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The Cyclical Behavior of Equilibrium Unemployment and Vacancies

By ROBERT SHIMER*

This paper argues that the textbook search and matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude. In the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the search model predicts that the two variables should have nearly the same volatility. A shock that changes average labor productivity primarily alters the present value of wages, generating only a small movement along a downward-sloping Beveridge curve (unemployment-vacancy locus). A shock to the separation rate generates a counterfactually positive correlation between unemployment and vacancies. In both cases, the model exhibits virtually no propagation. (JEL E24, E32, J41, J63, J64)

In recent years, the Mortensen-Pissarides search and matching model has become the standard theory of equilibrium unemployment (Dale Mortensen and Chris Pissarides, 1994; Pissarides, 2000). The model is attractive for a number of reasons: it offers an appealing description of how the labor market functions; it is analytically tractable; it has rich and generally intuitive comparative statics; and it can easily be adapted to study a number of labor market policy issues, such as unemployment insurance, firing restrictions, and mandatory advanced notification of layoffs. Given these successes, one might expect that there would be strong evidence that the model is consistent with key business cycle facts. On the contrary, I argue in this paper that the model cannot explain the

cyclical behavior of two of its central elements, unemployment and vacancies, which are both highly variable and strongly negatively correlated in U.S. data. Equivalently, the model cannot explain the strong procyclicality of the rate at which an unemployed worker finds a job.

I focus on two sources of shocks: changes in labor productivity and changes in the separation rate of employed workers from their job. In a one-sector model, a change in labor productivity is most easily interpreted as a technology or supply shock. But in a multi-sector model, a preference or demand shock changes the relative price of goods, which induces a change in real labor productivity as well. Thus these shocks represent a broad set of possible impulses.

An increase in labor productivity relative to the value of nonmarket activity and to the cost of advertising a job vacancy makes unemployment relatively expensive and vacancies relatively cheap.[1] The market substitutes toward vacancies, and the increased job-finding rate pulls down the unemployment rate, moving the economy along a downward sloping Beveridge curve (vacancy-unemployment or v-u locus). But the increase in hiring also shortens unemployment duration, raising workers' threat point in wage bargaining, and therefore raising the

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[1] The interpretation in this paragraph and its sequel builds on discussions with Robert Hall.

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expected present value of wages in new jobs. Higher wages absorb most of the productivity increase, eliminating the incentive for vacancy creation. As a result, fluctuations in labor productivity have little impact on the unemployment, vacancy, and job-finding rates.

An increase in the separation rate does not affect the relative value of unemployment and vacancies, and so leaves the v-u ratio essentially unchanged. Since the increase in separations reduces employment duration, the unemployment rate increases, and so therefore must vacancies. As a result, fluctuations in the separation rate induce a counterfactually positive correlation between unemployment and vacancies.

Section I presents the relevant U.S. business cycle facts: unemployment u is strongly countercyclical, vacancies v are equally strongly procyclical, and the correlation between the two variables is -0.89 at business cycle frequencies. As a result, the v-u ratio is procyclical and volatile, with a standard deviation around its trend equal to 0.38 log points. To provide further evidence in support of this finding, I examine the rate at which unemployed workers find jobs. If the process of pairing workers with jobs is well-described by an increasing, constant returns-to-scale matching function $m(u,v)$, as in Pissarides (1985), the job finding rate is $f = m(u,v)/u$, an increasing function of the v-u ratio. I use unemployment-duration data to measure the job-finding rate directly. The standard deviation of fluctuations in the job-finding rate around trend is 0.12 log points and the correlation with the v-u ratio is 0.95. Finally I look at the two proposed impulses. The separation rate is less correlated with the cycle and moderately volatile, with a standard deviation about trend equal to 0.08 log points. Average labor productivity is weakly procyclical and even more stable, with a standard deviation about trend of 0.02 log points.

In Section II, I extend the Pissarides's (1985) search and matching model to allow for aggregate fluctuations. I introduce two types of shocks: labor productivity shocks raise output in all matches but do not affect the rate at which employed workers lose their job; and separation shocks raise the rate at which employed workers become unemployed but do not affect the productivity in surviving matches. In equilibrium, there is only one real economic decision: firms'

choice of whether to open a new vacancy. The equilibrium vacancy rate depends on the unemployment rate, on labor market tightness, and on the expected present value of wages in new employment relationships. Wages, in turn, are determined by Nash bargaining, at least in new matches. In principle, the wage in old matches may be rebargained in the face of aggregate shocks or may be fixed by a long-term employment contract. Section II A describes the basic model, while Section II B derives a forward-looking equation for the v-u ratio in terms of model parameters.

Section II C performs simple analytical comparative statics in some special cases. For example, I show that the elasticity of the v-u ratio with respect to the difference between labor productivity and the value of nonmarket activity or "leisure" is barely in excess of 1 for reasonable parameter values. To reconcile this with the data, one must assume that the value of leisure is nearly equal to labor productivity, so market work provides little incremental utility. The separation rate has an even smaller impact on the v-u ratio, with an elasticity of -0.1 according to the comparative statics. Moreover, while shocks to labor productivity at least induce a negative correlation between unemployment and vacancies, separation shocks cause both variables to increase, which tends to generate a positive correlation between the two variables. Similar results obtain in some other special cases.

Section II D calibrates the stochastic model to match U.S. data along as many dimensions as possible, and Section II E presents the results. The exercise confirms the quantitative predictions of the comparative statics. If the economy is hit only by productivity shocks, it moves along a downward-sloping Beveridge curve, but empirically plausible movements in labor productivity result in tiny fluctuations in the v-u ratio. Moreover, labor productivity is perfectly correlated with the v-u ratio, indicating that the model has almost no internal propagation mechanism. If the economy is hit only by separation shocks, the v-u ratio is stable in the face of large unemployment fluctuations, so vacancies are countercyclical. Equivalently, the model-generated Beveridge curve is upward-sloping.

Section II F explores the extent to which the Nash bargaining solution is responsible for these results. First I examine the behavior of

wages in the face of labor productivity and separation shocks. An increase in labor productivity encourages firms to create vacancies. The resulting increase in the job-finding rate puts upward pressure on wages, soaking up virtually all of the shock. A decrease in the separation rate also induces firms to create more vacancies, again putting upward pressure on wages and minimizing the impact on the v-u ratio and job-finding rate. On the other hand, I examine a version of the model in which only workers' bargaining power is stochastic. Small fluctuations in bargaining power generate realistic movements in the v-u ratio while inducing only a moderately countercyclical real wage, with a standard deviation of 0.01 log points around trend.

Section III provides another angle from which to view the model's basic shortcoming. I consider a centralized economy in which a social planner decides how many vacancies to create in order to maximize the present value of market and nonmarket income net of vacancy creation costs. The decentralized and centralized economies behave identically if the matching function is Cobb-Douglas in unemployment and vacancies and workers' bargaining power is equal to the elasticity of the matching function with respect to the unemployment rate, a generalization of Arthur Hosios (1990). But if unemployment and vacancies are more substitutable, fluctuations are amplified in the centralized economy, essentially because the shadow wage is less procyclical. Empirically it is difficult to measure the substitutability of unemployment and vacancies in the matching function, and therefore difficult to tell whether observed fluctuations are optimal.

Section IV reconciles this paper with a number of existing studies that claim standard search and matching models are consistent with the business cycle behavior of labor markets. Finally, the paper concludes in Section V by suggesting some modifications to the model that might deliver rigid wages and thereby do a better job of matching the empirical evidence on vacancies and unemployment.

It is worth emphasizing one important—but standard—feature of the search and matching framework that I exploit throughout this paper: workers are risk-neutral and supply labor inelastically. In the absence of search frictions, employment would be constant even in the face of

productivity shocks. This distinguishes the present model from those based upon intertemporal labor supply decisions (Robert E. Lucas, Jr., and Leonard Rapping, 1969). Thus this paper explores the extent to which a combination of search frictions and aggregate shocks can generate plausible fluctuations in unemployment and vacancies if labor supply is inelastic. It suggests that search frictions per se scarcely amplify shocks. The paper does not examine whether a search model with an elastic labor supply can provide a satisfactory explanation for the observed fluctuations in these two variables.

I. U.S. Labor Market Facts

This section discusses the time series behavior of unemployment u, vacancies v, the job finding rate f, the separation rate s, and labor productivity p in the United States. Table 1 summarizes the detrended data.

A. Unemployment

The unemployment rate is the most commonly used cyclical indicator of job-search activity. In an average month from 1951 to 2003, 5.67 percent of the U.S. labor force was out of work, available for work, and actively seeking work. This time series exhibits considerable temporal variation, falling to as low as 2.6 percent in 1953 and 3.4 percent in 1968 and 1969, but reaching 10.8 percent in 1982 and 1983 (Figure 1). Some of these fluctuations are almost certainly due to demographic and other factors unrelated to business cycles. To highlight business-cycle-frequency fluctuations, I take the difference between the log of the unemployment level and an extremely low frequency trend, a Hodrick-Prescott (HP) filter with smoothing parameter 10^5 using quarterly data.[2] The difference between log unemployment and its trend has a standard deviation of 0.19, so unemployment is often as much as 38 percent above or below trend. Detrended unemployment also exhibits considerable persistence, with quarterly autocorrelation 0.94.

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[2] I use the level of unemployment rather than the rate to keep the units comparable to those of vacancies. A previous version of this paper used the unemployment rate, with no effect on the conclusions.

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation matrix	u	1	−0.894	−0.971	−0.949	0.709	−0.408
	v	—	1	0.975	0.897	−0.684	0.364
	v/u	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

Notes: Seasonally adjusted unemployment u is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job-finding rate f and separation rate s are constructed from seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS, as explained in equations (1) and (2). u, v, f, and s are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.
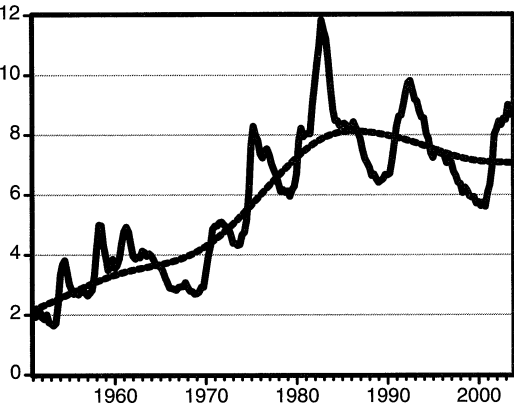

FIGURE 1. QUARTERLY U.S. UNEMPLOYMENT (IN MILLIONS)
AND TREND, 1951–2003

Notes: Unemployment is a quarterly average of the seasonally adjusted monthly series constructed by the BLS from the CPS, survey home page http://www.bls.gov/cps/. The trend is an HP filter of the quarterly data with smoothing parameter 10^5.

There is some question as to whether unemployment or the employment-population ratio is a better indicator of job-search activity. Advocates of the latter view, for example Harold Cole and Richard Rogerson (1999), argue that the number of workers moving directly into employment from out-of-the-labor force is as large as the number who move from unemployment to employment (Olivier Blanchard and Peter Diamond, 1990). On the other hand, there is ample evidence that unemployment and non-

participation are distinct economic conditions. Chinhui Juhn et al. (1991) show that almost all of the cyclical volatility in prime-aged male nonemployment is accounted for by unemployment. Christopher Flinn and James Heckman (1983) show that unemployed workers are significantly more likely to find a job than nonparticipants, although Stephen Jones and Craig Riddell (1999) argue that other variables also help to predict the likelihood of finding a job. In any case, since labor force participation is procyclical, the employment-population ratio is a more cyclical measure of job-search activity, worsening the problems highlighted in this paper.

