

CS311 Formal Language and Automata

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Pushdown Automata

CFG -- Example

Let G be the CFG having productions:

$$S \rightarrow aSa \mid bSb \mid c$$

Then G will generate the language

$$L = \{xcx^R \mid x \in \{a, b\}^*\}$$

There is no DFA that accepts L . Why?

If we want to design an automaton that accept L , we need memory in the automaton.

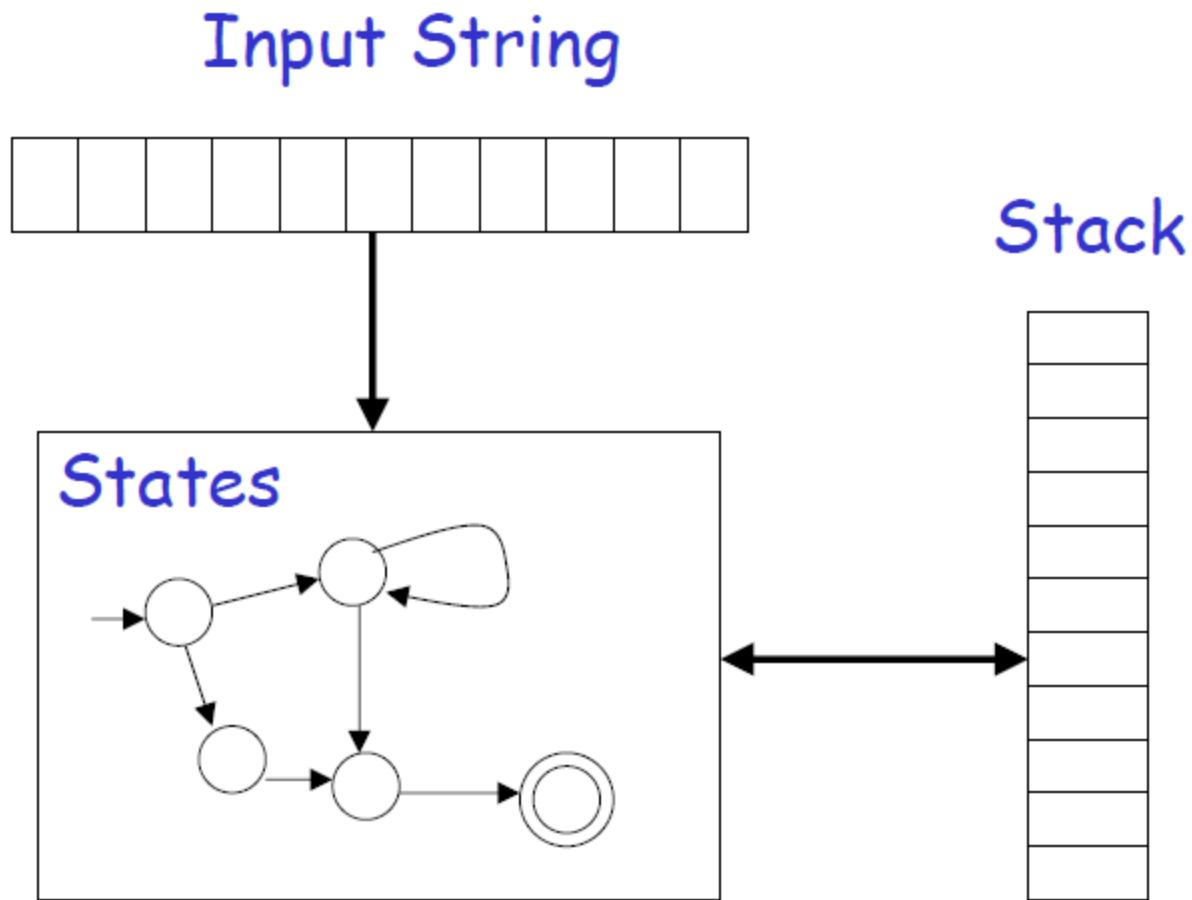
Memory

- What kind of memory do we need to be able to recognize strings in L, using a single left-to-right pass?
- Example: aaabbbcbbaaa
- We need to remember what we saw before the c
- We could push the first part of the string onto a stack and, when the c is encountered, start popping characters off of the stack and matching them with each character from the center of the string on to the end.
- If everything matches, this string is in the language.

