Chapter 2

Asset Pricing Theory

Building on the results of the previous chapter, we will now cover basic results in asset pricing theory. Section 2.1 derives asset pricing formulas for assets with arbitrary payoff structure. Section 2.2 studies conditions for market to be complete. Section 2.3 derives a fundamental result in corporate finance: the Modigliani Miller Theorem.

2.1 Equilibrium asset pricing

2.1.1 Fundamental values and (no) bubbles

Consider the following extension of the model we studied in the previous chapter. In addition to trading sequentially a complete set of Arrow securities, agents can also trade N long lived assets indexed by $n \in \{1, 2, \cdots, N\}$ and $N < \infty$. Without loss of generality, we normalize the number of shares of each long lived asset to one. At time zero, these shares are owned by agents – according to some distribution $k_{i,-1}^{(n)}$ such that $\sum_{i=1}^{I} k_{i,-1}^{(n)} = 1$. At time t after history s^t, asset n pays off aggregate dividend $d_t^{(n)}(s^t) \geq 0$ and is traded at the "ex-dividend" price $p_t^{(n)}(s^t)$. By "ex-dividend price", we mean the price after dividend have been paid in the current period. The stream of dividend is assumed to belong to ℓ_∞, i.e., it is bounded.

Question: Why does the normalization entail no loss of generality?

A recursive asset pricing formula. Agents solve the same problem as in the sequential trading economy by choosing, in addition to consumption $c_{it}(s^t)$ and Arrow-Debreu securities $a_{it+1}(s^t, s_{t+1})$, holdings of long-lived assets $k_{it}^{(n)}(s^t)$, subject to the following sequential budget constraint:

$$\begin{aligned} & c_{it}(s^t) + \sum_{s_{t+1}} Q_{t+1}(s_{t+1}|s^t) a_{it+1}(s^t, s_{t+1}) + \sum_n p_t^{(n)}(s^t) k_{it}^{(n)}(s^t) \\ \leq & y_{it}(s^t) + a_{it}(s^t) + \sum_n \left[p_t^{(n)}(s^t) + d_t^{(n)}(s^t) \right] k_{it-1}^{(n)}(s^{t-1}), \end{aligned}$$

the No Ponzi Game constraint and transversality conditions,

$$\lim_{T\to\infty} \sum_{s^T \succeq s^t} q_{0T}(s^T) \left\{ a_{iT}(s^T) + \sum_n \left[p_T^{(n)}(s^T) + d_T^{(n)}(s^T) \right] k_{iT-1}^{(n)}(s^{T-1}) \right\} = 0, \tag{2.1.1}$$

for all times $t \geq 0$ and histories $s^t \in S^t$, and given some initial asset holdings $a_{i0}(s_0) = 0$ and $k_{i,-1}^{(n)}$.

Let $\lambda_{it}(s^t)$ denote the multiplier on the sequential budget constraint at time t after history s^t. This multiplier is to be interpreted as the marginal value of wealth: more precisely, it represents the marginal value of relaxing the sequential budget constraint at time t and history s^t, by a marginal increase in financial wealth. Setting up the Lagragian of the agent's problem we obtain three first-order conditions. The first-order condition with respect to $c_{it}(s^t)$ is:

$$\beta^t \pi_{0t}(s^t) u'[c_{it}(s^t)] = \lambda_{it}(s^t).$$

It states that the marginal utility at time t after history s^t is equal to the marginal value of wealth. The first-order condition with respect to $a_{it+1}(s^t, s_{t+1})$ is:

$$\lambda_{it}(s^t) Q_{t+1}(s_{t+1}|s^t) = \lambda_{it+1}(s^{t+1}).$$

The left-hand side is the marginal cost of investing, at time t and after history s^t, in a marginal unit of Arrow security paying off if the state next period is s_{t+1}. The right-hand side is the marginal benefit. It is equal to the Lagrange multiplier $\lambda_{it+1}(s^{t+1})$, since that marginal unit of Arrow security create a marginal increase in wealth if next-period state is s_{t+1}. Finally, the first-order condition with respect to $k_t^{(n)}(s^t)$ is

$$\lambda_{it}(s^t) p_t^{(n)}(s^t) = \sum_{s_{t+1}} \lambda_{it+1}(s^t, s_{t+1}) \left[p_{t+1}^{(n)}(s^{t+1}) + d_{t+1}^{(n)}(s^{t+1}) \right].$$

