Assignment 2: Optimal Portfolio Construction

Strategy	Weights		
	Asset 01	Asset 05	Asset 11
Base Case	33.33%	33.33%	33.33%
Long Only	0%	6.00%	94.00%
Long Short	-20.00%	20.00%	100%

Table 1: Asset weights

Strategy	Bounds	Constraints
Long Only	Weights Between 0 to 1	$\sum i\ Wi=1$
Long Short	Weights Between -0.2 to 1	$\sum i\ Wi=1$

Table 2: Bounds and Constraints

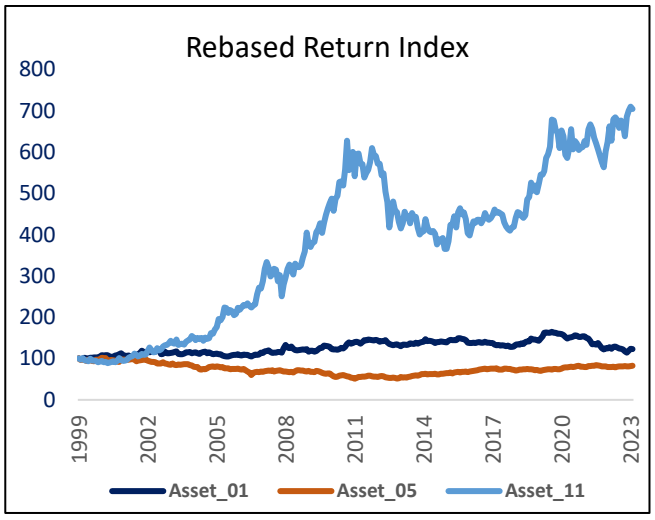

Chart 1 : Asset performance index

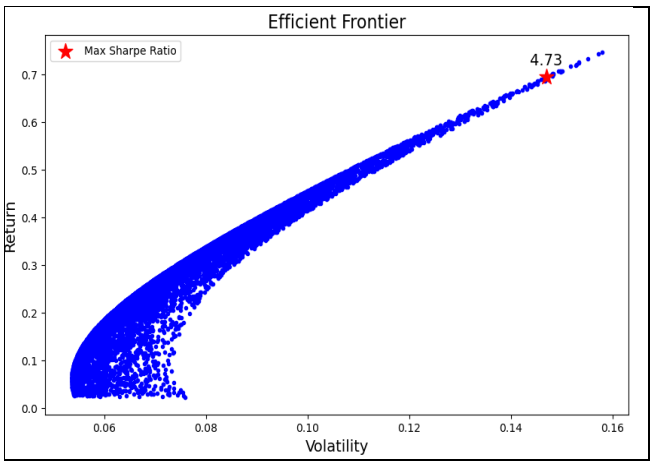

Chart 2: Efficient Frontier

Part A: Optimal Portfolio

Description: The optimization process aimed to construct an optimized portfolio with three assets (Asset_01, Asset_05, Asset_11) to maximize the risk-adjusted return, represented by the Sharpe Ratio. Constraints were set to ensure the portfolio's total weight sums up to 1, indicating a fully invested portfolio. Bounds were defined to confine the weights of each asset within the range of 0 to 1, signifying a long-only investment strategy. The optimization output indicated a maximum Sharpe Ratio of 4.73 for the portfolio. The corresponding weights assigned to each asset for this optimal portfolio were [0, 0.060, 0.940], indicating a significant allocation to Asset_11.

During the optimization process for constructing the optimal portfolio, a Long-Short strategy was also tested, yielding slightly better results. The output revealed a maximum Sharpe Ratio of 4.787 for the portfolio, with asset weights allocated as follows: Asset_01 (-0.20), Asset_05 (0.20), and Asset_11 (1.00). The bounds set for this optimal portfolio ensured that asset weights ranged from -0.2 to 1, while the constraint maintained the sum of weights at 1.

Function used: We defined a function that calculates the negative Sharpe Ratio, as our goal was to maximize it. This function takes the portfolio weights as inputs and returns the negative value of the Sharpe Ratio, as the optimizer minimizes the objective function by default.

