CS3110 Formal Language and Automata

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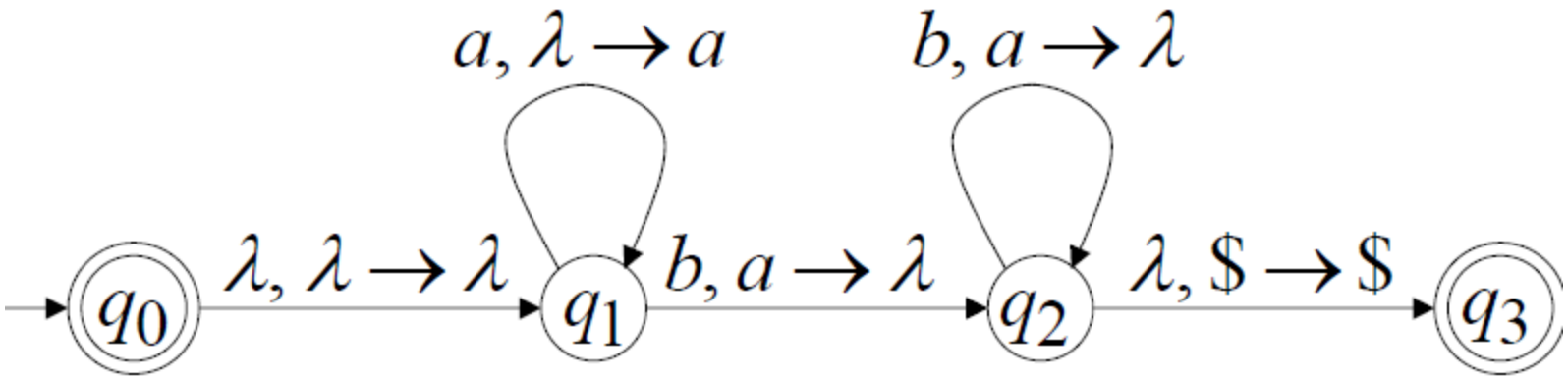
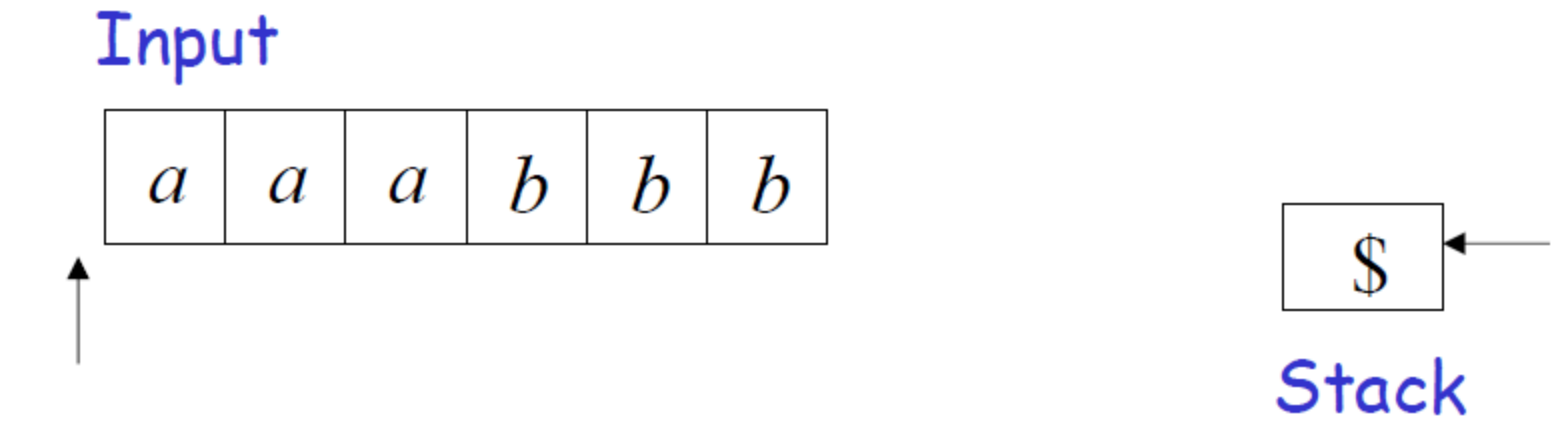
Example PDA

