1. $A = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}$

$$(4-\lambda)(5-\lambda) - 30 = 0$$
$$\lambda^2 - 9\lambda - 10 = 0$$
$$(\lambda - 10)(\lambda + 1) = 0$$
$$\lambda = 10, \ \lambda = -1$$

$$x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 11 \end{bmatrix} = 11\begin{bmatrix} 0.8181 \\ 1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 11 \end{bmatrix} = \begin{bmatrix} 91 \\ 109 \end{bmatrix} = 109\begin{bmatrix} 0.8349 \\ 1 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 91 \\ 109 \end{bmatrix} = \begin{bmatrix} 909 \\ 1091 \end{bmatrix} = 1091\begin{bmatrix} 0.8332 \\ 1 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 909 \\ 1091 \end{bmatrix} = \begin{bmatrix} 9091 \\ 10909 \end{bmatrix} = 10909\begin{bmatrix} 0.8333 \\ 1 \end{bmatrix}$$

4 iterations

$$\lambda = \frac{\lambda x \cdot x}{x \cdot x} = \frac{Ax \cdot x}{x \cdot x}$$

$$= \frac{\left(\begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 0.8333 \\ 1 \end{bmatrix}\right)^T \begin{bmatrix} 0.8333 \\ 1 \end{bmatrix}}{\begin{bmatrix} 0.8333 \\ 1 \end{bmatrix}^T \begin{bmatrix} 0.8333 \\ 1 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 8.3332 \\ 9.9998 \end{bmatrix}^T \begin{bmatrix} 0.8333 \\ 1 \end{bmatrix}}{1.6944}$$

$$= \frac{16.944}{1.6944}$$

$$= 10$$

The dominant eigenvalue is 10.

2.

x	1	2	3	4
y	0	1	1	2

$y = Cx + D$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \qquad x = \begin{bmatrix} C \\ D \end{bmatrix}$$

$$Ax = b$$

$$A^T A x = A^T b$$

$$A^T A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 10 \\ 10 & 4 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 10 \\ 10 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \end{bmatrix}$$

$$30C + 10D = 13$$
$$10C + 4D = 4$$

$$-2D = 1$$
$$D = -\frac{1}{2}$$

$$10C + (-2) = 4$$
$$C = \frac{3}{5}$$

$$y = \frac{3}{5}x - \frac{1}{2}$$

3.

x	-1	0	1
y	3.1	0.9	2.9

$y = ax^2 + b$

$e = (ax_k^2 + b - y_k)^2$

$\frac{\partial e}{\partial a} = 2(ax_k^2 + b - y_k)(x_k^2)$

$= 2ax_k^4 + 2bx_k^2 - 2x_k^2 y_k$

$\frac{\partial e}{\partial b} = 2(ax_k^2 + b - y_k)(1)$

$= 2ax_k^2 + 2b - 2y_k$

$\sum_{k=1}^{3} ax_k^4 + bx_k^2 - x_k^2 y_k = 0$

$\sum_{k=1}^{3} ax_k^2 + b - y_k = 0$

x	-1	0	1
y	3.1	0.9	2.9

$a((-1)^4 + 0^4 + 1^4) + b((-1)^2 + 0^2 + 1^2) - ((-1)^2 \cdot 3.1 + 0 + 1^2 \cdot 2.9) = 0$

$a((-1)^2 + 0^2 + 1^2) + b(1 + 1 + 1) - (3.1 + 0.9 + 2.9) = 0$

$2a + 2b - 6 = 0$

$a + b = 3$

$2a + 3b - 6.9 = 0$

$2a + 3b = 6.9$

$b = 0.9$

$a = 2.1$

$y = 2.1x^2 + 0.9$