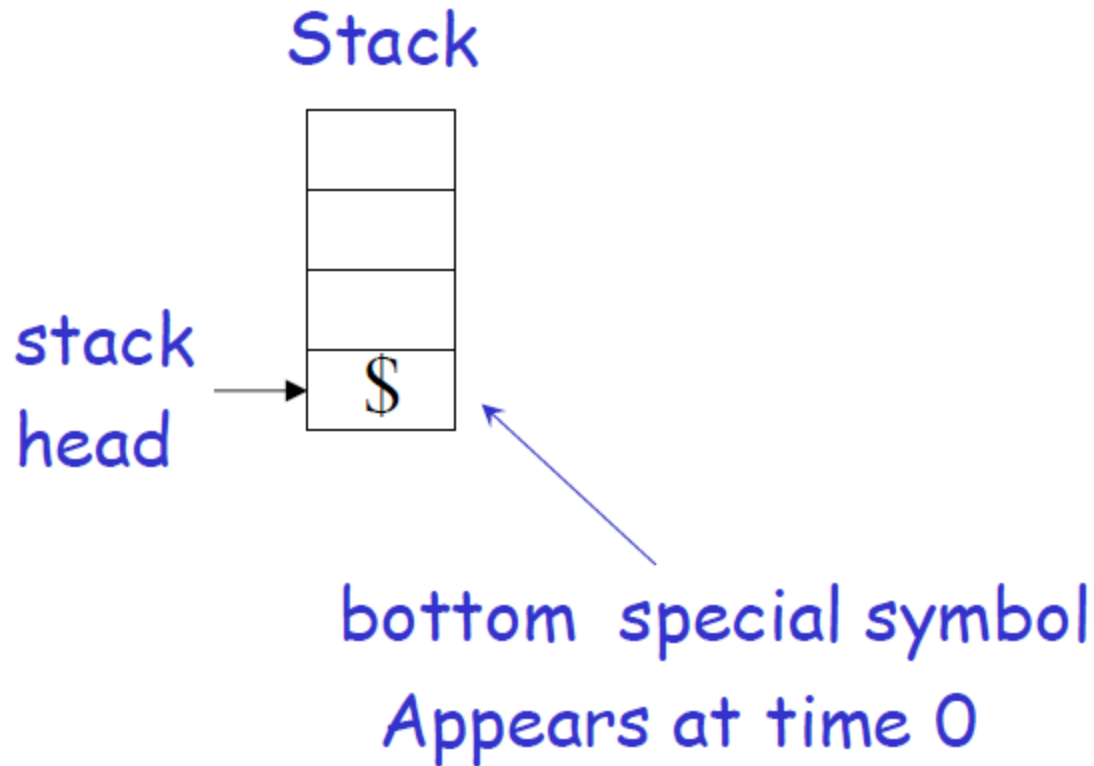
Counting

- We can also use a stack for counting out equal numbers of a's and b's on different sides of a center marker.
- Example: $L = a^n c b^n$ aaaacbbbbb
- Push the a's onto the stack until you see a c, then pop an a off and match it with a b whenever you see a b.
- If we finish processing the string successfully (and there are no more a's on our stack), then the string belongs to L.

