Test dataset

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Total optimal pinning sets: 1
Total minimal pinning sets: 3
Total pinning sets: 44
Pinning number: 4

Average optimal gonality: 2.25

Average minimal gonality: 2.42

Average overall gonality: 2.82

Table 1: Pinning sets/average gonality by cardinal

Cardinal	4	5	6	7	8	9	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	2
Nonminimal pinning sets	0	5	15	14	6	1	41
Average gonality	2.25	2.54	2.78	2.94	3.04	3.11	

Table 2: Pinning set data

Pinning set	Pindicator	Regions	Card	Gonality seq	Average gonality
A (optimal)	🔴	$\{1, 3, 4, 8\}$	4	[2, 3, 2, 2]	2.25
a (minimal)	🟢	$\{1, 2, 4, 7, 8\}$	5	[2, 3, 2, 3, 2]	2.4
b (minimal)	🟢	$\{1, 2, 4, 6, 8\}$	5	[2, 3, 2, 4, 2]	2.6

Figure 1: Snappy loop plot.

Figure 2: Minimal join semilattice of pinning sets.

2 [(1, 7, 2, 6), (4, 9, 5, 10), (2, 12, 3, 11), (7, 13, 8, 12), (18, 13, 1, 14), (3, 17, 4, 16), (5, 14, 6, 15), (8, 18, 9, 17), (10, 15, 11, 16)]

Total optimal pinning sets: 2
Total minimal pinning sets: 13
Total pinning sets: 395
Pinning number: 4

Average optimal gonality: 3.0

Average minimal gonality: 3.25

Average overall gonality: 3.23

Table 3: Pinning sets/average gonality by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	7	0	0	0	0	0	11
Nonminimal pinning sets	0	14	66	130	111	49	11	1	382
Average gonality	3.0	3.13	3.21	3.23	3.25	3.27	3.27	3.27	

