

# Pinning set data for all irreducible, indecomposable spherimultiloops with one component and at most 14 regions

Christopher-Lloyd Simon and Ben Stucky

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# 1 Introduction

This document presents exhaustive data about the pinning sets of certain unoriented multiloops in the unoriented sphere - pinning set data for all irreducible, indecomposable spherimultiloops with one component and at most 14 regions. The reader should refer to the authors' forthcoming paper [SS24] for an explanation of the terminology used throughout this catalog, and for a detailed description and analysis of the algorithms used to compute all the information displayed here.

Section 3 presents statistics for the dataset at a glance. We focus in particular on computing statistics related to degrees of regions appearing in pinning sets. The data is presented in tabular form by number of regions and by pinning number in subsections 3.1 and 3.2, and in graphical form in subsections 3.3 and 3.4. Note that the number of multiloops by region matches [OEI24, A264759].

Section 4 contains detailed visualizations of the pinning sets of every multiloop in the database with at most 12 regions, and tables describing some of their individual statistics, with emphasis on degree. For each multiloop, optimal pinning sets are labelled with capital letters and colored using shades of red, and the other minimal pinning sets are labelled with lowercase letters and colored using shades of green. For better visibility, we do not plot the entire pinning semi-lattice; rather, the sub-semi-lattice generated by (taking unions of) minimal pinning sets, together with the set of all regions. The heights of vertices in the semi-lattice (and the labels therein) correspond to their cardinals. A lighter edge emphasises a greater difference between its endpoint's cardinals.

The multiloops in this catalog were generated with `plantri` [BM] and drawn with `SnapPy` [CDGW]. For each multiloop, we include its `plantri` code and planar diagram (PD) code in case the reader wishes to study it using either program.

# 2 References

- [BM] Gunnar Brinkmann and Brendan McKay. Plantri and fullgen, programs for generating planar graphs of specified types. Available at <https://users.cecs.anu.edu.au/~bdm/plantri/> (08/04/2024).
- [CDGW] Marc Culler, Nathan M. Dunfield, Matthias Goerner, and Jeffrey R. Weeks. SnapPy, a computer program for studying the geometry and topology of 3-manifolds. Available at <http://snappy.computop.org> (08/04/2024).
- [OEI24] OEIS Foundation Inc. The On-Line Encyclopedia of Integer Sequences, 2024. Published electronically at <http://oeis.org>.
- [SS24] Christopher-Lloyd Simon and Ben Stucky. Pin the loop taut : a one-player topologame, 2024. Submitted for publication, arxiv version.

### 3 Statistical overview

#### 3.1 By number of regions - tabular data

Table 1: Statistical overview by number of regions (decimals shown to at most 6 significant figures).

Number of regions	Number of multiloops with this number of regions	Average pinning number	Average pinning number/number of regions	Average optimal pinning degree	Average minimal pinning degree	Average overall pinning set degree
5	1	3	0.6	2	2	2.225
6	1	4	0.666667	2.5	2.5	2.58095
7	2	4	0.571429	2	2	2.51205
8	3	4	0.5	2.25	2.275	2.71749
9	10	4.6	0.511111	2.23083	2.264	2.77804
10	27	4.62963	0.462963	2.28435	2.31396	2.87324
11	101	4.84158	0.440144	2.25362	2.29853	2.92965
12	364	5.07692	0.423077	2.24946	2.28814	2.98046
13	1610	5.3646	0.412661	2.23473	2.27936	3.02053
14	7202	5.65579	0.403985	2.23568	2.27817	3.05871

### 3.2 By pinning number - tabular data

Table 2: Statistical overview by pinning number (decimals shown to at most 6 significant figures).

Pinning number	Number of multiloops with this pinning number	Average number of regions	Average pinning number/number of regions	Average optimal pinning set degree	Average minimal pinning set degree	Average overall pinning set degree
3	18	11.9444	0.270036	2.01852	2.0321	2.99844
4	864	13.3414	0.302154	2.26378	2.35916	3.10884
5	3760	13.638	0.367719	2.27128	2.32555	3.07855
6	3403	13.799	0.435414	2.22464	2.25229	3.02908
7	1071	13.8459	0.506154	2.16135	2.17592	2.96864
8	171	13.8596	0.577726	2.09369	2.1006	2.90536
9	29	13.7586	0.655724	2.04598	2.04751	2.84439
10	4	14	0.714286	2.025	2.03523	2.81485
11	1	13	0.846154	2	2	2.72115

### 3.3 By number of regions - graphical data

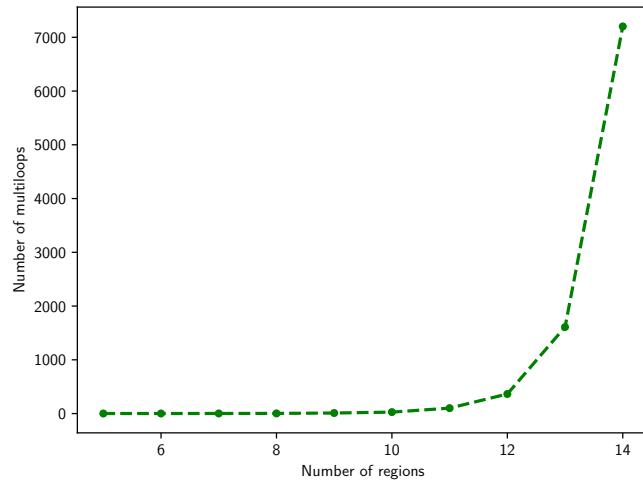


Figure 1: Number of multiloops by number of regions.

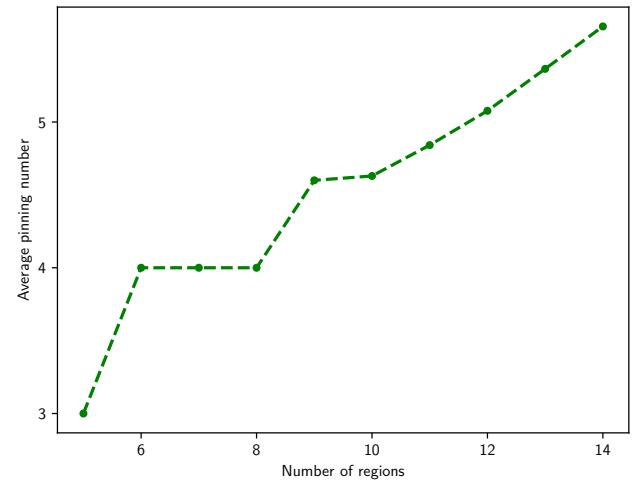


Figure 2: Average pinning number by number of regions.

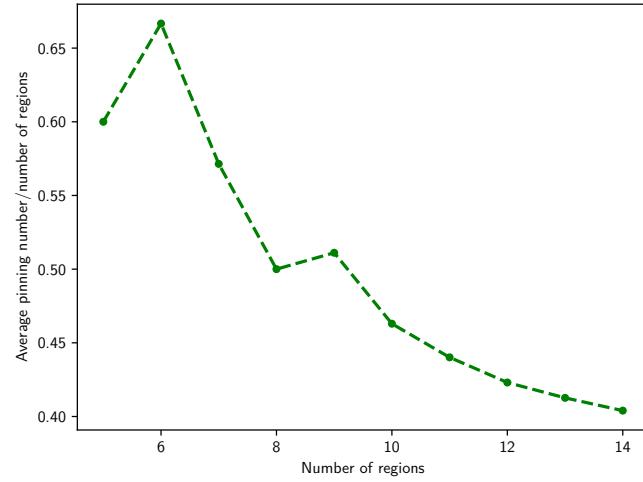


Figure 3: Average pinning number/number of regions by number of regions.

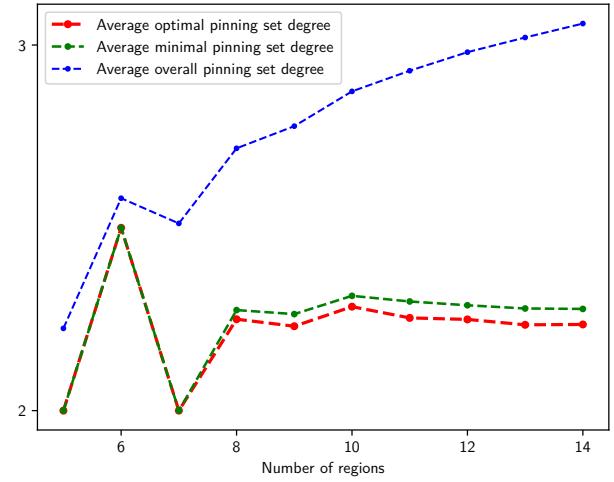


Figure 4: Average pinning set degree data by number of regions.

### 3.4 By pinning number - graphical data

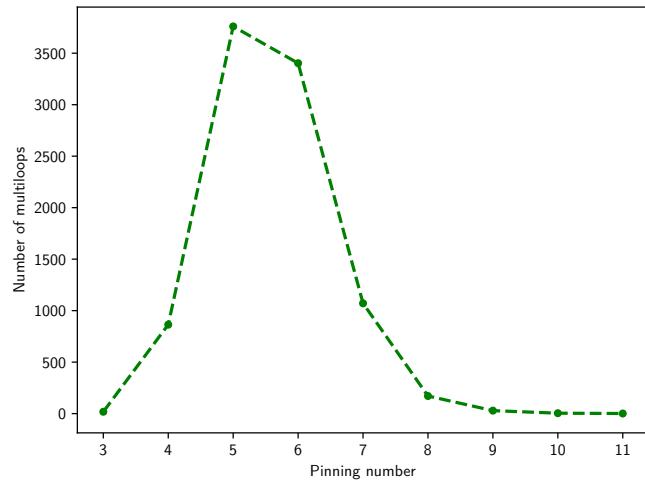


Figure 5: Number of multiloops by pinning number.

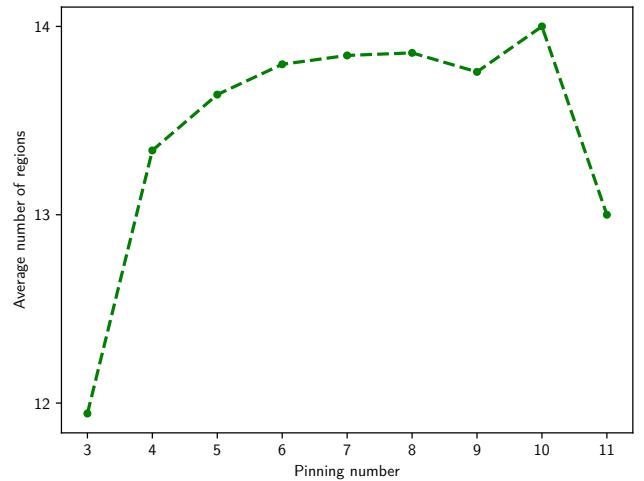


Figure 6: Average number of regions by pinning number.

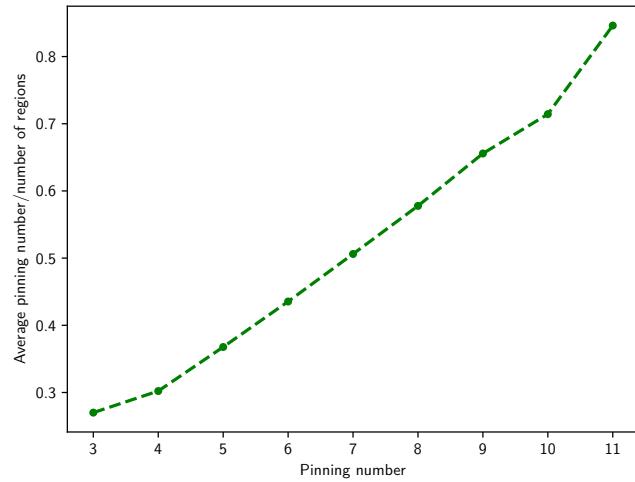


Figure 7: Average pinning number/number of regions by pinning number.

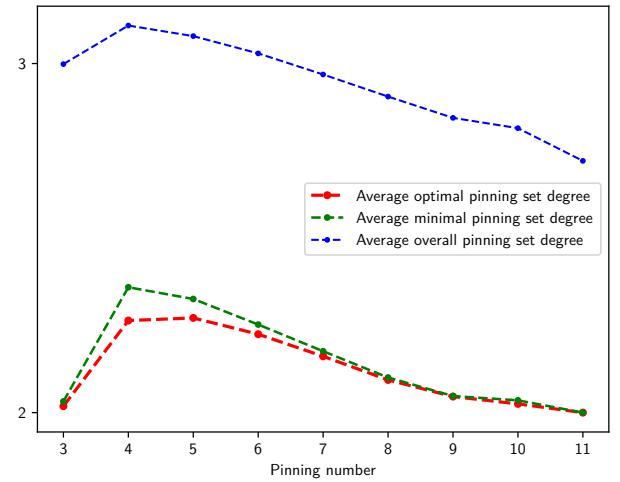


Figure 8: Average pinning set degree data by pinning number.

## 4 Spherimultiloops

### 4.1 5 regions

4.1.1  $[[3, 6, 4, 1], [5, 2, 6, 3], [4, 2, 5, 1]]$

PD code drawn by SnapPy:  $[(6, 3, 1, 4), (4, 1, 5, 2), (2, 5, 3, 6)]$

Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 2, 2, 0], [0, 1, 1, 0]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 4

**Average overall degree:** 2.23

Table 3: Pinning sets/average degree by cardinal

Cardinal	3	4	5	Total
Optimal pinning sets	1	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0
Nonminimal pinning sets	0	2	1	3
Average degree	2.0	2.25	2.4	

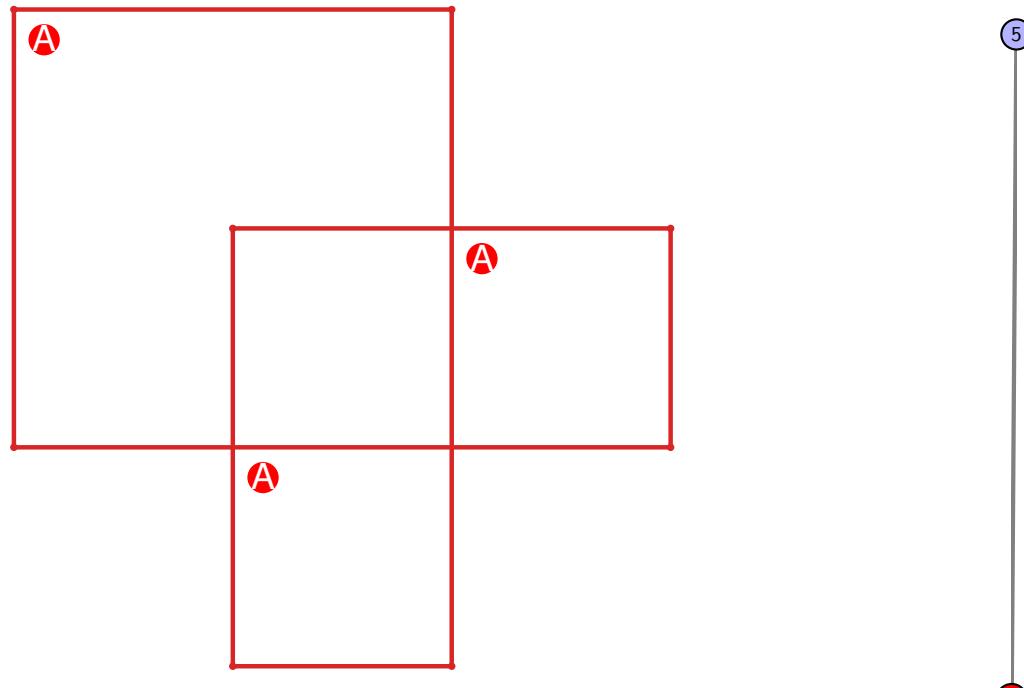


Figure 9: SnapPy multiloop plot.

Figure 10: Minimal join sub-semi-lattice of minimal pinning sets.

## 4.2 6 regions

4.2.1  $[[8, 3, 1, 4], [4, 7, 5, 8], [5, 2, 6, 3], [1, 6, 2, 7]]$

PD code drawn by SnapPy:  $[(3, 8, 4, 1), (6, 1, 7, 2), (7, 4, 8, 5), (2, 5, 3, 6)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 3, 3], [0, 2, 2, 1]]$

Total optimal pinning sets: 2

Average optimal degree: 2.5

Total minimal pinning sets: 2

Average minimal degree: 2.5

Total pinning sets: 7

Average overall degree: 2.58

Pinning number: 4

Table 4: Pinning sets/average degree by cardinal

Cardinal	4	5	6	Total
Optimal pinning sets	2	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0
Nonminimal pinning sets	0	4	1	5
Average degree	2.5	2.6	2.67	

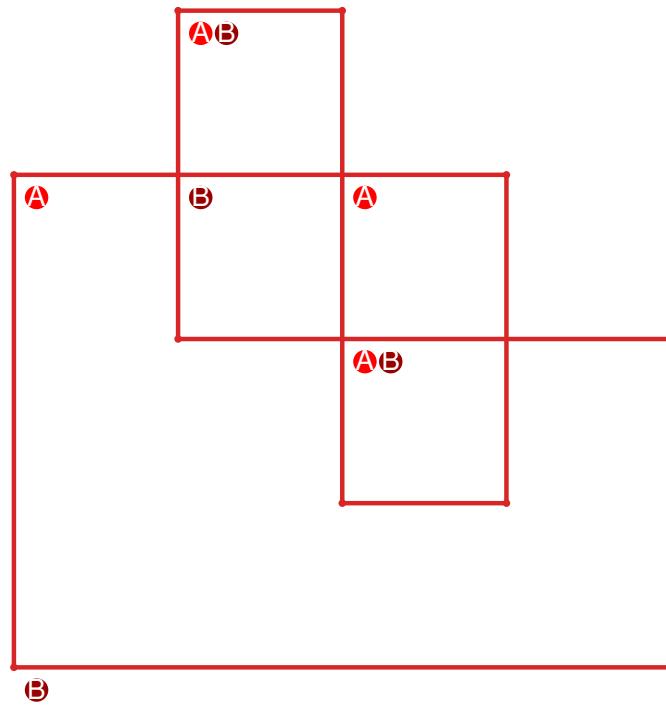


Figure 11: SnapPy multiloop plot.

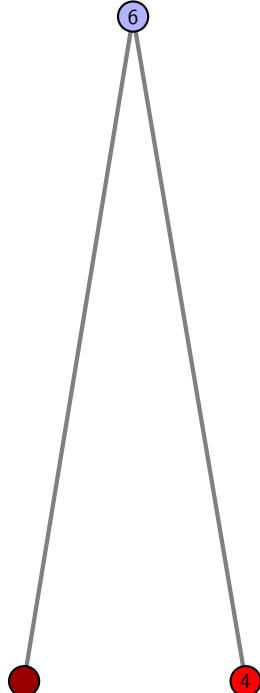


Figure 12: Minimal join sub-semi-lattice of minimal pinning sets.

### 4.3 7 regions

4.3.1  $[[5, 10, 6, 1], [9, 4, 10, 5], [6, 2, 7, 1], [3, 8, 4, 9], [2, 8, 3, 7]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (4, 9, 5, 10), (10, 5, 1, 6), (2, 7, 3, 8), (8, 3, 9, 4)]$

Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 4, 4, 1], [2, 3, 3, 2]]$

Total optimal pinning sets: 1

Average optimal degree: 2.0

Total minimal pinning sets: 1

Average minimal degree: 2.0

Total pinning sets: 4

Average overall degree: 2.46

Pinning number: 5

Table 5: Pinning sets/average degree by cardinal

Cardinal	5	6	7	Total
Optimal pinning sets	1	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0
Nonminimal pinning sets	0	2	1	3
Average degree	2.0	2.5	2.86	

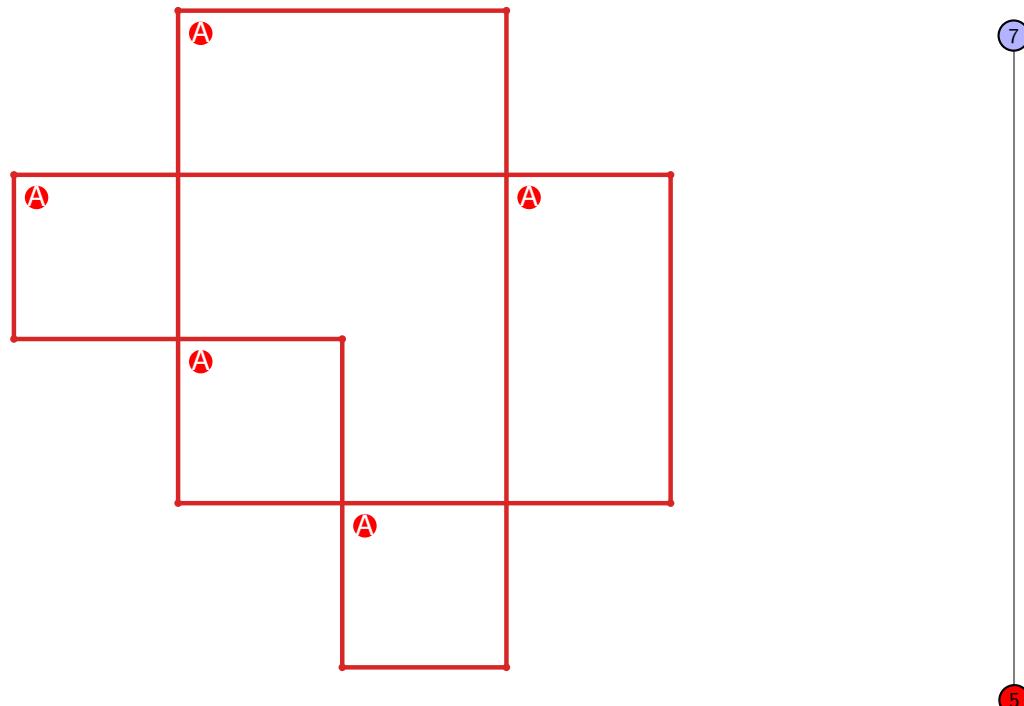


Figure 13: SnapPy multiloop plot.

Figure 14: Minimal join sub-semi-lattice of minimal pinning sets.

#### 4.3.2 [[5, 10, 6, 1], [9, 4, 10, 5], [6, 4, 7, 3], [1, 8, 2, 9], [7, 2, 8, 3]]

PD code drawn by `SnapPy`: [(10, 3, 1, 4), (6, 1, 7, 2), (8, 5, 9, 6), (2, 7, 3, 8), (4, 9, 5, 10)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 4, 4, 1], [2, 3, 3, 2]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.56

**Pinning number:** 3

Table 6: Pinning sets/average degree by cardinal

Cardinal	3	4	5	6	7	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.38	2.6	2.75	2.86	

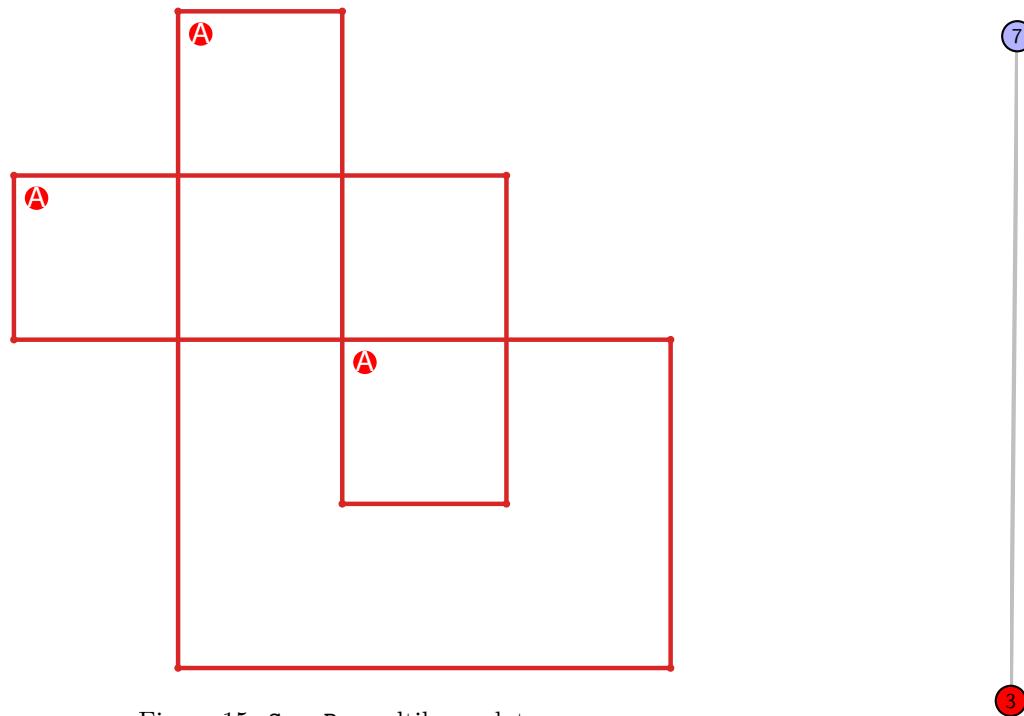


Figure 15: `SnapPy` multiloop plot.

Figure 16: Minimal join sub-semi-lattice of minimal pinning sets.

## 4.4 8 regions

4.4.1  $[[12, 5, 1, 6], [6, 11, 7, 12], [7, 4, 8, 5], [1, 10, 2, 11], [3, 8, 4, 9], [9, 2, 10, 3]]$

PD code drawn by SnapPy:  $[(4, 1, 5, 2), (9, 2, 10, 3), (12, 5, 1, 6), (10, 7, 11, 8), (3, 8, 4, 9), (6, 11, 7, 12)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 5, 1], [2, 5, 5, 2], [3, 4, 4, 3]]$

Total optimal pinning sets: 1

Average optimal degree: 2.0

Total minimal pinning sets: 1

Average minimal degree: 2.0

Total pinning sets: 16

Average overall degree: 2.63

Pinning number: 4

Table 7: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.4	2.67	2.86	3.0	

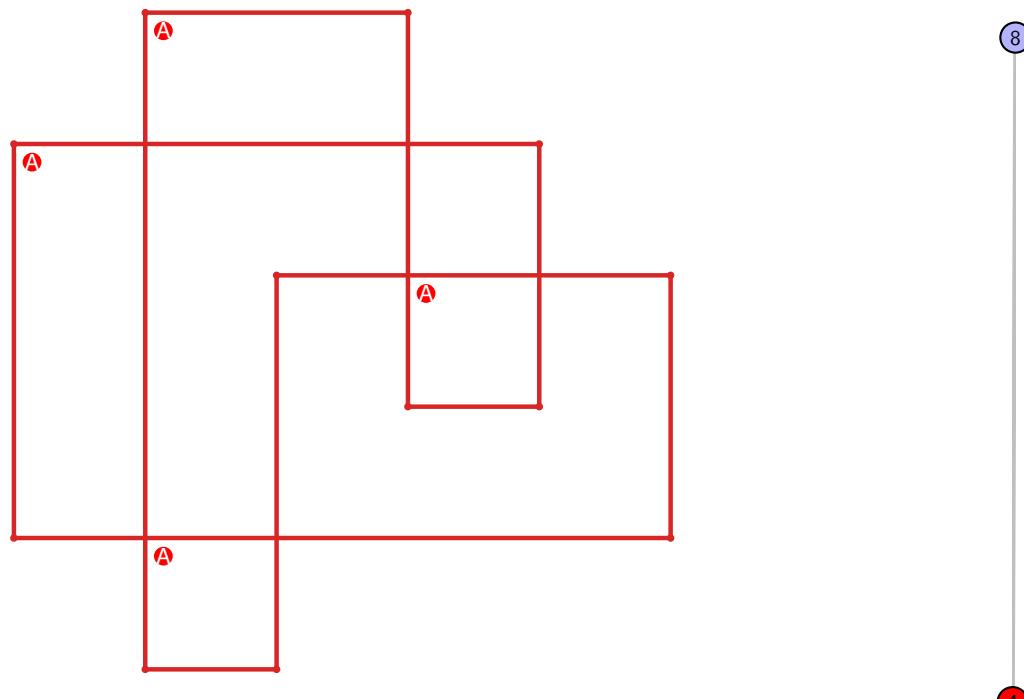


Figure 17: SnapPy multiloop plot.

Figure 18: Minimal join sub-semi-lattice of minimal pinning sets.

4.4.2 [[12, 7, 1, 8], [8, 4, 9, 3], [11, 2, 12, 3], [6, 1, 7, 2], [4, 10, 5, 9], [5, 10, 6, 11]]

PD code drawn by SnapPy: [(9, 12, 10, 1), (1, 6, 2, 7), (10, 5, 11, 6), (7, 2, 8, 3), (3, 8, 4, 9), (4, 11, 5, 12)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 3], [0, 2, 5, 0], [1, 5, 5, 1], [2, 4, 4, 3]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 20

**Average overall degree:** 2.72

**Pinning number:** 4

Table 8: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	1
Nonminimal pinning sets	0	4	8	5	1	18
Average degree	2.25	2.52	2.75	2.91	3.0	

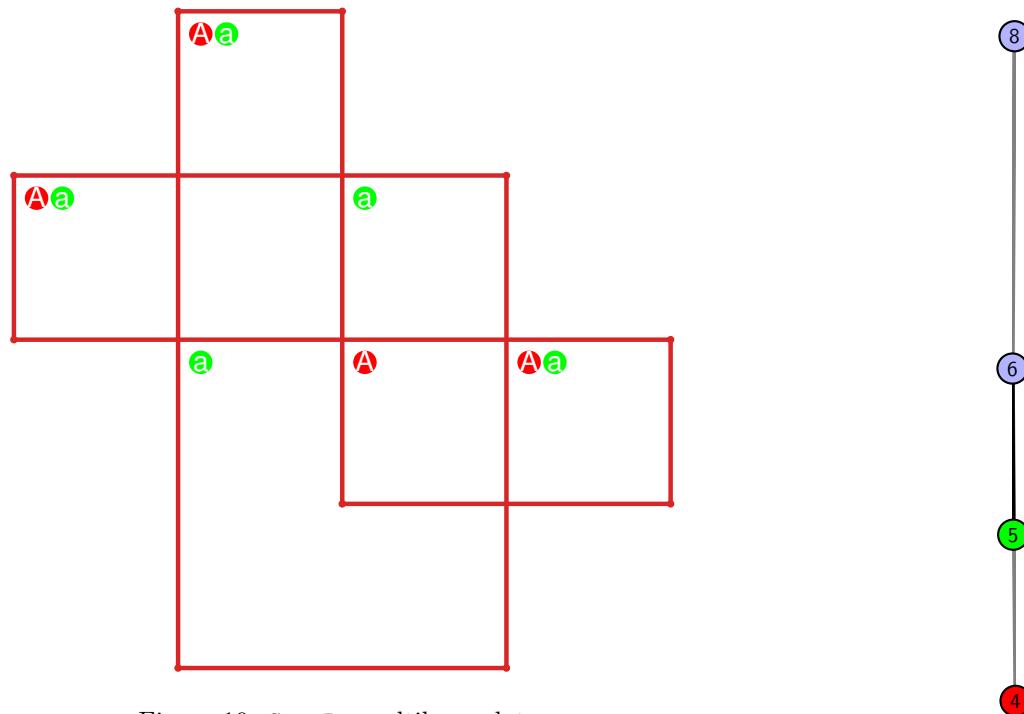


Figure 19: SnapPy multiloop plot.

Figure 20: Minimal join sub-semi-lattice of minimal pinning sets.

4.4.3 [[12, 3, 1, 4], [4, 10, 5, 9], [11, 8, 12, 9], [2, 7, 3, 8], [1, 7, 2, 6], [10, 6, 11, 5]]

PD code drawn by SnapPy: [(10, 1, 11, 2), (3, 8, 4, 9), (4, 11, 5, 12), (12, 5, 1, 6), (9, 6, 10, 7), (7, 2, 8, 3)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 4, 4], [0, 3, 3, 5], [1, 4, 2, 1]]

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.5

**Total pinning sets:** 32

**Average overall degree:** 2.8

**Pinning number:** 4

Table 9: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	Total
Optimal pinning sets	3	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	10	12	6	1	29
Average degree	2.5	2.72	2.86	2.95	3.0	

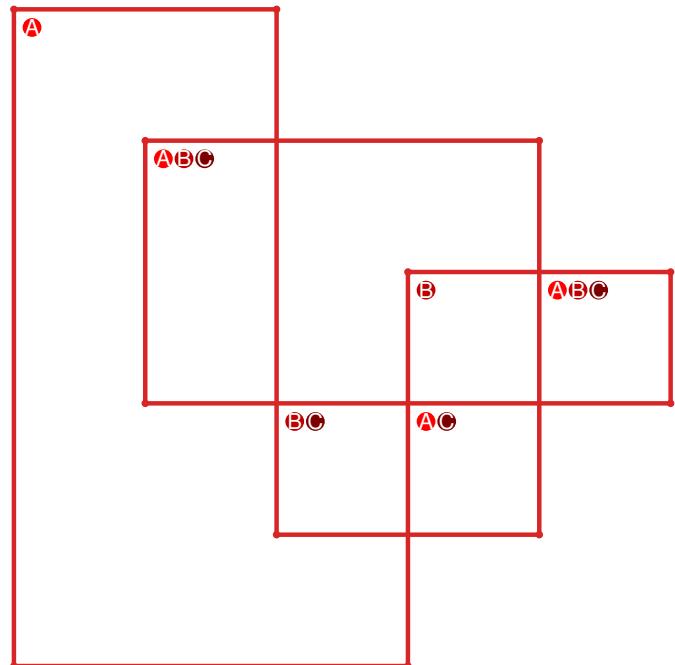


Figure 21: SnapPy multiloop plot.

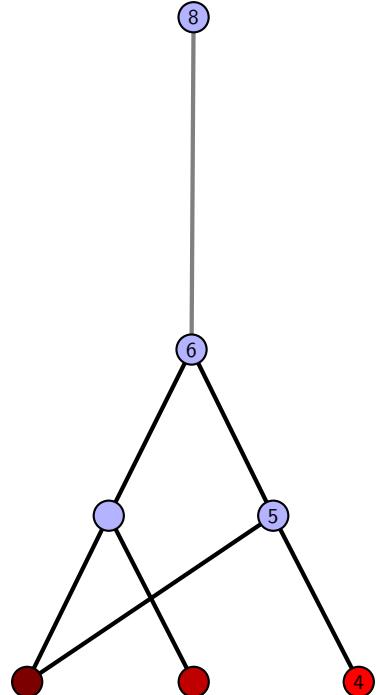


Figure 22: Minimal join sub-semi-lattice of minimal pinning sets.

## 4.5 9 regions

4.5.1  $[[7, 14, 8, 1], [13, 6, 14, 7], [8, 2, 9, 1], [5, 12, 6, 13], [2, 10, 3, 9], [11, 4, 12, 5], [10, 4, 11, 3]]$

PD code drawn by SnapPy:  $[(14, 7, 1, 8), (8, 1, 9, 2), (10, 3, 11, 4), (6, 13, 7, 14), (2, 9, 3, 10), (4, 11, 5, 12), (12, 5, 13, 6)]$   
 Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 5, 1], [2, 6, 6, 2], [3, 6, 6, 3], [4, 5, 5, 4]]$

Total optimal pinning sets: 1

Average optimal degree: 2.0

Total minimal pinning sets: 1

Average minimal degree: 2.0

Total pinning sets: 4

Average overall degree: 2.59

Pinning number: 7

Table 10: Pinning sets/average degree by cardinal

Cardinal	7	8	9	Total
Optimal pinning sets	1	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0
Nonminimal pinning sets	0	2	1	3
Average degree	2.0	2.62	3.11	

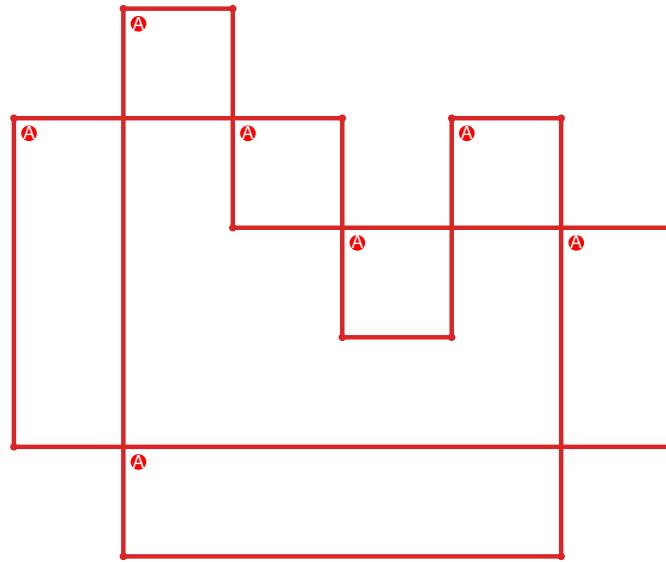


Figure 23: SnapPy multiloop plot.



Figure 24: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.2 [[7, 14, 8, 1], [13, 6, 14, 7], [8, 6, 9, 5], [1, 12, 2, 13], [9, 4, 10, 5], [11, 2, 12, 3], [3, 10, 4, 11]]

PD code drawn by SnapPy: [(9, 14, 10, 1), (7, 2, 8, 3), (3, 6, 4, 7), (11, 4, 12, 5), (1, 8, 2, 9), (13, 10, 14, 11), (5, 12, 6, 13)]  
 Planar representation generated by plantri: [[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 5, 1], [2, 6, 6, 2], [3, 6, 6, 3], [4, 5, 5, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.68

**Pinning number:** 5

Table 11: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.42	2.71	2.94	3.11	

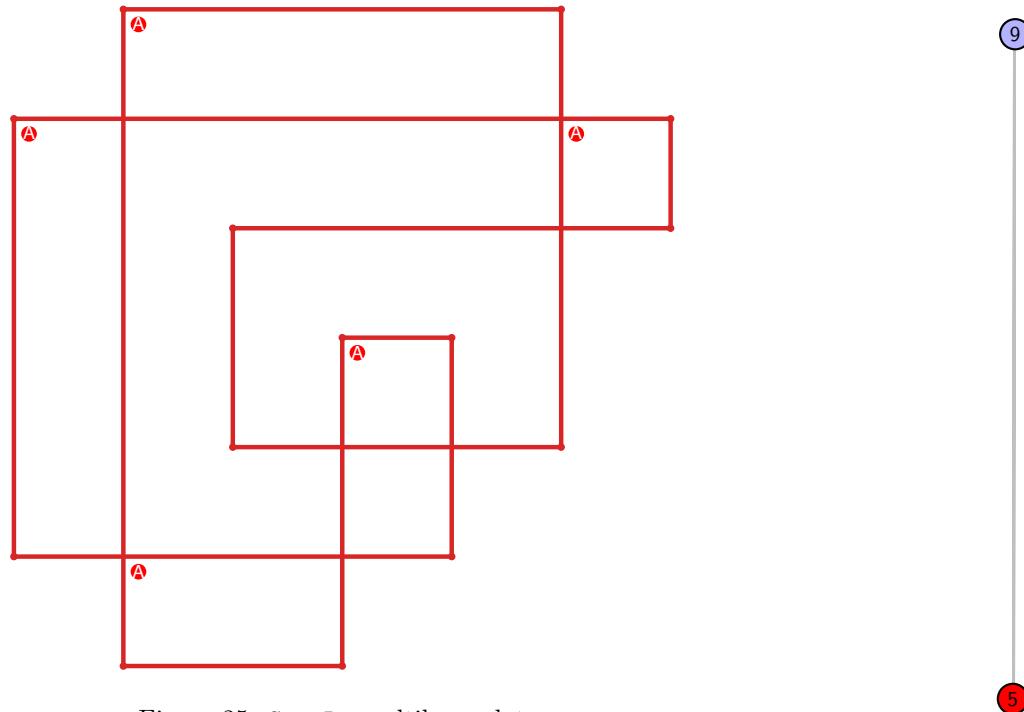


Figure 25: SnapPy multiloop plot.

Figure 26: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.3 [[7, 14, 8, 1], [13, 6, 14, 7], [8, 2, 9, 1], [5, 12, 6, 13], [2, 12, 3, 11], [9, 4, 10, 5], [3, 10, 4, 11]]

PD code drawn by SnapPy: [(5, 14, 6, 1), (9, 2, 10, 3), (13, 6, 14, 7), (7, 12, 8, 13), (1, 8, 2, 9), (3, 10, 4, 11), (11, 4, 12, 5)]  
 Planar representation generated by plantri: [[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 5, 0], [1, 5, 4, 1], [2, 3, 6, 6], [2, 6, 6, 3], [4, 5, 5, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.68

**Pinning number:** 5

Table 12: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.42	2.71	2.94	3.11	

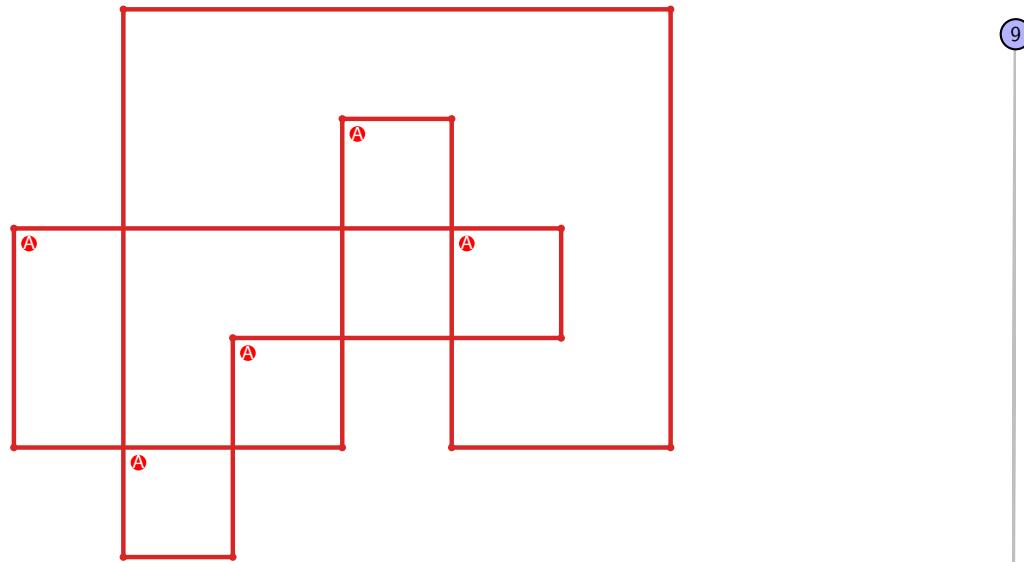


Figure 27: SnapPy multiloop plot.

Figure 28: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.4  $[[7, 14, 8, 1], [9, 6, 10, 7], [13, 2, 14, 3], [8, 2, 9, 1], [5, 12, 6, 13], [10, 4, 11, 3], [11, 4, 12, 5]]$

PD code drawn by SnapPy:  $[(14, 7, 1, 8), (8, 1, 9, 2), (6, 3, 7, 4), (2, 9, 3, 10), (4, 11, 5, 12), (12, 5, 13, 6), (10, 13, 11, 14)]$   
 Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 4, 3], [0, 2, 1, 0], [1, 2, 6, 6], [1, 6, 6, 2], [4, 5, 5, 4]]$

Total optimal pinning sets: 1

Average optimal degree: 2.0

Total minimal pinning sets: 1

Average minimal degree: 2.0

Total pinning sets: 64

Average overall degree: 2.79

Pinning number: 3

Table 13: Pinning sets/average degree by cardinal

Cardinal	3	4	5	6	7	8	9	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.42	2.67	2.83	2.95	3.04	3.11	

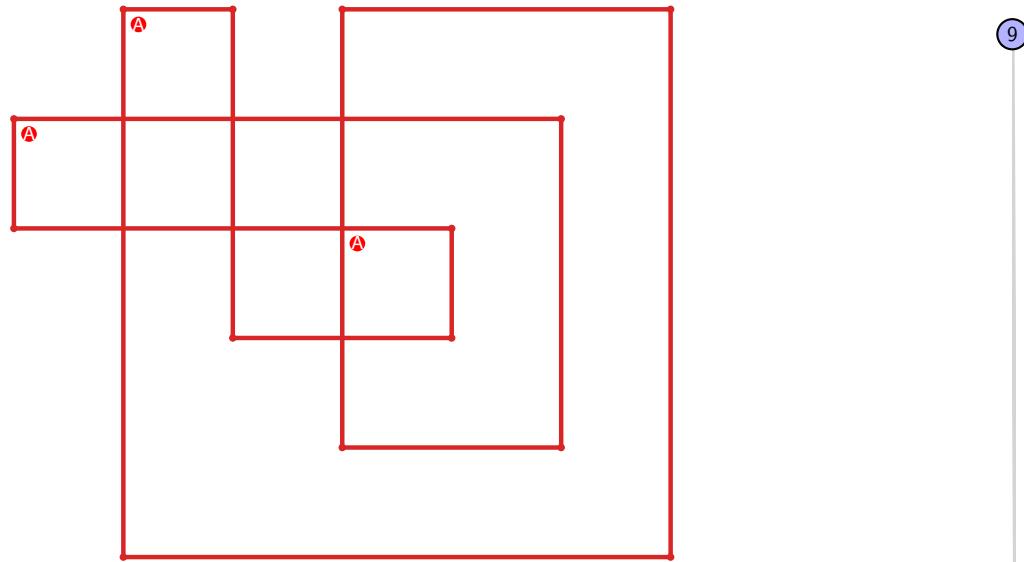


Figure 29: SnapPy multiloop plot.



Figure 30: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.5  $[[14, 7, 1, 8], [8, 6, 9, 5], [13, 2, 14, 3], [6, 1, 7, 2], [9, 13, 10, 12], [4, 11, 5, 12], [3, 11, 4, 10]]$

PD code drawn by SnapPy:  $[(2, 5, 3, 6), (11, 4, 12, 5), (7, 14, 8, 1), (1, 8, 2, 9), (9, 6, 10, 7), (3, 12, 4, 13), (10, 13, 11, 14)]$   
 Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 4, 3], [0, 2, 1, 0], [1, 2, 6, 5], [1, 4, 6, 6], [2, 5, 5, 4]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 5  
 Total pinning sets: 66  
 Pinning number: 4

Average optimal degree: 2.62  
 Average minimal degree: 2.61  
 Average overall degree: 2.92

Table 14: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	3
Nonminimal pinning sets	0	10	24	19	7	1	61
Average degree	2.62	2.75	2.91	3.01	3.07	3.11	

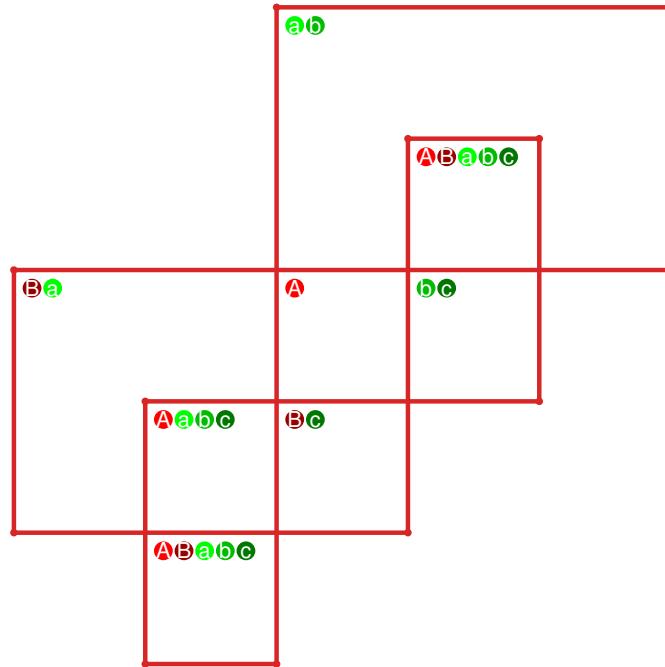


Figure 31: SnapPy multiloop plot.

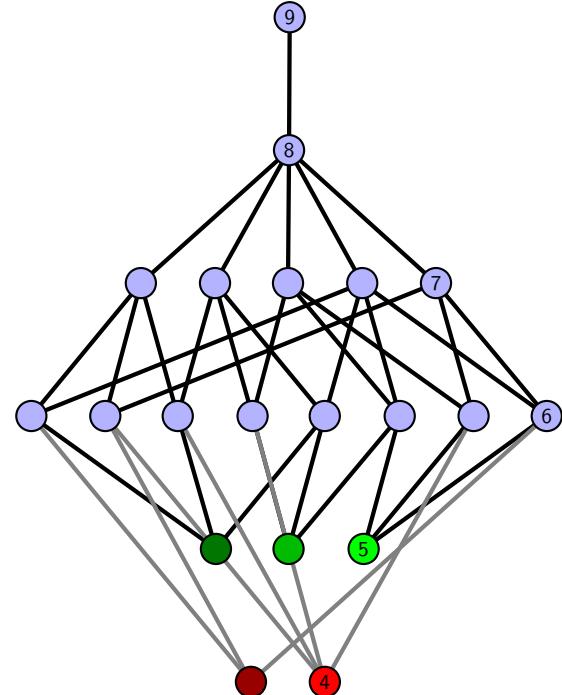


Figure 32: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.6 [[14, 5, 1, 6], [6, 13, 7, 14], [4, 11, 5, 12], [1, 11, 2, 10], [12, 7, 13, 8], [8, 3, 9, 4], [2, 9, 3, 10]]

PD code drawn by SnapPy: [(5, 14, 6, 1), (11, 2, 12, 3), (9, 4, 10, 5), (13, 6, 14, 7), (7, 12, 8, 13), (1, 8, 2, 9), (3, 10, 4, 11)]  
 Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 3], [0, 2, 6, 6], [1, 5, 2, 1], [2, 4, 6, 6], [3, 5, 5, 3]]

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.3

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.3

**Total pinning sets:** 30

**Average overall degree:** 2.75

**Pinning number:** 5

Table 15: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	Total
Optimal pinning sets	4	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	10	10	5	1	26
Average degree	2.3	2.67	2.86	3.0	3.11	

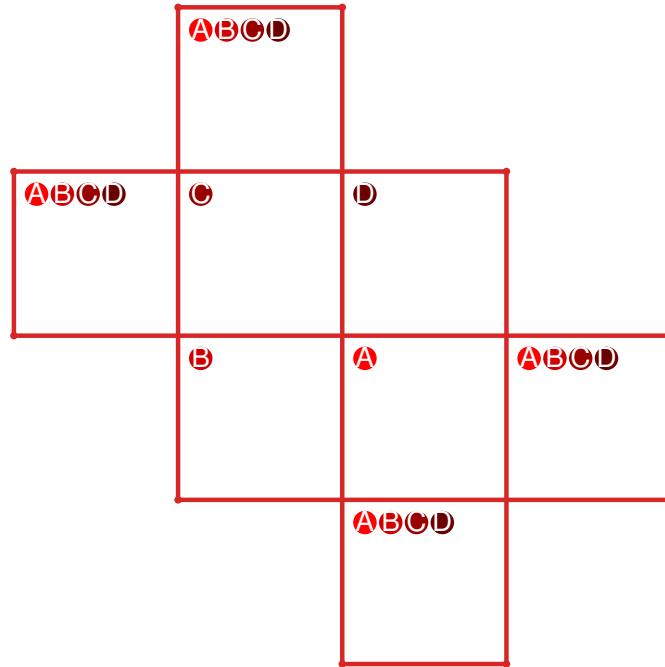


Figure 33: SnapPy multiloop plot.

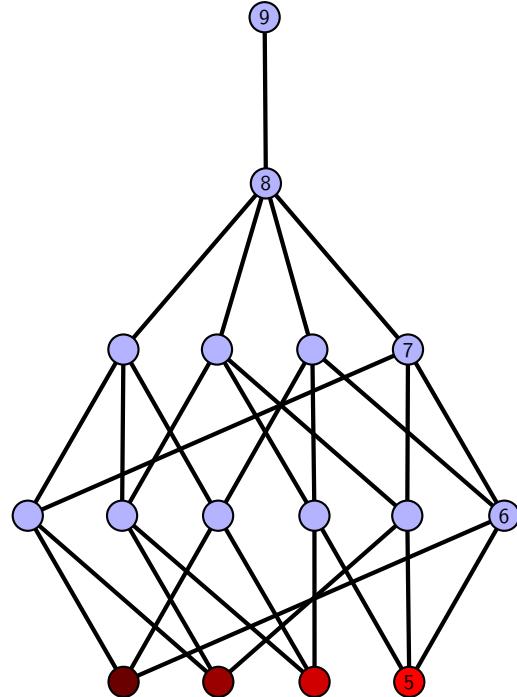


Figure 34: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.7  $[[7, 14, 8, 1], [6, 11, 7, 12], [13, 10, 14, 11], [8, 3, 9, 4], [1, 4, 2, 5], [12, 5, 13, 6], [2, 9, 3, 10]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (13, 2, 14, 3), (10, 3, 11, 4), (14, 7, 1, 8), (11, 8, 12, 9), (4, 9, 5, 10), (5, 12, 6, 13)]$   
 Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 6, 4], [0, 3, 6, 5], [1, 4, 2, 1], [2, 4, 3, 3]]$

Total optimal pinning sets: 2

Average optimal degree: 2.5

Total minimal pinning sets: 5

Average minimal degree: 2.68

Total pinning sets: 66

Average overall degree: 2.91

Pinning number: 4

Table 16: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	3
Nonminimal pinning sets	0	10	24	19	7	1	61
Average degree	2.5	2.74	2.9	3.01	3.07	3.11	

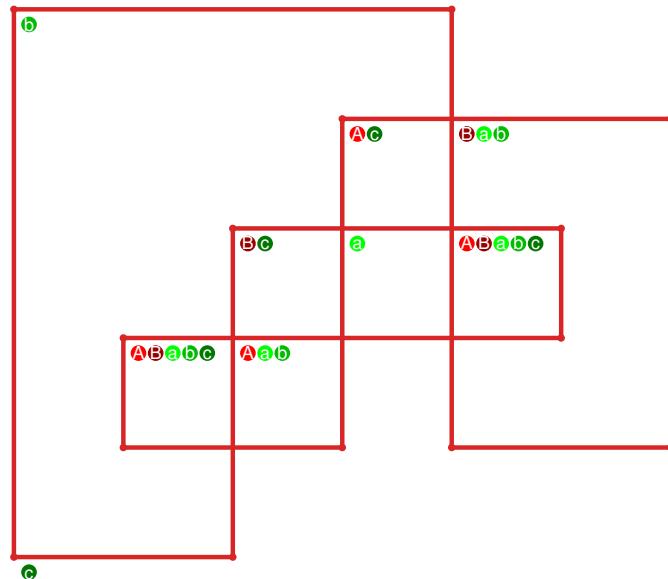


Figure 35: SnapPy multiloop plot.

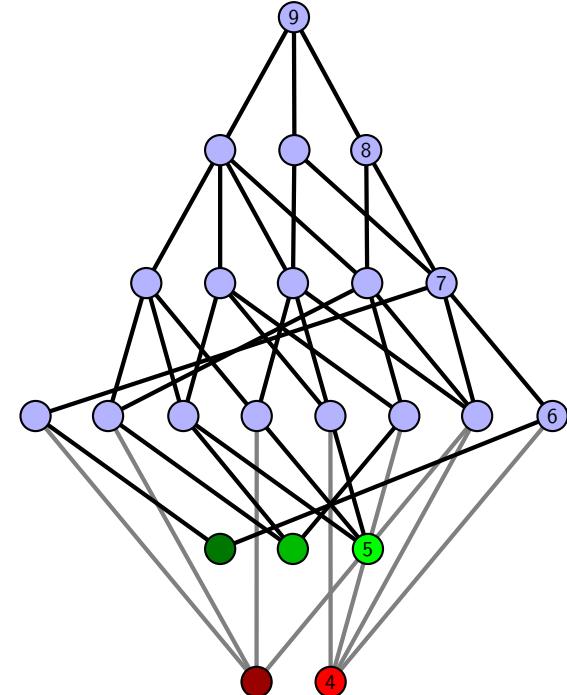


Figure 36: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.8 [[5, 14, 6, 1], [4, 11, 5, 12], [13, 10, 14, 11], [6, 10, 7, 9], [1, 9, 2, 8], [12, 3, 13, 4], [7, 3, 8, 2]]

PD code drawn by SnapPy: [(14, 5, 1, 6), (10, 1, 11, 2), (6, 13, 7, 14), (7, 4, 8, 5), (11, 8, 12, 9), (2, 9, 3, 10), (3, 12, 4, 13)]  
 Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 6, 4], [0, 3, 6, 6], [1, 6, 2, 1], [3, 5, 4, 4]]

Total optimal pinning sets: 6

Average optimal degree: 2.63

Total minimal pinning sets: 6

Average minimal degree: 2.63

Total pinning sets: 48

Average overall degree: 2.91

Pinning number: 5

Table 17: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	Total
Optimal pinning sets	6	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	17	17	7	1	42
Average degree	2.63	2.85	2.99	3.07	3.11	

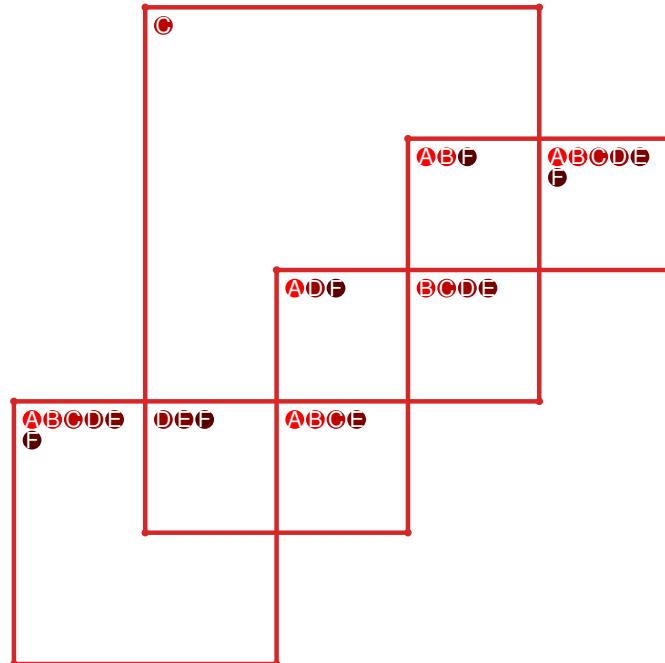


Figure 37: SnapPy multiloop plot.

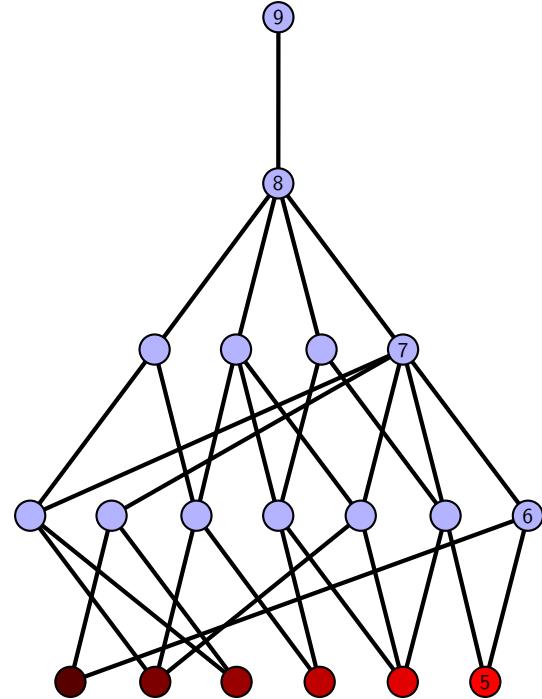


Figure 38: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.9 [[5, 14, 6, 1], [4, 7, 5, 8], [13, 6, 14, 7], [1, 10, 2, 11], [8, 3, 9, 4], [9, 12, 10, 13], [2, 12, 3, 11]]

PD code drawn by SnapPy: [(6, 1, 7, 2), (11, 4, 12, 5), (2, 5, 3, 6), (13, 8, 14, 9), (9, 14, 10, 1), (7, 10, 8, 11), (3, 12, 4, 13)]  
 Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 4, 2], [0, 1, 5, 0], [0, 5, 6, 6], [1, 6, 5, 1], [2, 4, 6, 3], [3, 5, 4, 3]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.42

**Total pinning sets:** 44

**Average overall degree:** 2.82

Table 18: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	2
Nonminimal pinning sets	0	5	15	14	6	1	41
Average degree	2.25	2.54	2.78	2.94	3.04	3.11	

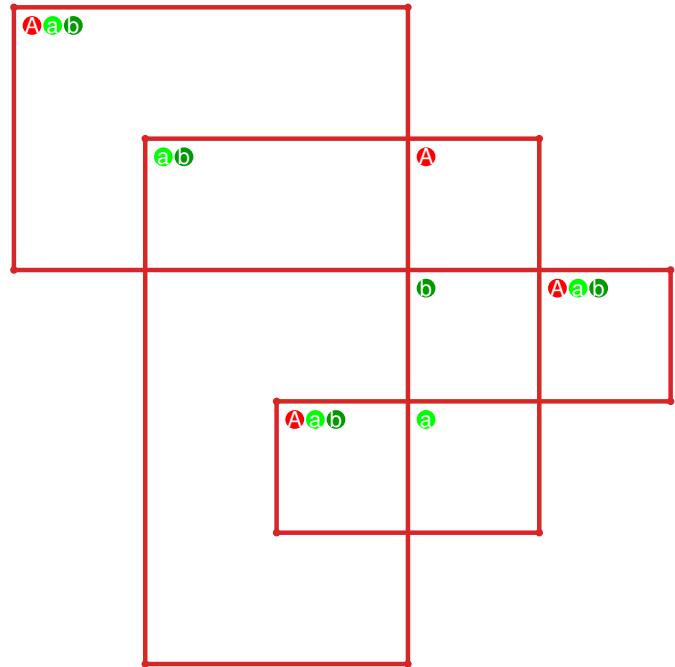


Figure 39: SnapPy multiloop plot.

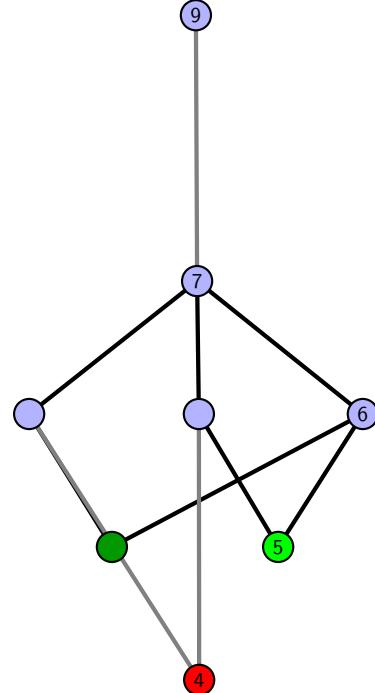


Figure 40: Minimal join sub-semi-lattice of minimal pinning sets.

4.5.10 [[5, 14, 6, 1], [11, 4, 12, 5], [13, 8, 14, 9], [6, 2, 7, 1], [3, 10, 4, 11], [12, 10, 13, 9], [7, 2, 8, 3]]

PD code drawn by `SnapPy`: [(8, 1, 9, 2), (10, 5, 11, 6), (6, 9, 7, 10), (14, 7, 1, 8), (2, 11, 3, 12), (12, 3, 13, 4), (4, 13, 5, 14)]  
 Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 4, 5], [0, 5, 5, 6], [0, 6, 6, 0], [1, 6, 5, 1], [1, 4, 2, 2], [2, 4, 3, 3]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.73

**Pinning number:** 4

Table 19: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	

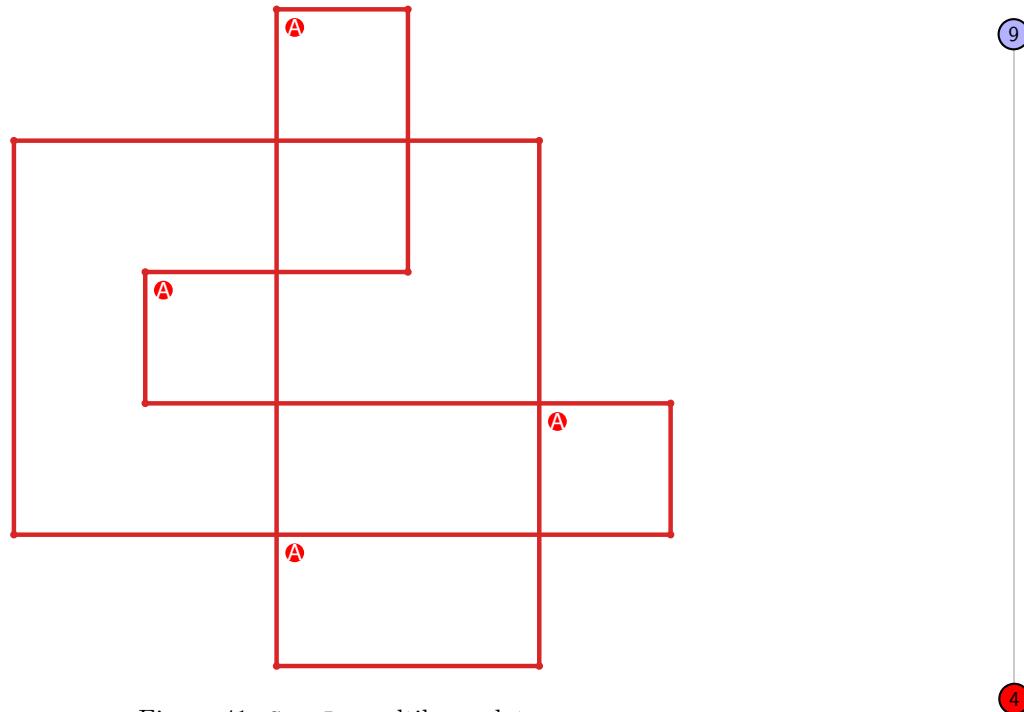


Figure 41: `SnapPy` multiloop plot.

Figure 42: Minimal join sub-semi-lattice of minimal pinning sets.

## 4.6 10 regions

4.6.1  $[[16, 7, 1, 8], [8, 15, 9, 16], [9, 6, 10, 7], [1, 14, 2, 15], [5, 10, 6, 11], [13, 2, 14, 3], [11, 4, 12, 5], [3, 12, 4, 13]]$

PD code drawn by `SnapPy`:  $[(11, 16, 12, 1), (9, 2, 10, 3), (7, 4, 8, 5), (14, 5, 15, 6), (3, 8, 4, 9), (1, 10, 2, 11), (15, 12, 16, 13), (6, 13, 7, 14)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 5, 1], [2, 6, 6, 2], [3, 7, 7, 3], [4, 7, 7, 4], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.71

**Pinning number:** 6

Table 20: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.43	2.75	3.0	3.2	

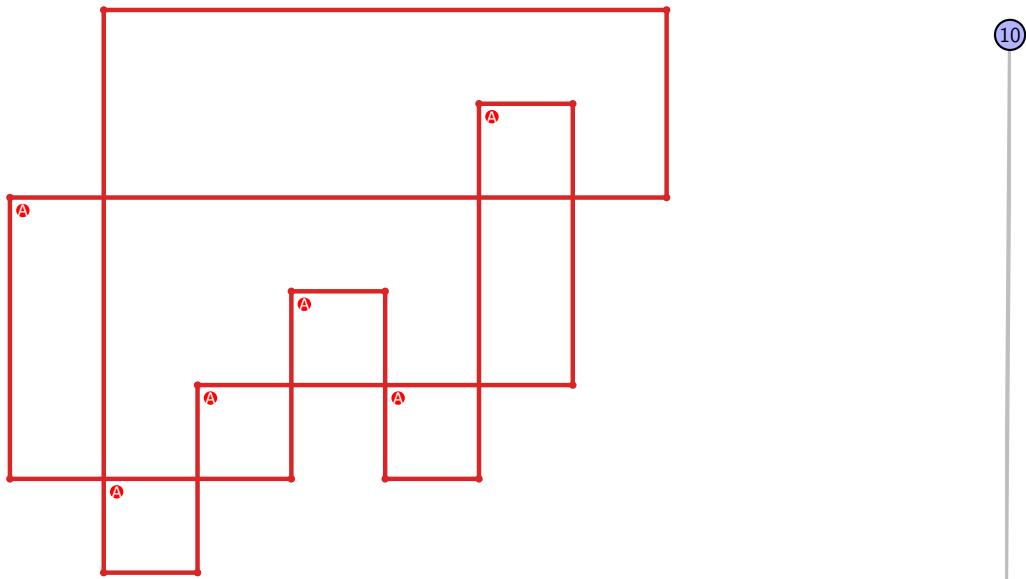


Figure 43: `SnapPy` multiloop plot.

Figure 44: Minimal join sub-semi-lattice of minimal pinning sets.

4.6.2 [[16, 11, 1, 12], [12, 8, 13, 7], [15, 4, 16, 5], [10, 1, 11, 2], [8, 3, 9, 4], [13, 6, 14, 7], [5, 14, 6, 15], [2, 9, 3, 10]]

PD code drawn by SnapPy: [(6, 1, 7, 2), (13, 2, 14, 3), (11, 4, 12, 5), (5, 10, 6, 11), (16, 7, 1, 8), (14, 9, 15, 10), (3, 12, 4, 13), (8, 15, 9, 16)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 4], [0, 7, 7, 0], [1, 7, 7, 2], [1, 6, 6, 1], [2, 5, 5, 2], [3, 4, 4, 3]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1  
**Total pinning sets:** 16  
**Pinning number:** 6

**Average optimal degree:** 2.0  
**Average minimal degree:** 2.0  
**Average overall degree:** 2.71

Table 21: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.43	2.75	3.0	3.2	

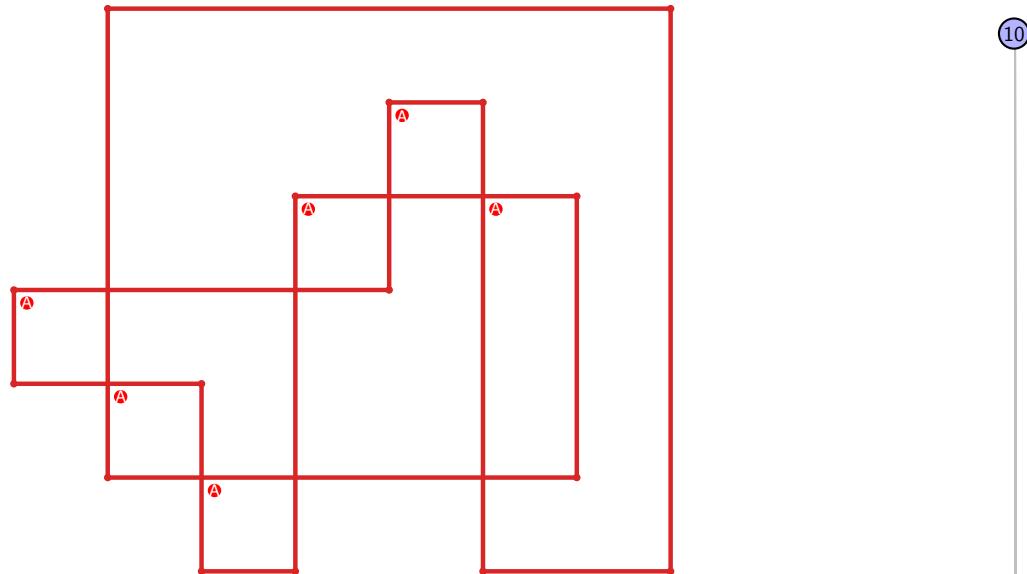


Figure 45: SnapPy multiloop plot.

6

Figure 46: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.3** [[16, 11, 1, 12], [12, 10, 13, 9], [15, 2, 16, 3], [10, 1, 11, 2], [13, 6, 14, 7], [8, 3, 9, 4], [5, 14, 6, 15], [7, 5, 8, 4]]

PD code drawn by `SnapPy`: [(13, 16, 14, 1), (6, 1, 7, 2), (2, 5, 3, 6), (11, 4, 12, 5), (7, 10, 8, 11), (14, 9, 15, 10), (3, 12, 4, 13), (8, 15, 9, 16)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 6, 3], [0, 2, 1, 0], [1, 6, 6, 7], [1, 7, 7, 2], [2, 7, 4, 4], [4, 6, 5, 5]]

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 114  
**Pinning number:** 4

**Average optimal degree:** 2.38  
**Average minimal degree:** 2.52  
**Average overall degree:** 2.92

Table 22: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	4
Nonminimal pinning sets	0	11	33	35	21	7	1	108
Average degree	2.38	2.64	2.85	2.98	3.07	3.14	3.2	

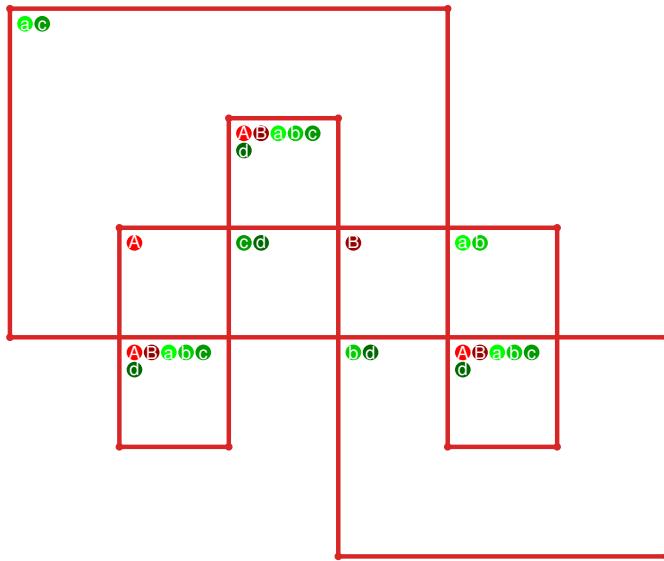


Figure 47: `SnapPy` multiloop plot.

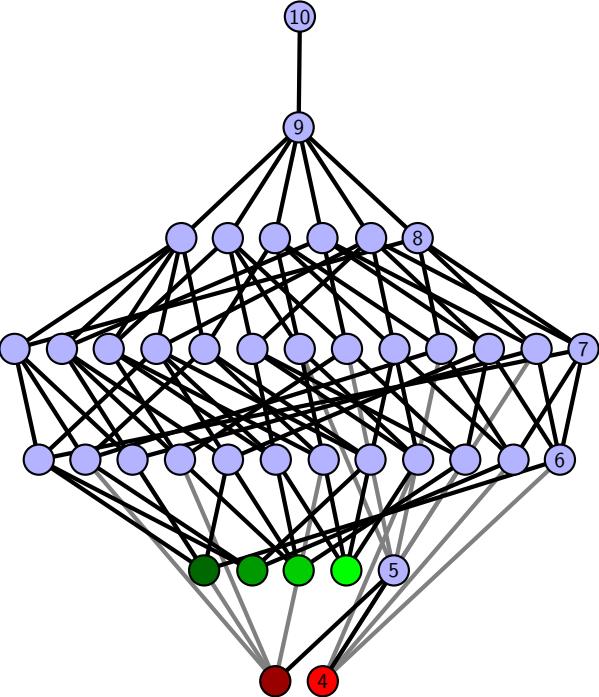


Figure 48: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.4** [[16, 9, 1, 10], [10, 8, 11, 7], [15, 2, 16, 3], [8, 1, 9, 2], [11, 4, 12, 5], [13, 6, 14, 7], [3, 14, 4, 15], [12, 6, 13, 5]]

PD code drawn by `SnapPy`: [(13, 16, 14, 1), (9, 2, 10, 3), (3, 10, 4, 11), (11, 4, 12, 5), (5, 8, 6, 9), (14, 7, 15, 8), (1, 12, 2, 13), (6, 15, 7, 16)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 6, 3], [0, 2, 1, 0], [1, 6, 7, 7], [1, 7, 7, 6], [2, 5, 4, 2], [4, 5, 5, 4]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1  
**Total pinning sets:** 64  
**Pinning number:** 4

**Average optimal degree:** 2.0  
**Average minimal degree:** 2.0  
**Average overall degree:** 2.82

Table 23: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	

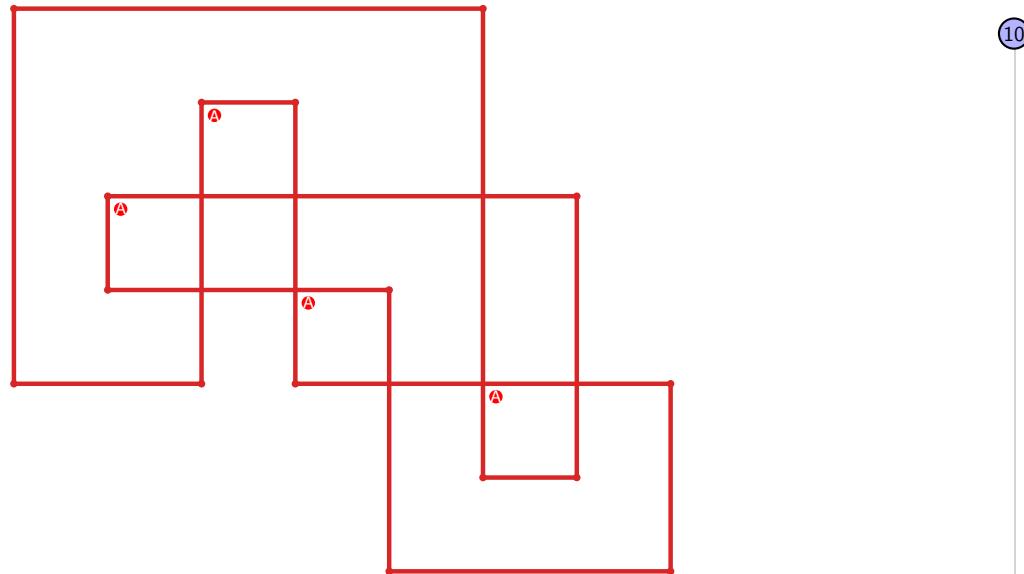


Figure 49: `SnapPy` multiloop plot.

Figure 50: Minimal join sub-semi-lattice of minimal pinning sets.

4.6.5 [[16, 9, 1, 10], [10, 8, 11, 7], [15, 2, 16, 3], [8, 1, 9, 2], [11, 15, 12, 14], [6, 3, 7, 4], [12, 6, 13, 5], [13, 4, 14, 5]]

PD code drawn by SnapPy: [(9, 16, 10, 1), (1, 8, 2, 9), (3, 6, 4, 7), (12, 5, 13, 6), (10, 7, 11, 8), (4, 13, 5, 14), (11, 14, 12, 15), (2, 15, 3, 16)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 4, 3], [0, 2, 1, 0], [1, 2, 6, 7], [1, 7, 6, 2], [4, 5, 7, 7], [4, 6, 6, 5]]

**Total optimal pinning sets:** 8  
**Total minimal pinning sets:** 8  
**Total pinning sets:** 207  
**Pinning number:** 4

**Average optimal degree:** 2.75  
**Average minimal degree:** 2.75  
**Average overall degree:** 3.01

Table 24: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	8	0	0	0	0	0	0	8
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	40	66	56	28	8	1	199
Average degree	2.75	2.9	3.0	3.07	3.12	3.17	3.2	

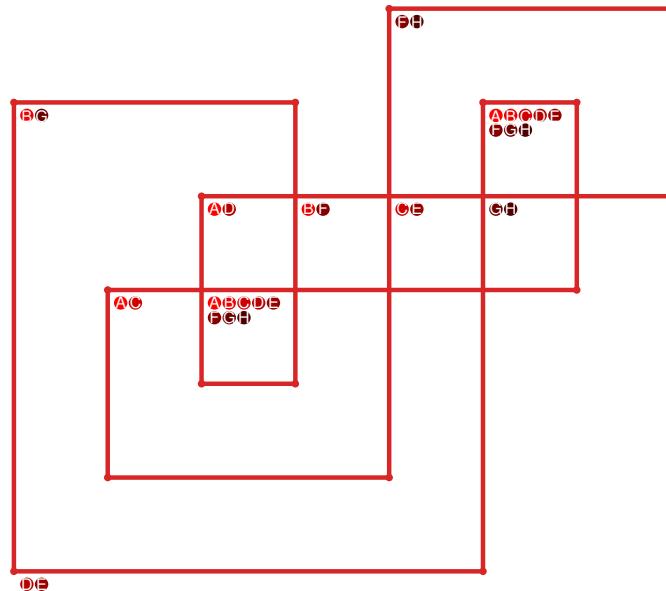


Figure 51: SnapPy multiloop plot.

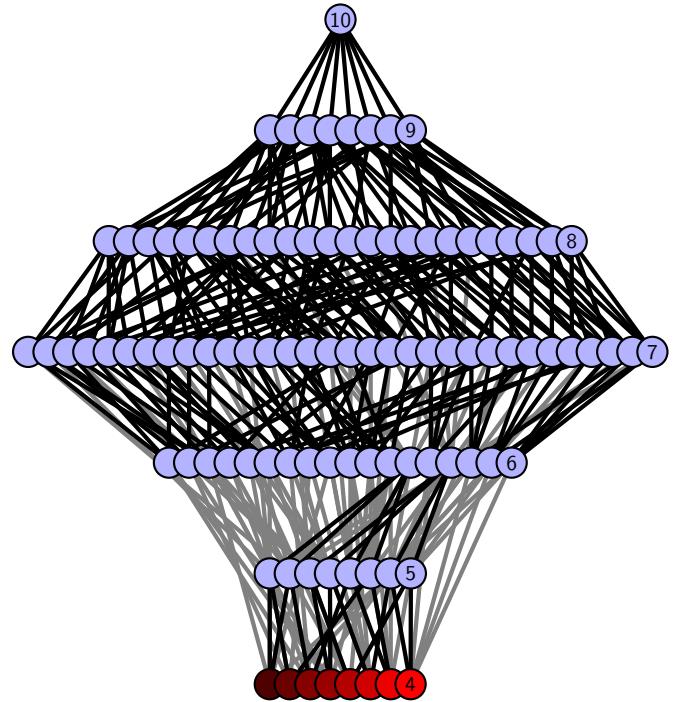


Figure 52: Minimal join sub-semi-lattice of minimal pinning sets.

4.6.6 [[16, 13, 1, 14], [14, 8, 15, 7], [15, 6, 16, 7], [12, 5, 13, 6], [1, 9, 2, 8], [2, 11, 3, 12], [4, 9, 5, 10], [10, 3, 11, 4]]

PD code drawn by SnapPy: [(8, 1, 9, 2), (10, 3, 11, 4), (13, 4, 14, 5), (5, 12, 6, 13), (6, 15, 7, 16), (2, 7, 3, 8), (16, 9, 1, 10), (14, 11, 15, 12)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 2, 2], [0, 1, 1, 3], [0, 2, 5, 6], [0, 6, 5, 1], [3, 4, 7, 7], [3, 7, 7, 4], [5, 6, 6, 5]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 80  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.33  
 Average overall degree: 2.9

Table 25: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.25	2.54	2.76	2.93	3.05	3.14	3.2	

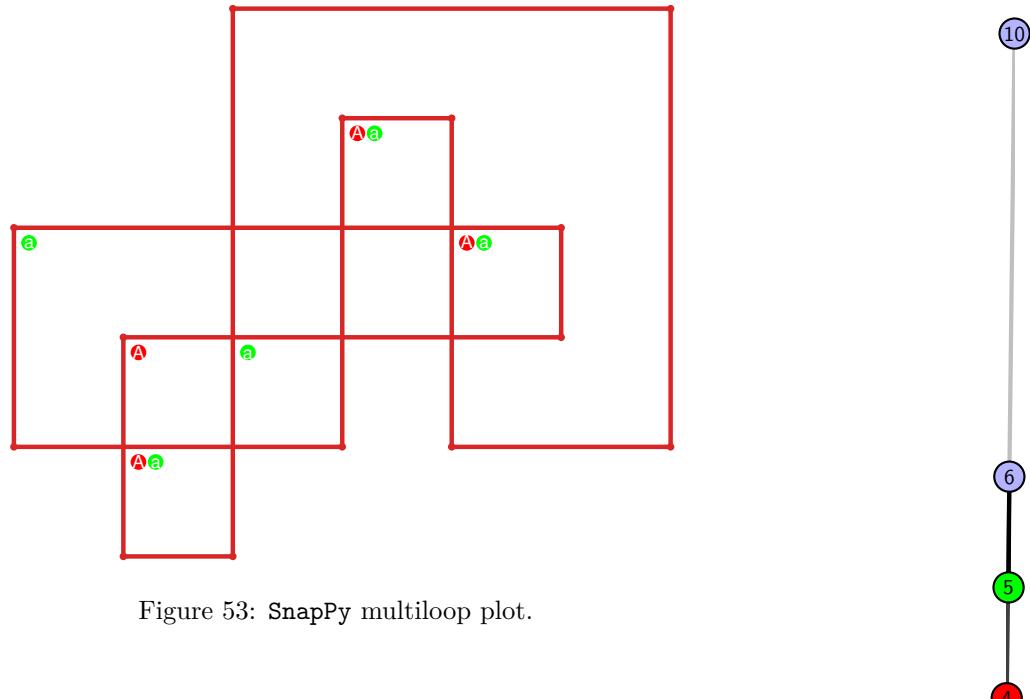


Figure 53: SnapPy multiloop plot.

Figure 54: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.7** `[[9, 16, 10, 1], [15, 8, 16, 9], [10, 8, 11, 7], [1, 14, 2, 15], [11, 5, 12, 4], [13, 6, 14, 7], [2, 6, 3, 5], [12, 3, 13, 4]]`

PD code drawn by `SnapPy`: `[(10, 1, 11, 2), (2, 9, 3, 10), (3, 16, 4, 1), (11, 4, 12, 5), (5, 8, 6, 9), (13, 6, 14, 7), (15, 12, 16, 13), (7, 14, 8, 15)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 5], [0, 5, 6, 1], [2, 6, 7, 7], [2, 7, 6, 3], [3, 5, 7, 4], [4, 6, 5, 4]]`

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.56

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.56

**Total pinning sets:** 144

**Average overall degree:** 2.99

**Pinning number:** 4

Table 26: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.56	2.79	2.94	3.04	3.12	3.17	3.2	

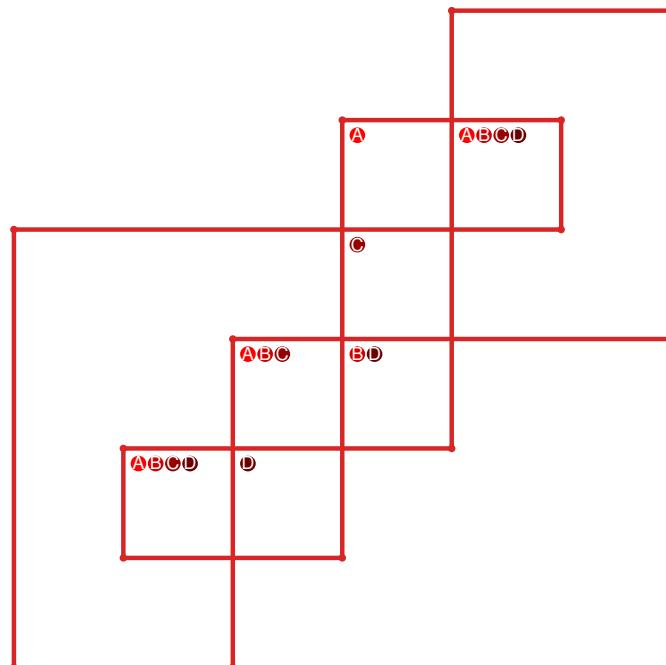


Figure 55: `SnapPy` multiloop plot.

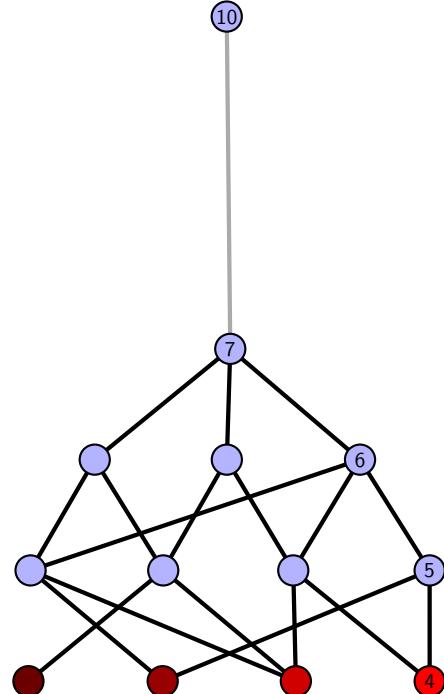


Figure 56: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.8**  $[[7, 16, 8, 1], [15, 6, 16, 7], [8, 6, 9, 5], [1, 14, 2, 15], [9, 4, 10, 5], [10, 13, 11, 14], [2, 11, 3, 12], [12, 3, 13, 4]]$

PD code drawn by `SnapPy`:  $[(10, 1, 11, 2), (7, 2, 8, 3), (3, 6, 4, 7), (13, 4, 14, 5), (16, 9, 1, 10), (8, 11, 9, 12), (15, 12, 16, 13), (5, 14, 6, 15)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 6, 1], [2, 7, 5, 2], [3, 4, 7, 6], [3, 5, 7, 7], [4, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 80

**Average overall degree:** 2.9

**Pinning number:** 4

Table 27: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.25	2.54	2.76	2.93	3.05	3.14	3.2	

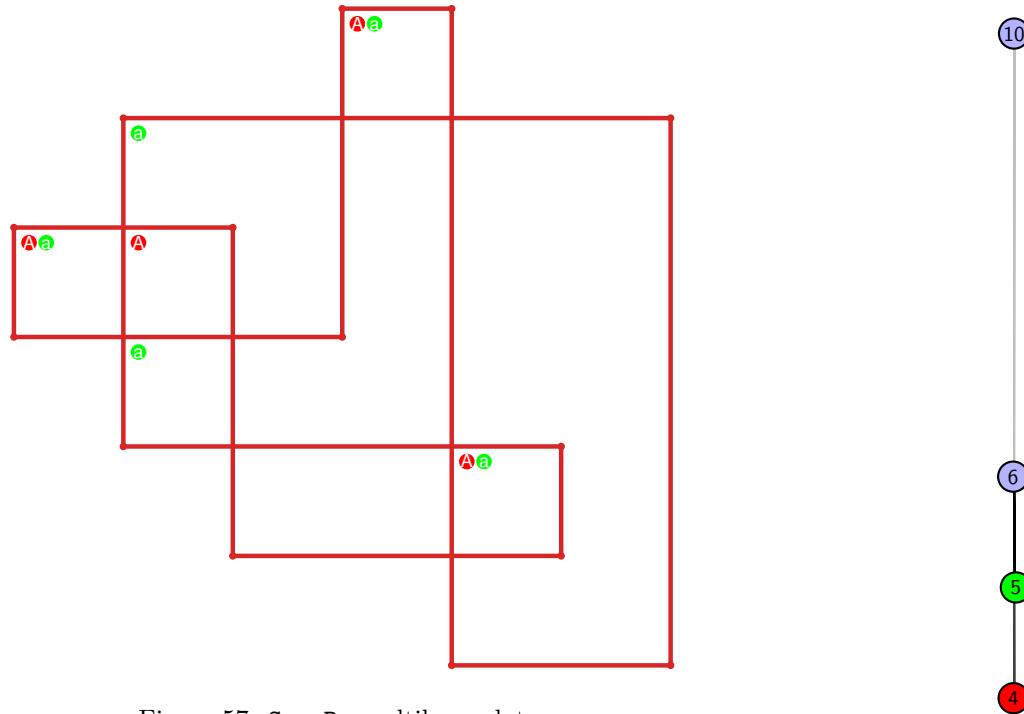


Figure 57: `SnapPy` multiloop plot.

Figure 58: Minimal join sub-semi-lattice of minimal pinning sets.

4.6.9  $[[7, 16, 8, 1], [15, 6, 16, 7], [8, 2, 9, 1], [5, 14, 6, 15], [2, 10, 3, 9], [4, 11, 5, 12], [13, 10, 14, 11], [3, 13, 4, 12]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (4, 13, 5, 14), (8, 5, 9, 6), (15, 6, 16, 7), (7, 14, 8, 15), (16, 9, 1, 10), (2, 11, 3, 12), (12, 3, 13, 4)]$

Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 6, 7, 2], [3, 7, 7, 6], [3, 5, 7, 4], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 20  
**Pinning number:** 6

**Average optimal degree:** 2.17  
**Average minimal degree:** 2.23  
**Average overall degree:** 2.78

Table 28: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	1
Nonminimal pinning sets	0	4	8	5	1	18
Average degree	2.17	2.49	2.81	3.07	3.2	

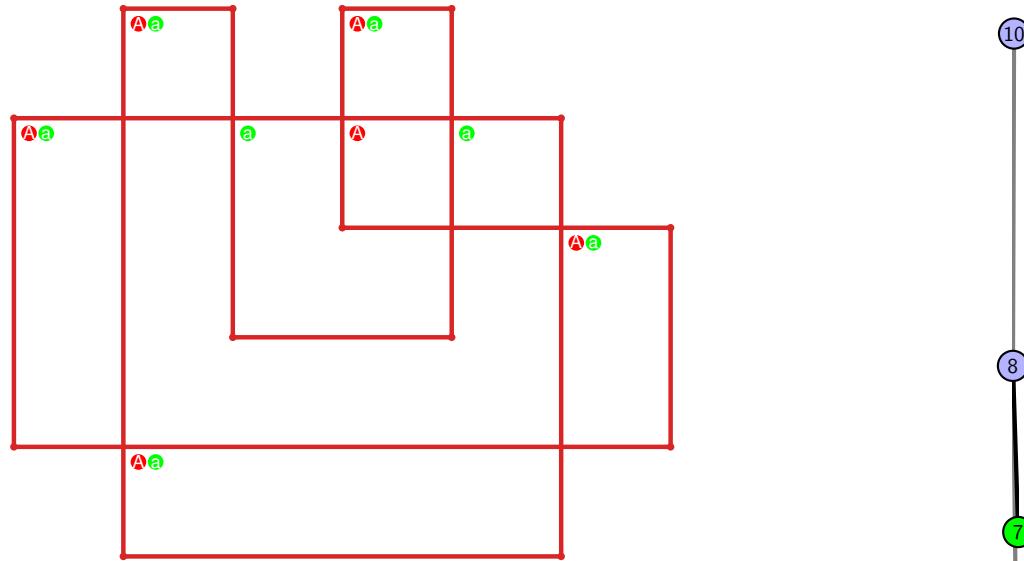


Figure 59: SnapPy multiloop plot.

Figure 60: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.10**  $[[16, 9, 1, 10], [10, 15, 11, 16], [8, 1, 9, 2], [5, 14, 6, 15], [11, 6, 12, 7], [2, 7, 3, 8], [13, 4, 14, 5], [12, 4, 13, 3]]$

PD code drawn by `SnapPy`:  $[(16, 9, 1, 10), (10, 1, 11, 2), (8, 3, 9, 4), (15, 4, 16, 5), (13, 6, 14, 7), (2, 11, 3, 12), (7, 12, 8, 13), (5, 14, 6, 15)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 5, 0], [1, 6, 6, 4], [1, 3, 7, 5], [2, 4, 7, 2], [3, 7, 7, 3], [4, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.76

**Pinning number:** 5

Table 29: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.69	2.9	3.07	3.2	

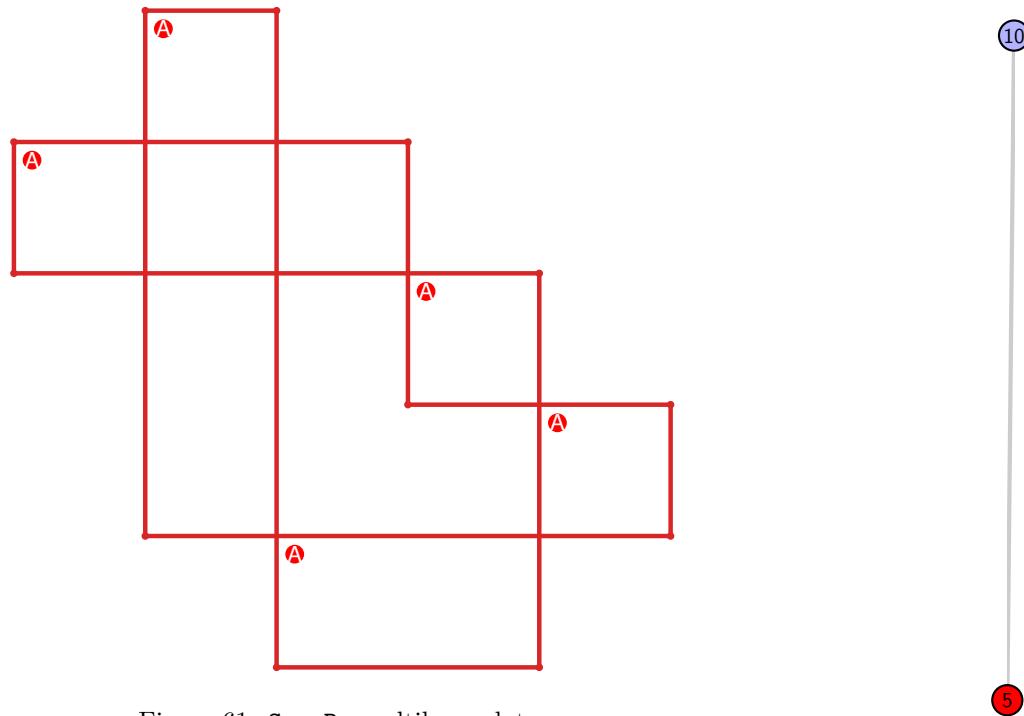


Figure 61: `SnapPy` multiloop plot.

Figure 62: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.11** `[[16, 5, 1, 6], [6, 14, 7, 13], [15, 12, 16, 13], [4, 11, 5, 12], [1, 9, 2, 8], [14, 8, 15, 7], [10, 3, 11, 4], [9, 3, 10, 2]]`

PD code drawn by `SnapPy`: `[(16, 7, 1, 8), (8, 1, 9, 2), (14, 3, 15, 4), (5, 12, 6, 13), (6, 15, 7, 16), (2, 9, 3, 10), (13, 10, 14, 11), (11, 4, 12, 5)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 7, 7, 5], [1, 4, 2, 1], [3, 7, 7, 3], [4, 6, 6, 4]]`

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 40  
**Pinning number:** 5

**Average optimal degree:** 2.2  
**Average minimal degree:** 2.27  
**Average overall degree:** 2.84

Table 30: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	1
Nonminimal pinning sets	0	5	13	13	6	1	38
Average degree	2.2	2.5	2.76	2.96	3.11	3.2	

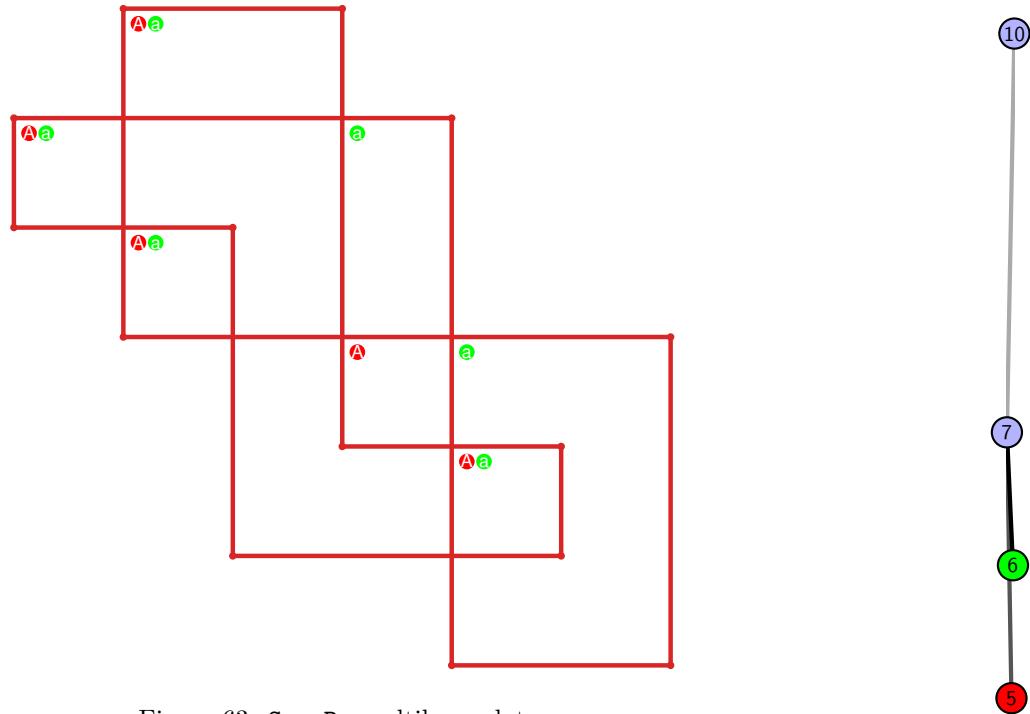


Figure 63: `SnapPy` multiloop plot.

Figure 64: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.12**  $[[16, 11, 1, 12], [12, 6, 13, 5], [15, 4, 16, 5], [10, 3, 11, 4], [1, 9, 2, 8], [6, 14, 7, 13], [7, 14, 8, 15], [2, 9, 3, 10]]$

PD code drawn by SnapPy:  $[(8, 1, 9, 2), (13, 2, 14, 3), (3, 10, 4, 11), (6, 15, 7, 16), (16, 7, 1, 8), (14, 9, 15, 10), (11, 4, 12, 5), (5, 12, 6, 13)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 7, 7], [0, 7, 7, 6], [1, 6, 6, 1], [2, 5, 5, 4], [3, 4, 4, 3]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 48

**Average overall degree:** 2.83

**Pinning number:** 5

Table 31: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.2	2.56	2.8	2.98	3.11	3.2	

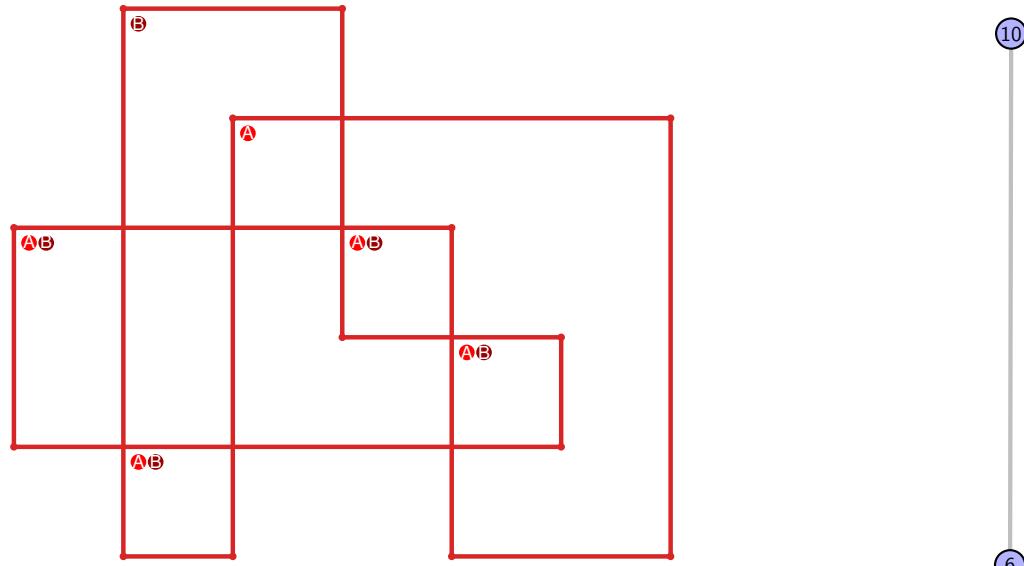


Figure 65: SnapPy multiloop plot.

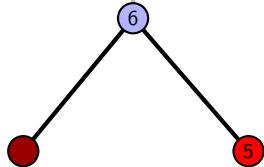


Figure 66: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.13**  $[[16, 5, 1, 6], [6, 14, 7, 13], [15, 12, 16, 13], [4, 9, 5, 10], [1, 9, 2, 8], [14, 8, 15, 7], [2, 11, 3, 12], [10, 3, 11, 4]]$

PD code drawn by `SnapPy`:  $[(7, 16, 8, 1), (13, 2, 14, 3), (4, 11, 5, 12), (5, 14, 6, 15), (1, 6, 2, 7), (15, 8, 16, 9), (12, 9, 13, 10), (10, 3, 11, 4)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 4], [0, 3, 6, 5], [1, 4, 2, 1], [2, 4, 7, 7], [3, 6, 6, 3]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.25

**Total pinning sets:** 96

**Average overall degree:** 2.9

**Pinning number:** 4

Table 32: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.25	2.58	2.8	2.95	3.06	3.14	3.2	

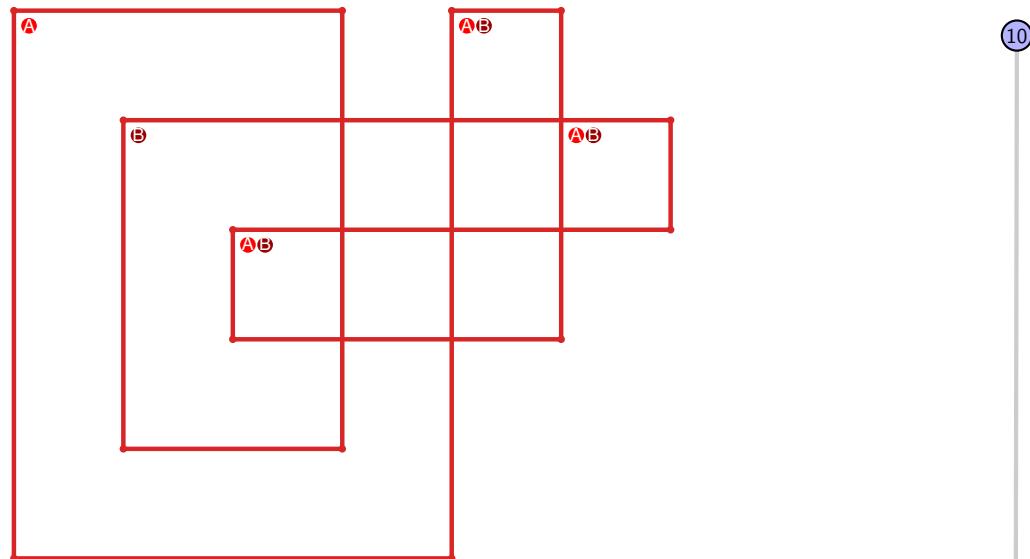


Figure 67: `SnapPy` multiloop plot.

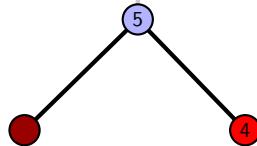


Figure 68: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.14** [[7, 16, 8, 1], [13, 6, 14, 7], [15, 10, 16, 11], [8, 3, 9, 4], [1, 4, 2, 5], [5, 12, 6, 13], [14, 12, 15, 11], [2, 9, 3, 10]]

PD code drawn by SnapPy: [(6, 1, 7, 2), (15, 2, 16, 3), (11, 4, 12, 5), (16, 7, 1, 8), (13, 8, 14, 9), (9, 12, 10, 13), (3, 10, 4, 11), (5, 14, 6, 15)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 6, 7], [0, 7, 7, 4], [0, 3, 7, 5], [1, 4, 6, 1], [1, 5, 2, 2], [2, 4, 3, 3]]

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 96  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.25  
**Average overall degree:** 2.9

Table 33: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.25	2.58	2.8	2.95	3.06	3.14	3.2	

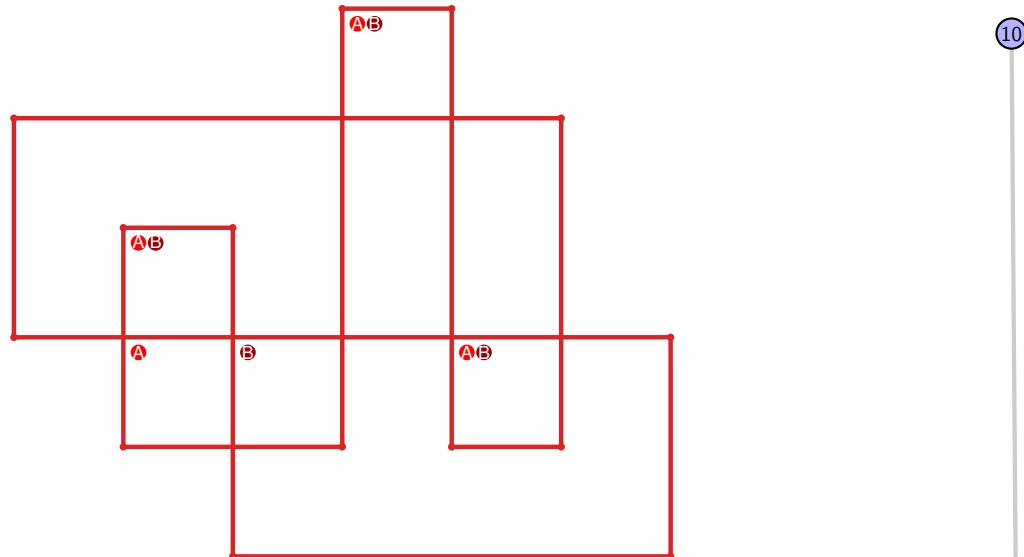


Figure 69: SnapPy multiloop plot.

