Mapping

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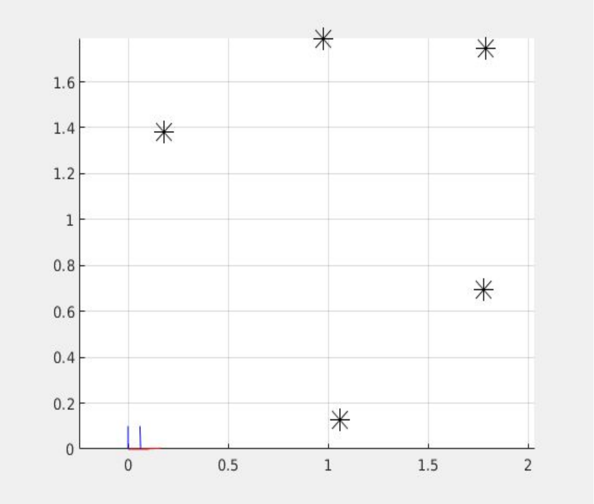
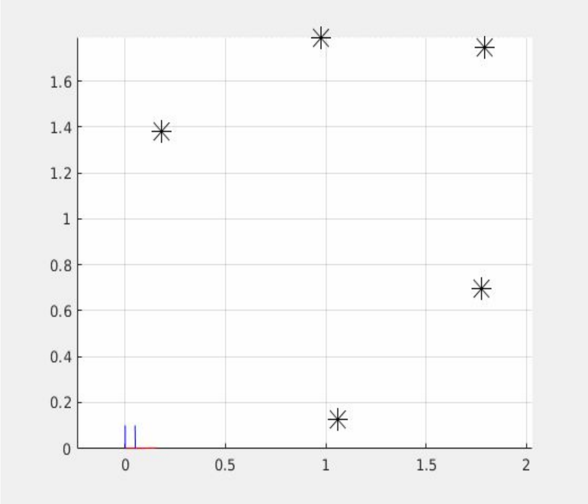
Learning objectives

- Mapping using an extended Kalman filter.

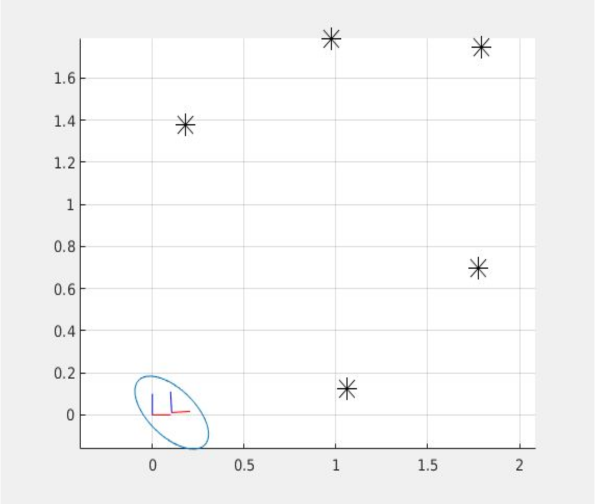
Lecture 9 Recap - Localization



Ground truth (unknown)



Odometry



EKF localization

Lecture 9 Recap

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Update step:

For each landmark $\mathbf{z}_t^i$ do:

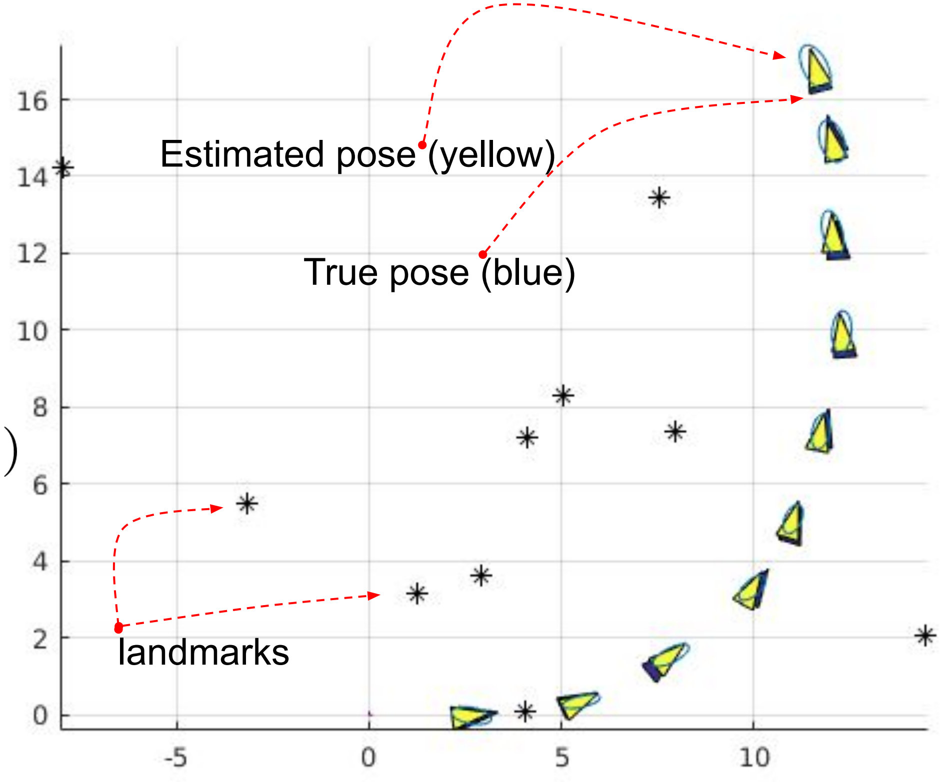
$$\bar{\mu}_t = \bar{\mu}_t + \mathbf{K}_t^i(\mathbf{z}_t^i - h(\bar{\mu}_t, i))$$

