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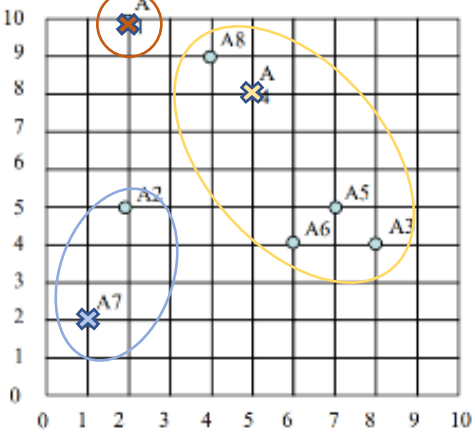
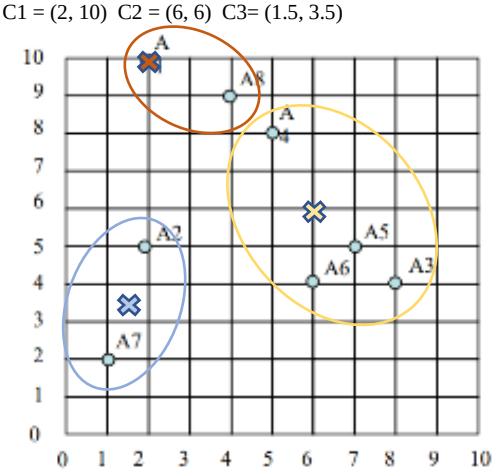
CS 4210.01 ASSIGNMENT #5

1. By considering the following 8 2D data points below do:
   a. Group the points into 3 clusters by using k-means algorithm with Euclidean distance. Show the intermediate clusters (**by drawing ellipses on this 2D space**) and centroids (**by drawing marks like X on this 2D**) in each iteration until convergence. Consider the initial centroids as: C1 = A1, C2 = A4, and C3 = A7.

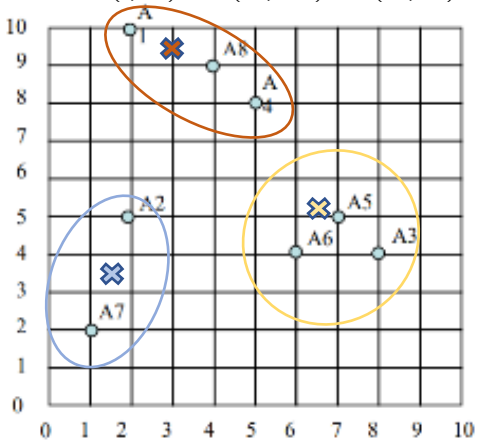
First Iteration:
C1 = (2, 10) C2 = (5, 8) C3= (1,2)



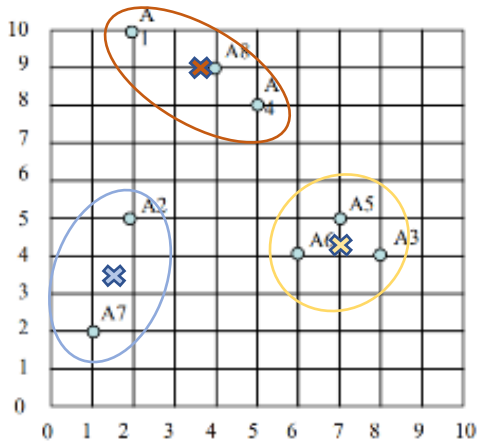
Second Iteration:
C1 = (2, 10) C2 = (6, 6) C3= (1.5, 3.5)



Third Iteration:
C1 = (3, 9.5) C2 = (6.5, 5.25) C3= (1.5, 3.5)



Fourth Iteration:
C1 = (3.67, 9) C2 = (7, 4.33) C3= (1.5, 3.5)



1st iteration								
Centroid: (C1, C2, C3)								
Instance	A1	A2	A3	A4	A5	A6	A7	A8
C1 dist.	0	5	8.4853	3.6056	7.0711	7.2111	8.0623	2.2361
C2 dist.	3.6056	4.2426	5	0	3.6056	4.1231	7.2111	1.4142
C3 dist.	8.0623	3.1623	7.2801	7.2111	6.7082	5.3852	0	7.6158
Cluster Assigned	C1	C3	C2	C2	C2	C2	C3	C2

