MMAE 411

Spacecraft Dynamics

Interplanetary transfer

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Outline

- Interplanetary transfer overview
- Hyperbolic orbits revisited
- Patched conics
  - Case I: Approach
  - Case II: Flyby
  - Case III: Departure

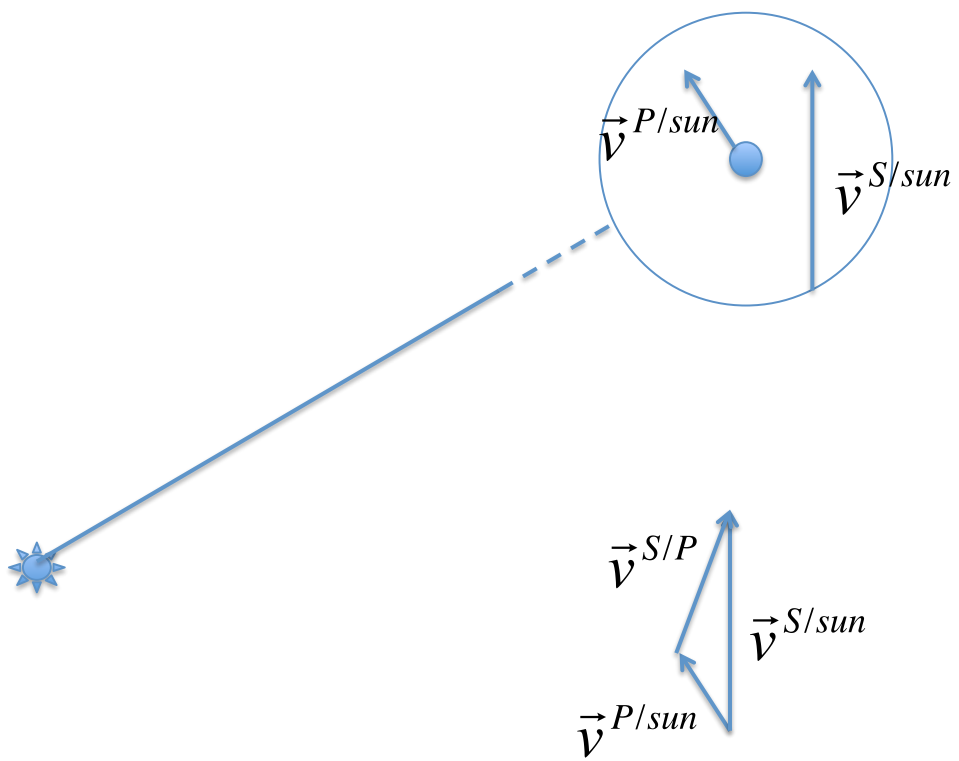
Overview

In the first several weeks of the class, we saw that we could write the equations of motion of two bodies gravitationally interacting. For anything more than two, there is no way to write a closed-form solution.

However, we can simplify the three-body problem into a sequence of two-body problems. This allows us to do interplanetary transfers.

Method: Sequence of separate 2-body problems

The sun is the main gravitational body in the solar system. But within a region around a planet, the "sphere of influence," the spacecraft orbits relative to the planet.