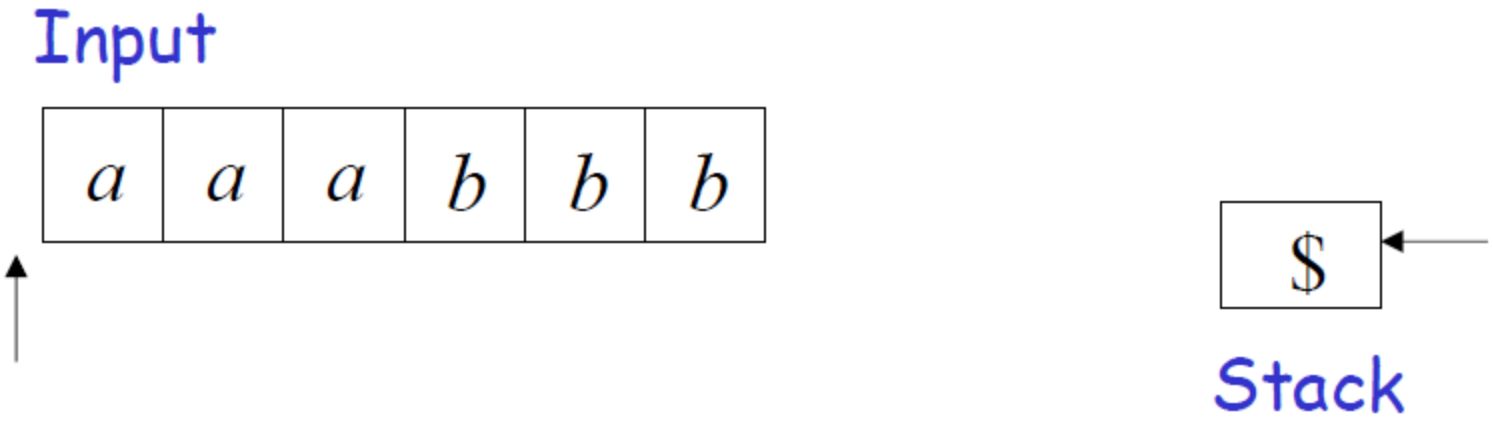
- PDA M, $L(M)=\{a^n b^n : n \geq 0\}$

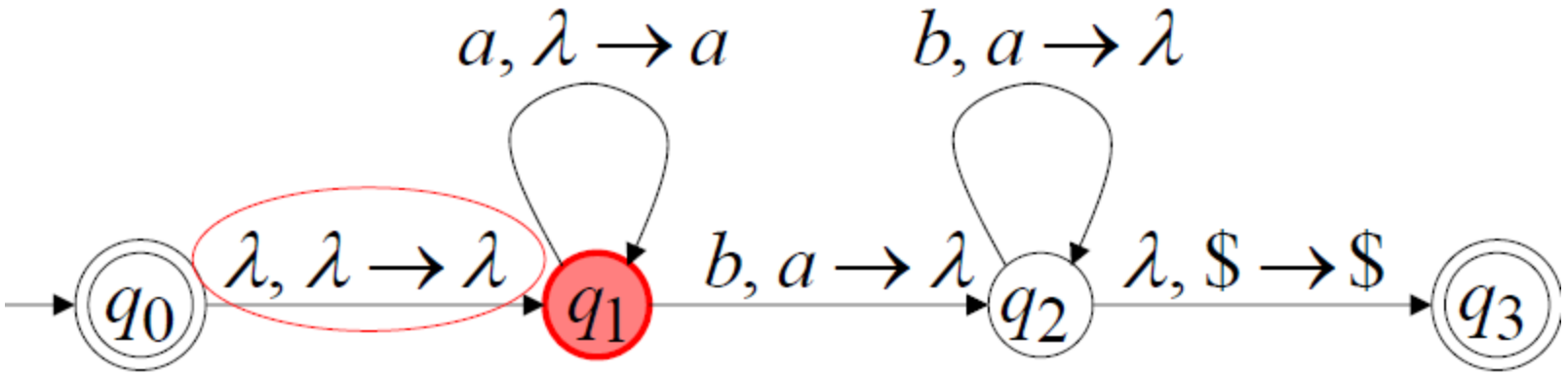


Execution Example: Time 0

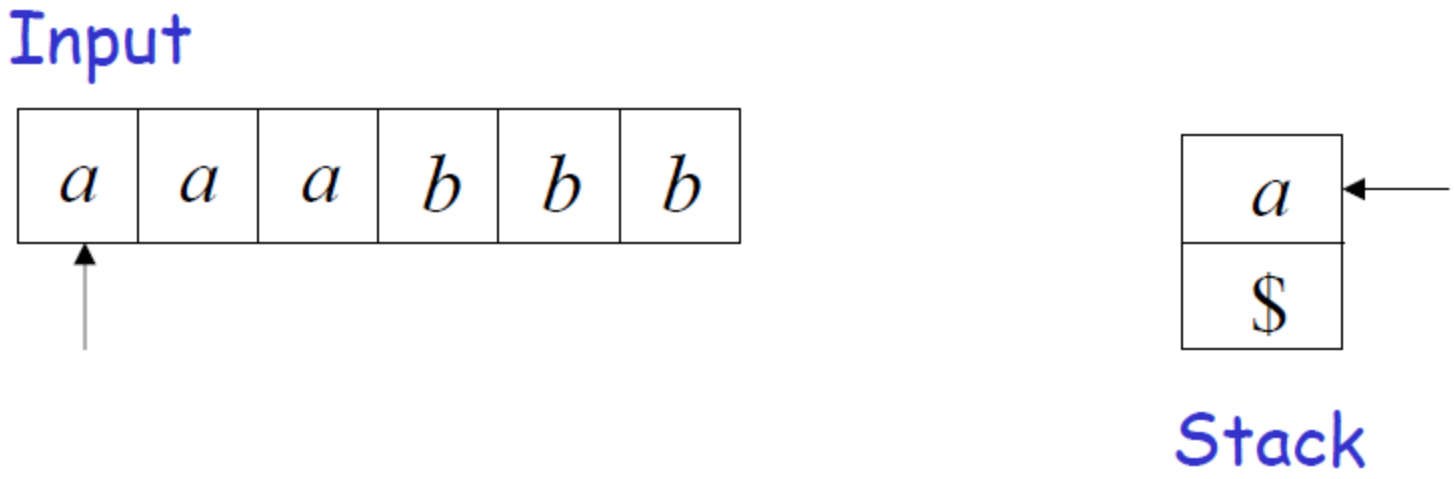


Execution Example: Time 1

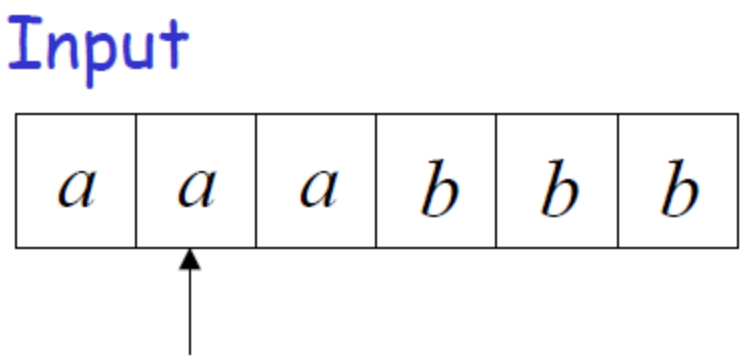


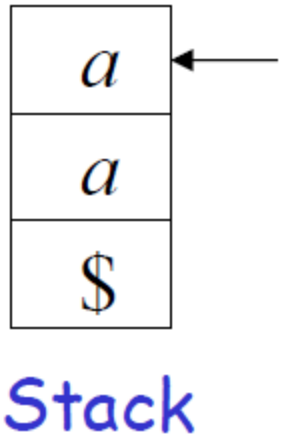


