CS 456: Quiz 5

Due on April 4, 2024

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Problem Description

Prove the following language is not regular, using the pumping lemma for regular languages

$$L = \{n_a(w) + 2 \leq n_b(w) : w \in \{a, b\}^*\}$$

Solution, Proof by contradiction

Assumption

Assume that L is a regular language. If L is regular, then all strings of length P or greater can be decomposed into xyz where the length of xy is less than or equal to P and the length of y is greater than 1. Then, $s = xy^iz$ is also in the language for all $i \geq 0$. P is the number of states used in the FA used to accept the languge and is a positive integer.

Counter Example

Assume that s is a string in the language L where $s = a^P b^{(P+2)}$. This string is in the language since there are 2 more b's than a's and the language mandates that the number of b's is at least two more than the number of a's.

String Decomposition

The string can be decomposed into an x, a y and a z value. The xy portion must be less than or equal to P so an acceptable value for xy is a^P. the y portion of this, can be any number k of the a's. The values for x, y, and z are shown below. It is also helpful to split the string into the portion that is included in y and the portion that is not included in y and this is also shown below.

$$x = a^{P-k},\ y = a^k,\ z = b^{P+2}$$

$$y = a^k,\ \textit{not}\ y = a^{P-k}b^{P+2}$$

Pumping the i value

In order for the language to be regular $s = xy^iz$ must be in the language for all $i \geq 0$. In order to show that the language is not regular, some i value must be found such that $s = xy^iz$ is not in the language. To find this, we will test a couple different values.

$i = 0, s = a^{P-k}b^{P+2}$	**Does not prove language is not regular,** $s_0 \in L$
$i = 1, s = a^P b^{P+2}$	**Does not prove language is not regular, if $s \in L$, then s_1 is always in L**
$i = 2, s = a^{P-k}a^{2k}b^{P+2} = a^{P+k}b^{P+2}$	$s_2 \notin L$ **In s_2, the number of b's is not at least 2 more than the number of a's**

Conclusion

By proof by contradiction, the language L is not a regular lanuage because it does not adhere to the pumping lemma. In order for a language to be regular, it must be possible to "pump" any string in the language. A counter example s in the language L was found that when "pumped" to s_2 was no longer in the language.

s_2 is not in the language since it has $P+2$ b's and $P+k$ a's. The number of b's is not at least 2 more than the number of a's since k is an integer greater than or equal to 1.