CS3110 Formal Language and Automata

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Prove a Language is Not Regular

- How can we prove that a language L is not regular?
  - Prove that there is no DFA that accepts L
  - This is not easy to prove…
  - Solution: the pumping Lemma!

The pigeonhole principle

- The "pigeonhole principle" states that if n + 1 items are placed into n pigeonholes, then at least 1 pigeonhole must end up with more than 1 item in it.

- In set notation:

  if $f : A \rightarrow B$

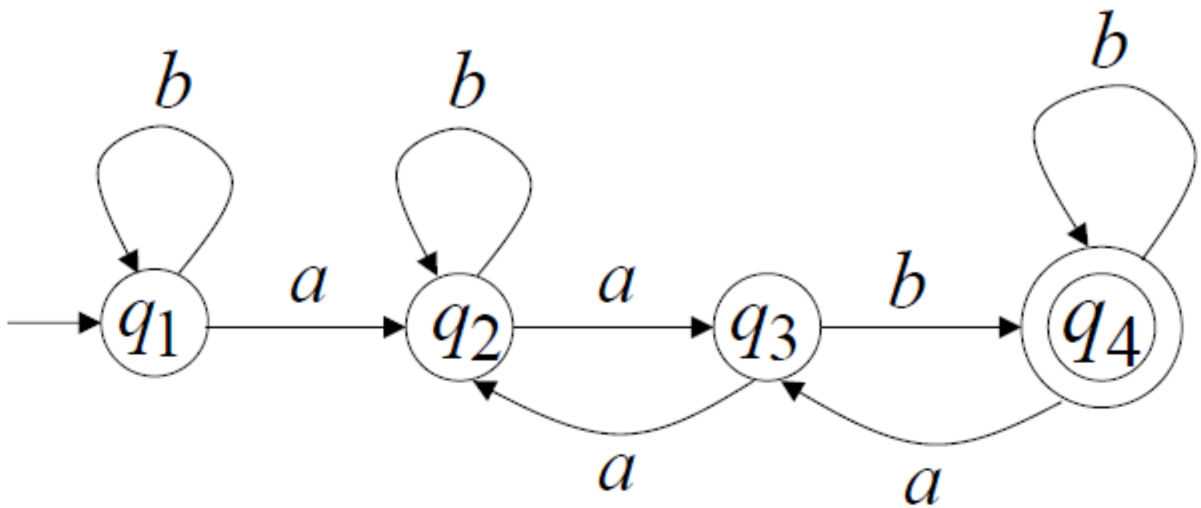
  $|A| = n + 1$

  $|B| = n$

  then f cannot be one-to-one

The pigeonhole principle and DFAs

- Example: a DFA with 4 states



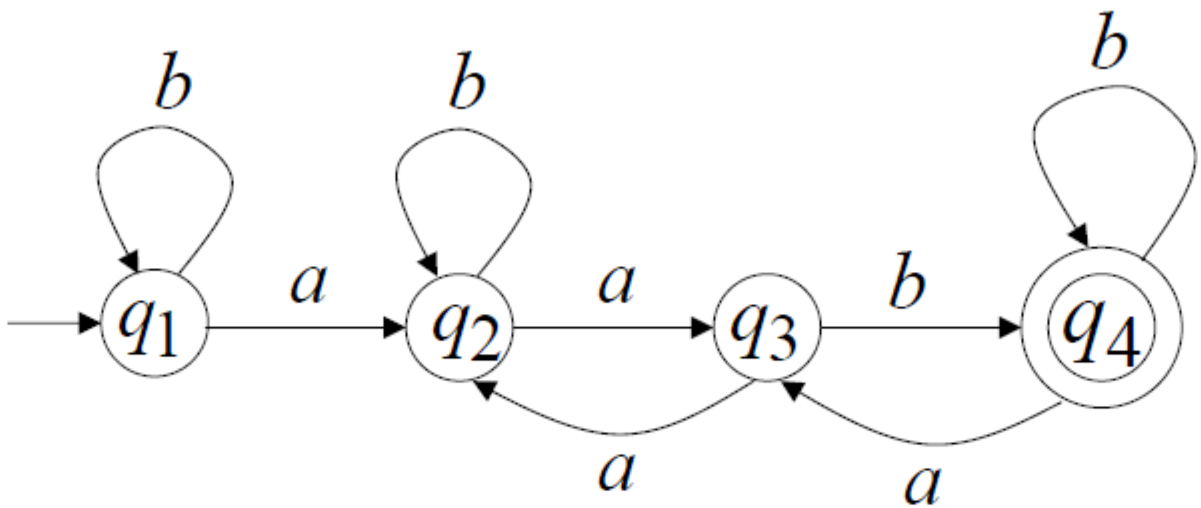
- In walks of some strings, no state is repeated