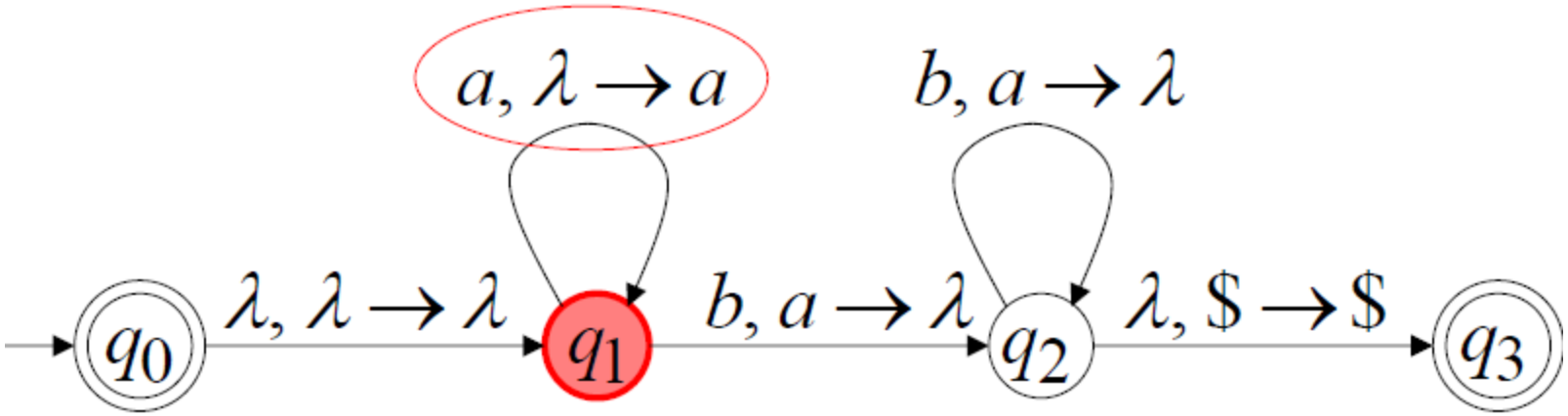
Execution Example: Time 2



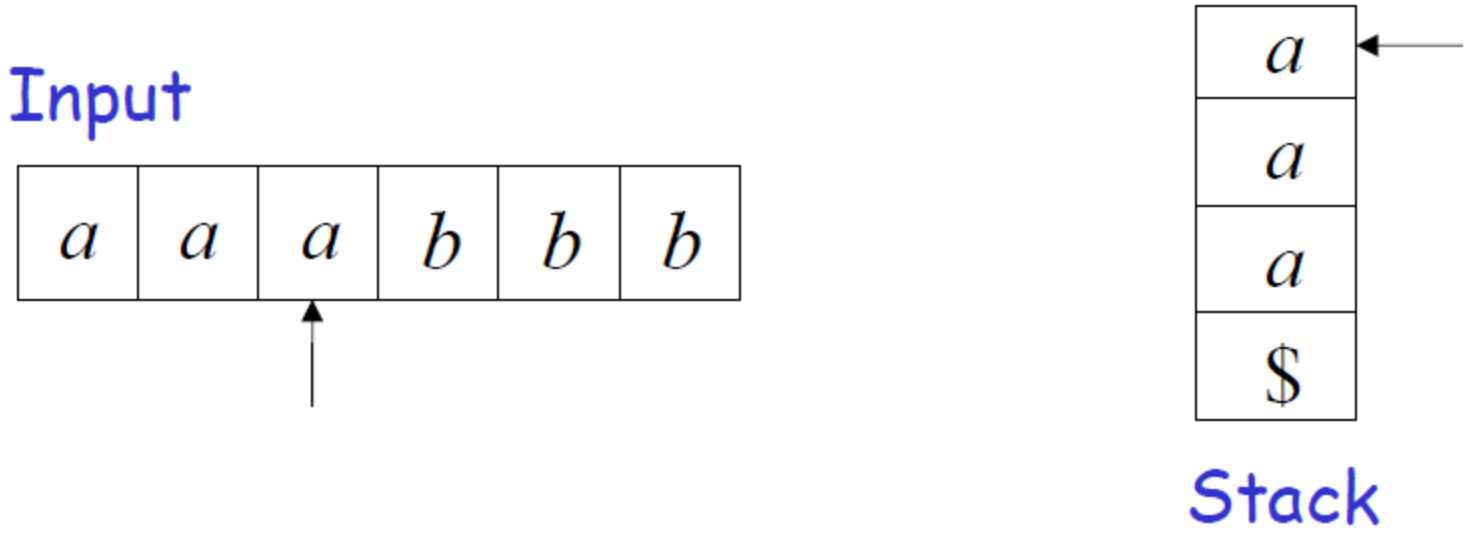
Execution Example: Time 3

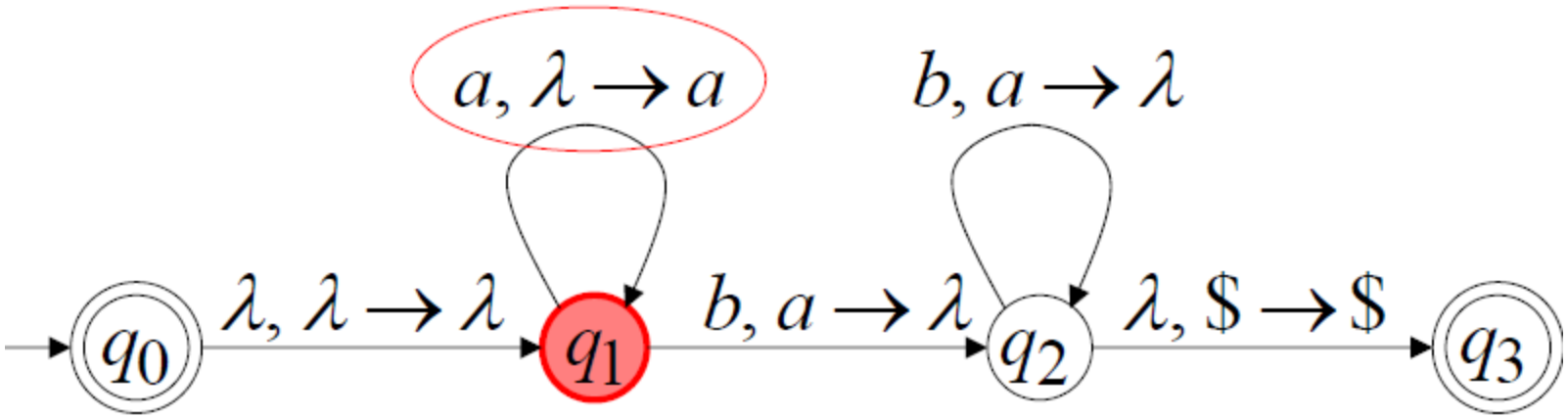




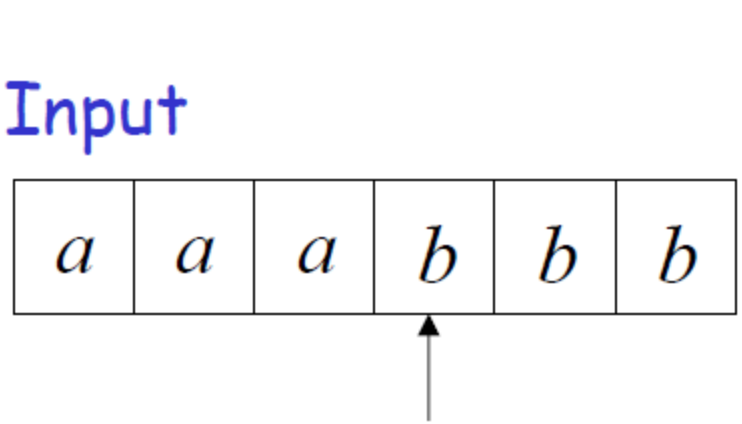


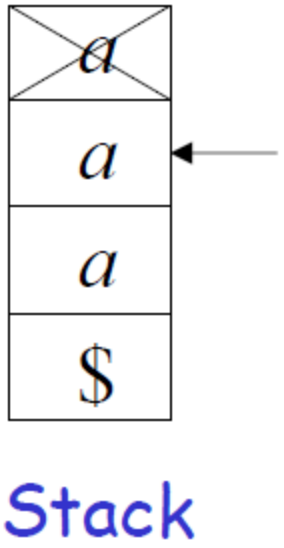
Execution Example: Time 4

