Problem Set 1. Econ 202B, 2021

1 Frictional wage dispersion

This problem is based on an important AER paper due to Hostein, Krusell, and Violante. Consider the McCall model we studied in class. There is a single risk neutral and infinitely-lived worker, who discounts the future at rate $\beta \in (0,1)$. The worker can be either employed, ("E") or unemployed ("U"). When employed the worker receives some wage, w, and may be fired with probability $\delta \in (0,1)$. When unemployed, the worker draws a wage offer with probability λ_U from a distribution with CDF $F(w)$. We have shown in class that the optimal policy of an unemployed worker is to accept all wage offers above the "reservation wage", $w^\star$, solving:

$$w^\star = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^\star}^{\bar{w}} [1 - F(w)]\, dw.$$

1. Let the employment state of the worker at time t is $s_t \in \{E, U\}$.

   (a) Argue that $\{s_t, t \geq 0\}$ is a Markov chain.

   (b) What is the transitionprobability matrix of this Markov chain?

   (c) What the stationary distribution of this Markov chain?

   (d) What is the probability that an unemployed workers finds and accept a job after $t = 1$ period of unemployment (the "job in job-finding probability")?

   (e) What is the probability that a worker finds and accepts a job after $t > 1$ period of unemployment?

   (f) What is the average duration of an unemployment spell?

2. Consider the reservation wage equation when $\beta = 1$ – i.e., when the worker is infinitely patient. Show that it can be simplified to:

$$w^\star = \frac{\delta}{\delta + \lambda[1 - F(w^\star)]} b + \frac{\lambda[1 - F(w^\star)]}{\delta + \lambda[1 - F(w^\star)]} \mathbb{E}[w \mid w \geq w^\star].$$

   Interpret this formula using your answer to question 1. Explain why the reservation wage is less than $\bar{w}$ even if the worker is infinitely patient. What happens to the reservation wage when $\delta \to 0$? Explain.

3. Assume that the unemployment benefit, b, replaces a fraction $\rho \in (0,1)$ of the average wage of an employed worker. That is $b = \rho \mathbb{E}[w \mid w \geq w^\star]$. Define the mean-min ratio to be

$$Mm \equiv \frac{\mathbb{E}[w \mid w \geq w^\star]}{w^\star},$$

the ratio of the observed average wage to the observed minimum wage. Use the reservation wage equation to show that the job-finding probability satisfies:

$$\lambda [1 - F(w^\star)] = \frac{1-\beta(1-\delta)}{\beta} \frac{1-\rho Mm}{Mm - 1}.$$

Researchers have argued that the observed amount of wage dispersion amongst observationally equivalent workers is very large, leading to empirical Mm ratios of about 1.5. Assume that firing probability is about $\delta = 3\%$ per month, the discount factor is $\beta = (0.95)^{1/12}$, and the replacement rate is $\rho = 0.4$ (all plausible parameters). What is the job-finding probability predicted by the McCall model? A good estimate of the average duration of unemployment in the US is about 3 months. Is it larger or smaller than the predicition of the McCall model? What creates such a discrepancy?

2 A risk-averse McCall worker

In this problem you will study a version of the McCall model for a risk-averse worker with inter-temporal utility

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t c_t^{1-\gamma}\right],$$

for some $\gamma \in [0,1)$. Each period the worker can either be employed at some wage w or unemployed. When employed, he may lose his job with probability $\delta \in (0,1)$. There are no unemployment benefit. When unemployed, the worker recives a job offer with probability λ. Conditional on getting a job offer, the wage offered w is drawn from a distribution with CDF $F(w)$. If an unemployed worker accepts the job offer, he will be employed in the next period. If he rejects the offer, he will remain unemployed. Finally, we assume that the worker can neither borrow nor save, so his consumption is $c_t = 0$ when unemployed, and $c_t = w$ when employed at wage w.

1. Write the Bellman equation for the value of an unemployed worker V_U and for a worker employed at wage w, $V_E(w)$.

2. Show that the worker's policy is characterized by a reservation wage $w^\star$ solving

$$(w^\star)^{1-\gamma} = \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^\star}^{\infty} (1-\gamma) w^{-\gamma} \left[1 - F(w)\right] dw. \tag{1}$$

For the remainder of the problem, assume that $F(w)$ has the following functional form:

$$\begin{aligned} 0 \le w < \underline{w}: &\quad F(w \,|\, \theta) = 0 \\ w \ge \underline{w}: &\quad F(w \,|\, \theta) = 1 - \left(\frac{w}{\underline{w}}\right)^{-\theta} \end{aligned}$$

where $\theta > 1$ and $\underline{w} \equiv \dfrac{\theta - 1}{\theta} m$. Note the support of the distribution has a lower bound equal to $\underline{w}$. It can be verifed that the mean of $F(w \,|\, \theta)$ is equal to m for all θ, and that an increase in θ makes the distribution $F(w \,|\, \theta)$ less risky in the sense of second-order stochastic dominance.

3. Consider $\theta < \theta'$. Sketch the two CDF $F(w \,|\, \theta)$ and $F(w \,|\, \theta')$ on the same graph for $w \in [0, \infty)$ (use the information provided to make an educated guess, no proof is needed).

4. What is the limit of the distribution $F(w \,|\, \theta)$ when $\theta \to \infty$?

5. Assume that $\gamma = 0$ (risk-neutrality) and $\theta \to \infty$. Show that the reservation wage to be less than m. Explain why this makes sense. What is the job finding probility of a worker?

6. Assume that $\gamma = 0$ (risk-neutrality) and $\theta \in (1, \infty)$. Find a condition on θ for the reservation wage is less than $\underline{w}$. Explain the relationship between this condition and the option value of search.

7. Assume that $\gamma = 0$ (risk-neutrality) and that the condition you derive above does not hold, so that $w^\star > \underline{w}$. Derive a formula for $w^\star$.

8. Assume that $\gamma > 0$ and that exogenous parameters ensure that $w^\star > \underline{w}$. Derive a formula for $w^\star$. Let $w^\star(\gamma)$ denote the reservation wage when risk aversion is γ. Is $w^\star(0) < w^\star(\gamma)$ or is $w^\star(0) > w^\star(\gamma)$? Provide intuition.

9. Assume that $\gamma > 0$ and that exogenous parameters ensure that $w^\star > \underline{w}$. Now consider an increase in θ (as explained above, this makes the distribution of wages less risky). Show analytically that an increase in θ has two effects on the reservation wage going in opposite direction, and explain why.