Class exercise 2.1 | Christopher Koepke

1) solve this using Gaussian elimination.

$$3x_1 + 4x_2 + 3x_3 = 1 \quad \leftarrow \text{1st pivot}$$
$$x_1 + 5x_2 - x_3 = 7 \qquad \text{factors: } \tfrac{1}{3}, \tfrac{6}{3}$$
$$6x_1 + 3x_2 + 7x_3 = 15$$

$\Downarrow$

$$3x_1 + 4x_2 + 3x_3 = 1$$
$$\tfrac{11}{3}x_2 - 2x_3 = \tfrac{20}{3} \quad \leftarrow \text{2nd pivot}$$
$$-5x_2 + x_3 = 13 \qquad \text{factors: } \frac{-5}{\frac{11}{3}} = \frac{-15}{11}$$

2nd eq $x_1 + 5x_2 - x_3 = 7$
$-(\tfrac{1}{3})(3x_1 + 4x_2 + 3x_3 = 1)$
$0x_1 + \tfrac{11}{3}x_2 - 2x_2 = \tfrac{20}{3}$

3rd eq $6x_1 + 3x_2 + 7x_3 = 15$
$-(\tfrac{6}{3})(3x_1 + 4x_2 + 3x_3 = 1)$
$0x_1 - 5x_2 + x_3 = 13$

3rd ed $-5x_2 + x_3 = 13$
$-(\frac{-15}{11})(\frac{11}{3}x_2 - 2x_3 = \frac{20}{3})$
$0x_2 - \frac{19}{11}x_3 = \frac{243}{11}$

$$3x_1 + 4x_2 + 3x_3 = 1$$
$$\tfrac{11}{3}x_2 - 2x_3 = \tfrac{20}{3}$$
$$-\tfrac{19}{11}x_3 = \tfrac{243}{11}$$

- $x_3 = -\frac{243}{19}$
- $\frac{11}{3}x_2 - 2(-\frac{243}{19}) = \frac{20}{3} \Rightarrow x_2 = -\frac{98}{19}$
- $3x_1 + 4(-\frac{98}{19}) + 3(-\frac{243}{19}) = 1 \Rightarrow x_1 = 20$

$$\bar{x} = \left[20, -\frac{98}{19}, -\frac{243}{19}\right]$$

2) Solve this using Gaussian elimination.

$$\begin{aligned} X_1 + 3X_2 + 2X_3 + X_4 &= -2 \\ 4X_1 + 2X_2 + X_3 + 2X_4 &= 2 \\ 2X_1 + X_2 + 2X_3 + 3X_4 &= 1 \\ X_1 + 2X_2 + 4X_3 + X_4 &= -1 \end{aligned}$$

1st pivot factors: $(\frac{4}{1}, \frac{2}{1}, \frac{1}{1})$

$$\begin{aligned} &4X_1 + 2X_2 + X_3 + 2X_4 = 2 \\ &-(\tfrac{4}{1})(X_1 + 3X_2 + 2X_3 + X_4 = -2) \\ \hline &0X_1 - 10X_2 - 7X_3 - 2X_4 = 10 \end{aligned}$$

$$\begin{aligned} &2X_1 + X_2 + 2X_3 + 3X_4 = 1 \\ &-(\tfrac{2}{1})(X_1 + 3X_2 + 2X_3 + X_4 = -2) \\ \hline &0X_1 - 5X_2 - 2X_3 + X_4 = 5 \end{aligned}$$

$$\begin{aligned} &X_1 + 2X_2 + 4X_3 + X_4 = -1 \\ &-(\tfrac{1}{1})(X_1 + 3X_2 + 2X_3 + X_4 = -2) \\ \hline &0X_1 - X_2 + 2X_3 + 0X_4 = 1 \end{aligned}$$

$$\begin{aligned} X_1 + 3X_2 + 2X_3 + X_4 &= -2 \\ -10X_2 - 7X_3 - 2X_4 &= 10 \\ -5X_2 - 2X_3 + X_4 &= 5 \\ -X_2 + 2X_3 + 0X_4 &= 1 \end{aligned}$$

2nd pivot factors $(\frac{-5}{-10}, \frac{-1}{-10})$

$$\begin{aligned} &-5X_2 - 2X_3 + X_4 = 5 \\ &-(\tfrac{5}{10})(-10X_2 - 7X_3 - 2X_4 = 10) \\ \hline &0X_2 + \tfrac{3}{2}X_3 + 2X_4 = 0 \end{aligned}$$

$$\begin{aligned} &-X_2 + 2X_3 + 0X_4 = 1 \\ &-(\tfrac{1}{10})(-10X_2 - 7X_3 - 2X_4 = 10) \\ \hline &0X_2 + \tfrac{27}{10}X_3 + \tfrac{2}{10}X_4 = 0 \end{aligned}$$

$$\begin{aligned} X_1 + 3X_2 + 2X_3 + X_4 &= -2 \\ -10X_2 - 7X_3 - 2X_4 &= 10 \\ \tfrac{3}{2}X_3 + 2X_4 &= 0 \\ \tfrac{27}{10}X_3 + \tfrac{1}{5}X_4 &= 0 \end{aligned}$$

3rd pivot factor $(\frac{27}{10} / \frac{3}{2})$

$$\begin{aligned} &\tfrac{27}{10}X_3 + \tfrac{1}{5}X_4 = 0 \\ &-(\tfrac{9}{5})(\tfrac{3}{2}X_3 + 2X_4 = 0) \\ \hline &0X_3 - \tfrac{17}{5}X_4 = 0 \end{aligned}$$

$$\begin{aligned} X_1 + 3X_2 + 2X_3 + X_4 &= -2 \\ -10X_2 - 7X_3 - 2X_4 &= 10 \\ \tfrac{3}{2}X_3 + 2X_4 &= 0 \\ -\tfrac{17}{5}X_4 &= 0 \end{aligned}$$

- $X_4 = 0$
- $\frac{3}{2}X_3 + 2(0) = 0 \Rightarrow X_3 = 0$
- $-10X_2 - 7(0) - 2(0) = 10 \Rightarrow X_2 = -1$
- $X_1 + 3(-1) + 2(0) + (0) = -2 \Rightarrow X_1 = 1$

$$\bar{X} = [1, -1, 0, 0]$$