{ii}}\right)^{\frac{1}{1-\sigma}} \end{aligned}$$

Therefore, the expression for welfare can be written as

$$W_i = \frac{1}{1-r_i}\left(\frac{w_i}{P_i}\right) = \frac{\lambda_{ii}^{\frac{1}{1-\sigma}} a_{ii}^{\frac{1}{\sigma-1}} A_i}{1-r_i}$$

Under autarky, $\lambda_{ii} = 1$, so assuming constant values of a_{ii} and A_i, the gains from trade are

$$\frac{W_i^A}{W_i} = \frac{a_{ii}^{\frac{1}{\sigma-1}} A_i}{\lambda_{ii}^{\frac{1}{1-\sigma}} a_{ii}^{\frac{1}{\sigma-1}} A_i(1-r_i)^{-1}} = (1-r_i)\lambda_{ii}^{\frac{1}{\sigma-1}}$$

The expression for welfare under iceberg costs is $W_i^I = \lambda_{ii}^{\frac{1}{1-\sigma}} a_{ii}^{\frac{1}{\sigma-1}} A_i$, so assuming constant values of a_{ii} and A_i, the gains from trade are

$$\frac{W_i^A}{W_i^I} = \lambda_{ii}^{\frac{1}{\sigma-1}}$$