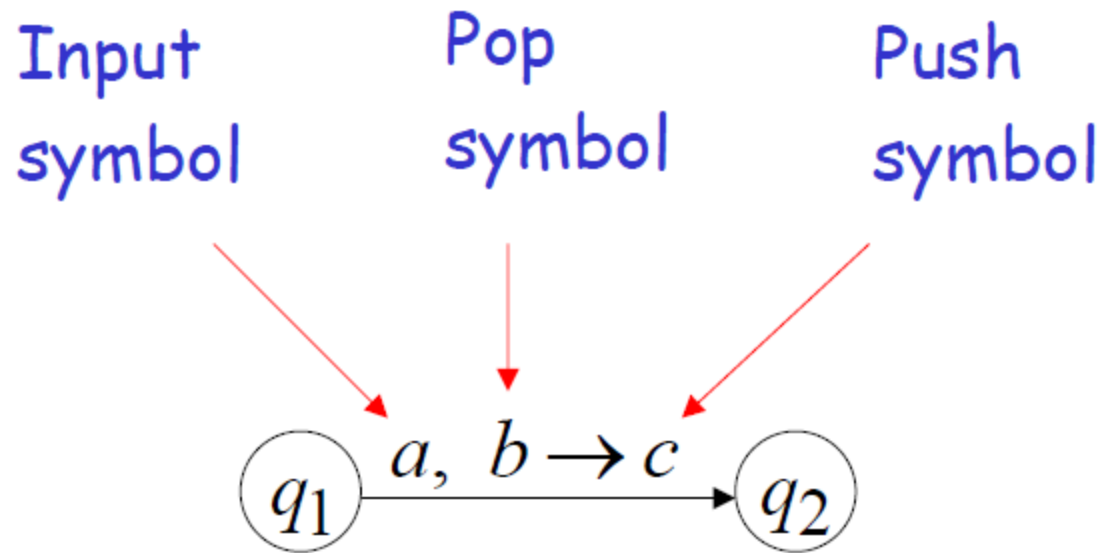
Pushdown Automaton -- PDA

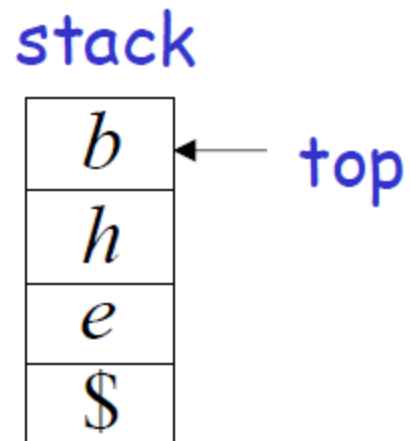
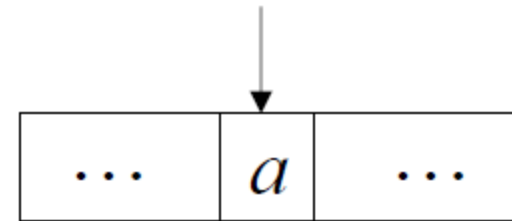
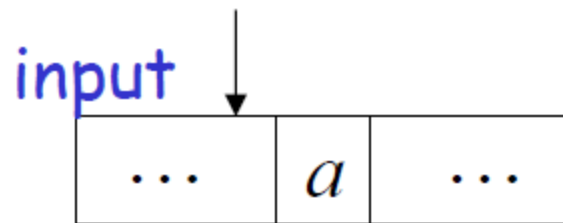
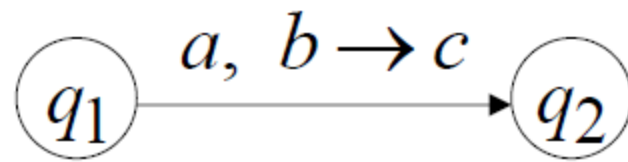


Initial Stack Symbol

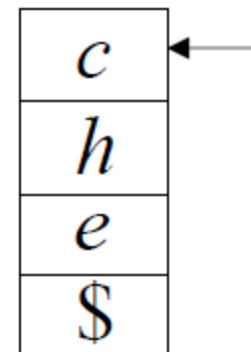


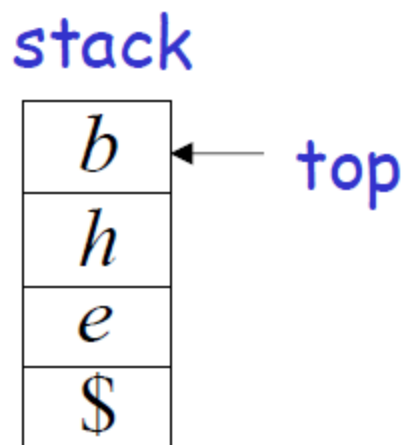
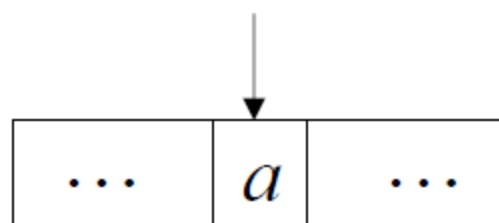
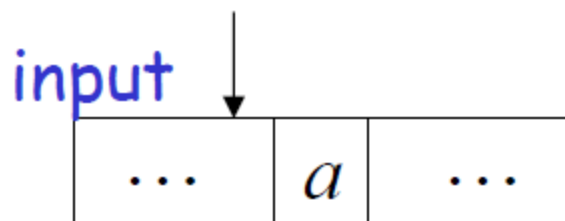
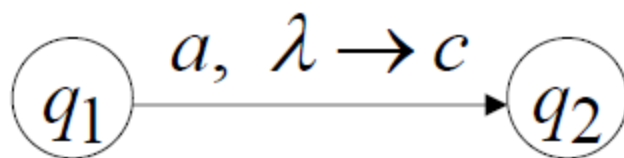
The States