Initial Values: These initial guesses for the portfolio weights provided a starting point for the optimization process. In this case, the weights were evenly distributed between the 3 assets, with each asset allocated an initial weight of 0.333.

Optimization Method: The optimization method used for the optimizer was Sequential Least Squares Programming (SLSQP), which is a numerical optimization algorithm designed for solving constrained nonlinear optimization problems.

Regression Analysis:

Feature	Value
R-Square	0.022
Adj R-Square	0.008
F-stat	1.590
Significance F	0.177

Table 3a: Regression output

Variables	Coefficients	P-value
Mkt-RF	0.059	0.331
Size	-0.026	0.767
Value**	-0.175**	0.025
RiskFree	1.603	0.357

Table 3b: Regression output

The portfolio returns were regressed against risk factors including Market Risk Premium (Mkt-RF), Size, Value, and Risk-Free Rate. The regression analysis revealed that the model only explains 2.2% of the total variance in the dependent variable, indicating a relatively weak explanatory power. The significance of the F-statistic, which tests the overall significance of the regression model, was found to be 0.177, suggesting a lack of significance in the regression model. Among the independent variables, only the Value factor was found to be statistically significant at a 95% confidence level, with a p-value of 0.025.

Portfolio	Portfolio Value at End	Profit Multiple
Base Case	31,134,478	3.11 X
Long Only	66,662,275	6.67 X
Long Short	69,581,433	6.96 X

Table 4: Portfolio Comparison

Portfolio	Sharpe Ratio	Historic Performance (Avg Annual Return)
Base Case	3.821	5.06%
Long Only	4.731	8.60%
Long Short	4.787	8.80%

Table 5: Historic Performance

Assets	Optimal Weights	ER
Asset_01	0.00	2.90%
Asset_05	0.06	2.20%
Asset_11	0.94	75.50%
Expected_ Portfolio_Return		71.10%

Table 6: Expected Performance

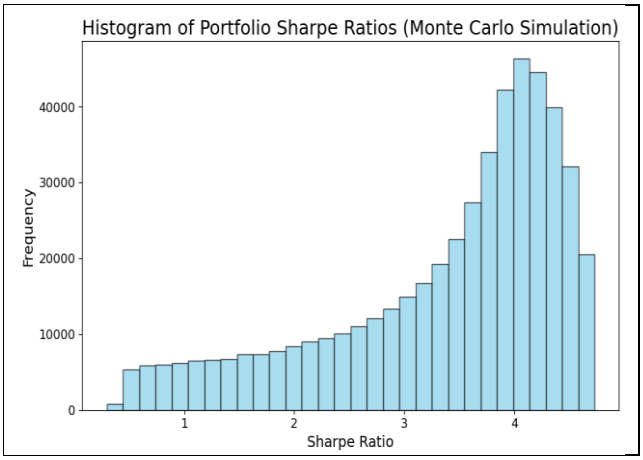

Chart 3: Montecarlo Simulation

Part A: Optimal Portfolio

Portfolio Comparison:

The initial investment for all portfolios was set at $10,000,000. Three different portfolios were constructed using varying strategies, employing the corresponding weights for each asset found in Table 1. Utilizing monthly return data spanning from January 2000 to January 2024, the portfolio values were calculated. The value of the Base Case portfolio after the 23-year period amounted to nearly $31 million. Conversely, the Long-Only portfolio was valued at $6.6 million, while the Long-Short Portfolio amassed a value close to $7 million. For detailed figures, please refer to Table 4. Chart 4 shows the month-wise portfolio values for each strategy.

Historic Performance:

The Average Annual Return (AAR) for the portfolio over the last 23 years of the portfolio can be found in table 5. With Base Case portfolio having the least AAR (5%), Long Only having an AAR of 8.6% and Long Short Portfolio having an AAR of 8.8.%. Sharpe Ratio for these portfolios was 3.82, 4.73 and 4.79 respectively. Additionally, the average annual standard deviation of the portfolio was 4.37%.

