CS311 Formal Language and Automata

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Chapter 3 Regular languages and expressions

Regular Expressions

- Regular expressions describe regular languages.

- Example: (a+b·c)* describes the language

  {a, bc}*={λ, a, bc, aa, abc, bca, …}

Regular Expressions – Recursive Definition

- Primitive regular expressions include $\varnothing$, λ , *a* .

- Given regular expressions r_1 and r_2

  $r_1 + r_2$

  $r_1 \cdot r_2$

  $r_1 *$

  (r_1)

  are all regular expressions.

Examples

- A regular expression: (a+bc)*(c+ $\varnothing$)

- Not a regular expression: (a+b+)

Languages of Regular Expressions

- L(r): language of regular expression r

- Example
    L((a+bc)*) = {λ, a, bc, aa, abc, bca, ...}

Definition

- For primitive regular expressions
    - L($\varnothing$)= $\varnothing$.
        - $\varnothing$ a regular expression corresponding to the language $\varnothing$.
    - L(λ) = {λ}.
        - λ is a regular expression corresponding to the language {λ}.
    - L(a) = {a}
        - For each symbol $a \in \Sigma$, a is a regular expression corresponding to the language {a}.

Definition

- For regular expressions r_1 and r_2

$$L(r_1 + r_2) = L(r_1) \cup L(r_2)$$
$$L(r_1 \cdot r_2) = L(r_1)L(r_2)$$
$$L(r_1{}^*) = (L(r_1))^*$$
$$L((r_1)) = L(r_1)$$

Example

- Regular expression: (a+b)a*
- L((a+b) · a*)= L((a+b))L(a*)

$$= L(a+b)L(a*)$$
$$= (L(a) \cup L(b))(L(a))*$$
$$= (\{a\} \cup \{b\})(\{a\})*$$
$$= \{a, b\} \{\lambda, a, aa, aaa, ...\}$$
$$= \{a, aa, aaa, ..., b, ba, baa, baaa, ...\}$$

Exercises

- Regular expression r=(a+b)*(a+bb)
L(r) = ?

- Regular expression r=(aa)*(bb)*b
L(r) = ?

Regular expression?

Σ = {a,b,c}

L(r)= {all strings with at least two consecutive aa}

r=?

r = (a+b+c)*aa(a+b+c)*

L(r)={all strings with no more than two a's}

r = (b + c)*(λ + a)(b + c)*(λ + a)(b + c)*

L(r) = {All strings in which all runs of a's have lengths that are multiples of three}

r = (aaa + b + c)*

L(r) = {all strings containing no more than three a's} ?

Hints for writing regular expressions

Assume Σ = {a, b, c}.

Zero or more a's: a*

One or more a's: aa*

Any string at all: (a + b + c)*

Any nonempty string: (a + b + c)(a + b + c)*

Any string that does not contain a: (b + c)*

Any string containing exactly one a: (b + c)*a(b + c)*

Do these strings match the regular expression?

Regular expression | String

$(01^* + 1)$ | 0101

$(a + \lambda)b$ | b

$(ab)^*a^*$ | λ

$(a + b)(ab)$ | bb