Chapter 11

Sequential Search

In a seminal paper on the economics of information, Stigler (1961) defines "search" as the process through which agents gather information in order to reduce their ignorance and make better decisions. A canonical economic example is that of workers who ignore the wage they would be offered for different jobs. They need to to engage in costly or time consuming search activity to locate the best job: they must study job offers, apply, and interview. In the last paragraph of his paper, Stigler notes that:

> "Ignorance is like subzero weather: by a sufficient expenditure its effect upon people can be kept within tolerable or even comfortable bound, but it would be wholly uneconomic entirely to eliminate all its effects. And just as an analysis of man's shelter and apparel would be somewhat incomplete if cold weather is ignored, so also our understanding of economic life will be incomplete if we do not systematically take account of the cold winds of ignorance"

Given that it is suboptimal to wholly eliminate the effects of subzero weather, one is left to wonder how to optimally reduce the effect of subzero weather. Or, going back to search, one is left to wonder how much search effort should economic agents exert in order to reduce their ignorance? McCall (1970) model, which we study in Section 11.1 addresses this question. It analyzes the manner in which an agent should search optimally in order to reduce but not fully eliminate its ignorance. This is a partial equilibrium model, in the sense that the distribution of wages is exogenously given. In Section 11.2 and 11.3 we go on and endogenize the distribution of wages, by studying the problem of firms who set their wage optimally, taking as given workers' optimal search behavior.

11.1 The McCall (1970) model

We start with a canonical model in the search-and-matching literature: the problem of a worker who sample sequentially job offers, and chooses which offer to accept or reject. We assume that the worker is risk-neutral, infinitely lived, with discount factor $\beta \in (0, 1)$, and so maximizes the expected present value of its consumption stream:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t c_t\right].$$

Each period a worker can either be employed at some wage w or unemployed. When employed, he may lose his job with probability $\delta \in (0,1)$. When unemployed, he receives unemployment benefit $b > 0$ and receives a job offer with probability λ. Conditional on getting a job offer, the wage offered w is drawn from a distribution with CDF $F(w)$ over support $[0, \bar{w}]$, where $\bar{w} > b$.[1] If an unemployed worker accepts the job offer, he will be employed in the next period. If he rejects the offer, he will remain unemployed. For now, we abstract from on-the-job search. In this section we take the distribution of wage offers, $F(w)$, as exogenous, but in the following two sections we will endogenize it by studying the problem of firms who post wages in order to attract workers.

11.1.1 Solving the worker's problem

A worker can be in either one of two states: unemployed, which we denote by "U", or employed, which we denote by "E". We let V_U denote the maximum attainable utility of a worker in the unemployment state and $V_E(w)$ the maximum attainable utility of a worker in the employment state, with wage w. The Bellman equations for V_U and $V_E(w)$ are:

$$V_U = b + \beta\left[\lambda\int_0^{\bar{w}}\max\left\{V_E(w), V_U\right\}dF(w) + (1-\lambda)V_U\right] \tag{11.1.1}$$

$$V_E(w) = w + \beta\left[(1-\delta)V_E(w) + \delta V_U\right]. \tag{11.1.2}$$

The value function V_U equals current period flow income b plus discounted continuation value, which consists of two parts. The first part is that, with probability λ, the worker receives a job offer, w, drawn from the wage distribution, $F(w)$. The worker then evaluates the value of being employed at that wage, $V_E(w)$, against the value of staying unemployed, V_U, and takes the maximum of the two. The second part is that, with probability $1-\lambda$, he does not receive any offer, in which case his value remains V_U. The Bellman equation for the employed worker, $V_E(w)$, can be interpreted along similar lines: it equals the current period flow income w plus discounted expected continuation value over remaining employed or transitioning to unemployment.

