MMAE 411
Spacecraft Dynamics

Reaction wheel and yo-yo mechanism

Recap 1

We examined the natural motion of the spacecraft by looking at its rotational equations of motion for two commonly encountered situations:

- torque–free motion
- non-spinning orbiting motion

In some cases the motion is oscillatory and in other cases unstable, as determined by the pitch EOM and by the roll-yaw EOM.

Recap 2

The approach in analyzing spacecraft dynamics is:

1. Set up your reference frames and coordinate systems.

2. Determine $\vec{\vec{I}}$ and $\vec{\omega}$ if not already given. Best to work in principal axis coordinates.

3. Determine $\vec{H}$, $\dot{\vec{H}}$, and $\vec{\omega} \times \vec{H}$.

4. Write the rotation EOM (Euler's equations) for your system.

5. Linearize them by neglecting terms that involve the product of small angles and/or small angular rates.

6. When equations are coupled, differentiate one and plug into the other to get a single ODE.

7. Look for the roots of the ODE in the complex space by taking the Laplace transform and solving for s.

After dynamics: control

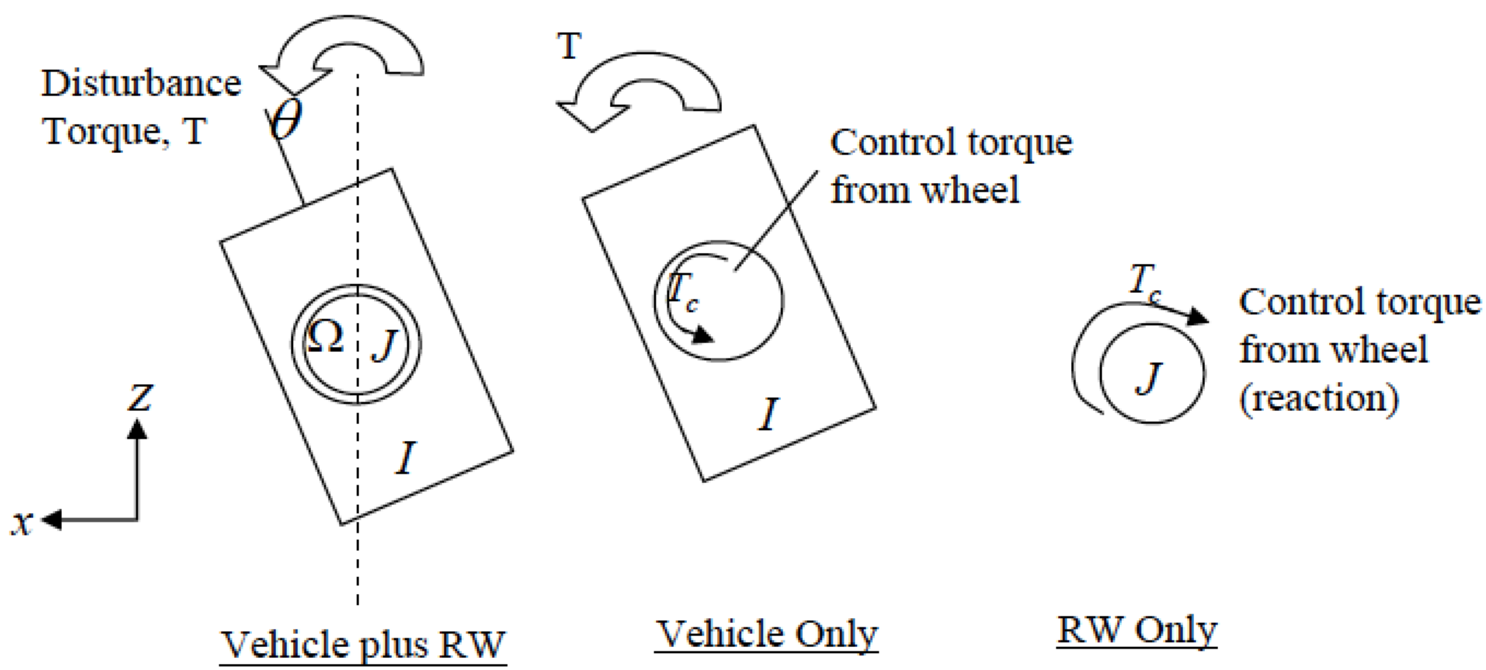
Once we've seen the EOM and how they respond to a disturbance, we can next look at ways of controlling the spacecraft in the presence of disturbances.