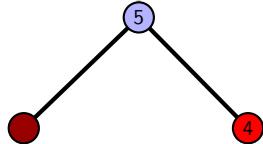


Figure 70: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.15**  $[[7, 16, 8, 1], [11, 6, 12, 7], [15, 8, 16, 9], [1, 4, 2, 5], [5, 10, 6, 11], [12, 10, 13, 9], [3, 14, 4, 15], [2, 14, 3, 13]]$

PD code drawn by `SnapPy`:  $[(15, 2, 16, 3), (9, 4, 10, 5), (13, 6, 14, 7), (7, 10, 8, 11), (3, 8, 4, 9), (11, 16, 12, 1), (1, 12, 2, 13), (5, 14, 6, 15)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 5, 6, 0], [0, 6, 7, 4], [1, 3, 5, 1], [1, 4, 7, 2], [2, 7, 7, 3], [3, 6, 6, 5]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3  
**Total pinning sets:** 112  
**Pinning number:** 4

**Average optimal degree:** 2.33  
**Average minimal degree:** 2.33  
**Average overall degree:** 2.91

Table 34: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	31	34	21	7	1	109
Average degree	2.33	2.64	2.84	2.97	3.07	3.14	3.2	

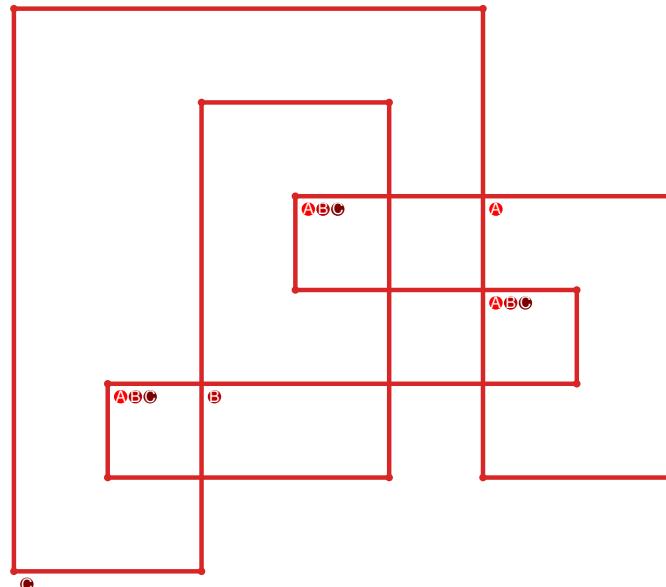


Figure 71: `SnapPy` multiloop plot.

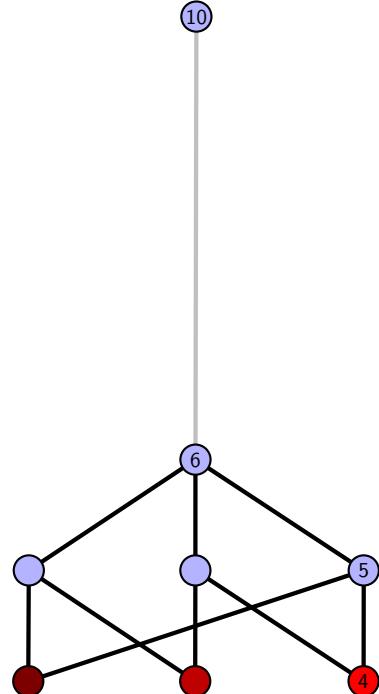


Figure 72: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.16** `[[16, 9, 1, 10], [10, 5, 11, 6], [6, 15, 7, 16], [8, 1, 9, 2], [13, 4, 14, 5], [11, 14, 12, 15], [7, 3, 8, 2], [3, 12, 4, 13]]`

PD code drawn by `SnapPy`: `[(10, 1, 11, 2), (7, 2, 8, 3), (13, 6, 14, 7), (16, 9, 1, 10), (8, 11, 9, 12), (3, 12, 4, 13), (5, 14, 6, 15), (15, 4, 16, 5)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 3], [0, 4, 5, 2], [0, 1, 5, 6], [0, 6, 6, 0], [1, 7, 7, 5], [1, 4, 7, 2], [2, 7, 3, 3], [4, 6, 5, 4]]`

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3  
**Total pinning sets:** 64  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.4  
**Average overall degree:** 2.9

Table 35: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	3	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	22	18	7	1	61
Average degree	2.4	2.69	2.9	3.04	3.14	3.2	

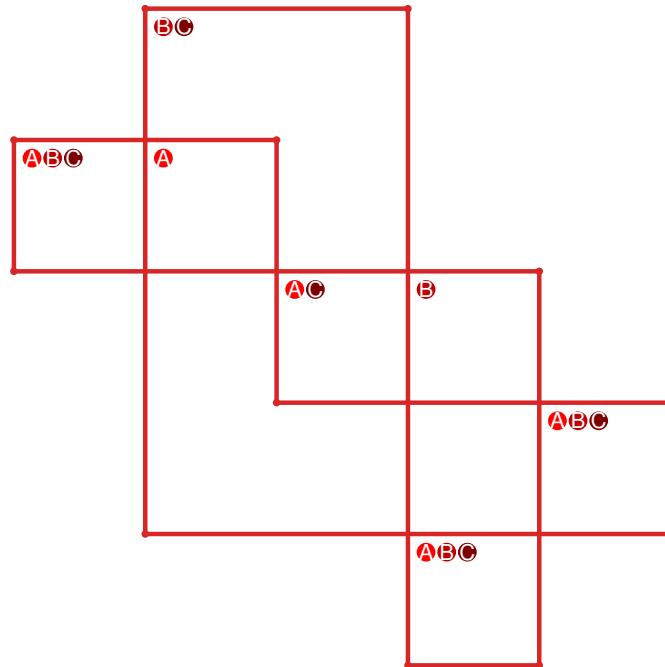


Figure 73: `SnapPy` multiloop plot.

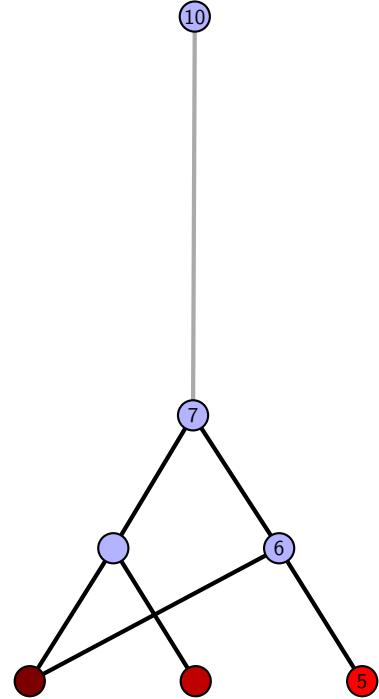


Figure 74: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.17**  $[[7, 16, 8, 1], [15, 6, 16, 7], [8, 6, 9, 5], [1, 13, 2, 12], [3, 14, 4, 15], [9, 4, 10, 5], [13, 10, 14, 11], [2, 11, 3, 12]]$

PD code drawn by `SnapPy`:  $[(7, 2, 8, 3), (3, 6, 4, 7), (13, 4, 14, 5), (11, 8, 12, 9), (16, 9, 1, 10), (10, 15, 11, 16), (1, 12, 2, 13), (5, 14, 6, 15)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 6, 7, 7], [1, 7, 6, 5], [2, 4, 6, 2], [3, 5, 4, 7], [3, 6, 4, 3]]$

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4

**Total pinning sets:** 72

**Pinning number:** 5

**Average optimal degree:** 2.45

**Average minimal degree:** 2.45

**Average overall degree:** 2.91

Table 36: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	4	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	16	25	19	7	1	68
Average degree	2.45	2.73	2.92	3.05	3.14	3.2	

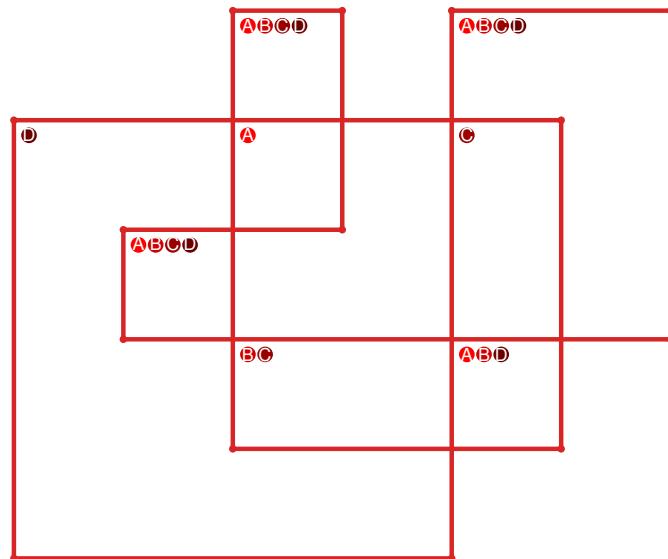


Figure 75: `SnapPy` multiloop plot.

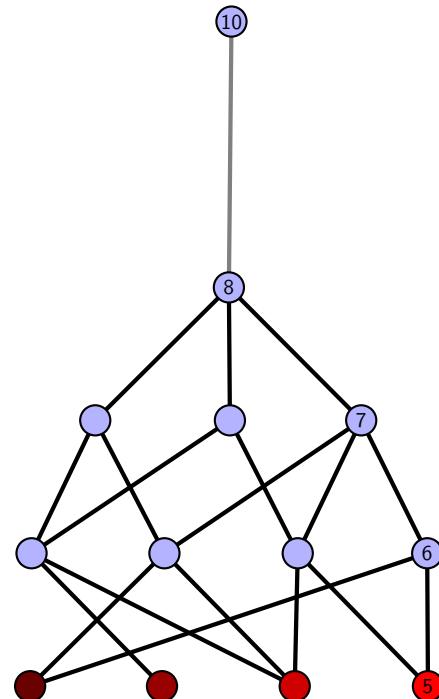


Figure 76: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.18**  $[[5, 16, 6, 1], [9, 4, 10, 5], [10, 15, 11, 16], [6, 11, 7, 12], [1, 12, 2, 13], [13, 8, 14, 9], [14, 3, 15, 4], [7, 3, 8, 2]]$

PD code drawn by `SnapPy`:  $[(16, 5, 1, 6), (12, 1, 13, 2), (7, 2, 8, 3), (11, 6, 12, 7), (4, 9, 5, 10), (15, 10, 16, 11), (8, 13, 9, 14), (3, 14, 4, 15)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 3], [0, 2, 7, 4], [0, 3, 7, 5], [1, 4, 7, 6], [1, 5, 7, 2], [3, 6, 5, 4]]$

**Total optimal pinning sets:** 10  
**Total minimal pinning sets:** 10  
**Total pinning sets:** 160  
**Pinning number:** 5

**Average optimal degree:** 3.04  
**Average minimal degree:** 3.04  
**Average overall degree:** 3.15

Table 37: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	10	0	0	0	0	0	10
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	42	60	37	10	1	150
Average degree	3.04	3.11	3.16	3.19	3.2	3.2	

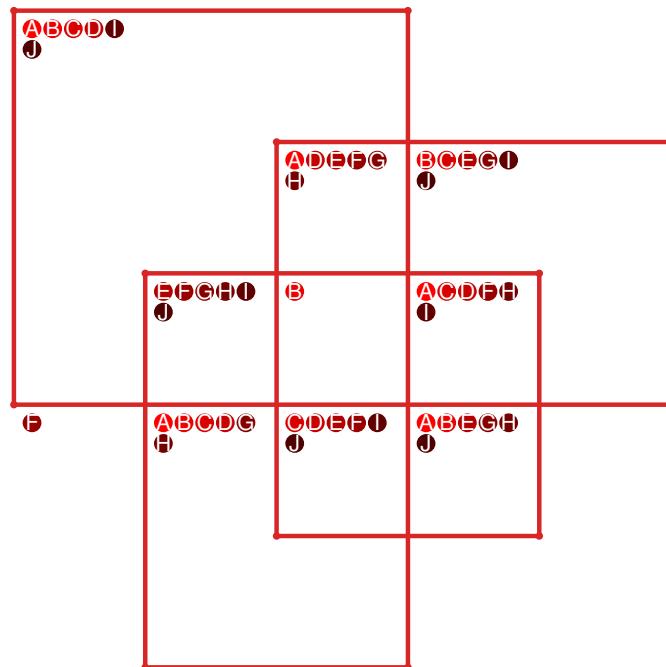


Figure 77: `SnapPy` multiloop plot.

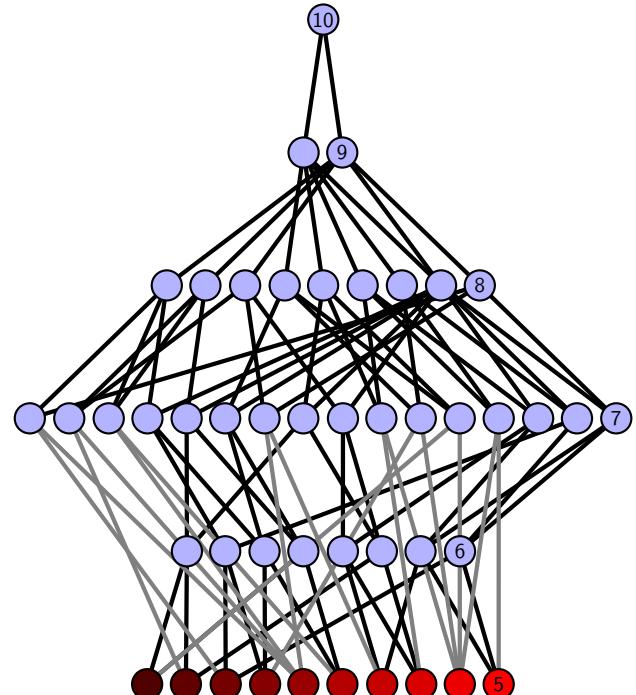


Figure 78: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.19** `[[16, 5, 1, 6], [6, 3, 7, 4], [4, 15, 5, 16], [1, 10, 2, 11], [11, 2, 12, 3], [7, 12, 8, 13], [9, 14, 10, 15], [8, 14, 9, 13]]`

PD code drawn by `SnapPy`: `[(9, 16, 10, 1), (4, 1, 5, 2), (14, 3, 15, 4), (15, 8, 16, 9), (5, 10, 6, 11), (11, 6, 12, 7), (7, 12, 8, 13), (2, 13, 3, 14)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 4, 5, 2], [0, 1, 6, 0], [0, 6, 4, 4], [1, 3, 3, 5], [1, 4, 7, 7], [2, 7, 7, 3], [5, 6, 6, 5]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.82

**Pinning number:** 4

Table 38: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	

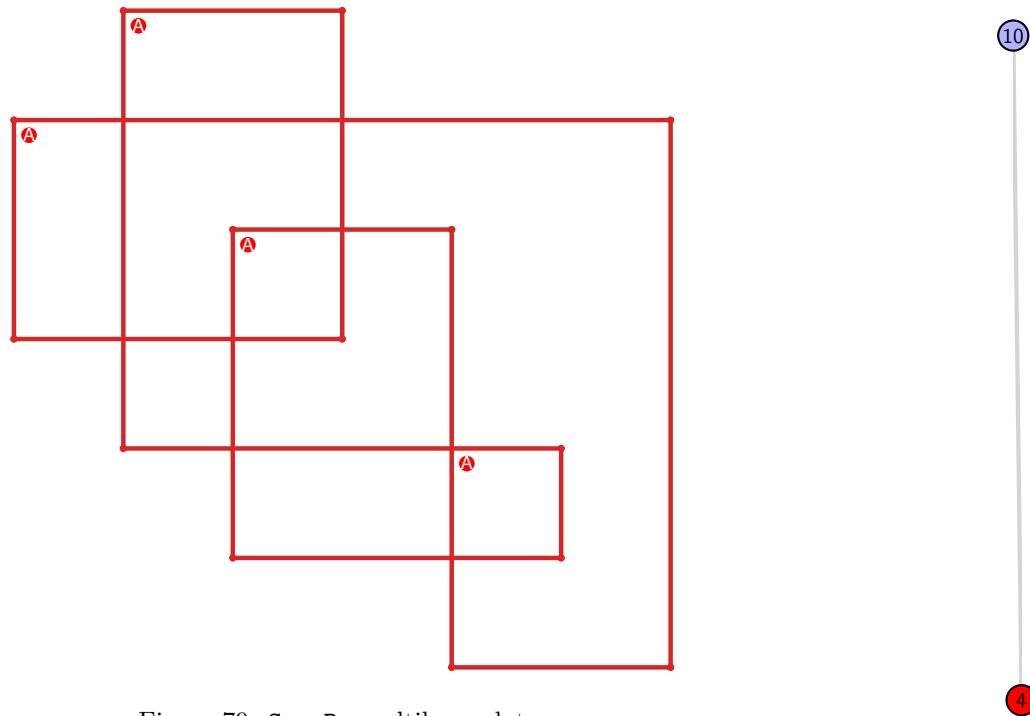


Figure 79: `SnapPy` multiloop plot.

Figure 80: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.20**  $[[16, 5, 1, 6], [6, 3, 7, 4], [4, 15, 5, 16], [1, 13, 2, 12], [2, 11, 3, 12], [7, 11, 8, 10], [14, 9, 15, 10], [13, 9, 14, 8]]$

PD code drawn by `SnapPy`:  $[(4, 1, 5, 2), (14, 3, 15, 4), (6, 11, 7, 12), (12, 7, 13, 8), (8, 5, 9, 6), (16, 9, 1, 10), (10, 15, 11, 16), (2, 13, 3, 14)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 2], [0, 1, 6, 0], [0, 7, 4, 4], [1, 3, 3, 5], [1, 4, 7, 6], [2, 5, 7, 7], [3, 6, 6, 5]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 104  
**Pinning number:** 4

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.48  
**Average overall degree:** 2.92

Table 39: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	0	0	0	0	0	5
Nonminimal pinning sets	0	6	29	34	21	7	1	98
Average degree	2.5	2.62	2.83	2.97	3.07	3.14	3.2	

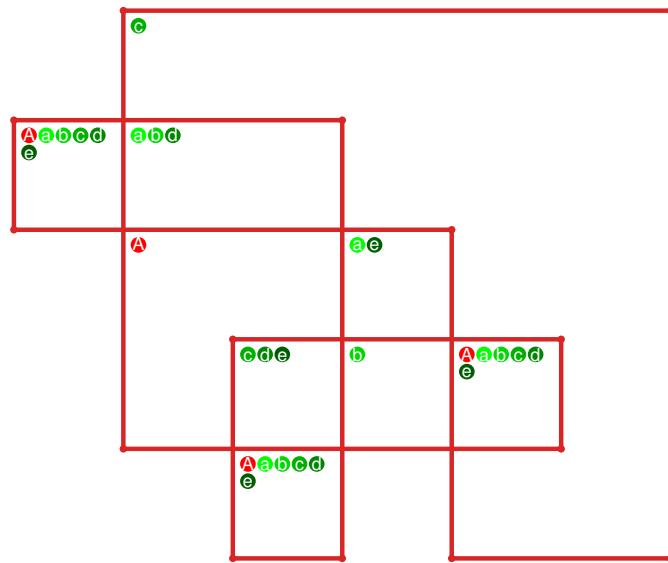


Figure 81: `SnapPy` multiloop plot.

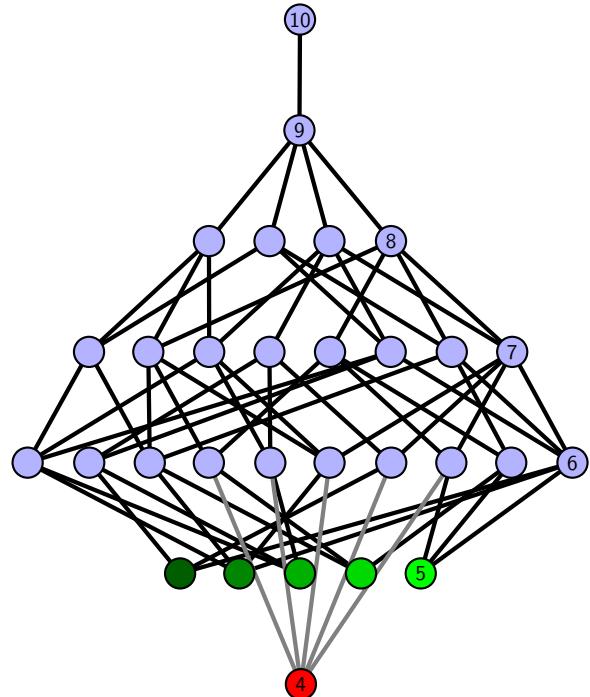


Figure 82: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.21**  $[[7, 16, 8, 1], [6, 13, 7, 14], [15, 12, 16, 13], [8, 2, 9, 1], [14, 5, 15, 6], [11, 2, 12, 3], [9, 4, 10, 5], [3, 10, 4, 11]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (11, 2, 12, 3), (16, 7, 1, 8), (12, 9, 13, 10), (3, 10, 4, 11), (4, 13, 5, 14), (14, 5, 15, 6), (8, 15, 9, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 5, 6, 0], [1, 6, 2, 1], [2, 7, 7, 3], [3, 7, 7, 4], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 48

**Average overall degree:** 2.83

**Pinning number:** 5

Table 40: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.2	2.56	2.8	2.98	3.11	3.2	

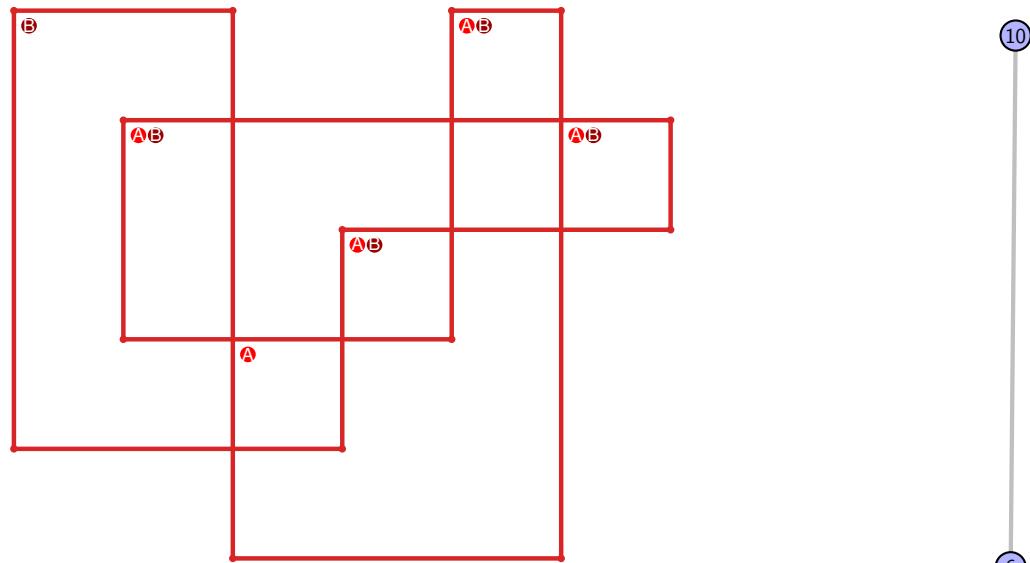


Figure 83: SnapPy multiloop plot.

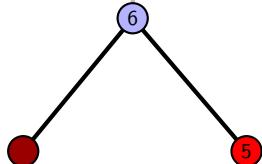


Figure 84: Minimal join sub-semi-lattice of minimal pinning sets.

4.6.22 [[16, 11, 1, 12], [12, 8, 13, 7], [15, 2, 16, 3], [10, 1, 11, 2], [8, 5, 9, 6], [13, 6, 14, 7], [3, 14, 4, 15], [4, 9, 5, 10]]

PD code drawn by SnapPy: [(5, 2, 6, 3), (10, 3, 11, 4), (4, 9, 5, 10), (1, 6, 2, 7), (13, 8, 14, 9), (16, 11, 1, 12), (7, 14, 8, 15), (12, 15, 13, 16)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 3], [0, 2, 7, 0], [1, 7, 7, 5], [1, 4, 6, 1], [2, 5, 7, 2], [3, 6, 4, 4]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 60  
**Pinning number:** 5

**Average optimal degree:** 2.3  
**Average minimal degree:** 2.3  
**Average overall degree:** 2.84

Table 41: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	4	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	14	20	15	6	1	56
Average degree	2.3	2.64	2.86	3.0	3.11	3.2	

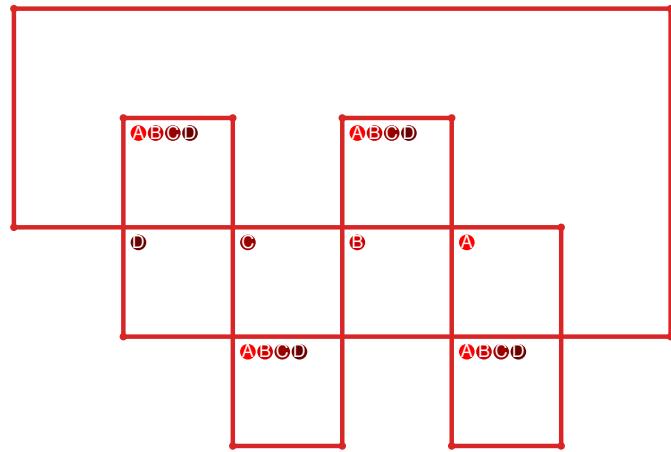


Figure 85: SnapPy multiloop plot.

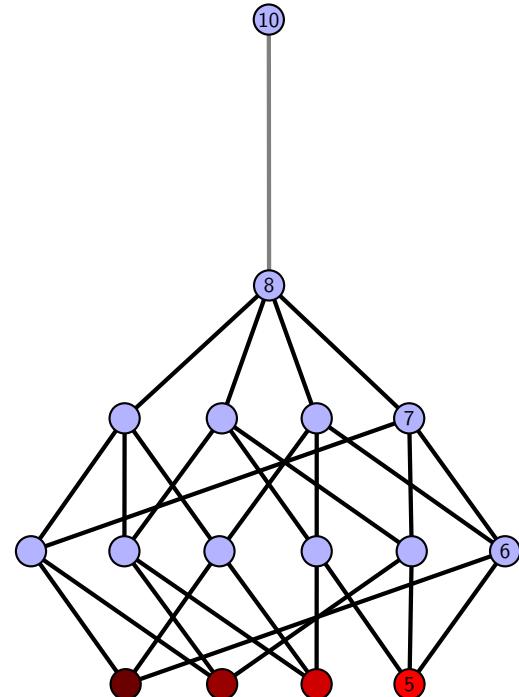


Figure 86: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.23**  $[[5, 16, 6, 1], [4, 11, 5, 12], [15, 2, 16, 3], [6, 2, 7, 1], [12, 9, 13, 10], [10, 3, 11, 4], [14, 7, 15, 8], [8, 13, 9, 14]]$

PD code drawn by `SnapPy`:  $[(10, 3, 11, 4), (8, 5, 9, 6), (16, 7, 1, 8), (4, 9, 5, 10), (1, 12, 2, 13), (13, 2, 14, 3), (11, 14, 12, 15), (6, 15, 7, 16)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 5], [0, 5, 6, 3], [0, 2, 6, 0], [1, 7, 7, 5], [1, 4, 2, 1], [2, 7, 7, 3], [4, 6, 6, 4]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 48

**Average overall degree:** 2.83

**Pinning number:** 5

Table 42: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.2	2.56	2.8	2.98	3.11	3.2	

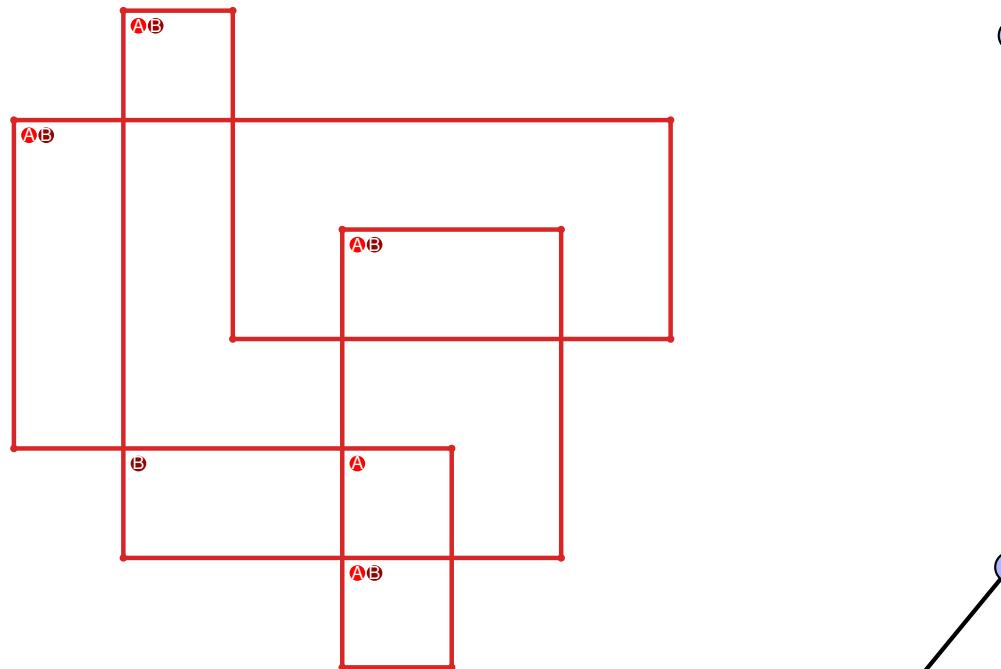


Figure 87: `SnapPy` multiloop plot.

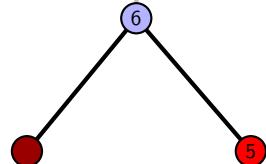


Figure 88: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.24**  $[[5, 16, 6, 1], [15, 4, 16, 5], [6, 14, 7, 13], [1, 11, 2, 10], [3, 14, 4, 15], [7, 12, 8, 13], [11, 8, 12, 9], [2, 9, 3, 10]]$

PD code drawn by SnapPy:  $[(13, 2, 14, 3), (9, 6, 10, 7), (16, 7, 1, 8), (8, 15, 9, 16), (5, 10, 6, 11), (11, 4, 12, 5), (1, 12, 2, 13), (3, 14, 4, 15)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 5], [0, 6, 7, 7], [1, 7, 2, 1], [2, 6, 6, 2], [3, 5, 5, 7], [3, 6, 4, 3]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.76

**Pinning number:** 5

Table 43: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.69	2.9	3.07	3.2	

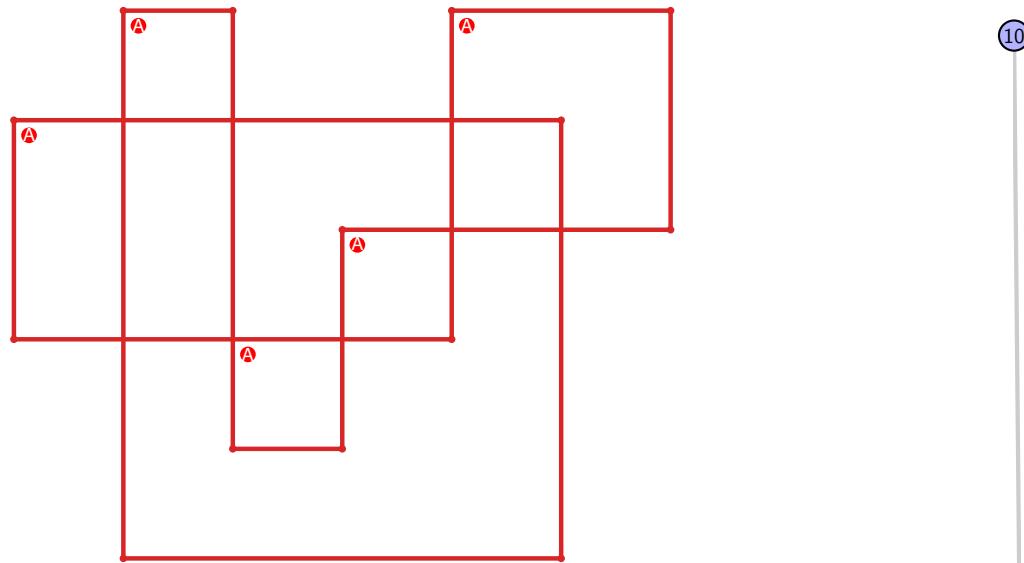


Figure 89: SnapPy multiloop plot.

Figure 90: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.25**  $[[5, 16, 6, 1], [4, 11, 5, 12], [15, 10, 16, 11], [6, 2, 7, 1], [12, 3, 13, 4], [9, 14, 10, 15], [2, 8, 3, 7], [13, 8, 14, 9]]$

PD code drawn by `SnapPy`:  $[(10, 1, 11, 2), (5, 16, 6, 1), (11, 6, 12, 7), (13, 8, 14, 9), (2, 9, 3, 10), (7, 12, 8, 13), (3, 14, 4, 15), (15, 4, 16, 5)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 5], [0, 6, 6, 0], [1, 6, 7, 1], [2, 7, 7, 2], [3, 7, 4, 3], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.76

**Pinning number:** 5

Table 44: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.69	2.9	3.07	3.2	

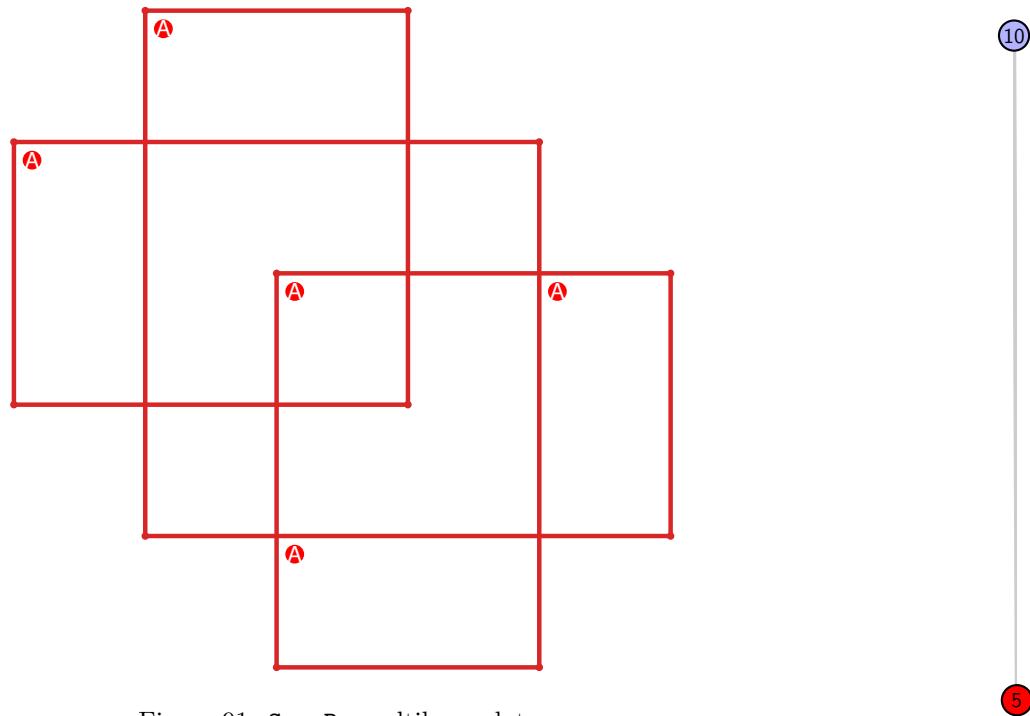


Figure 91: `SnapPy` multiloop plot.

Figure 92: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.26**  $[[16, 9, 1, 10], [10, 6, 11, 5], [15, 4, 16, 5], [8, 13, 9, 14], [1, 7, 2, 6], [11, 2, 12, 3], [3, 14, 4, 15], [12, 7, 13, 8]]$

PD code drawn by SnapPy:  $[(9, 16, 10, 1), (13, 2, 14, 3), (3, 8, 4, 9), (4, 15, 5, 16), (10, 5, 11, 6), (14, 7, 15, 8), (6, 11, 7, 12), (1, 12, 2, 13)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 6, 6], [0, 6, 7, 7], [0, 7, 5, 1], [1, 4, 7, 6], [2, 5, 3, 2], [3, 5, 4, 3]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.65

**Total pinning sets:** 124

**Average overall degree:** 2.98

**Pinning number:** 4

Table 45: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	12	34	40	25	8	1	120
Average degree	2.5	2.74	2.91	3.03	3.11	3.17	3.2	

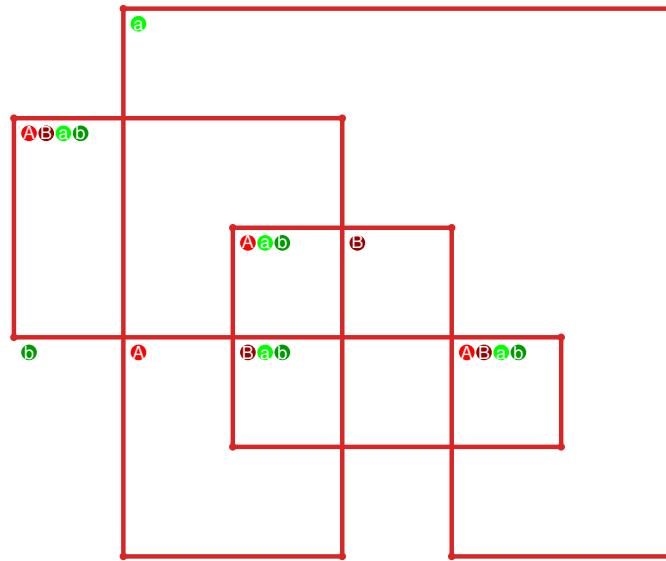


Figure 93: SnapPy multiloop plot.

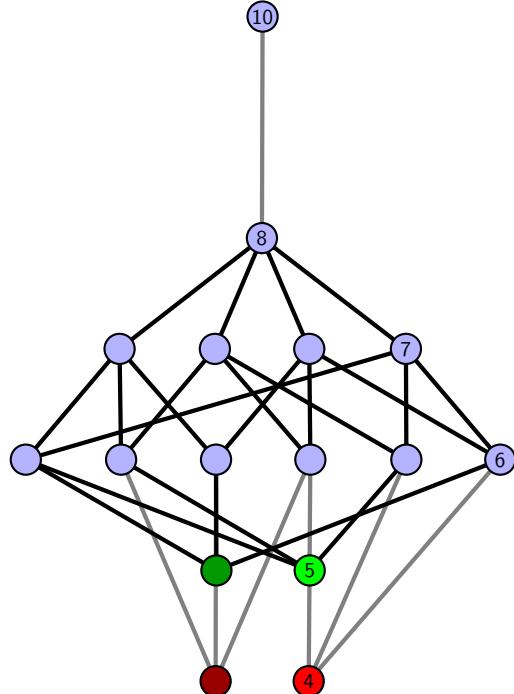


Figure 94: Minimal join sub-semi-lattice of minimal pinning sets.

**4.6.27**  $[[16, 5, 1, 6], [6, 12, 7, 11], [4, 15, 5, 16], [1, 13, 2, 12], [7, 2, 8, 3], [3, 10, 4, 11], [14, 9, 15, 10], [13, 9, 14, 8]]$

PD code drawn by `SnapPy`:  $[(5, 16, 6, 1), (14, 3, 15, 4), (6, 11, 7, 12), (12, 7, 13, 8), (1, 8, 2, 9), (9, 4, 10, 5), (10, 15, 11, 16), (2, 13, 3, 14)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 5, 6, 0], [0, 7, 4, 1], [1, 3, 7, 5], [1, 4, 6, 2], [2, 5, 7, 7], [3, 6, 6, 4]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 114  
**Pinning number:** 4

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.74  
**Average overall degree:** 3.0

Table 46: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	4
Nonminimal pinning sets	0	6	30	39	25	8	1	109
Average degree	2.5	2.76	2.91	3.03	3.11	3.17	3.2	

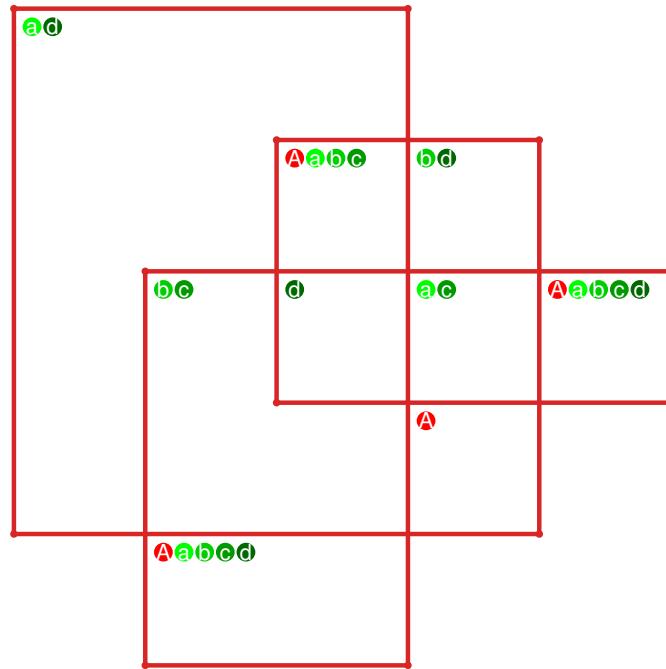


Figure 95: `SnapPy` multiloop plot.

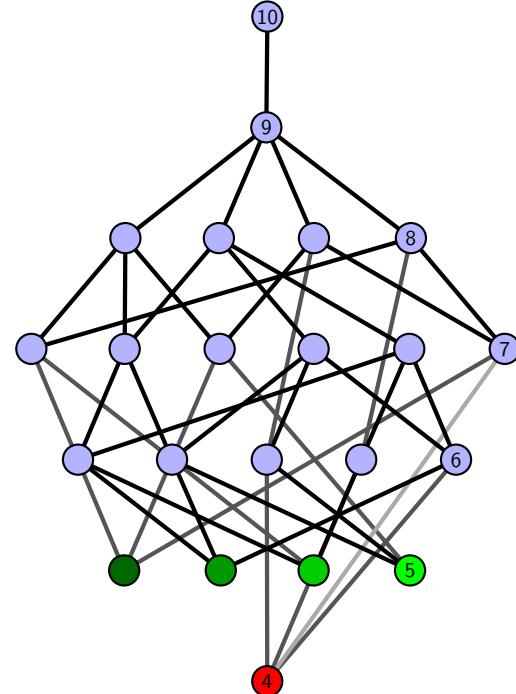


Figure 96: Minimal join sub-semi-lattice of minimal pinning sets.

## 4.7 11 regions

4.7.1  $[[9, 18, 10, 1], [17, 8, 18, 9], [10, 2, 11, 1], [7, 16, 8, 17], [2, 12, 3, 11], [15, 6, 16, 7], [12, 4, 13, 3], [5, 14, 6, 15], [4, 14, 5, 13]]$

PD code drawn by `SnapPy`:  $[(1, 10, 2, 11), (13, 4, 14, 5), (15, 6, 16, 7), (9, 18, 10, 1), (11, 2, 12, 3), (3, 12, 4, 13), (5, 14, 6, 15), (7, 16, 8, 17), (17, 8, 18, 9)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 5, 1], [2, 6, 6, 2], [3, 7, 7, 3], [4, 8, 8, 4], [5, 8, 8, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 4

**Average overall degree:** 2.67

**Pinning number:** 9

Table 47: Pinning sets/average degree by cardinal

Cardinal	9	10	11	Total
Optimal pinning sets	1	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0
Nonminimal pinning sets	0	2	1	3
Average degree	2.0	2.7	3.27	

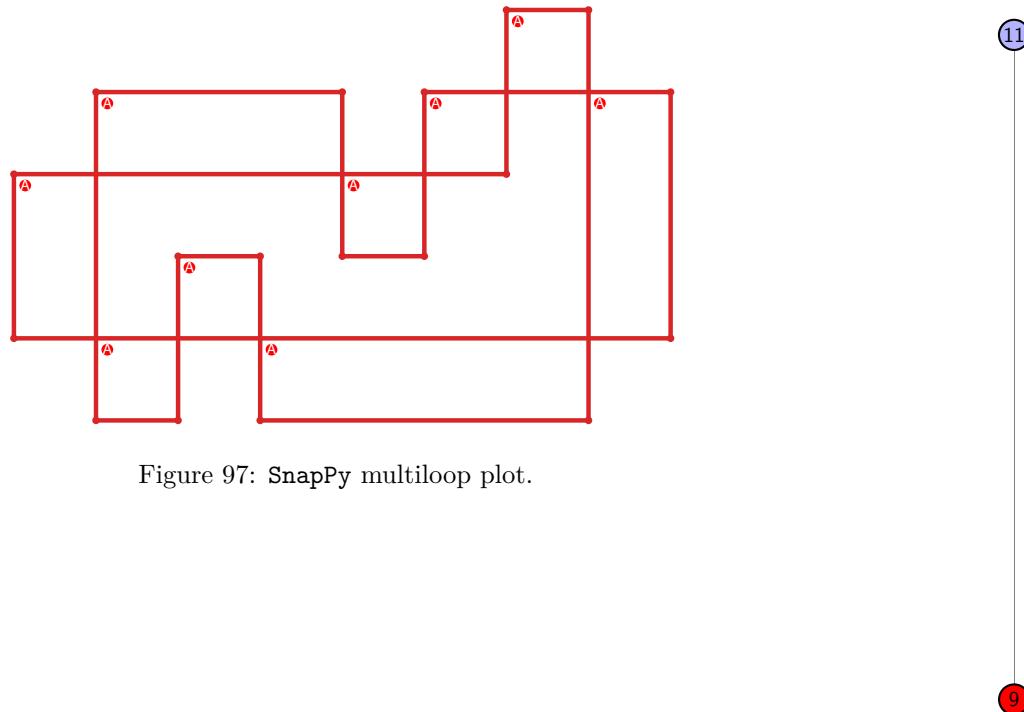


Figure 97: `SnapPy` multiloop plot.

Figure 98: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.2 [[9, 18, 10, 1], [17, 8, 18, 9], [10, 8, 11, 7], [1, 16, 2, 17], [11, 6, 12, 7], [15, 2, 16, 3], [5, 12, 6, 13], [3, 14, 4, 15], [13, 4, 14, 5]]

PD code drawn by SnapPy: [(13, 18, 14, 1), (11, 2, 12, 3), (9, 4, 10, 5), (5, 8, 6, 9), (15, 6, 16, 7), (3, 10, 4, 11), (1, 12, 2, 13), (17, 14, 18, 15), (7, 16, 8, 17)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 5, 1], [2, 6, 6, 2], [3, 7, 7, 3], [4, 8, 8, 4], [5, 8, 8, 5], [6, 7, 7, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.74

**Pinning number:** 7

Table 48: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.44	2.78	3.05	3.27	

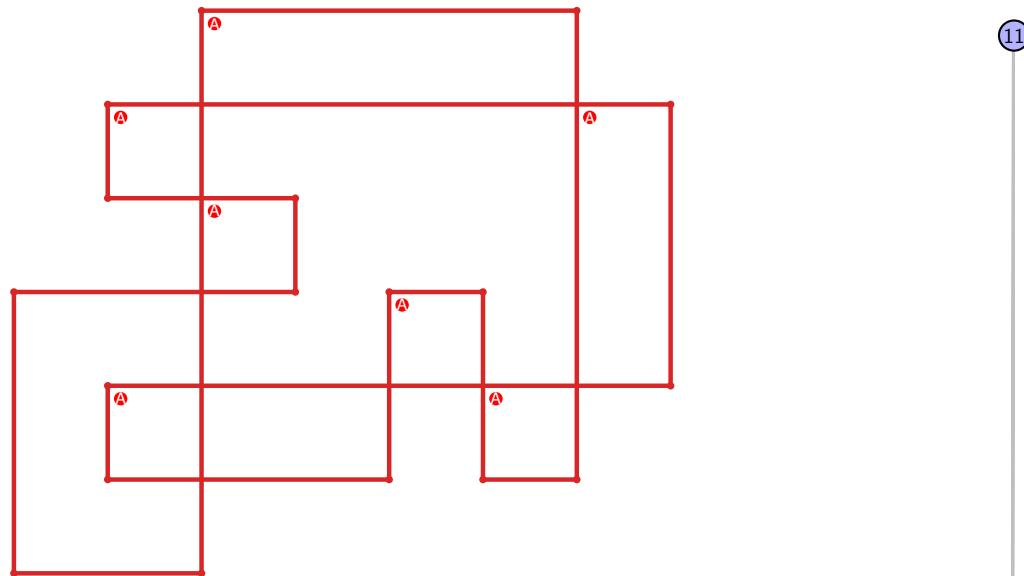


Figure 99: SnapPy multiloop plot.

11

7

Figure 100: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.3  $[[9, 18, 10, 1], [17, 8, 18, 9], [10, 2, 11, 1], [7, 16, 8, 17], [2, 12, 3, 11], [13, 6, 14, 7], [15, 4, 16, 5], [12, 4, 13, 3], [5, 14, 6, 15]]$

PD code drawn by SnapPy:  $[(1, 10, 2, 11), (13, 2, 14, 3), (11, 4, 12, 5), (15, 6, 16, 7), (9, 18, 10, 1), (3, 12, 4, 13), (5, 14, 6, 15), (7, 16, 8, 17), (17, 8, 18, 9)]$

Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 7, 7, 2], [3, 7, 8, 8], [3, 8, 8, 7], [4, 6, 5, 4], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.74

**Pinning number:** 7

Table 49: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.44	2.78	3.05	3.27	

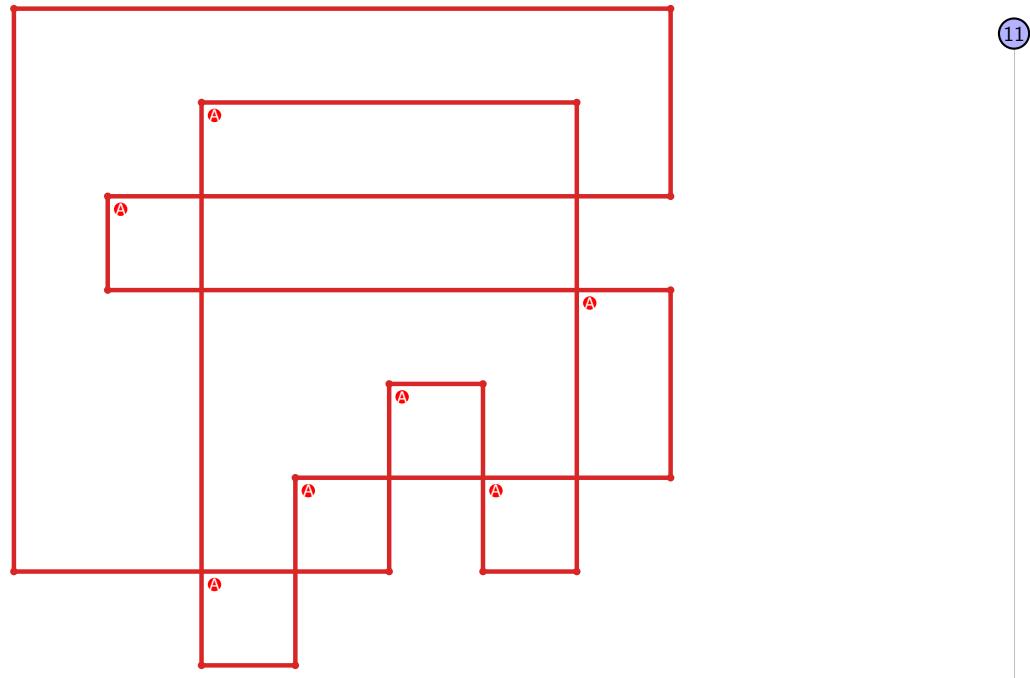


Figure 101: SnapPy multiloop plot.

11

7

Figure 102: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.4  $[[9, 18, 10, 1], [13, 8, 14, 9], [17, 4, 18, 5], [10, 2, 11, 1], [3, 12, 4, 13], [7, 14, 8, 15], [5, 16, 6, 17], [2, 12, 3, 11], [15, 6, 16, 7]]$

PD code drawn by `SnapPy`:  $[(14, 1, 15, 2), (12, 3, 13, 4), (10, 5, 11, 6), (8, 17, 9, 18), (18, 9, 1, 10), (4, 11, 5, 12), (2, 13, 3, 14), (6, 15, 7, 16), (16, 7, 17, 8)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 4], [0, 7, 7, 0], [1, 7, 7, 2], [1, 8, 8, 1], [2, 8, 8, 2], [3, 4, 4, 3], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.74

**Pinning number:** 7

Table 50: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.44	2.78	3.05	3.27	

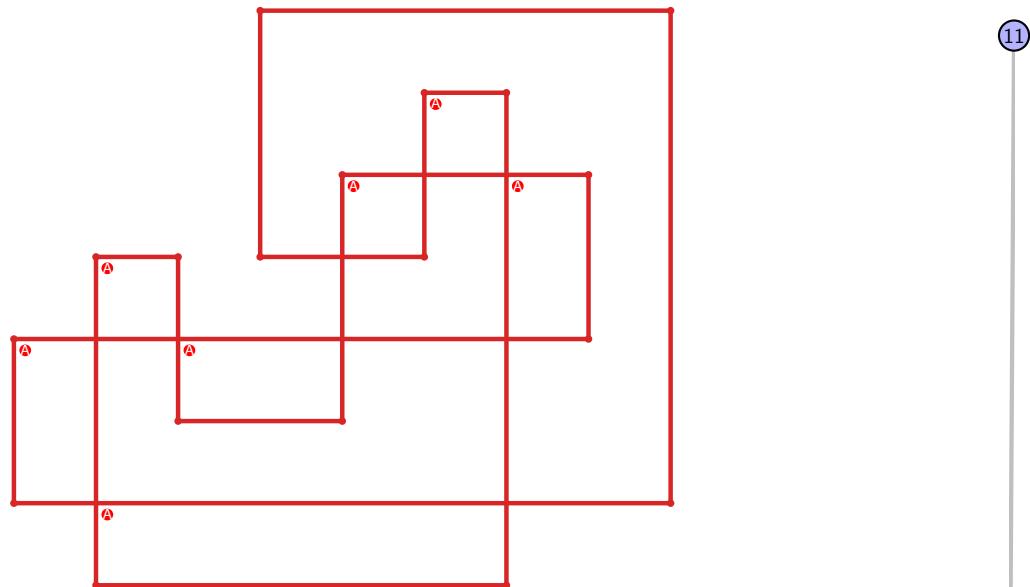


Figure 103: `SnapPy` multiloop plot.

7

Figure 104: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.5  $[[9, 18, 10, 1], [11, 8, 12, 9], [17, 2, 18, 3], [10, 2, 11, 1], [7, 16, 8, 17], [12, 4, 13, 3], [15, 6, 16, 7], [4, 14, 5, 13], [5, 14, 6, 15]]$

PD code drawn by `SnapPy`:  $[(18, 9, 1, 10), (12, 3, 13, 4), (8, 5, 9, 6), (10, 1, 11, 2), (2, 11, 3, 12), (4, 13, 5, 14), (6, 15, 7, 16), (16, 7, 17, 8), (14, 17, 15, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 4, 3], [0, 2, 1, 0], [1, 2, 6, 6], [1, 7, 7, 2], [4, 8, 8, 4], [5, 8, 8, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 51: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

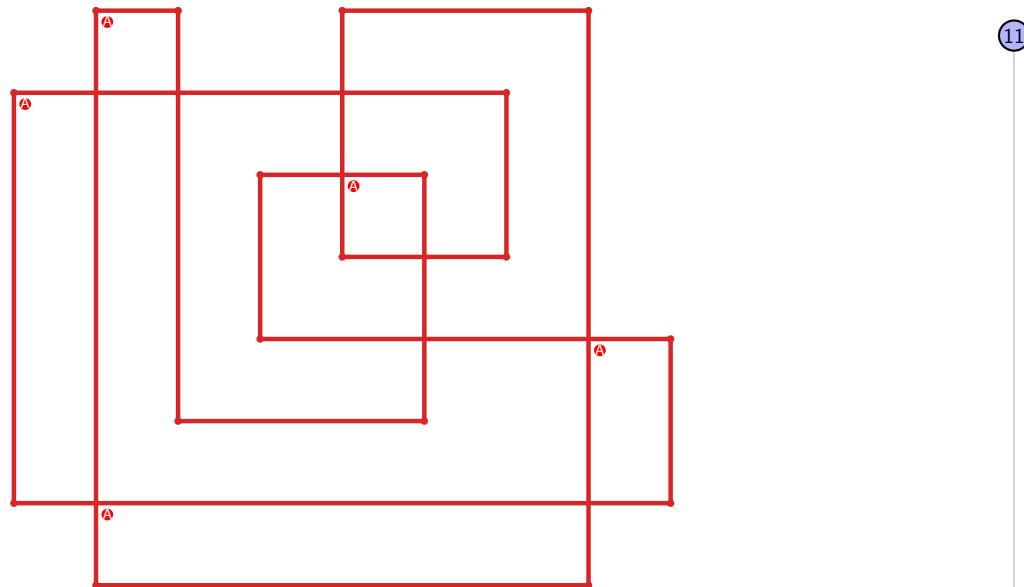


Figure 105: `SnapPy` multiloop plot.

(11)

Figure 106: Minimal join sub-semi-lattice of minimal pinning sets.

5

4.7.6  $[[9, 18, 10, 1], [11, 8, 12, 9], [17, 2, 18, 3], [10, 2, 11, 1], [7, 14, 8, 15], [12, 4, 13, 3], [5, 16, 6, 17], [15, 6, 16, 7], [13, 4, 14, 5]]$

PD code drawn by `SnapPy`:  $[(15, 18, 16, 1), (1, 10, 2, 11), (11, 2, 12, 3), (13, 4, 14, 5), (9, 6, 10, 7), (5, 12, 6, 13), (3, 14, 4, 15), (7, 16, 8, 17), (17, 8, 18, 9)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 6, 3], [0, 2, 1, 0], [1, 7, 7, 8], [1, 8, 8, 2], [2, 8, 7, 7], [4, 6, 6, 4], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 52: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

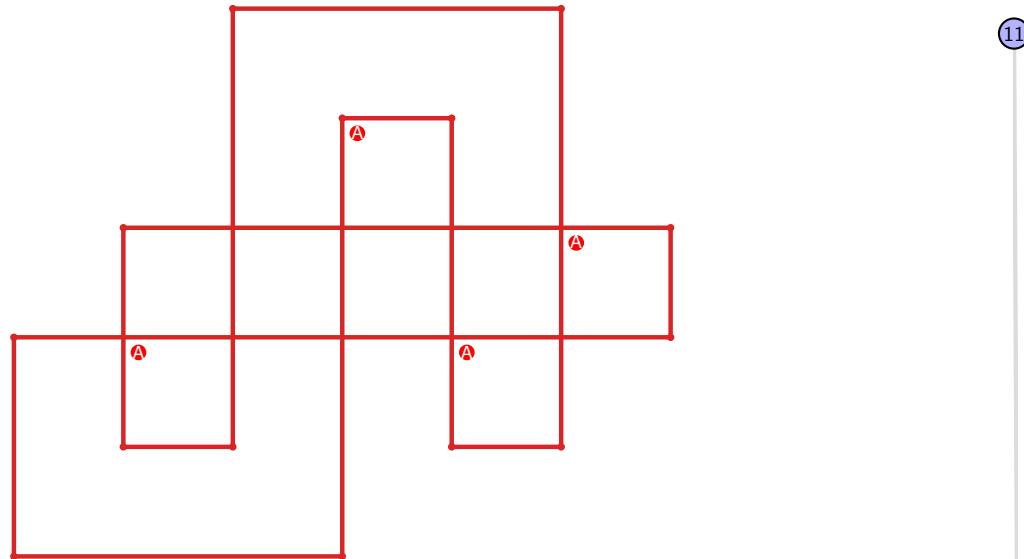


Figure 107: `SnapPy` multiloop plot.

4

Figure 108: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.7  $[[3, 18, 4, 1], [9, 2, 10, 3], [17, 4, 18, 5], [1, 8, 2, 9], [10, 8, 11, 7], [5, 14, 6, 15], [16, 11, 17, 12], [13, 6, 14, 7], [15, 13, 16, 12]]$

PD code drawn by SnapPy:  $[(7, 18, 8, 1), (15, 2, 16, 3), (11, 4, 12, 5), (5, 8, 6, 9), (17, 6, 18, 7), (14, 9, 15, 10), (10, 13, 11, 14), (3, 12, 4, 13), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 6, 7], [2, 7, 7, 8], [2, 8, 8, 4], [4, 8, 5, 5], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 53: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

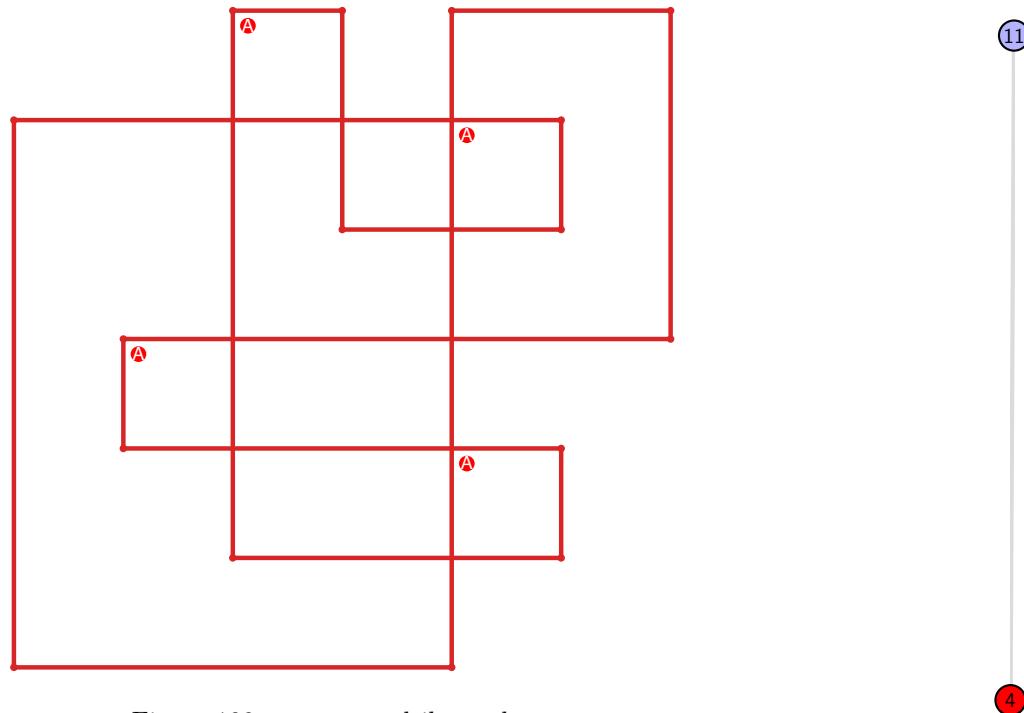


Figure 109: SnapPy multiloop plot.

Figure 110: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.8 [[3, 18, 4, 1], [2, 13, 3, 14], [8, 17, 9, 18], [4, 11, 5, 12], [1, 15, 2, 14], [15, 12, 16, 13], [16, 7, 17, 8], [9, 7, 10, 6], [10, 5, 11, 6]]

PD code drawn by SnapPy: [(7, 2, 8, 3), (3, 6, 4, 7), (13, 4, 14, 5), (1, 8, 2, 9), (12, 9, 13, 10), (17, 10, 18, 11), (11, 16, 12, 17), (5, 14, 6, 15), (18, 15, 1, 16)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 7], [0, 8, 8, 5], [0, 5, 1, 1], [1, 4, 3, 6], [2, 5, 7, 2], [2, 6, 8, 8], [3, 7, 7, 3]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 54: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

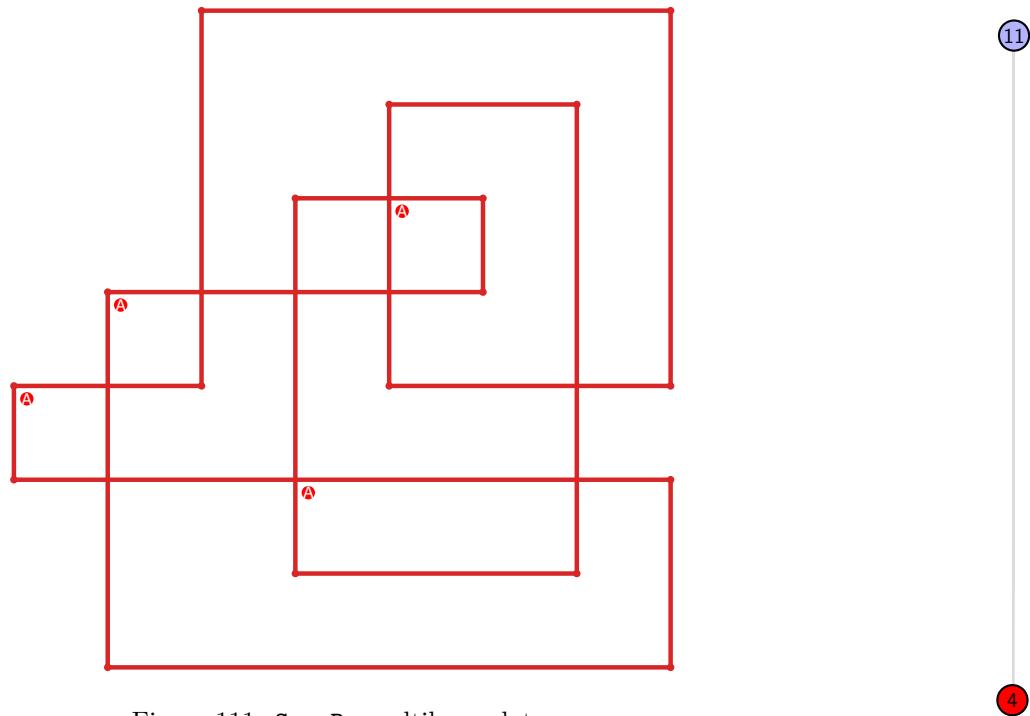


Figure 111: SnapPy multiloop plot.

Figure 112: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.9  $[[3, 18, 4, 1], [11, 2, 12, 3], [17, 6, 18, 7], [4, 16, 5, 15], [1, 10, 2, 11], [12, 10, 13, 9], [7, 14, 8, 15], [5, 16, 6, 17], [13, 8, 14, 9]]$

PD code drawn by SnapPy:  $[(9, 18, 10, 1), (15, 4, 16, 5), (11, 6, 12, 7), (7, 10, 8, 11), (17, 8, 18, 9), (5, 12, 6, 13), (13, 2, 14, 3), (3, 14, 4, 15), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 7], [0, 7, 7, 6], [0, 5, 1, 1], [1, 4, 8, 8], [2, 8, 8, 3], [2, 3, 3, 2], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 55: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

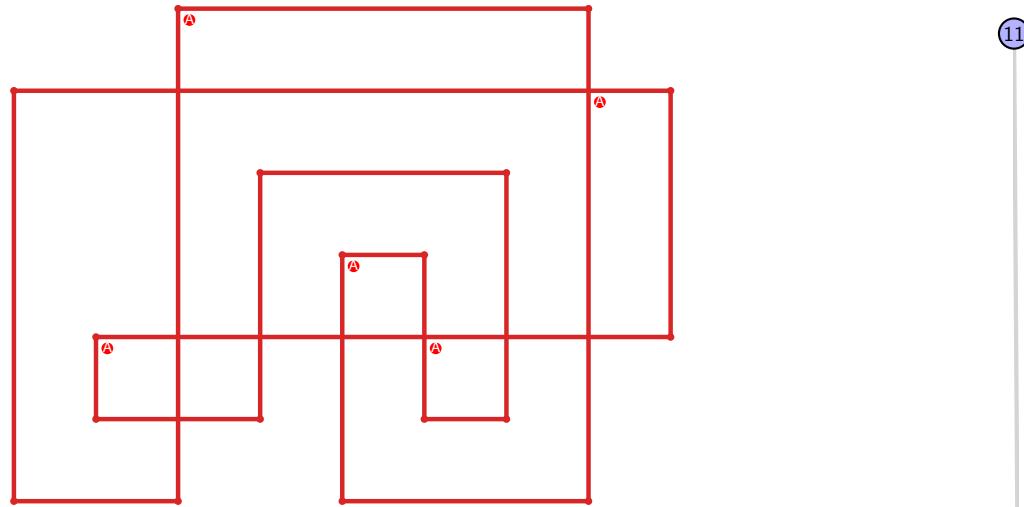


Figure 113: SnapPy multiloop plot.

5

Figure 114: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.10  $[[3, 18, 4, 1], [11, 2, 12, 3], [17, 4, 18, 5], [1, 10, 2, 11], [12, 10, 13, 9], [5, 14, 6, 15], [7, 16, 8, 17], [13, 8, 14, 9], [6, 16, 7, 15]]$

PD code drawn by SnapPy:  $[(9, 18, 10, 1), (15, 2, 16, 3), (11, 6, 12, 7), (7, 10, 8, 11), (17, 8, 18, 9), (3, 12, 4, 13), (13, 4, 14, 5), (5, 14, 6, 15), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 7], [2, 7, 8, 8], [2, 8, 8, 7], [4, 6, 5, 4], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 56: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

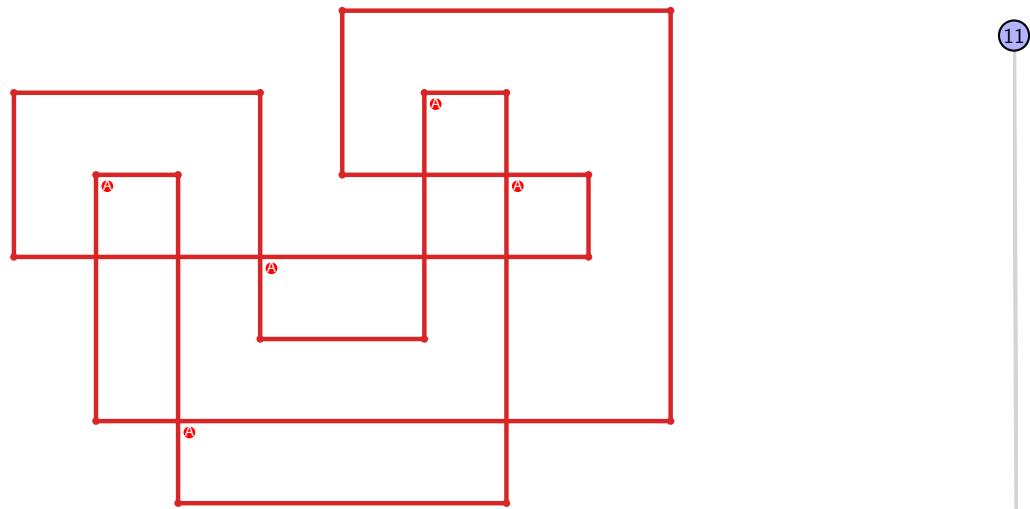


Figure 115: SnapPy multiloop plot.

5

Figure 116: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.11  $[[3, 18, 4, 1], [11, 2, 12, 3], [17, 8, 18, 9], [4, 14, 5, 13], [1, 10, 2, 11], [12, 10, 13, 9], [5, 16, 6, 17], [7, 14, 8, 15], [15, 6, 16, 7]]$

PD code drawn by SnapPy:  $[(9, 18, 10, 1), (13, 4, 14, 5), (15, 6, 16, 7), (7, 10, 8, 11), (17, 8, 18, 9), (11, 2, 12, 3), (5, 12, 6, 13), (3, 14, 4, 15), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 7], [0, 7, 6, 5], [0, 5, 1, 1], [1, 4, 3, 2], [2, 3, 8, 8], [2, 8, 8, 3], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 3

Table 57: Pinning sets/average degree by cardinal

Cardinal	3	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.44	2.7	2.87	3.0	3.09	3.17	3.22	3.27	

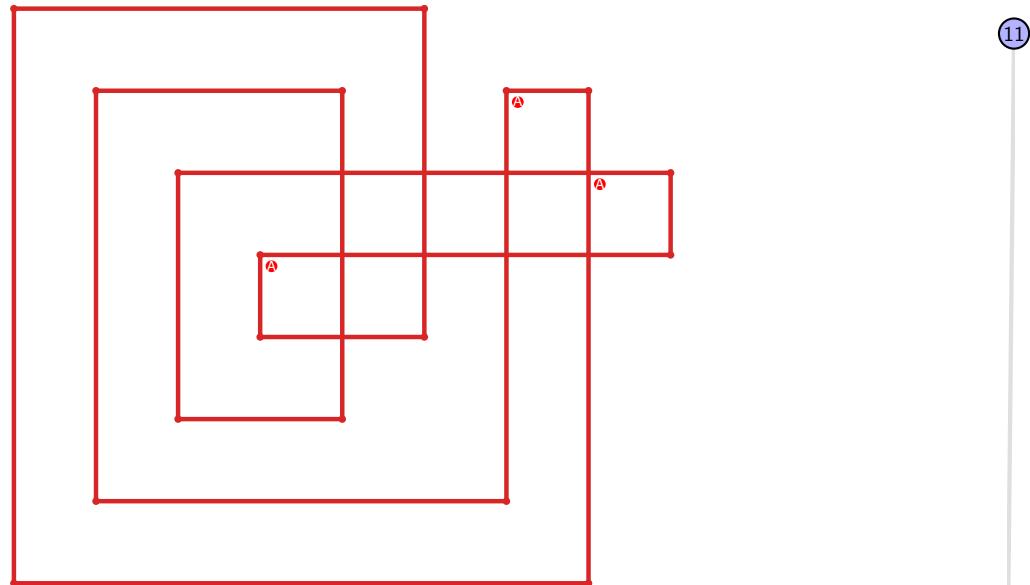


Figure 117: SnapPy multiloop plot.

11  
3

Figure 118: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.12  $[[3, 18, 4, 1], [2, 11, 3, 12], [14, 17, 15, 18], [4, 9, 5, 10], [1, 13, 2, 12], [13, 10, 14, 11], [16, 7, 17, 8], [15, 7, 16, 6], [8, 5, 9, 6]]$

PD code drawn by `SnapPy`:  $[(6, 3, 7, 4), (4, 13, 5, 14), (14, 5, 15, 6), (2, 7, 3, 8), (11, 8, 12, 9), (18, 9, 1, 10), (10, 17, 11, 18), (12, 15, 13, 16), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 7], [0, 8, 8, 5], [0, 5, 1, 1], [1, 4, 3, 2], [2, 8, 7, 7], [2, 6, 6, 8], [3, 7, 6, 3]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 3

Table 58: Pinning sets/average degree by cardinal

Cardinal	3	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.44	2.7	2.88	3.0	3.09	3.17	3.23	3.27	

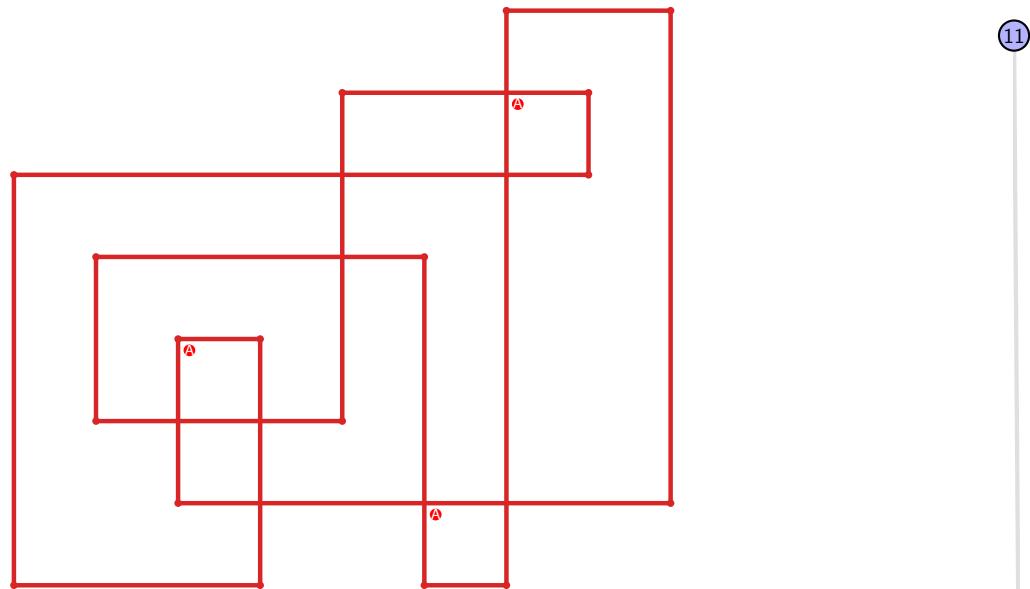


Figure 119: `SnapPy` multiloop plot.

Figure 120: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.13 [[5, 18, 6, 1], [13, 4, 14, 5], [17, 10, 18, 11], [6, 2, 7, 1], [3, 12, 4, 13], [14, 12, 15, 11], [9, 16, 10, 17], [2, 8, 3, 7], [15, 8, 16, 9]]

PD code drawn by `SnapPy`: [(11, 2, 12, 3), (5, 18, 6, 1), (13, 6, 14, 7), (15, 8, 16, 9), (9, 12, 10, 13), (1, 10, 2, 11), (7, 14, 8, 15), (3, 16, 4, 17), (17, 4, 18, 5)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 4, 5], [0, 5, 6, 6], [0, 7, 7, 0], [1, 7, 5, 1], [1, 4, 8, 2], [2, 8, 8, 2], [3, 8, 4, 3], [5, 7, 6, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 59: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

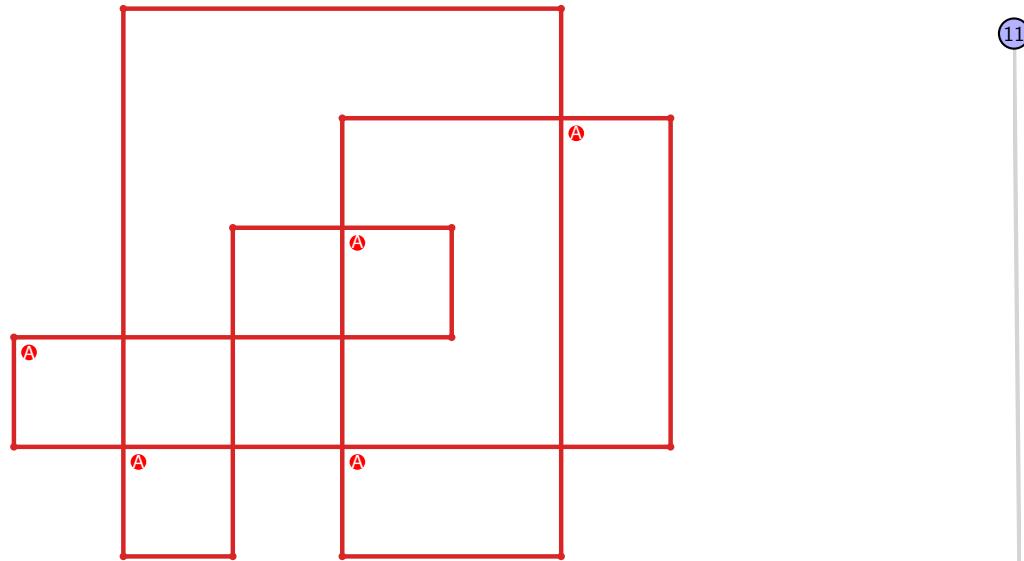


Figure 121: `SnapPy` multiloop plot.



Figure 122: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.14  $[[9, 18, 10, 1], [17, 8, 18, 9], [10, 2, 11, 1], [3, 16, 4, 17], [7, 12, 8, 13], [2, 12, 3, 11], [15, 6, 16, 7], [4, 14, 5, 13], [5, 14, 6, 15]]$

PD code drawn by `SnapPy`:  $[(13, 4, 14, 5), (15, 6, 16, 7), (9, 18, 10, 1), (5, 10, 6, 11), (11, 2, 12, 3), (3, 12, 4, 13), (1, 14, 2, 15), (7, 16, 8, 17), (17, 8, 18, 9)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 5, 0], [1, 5, 6, 7], [1, 7, 6, 5], [2, 4, 3, 2], [3, 4, 8, 8], [3, 8, 8, 4], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 60: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

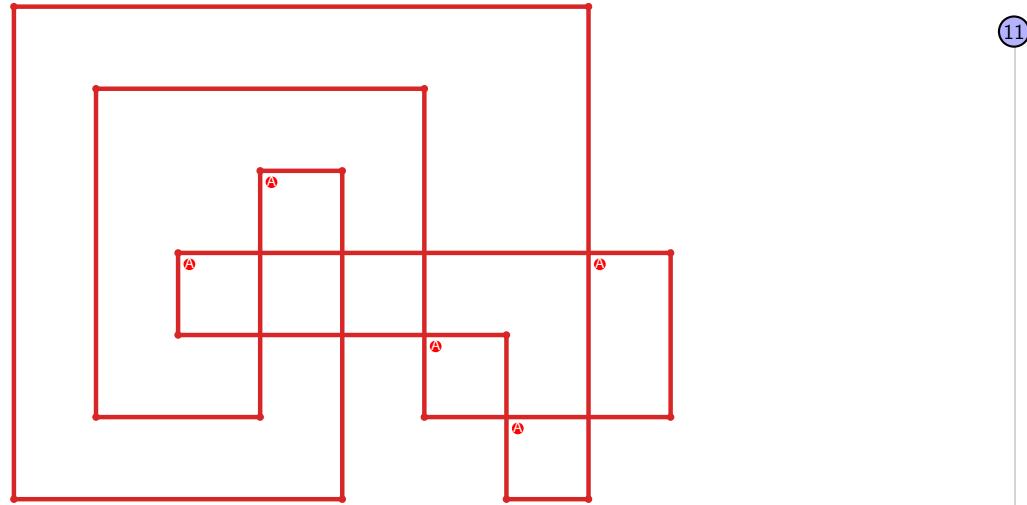


Figure 123: `SnapPy` multiloop plot.

(11)

5

Figure 124: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.15 [[18, 9, 1, 10], [10, 7, 11, 8], [8, 17, 9, 18], [1, 13, 2, 12], [6, 11, 7, 12], [16, 5, 17, 6], [13, 5, 14, 4], [2, 15, 3, 16], [14, 3, 15, 4]]

PD code drawn by SnapPy: [(18, 9, 1, 10), (12, 1, 13, 2), (6, 3, 7, 4), (14, 5, 15, 6), (2, 7, 3, 8), (8, 15, 9, 16), (16, 11, 17, 12), (4, 13, 5, 14), (10, 17, 11, 18)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 4, 2], [0, 1, 5, 0], [0, 6, 7, 4], [1, 3, 5, 1], [2, 4, 7, 6], [3, 5, 8, 8], [3, 8, 8, 5], [6, 7, 7, 6]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1  
**Total pinning sets:** 128  
**Pinning number:** 4

**Average optimal degree:** 2.0  
**Average minimal degree:** 2.0  
**Average overall degree:** 2.9

Table 61: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

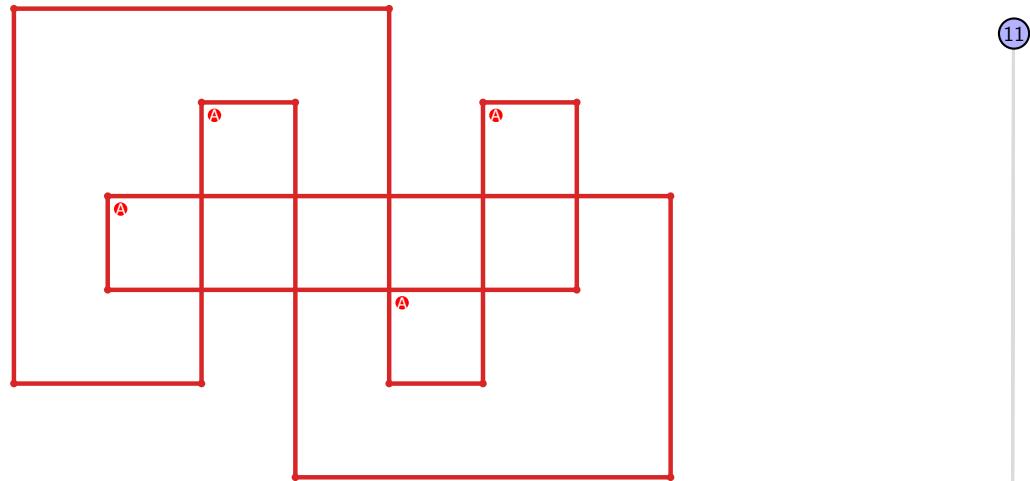


Figure 125: SnapPy multiloop plot.

4

Figure 126: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.16** `[[18, 7, 1, 8], [8, 17, 9, 18], [6, 15, 7, 16], [1, 11, 2, 10], [16, 9, 17, 10], [12, 5, 13, 6], [14, 3, 15, 4], [11, 3, 12, 2], [4, 13, 5, 14]]`

PD code drawn by `SnapPy`: `[(18, 9, 1, 10), (12, 1, 13, 2), (10, 3, 11, 4), (16, 5, 17, 6), (14, 7, 15, 8), (8, 17, 9, 18), (2, 11, 3, 12), (4, 13, 5, 14), (6, 15, 7, 16)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 6], [0, 7, 7, 4], [1, 3, 2, 1], [2, 7, 8, 8], [2, 8, 8, 7], [3, 6, 5, 3], [5, 6, 6, 5]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 62: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

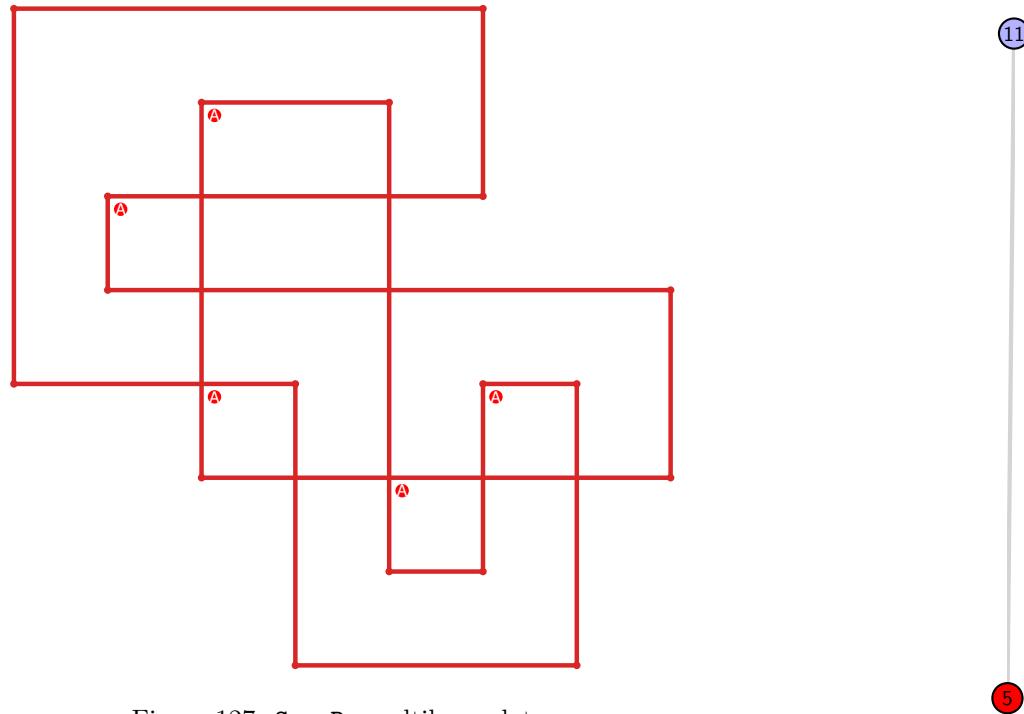


Figure 127: `SnapPy` multiloop plot.

Figure 128: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.17  $[[13, 18, 14, 1], [17, 12, 18, 13], [14, 6, 15, 5], [1, 10, 2, 11], [11, 16, 12, 17], [6, 16, 7, 15], [9, 4, 10, 5], [2, 8, 3, 7], [3, 8, 4, 9]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (2, 11, 3, 12), (4, 17, 5, 18), (18, 5, 1, 6), (16, 7, 17, 8), (14, 9, 15, 10), (12, 3, 13, 4), (8, 13, 9, 14), (10, 15, 11, 16)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 5, 6], [0, 6, 7, 4], [1, 3, 5, 1], [2, 4, 7, 2], [2, 8, 8, 3], [3, 8, 8, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 63: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

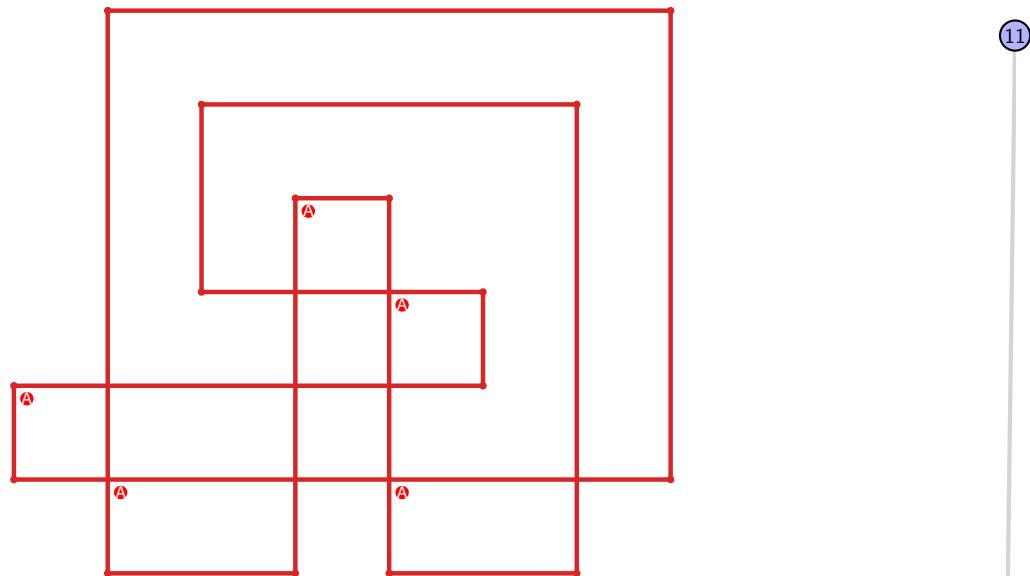


Figure 129: `SnapPy` multiloop plot.



Figure 130: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.18 [[9, 18, 10, 1], [15, 8, 16, 9], [17, 6, 18, 7], [10, 6, 11, 5], [1, 14, 2, 15], [7, 16, 8, 17], [11, 2, 12, 3], [13, 4, 14, 5], [12, 4, 13, 3]]

PD code drawn by `SnapPy`: [(18, 9, 1, 10), (10, 1, 11, 2), (2, 7, 3, 8), (14, 3, 15, 4), (12, 5, 13, 6), (16, 11, 17, 12), (4, 13, 5, 14), (6, 15, 7, 16), (8, 17, 9, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 5], [0, 5, 5, 3], [0, 2, 6, 7], [0, 7, 6, 1], [1, 2, 2, 1], [3, 4, 8, 8], [3, 8, 8, 4], [6, 7, 7, 6]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 120  
**Pinning number:** 5

**Average optimal degree:** 2.3  
**Average minimal degree:** 2.3  
**Average overall degree:** 2.91

Table 64: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	34	35	21	7	1	116
Average degree	2.3	2.63	2.85	3.0	3.11	3.2	3.27	

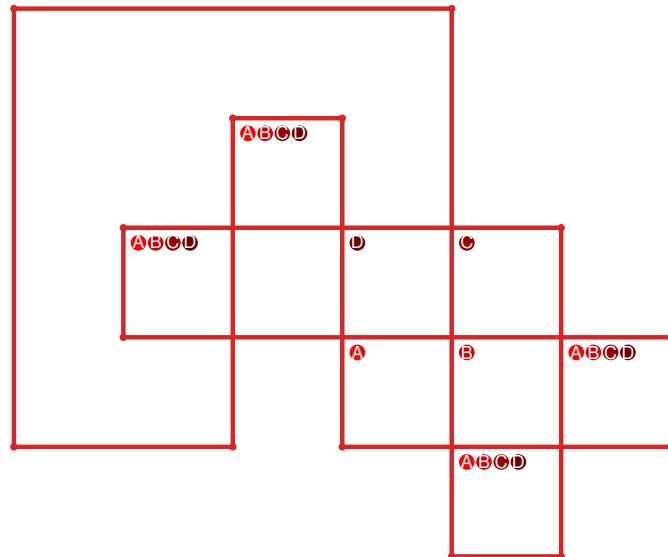


Figure 131: `SnapPy` multiloop plot.

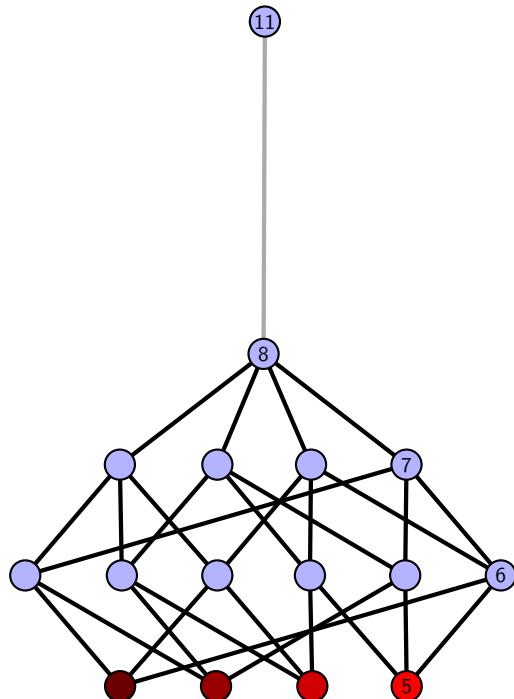


Figure 132: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.19  $[[18, 13, 1, 14], [14, 7, 15, 8], [12, 17, 13, 18], [1, 17, 2, 16], [6, 15, 7, 16], [8, 4, 9, 3], [11, 2, 12, 3], [5, 10, 6, 11], [4, 10, 5, 9]]$

PD code drawn by `SnapPy`:  $[(10, 3, 11, 4), (5, 18, 6, 1), (1, 6, 2, 7), (14, 9, 15, 10), (2, 11, 3, 12), (12, 15, 13, 16), (8, 13, 9, 14), (7, 16, 8, 17), (17, 4, 18, 5)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 3, 0], [0, 2, 6, 4], [1, 3, 7, 1], [1, 8, 8, 6], [2, 5, 7, 3], [4, 6, 8, 8], [5, 7, 7, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 96

**Average overall degree:** 2.9

**Pinning number:** 5

Table 65: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

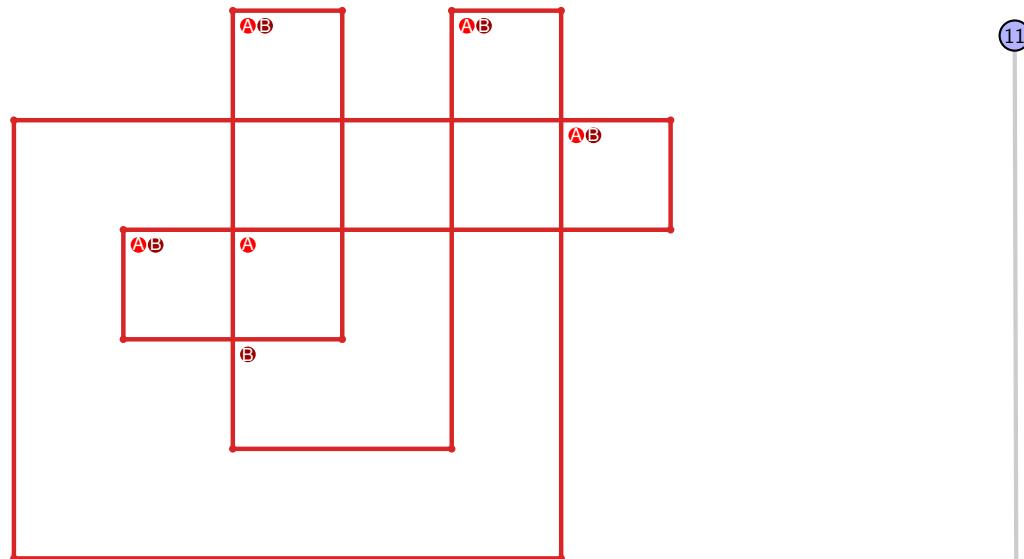


Figure 133: `SnapPy` multiloop plot.

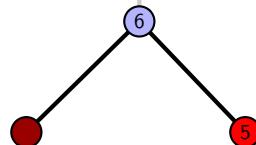


Figure 134: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.20 [[13, 18, 14, 1], [12, 9, 13, 10], [4, 17, 5, 18], [14, 7, 15, 8], [1, 11, 2, 10], [2, 11, 3, 12], [3, 8, 4, 9], [16, 5, 17, 6], [6, 15, 7, 16]]

PD code drawn by SnapPy: [(11, 18, 12, 1), (9, 2, 10, 3), (16, 3, 17, 4), (7, 4, 8, 5), (5, 14, 6, 15), (15, 6, 16, 7), (1, 10, 2, 11), (17, 12, 18, 13), (8, 13, 9, 14)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 7], [0, 8, 8, 6], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 3, 2], [2, 8, 8, 2], [3, 7, 7, 3]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 5

Table 66: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.2	2.5	2.74	2.94	3.09	3.2	3.27	

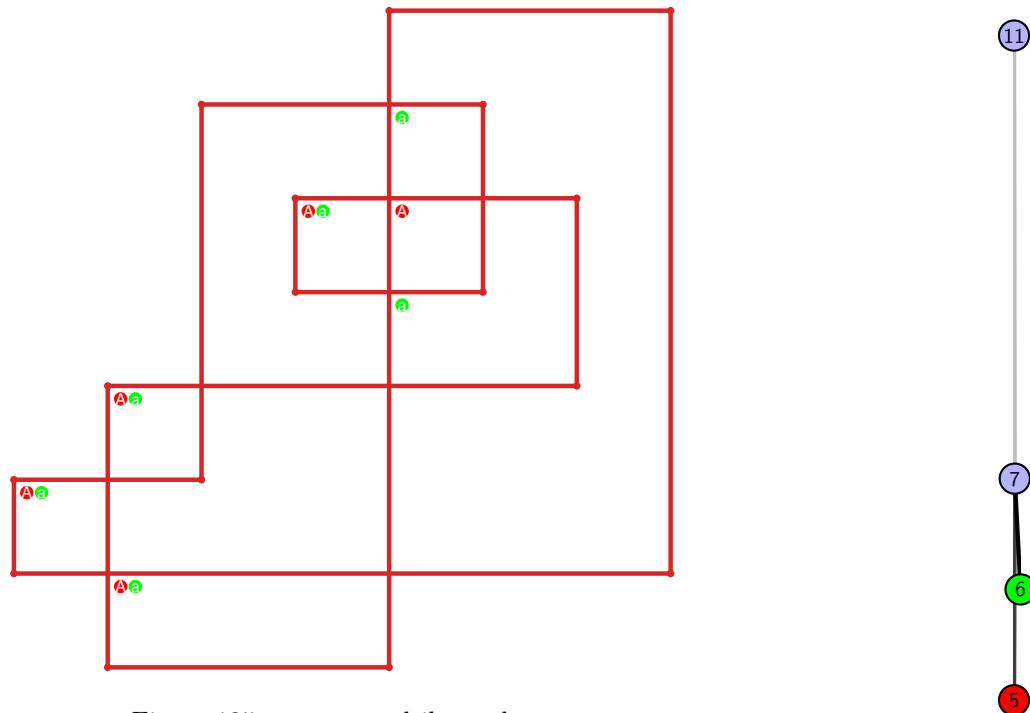


Figure 135: SnapPy multiloop plot.

Figure 136: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.21  $[[3, 18, 4, 1], [2, 9, 3, 10], [12, 17, 13, 18], [4, 13, 5, 14], [1, 11, 2, 10], [11, 8, 12, 9], [16, 7, 17, 8], [5, 15, 6, 14], [6, 15, 7, 16]]$

PD code drawn by SnapPy:  $[(14, 5, 15, 6), (9, 6, 10, 7), (18, 7, 1, 8), (8, 17, 9, 18), (2, 11, 3, 12), (12, 3, 13, 4), (4, 13, 5, 14), (10, 15, 11, 16), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 6, 2], [2, 5, 8, 8], [3, 8, 8, 3], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 100  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.35  
**Average overall degree:** 2.93

Table 67: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	4
Nonminimal pinning sets	0	6	27	33	21	7	1	95
Average degree	2.4	2.53	2.79	2.98	3.11	3.2	3.27	

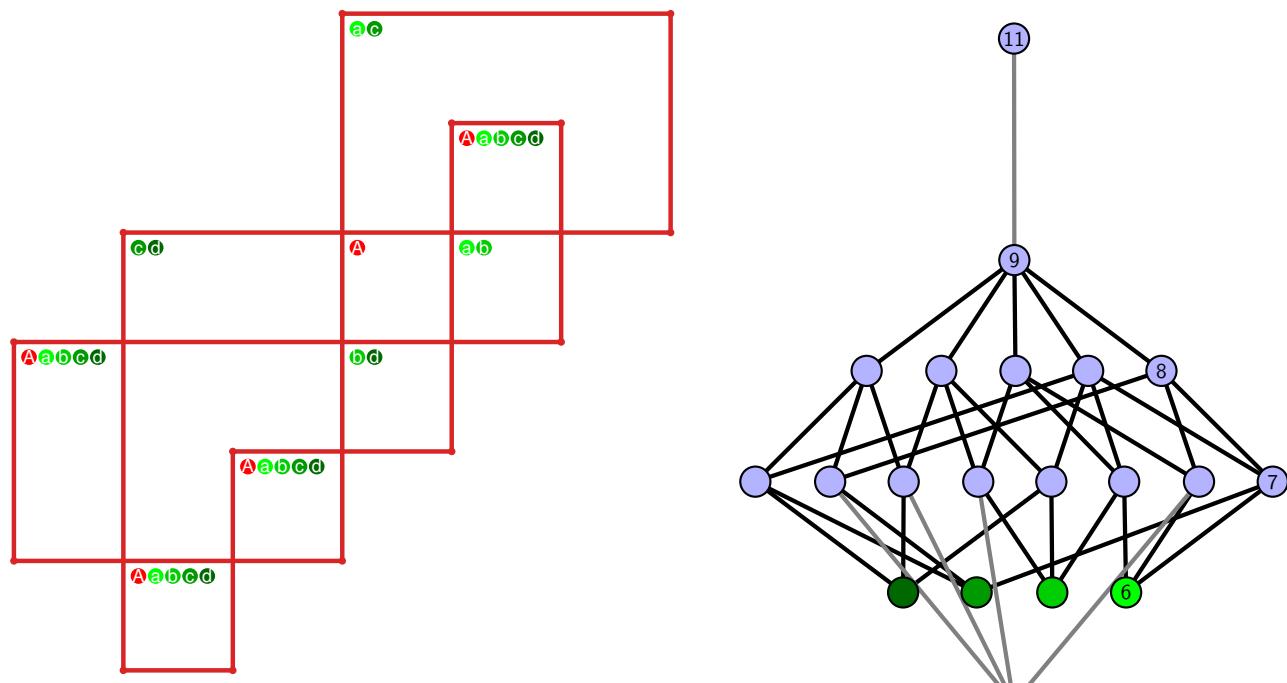


Figure 137: SnapPy multiloop plot.

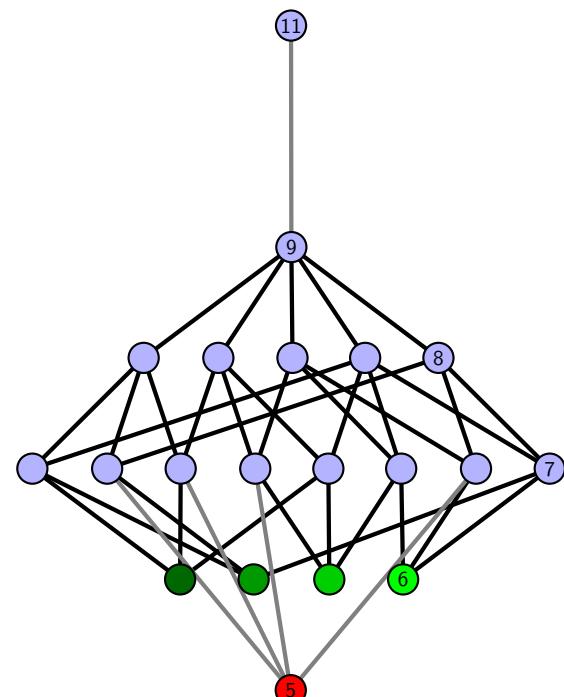


Figure 138: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.22  $[[3, 18, 4, 1], [11, 2, 12, 3], [17, 8, 18, 9], [4, 8, 5, 7], [1, 10, 2, 11], [12, 10, 13, 9], [13, 16, 14, 17], [5, 14, 6, 15], [15, 6, 16, 7]]$

PD code drawn by SnapPy:  $[(9, 18, 10, 1), (13, 4, 14, 5), (5, 2, 6, 3), (15, 6, 16, 7), (7, 10, 8, 11), (17, 8, 18, 9), (3, 12, 4, 13), (11, 14, 12, 15), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 6, 2], [2, 5, 8, 7], [3, 6, 8, 8], [3, 7, 7, 6]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 10  
 Total pinning sets: 320  
 Pinning number: 4

Average optimal degree: 2.75  
 Average minimal degree: 2.75  
 Average overall degree: 3.09

Table 68: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	8	0	0	0	0	0	0	8
Nonminimal pinning sets	0	14	69	101	80	36	9	1	310
Average degree	2.75	2.85	2.99	3.09	3.16	3.21	3.24	3.27	

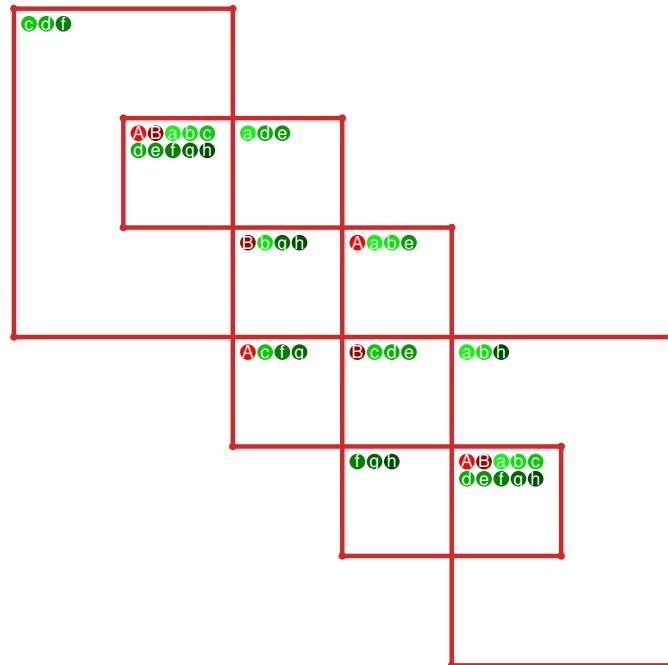


Figure 139: SnapPy multiloop plot.

