Polynomial Interpolation

CS3010

Numerical Methods

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Section 4.1

Lagrange Interpolating Polynomials

- The Lagrange interpolating polynomial is simply a reformulation of the Newton's polynomial that avoids the computation of divided differences
- The interpolating polynomial is a linear combination of cardinal polynomials $L_i(x)$. Each polynomial is order n and defined with property $L_i(x) = \delta_{ij} = 0\ (if\ i \neq j)\ or = 1\ (if\ i = j)$

$$L_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

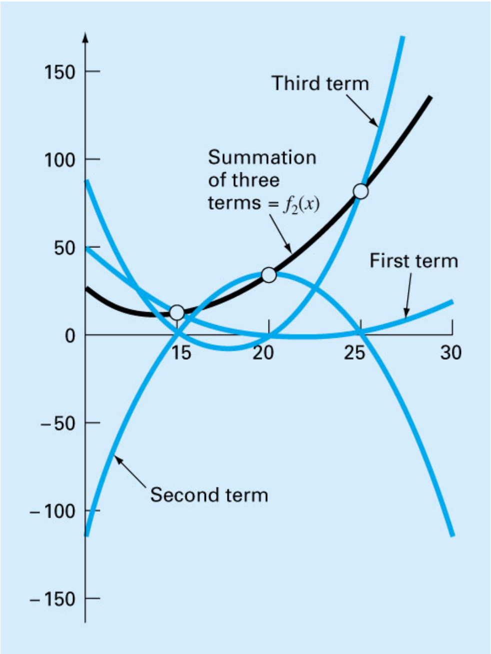
$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

Lagrange Interpolating Polynomials

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$f_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)((x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)((x_2 - x_1)} f(x_2)$$

- Note that $L_i(x)$ is a polynomial of degree n.
- When $L_i(x)$ is evaluated at $x = x_i$, each factor in the preceding equation becomes 1.
  - Hence $L_i(xi)$ = 1
- When $L_i(x)$ is evaluated at $x = x_j$, one of the factors in the above equation will be 0, and $L_i(x_j) = 0$ for $i \neq j$



Example : Lagrange Interpolating Polynomial

x	1/3	1/4	1
$f(x)$	2	-1	7

$$l_0(x) = \frac{\left(x - \frac{1}{4}\right)(x-1)}{\left(\frac{1}{3} - \frac{1}{4}\right)\left(\frac{1}{3} - 1\right)} = -18\left(x - \frac{1}{4}\right)(x-1)$$

$$l_1(x) = \frac{\left(x - \frac{1}{3}\right)(x-1)}{\left(\frac{1}{4} - \frac{1}{3}\right)\left(\frac{1}{4} - 1\right)} = 16\left(x - \frac{1}{3}\right)(x-1)$$

$$l_0(x) = \frac{\left(x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)}{\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)} = 2\left(x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)$$

$$p_2(x) = -36\left(x - \frac{1}{4}\right)(x-1) - 16\left(x - \frac{1}{3}\right)(x-1) + 14\left(x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)$$