CS3110 Formal Language and Automata

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Chapter 4. Properties of regular languages

- What happens when we perform operations on regular languages?
  - E.g., if we concatenate two regular languages, is the resulting language also regular?

- Can we decide whether a given language has a certain property or not?
  - E.g., Can we tell if a certain language is finite or not?

- Can we tell whether a given language is regular or not?

Closure properties of regular languages

- **Definition**: A regular language is any language that is accepted by a finite automaton

- **Theorem 4.1** : The class of regular languages is closed under the following operations (that is, performing these operations on regular languages creates other regular languages)
  - Union
  - Concatenation
  - Kleene star
  - Complement
  - Intersection
  - Difference

Using same NFA constructions as in the proof showing regular expressions correspond to regular languages

Unions, Intersections and Difference

Key idea of proof: construct FA that accepts intersections /complements

Suppose that

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ accepts language L_1, and

$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ accepts language L_2

Let M be an FA defined by $M = (Q, \Sigma, \delta, q_0, F)$ where

$Q = Q_1 \times Q_2$

$q_0 = (q_1, q_2)$

and the transition function δ is defined by:

$\delta((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$,

for any $p \in Q_1$, $q \in Q_2$, and $a \in \Sigma$

Unions, Intersections and Difference:

Then:

1. If $F = \{(p, q) \mid p \in F_1$ **or** $q \in F_2\}$, M accepts the language $L_1 \cup L_2$
2. If $F = \{(p, q) \mid p \in F_1$ **and** $q \in F_2\}$, M accepts the language $L_1 \cap L_2$
3. If $F = \{(p, q) \mid p \in F_1$ **and** $q \notin F_2\}$, M accepts the language $L_1 - L_2$

Complement

Consider the special case in which L_1 is Σ^*.

Here, $L_1 - L_2$ is actually L_2' (the complement of L_2)

Membership Question

Question: Given regular language L and string w, how can we check if $w \in L$?

Answer: Take the DFA that accepts L and check if w is accepted.

L is empty?

Question: Given regular language L, how can we check if L is empty: (L=f)?

Answer: Take the DFA that accepts L

Check if there is any path from the initial state to a final state.

L is finite?

Question: Given regular language L, how can we check if L is finite?

Answer: Take the DFA that accepts L

Check if there is a walk with cycle from the initial state to a final state.

Regular Languages and non-regular Languages

- Regular Languages
  - {a}*
  - L(b*c+a)
  - …

- Non-regular languages
  - $\{a^n b^n : n \geq 0\}$
  - $\{vv^R : v \in \{a,b\}^*\}$
  - …

Prove a Language is Not Regular

- How can we prove that a language L is not regular?
  - Prove that there is no DFA that accepts L
  - This is not easy to prove…
  - Solution: the pumping Lemma!