It is also conceivable that when unemployment rises, the amount of job-search activity per unemployed worker declines so much that aggregate search activity actually falls. There is both direct and indirect evidence against this hypothesis. As direct evidence, one would expect that a reduction in search intensity could be observed as a decline in the number of job-search methods used or a switch toward less time-intensive methods. An examination of Current Population Survey (CPS) data indicates no cyclical variation in the number or type of job-search methods utilized.[3] Indirect evidence comes from estimates of matching functions, which universally find that an increase in unemployment is associated with an increase in

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[3] Shimer (2004b) discusses this evidence in detail.

the number of matches (Barbara Petrongolo and Pissarides, 2001). If job-search activity declined sharply when unemployment increased, the matching function would be measured as decreasing in unemployment. I conclude that aggregate job search activity is positively correlated with unemployment.

B. Vacancies

The flip side of unemployment is job vacancies. The Job Openings and Labor Turnover Survey (JOLTS) provides an ideal empirical definition: "A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the Internet, posting help-wanted signs, accepting applications, or using similar methods."[4] Unfortunately, JOLTS began only in December 2000 and comparable data had never previously been collected in the United States. Although there are too few observations to look systematically at this time series, its behavior has been instructive. In the first month of the survey, the non-farm sector maintained a seasonally adjusted 4.6 million job openings. This number fell rapidly during 2001 and averaged just 2.9 million in 2002 and 2003. This decline in job openings, depicted by the solid line in Figure 2, coincided with a period of rising unemployment, suggesting that job vacancies are procyclical.

To obtain a longer time series, I use a standard proxy for vacancies, the Conference Board help-wanted advertising index, measured as the number of help-wanted advertisements in 51 major newspapers.[5] A potential shortcoming is
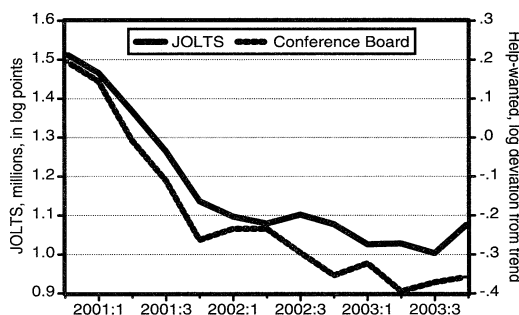

FIGURE 2. TWO MEASURES OF U.S. JOB VACANCIES, 2000Q4–2003Q4

Notes: The solid line shows the logarithm of the number of job openings in millions, measured by the BLS from the JOLTS, survey homepage http://www.bls.gov/jlt, quarterly averaged and seasonally adjusted. The dashed line shows the deviation from trend of the quarterly averaged, seasonally adjusted Conference Board help-wanted advertising index.

that help-wanted advertising is subject to low-frequency fluctuations that are related only tangentially to the labor market. In recent years, the Internet may have reduced firms' reliance on newspapers as a source of job advertising, while in the 1960s, newspaper consolidation may have increased advertising in surviving newspapers and Equal Employment Opportunity laws may have encouraged firms to advertise job openings more extensively. Fortunately, a low-frequency trend should remove the effect of these and other secular shifts. Figure 3 shows the help-wanted advertising index and its trend. Notably, the decline in the detrended help-wanted index closely tracks the decline in job openings measured directly from JOLTS during the period when the latter time series is available (Figure 2).

Figure 4 shows a scatter plot of the relationship between the cyclical component of unemployment and vacancies, the Beveridge curve. The correlation of the percentage deviation of unemployment and vacancies from trend is −0.89 between 1951 and 2003.[6] Moreover, the

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[4] This definition comes from the Bureau of Labor Statistics news release, July 30, 2002, available at http://www.bls.gov/jlt/jlt_nr1.pdf.

[5] Abraham (1987) discusses this measure in detail. From 1972 to 1981, Minnesota collected state-wide job vacancy data. Abraham compares this with Minnesota's help-wanted advertising index and shows that the two series track each other very closely through two business cycles and ten seasonal cycles.

[6] Abraham and Katz (1986) and Blanchard and Diamond (1989) discuss the U.S. Beveridge curve. Abraham and Katz (1986) argue that the negative correlation between unemployment and vacancies is inconsistent with Lilien's (1982) sectoral shifts hypothesis, and instead indicates that business cycles are driven by aggregate fluctuations. Blanchard and Diamond (1989) conclude that at business cycle
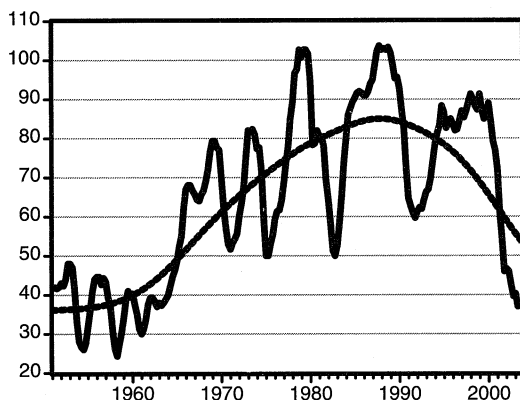
FIGURE 3. QUARTERLY U.S. HELP-WANTED ADVERTISING
INDEX AND TREND, 1951–2003

Notes: The help-wanted advertising index is a quarterly
average of the seasonally adjusted monthly series con-
structed by the Conference Board with normalization
1987 = 100. The data were downloaded from the Federal
Reserve Bank of St. Louis database at http://research.
stlouisfed.org/fred2/data/helpwant.txt. The trend is an HP
filter of the quarterly data with smoothing parameter 10^5.
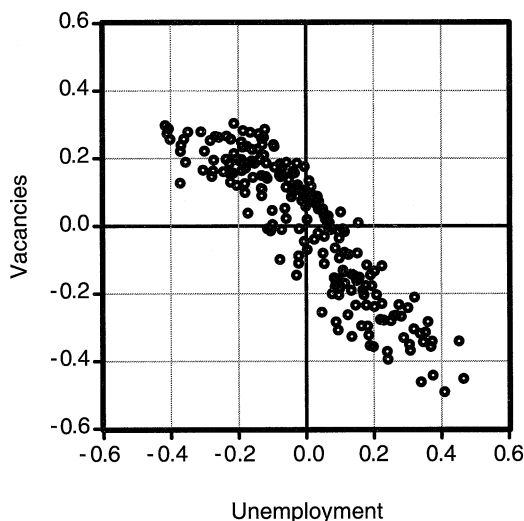

FIGURE 4. QUARTERLY U.S. BEVERIDGE CURVE,
1951–2003

Notes: Unemployment is constructed by the BLS from the
CPS. The help-wanted advertising index is constructed by
the Conference Board. Both are quarterly averages of sea-
sonally adjusted monthly series and are expressed as devi-
ations from an HP filter with smoothing parameter 10^5.

standard deviation of the cyclical variation in
unemployment and vacancies is almost identi-
cal, between 0.19 and 0.20, so the product of
unemployment and vacancies is nearly acyclic.
The v-u ratio is therefore extremely procyclical,
with a standard deviation of 0.38 around its
trend.

C Job-Finding Rate

An implication of the procyclicality of the
v-u ratio is that the hazard rate for an unem-
ployed worker of finding a job, his job-finding
rate, should be lower during a recession. As-
sume that the number of newly hired workers is
given by an increasing and constant returns-to-
scale matching function $m(u,v)$, depending on
the number of unemployed workers u and the
number of vacancies v. Then the probability that
any individual unemployed worker finds a job,
the average transition rate from unemployment
to employment, is $f \equiv m(u,v)/u = m(1,\theta)$, where
$\theta \equiv v/u$ is the v-u ratio. The job-finding rate f
should therefore move together with the v-u
ratio.

Gross worker flow data can be used to mea-
sure the job-finding rate directly, and indeed
both the unemployment-to-employment and
nonparticipation-to-employment transition rates
are strongly procyclical (Blanchard and Dia-
mond, 1990; Hoyt Bleakley et al., 1999; Ka-
tharine Abraham and Shimer, 2001). There are
two drawbacks to this approach. First, the req-
uisite public use dataset is available only since
1976, and so using these data would require
throwing away half of the available time series.
Second, measurement and classification error
lead a substantial overestimate of gross worker
flows (John Abowd and Arnold Zellner, 1985;
James Poterba and Lawrence Summers, 1986),
the magnitude of which cannot easily be com-
puted. Instead, I infer the job-finding rate from
the dynamic behavior of the unemployment
level and short-term unemployment level. Let
u_t^s denote the number of workers unemployed
for less than one month in month t. Then as-
suming all unemployed workers find a job with
probability f_t in month t and no unemployed
worker exits the labor force,

$$u_{t+1} = u_t(1 - f_t) + u_{t+1}^s.$$

frequencies, shocks generally drive the unemployment and
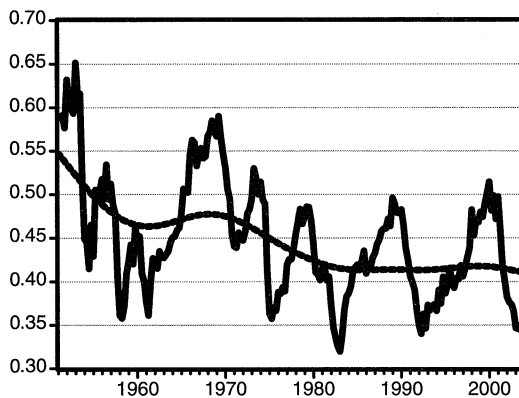vacancy rates in the opposite direction.

FIGURE 5. MONTHLY JOB-FINDING PROBABILITY FOR
UNEMPLOYED WORKERS, 1951–2003

Notes: The job-finding rate is computed using equation (1),
with unemployment and short-term unemployment data
constructed and seasonally adjusted by the BLS from the
CPS, survey home page http://www.bls.gov/cps/. It is ex-
pressed as a quarterly average of monthly data. The trend
is an HP filter of the quarterly data with smoothing param-
eter 10^5.

FIGURE 6. MONTHLY U.S. MATCHING FUNCTION,
1951–2003

Notes: The v-u ratio is constructed by the BLS from the
CPS and by the Conference Board. The job-finding rate is
constructed using equation (1) and BLS data from the CPS.
Both are quarterly averages of seasonally adjusted monthly
series and are expressed as deviations from an HP filter with
smoothing parameter 10^5.

Unemployment next month is the sum of the
number of unemployed workers this month who
fail to find a job and the number of newly
unemployed workers. Equivalently,

$$(1) \qquad f_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t}.$$

I use the unemployment level and the number of
workers unemployed for 0 to 4 weeks, both
constructed by the BLS from the CPS, to com-
pute f_t from 1951 to 2003.[7] Figure 5 shows the
results. The monthly hazard rate averaged 0.45
from 1951 to 2003. After detrending with the
usual low-frequency HP filter, the correlation
between f_t and θ_t at quarterly frequencies is
0.95, although the standard deviation of f_t is
about 31 percent that of θ_t. Given that both
measures are crudely yet independently con-
structed, this correlation is remarkable and
strongly suggests that a matching function is a
useful way to approach U.S. data.