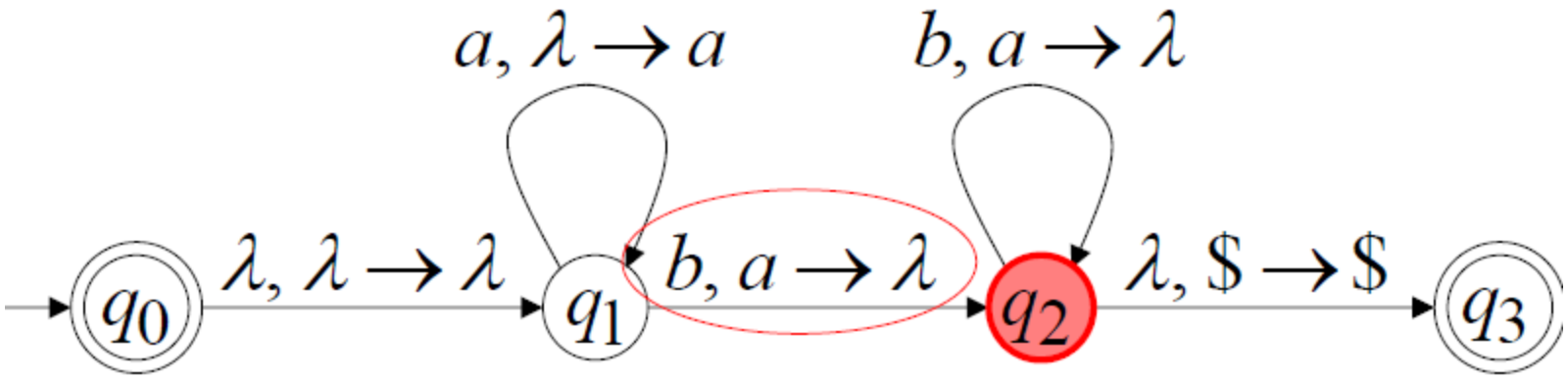




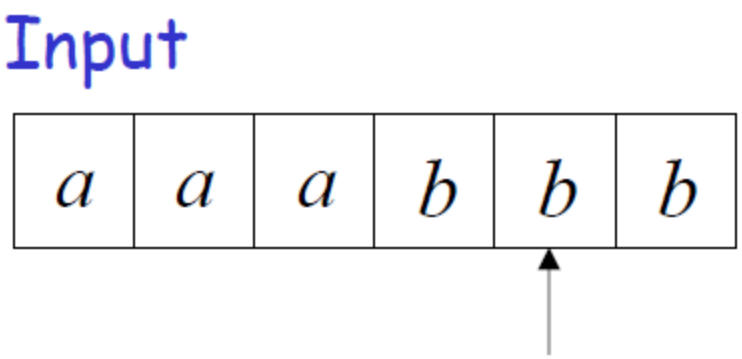
Execution Example: Time 5

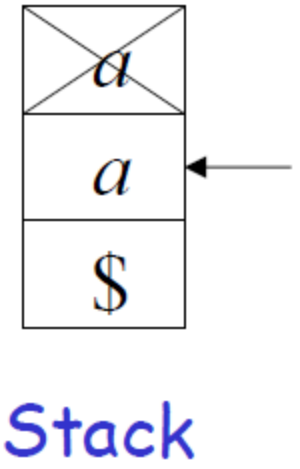


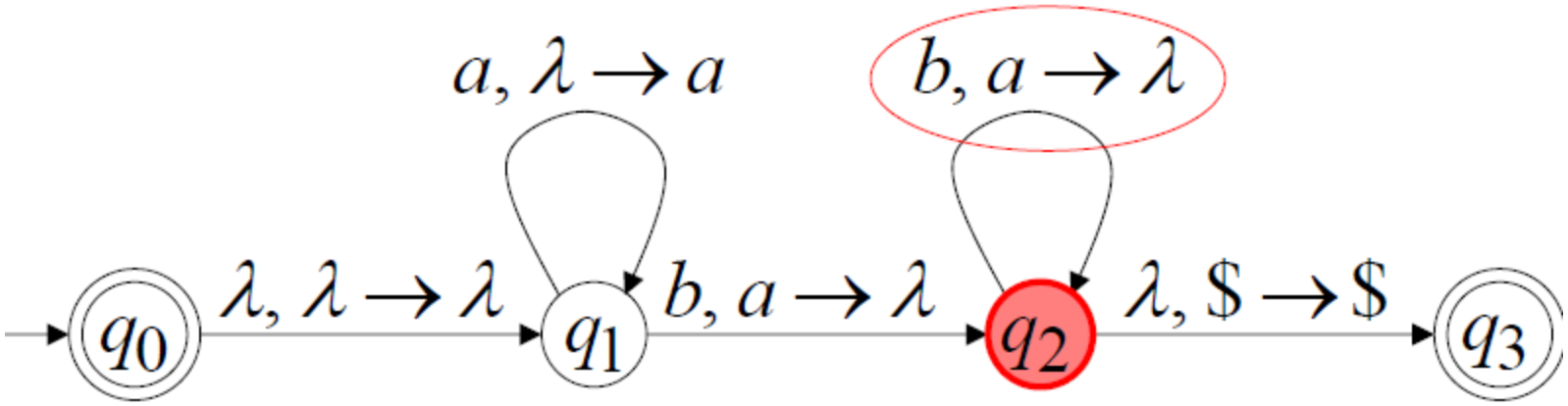




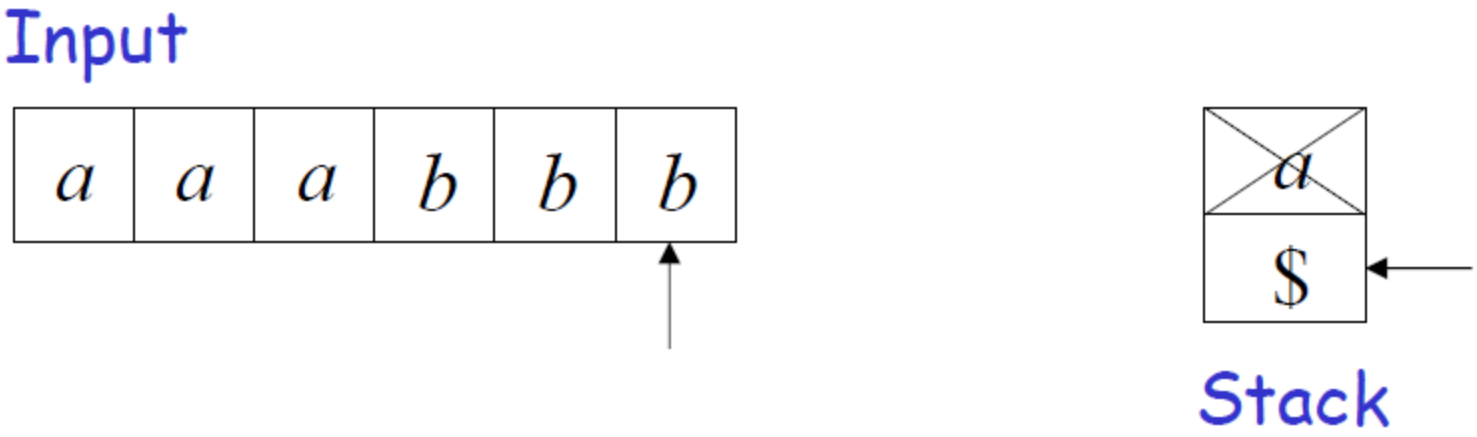
Execution Example: Time 6

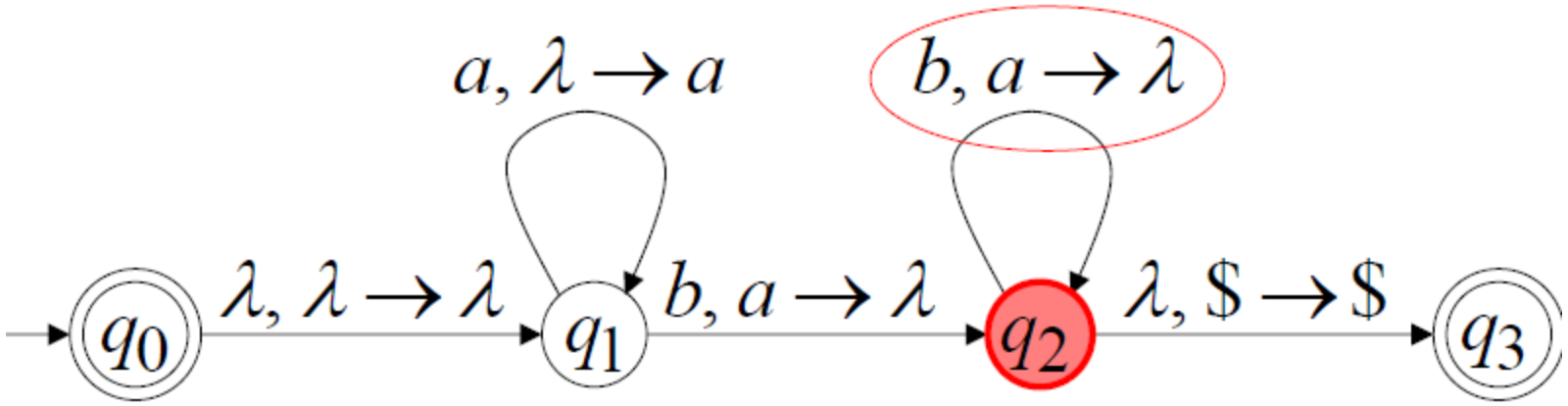




