CS3110 Homework 9 Total points: 10.
Section 4.3 #2, #5 b) c), #6 c).

Section 4.3

2. (2pts) Prove that the following language is not regular:

$L = \{a^n b^k c^n : n \geq 0, k \geq n\}$

**Answer**

Assume L is regular so the pumping lemma must hold. Given some positive integer m, we pick the string $w = a^m b^m c^m \in L$.

Since $|xy| \leq m$, y is all a's or $a^k$. (Do not confuse this k with the k describing L). The pumped strings will be

$w_i = a^{m+(i-1)k} b^m c^m \in L$, for i = 0, 1, 2, …

However, $w_2 = a^{m+k} b^m c^m$ is not in L, since $k \geq n$ (from definition of L) does not hold (there are more a's than b's). By contradiction, L is not regular.

5b. (2 pts) Prove that the following language is not regular:

$L = \{a^n b^l a^k : k \neq n + l\}$

**Answer**

Assume L is regular, so the pumping lemma must hold. $L' = \{ a^n b^l a^k \}$. By closure properties of regular languages, $L' - L = \{ a^n b^l a^k : k = n + l\}$. Given some positive integer m, we pick the string $w = a^m b^m a^{2m} \in L$.

Since $|xy| \leq m$, y is all a's or $a^k$. (Do not confuse this k with the k describing L). The pumped strings will be $w_i = a^{m+(i-1)k} b^m a^{2m} \in L$, for i = 0, 1, 2, …

However, $w_2 = a^{m+k} b^m a^{2m}$ is not in L, since $(m+k)+m \neq 2m$. By contradiction, L'-L is not regular. By closure properties of regular languages, L is also not regular.

5c. (3pts) Prove that the following language is not regular:

$L = \{a^n b^l a^k : n = l \text{ or } l \neq k\}$.

**Answer**

Suppose L is regular, so that the pumping lemma must hold. Given some positive integer m, we pick the string $w = a^m b^m a^m \in L$.

Since $|xy| \leq m$, y is all a's or $a^k$. (Do not confuse this k with the k describing L). The pumped strings will be

$w_i = a^{m+(i-1)k} b^m a^m \in L$, for i = 0, 1, 2, …

However, $w_2 = a^{m+k}b^m a^m$ which is not in L, since n = l (from definition of L) does not hold. By contradiction, L is not regular.

6c. (3pts) For $\Sigma = \{a\}$, determine whether or not $L = \{a^n : n = k^3 \text{ for some } k \geq 0\}$ is regular.

**Answer**

Suppose L is regular, so that the pumping lemma must hold. Given some positive integer m, we pick a string $w = a^{m\text{^}3} \in L$. Because of the constraint $|xy| \leq m$ and $|y| \geq 1$, y must be all a's, or $a^k$, where $1 \leq k \leq m$. Pump using i = 2. The resulting string $w_k = a^{m\text{^}3 + k}$ which is not in L because $m^3 < m^3 + k < (m + 1)^3$. By contradiction, L is not regular.