$$\bar{\mathbf{\Sigma}}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

end

$$\mu_t = \bar{\mu}_t$$

$$\mathbf{\Sigma}_t = \bar{\mathbf{\Sigma}}_t$$



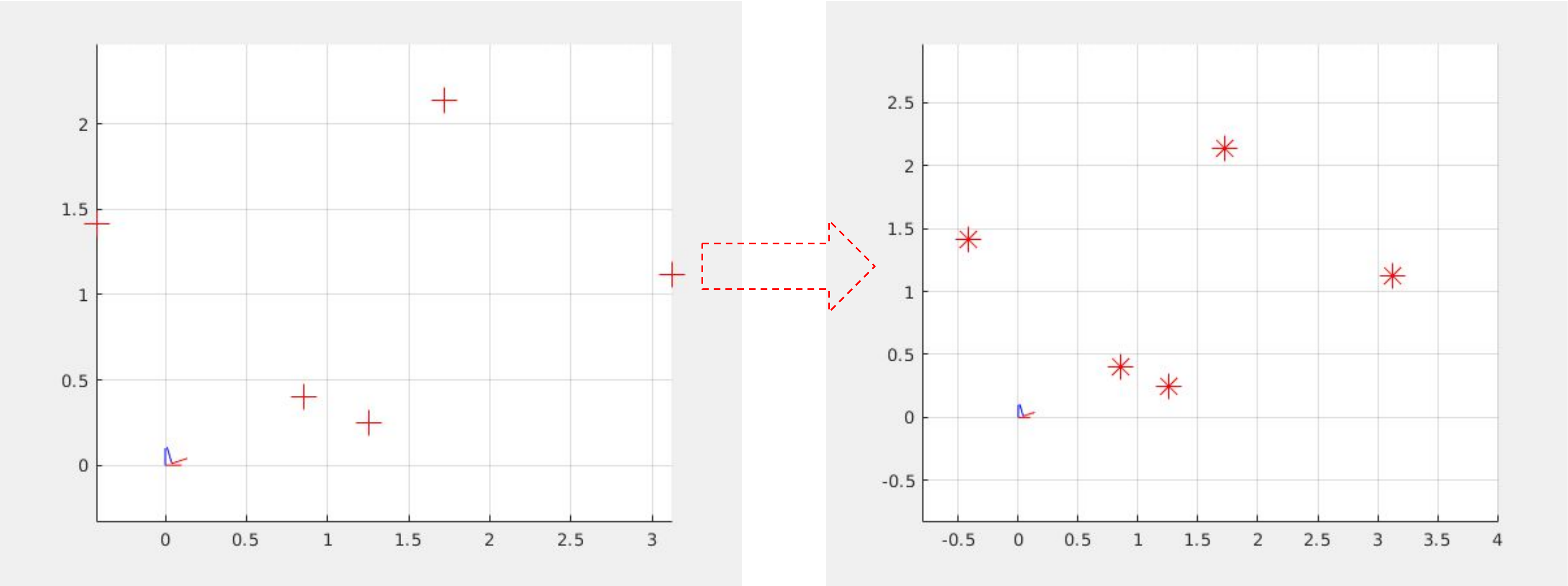
Mapping

Assumptions

- The robot knows its pose with absolute certainty.
- The robot is equipped with noisy range and bearing sensor.
- We have a way to associate the measurements with the already mapped landmarks when they appear in the view again.
- The state and the noise are Normally distributed.

The task

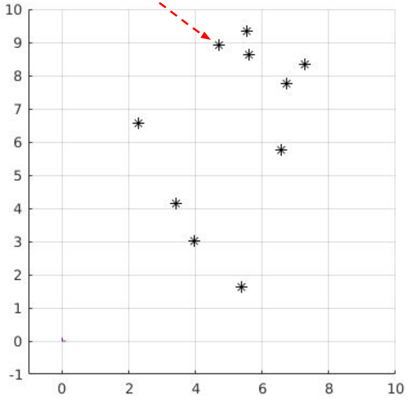
- Estimate the position of the landmarks in the map.


2
1.5
1
0.5
0
0 0.5 1 1.5 2 2.5 3
2.5
2
1.5
1
0.5
0
-0.5
-0.5 0 0.5 1 1.5 2 2.5 3 3.5 4

The state vector is the map

- The state vector is much larger than what we saw in the localization case.

$$\mathbf{M} = \begin{bmatrix} x_{l_1} \\ y_{l_1} \\ \vdots \\ x_{l_n} \\ y_{l_n} \end{bmatrix} \sim \mathcal{N}(\mu, \mathbf{\Sigma})$$



The covariance matrix

Is this matrix symmetric?

$$\Sigma_t = \begin{bmatrix} \Sigma_{l_{11}} & \Sigma_{l_{12}} & \dots & \Sigma_{l_{1n}} \\ \vdots & \vdots & \vdots & \vdots \\ \Sigma_{l_{n1}} & \Sigma_{l_{n2}} & \dots & \Sigma_{l_{nn}} \end{bmatrix}$$

The covariance matrix is much bigger and can be written in blocks. Each block tell us the correlation between two landmarks.

$$\Sigma_{l_{ij}} = \begin{bmatrix} \sigma_{x_i x_j} & \sigma_{x_i y_j} \\ \sigma_{y_i x_j} & \sigma_{y_i y_j} \end{bmatrix}$$

What if i == j?

Let's start from the EKF set of equations

Prediction step:

$$\bar{\mu}_t = f(\mu_{t-1}, \mathbf{u}_t)$$

$$\bar{\Sigma}_t = \mathbf{J}_{x_t} \mathbf{\Sigma}_{t-1} \mathbf{J}_{x_t}^T + \mathbf{J}_{u_t} \mathbf{R} \mathbf{J}_{u_t}^T$$

Given that the state vector only contain the positions of the landmarks, what is **f** and what are the the Jacobians matrices?