Rearranging the Bellman equation (11.1.2) for $V_E(w)$, we obtain:

$$V_E(w) = \frac{w + \beta\delta V_U}{1-\beta(1-\delta)}.$$

[1] Our definition allows $F(w)$ to be continuous, discrete, or a mixture of both. $F(w)$ is increasing in w, $F(0) = 0$, and $F(\bar{w}) = 1$. In particular, notice that the support $[0, \bar{w}]$ is the set of wages that have positive probability according to $F(w)$. Formally, w is in the support of F if $F(w+\varepsilon) - F(w-\varepsilon) > 0$ for all $\varepsilon > 0$.

After substituting this expression into the Bellman equation for V_U, we find:

$$\begin{aligned} V_U &= b + \beta\lambda \int_0^{\bar{w}} \max\left\{\frac{w+\beta\delta V_U}{1-\beta(1-\delta)}, V_U\right\} dF(w) + \beta(1-\lambda)V_U \\ &= b + \beta\lambda \int_0^{\bar{w}} \max\left\{\frac{w-(1-\beta)V_U}{1-\beta(1-\delta)}, 0\right\} dF(w) + \beta V_U. \end{aligned}$$

After rearranging:

$$(1-\beta)V_U = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_0^{\bar{w}} \max\{w-(1-\beta)V_U, 0\} dF(w). \tag{11.1.3}$$

This equation shows that a worker optimal policy function will be characterized by a reservation wage. Namely, the worker will accept any offer greater than $(1-\beta)V_U$, and reject otherwise. Precisely:

Proposition 11.1.1 (Reservation Wage)**.** *The optimal policy of an unemployed worker is to accept all wage offers above the reservation wage $w^* \equiv (1-\beta)V_U$, which is the unique solution of:*

$$w^* = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*}^{\bar{w}} [1-F(w)]\, dw. \tag{11.1.4}$$

Let us derive the reservation wage equation, (11.1.4). Substituting our notation w^* into equation (11.1.3), we obtain

$$w^* = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_0^{\bar{w}} \max(w-w^*, 0)\, dF(w) = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*}^{\bar{w}} (w-w^*)\, dF(w).$$

Integrating by parts we obtain

$$w^* = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \left\{-\left[(1-F(w))(w-w^*)\right]_{w^*}^{\bar{w}} + \int_{w^*}^{\bar{w}} [1-F(w)]\, dw\right\}.$$

Note that $\left[(1-F(w))(w-w^*)\right]_{w^*}^{\bar{w}} = 0$. Rearranging leads to the reservation wage equation (11.1.4).

Therefore finding w^* boils down to finding a fixed point of the reservation wage equation. We now show that the existence of a solution to the reservation wage equation. The right-hand side (RHS) of equation (11.1.4), $h(w^*) \equiv b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*}^{\bar{w}} [1-F(w)]\, dw$, is continuous and strictly decreasing in $w^* \in [0, w^*]$, reaching the value b when $b = \bar{w}$. The LHS, on the other hand, is the 45 degree line. As is clear from Figure 11.1, the LHS and RHS must intersect.

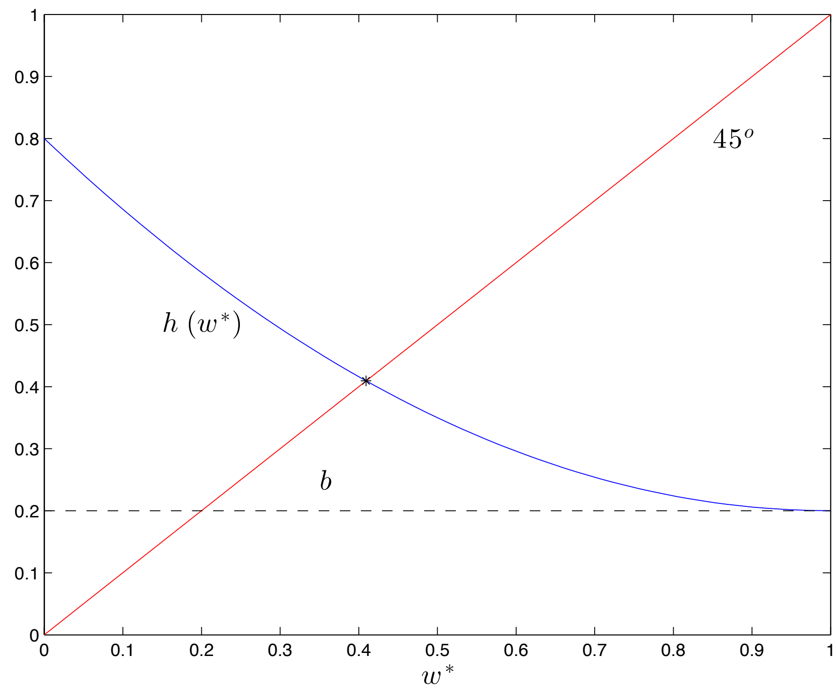
Note that the reservation wage is less than $\bar{w}$: although workers search for a good wage, they do not continue their search until they find the highest one, as this would be too time consuming. Instead, workers find it optimal to accept any wage higher than some threshold $w^\star$ which is less than $\bar{w}$.

