CS 3010.02 | Class exercise 3.1 | Christopher Koepke

1. Find a root for the following equation using bisection method. show at least 4 interations. $9x^4 + 18x^3 + 38x^2 - 57x + 14 = 0$ on the interval $[0, 1]$.

n	a_n	b_n	c_n	$f(a_n)$	$f(b_n)$	$f(c_n)$	error(ε_a)
0	0	1	0.5	14	22	-2.1875	
1	0.5	1	0.75	-2.1875	22	3.06640625	$0.333\bar{3}$
2	0.75	1	0.875	3.06640625	22	10.55297852	0.142857143
3	0.75	0.875	0.8125	3.06640625	10.55297852	6.350479126	-0.07692301
4	0.75	0.8125	0.78125	3.06640625	6.350479126	4.597939491	-0.04
5	0.75	0.78125	0.765625	3.0664063	4.59793949	3.805073321	-0.0240816

$$\varepsilon_a = \frac{c_n - c_{n-1}}{c_n} \qquad c = \frac{a+b}{2} \qquad \varepsilon_s \approx \frac{\left|a - \frac{a+b}{2}\right|}{\left|\frac{a+b}{2}\right|} \times 100\%$$

* if $f(a) \cdot f(c) < 0$ than $a = c$, find new c

if $f(a) \cdot f(c) > 0$ than $b = c$, find new c

2. Same equation, interval, interations as problem #1 but using the false position method. $9x^4 + 18x^3 + 38x^2 - 57x + 14 = 0$ on $[0,1]$

n	a_n	b_n	c_n	$f(a_n)$	$f(b_n)$	$f(c_n)$	error (ε_a)
0	0	1	$-\frac{7}{4}$	14	22	$\frac{55825}{256}$	
1	$\frac{55825}{256}$	1	0.9999997675	2.0540×10^{-10}	22	21.99997466	2.750000407
2	0.9999997675	1	0.7981650739	21.99997466	22	5.518580346	-0.2528733719
3	0.7981650739	1	0.7305833801	5.518580346	22	2.222454811	-0.0925037383
4	0.7305833801	1	0.7003083254	2.222454811	22	1.065703578	-0.0432310364
5	0.7003083254	1	0.685051902	1.065703578	22	0.5543026933	-0.0222704635

• does it converge faster? $\boxed{\text{yes, f(c) is closer to zero using the false position method.}}$

$$C = \frac{(b)f(a) - (a)f(b)}{f(a) - f(b)} \quad \text{OR} \quad C = \frac{(a)f(b) - (b)f(a)}{f(b) - f(a)}$$

• if $f(c) = 0$ or $|f(c)| < \varepsilon_a$ done

• if sign $f(c)$ = sign $f(a)$ than $a_{n+1} = c_n$
else, $b_{n+1} = c_n$

$$\varepsilon_a = \frac{c_n - c_{n-1}}{c_n}$$