Table 4: Pinning set data

| Pinning set	Pindicator	Regions	Card	Gonality seq	Average gonality
A (optimal)	●	$\{2, 6, 9, 10\}$	4	[3, 3, 3, 3]	3.0
B (optimal)	●	$\{1, 3, 4, 8\}$	4	[3, 3, 3, 3]	3.0
a (minimal)	●	$\{2, 4, 6, 8, 11\}$	5	[3, 3, 3, 3, 4]	3.2
b (minimal)	●	$\{2, 5, 7, 8, 11\}$	5	[3, 4, 4, 3, 4]	3.6
c (minimal)	●	$\{1, 2, 5, 8, 9\}$	5	[3, 3, 4, 3, 3]	3.2
d (minimal)	●	$\{2, 3, 7, 8, 10\}$	5	[3, 3, 4, 3, 3]	3.2
e (minimal)	●	$\{2, 6, 7, 8, 10, 11\}$	6	[3, 3, 4, 3, 3, 4]	3.33
f (minimal)	●	$\{2, 5, 6, 8, 9, 11\}$	6	[3, 4, 3, 3, 3, 4]	3.33
g (minimal)	●	$\{2, 5, 7, 8, 9, 10\}$	6	[3, 4, 4, 3, 3, 3]	3.33
h (minimal)	●	$\{1, 2, 4, 5, 8, 11\}$	6	[3, 3, 3, 4, 3, 4]	3.33
i (minimal)	●	$\{1, 3, 4, 6, 9, 10\}$	6	[3, 3, 3, 3, 3, 3]	3.0
j (minimal)	●	$\{2, 3, 4, 7, 8, 11\}$	6	[3, 3, 3, 4, 3, 4]	3.33
k (minimal)	●	$\{1, 2, 3, 5, 7, 8\}$	6	[3, 3, 3, 4, 4, 3]	3.33
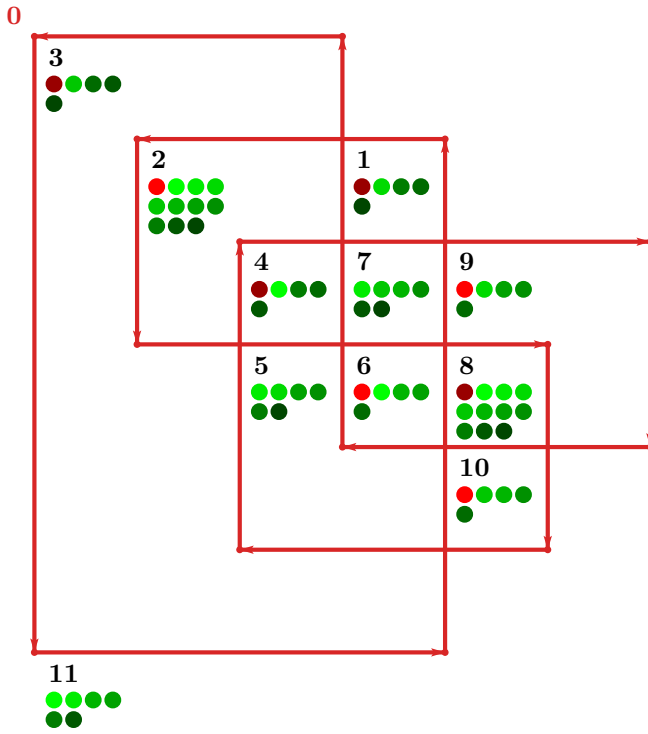
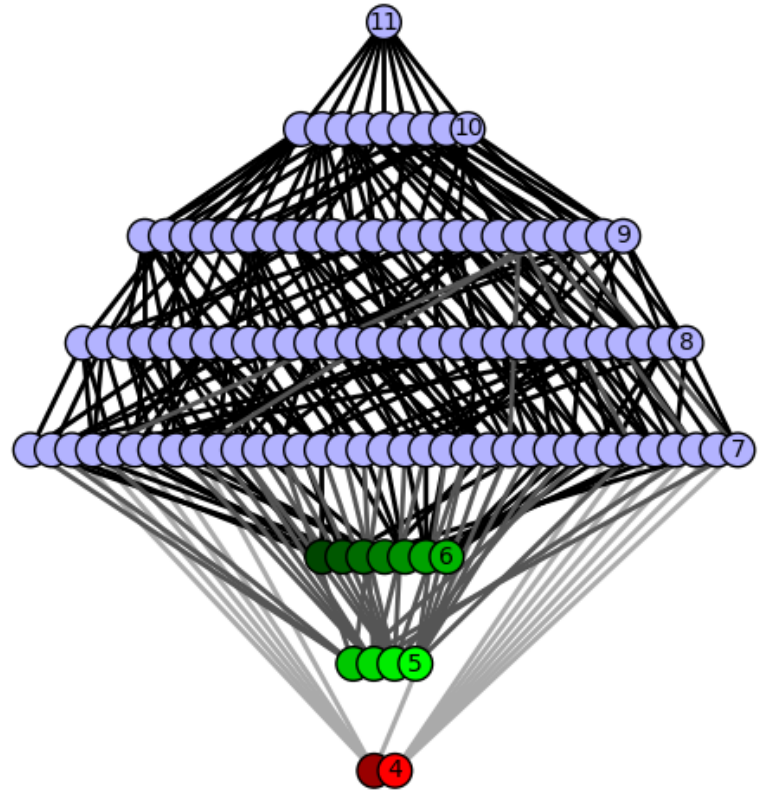Figure 3: Snappy loop plot.

Figure 4: Minimal join semilattice of pinning sets.

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3   [(1, 7, 2, 6), (3, 8, 4, 9), (5, 11, 6, 10), (16, 12, 1, 11), (2, 13, 3, 14), (4, 16, 5, 15), (7, 12, 8, 13), (9, 15, 10, 14)]

Total optimal pinning sets: 10
Total minimal pinning sets: 10
Total pinning sets: 160
Pinning number: 5

Average optimal gonality: 3.04

Average minimal gonality: 3.04

Average overall gonality: 3.15

Table 5: Pinning sets/average gonality by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	10	0	0	0	0	0	10
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	42	60	37	10	1	150
Average gonality	3.04	3.11	3.16	3.19	3.2	3.2	