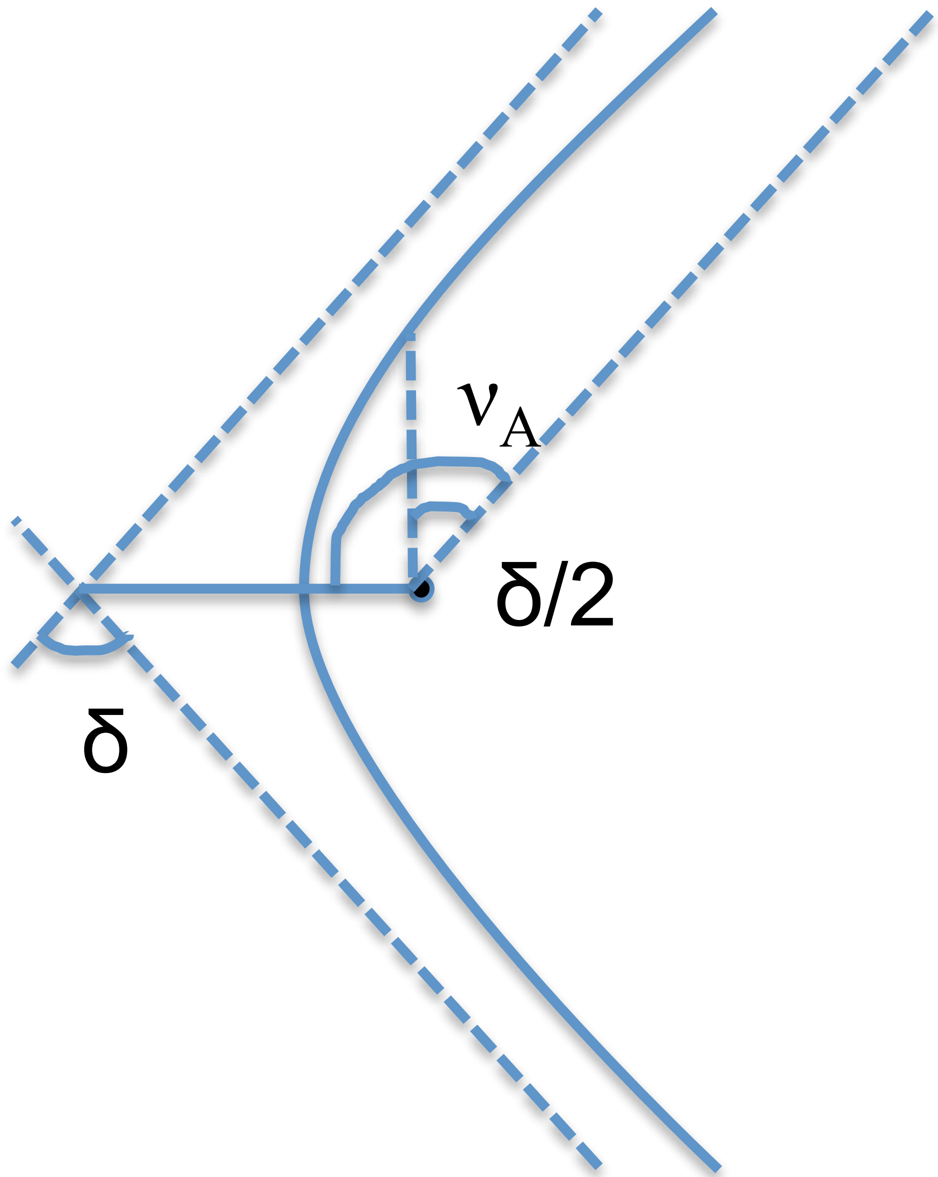
The idea: velocity vector triangles

1. Outside the sphere of influence, orbits of the spacecraft S and planet P are with respect to the sun, $\vec{v}^{S/\odot}$ and $\vec{v}^{P/\odot}$.