  a

  aa

  aab

The pigeonhole principle and DFAs



- In walks of some strings, at least a state is repeated

  aabb

  bbaa

  abbabb

  abbbabbabb…

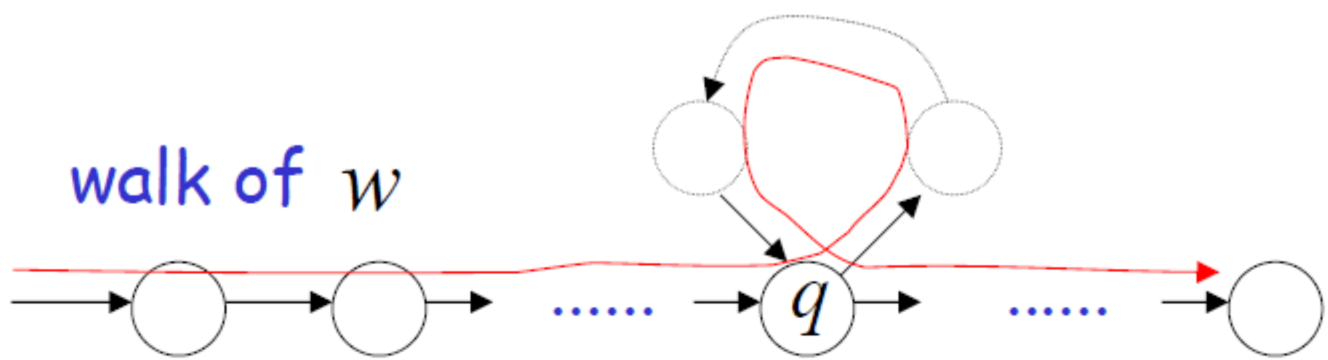
The string w has length $|w| \geq 4$

In General

- For any DFA:

  If String w has length ≥ number of states,

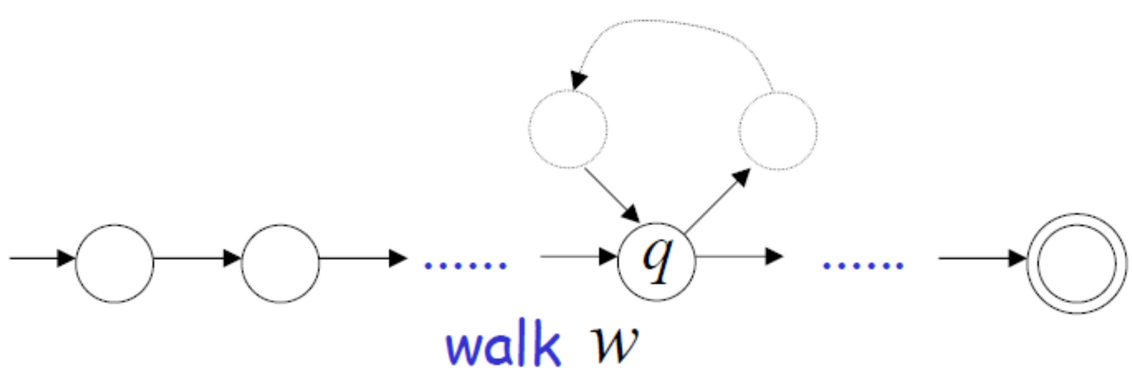
  then a state q must be repeated in the walk of w



