- Calculate a few interations of the power method with scaling to approximate a dominant Eigen Vector of the matrix A. How many interations before which successive approximations agree to three rounded decimal places? What is the dominant Eigen value using Rayleigh Quotient?

$$A = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}$$ * use $x_0 (1,1)$ as the initial approximation.

$$x^{(1)} = Ax^{(0)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 11 \end{bmatrix} = 11\begin{bmatrix} 1.222222 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.222 \\ 1 \end{bmatrix}$$

$$x^{(2)} = Ax^{(1)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 11 \end{bmatrix} = \begin{bmatrix} 91 \\ 109 \end{bmatrix} = 109\begin{bmatrix} 1.197802198 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.198 \\ 1 \end{bmatrix}$$

$$x^{(3)} = Ax^{(2)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 91 \\ 109 \end{bmatrix} = \begin{bmatrix} 909 \\ 1091 \end{bmatrix} = 1091\begin{bmatrix} 1.200220022 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.200 \\ 1 \end{bmatrix}$$ ✓

$$x^{(4)} = Ax^{(3)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 909 \\ 1091 \end{bmatrix} = \begin{bmatrix} 9091 \\ 10909 \end{bmatrix} = 10909\begin{bmatrix} 1.199978 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.200 \\ 1 \end{bmatrix}$$

$$x^{(5)} = Ax^{(4)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 9091 \\ 10909 \end{bmatrix} = \begin{bmatrix} 90909 \\ 109091 \end{bmatrix} = 109091\begin{bmatrix} 1.2000022 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.200 \\ 1 \end{bmatrix}$$

$$x^{(6)} = Ax^{(5)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 90909 \\ 109091 \end{bmatrix} = \begin{bmatrix} 90901 \\ 1090909 \end{bmatrix} = 1090909\begin{bmatrix} 1.19999978 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1.200 \\ 1 \end{bmatrix}$$

$$x^{(7)} = Ax^{(6)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 90901 \\ 1090909 \end{bmatrix} = \begin{bmatrix} 5818149 \\ 5999951 \end{bmatrix} = 5999951\begin{bmatrix} 1.031247395 \\ 1 \end{bmatrix}$$

$$x^{(8)} = Ax^{(7)} = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 5818149 \\ 5999951 \end{bmatrix} = \begin{bmatrix} 53272351 \\ 64908649 \end{bmatrix} = 64908649\begin{bmatrix} 1.218430345 \\ 1 \end{bmatrix}$$

- After three interations, each successive interation agree to the same approximation when rounded to three decimal places.

$$Ax = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1.199978 \\ 1 \end{bmatrix} = \begin{bmatrix} 9.799912 \\ 12.199868 \end{bmatrix}$$

$$Ax \cdot x = \begin{bmatrix} 9.799912 \\ 12.199868 \end{bmatrix}\begin{bmatrix} 1.199978 \\ 1 \end{bmatrix} = 23.9595468$$

$$x \cdot x = \begin{bmatrix} 1.199978 \\ 1 \end{bmatrix}\begin{bmatrix} 1.199978 \\ 1 \end{bmatrix} = 2.4399472$$

- Rayleigh quotient: $\lambda = \dfrac{Ax \cdot x}{x \cdot x} = \dfrac{23.9595468}{2.4399472} \approx 9.819698885 \approx \boxed{9.820}$