2. Inside the sphere of influence, the spacecraft orbit is relative to the planet, $\vec{v}^{S/P}$.

3. At the transition, we can use the velocity vector triangle to find the relationship $\vec{v}^{S/P}$.

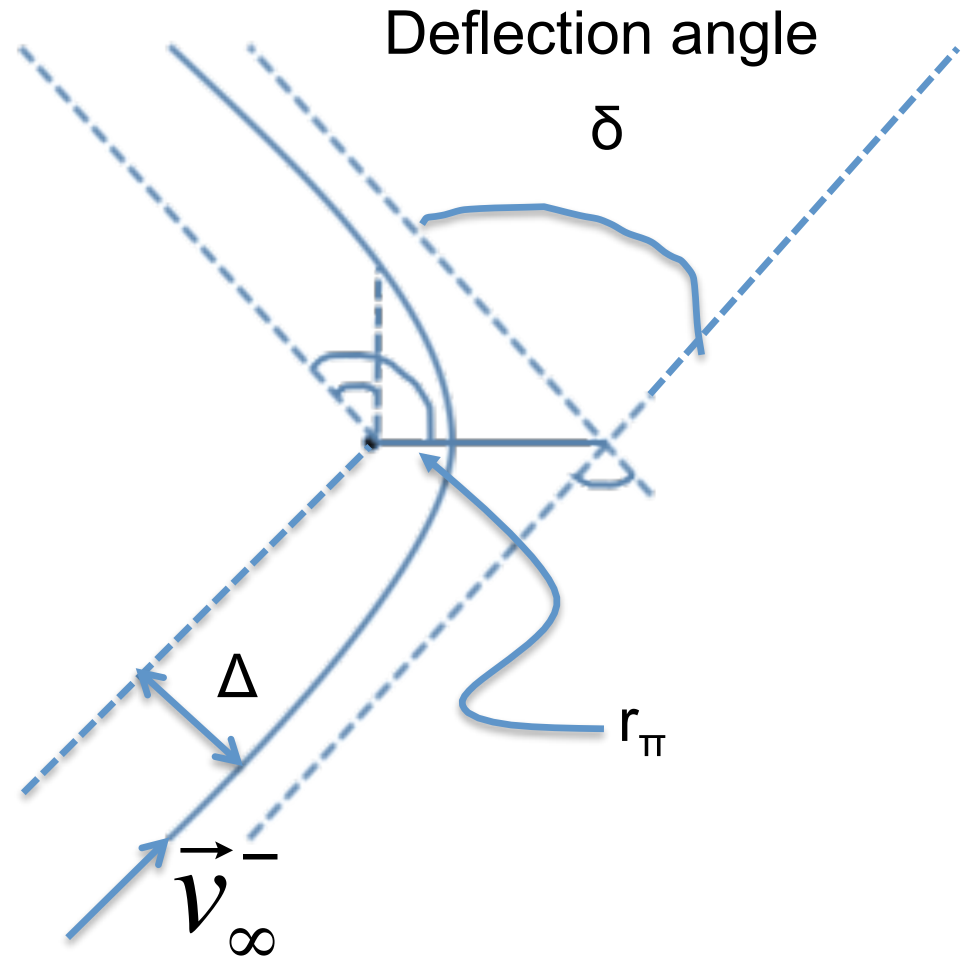
Hyperbolic orbits



Recall for a hyperbolic orbit

$$\begin{aligned} \mathcal{E} &> 0 && (1) \\ e &> 1 && (2) \\ a &< 0 && (3) \\ & && (4) \end{aligned}$$

