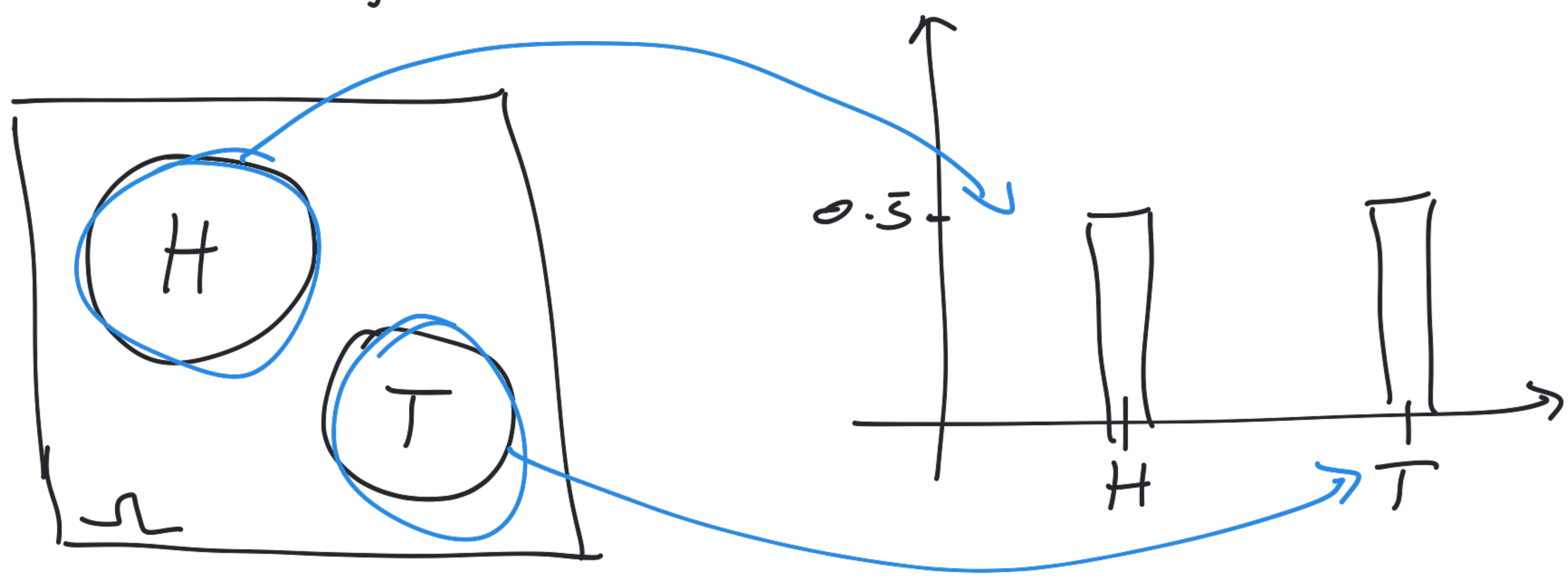
Probability Measure

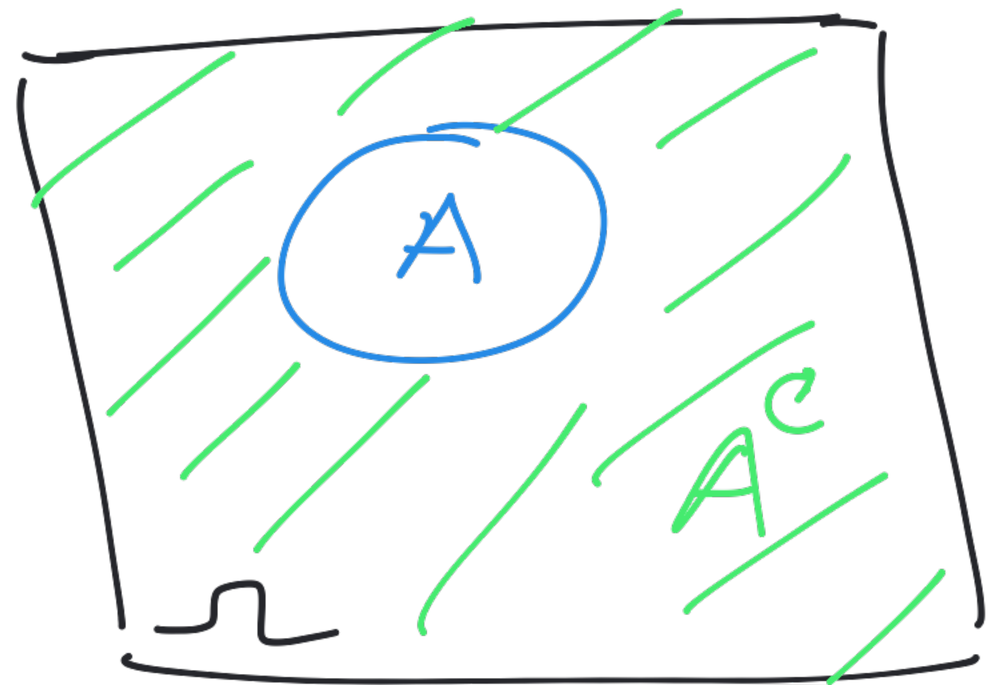
$\mathcal{F} \equiv$ flipping a fair coin $\qquad P : \mathcal{F} \to \mathbb{R}$