Proposition 11.1.2 (Comparative Statics)**.** *The reservation wage $w^*(b, \lambda, \delta, \beta, F(\cdot))$*

- *is increasing in b, λ, and β and is decreasing in δ;*
- *increases in response to a FOSD shift in $F(\cdot)$; and*

Figure 11.1: Finding the Reservation Wage

$$b = \frac{1}{5},\ \frac{\lambda\beta}{1-\beta(1-\delta)} = \frac{6}{5},\ F(w) = w \text{ over } [0,1]$$



- *decreases in response to a SOSD shift in $F(\cdot)$ while leaving the mean of the distribution unchanged.*

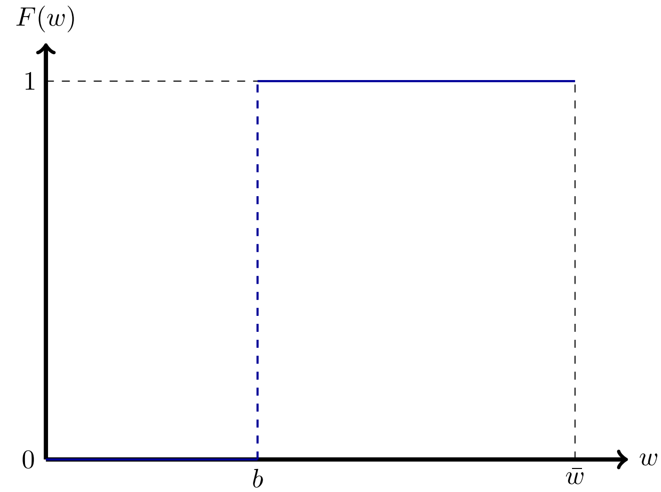
Increase in b, λ, or β. Graphically, the RHS shifts upward and therefore reservation wage increases. Workers are more willing to wait for the good jobs if unemployment benefit is more generous, if job finding probability is higher, or if they are more patient.

Increase in δ. Graphically, the RHS shifts downward and therefore reservation wage decreases. Less stable job markets are associated with lower reservation wage. What is going on here is that, by taking a job, one gives up the option value of search. When δ is large, the option value is very small because the worker expects to loose his job very quickly.

First-order stochastic dominance (FOSD) shift in the wage distribution (see Appendix B for a definition). $F(w) \underset{FOSD}{\succ} G(w)$. One sees from the figure that the CDF G has more probability mass on low values of w, while F has more probability mass on high values of w. So, in this sense, a worker drawing from F will draw higher wage than a worker drawing from G. The RHS shifts upward and reservation wage increases.

Second-order stochastic dominance (SOSD) shift in the wage distribution (see Appendix B for a definition). $F(w) \underset{SOSD}{\succ} G(w)$. F is less risky than G because lots of the probability mass is concentrated near the inflection point. For G, on the other hand, there is lots of probability mass near the tail (close to zero or to $\bar{w}$). In the mathematical definition of SOSD, these considerations

Figure 11.2: Equilibrium wage distribution in Diamond paradox



show up because the integrand $1-F(x)$ is greater than $1-G(x)$ for small x, since F is smaller than G. The integral must eventually be equal because the two distribution have the same mean. More intuitively, with G there are more chances of getting both higher and lower wages. Think for example of lowering the wage offers in the left-tail of the distribution, and raising the wage offers in the right tail. Lowering offers in the left tail does not hurt the worker, since the worker was not accepting these offers anyway. In contrast, the worker is accepting offers in the right-tail of the distribution: hence, raising these wage offers make the worker better off, increases the option value of continuing search, and ultimately increases the reservation wage.

