The Chinese University of Hong Kong

PHYS4061 Computational Physics, 1^{st} semester, 2019-2020

Homework 3

There are 3 options in this HW. You can choose A) finish Q1 and Q2, B) finish Q3, C) finish Q4. Q1 and Q2 have Java source code in lecture notes. For those who have taken PHYS3061 can reuse molecular dynamic code for Q4.

1. Suppose in a parallel world, all parameters remain unchanged, except that air resistance is $-\kappa|v|^{\frac{1}{3}}\vec{\boldsymbol{v}}$.
   Rewrite the java motor cyclist program in lecture notes 12 in C or Fortran to calculate this problem. Compare the difference between this model and the $-\kappa|v|\vec{\boldsymbol{v}}$ model of the original code.

2. Setup and run the java code of the relaxation method shown in lecture notes 15, you can also rewrite it in C/C++ or Fortran if you don't know how to run Java or you enjoy programming. Compare the convergence speed of 10 different p values of your choice, which one is better?
   Hint:
   a. Changing some parameters other than variable "p" related to convergence is needed.
   b. Check your understanding, what is meant by "converge"? It is not necessary to define "converge" in the same form as provided in the code.

3. Solving Tolman–Oppenheimer–Volkoff (TOV) equation using 4^{th} order Runge Kutta method.

   TOV equation is the hydrostatic equation of spherically symmetric body in static gravitational equilibrium in general relativity. The equation of state used is polytropic. TOV equations:

$$\frac{dp}{dr} = -G\left(\rho\left(1+\frac{\epsilon}{c^2}\right)+\frac{p}{c^2}\right)\frac{m+\frac{4\pi r^3 p}{c^2}}{r\left(r-\frac{2Gm}{c^2}\right)}$$

$$\frac{dm}{dr} = 4\pi r^2\rho\left(1+\frac{\epsilon}{c^2}\right)$$

$$p = K\rho^{\gamma}$$

$$\epsilon = \frac{p}{(\gamma - 1)\rho}$$

K = 1.982 x 10^{-6} (in CGS) or = 1.11456 x 10^{-15} (in Si unit), γ = 2.75, G is gravitational constant, c is speed of light. ϵ is specific internal energy.

What you need to do:

a. Find the radius and mass of the body using RK4, given initial condition ρ_c =5.0 x 10^{17} kg m^{-3}
b. Try using different ρ_c and plot the M-R curve (use km and mass of sun as units)

Find a reasonable max radius, step size, and cutoff condition for iteration. Submit the 1) Source code, 2) Output file with r, P(r) , M(r) and Density(r) for part a. ,3) Plot of 2, 4) M-R curve of B

Hint 1: in CGS unit all the numbers will have many digits, so the output formatting need more space.

Hint 2: Find a way to make the computer try different ρ_c automatically. Also test a wider range of ρ_c.

4. Three body simulation using gravitational force

Write a code for three body simulation of gravitational force. Implement Euler method and one predictor-corrector algorithm on same initial condition. Calculate the individual kinetic energy, total kinetic energy and total potential energy.

Submit the 1) source code, 2) Output .xyz files of trajectory, 3) plots of the KE vs time and PE vs time in two different integration methods you chosen, 4) A small report to explain the difference in energy between the two methods you used

Please comment your code, including how to compile and run your code.

Submit to ylfchong@phy.cuhk.edu.hk

Ask TA if you have questions regarding this HW.

Due date: 12 Nov 2019 Score will be deducted if the submission is late.