One can use the measured job-finding rate
and v-u ratio to estimate a matching function
$m(u,v)$. Data limitations force me to impose two
restrictions on the estimated function. First, be-
cause unemployment and vacancies are strongly
negatively correlated, it is difficult to tell em-
pirically whether $m(u,v)$ exhibits constant, in-
creasing, or decreasing returns to scale. But in
their literature survey, Petrongolo and Pissar-
ides (2001) conclude that most estimates of the
matching function cannot reject the null hypoth-
esis of constant returns; I therefore estimate $f_t =
f(\theta_t)$, consistent with a constant returns-to-scale
matching function. Figure 6 shows the raw data
for the job-finding rate f_t and the v-u ratio θ_t, a
nearly linear relationship when both variables
are expressed as deviations from log trend. Sec-
ond, I impose that the matching function is
Cobb-Douglas, $m(u,v) = \mu u^\alpha v^{1-\alpha}$, for some
unknown parameters α and μ. Again, the data
are not very informative as to whether this is a
reasonable restriction.[8] I estimate the matching

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[7] Abraham and Shimer (2001) argue that the redesign of
the CPS in January 1994, in particular the switch to depen-
dent interviewing, reduced measured short-term unemploy-
ment. They suggest some methods of dealing with this
discontinuity. In this paper, I simply inflate short-term un-
employment by 10 percent after the redesign took effect.

[8] Consider the CES matching function $\log f_t = \log \mu +
1/\rho \log (\alpha + (1 - \alpha)\theta_t^\rho)$. Cobb-Douglas corresponds to

function using detrended data on the job-finding rate and the v-u ratio. Depending on exactly how I control for autocorrelation in the residuals, I estimate values of α between 0.70 and 0.75. With a first-order autoregressive residual, I get $\alpha = 0.72$ with a standard error of 0.01.

One particularly crude aspect of this measure of the job-finding rate is the assumption that all workers are equally likely to find a job. Shimer (2004a) proves that when the unemployed are heterogeneous, f_t measures the mean job-finding rate in the unemployed population. That paper also compares my preferred measure of the job-finding rate with two alternatives. The first uses the unemployment level and mean unemployment duration to obtain a weighted average of the job-finding rate in the unemployed population, with weights proportional to each individual's unemployment duration.[9] The second follows Robert Hall (2004) and measures the job-finding rate of workers with short unemployment duration using the ratio of workers with 0 to 4 weeks of unemployment to workers with 5 to 14 weeks of unemployment. Since the job-finding rate declines with unemployment duration, I find that my preferred measure of the job-finding rate lies between these two alternatives. Hall measures an average job-finding rate of 0.48 per month, while unemployment duration data yield a job-finding rate of 0.34 per month. Nevertheless, all three measures are highly correlated, and so the choice of which measure to use does not qualitatively affect the conclusions of this study.

D. Separation Rate

I can also deduce the behavior of the separation rate from data on employment, short-term unemployment, and the hiring rate. Suppose first that whenever an employed worker loses her job, she becomes unemployed. Then the separation rate could simply be computed as the ratio of short-term unemployed workers next month, u^s_{t+1}, to employed workers this month, e_t. But this masks a significant time-aggregation
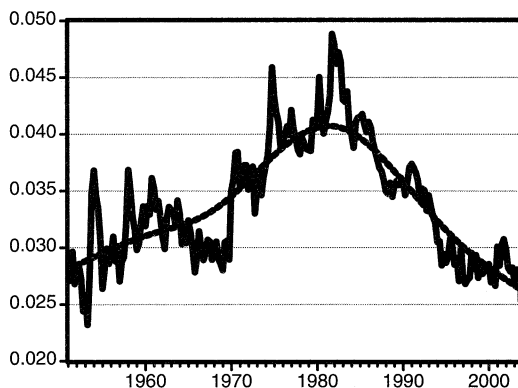

FIGURE 7. MONTHLY SEPARATION PROBABILITY FOR
EMPLOYED WORKERS, 1951–2003

Notes: The separation rate is computed using equation (2), with employment, unemployment, and short-term unemployment data constructed and seasonally adjusted by the BLS from the CPS, survey home page http://www.bls.gov/cps/. It is expressed as a quarterly average of monthly data. The trend is an HP filter of the quarterly data with smoothing parameter 10^5.

bias. When a worker loses her job, she has on average half a month to find a new job before she is recorded as unemployed. Accounting for this, the short-term unemployment rate the next month is approximately equal to

$$u^s_{t+1} = s_t e_t \left(1 - \tfrac{1}{2} f_t\right).$$

Ignoring the probability of finding another job within the month leads one to understate the separation rate. This problem is particularly acute when the job-finding rate is high, i.e., during expansions. I therefore measure the separation rate as

(2) $$s_t = \frac{u^s_{t+1}}{e_t \left(1 - \tfrac{1}{2} f_t\right)}.$$

Figure 7 shows the monthly separation rate thus constructed. It averaged 0.034 from 1951 to 2003, so jobs last on average for about 2.5 years. Fluctuations in the deviation of the log separation rate from trend are somewhat smaller than in the hiring rate, with a standard deviation of 0.08, and separations are countercyclical, so the correlation with the detrended v-u ratio is −0.72.

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limiting case of $\rho = 0$. When I estimate the CES function using nonlinear least squares and correct for first-order autocorrelation, I get a point estimate of $\rho = 0.06$ with a standard error of 0.38.

[9] A previous version of this paper relied on that measure of f_t. This had little effect on the results.
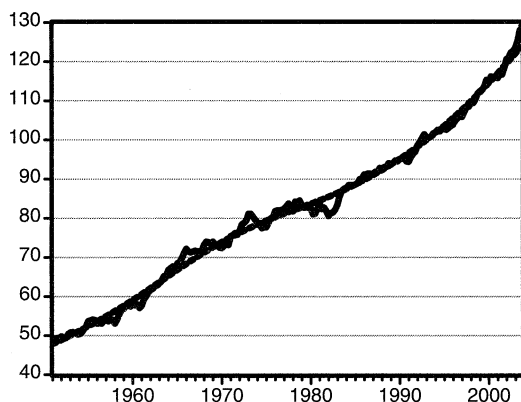
FIGURE 8. QUARTERLY U.S. AVERAGE LABOR
PRODUCTIVITY AND TREND, 1951–2003

Notes: Real output per person in the non-farm business
sector, constructed by the BLS Major Sector Productivity
and Costs program, survey home page http://www.bls.gov/
lpc/, 1992 = 100. The trend is an HP filter of the quarterly
data with smoothing parameter 10^5.

FIGURE 9. QUARTERLY U.S. VACANCY-UNEMPLOYMENT
RATIO AND AVERAGE LABOR PRODUCTIVITY, 1951–2003

Notes: Unemployment is constructed by the BLS from the
CPS. The help-wanted advertising index is constructed by
the Conference Board. Both are quarterly averages of sea-
sonally adjusted monthly series. Labor productivity is real
average output per worker in the non-farm business sector,
constructed by the BLS Major Sector Productivity and
Costs program. The v-u ratio and labor productivity are
expressed as deviations from an HP filter with smoothing
parameter 10^5.

The strong procyclicality of the job-finding
rate and relatively weak countercyclicality of
the separation rate might appear to contradict
Blanchard and Diamond's (1990) conclusion
that "the amplitude in fluctuations in the flow
out of employment is larger than that of the flow
into employment." This is easily reconciled.
Blanchard and Diamond look at the *number* of
people entering or exiting employment in a
given month, $f_t u_t$ or $s_t e_t$, while I focus on the
probability that an individual switches employ-
ment states, f_t and s_t. Although the probability
of entering employment f_t declines sharply in
recessions, this is almost exactly offset by the
increase in unemployment u_t, so that the num-
ber of people exiting unemployment is essen-
tially acyclic. Viewed through the lens of an
increasing matching function $m(u,v)$, this is
consistent with the independent evidence that
vacancies are strongly procyclical.

E. *Labor Productivity*

The final important empirical observation is
the weak procyclicality of labor productivity,
measured as real output per worker in the non-
farm business sector. The BLS constructs this
quarterly time series as part of its Major Sector
Productivity and Costs program. The output
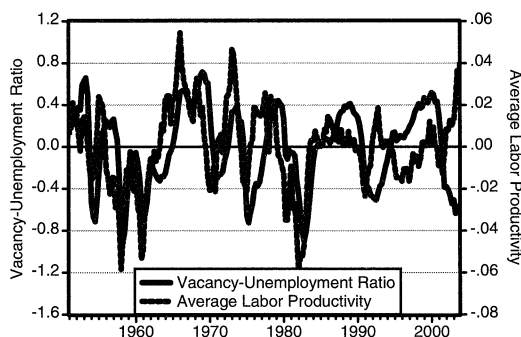measure is based on the National Income and

Product Accounts, while employment is con-
structed from the BLS establishment survey, the
Current Employment Statistics. This series of-
fers two advantages compared with total factor
productivity: it is available quarterly since
1948; and it better corresponds to the concept of
labor productivity in the subsequent models,
which do not include capital.

Figure 8 shows the behavior of labor produc-
tivity and Figure 9 compares the cyclical com-
ponents of the v-u ratio and labor productivity.
There is a positive correlation between the two
time series and some evidence that labor pro-
ductivity leads the v-u ratio by about one year,
with a maximum correlation of 0.56.[10] But the
most important fact is that labor productivity is
stable, never deviating by more than 6 per-
cent from trend. In contrast, the v-u ratio has
twice risen to 0.5 log points about its trend level
and six times has fallen by 0.5 log points below
trend.

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[10] From 1951 to 1985, the contemporaneous correlation
between detrended labor productivity and the v-u ratio was
0.57 and the peak correlation was 0.74. From 1986 to 2003,
however, the contemporaneous and peak correlations are
negative, −0.37 and −0.43, respectively. This has been
particularly noticeable during the last three years of data. An
exploration of the cause of this change goes beyond the
scope of this paper.

It is possible that the measured cyclicality of labor productivity is reduced by a composition bias, since less productive workers are more likely to lose their jobs in recessions. I offer two responses to this concern. First, there is a composition bias that points in the opposite direction: labor productivity is higher in more cyclical sectors of the economy, e.g., durable goods manufacturing. And second, a large literature on real wage cyclicality has reached a mixed conclusion about the importance of composition biases (Abraham and John Haltiwanger, 1995). Gary Solon et al. (1994) provide perhaps the strongest evidence that labor force composition is important for wage cyclicality, but even they argue that accounting for this might double the measured variability of real wages. This paper argues that the search and matching model cannot account for the cyclical behavior of vacancies and unemployment unless labor productivity is at least ten times as volatile as the data suggest, so composition bias is at best an incomplete explanation.

II. Search and Matching Model

I now examine whether a standard search and matching model can reconcile the strong procyclicality of the v-u ratio and the job-finding rate with the weak procyclicality of labor productivity and countercyclicality of the separation rate. The model I consider is essentially an aggregate stochastic version of Pissarides (1985, or 2000, Ch. 1).

A. Model

I start by describing the exogenous variables that drive fluctuations. Labor productivity p and the separation rate s follow a first-order Markov process in continuous time. A shock hits the economy according to a Poisson process with arrival rate λ, at which point a new pair (p', s') is drawn from a state dependent distribution. Let $\mathbb{E}_{p,s} X_{p',s'}$ denote the expected value of an arbitrary variable X following the next aggregate shock, conditional on the current state (p,s). I assume that this conditional expectation is finite, which is ensured if the state space is compact. At every point in time, the current values of productivity and the separation rate are common knowledge.