Let's begin by looking at a spacecraft we want to have not spin. It is equipped with a *reaction wheel*, or *null momentum wheel*, that can spin. The purpose of having the reaction wheel is to put unwanted spacecraft angular momentum in the wheel.

Reaction wheels effectively absorb the angular momentum of the satellite so that attitude motion can be controlled very precisely to 0. Basic idea: to spin a wheel on a spacecraft, there will be an equal and opposite torque on the s/c.

Free-body diagram with spinning wheel

Consider a spacecraft (s/c) in deep space, so no gravity. It is nominally non-spinning s/c but with a spinning wheel.



EOM with a spinning wheel

The wheel has angular momentum $\vec{H}_{wheel}$:

$$
\begin{aligned}
\vec{H}_{total} &= \vec{H}_{body} + \vec{H}_{wheel} && (1)\\
&\approx \vec{H}_{body} + J\Omega\hat{y} && (2)\\
\vec{M} &= \frac{{}^{I}d\vec{H}}{dt} && (3)\\
&= \frac{{}^{I}d\vec{H}_{body}}{dt} + \frac{{}^{I}d}{dt}\vec{H}_{wheel} && (4)
\end{aligned}
$$

The first term is our standard EOM.

Reaction wheel angular momentum

Let's look at the pitch EOM (angle θ about $\widehat{y}$). For the spacecraft:

$$\dot{\theta} = \delta\omega_y \tag{5}$$

$$\frac{{}^{I}d}{dt}\vec{H} = I_y\delta\dot{\omega}_y = I_y\ddot{\theta} = \underbrace{T_c}_{\text{motor torque}} + \underbrace{T_{dy}}_{\text{external disturbance torque}} \tag{6}$$

$$\tag{7}$$

Meanwhile, for the wheel the EOM is:

$$J\frac{{}^{I}d}{dt}(\Omega + \delta\dot{\omega}) = J(\dot{\Omega} + \ddot{\theta}) = -T_c \tag{8}$$

The motor torque is equal and opposite on the wheel, and the friction is equal and opposite also, relative to the spacecraft.

System angular momentum

Look at the 1-d physics of the overall system:

$$
\begin{aligned}
H &= I\dot{\theta} + J(\Omega + \dot{\theta}) && (9) \\
\frac{{}^{I}d}{dt}H &= I\ddot{\theta} + J(\dot{\Omega} + \ddot{\theta}) && (10) \\
&= T_{dy} && (11)
\end{aligned}
$$

Only external torques change the angular momentum. The internal torques cancel. In other words, you can't change the angular momentum of the system with just the internal torques of the reaction wheel.

However, you *can* use the motor to exchange the momentum between the wheel and the spacecraft.

Saturation

Let's assume your controller (generating an internal torque) does a perfect job of keeping $\delta\dot{\omega}_y = 0$ and $\delta\omega_y = 0$. Then,

$$\frac{{}^{I}d}{dt}H = J(\dot{\Omega} + \ddot{\theta}) = T_{dy} \tag{12}$$

So constant $T_{dy} \Rightarrow \Omega \uparrow \infty$. This is a fundamental problem with wheels: they saturate!

Eventually you need to dump the momentum. How? Apply a "negative" disturbance torque, that tells it to point in the wrong direction for awhile. Using:

- gravity moment (which we've seen)
- gas thrusters

Yo-yo mechanism

Another common scenario with spacecraft is that they may be launched and transfer orbits with angular momentum, but then for their final configuration they need a low or 0 spin. For these situations the *yo-yo* mechanism is a useful trick.

As with the reaction wheel, the goal is to transfer the unwanted angular momentum to another part of the system. In this case after the yo-yo masses are extended, they are cut from the spacecraft. They drift off, carrying the satellite's angular momentum away.

