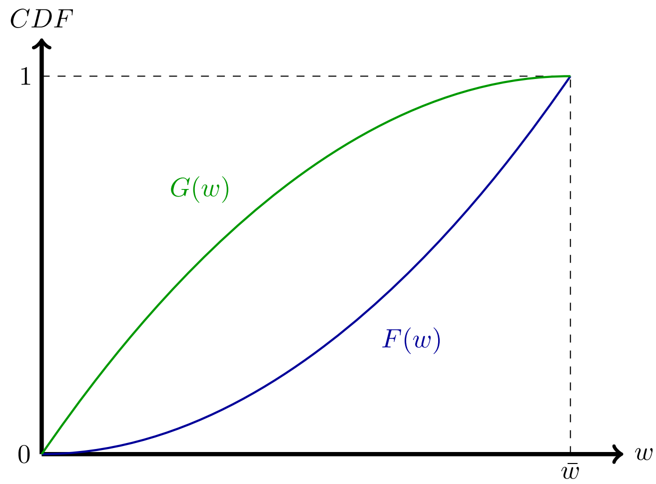

Figure 13.1: First order stochastic dominance

B Stochastic dominance

In this section we provide the formal definition of first-order and second-order stochastic dominance. We closely follow the presentation of Mas-Colell et al. (1995), Chapter 6.D.

B.1 First-order stochastic dominance.

The notion of first-order stochastic dominance is used to rank distribution in terms of their levels: a wage distribution first-order stochastically dominates another if it puts more probability mass on higher wages.

Definition B.1. Consider two cumulative distribution function (CDF), $F(\cdot)$ and $G(\cdot)$, over the support $[0, \bar{w}]$. The distribution $F(\cdot)$ is said to first-order stochastically dominates the distribution $G(\cdot)$ if, for all non-decreasing function $u : [0, \bar{w}] \to \mathbb{R}$,, we have

$$\int u(w)dF(w) \geq \int u(w)dG(w).$$

We have the following equivalent characterization:

Proposition B.1. *The distribution $F(\cdot)$ first-order stochastically dominates the distribution $G(\cdot)$ if and only if $F(w) \leq G(w)$ for all w.*

Figure 13.1 illustrates: on the picture, the CDF $F(\cdot)$ first-order stochastically dominates $G(\cdot)$. The CDF $F(\cdot)$ lies below $G(\cdot)$ because it puts more probability mass on higher wages and less probability mass on lower wages. Of course, at the upper bound of the support, we must have $F(w) = G(w) = 1$.

B.2 Second-order stochastic dominance

Next, we turn to the notion of second-order stochastic dominance which is used to rank distributions in terms of their riskiness.

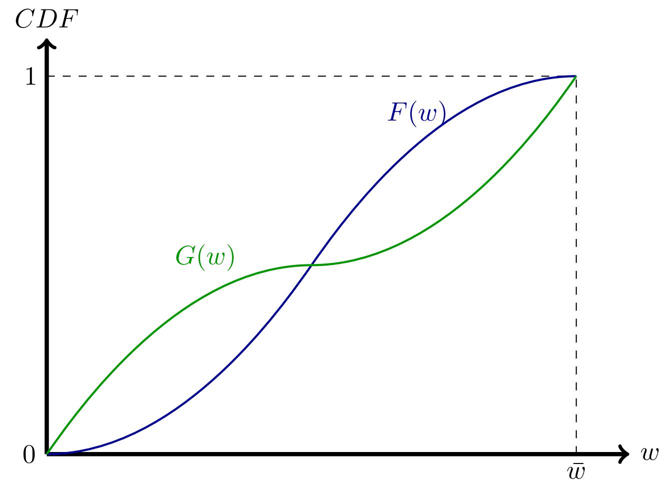

Figure 13.2: Second order stochastic dominance

Definition B.2. Consider two cumulative distribution function (CDF), $F(\cdot)$ and $G(\cdot)$, over the support $[0, \bar{w}]$, with identical mean. Then, the distribution $F(\cdot)$ is said to second-order stochastically dominates (or is less risky than) $G(\cdot)$ if, for all non-decreasing concave function $u : [0, \bar{w}] \to \mathbb{R}$, we have

$$\int u(w)dF(w) \geq \int u(w)dG(w).$$

In other word, $F(\cdot)$ dominates $G(\cdot)$if any risk-averse agent would prefer drawing wages from $F(\cdot)$ rather than from $G(\cdot)$. Figure 13.2 is an example of second-order stochastical dominance. If $F(\cdot)$ second-order stochastically dominates $G(\cdot)$, we have

$$\int_0^{\bar{w}} F(w)dw = \int_0^{\bar{w}} G(w)dw$$
$$\int_0^{w} G(x)dx \geq \int_0^{w} F(x)dx \text{ for all } w.$$