1. What straight line best fits the following data in least square sense?

$$X: \quad 1 \quad 2 \quad 3 \quad 4$$
$$y: \quad 0 \quad 1 \quad 1 \quad 2$$

$* \quad y = ax + b \quad$ straight line

$$\varphi(a,b) = \Sigma(ax_k + b_k - y_k)^2$$

1st eq: $(a)\Sigma(x_k^2) + (b)\Sigma(x_k) = \Sigma(y_k x_k)$

2nd eq: $(a)\Sigma(x_k) + (b)\Sigma(1) = \Sigma(y_k)$

$* \quad \Sigma(1) = \#$ of data

$$\Sigma(1) = 4, \quad \Sigma(x_k) = 10, \quad \Sigma(x_k^2) = 30, \quad \Sigma(y_k x_k) = 13$$
$$\Downarrow \qquad \Sigma(y_k) = 4$$

1st eq: $30a + 10b = 13$

2nd eq: $(10a + 4b = 4) \times 3 \implies$

$$\begin{aligned} 30a + 10b &= 13 \\ -(30a + 12b &= 12) \\ \hline -2b &= 1 \implies \boxed{b = -\tfrac{1}{2}} \end{aligned}$$

$$30a + 10(-\tfrac{1}{2}) = 13$$
$$30a = 13 + 5$$
$$\boxed{a = {}^{18}/_{30} = {}^3/_5}$$

$$\boxed{y = \left(\tfrac{3}{5}\right)x - \tfrac{1}{2}}$$

2. Find the equation of a parabola ($y = ax^2 + b$) that best represents the following data. use the method of least squares.

X: -1   0   1

y: 3.1   0.9   2.9

$$P(a,b) = \sum (ax_k^2 + b_k - y_k)^2$$

$$\frac{d}{dy} \sum (ax^2 + b - y)^2 = \begin{cases} \sum 2(ax^2 + b - y)x^2 \\ \sum 2(ax^2 + b - y) \end{cases}$$

$* \sum(1) = \#$ of data

1st eq: $(a)\sum(x_k^4) + (b)\sum(x_k^2) = \sum(y_k x_k^2)$

2nd eq: $(a)\sum(x_k^2) + (b)\sum(1) = \sum(y_k)$

$$\sum(1) = 3, \quad \sum(x_k^2) = 2, \quad \sum(x_k^4) = 2 \quad \sum(y_k x_k^2) = 6$$
$$\sum(y_k) = 6.9$$

1st eq: $2a + 2b = 6$

2nd eq: $2a + 3b = 6.9$

$\Rightarrow$
$$\begin{array}{r} 2a + 2b = 6 \\ -(2a + 3b = 6.9) \\ \hline -b = -0.9 \end{array} \quad \Rightarrow b = 0.9$$

$$2a + 2(0.9) = 6$$
$$2a = 6 - 1.8$$
$$a = \frac{4.2}{2} = 2.1$$

$$\boxed{y = (2.1)x^2 + (0.9)}$$