Update step:

For each landmark do:

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t(\mathbf{z}_t - h(\bar{\mu}_t))$$

$$\mathbf{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{G}_t)\bar{\mathbf{\Sigma}}_t$$

The prediction step

- The landmarks are static and do not change between time steps.

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\boldsymbol{\Sigma}}_t = \boldsymbol{\Sigma}_{t-1}$$

The same set of equations

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\mathbf{\Sigma}}_t = \mathbf{\Sigma}_{t-1}$$

Update step:

For each landmark do:

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t(\mathbf{z}_t - h(\bar{\mu}_t))$$

$$\mathbf{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{G}_t)\bar{\mathbf{\Sigma}}_t$$

In the context of mapping, what is the function **h**?

The same set of equations

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\Sigma}_t = \Sigma_{t-1}$$

The map

$$h(\bar{\mu}_t, \mathbf{x}_t^r)$$

The true pose of the robot (known)

Update step:

<u>For each observed landmark do:</u>

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t(\mathbf{z}_t - \cancel{h(\bar{\mu}_t)})$$

$$\Sigma_t = (\mathbf{I} - \mathbf{K}_t \mathbf{G}_t)\bar{\Sigma}_t$$

The same set of equations

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\mathbf{\Sigma}}_t = \mathbf{\Sigma}_{t-1}$$

We also assume known correspondences

Update step:

For each observed landmark do:

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t^i(\mathbf{z}_t^i - h(\bar{\mu}_t, i, \mathbf{x}_t^r))$$

$$\mathbf{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

$$\mathbf{z}_t^i = \begin{bmatrix} r^i \\ \beta^i \end{bmatrix}$$

The same measurement model we used for localization

$$h(\bar{\mu}_t, i, \mathbf{x}_t^r) = \begin{bmatrix} r^i \\ \beta^i \end{bmatrix} = \begin{bmatrix} \sqrt{(x_r - x_{l_i})^2 + (y_r - y_{l_i})^2} \\ atan2(y_{l_i} - y_r, x_{l_i} - x_r) - \theta_r \end{bmatrix}$$

Range and bearing with respect to robot's own frame of reference at time step t.

Coordinates of landmark i

The true pose of the robot

The same set of equations

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\mathbf{\Sigma}}_t = \mathbf{\Sigma}_{t-1}$$

Update step:

For each observed landmark do:

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t^i(\mathbf{z}_t^i - h(\bar{\mu}_t, i, \mathbf{x}_t^r))$$

$$\mathbf{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t^i \mathbf{G}_t^i)\bar{\mathbf{\Sigma}}_t$$

Given that we mapped **n** landmarks at time step **t**, what are the dimensions of these matrices?

$$\mathbf{K}_t = \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T (\mathbf{G}_t \bar{\mathbf{\Sigma}}_t \mathbf{G}_t^T + \mathbf{Q})^{-1}$$

The Jacobian matrix of the measurement function

$$\mathbf{G}_t^i = \frac{\partial h(\mu_t, i, \mathbf{x}_t^r)}{\partial \mu_t}$$

$$= \begin{bmatrix} 0 & \cdots & \frac{x_{l_i} - x_r}{r} & \frac{y_{l_i} - y_r}{r} & \cdots & 0 \\ 0 & \cdots & -\frac{y_{l_i} - y_r}{r^2} & \frac{x_{l_i} - x_r}{r^2} & \cdots & 0 \end{bmatrix}$$

The same set of equations

Prediction step:

$$\bar{\mu}_t = \mu_{t-1}$$

$$\bar{\boldsymbol{\Sigma}}_t = \boldsymbol{\Sigma}_{t-1}$$

Update step:

For each observed landmark do:

What if we observe a landmark for the first time (i.e it is not in our state vector yet).

