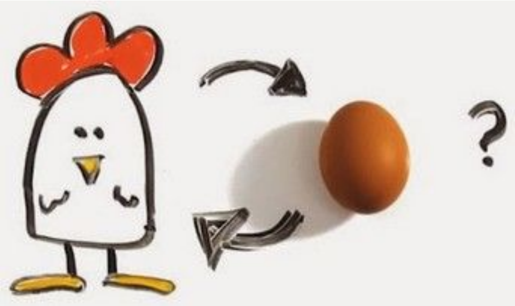
Simultaneous Localization And Mapping



Feras Dayoub

Student Perceptions of Teaching survey now open

SEMESTER 1 STUDENT PERCEPTIONS OF TEACHING SURVEY NOW OPEN

The *Student Perceptions of Teaching (SPOT) survey* is open from **Friday 10 May until midnight Monday 3 June.** The survey is designed to encourage constructive feedback and find out what students value about their learning experience. It can be accessed through a personalised link that will be sent directly to their student email.

Let me know what you like about the second have of the unit and what you think can make it better.

Report [5%]

Study the effects of the Q and R matrices on the pose estimation process during localization.
You are expected to make a set of observations supported by empirical data presented as graphs or tables.
Words limit [1500 words].

To be submitted via TurnitIn by 11pm Sunday 2 June.

2019 IEEE

International Conference on
Robotics and Automation

IEEE Robotics & Automation Society

IEEE

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Feras is away in Week 12

Guest lecture in Week 12

QUT

Dr David Hall

Science and Engineering Faculty,

Electrical Engineering, Computer Science,

Centre of Excellence (COE) in Robotic Vision

Learning objectives

- SLAM using an extended Kalman filter.

Lecture 9 - 10 recap

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each observed landmark do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i) \bar{\mathbf{\Sigma}}_t$$

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\mathbf{\Sigma}}_t = \mathbf{\Sigma}_{t-1}$$

Update step:

For each observed landmark do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\mu}_t, i, \mathbf{x}_t^r))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i) \bar{\mathbf{\Sigma}}_t$$

localization **mapping**

Assumptions

- The robot does not know its pose in the map.
- The wheel encoders are noisy.
- The robot does not know the position of the landmarks in the map.
- The sensor onboard the robot is noisy.
- The robot can associate the measurements with the landmarks.

The task

- The robot should **localize itself** inside a map using a set of landmarks and at the same time use its pose and sensor **to map** the positions of the landmarks.

SLAM: the chicken or egg problem

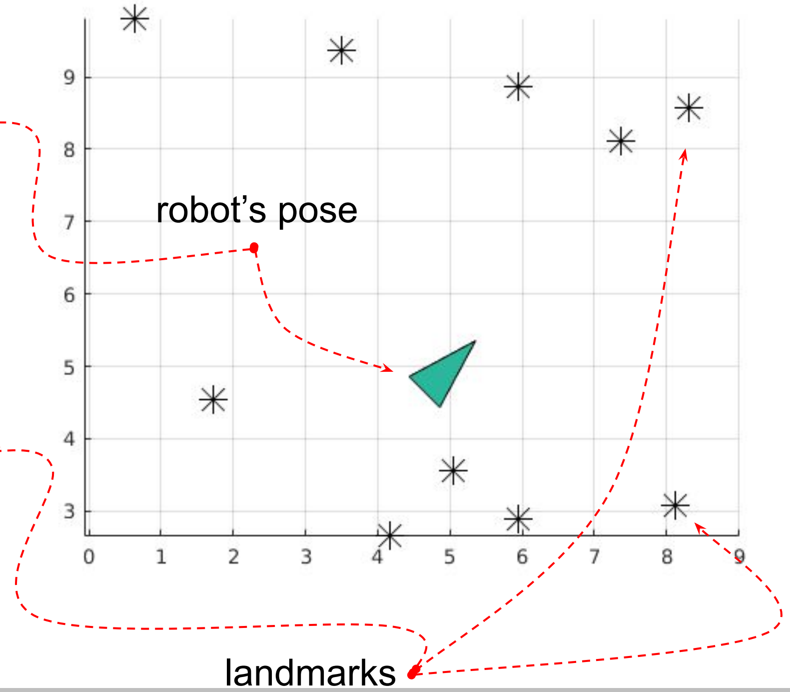
- As we saw in lecture 8, we need the position of the landmarks (i.e the map) to estimate the pose of the robot.
- And we saw in lecture 9 that in order to estimate the position of the landmarks in the map we need the true pose of the robot.
- In this lecture we are going to do the two above processes at the same time. This is called simultaneous localisation and mapping (SLAM).

Localize yourself in a map that you are building using the estimation of your pose in it!

The state vector

- The state vector contains both the pose of the robot and the positions of the landmarks in the map.