Just as in the first-order condition with respect to $a_{it+1}(s^t, s_{t+1})$, the left-hand side is the marginal cost of investing in the asset. The right-hand side is the marginal benefit, which is also very similar to the marginal benefit in the first-order condition with respect to $a_{it+1}(s^t, s_{t+1})$. There are two key differences. First, the first-order condition involves a sum of marginal benefits across the multiple states $s_{t+1} \in S$ in which asset n has positive cash flows. Second, the marginal benefit in a given state s_{t+1} has two components: the dividend $d_{t+1}^{(n)}$, and the re-sale value $p_{t+1}^{(n)}$. Combining the last two first-order conditions, we obtain:

Proposition 2.1.1. *The price of asset n must satisfy the recursive asset pricing formula:*

$$p_t^{(n)}(s^t) = \sum_{s_{t+1}} Q_{t+1}(s_{t+1}|s^t) \left[p_{t+1}^{(n)}(s^{t+1}) + d_{t+1}^{(n)}(s^{t+1}) \right]. \tag{2.1.2}$$

A derivation via no-arbitrage. In the previous paragraph we derived Proposition 2.1.1 by taking first-order conditions. It is important to note that it can also be derived via a no-arbitrage reasoning. Suppose indeed, by way of contradiction, that:

$$p_t^{(n)}(s^t) < \sum_{s_{t+1}} Q_{t+1}(s_{t+1}|s^t) \left[p_{t+1}^{(n)}(s^{t+1}) + d_{t+1}^{(n)}(s^{t+1}) \right].$$

This intuitively means that the price of asset n is too low and suggests that agent i could make profit by "buying-low and selling high". More formally, agent i can buy σ shares of asset n and sell $\sigma \left[p_{t+1}^{(n)}(s^{t+1}) + d_{t+1}^{(n)}(s^{t+1})\right]$ shares of Arrow-Debreu securities paying off 1 in state s_{t+1}, for each $s_{t+1} \in S$. This would increase her consumption by

$$\sigma \left\{ \sum_{s_{t+1}} Q_{t+1}(s_{t+1}|s^t) \left[p_{t+1}^{(n)}(s^{t+1}) + d_{t+1}^{(n)}(s^{t+1})\right] - p_t^{(n)}(s^t) \right\}$$

at (t, s^t) while keeping subsequent consumption unchanged. Since this is true for any $\sigma > 0$, this implies that the agent will have an infinite demand for asset n at (t, s^t), contradicting the market clearing condition. It cannot be part of an equilibrium. The same reasoning (with opposite trades) goes through with reverse inequality.

Fundamental Value and no bubble. Iterating equation (2.1.2) forward from time t to T, we obtain:

$$p_t^{(n)}(s^t) = \sum_{k=1}^{T-t} \sum_{s^{t+k} \succeq s^t} \frac{q_{0t+k}(s^{t+k})}{q_{0t}(s^t)} d_{t+k}^{(n)}(s^{t+k}) + \sum_{s^T \succeq s^t} \frac{q_{0T}(s^T)}{q_{0t}(s^t)} p_T^{(n)}(s^T), \tag{2.1.3}$$

where we used that, for $t \geq 1$:

$$\frac{q_{0t+k}(s^{t+k})}{q_{0t}(s^t)} = Q_{t+k}(s_{t+k} \mid s^{t+k-1}) \times Q_{t+k-1}(s_{t+k-1} \mid, s^{t+k-2}) \times \ldots \times Q_{t+1}(s_{t+1} \mid s^t).$$

Given that the stream of dividend is bounded, the first term on the right-side of (2.1.3) has a limit as $T \to \infty$. Since the left-side is fixed, this means that the second term on the right-side also has a limit. This allows us to take limits in both terms and obtain:

$$p_t^{(n)}(s^t) = \sum_{k=1}^{\infty} \sum_{s^{t+k} \succeq s^t} \frac{q_{0t+k}(s^{t+k})}{q_{0t}(s^t)} d_{t+k}^{(n)}(s^{t+k}) + \lim_{T \to \infty} \sum_{s^T \succeq s^t} \frac{q_{0T}(s^T)}{q_{0t}(s^t)} p_T^{(n)}(s^T).$$