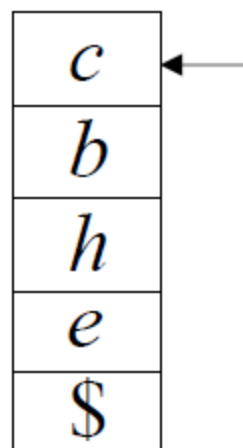


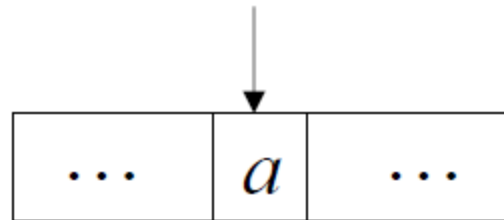
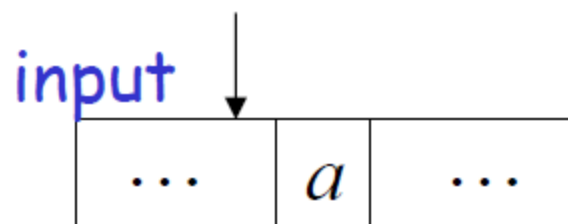
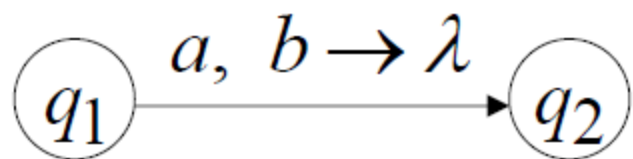
Replace



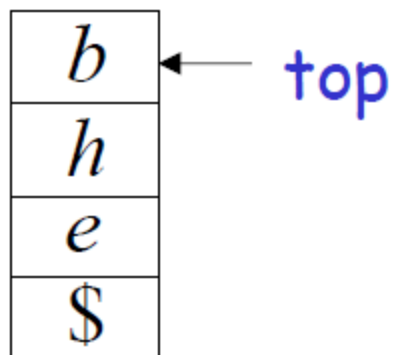


Push

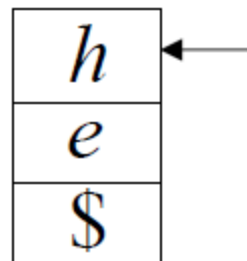


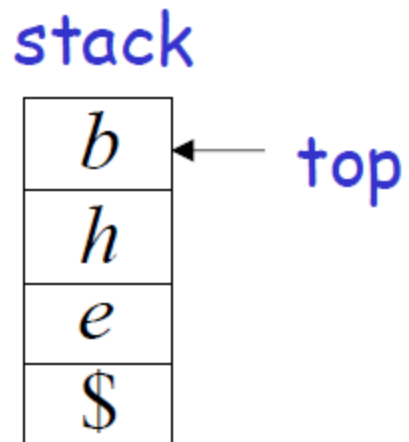
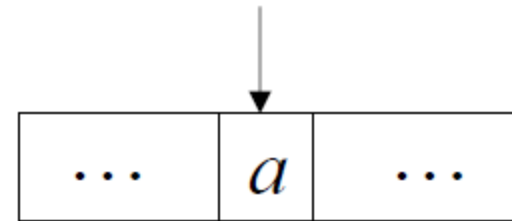
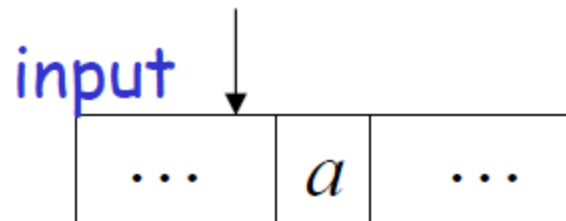
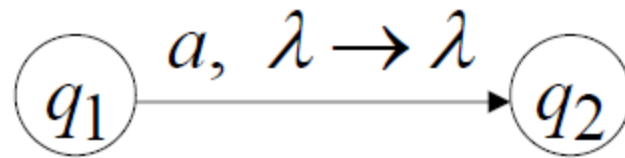