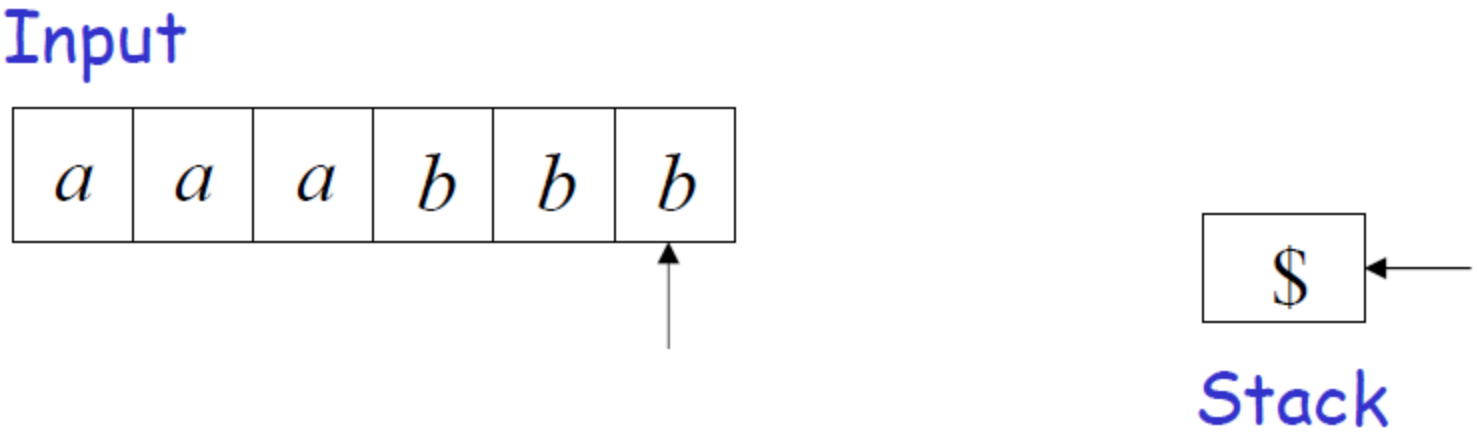


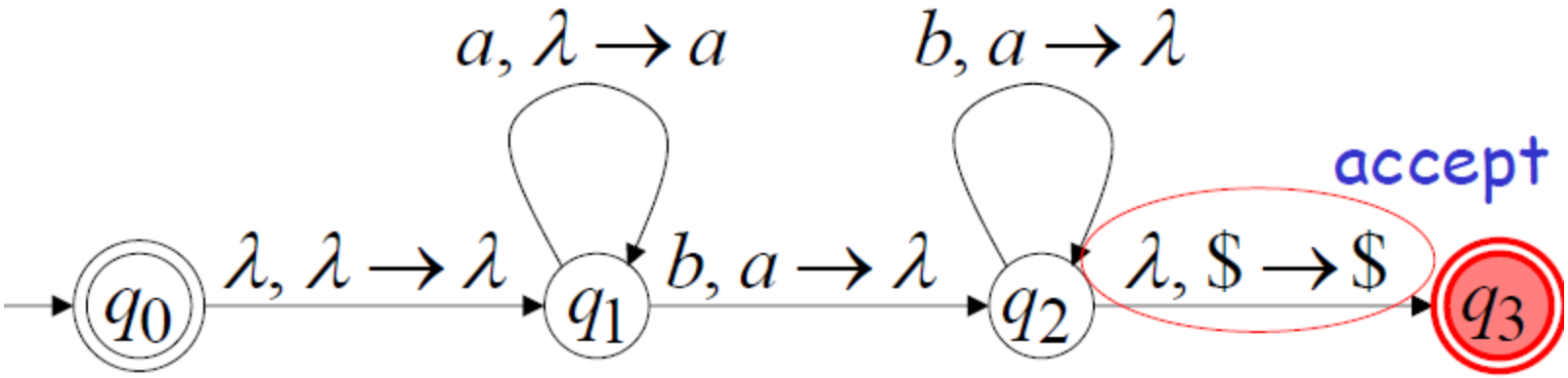
Execution Example: Time 7





Execution Example: Time 8





PDA – Accepting a string

- A string is accepted if there is a computation such that
  - All the input is consumed
  - And the last state is an accepting state