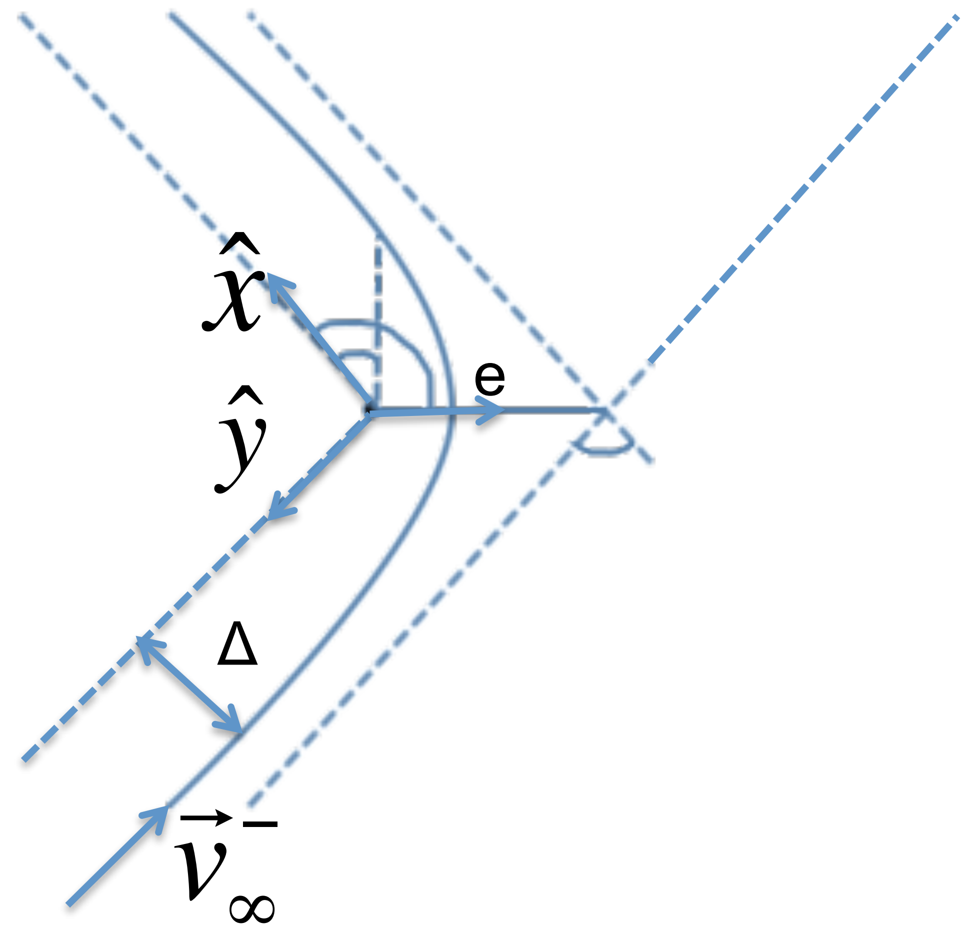
When the spacecraft enters the sphere of influence, assume that at that point it is "infinitely" far away such that $r = \infty$. Its velocity relative to the planet is treated as though it is $\vec{v}_\infty$.



$$\mathcal{E} = \frac{v_\infty^2}{2} - \cancelto{0}{\frac{\mu}{r}} = -\frac{\mu}{2a} \quad (5)$$

$$a = -\frac{\mu}{v_\infty^2} \quad (6)$$

Case I: Planetary approach



To find the vector solution, let's set up a convenient coordinate system, centered at the planet. $\hat{y}$ is anti-parallel to the approach velocity $\vec{v_\infty^-}$, $\hat{z}$ is perpendicular to the plane of the orbit, and $\hat{x}$ completes the righthand system.

In a planetary approach we might be interested in a number of parameters: the approach angle ν_A, the deflection angle δ and the point of closest approach r_π.

To find the approach angle, we need the eccentricity vector. Eccentricity is given by

$$\vec{e} \; = \; \frac{\vec{v} \times \vec{h}}{\mu} - \hat{r} \tag{7}$$

where $\hat{r}$ is a unit vector pointing to the spacecraft position at r_∞.

Eccentricity

$$\vec{h} = \begin{bmatrix} 0 \\ 0 \\ (v_\infty^-)\Delta \end{bmatrix} \quad (8)$$

$$\vec{v_\infty^-} = \begin{bmatrix} 0 \\ -v_\infty^- \\ 0 \end{bmatrix} \quad (9)$$

$$\vec{e} = \vec{v_\infty^-} \times \vec{h} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (10)$$

$$= \begin{bmatrix} -\frac{(v_\infty^-)^2\Delta}{\mu} \\ -1 \\ 0 \end{bmatrix} \quad (11)$$

We've neglected Δ in the unit vector $\hat{r}$ because it's so small compared to $|r|$.