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{x}^r \\ M \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \\ \theta_r \\ x_{l_1} \\ y_{l_1} \\ \vdots \\ x_{l_n} \\ y_{l_n} \end{bmatrix}$$



We still live in a Gaussian world!

$$\mathbf{x} \sim \mathcal{N}(\mu, \Sigma)$$

You used this in the localization case

You used this in the mapping case

$$\mu = \begin{bmatrix} \mu^r \\ \mu_{l1} \\ \vdots \\ \mu_{ln} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \boxed{\Sigma_{\mathbf{x}^r}} & \Sigma_{\mathbf{x}^r l_1} & \cdots & \Sigma_{\mathbf{x}^r l_n} \\ \Sigma_{l_1 \mathbf{x}^r} & \Sigma_{l_{11}} & \cdots & \Sigma_{l_{1n}} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{l_n \mathbf{x}^r} & \Sigma_{l_{n1}} & \cdots & \Sigma_{l_{nn}} \end{bmatrix}$$

The mean vector
of the robot pose

The same set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

- We predict both the pose of the robot and the positions of the landmarks.
- In the prediction step the robot moves and the landmarks stay static.

Update step:

For each landmark $\mathbf{z}_t^i$ do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i(\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

Prediction step:

$$\bar{\mu}_t = \begin{bmatrix} f_r(\mu^r_{t-1}, \mathbf{u}_t) \\ \mu_{l1_{t-1}} \\ \vdots \\ \mu_{ln_{t-1}} \end{bmatrix}$$

$\mathbf{J}_{x_t}$ The Jacobian matrix of **f** w.r.t the state vector.

$\mathbf{J}_{u_t}$ The Jacobian matrix of **f** w.r.t the odometry.

$$\bar{\Sigma}_t = \mathbf{J}_{x_t}\mathbf{\Sigma}_{t-1}\mathbf{J}^T_{x_t} + \mathbf{J}_{u_t}\mathbf{R}\mathbf{J}^T_{u_t}$$

If at time step **t** we have mapped **n** landmarks, what is the dimension of these matrices?

The prediction step

The same matrices from the localization case

$$\mathbf{J}_{x_t} = \begin{bmatrix} \mathbf{J}_{x_t^r} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix} \qquad \mathbf{J}_{u_t} = \begin{bmatrix} \mathbf{J}_{u_t^r} \\ 0_{2n\times 2} \end{bmatrix}$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t}\mathbf{\Sigma}_{t-1}\mathbf{J}_{x_t}^T + \mathbf{J}_{u_t}\mathbf{R}\mathbf{J}_{u_t}^T$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \boldsymbol{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

$$\begin{bmatrix} \mathbf{J}_{x_t^r} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix} \begin{bmatrix} \Sigma_{\mathbf{x}^r} & \Sigma_{\mathbf{x}^r l_1} & \cdots & \Sigma_{\mathbf{x}^r l_n} \\ \Sigma_{l_1 \mathbf{x}^r} & \Sigma_{l_{11}} & \cdots & \Sigma_{l_{1n}} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{l_n \mathbf{x}^r} & \Sigma_{l_{n1}} & \cdots & \Sigma_{l_{nn}} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{x_t^r} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix}^T$$

We use the same treatment with this term as well.

The same set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each landmark $\mathbf{z}_t^i$ do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i) \bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$

Similar to the mapping case but with the fact that the pose of the robot is now part of the state vector

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

The same measurement function we used for localization and for mapping.

$$h(\bar{\mu}_t, i) = \begin{bmatrix} r^i \\ \beta^i \end{bmatrix} = \begin{bmatrix} \sqrt{(x_r - x_{l_i})^2 + (y_r - y_{l_i})^2} \\ atan2(y_{l_i} - y_r, x_{l_i} - x_r) - \theta_r \end{bmatrix}$$

Range and bearing with respect to robot's own frame of reference at time step t.

Coordinates of landmark i from the vector $\bar{\mu}_t$

The predicted pose of the robot from the vector $\bar{\mu}_t$

The Jacobian matrix of the measurement function

$$\mathbf{G}_t^i = \frac{\partial h(\bar{\mu}_t, i)}{\partial \bar{\mu}_t}$$

$$= \begin{bmatrix} -\frac{x_{l_i} - x_r}{r} & -\frac{y_{l_i} - y_r}{r} & 0 & \dots & \frac{x_{l_i} - x_r}{r} & \frac{y_{l_i} - y_r}{r} & \dots \\ \frac{y_{l_i} - y_r}{r^2} & -\frac{x_{l_i} - x_r}{r^2} & -1 & \dots & -\frac{y_{l_i} - y_r}{r^2} & \frac{x_{l_i} - x_r}{r^2} & \dots \end{bmatrix}$$

zeros

The same set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each landmark $\mathbf{z}_t^i$ do:

$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t(\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$

But what if we see a landmark for the first time?