Next I turn to the economic agents in the economy, a measure 1 of risk-neutral, infinitely-lived workers and a continuum of risk-neutral, infinitely-lived firms. All agents discount future payoffs at rate $r > 0$.

Workers can either be unemployed or employed. An unemployed worker gets flow utility z from non-market activity ("leisure") and searches for a job. An employed worker earns an endogenous wage but may not search. I discuss wage determination shortly.

Firms have a constant returns-to-scale production technology that uses only labor, with labor productivity at time t given by the stochastic realization $p(t)$. In order to hire a worker, a firm must maintain an open vacancy at flow cost c. Free entry drives the expected present value of an open vacancy to zero. A worker and a firm separate according to a Poisson process with arrival rate governed by the stochastic separation rate $s(t)$, leaving the worker unemployed and the firm with nothing.

Let $u(t)$ denote the endogenous unemployment rate,[11] $v(t)$ denote the endogenous measure of vacancies in the economy, and $\theta(t) \equiv v(t)/u(t)$ denote the v-u ratio at time t. The flow of matches is given by a constant returns-to-scale function $m(u(t), v(t))$, increasing in both arguments. This implies that an unemployed worker finds a job according to a Poisson process with time-varying arrival rate $f(\theta(t)) \equiv m(1, \theta(t))$ and that a vacancy is filled according to a Poisson process with time-varying arrival rate $q(\theta(t)) \equiv m(\theta(t)^{-1}, 1) = f(\theta(t))/\theta(t)$.

I assume that in every state of the world, labor productivity $p(t)$ exceeds the value of leisure z, so there are bilateral gains from matching. There is no single compelling theory of wage determination in such an environment, and so I follow the literature and assume that when a worker and firm first meet, the expected gains from trade are split according to the Nash bargaining solution. The worker can threaten to become unemployed and the firm can threaten to end the job. The present value of surplus

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[11] With the population of workers constant and normalized to one, the unemployment rate and unemployment level are identical in this model. I therefore use these terms interchangeably.

beyond these threats is divided between the worker and firm, with the worker keeping a fraction $\beta \in (0, 1)$ of the surplus, her "bargaining power." I make almost no assumptions about what happens to wages after this initial agreement, except that the worker and firm manage to exploit all the joint gains from trade. For example, the wage may be re-bargained whenever the economy is hit with a shock. Alternatively, it may be fixed at its initial value until such time as the firm would prefer to fire the worker or the worker would prefer to quit, whereupon the pair resets the wage so as to avoid an unnecessary and inefficient separation.

B. *Characterization of Equilibrium*

I look for an equilibrium in which the v-u ratio depends only on the current value of p and s, $\theta_{p,s}$.[12] Given the state-contingent v-u ratio, the unemployment rate evolves according to a standard backward-looking differential equation,

$$(3) \quad \dot{u}(t) = s(t)(1 - u(t)) - f(\theta_{p(t),s(t)})u(t)$$

where $(p(t), s(t))$ is the aggregate state at time t. A flow $s(t)$ of the $1 - u(t)$ employed workers become unemployed, while a flow $f(\theta)$ of the $u(t)$ unemployed workers find a job. An initial condition pins down the unemployment rate and the aggregate state at some date t_0.

I characterize the v-u ratio using a recursive equation for the joint value to a worker and firm of being matched in excess of breaking up as a function of the current aggregate state, $V_{p,s}$.

$$(4) \quad rV_{p,s} = p - (z + f(\theta_{p,s})\beta V_{p,s}) - sV_{p,s}$$
$$+ \lambda(\mathbb{E}_{p,s}V_{p',s'} - V_{p,s}).$$

Appendix A derives this equation from more primitive conditions. The first two terms represent the current flow surplus from matching. If

the pair is matched, they produce p units of output. If they were to break up the match, free entry implies the firm would be left with nothing, while the worker would become unemployed, getting flow utility from leisure z and from the probability $f(\theta_{p,s})$ of contacting a firm, in which event the worker would keep a fraction β of the match value $V_{p,s}$. Next, there is a flow probability s that the worker and firm separate, destroying the match value. Finally, an aggregate shock arrives at rate λ, resulting in an expected change in match value $\mathbb{E}_{p,s}V_{p',s'} - V_{p,s}$.

Another critical equation for the match value comes from firms' free entry condition. The flow cost of a vacancy c must equal the flow probability that the vacancy contacts a worker times the resulting capital gain, which by Nash bargaining is equal to a fraction $1 - \beta$ of the match value $V_{p,s}$:

$$(5) \quad c = q(\theta_{p,s})(1 - \beta)V_{p,s}.$$

Eliminating current and future values of $V_{p,s}$ from (4) using (5) gives

$$(6) \quad \frac{r + s + \lambda}{q(\theta_{p,s})} + \beta\theta_{p,s}$$
$$= (1 - \beta)\frac{p - z}{c} + \lambda\mathbb{E}_{p,s}\frac{1}{q(\theta_{p',s'})}$$

which implicitly defines the v-u ratio as a function of the current state (p,s).[13] This equation can easily be solved numerically, even with a large state vector. This simple representation of the equilibrium of a stochastic version of the Pissarides (1985) model appears to be new to the literature.

C. *Comparative Statics*

In some special cases, equation (6) can be solved analytically to get a sense of the

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[12] It is straightforward to show in a deterministic version of this model that there is no other equilibrium, e.g., one in which θ also depends on the unemployment rate. See Pissarides (1985).

[13] A similar equation obtains in the presence of aggregate variation in the value of leisure z, the cost of a vacancy c, or workers' bargaining power β.

quantitative results implied by this analysis. First, suppose there are no aggregate shocks, $\lambda = 0$.[14] Then the state-contingent v-u ratio satisfies

$$\frac{r + s}{q(\theta_{p,s})} + \beta\theta_{p,s} = (1 - \beta)\frac{p - z}{c}.$$

The elasticity of the v-u ratio θ with respect to "net labor productivity" $p - z$ is

$$\frac{r + s + \beta f(\theta_{p,s})}{(r + s)(1 - \eta(\theta_{p,s})) + \beta f(\theta_{p,s})}$$

where $\eta(\theta) \in [0,1]$ is the elasticity of $f(\theta)$. This is large only if workers' bargaining power β is small and the elasticity η is close to one. But with reasonable parameter values, it is close to 1. For example, think of a time period as equal to one month, so the average job-finding rate is approximately 0.45 (Section I C), the elasticity $\eta(\theta)$ is approximately 0.28 (Section I C again), the average separation probability is approximately 0.034 (Section I D), and the interest rate is about 0.004. Then if workers' bargaining power β is equal to $1 - \eta(\theta)$, the so-called Hosios (1990) condition for efficiency,[15] the elasticity of the v-u ratio with respect to net labor productivity is 1.03. Lower values of β yield a slightly higher elasticity, say 1.15 when $\beta = 0.1$, but only at $\beta = 0$ does the elasticity of the v-u ratio with respect to $p - z$ rise appreciably, to 1.39. It would take implausible parameter values for this elasticity to exceed 2. This implies that unless the value of leisure is close to labor productivity, the v-u ratio is likely to be unresponsive to changes in the labor productivity.

I can similarly compute the elasticity of the v-u ratio with respect to the separation rate:

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[14] Shimer (2003) performs comparative statics exercises under much weaker assumptions. For example, in that paper the matching function can exhibit increasing or decreasing returns to scale and there can be an arbitrary idiosyncratic process for productivity, allowing for endogenous separations (Mortensen and Pissarides, 1994). I show that the results presented here generalize to such an environment if workers and firms are sufficiently patient relative to the search frictions.

[15] Section III shows that the Hosios condition carries over to the stochastic model.

$$\frac{-s}{(r + s)(1 - \eta(\theta_{p,s})) + \beta f(\theta_{p,s})}.$$

Substituting the same numbers into this expression gives -0.10. Doubling the separation rate would have a scarcely discernible impact on the v-u ratio.

Finally, one can examine the independent behavior of vacancies and unemployment. In steady state, equation (3) holds with $\dot{u} = 0$. If the matching function is Cobb-Douglas, $m(u,v) = \mu u^\alpha v^{1-\alpha}$, this implies

$$v_{p,s} = \left(\frac{s(1 - u_{p,s})}{\mu u^\alpha_{p,s}}\right)^{1/(1-\alpha)}.$$

For a given separation rate s, this describes a decreasing relationship between unemployment and vacancies, consistent with the Beveridge curve (Figure 4). An increase in labor productivity raises the v-u ratio which lowers the unemployment rate and hence raises the vacancy rate. Vacancies and unemployment should move in opposite directions in response to such shocks. But an increase in the separation rate scarcely affects the v-u ratio. Instead, it tends to raise both the unemployment and vacancy rates, an effect that is likely to produce a counterfactually positive correlation between unemployment and vacancies.

I can perform similar analytic exercises by making other simplifying assumptions. Suppose that each vacancy contacts an unemployed worker at a constant Poisson rate μ, independent of the unemployment rate, so $q(\theta) = \mu$. Given the risk-neutrality assumptions, this is equivalent to assuming that firms must pay a fixed cost c/μ in order to hire a worker. Then even with aggregate shocks, equation (6) yields a static equation for the v-u ratio:

$$\frac{r + s}{\mu} + \beta\theta_{p,s} = (1 - \beta)\frac{p - z}{c}.$$

In this case, the elasticity of the v-u ratio with respect to net labor productivity is

$$\frac{r + s + \beta\mu\theta}{\beta\mu\theta}$$

and the elasticity of the v-u ratio with respect to

the separation rate is $-s/\beta\mu\theta$. Since $f(\theta) = \mu\theta$, one can again pin down all the parameter values except workers' bargaining power β. Using the same parameter values as above, including $\beta = 0.72$, I obtain elasticities of 1.12 and -0.105, almost unchanged from the case with no shocks. More generally, unless β is nearly equal to zero, both elasticities are very small.

At the opposite extreme, suppose that each unemployed worker contacts a vacancy at a constant Poisson rate μ, independent of the vacancy rate, so $f(\theta) = \mu$ and $q(\theta) = \mu/\theta$. Also assume that the separation rate s is constant and average labor productivity p is a Martingale, $\mathbb{E}_p p' = p$. With this matching function, equation (6) is linear in current and future values of the v-u ratio:

$$\left(\frac{r + s + \lambda}{\mu} + \beta\right)\theta_p$$

$$= (1 - \beta)\frac{p - z}{c} + \frac{\lambda}{\mu}\mathbb{E}_p\theta_{p'}.$$

It is straightforward to verify that the v-u ratio is linear in productivity, and therefore $\mathbb{E}_p\theta_{p'} = \theta_p$, i.e.,

$$\left(\frac{r + s}{\mu} + \beta\right)\theta_p = (1 - \beta)\frac{p - z}{c}$$

so the elasticity of the v-u ratio with respect to net labor productivity is 1, regardless of workers' bargaining power. I conclude that with a wide range of parameterizations, the v-u ratio θ should be approximately proportional to net labor productivity $p - z$.