The first part of the asset formula is the present discounted value of the dividend. Because it only depends on the future dividends of the asset, it is called the "fundamental value" of the asset. The second part of the equation above is called the "bubble" component of asset n's price, because it is independent from the cash flow of the asset. An important question in asset pricing is to determine conditions under which bubbles can exist. In the present model we have that

Proposition 2.1.2. *In the model of Chapter 2, for each asset $n \in \{1, \ldots, N\}$, the bubble component is zero:*

$$\lim_{T \to \infty} \sum_{s^T \succeq s^t} \frac{q_{0T}(s^T)}{q_{0t}(s^t)} p_T^{(n)}(s^T) = 0.$$

Proof. The formal mathematical argument goes as follows. By summing the No Ponzi Game condition (equation (2.1.1)) across all agents, since Arrow-Debreu securities are in zero net supply and the trees are in supply of one, i.e. $\sum_i a_{iT}(s^T) = 0$ and $\sum_i k^{(n)}_{iT-1}(s^{T-1}) = 1$, we have

$$\lim_{T\to\infty} \sum_{s^T \succeq s^t} \frac{q_{0T}(s^T)}{q_{0t}(s^t)} \sum_n p^{(n)}_T(s^T) = 0.$$

By exchanging the limit and second summation term across the trees, we see that the sum of all asset bubbles must be zero. Since prices are non-negative, bubbles are non-negative as well. Hence, if the sum of all asset bubbles is zero, then it must be true that each asset bubble is zero. □

The economic intuition is that agents do not want to over accumulate wealth as $T \to \infty$. If there is a bubble, they would find it optimal to consume it by selling their asset holdings. Intuitively, this cannot be an equilibrium because assets must be held.

This is not to say that bubbles are ruled out in general. Several assumptions made in the present model play an important role in generating Proposition 2.1.2.

- Infinitely lived agents. Optimality with finite-lived agents only impose the formula of Proposition 2.1.1 – in particular, the transversality condition does not necessarily holds, and the argument in the proof of Proposition 2.1.2 may fail. In fact, there is a long tradition of studying bubbles in infinite horizon model with overlapping generation of finitely lived agents. This literature follows Samuelson (1958), who showed that, under some condition, fiat money may be valued in equilibrium. According to the above definition, money is a "pure bubble": since it pays no dividend, its fundamental value is zero.

- Asset in positive supply. The formal proof uses the argument that assets are in positive supply to generate an "aggregate" No Ponzi Game constraint. Kocherlakota (1992) shows that bubbles can arise for long-lived assets in zero net supply.

- No asset holding constraint. The intuitive explanation of the argument suggests that the absence of asset holding constraints is important: agents must be able to "sell" the bubble. Indeed, Kocherlakota (2008) shows that, if agents were in some way forced to hold on to their asset, then bubbles can arise.

2.1.2 Stochastic discount factor and risk neutral probabilities

The fundamental value equation (2.1.2) can be re-written as

$$p^{(n)}_t(s^t) = \mathbb{E}\left\{ M_{t+1}(s_{t+1} \mid s^t) \left[p^{(n)}_{t+1}(s^{t+1}) + d^{(n)}_{t+1}(s^{t+1}) \right] \,\middle|\, s^t \right\},$$

where

$$M_{t+1}(s_{t+1} \mid s^t) \equiv \frac{Q_{t+1}(s_{t+1} \mid s^t)}{\pi_{t+1}(s_{t+1} \mid s^t)}$$

is what is called the *stochastic discount factor*. From the first-order condition of agent i, it follows that the stochastic discount factor is equal to the marginal rate of substitution between goods at time

$t+1$ and time t, for all agents $i \in \{1, \ldots, I\}$:

$$M_{t+1}(s_{t+1} \mid s^t) = \beta \frac{u'\left[c_{it+1}(s^{t+1})\right]}{u'\left[c_{it}(s^t)\right]}.$$

A special case that has received a lot of attention in the literature is when all agents have CRRA utility with identical risk aversion. As we have shown in Example 1.3.2, in that case agent i consumption share is constant over time: that is, his consumption can be written as $\alpha_i y_t(s^t)$, for some fixed $\alpha_i \in [0,1]$. Together with the fact that $u'(c) = c^{-\gamma}$, this implies that the stochastic discount factor is:

$$M_{t+1}(s_{t+1}) \mid s^t) = \beta \left(\frac{y_{t+1}(s^{t+1})}{y_t(s^t)} \right)^{-\gamma}.$$