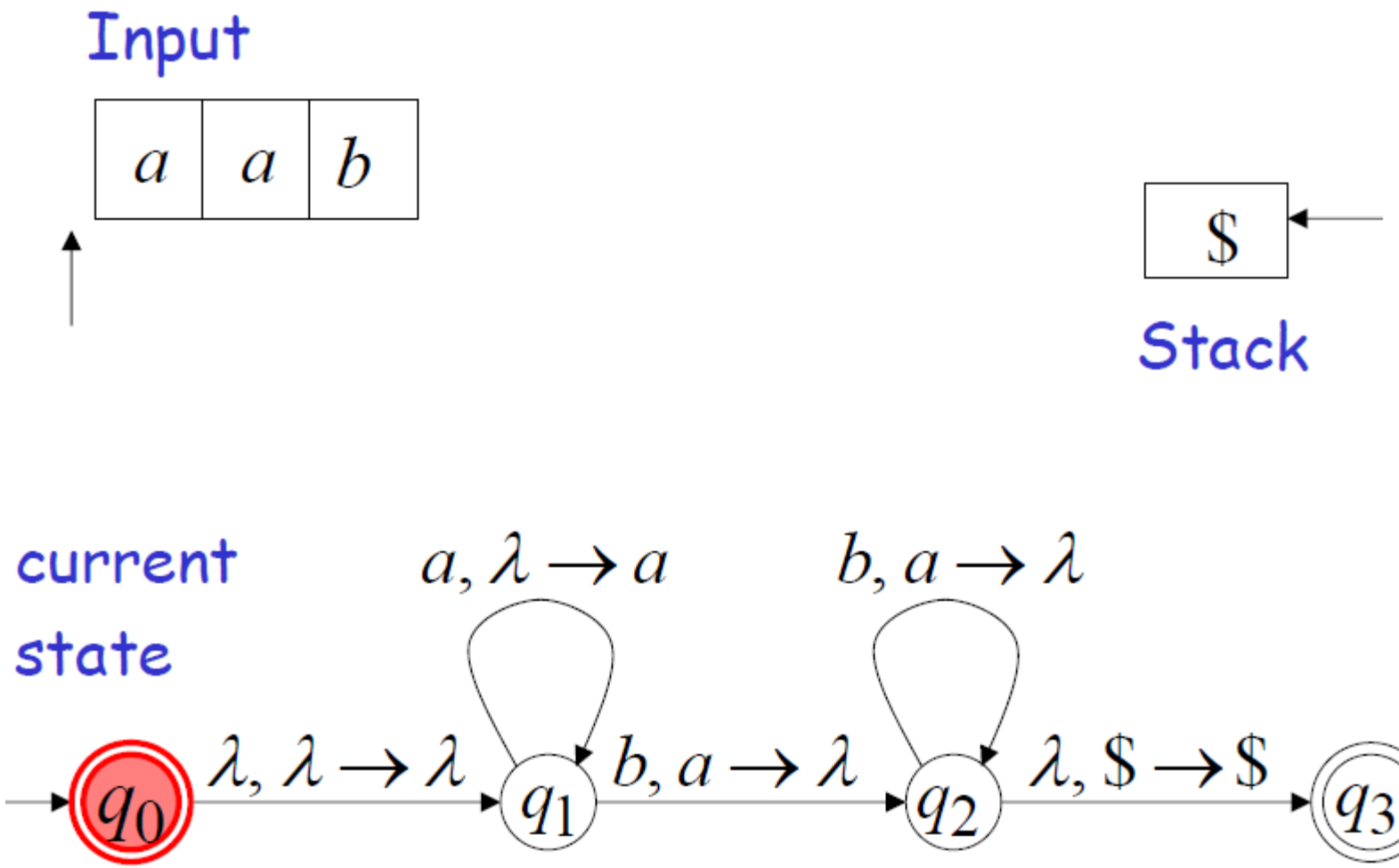
  - At the end of computation, we do not care about the stack contents. The stack can be empty at the last state.

PDA – Rejecting a string

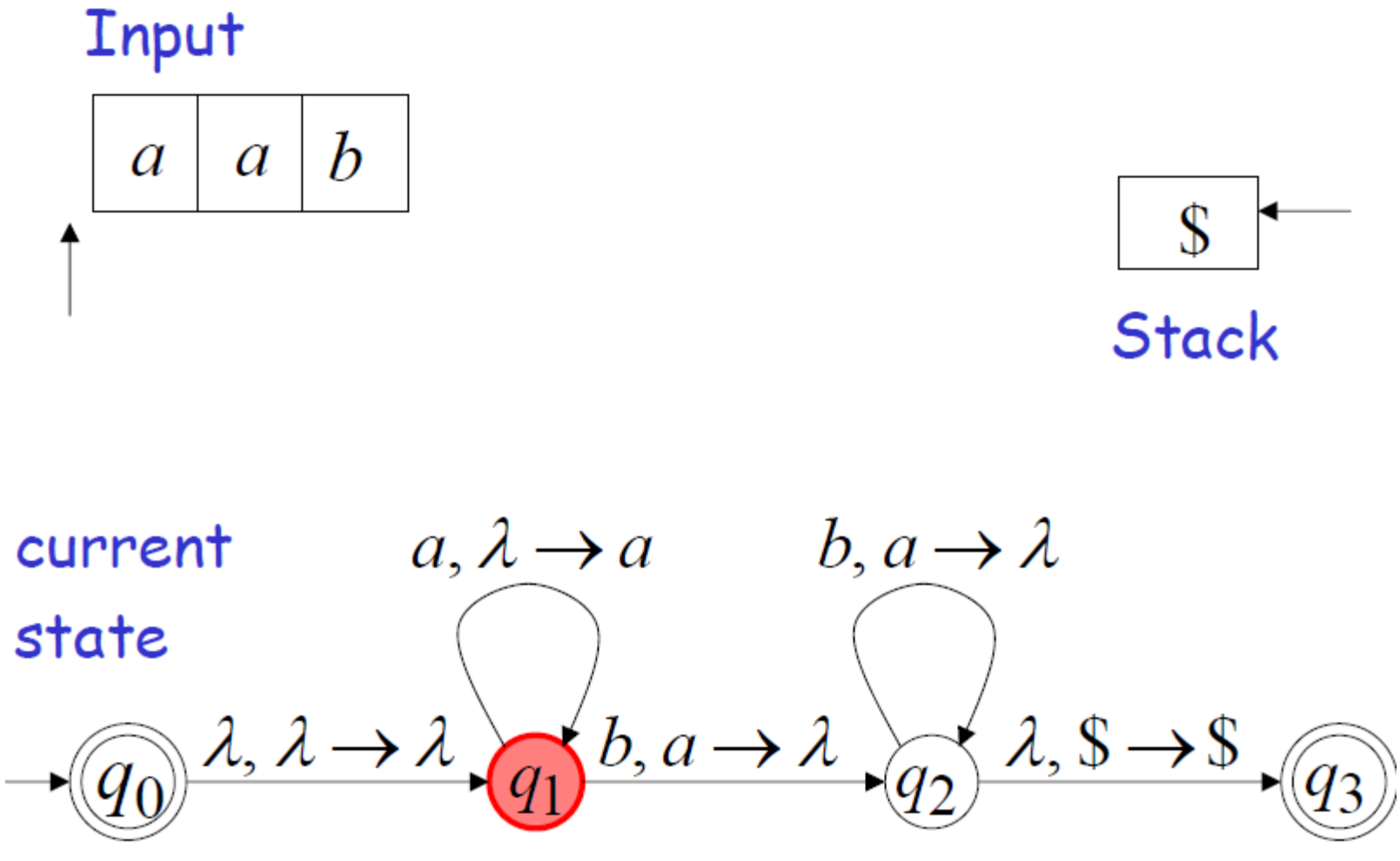
- A string is rejected if there is no computation such that
  - All the input is consumed
  - And the last state is an accepting state

  - At the end of computation, we do not care about the stack contents.