D. Calibration

This section parameterizes the model to match the time series behavior of the U.S. unemployment rate. The most important question is the choice of the Markov process for labor productivity and separations. Appendix C develops a discrete state space model which builds on a simple Poisson process corresponding to the theoretical analysis in Section II B. I define an underlying variable y that lies on a finite ordered set of points. When a Poisson shock hits, y either moves up or down by one point. The probability of moving up is itself decreasing in the current value of y, which ensures that

y is mean reverting. The stochastic variables are then expressed as functions of y.

Although I use the discrete state space model in my simulations as well, it is almost exactly correct and significantly easier to think about the behavior of the extrinsic shocks by discussing a related continuous state space model.[16] I express the state variables as functions of an Ornstein-Uhlenbeck process (see Howard Taylor and Samuel Karlin, 1998, Section 8.5). Let y satisfy

$$dy = -\gamma y dt + \sigma db$$

where b is a standard Brownian motion. Here $\gamma > 0$ is a measure of persistence, with higher values indicating faster mean reversion, and $\sigma > 0$ is the instantaneous standard deviation. This process has some convenient properties: y is conditionally and unconditionally normal; it is mean reverting, with expected value converging asymptotically to zero; and asymptotically its variance converges to $\sigma^2/2\gamma$.

I consider two different cases. In the first, the separation rate is constant and productivity satisfies $p = z + e^y(p^* - z)$, where y is an Ornstein-Uhlenbeck process with parameters γ and σ, and $p^* > z$ is a measure of long-run average productivity. Since $e^y > 0$, this ensures $p > z$. In the second case, productivity is constant and separations satisfy $s = e^y s^*$, where again y follows an Ornstein-Uhlenbeck process and now $s^* > 0$ is a measure of the long-run average separation rate. In both cases, the stochastic process is reduced to three parameters, γ, σ, and either p^* or s^*.

I now proceed to explain the choice of the other parameters, starting with the case of stochastic productivity. I follow the literature and assume that the matching function is Cobb-Douglas,

$$f(\theta) = \theta q(\theta) = \mu\theta^{1-\alpha}.$$

This reduces the calibration to ten parameters:

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[16] I work on a discrete grid with $2n + 1 = 2001$ points, which closely approximate Gaussian innovations. This implies that Poisson arrival rate of shocks is $\lambda = n\gamma = 4$ times per quarter in the model with labor productivity shocks and $\lambda = 220$ in the model with (less persistent) separation shocks.

the productivity parameter p^*, the value of leisure z, workers' bargaining power β, the discount rate r, the separation rate s, the two matching function parameters α and μ, the vacancy cost c, and the mean reversion and standard deviation of the stochastic process, γ and σ.

Without loss of generality, I normalize the productivity parameter to $p^* = 1$. I choose the standard deviation and persistence of the productivity process to match the empirical behavior of labor productivity. This requires setting $\sigma = 0.0165$ and $\gamma = 0.004$. An increase in the volatility of productivity σ has a nearly proportional effect on the volatility of other variables, while the persistence of the stochastic process γ scarcely affects the reported results. For example, suppose I reduce γ to 0.001, so productivity is more nearly a random walk. Because it is difficult to distinguish small values of γ in a finite dataset, after HP filtering the model-generated data, the persistence and magnitude of the impulse is virtually unchanged compared with the baseline parameterization. But reassuringly, the detrended behavior of unemployment and vacancies is also scarcely affected by increasing the persistence of labor productivity.

I set the value of leisure to $z = 0.4$. Since mean labor income in the model is 0.993, this lies at the upper end of the range of income replacement rates in the United States if interpreted entirely as an unemployment benefit.

I normalize a time period to be one quarter, and therefore set the discount rate to $r = 0.012$, equivalent to an annual discount factor of 0.953. The analysis in Section I D suggests a quarterly separation rate of $s = 0.10$, so jobs last for about 2.5 years on average. This is comparable to Abowd and Zellner's (1985) finding that 3.42 percent of employed workers exit employment during a typical month between 1972 and 1982, after correcting for classification and measurement error. It is also comparable to measured turnover rates in the JOLTS, although some separations in that survey reflect job-to-job transitions, a possibility that is absent from this model.

Using the matching function estimates from Section I C, I set the elasticity parameter to $\alpha = 0.72$. This lies toward the upper end of the range of estimates that Petrongolo and Pissarides (2001) report. I also set workers' bargaining power β to the same value, 0.72. Although the

TABLE 2—PARAMETER VALUES IN SIMULATIONS OF THE MODEL

Parameter	Source of shocks	
	Productivity	Separation
Productivity p	stochastic	1
Separation rate s	0.1	stochastic
Discount rate r	0.012	0.012
Value of leisure z	0.4	0.4
Matching function $q(\theta)$	$1.355\theta^{-0.72}$	$1.355\theta^{-0.72}$
Bargaining power β	0.72	0.72
Cost of vacancy c	0.213	0.213
Standard deviation σ	0.0165	0.0570
Autoregressive parameter γ	0.004	0.220
Grid size $2n + 1$	2001	2001

Note: The text provides details on the stochastic process for productivity and for the separation rate.

reported results are insensitive to the value of that parameter, I show in Section III that if $\alpha = \beta$, the "Hosios (1990) Rule," the decentralized equilibrium maximizes a well-posed social planner's problem.

I use the final two parameters, the matching function constant μ and the vacancy cost c, to pin down the average job-finding rate and the average v-u ratio. As reported in Section I C, a worker finds a job with a 0.45 probability per month, so the flow arrival rate of job offers $\mu\theta^{1-\alpha}$ should average approximately 1.35 on a quarterly basis. I do not have a long time series with the level of the v-u ratio, but fortunately the model offers one more normalization. Equation (6) implies that doubling c and multiplying μ by a factor $2^{1-\alpha}$ divides the v-u ratio θ in half, doubles the rate at which firms contact workers $q(\theta)$, but does not affect the rate at which workers find jobs. In other words, the average v-u ratio is intrinsically meaningless in the model. I choose to target a mean v-u ratio of 1, which requires setting $\mu = 1.355$ and $c = 0.213$.

In the case of shocks to the separation rate, I change only the stochastic process so as to match the empirical results discussed in Section I D. Productivity is constant and equal to 1, while the mean separation rate is $s^* = 0.10$. I set $\sigma = 0.057$ and $\gamma = 0.220$, a much less persistent stochastic process. This leaves the average v-u ratio and average job-finding rate virtually unchanged. Table 2 summarizes the parameter choices in the two simulations.

I use equation (6) to find the state-contingent v-u ratio $\theta_{p,s}$ and then simulate the model. That

TABLE 3—LABOR PRODUCTIVITY SHOCKS

		u	v	v/u	f	p
Standard deviation		0.009	0.027	0.035	0.010	0.020
		(0.001)	(0.004)	(0.005)	(0.001)	(0.003)
Quarterly autocorrelation		0.939	0.835	0.878	0.878	0.878
		(0.018)	(0.045)	(0.035)	(0.035)	(0.035)
	u	1	−0.927	−0.958	−0.958	−0.958
			(0.020)	(0.012)	(0.012)	(0.012)
	v	—	1	0.996	0.996	0.995
				(0.001)	(0.001)	(0.001)
Correlation matrix	v/u	—	—	1	1.000	0.999
					(0.000)	(0.001)
	f	—	—	—	1	0.999
						(0.001)
	p	—	—	—	—	1

Notes: Results from simulating the model with stochastic labor productivity. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for productivity.

is, starting with an initial unemployment rate and aggregate state at time 0, I use a pseudo-random number generator to calculate the arrival time of the first Poisson shock. I compute the unemployment rate when that shock arrives, generate a new aggregate state using the discrete-state-space mean-reverting stochastic process described in Appendix C, and repeat. At the end of each period (quarter), I record the aggregate state and the unemployment rate.

I throw away the first 1,000 "quarters" of data. I then use the model to generate 212 data points, corresponding to quarterly data from 1951 to 2003, and detrend the log of the model-generated data using an HP filter with the usual smoothing parameter 10^5. I repeat this 10,000 times, giving me good estimates of both the mean of the model-generated data and the standard deviation across model-generated observations. The latter provides a sense of how precisely the model predicts the value of a particular variable.

E. Results

Table 3 reports the results from simulations of the model with labor productivity shocks. Along some dimensions, notably the co-movement of unemployment and vacancies, the model performs remarkably well. The empirical correlation between these two variables is −0.89, the Beveridge curve. The model actually produces a stronger negative correlation, −0.93,

although the difference is insignificant. It is worth emphasizing that the negative correlation between unemployment and vacancies is a result, not a direct target of the calibration exercise. The model also generates the correct autocorrelation for unemployment, although the behavior of vacancies is somewhat off target. In the data, vacancies are as persistent and volatile as unemployment, while in the model the auto-correlation of vacancies is significantly lower than that of unemployment, while the standard deviation of vacancies is three times as large as the standard deviation of unemployment fluctuations around trend. It is likely that anything that makes vacancies a state variable, such as planning lags, an adjustment cost, or irreversibility in vacancy creation, would increase their persistence and reduce their volatility, bringing the model more in line with the data along these dimensions. Shigeru Fujita (2003) develops a model that adds these realistic features.

But the real problem with the model lies in the volatility of vacancies and unemployment or, more succinctly, in the volatility of the v-u ratio θ and the job-finding rate f. In a reasonably calibrated model, the v-u ratio is less than 10 percent as volatile as in U.S. data. This is exactly the result predicted from the deterministic comparative statics in Section II C. A 1-percent increase in labor productivity p from its average value of 1 raises net labor productivity $p - z$ by about 1.66 percent. Using the deterministic model, I argued before that the elasticity of the

TABLE 4—SEPARATION RATE SHOCKS

		u	v	v/u	f	s
Standard deviation		0.065	0.059	0.006	0.002	0.075
		(0.007)	(0.006)	(0.001)	(0.000)	(0.007)
Quarterly autocorrelation		0.864	0.862	0.732	0.732	0.733
		(0.026)	(0.026)	(0.048)	(0.048)	(0.048)
	u	1	0.999	−0.906	−0.906	0.908
			(0.000)	(0.017)	(0.017)	(0.017)
	v	—	1	−0.887	−0.887	0.888
				(0.020)	(0.020)	(0.021)
Correlation matrix	v/u	—	—	1	1.000	−0.999
					(0.000)	(0.000)
	f	—	—	—	1	−0.999
						(0.000)
	s	—	—	—	—	1

Notes: Results from simulating the model with a stochastic separation rate. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for the separation rate.

v-u ratio with respect to net labor productivity is about 1.03 with this choice of parameters, giving a total elasticity of θ with respect to p of approximately $1.66 \times 1.03 = 1.71$ percent. In fact, the standard deviation of log θ around trend is 1.75 times as large as the standard deviation of log p. Similarly, the job-finding rate is 12 times as volatile in the data as in the model.

Not only is there little amplification, but there is also no propagation of the labor productivity shock in the model. The contemporaneous correlation between labor productivity, the v-u ratio, and the job-finding rate is 1.00. In the data, the contemporaneous correlation between the first two variables is 0.40 and the v-u ratio lags labor productivity by about one year. The empirical correlation between labor productivity and the job-finding rate is similar.

Table 4 reports the results from the model with shocks to the separation rate. These introduce an almost perfectly positive correlation between unemployment and vacancies, an event that has essentially never been observed in the United States at business cycle frequencies (see Figure 3). As a result, separation shocks produce almost no variability in the v-u ratio or the job finding rate. Again, this is consistent with the back-of-the-envelope calculations performed in Section II C, where I argued that the elasticity of the v-u ratio with respect to the separation rate should be approximately −0.10. According to the model, the ratio of the stan-

dard deviations is about 0.08 and the two variables are strongly negatively correlated.