stack

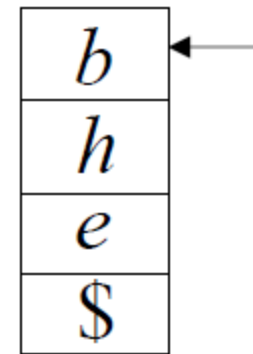


Pop

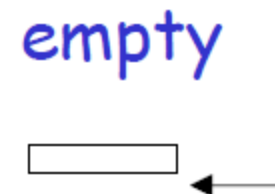
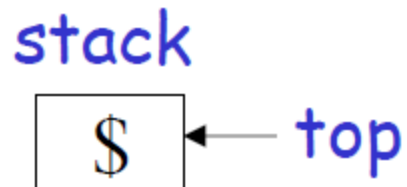
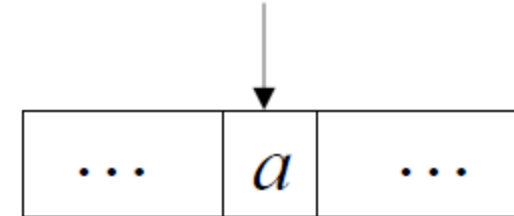
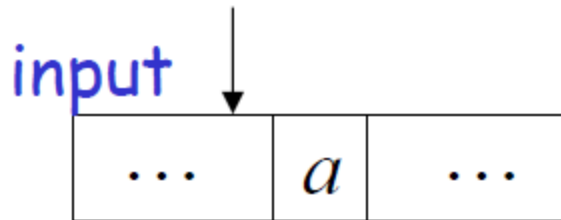
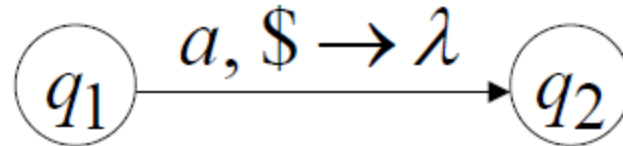




No Change

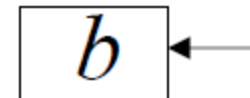
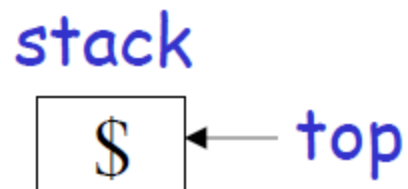
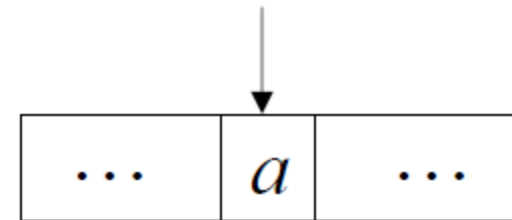
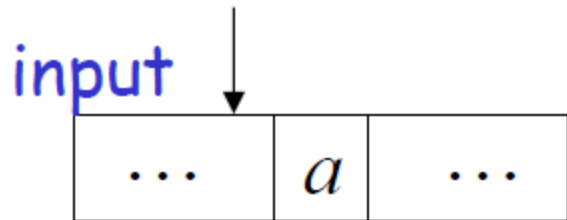
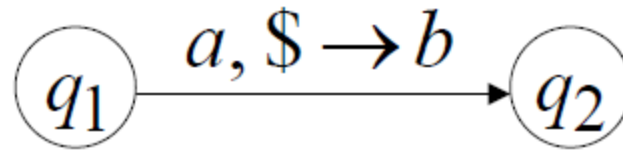


Empty Stack



The automaton halts. No possible transition after q_2

A Possible Transition



Example PDA

- PDA M , $L(M) = \{a^n b^n : n \geq 0\}$