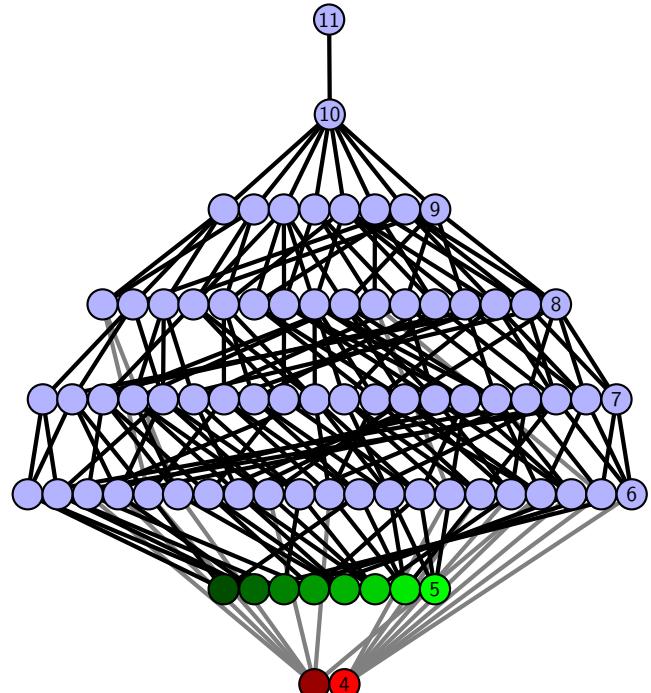


Figure 140: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.23**  $[[3, 18, 4, 1], [2, 9, 3, 10], [17, 4, 18, 5], [1, 11, 2, 10], [11, 8, 12, 9], [5, 12, 6, 13], [13, 16, 14, 17], [14, 7, 15, 8], [6, 15, 7, 16]]$

PD code drawn by `SnapPy`:  $[(15, 2, 16, 3), (12, 5, 13, 6), (9, 6, 10, 7), (18, 7, 1, 8), (8, 17, 9, 18), (3, 10, 4, 11), (4, 13, 5, 14), (11, 14, 12, 15), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 5], [2, 4, 8, 6], [2, 5, 8, 7], [4, 6, 8, 8], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 4

Table 69: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.25	2.55	2.77	2.93	3.06	3.15	3.23	3.27	

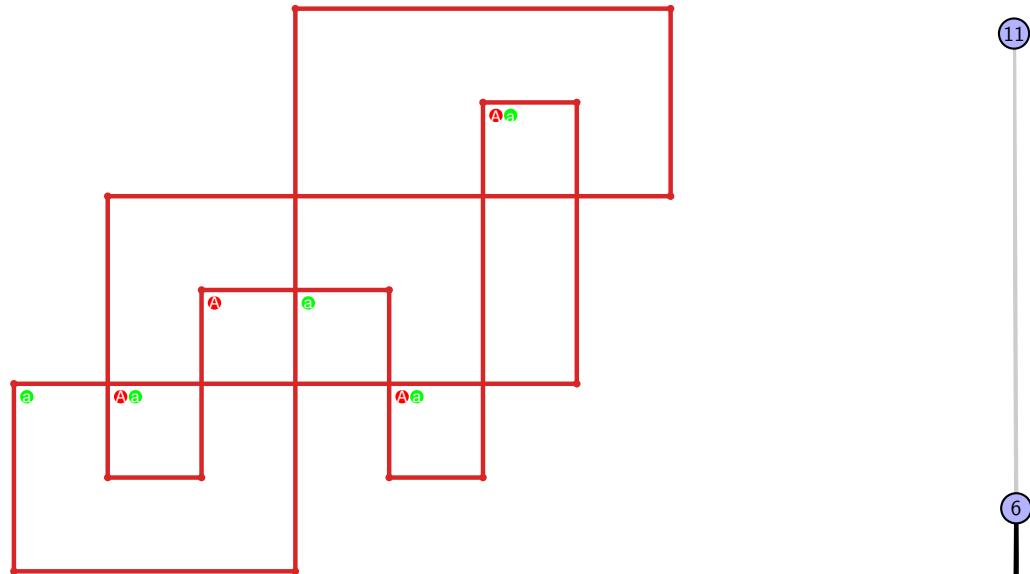


Figure 141: `SnapPy` multiloop plot.



Figure 142: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.24**  $[[18, 9, 1, 10], [10, 13, 11, 14], [4, 17, 5, 18], [5, 8, 6, 9], [1, 12, 2, 13], [11, 2, 12, 3], [14, 3, 15, 4], [7, 16, 8, 17], [6, 16, 7, 15]]$

PD code drawn by `SnapPy`:  $[(11, 18, 12, 1), (2, 5, 3, 6), (15, 6, 16, 7), (16, 9, 17, 10), (7, 10, 8, 11), (12, 3, 13, 4), (4, 13, 5, 14), (1, 14, 2, 15), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 3], [0, 2, 7, 8], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 8, 2], [2, 8, 8, 3], [3, 7, 7, 6]]$

**Total optimal pinning sets:** 5  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 184  
**Pinning number:** 5

**Average optimal degree:** 2.64  
**Average minimal degree:** 2.64  
**Average overall degree:** 3.05

Table 70: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	5	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	25	54	57	32	9	1	178
Average degree	2.64	2.85	3.0	3.11	3.19	3.24	3.27	

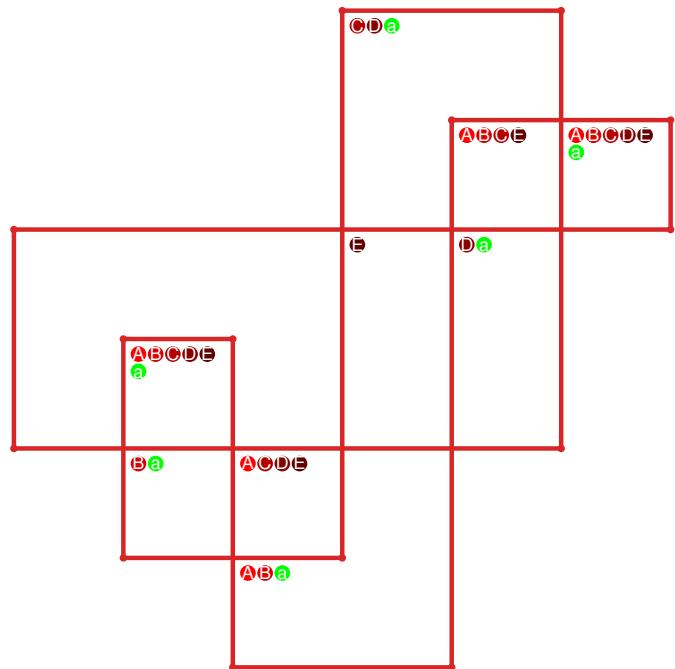


Figure 143: `SnapPy` multiloop plot.

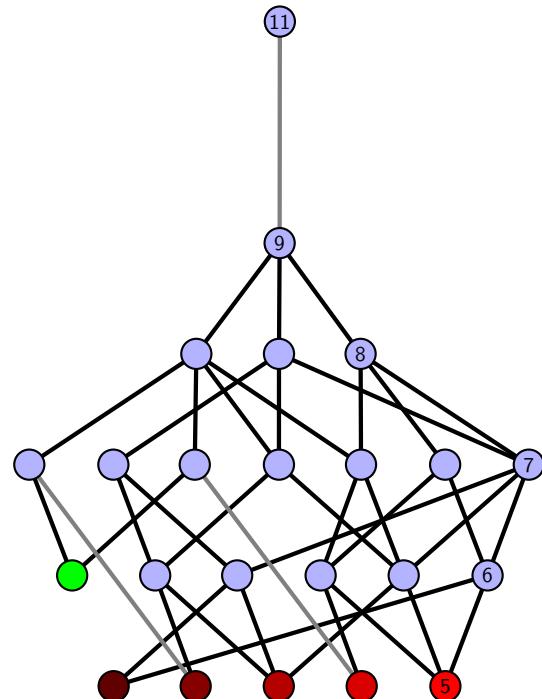


Figure 144: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.25  $[[18, 11, 1, 12], [12, 4, 13, 3], [6, 17, 7, 18], [7, 10, 8, 11], [1, 5, 2, 4], [13, 2, 14, 3], [14, 5, 15, 6], [9, 16, 10, 17], [8, 16, 9, 15]]$

PD code drawn by SnapPy:  $[(11, 18, 12, 1), (1, 10, 2, 11), (2, 17, 3, 18), (12, 3, 13, 4), (5, 8, 6, 9), (16, 9, 17, 10), (13, 6, 14, 7), (7, 14, 8, 15), (4, 15, 5, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 7, 8], [0, 6, 5, 1], [1, 4, 6, 1], [2, 5, 4, 8], [2, 8, 8, 3], [3, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.59

**Total pinning sets:** 200

**Average overall degree:** 3.04

Table 71: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	1	0	0	0	0	0	3
Nonminimal pinning sets	0	7	31	58	58	32	9	1	196
Average degree	2.5	2.71	2.88	3.01	3.12	3.19	3.24	3.27	

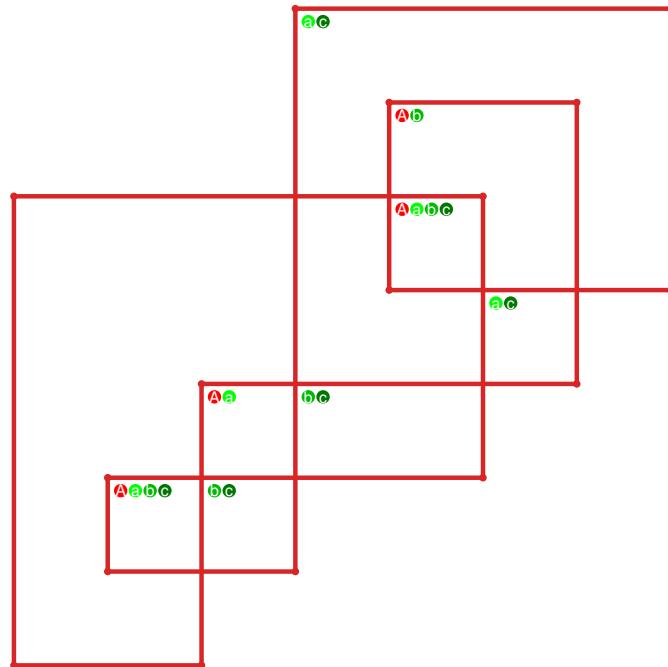


Figure 145: SnapPy multiloop plot.

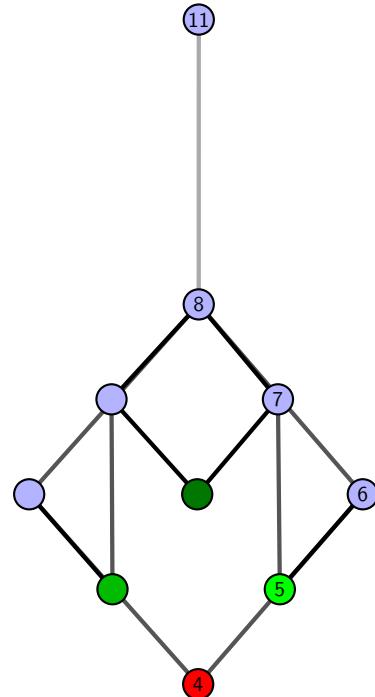


Figure 146: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.26  $[[18, 9, 1, 10], [10, 17, 11, 18], [11, 8, 12, 9], [1, 16, 2, 17], [7, 12, 8, 13], [15, 6, 16, 7], [2, 6, 3, 5], [13, 5, 14, 4], [14, 3, 15, 4]]$

PD code drawn by SnapPy:  $[(18, 9, 1, 10), (1, 16, 2, 17), (11, 2, 12, 3), (7, 4, 8, 5), (14, 5, 15, 6), (3, 8, 4, 9), (15, 12, 16, 13), (6, 13, 7, 14), (10, 17, 11, 18)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 6, 1], [2, 7, 5, 2], [3, 4, 8, 6], [3, 5, 8, 7], [4, 6, 8, 8], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 160  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.33  
**Average overall degree:** 2.97

Table 72: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.25	2.55	2.77	2.93	3.06	3.15	3.23	3.27	

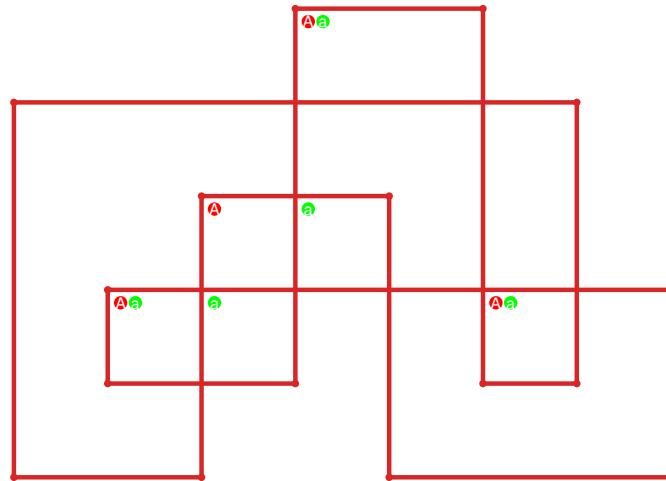


Figure 147: SnapPy multiloop plot.



Figure 148: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.27  $[[18, 7, 1, 8], [8, 17, 9, 18], [9, 6, 10, 7], [1, 16, 2, 17], [12, 5, 13, 6], [10, 15, 11, 16], [2, 11, 3, 12], [4, 13, 5, 14], [14, 3, 15, 4]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (5, 2, 6, 3), (14, 3, 15, 4), (16, 7, 17, 8), (18, 9, 1, 10), (6, 11, 7, 12), (15, 12, 16, 13), (4, 13, 5, 14), (10, 17, 11, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 5], [0, 5, 6, 1], [2, 6, 7, 7], [2, 8, 6, 3], [3, 5, 8, 4], [4, 8, 8, 4], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.42

**Total pinning sets:** 176

**Average overall degree:** 2.98

**Pinning number:** 4

Table 73: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.25	2.56	2.78	2.95	3.07	3.16	3.23	3.27	

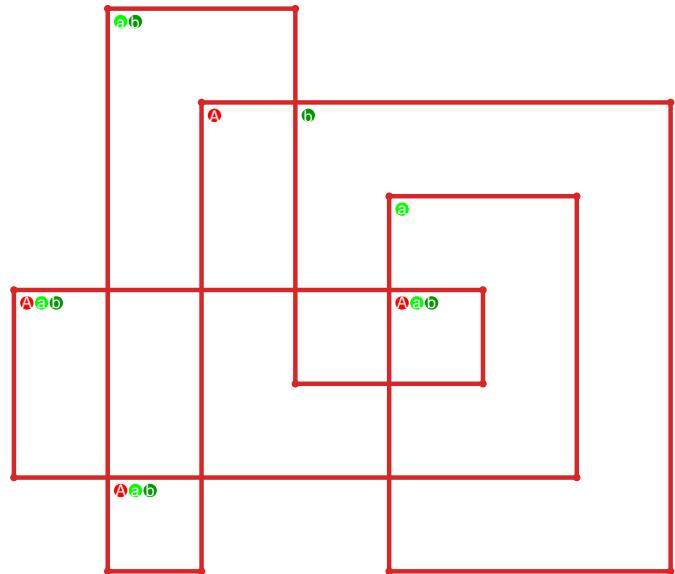


Figure 149: `SnapPy` multiloop plot.

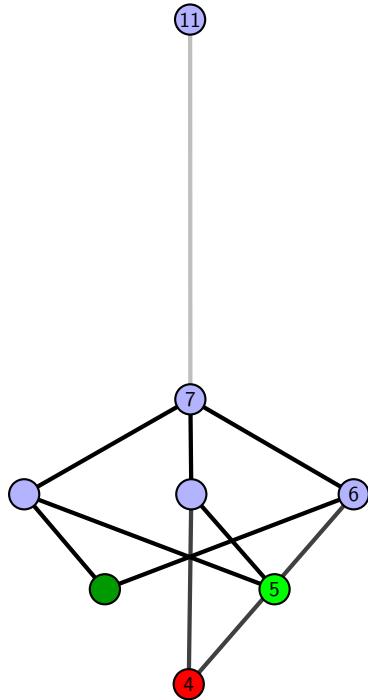


Figure 150: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.28**  $[[3, 18, 4, 1], [2, 13, 3, 14], [17, 8, 18, 9], [4, 8, 5, 7], [1, 15, 2, 14], [15, 12, 16, 13], [9, 16, 10, 17], [5, 10, 6, 11], [11, 6, 12, 7]]$

PD code drawn by `SnapPy`:  $[(9, 4, 10, 5), (5, 2, 6, 3), (15, 6, 16, 7), (3, 8, 4, 9), (13, 10, 14, 11), (18, 11, 1, 12), (12, 17, 13, 18), (7, 14, 8, 15), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 8, 6], [2, 5, 7, 2], [3, 6, 8, 8], [3, 7, 7, 5]]$

**Total optimal pinning sets:** 5  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 248  
**Pinning number:** 4

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.4  
**Average overall degree:** 2.98

Table 74: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	5	0	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	25	55	70	56	28	8	1	243
Average degree	2.4	2.69	2.87	3.0	3.09	3.17	3.23	3.27	

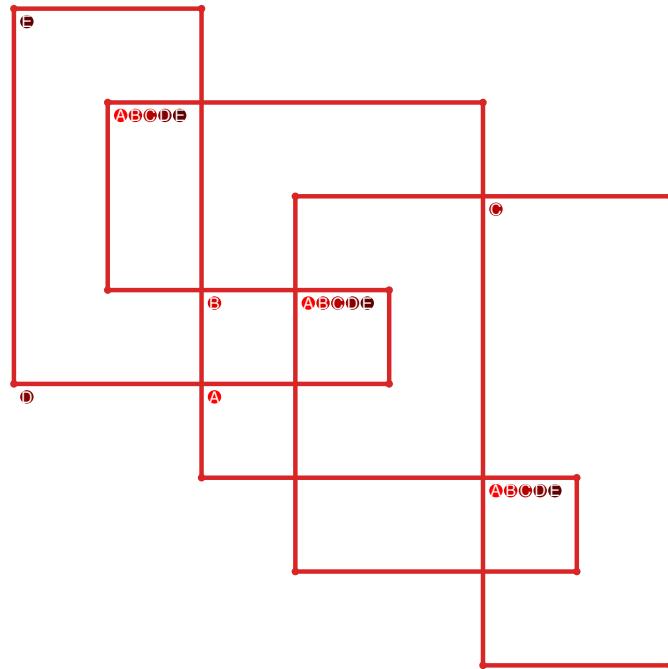


Figure 151: `SnapPy` multiloop plot.

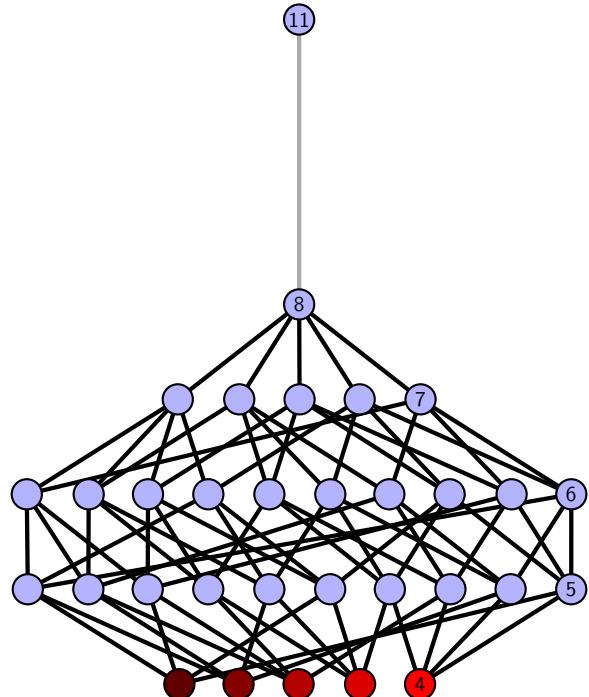


Figure 152: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.29  $[[3, 18, 4, 1], [13, 2, 14, 3], [17, 10, 18, 11], [4, 10, 5, 9], [1, 12, 2, 13], [14, 12, 15, 11], [7, 16, 8, 17], [5, 8, 6, 9], [15, 6, 16, 7]]$

PD code drawn by SnapPy:  $[(11, 18, 12, 1), (13, 4, 14, 5), (3, 6, 4, 7), (7, 2, 8, 3), (15, 8, 16, 9), (9, 12, 10, 13), (17, 10, 18, 11), (5, 14, 6, 15), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 8, 7], [3, 6, 8, 3], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 5

**Average minimal degree:** 2.53

**Total pinning sets:** 200

**Average overall degree:** 2.99

**Pinning number:** 4

Table 75: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	7	37	60	54	28	8	1	195
Average degree	2.25	2.58	2.82	2.98	3.09	3.17	3.23	3.27	

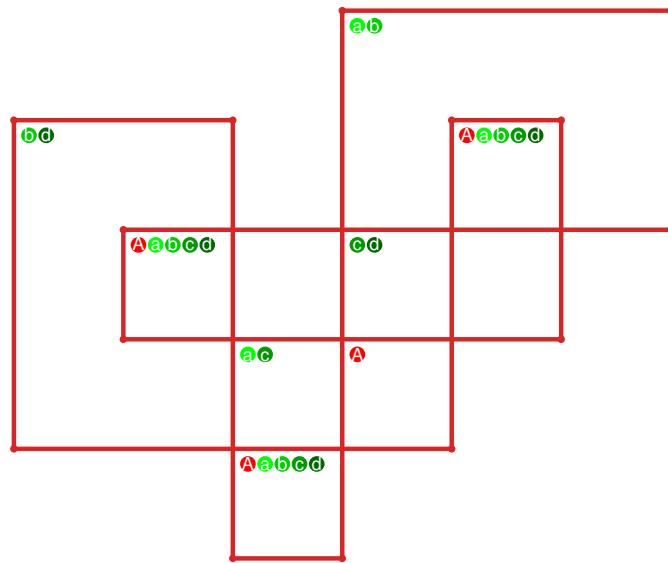


Figure 153: SnapPy multiloop plot.

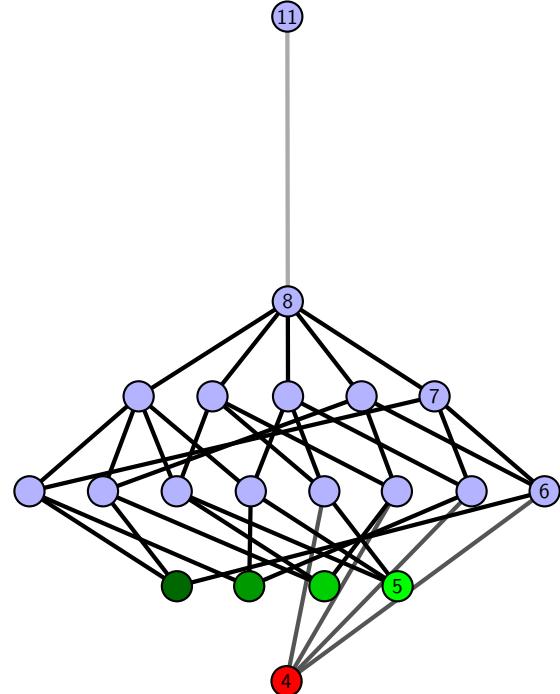


Figure 154: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.30  $[[3, 18, 4, 1], [2, 7, 3, 8], [12, 17, 13, 18], [4, 13, 5, 14], [1, 9, 2, 8], [9, 6, 10, 7], [16, 11, 17, 12], [5, 15, 6, 14], [10, 15, 11, 16]]$

PD code drawn by SnapPy:  $[(6, 3, 7, 4), (17, 4, 18, 5), (5, 16, 6, 17), (13, 8, 14, 9), (1, 10, 2, 11), (11, 2, 12, 3), (7, 12, 8, 13), (9, 14, 10, 15), (18, 15, 1, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 7, 8], [2, 8, 8, 2], [3, 8, 5, 3], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 76: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

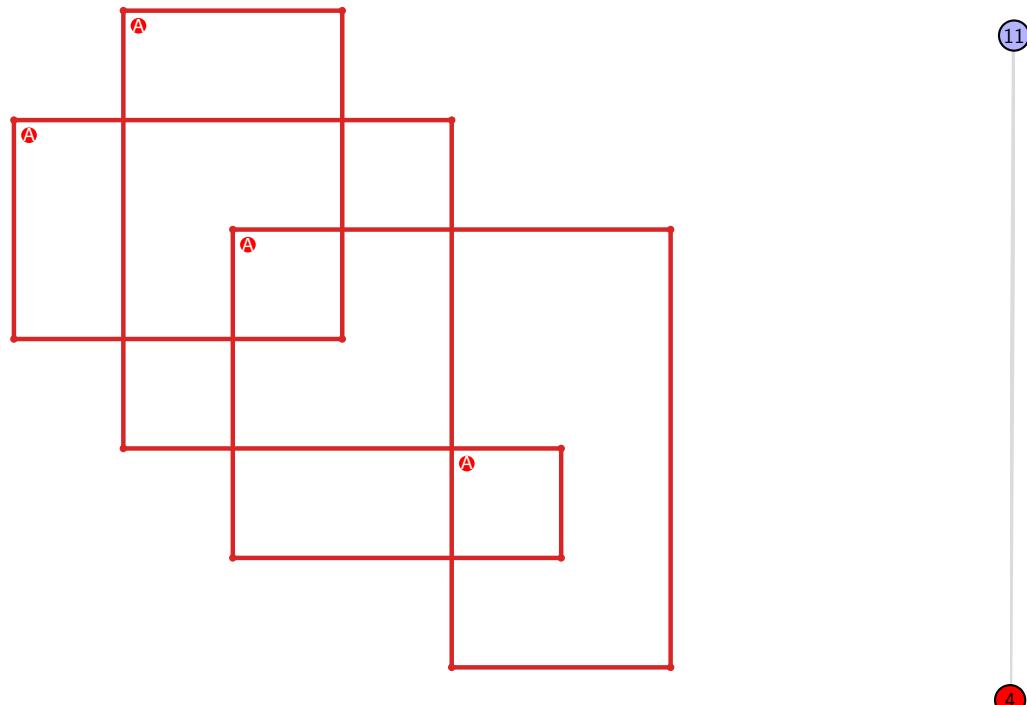


Figure 155: SnapPy multiloop plot.

Figure 156: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.31  $[[3, 18, 4, 1], [2, 11, 3, 12], [17, 4, 18, 5], [1, 13, 2, 12], [13, 10, 14, 11], [5, 9, 6, 8], [16, 7, 17, 8], [9, 14, 10, 15], [6, 15, 7, 16]]$

PD code drawn by SnapPy:  $[(15, 2, 16, 3), (14, 5, 15, 6), (6, 3, 7, 4), (11, 8, 12, 9), (18, 9, 1, 10), (10, 17, 11, 18), (7, 12, 8, 13), (4, 13, 5, 14), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 7], [2, 7, 8, 6], [2, 5, 8, 8], [4, 8, 5, 4], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 5

Table 77: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.2	2.5	2.74	2.94	3.09	3.2	3.27	

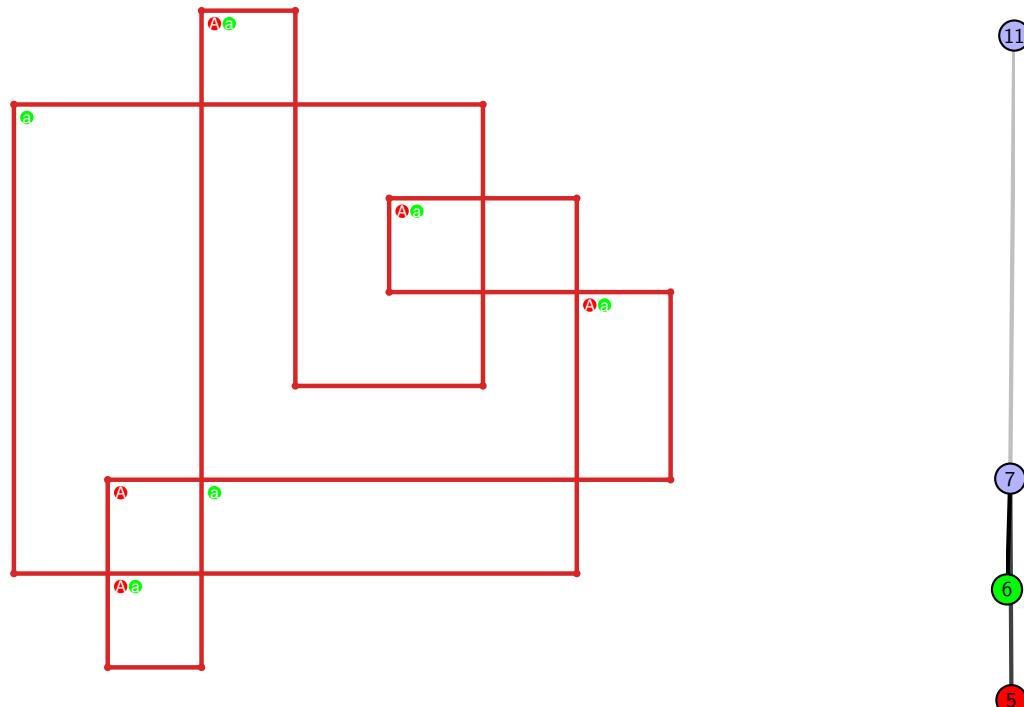


Figure 157: SnapPy multiloop plot.

Figure 158: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.32**  $[[3, 18, 4, 1], [2, 9, 3, 10], [14, 17, 15, 18], [4, 15, 5, 16], [1, 11, 2, 10], [11, 8, 12, 9], [13, 6, 14, 7], [16, 5, 17, 6], [7, 12, 8, 13]]$

PD code drawn by `SnapPy`:  $[(14, 3, 15, 4), (11, 4, 12, 5), (9, 6, 10, 7), (18, 7, 1, 8), (8, 17, 9, 18), (5, 10, 6, 11), (2, 13, 3, 14), (12, 15, 13, 16), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 8], [2, 8, 8, 7], [2, 6, 3, 3], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 5

Table 78: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.2	2.5	2.74	2.94	3.09	3.2	3.27	

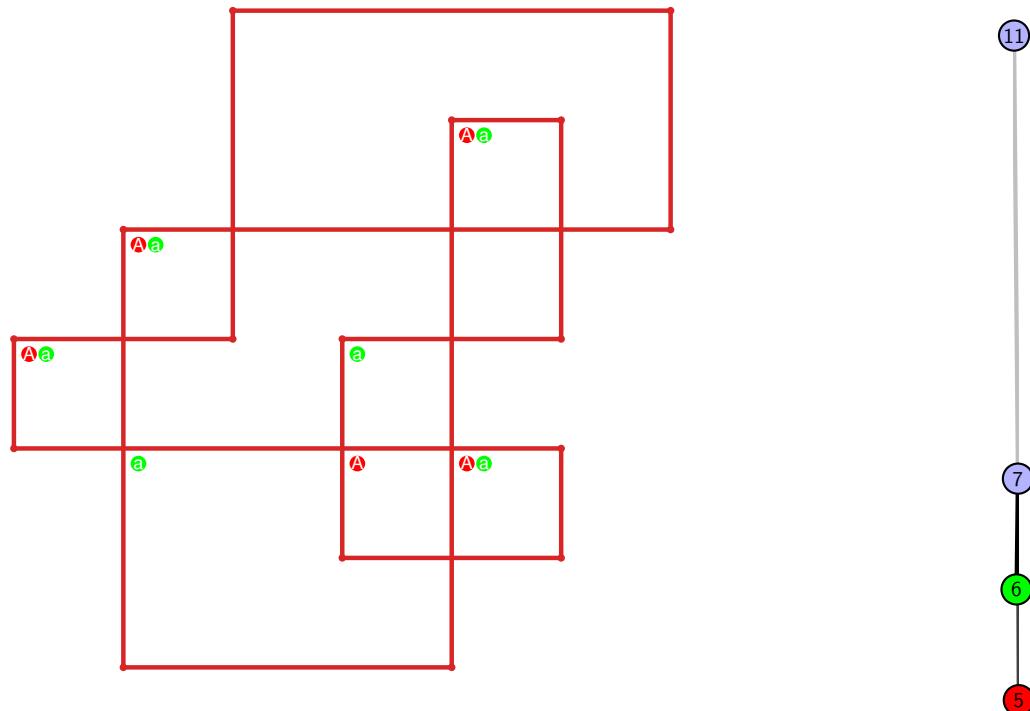


Figure 159: `SnapPy` multiloop plot.

Figure 160: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.33**  $[[5, 18, 6, 1], [4, 11, 5, 12], [14, 17, 15, 18], [6, 15, 7, 16], [1, 13, 2, 12], [10, 3, 11, 4], [13, 8, 14, 9], [16, 7, 17, 8], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(7, 18, 8, 1), (13, 4, 14, 5), (10, 5, 11, 6), (17, 6, 18, 7), (1, 8, 2, 9), (9, 16, 10, 17), (3, 12, 4, 13), (11, 14, 12, 15), (2, 15, 3, 16)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 7, 7], [0, 6, 8, 1], [1, 8, 8, 1], [2, 8, 4, 7], [2, 6, 3, 3], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 160  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.33  
**Average overall degree:** 2.97

Table 79: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.25	2.55	2.77	2.93	3.06	3.15	3.23	3.27	

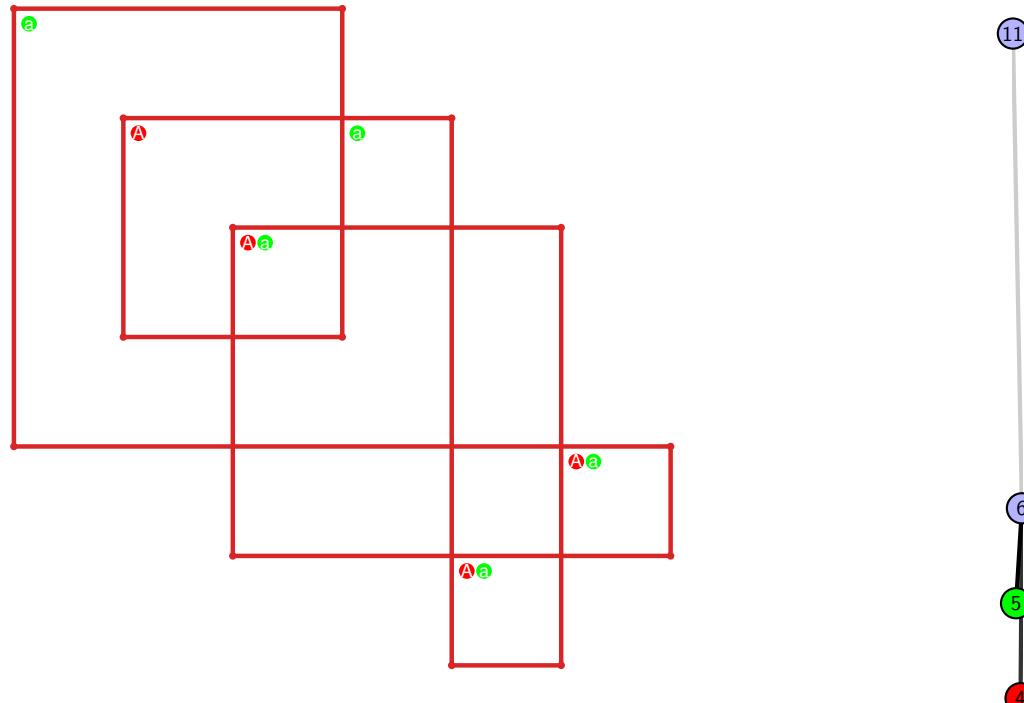


Figure 161: `SnapPy` multiloop plot.

Figure 162: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.34 [[18, 9, 1, 10], [10, 4, 11, 3], [14, 17, 15, 18], [15, 8, 16, 9], [1, 5, 2, 4], [11, 2, 12, 3], [6, 13, 7, 14], [7, 16, 8, 17], [5, 13, 6, 12]]

PD code drawn by SnapPy: [(18, 7, 1, 8), (1, 16, 2, 17), (12, 5, 13, 6), (15, 6, 16, 7), (9, 2, 10, 3), (3, 10, 4, 11), (4, 13, 5, 14), (11, 14, 12, 15), (8, 17, 9, 18)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 7, 7], [0, 8, 5, 1], [1, 4, 8, 1], [2, 8, 8, 7], [2, 6, 3, 3], [4, 6, 6, 5]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.49

**Total pinning sets:** 100

**Average overall degree:** 2.98

**Pinning number:** 5

Table 80: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	1	0	0	0	0	3
Nonminimal pinning sets	0	6	23	34	24	8	1	96
Average degree	2.4	2.62	2.82	3.0	3.14	3.23	3.27	

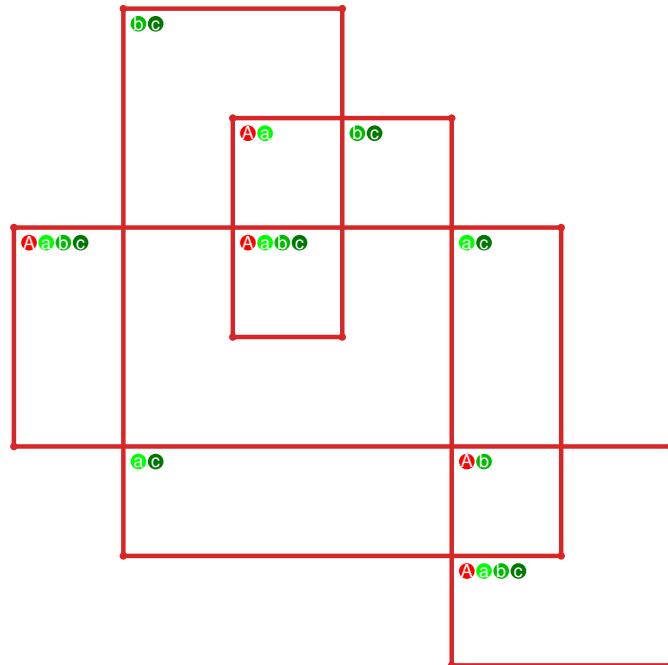


Figure 163: SnapPy multiloop plot.

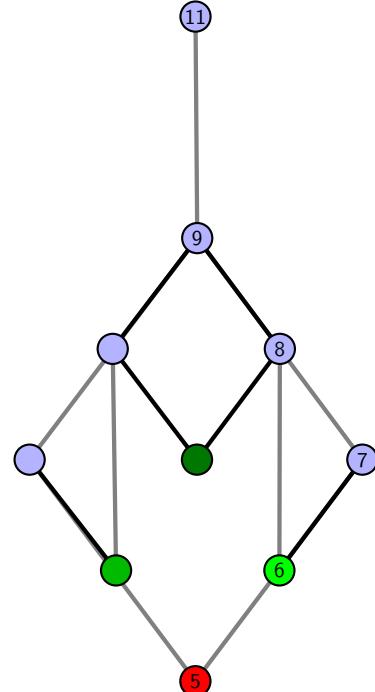


Figure 164: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.35  $[[5, 18, 6, 1], [4, 9, 5, 10], [14, 17, 15, 18], [6, 15, 7, 16], [1, 11, 2, 10], [12, 3, 13, 4], [13, 8, 14, 9], [16, 7, 17, 8], [11, 3, 12, 2]]$

PD code drawn by SnapPy:  $[(9, 18, 10, 1), (13, 4, 14, 5), (10, 5, 11, 6), (1, 6, 2, 7), (7, 16, 8, 17), (3, 12, 4, 13), (11, 14, 12, 15), (2, 15, 3, 16), (17, 8, 18, 9)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 3], [0, 2, 7, 7], [0, 8, 8, 1], [1, 8, 8, 6], [1, 5, 7, 2], [2, 6, 3, 3], [4, 5, 5, 4]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 8  
 Total pinning sets: 128  
 Pinning number: 5

Average optimal degree: 2.6  
 Average minimal degree: 2.55  
 Average overall degree: 3.0

Table 81: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	7
Nonminimal pinning sets	0	6	36	43	26	8	1	120
Average degree	2.6	2.67	2.89	3.05	3.15	3.23	3.27	

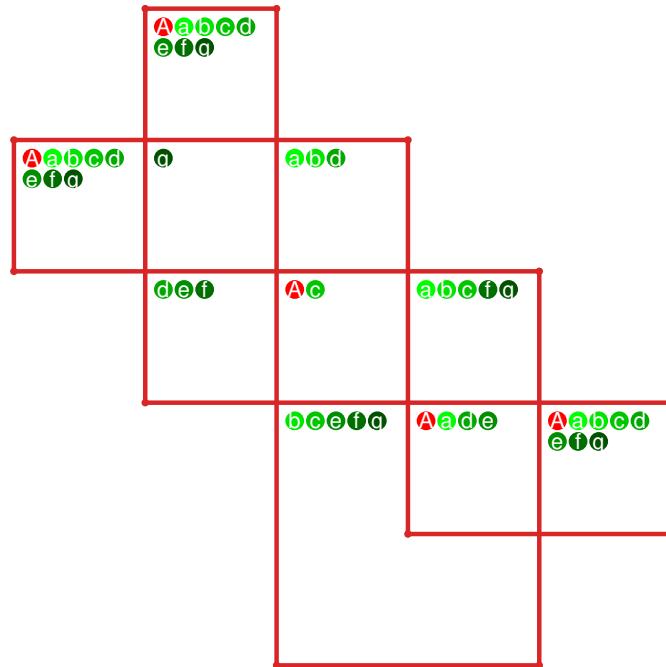


Figure 165: SnapPy multiloop plot.

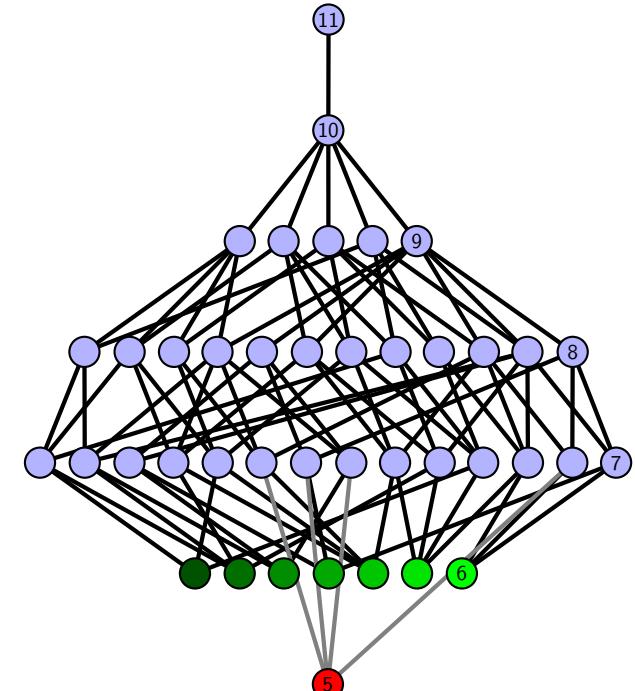


Figure 166: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.36**  $[[7, 18, 8, 1], [6, 15, 7, 16], [17, 8, 18, 9], [1, 17, 2, 16], [14, 5, 15, 6], [9, 3, 10, 2], [10, 13, 11, 14], [11, 4, 12, 5], [3, 12, 4, 13]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (11, 2, 12, 3), (14, 5, 15, 6), (18, 9, 1, 10), (7, 10, 8, 11), (3, 12, 4, 13), (13, 16, 14, 17), (4, 15, 5, 16), (17, 6, 18, 7)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 4], [0, 5, 3, 0], [0, 2, 5, 1], [1, 6, 7, 1], [2, 8, 6, 3], [4, 5, 8, 7], [4, 6, 8, 8], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.49

**Total pinning sets:** 100

**Average overall degree:** 2.98

**Pinning number:** 5

Table 82: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	1	0	0	0	0	3
Nonminimal pinning sets	0	6	23	34	24	8	1	96
Average degree	2.4	2.62	2.82	3.0	3.14	3.23	3.27	

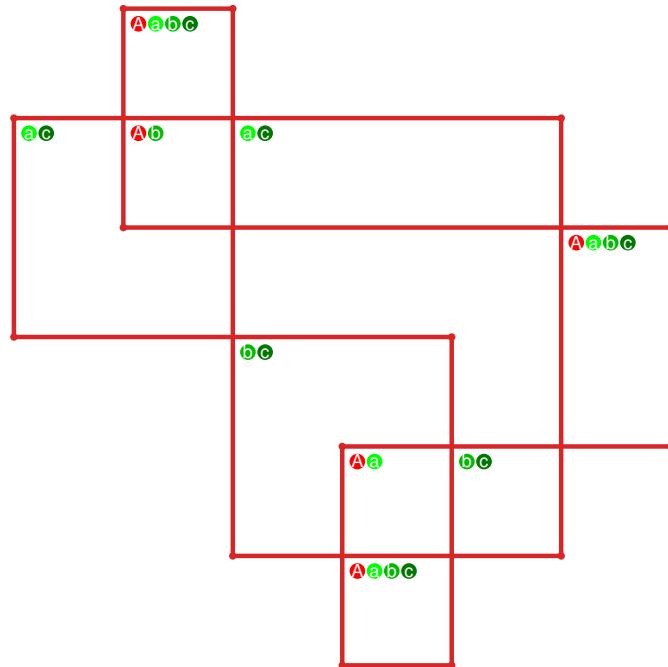


Figure 167: `SnapPy` multiloop plot.

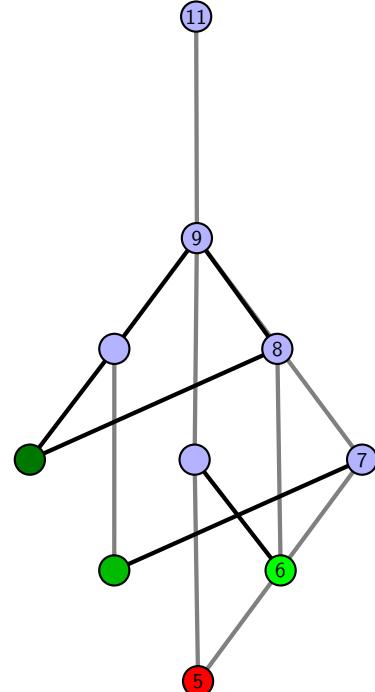


Figure 168: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.37**  $[[7, 18, 8, 1], [17, 6, 18, 7], [8, 11, 9, 12], [1, 4, 2, 5], [5, 16, 6, 17], [10, 15, 11, 16], [9, 15, 10, 14], [12, 3, 13, 4], [2, 13, 3, 14]]$

PD code drawn by `SnapPy`:  $[(10, 1, 11, 2), (17, 2, 18, 3), (15, 4, 16, 5), (6, 9, 7, 10), (18, 11, 1, 12), (12, 7, 13, 8), (8, 13, 9, 14), (3, 14, 4, 15), (5, 16, 6, 17)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 7], [0, 7, 8, 4], [1, 3, 5, 1], [2, 4, 6, 6], [2, 5, 5, 8], [2, 8, 8, 3], [3, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 83: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

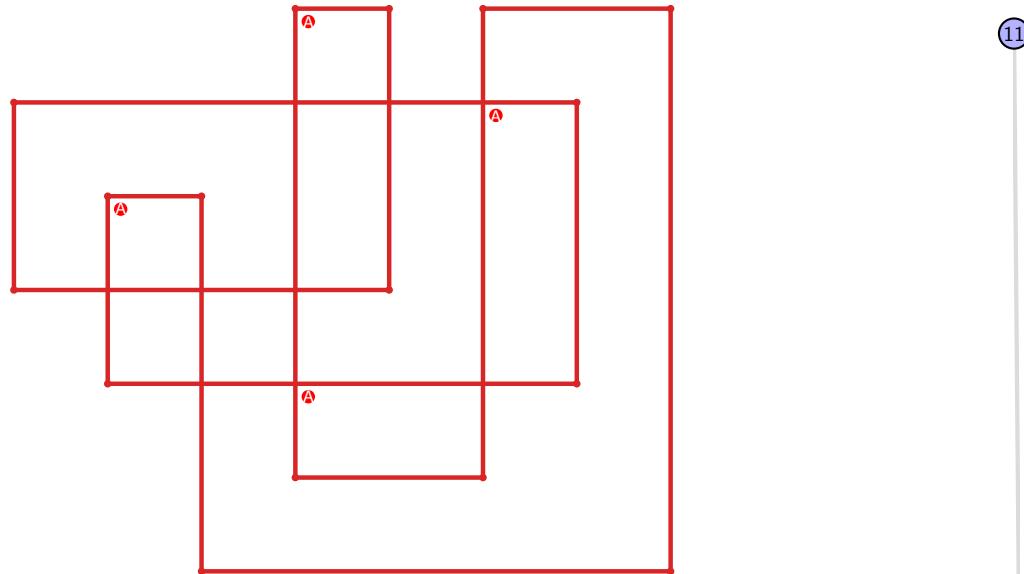


Figure 169: `SnapPy` multiloop plot.



Figure 170: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.38  $[[18, 5, 1, 6], [6, 16, 7, 15], [4, 17, 5, 18], [1, 17, 2, 16], [7, 11, 8, 10], [14, 9, 15, 10], [3, 12, 4, 13], [2, 12, 3, 11], [8, 13, 9, 14]]$

PD code drawn by SnapPy:  $[(15, 2, 16, 3), (11, 6, 12, 7), (7, 4, 8, 5), (8, 17, 9, 18), (18, 9, 1, 10), (5, 10, 6, 11), (12, 3, 13, 4), (13, 16, 14, 17), (1, 14, 2, 15)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 3, 0], [0, 2, 7, 1], [1, 7, 8, 5], [1, 4, 8, 8], [2, 8, 7, 7], [3, 6, 6, 4], [4, 6, 5, 5]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 144  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.4  
 Average overall degree: 2.97

Table 84: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.4	2.7	2.91	3.05	3.15	3.23	3.27	

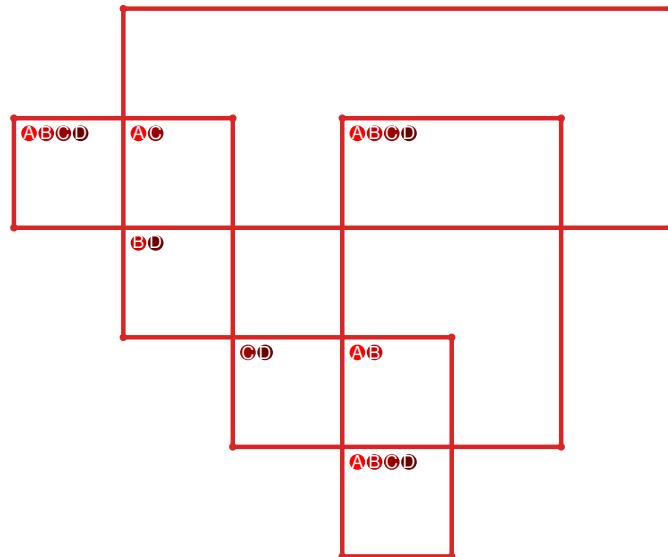


Figure 171: SnapPy multiloop plot.

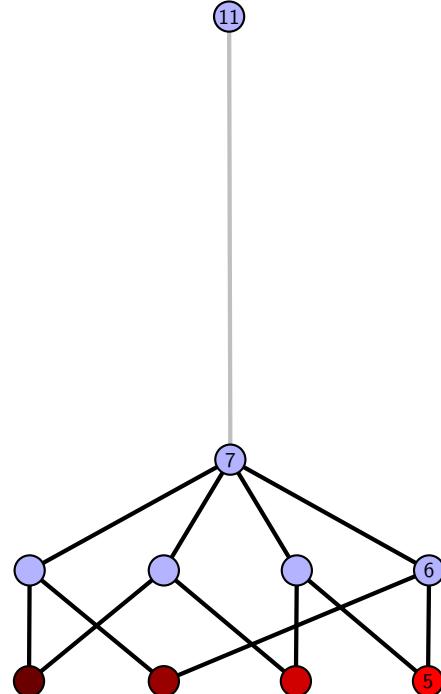


Figure 172: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.39  $[[18, 3, 1, 4], [4, 14, 5, 13], [17, 10, 18, 11], [2, 15, 3, 16], [1, 15, 2, 14], [5, 9, 6, 8], [12, 7, 13, 8], [11, 7, 12, 6], [9, 16, 10, 17]]$

PD code drawn by SnapPy:  $[(16, 1, 17, 2), (4, 9, 5, 10), (10, 5, 11, 6), (6, 3, 7, 4), (15, 8, 16, 9), (11, 2, 12, 3), (12, 17, 13, 18), (18, 13, 1, 14), (7, 14, 8, 15)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 8, 8], [0, 8, 4, 4], [0, 3, 3, 1], [1, 8, 7, 6], [1, 5, 7, 7], [2, 6, 6, 5], [2, 5, 3, 2]]$

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4

**Total pinning sets:** 144

**Pinning number:** 5

**Average optimal degree:** 2.4

**Average minimal degree:** 2.4

**Average overall degree:** 2.97

Table 85: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.4	2.7	2.91	3.05	3.15	3.23	3.27	

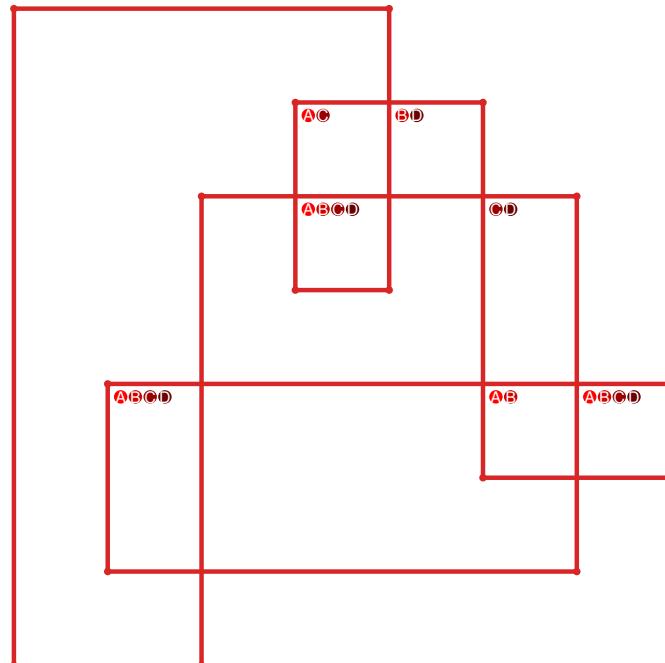


Figure 173: SnapPy multiloop plot.

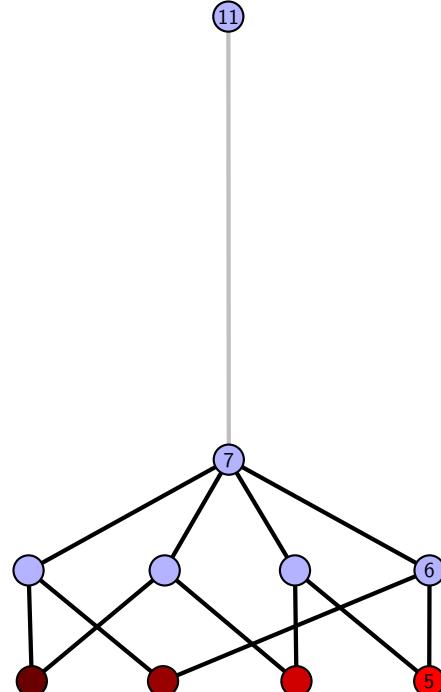


Figure 174: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.40  $[[9, 18, 10, 1], [17, 8, 18, 9], [10, 5, 11, 6], [1, 16, 2, 17], [4, 7, 5, 8], [11, 7, 12, 6], [12, 15, 13, 16], [2, 13, 3, 14], [14, 3, 15, 4]]$

PD code drawn by SnapPy:  $[(12, 1, 13, 2), (8, 3, 9, 4), (4, 9, 5, 10), (2, 5, 3, 6), (15, 6, 16, 7), (18, 11, 1, 12), (10, 13, 11, 14), (17, 14, 18, 15), (7, 16, 8, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 4, 0], [0, 4, 5, 5], [0, 6, 7, 1], [1, 8, 5, 2], [2, 4, 6, 2], [3, 5, 8, 7], [3, 6, 8, 8], [4, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.42

**Total pinning sets:** 176

**Average overall degree:** 2.98

**Pinning number:** 4

Table 86: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.25	2.56	2.78	2.95	3.07	3.16	3.23	3.27	

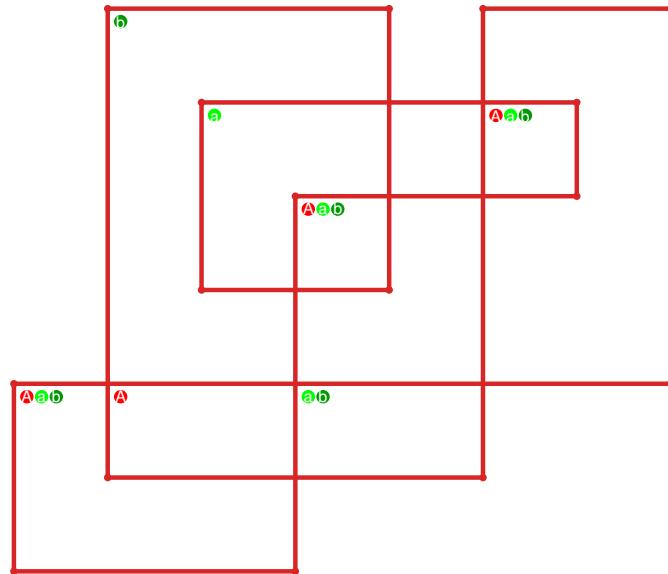


Figure 175: SnapPy multiloop plot.

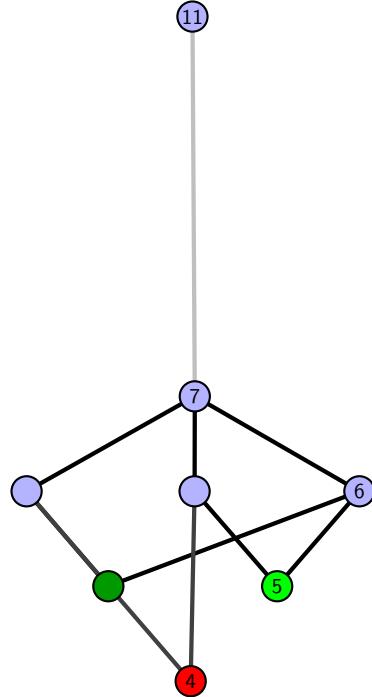


Figure 176: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.41**  $[[7, 18, 8, 1], [17, 6, 18, 7], [8, 11, 9, 12], [1, 16, 2, 17], [10, 5, 11, 6], [9, 5, 10, 4], [12, 15, 13, 16], [2, 13, 3, 14], [14, 3, 15, 4]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (2, 7, 3, 8), (8, 3, 9, 4), (15, 4, 16, 5), (6, 9, 7, 10), (18, 11, 1, 12), (10, 13, 11, 14), (17, 14, 18, 15), (5, 16, 6, 17)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 3, 4, 0], [0, 4, 5, 6], [0, 6, 7, 1], [1, 5, 5, 2], [2, 4, 4, 8], [2, 8, 7, 3], [3, 6, 8, 8], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.42

**Total pinning sets:** 176

**Average overall degree:** 2.98

**Pinning number:** 4

Table 87: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.25	2.56	2.78	2.95	3.07	3.16	3.23	3.27	

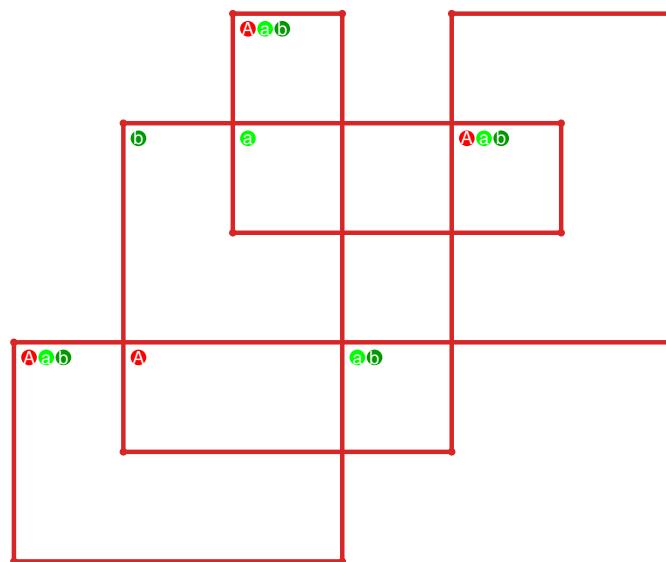


Figure 177: `SnapPy` multiloop plot.

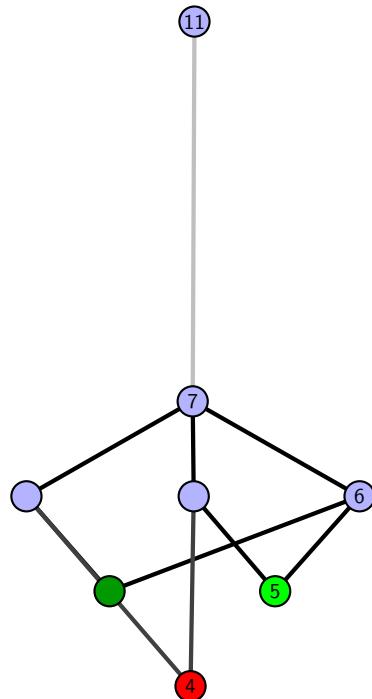


Figure 178: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.42 [[18, 7, 1, 8], [8, 17, 9, 18], [11, 6, 12, 7], [1, 16, 2, 17], [9, 4, 10, 5], [5, 10, 6, 11], [12, 15, 13, 16], [2, 13, 3, 14], [14, 3, 15, 4]]

PD code drawn by SnapPy: [(12, 1, 13, 2), (7, 2, 8, 3), (9, 4, 10, 5), (16, 5, 17, 6), (3, 8, 4, 9), (18, 11, 1, 12), (10, 13, 11, 14), (17, 14, 18, 15), (6, 15, 7, 16)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 3, 4, 0], [0, 5, 5, 6], [0, 6, 7, 1], [1, 8, 5, 5], [2, 4, 4, 2], [2, 8, 7, 3], [3, 6, 8, 8], [4, 7, 7, 6]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 96

**Average overall degree:** 2.9

**Pinning number:** 5

Table 88: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

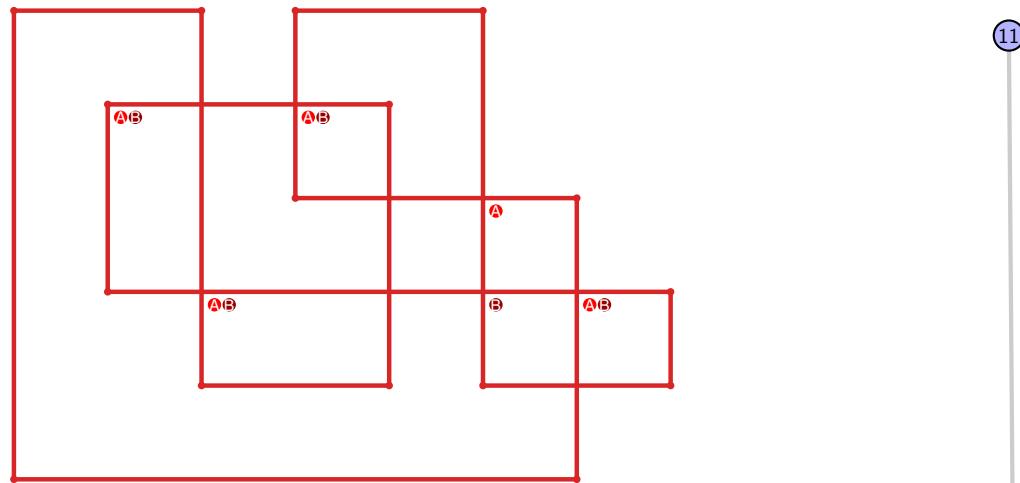


Figure 179: SnapPy multiloop plot.

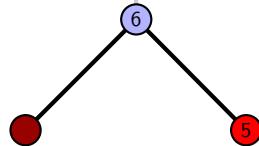


Figure 180: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.43 [[18, 9, 1, 10], [10, 17, 11, 18], [8, 1, 9, 2], [16, 11, 17, 12], [2, 7, 3, 8], [12, 5, 13, 6], [6, 15, 7, 16], [3, 15, 4, 14], [4, 13, 5, 14]]

PD code drawn by SnapPy: [(9, 2, 10, 3), (17, 4, 18, 5), (15, 6, 16, 7), (13, 8, 14, 9), (1, 10, 2, 11), (11, 18, 12, 1), (3, 12, 4, 13), (7, 14, 8, 15), (5, 16, 6, 17)]

Planar representation generated by plantri: [[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 6, 7, 2], [3, 8, 8, 6], [3, 5, 7, 4], [4, 6, 8, 8], [5, 7, 7, 5]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.79

**Pinning number:** 6

Table 89: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.7	2.93	3.12	3.27	

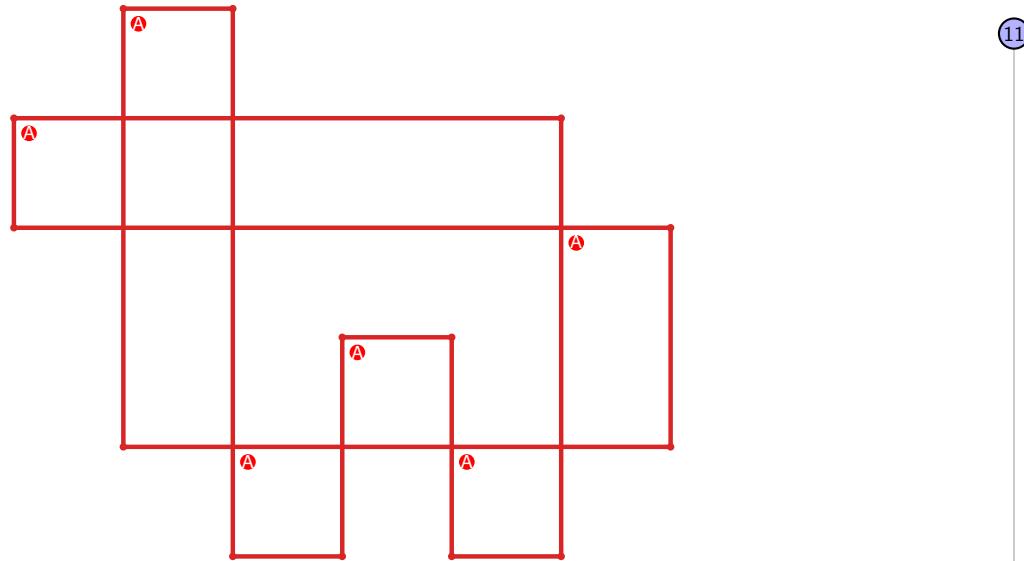


Figure 181: SnapPy multiloop plot.



Figure 182: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.44** `[[18, 9, 1, 10], [10, 17, 11, 18], [11, 8, 12, 9], [1, 7, 2, 6], [16, 5, 17, 6], [7, 12, 8, 13], [2, 15, 3, 16], [4, 13, 5, 14], [14, 3, 15, 4]]`

PD code drawn by `SnapPy`: `[(12, 1, 13, 2), (14, 3, 15, 4), (4, 17, 5, 18), (9, 6, 10, 7), (16, 7, 17, 8), (5, 10, 6, 11), (2, 11, 3, 12), (18, 13, 1, 14), (8, 15, 9, 16)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 4], [1, 3, 6, 7], [2, 7, 3, 2], [3, 8, 8, 4], [4, 8, 8, 5], [6, 7, 7, 6]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 90: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

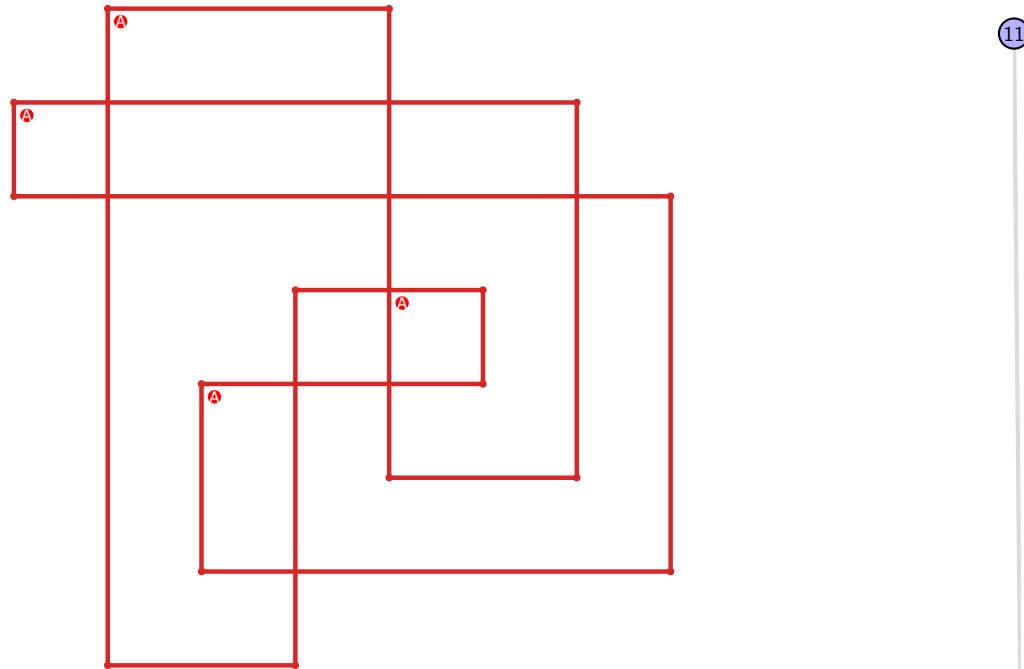


Figure 183: `SnapPy` multiloop plot.

Figure 184: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.45  $[[18, 7, 1, 8], [8, 17, 9, 18], [9, 6, 10, 7], [1, 10, 2, 11], [11, 16, 12, 17], [12, 5, 13, 6], [2, 15, 3, 16], [4, 13, 5, 14], [14, 3, 15, 4]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (5, 2, 6, 3), (14, 3, 15, 4), (15, 6, 16, 7), (18, 9, 1, 10), (16, 11, 17, 12), (7, 12, 8, 13), (4, 13, 5, 14), (10, 17, 11, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 4], [1, 3, 6, 5], [2, 4, 7, 7], [3, 8, 8, 4], [5, 8, 8, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 96

**Average overall degree:** 2.9

**Pinning number:** 5

Table 91: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

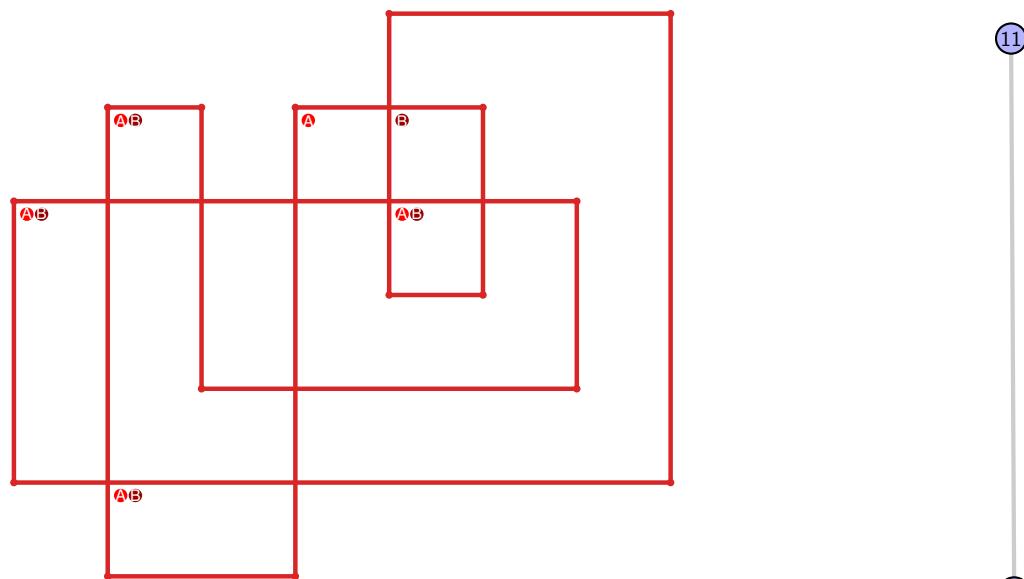


Figure 185: `SnapPy` multiloop plot.

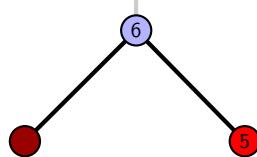


Figure 186: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.46**  $[[18, 11, 1, 12], [12, 17, 13, 18], [13, 10, 14, 11], [1, 14, 2, 15], [16, 5, 17, 6], [9, 4, 10, 5], [2, 8, 3, 7], [15, 7, 16, 6], [3, 8, 4, 9]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (13, 2, 14, 3), (3, 12, 4, 13), (4, 17, 5, 18), (18, 5, 1, 6), (10, 7, 11, 8), (15, 8, 16, 9), (16, 11, 17, 12), (9, 14, 10, 15)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 5], [2, 4, 8, 8], [3, 8, 8, 7], [3, 6, 4, 4], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 96  
**Pinning number:** 5

**Average optimal degree:** 2.2  
**Average minimal degree:** 2.2  
**Average overall degree:** 2.9

Table 92: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

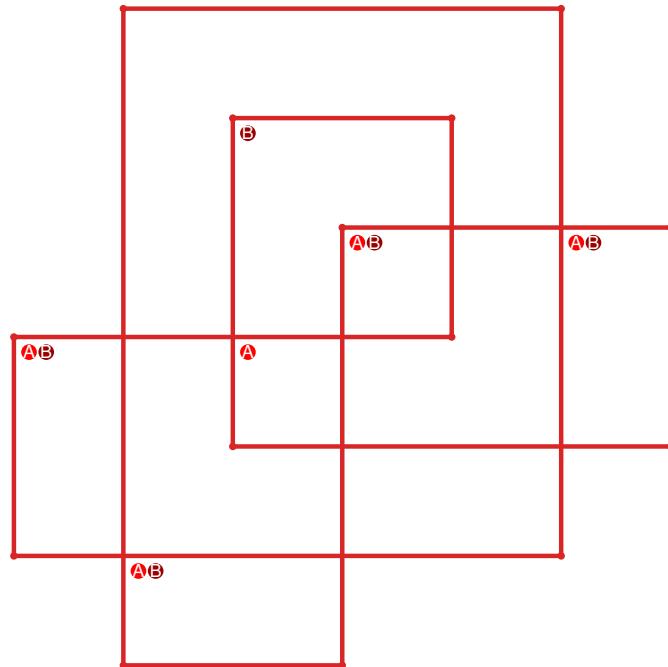


Figure 187: `SnapPy` multiloop plot.

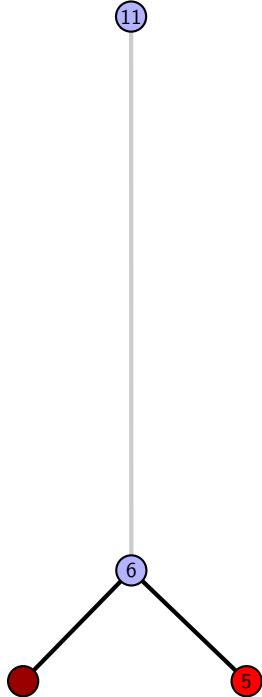


Figure 188: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.47**  $[[5, 18, 6, 1], [4, 9, 5, 10], [17, 8, 18, 9], [6, 16, 7, 15], [1, 12, 2, 13], [10, 3, 11, 4], [7, 16, 8, 17], [11, 14, 12, 15], [2, 14, 3, 13]]$

PD code drawn by `SnapPy`:  $[(13, 4, 14, 5), (18, 5, 1, 6), (15, 8, 16, 9), (9, 16, 10, 17), (7, 10, 8, 11), (11, 2, 12, 3), (3, 12, 4, 13), (1, 14, 2, 15), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 6], [0, 6, 6, 7], [0, 7, 8, 8], [1, 8, 7, 1], [2, 3, 3, 2], [3, 5, 8, 4], [4, 7, 5, 4]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 96

**Average overall degree:** 2.9

**Pinning number:** 5

Table 93: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

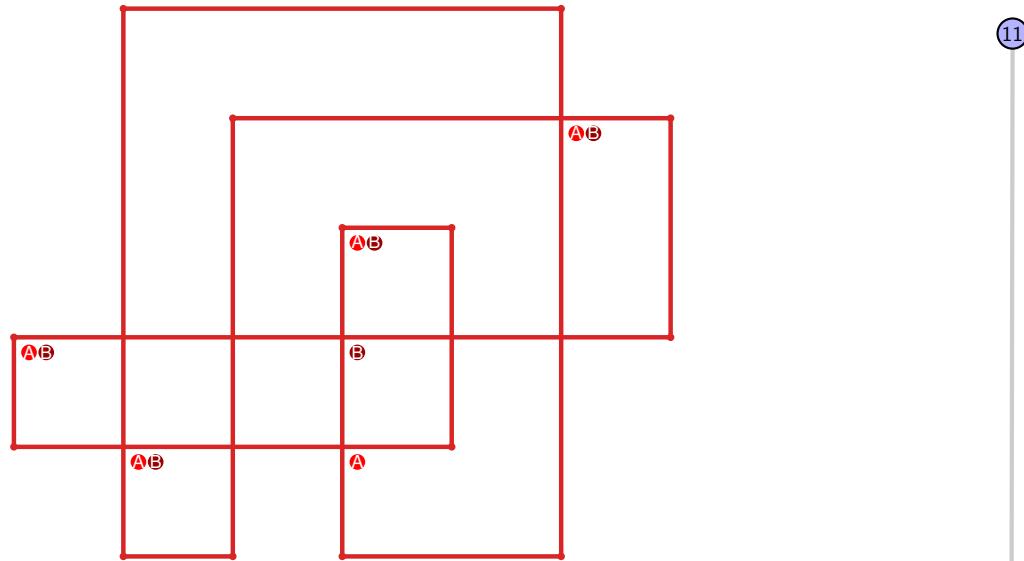


Figure 189: `SnapPy` multiloop plot.

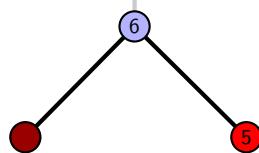


Figure 190: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.48** [[9, 18, 10, 1], [8, 15, 9, 16], [17, 14, 18, 15], [10, 3, 11, 4], [1, 6, 2, 7], [16, 7, 17, 8], [2, 13, 3, 14], [11, 5, 12, 4], [12, 5, 13, 6]]

PD code drawn by `SnapPy`: [(18, 7, 1, 8), (8, 1, 9, 2), (15, 2, 16, 3), (12, 3, 13, 4), (16, 9, 17, 10), (13, 10, 14, 11), (4, 11, 5, 12), (5, 14, 6, 15), (6, 17, 7, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 7], [0, 8, 6, 5], [1, 4, 2, 1], [2, 4, 8, 3], [3, 8, 8, 3], [4, 7, 7, 6]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 3

**Total pinning sets:** 176

**Pinning number:** 4

**Average optimal degree:** 2.25

**Average minimal degree:** 2.42

**Average overall degree:** 2.98

Table 94: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.25	2.56	2.78	2.95	3.07	3.16	3.23	3.27	

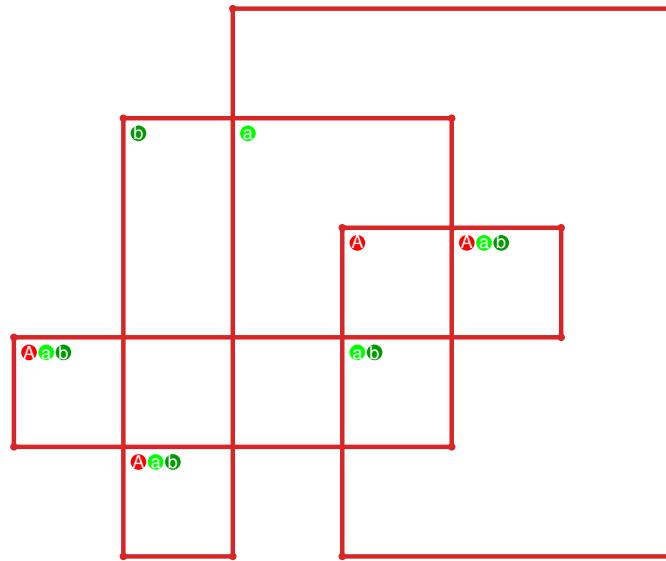


Figure 191: `SnapPy` multiloop plot.

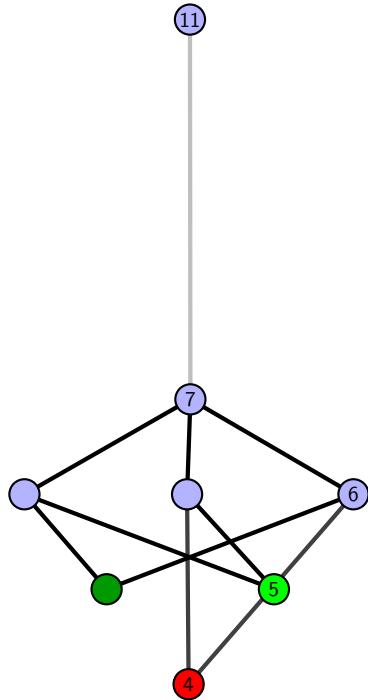


Figure 192: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.49  $[[18, 7, 1, 8], [8, 16, 9, 15], [17, 14, 18, 15], [3, 6, 4, 7], [1, 11, 2, 10], [16, 10, 17, 9], [2, 13, 3, 14], [5, 12, 6, 13], [4, 12, 5, 11]]$

PD code drawn by `SnapPy`:  $[(18, 3, 1, 4), (15, 4, 16, 5), (6, 13, 7, 14), (7, 16, 8, 17), (8, 1, 9, 2), (2, 9, 3, 10), (17, 10, 18, 11), (14, 11, 15, 12), (12, 5, 13, 6)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 8], [0, 8, 6, 5], [1, 4, 2, 1], [2, 4, 7, 3], [3, 6, 8, 8], [3, 7, 7, 4]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.58

**Total pinning sets:** 272

**Average overall degree:** 3.05

**Pinning number:** 4

Table 95: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.5	2.76	2.93	3.05	3.14	3.2	3.24	3.27	

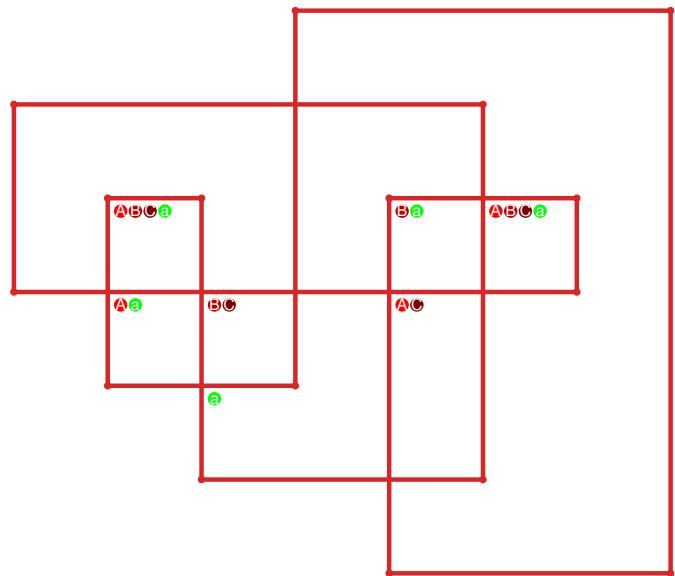


Figure 193: `SnapPy` multiloop plot.

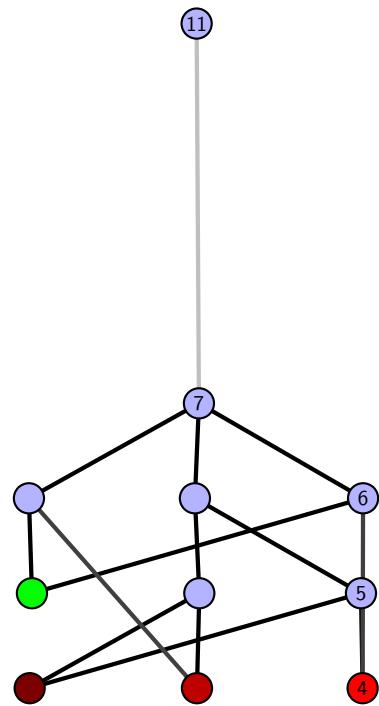


Figure 194: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.50  $[[18, 5, 1, 6], [6, 16, 7, 15], [17, 14, 18, 15], [11, 4, 12, 5], [1, 9, 2, 8], [16, 8, 17, 7], [2, 13, 3, 14], [3, 10, 4, 11], [12, 10, 13, 9]]$

PD code drawn by SnapPy:  $[(8, 1, 9, 2), (15, 2, 16, 3), (4, 13, 5, 14), (5, 16, 6, 17), (6, 9, 7, 10), (18, 7, 1, 8), (17, 10, 18, 11), (14, 11, 15, 12), (12, 3, 13, 4)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 8], [0, 8, 6, 5], [1, 4, 2, 1], [2, 4, 8, 7], [3, 6, 8, 3], [3, 7, 6, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 5

**Average minimal degree:** 2.62

**Total pinning sets:** 224

**Average overall degree:** 3.05

**Pinning number:** 4

Table 96: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	7	39	67	63	33	9	1	219
Average degree	2.5	2.71	2.89	3.03	3.13	3.2	3.24	3.27	

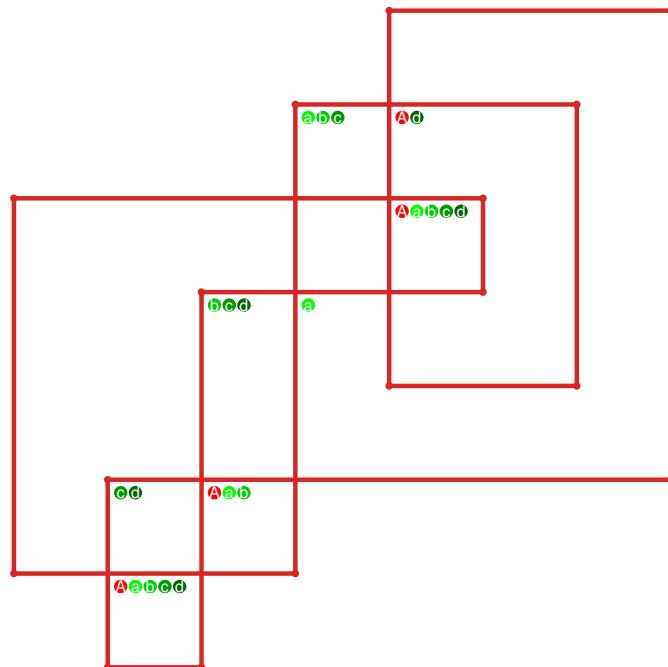


Figure 195: SnapPy multiloop plot.

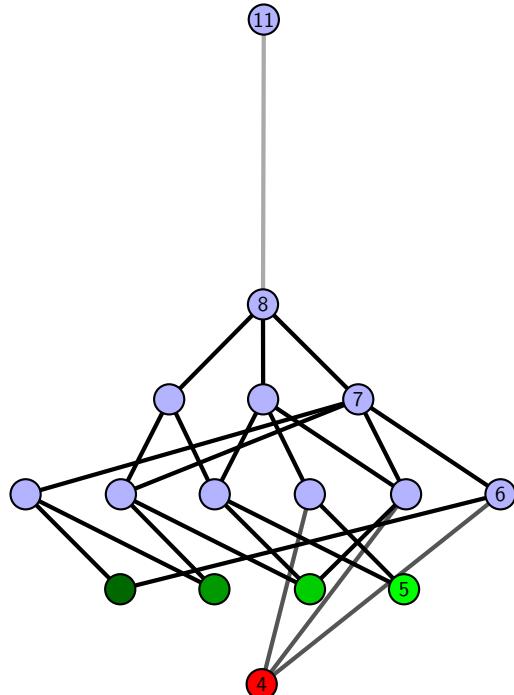


Figure 196: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.51  $[[7, 18, 8, 1], [6, 15, 7, 16], [17, 14, 18, 15], [8, 2, 9, 1], [16, 5, 17, 6], [10, 13, 11, 14], [2, 11, 3, 12], [9, 4, 10, 5], [12, 3, 13, 4]]$

PD code drawn by SnapPy:  $[(5, 18, 6, 1), (12, 1, 13, 2), (16, 7, 17, 8), (6, 9, 7, 10), (13, 10, 14, 11), (2, 11, 3, 12), (3, 14, 4, 15), (15, 4, 16, 5), (8, 17, 9, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 7, 0], [1, 7, 2, 1], [2, 7, 8, 6], [3, 5, 8, 8], [3, 8, 5, 4], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 136  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.47  
**Average overall degree:** 2.98

Table 97: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	16	38	43	26	8	1	132
Average degree	2.4	2.69	2.89	3.05	3.15	3.23	3.27	

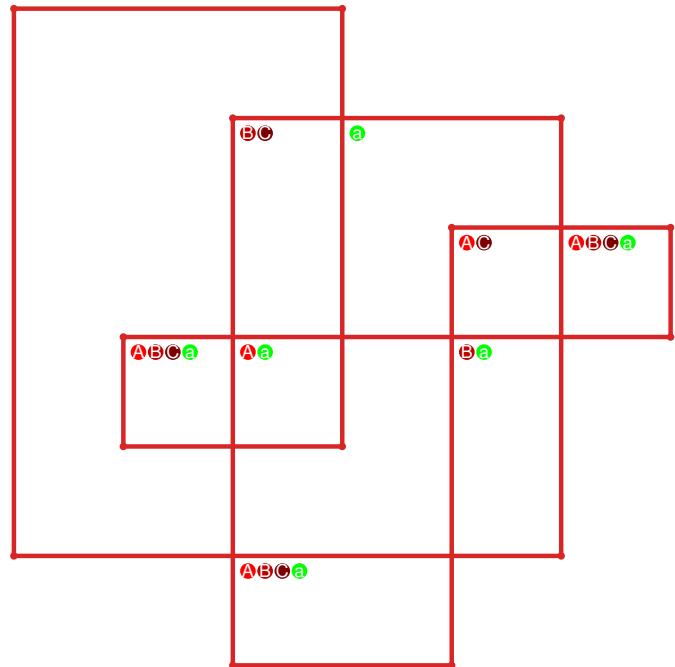


Figure 197: SnapPy multiloop plot.

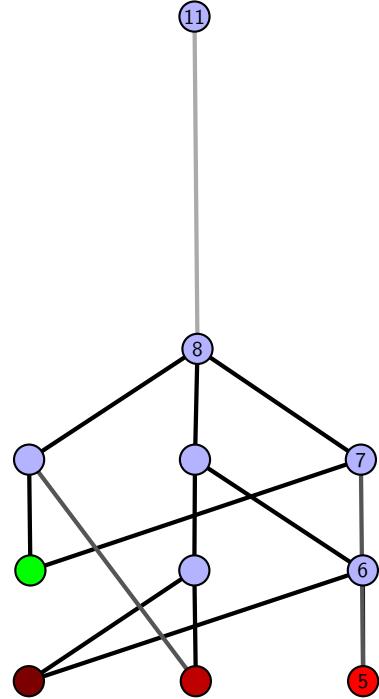


Figure 198: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.52 [[5, 18, 6, 1], [4, 15, 5, 16], [17, 14, 18, 15], [6, 11, 7, 12], [1, 8, 2, 9], [16, 3, 17, 4], [10, 13, 11, 14], [7, 13, 8, 12], [2, 10, 3, 9]]

PD code drawn by SnapPy: [(14, 1, 15, 2), (17, 6, 18, 7), (7, 18, 8, 1), (4, 9, 5, 10), (10, 5, 11, 6), (8, 11, 9, 12), (15, 12, 16, 13), (2, 13, 3, 14), (3, 16, 4, 17)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 7], [0, 7, 8, 8], [1, 8, 2, 1], [2, 8, 7, 3], [3, 6, 4, 3], [4, 6, 5, 4]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 3  
 Total pinning sets: 176  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.42  
 Average overall degree: 2.98

Table 98: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.25	2.56	2.78	2.95	3.07	3.16	3.23	3.27	

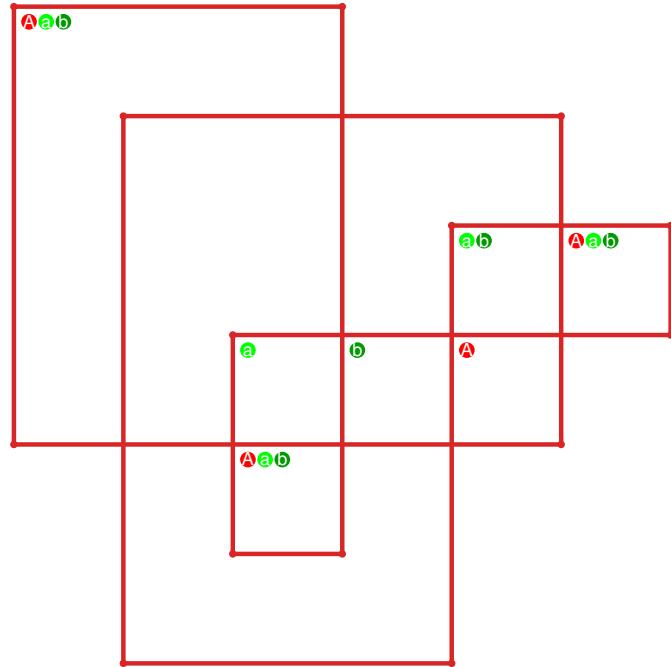


Figure 199: SnapPy multiloop plot.

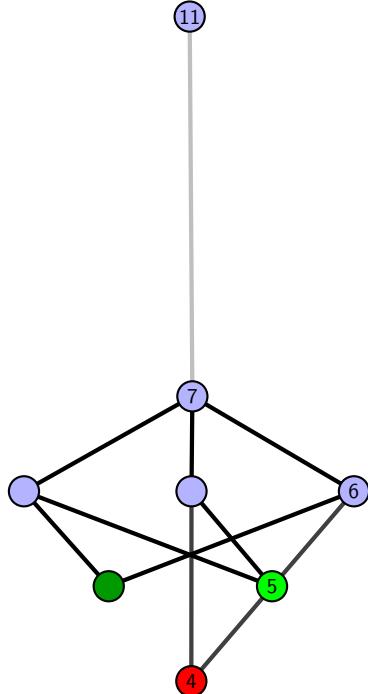


Figure 200: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.53**  $[[18, 9, 1, 10], [10, 3, 11, 4], [8, 17, 9, 18], [1, 12, 2, 13], [13, 2, 14, 3], [11, 14, 12, 15], [4, 7, 5, 8], [5, 16, 6, 17], [15, 6, 16, 7]]$

PD code drawn by `SnapPy`:  $[(11, 18, 12, 1), (8, 1, 9, 2), (13, 4, 14, 5), (2, 5, 3, 6), (16, 7, 17, 8), (17, 10, 18, 11), (9, 12, 10, 13), (3, 14, 4, 15), (6, 15, 7, 16)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 6, 7, 0], [0, 5, 4, 4], [1, 3, 3, 5], [1, 4, 3, 8], [1, 8, 7, 2], [2, 6, 8, 8], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.47

**Total pinning sets:** 136

**Average overall degree:** 2.98

**Pinning number:** 5

Table 99: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	16	38	43	26	8	1	132
Average degree	2.4	2.69	2.89	3.05	3.15	3.23	3.27	

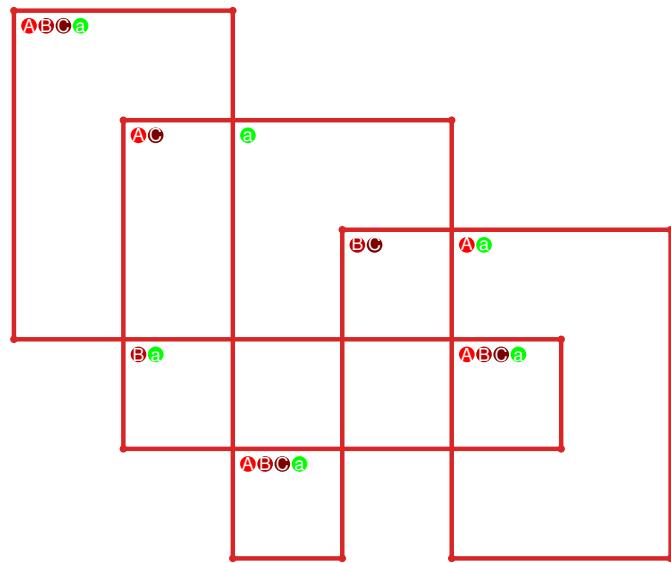


Figure 201: `SnapPy` multiloop plot.

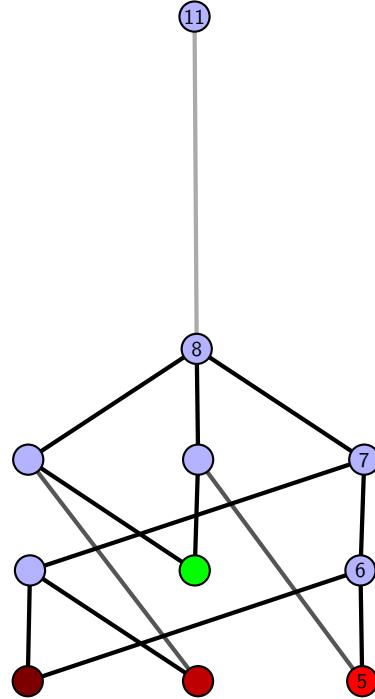


Figure 202: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.54**  $[[5, 18, 6, 1], [13, 4, 14, 5], [14, 17, 15, 18], [6, 15, 7, 16], [1, 11, 2, 10], [12, 9, 13, 10], [3, 8, 4, 9], [16, 7, 17, 8], [11, 3, 12, 2]]$

PD code drawn by `SnapPy`:  $[(18, 9, 1, 10), (11, 2, 12, 3), (14, 5, 15, 6), (1, 6, 2, 7), (10, 7, 11, 8), (8, 17, 9, 18), (4, 13, 5, 14), (12, 15, 13, 16), (3, 16, 4, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 7, 3], [0, 2, 7, 7], [0, 8, 8, 5], [1, 4, 8, 6], [1, 5, 8, 7], [2, 6, 3, 3], [4, 6, 5, 4]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 9

**Total pinning sets:** 168

**Pinning number:** 5

**Average optimal degree:** 2.6

**Average minimal degree:** 2.76

**Average overall degree:** 3.06

Table 100: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	7
Nonminimal pinning sets	0	12	49	56	32	9	1	159
Average degree	2.6	2.81	2.99	3.11	3.19	3.24	3.27	

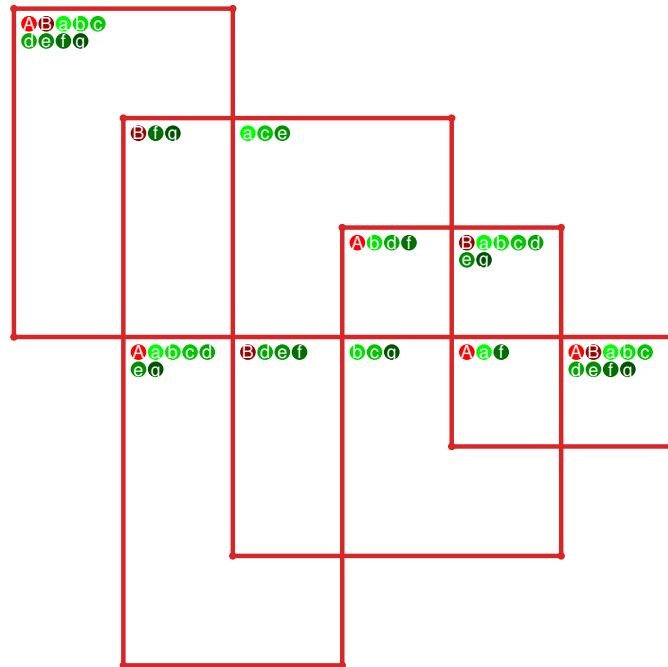


Figure 203: `SnapPy` multiloop plot.

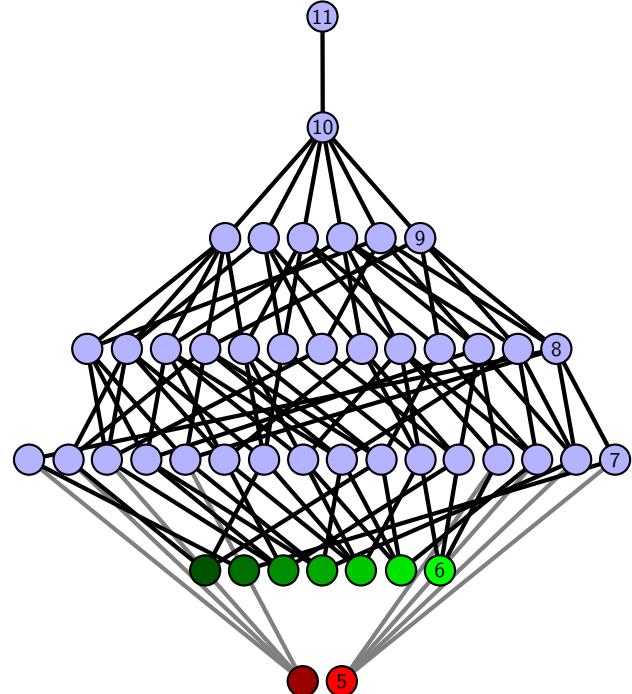


Figure 204: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.55  $[[18, 7, 1, 8], [8, 13, 9, 14], [14, 17, 15, 18], [15, 6, 16, 7], [1, 10, 2, 11], [3, 12, 4, 13], [9, 4, 10, 5], [5, 16, 6, 17], [2, 12, 3, 11]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (12, 3, 13, 4), (15, 4, 16, 5), (16, 7, 17, 8), (8, 17, 9, 18), (18, 9, 1, 10), (5, 10, 6, 11), (2, 13, 3, 14), (11, 14, 12, 15)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 7, 3], [0, 2, 7, 7], [0, 6, 8, 8], [1, 8, 8, 6], [1, 5, 4, 7], [2, 6, 3, 3], [4, 5, 5, 4]]$

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 144  
**Pinning number:** 5

**Average optimal degree:** 2.45  
**Average minimal degree:** 2.45  
**Average overall degree:** 2.98

Table 101: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.45	2.72	2.92	3.05	3.15	3.23	3.27	

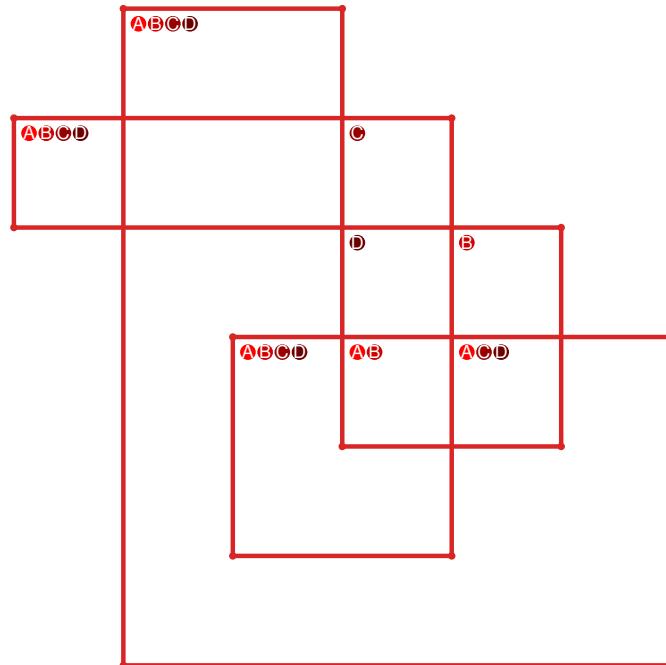


Figure 205: SnapPy multiloop plot.

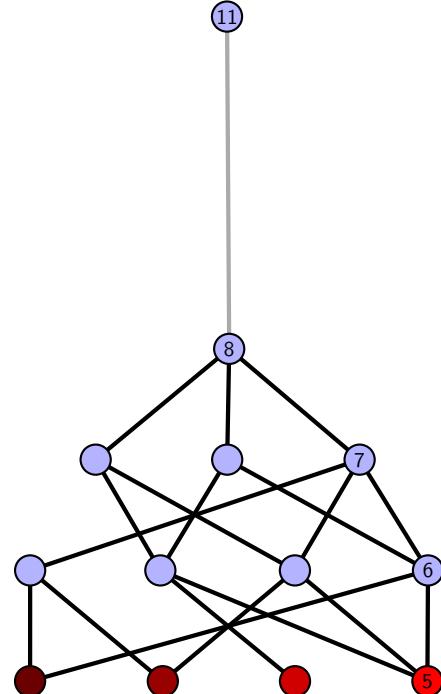


Figure 206: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.56** [[9, 18, 10, 1], [17, 8, 18, 9], [10, 8, 11, 7], [1, 7, 2, 6], [16, 5, 17, 6], [11, 14, 12, 15], [2, 15, 3, 16], [13, 4, 14, 5], [12, 4, 13, 3]]

PD code drawn by `SnapPy`: [(18, 9, 1, 10), (10, 1, 11, 2), (13, 2, 14, 3), (3, 16, 4, 17), (4, 7, 5, 8), (14, 5, 15, 6), (8, 11, 9, 12), (17, 12, 18, 13), (6, 15, 7, 16)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 4], [1, 3, 6, 7], [2, 7, 8, 6], [3, 5, 8, 4], [4, 8, 8, 5], [5, 7, 7, 6]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 8

**Total pinning sets:** 256

**Pinning number:** 4

**Average optimal degree:** 2.5

**Average minimal degree:** 2.71

**Average overall degree:** 3.06

Table 102: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	0	7
Nonminimal pinning sets	0	7	49	79	69	34	9	1	248
Average degree	2.5	2.74	2.93	3.05	3.14	3.2	3.24	3.27	

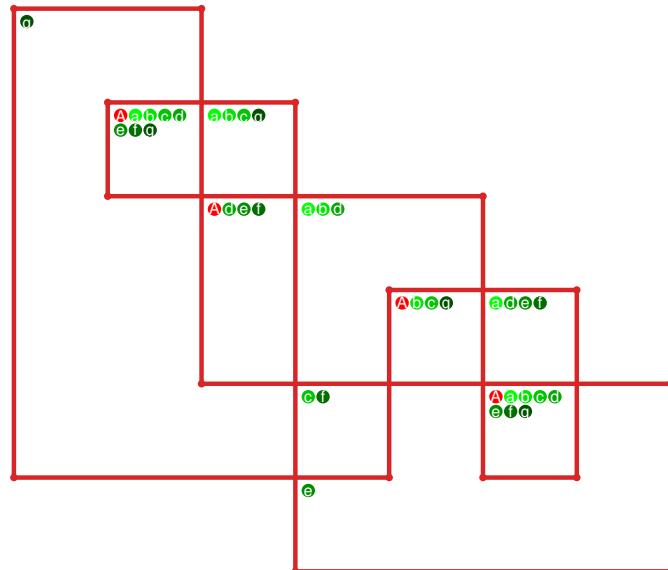


Figure 207: `SnapPy` multiloop plot.