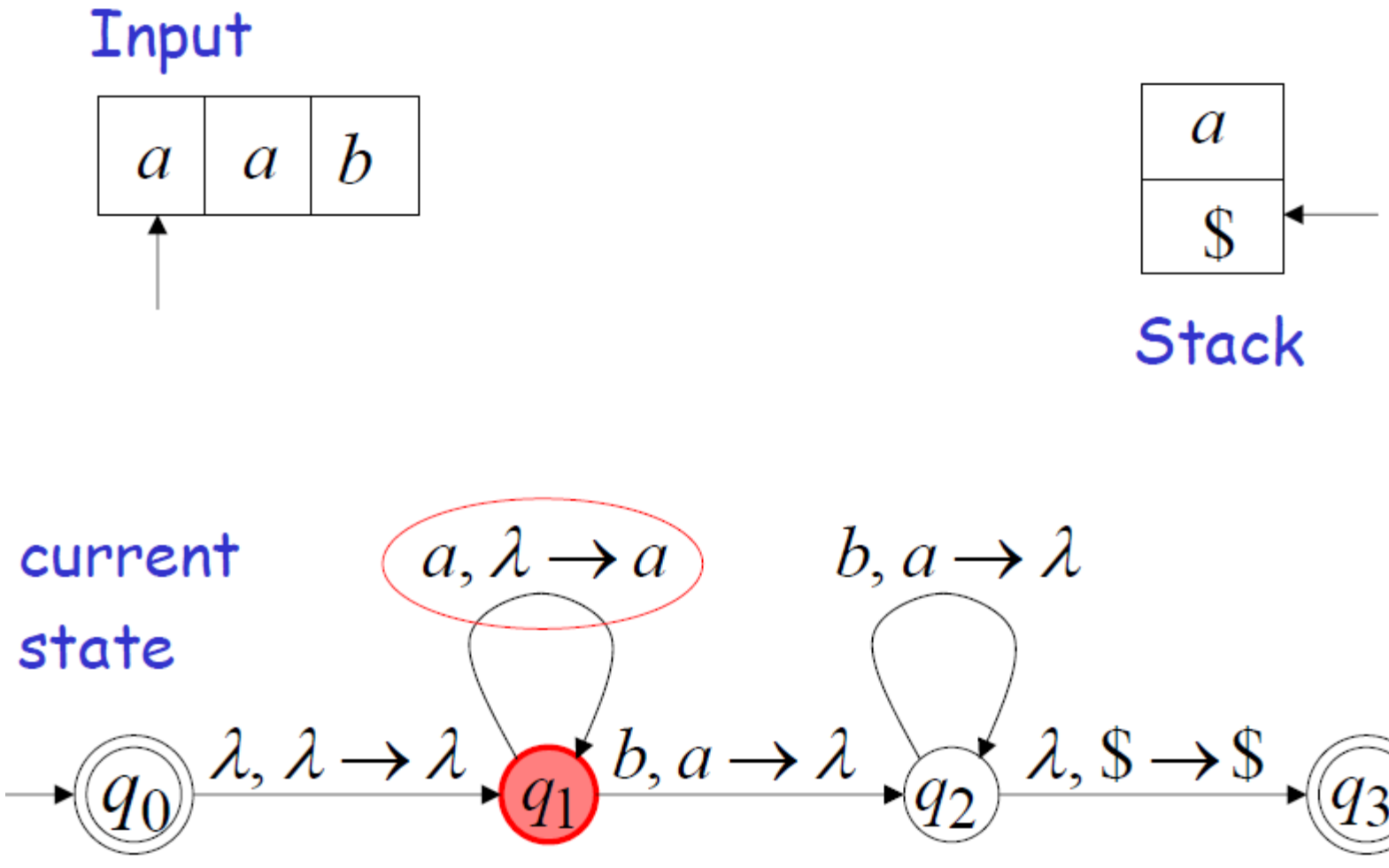
Rejection Example: Time 0



Rejection Example: Time 1



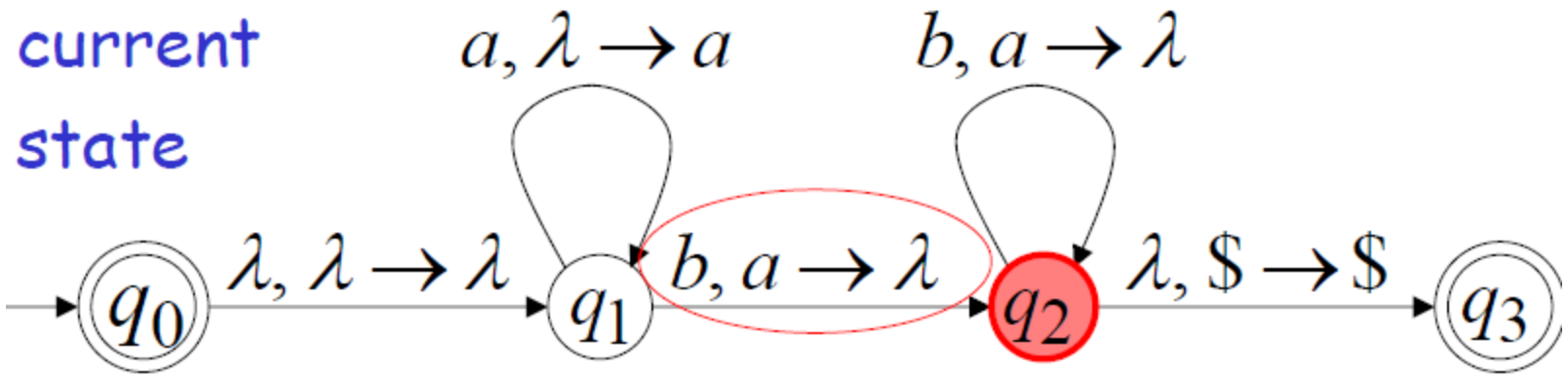
Rejection Example: Time 2



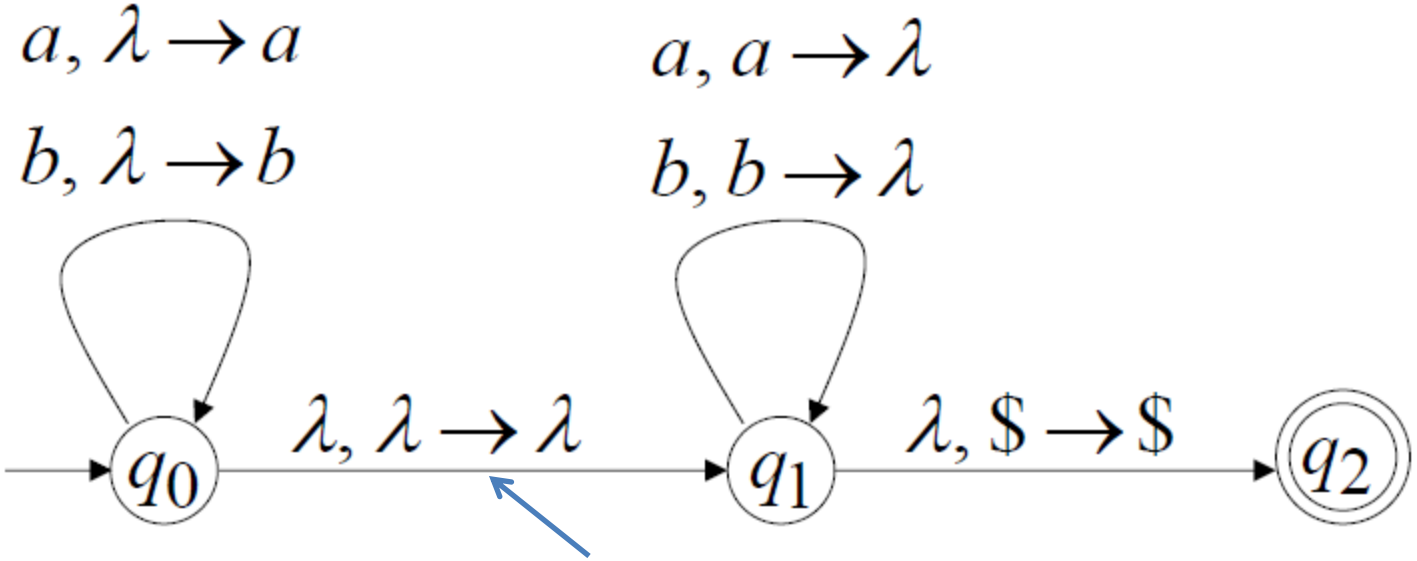
Rejection Example: Time 3

Rejection Example: Time 4

Reject



PDA -- Example

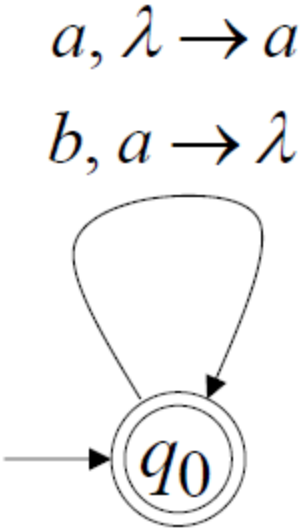


Guess the middle of string

$L(M)=\{vv^R: v \in \{a,b\}^*\}$

How the PDA executes, with input string abbb?

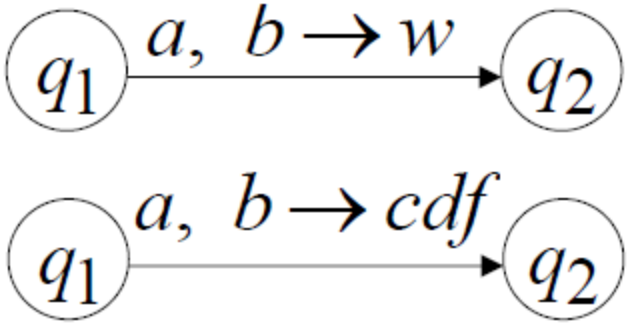
Exercise: L(M)=?

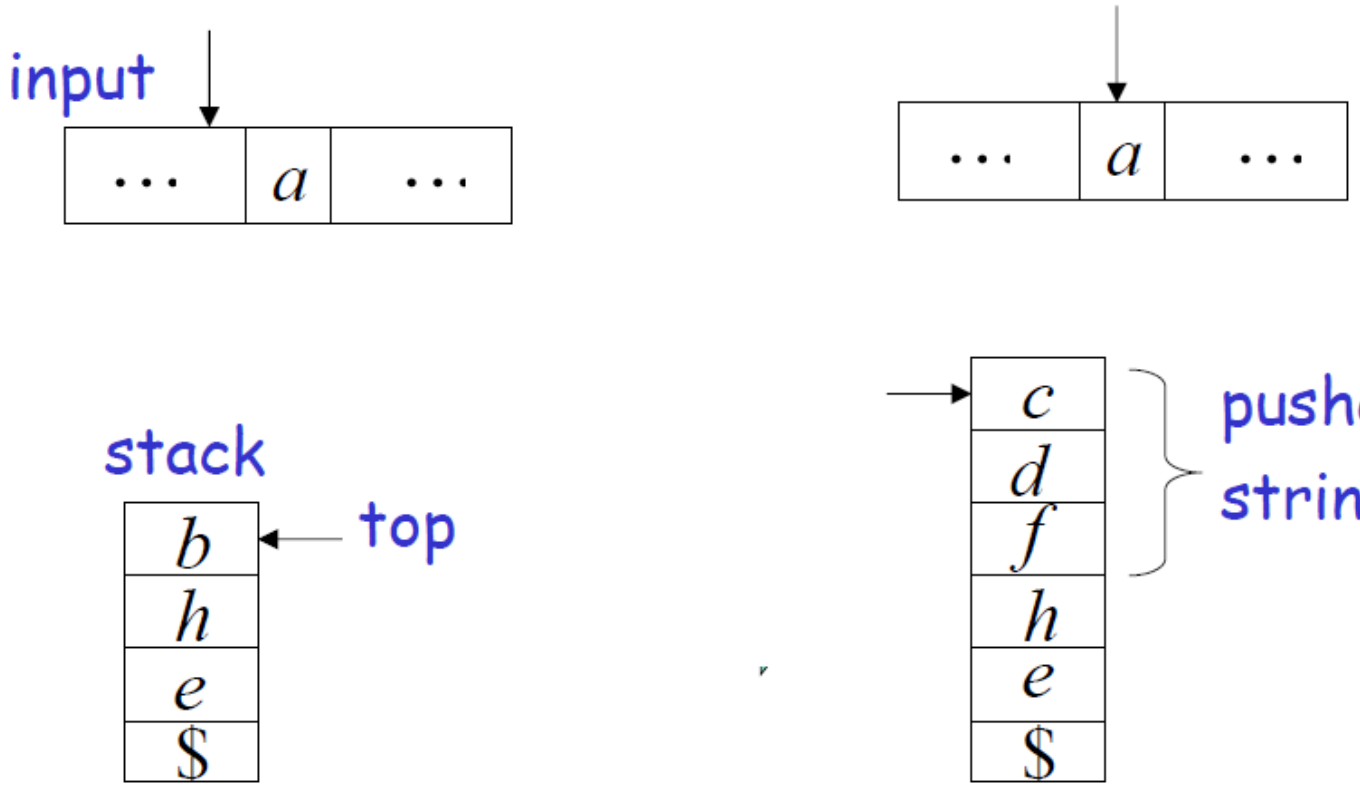


L(M)={w $\in$ {a,b}*: every prefix v, $n_a(v) \geq n_b(v)$}

Push Strings

Example