It is a function of the discount factor β and the growth rate of aggregate consumption, $y_{t+1}(s^{t+1})/y_t(s^t)$. According to the formula, asset prices tend to be larger if:

- The discount factor β is larger. This is because, in that case agents are more patient.
- The growth rate of consumption is lower. This is because, when the growth rate of aggregate consumption is low, the representative agent has a strong saving demand, which drives up the price of all saving instruments.
- The growth rate of consumption is negatively correlated with the payoff of the asset. This is because, in that case, the asset is a good hedging instrument and is thus more valuable.

Or alternatively, we can rewrite it in the risk-neutral expectation version:

$$p_t^{(n)}(s^t) = \frac{1}{1 + r_t(s^t)} \hat{\mathbb{E}} \left\{ \left[p_{t+1}^{(n)}(s^{t+1}) + d_{t+1}^{(n)}(s^{t+1}) \right] \,\middle|\, s^t \right\}$$

where the (hatted) $\hat{\mathbb{E}}\left[\cdot|s^t\right]$ represents expectation with respect to the "risk-neutral measure", defined as follows:

$$\hat{\pi}_{t+1}\left(s_{t+1}|s^t\right) = \frac{Q_{t+1}(s_{t+1} \mid s^t)}{\sum_{s_{t+1}} Q_{t+1}(s_{t+1} \mid s^t)},$$

and $r_t(s^t)$ is the risk free rate between period t and $t+1$ such that

$$\frac{1}{1 + r_t(s^t)} = \sum_{s_{t+1}} Q_{t+1}(s_{t+1}|s^t).$$

It is important to keep in mind that risk neutral probabilities are not "objective" probabilities. It is a common conceptual mistake and a common temptation to interpret risk-neutral probabilities as market beliefs about future outcome. But risk-neutral probabilities are prices, so they are more than just market beliefs: they also encode information about the marginal utility of households in various states of the world.

So why do we care so much about these "probabilities"? It turns out that, analytically, it is a very important observation. It allows researchers to use tools from probability theory and stochastic processes to solve complex valuation problems - this is especially true in continuous time models. Researchers would take as given that market are complete, that an equilibrium has been established, and say that this implies the existence of prices. They can make analytically convenient assumptions about prices and then reduce the problem of pricing various assets as a problem of calculating (often quite complex) conditional expectations.

2.2 Endogenously complete markets

In the real world, agents do not trade Arrow securities but long-lived assets such as stocks, bonds, housing, etc... In this section we shut down the market for Arrow securities, and allow agents to trade long-lived assets. We ask: under which condition is the market *endogenously complete*, in the sense that the market outcome is the same as with a complete set of Arrow securities? By "same market outcome", we mean: same consumption allocation, and same price for long-lived assets. The portfolio choices of agents will be different in general: as will be clear shortly, the reason market outcomes is the same is because agents can perfectly substitute their Arrow security positions by positions in long-lived assets.

The broad answer to this question is this: the market is endogenously complete when all agents' equilibrium consumption plan can be achieved by trading only long-lived asset, without trading any Arrow securities. Indeed, removing Arrow securities only makes the budget set smaller. Hence, the optimum remains the same as long as it remains feasible within this smaller budget set.

The specific answer to this question is harder: namely, it is not easy to give fully general joint conditions on preferences and on the asset structure for this to be true. But there are some important and natural cases.

Here we study the following sufficient condition for endogenously complete market: we ask whether it is possible to replicate the payoff of Arrow securities by creating portfolios of long-lived securities. Clearly, this implies that the budget set remains the same after the market for Arrow securities has been removed. If the budget set remains the same, it is clear that the optimum remains the same as well. To study this replication question formally, we first define the payoff matrix of long lived assets.

