CS 3010.02 | Class exercise 5.3 | Christopher Koepke

1. Find an approx. value of $\int_1^2 x^{-1}\,dx$ using composite simpson's rule using $h = 0.25$. Give a bound on the error (use the upper error formula that involves the 4^{th} derivative of the function). Also, find the true integral and make sure that the true absolute error is less than the upper bound on the error.

$$I \cong \frac{\frac{1}{4}}{3}\left[f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + f(2)\right]$$

$$I \cong \frac{1}{12}\left[1 + 4(0.8) + 2(0.6667) + 4(0.5714) + 0.5\right]$$

$$I \cong 0.6932539683$$

* True integral value ≈ 0.6931471806

* absolute error: $|0.6932539683 - 0.6932539683| \approx 1.06788 \times 10^{-4}$

$f'(x) = -\frac{1}{x^2}$ $\quad f''(x) = 2/x^3$ $\quad f'''(x) = -6/x^4$ $\quad f^{IV}(x) = 24/x^5$

* Max: $x = 1 \Rightarrow f^{IV}(1) = 24$

$x = 2 \Rightarrow f^{IV}(2) = 3/4$

Upper bound error; $(b-a)\cdot\frac{h^4}{180}\cdot f^{IV}(\xi)$

$$\frac{(1)(0.25)^4}{180}\cdot 24 \approx 5.208 \times 10^{-4}$$

absolute error < upper bound error ✓

0.000106788 < 0.0005208 ✓

2. what is the approx. integral if $n=6$? what is the reduction in the true relative error?

$$h = \frac{b-a}{n} = \frac{2-1}{6} \approx 0.166667$$

$* \left(1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}, 2\right)$

$$I \cong \frac{\frac{1}{6}}{3}\left[f(1) + 4f\left(\tfrac{7}{6}\right) + 2f\left(\tfrac{4}{3}\right) + 4f\left(\tfrac{3}{2}\right) + 2f\left(\tfrac{5}{3}\right) + 4f\left(\tfrac{11}{6}\right) + f(2)\right]$$

$$I \cong \frac{1}{18}\left[1 + 4\left(\tfrac{6}{7}\right) + 2\left(\tfrac{3}{4}\right) + 4\left(\tfrac{2}{3}\right) + 2\left(\tfrac{3}{5}\right) + 4\left(\tfrac{6}{11}\right) + \tfrac{1}{2}\right]$$

$$I \cong 0.6931697932$$

* True integral value ≈ 0.6931471806

absolute error: $|0.6931471806 - 0.6931697932| \approx 2.26126 \times 10^{-5}$

* abs error difference between #1 and #2 $= 8.41751 \times 10^{-5}$

* relative error of #1 $= 1.54062085 \times 10^{-4}$

relative error of #2 $= 3.26230859 \times 10^{-5}$

difference of relative error between #1 and #2 $= 1.21438991 \times 10^{-4}$