Homework #3

Problem #1

a) Trace the multiplication hardware when multiplying two 5-bit unsigned numbers 10101 x 01011

Iteration	Multiplicand	Multiplier	Product	Actions
0	0001 0101	01011	0000 0000	Initial values
1	0001 0101	01011	0001 0101	Prod = Prod + multiplicand Shift left multiplicand Shift right multiplier
2	0010 1010	00101	0011 1111	Prod = Prod + multiplicand Shift left multiplicand shift right multiplier
3	0101 0100	00010	0011 1111	NO operation on product Shift left muliplicand Shift right multiplier
4	1010 1000	00001	1110 0111	Prod = Prod + multiplicand shift left multiplicand Shift right multiplier
5	0101 0000	00000	1110 0111	NO operation on product End algorithm

* 10101 x 01011 = 0 1110 0111 ✓

b) Will this multiplication issue an overflow warning?

- NO, the product is less than 32-bits so the result will have leading zeros, 0000 0000 1110 0111.
- MIPS integer multiplication instructions also ignore overflow, thus software must perform the overflow checking.

Problem #2

1) What FP number is represented? $\boxed{-1.0 \times 2^0}$

1 0111 1111 0000 0000 0000 0000 0000 000

- sign = -1   Exponent = 127 - bias   Fraction = 0

2) What FP number is represented? $\boxed{+\infty}$

0 1111 1111 0000 0000 0000 0000 0000 000

- sign = +1   Exponent = 255 - bias   Fraction = 0

3) For double precision FP, how many bits for exponent? How many bits for the fraction?

- Double precision FP representation uses 11 bits for the exponent and 52 bits for the fraction.

4) What is the book's definition of FP number underflow?

$\boxed{\text{B.}}$ A negative exponent becomes too large to fit in the exponent field.

Problem #3

1) convert -126.625 to single precision FP number

$-126.625_{10} = -1111110.101_2 = -1.111110101_2 \times 2^6$

- $126/2 = 63$ R/0
- $63/2 = 31$ R/1
- $31/2 = 15$ R/1
- $15/2 = 7$ R/1
- $7/2 = 3$ R/1
- $3/2 = 1$ R/1
- $1/2 = 0$ R/1

*sign = 1   Exp = $2^{(133-127)} = 2^6$

1000 0101

FP representation of -126.625_{10}

sign	Exp	Mantessa
1	1000 0101	1111 1010 1000 0000 0000 000

$0.625 \times 2 = \underline{1}.25$

$0.25 \times 2 = \underline{0}.5$

$0.5 \times 2 = \underline{1}$

Problem # 3

2) convert 0.875 to single Precision FP number.

$0.875_{10} = 0.111_2 = 1.11 \times 2^{-1}$ * sign = 0 $Exp = 2^{(126-127)} = 2^{-1}$

0111 1110

$0.875 \times 2 = 1.75$

$0.75 \times 2 = 1.5$

$0.5 \times 2 = 1.0$

FP representation of 0.875_{10}

sign	Exp	Mantessa
0	0111 1110	1100 0000 0000 0000 0000 000

3) convert FP number 1100 1100 0011 0011 0000 0000 0000 0000 to base-10 decimal number.

sign	Exp	Mantessa
1	1001 1000	0110 0110 0000 0000 0000 000
(-1)	(152)	(1.0110011)
	152-127	2^{-2} 2^{-3} 2^{-6} 2^{-7} = 0.3984375

$\boxed{-1.3984375 \times 2^{25}}$

Problem # 4

1) Show each step of adding two base-10 FP numbers.

$(9.8942 \times 10^4) + (7.9529 \times 10^3)$

Step 1: Align exponents

9.8942×10^4

0.79529×10^4

Step 2: Add significants

9.89420×10^4

$+ 0.79529 \times 10^4$

10.68949×10^4

Step 3: Normalize sum

$10.68949 \times 10^4 \Rightarrow 1.068949 \times 10^5$

Step 4: Round sum (4th decimal num)

$1.068949 \times 10^5 \Rightarrow 1.0689 \times 10^5$

Final result: 1.0689×10^5

Problem # 4

2) Show each step of multiplying two base-10 FP numbers.

$(-1.2412 \times 10^{-5}) + (3.1002 \times 10^{9})$

Step 1: Determine sign

$S = S1 \text{ XOR } S2 = -1$

Step 2: Add exponents

$-5 + 9 = 4$

Step 3: Multiply significants

$-1.2412 \times 3.1002 = -3.84796824$

Step 4: Normalize sum

$3.84796824 \times 10^{4} \Rightarrow 3.84796824 \times 10^{4}$

Step 5: Round sum (4th decimal num)

$3.84796824 \times 10^{4} \Rightarrow -3.8480 \times 10^{4}$

Final result: -3.8480×10^{4}

3) For FP multiplication, how do we detect overflow? How do we detect underflow?

- From step #2, we add the resulting exponent to the FP's bias (127 for single precision). If the result is greater than 254, overflow will occur. If the result is less than 1, underflow occurs. We can also check if overflow or underflow occurs by checking the decimal range of Floating Point representation. If a number is less than $2.0_{ten} \times 10^{-38}$, underflow occurs. If a number is greater than $2.0_{ten} \times 10^{38}$, overflow occurs.