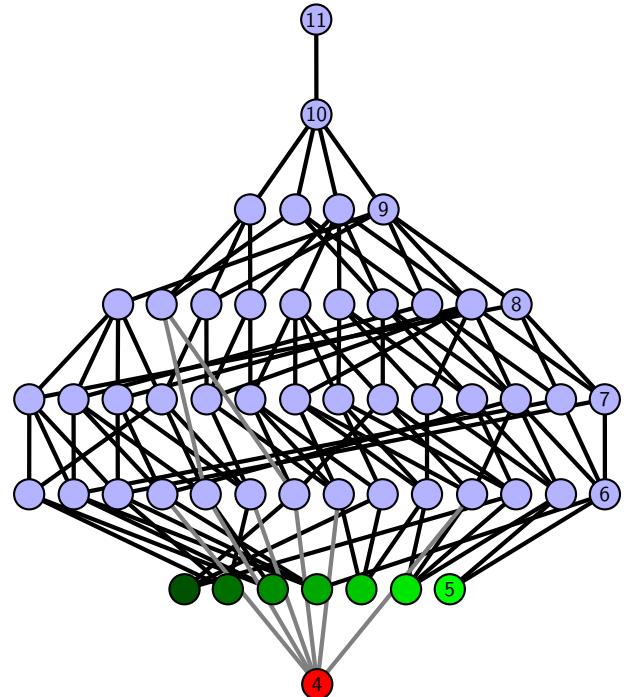


Figure 208: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.57**  $[[11, 18, 12, 1], [17, 10, 18, 11], [12, 10, 13, 9], [1, 9, 2, 8], [16, 7, 17, 8], [13, 4, 14, 5], [2, 15, 3, 16], [3, 6, 4, 7], [14, 6, 15, 5]]$

PD code drawn by `SnapPy`:  $[(11, 2, 12, 3), (3, 12, 4, 13), (1, 4, 2, 5), (14, 5, 15, 6), (6, 17, 7, 18), (7, 10, 8, 11), (15, 8, 16, 9), (18, 13, 1, 14), (9, 16, 10, 17)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 4], [1, 3, 6, 7], [2, 7, 8, 8], [3, 8, 7, 4], [4, 6, 8, 5], [5, 7, 6, 5]]$

**Total optimal pinning sets:** 6  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 192  
**Pinning number:** 5

**Average optimal degree:** 2.6  
**Average minimal degree:** 2.6  
**Average overall degree:** 3.04

Table 103: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	6	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	29	57	58	32	9	1	186
Average degree	2.6	2.84	3.0	3.12	3.19	3.24	3.27	

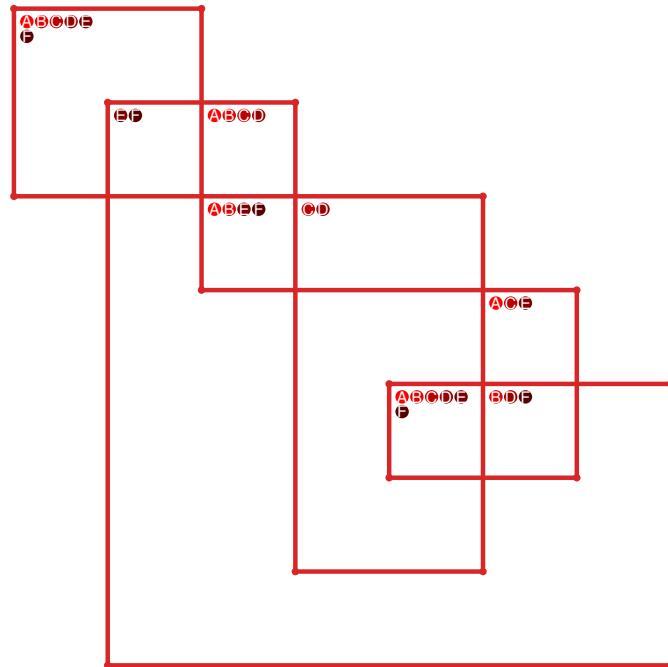


Figure 209: `SnapPy` multiloop plot.

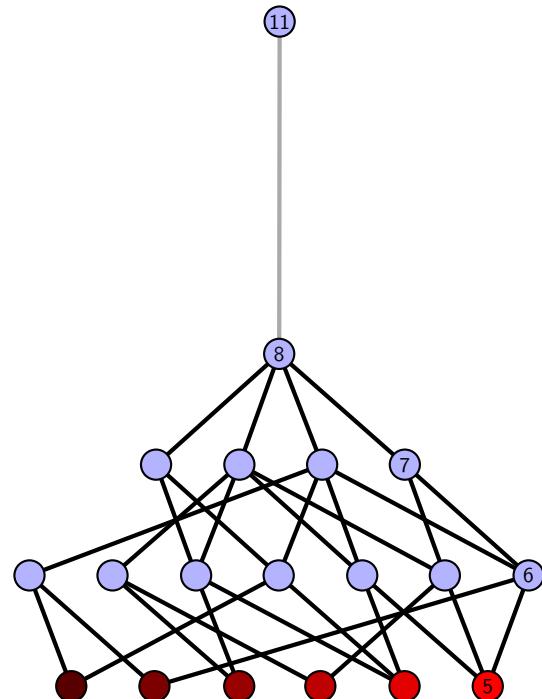


Figure 210: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.58  $[[18, 11, 1, 12], [12, 17, 13, 18], [13, 10, 14, 11], [1, 14, 2, 15], [7, 16, 8, 17], [4, 9, 5, 10], [2, 5, 3, 6], [15, 6, 16, 7], [8, 3, 9, 4]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (18, 3, 1, 4), (13, 4, 14, 5), (2, 7, 3, 8), (11, 8, 12, 9), (16, 9, 17, 10), (17, 12, 18, 13), (5, 14, 6, 15), (10, 15, 11, 16)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 8], [2, 8, 8, 6], [3, 5, 8, 7], [3, 6, 4, 4], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 6  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 168  
**Pinning number:** 5

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.5  
**Average overall degree:** 2.99

Table 104: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	6	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	27	50	49	27	8	1	162
Average degree	2.5	2.77	2.95	3.07	3.16	3.23	3.27	

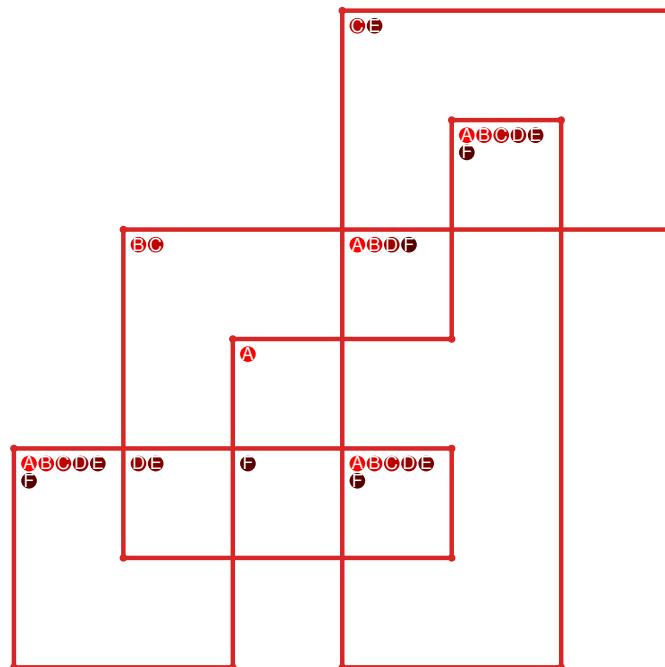


Figure 211: SnapPy multiloop plot.

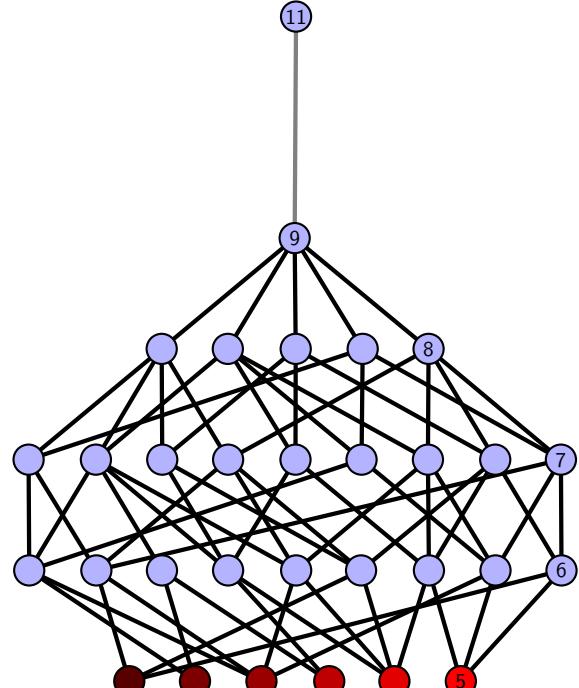


Figure 212: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.59  $[[5, 18, 6, 1], [13, 4, 14, 5], [14, 17, 15, 18], [6, 15, 7, 16], [1, 11, 2, 10], [3, 12, 4, 13], [16, 7, 17, 8], [11, 8, 12, 9], [2, 9, 3, 10]]$

PD code drawn by SnapPy:  $[(11, 2, 12, 3), (14, 5, 15, 6), (9, 6, 10, 7), (18, 7, 1, 8), (8, 17, 9, 18), (1, 10, 2, 11), (4, 13, 5, 14), (12, 15, 13, 16), (3, 16, 4, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 6, 6], [0, 7, 8, 8], [1, 8, 7, 1], [2, 7, 3, 3], [4, 6, 5, 8], [4, 7, 5, 4]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 120  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.45  
**Average overall degree:** 2.97

Table 105: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	11	32	39	25	8	1	116
Average degree	2.4	2.65	2.87	3.03	3.15	3.23	3.27	

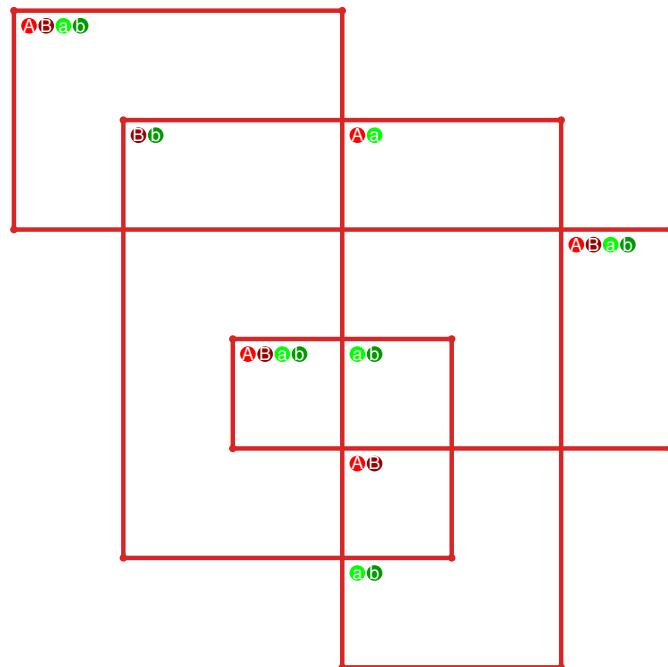


Figure 213: SnapPy multiloop plot.

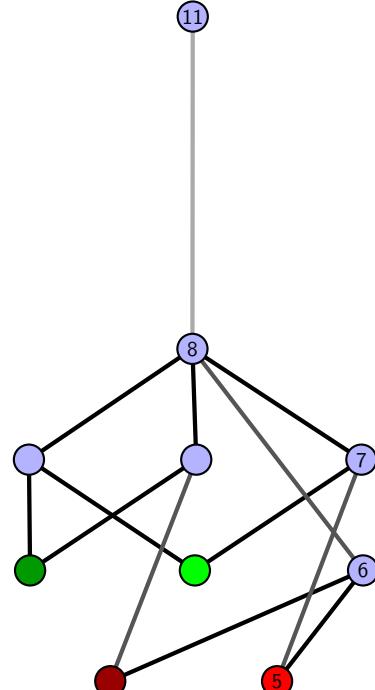


Figure 214: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.60 [[9, 18, 10, 1], [8, 15, 9, 16], [17, 14, 18, 15], [10, 5, 11, 6], [1, 6, 2, 7], [16, 7, 17, 8], [4, 13, 5, 14], [11, 3, 12, 2], [12, 3, 13, 4]]

PD code drawn by SnapPy: [(18, 9, 1, 10), (10, 1, 11, 2), (8, 3, 9, 4), (17, 4, 18, 5), (14, 5, 15, 6), (2, 11, 3, 12), (15, 12, 16, 13), (6, 13, 7, 14), (7, 16, 8, 17)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 4], [0, 3, 7, 5], [1, 4, 2, 1], [2, 8, 8, 3], [3, 8, 8, 4], [6, 7, 7, 6]]

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 5

**Average minimal degree:** 2.51

**Total pinning sets:** 140

**Average overall degree:** 2.98

**Pinning number:** 5

Table 106: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	16	40	44	26	8	1	135
Average degree	2.4	2.69	2.9	3.05	3.15	3.23	3.27	

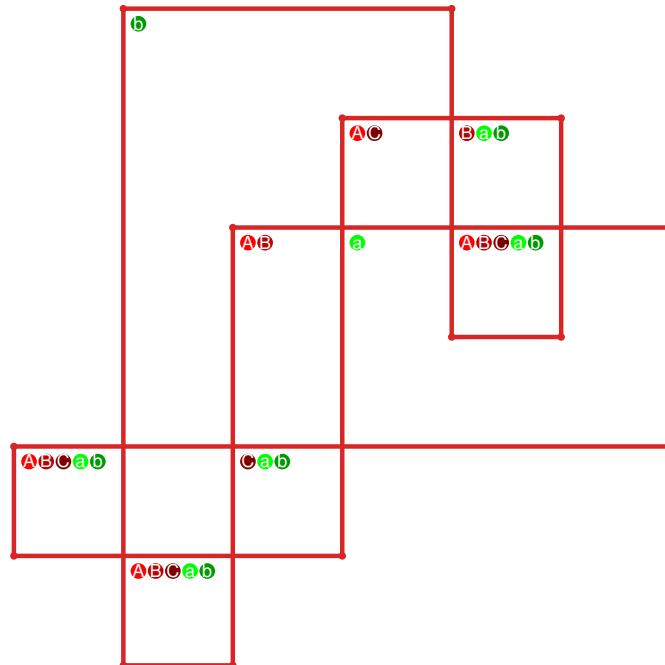


Figure 215: SnapPy multiloop plot.

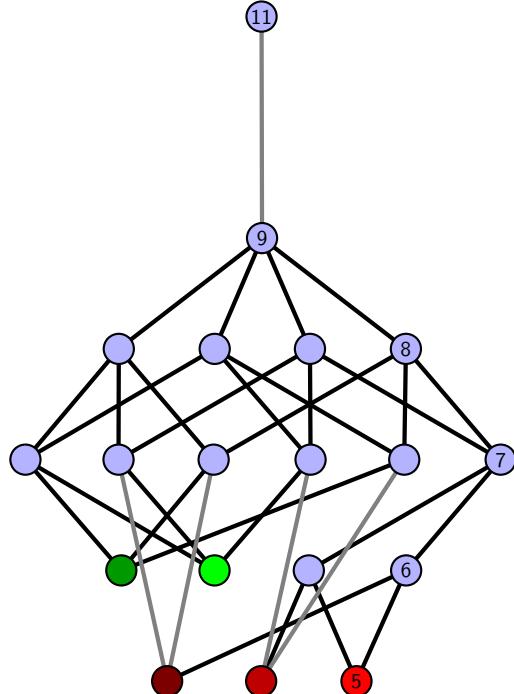


Figure 216: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.61 [[9, 18, 10, 1], [15, 8, 16, 9], [17, 12, 18, 13], [10, 4, 11, 3], [1, 6, 2, 7], [7, 14, 8, 15], [16, 14, 17, 13], [11, 4, 12, 5], [5, 2, 6, 3]]

PD code drawn by SnapPy: [(3, 18, 4, 1), (1, 8, 2, 9), (9, 2, 10, 3), (17, 4, 18, 5), (13, 6, 14, 7), (15, 10, 16, 11), (11, 14, 12, 15), (5, 12, 6, 13), (7, 16, 8, 17)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 6, 7], [0, 7, 7, 8], [0, 8, 8, 5], [1, 4, 6, 1], [1, 5, 2, 2], [2, 8, 3, 3], [3, 7, 4, 4]]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 107: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

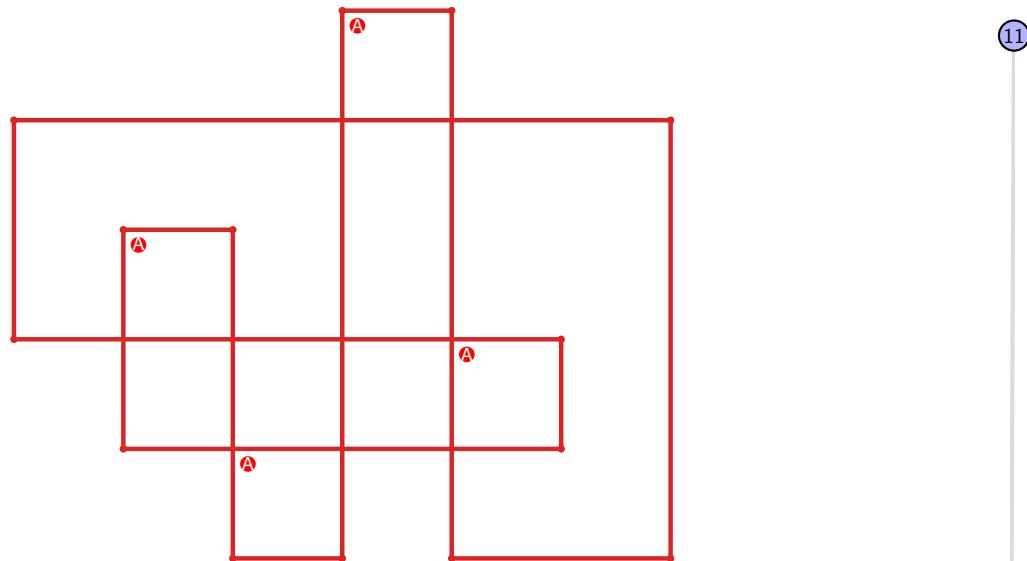


Figure 217: SnapPy multiloop plot.

4

Figure 218: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.62  $[[7, 18, 8, 1], [6, 13, 7, 14], [17, 10, 18, 11], [8, 3, 9, 4], [1, 4, 2, 5], [14, 5, 15, 6], [15, 12, 16, 13], [11, 16, 12, 17], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (17, 2, 18, 3), (12, 3, 13, 4), (18, 7, 1, 8), (15, 8, 16, 9), (13, 10, 14, 11), (4, 11, 5, 12), (9, 14, 10, 15), (5, 16, 6, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 7, 7, 8], [0, 8, 8, 4], [0, 3, 8, 5], [1, 4, 6, 1], [1, 5, 7, 7], [2, 6, 6, 2], [2, 4, 3, 3]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 96

**Average overall degree:** 2.9

**Pinning number:** 5

Table 108: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

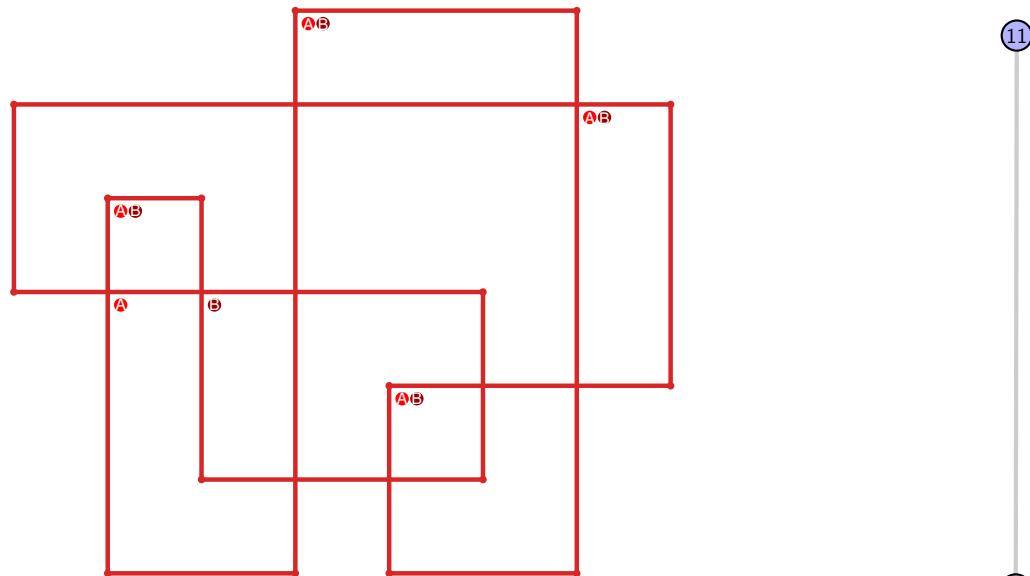


Figure 219: `SnapPy` multiloop plot.

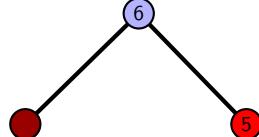


Figure 220: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.63 [[7, 18, 8, 1], [6, 13, 7, 14], [17, 8, 18, 9], [1, 4, 2, 5], [14, 5, 15, 6], [15, 12, 16, 13], [9, 16, 10, 17], [10, 3, 11, 4], [2, 11, 3, 12]]

PD code drawn by SnapPy: [(7, 18, 8, 1), (16, 1, 17, 2), (11, 2, 12, 3), (14, 5, 15, 6), (17, 8, 18, 9), (12, 9, 13, 10), (3, 10, 4, 11), (6, 13, 7, 14), (4, 15, 5, 16)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 6, 0], [0, 7, 8, 4], [1, 3, 5, 1], [1, 4, 8, 6], [2, 5, 7, 2], [3, 6, 8, 8], [3, 7, 7, 5]]

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.27

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.27

**Total pinning sets:** 112

**Average overall degree:** 2.91

**Pinning number:** 5

Table 109: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	31	34	21	7	1	109
Average degree	2.27	2.6	2.83	2.99	3.11	3.2	3.27	

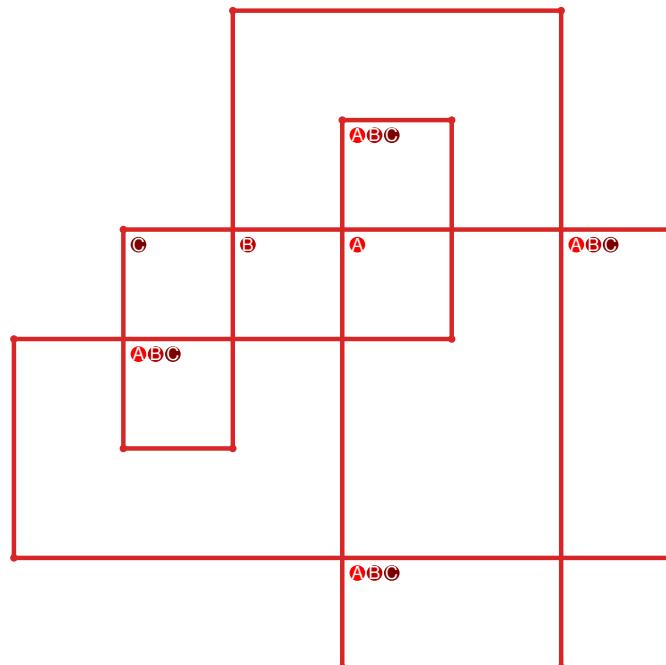


Figure 221: SnapPy multiloop plot.

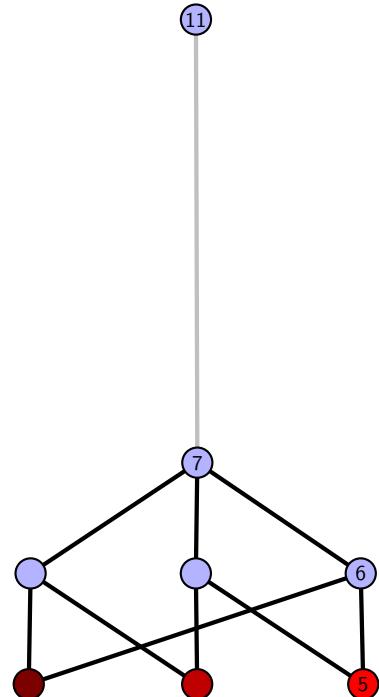


Figure 222: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.64 [[13, 18, 14, 1], [3, 12, 4, 13], [17, 14, 18, 15], [1, 6, 2, 7], [7, 2, 8, 3], [11, 4, 12, 5], [15, 11, 16, 10], [16, 9, 17, 10], [5, 8, 6, 9]]

PD code drawn by `SnapPy`: [(5, 18, 6, 1), (9, 2, 10, 3), (17, 4, 18, 5), (3, 6, 4, 7), (14, 7, 15, 8), (8, 13, 9, 14), (1, 10, 2, 11), (15, 12, 16, 13), (11, 16, 12, 17)]

Planar representation generated by `plantri`: [[1, 2, 2, 3], [0, 4, 5, 5], [0, 6, 7, 0], [0, 8, 4, 4], [1, 3, 3, 8], [1, 8, 6, 1], [2, 5, 7, 7], [2, 6, 6, 8], [3, 7, 5, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.9

**Pinning number:** 4

Table 110: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	

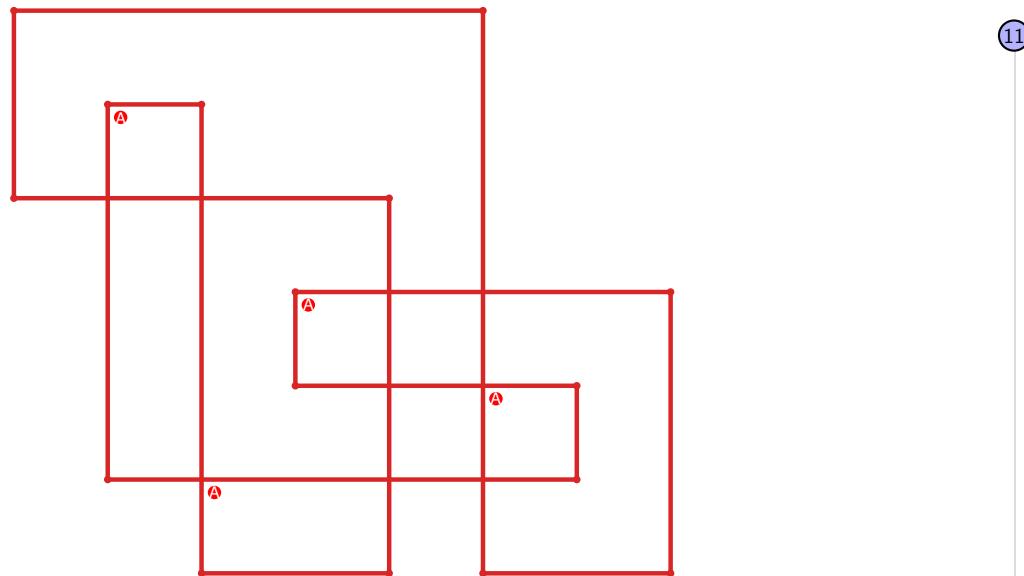


Figure 223: `SnapPy` multiloop plot.

4

Figure 224: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.65**  $[[15, 18, 16, 1], [14, 5, 15, 6], [17, 12, 18, 13], [16, 12, 17, 11], [1, 8, 2, 9], [6, 9, 7, 10], [4, 13, 5, 14], [10, 3, 11, 4], [7, 2, 8, 3]]$

PD code drawn by `SnapPy`:  $[(7, 18, 8, 1), (11, 2, 12, 3), (16, 5, 17, 6), (3, 6, 4, 7), (8, 13, 9, 14), (14, 9, 15, 10), (1, 10, 2, 11), (12, 15, 13, 16), (4, 17, 5, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 6], [0, 6, 3, 3], [0, 2, 2, 7], [0, 8, 8, 5], [1, 4, 8, 7], [1, 7, 2, 1], [3, 6, 5, 8], [4, 7, 5, 4]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.25

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 4

Table 111: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.25	2.58	2.81	2.96	3.08	3.16	3.22	3.27	

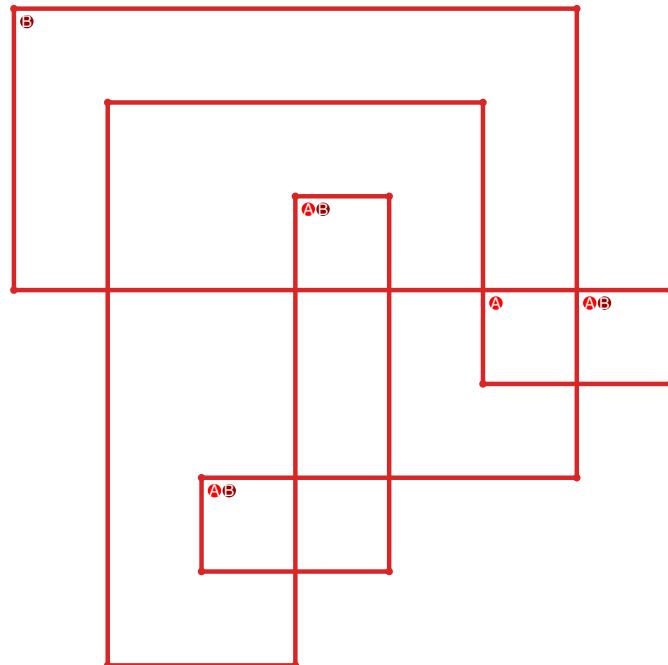


Figure 225: `SnapPy` multiloop plot.

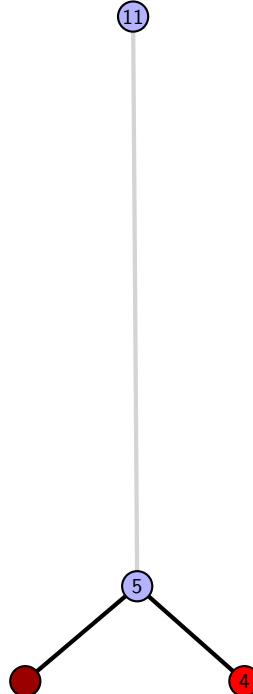


Figure 226: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.66  $[[7, 18, 8, 1], [17, 6, 18, 7], [8, 11, 9, 12], [1, 12, 2, 13], [13, 16, 14, 17], [10, 5, 11, 6], [9, 5, 10, 4], [2, 15, 3, 16], [14, 3, 15, 4]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (2, 7, 3, 8), (8, 3, 9, 4), (15, 4, 16, 5), (6, 9, 7, 10), (17, 10, 18, 11), (18, 13, 1, 14), (11, 14, 12, 15), (5, 16, 6, 17)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 5, 0], [0, 5, 6, 3], [0, 2, 7, 4], [1, 3, 7, 8], [1, 6, 6, 2], [2, 5, 5, 8], [3, 8, 8, 4], [4, 7, 7, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.25

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 4

Table 112: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.25	2.58	2.81	2.96	3.08	3.16	3.23	3.27	

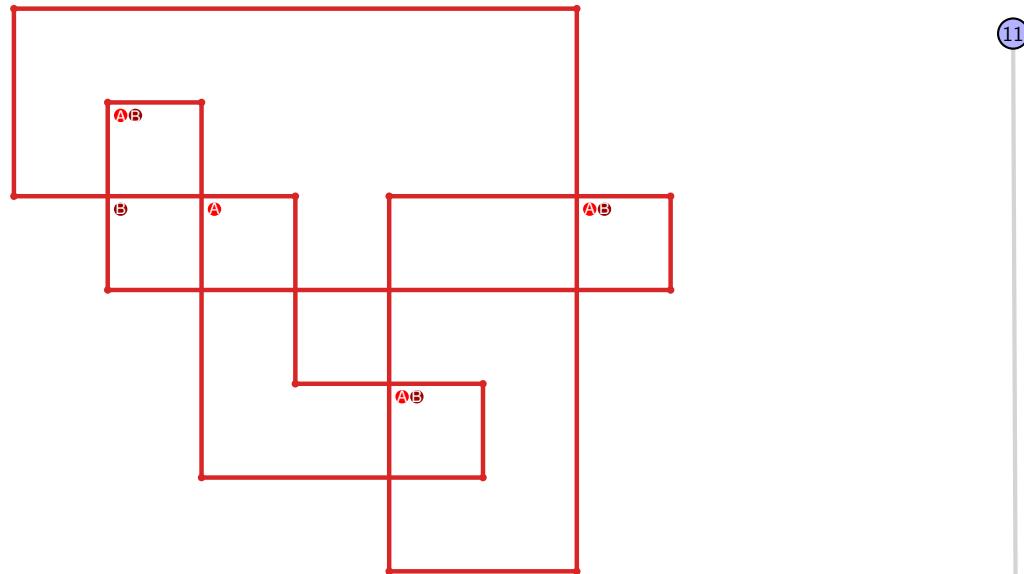


Figure 227: `SnapPy` multiloop plot.

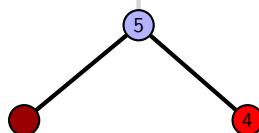


Figure 228: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.67  $[[15, 18, 16, 1], [3, 14, 4, 15], [4, 17, 5, 18], [16, 5, 17, 6], [1, 11, 2, 10], [2, 9, 3, 10], [13, 8, 14, 9], [6, 12, 7, 11], [7, 12, 8, 13]]$

PD code drawn by SnapPy:  $[(14, 3, 15, 4), (11, 6, 12, 7), (18, 7, 1, 8), (8, 17, 9, 18), (9, 4, 10, 5), (5, 10, 6, 11), (1, 12, 2, 13), (2, 15, 3, 16), (13, 16, 14, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 3, 3], [0, 2, 2, 7], [0, 7, 5, 5], [1, 4, 4, 6], [1, 5, 8, 8], [3, 8, 8, 4], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 96  
**Pinning number:** 5

**Average optimal degree:** 2.2

**Average minimal degree:** 2.2

**Average overall degree:** 2.9

Table 113: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.2	2.55	2.79	2.97	3.1	3.2	3.27	

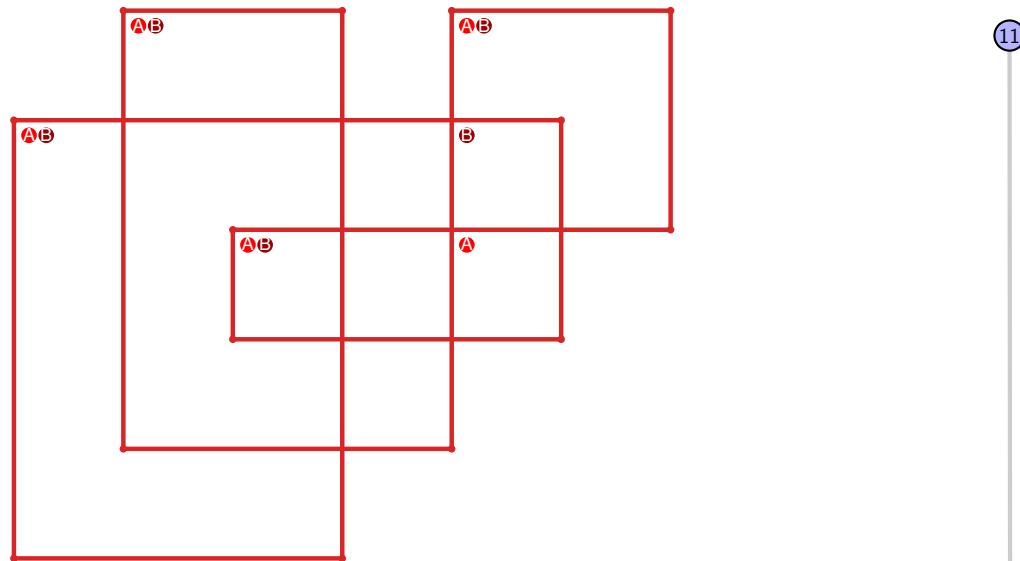


Figure 229: SnapPy multiloop plot.

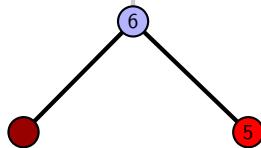


Figure 230: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.68**  $[[9, 18, 10, 1], [8, 15, 9, 16], [17, 14, 18, 15], [10, 2, 11, 1], [16, 7, 17, 8], [4, 13, 5, 14], [2, 12, 3, 11], [3, 6, 4, 7], [12, 5, 13, 6]]$

PD code drawn by `SnapPy`:  $[(10, 3, 11, 4), (1, 4, 2, 5), (14, 5, 15, 6), (9, 18, 10, 1), (2, 11, 3, 12), (15, 12, 16, 13), (6, 13, 7, 14), (7, 16, 8, 17), (17, 8, 18, 9)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 6, 0], [1, 7, 2, 1], [2, 7, 8, 8], [3, 8, 7, 3], [4, 6, 8, 5], [5, 7, 6, 5]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.33

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.39

**Total pinning sets:** 68

**Average overall degree:** 2.92

**Pinning number:** 6

Table 114: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	1
Nonminimal pinning sets	0	13	24	19	7	1	64
Average degree	2.33	2.65	2.91	3.09	3.2	3.27	

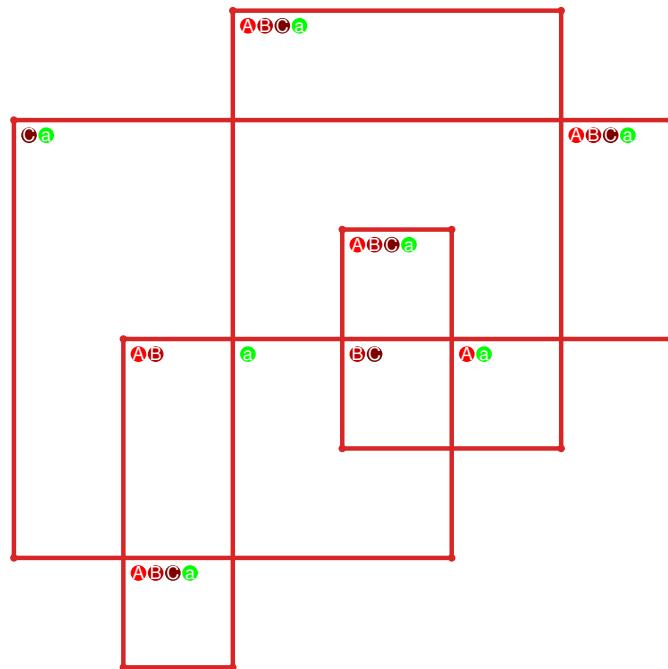


Figure 231: `SnapPy` multiloop plot.

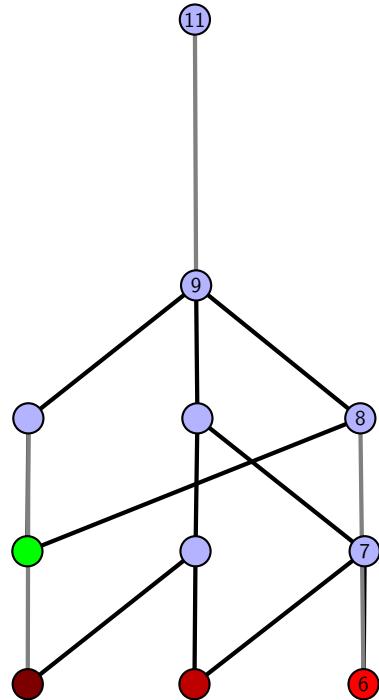


Figure 232: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.69  $[[18, 9, 1, 10], [10, 17, 11, 18], [11, 8, 12, 9], [1, 12, 2, 13], [16, 3, 17, 4], [7, 2, 8, 3], [13, 7, 14, 6], [4, 15, 5, 16], [14, 5, 15, 6]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (2, 9, 3, 10), (7, 4, 8, 5), (14, 5, 15, 6), (15, 8, 16, 9), (18, 11, 1, 12), (6, 13, 7, 14), (3, 16, 4, 17), (12, 17, 13, 18)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 5, 6], [1, 7, 7, 5], [2, 4, 6, 3], [3, 5, 8, 8], [4, 8, 8, 4], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.33

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.33

**Total pinning sets:** 64

**Average overall degree:** 2.91

**Pinning number:** 6

Table 115: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	22	18	7	1	61
Average degree	2.33	2.66	2.9	3.07	3.2	3.27	

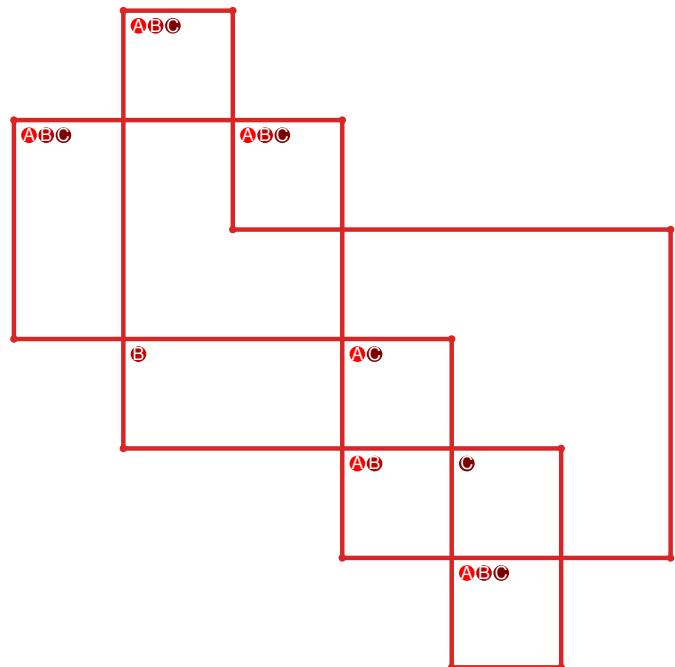


Figure 233: SnapPy multiloop plot.

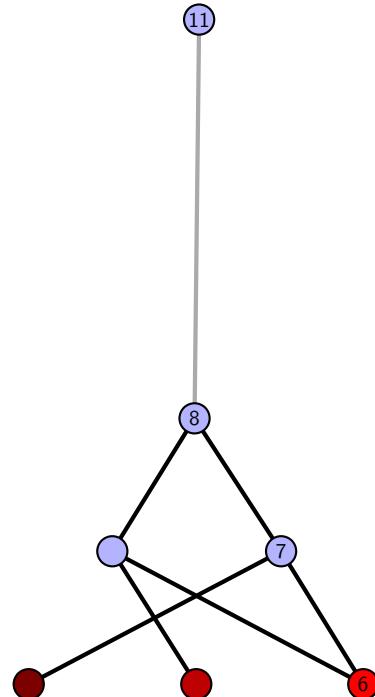


Figure 234: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.70  $[[18, 7, 1, 8], [8, 15, 9, 16], [6, 17, 7, 18], [1, 13, 2, 12], [3, 14, 4, 15], [9, 4, 10, 5], [16, 5, 17, 6], [13, 10, 14, 11], [2, 11, 3, 12]]$

PD code drawn by SnapPy:  $[(7, 2, 8, 3), (14, 3, 15, 4), (16, 5, 17, 6), (11, 8, 12, 9), (18, 9, 1, 10), (10, 17, 11, 18), (1, 12, 2, 13), (6, 13, 7, 14), (4, 15, 5, 16)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 6, 6, 0], [0, 7, 8, 8], [1, 8, 7, 5], [1, 4, 7, 6], [1, 5, 2, 2], [3, 5, 4, 8], [3, 7, 4, 3]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.48

**Total pinning sets:** 104

**Average overall degree:** 2.97

**Pinning number:** 5

Table 116: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	3
Nonminimal pinning sets	0	6	26	35	24	8	1	100
Average degree	2.4	2.61	2.84	3.01	3.14	3.23	3.27	

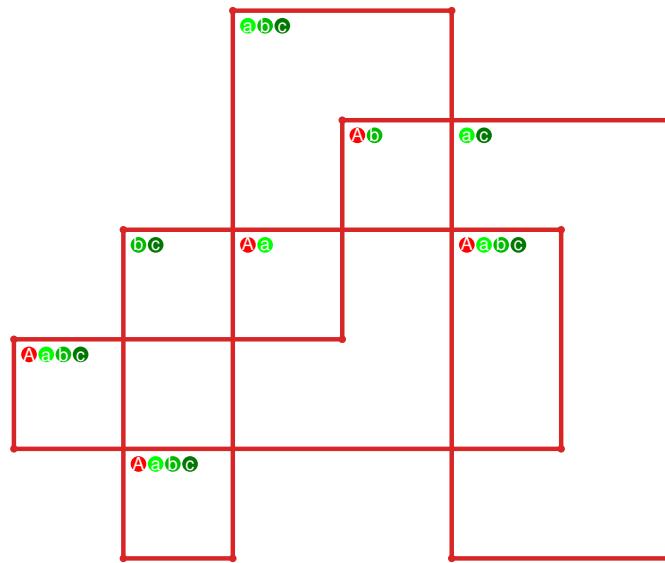


Figure 235: SnapPy multiloop plot.

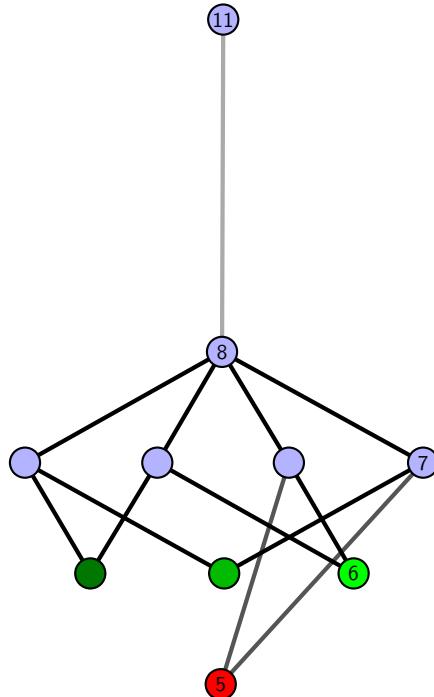


Figure 236: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.71  $[[11, 18, 12, 1], [10, 3, 11, 4], [17, 2, 18, 3], [12, 2, 13, 1], [4, 15, 5, 16], [16, 9, 17, 10], [13, 6, 14, 7], [7, 14, 8, 15], [5, 8, 6, 9]]$

PD code drawn by SnapPy:  $[(15, 18, 16, 1), (8, 1, 9, 2), (11, 4, 12, 5), (3, 6, 4, 7), (14, 7, 15, 8), (5, 12, 6, 13), (2, 13, 3, 14), (9, 16, 10, 17), (17, 10, 18, 11)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 5, 2], [0, 1, 5, 3], [0, 2, 6, 0], [1, 7, 8, 5], [1, 4, 8, 2], [3, 8, 7, 7], [4, 6, 6, 8], [4, 7, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.6

**Total minimal pinning sets:** 7

**Average minimal degree:** 2.66

**Total pinning sets:** 136

**Average overall degree:** 3.04

**Pinning number:** 5

Table 117: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	0	0	0	0	0	6
Nonminimal pinning sets	0	6	36	47	30	9	1	129
Average degree	2.6	2.74	2.93	3.08	3.19	3.24	3.27	

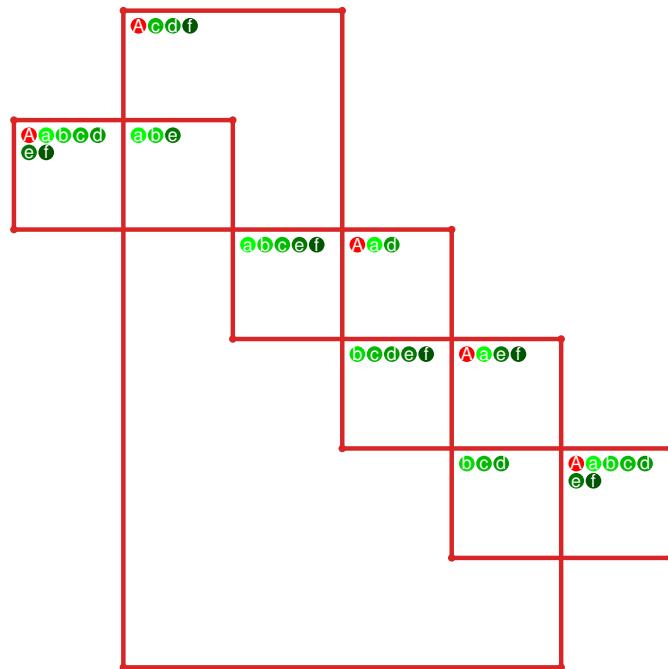


Figure 237: SnapPy multiloop plot.

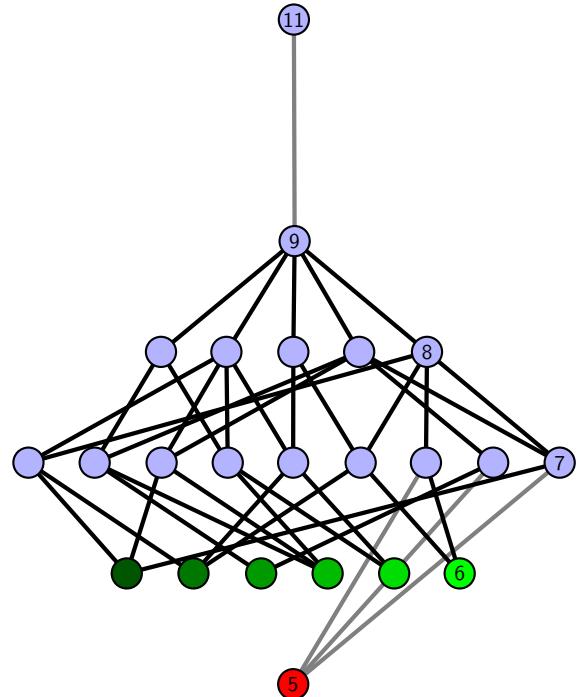


Figure 238: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.72  $[[18, 3, 1, 4], [4, 15, 5, 16], [10, 17, 11, 18], [11, 2, 12, 3], [1, 12, 2, 13], [7, 14, 8, 15], [5, 8, 6, 9], [16, 9, 17, 10], [13, 6, 14, 7]]$

PD code drawn by SnapPy:  $[(9, 18, 10, 1), (16, 1, 17, 2), (11, 4, 12, 5), (3, 6, 4, 7), (14, 7, 15, 8), (17, 10, 18, 11), (5, 12, 6, 13), (2, 13, 3, 14), (8, 15, 9, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 7, 3], [0, 2, 4, 4], [0, 3, 3, 8], [1, 8, 8, 6], [1, 5, 8, 7], [1, 6, 2, 2], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 5

**Average minimal degree:** 2.51

**Total pinning sets:** 112

**Average overall degree:** 2.98

**Pinning number:** 5

Table 118: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	4
Nonminimal pinning sets	0	6	29	38	25	8	1	107
Average degree	2.4	2.62	2.85	3.02	3.15	3.23	3.27	

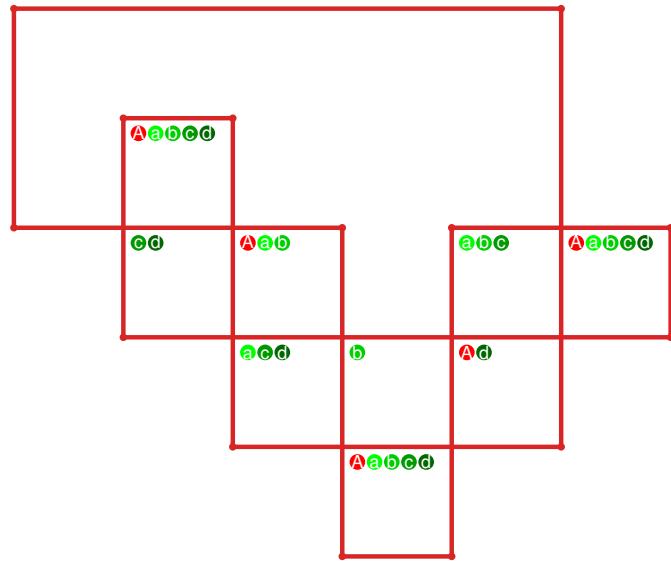


Figure 239: SnapPy multiloop plot.

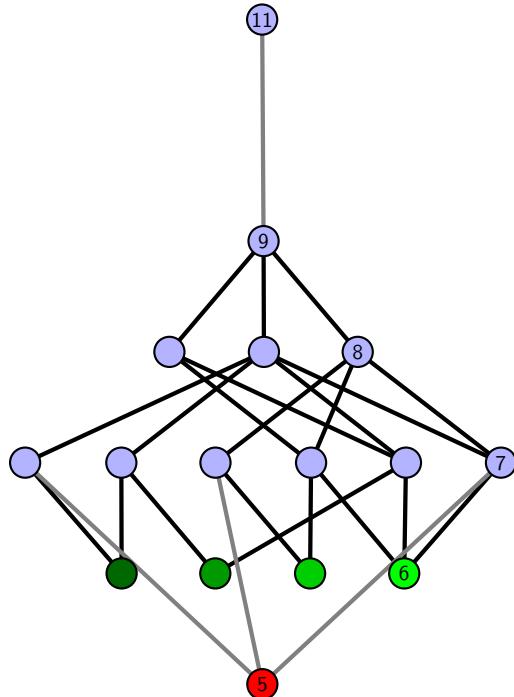


Figure 240: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.73 [[18, 9, 1, 10], [10, 15, 11, 16], [17, 2, 18, 3], [8, 1, 9, 2], [14, 5, 15, 6], [11, 5, 12, 4], [16, 4, 17, 3], [12, 7, 13, 8], [6, 13, 7, 14]]

PD code drawn by SnapPy: [(15, 18, 16, 1), (1, 14, 2, 15), (2, 9, 3, 10), (12, 3, 13, 4), (10, 5, 11, 6), (16, 7, 17, 8), (4, 11, 5, 12), (8, 13, 9, 14), (6, 17, 7, 18)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 5, 6], [0, 6, 6, 3], [0, 2, 7, 0], [1, 8, 8, 5], [1, 4, 7, 6], [1, 5, 2, 2], [3, 5, 8, 8], [4, 7, 7, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.34

**Total pinning sets:** 88

**Average overall degree:** 2.91

**Pinning number:** 5

Table 119: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	6	22	29	20	7	1	85
Average degree	2.2	2.5	2.76	2.96	3.1	3.2	3.27	

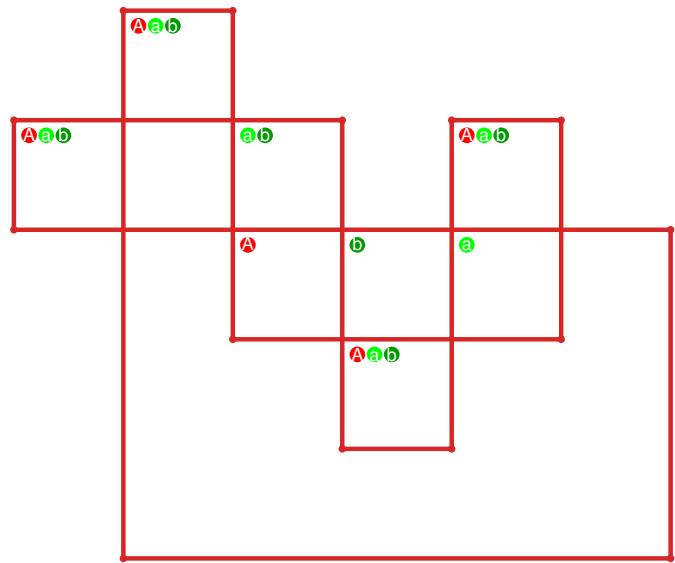


Figure 241: SnapPy multiloop plot.

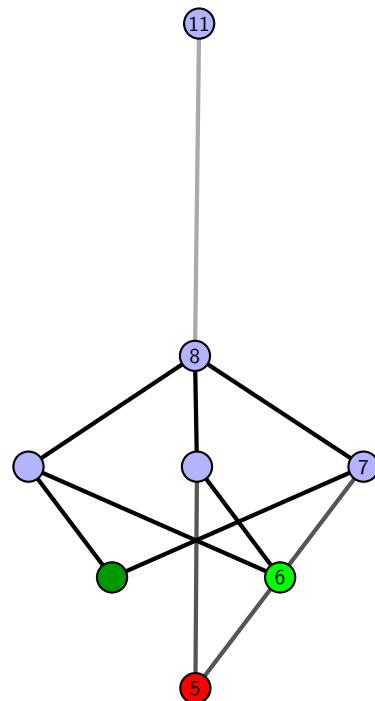


Figure 242: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.74 [[11, 18, 12, 1], [17, 10, 18, 11], [12, 10, 13, 9], [1, 14, 2, 15], [5, 16, 6, 17], [13, 8, 14, 9], [2, 8, 3, 7], [15, 4, 16, 5], [6, 4, 7, 3]]

PD code drawn by SnapPy: [(18, 3, 1, 4), (12, 1, 13, 2), (4, 17, 5, 18), (10, 5, 11, 6), (6, 9, 7, 10), (14, 7, 15, 8), (16, 11, 17, 12), (2, 13, 3, 14), (8, 15, 9, 16)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 7], [1, 7, 7, 8], [2, 6, 3, 2], [3, 5, 8, 8], [3, 8, 4, 4], [4, 7, 6, 6]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 5  
 Total pinning sets: 100  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.44  
 Average overall degree: 2.92

Table 120: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	4
Nonminimal pinning sets	0	6	27	33	21	7	1	95
Average degree	2.2	2.52	2.79	2.98	3.11	3.2	3.27	

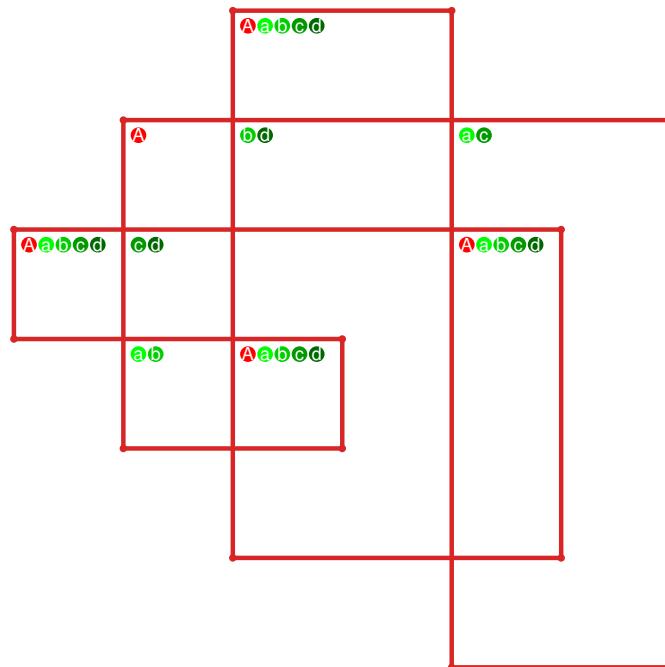


Figure 243: SnapPy multiloop plot.

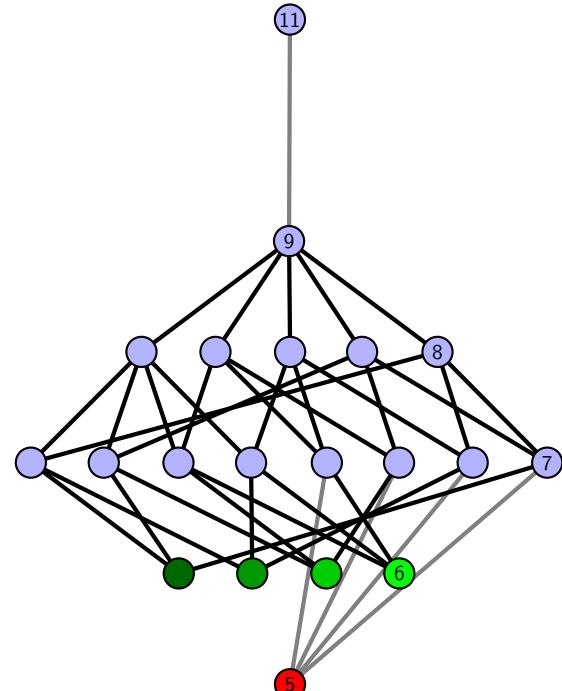


Figure 244: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.75  $[[5, 18, 6, 1], [4, 15, 5, 16], [17, 14, 18, 15], [6, 14, 7, 13], [1, 11, 2, 10], [16, 3, 17, 4], [7, 12, 8, 13], [11, 8, 12, 9], [2, 9, 3, 10]]$

PD code drawn by SnapPy:  $[(13, 18, 14, 1), (8, 5, 9, 6), (17, 6, 18, 7), (7, 16, 8, 17), (4, 9, 5, 10), (10, 3, 11, 4), (14, 11, 15, 12), (1, 12, 2, 13), (2, 15, 3, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 7, 8, 8], [1, 8, 2, 1], [3, 7, 7, 3], [4, 6, 6, 8], [4, 7, 5, 4]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 80  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.91

Table 121: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.2	2.5	2.74	2.94	3.09	3.2	3.27	

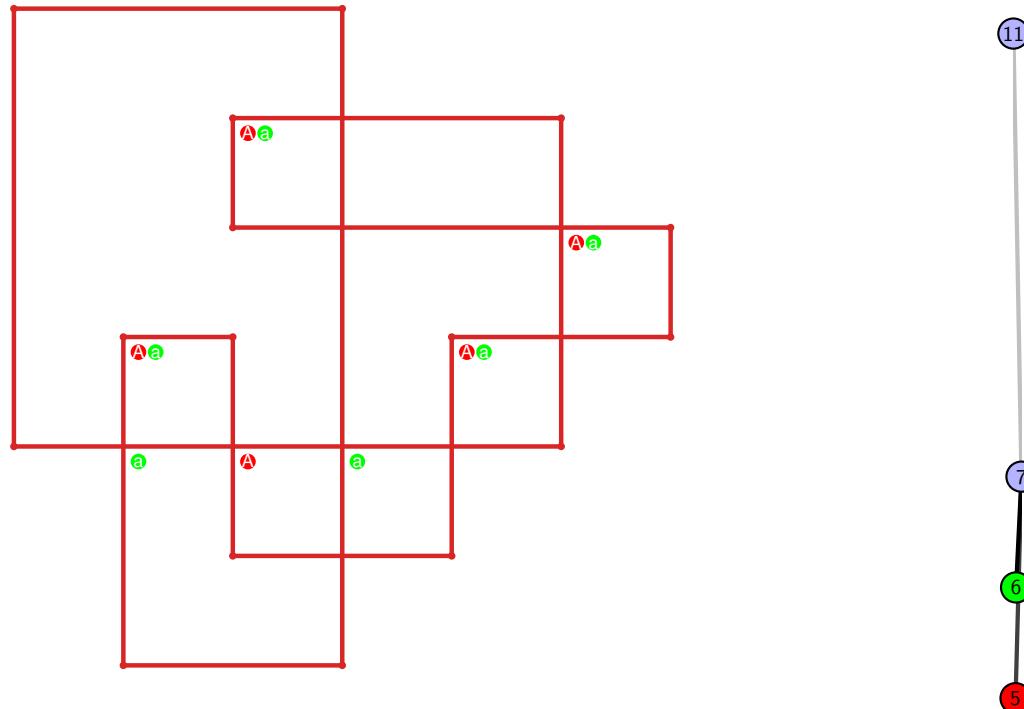


Figure 245: SnapPy multiloop plot.

Figure 246: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.76  $[[18, 9, 1, 10], [10, 4, 11, 3], [17, 2, 18, 3], [8, 1, 9, 2], [4, 12, 5, 11], [13, 16, 14, 17], [14, 7, 15, 8], [12, 6, 13, 5], [6, 15, 7, 16]]$

PD code drawn by SnapPy:  $[(15, 18, 16, 1), (1, 8, 2, 9), (3, 10, 4, 11), (12, 5, 13, 6), (16, 7, 17, 8), (9, 2, 10, 3), (4, 13, 5, 14), (11, 14, 12, 15), (6, 17, 7, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 3], [0, 2, 6, 0], [1, 7, 7, 1], [2, 7, 8, 6], [3, 5, 8, 8], [4, 8, 5, 4], [5, 7, 6, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 68  
 Pinning number: 6

Average optimal degree: 2.33  
 Average minimal degree: 2.39  
 Average overall degree: 2.92

Table 122: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	1
Nonminimal pinning sets	0	13	24	19	7	1	64
Average degree	2.33	2.65	2.91	3.09	3.2	3.27	

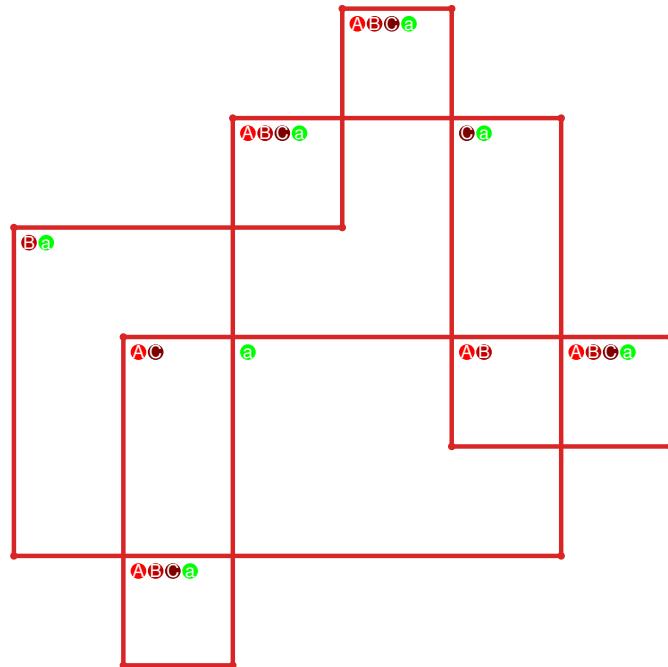


Figure 247: SnapPy multiloop plot.

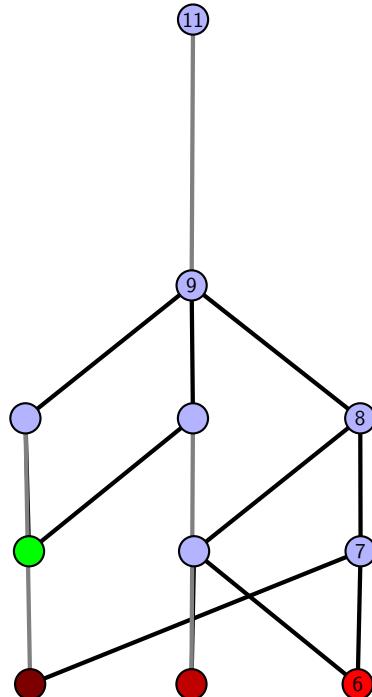


Figure 248: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.77  $[[7, 18, 8, 1], [6, 11, 7, 12], [17, 10, 18, 11], [8, 2, 9, 1], [12, 5, 13, 6], [13, 16, 14, 17], [9, 2, 10, 3], [15, 4, 16, 5], [14, 4, 15, 3]]$

PD code drawn by SnapPy:  $[(18, 11, 1, 12), (12, 1, 13, 2), (9, 2, 10, 3), (14, 7, 15, 8), (3, 8, 4, 9), (10, 13, 11, 14), (4, 15, 5, 16), (16, 5, 17, 6), (6, 17, 7, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 6], [0, 6, 6, 0], [1, 7, 5, 1], [2, 4, 7, 8], [2, 8, 3, 3], [4, 8, 8, 5], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.34

**Total pinning sets:** 88

**Average overall degree:** 2.91

**Pinning number:** 5

Table 123: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	6	22	29	20	7	1	85
Average degree	2.2	2.5	2.76	2.96	3.1	3.2	3.27	

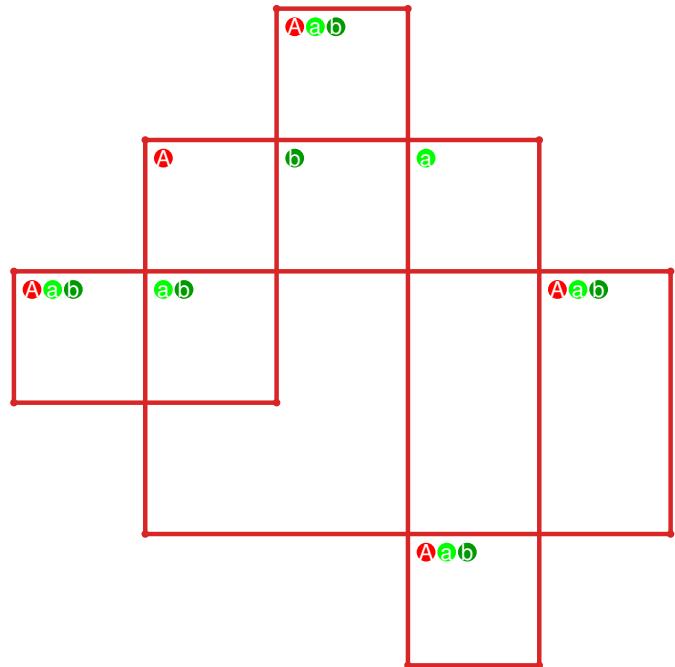


Figure 249: SnapPy multiloop plot.

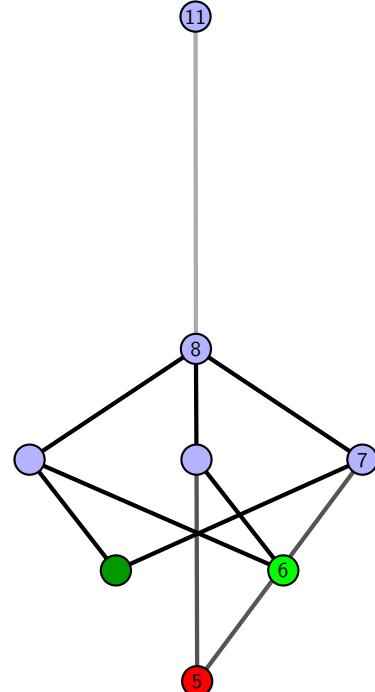


Figure 250: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.78  $[[5, 18, 6, 1], [4, 13, 5, 14], [17, 6, 18, 7], [1, 11, 2, 10], [14, 10, 15, 9], [3, 8, 4, 9], [12, 7, 13, 8], [16, 11, 17, 12], [2, 16, 3, 15]]$

PD code drawn by SnapPy:  $[(15, 4, 16, 5), (10, 5, 11, 6), (1, 6, 2, 7), (7, 12, 8, 13), (8, 17, 9, 18), (14, 9, 15, 10), (2, 11, 3, 12), (13, 18, 14, 1), (3, 16, 4, 17)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 6, 7, 0], [0, 7, 8, 4], [1, 3, 8, 5], [1, 4, 8, 6], [1, 5, 7, 2], [2, 6, 8, 3], [3, 7, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 3.0

**Total minimal pinning sets:** 13

**Average minimal degree:** 2.95

**Total pinning sets:** 326

**Average overall degree:** 3.15

**Pinning number:** 4

Table 124: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	8	4	0	0	0	0	0	12
Nonminimal pinning sets	0	7	59	106	89	41	10	1	313
Average degree	3.0	2.97	3.06	3.14	3.2	3.24	3.26	3.27	

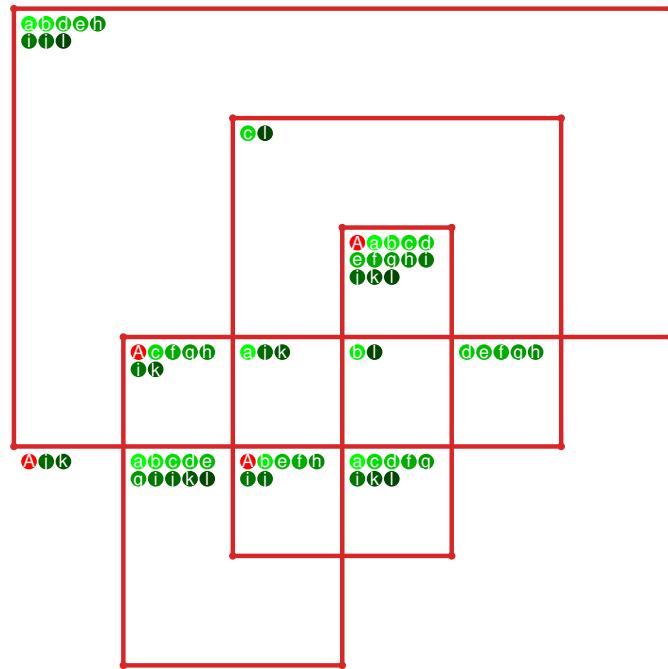


Figure 251: SnapPy multiloop plot.

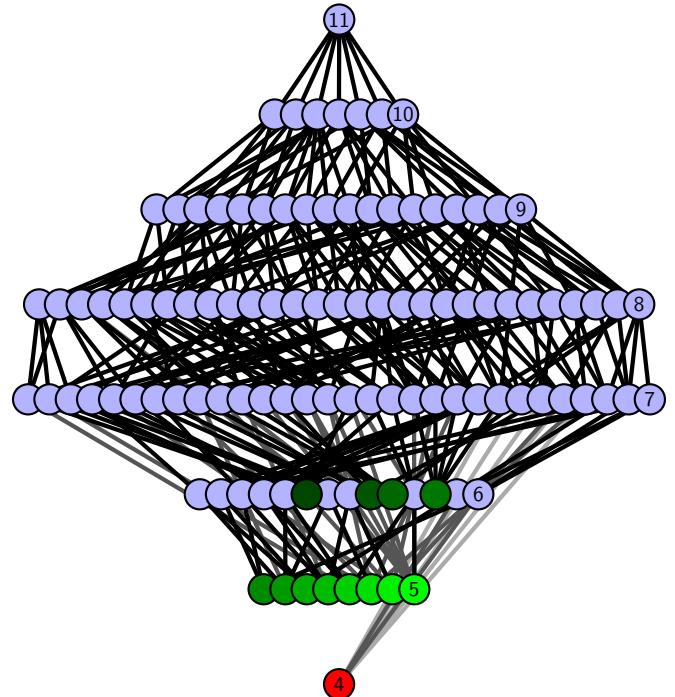


Figure 252: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.79  $[[18, 3, 1, 4], [4, 13, 5, 14], [14, 17, 15, 18], [9, 2, 10, 3], [1, 10, 2, 11], [12, 5, 13, 6], [7, 16, 8, 17], [15, 8, 16, 9], [11, 7, 12, 6]]$

PD code drawn by SnapPy:  $[(8, 1, 9, 2), (2, 7, 3, 8), (11, 4, 12, 5), (15, 6, 16, 7), (18, 9, 1, 10), (3, 12, 4, 13), (10, 13, 11, 14), (5, 16, 6, 17), (14, 17, 15, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 7, 4, 4], [0, 3, 3, 8], [1, 8, 8, 1], [2, 8, 7, 7], [2, 6, 6, 3], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.27

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.27

**Total pinning sets:** 112

**Average overall degree:** 2.91

**Pinning number:** 5

Table 125: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	31	34	21	7	1	109
Average degree	2.27	2.6	2.83	2.99	3.11	3.2	3.27	

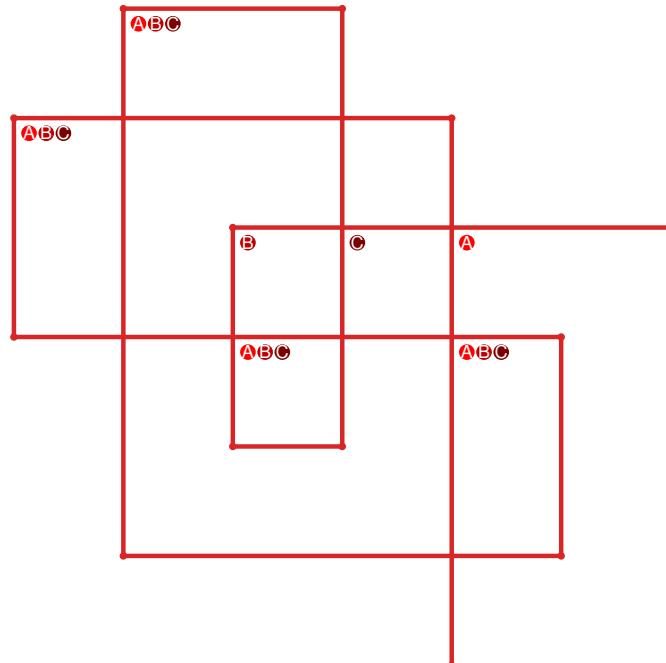


Figure 253: SnapPy multiloop plot.

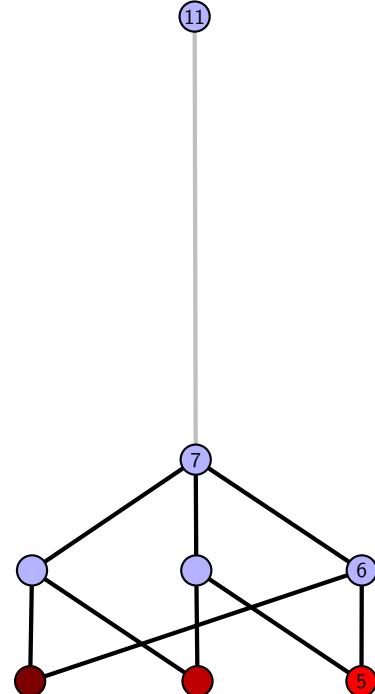


Figure 254: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.80 [[18, 13, 1, 14], [14, 10, 15, 9], [4, 17, 5, 18], [12, 1, 13, 2], [10, 7, 11, 8], [15, 8, 16, 9], [16, 3, 17, 4], [5, 3, 6, 2], [6, 11, 7, 12]]

PD code drawn by SnapPy: [(13, 18, 14, 1), (7, 4, 8, 5), (12, 5, 13, 6), (6, 11, 7, 12), (3, 8, 4, 9), (15, 10, 16, 11), (1, 14, 2, 15), (9, 16, 10, 17), (17, 2, 18, 3)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 7], [0, 7, 8, 0], [1, 8, 8, 5], [1, 4, 6, 1], [2, 5, 7, 2], [2, 6, 8, 3], [3, 7, 4, 4]]

**Total optimal pinning sets:** 5  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 124  
**Pinning number:** 5

**Average optimal degree:** 2.32  
**Average minimal degree:** 2.32  
**Average overall degree:** 2.91

Table 126: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	5	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	35	35	21	7	1	119
Average degree	2.32	2.65	2.86	3.0	3.11	3.2	3.27	

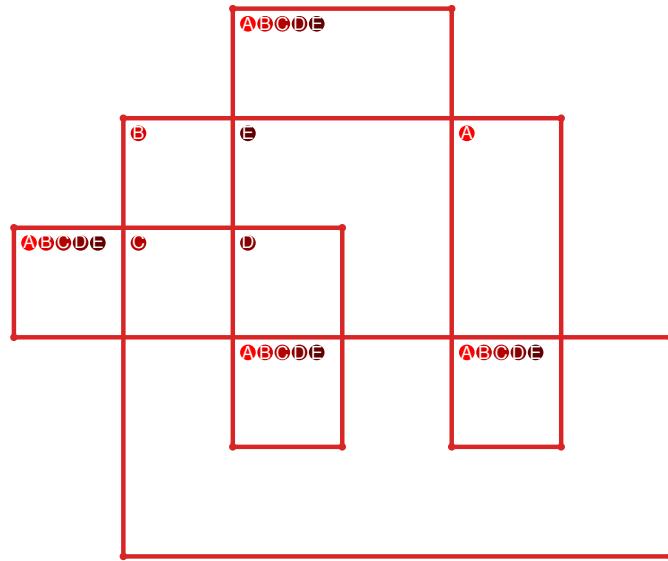


Figure 255: SnapPy multiloop plot.

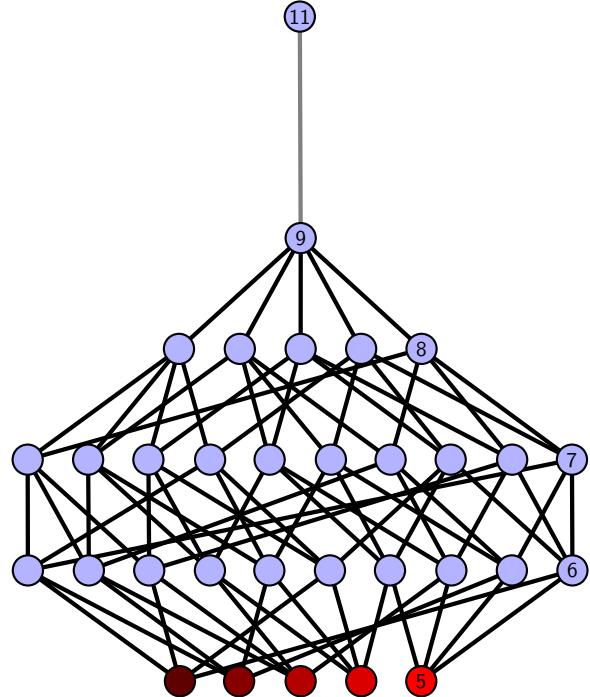


Figure 256: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.81 [[11, 18, 12, 1], [17, 10, 18, 11], [12, 8, 13, 7], [1, 7, 2, 6], [3, 16, 4, 17], [9, 14, 10, 15], [8, 14, 9, 13], [2, 5, 3, 6], [15, 4, 16, 5]]

PD code drawn by SnapPy: [(13, 2, 14, 3), (18, 3, 1, 4), (4, 17, 5, 18), (5, 10, 6, 11), (15, 8, 16, 9), (11, 6, 12, 7), (7, 12, 8, 13), (1, 14, 2, 15), (9, 16, 10, 17)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 5, 0], [0, 6, 6, 3], [0, 2, 7, 7], [1, 7, 8, 8], [1, 8, 6, 6], [2, 5, 5, 2], [3, 8, 4, 3], [4, 7, 5, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 127: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

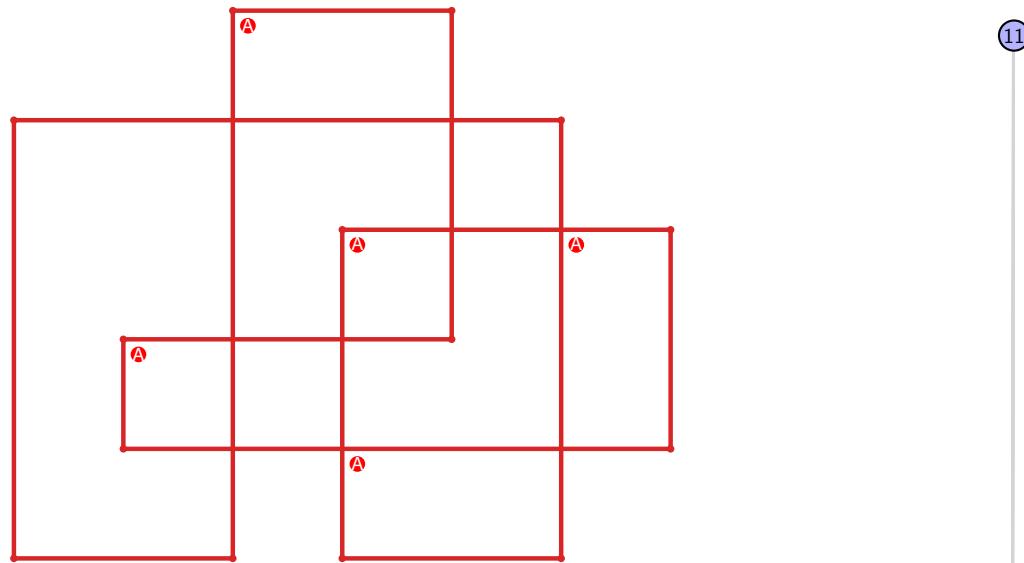


Figure 257: SnapPy multiloop plot.

11  
5

Figure 258: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.82  $[[18, 7, 1, 8], [8, 16, 9, 15], [17, 14, 18, 15], [6, 1, 7, 2], [16, 10, 17, 9], [13, 2, 14, 3], [5, 10, 6, 11], [3, 12, 4, 13], [11, 4, 12, 5]]$

PD code drawn by SnapPy:  $[(8, 1, 9, 2), (15, 4, 16, 5), (6, 13, 7, 14), (2, 7, 3, 8), (18, 9, 1, 10), (14, 11, 15, 12), (12, 5, 13, 6), (3, 16, 4, 17), (10, 17, 11, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 5, 6, 0], [1, 6, 2, 1], [2, 7, 7, 3], [3, 8, 8, 4], [5, 8, 8, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 48

**Average overall degree:** 2.85

**Pinning number:** 6

Table 128: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.17	2.54	2.81	3.02	3.17	3.27	

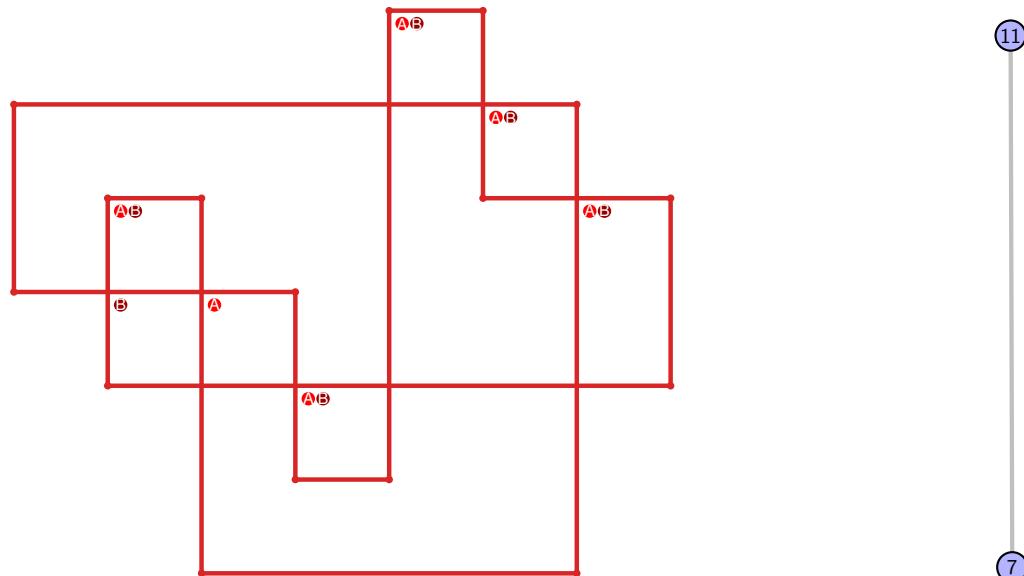


Figure 259: SnapPy multiloop plot.

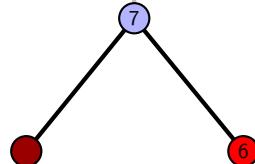


Figure 260: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.83 [[18, 13, 1, 14], [14, 7, 15, 8], [8, 17, 9, 18], [12, 1, 13, 2], [6, 15, 7, 16], [16, 5, 17, 6], [9, 3, 10, 2], [4, 11, 5, 12], [3, 11, 4, 10]]

PD code drawn by SnapPy: [(13, 2, 14, 3), (11, 4, 12, 5), (7, 18, 8, 1), (1, 8, 2, 9), (15, 10, 16, 11), (3, 12, 4, 13), (5, 14, 6, 15), (9, 16, 10, 17), (17, 6, 18, 7)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 6], [0, 6, 7, 0], [1, 5, 5, 1], [2, 4, 4, 7], [2, 8, 8, 3], [3, 8, 8, 5], [6, 7, 7, 6]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1

**Total pinning sets:** 64  
**Pinning number:** 5

**Average optimal degree:** 2.0

**Average minimal degree:** 2.0

**Average overall degree:** 2.84

Table 129: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

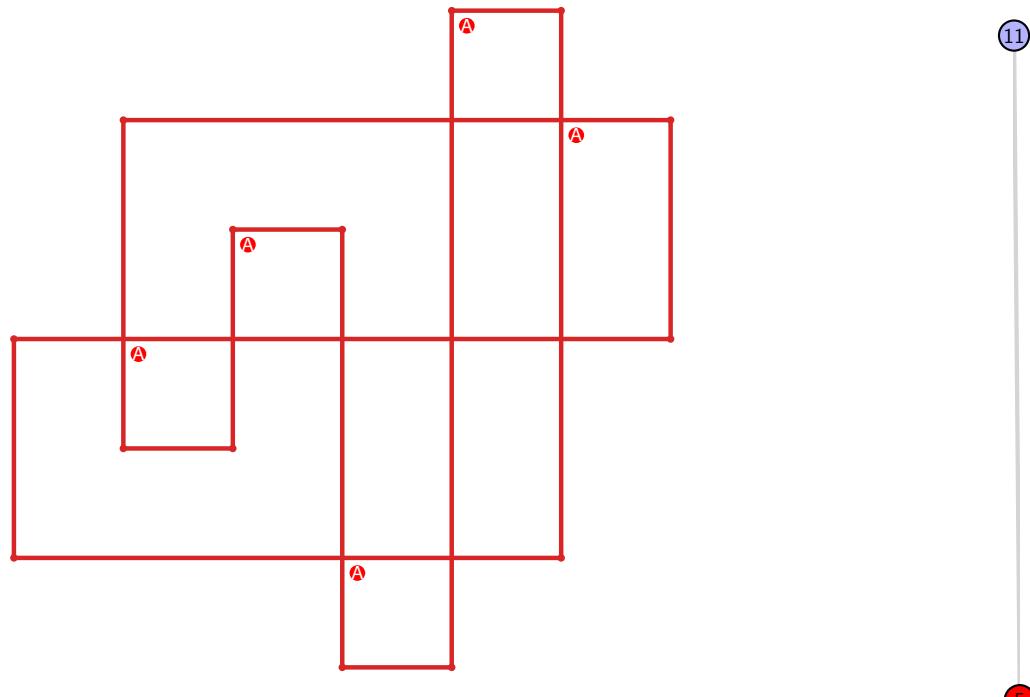


Figure 261: SnapPy multiloop plot.

11

5

Figure 262: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.84** [[11, 18, 12, 1], [17, 10, 18, 11], [12, 8, 13, 7], [1, 7, 2, 6], [9, 16, 10, 17], [8, 16, 9, 15], [13, 4, 14, 5], [2, 5, 3, 6], [3, 14, 4, 15]]

PD code drawn by `SnapPy`: [(10, 1, 11, 2), (17, 2, 18, 3), (3, 16, 4, 17), (4, 9, 5, 10), (14, 7, 15, 8), (18, 11, 1, 12), (12, 5, 13, 6), (6, 13, 7, 14), (8, 15, 9, 16)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 3], [0, 2, 7, 7], [1, 5, 5, 1], [2, 4, 4, 8], [2, 8, 8, 7], [3, 6, 8, 3], [5, 7, 6, 6]]

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3

**Total pinning sets:** 56  
**Pinning number:** 6

**Average optimal degree:** 2.22  
**Average minimal degree:** 2.22  
**Average overall degree:** 2.86

Table 130: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	12	19	15	6	1	53
Average degree	2.22	2.6	2.86	3.04	3.17	3.27	

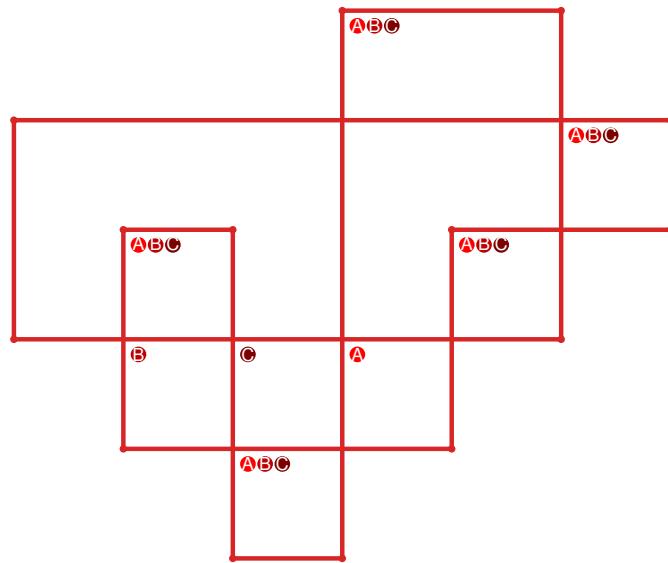


Figure 263: `SnapPy` multiloop plot.

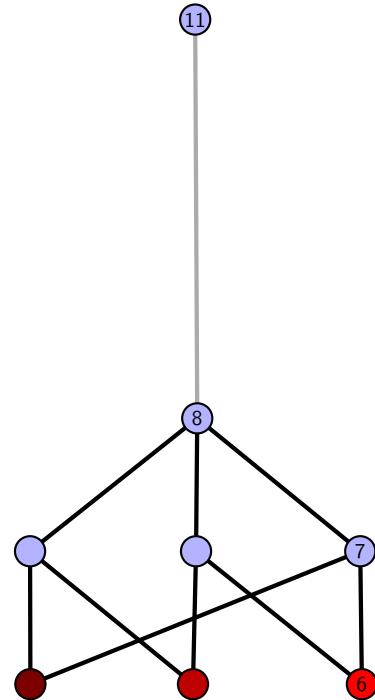


Figure 264: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.85  $[[11, 18, 12, 1], [17, 10, 18, 11], [12, 10, 13, 9], [1, 7, 2, 6], [3, 16, 4, 17], [13, 8, 14, 9], [7, 14, 8, 15], [2, 5, 3, 6], [15, 4, 16, 5]]$

PD code drawn by SnapPy:  $[(13, 2, 14, 3), (18, 3, 1, 4), (4, 17, 5, 18), (11, 6, 12, 7), (7, 10, 8, 11), (15, 8, 16, 9), (5, 12, 6, 13), (1, 14, 2, 15), (9, 16, 10, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 6, 7, 7], [1, 7, 8, 8], [2, 6, 6, 2], [3, 5, 5, 8], [3, 8, 4, 3], [4, 7, 6, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 131: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

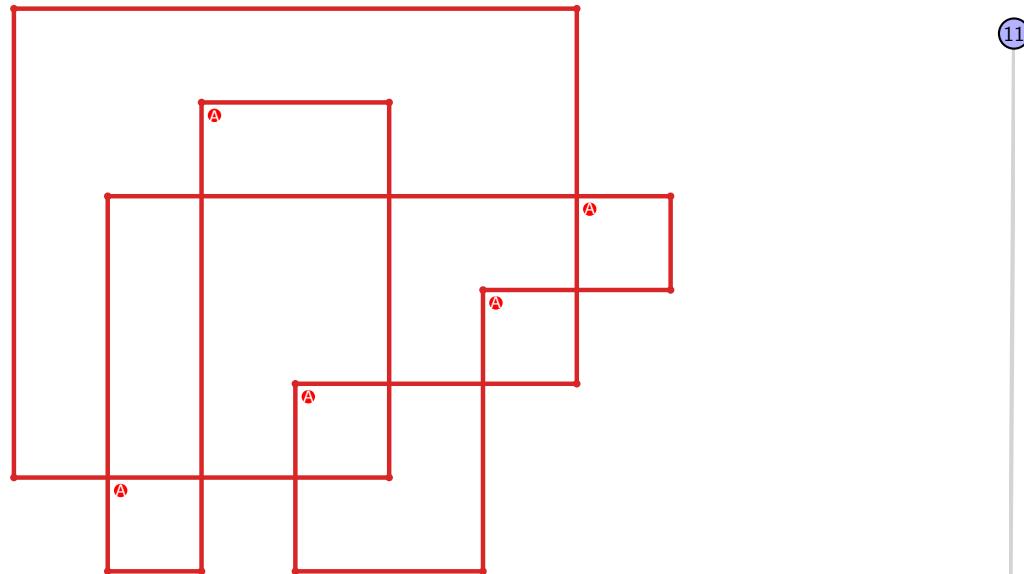


Figure 265: SnapPy multiloop plot.



Figure 266: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.86** `[[18, 9, 1, 10], [10, 7, 11, 8], [8, 17, 9, 18], [1, 15, 2, 14], [6, 11, 7, 12], [16, 5, 17, 6], [15, 5, 16, 4], [2, 13, 3, 14], [12, 3, 13, 4]]`

PD code drawn by `SnapPy`: `[(9, 18, 10, 1), (1, 12, 2, 13), (13, 2, 14, 3), (7, 4, 8, 5), (15, 6, 16, 7), (3, 8, 4, 9), (17, 10, 18, 11), (11, 16, 12, 17), (5, 14, 6, 15)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 4, 4, 2], [0, 1, 5, 0], [0, 6, 7, 7], [1, 8, 5, 1], [2, 4, 6, 6], [3, 5, 5, 8], [3, 8, 8, 3], [4, 7, 7, 6]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.84

**Pinning number:** 5

Table 132: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.39	2.67	2.88	3.04	3.17	3.27	

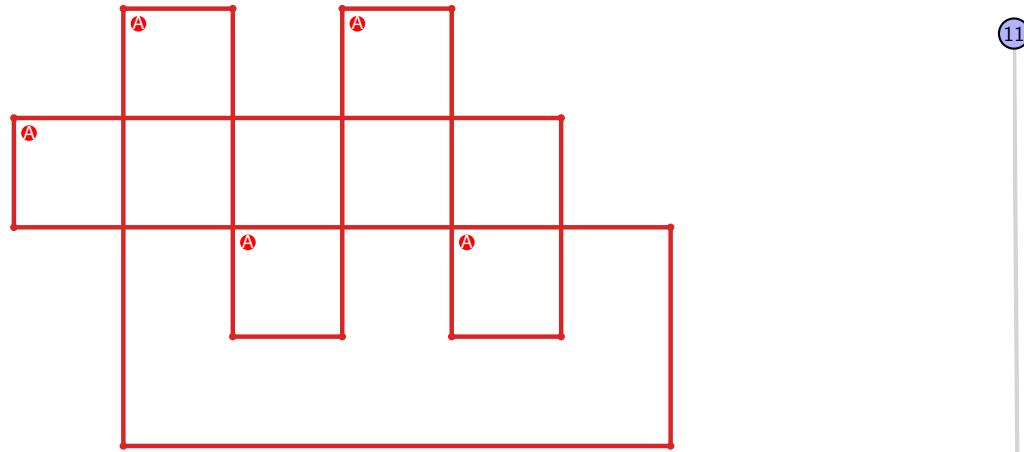


Figure 267: `SnapPy` multiloop plot.

5

Figure 268: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.87** `[[13, 18, 14, 1], [17, 12, 18, 13], [14, 12, 15, 11], [1, 11, 2, 10], [5, 16, 6, 17], [15, 6, 16, 7], [2, 9, 3, 10], [4, 7, 5, 8], [8, 3, 9, 4]]`

PD code drawn by `SnapPy`: `[(10, 1, 11, 2), (17, 2, 18, 3), (15, 4, 16, 5), (5, 14, 6, 15), (6, 9, 7, 10), (12, 7, 13, 8), (18, 11, 1, 12), (8, 13, 9, 14), (3, 16, 4, 17)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 6], [1, 7, 5, 5], [2, 4, 4, 7], [3, 8, 8, 3], [4, 8, 8, 5], [6, 7, 7, 6]]`

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 48

**Average overall degree:** 2.85

**Pinning number:** 6

Table 133: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.17	2.54	2.81	3.02	3.17	3.27	

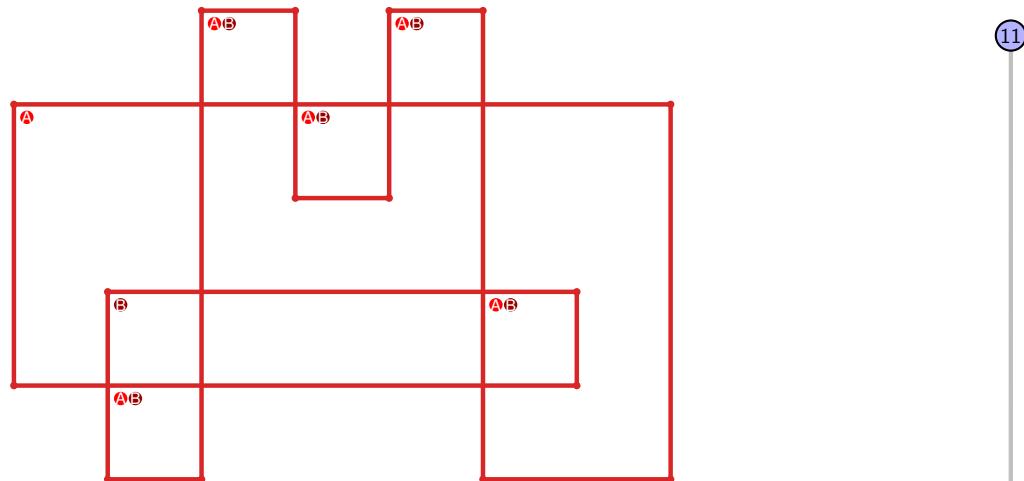


Figure 269: `SnapPy` multiloop plot.

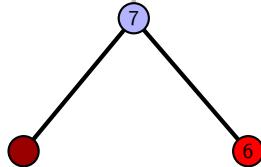


Figure 270: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.88**  $[[7, 18, 8, 1], [17, 6, 18, 7], [8, 2, 9, 1], [5, 16, 6, 17], [2, 10, 3, 9], [13, 4, 14, 5], [15, 10, 16, 11], [3, 12, 4, 13], [14, 12, 15, 11]]$

PD code drawn by `SnapPy`:  $[(11, 2, 12, 3), (15, 4, 16, 5), (7, 18, 8, 1), (13, 8, 14, 9), (9, 12, 10, 13), (1, 10, 2, 11), (3, 14, 4, 15), (5, 16, 6, 17), (17, 6, 18, 7)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 6, 7, 2], [3, 7, 7, 8], [3, 8, 8, 4], [4, 8, 5, 5], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.79

**Pinning number:** 6

Table 134: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.7	2.93	3.12	3.27	

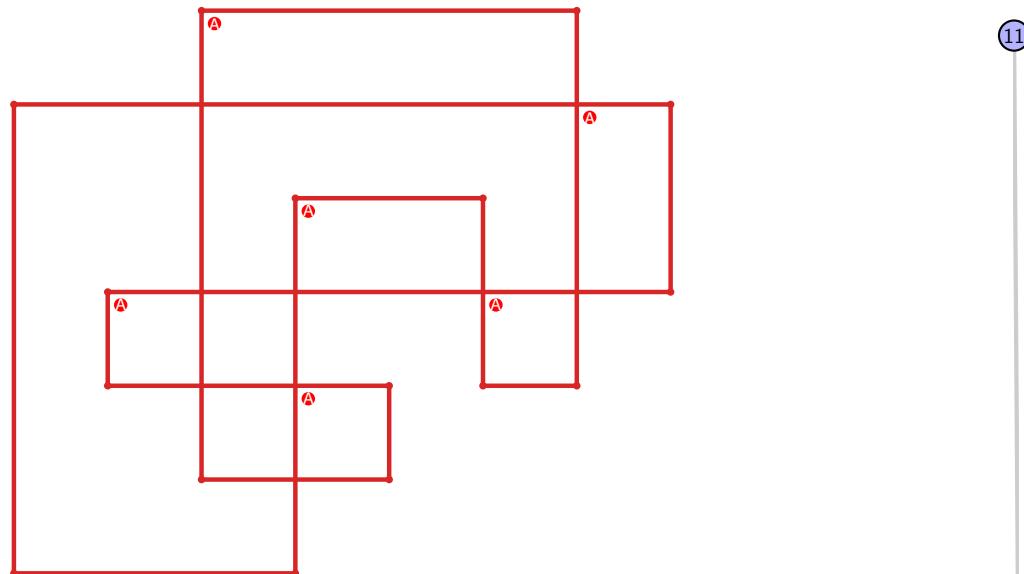


Figure 271: `SnapPy` multiloop plot.



Figure 272: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.89  $[[5, 18, 6, 1], [17, 4, 18, 5], [6, 16, 7, 15], [1, 10, 2, 11], [3, 16, 4, 17], [7, 14, 8, 15], [9, 12, 10, 13], [2, 12, 3, 11], [13, 8, 14, 9]]$

PD code drawn by SnapPy:  $[(14, 1, 15, 2), (10, 5, 11, 6), (16, 7, 17, 8), (8, 17, 9, 18), (6, 9, 7, 10), (4, 11, 5, 12), (12, 3, 13, 4), (18, 13, 1, 14), (2, 15, 3, 16)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 5], [0, 6, 7, 7], [1, 7, 2, 1], [2, 8, 8, 2], [3, 8, 8, 7], [3, 6, 4, 3], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.79

**Pinning number:** 6

Table 135: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.7	2.93	3.12	3.27	

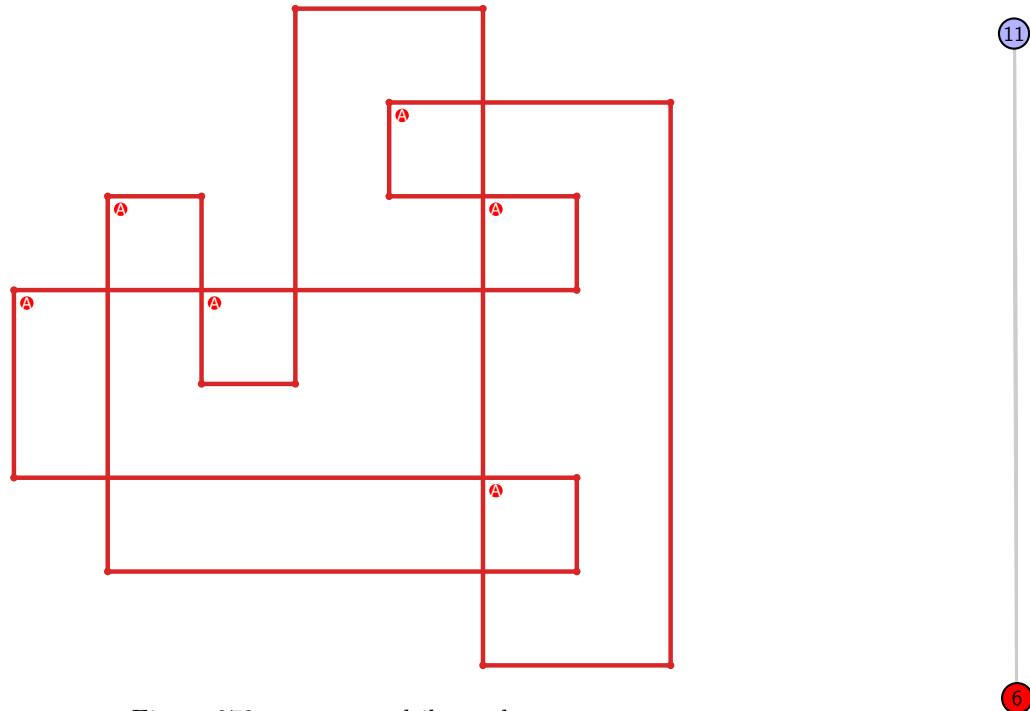


Figure 273: SnapPy multiloop plot.

Figure 274: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.90  $[[18, 11, 1, 12], [12, 5, 13, 6], [8, 17, 9, 18], [10, 1, 11, 2], [4, 13, 5, 14], [6, 15, 7, 16], [16, 7, 17, 8], [9, 3, 10, 2], [14, 3, 15, 4]]$

PD code drawn by SnapPy:  $[(7, 18, 8, 1), (11, 2, 12, 3), (13, 6, 14, 7), (17, 8, 18, 9), (1, 10, 2, 11), (3, 12, 4, 13), (5, 14, 6, 15), (15, 4, 16, 5), (9, 16, 10, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 5], [0, 6, 6, 7], [0, 7, 7, 0], [1, 8, 8, 1], [1, 8, 6, 6], [2, 5, 5, 2], [2, 8, 3, 3], [4, 7, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.79

**Pinning number:** 6

Table 136: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.7	2.93	3.12	3.27	

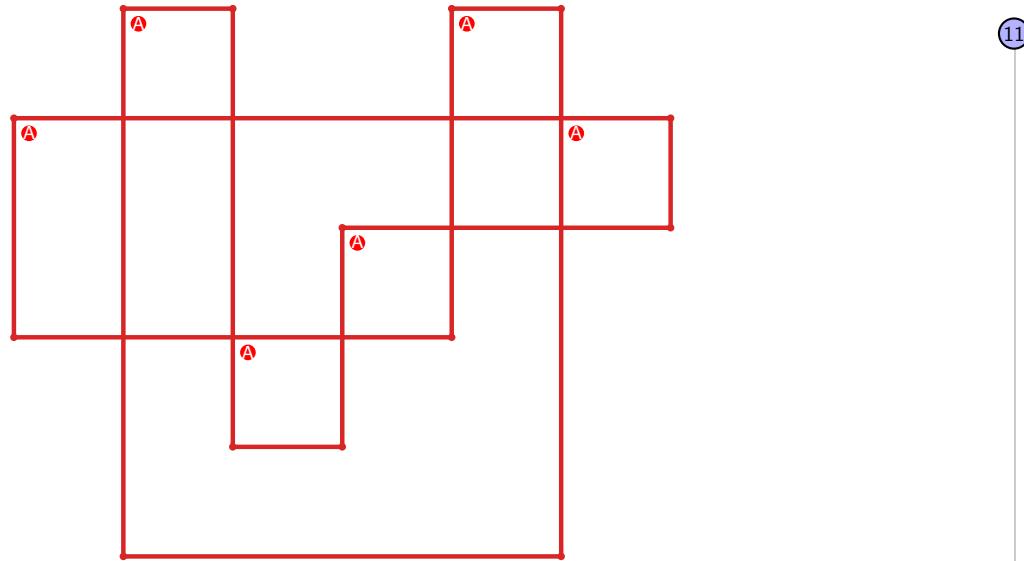


Figure 275: SnapPy multiloop plot.