The Pumping Lemma – Basic Idea

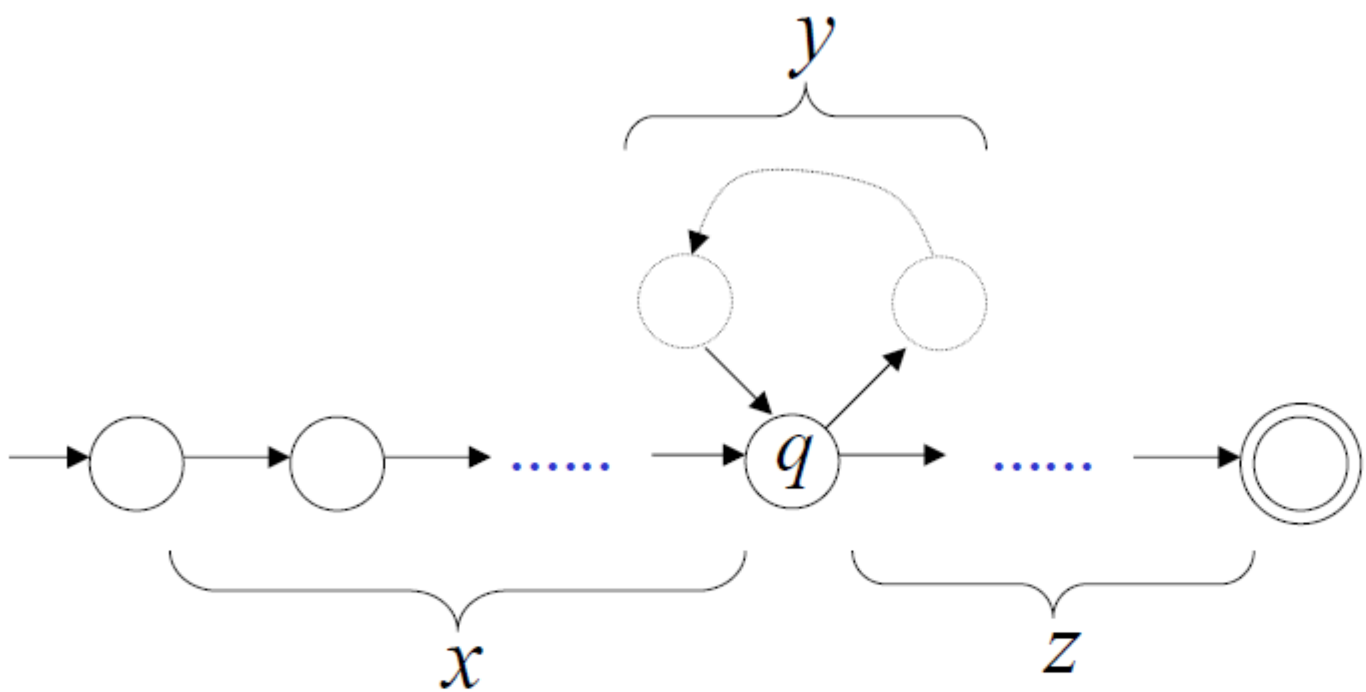
- Take an infinite regular language L, there exists a DFA that accepts L.
- Assume the DFA has m states.
- Take a string $w \in L$, if string w has length $|w| \geq m$, then from the pigeonhole principle, a state is repeated in the walk w.

  Let q be the first state repeated once in the walk of w.