One might be concerned that the disjoint analysis of labor productivity and separation shocks masks some important interaction between the two impulses. Modeling an endogenous increase in the separation rate due to low labor productivity, as in Mortensen and Pissarides (1994), goes beyond the scope of this paper. Instead, I introduce perfectly negatively correlated labor productivity and separation shocks into the basic model. More precisely, I assume $p = z + e^y(p^* - z)$ and $s = e^{-\sigma_s y}s^*$, both nonlinear functions of the same latent variable y. The parameter $\sigma_s > 0$ permits a different volatility for p and s.

I start with the parameterization of the model with only labor productivity shocks and introduce volatility in the separation rate. Table 5 shows the results from a calibration with equal standard deviations in the deviation from trend of the separation rate and labor productivity ($\sigma_s = 1 - z$). The behavior of vacancies in the model is now far from the data, with an autocorrelation of 0.29 (compared to 0.94 empirically) and a correlation with unemployment of −0.43 (−0.89). The difference between model and data is highly significant both economically and statistically. Moreover, although cyclical fluctuations in the separation rate boost the volatility of unemployment considerably, they have a small effect on the cyclical volatility of the v-u ratio and job-finding rate, which remain

TABLE 5—LABOR PRODUCTIVITY AND SEPARATION RATE SHOCKS

		u	v	v/u	f	s	p
Standard deviation		0.031	0.011	0.037	0.014	0.020	0.020
		(0.005)	(0.001)	(0.006)	(0.002)	(0.003)	(0.003)
Quarterly autocorrelation		0.933	0.291	0.878	0.878	0.878	0.878
		(0.020)	(0.085)	(0.035)	(0.035)	(0.035)	(0.035)
	u	1	−0.427	−0.964	−0.964	0.964	−0.964
			(0.068)	(0.011)	(0.011)	(0.011)	(0.011)
	v	—	1	0.650	0.650	−0.649	0.648
				(0.042)	(0.042)	(0.042)	(0.042)
Correlation matrix	v/u	—	—	1	1.000	−1.000	0.999
					(0.000)	(0.000)	(0.001)
	f	—	—	—	1	−1.000	0.999
						(0.000)	(0.001)
	s	—	—	—	—	1	−0.999
							(0.001)
	p	—	—	—	—	—	1

Notes: Results from simulating the model with stochastic but perfectly correlated labor productivity and separations. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process.

at around 10 percent of their empirical values. Smaller fluctuations in the separation rate naturally have a smaller effect, while realistically large fluctuations in the separation rate induce a strong positive correlation between unemployment and vacancies, even in the presence of correlated productivity shocks.

To summarize, the stochastic version of the Pissarides (1985) model confirms that separation shocks induce a positive correlation between unemployment and vacancies. It also confirms that, while labor productivity shocks are qualitatively consistent with a downward-sloping Beveridge curve, the search model does not substantially amplify the extrinsic shocks and so labor productivity shocks induce only very small movements along the curve.

F. Wages

Until this point, I have assumed that the surplus in new matches is divided according to a generalized Nash bargaining solution but have made no assumption about the division of surplus in old matches. Although this is sufficient for determining the response of unemployment and vacancies to exogenous shocks, it does not pin down the timing of wage payments. In this section, I introduce an additional assumption, that the surplus in all matches, new or old, is always divided according to the Nash bargain-

ing solution, as would be the case if wages were renegotiated following each aggregate shock. This stronger restriction pins down the wage as a function of the aggregate state, $w_{p,s}$. This facilitates a more detailed discussion of wages, which serves two purposes. First, modeling wages illustrates that flexibility of the present value of wage payments is critical for many of the results emphasized in this paper. And second, it enables me to relate this paper to a literature that examines whether search models can generate rigid wages. Appendix B proves that a continually renegotiated wage solves

$$(7) \qquad w_{p,s} = (1 - \beta)z + \beta(p + c\theta_{p,s}).$$

This generalizes equation (1.20) in Pissarides (2000) to a stochastic environment.

Consider first the effect of a separation shock on the wage. An increase in the separation rate s induces a slight decline in the v-u ratio (see Table 4), which in turn, by equation (7), reduces wages slightly. Although the direct effect of the shock lowers firms' profits by shortening the duration of matches, the resulting decline in wages partially offsets this, so the drop in the v-u ratio is small.

Second, consider a productivity shock. A 1-percent increase in net labor productivity $p - z$ raises the v-u ratio by about 1 percent (see Table 3). Equation (7) then implies that the net wage $w - z$ increases by about 1 percent,

TABLE 6—BARGAINING POWER SHOCKS

		u	v	v/u	f	w
Standard deviation		0.091	0.294	0.379	0.106	0.011
		(0.018)	(0.086)	(0.099)	(0.028)	(0.015)
Quarterly autocorrelation		0.940	0.837	0.878	0.878	0.864
		(0.023)	(0.046)	(0.036)	(0.036)	(0.047)
	u	1	−0.915	−0.949	−0.949	0.818
			(0.045)	(0.032)	(0.032)	(0.112)
	v	—	1	0.995	0.995	−0.827
				(0.001)	(0.001)	(0.128)
Correlation matrix	v/u	—	—	1	1.000	−0.838
					(0.000)	(0.124)
	f	—	—	—	1	−0.838
						(0.124)
	w	—	—	—	—	1

Notes: Results from simulating the model with stochastic bargaining power. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for the workers' bargaining power.

soaking up most of the productivity shock and giving firms little incentive to create new vacancies. Hence there is a modest increase in vacancies and decrease in unemployment in response to a large productivity shock.

To understand fully the importance of wages for the v-u ratio, it is useful to consider a version of the model in which labor productivity and the separation rate are constant at $p = 1$ and $s = 0.1$, but workers' bargaining power β changes stochastically. An increase in β reduces the profit from creating vacancies, which puts downward pressure on the v-u ratio. It is difficult to know exactly how much variability in β is reasonable, but one can ask how much wage variability is required to generate the observed volatility in the v-u ratio. I assume β is a function of the latent variable y, $\beta = \Phi(y + \Phi^{-1}(\alpha))$, where Φ is the cumulative standard normal distribution. If y were constant at zero, this implies $\beta = \alpha$, but more generally β is simply bounded between 0 and 1. I set the standard deviation of y to $\sigma = 0.099$ and the mean reversion parameter to $\gamma = 0.004$. Although this implies very modest fluctuations in wages—the standard deviation of detrended log wages, computed as in equation (7), is just 0.01—the calibrated model generates the observed volatility in the v-u ratio, with persistence similar to that in the model with labor productivity shocks. Table 6 shows the complete results. Since bargaining power is the only

driving force, wages are counterfactually countercyclical (Abraham and Haltiwanger, 1995). Nevertheless, it seems plausible that a model with a combination of wage and labor productivity shocks could generate the observed behavior of unemployment, vacancies, and real wages. Of course, the unanswered question is what exactly a wage shock is.

If wages are bargained in new matches but then not continually renegotiated, this analysis is inapplicable. Nevertheless, one can prove that the frequency of wage negotiation does not affect the expected present value of wage payments in new matches, but only changes the timing of wage payments. An increase in productivity or decrease in separations raises the present value of wage payments in new jobs and therefore has little effect on the v-u ratio. An increased workers' bargaining power in a new employment relationship induces a large reduction in vacancies and in the v-u ratio.

III. Optimal V-U Fluctuations

Another way to highlight the role played by the Nash bargaining assumption is to examine a centralized economy in which it is possible to sidestep the wage-setting issue entirely.[17] Con-

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[17] A number of papers examine a "competitive search economy," in which firms can commit to wages before

sider a hypothetical social planner who chooses a state-contingent v-u ratio in order to maximize the present discounted value of output net of vacancy creation costs. The planner's problem is represented recursively as

$$rW(p, s, u) = \max_{\theta}\big(zu + p(1 - u) - cu\theta$$

$$+ W_u(p, s, u)(s(1 - u) - uf(\theta))$$

$$+ \lambda\mathbb{E}_{p,s}(W(p', s', u) - W(p, s, u))\big).$$

Instantaneous output is equal to z times the unemployment rate u plus p times the employment rate minus c times the number of vacancies $v \equiv u\theta$. The value changes gradually as the unemployment rate adjusts, with $\dot{u} = s(1 - u) - uf(\theta)$, and suddenly when an aggregate shock changes the state from (p,s) to (p',s') at rate λ.

It is straightforward to verify that the Bellman value W is linear in the unemployment rate, $W_u(p,s,u) = -c/f'(\theta_{p,s})$, and the v-u ratio satisfies

$$\frac{r + s + \lambda}{f'(\theta_{p,s})} - \theta_{p,s}\left(1 - \frac{f(\theta_{p,s})}{\theta_{p,s}f'(\theta_{p,s})}\right)$$

$$= \frac{p - z}{c} + \lambda\mathbb{E}_{p,s}\left(\frac{1}{f'(\theta_{p',s'})}\right).$$

This implicitly defines the optimal $\theta_{p,s}$, independent of the unemployment rate.

With a Cobb-Douglas matching function $m(u,v) = \mu u^{\alpha}v^{1-\alpha}$, this reduces to

$$\frac{r + s + \lambda}{q(\theta_{p,s})} + \alpha\theta_{p,s}$$

$$= (1 - \alpha)\frac{p - z}{c} + \lambda\mathbb{E}_{p,s}\left(\frac{1}{q(\theta_{p',s'})}\right)$$

hiring workers and can increase their hiring rate by promising higher wages (Peters, 1991; Montgomery, 1991; Moen, 1997; Shimer, 1996; Burdett et al., 2001). It is by now well-known that a competitive search equilibrium maximizes output, essentially by creating a market for job applications. This discussion of output-maximizing search behavior therefore also pertains to these models.

a special case of equation (6), with workers' bargaining power β equal to the elasticity α. This generalizes the Hosios (1990) condition for efficiency of the decentralized equilibrium to an economy with stochastic productivity and separation rates. Since the numerical example in Section II E assumes a Cobb-Douglas matching function with $\alpha = \beta$, the equilibrium allocation described in that section solves the social planner's problem. Conversely, if those parameter values describe the U.S. economy, the observed degree of wage rigidity is inconsistent with output maximization.

With other matching functions, the link between the equilibrium with wage bargaining and the solution to the planner's problem is broken. At one extreme, if unemployment and vacancies are perfect substitutes, i.e., $f(\theta) = \alpha_u + \alpha_v\theta$, then the output-maximizing v-u ratio is infinite whenever $\alpha_v(p - z) > c(r + s + \alpha_u)$ and is zero if the inequality is reversed. With near-perfect substitutability, the output-maximizing v-u ratio is very sensitive to current productivity. On the other hand, if unemployment and vacancies are perfect complements, $f(\theta) = \min\langle\alpha_u,\alpha_v\theta\rangle$, the v-u ratio never strays from the efficient ratio α_u/α_v. With imperfect complements, the impact of productivity shocks on the v-u ratio is muffled but not eliminated.

The economics behind these theoretical findings is simple. An increase in labor productivity relative to the value of non-market activity and the cost of advertising a vacancy induces a switch away from the expensive activity, unemployment, and toward the relatively cheap activity, vacancies. The magnitude of the switch depends on how substitutable unemployment and vacancies are in the matching function. If they are strong complements, substitution is nearly impossible and the v-u ratio barely changes. If they are strong substitutes, substitution is nearly costless, and the v-u ratio is highly procyclical.

