DATA STRUCTURES AND ALGORITHM ANALYSIS
COMP 3804

Assignment 1

Date Due: Sept 29, 2017
Time Due: 11:00am
Your assignment should preferably be typed and should be submitted online on CuLearn.

Practice with Order

1. (16 pts) Prove or disprove the following:
   (a) (4 pts) $n^3/5 - 4n^2 - n + 1$ is $\theta(n^3)$
   (b) (2 pts) $n^2/3 + 17n$ is $O(n^3)$
   (c) (2 pts) $3n^2 - 2n - 14$ is $\Omega(n^2)$
   (d) (2 pts) $2n^3 \log n - 3n^2 \log n$ is $\Omega(n^2)$
   (e) (2 pts) 3^n is $\Omega(3^{n+1})$
   (f) (2 pts) $7n^3$ is $O(n^2)$
   (g) (2 pts) $4n^2$ is $\Omega(n^3)$

Practice with Proofs

2. Consider $n \geq 3$ lines in general position (i.e. a set of lines is in general position when no two lines are parallel and no three lines intersect at a point.). This set of lines partitions the plane into regions.
   (a) (4 pts) Prove that at least one of these regions is a triangle.
   (b) (Optional Bonus Challenge: 4 pts) Prove that at least $n - 2$ of these regions are triangles.

3. (4 pts) Prove by induction that $\sum_{i=1}^{n} 1/(n+i) < 13/24$.

Practice with Recurrences

4. (8 pts) The following questions are based on the following recurrence:
   $T(0) = 0, T(1) = 1$, and $T(n) = T(n-1) + T(n-2),\ \forall n \geq 2$
   (a) (4 pts) Use the fact that $T(2n) = T(n-1)T(n) + T(n)T(n+1), \forall n \geq 1$ to prove by induction that $T(2n) = T(n+1)^2 - T(n-1)^2, \forall n \geq 1$
   (b) (4 pts) Use the fact proved above that $T(2n) = T(n+1)^2 - T(n-1)^2, \forall n \geq 1$ to prove by induction that $T(2n+1) = T(n)^2 + T(n+1)^2, \forall n \geq 1$

5. (18 pts)
   (a) (6 pts) Let $n = 5^k$. Resolve the following recurrence. $T(1) = 1$ and $T(n) = T(n/5) + n, \forall n \geq 5$. Use induction on k. Notice that the base case is $k = 0$.
   (b) (6 pts) $T(0) = 0, T(1) = 1$ and $T(n) = 2T(n-1) - T(n-2), \forall n \geq 2$. Hint: Write out the first few values to find a pattern. Then, try to prove that your pattern is correct using induction.
   (c) (6 pts) Let $T(0) = 0, T(1) = 1$ and $T(n) = 6T(n-1) - 9T(n-2),\ \forall n \geq 2$. Use constructive induction to find an upper bound on the recurrence (i.e. assume that $T(n) \leq \alpha c^n$, where $\alpha > 0$ and $c > 1$). Both α and c are unknown, but use induction to figure out suitable values for these variables.