Yo-yo system E and H

Consider a satellite of radius R spinning at rate ω_0 about an axis with moment of inertia I. Before deployment, the 2 yo-yo masses m are part of the system and spinning at the same rate, locked to the rim of the satellite. Energy and angular momentum of the system:

$$E = \frac{1}{2}I\omega_0^2 + \frac{1}{2}(2m)(R\omega_0)^2 \quad (13)$$

$$H = I\omega_0 + 2mR^2\omega_0 \quad (14)$$

As the cords of the yo-yo's unwind, they no longer spin rigidly with the spacecraft.

Yo-yo frame after release

Attach a frame and coordinate system C to the center of the spacecraft whose y axis $\widehat{c}_y$ always points to the tangent point of the yo-yo cord. In this cord frame the mass m is moving out along the x axis at a rate dl/dt, where l is the length that has unwound.

This frame has rotated by an angle ψ since the cord was released. This angle is related to the amount of unspooled cord by $l = \psi R$. The frame has an angular velocity

$$
\begin{aligned}
{}^{I}\vec{\omega}^{C} &= (\omega + d\psi/dt)\,\widehat{c}_z && (15)\\
&= \left(\omega + \frac{1}{R}\frac{dl}{dt}\right)\widehat{c}_z && (16)
\end{aligned}
$$

Yo-yo E and H after release

With a frame and coordinate system set up, we can write velocity and then H and E.

$$\vec{r} = l\hat{c}_x + R\hat{c}_y \tag{17}$$

$$\vec{v} = \frac{{}^I d}{dt}\vec{r} = \frac{{}^C d}{dt}\vec{r} + {}^I\vec{\omega}^C \times \vec{r} \tag{18}$$

$$= \dot{l}\hat{c}_x + \left(\omega + \frac{\dot{l}}{R}\right)\hat{c}_z \times (l\hat{c}_x + R\hat{c}_y) \tag{19}$$

$$= -R\omega\hat{c}_x + l\left(\omega + \frac{\dot{l}}{R}\right)\hat{c}_y \tag{20}$$

$$\vec{H}_m = 2m\vec{r} \times \vec{v} = 2m\left[\omega(R^2 + l^2) + l^2\frac{\dot{l}}{R}\right]\hat{c}_z \tag{21}$$

$$\tag{22}$$

$$E_m = \frac{1}{2}m\vec{v}\cdot\vec{v} = m\left[\omega^2 R^2 + l^2\left(\omega + \frac{\dot{l}}{R}\right)^2\right] \tag{23}$$

Length of cord required for satellite de-spin

We can ask: how long do the yo-yo cords l need to be in order to de-spin the satellite from ω to 0? The answer can be found from realizing that E and H are conserved. So the values before yo-yo deployment are the same as the values during deployment:

$$H = (I + 2mR^2)\omega_0 = I\omega + 2m\left[\omega(R^2 + l^2) + l^2\frac{\dot{l}}{R}\right] \quad (24)$$

$$E = \frac{1}{2}(I + 2mR^2)\omega_0^2 \quad (25)$$

Length of cord required for satellite de-spin, part 2

The H equation can be rearranged:

$$K(\omega_0 - \omega) = \frac{l^2}{R^2}\left(\omega + \frac{\dot{l}}{R}\right) \quad (26)$$

$$K \equiv 1 + \frac{I}{2mR^2} \quad (27)$$

And the energy equation is:

$$K(\omega_0^2 - \omega^2) = \frac{l^2}{R^2}\left(\omega + \frac{\dot{l}}{R}\right)^2 \quad (28)$$

Since $(\omega_0^2 - \omega^2) = (\omega_0 + \omega)(\omega_0 - \omega)$ we can divide these two equations:

$$\omega_0 + \omega = \omega + \frac{\dot{l}}{R} \quad (29)$$

The unwrap rate is constant at $\dot{l} = R\omega_0$.

Length of cord required for satellite de-spin, part 3

Plug back in for $\dot{l}$ to get:

$$\omega = \omega_0 \frac{KR^2 - l^2}{KR^2 + l^2} \tag{30}$$

The required final cord length l_f will be:

$$l_f = R\left(K\frac{\omega_0 - \omega_f}{\omega_0 + \omega_f}\right)^{1/2} \tag{31}$$

If we want to completely de-spin the satellite, then:

$$l_f = R\sqrt{K} = \sqrt{R^2 + \frac{I}{2m}} \tag{32}$$

The cord length required does *not* depend on the initial spin of the satellite!