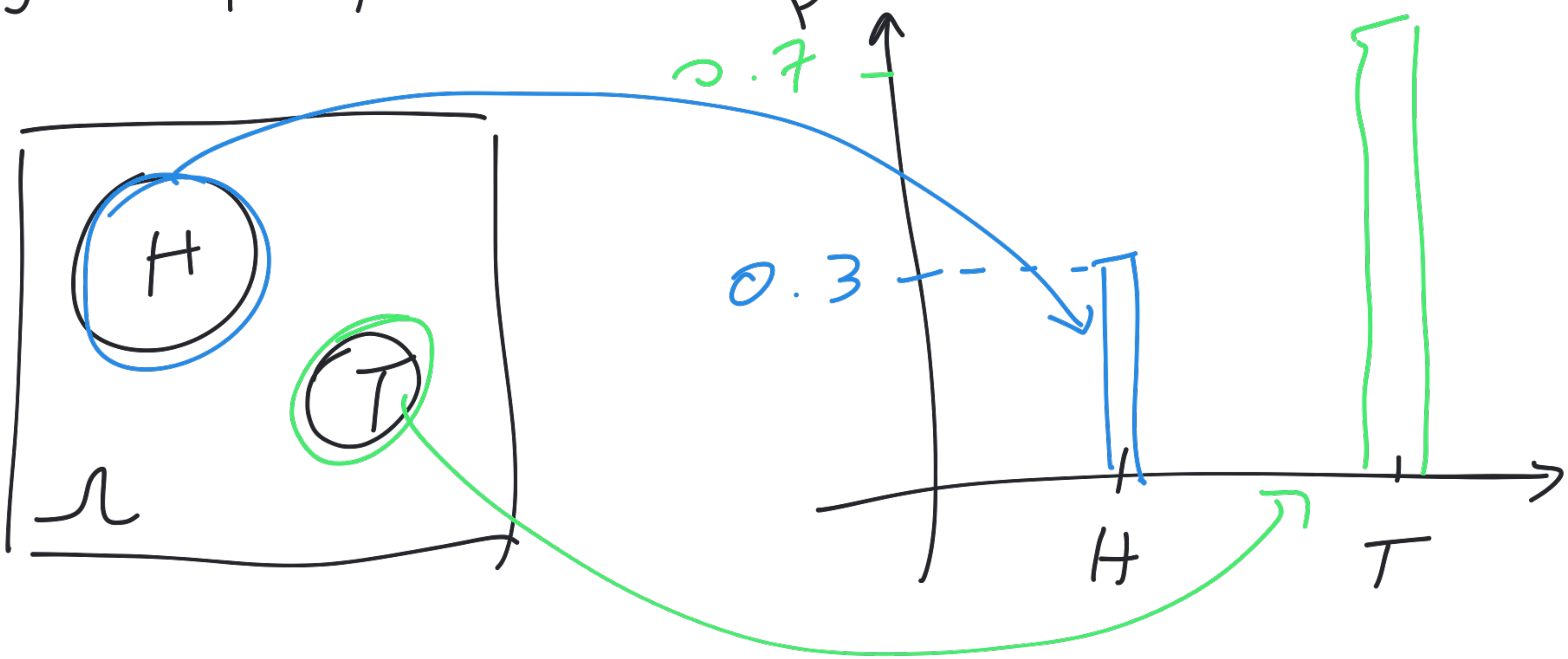
Probability Measure

$P : \mathcal{F} \longrightarrow \mathbb{R}$

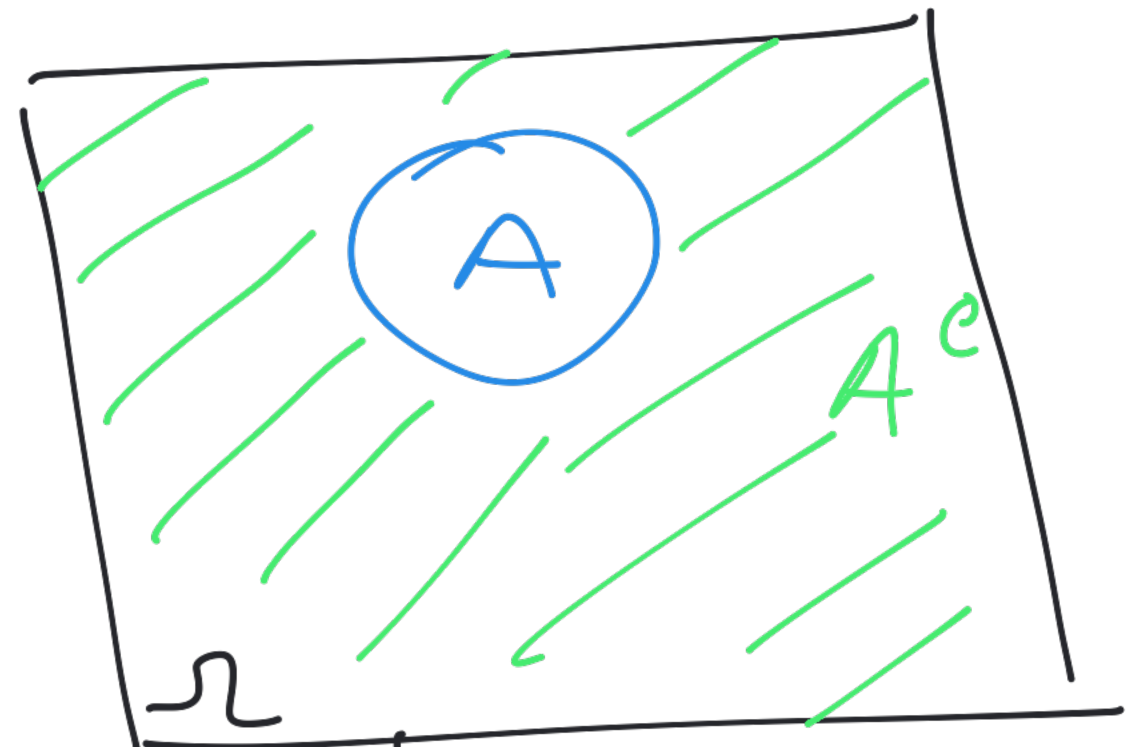
$\mathcal{F}$ = flipping a coin



Corollaries

1. $P(A^c) = P(\overline{A}) = 1 - P(A)$

$\Omega = A \cup A^c$



$P(\Omega) = 1$

axiom 1

$P(A \cup A^c) = 1 \iff P(A) + P(A^c) = 1$

$\iff P(A^c) = 1 - P(A)$

A and A^c are M.E.