Expected Performance:

Based on the expected performance of each asset and their corresponding optimal weights, the portfolio is projected to achieve a return of approximately 71.10%.

Efficient Frontier:

An Efficient frontier with the maximum Sharpe ratio was plotted (Chart 2) which also confirms the optimal weights that was generated by the optimizer. Both the optimizer and shape ratio plot suggest that optimum weights for the assets [0,0.06,0.94] and fetches the same max Sharpe ratio 4.731.

Montecarlo Simulation: A Monte Carlo simulation comprising 100,000 iterations was conducted, each with unique weight combinations for the assets. The simulation revealed a maximum Sharpe ratio of 4.731, consistent with our earlier findings and reinforcing our previous analysis.

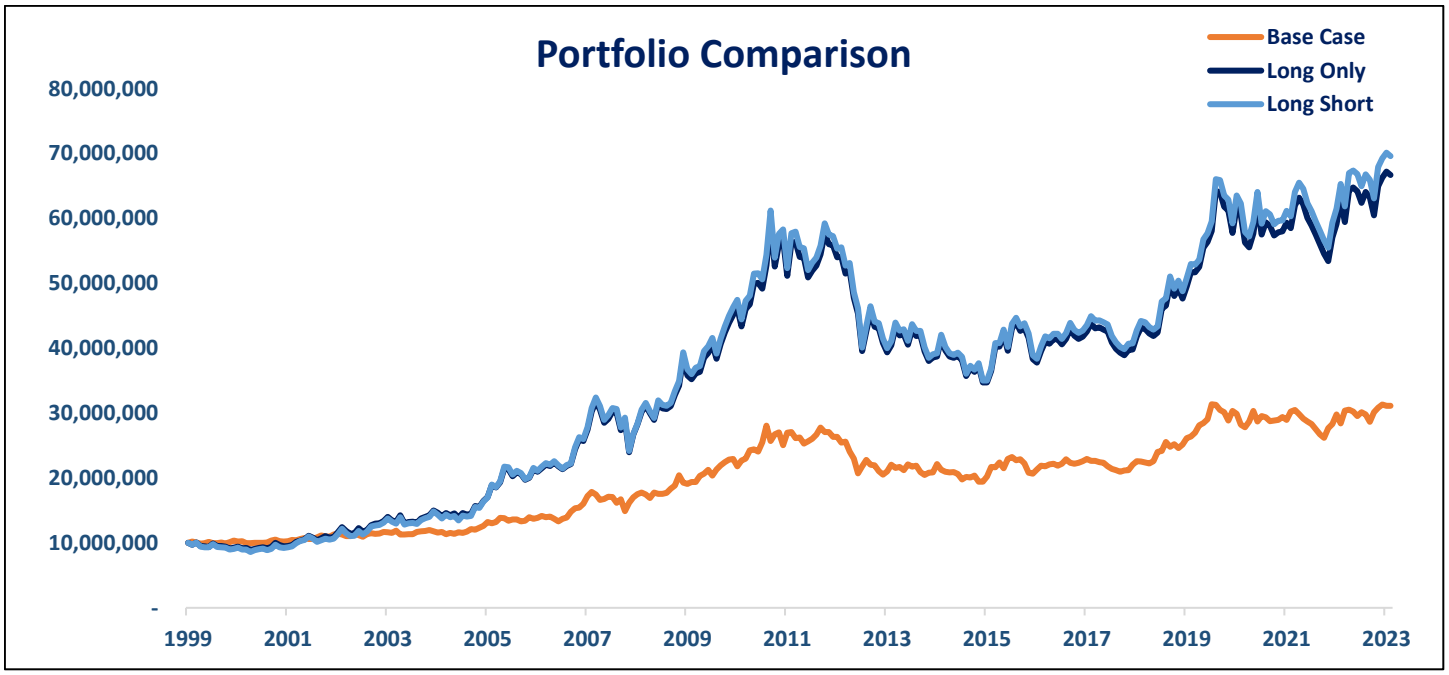

Chart 4: Portfolio Comparison

Date	Rebased Index Asset 01	Return Asset 01
31/12/1999	108.555	-
31/01/2000	106.804	-1.61%
29/02/2000	107.783	0.92%
31/03/2000	110.93	2.92%
28/04/2000	109.322	-1.45%