2st iteration								
Centroid: (C1, C2, C3)								
Instance	A1	A2	A3	A4	A5	A6	A7	A8
C1 dist.	0	5	8.4853	3.6056	7.0711	7.2111	8.0623	2.2361
C2 dist.	5.6569	4.1231	2.8284	2.2361	1.1412	2	6.4031	3.6056
C3 dist.	6.5192	1.5811	6.5192	5.7009	5.7009	4.5277	1.5811	6.0415
Cluster Assigned	C1	C3	C2	C2	C2	C2	C3	C1

3st iteration								
Centroid: (C1, C2, C3)								
Instance	A1	A2	A3	A4	A5	A6	A7	A8
C1 dist.	1.1180	4.6098	7.4330	2.5	6.0208	6.2650	7.7621	1.1180
C2 dist.	6.5431	4.5069	1.9526	3.1325	0.5590	1.3463	6.3885	4.5070
C3 dist.	6.5192	1.5811	6.5192	5.7009	5.7009	4.5277	1.5811	6.0415
Cluster Assigned	C1	C3	C2	C1	C2	C2	C3	C1

4st iteration								
Centroid: (C1, C2, C3)								
Instance	A1	A2	A3	A4	A5	A6	A7	A8
C1 dist.	1.9465	4.3346	6.6143	1.6640	5.2047	5.5162	7.4919	0.33
C2 dist.	7.5597	2.1092	1.0530	4.1796	0.67	1.0530	6.4365	5.5506
C3 dist.	6.5192	1.5811	6.5192	5.7009	5.7009	4.5277	1.5811	6.0415
Cluster Assigned	C1	C3	C2	C1	C2	C2	C3	C1

Note: Clusters do not move after the 4th iteration, thus convergence.

b. Calculate the SSE (Sum of Square Errors) of the final clustering.

$$SSE = \sum_{i} \sum_{x \in C_i} (x - m_i)^2$$

$$SSE = \sum_{1} (2 - 3.67)^2 + (4 - 3.67)^2 + (5 - 3.67)^2 + (10 - 9)^2 + (9 - 9)^2 + (8 - 9)^2 = 6.7678$$

$$SSE = \sum_{2} (6 - 7)^2 + (7 - 7)^2 + (8 - 7)^2 + (4 - 4.33)^2 + (5 - 4.33)^2 + (4 - 4.33)^2 = 2.6667$$

$$SSE = \sum_{3} (1 - 1.5)^2 + (2 - 1.5)^2 + (2 - 3.5)^2 + (5 - 3.5)^2 = 5$$

$$SSE = 6.7678 + 2.6667 + 5 = 14.4345$$

2. Complete the Python program (clustering.py) that will read the file training_data.csv to cluster the data. Your goal is to run k-means multiple times and check which k value maximizes the Silhouette coefficient. You also need to plot the values of k and their corresponding Silhouette coefficients so that we can visualize and confirm the best k value found. Next, you will calculate and print the Homogeneity score (the formula of this evaluation metric is provided in the template) of this best k clustering task by using the testing_data.csv, which is a file that includes ground truth data (classes).

https://github.com/chris-k87/CS_4210.01/tree/main/Assignment_5/Clustering

3. The dataset below presents the user ratings on a 1-3 scale for 6 different rock bands.

	Bon Jovi	Metallica	Scorpions	AC/DC	Kiss	Guns n' Roses
Fred	1	3	-	3	1	3
Lillian	3	-	2	2	3	1
Cathy	2	2	2	3	-	2
John	3	2	2	2	?	?

a. Apply **user-based** collaborative filtering on the dataset to decide about recommending the bands Kiss and Guns n' Roses to John. You should make a recommendation when the predicted rating is greater than or equal to 2.0. Use cosine similarity, a neutral value (1.5) for missing values, and the top 2 similar neighbors to build your model.