In the decentralized economy, the extent of substitution between unemployment and vacancies is governed not only by the matching function but also by the bargaining solution, as shown by the comparative statics exercises in Section II C. The Nash bargaining solution effectively corresponds to a moderate degree of substitutability, the Cobb-Douglas case. If wages were more rigid, an increase in productivity would induce more vacancy creation and

less unemployment, analogous to a centralized environment with a high elasticity of substitution in the matching function.

The substitutability of unemployment and vacancies is an empirical issue. Blanchard and Diamond (1989) use nonlinear least squares to estimate a Constant Elasticity of Substitution (CES) matching function on U.S. data. Their point estimate for the elasticity of substitution is 0.74, i.e., slightly less substitutable than the Cobb-Douglas case, although they cannot reject the Cobb-Douglas elasticity of 1. As footnote 8 describes, my data suggest an elasticity slightly in excess of 1, although my point estimate is imprecise. Whether the observed movements in unemployment and vacancies are optimal when viewed through the lens of the textbook search and matching model, therefore, remains an open question.

IV. Related Literature

There is a large literature that explores whether the search model is consistent with the cyclical behavior of labor markets. Some papers look at the implications of the model for the behavior of various stocks and flows, including the unemployment and vacancy rates, but do not examine the implicit magnitude of the exogenous impulses. Others assume that business cycles are driven by fluctuations in the separation rate s. These papers either impose exogenously or derive within the model a counterfactually constant v-u ratio θ. A third group of papers has tried but failed to reconcile the procyclicality of the v-u ratio with extrinsic shocks of a plausible magnitude.

Papers by Abraham and Lawrence Katz (1986), Blanchard and Diamond (1989), and Cole and Rogerson (1999) fit into the first category, matching the behavior of labor market stocks and flows by sidestepping the magnitude of impulses. For example, Abraham and Katz (1986) argue that the downward-sloping Beveridge curve is inconsistent with models in which unemployment is driven by fluctuations in the separation rate, notably David Lilien's (1982) sectoral shifts model. That leads them to advocate an alternative in which unemployment fluctuations are driven by aggregate disturbances, e.g., productivity shocks. Unfortunately, they fail to examine the magnitude of shocks needed to deliver the observed shifts along the Beveridge curve. Blanchard and Dia-

mond (1989) also focus on the negative correlation between unemployment and vacancies, but they do not model the supply of jobs and hence do not explain why there are so few vacancies during recessions. Instead, they assume the total stock of jobs follows an exogenous stochastic process. This paper pushes the cyclicality of the v-u ratio to the front of the picture. Likewise, Cole and Rogerson (1999) argue that the Mortensen and Pissarides (1994) model can match a variety of business cycle facts, but they do so in a reduced form model that treats fluctuations in the job-finding rate, and hence implicitly in the v-u ratio, as exogenous.

The second group of papers, including work by Michael Pries (2004), Gary Ramey and Joel Watson (1997), Wouter Den Haan et al. (2000), and Joao Gomes et al. (2001), assumes that employment fluctuations are largely due to time-variation in the separation rate, minimizing the role played by the observed cyclicality of the v-u ratio. These papers typically deliver rigid wages from a search model, consistent with the findings in Section II E. Building on the ideas in Hall (1995), Pries (2004) shows that a brief adverse shock that destroys some old employment relationships can generate a long transition period of high unemployment, as the displaced workers move through a number of short-term jobs before eventually finding their way back into long-term relationships. During this transition process, the v-u ratio remains constant, since aggregate economic conditions have returned to normal. Equivalently, the economy moves along an upward-sloping Beveridge curve during the transition period, in contradiction to the evidence. Ramey and Watson (1997) argue that two-sided asymmetric information generates rigid wages in a search model. But in their model, shocks to the separation rate are the only source of fluctuations in unemployment. The job-finding rate $f(\theta)$ is exogenous and constant, which is equivalent to assuming that vacancies are proportional to unemployment. This is probably an important part of the explanation for why their model produces rigid wages. Den Haan et al. (2000) show that fluctuations in the separation rate amplify productivity shocks in a model similar to the one examined here; however, they do not discuss the cyclical behavior of the v-u ratio. Similarly, Gomes et al. (2001) sidestep the v-u issue by looking at a model in which the job-finding rate

is exogenous and constant, i.e., vacancies are proportional to unemployment. Again, this helps keep wages relatively rigid in their model.

Mortensen and Pissarides (1994) have probably the best known paper in this literature. In their three-state "illustrative simulation," the authors introduce, without comment, enormous productivity or leisure shocks into their model. Average labor productivity minus the value of leisure $p - z$ is approximately three times as high in the good state as in the bad state.[18] This paper confirms that in response to such large shocks, the v-u ratio should also be about three times as large in the good state as in the bad state, but argues that there is no evidence for these large shocks in the data. Even if one accepts the magnitude of the implied impulses, Mortensen and Pissarides (1994) still deliver only a correlation of -0.26 between unemployment and vacancies, far lower than the empirical value of -0.88. This is probably because of the tension between productivity shocks, which put the economy on a downward-sloping Beveridge curve, and endogenous movements in the separation rate, which have the opposite effect. Monika Merz (1995) and David Andolfatto (1996) both put the standard search model into a real business cycle framework with intertemporal substitution of leisure, capital accumulation, and other extensions. Neither paper can match the negative correlation between unemployment and vacancies, and both papers generate real wages that are too flexible in response to productivity shocks. Thus these papers encounter the problem I highlight in this paper, although they do not emphasize this shortcoming of the search model. Finally, Hall (2003), building on an earlier version of this paper, discusses some of the same issues. Hall (2005) proposes one possible solution: real wages are determined by a social norm that does not change over the business cycle.

V. Conclusion

I have argued in this paper that a search and matching model in which wages are determined by Nash bargaining cannot generate substantial movements along a downward-sloping Beveridge curve in response to shocks of a plausible magnitude. A labor productivity shock results primarily in higher wages, with little effect on the v-u ratio. A separation shock generates an increase in both unemployment and vacancies. It is important to stress that this is not an attack on the search approach to labor markets, but rather a critique of the commonly-used Nash bargaining assumption for wage determination. An alternative wage determination mechanism that generates more rigid wages in new jobs, measured in present value terms, will amplify the effect of productivity shocks on the v-u ratio, helping to reconcile the evidence and theory. Countercyclical movements in workers' bargaining power provide one such mechanism, at least in a reduced-form sense.

If the matching function is Cobb-Douglas, the observed behavior of the v-u ratio is not socially optimal for plausible parameterizations of the model, but this conclusion could be overturned if the elasticity of substitution between unemployment and vacancies in the matching function is sufficiently large. Estimates of a CES matching function are imprecise, so it is unclear whether observed wages are "too rigid."

One way to generate more rigid wages in a theoretical model is to introduce considerations whereby wages affect the worker turnover rate. For example, in the Burdett and Mortensen (1998) model of on-the-job search, firms have an incentive to offer high wages in order to attract workers away from competitors and to reduce employees' quit rate. The distribution of productivity affects an individual firm's wage offer and vacancy creation decisions in complex ways, breaking the simple link between average labor productivity and the v-u ratio in the Pissarides (1985) model. In particular, a shift in the productivity distribution that leaves average labor productivity unchanged may appreciably affect average wages and hence the equilibrium v-u ratio.

Another possibility is to drop some of the informational assumptions in the standard search model.[19] Suppose that when a worker

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[18] This calculation would be easy in the absence of heterogeneity, i.e., if their parameter σ were equal to zero. Then $\bar{p} - z$ would take on three possible values: 0.022, 0.075, and 0.128, for a six-fold difference in $\bar{p} - z$ between the high and low states.

[19] Ramey and Watson (1997) develop a search model with two-sided asymmetric information. Because they assume workers' job finding rate is exogenous and acyclic,

and firm meet, they draw an idiosyncratic match-specific productivity level from some distribution F. Workers and firms know about aggregate variables, including the unemployment rate and the distribution F, but only the firm knows the realized productivity level. Bargaining proceeds as follows: with probability $\beta \in (0,1)$, a worker makes a take-it-or-leave-it wage demand, and otherwise the firm makes a take-it-or-leave-it offer. Obviously the firm extracts all the rents from the employment relationship when it makes an offer. But if the uninformed worker makes the offer, she faces a tradeoff between demanding a higher wage and reducing her risk of unemployment, so the wage depends on the hazard rate of the distribution F. This again breaks the link between average labor productivity and the equilibrium v-u ratio. Exploring whether either of these models, or some related model, deliver substantial fluctuations in the v-u ratio in response to plausible impulses remains a topic for future research.

APPENDIX A: DERIVATION OF THE EQUATION FOR SURPLUS (4)

For notational simplicity alone, assume the wage payment depends only on the aggregate state, $w_{p,s}$, not on the history of the match. I return to this issue at the end of this section. Define $U_{p,s}$, $E_{p,s}$, and $J_{p,s}$ to be the state-contingent present value of an unemployed worker, employed worker, and filled job, respectively. They are linked recursively by:

$$(8) \qquad rU_{p,s} = z + f(\theta_{p,s})(E_{p,s} - U_{p,s})$$
$$+ \lambda(\mathbb{E}_{p,s}U_{p',s'} - U_{p,s})$$

$$(9) \qquad rE_{p,s} = w_{p,s} - s(E_{p,s} - U_{p,s})$$
$$+ \lambda(\mathbb{E}_{p,s}E_{p',s'} - E_{p,s})$$

$$(10) \qquad rJ_{p,s} = p - w_{p,s} - sJ_{p,s}$$
$$+ \lambda(\mathbb{E}_{p,s}J_{p',s'} - J_{p,s}).$$

Equation (8) states that the flow value of an unemployed worker is equal to her value of

leisure z plus the probability she finds a job $f(\theta_{p,s})$ times the resulting capital gain $E - U$ plus the probability of an aggregate shock times that capital gain. Equation (9) expresses a similar idea for an employed worker, who receives a wage payment $w_{p,s}$ but loses her job at rate s. Equation (10) provides an analogous recursive formulation for the value of a filled job. Note that a firm is left with nothing when a filled job ends.

Sum equations (9) and (10) and then subtract equation (8), defining $V_{p,s} \equiv J_{p,s} + E_{p,s} - U_{p,s}$:

$$(11) \qquad rV_{p,s} = p - z - f(\theta_{p,s})(E_{p,s} - U_{p,s})$$
$$- sV_{p,s} + \lambda(\mathbb{E}_{p,s}V_{p',s'} - V_{p,s}).$$

In addition, the Nash bargaining solution implies that the wage is set so as to maximize the Nash product $(E_{p,s} - U_{p,s})^\beta J_{p,s}^{1-\beta}$, which gives

$$(12) \qquad \frac{E_{p,s} - U_{p,s}}{\beta} = V_{p,s} = \frac{J_{p,s}}{1-\beta}.$$

Substituting for $E - U$ in equation (11) yields equation (4).

If I allow wages to depend in an arbitrary manner on the history of the match, this would affect the Bellman values E and J; however, the wage, and therefore the history-dependence, would drop out when summing the Bellman equations for E and J. In other words, the match surplus V is unaffected by the frequency of wage renegotiation.