Table 7: Return Calculation

Date	Asset 01	Asset 05	Asset 11	Portfolio Return	Portfolio Value
31/12/1999	-	-	-	0.00%	10,000,000.00
31/01/2000	-1.61%	-3.17%	-2.73%	-2.50%	9,749,583.37
29/02/2000	0.92%	1.91%	3.78%	2.20%	9,964,252.38
31/03/2000	2.92%	-1.83%	-5.55%	-1.49%	9,816,229.95
28/04/2000	-1.45%	-3.05%	-0.79%	-1.77%	9,642,914.40
31/05/2000	-0.52%	3.48%	-0.94%	0.67%	9,707,584.24

Table 8: Portfolio Construction

Portfolio Value at Start	10,000,000.00
Portfolio Value at End	66,662,274.95
Profit Multiple	6.67 X
HPR	567%
AAR	8.60%

Table 9: Historic performance

Assets	Optimal Weights	ER
Asset_01	0.00	2.90%
Asset_05	0.06	2.20%
Asset_11	0.94	75.50%
Expected_ Portfolio_Return		71.10%

Table 10: Expected Performance

Date	Portfolio Return	Mkt-RF	Size	Value	Risk Free
31/01/2000	-2.76%	-4.74%	5.77%	-1.88%	0.41%
29/02/2000	3.67%	2.45%	21.36%	-9.59%	0.43%
31/03/2000	-5.33%	5.20%	-17.20%	8.13%	0.47%
28/04/2000	-0.93%	-6.40%	-6.68%	7.26%	0.46%
31/05/2000	-0.68%	-4.42%	-6.05%	4.75%	0.50%
30/06/2000	5.55%	4.64%	12.84%	-8.42%	0.40%

Table 11: Regression inputs

Part B: Important Calculations

Return of Assets:
The returns for each asset were computed using the formula $Return = \frac{V_t}{V_{t-1}} - 1$, as illustrated in Table 7.

Rebased Index Calculation: The Rebased Index was derived by setting the initial value of all assets at 100 and applying the returns for each asset to calculate the subsequent month's Rebased Price. These calculations are essential for evaluating the performance and growth of the assets over time.

Portfolio Construction:
The portfolio was assembled based on the optimal weights and respective asset returns. Subsequently, the portfolio return was computed using the sum-product of the weights and asset returns. This return was then utilized to determine the portfolio's value for the given month, with these steps being repeated monthly.

Historic portfolio Performance:
By extracting the initial and final portfolio values, the Profit Multiple is determined by dividing the portfolio value at the end by the value at start, reflecting the growth of the initial investment. An impressive Profit Multiple of 6.7X signifies a substantial increase in the portfolio's value over time. Additionally, the Holding Period Return (HPR) is calculated using a standard return formula. Since the HPR is measured on a monthly basis, the Average Annual Return (AAR) is computed using the formula $\boldsymbol{AAR = (1 + HPR)^{\frac{1}{23}}}$ **-1**. As the historic data taken in this case is over 23 years the above formula is used to calculate AAR from HPR in monthly terms.

Expected Portfolio Performance:
To anticipate the portfolio's performance, the expected return for each asset was determined using the sum product of weights and expected returns, as depicted in Table 10.

Regression:
A regression analysis was conducted to evaluate the relationship between portfolio returns and various risk factors, including Mkt-RF, Size, Value, and Risk Free. The results of the regression analysis are presented in tables 3a and 3b. Among the four risk factors examined, only the Value factor exhibited statistical significance, indicating its influence on portfolio returns. This finding highlights the importance of considering value-related metrics in portfolio management and decision-making processes.