MMAE 411

Spacecraft Dynamics

Orbit maneuvers

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Outline

- Hohmann transfer

- General coplanar transfer

In-plane orbit change

Suppose a spacecraft is in an orbit about a central body. Its orbit defines an orbital plane. Two types of changes can be done to the orbit from here:

- co-planar transfer: a change of the orbit to a new orbit lying in the same plane

- out-of-plane transfer: an orbit that is in a different plane.

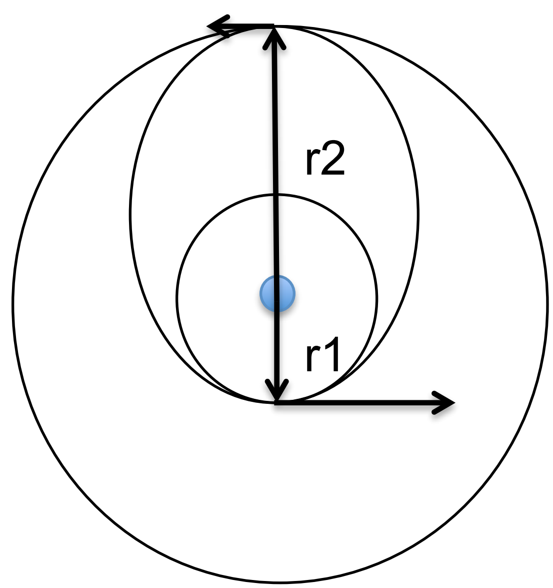
We consider today the changes that change to a different orbit *in the plane.*

The orbit can be changed by adding or subtracting energy. Energy can be added or removed by using fuel to change the velocity (speed and/or direction).

Hohmann Transfer

The Hohmann transfer is an in-plane orbit change, from one circular orbit to a co-planar circular orbit with a different radius.

This is the most efficient way (least fuel spent, i.e., least amount of δv) to change orbits. However, it also takes longer than any other kind of transfer.



Two δv's in Hohmann Transfer

The Hohmann transfer orbit is an ellipse that is tangential to both the starting orbit and ending orbit. It requires two changes of velocity (i.e., energy):

- Δv_1 to go from circle of radius r_1 to the ellipse

- Δv_2 to go from the ellipse to the circle of radius r_2

The direction of the velocity does not change. Only the speed changes. It increases to go from a small circle r_1 to an ellipse, and then it increases again from the ellipse to stay at the larger radius r_2.

The Hohmann transfer can also be used to go from a large circle to a small one, by slowing the sv down twice. (You still have to spend fuel to do it, though).

Δv_1 in Hohmann Transfer

To compute Δv_1, which changes the orbit from a circle of radius r_1 to the ellipse, we compute the speed of the circular orbit.

$$\begin{aligned} \Delta v_1 &= v_1 - v_{r_1} && (1) \\ v_{r1} &= \sqrt{\frac{\mu}{r_1}} && (2) \end{aligned}$$

The speed of the sv in the elliptical orbit can be computed from the energy:

$$\begin{aligned} \mathcal{E} &= -\frac{\mu}{2a} = \frac{v_\pi^2}{2} - \frac{\mu}{r_1} && (3) \\ v_1 &= \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_t}} && (4) \\ a_t &= \frac{r_1 + r_2}{2} && (5) \end{aligned}$$

Δv_1 and Δv_2 in Hohmann Transfer

So

$$\Delta v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_t}} - \sqrt{\frac{\mu}{r_1}} \tag{6}$$

A similar process can be used to compute the Δv_2 required to transfer from the ellipse to the circular orbit of radius r_2:

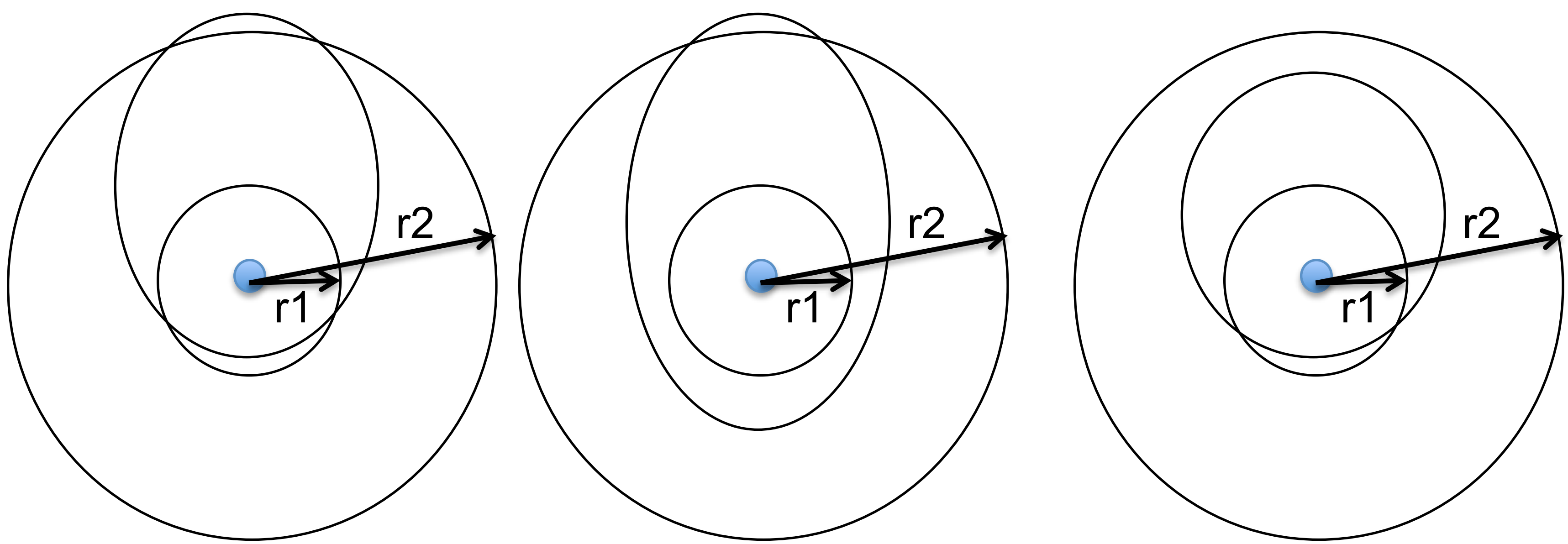
$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_t}} \tag{7}$$

The time of flight to transfer between one circle and the other is just half the period of the elliptical orbit:

$$\boxed{\Delta t = \frac{\mathcal{P}}{2} = \pi\sqrt{\frac{a^3}{\mu}}}$$

General co-planar transfer between orbits

The Hohmann transfer, while the most efficient, is not the only way to transfer between coplanar orbits. The transfer orbit only has to intersect (or at minimum be tangential to) both orbits.



Possible because $r_\pi < r_1$ and $r_\alpha > r_2$

Impossible: $r_\pi > r_1$

Impossible: $r_\alpha < r_2$

General transfer between circular orbits, part 1

A non-Hohmann transfer will generally use more fuel because v must be used to change speed *and* direction. If you know the transfer ellipse's a and e that let it intersect both orbits, you can compute the Δv_{tot} required.

First note the energy and magnitude of angular momentum of the transfer ellipse:

$$\mathcal{E}_t = -\frac{\mu}{2a} = \frac{v_t^2}{2} - \frac{\mu}{r_t} \tag{8}$$

$$h_t = \sqrt{\mu p} \tag{9}$$

$$= \sqrt{\mu a(1-e^2)} \tag{10}$$

Next, proceed just like we did for the Hohmann transfer: solve (8) for the speed at point 1 in the transfer orbit:

$$v_1 = \sqrt{2\left(\frac{\mu}{r_1} + \mathcal{E}_t\right)} \tag{11}$$

General transfer between circular orbits, part 2

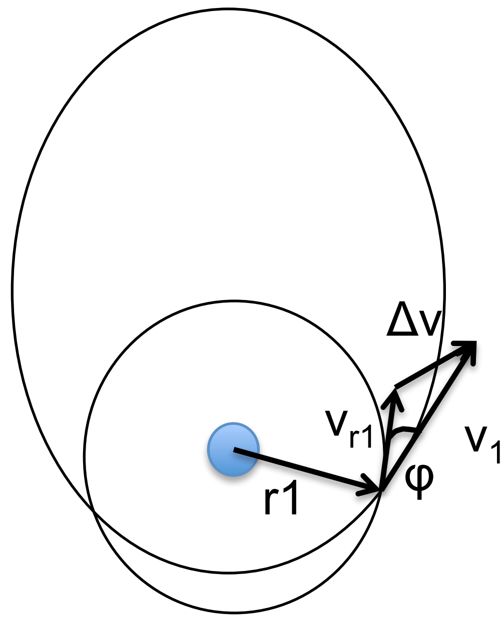
The speed in the circular orbit before the transfer begins is $v_{r1} = \sqrt{\frac{\mu}{r}}$. However, for a general transfer, we can't just difference the speeds. We have to take into account the angle between $\vec{v}_1$ and $\vec{v}_{r1}$.

That angle is the flight path angle ϕ, given by:

$$h = rv \cos\phi \tag{12}$$

$$\phi = \arccos \frac{h_t}{r_1 v_1} \tag{13}$$

General transfer between circular orbits, part 3



Looking at the geometry of the vector diagram, we can use the law of cosines to solve for the amount of Δv required:

$$\Delta v_1^2 \;=\; v_1^2 + v_{r1}^2 - 2 v_1 v_{r1} \cos\phi_1 \qquad (14)$$

The speed change required at point 2 (arriving at the outer orbit) can be computed in a similar way. The Hohmann transfer is just a special case of (14), with $\phi_1 = 0$.