APPENDIX B: DERIVATION OF THE WAGE EQUATION

Assume that wages are continually renegotiated, so the wage depends only on the current aggregate state (p,s). Eliminate current and future values of J from equation (10) using equation (12):

$$w_{p,s} = p - (r + s + \lambda)(1 - \beta)V_{p,s}$$
$$+ \lambda\mathbb{E}_{p,s}(1 - \beta)V_{p',s'}.$$

Similarly, eliminate current and future values of V using (5):

$$w_{p,s} = p - \frac{(r + s + \lambda)c}{q(\theta_{p,s})} + \lambda\mathbb{E}_{p,s}\frac{c}{q(\theta_{p',s'})}.$$

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their results are not directly applicable to this analysis, although their methodology may prove useful.

Finally, replace the last two terms using equation (6) to get equation (7).

Appendix C: The Stochastic Process

The text describes a continuous state space approximation to the discrete state space model used in both the theory and simulations. Here I describe the discrete state space model and show that it asymptotes to an Ornstein-Uhlenbeck process.

Consider a random variable y that is hit with shocks according to a Poisson process with arrival rate λ. The initial value of y lies on a discrete grid,

$$y \in Y \equiv \{-n\Delta, -(n-1)\Delta, \dots ,$$

$$0, \dots, (n-1)\Delta, n\Delta\}$$

where $\Delta > 0$ is the step size and $2n + 1 \geq 3$ is the number of grid points. When a shock hits, the new value y' either moves up or down by one grid point:

$$y' = \begin{cases} y + \Delta \\ y - \Delta \end{cases} \text{ with probability } \begin{cases} \dfrac{1}{2}\left(1 - \dfrac{y}{n\Delta}\right) \\ \dfrac{1}{2}\left(1 + \dfrac{y}{n\Delta}\right) \end{cases}.$$

Note that although the step size is constant, the probability that $y' = y + \Delta$ is smaller when y is larger, falling from 1 at $y = -n\Delta$ to zero at $y = n\Delta$.

It is trivial to confirm that $y' \in Y$, so the state space is discrete. To proceed further, define $\gamma \equiv \lambda/n$ and $\sigma \equiv \sqrt{\lambda}\Delta$. For any fixed $y(t)$, I examine the behavior of $y(t + h)$ over an arbitrarily short time period h. For sufficiently short h, the probability that two Poisson shocks arrive is negligible, and so $y(t + h)$ is equal to $y(t)$ with probability $1 - h\lambda$, has increased by Δ with probability $h\lambda(1 - y(t)/n\Delta)/2$, and has decreased by Δ with probability $h\lambda (1 + y(t)/n\Delta)/2$. Adding this together shows

$$\mathbb{E}\left[y(t + h) - y(t)|y(t)\right] = -\frac{h\lambda}{n}\,y(t) = -h\gamma y(t).$$

Next, the conditional variance of $y(t + h) - y(t)$ can be decomposed into

$$\text{Var}[y(t + h) - y(t)|y(t)]$$

$$= \mathbb{E}[(y(t+h) - y(t))^2|y(t)]$$

$$- (\mathbb{E}[y(t + h) - y(t)|y(t)])^2.$$

The first term evaluates to $h\lambda\Delta^2$ over a sufficiently short time interval h, since it is equal to Δ^2 if a shock, positive or negative, arrives and zero otherwise. The second term is $(h\gamma y(t))^2$, and so is negligible over a short time interval h. Thus

$$\text{Var}[y(t + h) - y(t)|y(t)] = h\lambda\Delta^2 = h\sigma^2.$$

Putting this together, we can represent the stochastic process for y as

$$dy = -\gamma y\,dt + \sigma dx$$

where for $t > 0$, the expected value of $x(t)$ given $x(0)$ is $x(0)$ and the conditional variance is t. This is similar to a Brownian motion, except that the innovations in x are not Gaussian, since y is constrained to lie on a discrete grid.

Now suppose one changes the three parameters of the stochastic process, the step size, arrival rate of shocks, and number of steps, from (Δ,λ,n) to $(\Delta\sqrt{\varepsilon}, \lambda/\varepsilon, n/\varepsilon)$ for any $\varepsilon > 0$. It is easy to verify that this does not change either the autocorrelation parameter $\gamma = \lambda/n$ or the instantaneous variance $\sigma = \sqrt{\lambda}\Delta$. But as $\varepsilon \to 0$, the distribution of the innovation process x converges to a normal by the Central Limit Theorem. Equivalently, y converges to an Ornstein-Uhlenbeck process.[20] This observation is also useful for computation. It is possible to find a solution on a coarse grid and then to refine the grid by decreasing ε without substantially changing the results.

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[20] Notably, for large n it is extraordinarily unlikely that the state variable reaches its limiting values of $\pm n\Delta$. The unconditional distribution of the state variable is approximately normal with mean zero and standard deviation $\sigma/\sqrt{2\gamma} = \Delta\sqrt{n/2}$. The limiting values of the state variables therefore lie $\sqrt{2n}$ standard deviations above and below the mean. If $n = 1000$, as is the case in the simulations, one should expect to observe such values approximately once in 10^{436} periods.

REFERENCES

Abowd, John M. and Zellner, Arnold. "Estimating Gross Labor-Force Flows." *Journal of Business and Economic Statistics*, 1985, *3*(3), pp. 254–83.

Abraham, Katharine G. and Katz, Lawrence F. "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?" *Journal of Political Economy*, 1986, *94*(3), pp. 507–22.

Abraham, Katherine G. "Help-Wanted Advertising, Job Vacancies, and Unemployment." *Brookings Papers on Economic Activity*, 1987, *0*(1), pp. 207–43.

Abraham, Katharine G. and Haltiwanger, John C. "Real Wages and the Business Cycle." *Journal of Economic Literature*, 1995, *33*(3), pp. 1215–64.

Abraham, Katharine G. and Shimer, Robert. "Changes in Unemployment Duration and Labor-Force Attachment," in Alan B. Krueger and Robert M. Solow, eds., *The roaring nineties: Can full employment be sustained?* New York: Russell Sage Foundation, 2001, pp. 367–420.

Andolfatto, David. "Business Cycles and Labor-Market Search." *American Economic Review*, 1996, *86*(1), pp. 112–32.

Blanchard, Olivier Jean and Diamond, Peter. "The Beveridge Curve." *Brookings Papers on Economic Activity*, 1989, *0*(1), pp. 1–60.

Blanchard, Olivier Jean and Diamond, Peter. "The Cyclical Behavior of the Gross Flows of U.S. Workers." *Brookings Papers on Economic Activity*, 1990, *0*(2), pp. 85–143.

Bleakley, Hoyt; Ferris, Ann E. and Fuhrer, Jeffrey C. "New Data on Worker Flows during Business Cycles." *New England Economic Review*, 1999, *0*(0), pp. 49–76.

Burdett, Kenneth and Mortensen, Dale. "Equilibrium Wage Differentials and Employer Size." *International Economic Review*, 1998, *39*(2), pp. 257–74.

Burdett, Kenneth; Shouyong, Shi and Wright, Randall. "Pricing and Matching with Frictions." *Journal of Political Economy*, 2001, *105*(5), pp. 1060–85.

Cole, Harold L. and Rogerson, Richard. "Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?" *International Economic Review*, 1999, *40*(4), pp. 933–59.

den Haan, Wouter J.; Ramey, Garey and Watson, Joel. "Job Destruction and Propagation of Shocks." *American Economic Review*, 2000, *90*(3), pp. 482–98.

Flinn, Christopher J. and Heckman, James J. "Are Unemployment and Out of the Labor Force Behaviorally Distinct Labor Force States?" *Journal of Labor Economics*, 1983, *1*(1), pp. 28–42.

Fujita, Shigeru. "The Beveridge Curve, Job Creation and the Propagation of Shocks." Unpublished Paper, 2003.

Gomes, Joao; Greenwood, Jeremy and Rebelo, Sergio. "Equilibrium Unemployment." *Journal of Monetary Economics*, 2001, *48*(1), pp. 109–52.

Hall, Robert E. "Lost Jobs." *Brookings Papers on Economic Activity*, 1995, *0*(1), pp. 221–56.

Hall, Robert E. "Modern Theory of Unemployment Fluctuations: Empirics and Policy Applications." *American Economic Review*, 2003, *93*(2), pp. 145–50.

Hall, Robert E. "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market." Unpublished Paper, 2004.

Hall, Robert E. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 2005, *95*(1), pp. 53–69.

Hosios, Arthur J. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies*, 1990, *57*(2), pp. 279–98.

Jones, Stephen R. G. and Riddell, W. Craig. "The Measurement of Unemployment: An Empirical Approach." *Econometrica*, 1999, *67*(1), pp. 147–61.

Juhn, Chinhui; Murphy, Kevin M. and Topel, Robert H. "Why Has the Natural Rate of Unemployment Increased over Time?" *Brookings Papers on Economic Activity*, 1991, *0*(2), pp. 75–126.

Lilien, David M. "Sectoral Shifts and Cyclical Unemployment." *Journal of Political Economy*, 1982, *90*(4), pp. 777–93.

Lucas, Robert E., Jr. and Rapping, Leonard A. "Real Wages, Employment, and Inflation." *Journal of Political Economy*, 1969, *77*(5), pp. 721–54.

Merz, Monika. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics*, 1995, *36*(2), pp. 269–300.

Moen, Espen R. "Competitive Search Equilibrium." *Journal of Political Economy*, 1997, *105*(2), pp. 385–411.

Montgomery, James. "Equilibrium Wage Dispersion and Interindustry Wage Differentials." *Quarterly Journal of Economics*, 1991, *106*(1), pp. 163–79.

Mortensen, Dale T. and Pissarides, Christopher A. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, 1994, *61*(3), pp. 397–415.

Peters, Michael. "Ex Ante Price Offers in Matching Games: Non-Steady States." *Econometrica*, 1991, *59*(5), pp. 1425–54.

Petrongolo, Barbara and Pissarides, Christopher A. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 2001, *39*(2), pp. 390–431.

Pissarides, Christopher A. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *American Economic Review*, 1985, *75*(4), pp. 676–90.

Pissarides, Christopher A. *Equilibrium unemployment theory*, 2nd ed. Cambridge, MA: MIT Press, 2001.

Poterba, James M. and Summers, Lawrence H. "Reporting Errors and Labor Market Dynamics." *Econometrica*, 1986, *54*(6), pp. 1319–38.

Pries, Michael J. "Persistence of Employment Fluctuations: A Model of Recurring Job Loss." *Review of Economic Studies*, 2004, *71*(1), pp. 193–215.

Ramey, Garey and Watson, Joel. "Contractual Fragility, Job Destruction, and Business Cycles." *Quarterly Journal of Economics*, 1997, *112*(3), pp. 873–911.

Shimer, Robert. "Contracts in a Frictional Labor Market." Unpublished Paper, 1996.

Shimer, Robert. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies: Evidence and Theory." National Bureau of Economic Research, Inc., NBER Working Papers: 9536, 2003.

Shimer, Robert. "Search Intensity." Unpublished Paper, 2004.

Solon, Gary; Barsky, Robert and Parker, Jonathan A. "Measuring the Cyclicality of Real Wages: How Important Is Composition Bias?" *Quarterly Journal of Economics*, 1994, *109*(1), pp. 1–25.

Taylor, Howard, and Samuel Karlin. *An introduction to stochastic modeling*. San Diego: Academic Press, 3rd ed., 1998.