$\overline{r_{Fred}} = \frac{1+3+1.5+3}{4} \approx 2.125$ $\overline{r_{Lillian}} = \frac{3+2+2+2}{4} \approx 2.125$ $\overline{r_{Cathy}} = \frac{2+2+2+3}{4} \approx 2.25$ $\overline{r_{John}} = \frac{3+2+2+2}{4} \approx 2.25$

John [3, 2, 2, 2]

Fred [1, 3, 1.5, 3] $Cos(John, Fred) = \frac{(3*1)+(2*3)+(2*1.5)+(2*3)}{\sqrt{(3^2+2^2+2^2+2^2)*(1^2+3^2+1.5^2+3^2)}} = \frac{18}{\sqrt{(21)*(21.25)}} \approx 0.8521$

Lillian [3, 1.5, 2, 2] $Cos(John, Lillian) = \frac{(3*3)+(2*1.5)+(2*2)+(2*2)}{\sqrt{(3^2+2^2+2^2+2^2)*(3^2+1.5^2+2^2+2^2)}} = \frac{20}{\sqrt{(21)*(19.25)}} \approx 0.9947$

Cathy [2, 2, 2, 3] $Cos(John, Cathy) = \frac{(3*2)+(2*2)+(2*2)+(2*3)}{\sqrt{(3^2+2^2+2^2+2^2)*(2^2+2^2+2^2+3^2)}} = \frac{20}{\sqrt{(21)*(21)}} \approx 0.9524$

$$r_{i,j} = \overline{r_i} + \frac{\sum_k Cos(u_i, u_k)(r_{k,j} - \overline{r_k})}{\sum_k |Cos(u_i, u_k)|}$$

- $r_{John,Kiss} = 2.25 + \frac{\sum_k Cos(u_{John}, u_k)(r_{k,Kiss} - \overline{r_k})}{\sum_k |Cos(u_{John}, u_k)|} = 2.25 + \frac{(0.9947)*(3-2.125)+(0.9524)*(1.5-2.25)}{|(0.9947)+(0.9524)|} \approx 2.3302$
- $r_{John,Gn'R} = 2.25 + \frac{\sum_k Cos(u_{John}, u_k)(r_{k,Gn'R} - \overline{r_k})}{\sum_k |Cos(u_{John}, u_k)|} = 2.25 + \frac{(0.9947)*(1-2.125)+(0.9524)*(2-2.25)}{|(0.9947)+(0.9524)|} \approx 1.5530$

b. Now, apply **item-based** collaborative filtering to make the same decision. Use the same parameters defined before to build your model.

$\overline{r_{Kiss}} = \frac{1+3+1.5}{3} \approx 1.8333$ $\overline{r_{Gn'R}} = \frac{3+1+2}{3} \approx 2.0000$ $\overline{r_{Bon\ Jovi}} = \frac{1+3+2}{3} \approx 2.0000$

$\overline{r_{Metallica}} = \frac{3+1.5+2}{3} \approx 2.1667$ $\overline{r_{Scorpions}} = \frac{1.5+2+2}{3} \approx 1.8333$ $\overline{r_{AC/DC}} = \frac{3+2+3}{3} \approx 2.6667$

Kiss [1, 3, 1.5] Guns n' Roses[3, 1, 2]

Bon Jovi [1, 3, 2] $Cos(Kiss, Bon\ Jovi) = \frac{(1*1)+(3*3)+(1.5*2)}{\sqrt{(1^2+3^2+1.5^2)*(1^2+3^2+2^2)}} = \frac{13}{\sqrt{(12.25)*(14)}} \approx 0.9927$

$Cos(Gn'R, Bon\ Jovi) = \frac{(3*1)+(1*3)+(2*2)}{\sqrt{(3^2+1^2+2^2)*(1^2+3^2+2^2)}} = \frac{10}{\sqrt{(14)*(14)}} \approx 0.7143$

Metallica [3, 1.5, 2] $Cos(Kiss, \text{Metallica}) = \frac{(1*3)+(3*1.5)+(1.5*2)}{\sqrt{(1^2+3^2+1.5^2)*(3^2+1.5^2+2^2)}} = \frac{10.5}{\sqrt{(12.25)*(15.25)}} \approx 0.7682$