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t^i(\mathbf{z}_t^i - h(\bar{\mu}_t, i, \mathbf{x}_t^r))$$

$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t^i\mathbf{G}_t^i)\bar{\boldsymbol{\Sigma}}_t$$

Landmark initialization

$$\bar{\mu_t}^* = \begin{bmatrix} \bar{\mu_t} \\ l_{new} \end{bmatrix} = \begin{bmatrix} \bar{\mu_t} \\ x_{l_{new}} \\ y_{l_{new}} \end{bmatrix}$$

Given that the robot observes range and bearing to a landmark in its own frame of reference, how can we find the coordinates of the new landmark in the map frame?

Simply expand the state vector with the coordinates of the new landmark in the map.

The landmark initialization function

$$\mathbf{z} = \begin{bmatrix} r \\ \beta \end{bmatrix}$$

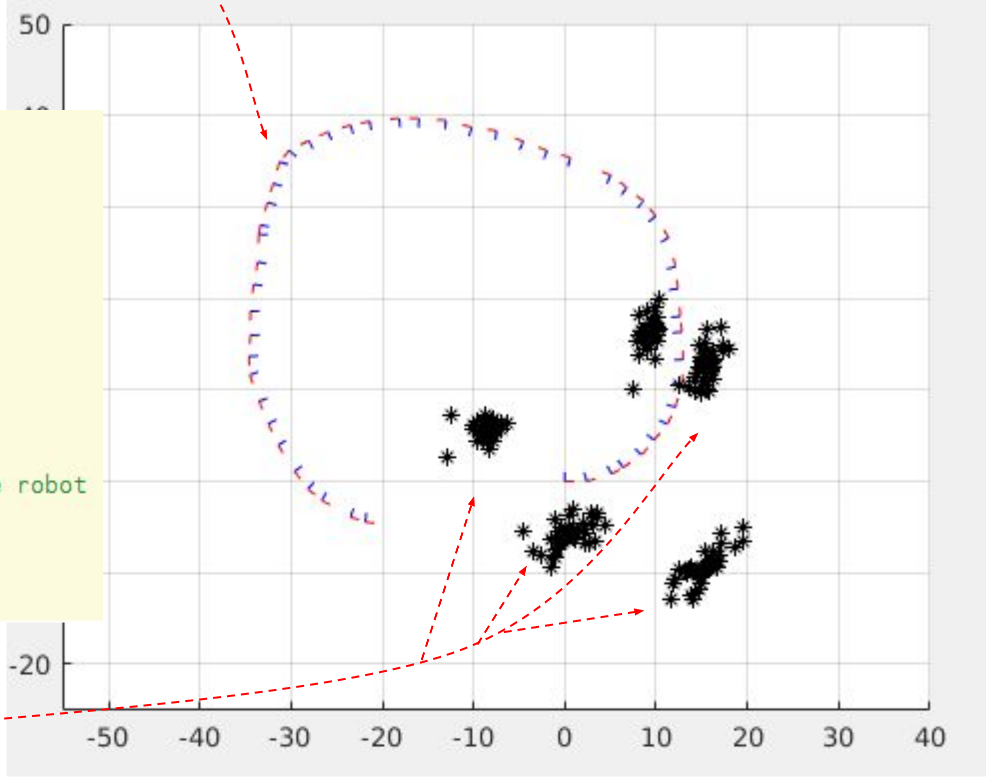
$$l_{new} = q(\mathbf{z}_t^{new}, \mathbf{x}_t^r)$$

Range and bearing to a never seen before landmark.

$$l_{new} = \begin{bmatrix} x_r + r \times cos(\theta_r + \beta) \\ y_r + r \times sin(\theta_r + \beta) \end{bmatrix}$$

True pose of the robot

```
%%
load_data()
% this simulator runs for 50 steps
nsteps = 50;
for k = 1:nsteps
    % the true pose of the robot is known
    xr           = get_pose(k);
    plot_robot(xr)
    % set of ranges and bearings to the landmarks
    z           = sense(k);
    for i=1:length(z)
       zi  = z(i,:);
       % plot the (x,y) of each landmark based on the pose of the robot
       l   = initL(zi,xr);
       scatter (l(1),l(2),'k*');
    end
end
```



Running the initialisation function
after each time step. No filtering

What about the covariance matrix?