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

Landmark initialization

$$\bar{\mu_t}^* = \begin{bmatrix} \bar{\mu_t} \\ l_{new} \end{bmatrix} = \begin{bmatrix} \bar{\mu_t} \\ x_{l_{new}} \\ y_{l_{new}} \end{bmatrix}$$

You already know how to find these as we already encountered them in the mapping case during last lecture.

Simply expand the state vector with the coordinates of the new landmark in the map!

The landmark initialisation function

$$\mathbf{z} = \begin{bmatrix} r \\ \beta \end{bmatrix}$$

$$l_{new} = q(\mathbf{z}_t^{new}, \bar{\mu}_t)$$

Sensor measurement to a never seen before landmark.

$$l_{new} = \begin{bmatrix} x_r + r \times cos(\theta_r + \beta) \\ y_r + r \times sin(\theta_r + \beta) \end{bmatrix}$$

The first three components of $\bar{\mu}_t$

The Jacobian of the landmark initialisation function w.r.t the **z**

$$\mathbf{L}_z = \frac{\partial q(\mathbf{z}, \bar{\mu}_t)}{\partial \mathbf{z}}$$

$$= \begin{bmatrix} cos(\theta_r + \beta) & -r \times sin(\theta_r + \beta) \\ sin(\theta_r + \beta) & r \times cos(\theta_r + \beta) \end{bmatrix}$$

$$\mathbf{z} = \begin{bmatrix} r \\ \beta \end{bmatrix}$$

What about the covariance matrix?

$$\bar{\Sigma}_t^* = \begin{bmatrix} \bar{\Sigma}_t & 0 \\ 0 & \mathbf{L_z Q L_z^T} \end{bmatrix}$$

The covariance matrix expands as well!

Zero matrices!

The covariance of the sensor noise.

Putting it all together

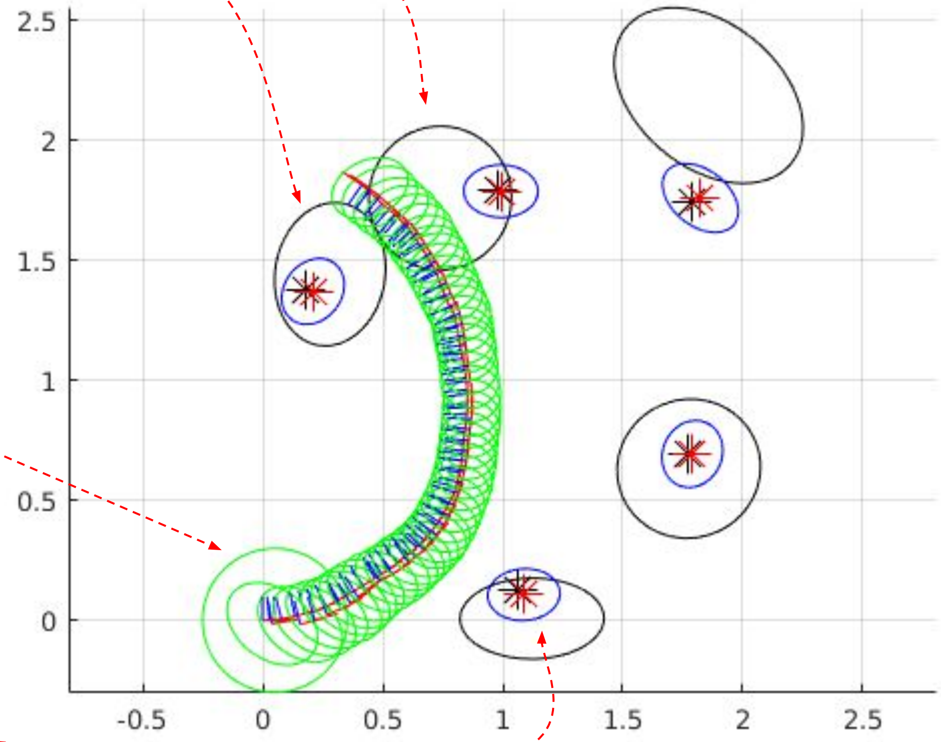
1. Move.
2. Perform the **prediction step** which updates the mean and covariance.
3. Make a new Measurement (range and bearing to a landmark).
4. if we have not seen the landmark before:
    - Do landmark **initialization** based on the robot estimated pose and expand the mean and the covariance.

    else

    - Perform the **update step** and update the mean and the covariance.
5. Go to 1.

The uncertainty on the positions of the landmarks on initialization.

The uncertainty on the pose of the robot (green ellipses)



The uncertainty on the position of the landmarks after 50 steps (**blue ellipses**)

Week 13 Lecture

What are the strengths and weaknesses of EKF-SLAM and what are the other flavours of SLAM algorithms?