So in the end,

$$e^2 = 1 + \frac{v_\infty^4 \Delta^2}{\mu^2} \quad (12)$$

Approach angle

The asymptotic approach angle ν_A can be found by:

$$
\begin{aligned}
e \cos \nu_A &= \vec{e} \cdot \widehat{r} && (13) \\
&= \begin{bmatrix} -\frac{(v_\infty^-)^2 \Delta}{\mu} \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} && (14) \\
\nu_A &= \pm \arccos\left(\frac{-1}{e}\right) && (15)
\end{aligned}
$$

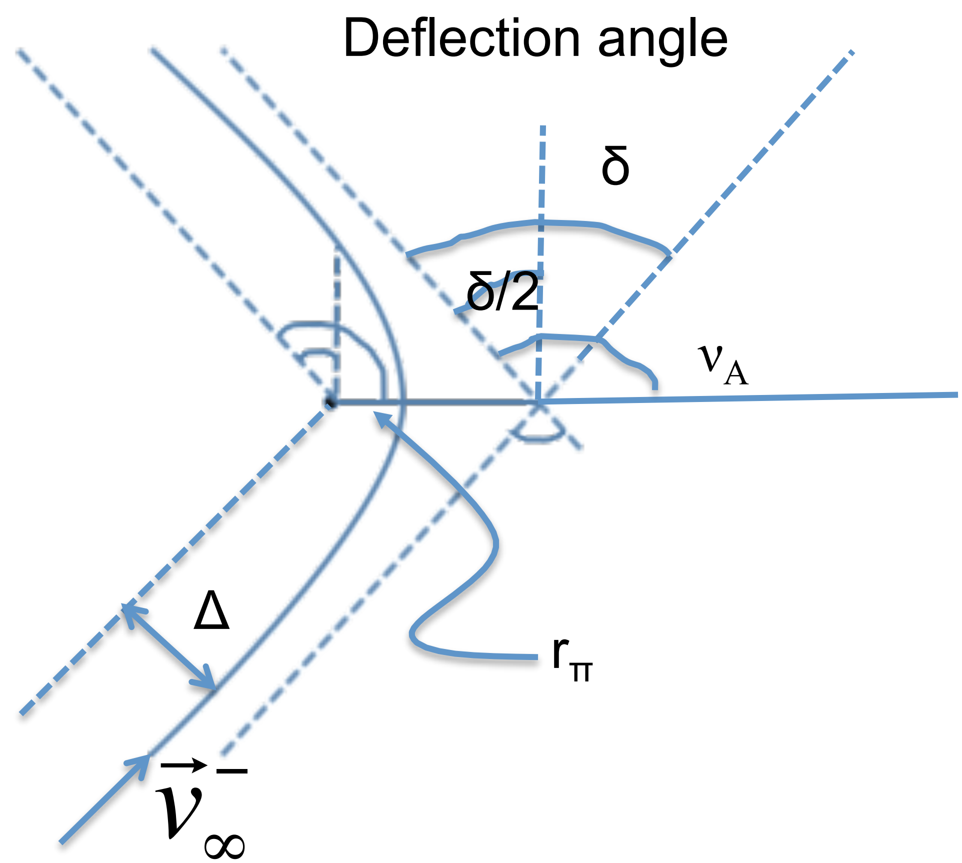
Alternatively, ν_A can be found by taking the limit as $r \to \infty$ of

$$
r = \frac{a(1 - e^2)}{1 + e \cos \nu} \qquad (16)
$$

Note which side of the e vector the spacecraft is on.

- If position is in the third quadrant $\to$ approaching.
- Second quadrant $\to$ departing.

Deflection angle



From the geometry we can see that the deflection angle δ has the following relationship:

$$
\begin{aligned}
\nu_A &= \frac{\pi}{2} + \frac{\delta}{2} && (17)\\
\cos \nu_A &= -\frac{1}{e} && (18)\\
\cos\left(\frac{\pi}{2} + \frac{\delta}{2}\right) &= -\frac{1}{e} && (19)\\
-\sin\left(\frac{\delta}{2}\right) &= -\frac{1}{e} && (20)\\
\delta &= 2\arcsin\left(\frac{1}{e}\right) && (21)\\
&= 2\arcsin\left(\frac{\mu}{\sqrt{\mu^2 + v_\infty^4 \Delta^2}}\right) && (22)
\end{aligned}
$$

Periapsis

The big question in planetary approach is: how close does the spacecraft come to the planet, i.e., what is r_π?

