Flype conjecture

Christopher-Lloyd Simon and Ben Stucky

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Contents

1 Introduction

This document presents exhaustive data about the pinning sets of certain unoriented multiloops in the unoriented sphere - flype conjecture. The reader should refer to the authors' forthcoming paper [SS24] for an explanation of the terminology used throughout this catalog, and for a detailed description and analysis of the algorithms used to compute all the information displayed here.

Section 3 presents statistics for the dataset at a glance. We focus in particular on computing statistics related to degrees of regions appearing in pinning sets. The data is presented in tabular form by number of regions and by pinning number in subsections 3.1 and 3.2, and in graphical form in subsections 3.3 and 3.4. Note that the number of multiloops by region matches [OEI24, unknown].

Section 4 contains detailed visualizations of the pinning sets of every multiloop in the database with at most None regions, and tables describing some of their individual statistics, with emphasis on degree. For each multiloop, optimal pinning sets are labelled with capital letters and colored using shades of red, and the other minimal pinning sets are labelled with lowercase letters and colored using shades of green. For better visibility, we do not plot the entire pinning semi-lattice; rather, the sub-semi-lattice generated by (taking unions of) minimal pinning sets, together with the set of all regions. The heights of vertices in the semi-lattice (and the labels therein) correspond to their cardinals. A lighter edge emphasises a greater difference between its endpoint's cardinals.

The multiloops in this catalog were generated with `plantri` [BM] and drawn with `SnapPy` [CDGW]. For each multiloop, we include its `plantri` code and planar diagram (PD) code in case the reader wishes to study it using either program.

2 References

[BM] Gunnar Brinkmann and Brendan McKay. Plantri and fullgen, programs for generating planar graphs of specified types. Available at `https://users.cecs.anu.edu.au/~bdm/plantri/`(08/04/2024).

[CDGW] Marc Culler, Nathan M. Dunfield, Matthias Goerner, and Jeffrey R. Weeks. SnapPy, a computer program for studying the geometry and topology of 3-manifolds. Available at `http://snappy.computop.org` (08/04/2024).

[OEI24] OEIS Foundation Inc. The On-Line Encyclopedia of Integer Sequences, 2024. Published electronically at `http://oeis.org`.

[SS24] Christopher-Lloyd Simon and Ben Stucky. Pin the loop taut : a one-player topologame, 2024. Submitted for publication, arxiv version.

3 Statistical overview

3.1 By number of regions - tabular data

Table 1: Statistical overview by number of regions (decimals shown to at most 6 significant figures).

Number of regions	Number of multiloops with this number of regions	Average pinning number	Average pinning number/number of regions	Average optimal pinning set degree	Average minimal pinning set degree	Average overall pinning set degree
5	2	3.5	0.5	2	2.1	2.52851

3.2 By pinning number - tabular data

Table 2: Statistical overview by pinning number (decimals shown to at most 6 significant figures).

Pinning number	Number of multiloops with this pinning number	Average number of regions	Average pinning number/number of regions	Average optimal pinning set degree	Average minimal pinning set degree	Average overall pinning set degree
3	1	5	0.428571	2	2.2	2.56614
4	1	5	0.571429	2	2	2.49087

3.3 By number of regions - graphical data

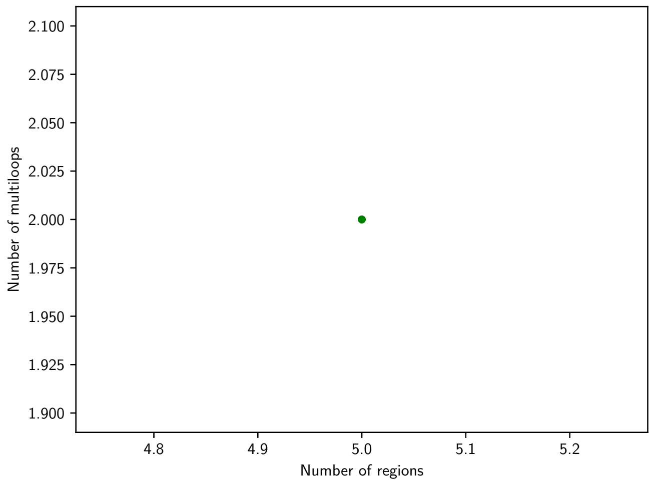

Figure 1: Number of multiloops by number of regions.