11.2 The Diamond (1971) paradox

Where does the distribution of wage, $F(w)$, come from? To answer this question, we start with what is perhaps the simplest equilibrium setting that could generate an equilibrium wage distribution. Namely, we assume that there is a continuum of firms with constant return to scale technology: each worker a firm employs produces $\bar{w}$ units of output. Unemployed workers contact a randomly chosen firm with probability λ. When meeting a worker, firms decide what wage to offer and make take-it-or-leave-it offers. Given the equilibrium distribution of wage, $F(w)$, the optimal policy of a worker is characterized by the reservation wage w^*, as we previously discussed. Given the reservation wage policy of workers, the problem of a single firm is

$$\max_{w}\,(\bar{w}-w)\,\mathbb{I}_{\{w\geq w^*\}},$$

where the worker's acceptance policy is captured by the indicator function.

Proposition 11.2.1. *All firms offer the same wage, equal to the unemployment benefit, b. Consequently, the equilibrium wage distribution is degenerate, $F(w)=\mathbb{I}_{\{w\geq b\}}$.*

The firm's profit is clearly maximized by offering the reservation wage. This implies that the support of the wage distribution function collapses to a unit mass at w^*. But using equation (11.1.4), we immediately see that the workers' reservation wage is equal to the unemployment benefit, $w^* = b$.

The key insight is that "local" monopolies result in monopoly-like prices. That is, the equilibrium wage is established as if there were a single monopoly firm in the economy. This holds regardless of the model parameters. In particular, this holds even if workers can contact alternative firms with high probability (high λ), and even if workers are very patient and so are willing to wait a long time in order to find a better wage (high β). Why do we obtain this surprising result? Think of a conversation between a firm and a worker who can search very fast. The worker says to the firm "you must increase the wage because I can find a new offer very quickly, there is tough competition out there". The firm answers: "be my guest, go ahead and continue your search, but you will get the same low wage". Because the worker will meet firms sequentially - he will never be able to get the firms to compete in price to hire him. Once the worker has rejected an offer, he is back into the same bilateral bargaining problem.

11.3 Burdett and Judd (1983) pricing

The Diamond's paradox relies crucially on the assumption that all trades are strictly bilateral. A worker cannot use a previous offer to receive better terms when he encounters a new firm. Put differently, at any point in time, firms never compete for the same worker. Clearly, this is extreme. For example, when worker search on the job and interviews with a new firm, he presumably benefits from the competition between his current and prospective employer. In this Section we will see that, whenever workers can receive simultaneous offers, the Diamond paradox breaks down. We will assume that each worker receives k wage offers with probability q_k:

$$k \in \{1, 2, 3, ...\}\,; \sum_{k=1}^{\infty} q_k = 1.$$

We will show that, quite intuitively, when workers receive simultaneous offers with probability one, $q_1 = 0$, then workers receive competitive wages, equal to their productivity. More interestingly, suppose that workers receive simultaneous offers with probability less than one, $q_1 \in (0, 1)$: sometimes they only receive one offer, and some other times they receive more than one offer. Then, we will obtain an outcome intermediate between monopoly wages (Diamond's paradox) and competitive wages. Moreover, the distribution of wage will be non-degenerate: identical workers may receive different wage offers. This is a result that the literature has found particularly interesting because, in reality as in this model, workers with identical observable characteristics appear to earn very different wages.