2. $P(A) \leq 1$

$$P(A) = 1 - \underbrace{P(A^c)}_{\geq 0,\ \text{axiom } 1}$$

$$\leq 1$$

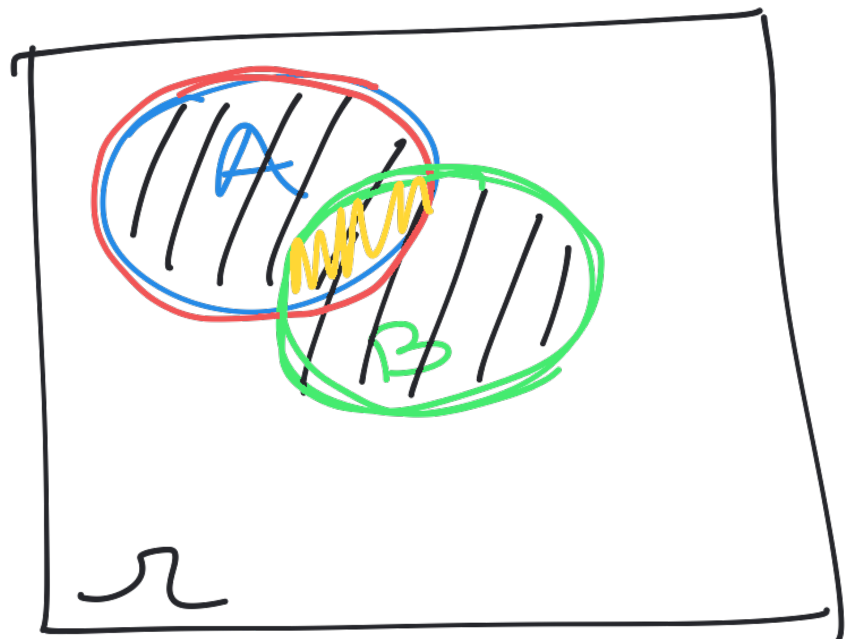
3 $P(\emptyset) = 0$

$\emptyset = \overline{\Omega}$

$$P(\emptyset) = 1 - P(\overline{\emptyset}) = 1 - P(\Omega)$$
$$= 1 - 1$$
$$= 0$$

4 By induction

5 $\underline{P(A \cup B)} = \underline{P(A)} + \underline{P(B)} - \underline{P(A \cap B)}$



First roll

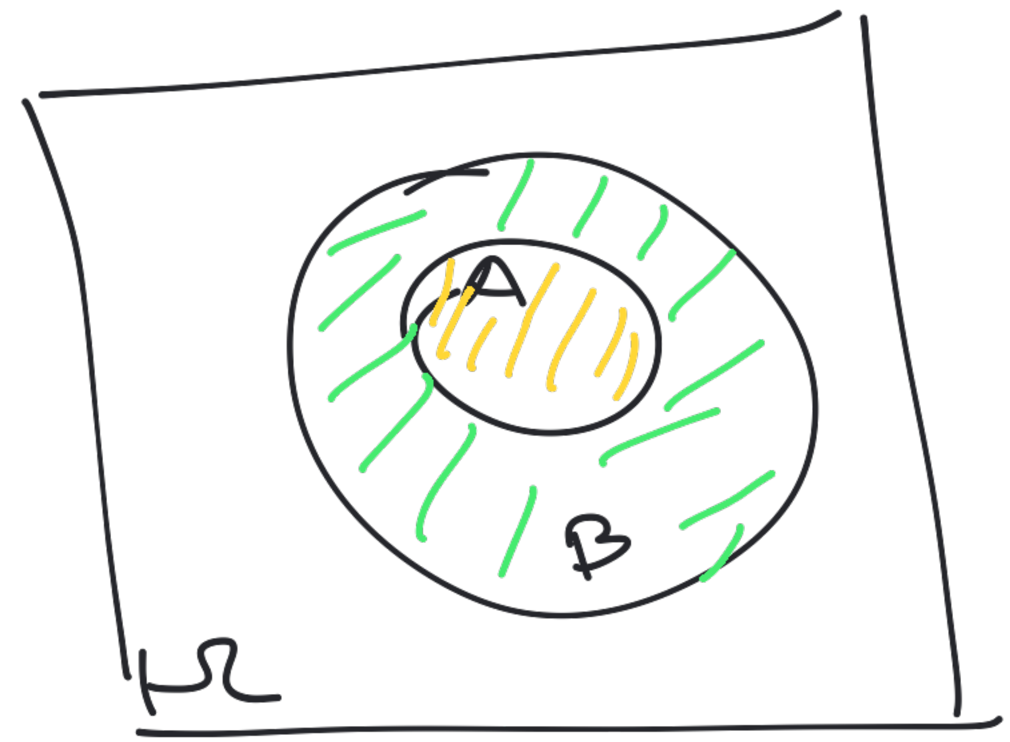
2nd roll	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	…		
2	(2,1)	(2,2)	…			
3	⋮					
4						
5						
6						

$E_i \equiv$ 1 OR 2 on Roll i

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \frac{1}{3} + \frac{1}{3} - \frac{4}{36}$$

6. by induction

7. If $A \subset B$ then $P(A) \le P(B)$



$$B = A \cup (B \cap \bar{A})$$

M.E.

$$P(B) = P(A \cup (B \cap \bar{A}))$$
$$= P(A) + \underbrace{P(B \cap \bar{A})}_{\geq 0} \Rightarrow P(B) \le P(A)$$