Homework Module #9 Sec 4.3 # 2, 5b, 5c, 6c

2) Show that the language $L = \{a^n b^k c^n : n \geq 0, k \geq n\}$ is not regular.

- Assume L is regular and m is a constant, then we get $w = a^m b^m c^m$ where $w \in L$.
- We take $w = xyz$ where $|xy| \leq m$ and $|y| \geq 1$ thus $x = \varnothing$, $y = a^{m+}$ and $z = b^m c^m$.
- From the pumping lemma, we get $w = y^i z$, where $i = 0, 1, 2 \ldots$
- For $i = 0$ we get $w = b^m c^m$ where $w \notin L$.
  For $i = 1$ we get $w = a^m b^m c^m$ where $w \in L$
  For $i = 2$ we get $w = a^{2m} b^m c^m$ where $w \notin L$
- This is a contradiction, thus L is NOT regular.

5) Prove that the following languages are not regular.

b) $L = \{a^n b^l a^k : k \neq n + l\}$

- Assume that L is regular and m is a constant, we get $w = a^m b^m a^{2m+1}$ and $w \in L$.
- We take $w = xyz$, where $|xy| \leq m$ and $|y| \geq 1$, thus $x = \varnothing$, $y = a^m$ and $z = b^m a^{2m+1}$
- From the pumping lemma, we get $w = y^i z$, where $i = 0, 1, 2 \ldots$
- For $i = 0$, we get $w = b^m a^{2m+1}$ where $w \notin L$
  For $i = 1$, we get $w = a^m b^m a^{2m+1}$ where $w \in L$
  For $i = 2$, we get $w = a^{2m} b^m a^{2m+1}$ where $w \notin L$
- This is a contradiction, thus L is NOT regular.

5)

c) $L = \{a^n b^l a^k : n = l \text{ OR } l \neq k\}$

- Assume L is regular and m is a constant, we get $w = a^m b^m a^m$ for the condition $n = l$.
- We take $w = xyz$ where $|xy| \leq m$ and $|y| \geq 1$, thus $x = \varnothing$, $y = a^m$, and $z = b^m a^m$
- From the pumping lemma, we get $w = y^i z$.
- For $i = 0$ we get $w = b^m a^m$ where $w \notin L$
- This is a contradiction, thus L is NOT regular.

6) Determine whether or not the following languages on $\Sigma = \{a\}$ are regular:

c) $L = \{a^n : n = k^3 \text{ for some } k \geq 0\}$

- Assume L is regular and m is a constant, we get $w = a^{m^3}$.
- We take $w = xyz$, where $|xy| \leq m$, and $|y| \geq 1$, thus $x = \varnothing$ $y = a^{m^3}$ and $z = \varnothing$.
- From the pumping lemma, we get $w = y^i$.
- For $i = 2$, we get $w = (a^{m^3})^2 = a^{(m^3+2)}$ where $w \notin L$
- This is a contradiction, thus L is NOT regular.