Outcomes H an T are m. E.

Corollary

① $P(A^c) = 1 - P(A)$



$\boxed{A \cup A^c = \Omega}$

$A \cap A^c = \emptyset$

$P(\Omega) = 1$, using axiom 1

$P(A \cup A^c) = 1$

using axiom 3,

$P(A) + P(A^c) = 1 \iff P(A^c) = 1 - P(A)$

2. $P(A) \leq 1$

$$P(A) = 1 - \underbrace{P(A^c)}_{\geq 0}$$

≤ 1

3 $P(\emptyset) = 0$

$\emptyset = \overline{\Omega} \equiv \Omega^c$

$P(\emptyset) = 1 - P(\overline{\emptyset}) = 1 - P(\Omega)$

$= 1 - 1$

$= 0$

4 by induction

5 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Let $E_i \equiv$ 1 OR 2 on Roll i

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

(1,1), (1,2)
(2,1), (2,2)

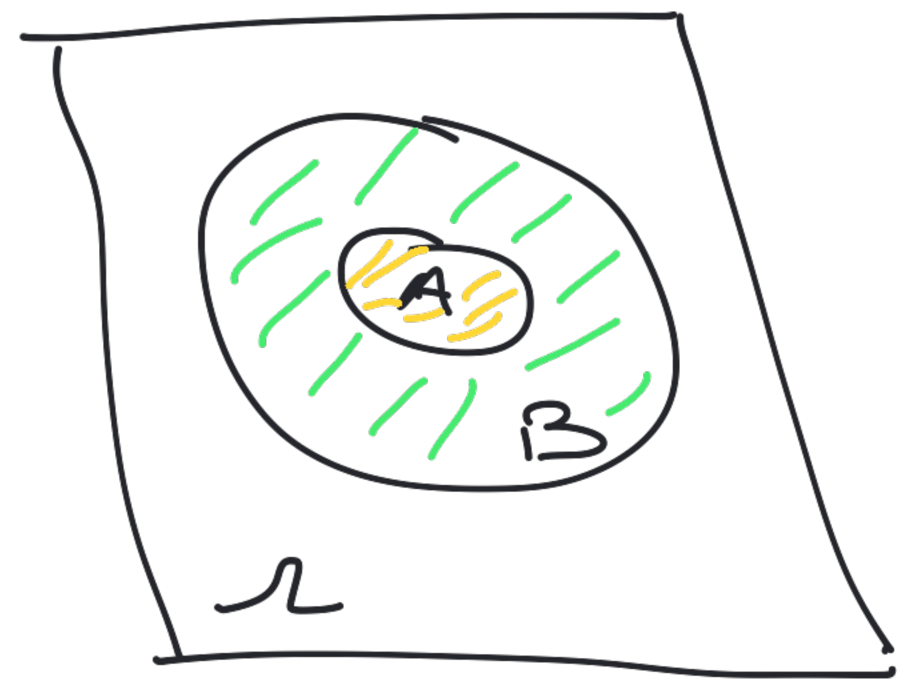
$$= \frac{1}{3} + \frac{1}{3} - \frac{4}{36}$$

⑥ by induction

⑦ If $A \subset B$, then $P(A) \leq P(B)$

Let $B = A \cup (B \cap \bar{A})$

M.E.



Then, $P(B) = P(A \cup (B \cap \bar{A}))$

$$= P(A) + \underbrace{P(B \cap \bar{A})}_{\geq 0}$$

M.E.

$$\Longrightarrow P(B) \geq P(A) \checkmark$$