Definition 2.2.1. The one-period ahead payoff matrix of long-lived assets at time t after history s^t is:

$$\Pi_t(s^t) = \left[p^{(n)}_{t+1}(s^t, s_{t+1}) + d^{(n)}_{t+1}(s^t, s_{t+1})\right]_{s_{t+1}\in S(row),\ n\in N(column).} \tag{2.2.1}$$

Suppose that, at time t and after history s^t, we seek to replicate the payoff of some Arrow security with a portfolio of long lived assets. That is, we seek portfolio weights $\{w_t^{(n)}(s^t, s)\}_{n\in N}$ such that

$$\sum_n w_t^{(n)}(s^t, s)\left[p^{(n)}_{t+1}(s^t, s_{t+1}) + d^{(n)}_{t+1}(s^t, s_{t+1})\right] = \begin{cases} 1, & \text{if } s_{t+1} = s \\ 0, & \text{if } s_{t+1} \neq s. \end{cases}$$

In matrix form, this can be written:

$$\Pi_t(s^t)w_t(s^t, s) = \begin{pmatrix} \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$$

where the "1" is placed in row s, and all other rows are zero. Thus, the problem of finding the portfolio is the same as inverting the matrix $\Pi_t(s^t)$. Put differently, the matrix of portfolio weights required to replicate Arrow securities with long lived assets is the inverse of the payoff matrix. This observation implies the following proposition.

Proposition 2.2.1. *A sufficient condition for the market to be endogenously complete is that Rank* $[\Pi_t(s^t)] = \#S$ *at all times* $t \geq 0$ *and after all histories* $s^t \in S^t$*.*

Proof. Consider an equilibrium with both long-lived assets and Arrow securities. Shut down the market for Arrow securities and keep the prices of long-lived asset the same. Then, the following must be true, with or without the rank condition stated in the proposition. First, the set of budget feasible consumption plans with long lived assets only is included in the set of budget feasible consumption plans with both Arrow securities and long lived assets. Second, the maximum utility obtained with long lived assets is no greater than the maximum utility obtained with both Arrow-Debreu securities and long lived assets.

If the rank condition stated in the proposition holds, then we obtain another result: the optimum with both Arrow securities and long lived assets becomes attainable with long lived assets only. Indeed, every agent can substitute its demand for Arrow securities by a demand for the corresponding replicating portfolio, i.e., the portfolio of long-lived assets that replicates its payoff. This implies that the optimal consumption plan remains the same after the market for Arrow security has been shut down.

The only thing we need to verify is that the markets clear. This is clearly true for the market for consumption good. What about the market for long-lived securities? Here we note that, since the markets for Arrow security clear, the demand for "replicating portfolio" net out to zero, and so that the market for long lived asset clear (it is useful to go through some algebra to convince you that this is indeed true). □

2.3 The Modigliani Miller Proposition

The proposition of Modigliani and Miller (1958) states conditions such that the decision of whether to undertake an investment project is only a matter of calculating net present value: if the present value of the cash flow is positive, then the project should be undertaken. This means that financing choices are irrelevant: the choice of internal financing vs. debt or vs equity has no impact on the value of the firm. Miller (1991) explains this as follows:

> "Think of the firm as a gigantic tub of whole milk. The farmer can sell the whole milk as it is. Or he can separate out the cream, and sell it at a considerably higher price than the whole milk would bring. The

> Modigliani-Miller proposition says that if there were no costs of separation, (and, of course, no government dairy support program), the cream plus the skim milk would bring the same price as the whole milk."

The broad conditions for the Modigliani Miller proposition to apply are: efficient markets, symmetric information, no taxes, no bankprutcy cost (see Villamil, 2008). In what follows I will derive a version of the proposition in the complete market infinite-horizon framework we have been working with so far.

Imagine one single agent in the economy with some initial wealth W_0 and some income stream $\{y_t(s^t) : t \geq 0, s^t \in S^t\}$. At time $t = 0$, the agent must decide whether or not to undertake a project. The cost of the project is K_0, and its cash flow stream is $\{Y_t(s^t) : t \geq 0, s^t \in S^t\}$.

If she undertakes the project, the agent has several financing choices to make. It can use its own wealth (what is often referred to as "internal finance"), raise equity, or raise debt (equity and debt would be referred to as "external finance"), subject to the constraint:

$$K_0 + a_0 = W_0 + E_0 + D_0. \tag{2.3.1}$$

The left-side of the budget constraint is the investment cost, K_0, and the wealth that is left after investing, a_0. The right-side is the initial wealth, W_0, the time zero price of all equity issued, E_0, and the time zero price of all debt issued, D_0. Both E_0 and D_0 are determined in financial market, as the present value of their corresponding income stream evaluated at time-zero prices $q_{0t}(s^t)$.