What is this matrix?

$$\bar{\Sigma}_t^* = \begin{bmatrix} \bar{\Sigma}_t & 0 \\ 0 & \mathbf{L_z Q L_z^T} \end{bmatrix}$$

The covariance matrix expands as well!

Zero matrices!

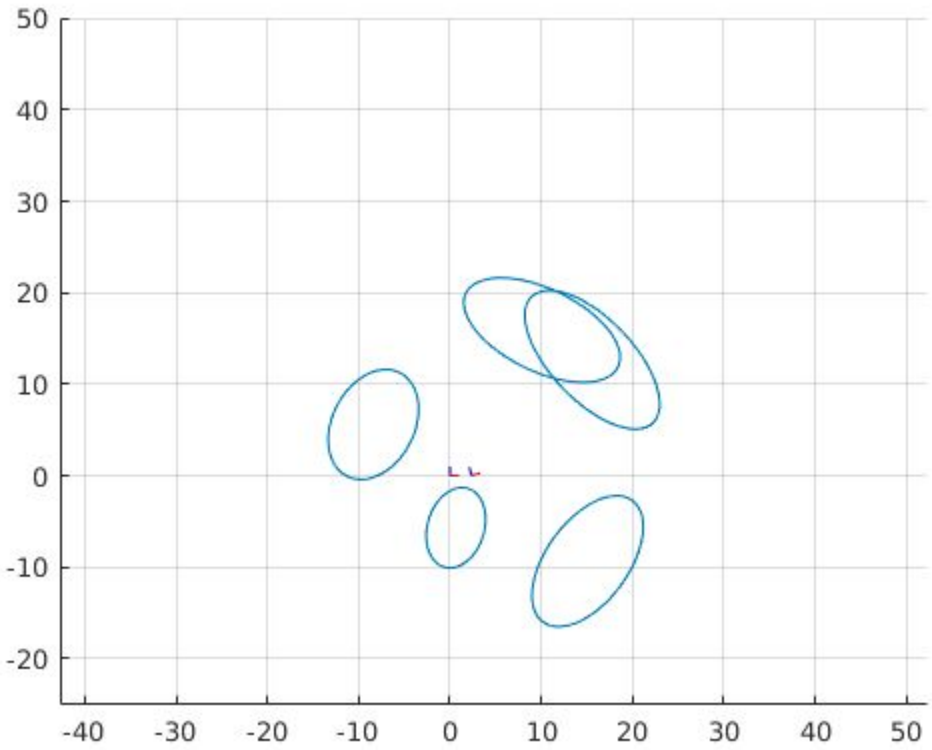
The covariance of the sensor noise.

The Jacobian of the landmark initialisation function

$$\mathbf{L}_z = \frac{\partial q(\mathbf{z}, \bar{\mu}_t)}{\partial \mathbf{z}}$$

$$= \boxed{?}$$

At time step t=1 in the case where we have observed all the landmarks for the first time