$Cos(Gn'R, \text{Metallica}) = \frac{(3*3)+(1*1.5)+(2*2)}{\sqrt{(3^2+1^2+2^2)*(3^2+1.5^2+2^2)}} = \frac{14.5}{\sqrt{(14)*(15.25)}} \approx 0.9924$

Scorpions [1.5, 2, 2] $Cos(Kiss, \text{Scorpions}) = \frac{(1*1.5)+(3*2)+(1.5*2)}{\sqrt{(1^2+3^2+1.5^2)*(1.5^2+2^2+2^2)}} = \frac{10.5}{\sqrt{(12.25)*(10.25)}} \approx 0.9370$

$Cos(Gn'R, \text{Scorpions}) = \frac{(3*1.5)+(1*2)+(2*2)}{\sqrt{(3^2+1^2+2^2)*(1.5^2+2^2+2^2)}} = \frac{10.5}{\sqrt{(14)*(10.25)}} \approx 0.8765$

AC/DC [3, 2, 3] $Cos(Kiss, \text{AC/DC}) = \frac{(1*3)+(3*2)+(1.5*3)}{\sqrt{(1^2+3^2+1.5^2)*(3^2+2^2+3^2)}} = \frac{13.5}{\sqrt{(12.25)*(22)}} \approx 0.8223$

$Cos(Gn'R, \text{AC/DC}) = \frac{(3*3)+(1*2)+(2*3)}{\sqrt{(3^2+1^2+2^2)*(3^2+2^2+3^2)}} = \frac{17}{\sqrt{(14)*(22)}} \approx 0.9687$

- $r_{Kiss,John} = 1.8333 + \frac{\sum_k Cos(u_{Kiss},u_k)(r_{k,Kiss}-\overline{r_k})}{\sum_k|Cos(u_{Kiss},u_k)|}$
  $= 1.8333 + \frac{(0.9927)*(3-2)+(0.9370)*(2-1.8333)}{|(0.9927)+(0.9370)|} \approx 2.4511$
- $r_{Gn'R,John} = 2 + \frac{\sum_k Cos(u_{Gn'R},u_k)(r_{k,Gn'R}-\overline{r_k})}{\sum_k|Cos(u_{Gn'R},u_k)|}$
  $= 2 + \frac{(0.9924)*(2-2.1667)+(0.9687)*(2-2.1667)}{|(0.9924)+(0.9687)|} \approx 1.8333$

4. Consider the following transaction dataset. Suppose that minimum support is set to 30% (*minsup*) and minimum confidence is set to 60%.

Transaction ID	Items Bought
1	$\{a,b,d,e\}$
2	$\{b,c,d\}$
3	$\{a,b,d,e\}$
4	$\{a,c,d,e\}$
5	$\{b,c,d,e\}$
6	$\{b,d,e\}$
7	$\{c,d\}$
8	$\{a,b,c\}$
9	$\{a,d,e\}$
10	$\{b,d\}$

a. Rank all frequent itemsets according to their support (list their support values).

1-Itemset	Support Count (σ)	Support
{a}	5	5/10 = 0.5
{b}	7	7/10 = 0.7
{c}	5	5/10 = 0.5
{d}	9	9/10 = 0.9
{e}	6	6/10 = 0.6

2-Itemset	Support Count (σ)	Support
{a,b}	3	3/10 = 0.3
{a,c}	2	2/10 = 0.2
{a,d}	4	4/10 = 0.4
{a,e}	4	4/10 = 0.4
{b,c}	3	3/10 = 0.3
{b,d}	6	6/10 = 0.6
{b,e}	4	4/10 = 0.4
{c,d}	4	4/10 = 0.4
{c,e}	2	2/10 = 0.2
{d,e}	6	6/10 = 0.6

3-Itemset	Support Count (σ)	Support
{a,b,c}	1	1/10 = 0.1
{a,b,d}	2	2/10 = 0.2
{a,b,e}	2	2/10 = 0.2
{a,c,d}	1	1/10 = 0.1
{a,c,e}	1	1/10 = 0.1
{a,d,e}	4	4/10 = 0.4
{b,c,d}	2	2/10 = 0.2
{b,c,e}	1	1/10 = 0.1
{b,d,e}	4	4/10 = 0.4
{c,d,e}	2	2/10 = 0.2