6

Figure 276: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.91 [[9, 18, 10, 1], [17, 8, 18, 9], [10, 2, 11, 1], [5, 16, 6, 17], [7, 12, 8, 13], [2, 12, 3, 11], [15, 4, 16, 5], [6, 14, 7, 13], [3, 14, 4, 15]]

PD code drawn by SnapPy: [(12, 1, 13, 2), (14, 5, 15, 6), (8, 17, 9, 18), (4, 9, 5, 10), (10, 3, 11, 4), (18, 11, 1, 12), (2, 13, 3, 14), (6, 15, 7, 16), (16, 7, 17, 8)]

Planar representation generated by plantri: [[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 5, 0], [1, 6, 6, 7], [1, 7, 7, 5], [2, 4, 8, 2], [3, 8, 8, 3], [3, 8, 4, 4], [5, 7, 6, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.79

**Pinning number:** 6

Table 137: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.7	2.93	3.12	3.27	

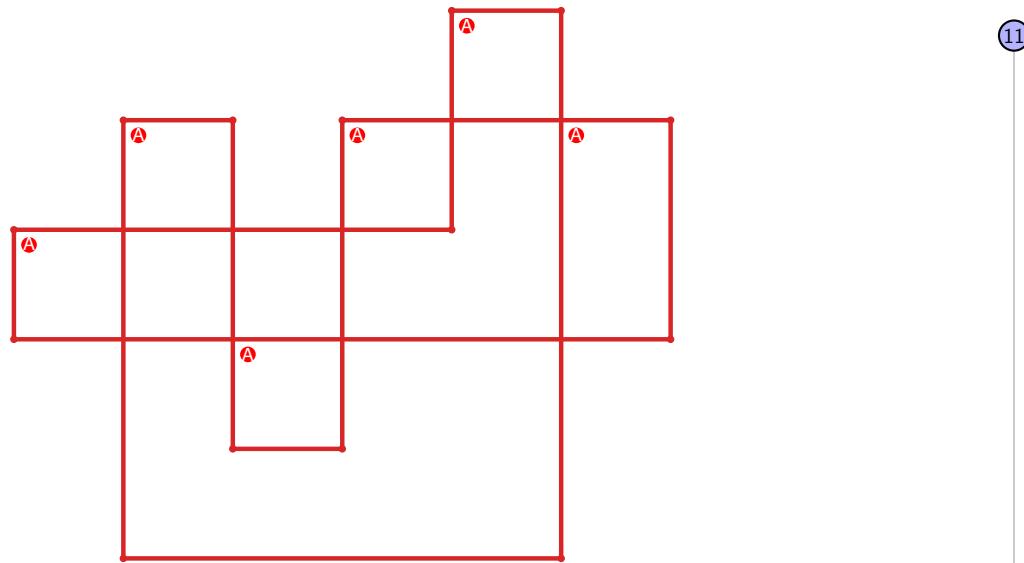


Figure 277: SnapPy multiloop plot.

11  
6

Figure 278: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.92**  $[[18, 11, 1, 12], [12, 5, 13, 6], [6, 17, 7, 18], [10, 1, 11, 2], [4, 13, 5, 14], [16, 7, 17, 8], [2, 9, 3, 10], [14, 3, 15, 4], [8, 15, 9, 16]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (10, 3, 11, 4), (14, 5, 15, 6), (18, 7, 1, 8), (16, 9, 17, 10), (2, 11, 3, 12), (6, 13, 7, 14), (4, 15, 5, 16), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 5], [0, 6, 6, 0], [1, 7, 7, 1], [2, 8, 8, 2], [3, 8, 7, 3], [4, 6, 8, 4], [5, 7, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.79

**Pinning number:** 6

Table 138: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.7	2.93	3.12	3.27	

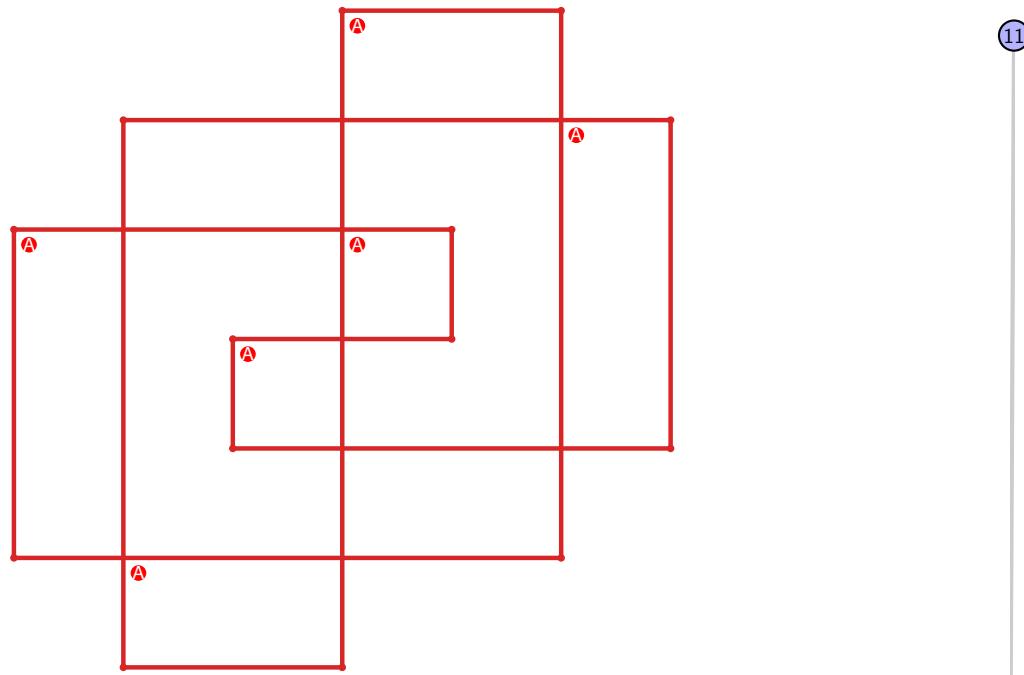


Figure 279: `SnapPy` multiloop plot.

Figure 280: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.93  $[[18, 5, 1, 6], [6, 13, 7, 14], [4, 17, 5, 18], [1, 12, 2, 13], [7, 2, 8, 3], [14, 3, 15, 4], [16, 9, 17, 10], [11, 8, 12, 9], [15, 11, 16, 10]]$

PD code drawn by SnapPy:  $[(5, 18, 6, 1), (12, 1, 13, 2), (16, 3, 17, 4), (9, 6, 10, 7), (14, 7, 15, 8), (8, 13, 9, 14), (17, 10, 18, 11), (4, 11, 5, 12), (2, 15, 3, 16)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 5, 6, 0], [0, 7, 4, 1], [1, 3, 7, 5], [1, 4, 8, 2], [2, 8, 8, 7], [3, 6, 8, 4], [5, 7, 6, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 256  
 Pinning number: 4

Average optimal degree: 2.58  
 Average minimal degree: 2.59  
 Average overall degree: 3.06

Table 139: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	18	51	75	65	33	9	1	252
Average degree	2.58	2.8	2.95	3.06	3.14	3.2	3.24	3.27	

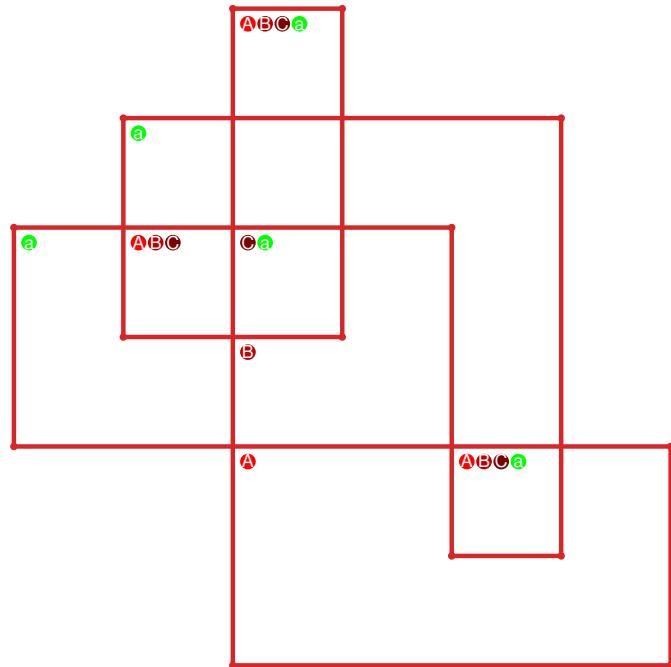


Figure 281: SnapPy multiloop plot.

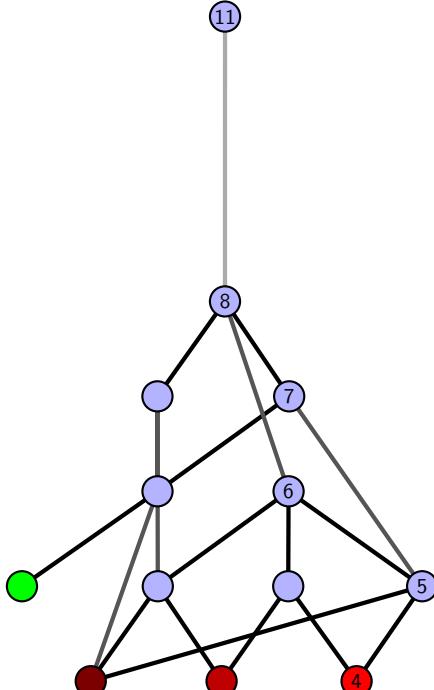


Figure 282: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.94 [[18, 7, 1, 8], [8, 3, 9, 4], [4, 17, 5, 18], [6, 11, 7, 12], [1, 14, 2, 15], [15, 2, 16, 3], [9, 16, 10, 17], [5, 13, 6, 12], [13, 10, 14, 11]]

PD code drawn by `SnapPy`: [(13, 18, 14, 1), (6, 1, 7, 2), (16, 5, 17, 6), (11, 8, 12, 9), (4, 9, 5, 10), (10, 3, 11, 4), (17, 12, 18, 13), (7, 14, 8, 15), (2, 15, 3, 16)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 7], [0, 7, 7, 8], [0, 8, 5, 5], [1, 4, 4, 6], [1, 5, 8, 2], [2, 8, 3, 3], [3, 7, 6, 4]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 288  
**Pinning number:** 4

**Average optimal degree:** 2.62  
**Average minimal degree:** 2.65  
**Average overall degree:** 3.06

Table 140: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	22	61	85	70	34	9	1	282
Average degree	2.62	2.82	2.96	3.07	3.14	3.2	3.24	3.27	

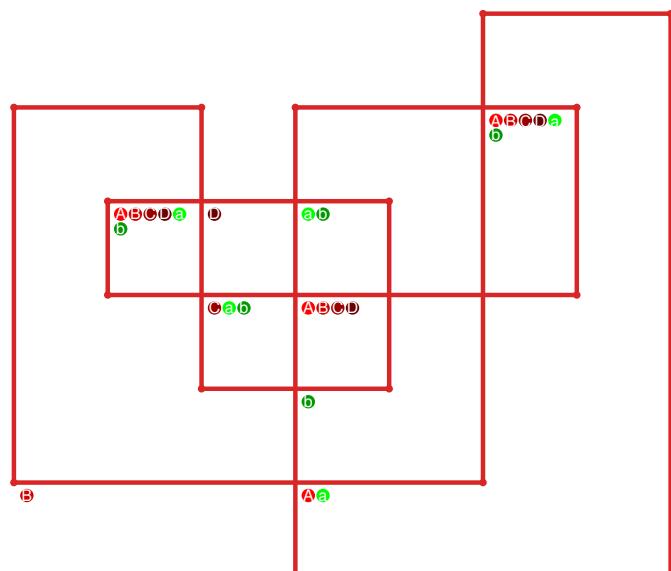


Figure 283: `SnapPy` multiloop plot.

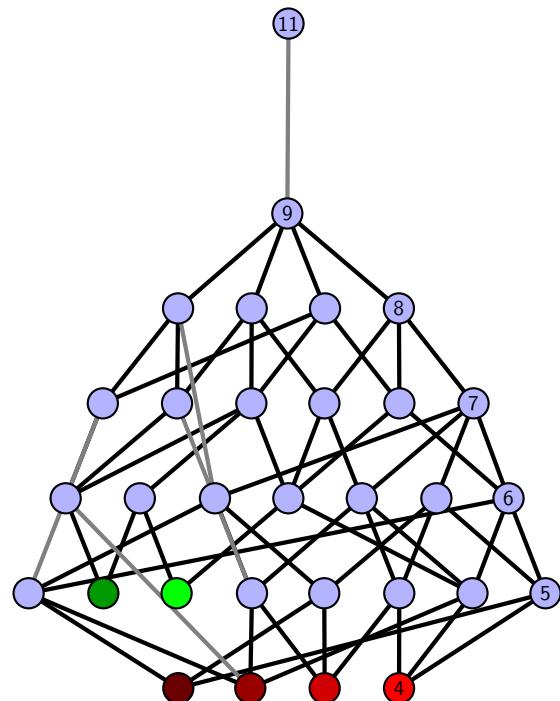


Figure 284: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.95  $[[18, 9, 1, 10], [10, 6, 11, 5], [17, 14, 18, 15], [8, 13, 9, 14], [1, 13, 2, 12], [6, 12, 7, 11], [4, 15, 5, 16], [16, 3, 17, 4], [7, 2, 8, 3]]$

PD code drawn by SnapPy:  $[(5, 18, 6, 1), (14, 1, 15, 2), (3, 8, 4, 9), (16, 7, 17, 8), (9, 4, 10, 5), (10, 17, 11, 18), (6, 11, 7, 12), (15, 12, 16, 13), (2, 13, 3, 14)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 3], [0, 2, 8, 4], [0, 3, 8, 5], [1, 4, 8, 1], [1, 7, 7, 2], [2, 6, 6, 8], [3, 7, 5, 4]]$

**Total optimal pinning sets:** 8  
**Total minimal pinning sets:** 9  
**Total pinning sets:** 222  
**Pinning number:** 5

**Average optimal degree:** 2.7  
**Average minimal degree:** 2.73  
**Average overall degree:** 3.06

Table 141: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	8	0	0	0	0	0	0	8
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	37	69	64	33	9	1	213
Average degree	2.7	2.9	3.04	3.13	3.2	3.24	3.27	

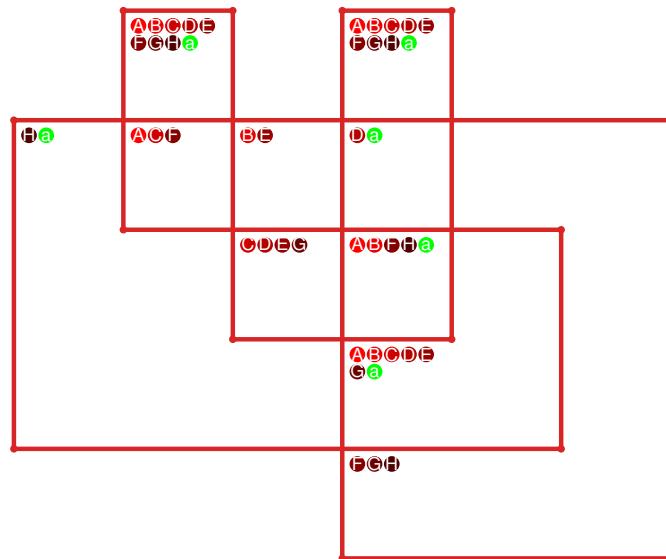


Figure 285: SnapPy multiloop plot.

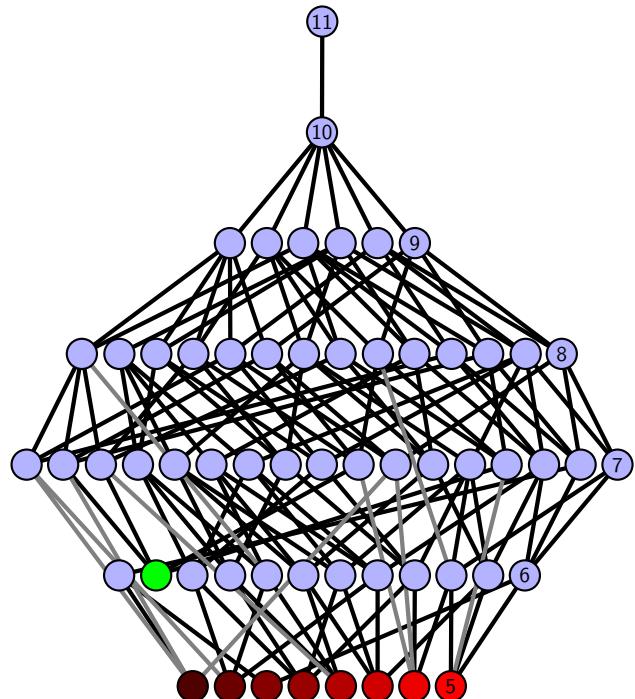


Figure 286: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.96  $[[18, 5, 1, 6], [6, 15, 7, 16], [12, 17, 13, 18], [13, 4, 14, 5], [1, 14, 2, 15], [7, 10, 8, 11], [16, 11, 17, 12], [8, 3, 9, 4], [2, 9, 3, 10]]$

PD code drawn by SnapPy:  $[(5, 18, 6, 1), (10, 1, 11, 2), (15, 2, 16, 3), (17, 6, 18, 7), (4, 7, 5, 8), (13, 8, 14, 9), (16, 11, 17, 12), (3, 12, 4, 13), (9, 14, 10, 15)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 3], [0, 2, 7, 4], [0, 3, 8, 1], [1, 8, 7, 6], [1, 5, 2, 2], [3, 5, 8, 8], [4, 7, 7, 5]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 8  
 Total pinning sets: 254  
 Pinning number: 4

Average optimal degree: 2.5  
 Average minimal degree: 2.78  
 Average overall degree: 3.07

Table 142: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	1	0	0	0	0	0	7
Nonminimal pinning sets	0	7	47	79	69	34	9	1	246
Average degree	2.5	2.75	2.93	3.06	3.14	3.2	3.24	3.27	

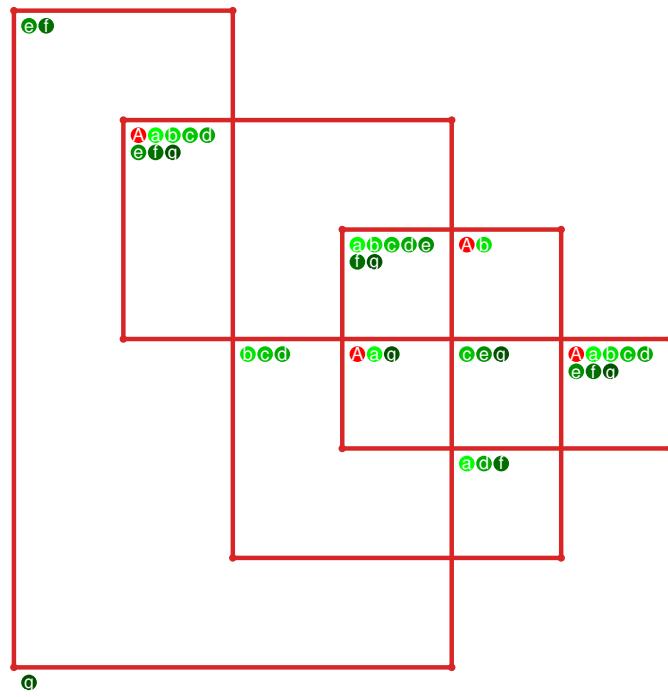


Figure 287: SnapPy multiloop plot.

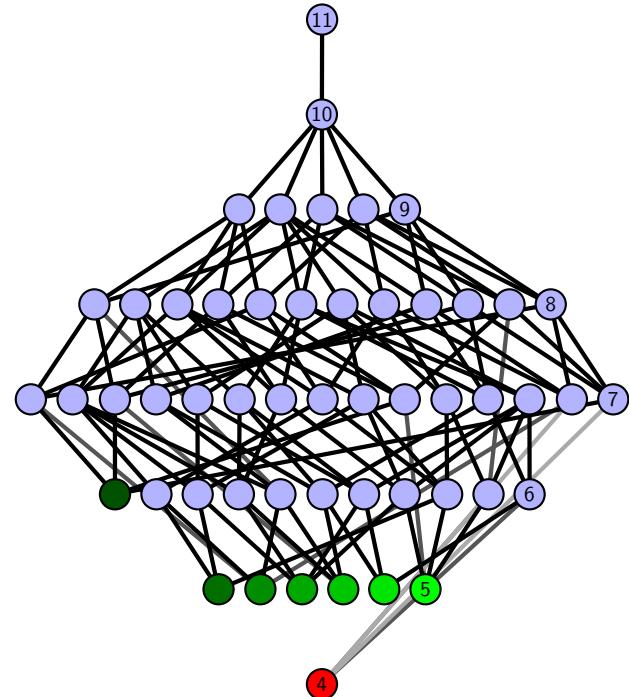


Figure 288: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.97  $[[18, 5, 1, 6], [6, 11, 7, 12], [12, 17, 13, 18], [13, 4, 14, 5], [1, 10, 2, 11], [7, 16, 8, 17], [8, 3, 9, 4], [14, 9, 15, 10], [2, 15, 3, 16]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (11, 2, 12, 3), (16, 3, 17, 4), (12, 7, 13, 8), (17, 8, 18, 9), (4, 9, 5, 10), (18, 13, 1, 14), (5, 14, 6, 15), (10, 15, 11, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 3], [0, 2, 6, 7], [0, 7, 8, 1], [1, 8, 6, 2], [3, 5, 8, 7], [3, 6, 8, 4], [4, 7, 6, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 3.0

**Total minimal pinning sets:** 13

**Average minimal degree:** 3.25

**Total pinning sets:** 395

**Average overall degree:** 3.23

**Pinning number:** 4

Table 143: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	7	0	0	0	0	0	11
Nonminimal pinning sets	0	14	66	130	111	49	11	1	382
Average degree	3.0	3.13	3.21	3.23	3.25	3.27	3.27	3.27	

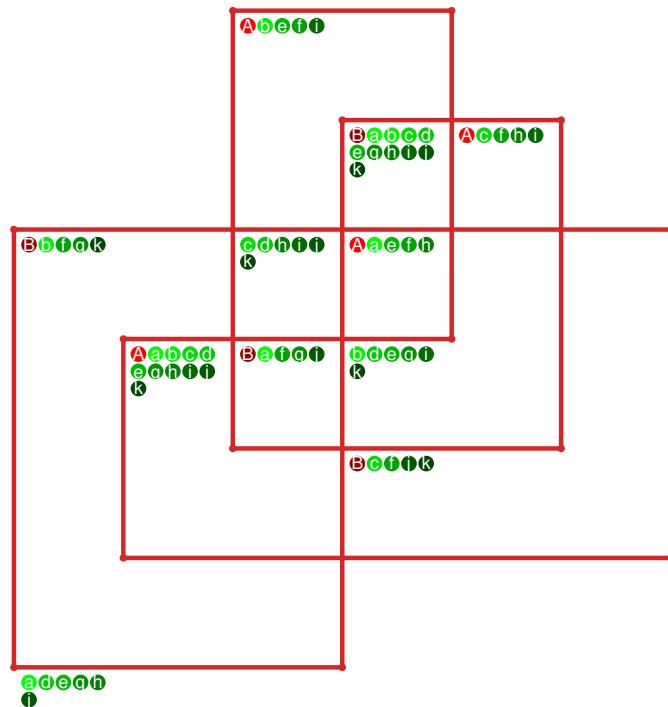


Figure 289: SnapPy multiloop plot.

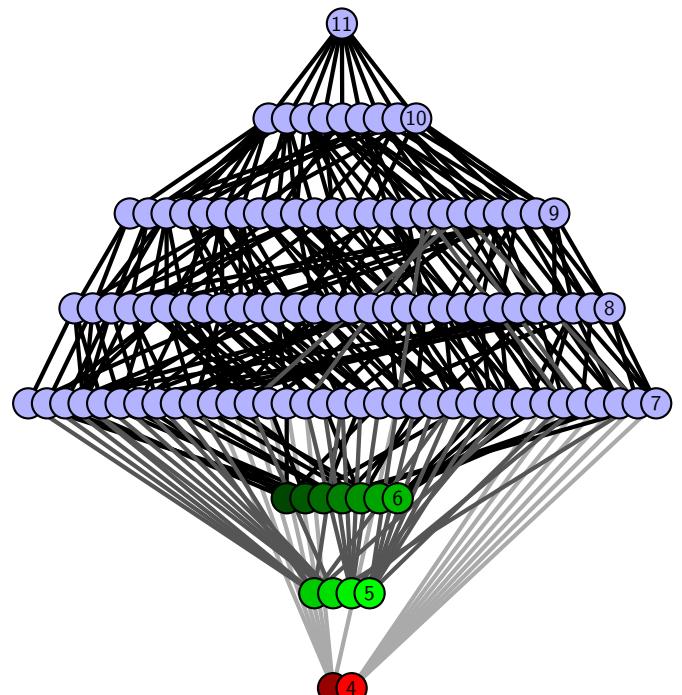


Figure 290: Minimal join sub-semi-lattice of minimal pinning sets.

**4.7.98**  $[[5, 18, 6, 1], [15, 4, 16, 5], [17, 8, 18, 9], [6, 13, 7, 14], [1, 14, 2, 15], [3, 10, 4, 11], [16, 10, 17, 9], [12, 7, 13, 8], [2, 12, 3, 11]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (9, 4, 10, 5), (14, 5, 15, 6), (6, 13, 7, 14), (18, 7, 1, 8), (8, 17, 9, 18), (3, 10, 4, 11), (16, 11, 17, 12), (2, 15, 3, 16)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 7], [0, 7, 7, 4], [0, 3, 8, 1], [1, 8, 8, 6], [1, 5, 2, 2], [2, 8, 3, 3], [4, 7, 5, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.46

**Total pinning sets:** 184

**Average overall degree:** 2.98

**Pinning number:** 4

Table 144: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	7	33	54	50	27	8	1	180
Average degree	2.25	2.56	2.79	2.96	3.08	3.16	3.23	3.27	

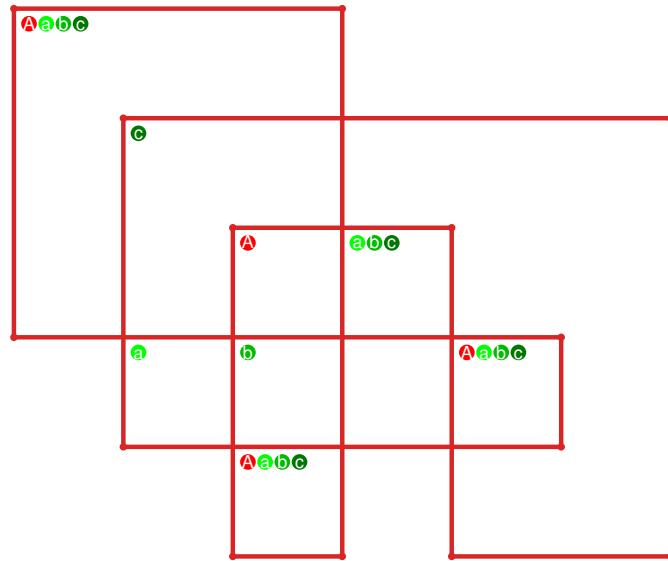


Figure 291: `SnapPy` multiloop plot.

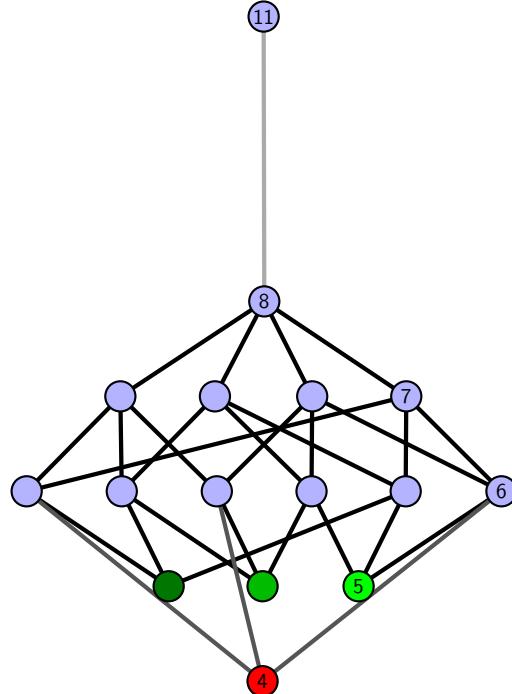


Figure 292: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.99  $[[18, 5, 1, 6], [6, 11, 7, 12], [12, 17, 13, 18], [13, 4, 14, 5], [1, 10, 2, 11], [7, 16, 8, 17], [3, 14, 4, 15], [9, 2, 10, 3], [15, 8, 16, 9]]$

PD code drawn by SnapPy:  $[(7, 18, 8, 1), (11, 2, 12, 3), (16, 3, 17, 4), (13, 6, 14, 7), (17, 8, 18, 9), (4, 9, 5, 10), (1, 12, 2, 13), (5, 14, 6, 15), (10, 15, 11, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 7, 7, 1], [1, 8, 8, 2], [3, 8, 7, 3], [4, 6, 8, 4], [5, 7, 6, 5]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 144  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.38  
**Average overall degree:** 2.97

Table 145: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	7	21	39	41	25	8	1	142
Average degree	2.25	2.57	2.77	2.92	3.05	3.15	3.23	3.27	

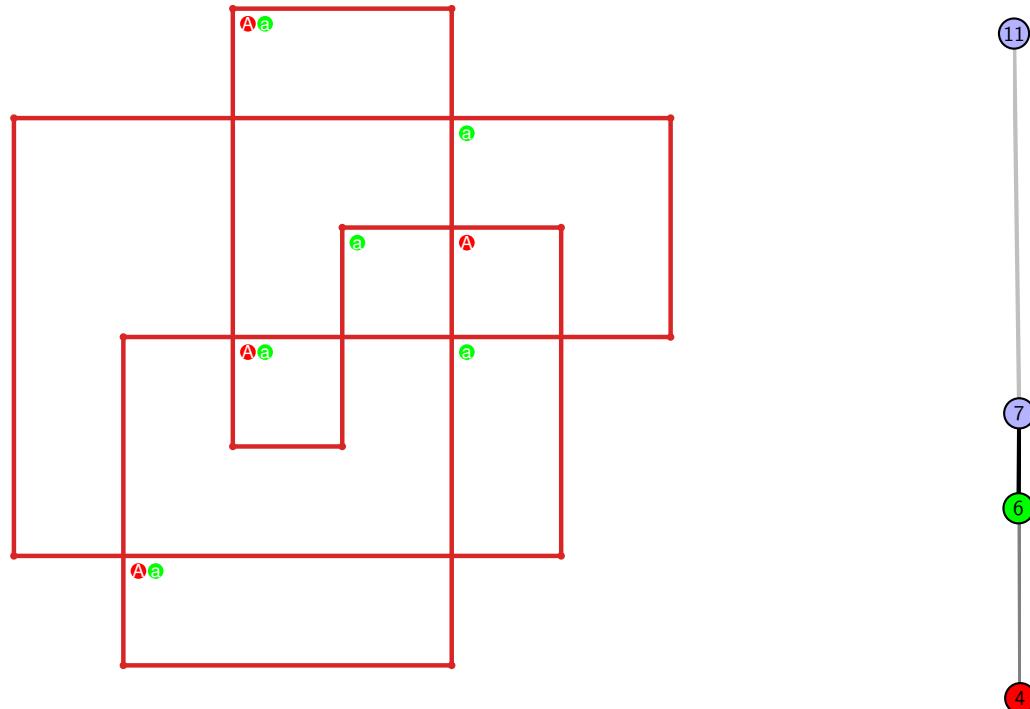


Figure 293: SnapPy multiloop plot.

Figure 294: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.100  $[[5, 18, 6, 1], [13, 4, 14, 5], [17, 6, 18, 7], [1, 12, 2, 13], [3, 8, 4, 9], [14, 8, 15, 7], [11, 16, 12, 17], [2, 10, 3, 9], [15, 10, 16, 11]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (14, 3, 15, 4), (4, 11, 5, 12), (18, 5, 1, 6), (6, 17, 7, 18), (12, 7, 13, 8), (16, 9, 17, 10), (8, 13, 9, 14), (2, 15, 3, 16)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 5, 6, 0], [0, 6, 7, 1], [1, 7, 7, 5], [1, 4, 8, 2], [2, 8, 8, 3], [3, 8, 4, 4], [5, 7, 6, 6]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 240  
 Pinning number: 4

Average optimal degree: 2.38  
 Average minimal degree: 2.38  
 Average overall degree: 2.98

Table 146: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	22	52	69	56	28	8	1	236
Average degree	2.38	2.67	2.87	3.0	3.09	3.17	3.22	3.27	

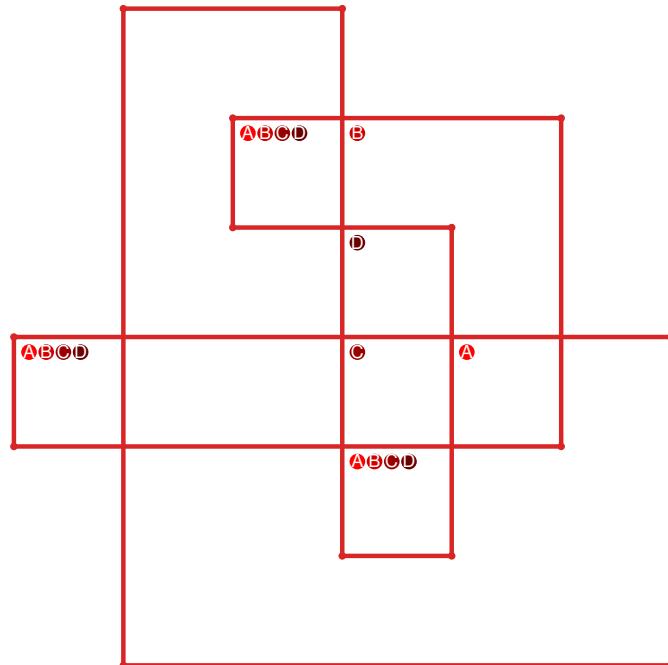


Figure 295: SnapPy multiloop plot.

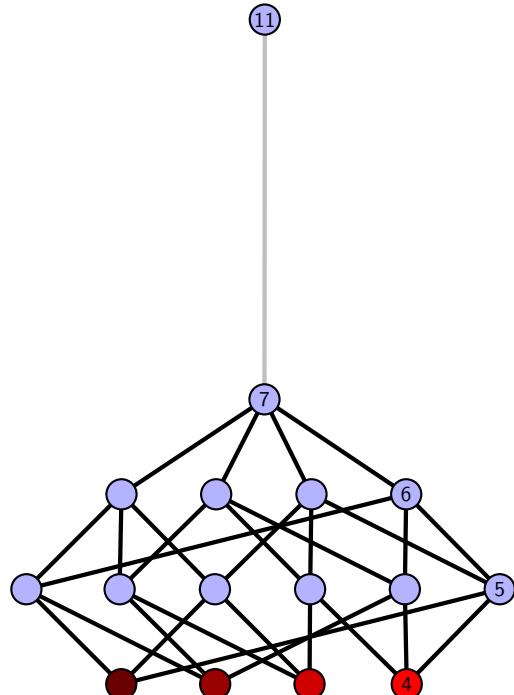


Figure 296: Minimal join sub-semi-lattice of minimal pinning sets.

4.7.101  $[[5, 18, 6, 1], [4, 9, 5, 10], [17, 6, 18, 7], [1, 11, 2, 10], [3, 14, 4, 15], [8, 13, 9, 14], [7, 13, 8, 12], [16, 11, 17, 12], [2, 16, 3, 15]]$

PD code drawn by SnapPy:  $[(14, 3, 15, 4), (1, 6, 2, 7), (7, 16, 8, 17), (13, 8, 14, 9), (9, 4, 10, 5), (5, 10, 6, 11), (18, 11, 1, 12), (12, 17, 13, 18), (2, 15, 3, 16)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 7, 0], [0, 7, 8, 1], [1, 8, 8, 5], [1, 4, 6, 6], [2, 5, 5, 7], [2, 6, 8, 3], [3, 7, 4, 4]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 8  
 Total pinning sets: 188  
 Pinning number: 5

Average optimal degree: 2.53  
 Average minimal degree: 2.65  
 Average overall degree: 3.0

Table 147: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	Total
Optimal pinning sets	6	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	30	59	54	28	8	1	180
Average degree	2.53	2.79	2.97	3.09	3.17	3.23	3.27	

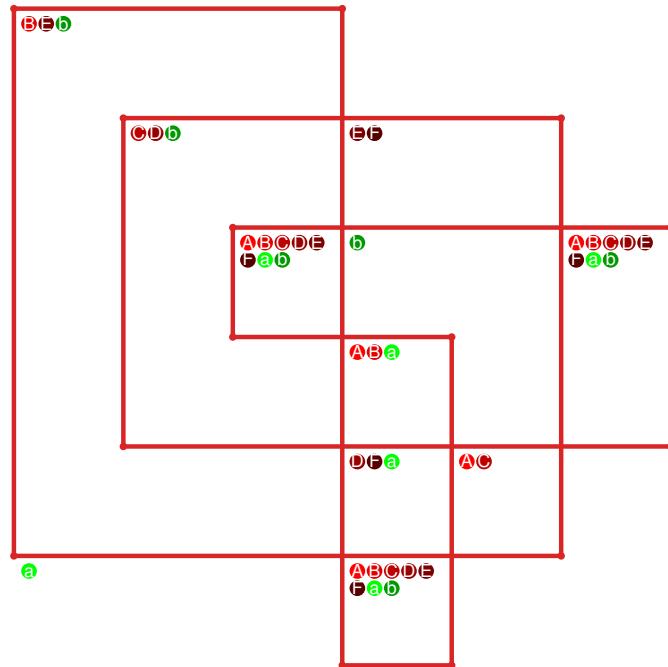


Figure 297: SnapPy multiloop plot.

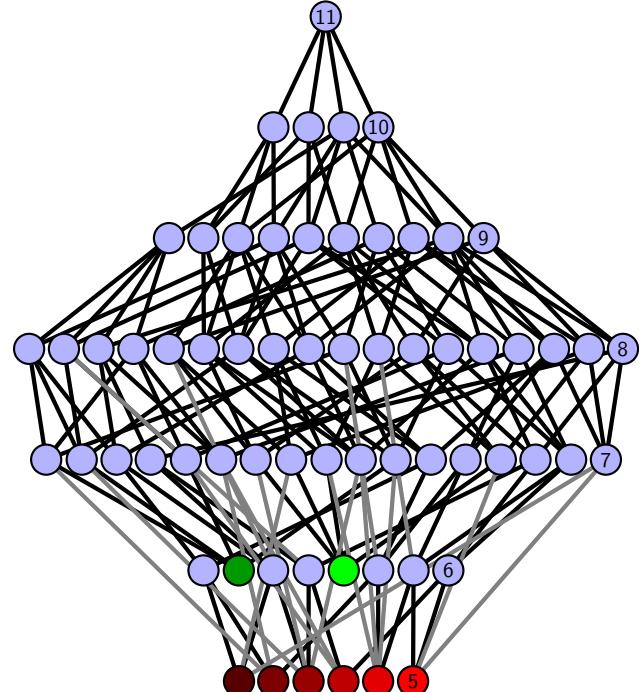


Figure 298: Minimal join sub-semi-lattice of minimal pinning sets.

## 4.8 12 regions

4.8.1 [[20, 9, 1, 10], [10, 19, 11, 20], [11, 8, 12, 9], [1, 18, 2, 19], [7, 12, 8, 13], [17, 2, 18, 3], [13, 6, 14, 7], [3, 16, 4, 17], [5, 14, 6, 15], [15, 4, 16, 5]]

PD code drawn by SnapPy: [(12, 1, 13, 2), (10, 3, 11, 4), (8, 5, 9, 6), (17, 6, 18, 7), (4, 9, 5, 10), (2, 11, 3, 12), (20, 13, 1, 14), (18, 15, 19, 16), (7, 16, 8, 17), (14, 19, 15, 20)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 5, 1], [2, 6, 6, 2], [3, 7, 7, 3], [4, 8, 8, 4], [5, 9, 9, 5], [6, 9, 9, 6], [7, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.77

**Pinning number:** 8

Table 148: Pinning sets/average degree by cardinal

Cardinal	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.44	2.8	3.09	3.33	

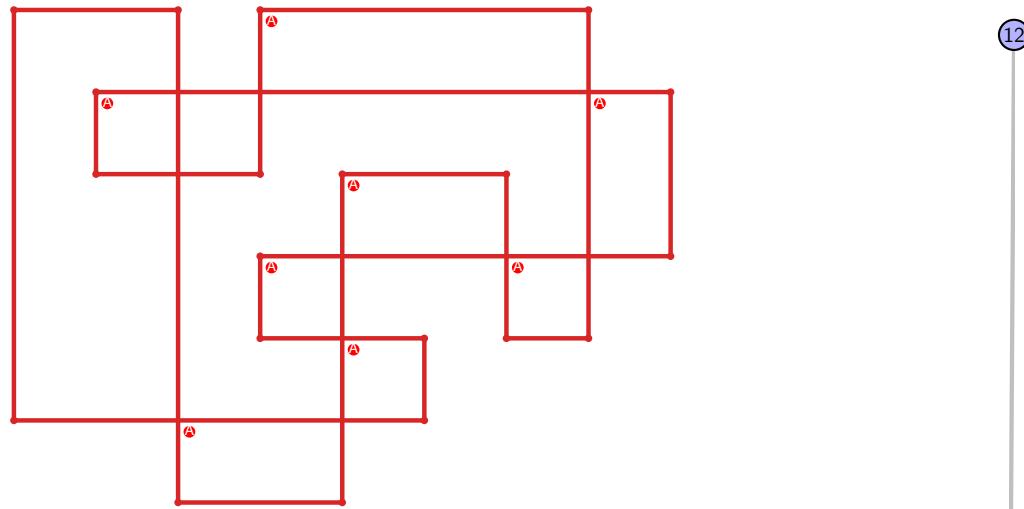


Figure 299: SnapPy multiloop plot.



Figure 300: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.2 [[20, 13, 1, 14], [14, 10, 15, 9], [19, 4, 20, 5], [12, 1, 13, 2], [10, 3, 11, 4], [15, 8, 16, 9], [5, 18, 6, 19], [2, 11, 3, 12], [7, 16, 8, 17], [17, 6, 18, 7]]

PD code drawn by `SnapPy`: [(7, 20, 8, 1), (16, 1, 17, 2), (14, 3, 15, 4), (12, 5, 13, 6), (6, 11, 7, 12), (19, 8, 20, 9), (17, 10, 18, 11), (4, 13, 5, 14), (2, 15, 3, 16), (9, 18, 10, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 4], [0, 7, 7, 0], [1, 7, 7, 2], [1, 8, 8, 1], [2, 9, 9, 2], [3, 4, 4, 3], [5, 9, 9, 5], [6, 8, 8, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 16

**Average overall degree:** 2.77

**Pinning number:** 8

Table 149: Pinning sets/average degree by cardinal

Cardinal	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0
Nonminimal pinning sets	0	4	6	4	1	15
Average degree	2.0	2.44	2.8	3.09	3.33	

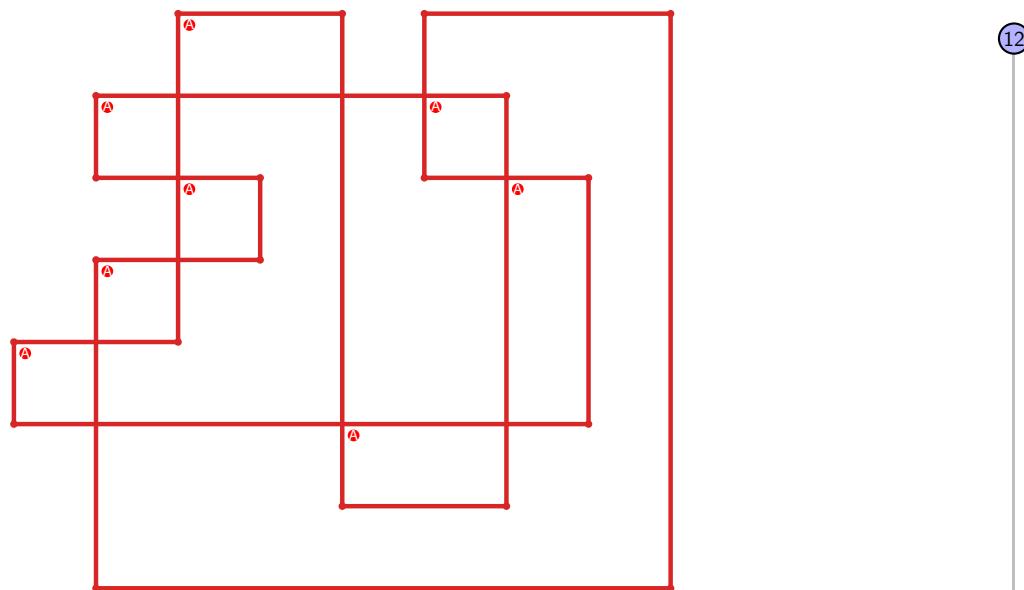


Figure 301: `SnapPy` multiloop plot.

8

Figure 302: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.3  $[[20, 11, 1, 12], [12, 10, 13, 9], [19, 2, 20, 3], [10, 1, 11, 2], [13, 4, 14, 5], [17, 8, 18, 9], [3, 18, 4, 19], [14, 6, 15, 5], [7, 16, 8, 17], [6, 16, 7, 15]]$

PD code drawn by `SnapPy`:  $[(17, 20, 18, 1), (11, 2, 12, 3), (3, 12, 4, 13), (15, 6, 16, 7), (7, 10, 8, 11), (18, 9, 19, 10), (13, 4, 14, 5), (5, 14, 6, 15), (1, 16, 2, 17), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 6, 3], [0, 2, 1, 0], [1, 6, 7, 7], [1, 8, 8, 6], [2, 5, 4, 2], [4, 9, 9, 4], [5, 9, 9, 5], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 150: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

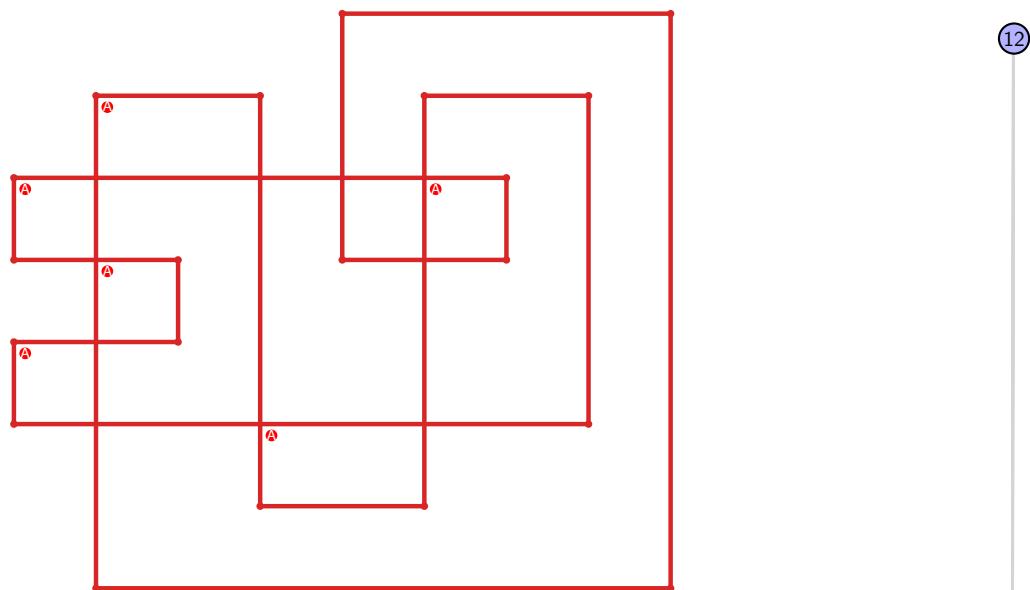


Figure 303: `SnapPy` multiloop plot.

6

Figure 304: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.4  $[[3, 20, 4, 1], [2, 13, 3, 14], [19, 6, 20, 7], [4, 11, 5, 12], [1, 15, 2, 14], [15, 12, 16, 13], [7, 18, 8, 19], [10, 5, 11, 6], [16, 10, 17, 9], [17, 8, 18, 9]]$

PD code drawn by `SnapPy`:  $[(8, 3, 9, 4), (17, 4, 18, 5), (15, 6, 16, 7), (7, 14, 8, 15), (2, 9, 3, 10), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (5, 16, 6, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 7], [0, 7, 7, 5], [0, 5, 1, 1], [1, 4, 3, 8], [2, 9, 9, 2], [2, 8, 3, 3], [5, 7, 9, 9], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 151: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

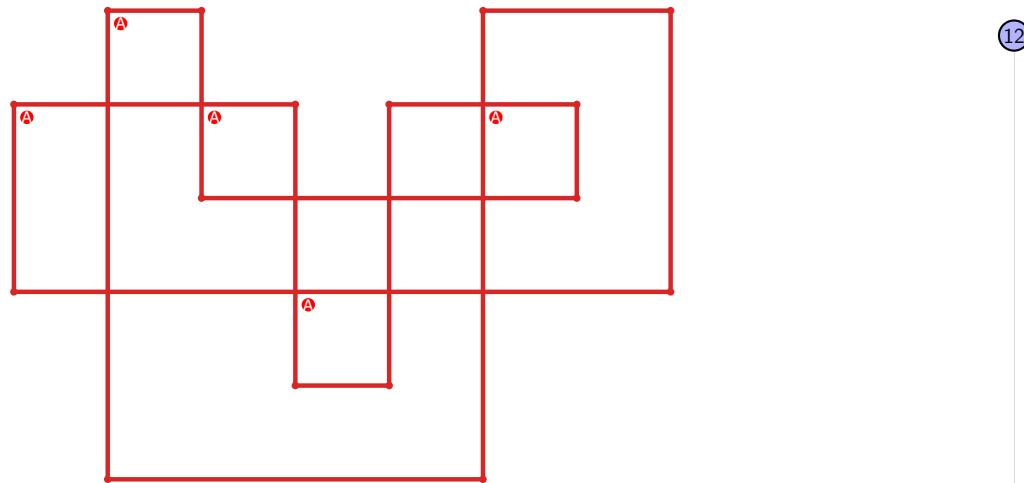


Figure 305: `SnapPy` multiloop plot.

5

Figure 306: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.5  $[[3, 20, 4, 1], [2, 11, 3, 12], [19, 4, 20, 5], [1, 13, 2, 12], [13, 10, 14, 11], [5, 16, 6, 17], [9, 18, 10, 19], [14, 8, 15, 7], [15, 6, 16, 7], [17, 8, 18, 9]]$

PD code drawn by SnapPy:  $[(17, 2, 18, 3), (13, 4, 14, 5), (5, 12, 6, 13), (15, 6, 16, 7), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (3, 14, 4, 15), (7, 16, 8, 17), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 6, 7], [2, 8, 8, 9], [2, 9, 9, 4], [4, 9, 8, 8], [5, 7, 7, 5], [5, 7, 6, 6]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 1  
 Total pinning sets: 128  
 Pinning number: 5

Average optimal degree: 2.0  
 Average minimal degree: 2.0  
 Average overall degree: 2.91

Table 152: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

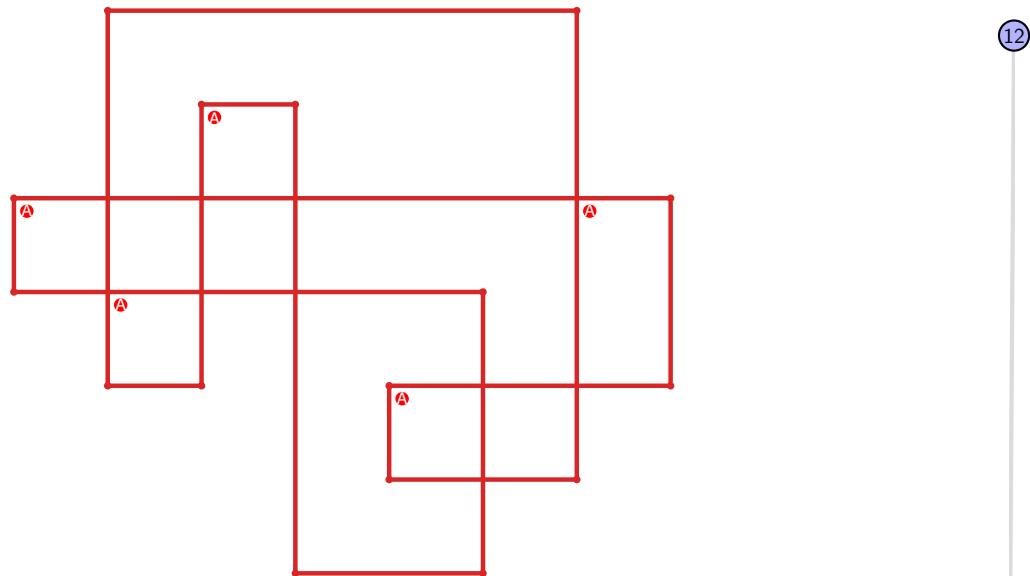


Figure 307: SnapPy multiloop plot.



Figure 308: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.6  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 11, 7, 12], [1, 15, 2, 14], [3, 18, 4, 19], [10, 7, 11, 8], [12, 16, 13, 15], [2, 13, 3, 14], [17, 8, 18, 9], [9, 16, 10, 17]]$

PD code drawn by SnapPy:  $[(17, 2, 18, 3), (8, 5, 9, 6), (15, 6, 16, 7), (4, 9, 5, 10), (20, 11, 1, 12), (12, 19, 13, 20), (13, 10, 14, 11), (7, 14, 8, 15), (1, 16, 2, 17), (3, 18, 4, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 5, 6], [0, 6, 7, 7], [1, 7, 8, 1], [2, 8, 9, 2], [2, 9, 7, 3], [3, 6, 4, 3], [4, 9, 9, 5], [5, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 153: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

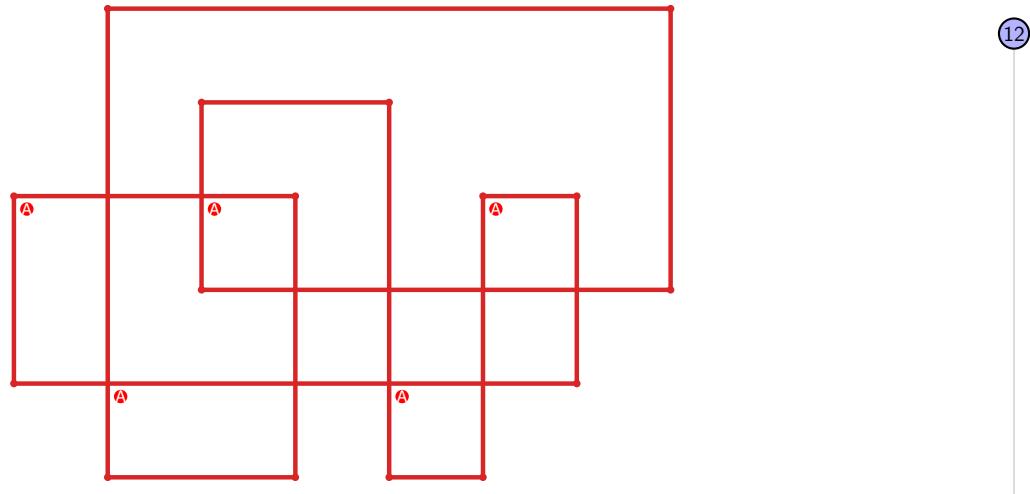


Figure 309: SnapPy multiloop plot.

5

Figure 310: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.7  $[[3, 20, 4, 1], [2, 15, 3, 16], [19, 8, 20, 9], [4, 11, 5, 12], [1, 17, 2, 16], [17, 14, 18, 15], [9, 18, 10, 19], [10, 7, 11, 8], [5, 13, 6, 12], [6, 13, 7, 14]]$

PD code drawn by `SnapPy`:  $[(8, 5, 9, 6), (17, 6, 18, 7), (2, 9, 3, 10), (10, 3, 11, 4), (4, 11, 5, 12), (15, 12, 16, 13), (20, 13, 1, 14), (14, 19, 15, 20), (7, 16, 8, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 7], [0, 7, 8, 8], [0, 5, 1, 1], [1, 4, 9, 6], [2, 5, 7, 2], [2, 6, 9, 3], [3, 9, 9, 3], [5, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 154: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

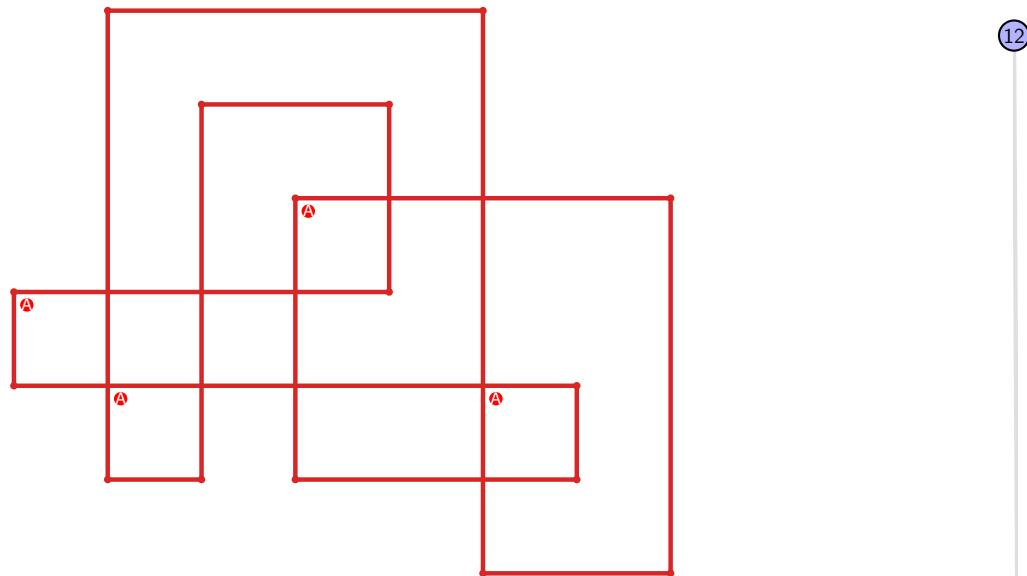


Figure 311: `SnapPy` multiloop plot.

Figure 312: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.8  $[[3, 20, 4, 1], [2, 9, 3, 10], [19, 4, 20, 5], [1, 11, 2, 10], [11, 8, 12, 9], [5, 16, 6, 17], [18, 13, 19, 14], [7, 12, 8, 13], [15, 6, 16, 7], [17, 15, 18, 14]]$

PD code drawn by `SnapPy`:  $[(17, 2, 18, 3), (13, 4, 14, 5), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (5, 10, 6, 11), (16, 11, 17, 12), (12, 15, 13, 16), (3, 14, 4, 15), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 7], [2, 8, 8, 9], [2, 9, 9, 7], [4, 6, 8, 4], [5, 7, 9, 5], [5, 8, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 155: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

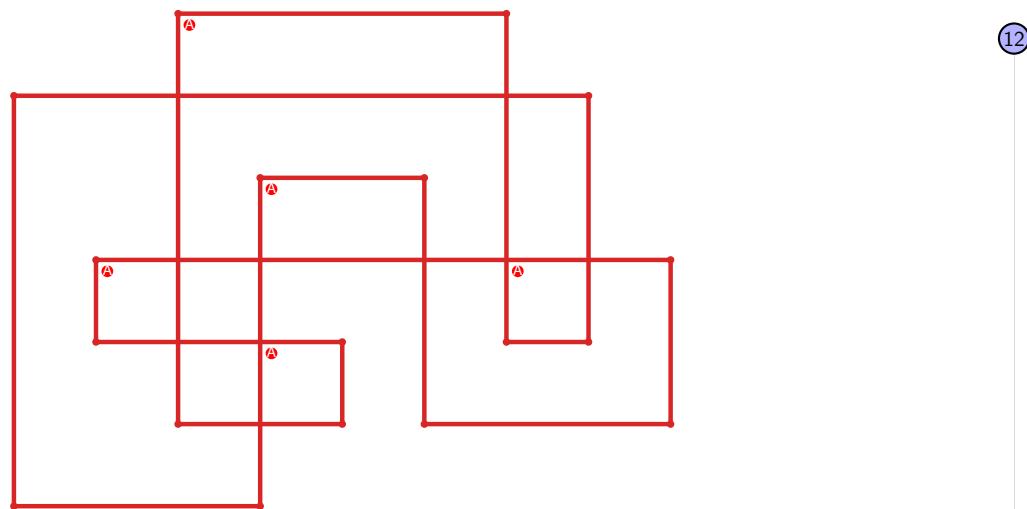


Figure 313: `SnapPy` multiloop plot.

5

Figure 314: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.9  $[[3, 20, 4, 1], [2, 13, 3, 14], [19, 4, 20, 5], [1, 15, 2, 14], [15, 12, 16, 13], [5, 18, 6, 19], [6, 11, 7, 12], [16, 10, 17, 9], [17, 8, 18, 9], [10, 7, 11, 8]]$

PD code drawn by `SnapPy`:  $[(17, 2, 18, 3), (8, 5, 9, 6), (15, 6, 16, 7), (7, 14, 8, 15), (4, 9, 5, 10), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (3, 16, 4, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 5, 0], [0, 4, 1, 1], [1, 3, 6, 7], [2, 8, 6, 2], [4, 5, 9, 9], [4, 9, 8, 8], [5, 7, 7, 9], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 156: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

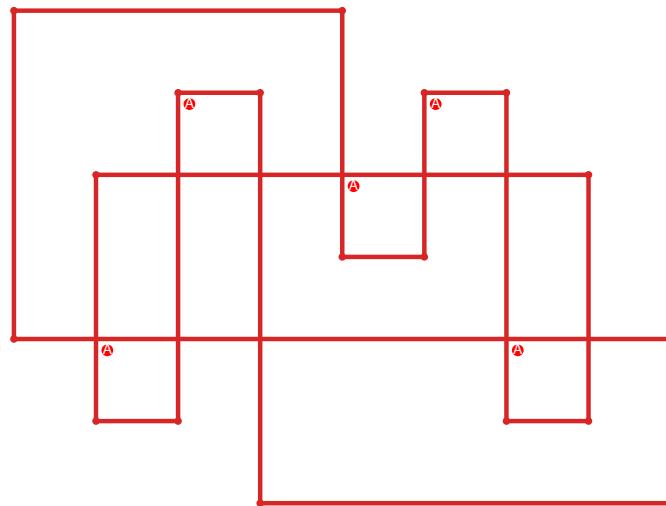


Figure 315: `SnapPy` multiloop plot.

(12)

5

Figure 316: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.10  $[[3, 20, 4, 1], [2, 15, 3, 16], [19, 8, 20, 9], [4, 13, 5, 14], [1, 17, 2, 16], [17, 14, 18, 15], [9, 18, 10, 19], [10, 7, 11, 8], [12, 5, 13, 6], [6, 11, 7, 12]]$

PD code drawn by SnapPy:  $[(9, 2, 10, 3), (7, 4, 8, 5), (16, 5, 17, 6), (3, 8, 4, 9), (1, 10, 2, 11), (14, 11, 15, 12), (19, 12, 20, 13), (13, 18, 14, 19), (6, 15, 7, 16), (20, 17, 1, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 7], [0, 8, 8, 5], [0, 5, 1, 1], [1, 4, 3, 6], [2, 5, 7, 2], [2, 6, 9, 9], [3, 9, 9, 3], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 157: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

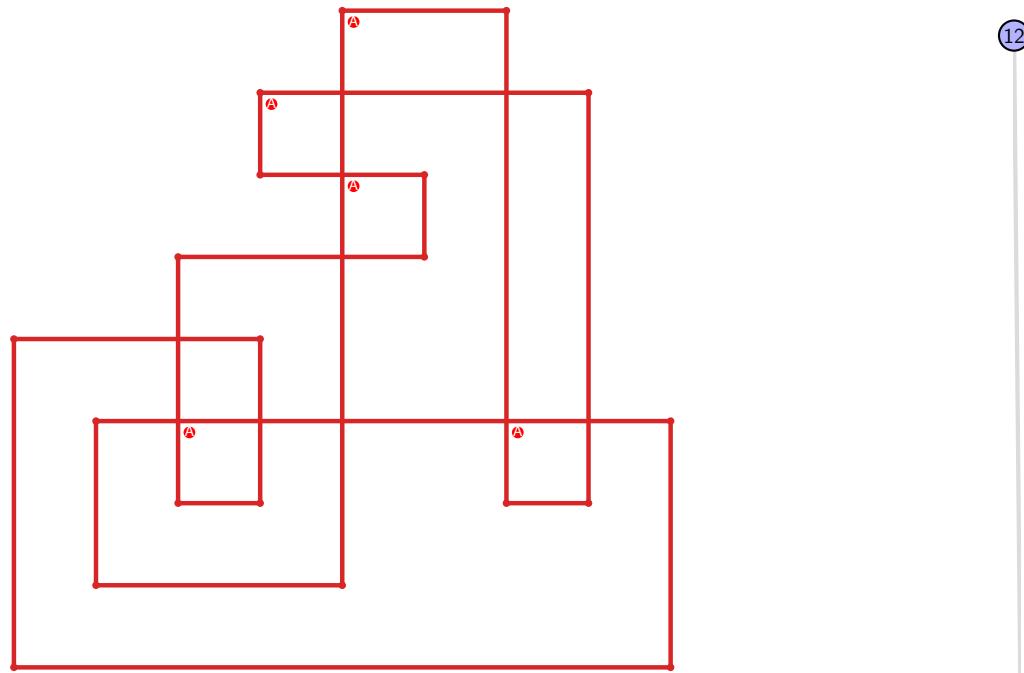


Figure 317: SnapPy multiloop plot.

Figure 318: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.11  $[[3, 20, 4, 1], [9, 2, 10, 3], [19, 4, 20, 5], [1, 8, 2, 9], [10, 8, 11, 7], [5, 15, 6, 14], [18, 11, 19, 12], [6, 15, 7, 16], [16, 13, 17, 14], [12, 17, 13, 18]]$

PD code drawn by SnapPy:  $[(16, 1, 17, 2), (4, 7, 5, 8), (18, 5, 19, 6), (15, 8, 16, 9), (13, 10, 14, 11), (11, 2, 12, 3), (3, 12, 4, 13), (9, 14, 10, 15), (20, 17, 1, 18), (6, 19, 7, 20)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 6, 7], [2, 7, 7, 8], [2, 9, 9, 4], [4, 8, 5, 5], [5, 7, 9, 9], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1

**Total pinning sets:** 128

**Pinning number:** 5

**Average optimal degree:** 2.0

**Average minimal degree:** 2.0

**Average overall degree:** 2.91

Table 158: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

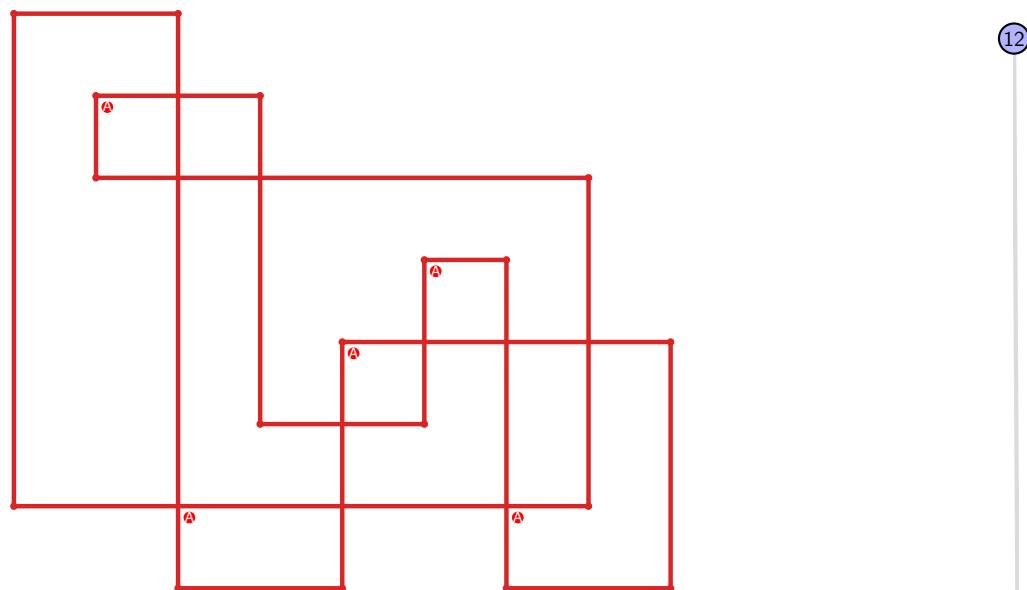


Figure 319: SnapPy multiloop plot.



Figure 320: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.12**  $[[3, 20, 4, 1], [2, 11, 3, 12], [19, 6, 20, 7], [4, 18, 5, 17], [1, 13, 2, 12], [13, 10, 14, 11], [7, 16, 8, 17], [5, 18, 6, 19], [9, 14, 10, 15], [15, 8, 16, 9]]$

PD code drawn by `SnapPy`:  $[(17, 4, 18, 5), (13, 6, 14, 7), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (7, 12, 8, 13), (5, 14, 6, 15), (15, 2, 16, 3), (3, 16, 4, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 7], [0, 7, 7, 6], [0, 5, 1, 1], [1, 4, 8, 8], [2, 9, 9, 3], [2, 3, 3, 2], [5, 9, 9, 5], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 159: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

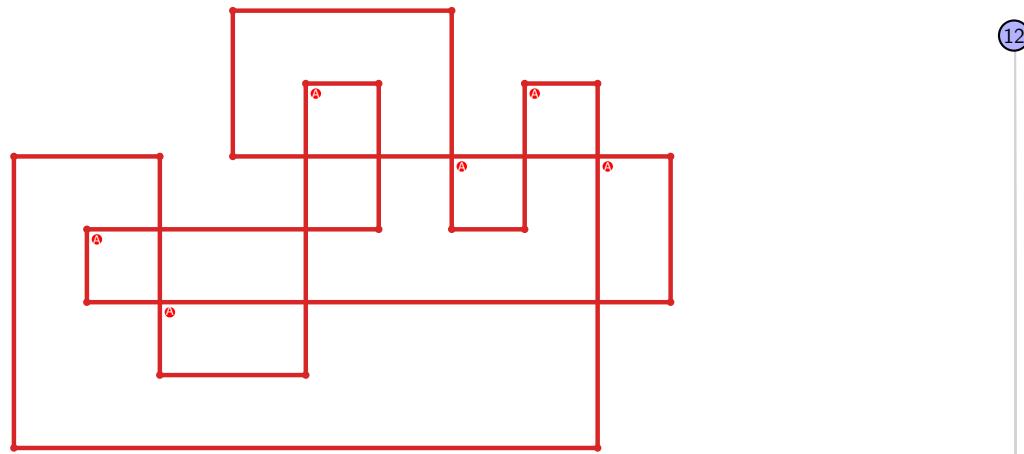


Figure 321: `SnapPy` multiloop plot.

6

Figure 322: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.13  $[[3, 20, 4, 1], [2, 11, 3, 12], [19, 4, 20, 5], [1, 13, 2, 12], [13, 10, 14, 11], [5, 16, 6, 17], [7, 18, 8, 19], [9, 14, 10, 15], [15, 8, 16, 9], [6, 18, 7, 17]]$

PD code drawn by `SnapPy`:  $[(17, 2, 18, 3), (13, 6, 14, 7), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (7, 12, 8, 13), (3, 14, 4, 15), (15, 4, 16, 5), (5, 16, 6, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 7], [2, 8, 9, 9], [2, 9, 9, 8], [4, 8, 8, 4], [5, 7, 7, 6], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 160: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

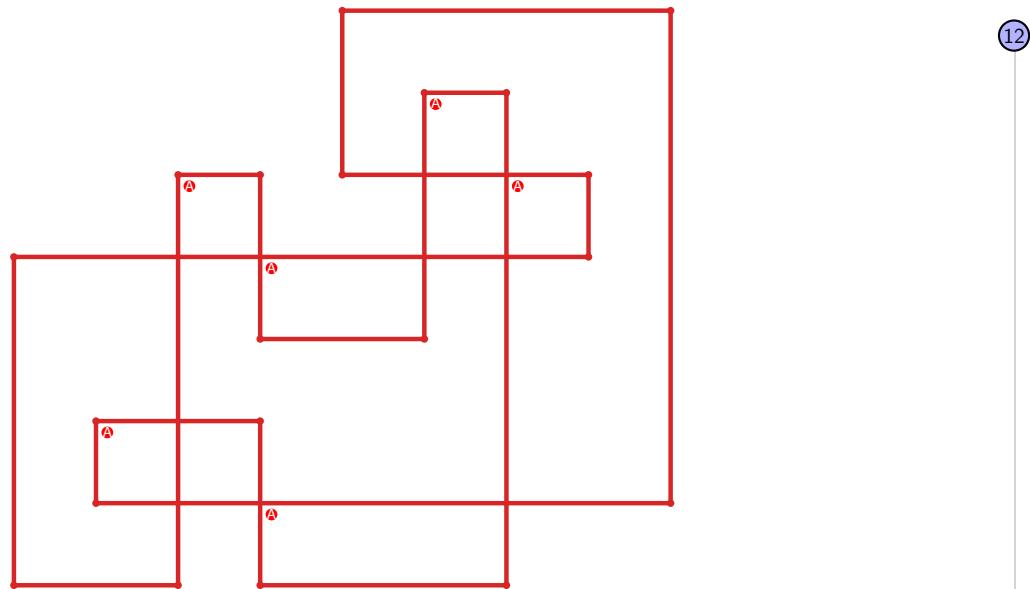


Figure 323: `SnapPy` multiloop plot.

6

Figure 324: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.14  $[[3, 20, 4, 1], [2, 11, 3, 12], [14, 19, 15, 20], [4, 9, 5, 10], [1, 13, 2, 12], [13, 10, 14, 11], [5, 18, 6, 19], [15, 8, 16, 9], [17, 6, 18, 7], [7, 16, 8, 17]]$

PD code drawn by SnapPy:  $[(13, 2, 14, 3), (11, 4, 12, 5), (20, 5, 1, 6), (9, 6, 10, 7), (18, 7, 19, 8), (8, 17, 9, 18), (3, 12, 4, 13), (1, 14, 2, 15), (10, 15, 11, 16), (19, 16, 20, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 7], [0, 7, 6, 5], [0, 5, 1, 1], [1, 4, 3, 2], [2, 3, 8, 8], [2, 9, 9, 3], [6, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 161: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

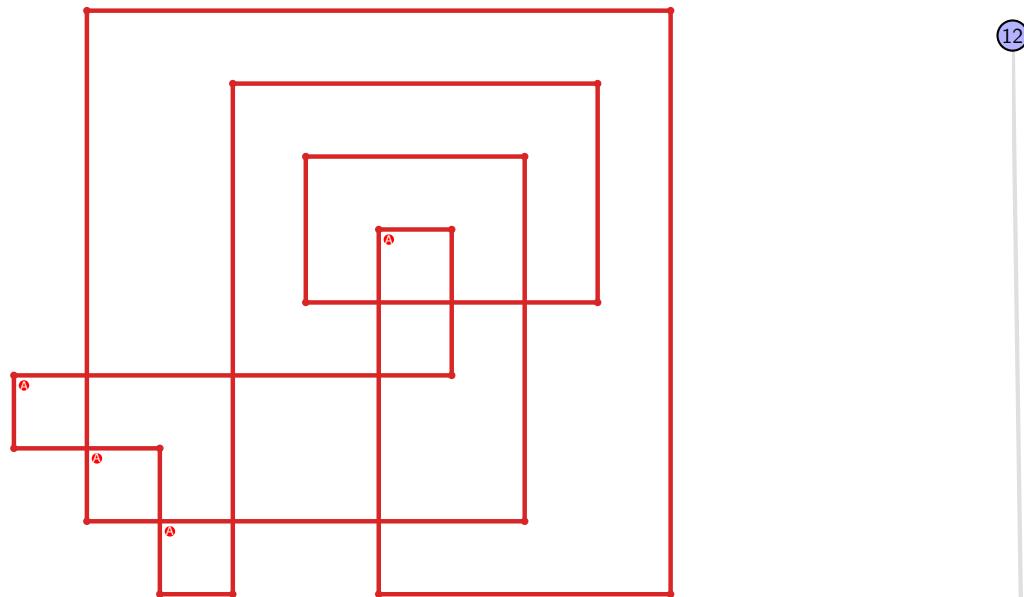


Figure 325: SnapPy multiloop plot.



Figure 326: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.15**  $[[3, 20, 4, 1], [2, 7, 3, 8], [10, 19, 11, 20], [4, 16, 5, 15], [1, 9, 2, 8], [9, 6, 10, 7], [18, 13, 19, 14], [11, 17, 12, 16], [5, 14, 6, 15], [12, 17, 13, 18]]$

PD code drawn by `SnapPy`:  $[(7, 4, 8, 5), (20, 5, 1, 6), (6, 19, 7, 20), (16, 11, 17, 12), (3, 12, 4, 13), (13, 2, 14, 3), (14, 9, 15, 10), (10, 15, 11, 16), (8, 17, 9, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 7], [0, 7, 8, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 9], [2, 9, 9, 3], [3, 6, 5, 3], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 162: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

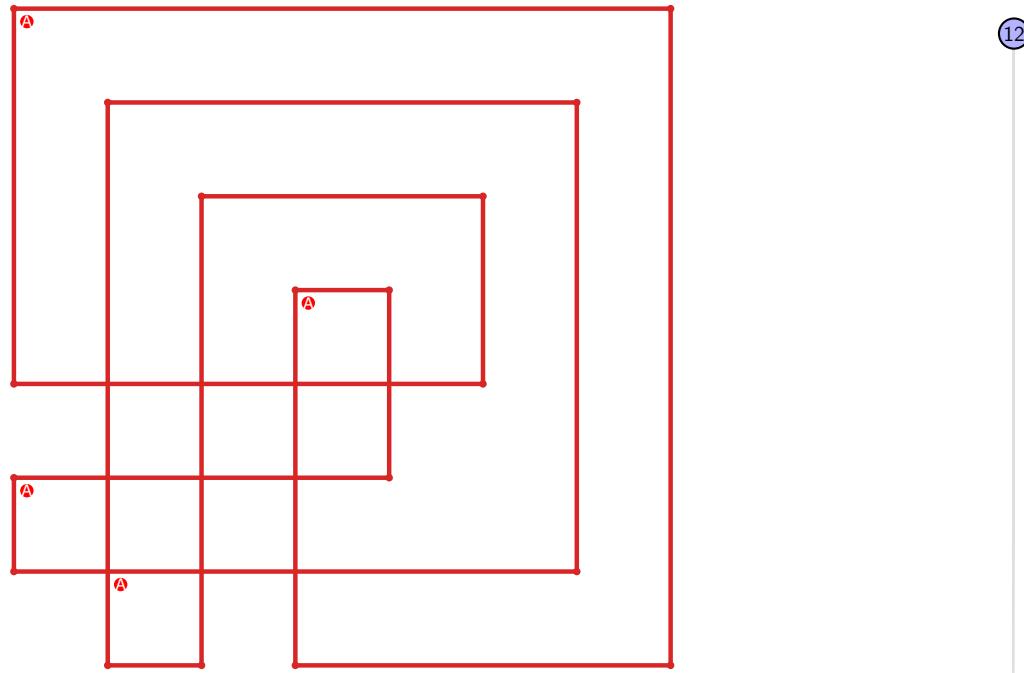


Figure 327: `SnapPy` multiloop plot.

Figure 328: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.16  $[[3, 20, 4, 1], [2, 11, 3, 12], [19, 4, 20, 5], [1, 13, 2, 12], [13, 10, 14, 11], [5, 14, 6, 15], [9, 18, 10, 19], [6, 18, 7, 17], [15, 8, 16, 9], [7, 16, 8, 17]]$

PD code drawn by `SnapPy`:  $[(17, 2, 18, 3), (13, 6, 14, 7), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (3, 12, 4, 13), (5, 14, 6, 15), (15, 4, 16, 5), (7, 16, 8, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 6, 5], [2, 4, 7, 8], [2, 8, 7, 4], [5, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 163: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

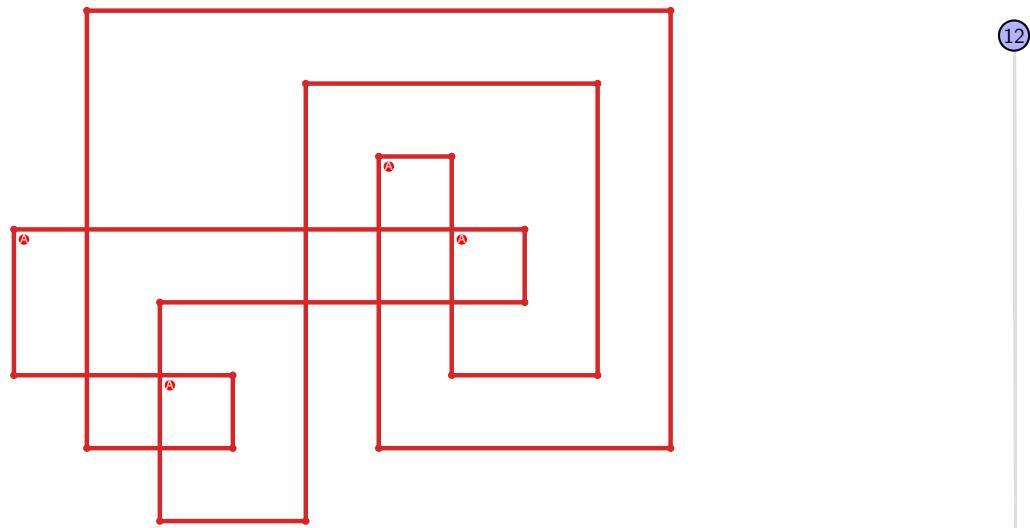


Figure 329: `SnapPy` multiloop plot.



Figure 330: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.17  $[[20, 3, 1, 4], [4, 13, 5, 14], [10, 19, 11, 20], [11, 2, 12, 3], [1, 12, 2, 13], [5, 19, 6, 18], [14, 9, 15, 10], [6, 15, 7, 16], [8, 17, 9, 18], [7, 17, 8, 16]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (18, 1, 19, 2), (12, 7, 13, 8), (19, 10, 20, 11), (2, 11, 3, 12), (4, 13, 5, 14), (14, 5, 15, 6), (6, 15, 7, 16), (16, 3, 17, 4), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 5, 3], [0, 2, 4, 4], [0, 3, 3, 1], [1, 2, 7, 8], [1, 8, 7, 2], [5, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 320

**Average overall degree:** 3.03

**Pinning number:** 4

Table 164: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

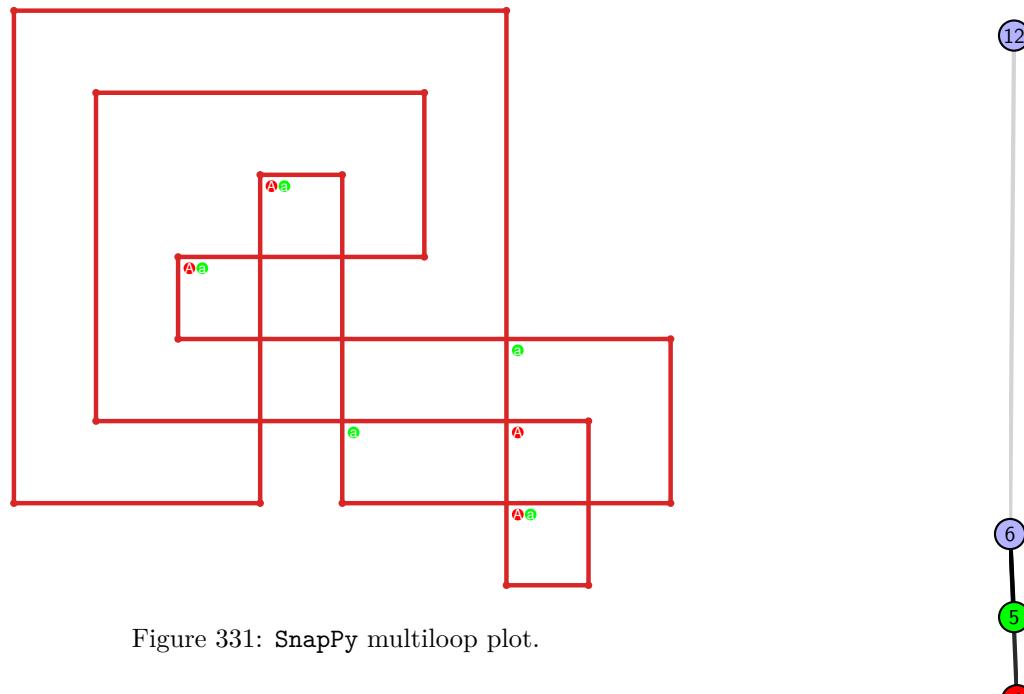


Figure 331: `SnapPy` multiloop plot.

Figure 332: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.18** [[20, 11, 1, 12], [12, 10, 13, 9], [19, 2, 20, 3], [10, 1, 11, 2], [13, 19, 14, 18], [8, 3, 9, 4], [14, 6, 15, 5], [17, 4, 18, 5], [7, 16, 8, 17], [6, 16, 7, 15]]

PD code drawn by `SnapPy`: [(5, 20, 6, 1), (14, 1, 15, 2), (2, 11, 3, 12), (6, 9, 7, 10), (17, 8, 18, 9), (12, 3, 13, 4), (4, 13, 5, 14), (15, 10, 16, 11), (7, 18, 8, 19), (16, 19, 17, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 4, 3], [0, 2, 1, 0], [1, 2, 6, 7], [1, 7, 8, 2], [4, 9, 9, 7], [4, 6, 8, 5], [5, 7, 9, 9], [6, 8, 8, 6]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 8  
**Total pinning sets:** 436  
**Pinning number:** 4

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.56  
**Average overall degree:** 3.06

Table 165: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	1	0	0	0	0	0	0	7
Nonminimal pinning sets	0	8	56	111	123	84	36	9	1	428
Average degree	2.5	2.64	2.85	3.0	3.11	3.19	3.24	3.29	3.33	

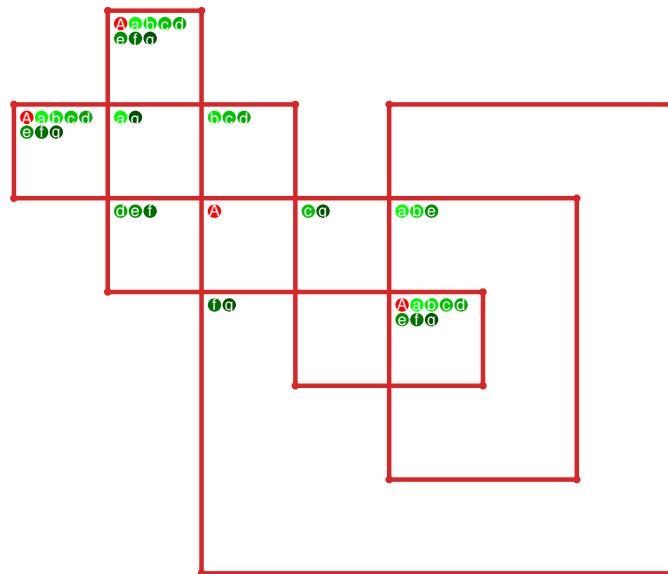


Figure 333: `SnapPy` multiloop plot.

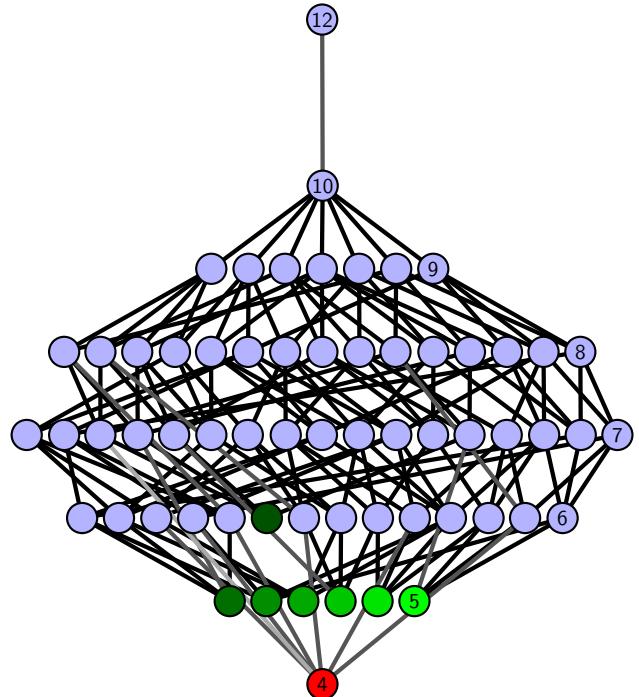


Figure 334: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.19  $[[9, 20, 10, 1], [11, 8, 12, 9], [19, 2, 20, 3], [10, 2, 11, 1], [7, 18, 8, 19], [12, 4, 13, 3], [6, 15, 7, 16], [17, 4, 18, 5], [13, 17, 14, 16], [14, 5, 15, 6]]$

PD code drawn by SnapPy:  $[(20, 9, 1, 10), (14, 1, 15, 2), (11, 2, 12, 3), (8, 5, 9, 6), (3, 12, 4, 13), (13, 10, 14, 11), (4, 15, 5, 16), (6, 17, 7, 18), (18, 7, 19, 8), (16, 19, 17, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 4, 3], [0, 2, 1, 0], [1, 2, 6, 7], [1, 7, 8, 2], [4, 8, 9, 9], [4, 9, 8, 5], [5, 7, 9, 6], [6, 8, 7, 6]]$

Total optimal pinning sets: 5  
 Total minimal pinning sets: 5  
 Total pinning sets: 608  
 Pinning number: 4

Average optimal degree: 2.6  
 Average minimal degree: 2.6  
 Average overall degree: 3.12

Table 166: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	33	95	156	160	105	43	10	1	603
Average degree	2.6	2.83	2.98	3.09	3.17	3.23	3.27	3.31	3.33	

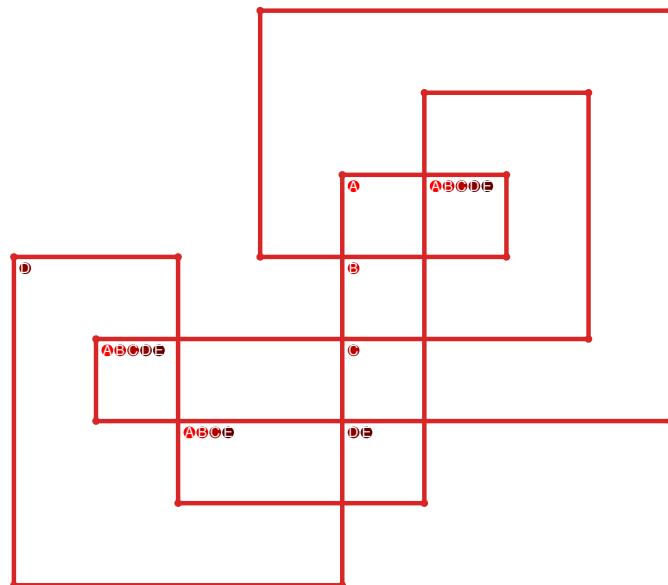


Figure 335: SnapPy multiloop plot.

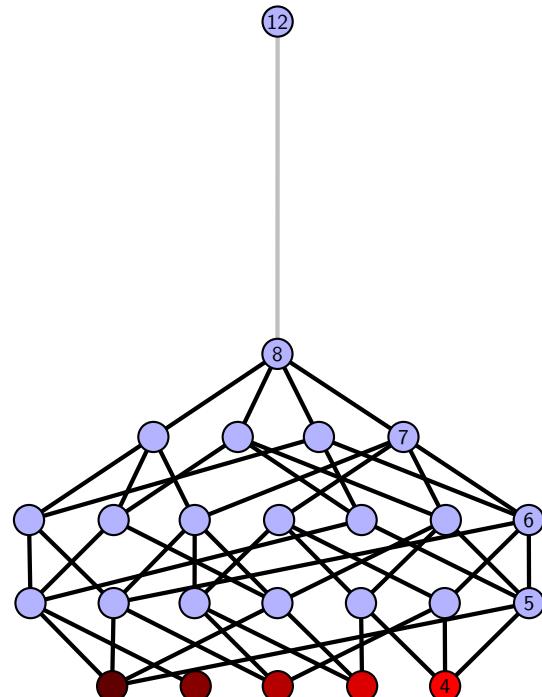


Figure 336: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.20  $[[20, 15, 1, 16], [16, 14, 17, 13], [19, 2, 20, 3], [14, 1, 15, 2], [17, 9, 18, 8], [12, 3, 13, 4], [18, 9, 19, 10], [7, 4, 8, 5], [11, 6, 12, 7], [10, 6, 11, 5]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (4, 1, 5, 2), (2, 7, 3, 8), (8, 3, 9, 4), (5, 14, 6, 15), (15, 6, 16, 7), (10, 13, 11, 14), (17, 12, 18, 13), (11, 18, 12, 19), (16, 19, 17, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 6, 3], [0, 2, 1, 0], [1, 6, 6, 7], [1, 7, 8, 2], [2, 9, 4, 4], [4, 9, 8, 5], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.25

**Total pinning sets:** 384

**Average overall degree:** 3.03

**Pinning number:** 4

Table 167: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

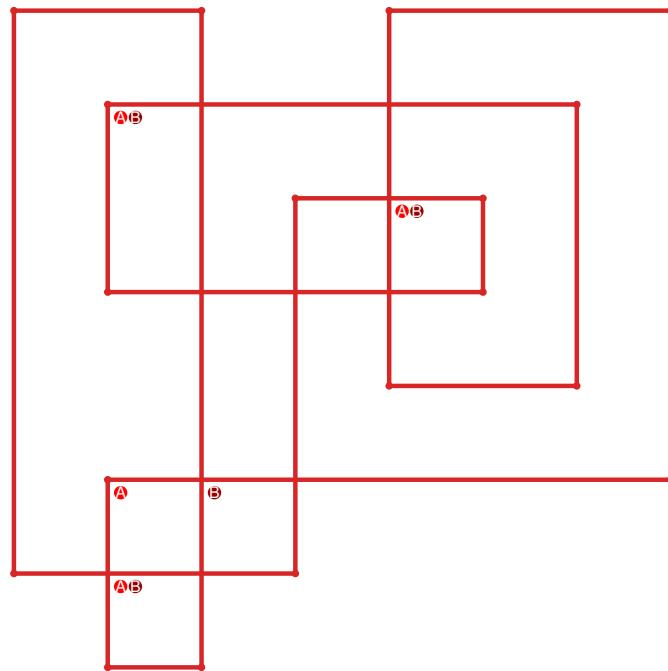


Figure 337: SnapPy multiloop plot.

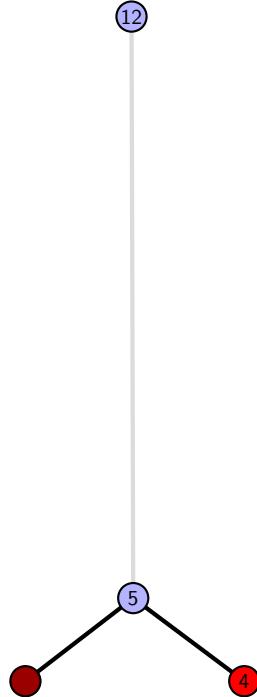


Figure 338: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.21  $[[11, 20, 12, 1], [13, 10, 14, 11], [19, 2, 20, 3], [12, 2, 13, 1], [9, 18, 10, 19], [14, 4, 15, 3], [17, 8, 18, 9], [4, 8, 5, 7], [15, 7, 16, 6], [16, 5, 17, 6]]$

PD code drawn by SnapPy:  $[(20, 11, 1, 12), (13, 2, 14, 3), (3, 12, 4, 13), (4, 1, 5, 2), (14, 5, 15, 6), (10, 7, 11, 8), (6, 15, 7, 16), (8, 17, 9, 18), (18, 9, 19, 10), (16, 19, 17, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 5, 4, 3], [0, 2, 1, 0], [1, 2, 6, 6], [1, 7, 8, 2], [4, 9, 7, 4], [5, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 320

**Average overall degree:** 3.03

**Pinning number:** 4

Table 168: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

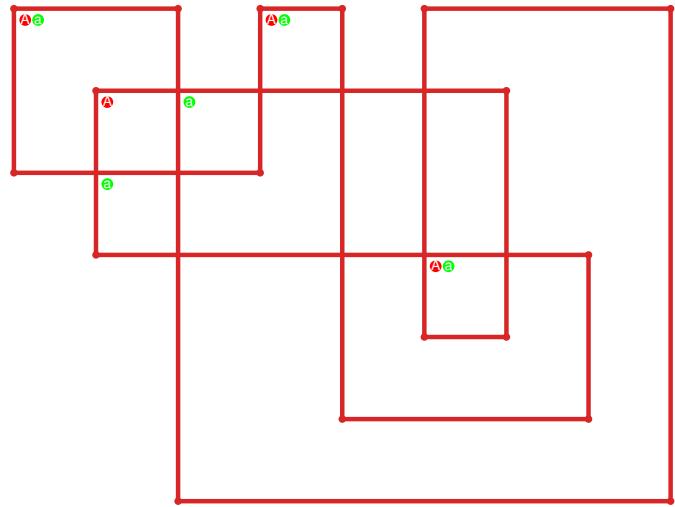


Figure 339: SnapPy multiloop plot.

(12)

(6)

(5)

(4)

Figure 340: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.22  $[[15, 20, 16, 1], [14, 11, 15, 12], [4, 19, 5, 20], [16, 9, 17, 10], [1, 13, 2, 12], [2, 13, 3, 14], [3, 10, 4, 11], [7, 18, 8, 19], [5, 8, 6, 9], [17, 6, 18, 7]]$

PD code drawn by SnapPy:  $[(13, 2, 14, 3), (1, 4, 2, 5), (12, 5, 13, 6), (19, 6, 20, 7), (10, 7, 11, 8), (8, 17, 9, 18), (18, 9, 19, 10), (3, 14, 4, 15), (20, 15, 1, 16), (11, 16, 12, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 8], [0, 8, 9, 6], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 3, 2], [2, 9, 9, 8], [2, 7, 9, 3], [3, 8, 7, 7]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 7  
 Total pinning sets: 384  
 Pinning number: 5

Average optimal degree: 2.67  
 Average minimal degree: 2.67  
 Average overall degree: 3.11

Table 169: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	34	86	115	90	41	10	1	377
Average degree	2.67	2.87	3.01	3.12	3.21	3.27	3.31	3.33	

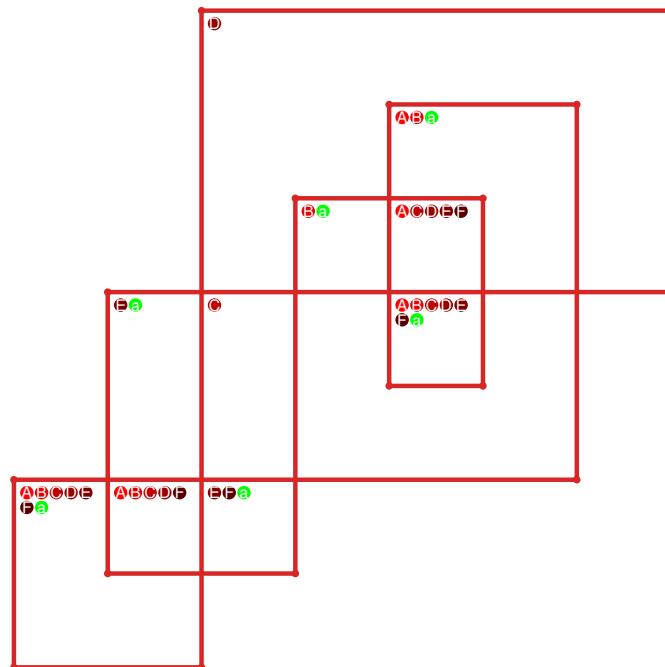


Figure 341: SnapPy multiloop plot.

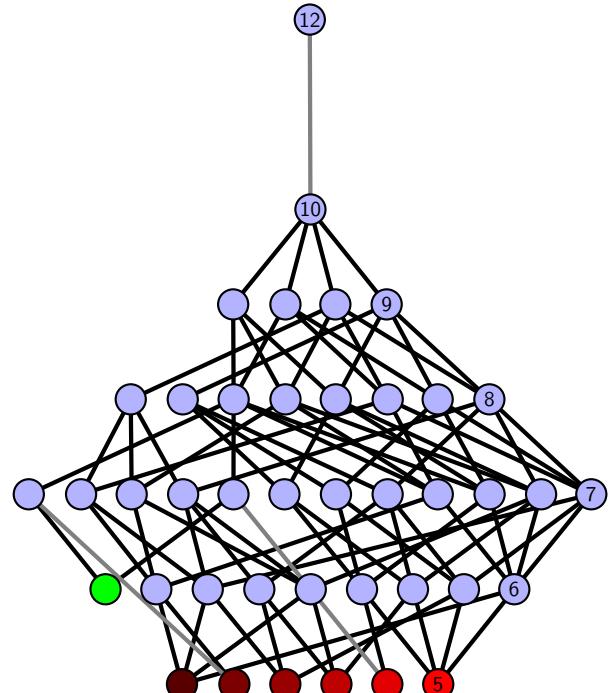


Figure 342: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.23  $[[3, 20, 4, 1], [2, 13, 3, 14], [16, 19, 17, 20], [4, 11, 5, 12], [1, 15, 2, 14], [15, 12, 16, 13], [7, 18, 8, 19], [17, 8, 18, 9], [10, 5, 11, 6], [6, 9, 7, 10]]$

PD code drawn by SnapPy:  $[(8, 3, 9, 4), (15, 6, 16, 7), (4, 7, 5, 8), (2, 9, 3, 10), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (5, 16, 6, 17), (14, 17, 15, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 7], [0, 8, 8, 5], [0, 5, 1, 1], [1, 4, 3, 2], [2, 9, 7, 7], [2, 6, 6, 9], [3, 9, 9, 3], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 170: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

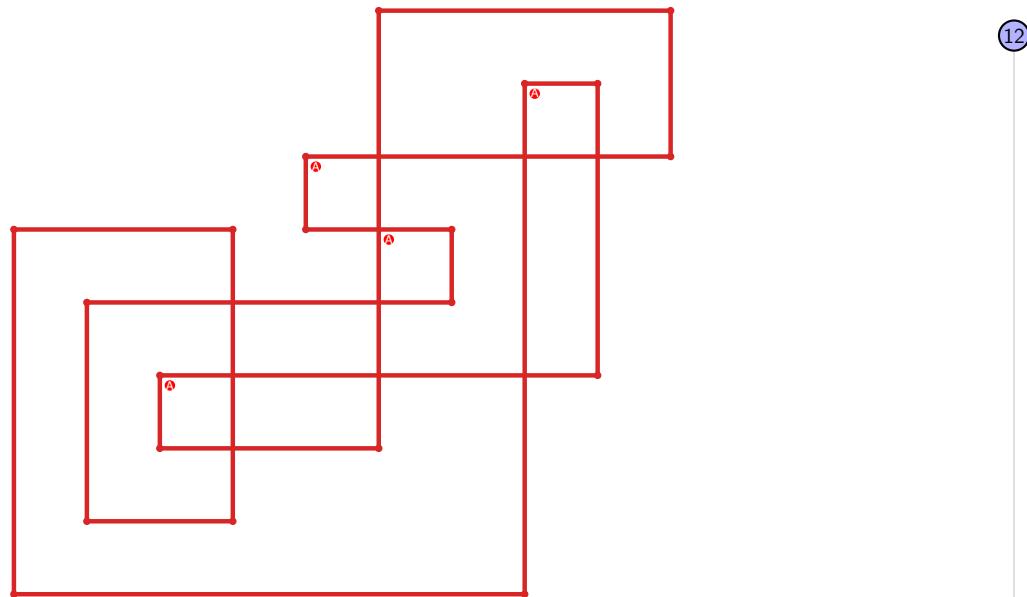


Figure 343: SnapPy multiloop plot.

4

Figure 344: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.24  $[[3, 20, 4, 1], [11, 2, 12, 3], [19, 4, 20, 5], [1, 10, 2, 11], [12, 10, 13, 9], [5, 9, 6, 8], [18, 13, 19, 14], [6, 15, 7, 16], [16, 7, 17, 8], [14, 17, 15, 18]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (17, 2, 18, 3), (14, 5, 15, 6), (6, 3, 7, 4), (7, 10, 8, 11), (19, 8, 20, 9), (16, 11, 17, 12), (4, 13, 5, 14), (12, 15, 13, 16), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 6, 5], [2, 4, 7, 8], [2, 9, 9, 4], [5, 9, 8, 8], [5, 7, 7, 9], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 171: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

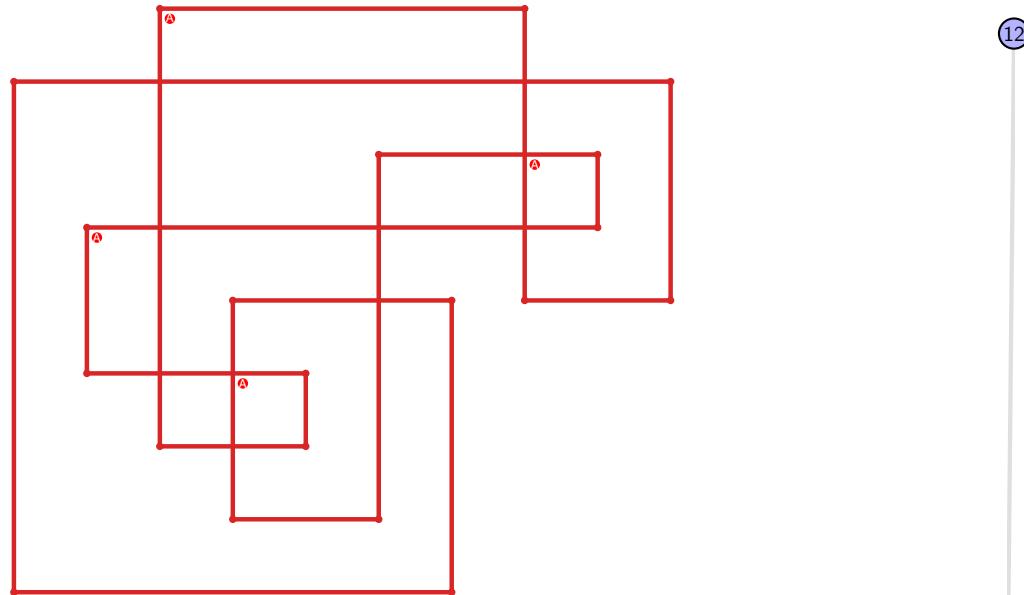


Figure 345: SnapPy multiloop plot.



Figure 346: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.25  $[[3, 20, 4, 1], [2, 11, 3, 12], [14, 19, 15, 20], [4, 9, 5, 10], [1, 13, 2, 12], [13, 10, 14, 11], [18, 15, 19, 16], [8, 5, 9, 6], [16, 8, 17, 7], [17, 6, 18, 7]]$

PD code drawn by SnapPy:  $[(6, 3, 7, 4), (15, 4, 16, 5), (5, 14, 6, 15), (2, 7, 3, 8), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (16, 13, 17, 14), (12, 17, 13, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 6], [0, 7, 7, 5], [0, 5, 1, 1], [1, 4, 3, 2], [2, 8, 9, 2], [3, 9, 8, 3], [6, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 1  
 Total pinning sets: 256  
 Pinning number: 4

Average optimal degree: 2.0  
 Average minimal degree: 2.0  
 Average overall degree: 2.97

Table 172: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

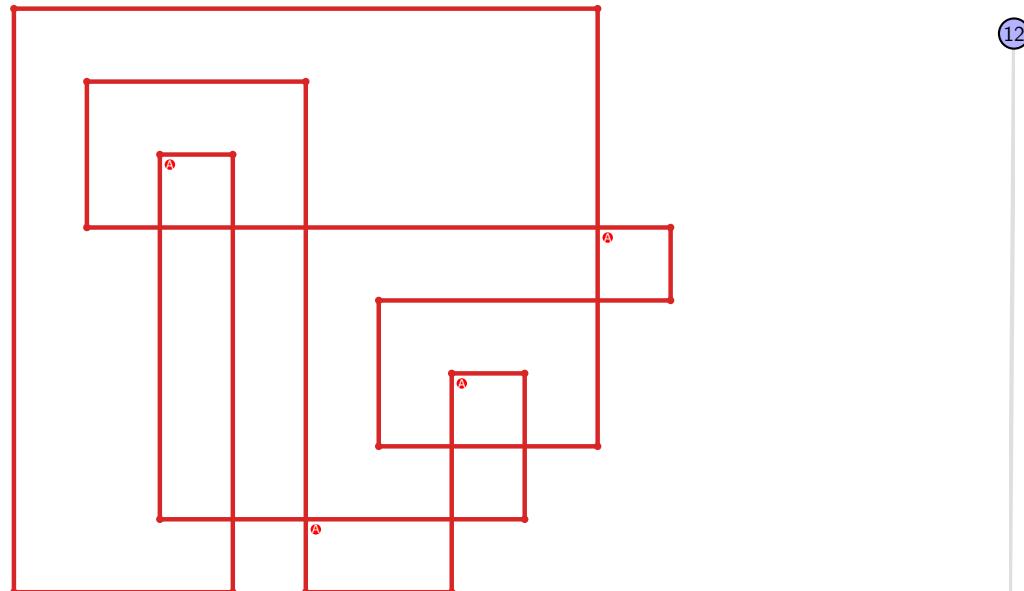


Figure 347: SnapPy multiloop plot.