Table 6: Pinning set data

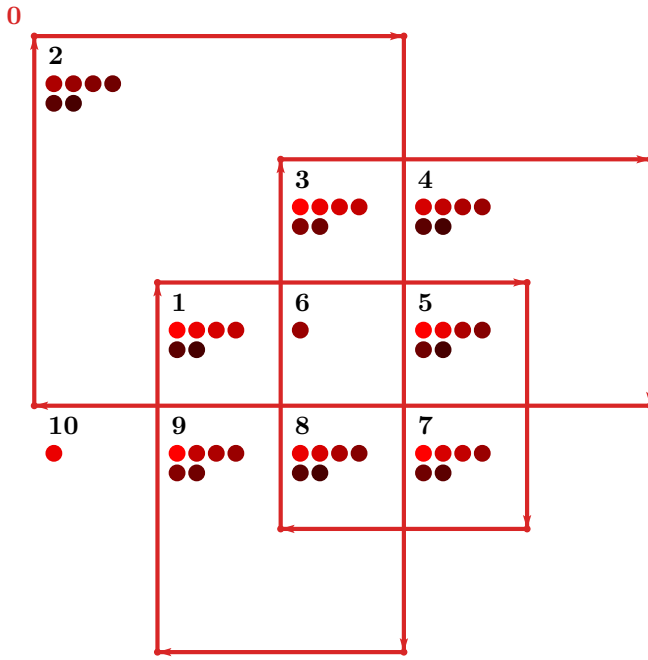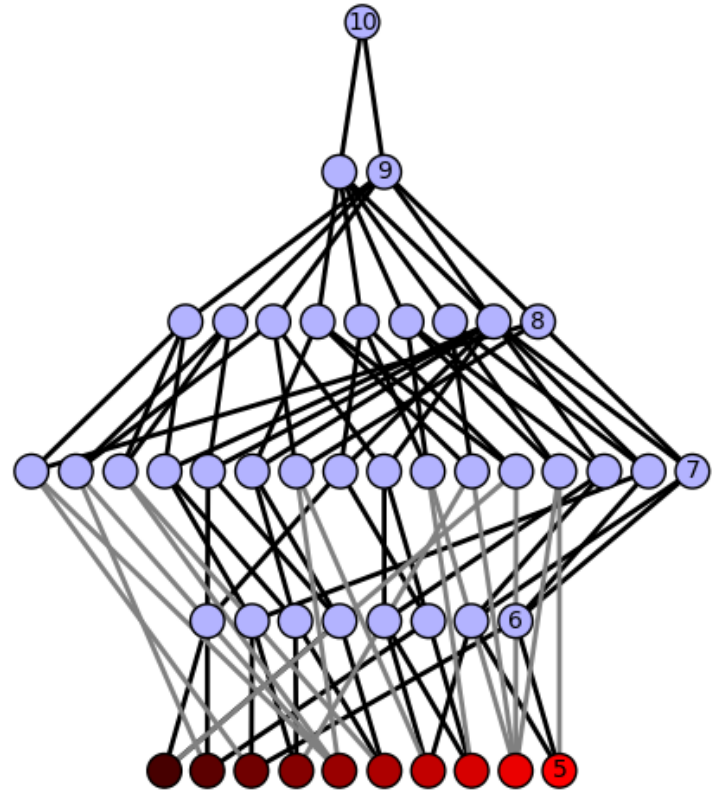| Pinning set	Pindicator	Regions	Card	Gonality seq	Average gonality
A (optimal)	●	$\{1, 3, 5, 7, 9\}$	5	$[3, 3, 3, 3, 3]$	3.0
B (optimal)	●	$\{1, 3, 5, 8, 10\}$	5	$[3, 3, 3, 3, 4]$	3.2
C (optimal)	●	$\{1, 3, 4, 7, 8\}$	5	$[3, 3, 3, 3, 3]$	3.0
D (optimal)	●	$\{1, 3, 4, 7, 9\}$	5	$[3, 3, 3, 3, 3]$	3.0
E (optimal)	●	$\{2, 4, 5, 8, 9\}$	5	$[3, 3, 3, 3, 3]$	3.0
F (optimal)	●	$\{2, 4, 6, 7, 9\}$	5	$[3, 3, 4, 3, 3]$	3.2
G (optimal)	●	$\{2, 3, 5, 8, 9\}$	5	$[3, 3, 3, 3, 3]$	3.0
H (optimal)	●	$\{2, 3, 5, 7, 9\}$	5	$[3, 3, 3, 3, 3]$	3.0
I (optimal)	●	$\{1, 2, 4, 7, 8\}$	5	$[3, 3, 3, 3, 3]$	3.0
J (optimal)	●	$\{1, 2, 4, 5, 8\}$	5	$[3, 3, 3, 3, 3]$	3.0

Figure 5: Snappy loop plot.

Figure 6: Minimal join semilattice of pinning sets.