The model works as follows. There is a continuum of measure one of workers and a continuum of measure one of firms. Each firm has a constant return to scale technology: each worker it employs produces $\bar{w}$ units of output. Workers are as in the McCall's model: they are infinitely lived, risk neutral, have a discount rate β, and they alternate between periods of employment and unemployment. When employed, a worker looses its job with probability δ. When unemployed, workers receive the benefit $b \leq \bar{w}$ and they draw wage offers. Precisely, we assume that each unemployed worker can access the labor market with probability λ. Conditional on accessing the market, an unemployed worker is able to send k wage requests with probability q_k, to a random set of k firms. Each firm sends back a wage offer.

The worker's problem is exactly the same as before, with a small twist to calculate the distribution of offers faced by an individual worker: the $F(w)$ that enters the reservation wage equation is the distribution of the best offer received by the worker. Namely, if we let $G(w)$ denote the equilibrium distribution of wages posted by firms then the distribution of the best offer is $F(w) = \sum_{k=1}^{\infty} q_k [G(w)]^k$. Namely, a worker receives k job offers with probability q_k, and the probability distribution of the best of these k offer is $G(w)^k$ – indeed, the probability that the best of k offer is less than w has to be equal to the probability that all of k offer is less than w. The worker's optimal strategy is to accept all offers greater than w^*, where w^* solve the reservation wage equation (11.1.4)

Having discussed the worker's problem, we turn to the problem of an individual firm. When it decides which wage to offer to a worker, a firm anticipates that this worker may have received simultaneous offers from other firms. What is the probability that a firm sends an offer to a worker who has (in total) k simultaneous offers? To answer this question, it is helpful to be more explicit about the process through which workers send wage requests and receive offers. Namely, imagine that each worker sends CV by email to a random number of firms: the email client sends k message with probability q_k. We can organize the pool of all workers' emails into bins indexed by $k \in 1, 2, \ldots$. If a worker has sent k emails, then all emails go in the k^{th} pile. Hence, the number of email in the k^{th} pile is kq_k. Now, from the point of view of a single firm, the probability of drawing an email from the k^{th} pile is:

$$\phi_k = \frac{kq_k}{\sum_{\ell=1}^{\infty} \ell q_\ell}$$

Since a worker receives k offers if and only if his email have been put in the k^{th} pile, ϕ_k also represents the probability that a firm sends an offer to a worker who has (in total) k simultaneous offers. Hence, the expected profit for a firm offering wage w to a worker upon receiving its CV is:

$$\Pi(w) = (\bar{w} - w) \sum_{k=1}^{\infty} \phi_k G^{k-1}(w) \left\{ \sum_{j=0}^{k-1} \binom{k-1}{j} \left[\frac{G(w) - G(w-)}{G(w)}\right]^j \left[\frac{G(w-)}{G(w)}\right]^{k-1-j} \frac{1}{j+1} \right\} \mathbb{I}_{\{w > w^*\}}$$

where $G(w-) = \lim_{\tilde{w} \uparrow w} G(\tilde{w})$. In the formula, $\bar{w} - w$ is the profit conditional on hiring the worker at wage w. This profit is multipled by the probability of hiring the worker at wage w. First, with probability ϕ_k, the offer is received by a worker who has (in total), k simultaneous offers. The probability that all other simultaneous offers are weakly less than w is $G(w)^k$. Conditional on an offer being weakly less than w, the offer is exactly equal to w with probability $\frac{G(w)-G(w-)}{G(w)}$, and it is strictly less than w with probability $\frac{G(w-)}{G(w)}$. Hence, the probability of j other offers equal to w and $k-1-j$ other offers strictly less than w is given by the Binomial formula