Figure 348: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.26  $[[3, 20, 4, 1], [2, 9, 3, 10], [14, 19, 15, 20], [4, 7, 5, 8], [1, 11, 2, 10], [11, 8, 12, 9], [18, 13, 19, 14], [15, 6, 16, 7], [5, 16, 6, 17], [12, 17, 13, 18]]$

PD code drawn by `SnapPy`:  $[(13, 4, 14, 5), (2, 5, 3, 6), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (16, 11, 17, 12), (3, 14, 4, 15), (10, 15, 11, 16), (12, 17, 13, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 7], [0, 7, 8, 5], [0, 5, 1, 1], [1, 4, 3, 9], [2, 9, 9, 2], [2, 8, 8, 3], [3, 7, 7, 9], [5, 8, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 173: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

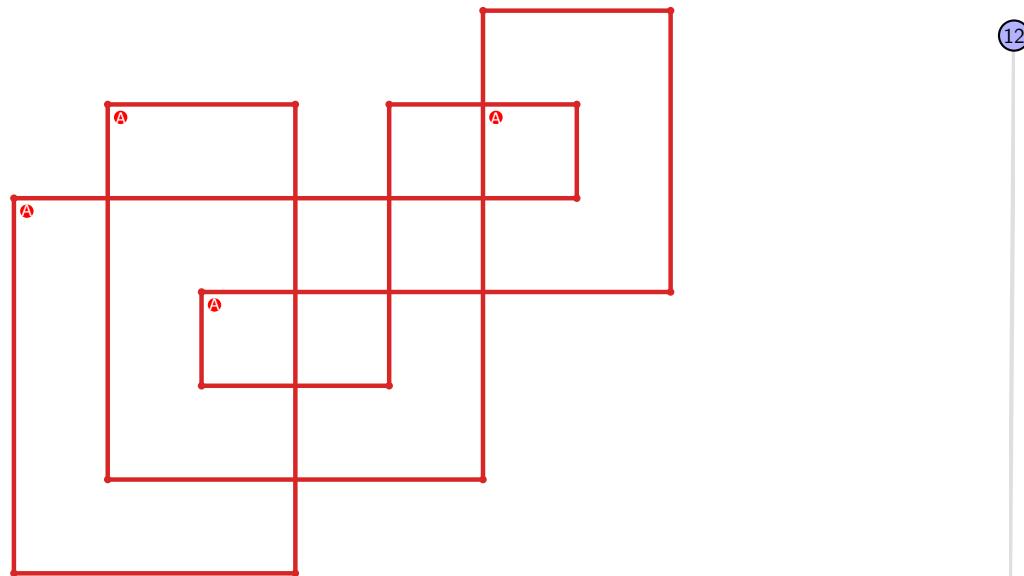


Figure 349: `SnapPy` multiloop plot.

Figure 350: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.27  $[[3, 20, 4, 1], [2, 9, 3, 10], [19, 14, 20, 15], [4, 7, 5, 8], [1, 11, 2, 10], [11, 8, 12, 9], [15, 12, 16, 13], [13, 18, 14, 19], [6, 17, 7, 18], [5, 17, 6, 16]]$

PD code drawn by SnapPy:  $[(2, 5, 3, 6), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (13, 10, 14, 11), (17, 12, 18, 13), (14, 3, 15, 4), (4, 15, 5, 16), (11, 16, 12, 17), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 7], [0, 8, 9, 5], [0, 5, 1, 1], [1, 4, 3, 6], [2, 5, 9, 7], [2, 6, 8, 2], [3, 7, 9, 9], [3, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 512

**Average overall degree:** 3.03

**Pinning number:** 3

Table 174: Pinning sets/average degree by cardinal

Cardinal	3	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	36	84	126	126	84	36	9	1	511
Average degree	2.0	2.44	2.71	2.89	3.02	3.11	3.19	3.24	3.29	3.33	

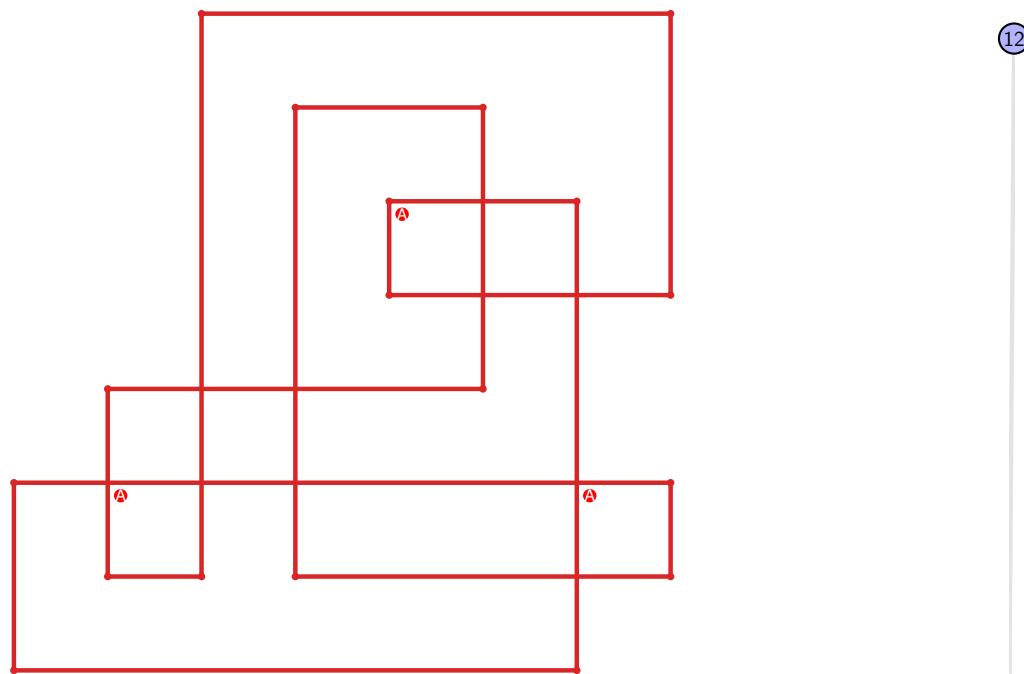


Figure 351: SnapPy multiloop plot.

Figure 352: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.28  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 16, 7, 15], [1, 13, 2, 12], [3, 18, 4, 19], [16, 8, 17, 7], [9, 14, 10, 15], [13, 10, 14, 11], [2, 11, 3, 12], [17, 8, 18, 9]]$

PD code drawn by `SnapPy`:  $[(15, 20, 16, 1), (3, 12, 4, 13), (13, 4, 14, 5), (9, 6, 10, 7), (18, 7, 19, 8), (8, 17, 9, 18), (5, 10, 6, 11), (11, 2, 12, 3), (19, 14, 20, 15), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 5, 6], [0, 7, 8, 8], [1, 8, 9, 1], [2, 9, 9, 2], [2, 9, 7, 7], [3, 6, 6, 8], [3, 7, 4, 3], [4, 6, 5, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 175: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

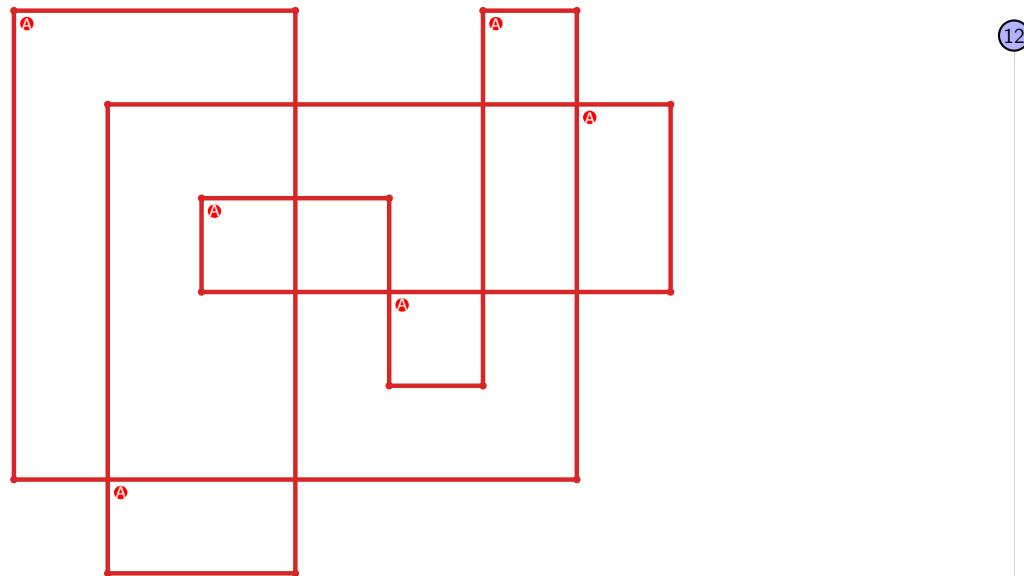


Figure 353: `SnapPy` multiloop plot.

6

Figure 354: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.29  $[[20, 7, 1, 8], [8, 19, 9, 20], [11, 6, 12, 7], [1, 16, 2, 17], [18, 9, 19, 10], [10, 17, 11, 18], [5, 14, 6, 15], [12, 4, 13, 3], [15, 2, 16, 3], [13, 4, 14, 5]]$

PD code drawn by `SnapPy`:  $[(20, 9, 1, 10), (10, 1, 11, 2), (8, 3, 9, 4), (17, 4, 18, 5), (15, 6, 16, 7), (2, 11, 3, 12), (18, 13, 19, 14), (7, 14, 8, 15), (5, 16, 6, 17), (12, 19, 13, 20)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 7], [0, 8, 8, 5], [1, 5, 5, 1], [2, 4, 4, 3], [2, 8, 9, 9], [2, 9, 9, 8], [3, 7, 6, 3], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 176: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

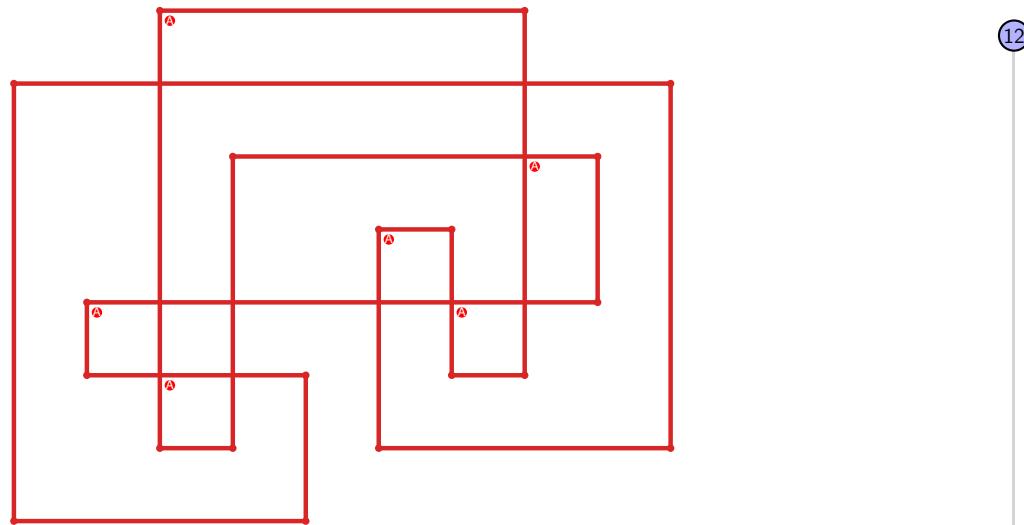


Figure 355: `SnapPy` multiloop plot.

6

Figure 356: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.30  $[[13, 20, 14, 1], [12, 15, 13, 16], [19, 14, 20, 15], [1, 10, 2, 11], [16, 11, 17, 12], [18, 5, 19, 6], [9, 4, 10, 5], [2, 8, 3, 7], [17, 7, 18, 6], [3, 8, 4, 9]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (15, 2, 16, 3), (3, 14, 4, 15), (4, 19, 5, 20), (20, 5, 1, 6), (18, 7, 19, 8), (13, 8, 14, 9), (16, 11, 17, 12), (9, 12, 10, 13), (10, 17, 11, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 4, 2], [0, 1, 5, 0], [0, 6, 7, 4], [1, 3, 8, 1], [2, 8, 8, 6], [3, 5, 9, 9], [3, 9, 9, 8], [4, 7, 5, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 177: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

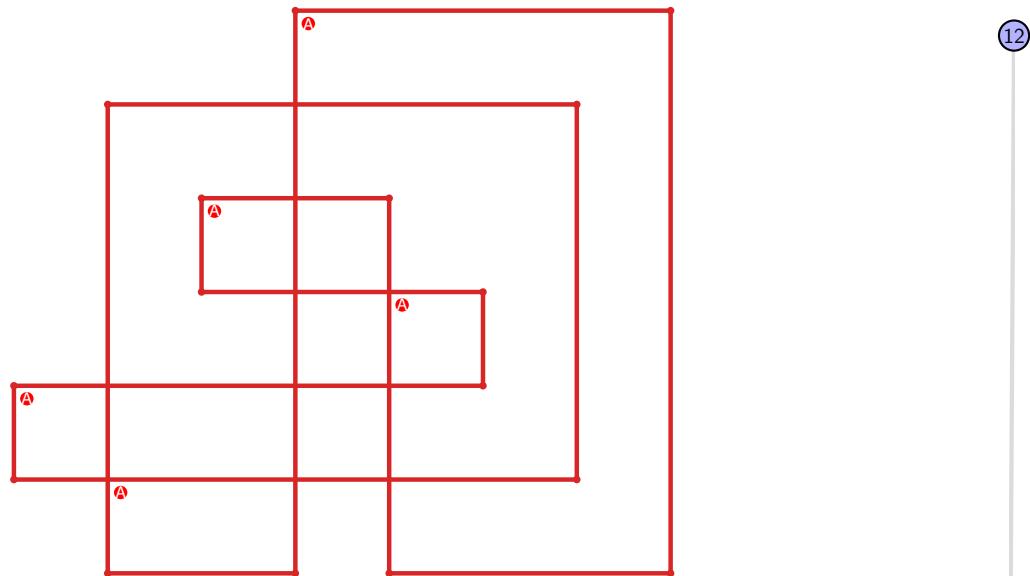


Figure 357: SnapPy multiloop plot.



Figure 358: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.31**  $[[9, 20, 10, 1], [8, 11, 9, 12], [19, 10, 20, 11], [1, 6, 2, 7], [12, 7, 13, 8], [13, 18, 14, 19], [14, 5, 15, 6], [2, 17, 3, 18], [4, 15, 5, 16], [16, 3, 17, 4]]$

PD code drawn by `SnapPy`:  $[(10, 1, 11, 2), (17, 2, 18, 3), (8, 3, 9, 4), (15, 6, 16, 7), (4, 7, 5, 8), (20, 11, 1, 12), (18, 13, 19, 14), (9, 14, 10, 15), (5, 16, 6, 17), (12, 19, 13, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 2], [0, 1, 5, 0], [0, 6, 7, 4], [1, 3, 5, 1], [2, 4, 7, 6], [3, 5, 8, 8], [3, 9, 9, 5], [6, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 178: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

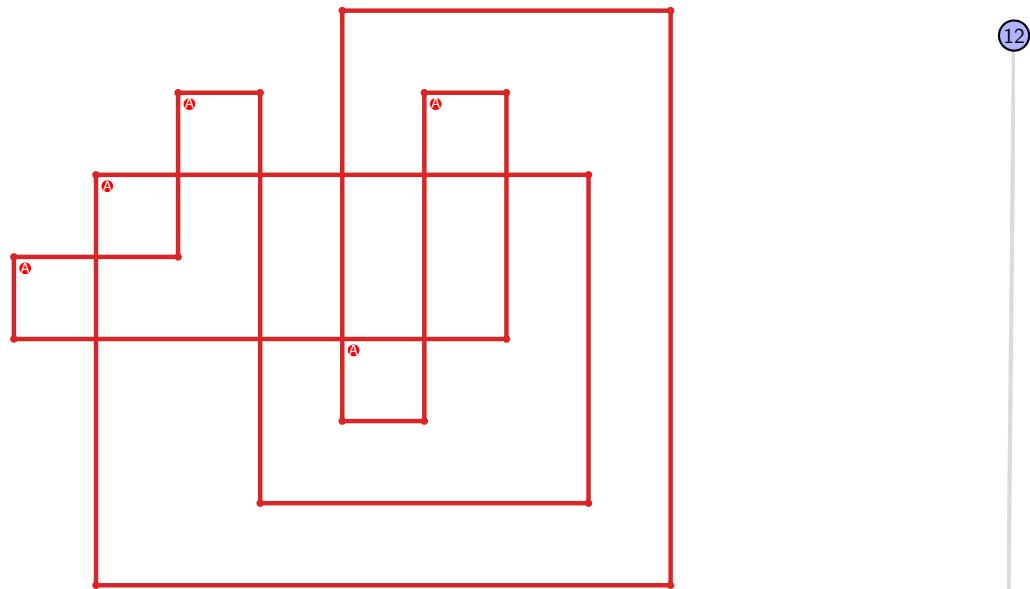


Figure 359: `SnapPy` multiloop plot.

12  
5

Figure 360: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.32  $[[20, 3, 1, 4], [4, 19, 5, 20], [11, 2, 12, 3], [1, 12, 2, 13], [18, 5, 19, 6], [10, 17, 11, 18], [13, 7, 14, 6], [14, 9, 15, 10], [16, 7, 17, 8], [8, 15, 9, 16]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (18, 1, 19, 2), (16, 3, 17, 4), (4, 11, 5, 12), (14, 5, 15, 6), (12, 7, 13, 8), (19, 10, 20, 11), (6, 13, 7, 14), (8, 15, 9, 16), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 3, 3], [0, 2, 2, 6], [1, 6, 5, 1], [2, 4, 7, 8], [3, 8, 7, 4], [5, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 179: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

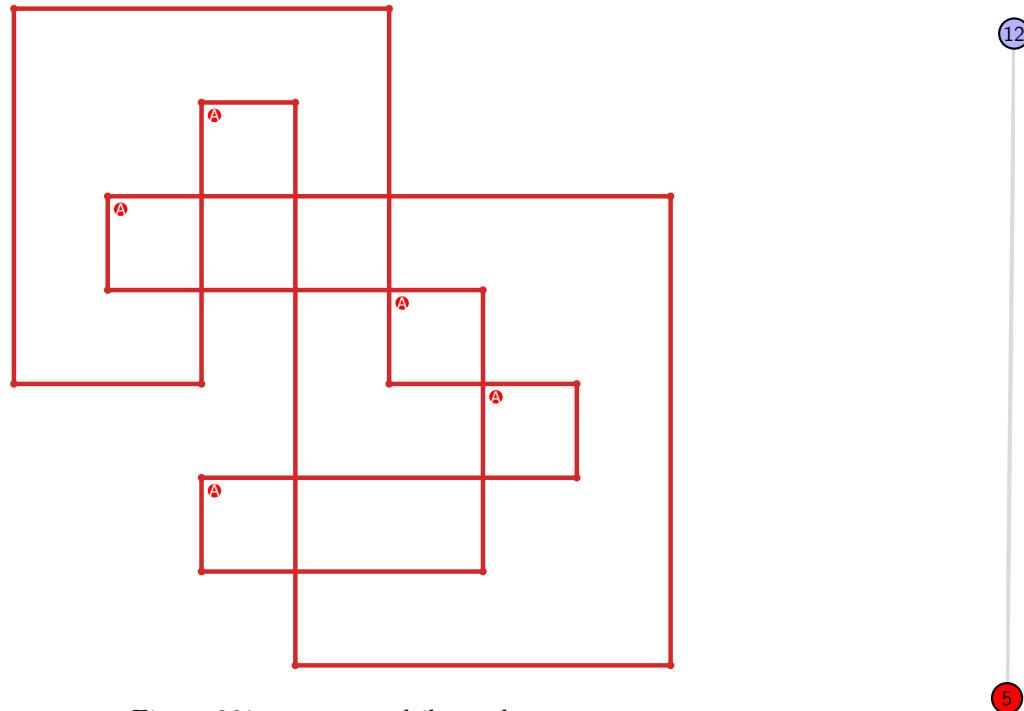


Figure 361: SnapPy multiloop plot.

12

5

Figure 362: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.33  $[[15, 20, 16, 1], [19, 14, 20, 15], [16, 7, 17, 8], [1, 12, 2, 13], [13, 18, 14, 19], [6, 17, 7, 18], [8, 4, 9, 3], [11, 2, 12, 3], [5, 10, 6, 11], [4, 10, 5, 9]]$

PD code drawn by SnapPy:  $[(12, 3, 13, 4), (5, 20, 6, 1), (1, 6, 2, 7), (17, 8, 18, 9), (15, 10, 16, 11), (2, 13, 3, 14), (9, 14, 10, 15), (11, 16, 12, 17), (7, 18, 8, 19), (19, 4, 20, 5)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 5, 6], [0, 7, 7, 4], [1, 3, 5, 1], [2, 4, 8, 2], [2, 9, 9, 7], [3, 6, 8, 3], [5, 7, 9, 9], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1

**Total pinning sets:** 64  
**Pinning number:** 6

**Average optimal degree:** 2.0

**Average minimal degree:** 2.0

**Average overall degree:** 2.85

Table 180: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

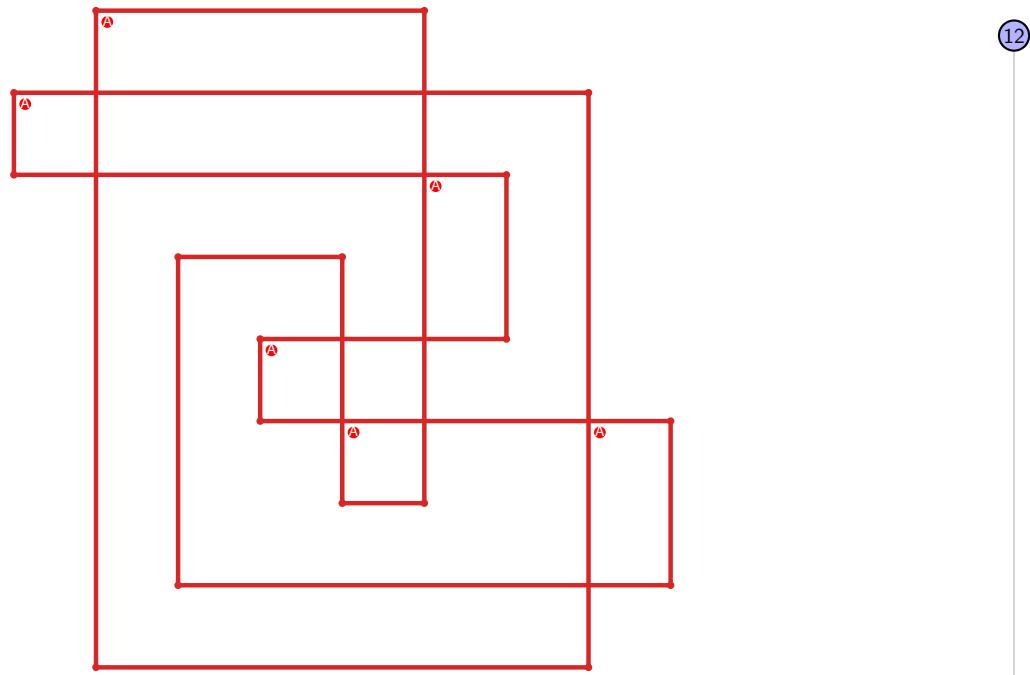


Figure 363: SnapPy multiloop plot.

Figure 364: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.34  $[[20, 15, 1, 16], [16, 10, 17, 9], [19, 6, 20, 7], [14, 5, 15, 6], [1, 11, 2, 10], [17, 8, 18, 9], [7, 18, 8, 19], [2, 13, 3, 14], [4, 11, 5, 12], [12, 3, 13, 4]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (12, 3, 13, 4), (17, 4, 18, 5), (15, 6, 16, 7), (7, 14, 8, 15), (8, 19, 9, 20), (2, 9, 3, 10), (20, 11, 1, 12), (18, 13, 19, 14), (5, 16, 6, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 8, 7, 1], [1, 6, 6, 1], [2, 5, 5, 2], [3, 4, 9, 9], [3, 9, 9, 4], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 181: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

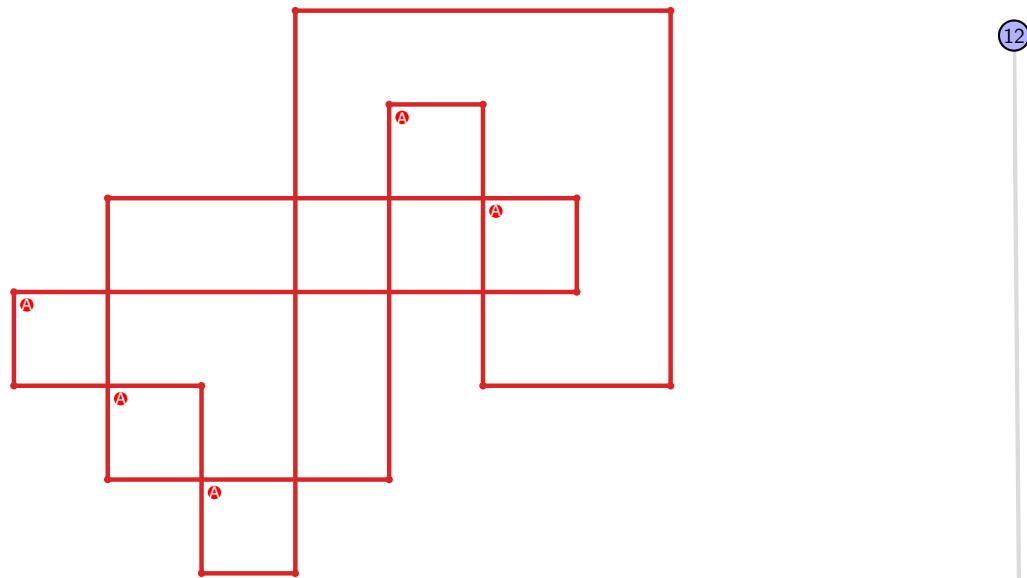


Figure 365: SnapPy multiloop plot.

5

Figure 366: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.35**  $[[20, 7, 1, 8], [8, 17, 9, 18], [10, 19, 11, 20], [11, 6, 12, 7], [1, 16, 2, 17], [9, 19, 10, 18], [5, 14, 6, 15], [12, 4, 13, 3], [15, 2, 16, 3], [13, 4, 14, 5]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (1, 10, 2, 11), (11, 2, 12, 3), (9, 4, 10, 5), (18, 5, 19, 6), (3, 12, 4, 13), (19, 14, 20, 15), (6, 15, 7, 16), (16, 7, 17, 8), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 5, 5, 3], [0, 2, 6, 7], [0, 8, 8, 1], [1, 2, 2, 1], [3, 8, 9, 9], [3, 9, 9, 8], [4, 7, 6, 4], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 182: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

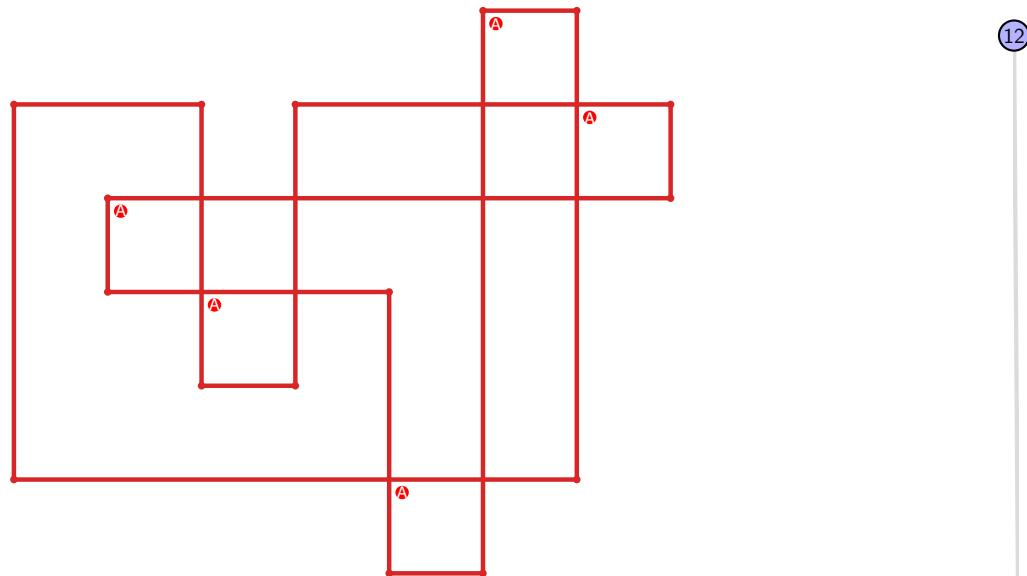


Figure 367: `SnapPy` multiloop plot.

Figure 368: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.36**  $[[20, 15, 1, 16], [16, 8, 17, 7], [14, 19, 15, 20], [1, 19, 2, 18], [8, 18, 9, 17], [11, 6, 12, 7], [13, 2, 14, 3], [9, 4, 10, 5], [5, 10, 6, 11], [12, 4, 13, 3]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (1, 10, 2, 11), (7, 2, 8, 3), (9, 4, 10, 5), (5, 14, 6, 15), (3, 8, 4, 9), (18, 13, 19, 14), (15, 6, 16, 7), (16, 19, 17, 20), (12, 17, 13, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 3, 0], [0, 2, 6, 4], [1, 3, 7, 1], [1, 8, 8, 9], [2, 9, 9, 3], [4, 9, 8, 8], [5, 7, 7, 5], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 183: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

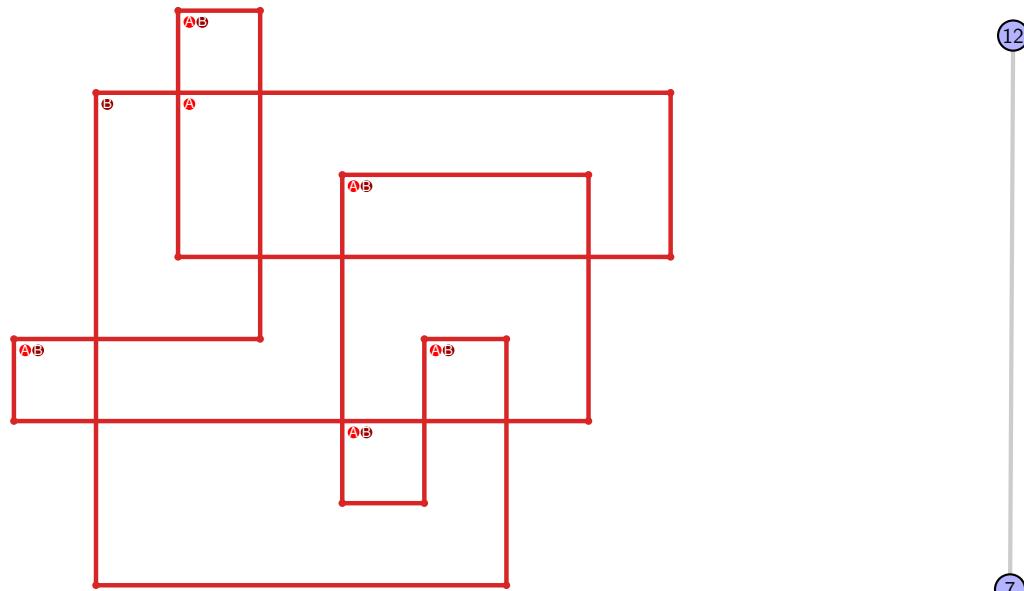


Figure 369: `SnapPy` multiloop plot.

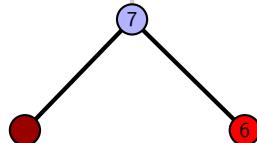


Figure 370: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.37**  $[[9, 20, 10, 1], [8, 15, 9, 16], [19, 10, 20, 11], [1, 19, 2, 18], [16, 7, 17, 8], [14, 3, 15, 4], [11, 3, 12, 2], [6, 17, 7, 18], [4, 13, 5, 14], [12, 5, 13, 6]]$

PD code drawn by `SnapPy`:  $[(20, 9, 1, 10), (12, 1, 13, 2), (10, 3, 11, 4), (14, 5, 15, 6), (17, 8, 18, 9), (2, 11, 3, 12), (6, 13, 7, 14), (4, 15, 5, 16), (16, 19, 17, 20), (7, 18, 8, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 3, 0], [0, 2, 6, 7], [1, 7, 7, 1], [1, 8, 8, 6], [2, 5, 9, 3], [3, 9, 4, 4], [5, 9, 9, 5], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 184: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

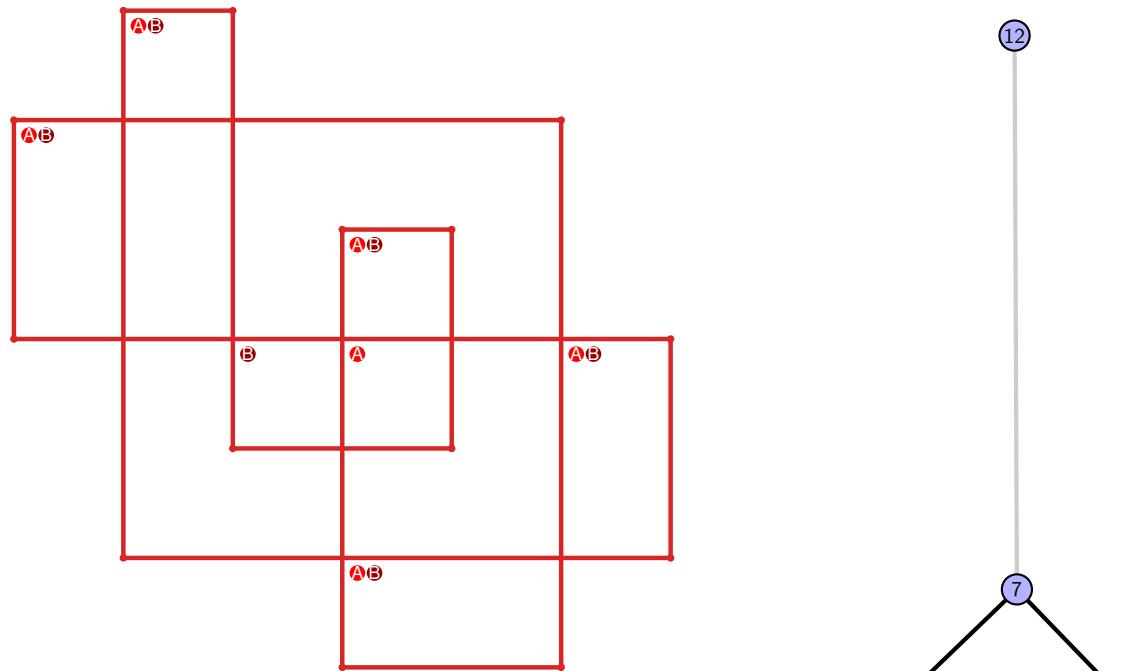


Figure 371: `SnapPy` multiloop plot.

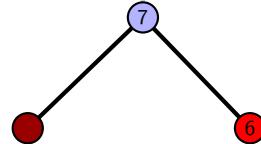


Figure 372: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.38**  $[[9, 20, 10, 1], [8, 5, 9, 6], [14, 19, 15, 20], [10, 2, 11, 1], [6, 11, 7, 12], [12, 7, 13, 8], [13, 4, 14, 5], [18, 15, 19, 16], [2, 18, 3, 17], [3, 16, 4, 17]]$

PD code drawn by `SnapPy`:  $[(10, 1, 11, 2), (5, 2, 6, 3), (9, 4, 10, 5), (3, 8, 4, 9), (15, 12, 16, 13), (20, 13, 1, 14), (14, 19, 15, 20), (11, 16, 12, 17), (6, 17, 7, 18), (18, 7, 19, 8)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 6], [0, 6, 7, 7], [0, 8, 4, 0], [1, 3, 5, 5], [1, 4, 4, 6], [1, 5, 9, 2], [2, 9, 8, 2], [3, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 185: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

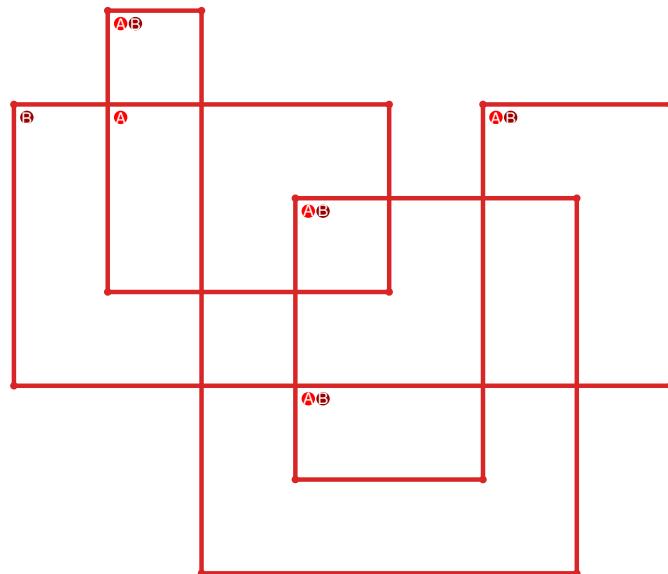


Figure 373: `SnapPy` multiloop plot.

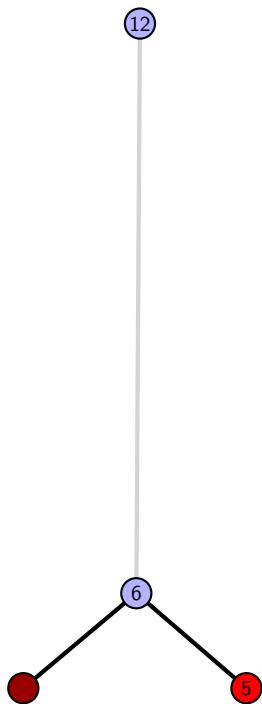


Figure 374: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.39  $[[20, 9, 1, 10], [10, 13, 11, 14], [4, 19, 5, 20], [8, 15, 9, 16], [1, 12, 2, 13], [11, 2, 12, 3], [14, 3, 15, 4], [18, 5, 19, 6], [16, 7, 17, 8], [6, 17, 7, 18]]$

PD code drawn by SnapPy:  $[(13, 20, 14, 1), (11, 2, 12, 3), (15, 4, 16, 5), (16, 7, 17, 8), (5, 8, 6, 9), (3, 10, 4, 11), (1, 12, 2, 13), (19, 14, 20, 15), (6, 17, 7, 18), (9, 18, 10, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 7], [0, 8, 8, 6], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 3, 2], [2, 9, 9, 2], [3, 9, 9, 3], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 186: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

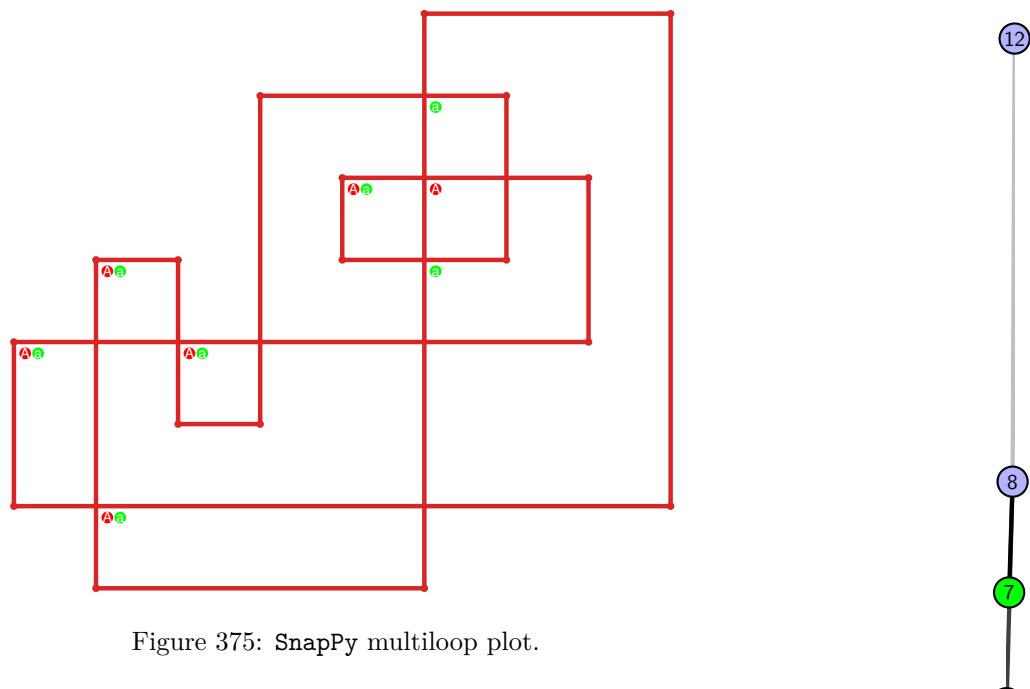


Figure 375: SnapPy multiloop plot.

Figure 376: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.40  $[[13, 20, 14, 1], [12, 9, 13, 10], [19, 14, 20, 15], [1, 11, 2, 10], [2, 11, 3, 12], [3, 8, 4, 9], [15, 4, 16, 5], [7, 18, 8, 19], [16, 6, 17, 5], [17, 6, 18, 7]]$

PD code drawn by SnapPy:  $[(12, 1, 13, 2), (2, 13, 3, 14), (14, 3, 15, 4), (20, 5, 1, 6), (9, 6, 10, 7), (7, 18, 8, 19), (19, 8, 20, 9), (16, 11, 17, 12), (4, 15, 5, 16), (10, 17, 11, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 7, 0], [0, 4, 4, 1], [1, 3, 3, 5], [1, 4, 7, 6], [2, 5, 8, 8], [2, 9, 9, 5], [6, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 187: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

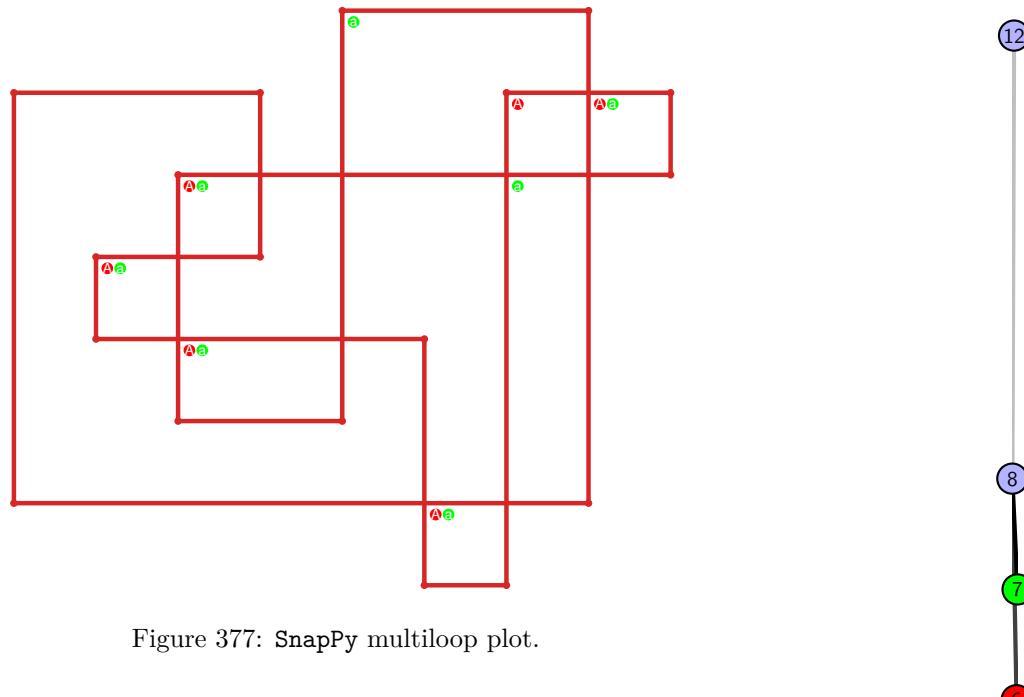


Figure 377: SnapPy multiloop plot.

Figure 378: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.41  $[[20, 7, 1, 8], [8, 13, 9, 14], [4, 19, 5, 20], [6, 17, 7, 18], [1, 12, 2, 13], [9, 15, 10, 14], [16, 3, 17, 4], [18, 5, 19, 6], [11, 2, 12, 3], [15, 11, 16, 10]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (14, 1, 15, 2), (16, 3, 17, 4), (18, 5, 19, 6), (8, 13, 9, 14), (19, 10, 20, 11), (6, 11, 7, 12), (12, 7, 13, 8), (4, 15, 5, 16), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 7], [0, 7, 7, 6], [0, 8, 8, 1], [1, 9, 9, 1], [2, 9, 8, 3], [2, 3, 3, 2], [4, 6, 9, 4], [5, 8, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 188: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

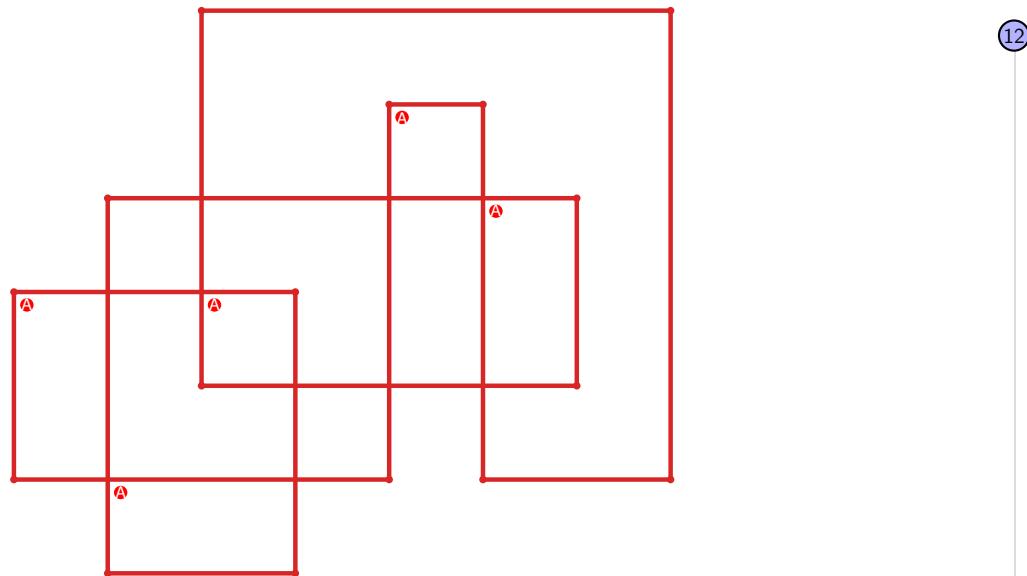


Figure 379: `SnapPy` multiloop plot.

5

Figure 380: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.42**  $[[20, 9, 1, 10], [10, 13, 11, 14], [8, 19, 9, 20], [1, 12, 2, 13], [11, 2, 12, 3], [14, 3, 15, 4], [16, 7, 17, 8], [18, 5, 19, 6], [15, 5, 16, 4], [6, 17, 7, 18]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (1, 12, 2, 13), (15, 2, 16, 3), (13, 4, 14, 5), (17, 6, 18, 7), (18, 9, 19, 10), (7, 10, 8, 11), (3, 14, 4, 15), (5, 16, 6, 17), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 7, 0], [0, 4, 4, 1], [1, 3, 3, 5], [1, 4, 8, 8], [2, 8, 9, 9], [2, 9, 9, 8], [5, 7, 6, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 189: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

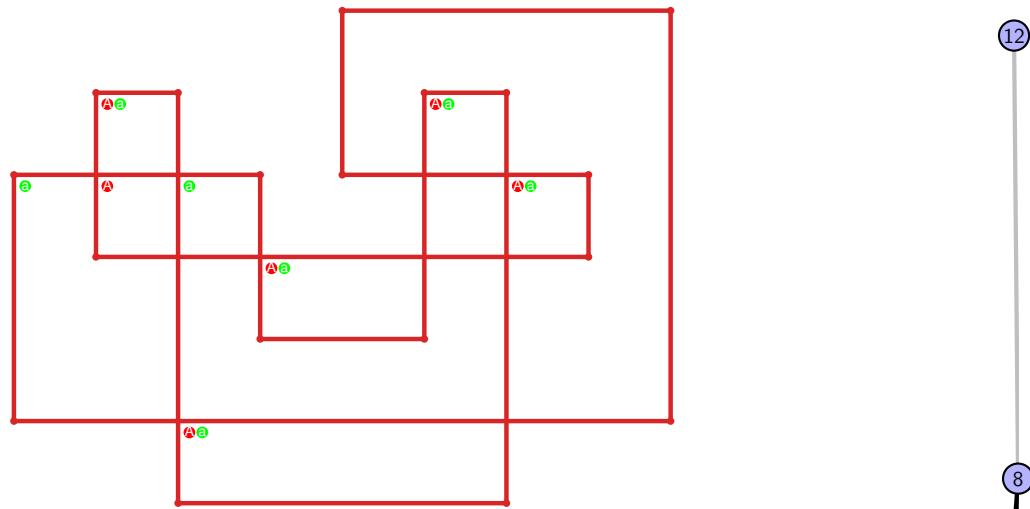


Figure 381: `SnapPy` multiloop plot.



Figure 382: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.43**  $[[20, 9, 1, 10], [10, 13, 11, 14], [8, 19, 9, 20], [1, 12, 2, 13], [11, 2, 12, 3], [14, 3, 15, 4], [18, 7, 19, 8], [15, 7, 16, 6], [4, 17, 5, 18], [16, 5, 17, 6]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (13, 2, 14, 3), (3, 12, 4, 13), (15, 4, 16, 5), (17, 6, 18, 7), (18, 9, 19, 10), (7, 10, 8, 11), (1, 14, 2, 15), (5, 16, 6, 17), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 6, 0], [0, 4, 4, 1], [1, 3, 3, 5], [1, 4, 7, 8], [2, 8, 7, 2], [5, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 190: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

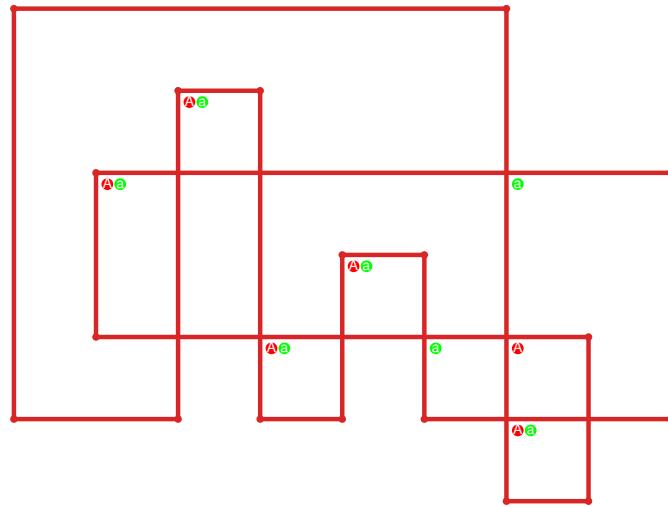


Figure 383: `SnapPy` multiloop plot.



Figure 384: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.44 [[20, 9, 1, 10], [10, 13, 11, 14], [6, 19, 7, 20], [8, 15, 9, 16], [1, 12, 2, 13], [11, 2, 12, 3], [14, 3, 15, 4], [18, 5, 19, 6], [7, 17, 8, 16], [4, 17, 5, 18]]

PD code drawn by `SnapPy`: [(9, 20, 10, 1), (12, 1, 13, 2), (16, 3, 17, 4), (18, 7, 19, 8), (19, 10, 20, 11), (8, 11, 9, 12), (6, 13, 7, 14), (14, 5, 15, 6), (2, 15, 3, 16), (4, 17, 5, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 7, 8], [0, 8, 8, 6], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 3, 9], [2, 9, 9, 2], [2, 9, 3, 3], [6, 8, 7, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 191: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

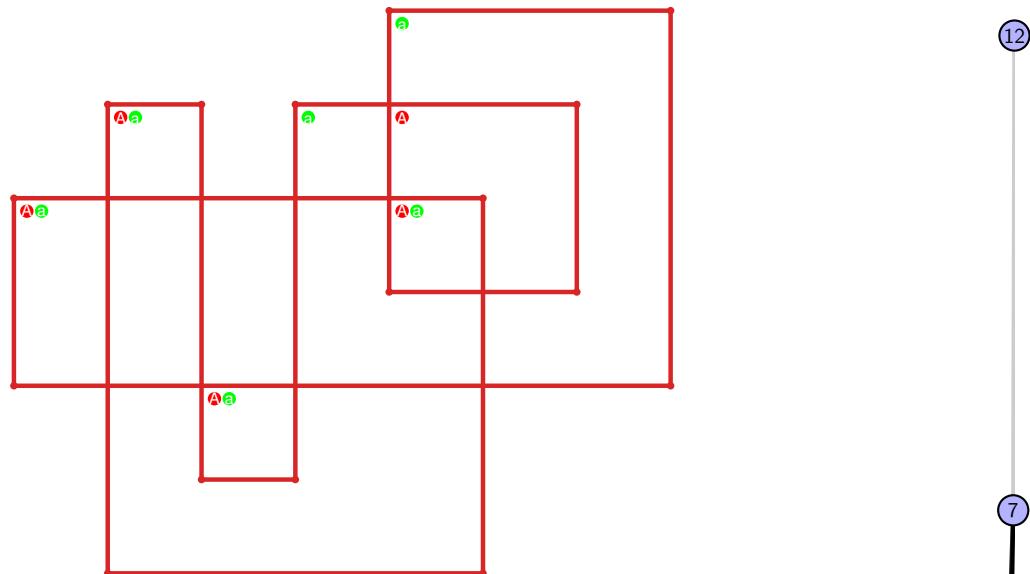


Figure 385: `SnapPy` multiloop plot.

Figure 386: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.45**  $[[13, 20, 14, 1], [12, 9, 13, 10], [19, 4, 20, 5], [14, 2, 15, 1], [10, 15, 11, 16], [16, 11, 17, 12], [17, 8, 18, 9], [5, 18, 6, 19], [6, 3, 7, 4], [2, 7, 3, 8]]$

PD code drawn by `SnapPy`:  $[(4, 1, 5, 2), (17, 2, 18, 3), (20, 5, 1, 6), (15, 6, 16, 7), (10, 7, 11, 8), (14, 9, 15, 10), (8, 13, 9, 14), (3, 16, 4, 17), (11, 18, 12, 19), (19, 12, 20, 13)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 6], [0, 7, 7, 8], [0, 9, 4, 0], [1, 3, 5, 5], [1, 4, 4, 6], [1, 5, 9, 7], [2, 6, 8, 2], [2, 7, 9, 9], [3, 8, 8, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 192: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

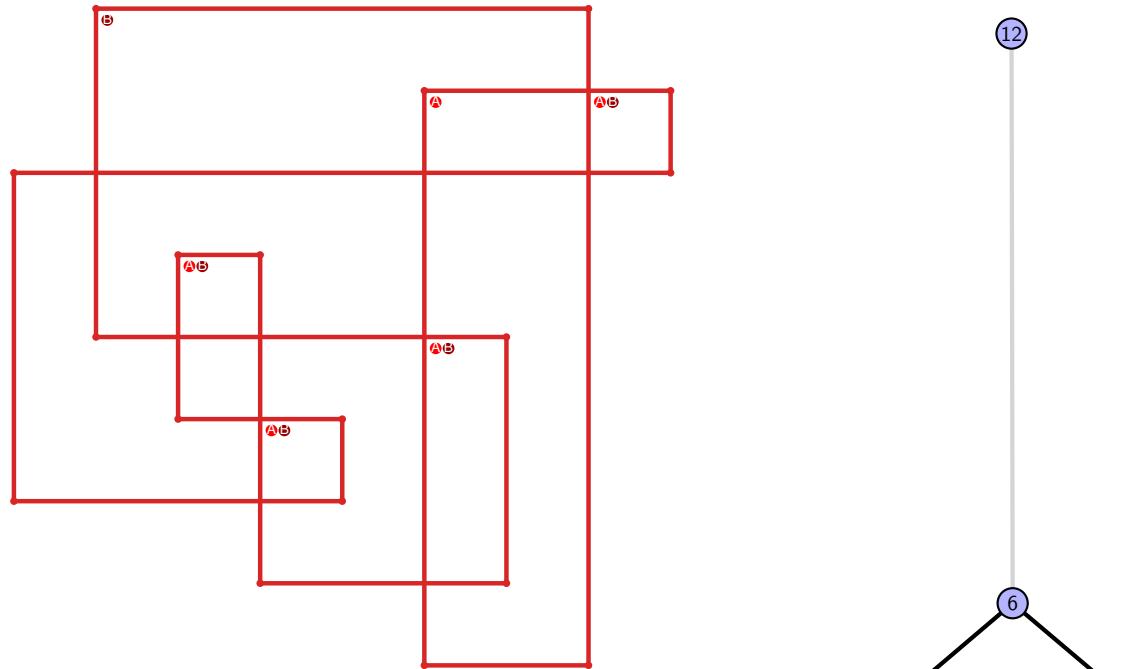
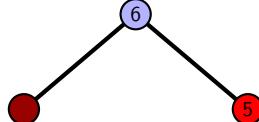


Figure 387: `SnapPy` multiloop plot.

Figure 388: Minimal join sub-semi-lattice of minimal pinning sets.



4.8.46  $[[20, 7, 1, 8], [8, 11, 9, 12], [6, 19, 7, 20], [1, 10, 2, 11], [9, 2, 10, 3], [12, 3, 13, 4], [16, 5, 17, 6], [18, 13, 19, 14], [4, 15, 5, 16], [17, 15, 18, 14]]$

PD code drawn by `SnapPy`:  $[(14, 1, 15, 2), (15, 4, 16, 5), (2, 5, 3, 6), (12, 7, 13, 8), (8, 11, 9, 12), (18, 9, 19, 10), (20, 13, 1, 14), (3, 16, 4, 17), (6, 17, 7, 18), (10, 19, 11, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 7, 0], [0, 4, 4, 1], [1, 3, 3, 5], [1, 4, 7, 8], [2, 8, 8, 9], [2, 9, 9, 5], [5, 9, 6, 6], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 193: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

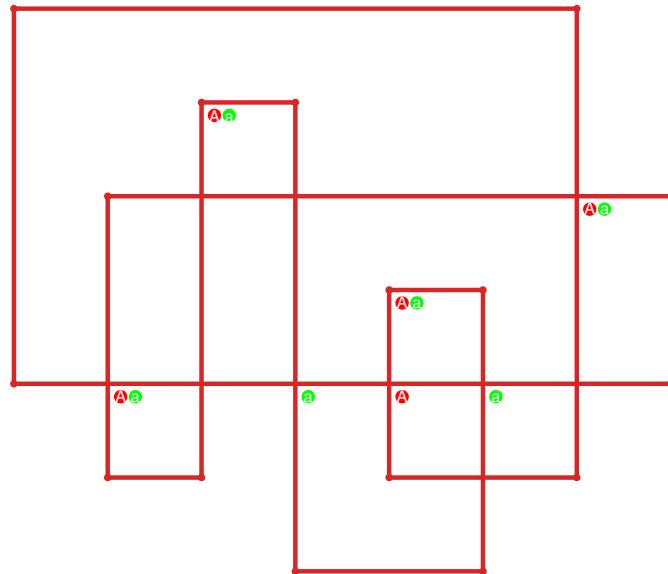


Figure 389: `SnapPy` multiloop plot.



Figure 390: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.47**  $[[5, 20, 6, 1], [4, 13, 5, 14], [19, 8, 20, 9], [6, 11, 7, 12], [1, 15, 2, 14], [16, 3, 17, 4], [17, 12, 18, 13], [9, 18, 10, 19], [10, 7, 11, 8], [15, 3, 16, 2]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (7, 4, 8, 5), (16, 5, 17, 6), (3, 8, 4, 9), (14, 9, 15, 10), (1, 10, 2, 11), (11, 18, 12, 19), (6, 15, 7, 16), (2, 17, 3, 18), (19, 12, 20, 13)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 7, 8], [0, 8, 8, 6], [0, 9, 9, 1], [1, 9, 9, 6], [1, 5, 3, 7], [2, 6, 8, 2], [2, 7, 3, 3], [4, 5, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 194: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

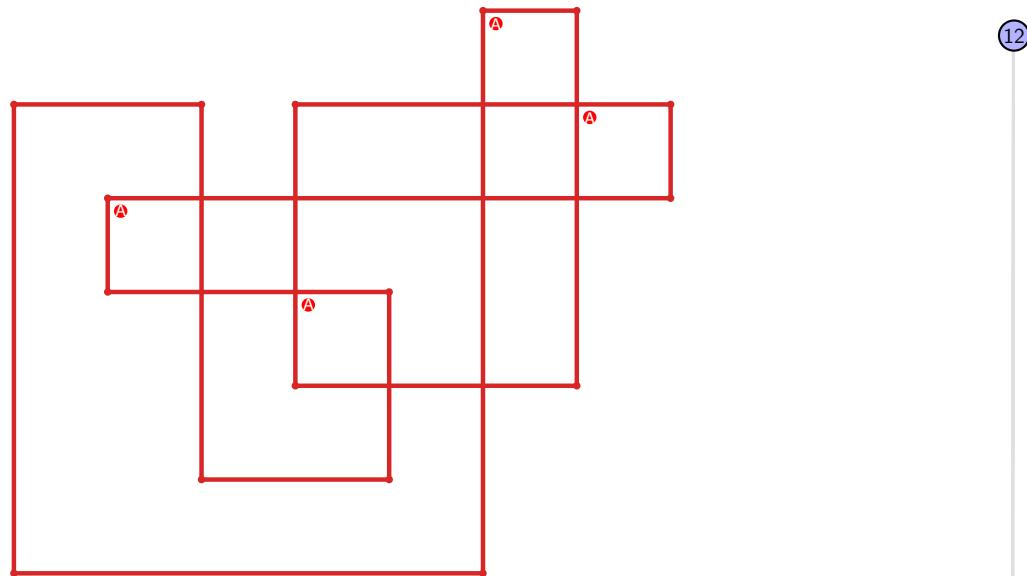


Figure 391: `SnapPy` multiloop plot.

Figure 392: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.48**  $[[20, 9, 1, 10], [10, 13, 11, 14], [4, 19, 5, 20], [8, 17, 9, 18], [1, 12, 2, 13], [11, 2, 12, 3], [14, 3, 15, 4], [18, 5, 19, 6], [16, 7, 17, 8], [15, 7, 16, 6]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (15, 2, 16, 3), (3, 12, 4, 13), (13, 4, 14, 5), (17, 6, 18, 7), (18, 9, 19, 10), (7, 10, 8, 11), (5, 14, 6, 15), (1, 16, 2, 17), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 7], [0, 7, 8, 8], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 9, 2], [2, 9, 3, 2], [3, 9, 9, 3], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 195: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

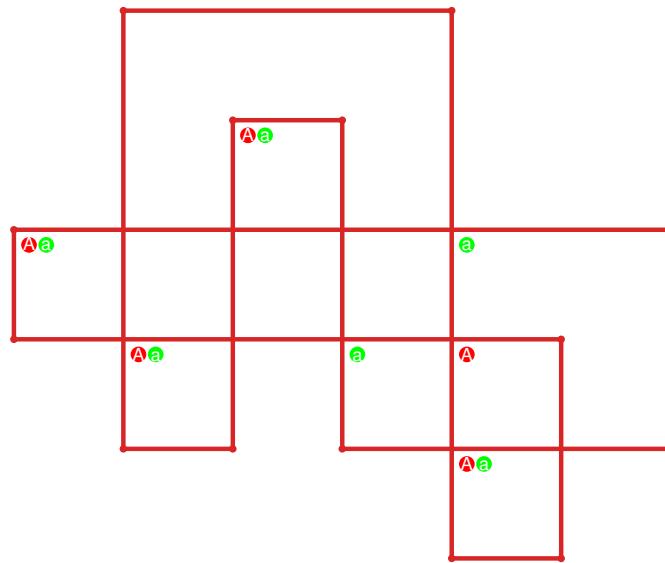


Figure 393: `SnapPy` multiloop plot.



Figure 394: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.49  $[[20, 7, 1, 8], [8, 11, 9, 12], [14, 19, 15, 20], [6, 17, 7, 18], [1, 10, 2, 11], [9, 2, 10, 3], [12, 3, 13, 4], [4, 13, 5, 14], [18, 15, 19, 16], [16, 5, 17, 6]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (13, 2, 14, 3), (17, 4, 18, 5), (18, 7, 19, 8), (5, 8, 6, 9), (1, 10, 2, 11), (15, 12, 16, 13), (3, 14, 4, 15), (11, 16, 12, 17), (6, 19, 7, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 8, 8], [0, 8, 9, 9], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 7, 7], [2, 6, 6, 9], [2, 9, 3, 2], [3, 8, 7, 3]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 196: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

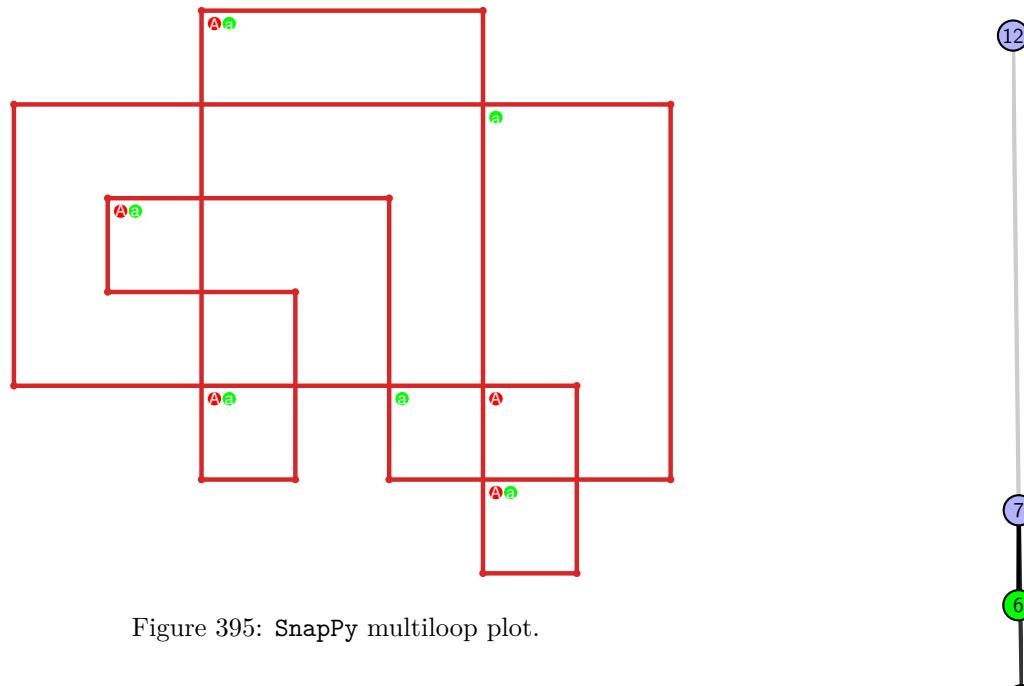


Figure 395: `SnapPy` multiloop plot.

Figure 396: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.50  $[[5, 20, 6, 1], [4, 9, 5, 10], [14, 19, 15, 20], [6, 18, 7, 17], [1, 11, 2, 10], [12, 3, 13, 4], [13, 8, 14, 9], [18, 15, 19, 16], [7, 16, 8, 17], [11, 3, 12, 2]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (10, 5, 11, 6), (1, 6, 2, 7), (7, 18, 8, 19), (15, 12, 16, 13), (4, 13, 5, 14), (14, 3, 15, 4), (11, 16, 12, 17), (2, 17, 3, 18), (19, 8, 20, 9)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 7], [0, 7, 8, 8], [0, 9, 9, 1], [1, 9, 9, 6], [1, 5, 8, 2], [2, 8, 3, 2], [3, 7, 6, 3], [4, 5, 5, 4]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 8  
 Total pinning sets: 218  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.47  
 Average overall degree: 2.99

Table 197: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	1	0	0	0	0	0	7
Nonminimal pinning sets	0	7	43	67	56	28	8	1	210
Average degree	2.4	2.55	2.8	2.99	3.11	3.2	3.27	3.33	

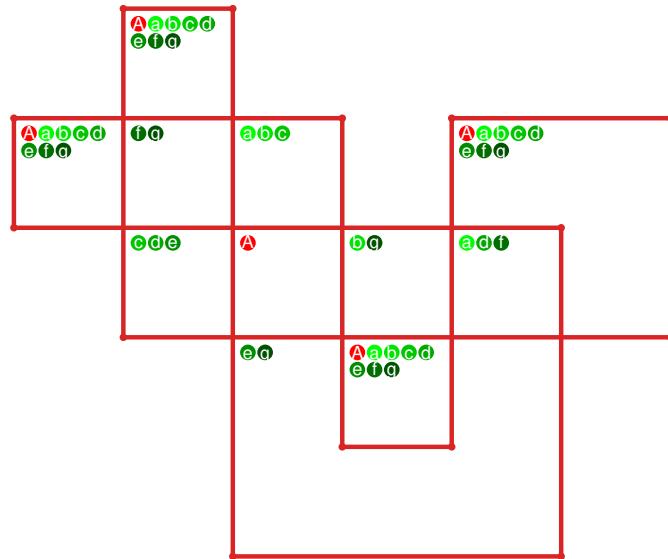


Figure 397: SnapPy multiloop plot.

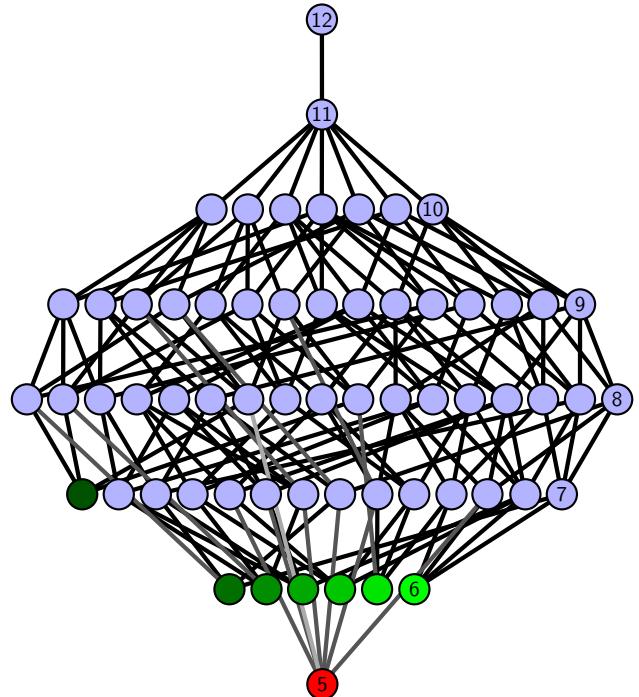


Figure 398: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.51  $[[3, 20, 4, 1], [11, 2, 12, 3], [14, 19, 15, 20], [4, 15, 5, 16], [1, 10, 2, 11], [12, 10, 13, 9], [13, 8, 14, 9], [18, 7, 19, 8], [5, 17, 6, 16], [6, 17, 7, 18]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (16, 5, 17, 6), (11, 6, 12, 7), (7, 10, 8, 11), (19, 8, 20, 9), (2, 13, 3, 14), (14, 3, 15, 4), (4, 15, 5, 16), (12, 17, 13, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 3], [0, 2, 8, 8], [0, 5, 1, 1], [1, 4, 6, 6], [2, 5, 5, 7], [2, 6, 9, 9], [3, 9, 9, 3], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 198: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

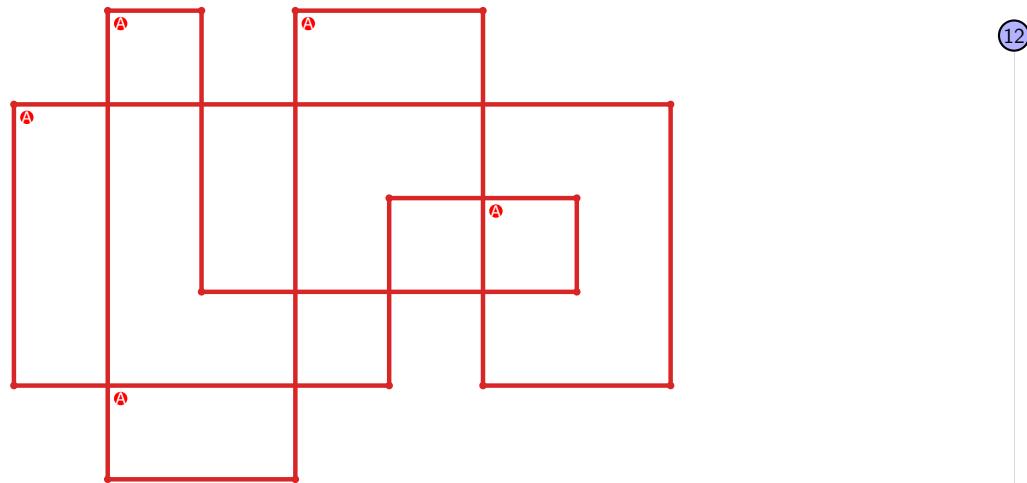


Figure 399: SnapPy multiloop plot.

5

Figure 400: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.52  $[[3, 20, 4, 1], [2, 11, 3, 12], [14, 19, 15, 20], [4, 15, 5, 16], [1, 13, 2, 12], [13, 10, 14, 11], [18, 9, 19, 10], [5, 9, 6, 8], [16, 8, 17, 7], [17, 6, 18, 7]]$

PD code drawn by SnapPy:  $[(15, 4, 16, 5), (5, 14, 6, 15), (6, 3, 7, 4), (16, 7, 17, 8), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (2, 13, 3, 14), (12, 17, 13, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 6, 2], [2, 5, 9, 7], [3, 6, 9, 8], [3, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 14  
 Total pinning sets: 418  
 Pinning number: 5

Average optimal degree: 2.8  
 Average minimal degree: 2.77  
 Average overall degree: 3.13

Table 199: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	10	1	0	0	0	0	0	11
Nonminimal pinning sets	0	21	94	134	101	43	10	1	404
Average degree	2.8	2.89	3.03	3.15	3.22	3.27	3.31	3.33	

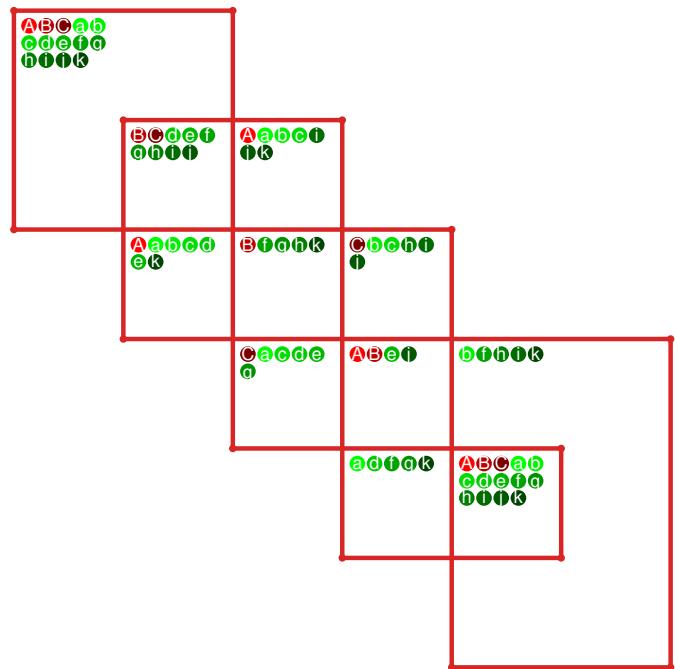


Figure 401: SnapPy multiloop plot.

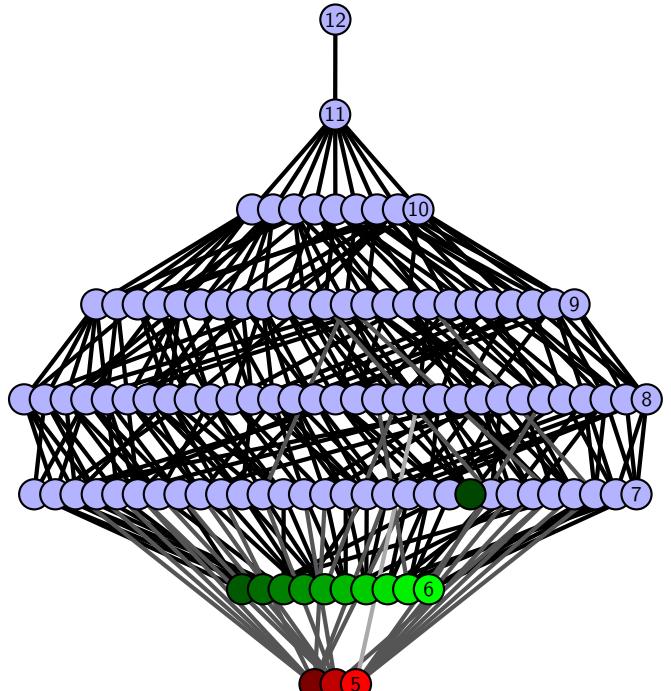


Figure 402: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.53  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 10, 20, 11], [4, 10, 5, 9], [1, 12, 2, 13], [14, 12, 15, 11], [15, 18, 16, 19], [5, 8, 6, 9], [6, 17, 7, 18], [16, 7, 17, 8]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (14, 5, 15, 6), (3, 6, 4, 7), (7, 2, 8, 3), (17, 8, 18, 9), (9, 12, 10, 13), (19, 10, 20, 11), (4, 15, 5, 16), (13, 16, 14, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 6, 2], [2, 5, 8, 9], [3, 9, 8, 3], [6, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 396  
**Pinning number:** 4

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.51  
**Average overall degree:** 3.06

Table 200: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	2	0	0	0	0	0	0	5
Nonminimal pinning sets	0	8	44	96	114	82	36	9	1	390
Average degree	2.5	2.65	2.83	2.98	3.1	3.18	3.24	3.29	3.33	

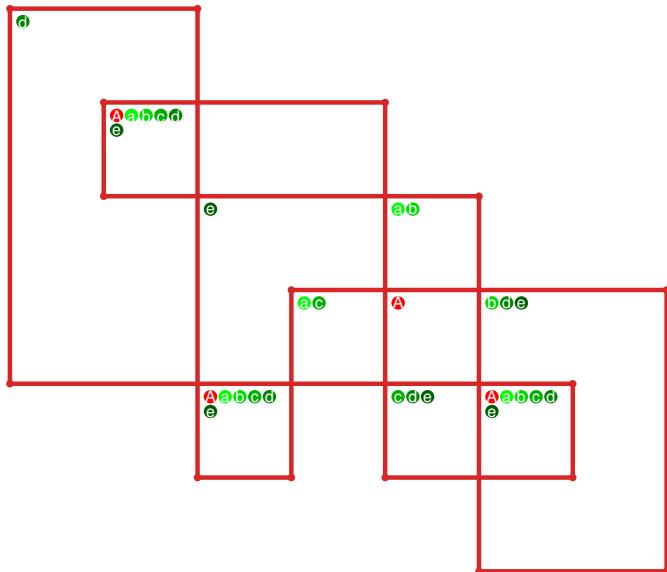


Figure 403: `SnapPy` multiloop plot.

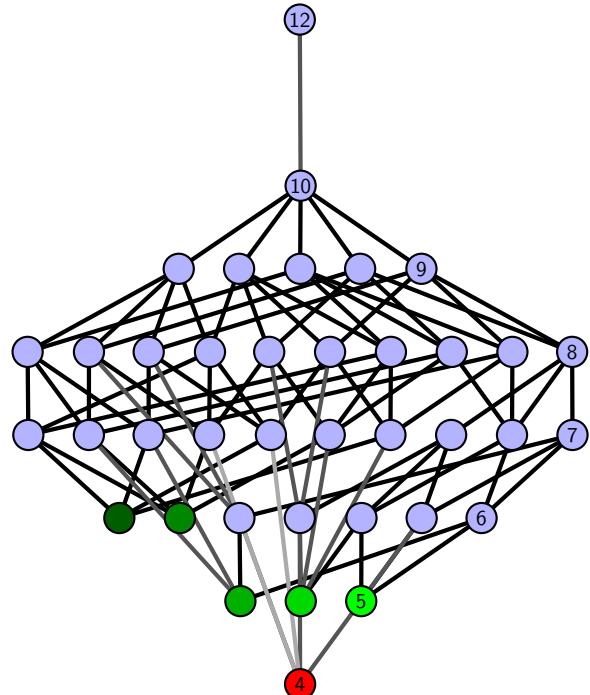


Figure 404: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.54  $[[3, 20, 4, 1], [2, 9, 3, 10], [12, 19, 13, 20], [4, 13, 5, 14], [1, 11, 2, 10], [11, 8, 12, 9], [18, 7, 19, 8], [5, 16, 6, 17], [14, 17, 15, 18], [15, 6, 16, 7]]$

PD code drawn by SnapPy:  $[(13, 4, 14, 5), (16, 5, 17, 6), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (2, 11, 3, 12), (3, 14, 4, 15), (12, 15, 13, 16), (10, 17, 11, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 6, 2], [2, 5, 8, 9], [3, 9, 9, 8], [3, 7, 9, 6], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.75

**Total minimal pinning sets:** 10

**Average minimal degree:** 2.66

**Total pinning sets:** 500

**Average overall degree:** 3.13

**Pinning number:** 4

Table 201: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	4	0	0	0	0	0	0	9
Nonminimal pinning sets	0	8	54	124	147	103	43	10	1	490
Average degree	2.75	2.82	2.94	3.06	3.16	3.23	3.27	3.31	3.33	

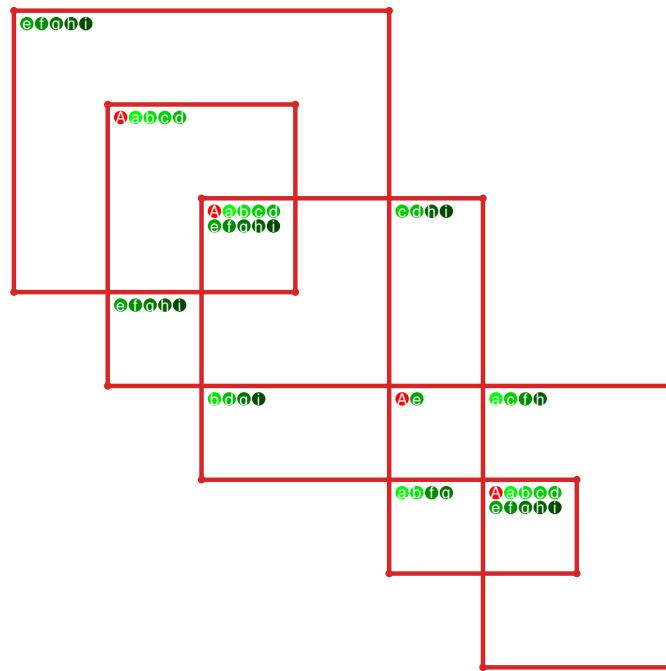


Figure 405: SnapPy multiloop plot.

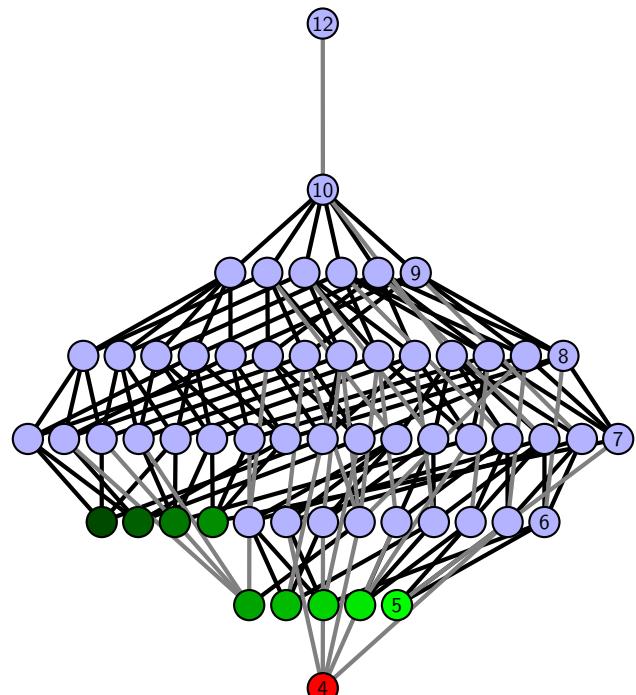


Figure 406: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.55  $[[3, 20, 4, 1], [2, 15, 3, 16], [19, 8, 20, 9], [4, 8, 5, 7], [1, 17, 2, 16], [17, 14, 18, 15], [9, 18, 10, 19], [5, 13, 6, 12], [6, 11, 7, 12], [13, 10, 14, 11]]$

PD code drawn by SnapPy:  $[(3, 20, 4, 1), (15, 4, 16, 5), (9, 6, 10, 7), (2, 7, 3, 8), (8, 1, 9, 2), (13, 10, 14, 11), (18, 11, 19, 12), (12, 17, 13, 18), (5, 14, 6, 15), (19, 16, 20, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 9, 6], [2, 5, 9, 2], [3, 9, 8, 8], [3, 7, 7, 9], [5, 8, 7, 6]]$

Total optimal pinning sets: 5  
 Total minimal pinning sets: 9  
 Total pinning sets: 505  
 Pinning number: 4

Average optimal degree: 2.6  
 Average minimal degree: 2.51  
 Average overall degree: 3.05

Table 202: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	0	4
Nonminimal pinning sets	0	30	84	126	126	84	36	9	1	496
Average degree	2.6	2.73	2.89	3.02	3.11	3.19	3.24	3.29	3.33	

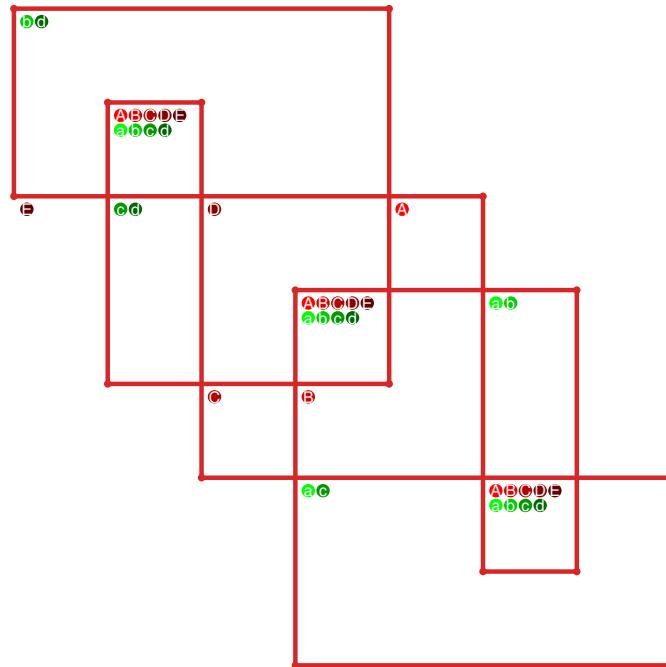


Figure 407: SnapPy multiloop plot.

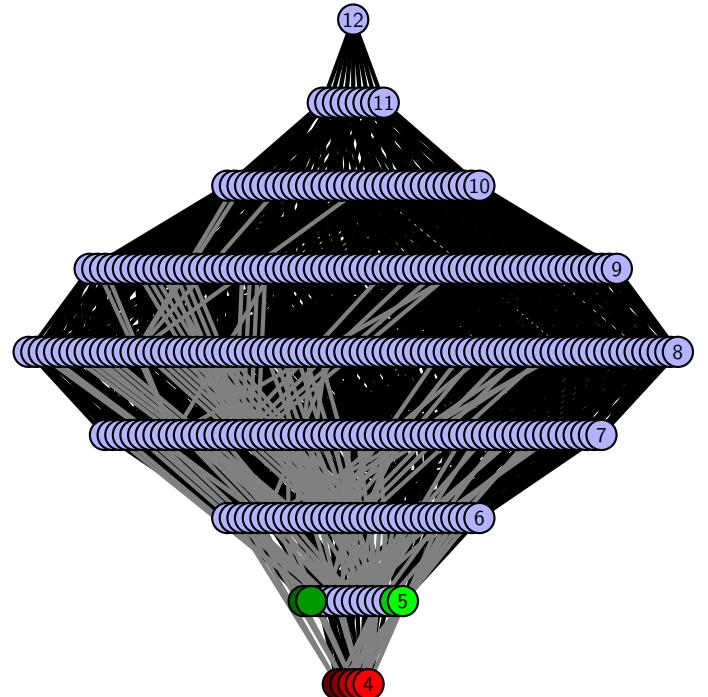


Figure 408: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.56**  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 4, 20, 5], [1, 12, 2, 13], [14, 12, 15, 11], [5, 18, 6, 19], [15, 9, 16, 8], [17, 10, 18, 11], [6, 10, 7, 9], [16, 7, 17, 8]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (17, 2, 18, 3), (14, 5, 15, 6), (6, 13, 7, 14), (7, 4, 8, 5), (15, 8, 16, 9), (9, 12, 10, 13), (19, 10, 20, 11), (3, 16, 4, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 5, 0], [0, 4, 1, 1], [1, 3, 6, 7], [2, 7, 8, 2], [4, 8, 9, 9], [4, 9, 8, 5], [5, 7, 9, 6], [6, 8, 7, 6]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 160  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.97

Table 203: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

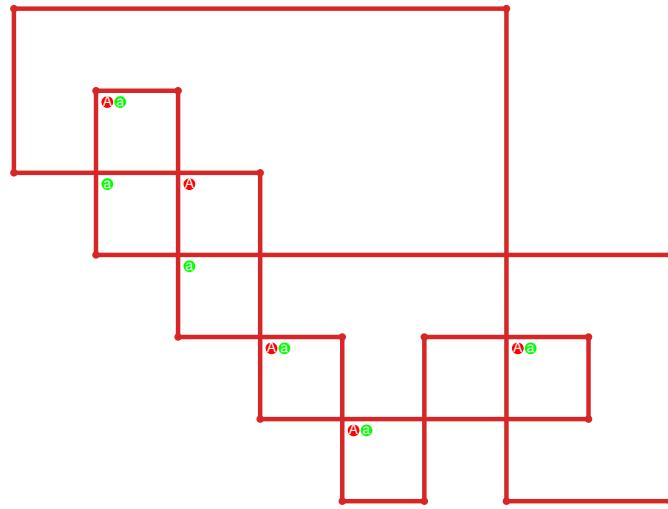


Figure 409: `SnapPy` multiloop plot.



Figure 410: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.57**  $[[3, 20, 4, 1], [11, 2, 12, 3], [19, 4, 20, 5], [1, 10, 2, 11], [12, 10, 13, 9], [5, 14, 6, 15], [15, 18, 16, 19], [13, 8, 14, 9], [6, 17, 7, 18], [16, 7, 17, 8]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (17, 2, 18, 3), (14, 5, 15, 6), (11, 6, 12, 7), (7, 10, 8, 11), (19, 8, 20, 9), (3, 12, 4, 13), (4, 15, 5, 16), (13, 16, 14, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 7], [2, 7, 8, 6], [2, 5, 8, 9], [4, 9, 5, 4], [5, 9, 9, 6], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 204: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

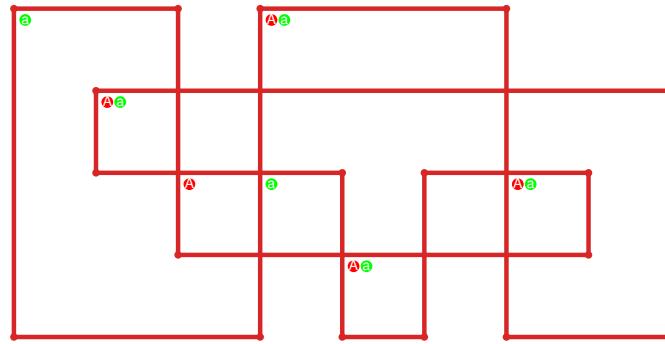


Figure 411: `SnapPy` multiloop plot.



Figure 412: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.58**  $[[5, 20, 6, 1], [15, 4, 16, 5], [19, 10, 20, 11], [6, 10, 7, 9], [1, 14, 2, 15], [3, 12, 4, 13], [16, 12, 17, 11], [18, 7, 19, 8], [8, 17, 9, 18], [13, 2, 14, 3]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (15, 4, 16, 5), (6, 3, 7, 4), (16, 7, 17, 8), (8, 13, 9, 14), (20, 9, 1, 10), (18, 11, 19, 12), (5, 14, 6, 15), (2, 17, 3, 18), (10, 19, 11, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 3], [0, 2, 7, 8], [0, 9, 9, 1], [1, 9, 9, 6], [1, 5, 8, 2], [2, 8, 8, 3], [3, 7, 7, 6], [4, 5, 5, 4]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 8  
 Total pinning sets: 276  
 Pinning number: 5

Average optimal degree: 2.5  
 Average minimal degree: 2.56  
 Average overall degree: 3.05

Table 205: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	6	0	0	0	0	0	0	6
Nonminimal pinning sets	0	13	57	84	70	34	9	1	268
Average degree	2.5	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

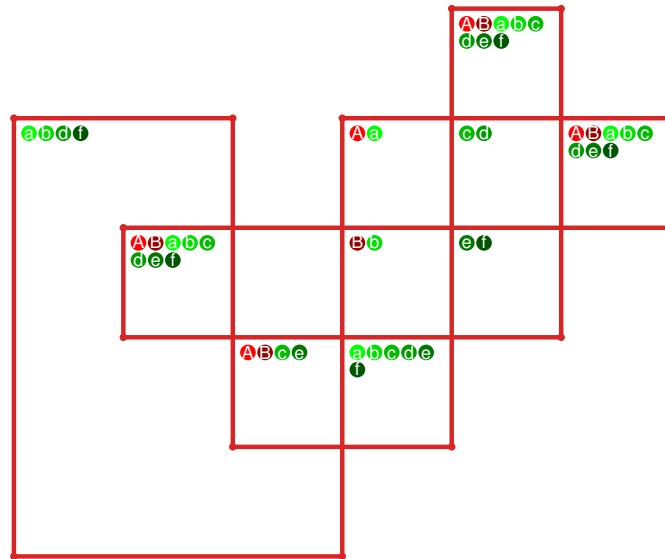


Figure 413: `SnapPy` multiloop plot.

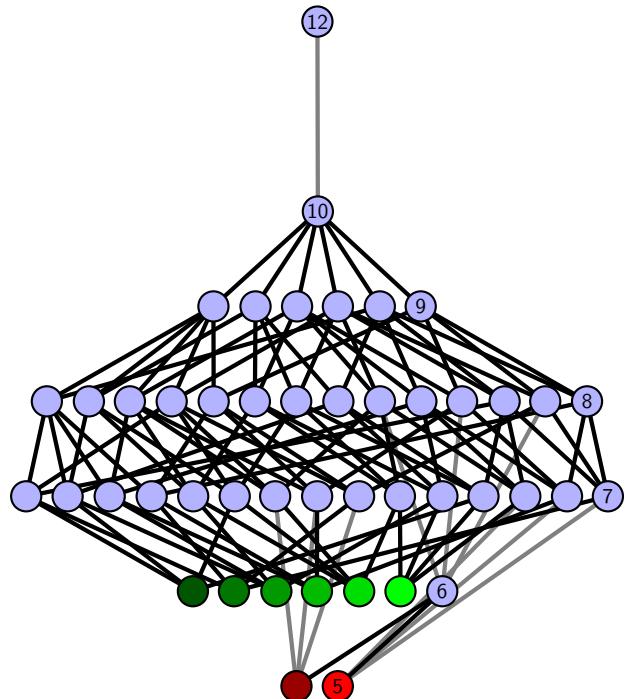


Figure 414: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.59**  $[[7, 20, 8, 1], [6, 17, 7, 18], [19, 8, 20, 9], [1, 19, 2, 18], [10, 5, 11, 6], [11, 16, 12, 17], [9, 3, 10, 2], [4, 13, 5, 14], [15, 12, 16, 13], [3, 15, 4, 14]]$

PD code drawn by `SnapPy`:  $[(7, 2, 8, 3), (16, 5, 17, 6), (12, 9, 13, 10), (1, 10, 2, 11), (11, 20, 12, 1), (8, 13, 9, 14), (3, 14, 4, 15), (15, 18, 16, 19), (4, 17, 5, 18), (19, 6, 20, 7)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 3, 0], [0, 2, 6, 1], [1, 6, 7, 5], [1, 4, 8, 8], [2, 9, 4, 3], [4, 9, 9, 8], [5, 7, 9, 5], [6, 8, 7, 7]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 8  
 Total pinning sets: 256  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.59  
 Average overall degree: 3.05

Table 206: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	0	7
Nonminimal pinning sets	0	7	49	79	69	34	9	1	248
Average degree	2.4	2.64	2.87	3.04	3.15	3.24	3.29	3.33	

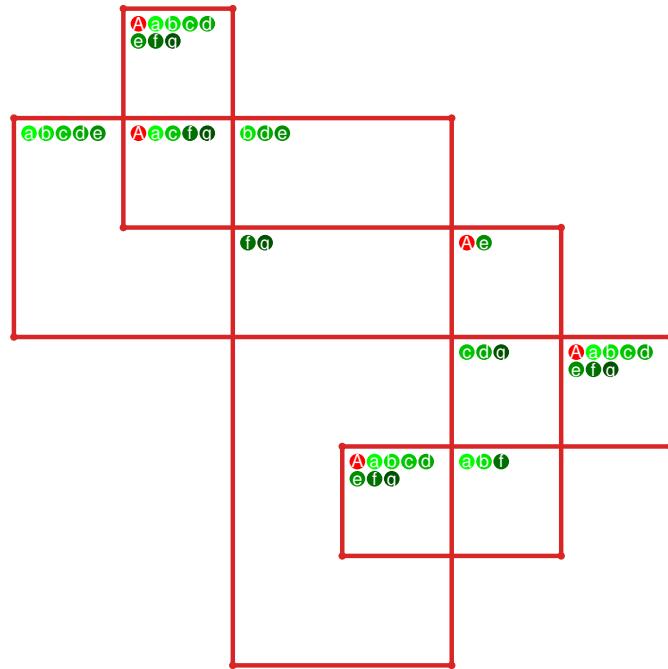


Figure 415: `SnapPy` multiloop plot.

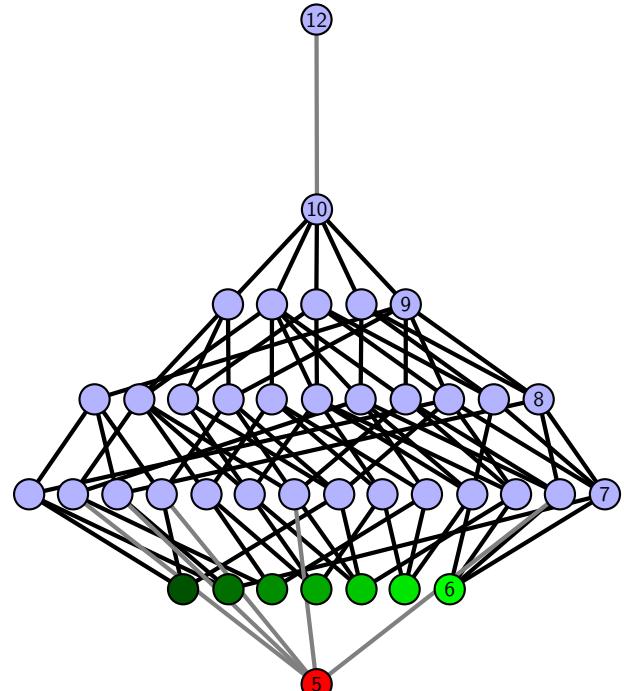


Figure 416: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.60  $[[20, 9, 1, 10], [10, 18, 11, 17], [8, 19, 9, 20], [1, 19, 2, 18], [11, 7, 12, 6], [16, 5, 17, 6], [7, 2, 8, 3], [12, 15, 13, 16], [13, 4, 14, 5], [3, 14, 4, 15]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (13, 2, 14, 3), (3, 8, 4, 9), (17, 6, 18, 7), (20, 11, 1, 12), (9, 12, 10, 13), (14, 7, 15, 8), (15, 18, 16, 19), (5, 16, 6, 17), (4, 19, 5, 20)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 3, 0], [0, 2, 6, 1], [1, 6, 7, 5], [1, 4, 7, 8], [2, 9, 4, 3], [4, 9, 8, 5], [5, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 11  
Total minimal pinning sets: 12

Total pinning sets: 286

Pinning number: 6

Average optimal degree: 2.73

Average minimal degree: 2.74

Average overall degree: 3.11

Table 207: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	11	0	0	0	0	0	0	11
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	50	91	82	40	10	1	274
Average degree	2.73	2.93	3.09	3.19	3.26	3.31	3.33	

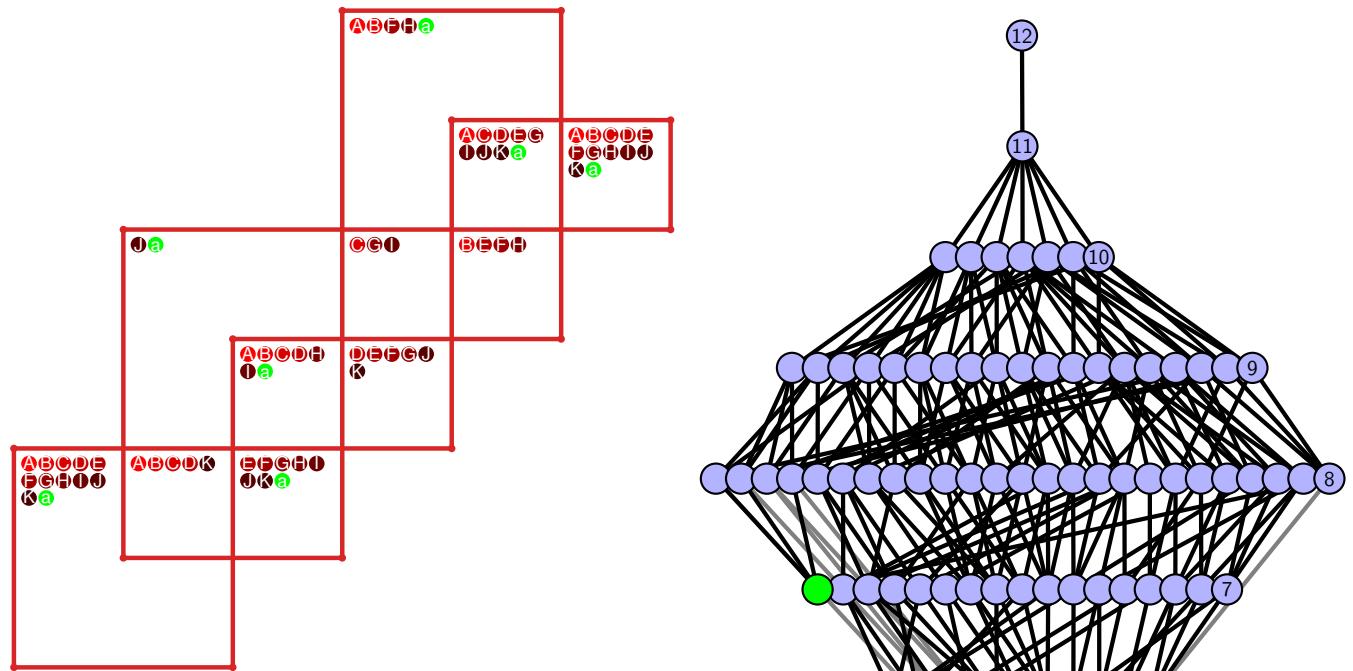


Figure 417: SnapPy multiloop plot.

Figure 418: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.61 [[9, 20, 10, 1], [13, 8, 14, 9], [19, 4, 20, 5], [10, 4, 11, 3], [1, 12, 2, 13], [7, 18, 8, 19], [14, 18, 15, 17], [5, 17, 6, 16], [11, 2, 12, 3], [6, 15, 7, 16]]

PD code drawn by `SnapPy`: [(9, 20, 10, 1), (1, 8, 2, 9), (17, 2, 18, 3), (11, 6, 12, 7), (19, 10, 20, 11), (4, 13, 5, 14), (14, 3, 15, 4), (15, 12, 16, 13), (5, 16, 6, 17), (7, 18, 8, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 5, 3], [0, 2, 8, 8], [0, 8, 8, 1], [1, 2, 9, 6], [1, 5, 9, 7], [2, 6, 9, 9], [3, 4, 4, 3], [5, 7, 7, 6]]

**Total optimal pinning sets:** 6  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 312  
**Pinning number:** 5

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.5  
**Average overall degree:** 3.04

Table 208: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	31	70	90	71	34	9	1	306
Average degree	2.5	2.76	2.93	3.06	3.16	3.24	3.29	3.33	

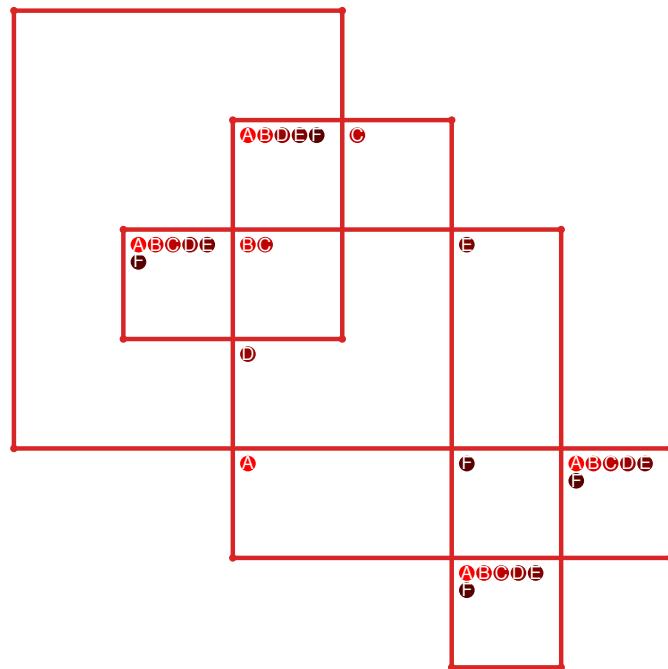


Figure 419: `SnapPy` multiloop plot.