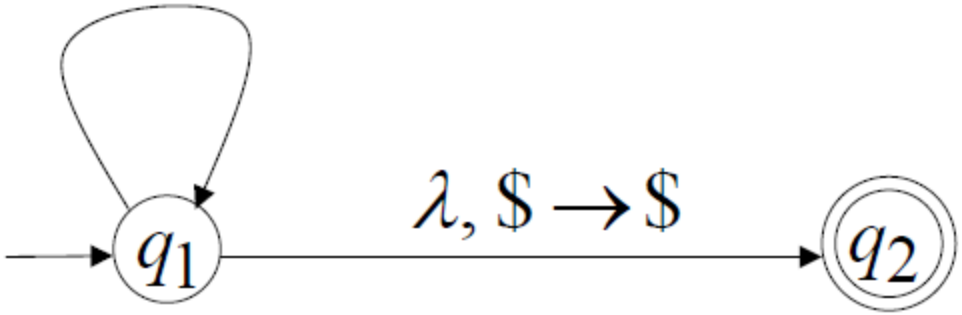


Exercise: L(M)=?

$a, \$ \to 0\$ \qquad b, \$ \to 1\$$

$a, 0 \to 00 \qquad b, 1 \to 11$

$a, 1 \to \lambda \qquad b, 0 \to \lambda$



Pushdown Automaton: Formal Definition

A nondeterministic pushdown automaton (NPDA) is a 7-tuple

Q is a finite set of states

Σ is the input alphabet (a finite set)

Γ is the stack alphabet (a finite set)

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \rightarrow$ (finite subsets of $Q \times \Gamma^*$)

is the transition function

$q_0 \in Q$ is the start state

$z \in \Gamma$ is the initial stack symbol

$F \subseteq Q$ is the set of accepting states