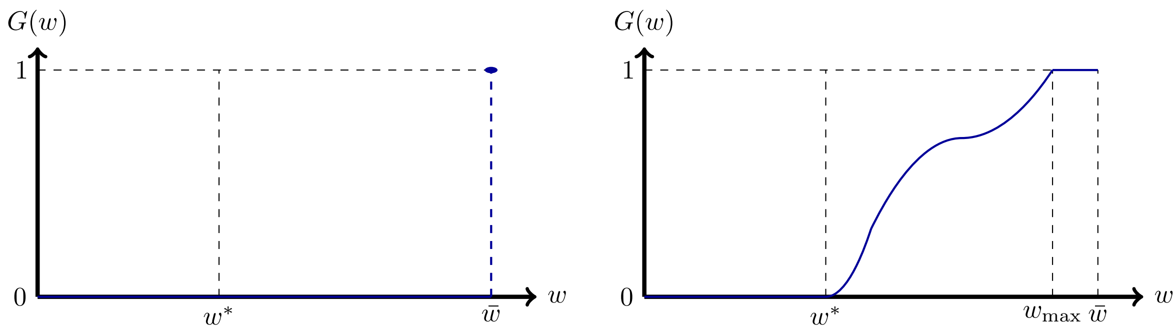
$$\binom{k-1}{j} \left[\frac{G(w) - G(w-)}{G(w)}\right]^j \left[\frac{G(w-)}{G(w)}\right]^{k-1-j}.$$

If there are j other offers equal to w, the worker picks a job at random. Thus, the firm under consideration hires the worker with probability $1/(j+1)$.

Definition 11.3.1. An equilibrium consists of a distribution of wages $G(w)$ and a reservation wage

The first condition ensures that the worker's accept/reject decisions are optimal given the the distribution of wages. In the second condition the "support" of the distribution refers to the set of wages that are posted with positive probability – formally, the set of wages such that $G(w+\varepsilon)-G(w-\varepsilon)>0$ for all $\varepsilon>0$ small enough (see Chung, 2001, page 10). Hence, the second condition means that all posted wages maximize profits.

Figure 11.3: Equilibrium wage distribution in Burdett-Judd Model



(a) when $q_0 = 1$ **(b)** when $q_1 \in (0,1)$

w^* such that

1. w^* solves the reservation wage equation (11.1.4) given $F(w)=\sum_{k=1}^{\infty} q_k G(w)^k$; and

2. $\forall w \in \text{support}(G)$, $w \in \underset{w'}{\text{argmax}}\, \Pi\left(w'\right)$.

We solve for an equilibrium in a few steps.

Proposition 11.3.1. *If $q_1 < 1$, the equilibrium can be either of the following two cases:*

1. support $(G)=\{\bar{w}\}$; or

2. $G(w)$ *is continuous with over support* $[w^*, w_{\max}]$ *, where* $b \leq w^* < w_{\max} < \bar{w}$.

Proof. Suppose that support $(G) \neq \{\bar{w}\}$. Given that firm cannot find it optimal to post wage strictly greater than $\bar{w}$, there must exists some $w<\bar{w}$ such that $G(w)>0$.

First, we argue that the upper bound of the support, which we denote by $w_{\max}$, is strictly less than $\bar{w}$. Indeed, by our maintained assumption that $G(w)>0$ for some $w<\bar{w}$, it follows that $\Pi(w)>0$, and so that the firm makes strictly positive profit. Since posting wages close to $\bar{w}$ would lead to vanishingly small profits, wages near $\bar{w}$ cannot be in the support of the wage distribution.

Second, we argue that $G(\cdot)$ must be continuous, that is, it cannot have any mass point. We proceed by contradiction. Suppose that there is discontinuity at some w, i.e $G(w)-G(w-)>0$. Then we show that offering w cannot be optimal, $w \notin \text{argmax}_{w^*} \Pi(w^*)$, and so $G(\cdot)$ cannot be an equilibrium wage distribution. To see why offering w cannot be optimal, we note that, since there is a positive measure of firms offering w, there is a positive probability of ties between these firms. By increasing its wage offer by some small positive amount above w but below $\bar{w}$, which is possible because $w \leq w_{\max} < \bar{w}$, a firm makes strictly larger profits: it eliminates all ties with other firms posting w and so it hires with discretely higher probability, while only making a small profit loss.