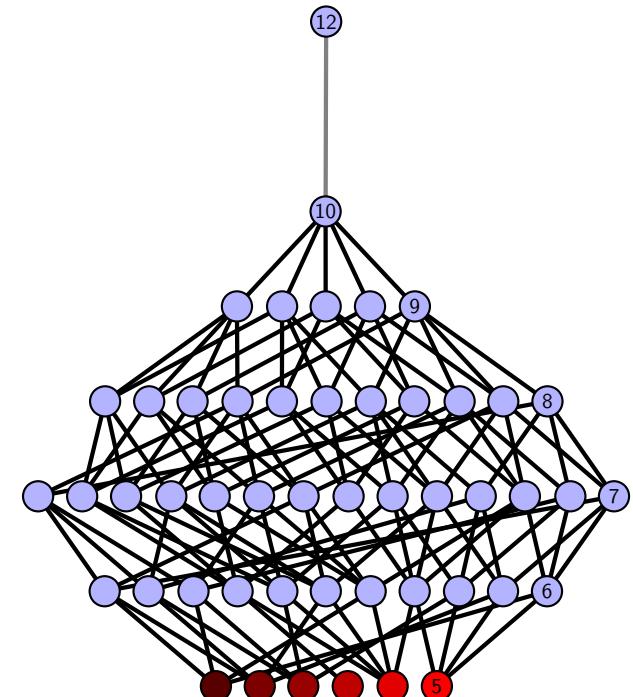


Figure 420: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.62  $[[20, 15, 1, 16], [16, 13, 17, 14], [14, 19, 15, 20], [1, 12, 2, 13], [17, 9, 18, 8], [18, 7, 19, 8], [11, 6, 12, 7], [2, 6, 3, 5], [9, 5, 10, 4], [10, 3, 11, 4]]$

PD code drawn by SnapPy:  $[(20, 5, 1, 6), (1, 18, 2, 19), (7, 2, 8, 3), (14, 3, 15, 4), (4, 13, 5, 14), (17, 8, 18, 9), (12, 9, 13, 10), (16, 11, 17, 12), (10, 15, 11, 16), (6, 19, 7, 20)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 2], [0, 1, 5, 0], [0, 6, 7, 1], [1, 8, 5, 5], [2, 4, 4, 6], [3, 5, 9, 7], [3, 6, 9, 8], [4, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.45

**Total pinning sets:** 240

**Average overall degree:** 3.03

**Pinning number:** 5

Table 209: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	13	45	71	64	33	9	1	236
Average degree	2.4	2.66	2.86	3.02	3.14	3.23	3.29	3.33	

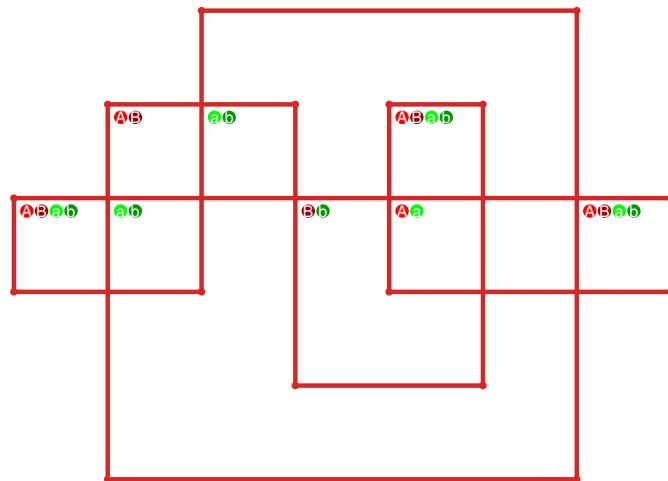


Figure 421: SnapPy multiloop plot.

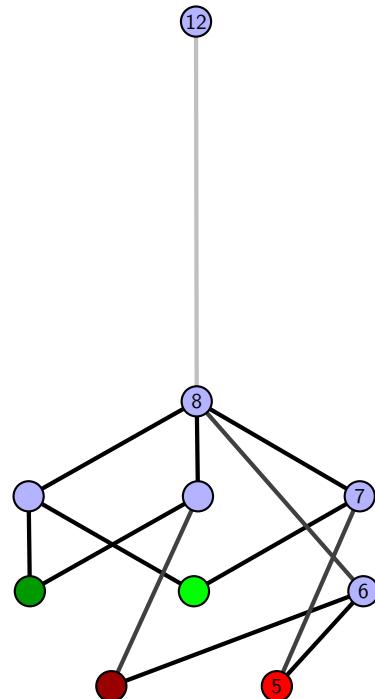


Figure 422: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.63 [[20, 11, 1, 12], [12, 9, 13, 10], [10, 19, 11, 20], [1, 8, 2, 9], [13, 18, 14, 19], [14, 7, 15, 8], [2, 6, 3, 5], [17, 4, 18, 5], [6, 15, 7, 16], [3, 16, 4, 17]]

PD code drawn by SnapPy: [(13, 20, 14, 1), (1, 18, 2, 19), (10, 3, 11, 4), (17, 4, 18, 5), (8, 5, 9, 6), (16, 7, 17, 8), (2, 11, 3, 12), (19, 12, 20, 13), (9, 14, 10, 15), (6, 15, 7, 16)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 3, 4, 2], [0, 1, 4, 0], [0, 5, 6, 1], [1, 7, 5, 2], [3, 4, 8, 8], [3, 8, 9, 7], [4, 6, 9, 9], [5, 9, 6, 5], [6, 8, 7, 7]]

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 240  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.45  
**Average overall degree:** 3.03

Table 210: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	13	45	71	64	33	9	1	236
Average degree	2.4	2.66	2.86	3.02	3.14	3.23	3.29	3.33	

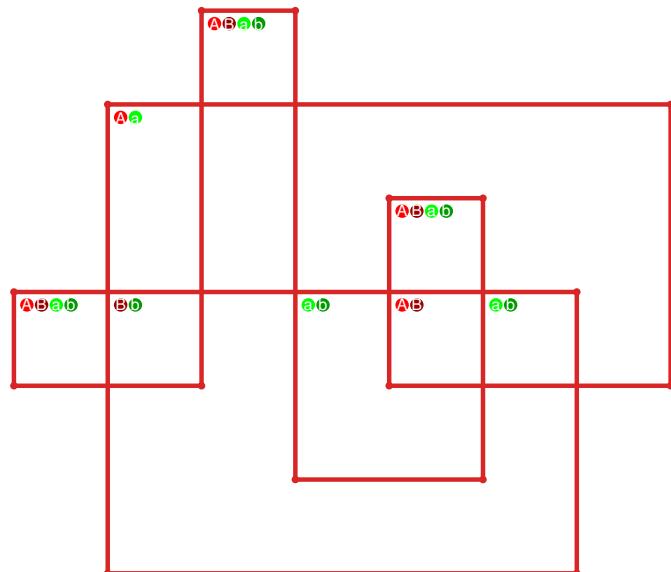


Figure 423: SnapPy multiloop plot.

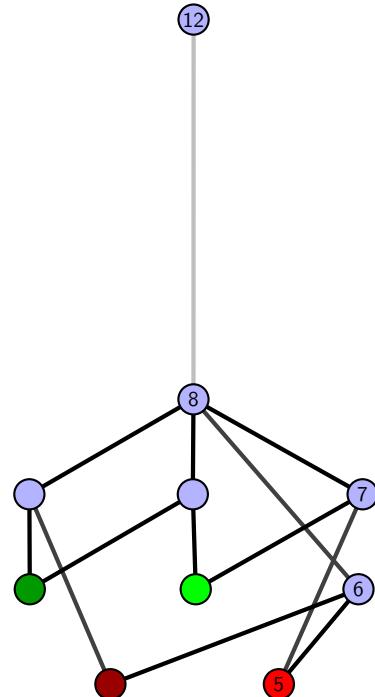


Figure 424: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.64  $[[11, 20, 12, 1], [10, 13, 11, 14], [19, 12, 20, 13], [1, 15, 2, 14], [18, 9, 19, 10], [15, 9, 16, 8], [2, 8, 3, 7], [4, 17, 5, 18], [16, 5, 17, 6], [3, 6, 4, 7]]$

PD code drawn by `SnapPy`:  $[(14, 3, 15, 4), (1, 4, 2, 5), (5, 20, 6, 1), (6, 13, 7, 14), (16, 7, 17, 8), (17, 10, 18, 11), (8, 11, 9, 12), (12, 19, 13, 20), (2, 15, 3, 16), (9, 18, 10, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 2], [0, 1, 4, 0], [0, 5, 6, 1], [1, 7, 5, 2], [3, 4, 8, 6], [3, 5, 9, 9], [4, 9, 8, 8], [5, 7, 7, 9], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 280  
**Pinning number:** 5

**Average optimal degree:** 2.47  
**Average minimal degree:** 2.51  
**Average overall degree:** 3.05

Table 211: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	18	58	84	70	34	9	1	274
Average degree	2.47	2.71	2.9	3.05	3.16	3.24	3.29	3.33	

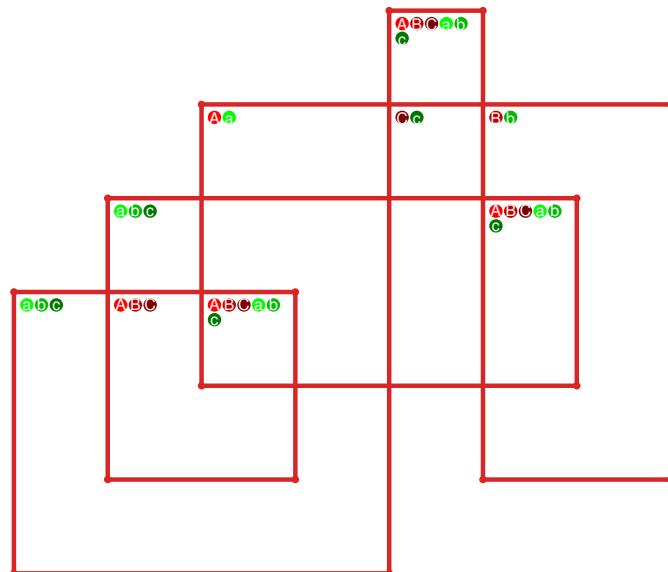


Figure 425: `SnapPy` multiloop plot.

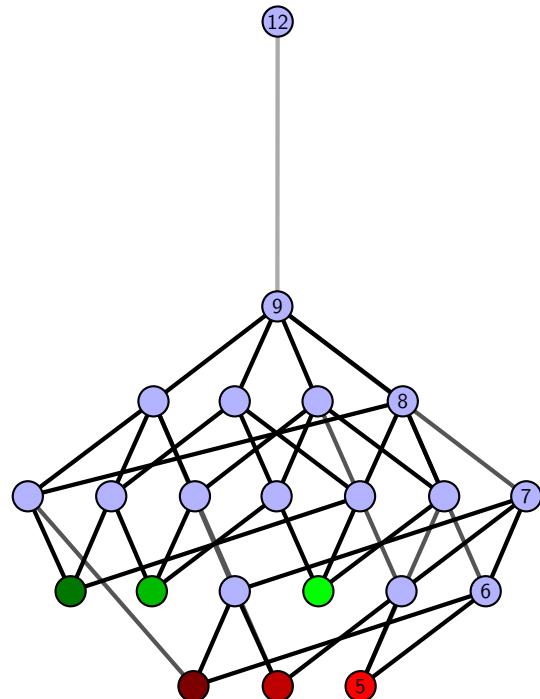


Figure 426: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.65**  $[[20, 11, 1, 12], [12, 9, 13, 10], [10, 19, 11, 20], [1, 8, 2, 9], [13, 18, 14, 19], [14, 7, 15, 8], [2, 15, 3, 16], [17, 4, 18, 5], [6, 3, 7, 4], [16, 6, 17, 5]]$

PD code drawn by `SnapPy`:  $[(4, 1, 5, 2), (15, 2, 16, 3), (3, 14, 4, 15), (12, 5, 13, 6), (19, 6, 20, 7), (10, 7, 11, 8), (18, 9, 19, 10), (20, 13, 1, 14), (11, 16, 12, 17), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 2], [0, 1, 4, 0], [0, 5, 6, 1], [1, 7, 5, 2], [3, 4, 8, 6], [3, 5, 8, 9], [4, 9, 9, 8], [5, 7, 9, 6], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 6

**Total pinning sets:** 320

**Pinning number:** 5

**Average optimal degree:** 2.6  
**Average minimal degree:** 2.63

**Average overall degree:** 3.1

Table 212: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	19	64	97	83	40	10	1	314
Average degree	2.6	2.8	2.97	3.1	3.2	3.27	3.31	3.33	

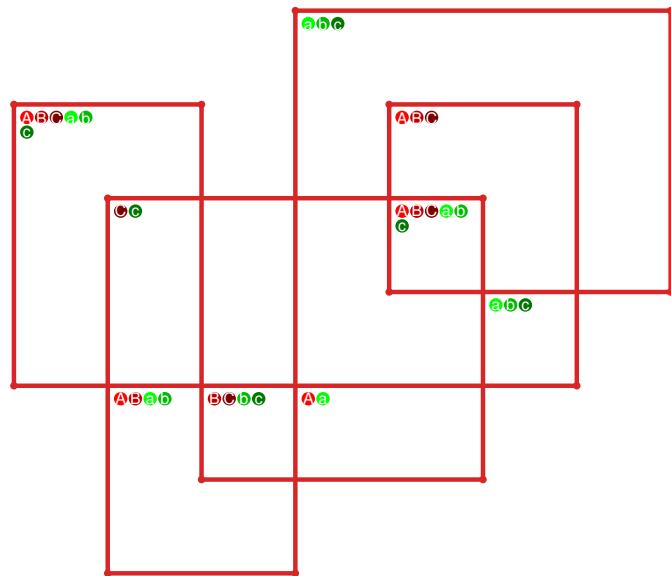


Figure 427: `SnapPy` multiloop plot.

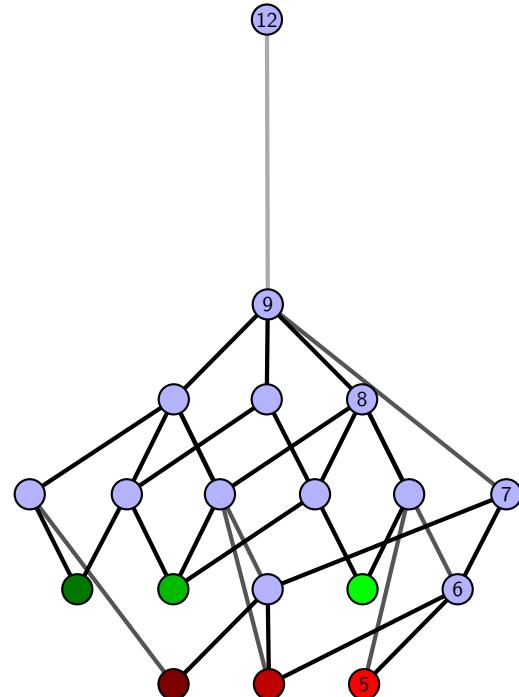


Figure 428: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.66  $[[20, 11, 1, 12], [12, 9, 13, 10], [10, 19, 11, 20], [1, 15, 2, 14], [8, 13, 9, 14], [18, 7, 19, 8], [15, 7, 16, 6], [2, 6, 3, 5], [17, 4, 18, 5], [16, 4, 17, 3]]$

PD code drawn by SnapPy:  $[(1, 14, 2, 15), (15, 2, 16, 3), (3, 20, 4, 1), (4, 13, 5, 14), (16, 5, 17, 6), (10, 7, 11, 8), (18, 9, 19, 10), (6, 11, 7, 12), (12, 19, 13, 20), (8, 17, 9, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 4, 2], [0, 1, 5, 0], [0, 6, 7, 4], [1, 3, 5, 1], [2, 4, 8, 6], [3, 5, 9, 7], [3, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 320  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.33  
**Average overall degree:** 3.03

Table 213: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

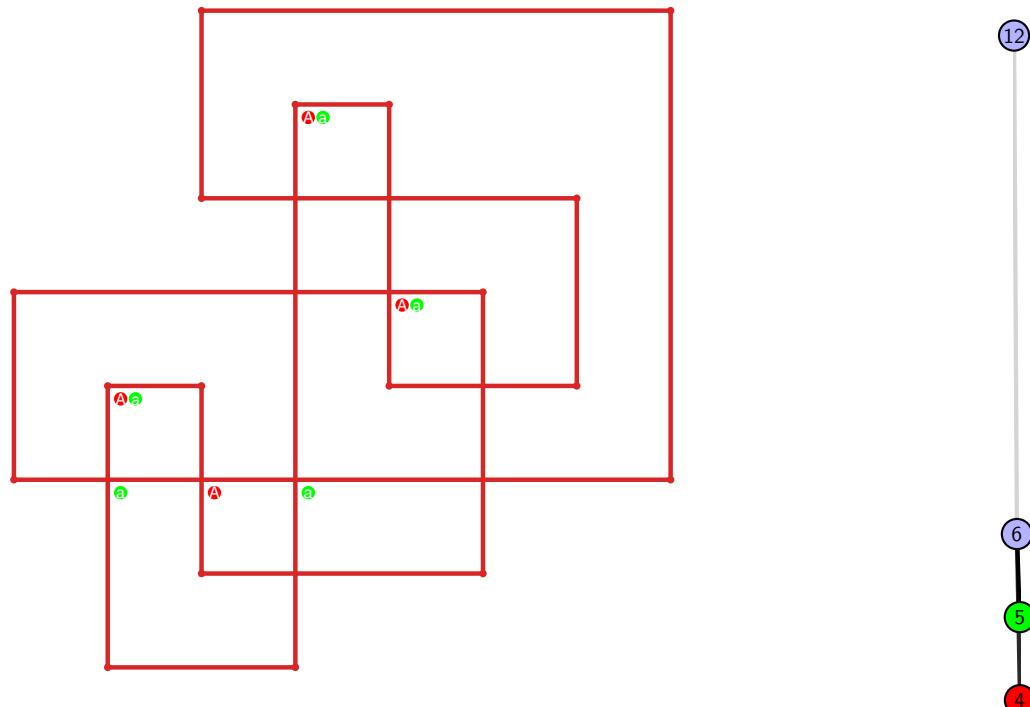


Figure 429: SnapPy multiloop plot.

Figure 430: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.67  $[[20, 7, 1, 8], [8, 19, 9, 20], [6, 17, 7, 18], [1, 11, 2, 10], [18, 9, 19, 10], [5, 14, 6, 15], [16, 3, 17, 4], [11, 3, 12, 2], [15, 12, 16, 13], [13, 4, 14, 5]]$

PD code drawn by SnapPy:  $[(20, 9, 1, 10), (14, 1, 15, 2), (11, 2, 12, 3), (18, 5, 19, 6), (16, 7, 17, 8), (8, 19, 9, 20), (13, 10, 14, 11), (3, 12, 4, 13), (4, 15, 5, 16), (6, 17, 7, 18)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 6], [0, 7, 7, 4], [1, 3, 2, 1], [2, 8, 9, 9], [2, 9, 8, 7], [3, 6, 8, 3], [5, 7, 6, 9], [5, 8, 6, 5]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2

**Total pinning sets:** 160

**Pinning number:** 5

**Average optimal degree:** 2.2  
**Average minimal degree:** 2.27

**Average overall degree:** 2.97

Table 214: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

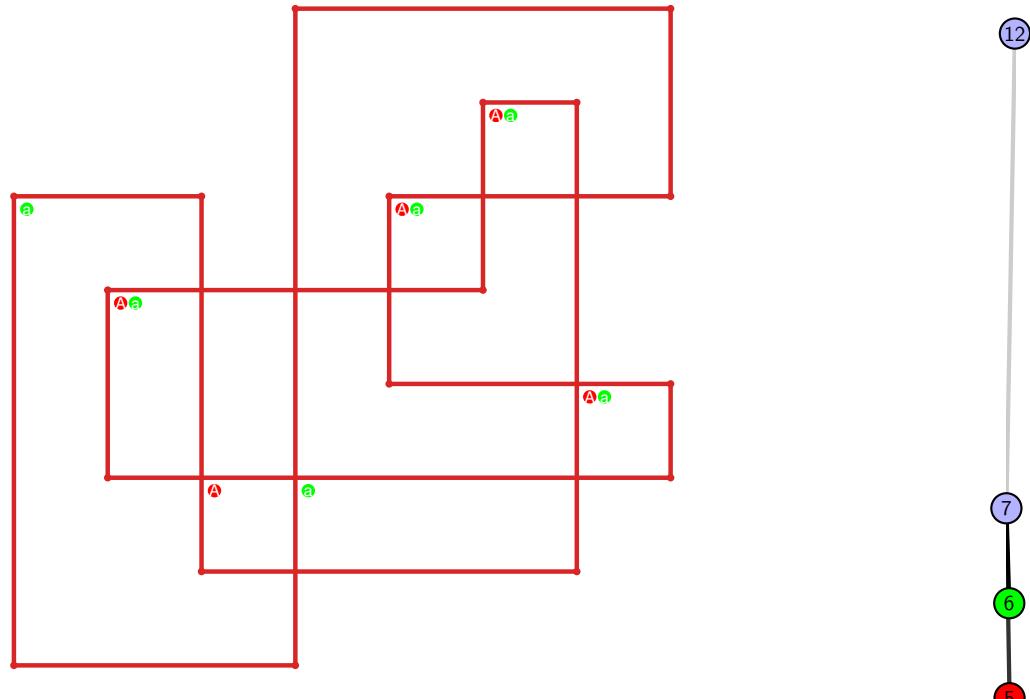


Figure 431: SnapPy multiloop plot.



Figure 432: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.68**  $[[20, 15, 1, 16], [16, 19, 17, 20], [14, 7, 15, 8], [1, 11, 2, 10], [18, 9, 19, 10], [17, 9, 18, 8], [13, 4, 14, 5], [6, 11, 7, 12], [2, 6, 3, 5], [3, 12, 4, 13]]$

PD code drawn by `SnapPy`:  $[(11, 2, 12, 3), (3, 20, 4, 1), (12, 5, 13, 6), (6, 15, 7, 16), (16, 7, 17, 8), (8, 19, 9, 20), (4, 9, 5, 10), (1, 10, 2, 11), (18, 13, 19, 14), (14, 17, 15, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 5, 0], [0, 5, 6, 7], [0, 7, 8, 4], [1, 3, 5, 5], [1, 4, 4, 2], [2, 8, 9, 9], [2, 9, 8, 3], [3, 7, 9, 6], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 5  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 304  
**Pinning number:** 5

**Average optimal degree:** 2.48  
**Average minimal degree:** 2.48  
**Average overall degree:** 3.04

Table 215: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	28	67	89	71	34	9	1	299
Average degree	2.48	2.74	2.93	3.06	3.16	3.24	3.29	3.33	

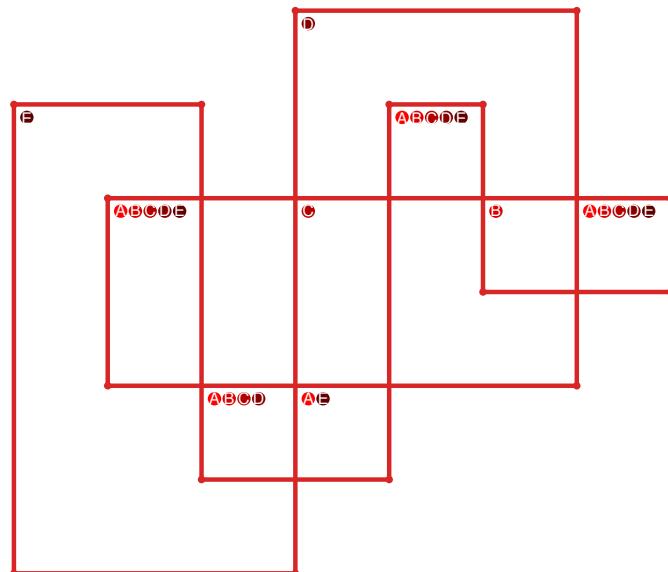


Figure 433: `SnapPy` multiloop plot.

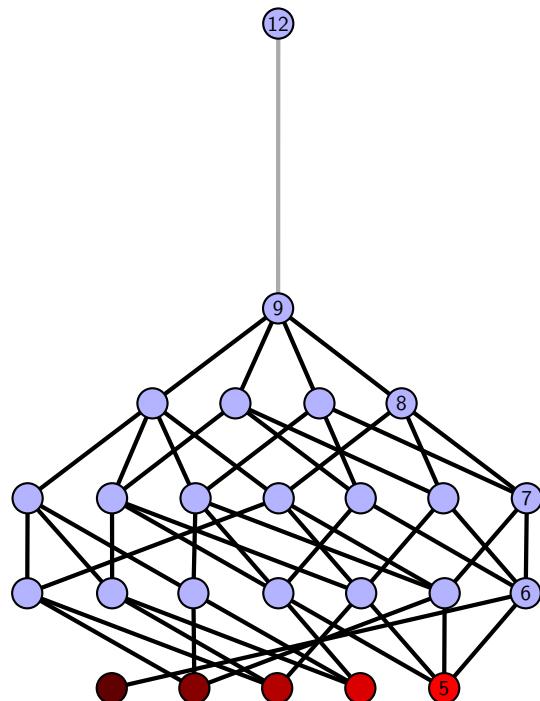


Figure 434: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.69  $[[20, 11, 1, 12], [12, 19, 13, 20], [10, 1, 11, 2], [18, 9, 19, 10], [13, 3, 14, 2], [4, 17, 5, 18], [5, 8, 6, 9], [3, 15, 4, 14], [7, 16, 8, 17], [6, 16, 7, 15]]$

PD code drawn by `SnapPy`:  $[(20, 11, 1, 12), (3, 6, 4, 7), (16, 7, 17, 8), (18, 9, 19, 10), (12, 1, 13, 2), (13, 4, 14, 5), (5, 14, 6, 15), (2, 15, 3, 16), (10, 17, 11, 18), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 4, 0], [0, 4, 3, 0], [1, 2, 5, 6], [1, 7, 7, 2], [3, 7, 8, 6], [3, 5, 8, 9], [4, 9, 5, 4], [5, 9, 9, 6], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 216: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

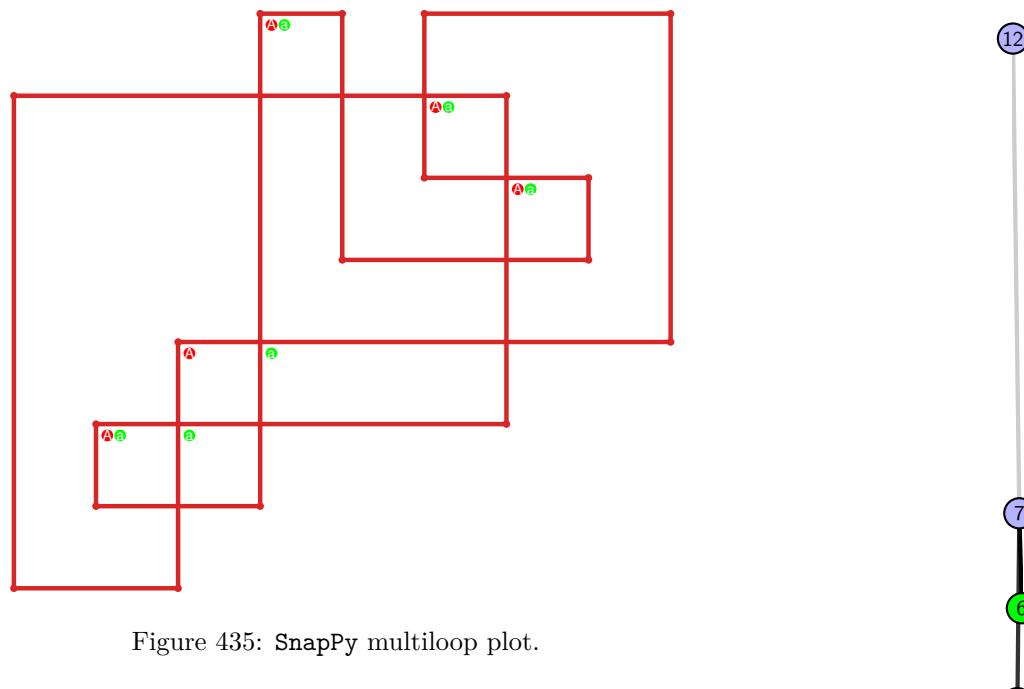


Figure 435: `SnapPy` multiloop plot.

12

7

6

5

Figure 436: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.70**  $[[20, 9, 1, 10], [10, 19, 11, 20], [8, 1, 9, 2], [18, 7, 19, 8], [11, 3, 12, 2], [12, 17, 13, 18], [13, 6, 14, 7], [3, 14, 4, 15], [5, 16, 6, 17], [4, 16, 5, 15]]$

PD code drawn by SnapPy:  $[(10, 3, 11, 4), (15, 4, 16, 5), (17, 6, 18, 7), (20, 11, 1, 12), (12, 1, 13, 2), (2, 13, 3, 14), (9, 14, 10, 15), (7, 16, 8, 17), (5, 18, 6, 19), (19, 8, 20, 9)]$

Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 3, 4, 0], [0, 4, 3, 0], [1, 2, 5, 6], [1, 7, 5, 2], [3, 4, 8, 6], [3, 5, 8, 7], [4, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 217: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

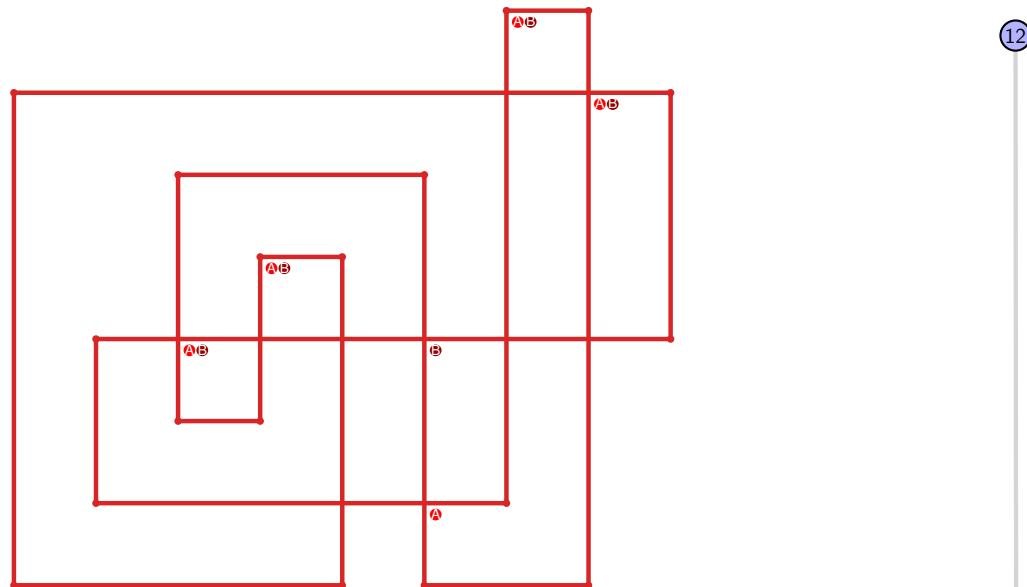


Figure 437: SnapPy multiloop plot.

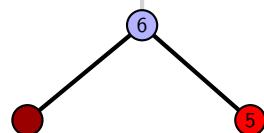


Figure 438: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.71  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 2, 7, 1], [7, 18, 8, 19], [3, 14, 4, 15], [2, 14, 3, 13], [17, 12, 18, 13], [8, 12, 9, 11], [15, 11, 16, 10], [16, 9, 17, 10]]$

PD code drawn by `SnapPy`:  $[(16, 1, 17, 2), (13, 6, 14, 7), (7, 12, 8, 13), (8, 5, 9, 6), (14, 9, 15, 10), (10, 19, 11, 20), (20, 11, 1, 12), (4, 15, 5, 16), (2, 17, 3, 18), (18, 3, 19, 4)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 3, 0], [1, 2, 6, 7], [1, 8, 5, 5], [2, 4, 4, 6], [3, 5, 9, 7], [3, 6, 9, 8], [4, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 218: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

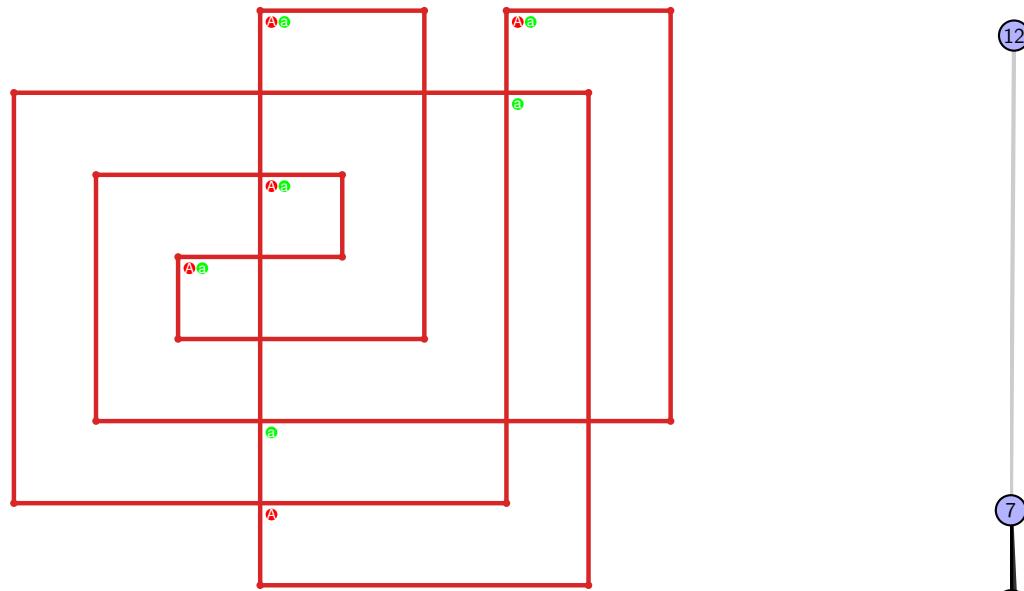


Figure 439: `SnapPy` multiloop plot.



Figure 440: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.72 [[9, 20, 10, 1], [19, 8, 20, 9], [10, 2, 11, 1], [7, 18, 8, 19], [2, 18, 3, 17], [11, 6, 12, 7], [3, 15, 4, 14], [5, 16, 6, 17], [12, 16, 13, 15], [4, 13, 5, 14]]

PD code drawn by SnapPy: [(8, 3, 9, 4), (16, 5, 17, 6), (1, 10, 2, 11), (11, 20, 12, 1), (12, 9, 13, 10), (2, 13, 3, 14), (14, 19, 15, 20), (4, 15, 5, 16), (6, 17, 7, 18), (18, 7, 19, 8)]

Planar representation generated by plantri: [[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 5, 0], [1, 5, 4, 1], [2, 3, 6, 7], [2, 7, 8, 3], [4, 8, 9, 9], [4, 9, 8, 5], [5, 7, 9, 6], [6, 8, 7, 6]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 160  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.97

Table 219: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

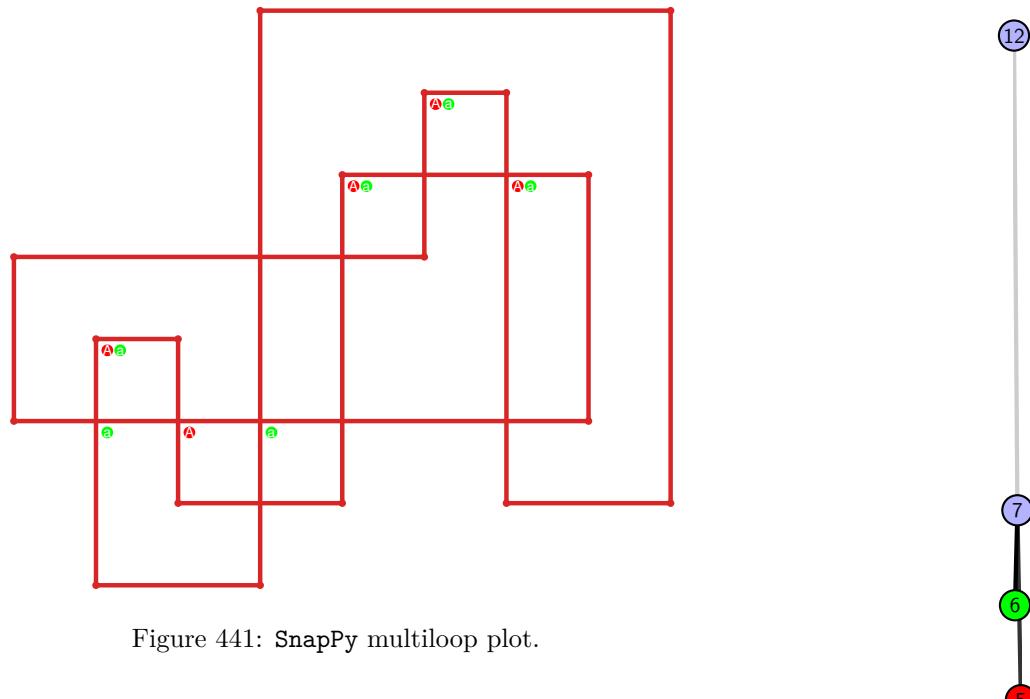


Figure 441: SnapPy multiloop plot.



Figure 442: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.73**  $[[5, 20, 6, 1], [13, 4, 14, 5], [19, 8, 20, 9], [6, 2, 7, 1], [3, 12, 4, 13], [14, 18, 15, 17], [9, 17, 10, 16], [11, 18, 12, 19], [7, 2, 8, 3], [15, 11, 16, 10]]$

PD code drawn by `SnapPy`:  $[(14, 3, 15, 4), (5, 10, 6, 11), (18, 7, 19, 8), (11, 4, 12, 5), (12, 9, 13, 10), (6, 13, 7, 14), (20, 15, 1, 16), (16, 1, 17, 2), (2, 17, 3, 18), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 4, 5], [0, 6, 7, 8], [0, 8, 8, 0], [1, 8, 7, 1], [1, 7, 9, 6], [2, 5, 9, 9], [2, 9, 5, 4], [2, 4, 3, 3], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 220: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

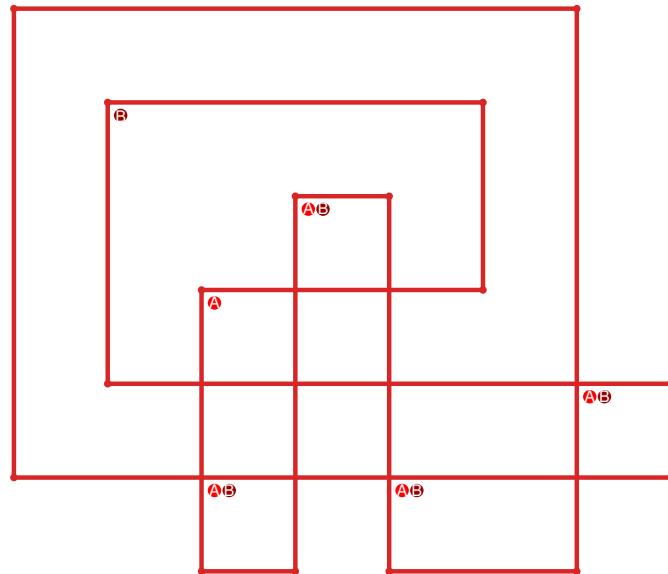


Figure 443: `SnapPy` multiloop plot.

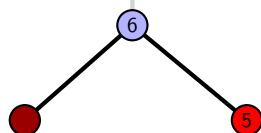


Figure 444: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.74  $[[13, 20, 14, 1], [3, 12, 4, 13], [19, 8, 20, 9], [14, 18, 15, 17], [1, 10, 2, 11], [11, 2, 12, 3], [4, 10, 5, 9], [7, 18, 8, 19], [15, 7, 16, 6], [16, 5, 17, 6]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (14, 3, 15, 4), (4, 13, 5, 14), (17, 6, 18, 7), (7, 2, 8, 3), (19, 8, 20, 9), (1, 10, 2, 11), (15, 12, 16, 13), (5, 16, 6, 17), (11, 18, 12, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 7], [0, 7, 8, 9], [0, 6, 5, 5], [1, 4, 4, 1], [1, 4, 9, 2], [2, 8, 3, 2], [3, 7, 9, 9], [3, 8, 8, 6]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 192  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.2  
 Average overall degree: 2.97

Table 221: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

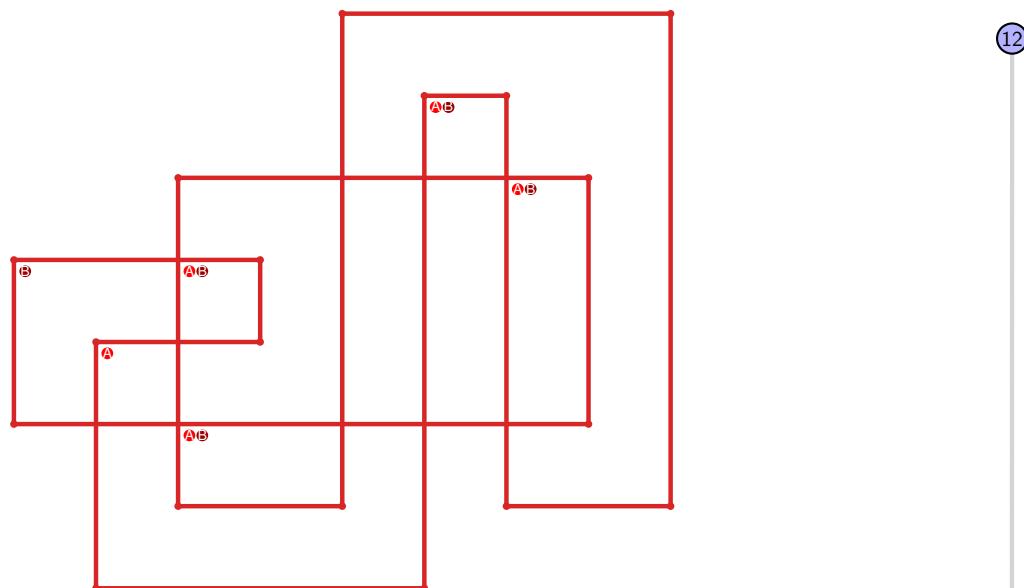


Figure 445: SnapPy multiloop plot.

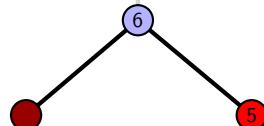


Figure 446: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.75**  $[[20, 13, 1, 14], [14, 3, 15, 4], [8, 19, 9, 20], [9, 12, 10, 13], [1, 17, 2, 16], [2, 15, 3, 16], [4, 17, 5, 18], [18, 7, 19, 8], [11, 6, 12, 7], [10, 6, 11, 5]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (3, 8, 4, 9), (9, 4, 10, 5), (16, 5, 17, 6), (7, 10, 8, 11), (18, 11, 19, 12), (20, 13, 1, 14), (14, 19, 15, 20), (2, 15, 3, 16), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 7, 7, 3], [0, 2, 8, 9], [0, 6, 5, 5], [1, 4, 4, 1], [1, 4, 9, 7], [2, 6, 8, 2], [3, 7, 9, 9], [3, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 222: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

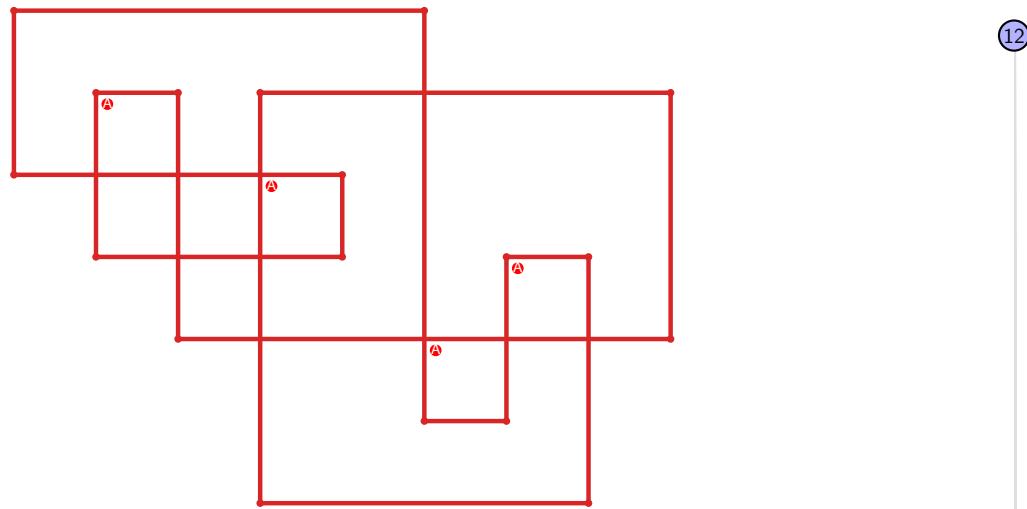


Figure 447: `SnapPy` multiloop plot.

4

Figure 448: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.76  $[[13, 20, 14, 1], [3, 12, 4, 13], [19, 8, 20, 9], [14, 8, 15, 7], [1, 10, 2, 11], [11, 2, 12, 3], [4, 10, 5, 9], [18, 15, 19, 16], [6, 17, 7, 18], [5, 17, 6, 16]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (5, 12, 6, 13), (17, 6, 18, 7), (7, 2, 8, 3), (19, 8, 20, 9), (1, 10, 2, 11), (16, 13, 17, 14), (14, 3, 15, 4), (4, 15, 5, 16), (11, 18, 12, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 3], [0, 2, 7, 8], [0, 6, 5, 5], [1, 4, 4, 1], [1, 4, 9, 2], [2, 9, 8, 3], [3, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.4

**Total pinning sets:** 256

**Average overall degree:** 3.03

**Pinning number:** 5

Table 223: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	19	51	75	65	33	9	1	253
Average degree	2.4	2.68	2.89	3.03	3.15	3.23	3.29	3.33	

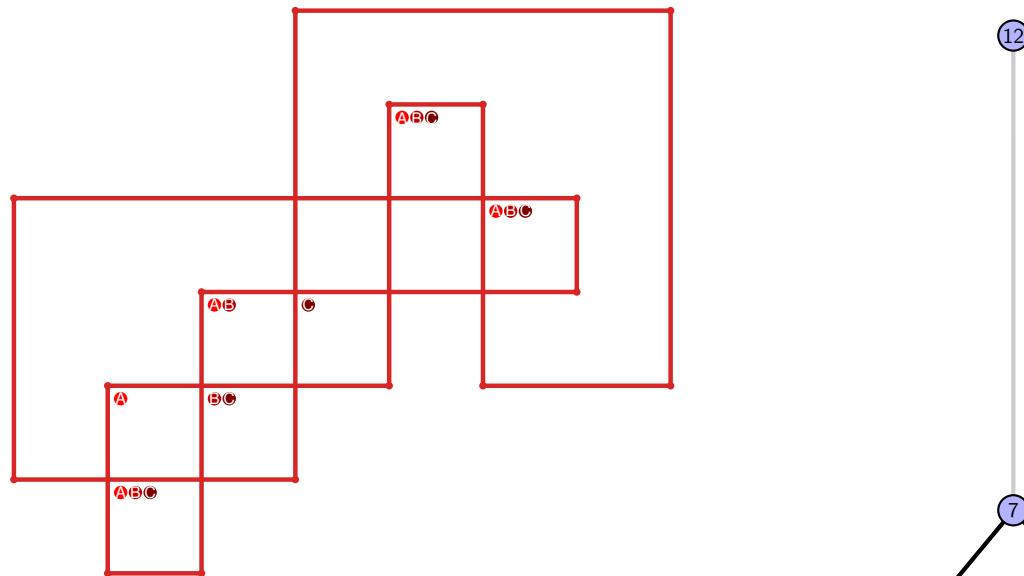


Figure 449: `SnapPy` multiloop plot.

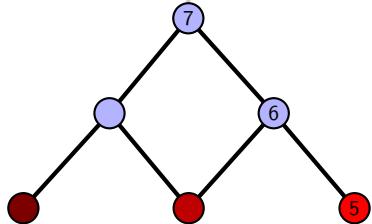


Figure 450: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.77  $[[11, 20, 12, 1], [19, 10, 20, 11], [12, 10, 13, 9], [1, 18, 2, 19], [13, 8, 14, 9], [17, 6, 18, 7], [2, 6, 3, 5], [7, 14, 8, 15], [16, 3, 17, 4], [4, 15, 5, 16]]$

PD code drawn by SnapPy:  $[(1, 18, 2, 19), (13, 2, 14, 3), (9, 4, 10, 5), (5, 8, 6, 9), (15, 6, 16, 7), (3, 10, 4, 11), (20, 11, 1, 12), (17, 14, 18, 15), (7, 16, 8, 17), (12, 19, 13, 20)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 6, 1], [2, 7, 7, 2], [3, 7, 8, 6], [3, 5, 8, 9], [4, 9, 5, 4], [5, 9, 9, 6], [6, 8, 8, 7]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 160  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.97

Table 224: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

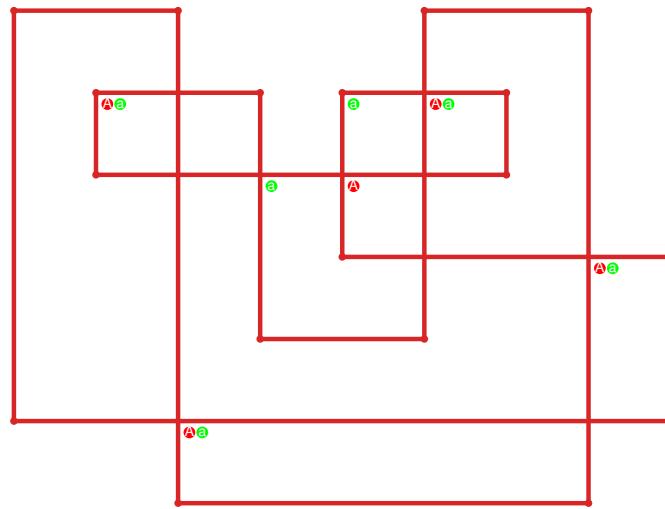


Figure 451: SnapPy multiloop plot.



Figure 452: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.78  $[[13, 20, 14, 1], [19, 12, 20, 13], [14, 12, 15, 11], [1, 18, 2, 19], [15, 7, 16, 6], [17, 10, 18, 11], [2, 8, 3, 7], [16, 5, 17, 6], [9, 4, 10, 5], [8, 4, 9, 3]]$

PD code drawn by SnapPy:  $[(20, 5, 1, 6), (6, 1, 7, 2), (13, 2, 14, 3), (3, 12, 4, 13), (4, 19, 5, 20), (14, 7, 15, 8), (8, 11, 9, 12), (16, 9, 17, 10), (18, 15, 19, 16), (10, 17, 11, 18)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 5], [0, 5, 6, 1], [2, 6, 7, 7], [2, 7, 8, 3], [3, 9, 9, 4], [4, 8, 5, 4], [5, 7, 9, 9], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 225: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

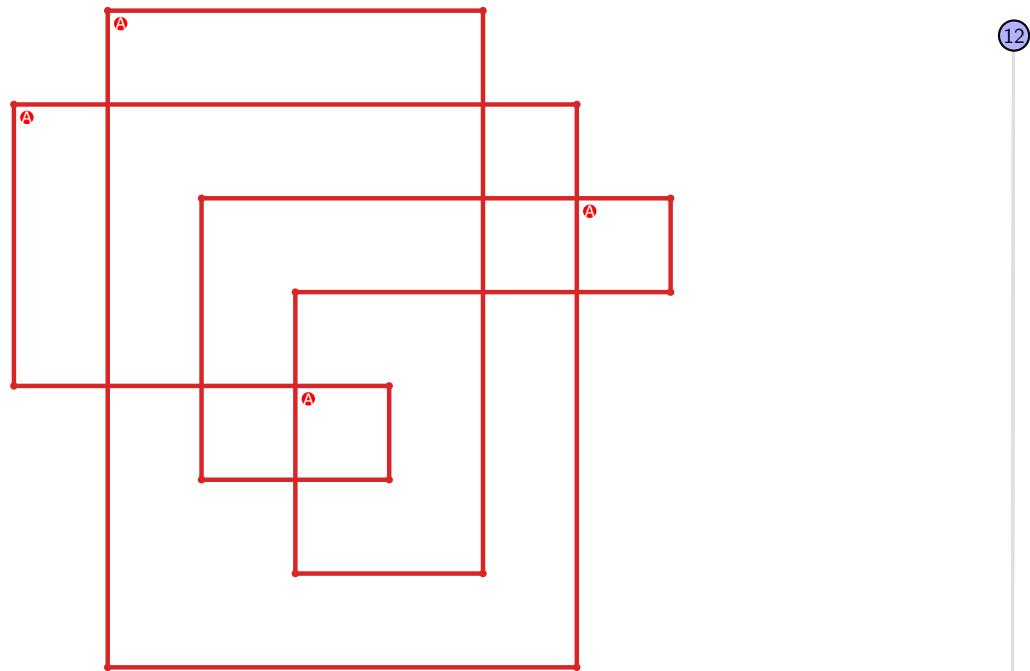


Figure 453: SnapPy multiloop plot.

12

4

Figure 454: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.79  $[[13, 20, 14, 1], [19, 12, 20, 13], [14, 12, 15, 11], [1, 18, 2, 19], [15, 4, 16, 5], [17, 10, 18, 11], [2, 7, 3, 8], [8, 3, 9, 4], [16, 6, 17, 5], [6, 9, 7, 10]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (13, 2, 14, 3), (3, 14, 4, 15), (20, 5, 1, 6), (4, 7, 5, 8), (15, 8, 16, 9), (9, 12, 10, 13), (17, 10, 18, 11), (19, 16, 20, 17), (11, 18, 12, 19)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 5], [0, 5, 6, 1], [2, 7, 8, 8], [2, 8, 9, 3], [3, 9, 7, 7], [4, 6, 6, 9], [4, 9, 5, 4], [5, 8, 7, 6]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 456  
**Pinning number:** 4

**Average optimal degree:** 2.38  
**Average minimal degree:** 2.52  
**Average overall degree:** 3.05

Table 226: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	0	4
Nonminimal pinning sets	0	15	65	116	124	84	36	9	1	450
Average degree	2.38	2.65	2.86	3.0	3.11	3.19	3.24	3.29	3.33	

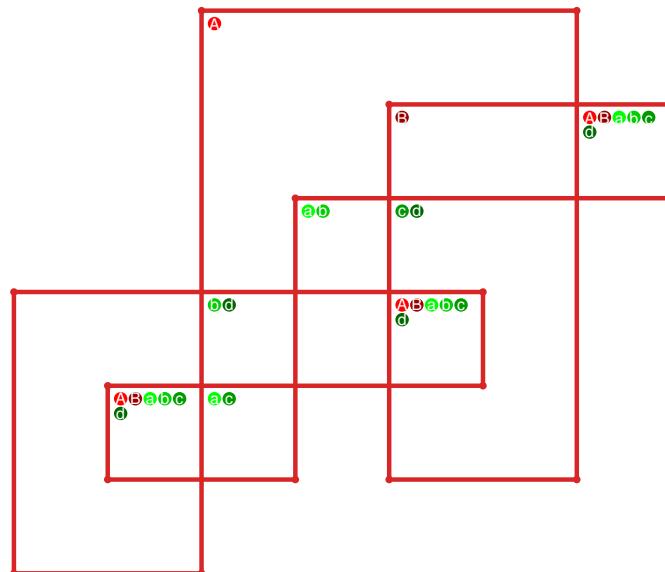


Figure 455: `SnapPy` multiloop plot.

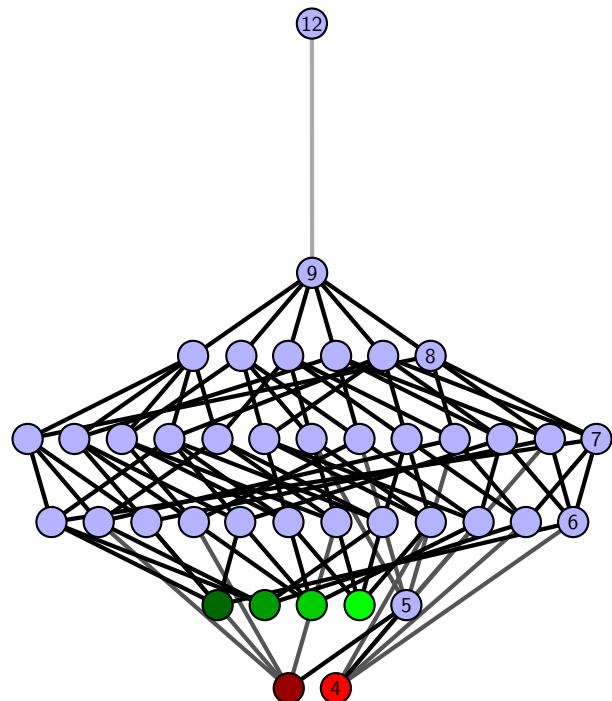


Figure 456: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.80  $[[11, 20, 12, 1], [19, 10, 20, 11], [12, 10, 13, 9], [1, 18, 2, 19], [13, 7, 14, 6], [8, 3, 9, 4], [17, 2, 18, 3], [7, 17, 8, 16], [14, 5, 15, 6], [4, 15, 5, 16]]$

PD code drawn by SnapPy:  $[(15, 20, 16, 1), (6, 1, 7, 2), (12, 3, 13, 4), (4, 11, 5, 12), (7, 10, 8, 11), (17, 8, 18, 9), (2, 13, 3, 14), (14, 5, 15, 6), (19, 16, 20, 17), (9, 18, 10, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 5], [0, 6, 6, 1], [2, 7, 8, 8], [2, 9, 7, 6], [3, 5, 7, 3], [4, 6, 5, 9], [4, 9, 9, 4], [5, 8, 8, 7]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 192

**Pinning number:** 5

**Average optimal degree:** 2.2

**Average minimal degree:** 2.2

**Average overall degree:** 2.97

Table 227: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

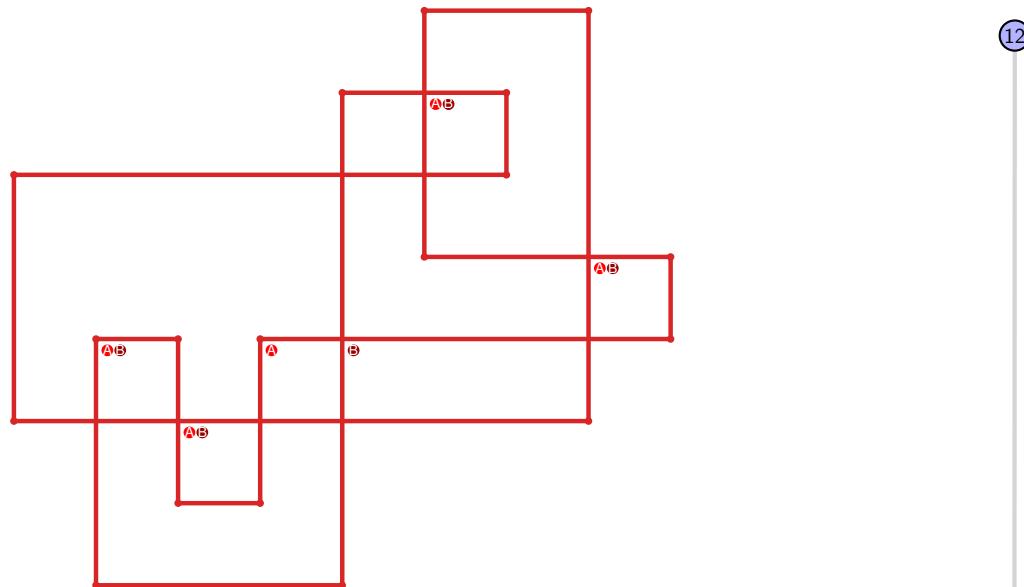


Figure 457: SnapPy multiloop plot.

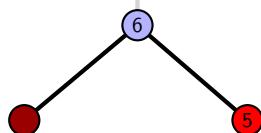


Figure 458: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.81  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 8, 11, 7], [1, 18, 2, 19], [11, 6, 12, 7], [12, 17, 13, 18], [2, 13, 3, 14], [14, 5, 15, 6], [16, 3, 17, 4], [4, 15, 5, 16]]$

PD code drawn by SnapPy:  $[(14, 1, 15, 2), (12, 3, 13, 4), (9, 4, 10, 5), (5, 8, 6, 9), (17, 6, 18, 7), (20, 11, 1, 12), (2, 13, 3, 14), (10, 15, 11, 16), (19, 16, 20, 17), (7, 18, 8, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 4], [0, 5, 6, 1], [2, 7, 5, 2], [3, 4, 8, 6], [3, 5, 8, 7], [4, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 228: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

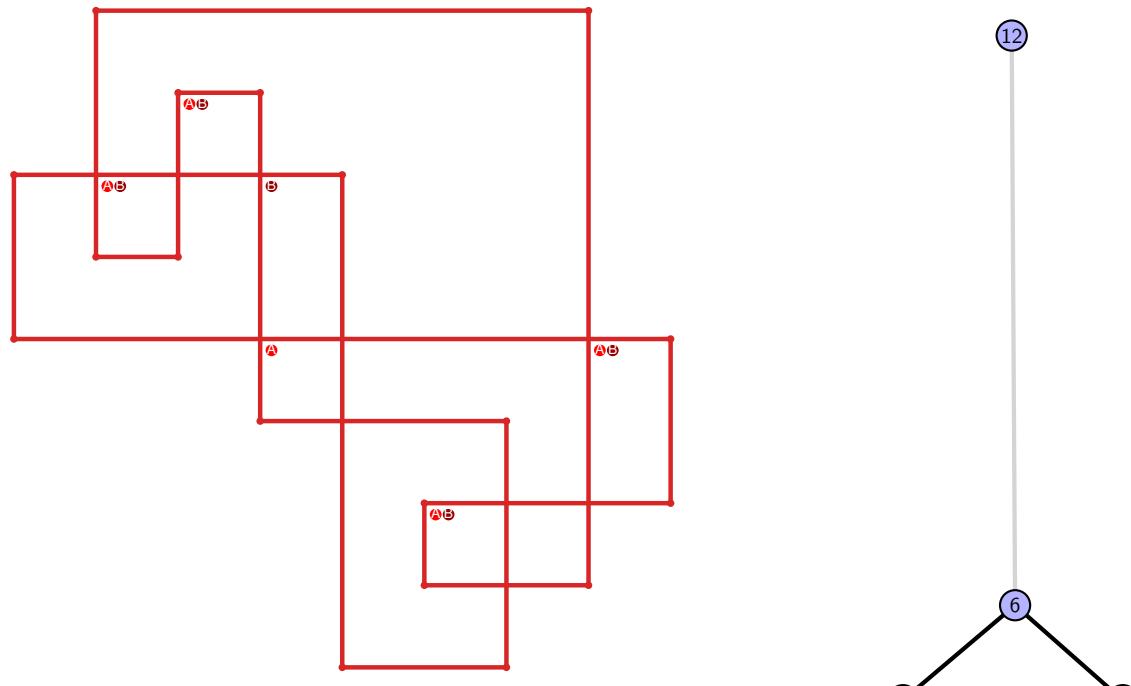


Figure 459: SnapPy multiloop plot.

Figure 460: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.82  $[[11, 20, 12, 1], [19, 10, 20, 11], [12, 10, 13, 9], [1, 18, 2, 19], [13, 7, 14, 6], [17, 8, 18, 9], [2, 8, 3, 7], [14, 5, 15, 6], [16, 3, 17, 4], [4, 15, 5, 16]]$

PD code drawn by SnapPy:  $[(14, 1, 15, 2), (12, 3, 13, 4), (4, 11, 5, 12), (5, 20, 6, 1), (15, 6, 16, 7), (7, 10, 8, 11), (17, 8, 18, 9), (2, 13, 3, 14), (19, 16, 20, 17), (9, 18, 10, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 2, 0], [0, 1, 4, 5], [0, 5, 6, 1], [2, 6, 7, 7], [2, 8, 6, 3], [3, 5, 8, 4], [4, 9, 9, 4], [5, 9, 9, 6], [7, 8, 8, 7]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 3  
 Total pinning sets: 176  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.34  
 Average overall degree: 2.98

Table 229: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.2	2.5	2.75	2.94	3.08	3.19	3.27	3.33	

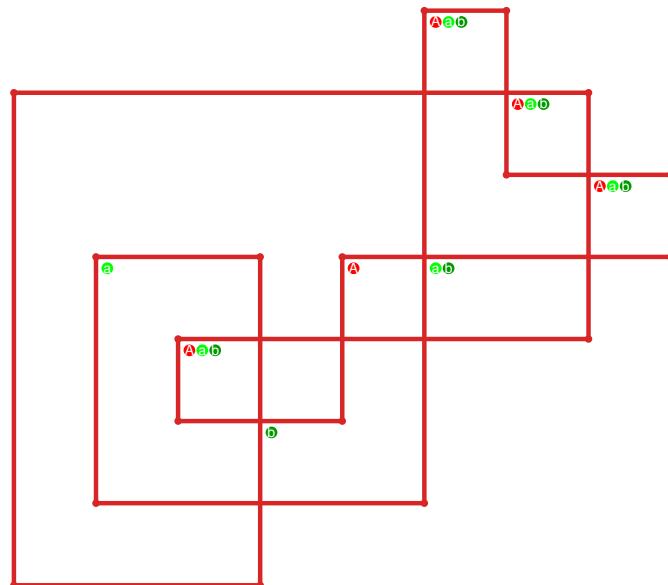


Figure 461: SnapPy multiloop plot.

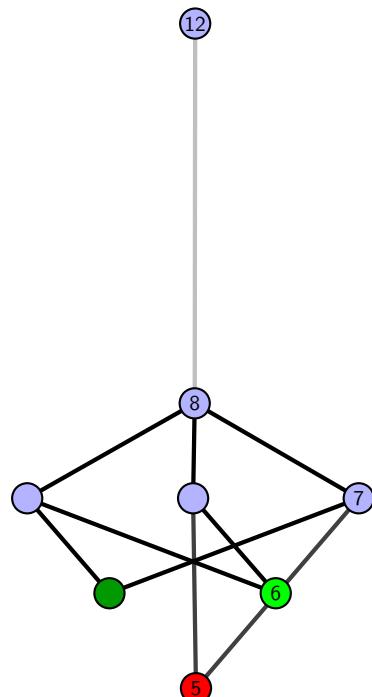


Figure 462: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.83  $[[20, 17, 1, 18], [18, 13, 19, 14], [14, 19, 15, 20], [16, 7, 17, 8], [1, 12, 2, 13], [15, 9, 16, 8], [11, 6, 12, 7], [2, 6, 3, 5], [9, 5, 10, 4], [10, 3, 11, 4]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (2, 7, 3, 8), (3, 20, 4, 1), (9, 4, 10, 5), (14, 5, 15, 6), (6, 13, 7, 14), (19, 10, 20, 11), (16, 11, 17, 12), (18, 15, 19, 16), (12, 17, 13, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 2, 2], [0, 1, 1, 5], [0, 5, 5, 6], [0, 6, 7, 1], [2, 8, 3, 3], [3, 9, 7, 4], [4, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.45

**Total pinning sets:** 240

**Average overall degree:** 3.03

**Pinning number:** 5

Table 230: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	13	45	71	64	33	9	1	236
Average degree	2.4	2.66	2.86	3.02	3.14	3.23	3.29	3.33	

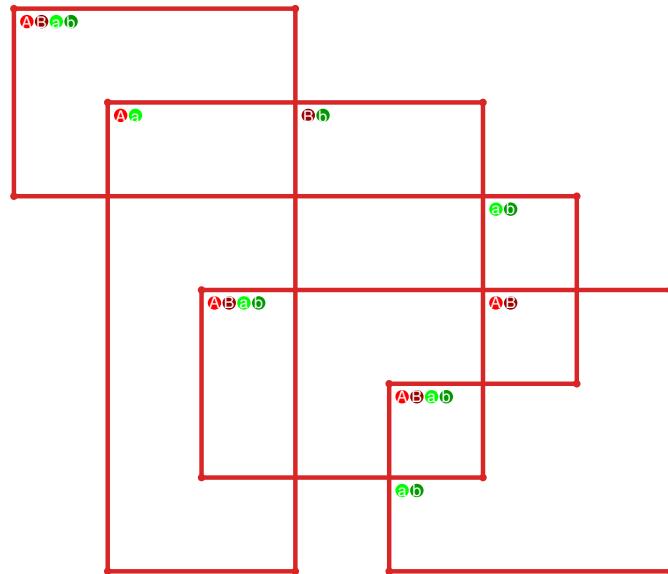


Figure 463: `SnapPy` multiloop plot.

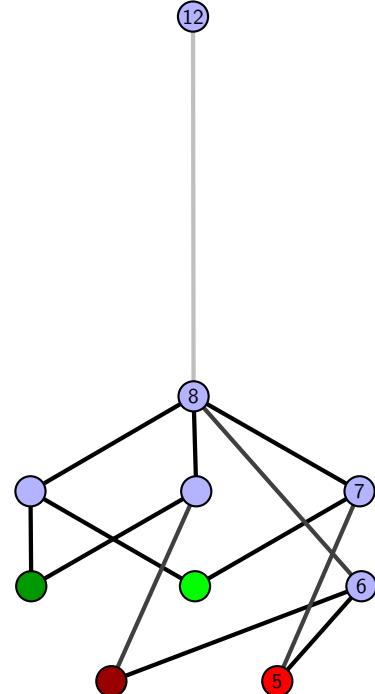


Figure 464: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.84 [[20, 11, 1, 12], [12, 19, 13, 20], [10, 7, 11, 8], [1, 18, 2, 19], [13, 9, 14, 8], [14, 9, 15, 10], [17, 6, 18, 7], [2, 6, 3, 5], [15, 5, 16, 4], [16, 3, 17, 4]]

PD code drawn by `SnapPy`: [(14, 1, 15, 2), (2, 13, 3, 14), (3, 20, 4, 1), (15, 4, 16, 5), (8, 5, 9, 6), (6, 11, 7, 12), (12, 7, 13, 8), (18, 9, 19, 10), (19, 16, 20, 17), (10, 17, 11, 18)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 3, 4, 0], [0, 4, 5, 6], [0, 6, 7, 1], [1, 5, 5, 2], [2, 4, 4, 8], [2, 9, 7, 3], [3, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 320

**Average overall degree:** 3.03

**Pinning number:** 4

Table 231: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

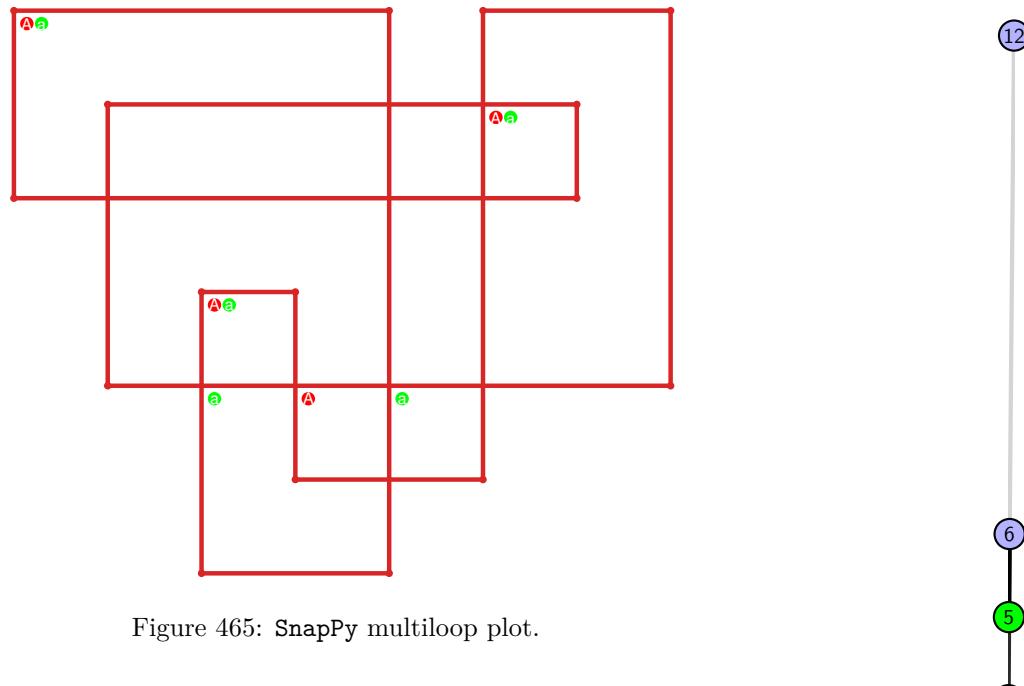


Figure 465: `SnapPy` multiloop plot.

Figure 466: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.85  $[[20, 9, 1, 10], [10, 19, 11, 20], [8, 13, 9, 14], [1, 18, 2, 19], [11, 15, 12, 14], [12, 7, 13, 8], [17, 6, 18, 7], [2, 6, 3, 5], [15, 5, 16, 4], [16, 3, 17, 4]]$

PD code drawn by SnapPy:  $[(20, 11, 1, 12), (1, 18, 2, 19), (13, 2, 14, 3), (8, 3, 9, 4), (16, 5, 17, 6), (4, 9, 5, 10), (10, 7, 11, 8), (17, 14, 18, 15), (6, 15, 7, 16), (12, 19, 13, 20)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 3, 4, 0], [0, 4, 5, 5], [0, 6, 7, 1], [1, 8, 5, 2], [2, 4, 6, 2], [3, 5, 9, 7], [3, 6, 9, 8], [4, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 320  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.33  
 Average overall degree: 3.03

Table 232: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

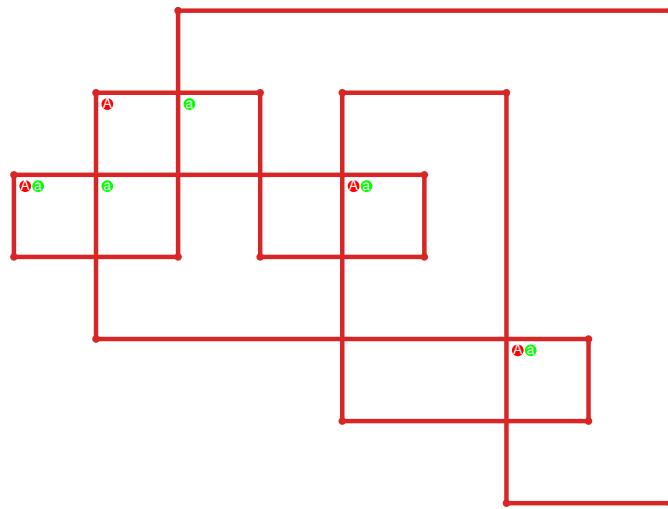


Figure 467: SnapPy multiloop plot.



Figure 468: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.86**  $[[11, 20, 12, 1], [19, 10, 20, 11], [12, 8, 13, 7], [1, 18, 2, 19], [9, 14, 10, 15], [8, 14, 9, 13], [17, 6, 18, 7], [2, 6, 3, 5], [15, 5, 16, 4], [16, 3, 17, 4]]$

PD code drawn by `SnapPy`:  $[(20, 11, 1, 12), (1, 18, 2, 19), (13, 2, 14, 3), (3, 8, 4, 9), (15, 6, 16, 7), (9, 4, 10, 5), (5, 10, 6, 11), (17, 14, 18, 15), (7, 16, 8, 17), (12, 19, 13, 20)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 3, 4, 0], [0, 5, 5, 6], [0, 6, 7, 1], [1, 8, 5, 5], [2, 4, 4, 2], [2, 9, 7, 3], [3, 6, 9, 8], [4, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 233: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

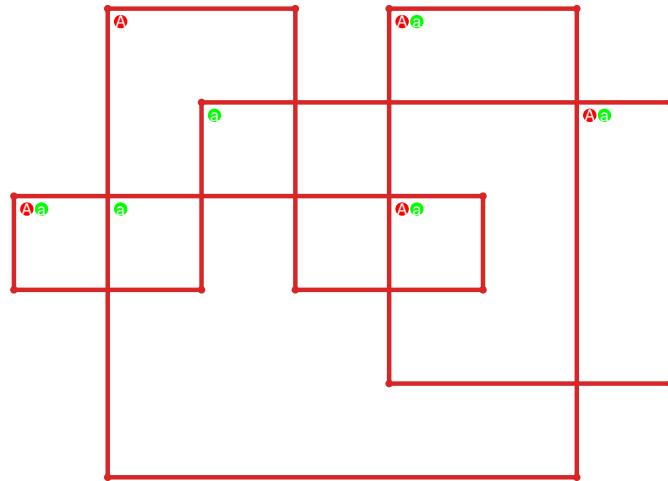


Figure 469: `SnapPy` multiloop plot.



Figure 470: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.87  $[[20, 5, 1, 6], [6, 3, 7, 4], [4, 19, 5, 20], [1, 15, 2, 14], [2, 13, 3, 14], [7, 18, 8, 19], [15, 8, 16, 9], [9, 12, 10, 13], [10, 17, 11, 18], [16, 11, 17, 12]]$

PD code drawn by SnapPy:  $[(4, 1, 5, 2), (18, 3, 19, 4), (13, 6, 14, 7), (14, 9, 15, 10), (7, 10, 8, 11), (20, 11, 1, 12), (12, 19, 13, 20), (8, 15, 9, 16), (5, 16, 6, 17), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 5, 2], [0, 1, 5, 0], [0, 6, 4, 4], [1, 3, 3, 7], [1, 8, 6, 2], [3, 5, 9, 7], [4, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 288  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.4  
 Average overall degree: 3.03

Table 234: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

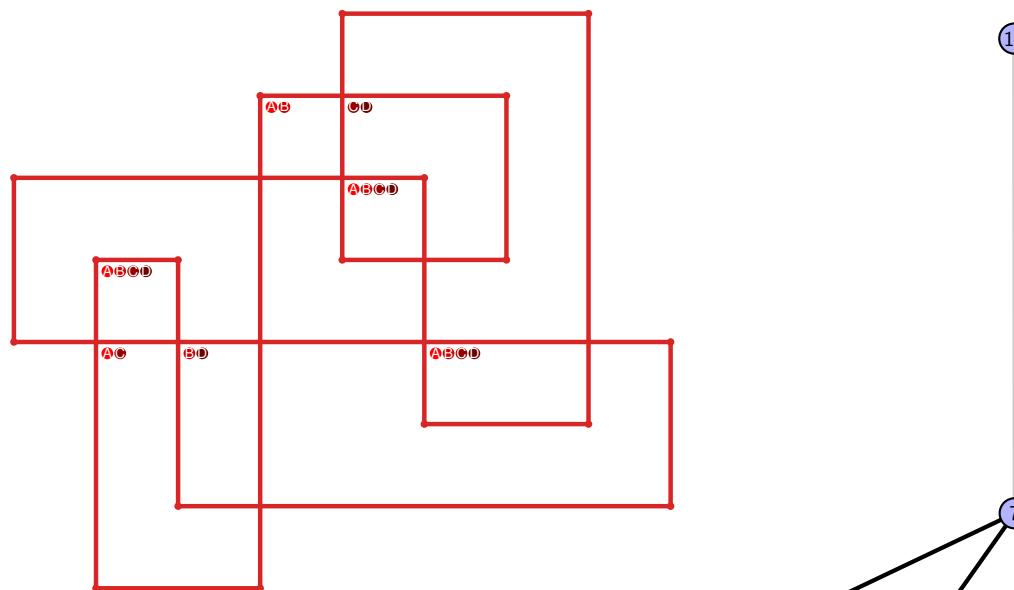


Figure 471: SnapPy multiloop plot.

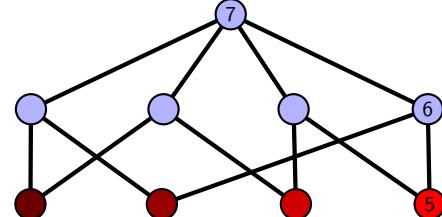


Figure 472: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.88  $[[20, 7, 1, 8], [8, 19, 9, 20], [9, 6, 10, 7], [1, 17, 2, 16], [18, 15, 19, 16], [12, 5, 13, 6], [10, 4, 11, 3], [17, 3, 18, 2], [11, 14, 12, 15], [4, 13, 5, 14]]$

PD code drawn by `SnapPy`:  $[(3, 20, 4, 1), (14, 1, 15, 2), (18, 7, 19, 8), (5, 8, 6, 9), (12, 9, 13, 10), (10, 15, 11, 16), (2, 13, 3, 14), (16, 11, 17, 12), (17, 4, 18, 5), (6, 19, 7, 20)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 6], [0, 7, 7, 4], [1, 3, 7, 8], [2, 8, 9, 9], [2, 9, 8, 7], [3, 6, 4, 3], [4, 6, 9, 5], [5, 8, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.42

**Total pinning sets:** 352

**Average overall degree:** 3.04

**Pinning number:** 4

Table 235: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	0	2
Nonminimal pinning sets	0	8	39	81	100	76	35	9	1	349
Average degree	2.25	2.56	2.79	2.95	3.08	3.17	3.24	3.29	3.33	

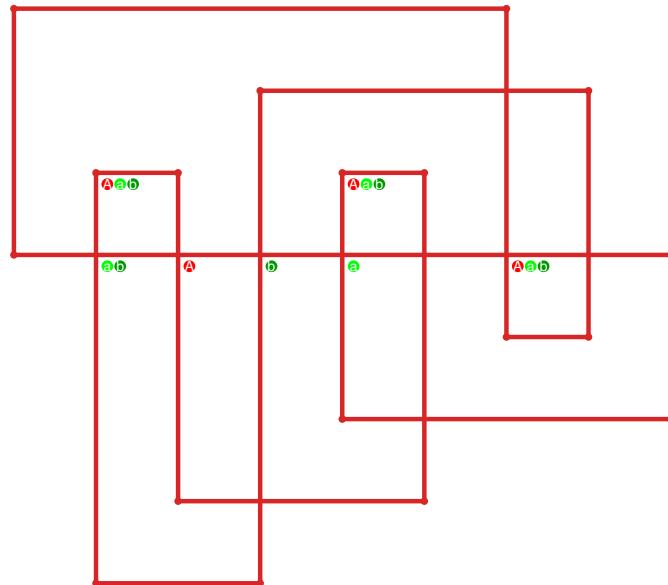


Figure 473: `SnapPy` multiloop plot.

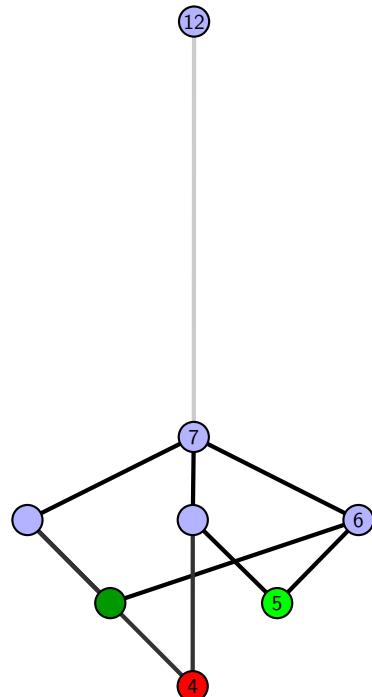


Figure 474: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.89  $[[20, 9, 1, 10], [10, 19, 11, 20], [11, 8, 12, 9], [1, 5, 2, 4], [18, 3, 19, 4], [14, 7, 15, 8], [12, 6, 13, 5], [2, 17, 3, 18], [13, 16, 14, 17], [6, 15, 7, 16]]$

PD code drawn by SnapPy:  $[(15, 20, 16, 1), (1, 18, 2, 19), (2, 9, 3, 10), (11, 4, 12, 5), (8, 5, 9, 6), (17, 6, 18, 7), (3, 12, 4, 13), (10, 13, 11, 14), (19, 14, 20, 15), (7, 16, 8, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 6], [0, 6, 7, 4], [1, 3, 7, 7], [2, 8, 9, 9], [2, 9, 8, 3], [3, 8, 4, 4], [5, 7, 6, 9], [5, 8, 6, 5]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 3  
 Total pinning sets: 352  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.42  
 Average overall degree: 3.04

Table 236: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	0	2
Nonminimal pinning sets	0	8	39	81	100	76	35	9	1	349
Average degree	2.25	2.56	2.79	2.95	3.08	3.17	3.24	3.29	3.33	

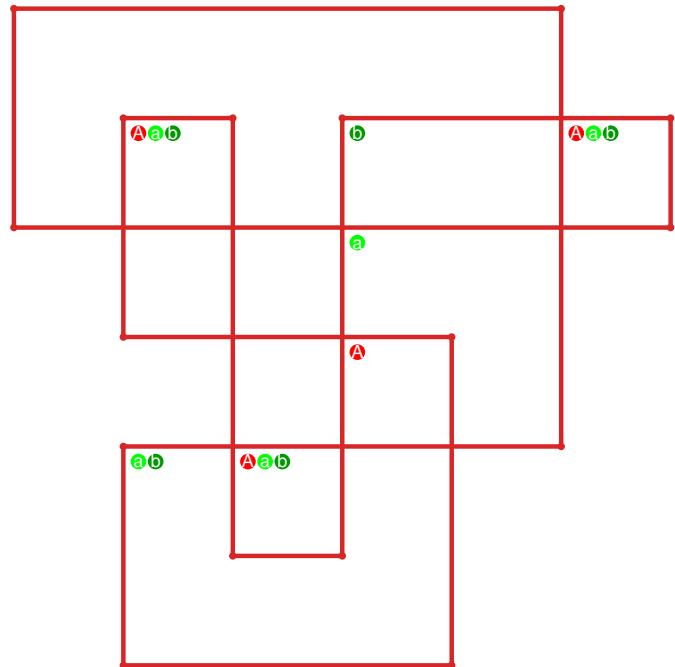


Figure 475: SnapPy multiloop plot.

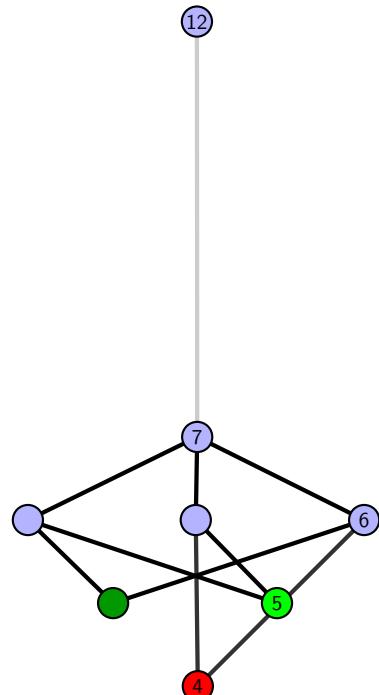


Figure 476: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.90  $[[20, 17, 1, 18], [18, 3, 19, 4], [4, 19, 5, 20], [5, 16, 6, 17], [1, 13, 2, 12], [2, 11, 3, 12], [8, 15, 9, 16], [6, 14, 7, 13], [7, 10, 8, 11], [14, 9, 15, 10]]$

PD code drawn by SnapPy:  $[(15, 20, 16, 1), (11, 6, 12, 7), (4, 7, 5, 8), (19, 8, 20, 9), (9, 18, 10, 19), (10, 3, 11, 4), (5, 12, 6, 13), (2, 13, 3, 14), (17, 14, 18, 15), (1, 16, 2, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 2, 2], [0, 1, 1, 3], [0, 2, 6, 7], [0, 7, 5, 5], [1, 4, 4, 8], [3, 8, 9, 9], [3, 9, 8, 4], [5, 7, 9, 6], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.4

**Total pinning sets:** 288

**Average overall degree:** 3.03

**Pinning number:** 5

Table 237: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

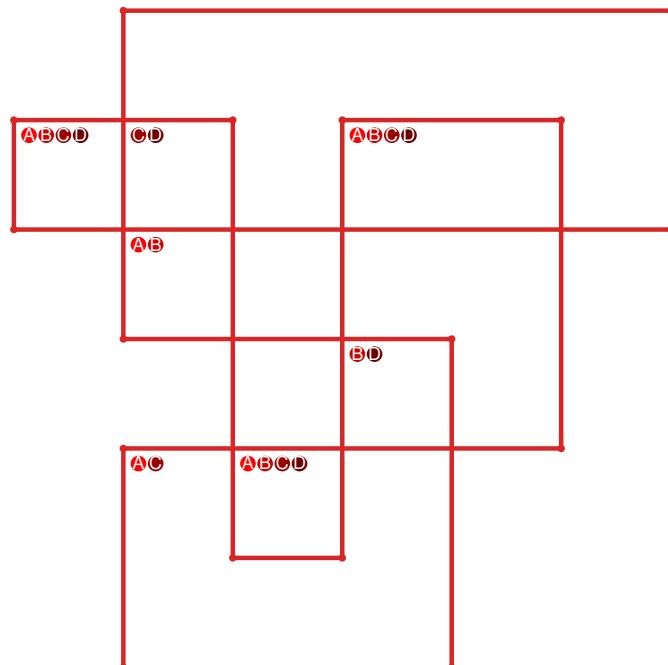


Figure 477: SnapPy multiloop plot.

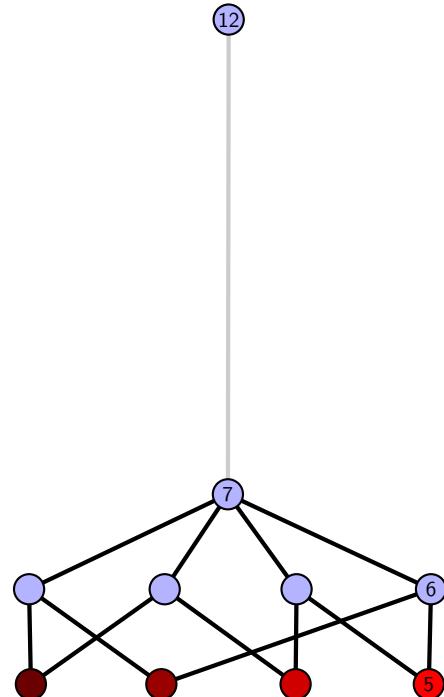


Figure 478: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.91  $[[11, 20, 12, 1], [19, 10, 20, 11], [12, 10, 13, 9], [1, 16, 2, 17], [3, 18, 4, 19], [13, 7, 14, 6], [15, 8, 16, 9], [2, 18, 3, 17], [4, 8, 5, 7], [14, 5, 15, 6]]$

PD code drawn by SnapPy:  $[(12, 3, 13, 4), (4, 11, 5, 12), (5, 2, 6, 3), (13, 6, 14, 7), (7, 10, 8, 11), (17, 8, 18, 9), (19, 14, 20, 15), (15, 20, 16, 1), (1, 16, 2, 17), (9, 18, 10, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 6], [0, 6, 7, 7], [1, 7, 7, 8], [2, 8, 9, 9], [2, 9, 8, 3], [3, 4, 4, 3], [4, 6, 9, 5], [5, 8, 6, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 238: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

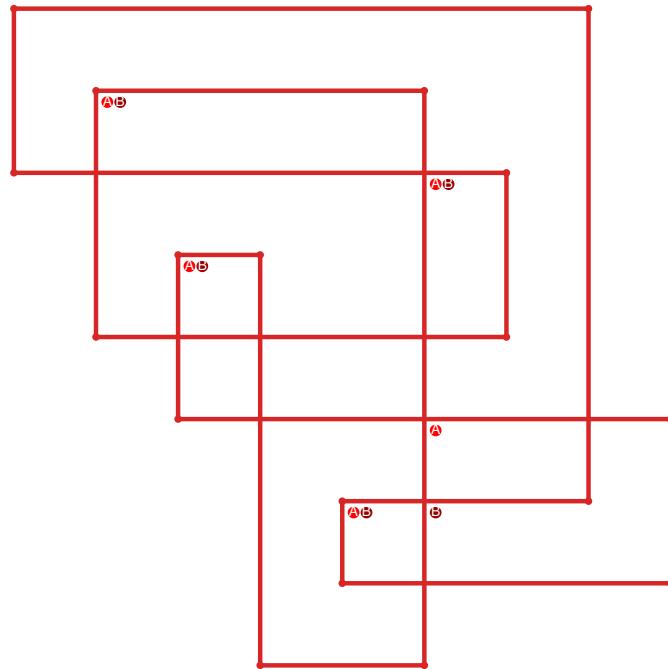


Figure 479: SnapPy multiloop plot.

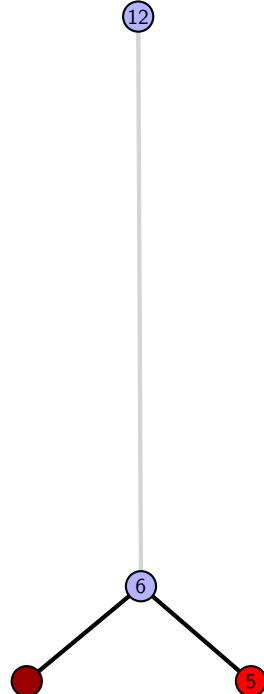


Figure 480: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.92  $[[13, 20, 14, 1], [12, 9, 13, 10], [4, 19, 5, 20], [14, 18, 15, 17], [1, 11, 2, 10], [2, 11, 3, 12], [3, 8, 4, 9], [18, 5, 19, 6], [15, 6, 16, 7], [7, 16, 8, 17]]$

PD code drawn by SnapPy:  $[(15, 2, 16, 3), (12, 3, 13, 4), (20, 5, 1, 6), (9, 6, 10, 7), (7, 18, 8, 19), (19, 8, 20, 9), (4, 13, 5, 14), (14, 11, 15, 12), (1, 16, 2, 17), (10, 17, 11, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 7], [0, 7, 8, 9], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 9, 2], [2, 8, 3, 2], [3, 7, 9, 9], [3, 8, 8, 6]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 232  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.54  
**Average overall degree:** 3.04

Table 239: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	0	0	0	0	0	0	5
Nonminimal pinning sets	0	7	42	70	64	33	9	1	226
Average degree	2.4	2.62	2.85	3.02	3.14	3.23	3.29	3.33	

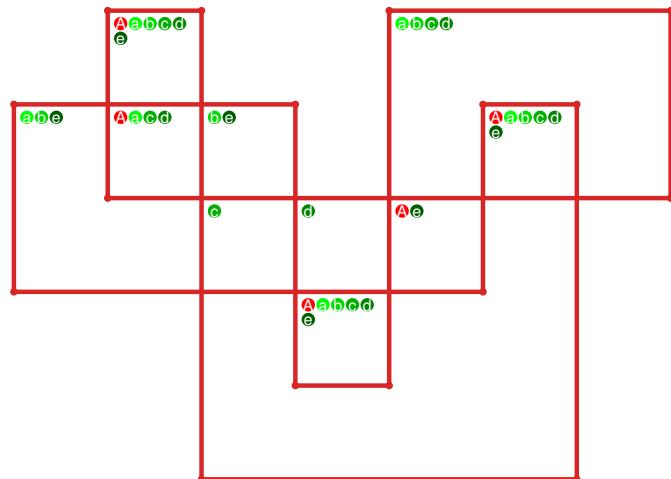


Figure 481: SnapPy multiloop plot.

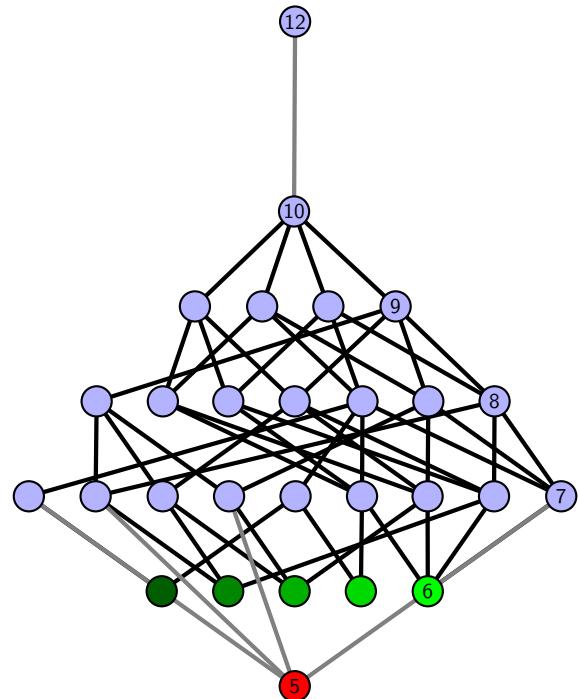


Figure 482: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.93**  $[[3, 20, 4, 1], [2, 9, 3, 10], [12, 19, 13, 20], [4, 18, 5, 17], [1, 11, 2, 10], [11, 8, 12, 9], [18, 13, 19, 14], [5, 14, 6, 15], [7, 16, 8, 17], [6, 16, 7, 15]]$

PD code drawn by `SnapPy`:  $[(9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (16, 11, 17, 12), (3, 12, 4, 13), (13, 4, 14, 5), (5, 14, 6, 15), (15, 2, 16, 3), (10, 17, 11, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 6], [0, 6, 7, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 7, 3, 2], [3, 6, 9, 9], [3, 9, 9, 5], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.3

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.3

**Total pinning sets:** 240

**Average overall degree:** 2.98

**Pinning number:** 5

Table 240: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	22	52	69	56	28	8	1	236
Average degree	2.3	2.62	2.84	3.0	3.11	3.2	3.27	3.33	

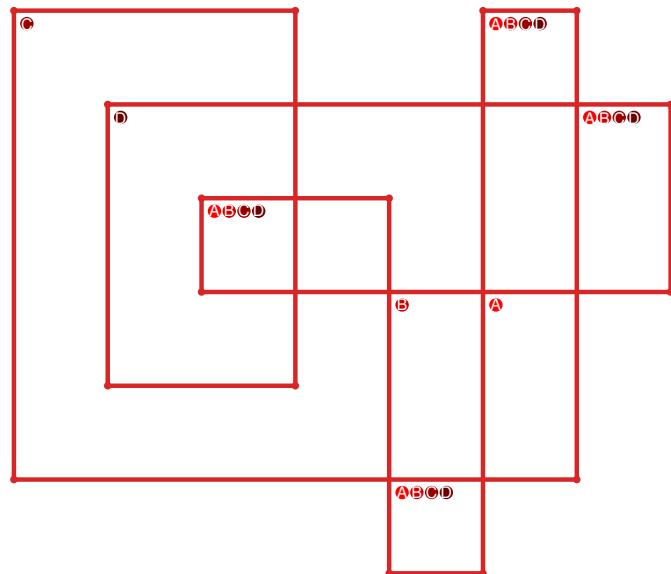


Figure 483: `SnapPy` multiloop plot.

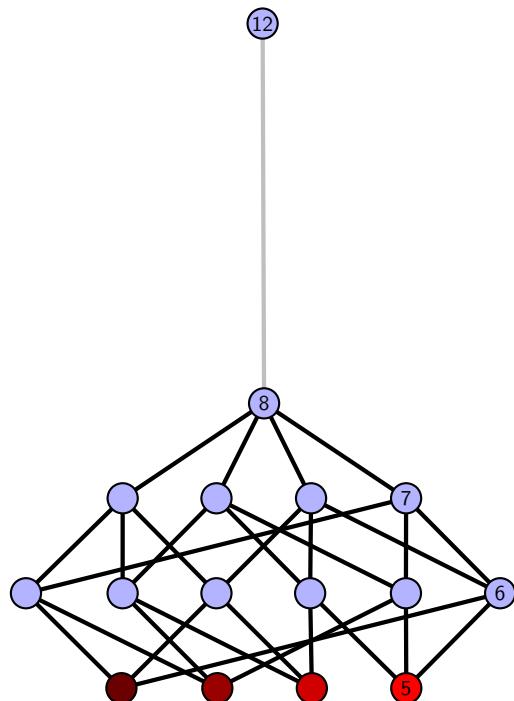


Figure 484: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.94  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 10, 20, 11], [4, 8, 5, 7], [1, 12, 2, 13], [14, 12, 15, 11], [9, 18, 10, 19], [8, 18, 9, 17], [5, 17, 6, 16], [6, 15, 7, 16]]$

PD code drawn by SnapPy:  $[(1, 18, 2, 19), (13, 4, 14, 5), (5, 8, 6, 9), (15, 6, 16, 7), (20, 9, 1, 10), (10, 19, 11, 20), (11, 2, 12, 3), (3, 12, 4, 13), (17, 14, 18, 15), (7, 16, 8, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 6], [0, 7, 8, 9], [0, 5, 1, 1], [1, 4, 9, 2], [2, 7, 7, 2], [3, 6, 6, 8], [3, 7, 9, 9], [3, 8, 8, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 241: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

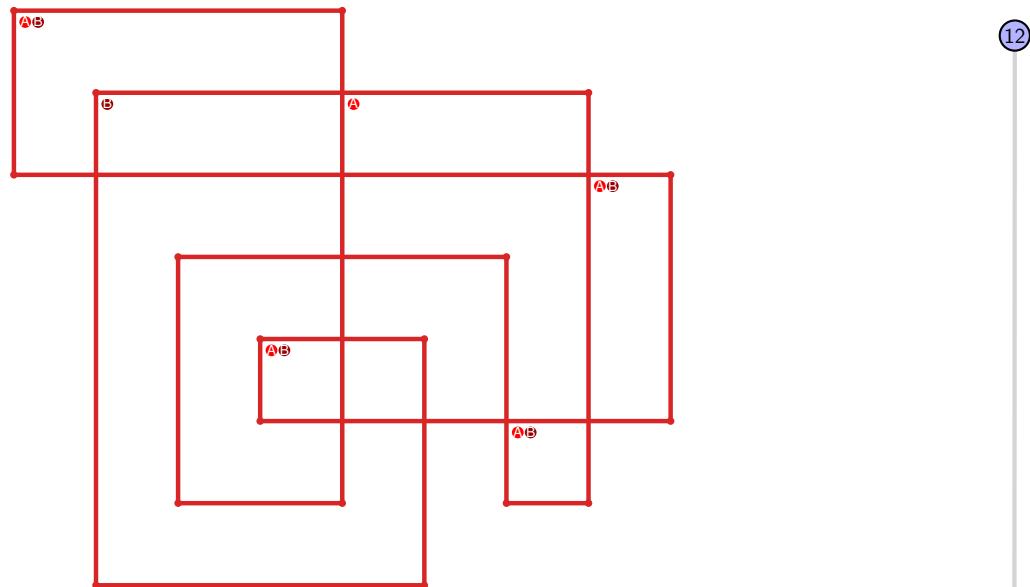


Figure 485: SnapPy multiloop plot.

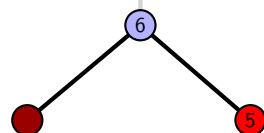


Figure 486: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.95  $[[3, 20, 4, 1], [2, 15, 3, 16], [10, 19, 11, 20], [4, 8, 5, 7], [1, 17, 2, 16], [17, 14, 18, 15], [18, 9, 19, 10], [11, 9, 12, 8], [5, 12, 6, 13], [13, 6, 14, 7]]$

PD code drawn by `SnapPy`:  $[(11, 4, 12, 5), (5, 2, 6, 3), (6, 9, 7, 10), (16, 7, 17, 8), (3, 10, 4, 11), (15, 12, 16, 13), (20, 13, 1, 14), (14, 19, 15, 20), (8, 17, 9, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 7], [0, 7, 8, 9], [0, 5, 1, 1], [1, 4, 9, 6], [2, 5, 7, 2], [2, 6, 8, 3], [3, 7, 9, 9], [3, 8, 8, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.25

**Total pinning sets:** 384

**Average overall degree:** 3.03

**Pinning number:** 4

Table 242: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

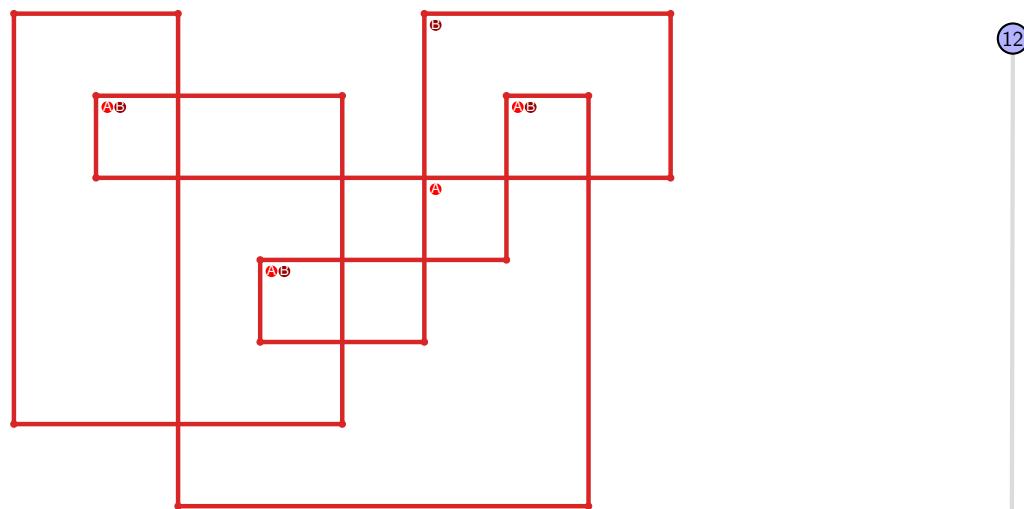


Figure 487: `SnapPy` multiloop plot.

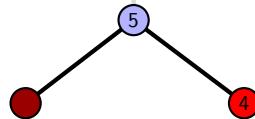


Figure 488: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.96**  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 10, 20, 11], [4, 18, 5, 17], [1, 12, 2, 13], [14, 12, 15, 11], [9, 18, 10, 19], [5, 9, 6, 8], [16, 7, 17, 8], [15, 7, 16, 6]]$

PD code drawn by `SnapPy`:  $[(2, 11, 3, 12), (12, 3, 13, 4), (4, 1, 5, 2), (15, 6, 16, 7), (7, 10, 8, 11), (17, 8, 18, 9), (13, 20, 14, 1), (5, 14, 6, 15), (19, 16, 20, 17), (9, 18, 10, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 6], [0, 6, 7, 8], [0, 5, 1, 1], [1, 4, 9, 2], [2, 7, 3, 2], [3, 6, 9, 8], [3, 7, 9, 9], [5, 8, 8, 7]]$

**Total optimal pinning sets:** 5  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 304  
**Pinning number:** 5

**Average optimal degree:** 2.48  
**Average minimal degree:** 2.48  
**Average overall degree:** 3.04

Table 243: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	28	67	89	71	34	9	1	299
Average degree	2.48	2.74	2.93	3.06	3.16	3.24	3.29	3.33	

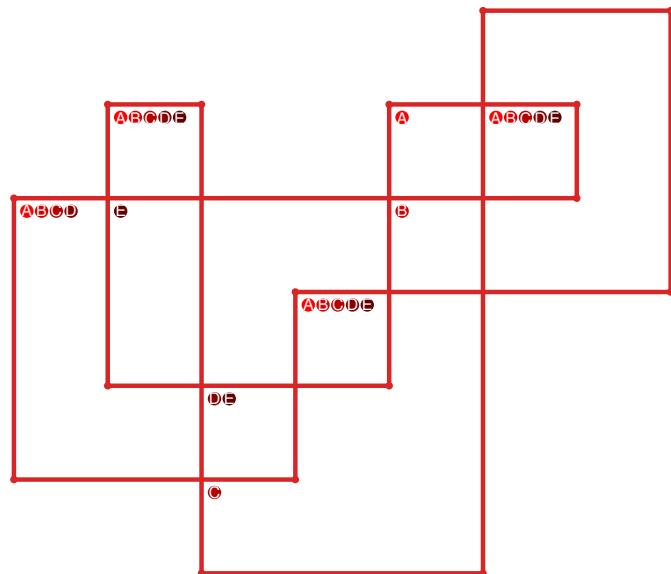


Figure 489: `SnapPy` multiloop plot.