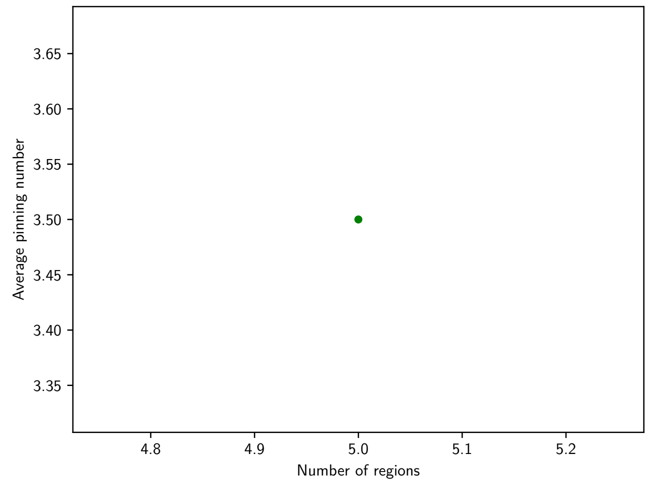

Figure 2: Average pinning number by number of regions.

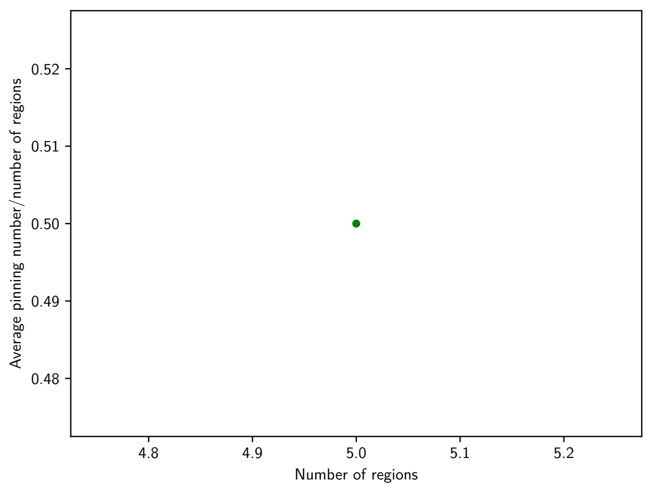

Figure 3: Average pinning number/number of regions by number of regions.

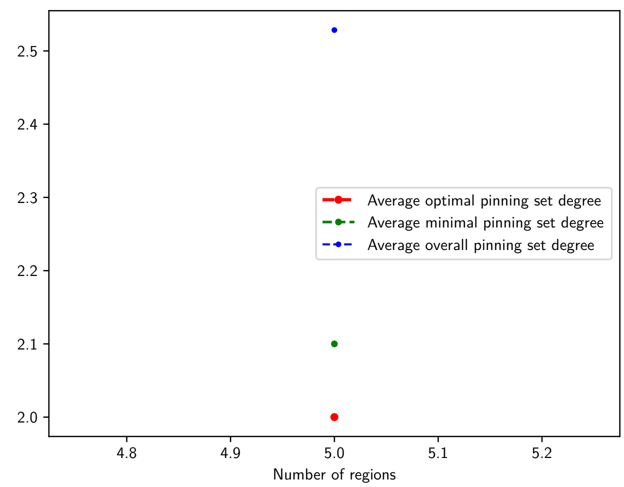

Figure 4: Average pinning set degree data by number of regions.

3.4 By pinning number - graphical data

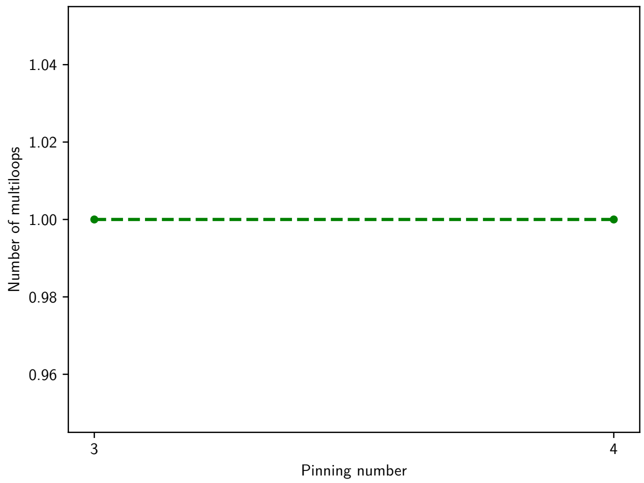

Figure 5: Number of multiloops by pinning number.