Third, we argue that there are no "holes" in the support of the wage distribution – mathematically, the support must be the interval $[w^*, w_{\max}]$. As before we proceed by contradiction. Consider any $w^\star \le w_1 < w_{\max}$ and let $w_2 = \sup\{w > w_1 : G(w) = G(w_1)\}$. Since $G(w)$ is continuous, we have that $G(w_2) = G(w_1)$. Since $G(w_{\max}) = 1$, we also have that $w_2 < w_{\max}$. Now suppose that $w_2 > w_1$, that is, suppose there is a "hole" in the distribution between w_1 and w_2. Then any firm posting a wage in a right neighborhood of w_2 will find it optimal to lower its wage to w_1. Indeed, this creates a small decrease in hiring probability but a discretely large increase in profit conditional on hiring. □

Proposition 11.3.2. *The equilibrium wage distribution can be either of the following three cases:*

1. Diamond paradox. *If $q_1 = 1$, then $G(w) = \mathbb{I}_{\{w \ge b\}}$;*
2. Bertrand. *If $q_1 = 0$, then $G(w) = \mathbb{I}_{\{w \ge \bar{w}\}}$;*
3. Price Dispersion. *If $q_1 \in (0,1)$, $G(\cdot)$ is continuous over some support $[w^*, w_{\max}]$. Moreover, $w_{\max} = (1-\phi_1)\bar{w} + \phi_1 w^*$ and $G(w) = 1 - H\left(\dfrac{\bar{w} - w}{\bar{w} - w^*}\right)$, where $H(x)$ denotes the solution y to $\phi_1 = x \sum_{k=1}^{\infty} \phi_k (1-y)^{k-1}$, $\forall x \in [\phi_1, 1]$; $H(x) = 0$, $\forall x < \phi_1$, and $H(x) = 1$, $\forall x > 1$.*

Proof. The first case has been studied in Section (11.2). All firm make the same offer, equal to the monopoly price, b.

In the second case, workers always receive at least two simultaneous offers. Then, we show by contradiction that $G(\cdot)$ is concentrated at $\bar{w}$. Suppose it is not. Then, from the previous proposition, we know $G(\cdot)$ is continuous over some support $[w^*, w_{\max}]$, where $w_{\max} < \bar{w}$. Then, a firm's profit is zero when it offers w^*, since the worker would receive a strictly higher offer with probability one. At the same time, the firm's profit is strictly positive when it offers $w_{\max}$, since it hires the worker with probability one and $w_{\max} < \bar{w}$. Hence, the firm is not indifferent between two offers in the support of $G(\cdot)$, w^* and $\bar{w}$, so we have reached a contradiction.

Now consider the third case. A distribution concentrated at $\bar{w}$ cannot be an equilibrium. Indeed, one can verify from equation (11.1.4) that the reservation wage, w^*, is strictly less than $\bar{w}$. Clearly, a firm would find it optimal to post the reservation wage, which yield strictly positive profit in case it is the only offer received by the worker, rather than $\bar{w}$, which yield zero profits. Hence, in the third case, we know that the distribution of wage is continuous over some support $[w^*, \bar{w}]$. We can go a step further and characterize the wage distribution explicitly.

To do so, it turns out convenient to write the wage in terms of the surplus share of the firm. That is, we write $w = mw^* + (1-m)\bar{w}$, so that the firm's profit conditional on hiring at wage w is $m(\bar{w} - w^\star)$. We denote the equilibrium distribution of m by $H(m)$. The equilibrium distribution of w is $G(w) = 1 - H\left(\dfrac{\bar{w} - w}{\bar{w} - w^*}\right)$ as stated in the proposition. With this new notation in mind, we can re-write the condition that firms' profits are equalized in the support of the m distribution as:

Since firms' profits are equalized for all $w \in [w^*, w_{\max}]$:

$$\Pi(1) = \phi_1 (\bar{w} - w^*) = \Pi(m) = m(\bar{w} - w^*) \sum_{k=1}^{\infty} \phi_k [1 - H(m)]^{k-1}.$$

Canceling $\bar{w} - w^*$ from both sides we obtain

$$\phi_1 = m \sum_{k=1}^{\infty} \phi_k \left[1 - H(m)\right]^{k-1}$$

When $H(m) = 0$ the right-hand side is equal to m, which implies that the lower bound of the support of the m distribution is ϕ_1. For $m \in (\phi_1, 1)$, the function $H(m)$ is the unique solution of $\phi_1 = x \sum_{k=1}^{\infty} \phi_k (1-y)^{k-1}$, as stated in the proposition. □

To fully solve the model, we need to solve for the reservation wage. Recall that the reservation wage equation (11.1.4) is:

$$w^* = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*}^{\bar{w}} (1 - F(w))\, dw = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w^*}^{\bar{w}} (w - w^*)\, dF(w).$$

Hence, we need to calculate the expected net wage, $w - w^\star$ of workers.