From angular momentum conservation,

$$v_\infty \cdot \boldsymbol{\Delta} = v_\pi \cdot r_\pi \tag{23}$$

From energy conservation,

$$\frac{v_\pi^2}{2} - \frac{\mu}{r_\pi} = \frac{v_\infty^2}{2} \tag{24}$$

$$v_\pi = \sqrt{v_\infty^2 + \frac{2\mu}{r_\pi}} \tag{25}$$

From practical concerns, you will usually have a constraint on r_π (not to get too close). Because it is related to $v_\infty \cdot \boldsymbol{\Delta}$, this constraint will affect the deflection angle δ.

Asymptotic offset distance Δ

Combine the h equation and the $\mathcal{E}$ equation:

$$\Delta = \frac{r_\pi}{v_\infty}\sqrt{v_\infty^2 + \frac{2\mu}{r_\pi}} \tag{26}$$

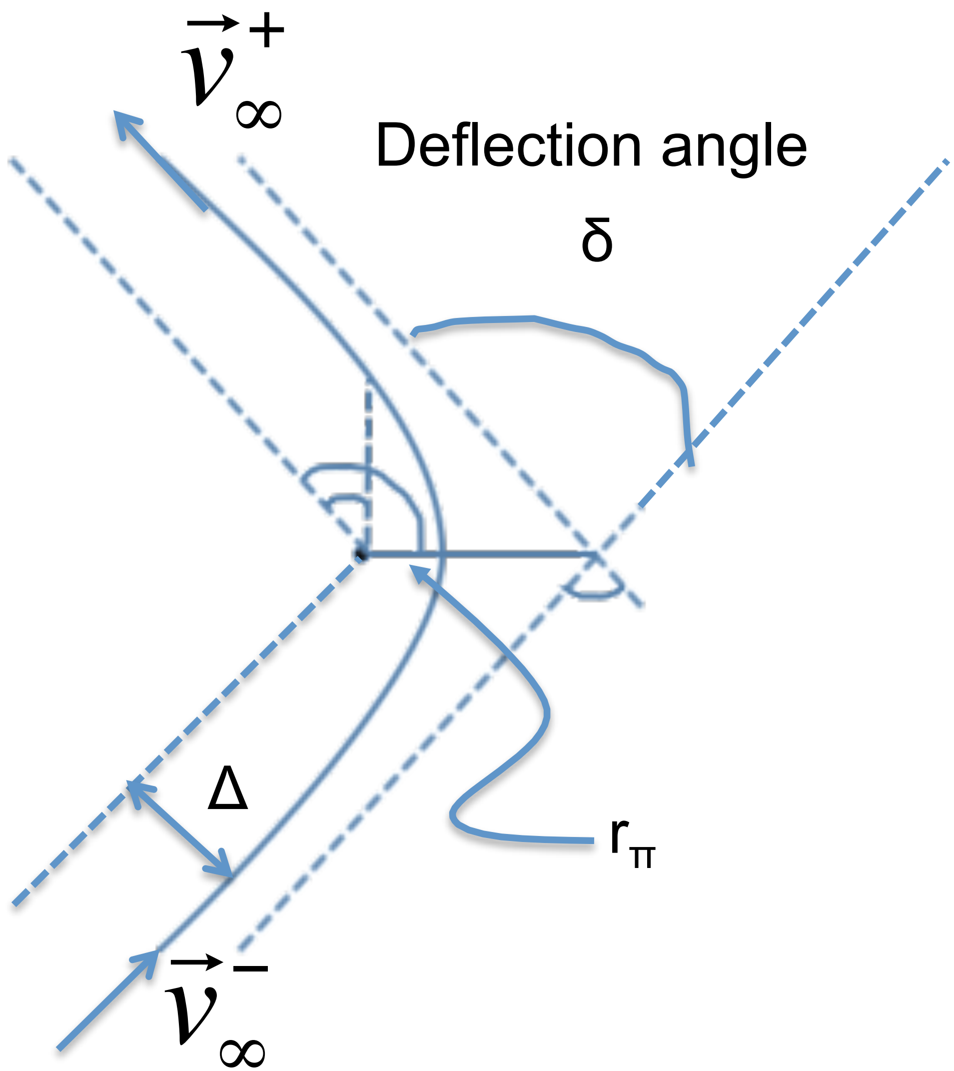
or

$$\boxed{\Delta = r_\pi\sqrt{1 + \frac{2\mu}{r_\pi} \cdot \frac{1}{v_\infty^2}}}$$

What's the use of this?

- Define "how close" is acceptable $\Rightarrow = r_\pi$. Then find $\Delta = f(r_\pi)$.
- At a *distance*, changing Δ is fairly easy.

Case II: swing by (flyby)



For flyby, we may have a desired deflection angle, δ. Just as v_∞^- is the velocity at initial approach into the sphere of influence, v_∞^+ will be the velocity upon departure.

Use relation between δ and e and the e^2 expression:

$$\begin{aligned} e &= \frac{1}{\sin\left(\frac{\delta}{2}\right)} && (27) \\ &= \sqrt{1 + \frac{v_\infty^4 \Delta^2}{\mu^2}} && (28) \end{aligned}$$

$$\boxed{\Delta^2 = \frac{\mu^2}{v_\infty^4}\left(\frac{1}{\sin^2\left(\frac{\delta}{2}\right)} - 1\right)}$$