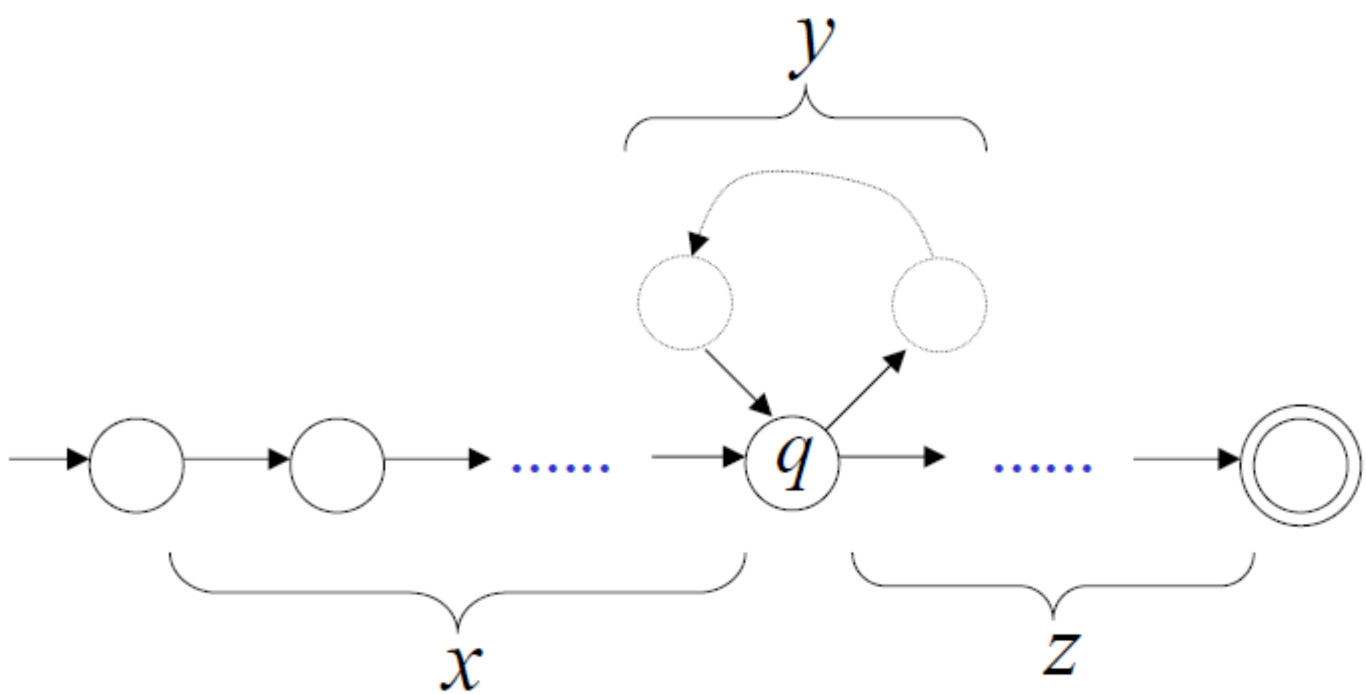
The Pumping Lemma – Basic Idea

- There must be cycle around q, the walk of the cycle is y.
- Write w=xyz



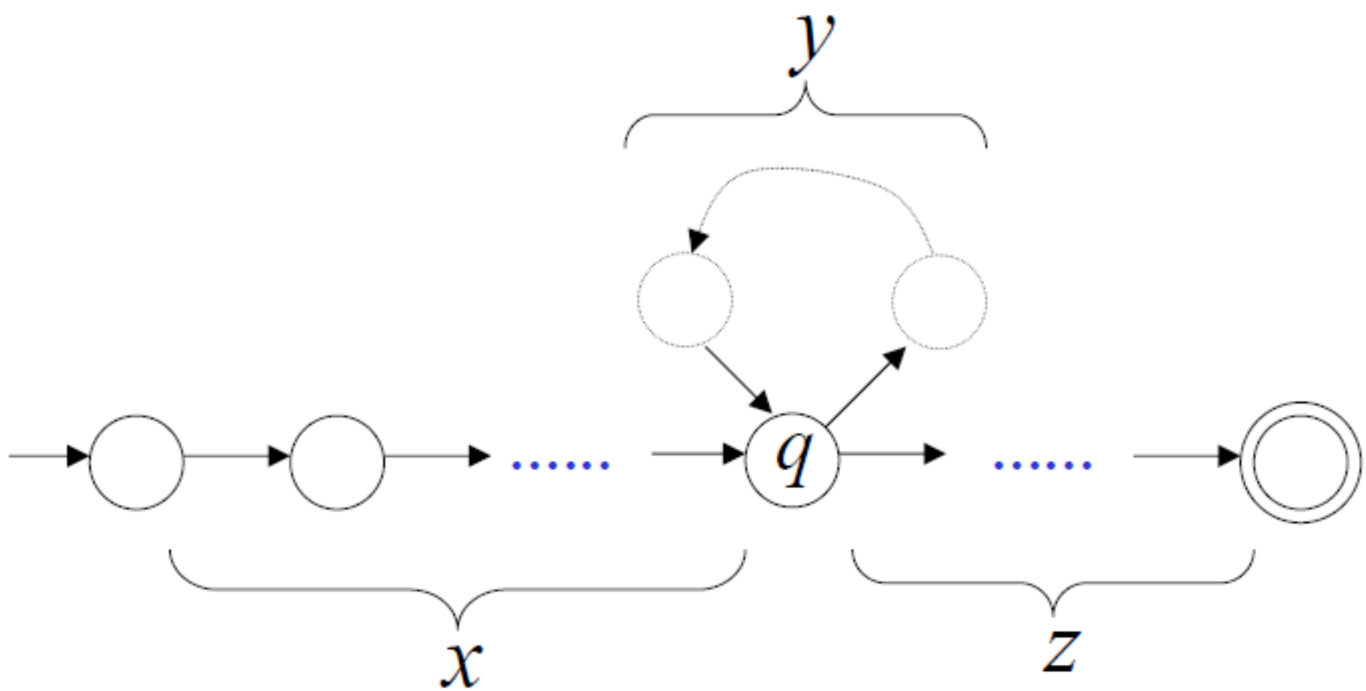
The Pumping Lemma – Basic Idea

- w = xyz
- Observations:
  - |xy|<= m (the number of states in the DFA)
  - |y|>=1.



The Pumping Lemma – Basic Idea

- Strings xz, xyz, xyyz, xyyyz,… are accepted.
- In general , xy^iz is accepted, i = 0, 1, 2, …



The Pumping Lemma – Formally

- Given an infinite regular language L
- There exists an integer m
- For any string $w \in L$, with length $|w| \geq m$
- we can write $w=xyz$
- with $|xy|<=m$ and $|y| \geq 1$,
- such that $xy^iz \in L$, $i=0, 1, 2\ldots$

The Pumping Lemma – Application

- Example: Prove that the language $L = \{a^n b^n \mid n \geq 0\}$ is not regular.

- The proof is by contradiction, and using pumping lemma
- If L is regular, it must be accepted by some DFA.
- Let m be the number of states of the DFA and consider some $w \in L$ such that $|w| \geq m$.

The Pumping Lemma – Application

- Example: Prove that the language $L = \{a^n b^n \mid n \geq 0\}$ is not regular.

- By the pumping lemma, we can split w into three pieces, w = xyz, such that for any $i \geq 0$, the string xy^iz is in L.

- So let $w = a^m b^m$. (since n can be any non-negative integer)
- Because $|xy| \leq m$, y must consist of all a's.
- But then xy^2z will contain more a's than b's. It cannot be accepted.
- This is a contradiction.

Exercise

Prove that $L=\{ww^R , w \in\{a,b\}^*\}$ is not regular.