To do so it is helpful to appeal (informally) to the law of large numbers and notice that the expected net wage for an individual worker is equal to the average net wage in a measure-one cross section of workers. In addition, the average net wage can be written:

$$\text{average}\left[(\bar{w} - w^\star) - (\bar{w} - w)\right] = \bar{w} - w^\star - \text{average}\left[\text{firm profit}\right].$$

That is, the average net wage is equal to the surplus, $\bar{w} - w^\star$, less firms average profits from the cross section of workers. But firms' average profits are:

$$\left(\sum \ell q_\ell\right) \times \phi_1 = q_1.$$

Indeed, firm receives a total of $\sum \ell q_\ell$ quote requests from workers, and the expected profit per request is ϕ_1. The last equality then follows from the definition of ϕ_1. Taken together, we obtain that the average net wage of worker has the remarkably simple expression $q_1 w^\star + (1-q_1)\bar{w}$.

The brute force calculation is also instructive and useful. The best m received by the worker is the lowest one offered by firms, with CDF is $\sum_{k=1}^{\infty} q_k \left\{1 - \left[1 - H(m)\right]^k\right\}$. The average of these best m is:

$$\begin{aligned}
\int_{\phi_1}^{1} m \sum_{k=1}^{\infty} q_k \, d\left\{1 - \left[1 - H(m)\right]^k\right\} &= \int_{\phi_1}^{1} \sum_{k=1}^{\infty} q_k k \left[1 - H(m)\right]^{k-1} \, dH(m) \\
&= \left(\sum_{\ell} q_\ell \ell\right) \int_{\phi_1}^{1} \sum_{k=1}^{\infty} \phi_k \left[1 - H(m)\right]^{k-1} \, dH(m) \\
&= \left(\sum_{\ell} q_\ell \ell\right) \phi_1 = q_1,
\end{aligned}$$

where the equality on the second line follows because, by its definition, $\phi_k = q_k k / \left(\sum_\ell q_\ell \ell\right)$, and the equality on the third line follows from the firm's indifference condition.

Using the above calculations then leads to the reservation wage equation:

$$w^\star = b + \frac{\beta\lambda}{1-\beta(1-\delta)}(1-q_1)(\bar{w}-w^\star)$$

allowing to solve for $w^\star$.

Some notes about this result. First of all, one has to admit that it is remarkably elegant. Going beyond that observation (!) one may wonder how is that result meaningful economically. The reason that applied researchers have found the result interesting is because it helps rationalize wage dispersion amongst observationally identical workers. Even if many workers have the same productivity (here, output at the firm, $\bar{w}$) they end up having potentially very different wages. This is in line with many observations that workers are, in reality, employed at very different wages, even if they have very similar levels of education. Finally, we shall also note this point goes beyond this particular model. Namely, the broader point is that search frictions naturally create wage dispersion in the labor market. In the present model, wage dispersion arises because a worker may randomly draw several offers at the same time. But more generally wage dispersion arise in search models because workers have different employment and search histories, which creates dispersed outside options and ultimately dispersed wages. For example, in Burdett and Mortensen (1998), workers draw only one offer at a time but they may search on the job. Hence, workers who already have already have a job, and so they compare the wage offer they draw with the wage they have in their current job. Hence, worker who already have a job are similar to worker who has two offers in hand in the model of Burdett and Judd (1983) we just studied. This again creates wage dispersion. In Postel-Vinay and Robin (2002), workers who already have a job use their current wage to bargain with prospective employer. Hence, identical workers at identical firms may have different wages because they had different history of outside offers.