With this, we can answer the practical question: given a desired deflection δ, what is the appropriate offset, Δ? Then we can use small lateral adjustments early on to "fine tune" Δ.

Case III: Departing from planet

Assume in this situation, we're initially in *circular* orbit $\Rightarrow r_\pi$ is given.

Heading into "solar" space, we have a desired v_∞. We just found

$$v_\pi = \sqrt{v_\infty^2 + \frac{2\mu}{r_\pi}} \tag{29}$$

Q: But what is the best position/time to apply Δv?
A: From e, get ν_A.

$$e^2 = 1 + \frac{v_\infty^4 \Delta^2}{\mu^2} \tag{30}$$

Case III: Departure asymptote angle ν_A

Squaring the expression for Δ and plugging it into e^2 :

$$
\begin{aligned}
\Delta^2 &= r_\pi^2 \left(1 + \frac{2\mu/r_\pi}{v_\infty^2}\right) && (31)\\
e^2 &= 1 + \frac{v_\infty^4}{\mu^2} r_\pi^2 \left(1 + \frac{2\mu/r_\pi}{v_\infty^2}\right) && (32)\\
&= 1 + \frac{v_\infty^4 r_\pi^2}{\mu^2} + \frac{2 v_\infty^2 r_\pi}{\mu} && (33)\\
&= \left(1 + \frac{v_\infty^2 r_\pi}{\mu}\right)^2 && (34)
\end{aligned}
$$

So

$$\boxed{e = 1 + \frac{v_\infty^2 r_\pi}{\mu}}$$

And

$$\boxed{\nu_A = \arccos\left(-\frac{1}{e}\right) = \arccos\left(\frac{-1}{1+\frac{v_\infty^2 r_\pi}{\mu}}\right)}$$

Summary

Approaching a planet we are "given" v_∞.

- Adjust Δ (use small lateral thrust) to obtain a desired r_π or deflection δ.

Departing from a parking orbit of radius r_π, we have a desired v_∞ and direction at the sphere of influence boundary.

- Find burn position r_π from ν_A equation above.