Problem Set 3. Econ 202B, 2021

updated on January 24th, 2021

In his celebrated 2005 AER paper, Robert Shimer studies the business cycle implications of the DMP model. He first shows that, in the data, fluctuations in market tightness, $\theta = v/u$, are about 20 time larger than fluctuations in labor productivity. He then sets up, solves and calibrates a stochastic version of the DMP model to study whether labor productivity shocks can be amplified by a factor of 20, as suggested by the data. He finds that, in equilibrium, market tightness θ does responds endogenously to labor productivity shocks – simply because firms have less incentives to post vacancy when productivity is low and vice versa when it is high, just as in the comparative static of θ with respect to $y - z$ we studied in class. However, there is almost no amplification: fluctuations in market tightness are about the same as fluctuations in productivity, instead of 20 larger in the data. This observation is often called the "Shimer puzzle" and was the starting point of a very active literature trying to reconcile the quantitative implications of the model with the data. Our objective in this homework is to study the argument in Shimer as well as the quantitative resolution proposed by Hagerdon and Manovskii in 2008.

1. Read the introduction and Section I (about labor market facts) of Shimer's paper.

2. Here is a preliminary question about Markov chains. Consider a finite-state Markov chain with state denoted by s_t at time t. Assume that, every period, the state either remains the same ($s_{t+1} = s_t$) with probability $\rho \in (0, 1)$ or, with probability $1 - \rho$, a new state $s_{t+1} = s'$ is drawn with probability $\pi(s')$.

   (a) Show that the stationary distribution of the Markov chain is $\pi(s)$.

   (b) Consider the Markov chain $x_t \equiv x(s_t)$. Shows that the first order autocorrelation of x_t is ρ. Shows that the k-th order autocorrelation is ρ^k.

3. Now consider the DMP model we studied in class except that labor productivity is stochastic: $y_t = y(s_t)$, where s_t is the Markov chain described above. Assume the following timing within period t:

   - At the beginning of period t, a new state s_t realizes;
   - Firms and workers negotiate wage and produce according to a labor productivity $y(s_t)$;

- Vacant firms post vacancies resulting in Market tightness $\theta(s_t)$;
- Unemployed workers and vacancies are matched to start production next period.

(a) Write the Bellman equation for the value of an employed worker, an unemployed worker, a filled job, and a vacancy at time t.

(b) Show that the surplus equation and the free-entry conditions we derived in class now takes the form:

$$\Sigma(s) = y(s) - z + \beta\left[1 - \delta - \phi\,\theta(s)q(\theta(s))\right]\mathbb{E}\left[\Sigma(s') \mid s\right]$$
$$c = \beta(1-\phi)q(\theta(s))\mathbb{E}\left[\Sigma(s') \mid s\right],$$

where s' denote next-period state and where expectations are taken conditional on the current state, s.

4. Now consider a first-order approximation of this system of equation by assuming that

$$y(s) = y + h\,\Delta y(s),$$

where y is the mean productivity, h is a small real number, and the $\Delta y(s)$ represent mean zero shocks around the mean productivity, that is $\mathbb{E}\left[\Delta y(s) = 0\right]$. Guess that, up to second order terms (i.e., in order h^2 or higher):

$$\Sigma(s) = \Sigma + h\,\Delta\Sigma(s)$$
$$\theta(s) = \theta + h\,\Delta\theta(s).$$

(a) Using a first-order Taylor expansion of the equilibrium equation in $h \to 0$, derive a system of equation for Σ, θ, $\Delta\Sigma(s)$, and $\Delta\theta(s)$. Show that Σ and θ solve the same equation as in the deterministic DMP model we studied in class.

(b) Argue that $\mathbb{E}\left[\Delta\Sigma(s\right] = \mathbb{E}\left[\Delta\theta(s\right] = 0$.

(c) Show using the free entry condition that:

$$\frac{\Delta\theta(s)}{\theta} = \frac{\rho}{\alpha}\frac{\Delta\Sigma(s)}{\Sigma} \text{ where } \alpha \equiv -\frac{\theta}{q}\frac{dq}{d\theta}.$$

Explain why fluctuations in surplus (on the right-hand side) matter for fluctuations in market tightness (on the left-hand side). Explain why the elasticity of the vacancy-filling rate, α, and the first-order autocorrelation, ρ, enters in the equation.

(d) Derive a closed form solution for $\Delta\Sigma(s)$ as a function of $\Delta y(s)$ and show that

$$\frac{\Delta\theta(s)}{\theta} = \Gamma \times \frac{\Delta y(s)}{y} \text{ where } \Gamma \equiv \frac{y/z}{y/z-1}\frac{\rho}{\alpha}\frac{1-\beta\left(1-\delta-\phi\theta q(\theta)\right)}{1-\beta\rho\left(1-\delta-\phi\theta q(\theta)/\alpha\right)}.$$

For this derivation, it will be useful to keep in mind that the elasticity of the job finding rate, $\theta q(\theta)$, is equal to $1-\alpha$ (why?). What does Γ represent? Why is this object crucial to understanding the amplification of labor productivity shock into changes in labor market tightness? How does Γ depends on y/z and ρ, and why?

5. Assume that the length of a period is one month. Use the table and figures offered by Shimer in Section 1 to set reasonable values for the parameters α, δ, and $\theta q(\theta)$. Set as Shimer does, $y/z = 2.5$, because this corresponds to the replacement rate of unemployment benefits in the US. Assume as well that $\phi = \alpha$ (the Hosios condition). Finally, use Table 1 to set the value of the monthly autocorrelation of labor productivity, ρ (keep in mind the the number in Table 1 is a quarterly autocorrelation).

6. Now keep α, δ, $\theta q(\theta)$ and ρ as above, and graph the set of y/z and ϕ that would be consistent with $\Gamma = 20$. Explain how your figure relates to the argument made in the introduction of Hagedorn and Manovskii.