

- Find the Eigen values & vectors for the matrix  $A$ .

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\text{determinant}(Ax - \lambda I) = 0$$

$$\bullet \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\det \left\{ \begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix} \right\} = 0$$

$$(1-\lambda)(4-\lambda) - (2 \cdot 2) = 0$$

$$4 - \lambda - 4\lambda + \lambda^2 - 4 = 0$$

$$-\lambda^2 - 5\lambda - 0 = 0$$

$$\lambda(\lambda + 5) = 0$$

$$\lambda = 0 \text{ OR } \lambda = 5$$

$$\bullet \lambda = 0$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$1x_1 + 2x_2 = 0x_1$$

$$2x_1 + 4x_2 = 0x_2$$

$$(x_1, x_2)^T = (-2, 1)^T$$

$$\bullet \lambda = 5$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 5 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$1x_1 + 2x_2 = 5x_1$$

$$2x_1 + 4x_2 = 5x_2$$

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$$-4x_1 + 2x_2 = 0$$

$$2x_1 - 1x_2 = 0$$

$$(x_1, x_2)^T = (1, 2)^T$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\lambda = 0 \quad \bar{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad \checkmark$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = 5 \quad \bar{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \checkmark$$