## 1 Pinning poset changes drastically under Reidemeister III

 $1.0.1 \quad [(1,\,7,\,2,\,6),\,(24,\,7,\,1,\,8),\,(5,\,10,\,6,\,11),\,(4,\,12,\,5,\,11),\,(9,\,15,\,10,\,14),\,(8,\,15,\,9,\,16),\,(13,\,18,\,14,\,19),\,(12,\,20,\,13,\,19),\,(2,\,22,\,3,\,21),\,(3,\,20,\,4,\,21),\,(17,\,23,\,18,\,22),\,(16,\,23,\,17,\,24)]$ 

PD code drawn by SnapPy: [(3, 22, 4, 23), (21, 4, 22, 5), (2, 7, 3, 8), (8, 1, 9, 2), (11, 6, 12, 7), (5, 12, 6, 13), (10, 15, 11, 16), (16, 9, 17, 10), (24, 17, 1, 18), (18, 23, 19, 24), (19, 14, 20, 15), (13, 20, 14, 21)]Planar representation generated by plantri: -

Total optimal pinning sets: 1 Total minimal pinning sets: 1 Total pinning sets: 256

Pinning number: 6

Average minimal degree: 2.0 Average minimal degree: 2.0

Average overall degree: 2.97

Table 1: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	13	14	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.36	2.62	2.83	3.0	3.14	3.25	3.35	3.43	

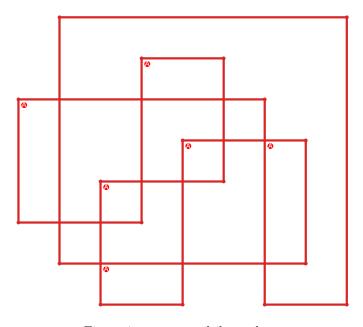


Figure 1: SnapPy multiloop plot.



Figure 2: Minimal join sub-semi-lattice of minimal pinning sets.

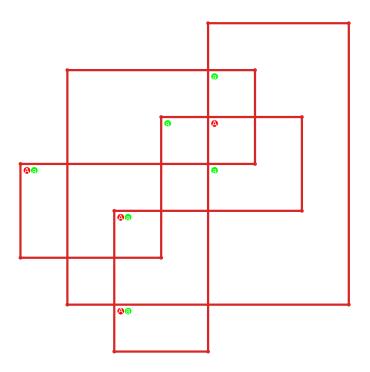
 $1.0.2 \quad [(1,\,7,\,2,\,6),\,(24,\,7,\,1,\,8),\,(5,\,10,\,6,\,11),\,(3,\,13,\,4,\,12),\,(9,\,15,\,10,\,14),\,(8,\,15,\,9,\,16),\,(13,\,18,\,14,\,19),\,(4,\,19,\,5,\,20),\,(2,\,22,\,3,\,21),\,(11,\,21,\,12,\,20),\,(17,\,23,\,18,\,22),\,(16,\,23,\,17,\,24)]$ 

PD code drawn by SnapPy: [(4, 23, 5, 24), (22, 5, 23, 6), (3, 8, 4, 9), (10, 1, 11, 2), (12, 7, 13, 8), (6, 13, 7, 14), (11, 16, 12, 17), (2, 17, 3, 18), (18, 9, 19, 10), (19, 24, 20, 1), (20, 15, 21, 16), (14, 21, 15, 22)] Planar representation generated by plantri: -

Total optimal pinning sets: 1 Total minimal pinning sets: 2 Total pinning sets: 1152 Pinning number: 4 Average optimal degree: 2.25 Average minimal degree: 2.38 Average overall degree: 3.14

Table 2: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	13	14	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	1	0	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	10	45	127	231	287	245	141	52	11	1	1150
Average degree	2.25	2.58	2.79	2.95	3.06	3.16	3.24	3.3	3.36	3.4	3.43	



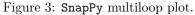




Figure 4: Minimal join sub-semi-lattice of minimal pinning sets.