CS 3010-02 | Class exercise 5.1 | Christopher Koepke

1. approximate $\int_1^2 f(x)dx$ given the table of values

x	1	5/4	3/2	7/4	2
$f(x)$	10	8	7	6	5

compute an estimate by the composite trapezoid rule.

* $h = b - a$ where $b-a$ is uniform at 0.25 between each interval

$$I = \left(\frac{h}{2}\right)\left[f(1) + f(2)\right] + h\left[f(5/4) + f(3/2) + F(7/4)\right]$$

$$= \left(\frac{0.25}{2}\right)\left[10 + 5\right] + (0.25)\left[8 + 7 + 6\right]$$

$$= (0.125)[15] + (0.25)[21]$$

$$\boxed{I = 7.125}$$

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2. compute an approx. value of $\int_0^1 (1+x^2)^{-1}dx$ by using the composite trapezoid rule with 3 points. Then compare with the actual value of the integral. Next use the error formula and numerically verify an upper bound on it. IE; calculate this maximum possible error from the error formula. show that the true absolute error less than the maximum error.

$f(x) = \frac{1}{1+x^2}$ on $[0,1]$ with points $x=0$, $x=\frac{1}{2}$ and $x=1$

- $f(0) = 1$ $\quad f(\frac{1}{2}) = \frac{4}{5}$ $\quad f(1) = \frac{1}{2}$

* $h = b - a$ where $b-a$ is uniform at 0.5 between each interval

$$I = \left(\frac{h}{2}\right)\left[f(0) + f(1)\right] + (h)f(\tfrac{1}{2})$$

$$= \left(\frac{0.5}{2}\right)\left[1 + \frac{1}{2}\right] + (0.5)(4/5)$$

$$\boxed{I = 0.775}$$

- Actual value of $\int_0^1 (1+x^2)^{-1}dy$ using an online calculator

$$\boxed{\int_0^1 (1+x^2)^{-1}dx = \left[\arctan(x)\right]\Big|_0^1 = \frac{\pi}{4} \approx 0.7853981634}$$

• upper bound on the error

• $f(x) = (1+x^2)^{-1}$ $\quad f'(x) = -\dfrac{2x}{(1+x^2)^2}$ $\quad f''(x) = -\dfrac{2(-3x^2+1)}{(1+x^2)^3}$

• at $x=0$ $|f''(x)| = |-2|$ $\quad$ at $x=1$ $|f''(x)| = |\frac{1}{2}|$ $\quad$ *max $f''(x) = |2|$

$$\text{error} \leq \left| \frac{K(b-a)^3}{12n^2} \right|$$

where $|f''(x)| \leq K$ for $a \leq x \leq b$
and $n = 3$

$$\text{error} \leq \left| \frac{2(1)^3}{12(3)^2} \right|$$

$$\boxed{\text{error} \leq \left| \frac{1}{6(3)^2} \right| \approx 0.0185185\overline{185}}$$

*computer - Trap

• absolute error : $\left| \frac{\pi}{4} - 0.775 \right| = 0.0103981634$

absolute error < upper bound error ✓