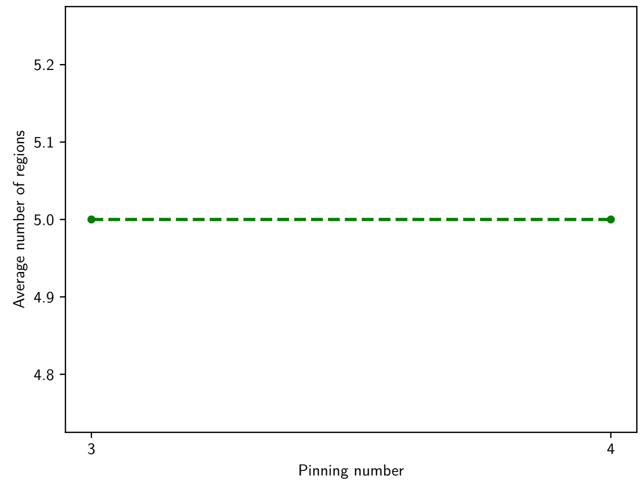

Figure 6: Average number of regions by pinning number.

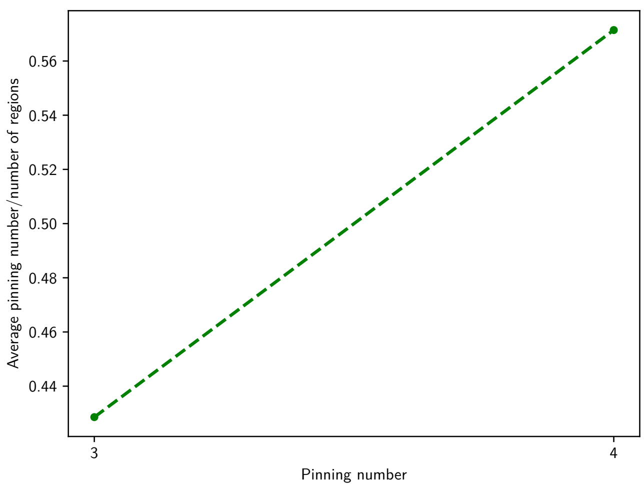

Figure 7: Average pinning number/number of regions by pinning number.

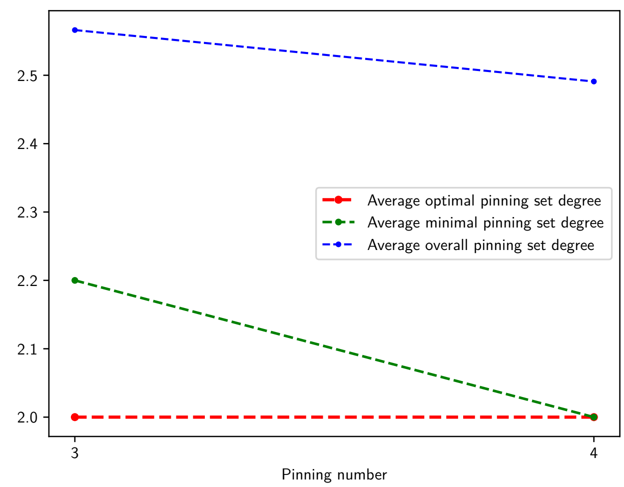

Figure 8: Average pinning set degree data by pinning number.

4 Spherimultiloops

4.1 5 regions

4.1.1 [(1, 10, 2, 1), (5, 2, 6, 3), (3, 9, 4, 8), (7, 5, 8, 4), (9, 6, 10, 7)]

PD code drawn by `SnapPy`: $[(10,3,1,4),(5,2,6,3),(4,9,5,10),(7,6,8,7),(1,8,2,9)]$
Planar representation generated by `plantri`: –

Total optimal pinning sets: 1
Total minimal pinning sets: 2
Total pinning sets: 18
Pinning number: 3

Average optimal degree: 2.0
Average minimal degree: 2.2
Average overall degree: 2.57

Table 3: Pinning sets/average degree by cardinal

Cardinal	3	4	5	6	7	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	1	0	0	1
Nonminimal pinning sets	0	4	6	5	1	16
Average degree	2.0	2.38	2.57	2.77	2.86	

