CS 3010.02 | Class exercise 3.2 and 3.3 | Christopher Koepke

1. Find the two positive roots using the Newton Rapson Method for $g(x) = x^4 + 2x^3 - 7x^2 + 3$.

$g'(x) = 4x^3 + 6x^2 - 14x$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_0 = 1$

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $x_{n+1}$ | approx. error $= \frac{\lvert x_{n+1} - x_n \rvert}{\lvert x_{n+1} \rvert}$ |
|---|---|---|---|---|---|
| 0 | 1 | $-1$ | $-4$ | $3/4$ | $\frac{1}{3}$ |
| 1 | $3/4$ | $57/256$ | $-87/16$ | $367/464$ | $19/367$ |
| 2 | $367/464$ | 0.0018132801 | $-5.3040414456$ | 0.7912901435 | $4.320383 \times 10^{-4}$ |
| 3 | 0.7912901435 | $-1.22594 \times 10^{-5}$ | $-5.33939$ | 0.7912879282 | $2.79961304 \times 10^{-6}$ *round error |

$x_0 = 2$

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $x_{n+1}$ | approx. error |
|---|---|---|---|---|---|
| 0 | 2 | 7 | 28 | $7/4$ | $1/7$ |
| 1 | $7/4$ | $425/256$ | $245/16$ | $1287/784$ | $85/1287$ |
| 2 | $1287/784$ | 0.2457991091 | 10.88147144 | 1.6189928858 | 0.0139523621 |
| 3 | 1.6189928858 | 0.0096056326 | 10.0353423 | 1.618035678 | $5.91569 \times 10^{-4}$ |

Two positive roots:

$x \approx 0.7912$ and $x \approx 1.6181$

2. Use the secant method to find the root for $f(x) = x^5 + x^3 + 3$ given $x_0 = 1$ and $x_1 = -1$, go up to 8 interations.

$$x_{n+1} = x_n - \left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right] \cdot f(x_n)$$

$$\text{approx. error} = \frac{|x_n - x_{n-1}|}{|x_n|}$$

| $n$ | $x_n$ | $x_{n-1}$ | $x_{n+1}$ | $f(x_n)$ | $f(x_{n-1})$ | $f(x_{n+1})$ | error |
|---|---|---|---|---|---|---|---|
| 1 | $-1$ | $1$ | $-\frac{3}{2}$ | $1$ | $5$ | $-255/32$ | $2$ |
| 2 | $-\frac{3}{2}$ | $-1$ | $-\frac{303}{287}$ | $-255/32$ | $1$ | 0.511648653 | $1/3$ |
| 3 | $-\frac{303}{287}$ | $-\frac{3}{2}$ | -1.084647581 | 0.511648653 | $-\frac{255}{32}$ | 0.2227388388 | $\frac{85}{202}$ |
| 4 | -1.084647581 | $-\frac{303}{287}$ | -1.106927224 | 0.2227388388 | 0.511648653 | -0.0181679883 | 0.026643172 |
| 5 | -1.106927224 | -1.084647581 | -1.105203327 | -0.018679883 | 0.2227388388 | 0.0010594088 | 0.0201274687 |
| 6 | -1.105203327 | -1.106927224 | -1.105295849 | 0.0010594088 | -0.018679883 | $3.00111492 \times 10^{-5}$ | -0.0015598008 |
| 7 | -1.105295849 | -1.105203327 | -1.105298546 | $3.00111492 \times 10^{-5}$ | 0.0010594088 | $6.87 \times 10^{-11}$ | $8.3707905 \times 10^{-5}$ |
| 8 | -1.105298546 | -1.105295849 | -1.105298546 | $6.87 \times 10^{-11}$ | $3.00111492 \times 10^{-5}$ | $6.87 \times 10^{-11}$ | $2.440064732 \times 10^{-6}$ |

$\boxed{\text{function root at } x \approx -1.105298546}$