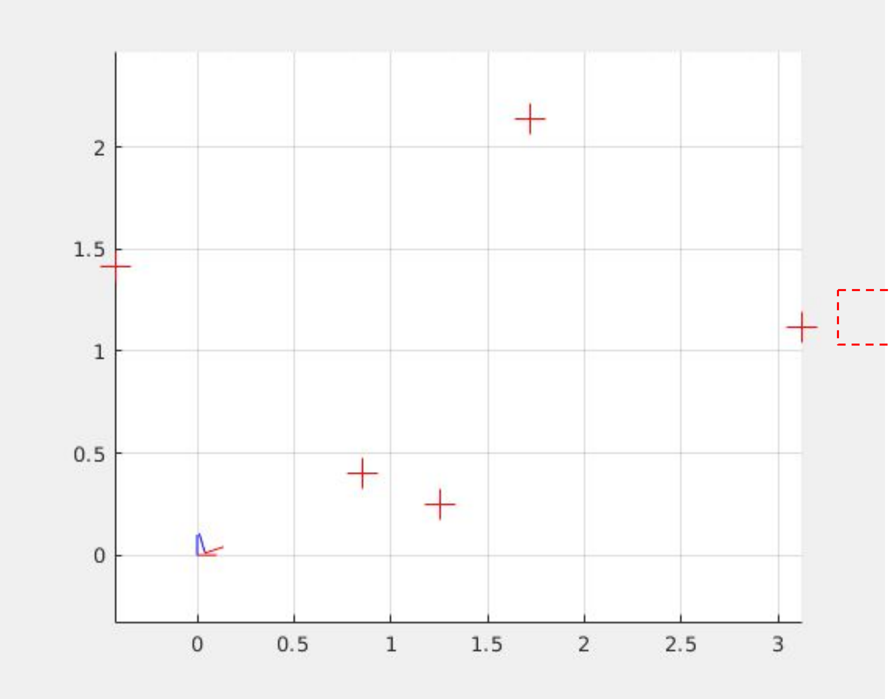


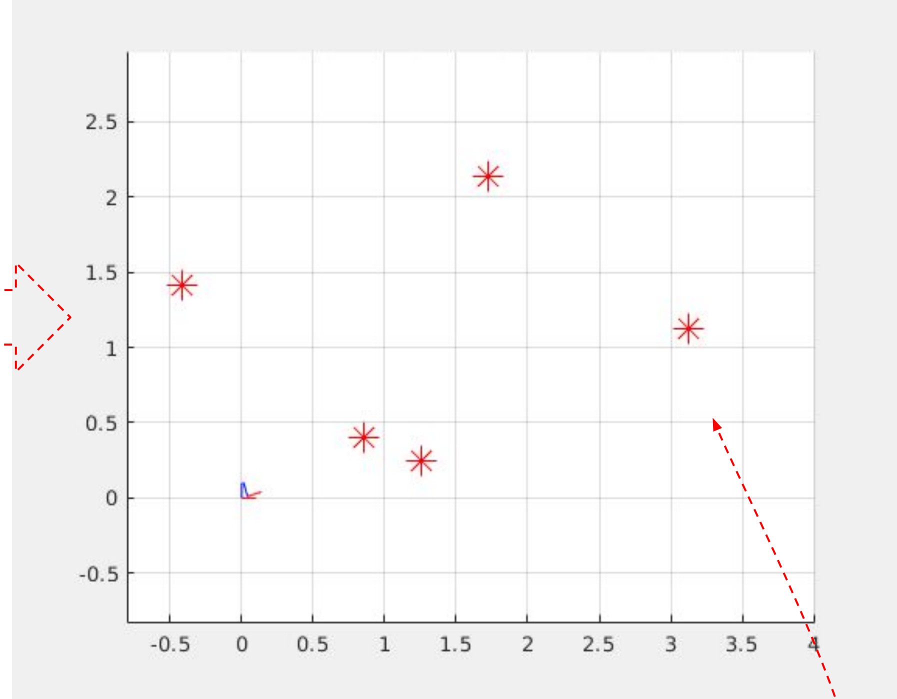
Putting it all together

1. Make a new Measurement (range and bearing to a landmark).
2. if we have not seen the landmark before:
    - Do landmark initialization based on the robot current pose.

    else

    - Predict the landmark position based on the robot current pose.
3. Update the state vector and the covariance.
4. Move.
5. Go to 1.





The ellipses are our 3-sigma bounds confidence of the position of the landmarks.
The red stars are the true (unknown) position of the landmarks and the black stars are our estimate.

Next lecture

Simultaneous
Localization
And
Mapping