4-Itemset	Support Count (σ)	Support
{a,b,c,d}	0	0/10 = 0.0
{a,b,c,e}	0	0/10 = 0.0
{a,b,d,e}	2	2/10 = 0.2
{b,c,d,e}	1	1/10 = 0.1

Note: Grey highlighted itemsets indicated failed *minsup* at 30%

b. For all frequent 3-itemsets, rank all association rules - according to their confidence values - which satisfy the requirements on minimum support and minimum confidence (list their confidence values).

Itemset: {a,d,e}

Association Rule	Support	Confidence
be->d	0.4	1.0
bd->e	0.4	0.67
de->b	0.4	0.67
e->bd	0.4	0.67
b->de	0.4	0.57
d->be	0.4	0.44

Itemset: {b,d,e}

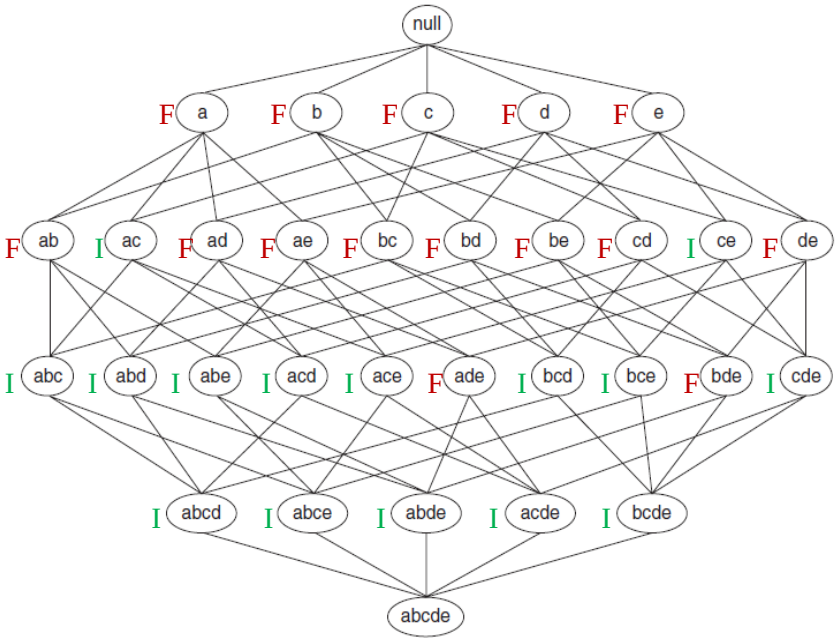
Association Rule	Support	Confidence
ad->e	0.4	1.0
ae->d	0.4	1.0
a->de	0.4	0.8
de->a	0.4	0.67
e->ad	0.4	0.67
d->ae	0.4	0.44

Note: Grey highlighted association rule failed *minconf* at 60%

c. Show how the 3-itemsets candidates can be generated by the F_{k-1} X F_{k-1} method and if these candidates will be pruned or not.

- F_2 = {ab, ad, ae, bc, bd, be, cd, de}
  - Merge: {ab, ad} = abd Pruned
  - Merge: {ab, ae} = abe Pruned
  - Merge: {ad, ae} = ade Not Pruned
  - Merge: {bc, bd} = bcd Pruned
  - Merge: {bc, be} = bce Pruned
  - Merge: {bd, be} = bde Not Pruned

d. Consider the lattice structure given below. Label each node with the following letter(s): *F* if it is frequent and *I* if it is infrequent.



5. Complete the Python program (association_rule_mining.py) that will read the file retail_dataset.csv to find strong rules related to supermarket products. You will need to install a python library this time. Just use your terminal to type: pip install mlxtend. Your goal is to output the rules that satisfy *minsup* = 0.2 and *minconf* = 0.6, as well as the priors and probability gains of the rule consequents when conditioned to the antecedents. The formulas for this math are given in the template.