When it raises equity, the agent can choose the dividend stream $\{d_t(s^t) : t \geq 0, s^t \in S^t\}$. When it raises debt, I will assume for simplicity that the agent promises a fixed coupon payment b forever. Together with the dividend stream, the size of the coupon payment determines the amount of leverage of the agent. I will assume as well that the agent may default on the debt, in which case the cash flows of the project are transferred to debt holders. I let $\alpha_t(s^t) = 1$ if the agent has not defaulted by time t after history s^t, and $\alpha_t(s^t) = 0$ otherwise. Taken together, we see that the agent has many financing decisions to make. In addition to choosing the amount of debt vs. equity issued, the agent chooses its dividend policy, the coupon payment on its debt, and its optimal default strategy. For example, the agent may want to choose more debt than equity because it is cheaper. Or, it may want to "hedge" so that the stream of dividend is less risky. The Modigliani-Miller proposition below shows that these financing choices are, in fact, all irrelevant.

To see this, I first state the sequential budget constraint of the agent:

$$c_t(s^t) + \sum_{s_{t+1}} Q_{t+1}(s_{t+1} \,|\, s^t) a_{t+1}(s^t, s_{t+1}) \leq a_t(s^t) + y_t(s^t) + \alpha_t(s^t) \left[Y_t(s^t) - d_t(s^t) - b \right]. \tag{2.3.2}$$

On the right-side of the budget constraint the last term is the net payoff associated with the project. If $\alpha_t(s^t) = 1$, the agent has not defaulted. He receives the cash flow $Y_t(s^t)$, and makes the dividend payment $d_t(s^t)$ to its equity holders, and the coupon payment b to its debt holders.

We now appeal to the logic of Proposition 1.5.1: the problem maximizing utility subject to the sequential budget constraints (2.3.1) and (2.3.2) is equivalent to the problem of maximizing utility subject to a single inter-temporal budget constraint. To derive this constraint, we first multiply (2.3.2) by the time-zero price $q_{0t}(s^t)$, we sum across all times and histories, and we use the No Ponzi Game constraint. This gives:

$$\sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)c_t(s^t) = a_0 + \sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)\left\{y_t(s^t) + \alpha_t(s^t)\left[Y_t(s^t) - d_t(s^t) - b\right]\right\}. \tag{2.3.3}$$

Notice that:

$$\alpha_t(s^t)\left[Y_t(s^t) - d_t(s^t) - b\right] = \underbrace{Y_t(s^t)}_{\text{payoff of project}} - \underbrace{\alpha_t(s^t)d_t(s^t)}_{\text{payoff of equity}} - \underbrace{\left\{\alpha_t(s^t)b + \left[1-\alpha_t(s^t)\right]Y_t(s^t)\right\}}_{\text{payoff of debt}}$$

The first term is the pay off of the project. The second term is the payoff of equity. The third term is the payoff of debt: the coupon payment b before default, and the cash flow of the project $Y_t(s^t)$ after default, since the project has been transferred to the debt holders. Now the market price of equity and debt must be equal to the present value of their respective payoffs:

$$E_0 = \sum_{t\geq 0}\sum_{s^t\in S^t} \alpha_t(s^t)q_{0t}(s^t)d_t(s^t)$$
$$D_0 = \sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)\left\{\alpha_t(s^t)b + \left[1-\alpha_t(s^t)\right]Y_t(s^t)\right\}.$$

Substituting back into (2.3.3) and using from (2.3.1) that $a_0 = W_0 + E_0 + D_0 - K_0$, we obtain:

$$\sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)c_t(s^t) = \sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)y_t(s^t) + \underbrace{\sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)Y_t(s^t) - K_0}_{\text{net present value}} + W_0. \tag{2.3.4}$$

Notice that the right-side of the inter-temporal budget constraint only depends on the net present value of the project. It does not depend on the dividend stream, on the size of the coupon payment, nor on the default policy. This implies

Proposition 2.3.1. *The optimal decision of the agent is to undertake the project if and only if its net present value is positive:* $\sum_{t\geq 0}\sum_{s^t\in S^t} q_{0t}(s^t)Y_t(s^t) - K_0 \geq 0$. *Conditional on undertaking the project, financing choices have no impact on the value of the project.*