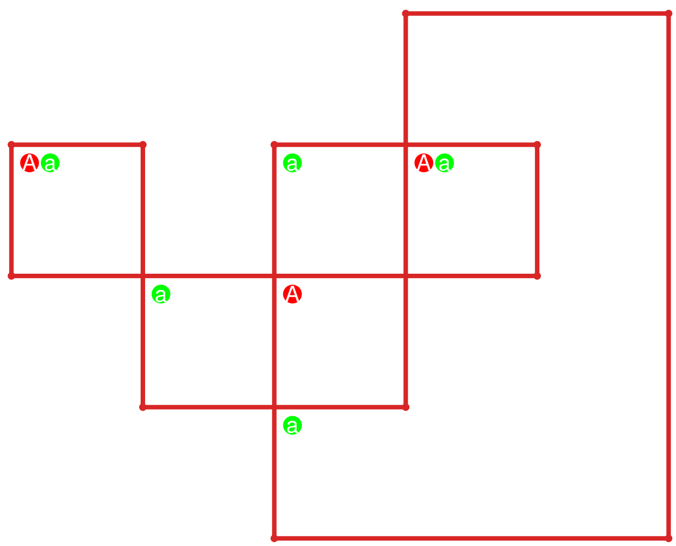

Figure 9: `SnapPy` multiloop plot.

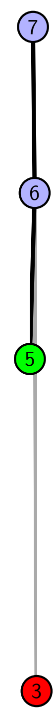

Figure 10: Minimal join sub-semi-lattice of minimal pinning sets.

4.1.2 [(2, 5, 3, 6), (6, 1, 7, 2), (8, 4, 9, 3), (4, 8, 5, 7), (10, 9, 1, 10)]

PD code drawn by `SnapPy`: $[(10,5,1,6),(9,2,10,3),(4,1,5,2),(7,6,8,7),(3,8,4,9)]$
Planar representation generated by `plantri`: $-$

Total optimal pinning sets: 2
Total minimal pinning sets: 2
Total pinning sets: 12
Pinning number: 4

Average optimal degree: 2.0
Average minimal degree: 2.0
Average overall degree: 2.49

Table 4: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	Total
Optimal pinning sets	2	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0
Nonminimal pinning sets	0	5	4	1	10
Average degree	2.0	2.44	2.71	2.86	

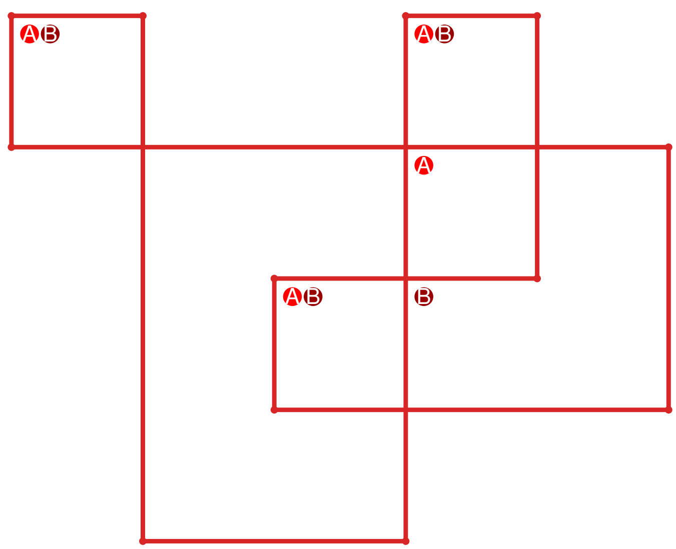

Figure 11: `SnapPy` multiloop plot.

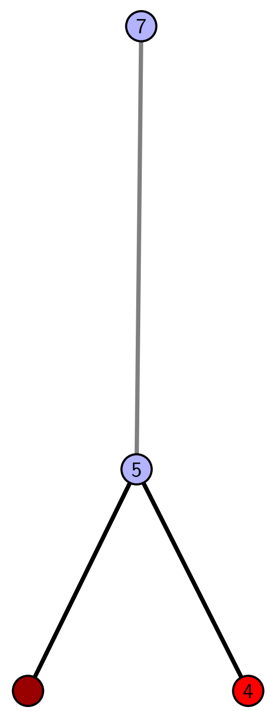

Figure 12: Minimal join sub-semi-lattice of minimal pinning sets.