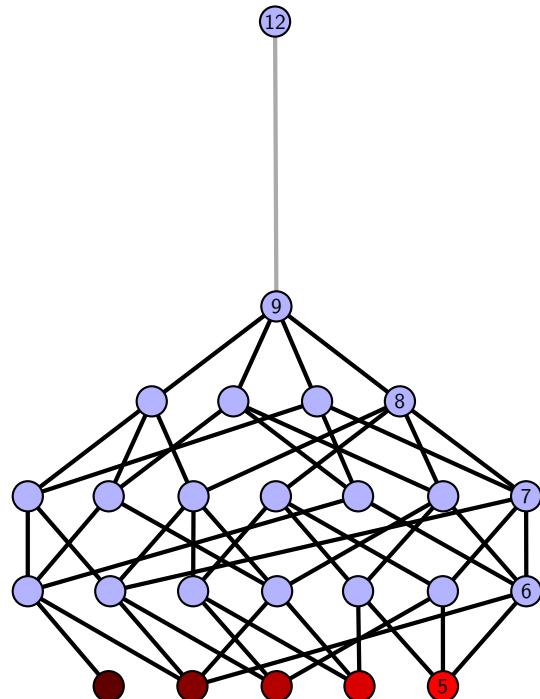


Figure 490: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.97  $[[3, 20, 4, 1], [9, 2, 10, 3], [19, 4, 20, 5], [1, 8, 2, 9], [10, 8, 11, 7], [5, 14, 6, 15], [15, 18, 16, 19], [11, 16, 12, 17], [13, 6, 14, 7], [17, 12, 18, 13]]$

PD code drawn by SnapPy:  $[(7, 20, 8, 1), (17, 2, 18, 3), (11, 4, 12, 5), (5, 8, 6, 9), (19, 6, 20, 7), (15, 10, 16, 11), (3, 12, 4, 13), (9, 14, 10, 15), (13, 16, 14, 17), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 8], [2, 8, 8, 6], [2, 5, 9, 7], [4, 6, 9, 9], [4, 9, 5, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 244: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

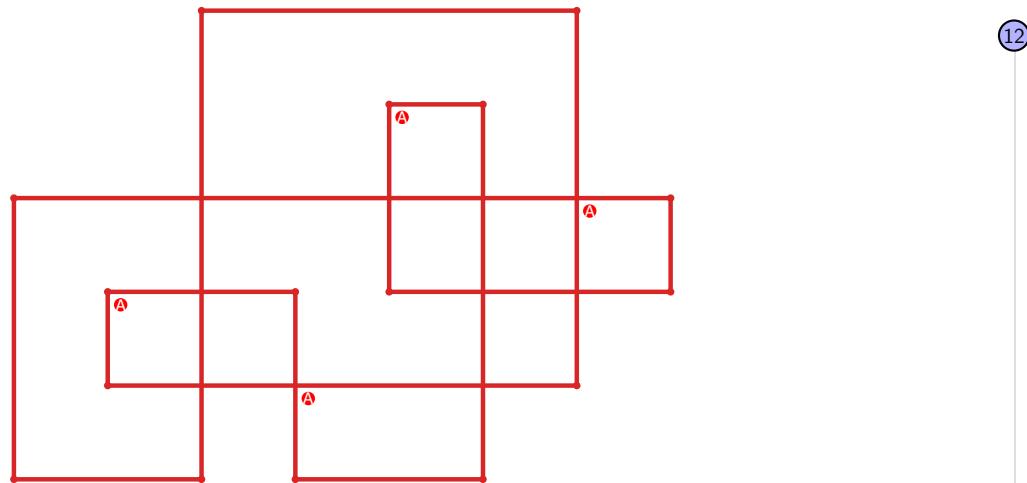


Figure 491: SnapPy multiloop plot.

4

Figure 492: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.98  $[[3, 20, 4, 1], [15, 2, 16, 3], [19, 8, 20, 9], [4, 8, 5, 7], [1, 14, 2, 15], [16, 14, 17, 13], [9, 18, 10, 19], [5, 10, 6, 11], [11, 6, 12, 7], [17, 12, 18, 13]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (9, 4, 10, 5), (5, 2, 6, 3), (17, 6, 18, 7), (3, 8, 4, 9), (15, 10, 16, 11), (11, 14, 12, 15), (19, 12, 20, 13), (7, 16, 8, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 9, 9], [2, 9, 7, 2], [3, 6, 8, 8], [3, 7, 7, 9], [5, 8, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 245: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

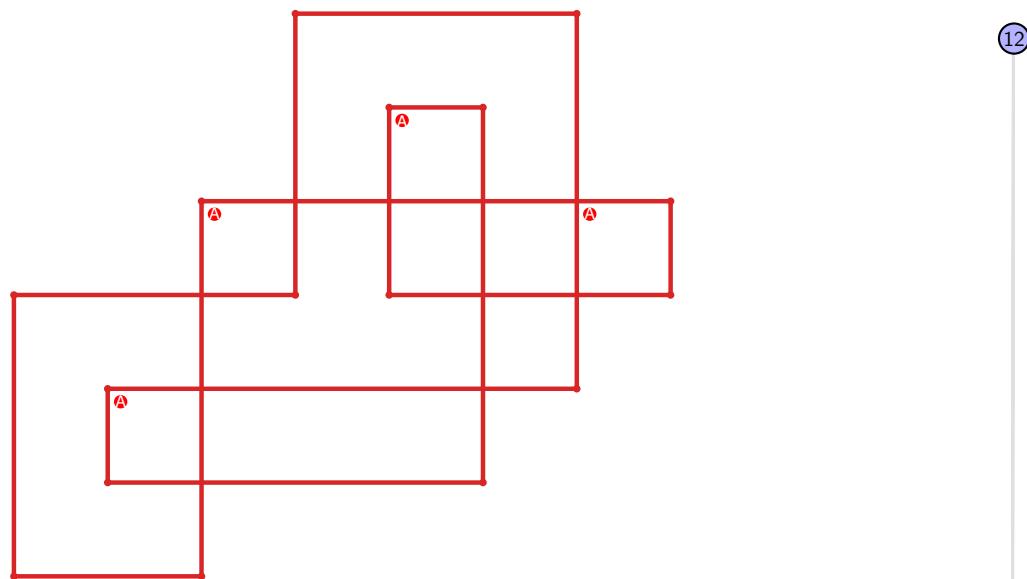


Figure 493: `SnapPy` multiloop plot.

12  
4

Figure 494: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.99  $[[3, 20, 4, 1], [2, 15, 3, 16], [19, 10, 20, 11], [4, 10, 5, 9], [1, 17, 2, 16], [17, 14, 18, 15], [11, 18, 12, 19], [5, 8, 6, 9], [6, 13, 7, 14], [12, 7, 13, 8]]$

PD code drawn by `SnapPy`:  $[(8, 3, 9, 4), (1, 4, 2, 5), (5, 20, 6, 1), (15, 6, 16, 7), (2, 9, 3, 10), (13, 10, 14, 11), (18, 11, 19, 12), (12, 17, 13, 18), (7, 14, 8, 15), (19, 16, 20, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 6], [2, 5, 9, 2], [3, 9, 8, 3], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1  
**Total pinning sets:** 256  
**Pinning number:** 4

**Average optimal degree:** 2.0  
**Average minimal degree:** 2.0  
**Average overall degree:** 2.97

Table 246: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

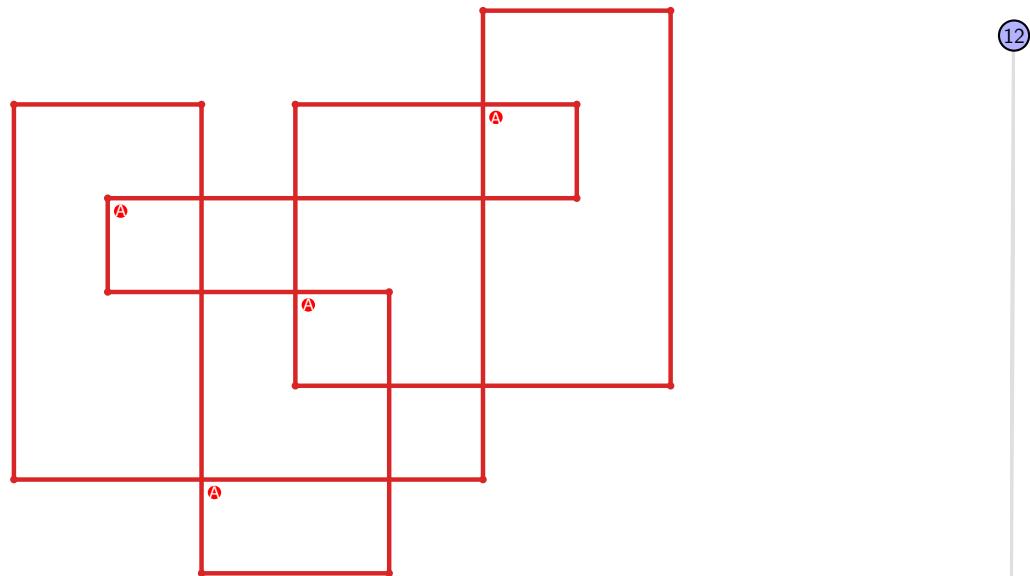


Figure 495: `SnapPy` multiloop plot.

4

Figure 496: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.100** `[[3, 20, 4, 1], [13, 2, 14, 3], [19, 8, 20, 9], [4, 8, 5, 7], [1, 12, 2, 13], [14, 12, 15, 11], [9, 18, 10, 19], [5, 17, 6, 16], [6, 15, 7, 16], [17, 10, 18, 11]]`

PD code drawn by `SnapPy`: `[(11, 20, 12, 1), (5, 2, 6, 3), (17, 6, 18, 7), (15, 8, 16, 9), (9, 12, 10, 13), (19, 10, 20, 11), (4, 13, 5, 14), (14, 3, 15, 4), (7, 16, 8, 17), (1, 18, 2, 19)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 8, 9], [2, 9, 9, 2], [3, 9, 8, 8], [3, 7, 7, 5], [5, 7, 6, 6]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 247: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

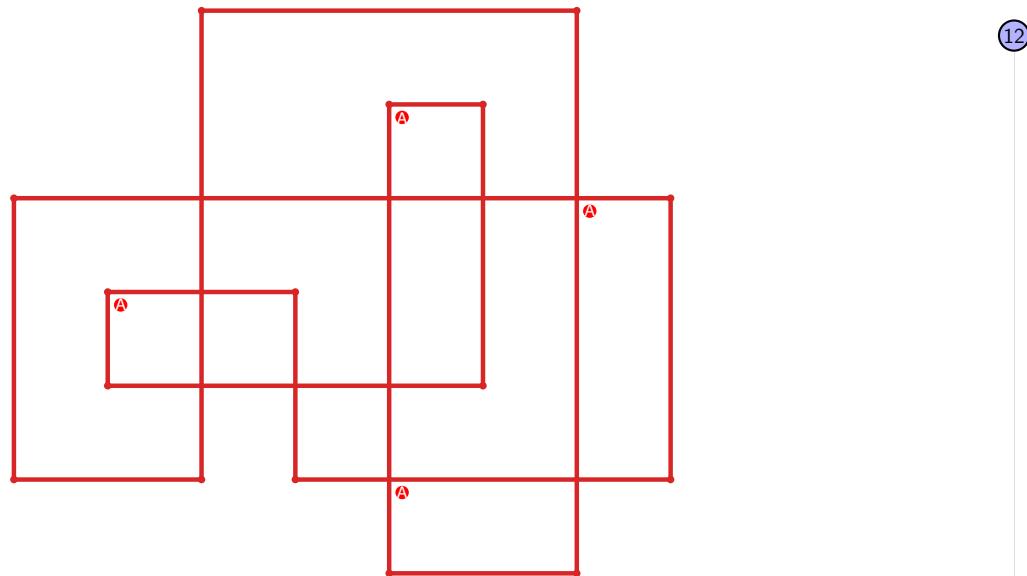


Figure 497: `SnapPy` multiloop plot.



Figure 498: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.101  $[[3, 20, 4, 1], [2, 15, 3, 16], [10, 19, 11, 20], [4, 11, 5, 12], [1, 17, 2, 16], [17, 14, 18, 15], [18, 9, 19, 10], [5, 9, 6, 8], [12, 8, 13, 7], [13, 6, 14, 7]]$

PD code drawn by SnapPy:  $[(11, 4, 12, 5), (5, 10, 6, 11), (6, 3, 7, 4), (16, 7, 17, 8), (2, 9, 3, 10), (15, 12, 16, 13), (20, 13, 1, 14), (14, 19, 15, 20), (8, 17, 9, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 9, 6], [2, 5, 7, 2], [3, 6, 9, 8], [3, 7, 9, 9], [5, 8, 8, 7]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 384  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.25  
**Average overall degree:** 3.03

Table 248: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

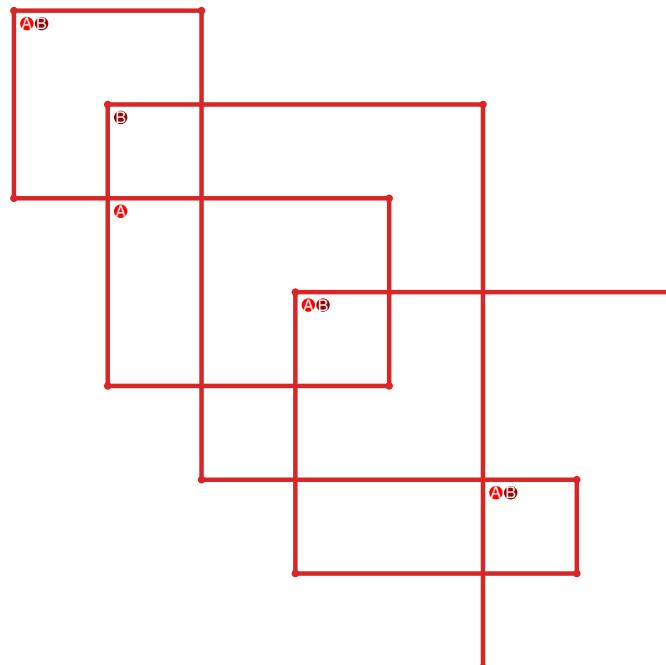


Figure 499: SnapPy multiloop plot.

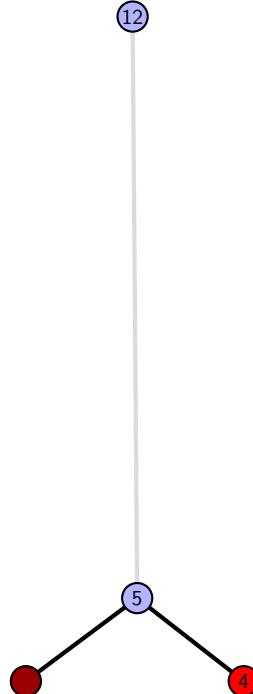


Figure 500: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.102**  $[[3, 20, 4, 1], [2, 9, 3, 10], [19, 4, 20, 5], [1, 11, 2, 10], [11, 8, 12, 9], [5, 12, 6, 13], [13, 18, 14, 19], [7, 16, 8, 17], [6, 16, 7, 15], [17, 14, 18, 15]]$

PD code drawn by `SnapPy`:  $[(17, 2, 18, 3), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (3, 10, 4, 11), (15, 12, 16, 13), (13, 4, 14, 5), (5, 14, 6, 15), (11, 16, 12, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 5], [2, 4, 8, 6], [2, 5, 9, 9], [4, 9, 8, 8], [5, 7, 7, 9], [6, 8, 7, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 224  
 Pinning number: 5

Average optimal degree: 2.27  
 Average minimal degree: 2.27  
 Average overall degree: 2.98

Table 249: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

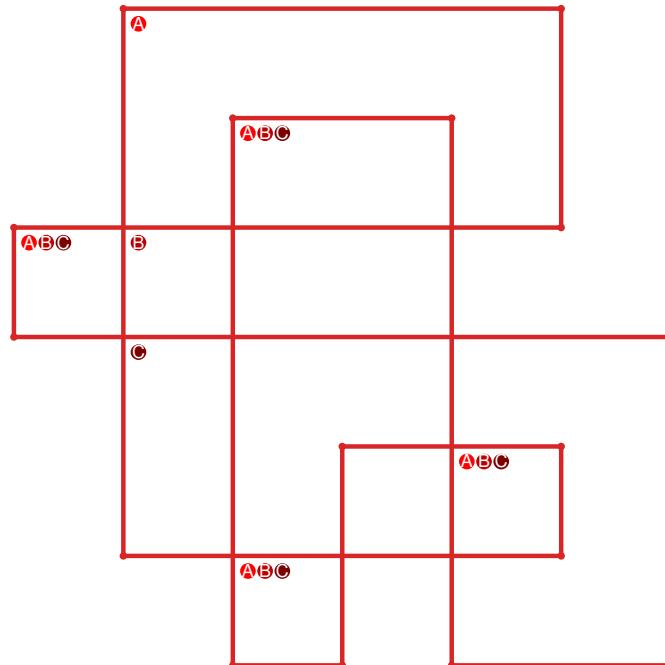


Figure 501: `SnapPy` multiloop plot.

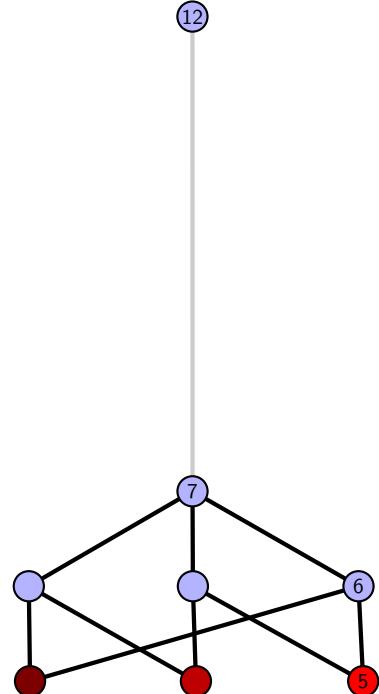


Figure 502: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.103**  $[[3, 20, 4, 1], [2, 13, 3, 14], [19, 10, 20, 11], [4, 10, 5, 9], [1, 15, 2, 14], [15, 12, 16, 13], [11, 16, 12, 17], [7, 18, 8, 19], [5, 8, 6, 9], [17, 6, 18, 7]]$

PD code drawn by `SnapPy`:  $[(15, 4, 16, 5), (3, 6, 4, 7), (7, 2, 8, 3), (17, 8, 18, 9), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (9, 14, 10, 15), (5, 16, 6, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 3], [0, 2, 8, 8], [0, 5, 1, 1], [1, 4, 6, 6], [2, 5, 5, 9], [2, 9, 9, 8], [3, 7, 9, 3], [6, 8, 7, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 224  
 Pinning number: 5

Average optimal degree: 2.27  
 Average minimal degree: 2.27  
 Average overall degree: 2.98

Table 250: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

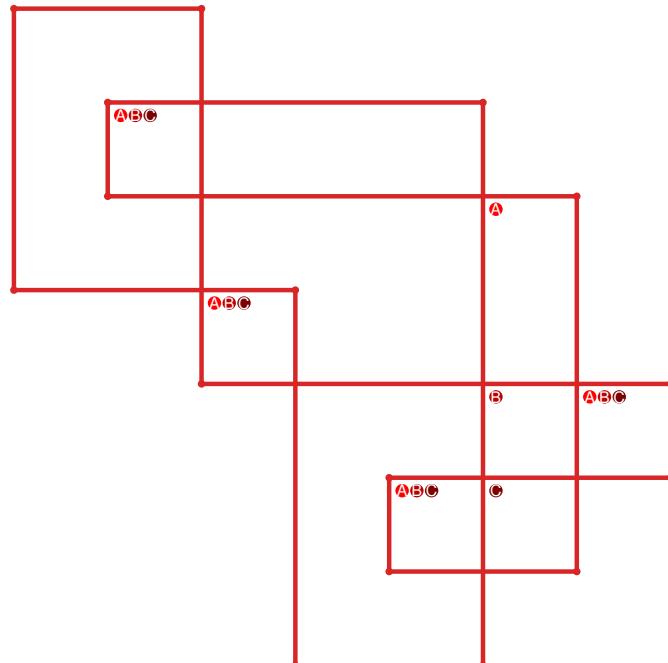


Figure 503: `SnapPy` multiloop plot.

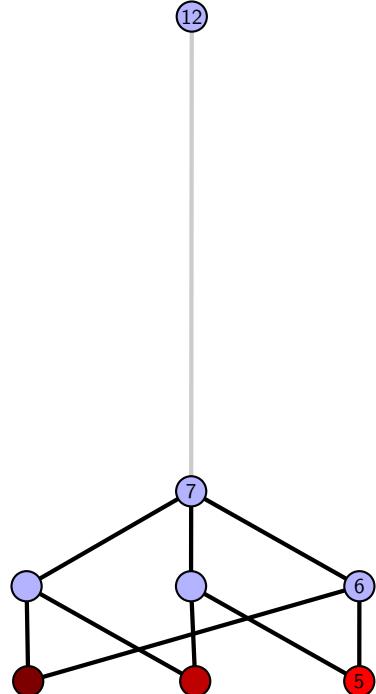


Figure 504: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.104  $[[3, 20, 4, 1], [15, 2, 16, 3], [19, 12, 20, 13], [4, 12, 5, 11], [1, 14, 2, 15], [16, 14, 17, 13], [18, 7, 19, 8], [5, 10, 6, 11], [17, 9, 18, 8], [9, 6, 10, 7]]$

PD code drawn by SnapPy:  $[(3, 20, 4, 1), (12, 1, 13, 2), (2, 11, 3, 12), (19, 4, 20, 5), (5, 18, 6, 19), (13, 6, 14, 7), (7, 10, 8, 11), (15, 8, 16, 9), (17, 14, 18, 15), (9, 16, 10, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 8, 9], [3, 9, 9, 3], [5, 9, 6, 6], [6, 8, 7, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 224  
 Pinning number: 5

Average optimal degree: 2.27  
 Average minimal degree: 2.27  
 Average overall degree: 2.98

Table 251: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

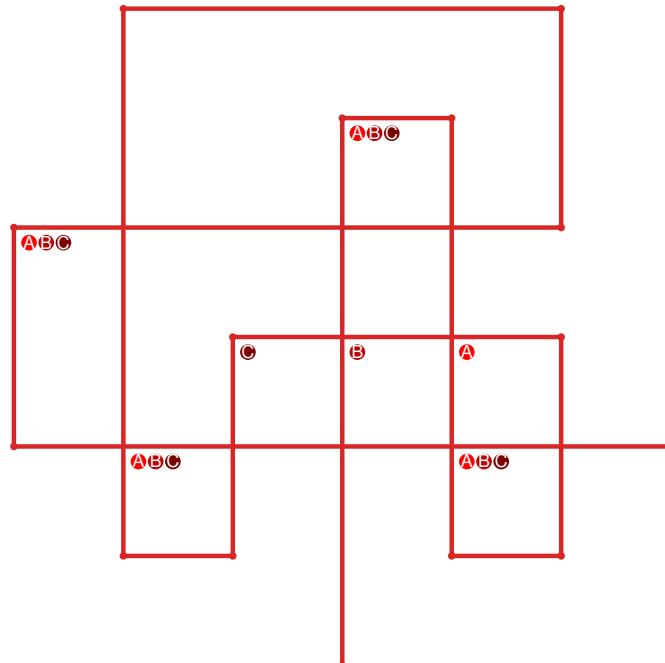


Figure 505: SnapPy multiloop plot.

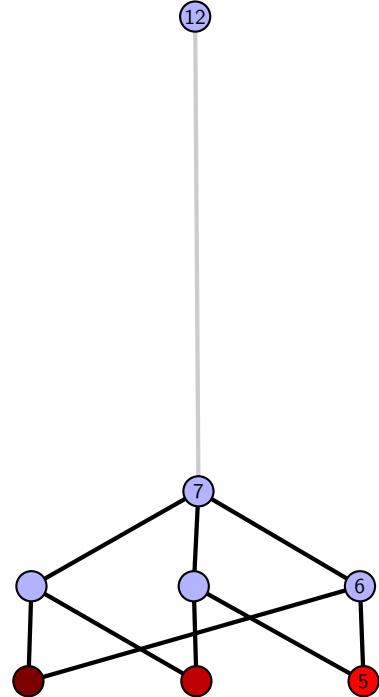


Figure 506: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.105  $[[3, 20, 4, 1], [2, 11, 3, 12], [14, 19, 15, 20], [4, 15, 5, 16], [1, 13, 2, 12], [13, 10, 14, 11], [18, 5, 19, 6], [16, 8, 17, 7], [9, 6, 10, 7], [17, 8, 18, 9]]$

PD code drawn by `SnapPy`:  $[(16, 3, 17, 4), (7, 4, 8, 5), (5, 14, 6, 15), (15, 6, 16, 7), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (2, 13, 3, 14), (12, 17, 13, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 6, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 3], [3, 9, 9, 8], [5, 7, 9, 6], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 10

**Average minimal degree:** 2.75

**Total pinning sets:** 496

**Average overall degree:** 3.12

**Pinning number:** 4

Table 252: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	5	0	0	0	0	0	0	9
Nonminimal pinning sets	0	8	51	123	147	103	43	10	1	486
Average degree	2.5	2.75	2.92	3.05	3.16	3.23	3.27	3.31	3.33	

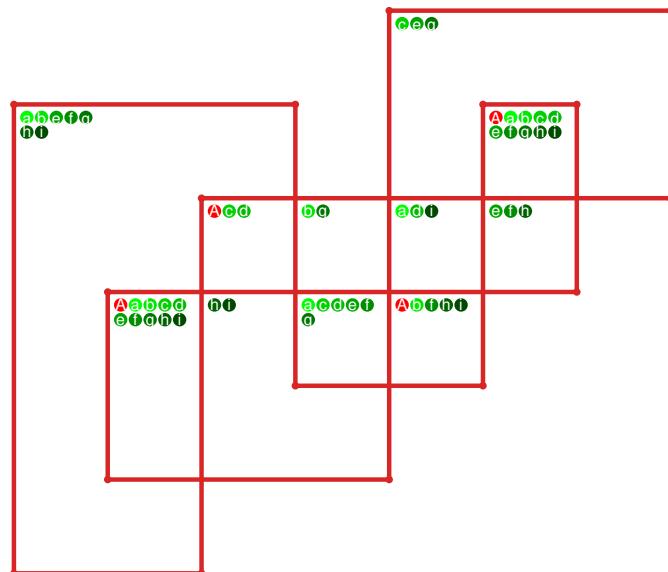


Figure 507: `SnapPy` multiloop plot.

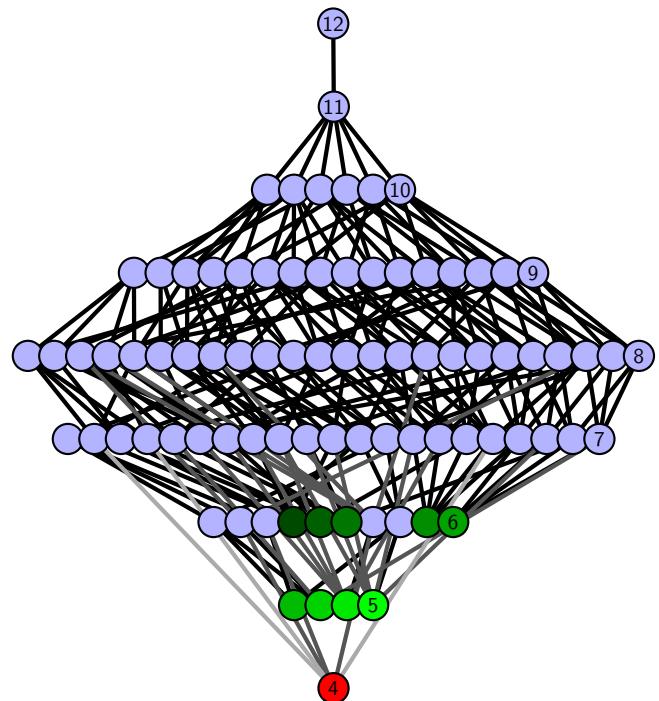


Figure 508: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.106**  $[[3, 20, 4, 1], [2, 9, 3, 10], [12, 19, 13, 20], [4, 13, 5, 14], [1, 11, 2, 10], [11, 8, 12, 9], [18, 5, 19, 6], [14, 18, 15, 17], [7, 16, 8, 17], [6, 16, 7, 15]]$

PD code drawn by `SnapPy`:  $[(16, 3, 17, 4), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (2, 11, 3, 12), (13, 4, 14, 5), (5, 14, 6, 15), (15, 12, 16, 13), (10, 17, 11, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 6, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 9, 7, 3], [3, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 5  
 Total minimal pinning sets: 7  
 Total pinning sets: 376  
 Pinning number: 5

Average optimal degree: 2.64  
 Average minimal degree: 2.67  
 Average overall degree: 3.11

Table 253: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	30	83	114	90	41	10	1	369
Average degree	2.64	2.85	3.01	3.12	3.21	3.27	3.31	3.33	

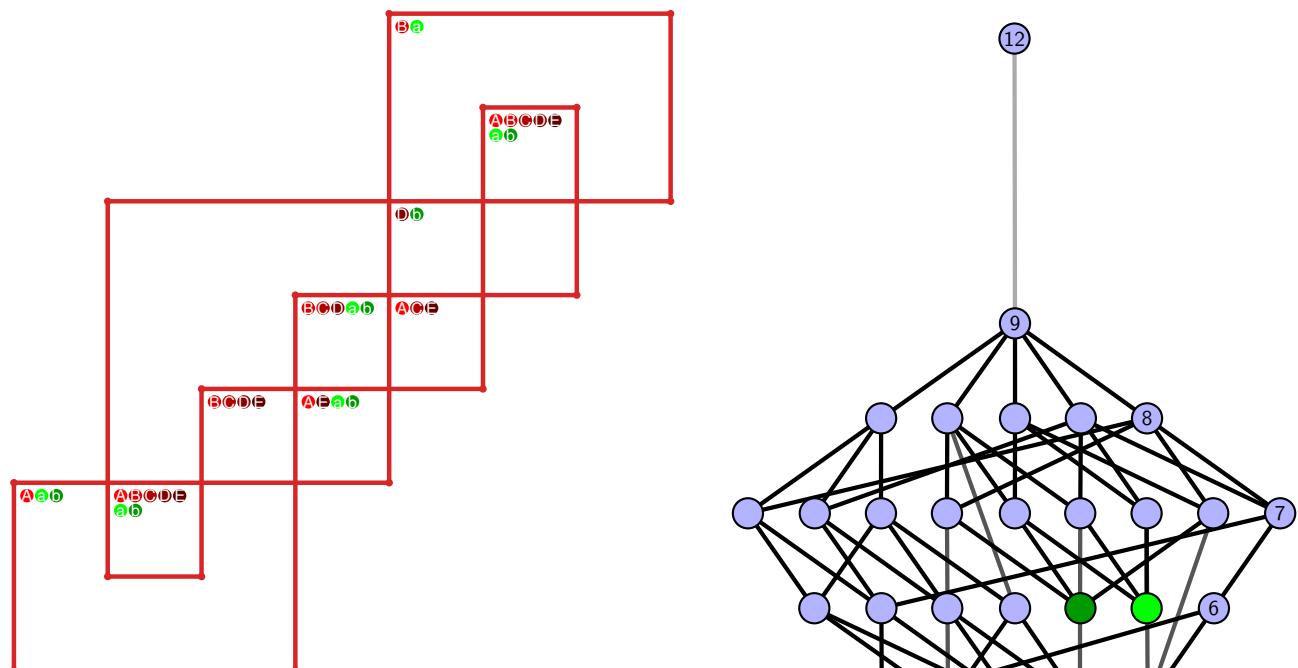


Figure 509: `SnapPy` multiloop plot.

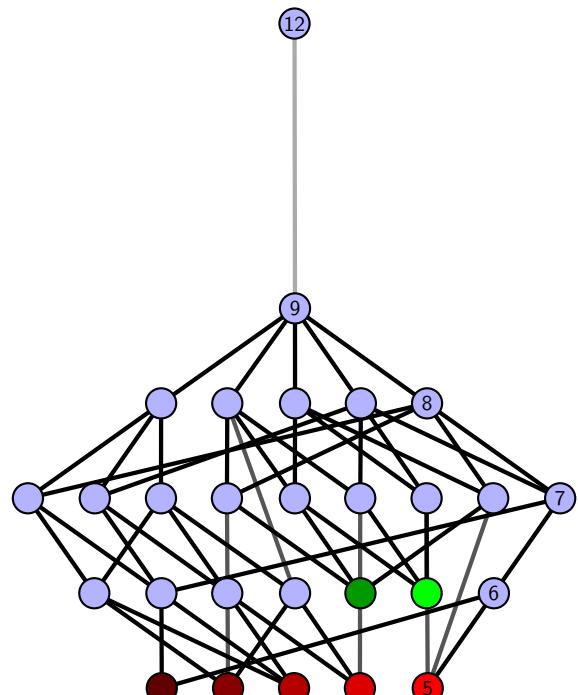


Figure 510: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.107  $[[3, 20, 4, 1], [2, 9, 3, 10], [12, 19, 13, 20], [4, 13, 5, 14], [1, 11, 2, 10], [11, 8, 12, 9], [18, 15, 19, 16], [5, 15, 6, 14], [7, 16, 8, 17], [17, 6, 18, 7]]$

PD code drawn by `SnapPy`:  $[(15, 4, 16, 5), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (2, 11, 3, 12), (12, 3, 13, 4), (16, 13, 17, 14), (5, 14, 6, 15), (10, 17, 11, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 7], [3, 6, 9, 3], [5, 9, 9, 6], [6, 8, 8, 7]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 9  
 Total pinning sets: 300  
 Pinning number: 5

Average optimal degree: 2.5  
 Average minimal degree: 2.59  
 Average overall degree: 3.06

Table 254: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	0	7
Nonminimal pinning sets	0	14	64	93	75	35	9	1	291
Average degree	2.5	2.7	2.91	3.06	3.17	3.24	3.29	3.33	

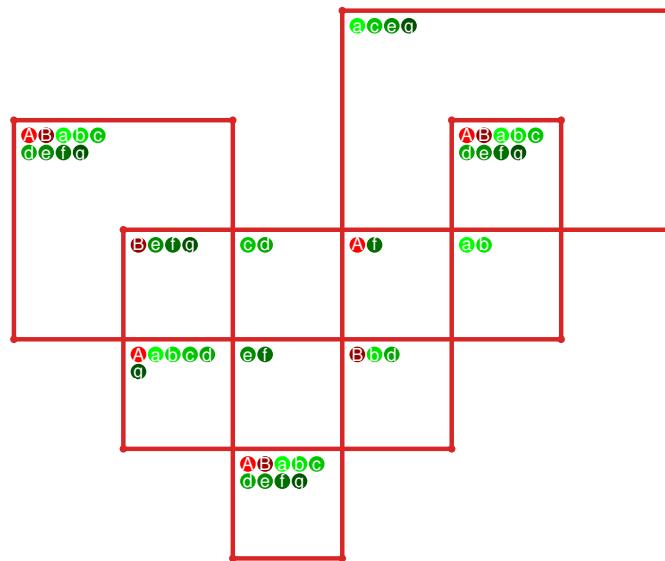


Figure 511: `SnapPy` multiloop plot.

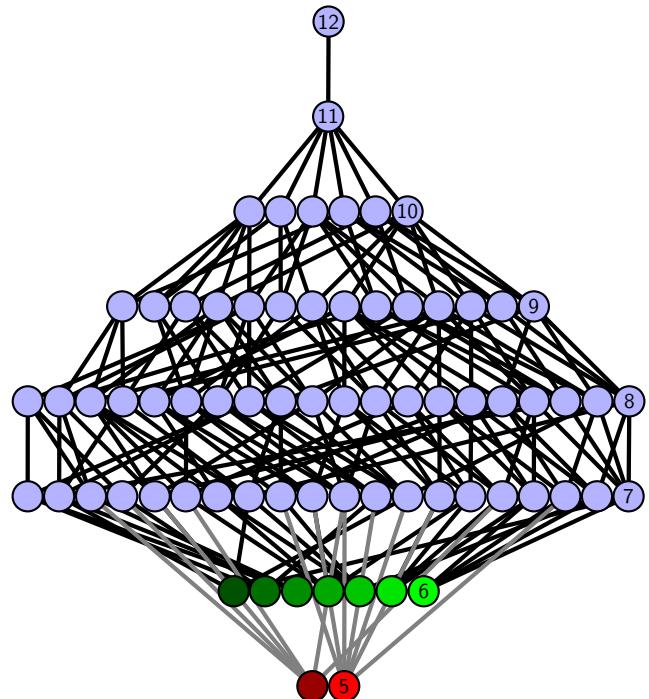


Figure 512: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.108**  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 10, 20, 11], [4, 10, 5, 9], [1, 12, 2, 13], [14, 12, 15, 11], [18, 5, 19, 6], [8, 17, 9, 18], [15, 7, 16, 6], [16, 7, 17, 8]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (16, 3, 17, 4), (4, 13, 5, 14), (14, 5, 15, 6), (7, 2, 8, 3), (17, 8, 18, 9), (9, 12, 10, 13), (19, 10, 20, 11), (6, 15, 7, 16), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 6, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 7, 3], [3, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 288  
**Pinning number:** 5

**Average optimal degree:** 2.45  
**Average minimal degree:** 2.45  
**Average overall degree:** 3.04

Table 255: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.45	2.72	2.91	3.05	3.16	3.24	3.29	3.33	

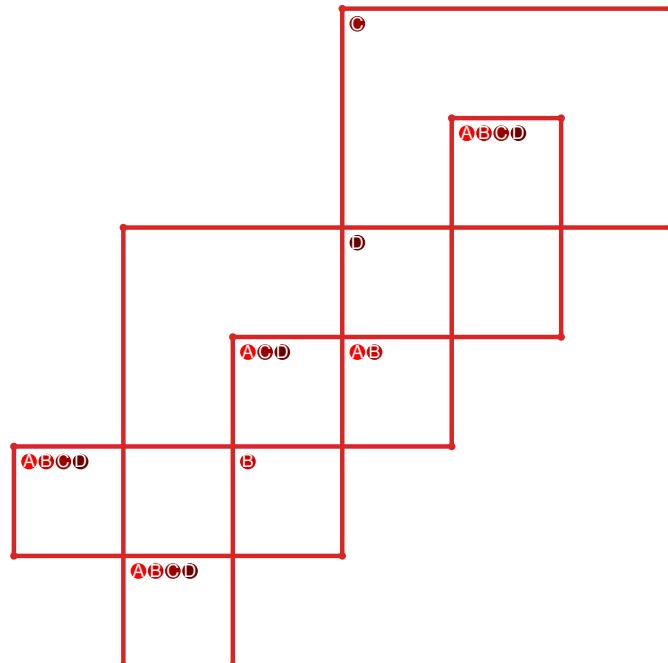


Figure 513: `SnapPy` multiloop plot.

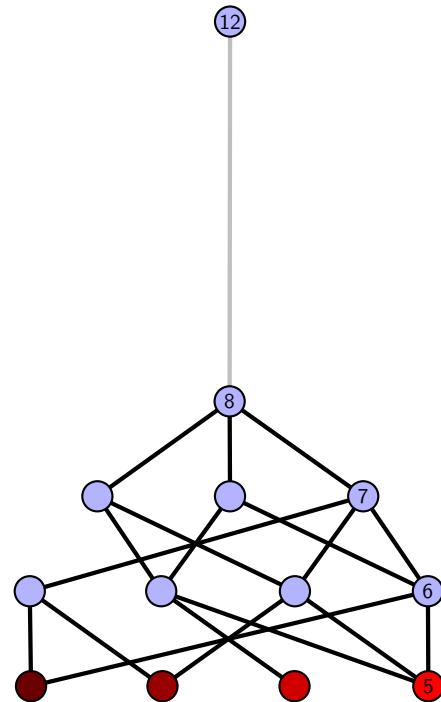


Figure 514: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.109 [[15, 20, 16, 1], [14, 11, 15, 12], [4, 19, 5, 20], [16, 5, 17, 6], [1, 13, 2, 12], [2, 13, 3, 14], [3, 10, 4, 11], [7, 18, 8, 19], [17, 8, 18, 9], [6, 9, 7, 10]]

PD code drawn by SnapPy: [(13, 2, 14, 3), (14, 5, 15, 6), (3, 6, 4, 7), (20, 7, 1, 8), (11, 8, 12, 9), (9, 18, 10, 19), (19, 10, 20, 11), (4, 15, 5, 16), (1, 16, 2, 17), (12, 17, 13, 18)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 3], [0, 2, 8, 9], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 9, 2], [2, 9, 8, 8], [3, 7, 7, 9], [3, 8, 7, 6]]

Total optimal pinning sets: 6  
 Total minimal pinning sets: 9  
 Total pinning sets: 420  
 Pinning number: 5

Average optimal degree: 2.67  
 Average minimal degree: 2.7  
 Average overall degree: 3.12

Table 256: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	36	98	128	96	42	10	1	411
Average degree	2.67	2.87	3.02	3.14	3.22	3.27	3.31	3.33	

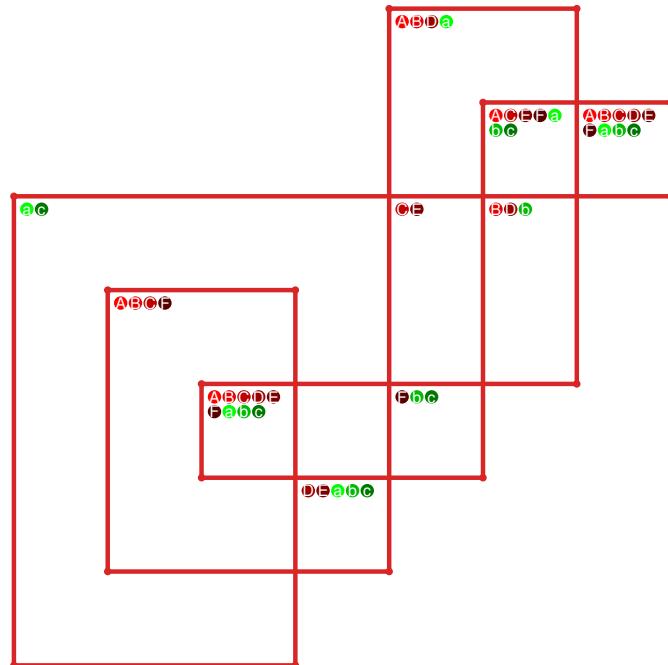


Figure 515: SnapPy multiloop plot.

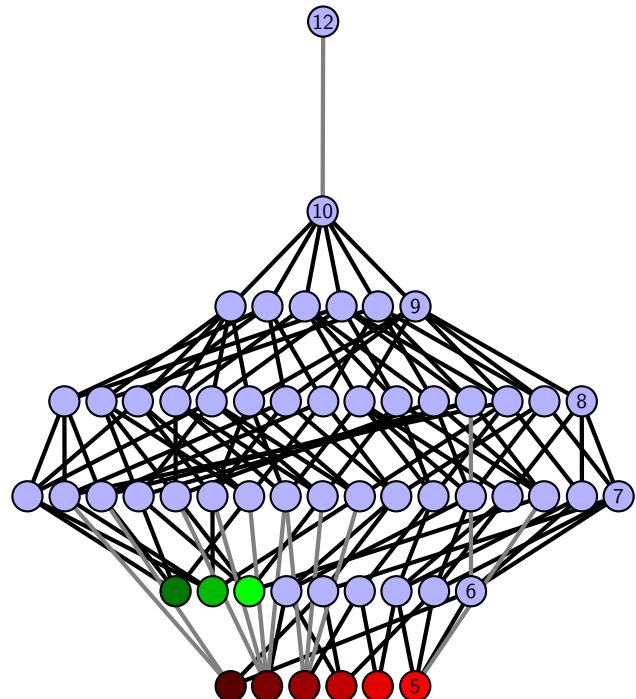


Figure 516: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.110  $[[15, 20, 16, 1], [14, 3, 15, 4], [6, 19, 7, 20], [16, 7, 17, 8], [1, 5, 2, 4], [2, 13, 3, 14], [5, 12, 6, 13], [9, 18, 10, 19], [17, 10, 18, 11], [8, 11, 9, 12]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (2, 19, 3, 20), (14, 5, 15, 6), (15, 8, 16, 9), (6, 9, 7, 10), (3, 10, 4, 11), (20, 11, 1, 12), (7, 16, 8, 17), (4, 17, 5, 18), (13, 18, 14, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 8, 9], [0, 6, 5, 1], [1, 4, 6, 1], [2, 5, 4, 9], [2, 9, 8, 8], [3, 7, 7, 9], [3, 8, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 7

**Average minimal degree:** 2.67

**Total pinning sets:** 472

**Average overall degree:** 3.11

**Pinning number:** 4

Table 257: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	0	0	0	0	0	0	0	6
Nonminimal pinning sets	0	8	57	115	135	97	42	10	1	465
Average degree	2.5	2.73	2.91	3.05	3.14	3.22	3.27	3.31	3.33	

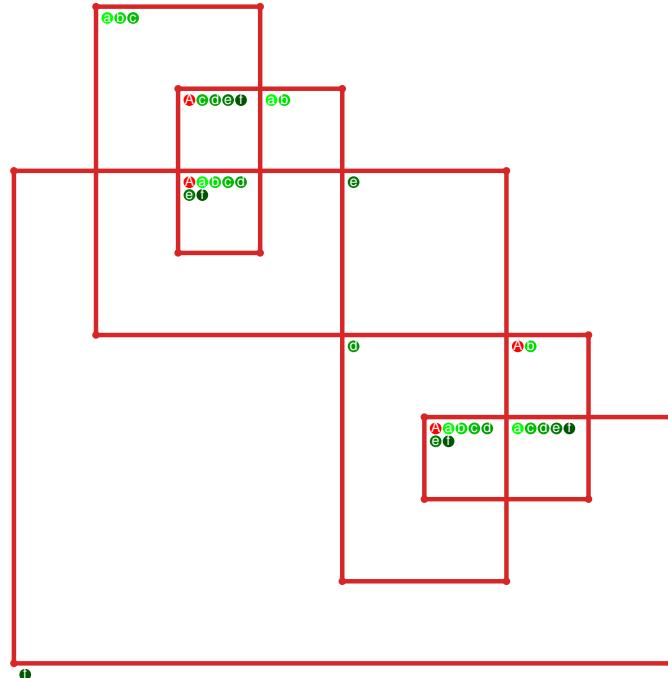


Figure 517: `SnapPy` multiloop plot.

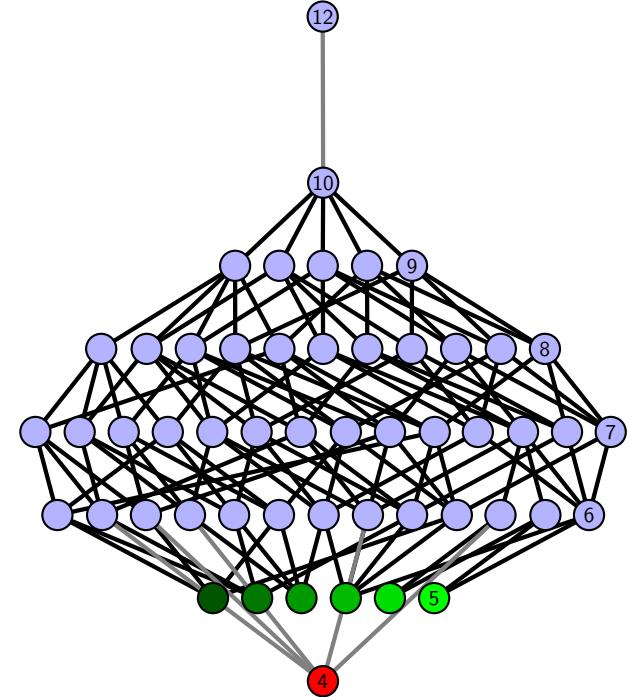


Figure 518: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.111  $[[15, 20, 16, 1], [3, 14, 4, 15], [19, 10, 20, 11], [16, 10, 17, 9], [1, 12, 2, 13], [13, 2, 14, 3], [4, 12, 5, 11], [18, 7, 19, 8], [17, 7, 18, 6], [8, 5, 9, 6]]$

PD code drawn by SnapPy:  $[(11, 20, 12, 1), (6, 3, 7, 4), (4, 15, 5, 16), (16, 5, 17, 6), (7, 14, 8, 15), (17, 8, 18, 9), (9, 2, 10, 3), (19, 10, 20, 11), (1, 12, 2, 13), (13, 18, 14, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 3], [0, 2, 8, 9], [0, 6, 5, 5], [1, 4, 4, 1], [1, 4, 9, 2], [2, 9, 8, 8], [3, 7, 7, 9], [3, 8, 7, 6]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 384  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.25  
**Average overall degree:** 3.03

Table 258: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

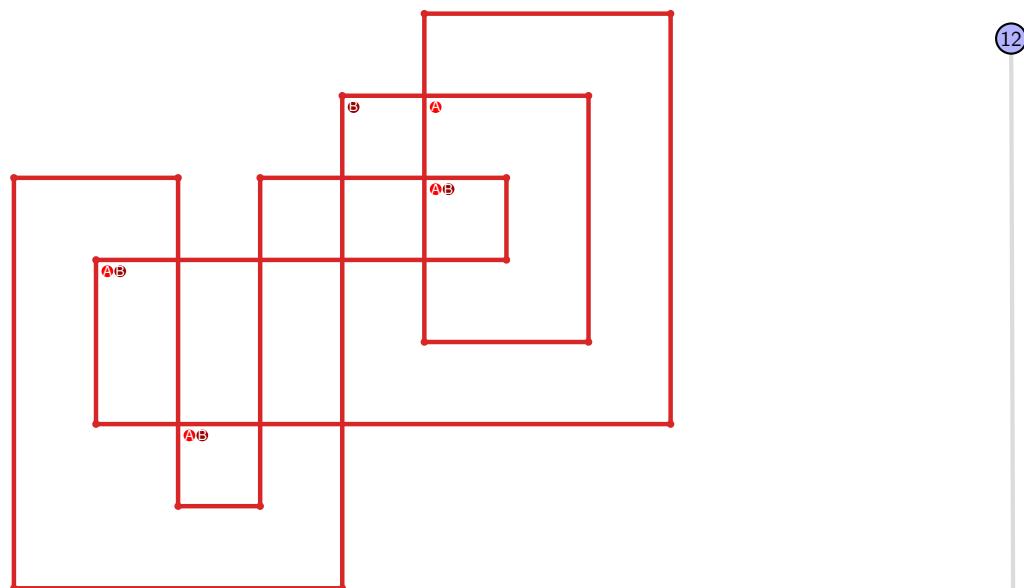


Figure 519: SnapPy multiloop plot.

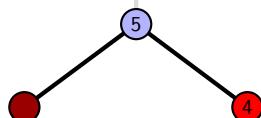


Figure 520: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.112**  $[[3, 20, 4, 1], [9, 2, 10, 3], [19, 6, 20, 7], [4, 16, 5, 15], [1, 8, 2, 9], [10, 8, 11, 7], [18, 13, 19, 14], [5, 16, 6, 17], [14, 11, 15, 12], [12, 17, 13, 18]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (17, 4, 18, 5), (5, 8, 6, 9), (19, 6, 20, 7), (12, 9, 13, 10), (16, 11, 17, 12), (13, 2, 14, 3), (3, 14, 4, 15), (10, 15, 11, 16), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 7], [0, 7, 7, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 9], [2, 9, 3, 3], [3, 9, 6, 5], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.38

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.38

**Total pinning sets:** 480

**Average overall degree:** 3.05

**Pinning number:** 4

Table 259: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	26	74	121	125	84	36	9	1	476
Average degree	2.38	2.68	2.87	3.01	3.11	3.19	3.24	3.29	3.33	

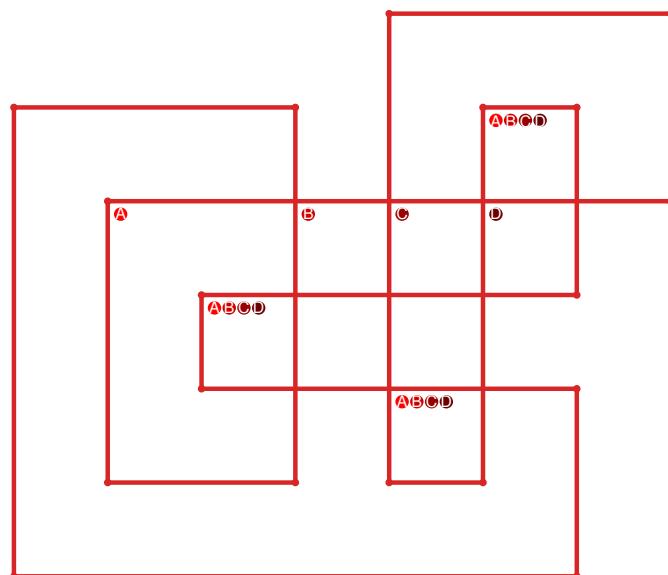


Figure 521: `SnapPy` multiloop plot.

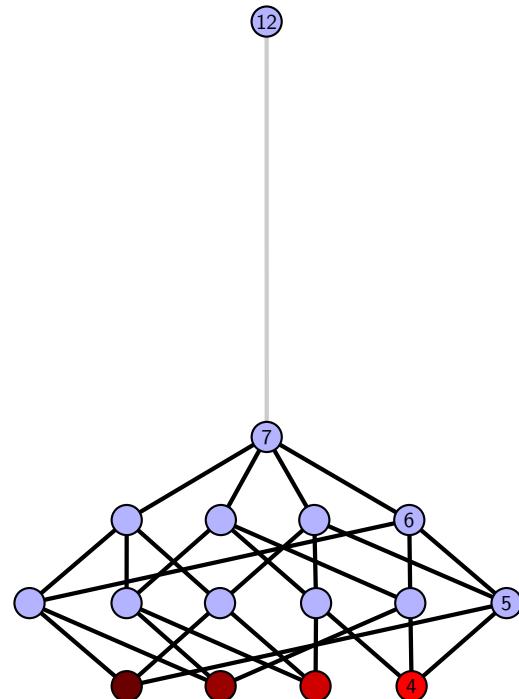


Figure 522: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.113  $[[3, 20, 4, 1], [11, 2, 12, 3], [19, 8, 20, 9], [4, 18, 5, 17], [1, 10, 2, 11], [12, 10, 13, 9], [7, 18, 8, 19], [5, 15, 6, 14], [16, 13, 17, 14], [6, 15, 7, 16]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (17, 6, 18, 7), (7, 10, 8, 11), (19, 8, 20, 9), (14, 11, 15, 12), (12, 3, 13, 4), (4, 13, 5, 14), (15, 2, 16, 3), (5, 16, 6, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 6], [0, 6, 7, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 9, 3, 2], [3, 9, 9, 8], [3, 7, 9, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 384  
**Pinning number:** 4

**Average optimal degree:** 2.25  
**Average minimal degree:** 2.25  
**Average overall degree:** 3.03

Table 260: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

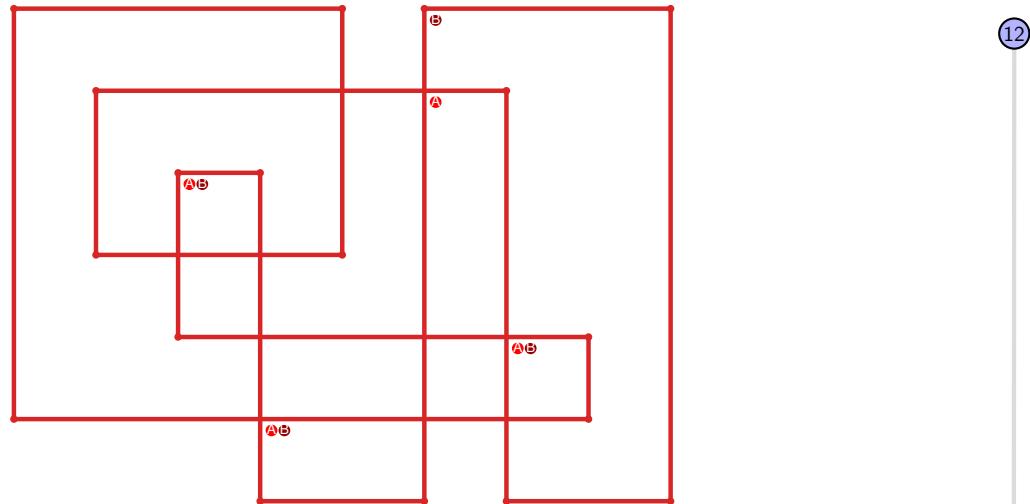


Figure 523: `SnapPy` multiloop plot.

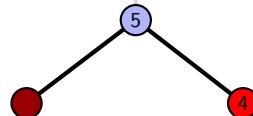


Figure 524: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.114 [[3, 20, 4, 1], [2, 11, 3, 12], [14, 19, 15, 20], [4, 15, 5, 16], [1, 13, 2, 12], [13, 10, 14, 11], [7, 18, 8, 19], [5, 17, 6, 16], [6, 9, 7, 10], [17, 8, 18, 9]]

PD code drawn by `SnapPy`: [(15, 6, 16, 7), (4, 7, 5, 8), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (2, 13, 3, 14), (14, 3, 15, 4), (5, 16, 6, 17), (12, 17, 13, 18), (1, 18, 2, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 9], [3, 9, 8, 3], [5, 7, 9, 6], [6, 8, 7, 6]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 8  
**Total pinning sets:** 324  
**Pinning number:** 5

**Average optimal degree:** 2.5  
**Average minimal degree:** 2.54  
**Average overall degree:** 3.05

Table 261: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	24	73	98	76	35	9	1	316
Average degree	2.5	2.73	2.93	3.07	3.17	3.24	3.29	3.33	

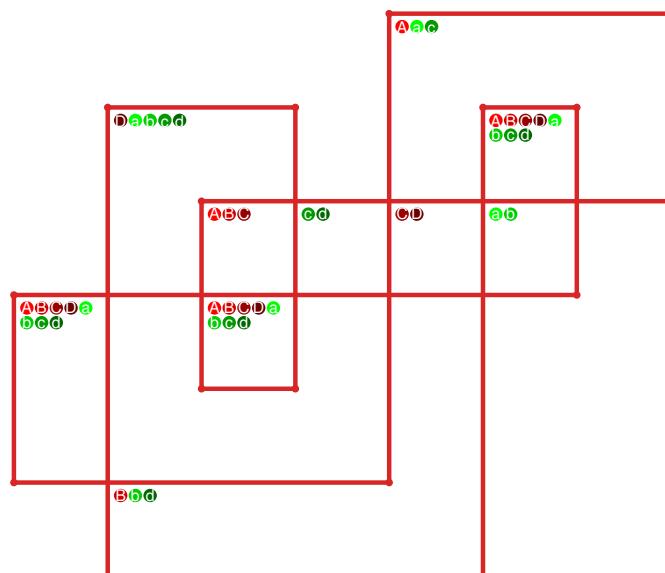


Figure 525: `SnapPy` multiloop plot.

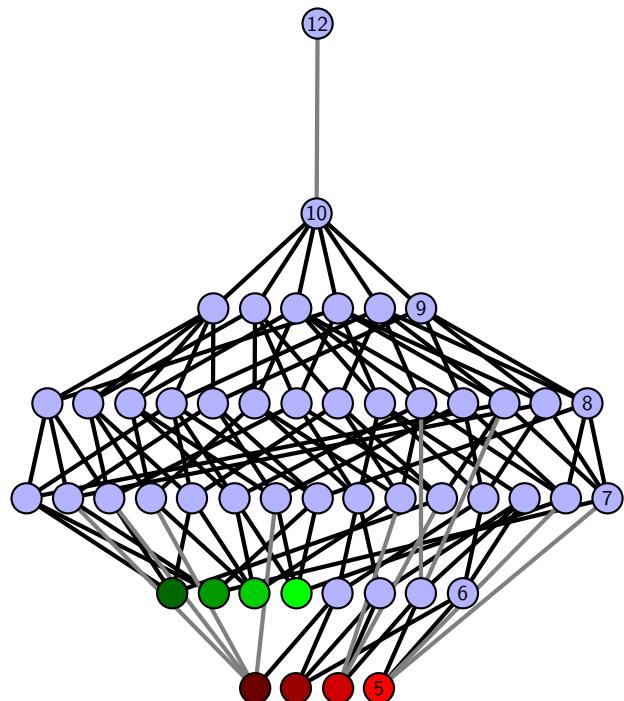


Figure 526: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.115  $[[3, 20, 4, 1], [2, 15, 3, 16], [8, 19, 9, 20], [4, 9, 5, 10], [1, 17, 2, 16], [17, 14, 18, 15], [18, 7, 19, 8], [5, 12, 6, 13], [10, 13, 11, 14], [11, 6, 12, 7]]$

PD code drawn by `SnapPy`:  $[(9, 4, 10, 5), (16, 5, 17, 6), (2, 7, 3, 8), (3, 10, 4, 11), (8, 11, 9, 12), (15, 12, 16, 13), (20, 13, 1, 14), (14, 19, 15, 20), (6, 17, 7, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 8, 6], [2, 5, 9, 2], [3, 9, 9, 8], [3, 7, 9, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.25

**Total pinning sets:** 384

**Average overall degree:** 3.03

**Pinning number:** 4

Table 262: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

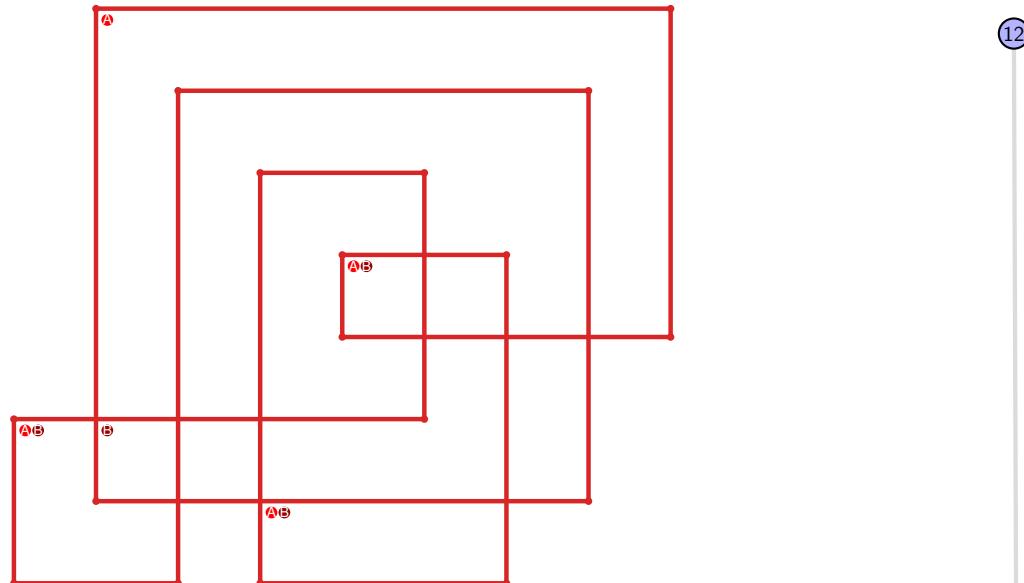


Figure 527: `SnapPy` multiloop plot.

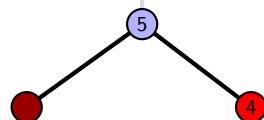


Figure 528: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.116**  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 10, 20, 11], [4, 10, 5, 9], [1, 12, 2, 13], [14, 12, 15, 11], [18, 7, 19, 8], [5, 17, 6, 16], [8, 15, 9, 16], [6, 17, 7, 18]]$

PD code drawn by `SnapPy`:  $[(15, 4, 16, 5), (6, 1, 7, 2), (16, 7, 17, 8), (8, 11, 9, 12), (18, 9, 19, 10), (5, 12, 6, 13), (13, 2, 14, 3), (3, 14, 4, 15), (20, 17, 1, 18), (10, 19, 11, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 9], [3, 9, 9, 8], [3, 7, 6, 5], [6, 7, 7, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 448  
 Pinning number: 4

Average optimal degree: 2.33  
 Average minimal degree: 2.33  
 Average overall degree: 3.05

Table 263: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	21	64	111	120	83	36	9	1	445
Average degree	2.33	2.65	2.85	3.0	3.1	3.18	3.24	3.29	3.33	

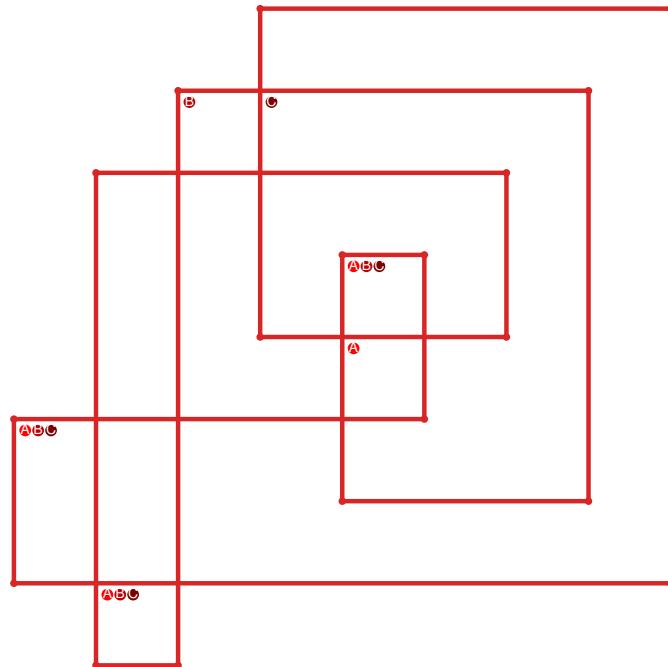


Figure 529: `SnapPy` multiloop plot.

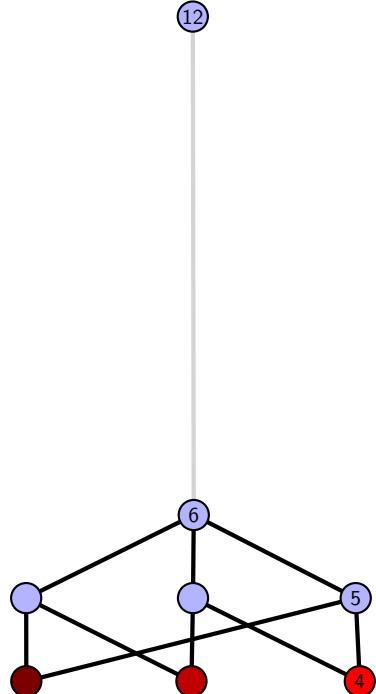


Figure 530: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.117  $[[3, 20, 4, 1], [2, 11, 3, 12], [14, 19, 15, 20], [4, 15, 5, 16], [1, 13, 2, 12], [13, 10, 14, 11], [7, 18, 8, 19], [5, 8, 6, 9], [16, 9, 17, 10], [17, 6, 18, 7]]$

PD code drawn by SnapPy:  $[(15, 4, 16, 5), (3, 6, 4, 7), (14, 7, 15, 8), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (2, 13, 3, 14), (5, 16, 6, 17), (12, 17, 13, 18), (1, 18, 2, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 9, 9, 7], [3, 6, 9, 8], [3, 7, 9, 5], [6, 8, 7, 6]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 8  
 Total pinning sets: 508  
 Pinning number: 4

Average optimal degree: 2.5  
 Average minimal degree: 2.76  
 Average overall degree: 3.12

Table 264: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	1	0	0	0	0	0	0	7
Nonminimal pinning sets	0	8	60	127	148	103	43	10	1	500
Average degree	2.5	2.74	2.93	3.07	3.16	3.23	3.27	3.31	3.33	

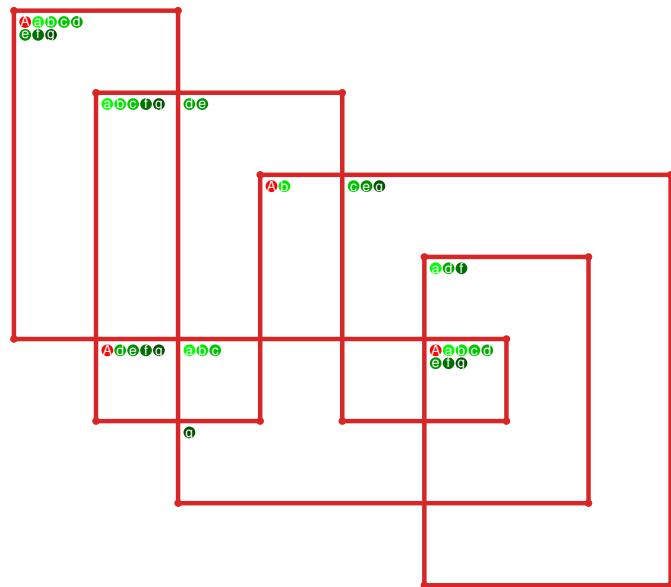


Figure 531: SnapPy multiloop plot.

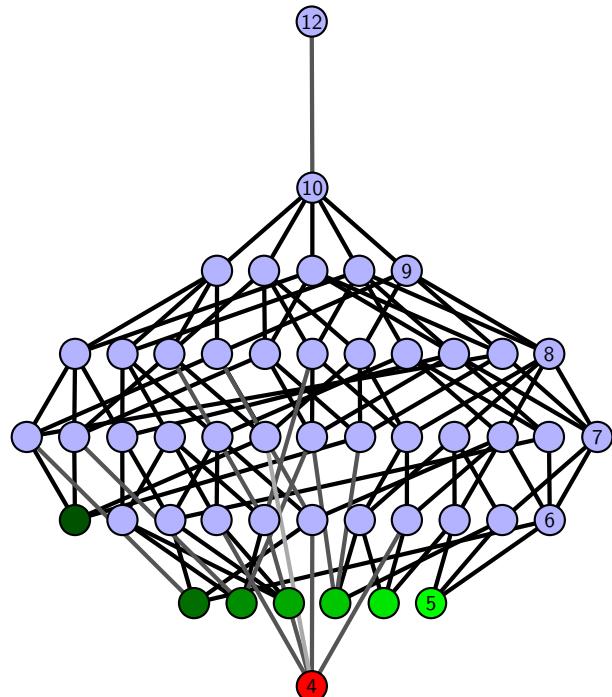


Figure 532: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.118  $[[3, 20, 4, 1], [2, 9, 3, 10], [12, 19, 13, 20], [4, 13, 5, 14], [1, 11, 2, 10], [11, 8, 12, 9], [15, 18, 16, 19], [5, 16, 6, 17], [14, 7, 15, 8], [17, 6, 18, 7]]$

PD code drawn by `SnapPy`:  $[(15, 4, 16, 5), (12, 5, 13, 6), (9, 6, 10, 7), (20, 7, 1, 8), (8, 19, 9, 20), (2, 11, 3, 12), (3, 14, 4, 15), (13, 16, 14, 17), (10, 17, 11, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 5, 6, 3], [0, 2, 7, 8], [0, 5, 1, 1], [1, 4, 8, 2], [2, 8, 9, 7], [3, 6, 9, 9], [3, 9, 6, 5], [6, 8, 7, 7]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 9  
 Total pinning sets: 600  
 Pinning number: 4

Average optimal degree: 2.62  
 Average minimal degree: 2.72  
 Average overall degree: 3.13

Table 265: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	0	0	7
Nonminimal pinning sets	0	16	85	157	168	110	44	10	1	591
Average degree	2.62	2.81	2.97	3.09	3.17	3.23	3.28	3.31	3.33	

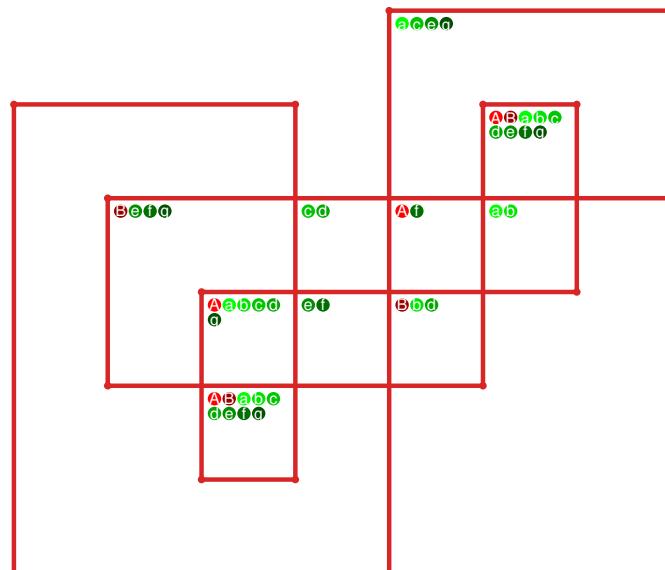


Figure 533: `SnapPy` multiloop plot.

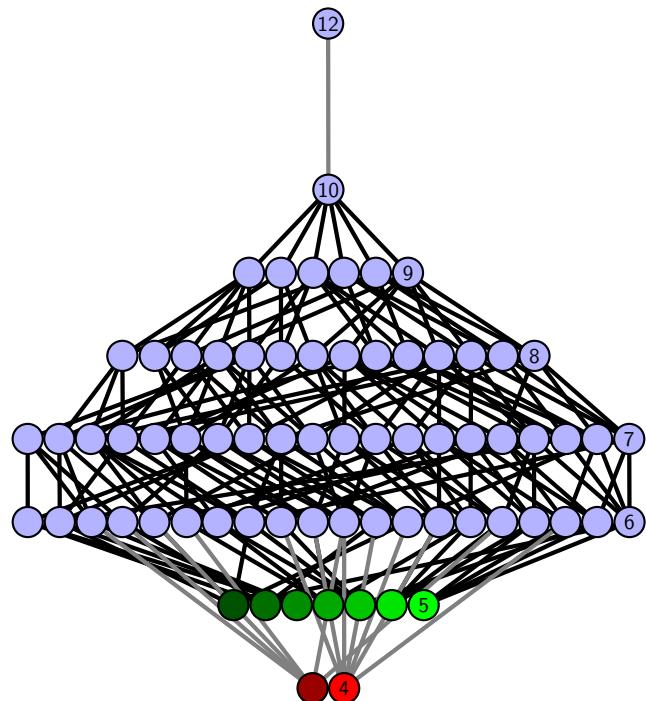


Figure 534: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.119**  $[[3, 20, 4, 1], [15, 2, 16, 3], [19, 4, 20, 5], [1, 14, 2, 15], [16, 14, 17, 13], [5, 11, 6, 10], [18, 9, 19, 10], [17, 9, 18, 8], [12, 7, 13, 8], [11, 7, 12, 6]]$

PD code drawn by `SnapPy`:  $[(16, 1, 17, 2), (3, 8, 4, 9), (9, 4, 10, 5), (5, 14, 6, 15), (15, 6, 16, 7), (7, 2, 8, 3), (10, 13, 11, 14), (18, 11, 19, 12), (20, 17, 1, 18), (12, 19, 13, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 8], [2, 9, 9, 6], [2, 5, 7, 7], [4, 6, 6, 8], [4, 7, 9, 9], [5, 8, 8, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 266: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

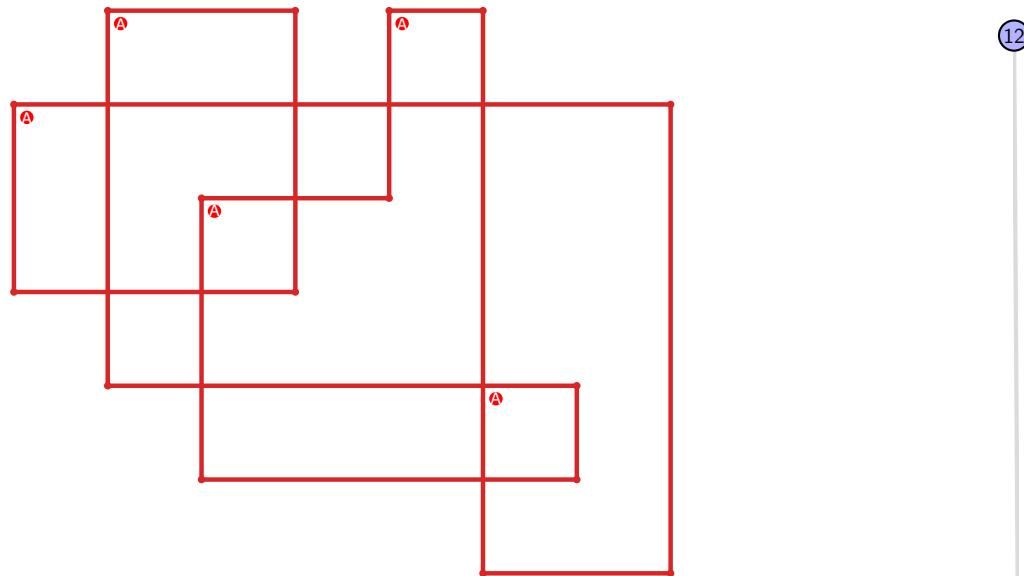


Figure 535: `SnapPy` multiloop plot.



Figure 536: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.120**  $[[3, 20, 4, 1], [13, 2, 14, 3], [19, 4, 20, 5], [1, 12, 2, 13], [14, 12, 15, 11], [5, 9, 6, 8], [18, 7, 19, 8], [15, 10, 16, 11], [9, 16, 10, 17], [6, 17, 7, 18]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (17, 2, 18, 3), (16, 5, 17, 6), (6, 3, 7, 4), (13, 8, 14, 9), (9, 12, 10, 13), (19, 10, 20, 11), (7, 14, 8, 15), (4, 15, 5, 16), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 3, 4], [0, 5, 6, 0], [0, 4, 1, 1], [1, 3, 7, 7], [2, 8, 9, 6], [2, 5, 9, 9], [4, 8, 8, 4], [5, 7, 7, 9], [5, 8, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 267: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

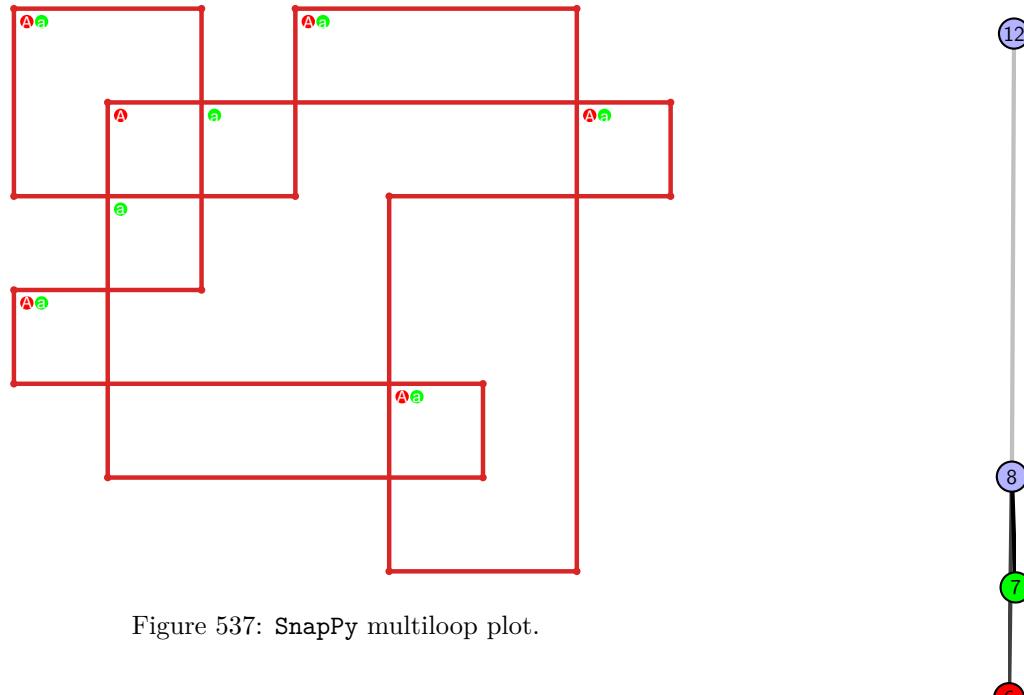


Figure 537: `SnapPy` multiloop plot.

Figure 538: Minimal join sub-semi-lattice of minimal pinning sets.

$$4.8.121 \quad [[3, 20, 4, 1], [11, 2, 12, 3], [16, 19, 17, 20], [4, 17, 5, 18], [1, 10, 2, 11], [12, 10, 13, 9], [15, 6, 16, 7], [18, 5, 19, 6], [13, 8, 14, 9], [7, 14, 8, 15]]$$

PD code drawn by SnapPy: [(9, 20, 10, 1), (16, 3, 17, 4), (13, 4, 14, 5), (11, 6, 12, 7), (7, 10, 8, 11), (19, 8, 20, 9), (5, 12, 6, 13), (2, 15, 3, 16), (14, 17, 15, 18), (1, 18, 2, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 3], [0, 2, 7, 7], [0, 5, 1, 1], [1, 4, 8, 8], [2, 9, 9, 7], [2, 6, 3, 3], [5, 9, 9, 5], [6, 8, 8, 6]]

Total optimal pinning sets: 1  
 Total pinned pinning sets: 2

Average optimal degree: 2.17

Total minimal pinning sets: 2

Average minimal degree: 2.23

Total pinning sets: 80

Average minimal degree: 2.2

Table 268: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

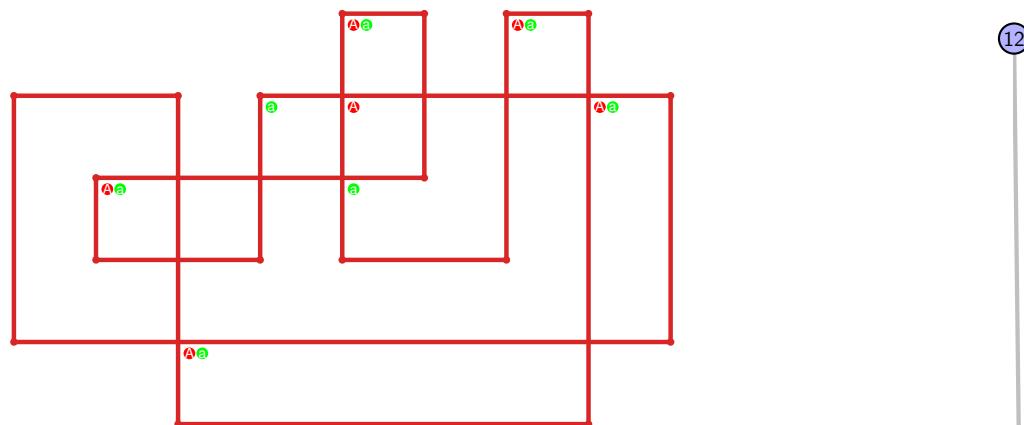


Figure 539: SnapPy multiloop plot.



Figure 540: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.122**  $[[5, 20, 6, 1], [11, 4, 12, 5], [16, 19, 17, 20], [6, 17, 7, 18], [1, 10, 2, 11], [3, 12, 4, 13], [15, 8, 16, 9], [18, 7, 19, 8], [9, 14, 10, 15], [2, 14, 3, 13]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (15, 4, 16, 5), (12, 5, 13, 6), (18, 7, 19, 8), (20, 9, 1, 10), (10, 19, 11, 20), (6, 11, 7, 12), (3, 14, 4, 15), (13, 16, 14, 17), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 7, 7], [0, 8, 9, 1], [1, 9, 9, 1], [2, 8, 8, 7], [2, 6, 3, 3], [4, 6, 6, 9], [4, 8, 5, 5]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 160  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.97

Table 269: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

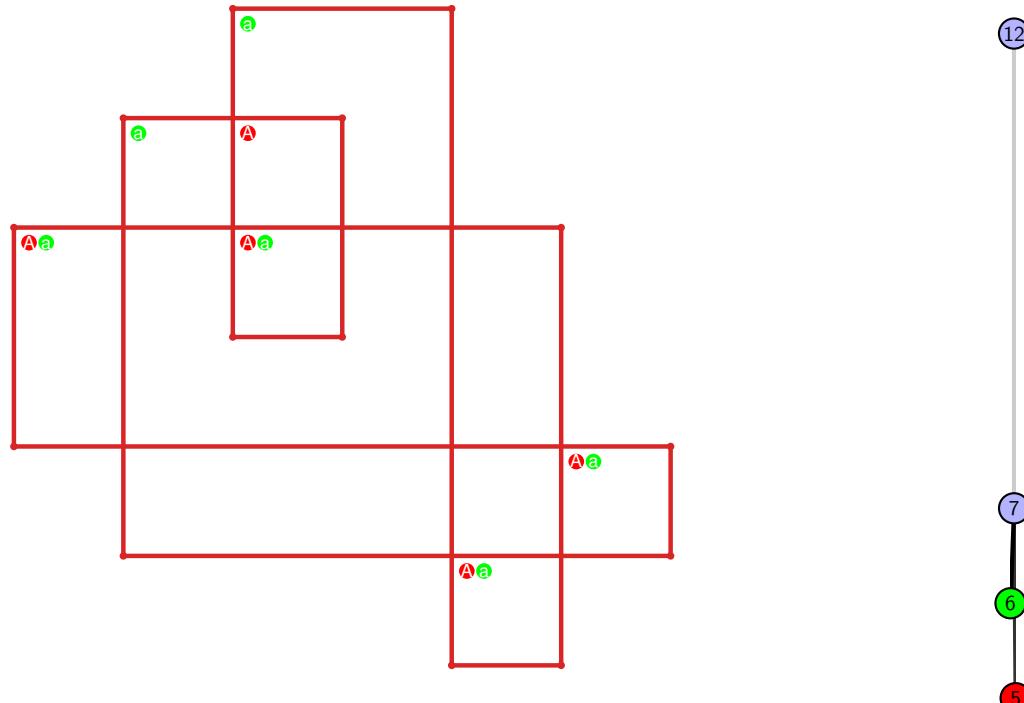


Figure 541: `SnapPy` multiloop plot.

Figure 542: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.123  $[[7, 20, 8, 1], [11, 6, 12, 7], [19, 8, 20, 9], [1, 10, 2, 11], [5, 12, 6, 13], [9, 18, 10, 19], [2, 18, 3, 17], [13, 17, 14, 16], [4, 15, 5, 16], [3, 15, 4, 14]]$

PD code drawn by SnapPy:  $[(8, 3, 9, 4), (16, 5, 17, 6), (18, 7, 19, 8), (2, 9, 3, 10), (11, 20, 12, 1), (1, 12, 2, 13), (13, 10, 14, 11), (14, 19, 15, 20), (6, 15, 7, 16), (4, 17, 5, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 3, 4, 4], [0, 5, 5, 0], [0, 5, 6, 1], [1, 7, 8, 1], [2, 6, 3, 2], [3, 5, 9, 7], [4, 6, 9, 8], [4, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 6  
 Total pinning sets: 156  
 Pinning number: 6

Average optimal degree: 2.42  
 Average minimal degree: 2.42  
 Average overall degree: 2.98

Table 270: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	25	45	45	26	8	1	150
Average degree	2.42	2.72	2.92	3.07	3.18	3.27	3.33	

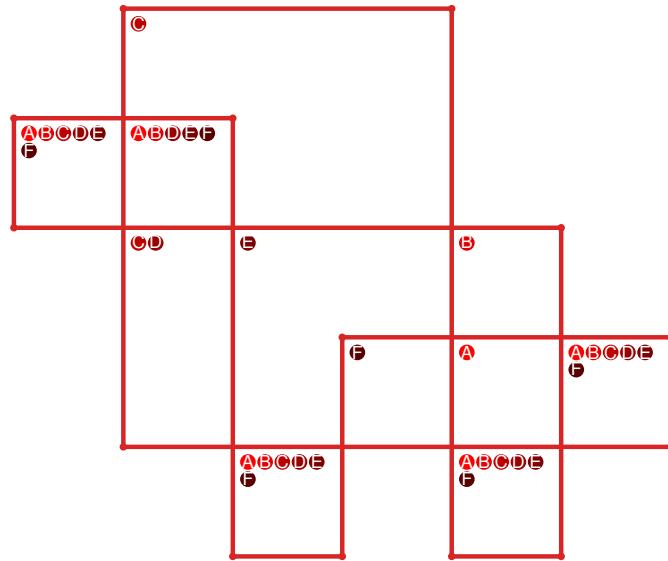


Figure 543: SnapPy multiloop plot.

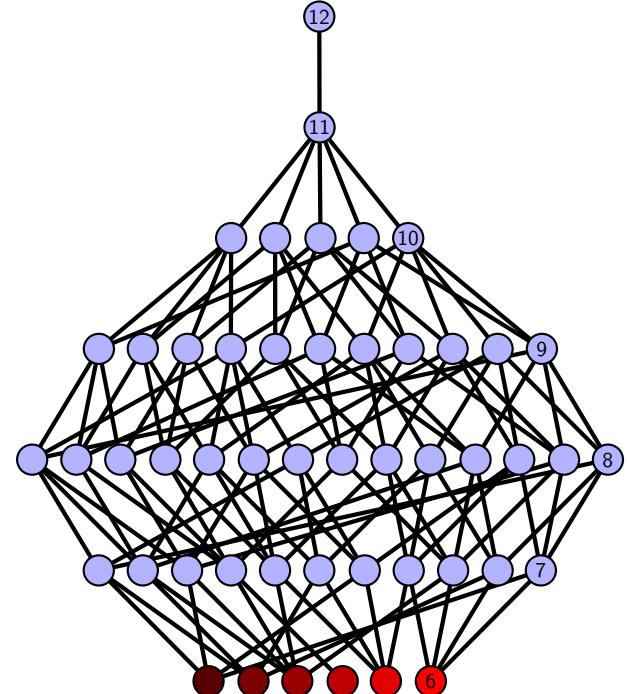


Figure 544: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.124**  $[[3, 20, 4, 1], [11, 2, 12, 3], [19, 6, 20, 7], [4, 18, 5, 17], [1, 10, 2, 11], [12, 10, 13, 9], [7, 15, 8, 14], [5, 18, 6, 19], [16, 13, 17, 14], [8, 15, 9, 16]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (17, 4, 18, 5), (7, 10, 8, 11), (19, 8, 20, 9), (14, 11, 15, 12), (12, 5, 13, 6), (6, 13, 7, 14), (15, 2, 16, 3), (3, 16, 4, 17), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 7], [0, 7, 7, 8], [0, 5, 1, 1], [1, 4, 8, 9], [2, 9, 9, 8], [2, 3, 3, 2], [3, 6, 9, 5], [5, 8, 6, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 271: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

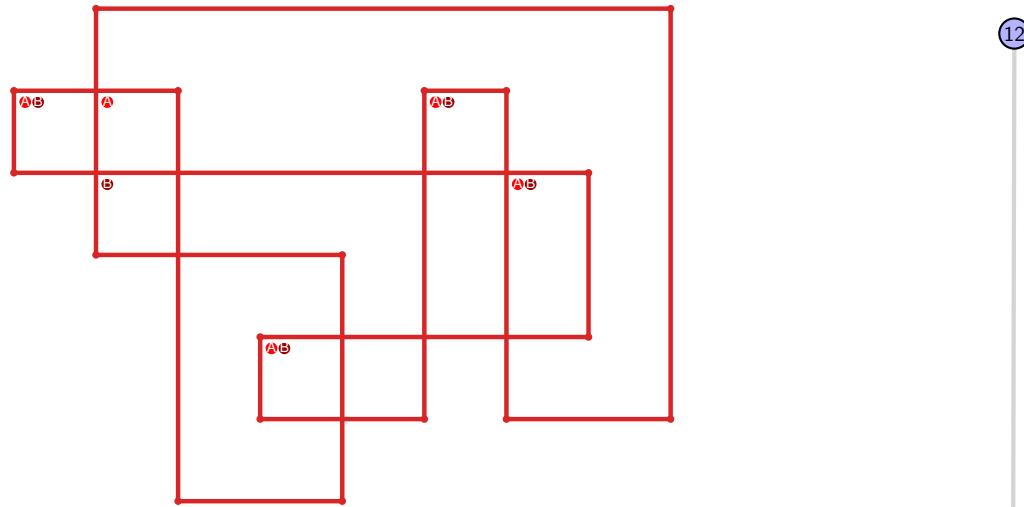


Figure 545: `SnapPy` multiloop plot.

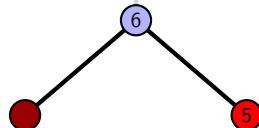


Figure 546: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.125  $[[3, 20, 4, 1], [2, 11, 3, 12], [19, 6, 20, 7], [4, 18, 5, 17], [1, 13, 2, 12], [13, 10, 14, 11], [7, 14, 8, 15], [5, 18, 6, 19], [9, 16, 10, 17], [8, 16, 9, 15]]$

PD code drawn by SnapPy:  $[(16, 3, 17, 4), (10, 7, 11, 8), (19, 8, 20, 9), (9, 18, 10, 19), (4, 11, 5, 12), (12, 5, 13, 6), (6, 13, 7, 14), (14, 1, 15, 2), (2, 15, 3, 16), (20, 17, 1, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 7, 7], [0, 7, 7, 8], [0, 5, 1, 1], [1, 4, 8, 6], [2, 5, 9, 9], [2, 3, 3, 2], [3, 9, 9, 5], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 272: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

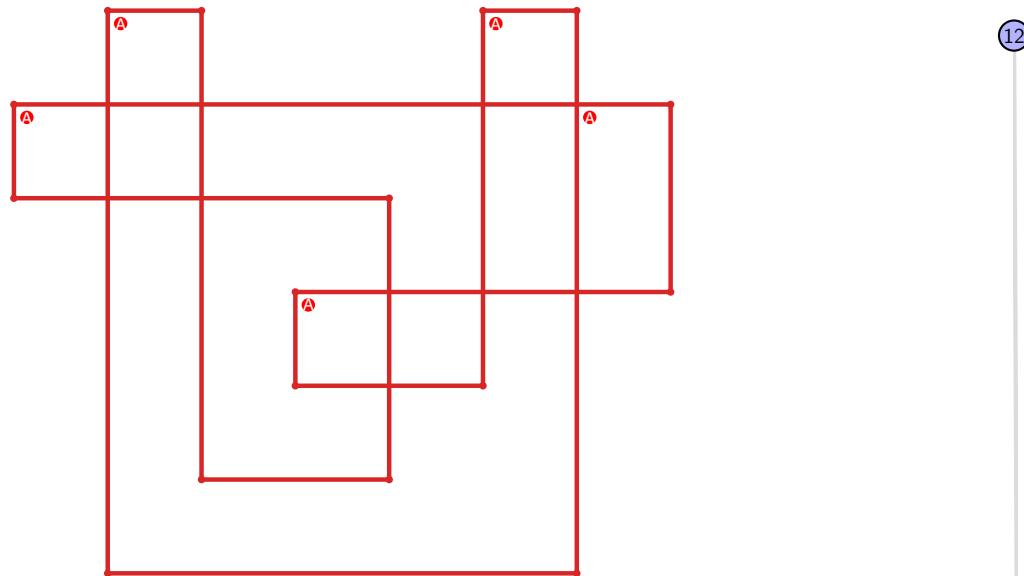


Figure 547: SnapPy multiloop plot.



Figure 548: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.126**  $[[11, 20, 12, 1], [10, 7, 11, 8], [19, 14, 20, 15], [12, 18, 13, 17], [1, 9, 2, 8], [2, 9, 3, 10], [3, 6, 4, 7], [15, 4, 16, 5], [13, 18, 14, 19], [5, 16, 6, 17]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (20, 3, 1, 4), (7, 4, 8, 5), (5, 18, 6, 19), (19, 6, 20, 7), (16, 11, 17, 12), (2, 13, 3, 14), (14, 9, 15, 10), (10, 15, 11, 16), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 8, 8], [0, 8, 8, 9], [0, 5, 5, 1], [1, 4, 4, 6], [1, 5, 9, 7], [2, 6, 9, 9], [2, 3, 3, 2], [3, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 273: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

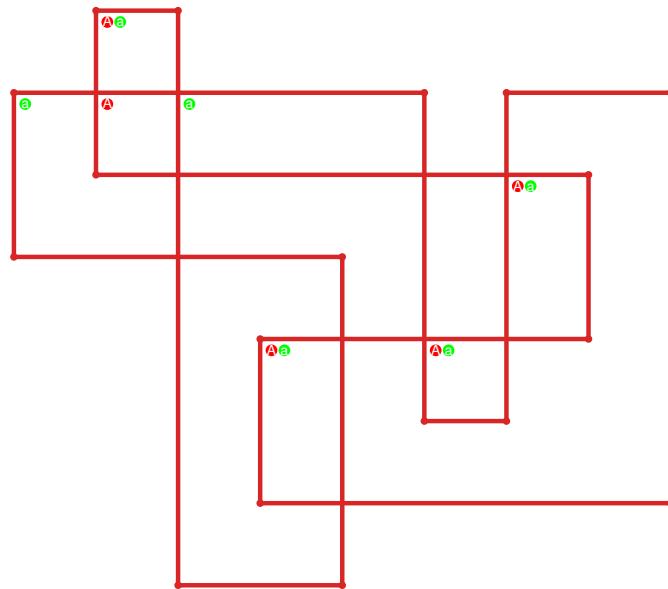


Figure 549: `SnapPy` multiloop plot.



Figure 550: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.127**  $[[3, 20, 4, 1], [2, 13, 3, 14], [6, 19, 7, 20], [4, 7, 5, 8], [1, 15, 2, 14], [15, 12, 16, 13], [18, 5, 19, 6], [8, 11, 9, 12], [16, 9, 17, 10], [10, 17, 11, 18]]$

PD code drawn by `SnapPy`:  $[(16, 3, 17, 4), (2, 5, 3, 6), (14, 7, 15, 8), (6, 9, 7, 10), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (8, 15, 9, 16), (4, 17, 5, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 6, 7], [0, 5, 1, 1], [1, 4, 7, 8], [2, 9, 3, 2], [3, 9, 8, 5], [5, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 272  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.47  
 Average overall degree: 3.04

Table 274: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.4	2.68	2.89	3.04	3.15	3.24	3.29	3.33	

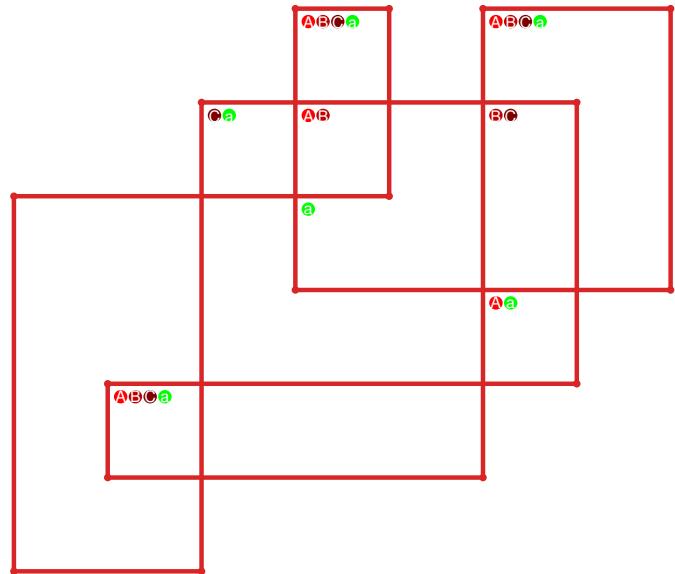


Figure 551: `SnapPy` multiloop plot.

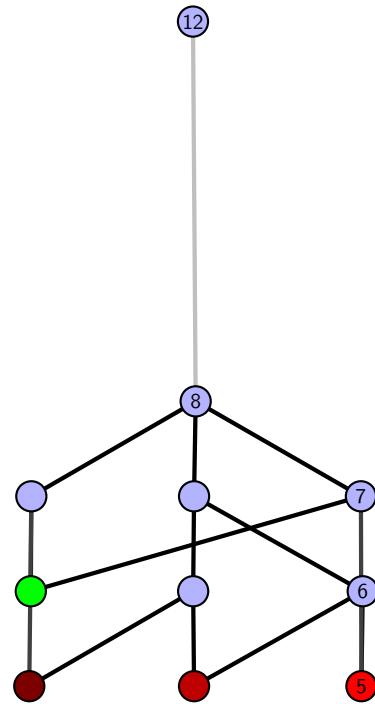


Figure 552: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.128  $[[3, 20, 4, 1], [2, 11, 3, 12], [6, 19, 7, 20], [4, 7, 5, 8], [1, 13, 2, 12], [13, 10, 14, 11], [18, 5, 19, 6], [8, 15, 9, 16], [16, 9, 17, 10], [14, 17, 15, 18]]$

PD code drawn by `SnapPy`:  $[(16, 3, 17, 4), (2, 5, 3, 6), (14, 7, 15, 8), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (6, 13, 7, 14), (12, 15, 13, 16), (4, 17, 5, 18), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 6, 7], [0, 5, 1, 1], [1, 4, 8, 9], [2, 9, 3, 2], [3, 9, 8, 8], [5, 7, 7, 9], [5, 8, 7, 6]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.47

**Total pinning sets:** 272

**Average overall degree:** 3.04

**Pinning number:** 5

Table 275: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.4	2.68	2.89	3.04	3.15	3.24	3.29	3.33	

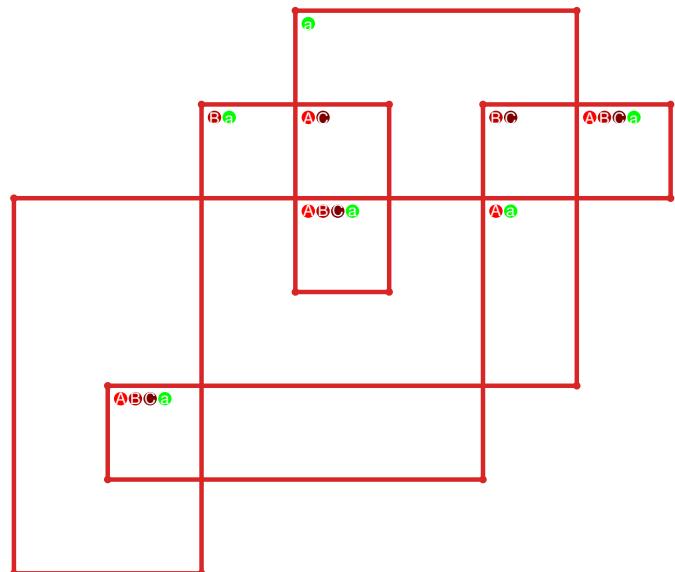


Figure 553: `SnapPy` multiloop plot.

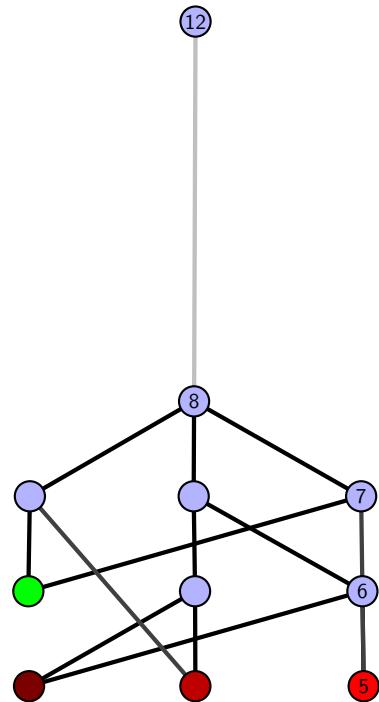


Figure 554: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.129**  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 18, 9, 17], [1, 4, 2, 5], [5, 18, 6, 19], [9, 14, 10, 15], [11, 16, 12, 17], [12, 3, 13, 4], [2, 13, 3, 14], [10, 16, 11, 15]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (18, 1, 19, 2), (16, 3, 17, 4), (19, 10, 20, 11), (6, 11, 7, 12), (12, 7, 13, 8), (8, 13, 9, 14), (14, 5, 15, 6), (2, 15, 3, 16), (4, 17, 5, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 6], [0, 7, 8, 4], [1, 3, 2, 1], [2, 8, 9, 9], [2, 9, 9, 7], [3, 6, 8, 8], [3, 7, 7, 5], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 276: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

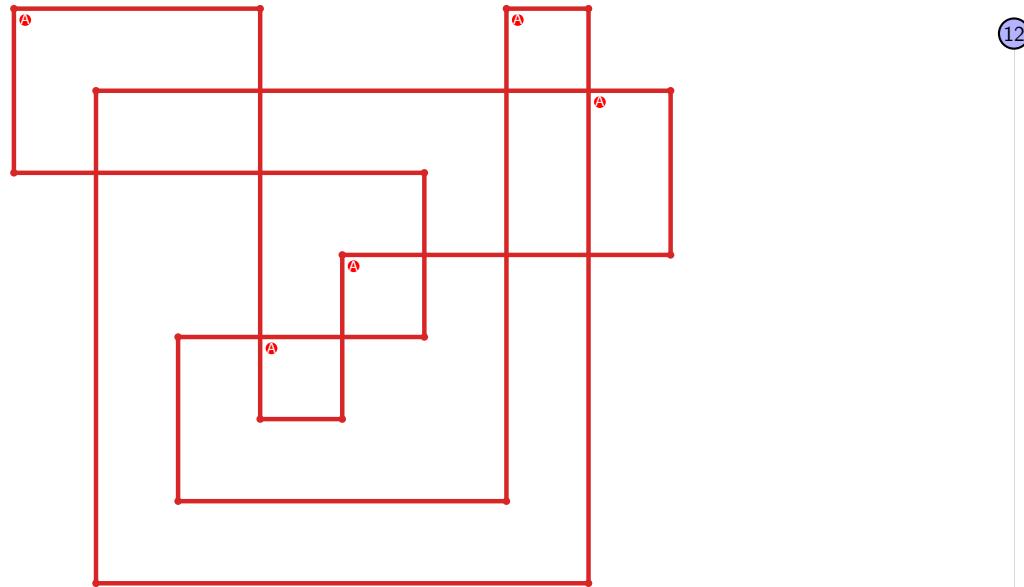


Figure 555: `SnapPy` multiloop plot.



Figure 556: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.130  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 18, 9, 17], [1, 4, 2, 5], [5, 18, 6, 19], [9, 15, 10, 14], [16, 13, 17, 14], [3, 12, 4, 13], [2, 12, 3, 11], [15, 11, 16, 10]]$

PD code drawn by SnapPy:  $[(19, 2, 20, 3), (17, 4, 18, 5), (8, 13, 9, 14), (9, 20, 10, 1), (1, 10, 2, 11), (14, 11, 15, 12), (12, 7, 13, 8), (15, 6, 16, 7), (3, 16, 4, 17), (5, 18, 6, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 6], [0, 7, 8, 4], [1, 3, 2, 1], [2, 9, 9, 6], [2, 5, 9, 7], [3, 6, 8, 8], [3, 7, 7, 9], [5, 8, 6, 5]]$

Total optimal pinning sets: 2  
Total minimal pinning sets: 2

Total pinning sets: 192

Pinning number: 5

Average optimal degree: 2.2

Average minimal degree: 2.2

Average overall degree: 2.97

Table 277: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

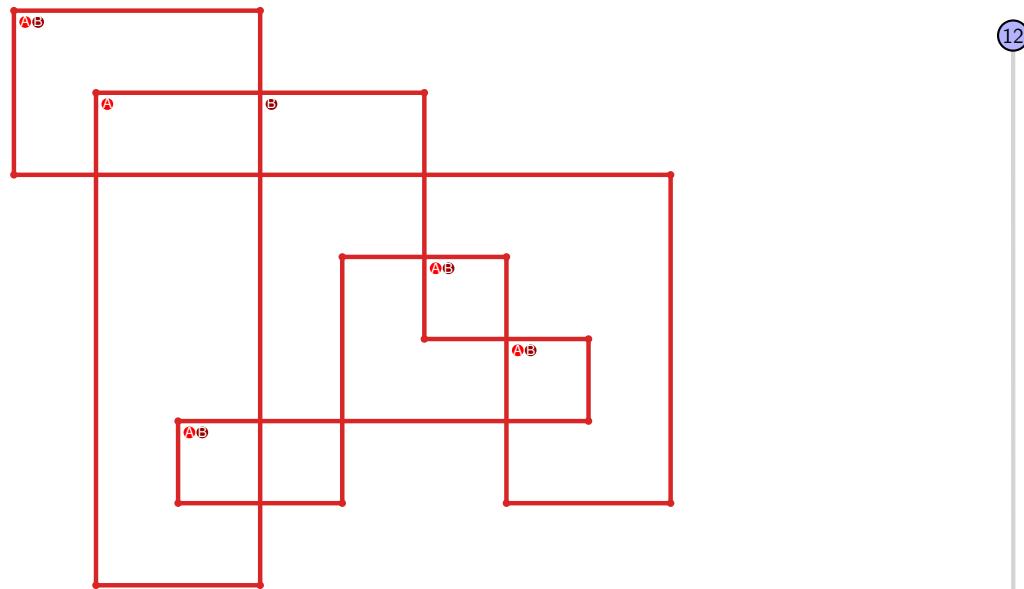


Figure 557: SnapPy multiloop plot.

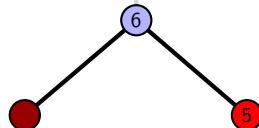


Figure 558: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.131**  $[[20, 7, 1, 8], [8, 18, 9, 17], [6, 19, 7, 20], [1, 19, 2, 18], [9, 15, 10, 14], [16, 13, 17, 14], [5, 2, 6, 3], [15, 11, 16, 10], [12, 3, 13, 4], [4, 11, 5, 12]]$

PD code drawn by `SnapPy`:  $[(3, 20, 4, 1), (10, 1, 11, 2), (18, 5, 19, 6), (8, 13, 9, 14), (2, 9, 3, 10), (14, 11, 15, 12), (12, 7, 13, 8), (15, 6, 16, 7), (16, 19, 17, 20), (4, 17, 5, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 3, 0], [0, 2, 6, 1], [1, 7, 7, 5], [1, 4, 7, 8], [2, 8, 9, 3], [4, 9, 5, 4], [5, 9, 9, 6], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.45

**Total pinning sets:** 240

**Average overall degree:** 3.03

**Pinning number:** 5

Table 278: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	13	45	71	64	33	9	1	236
Average degree	2.4	2.66	2.86	3.02	3.14	3.23	3.29	3.33	

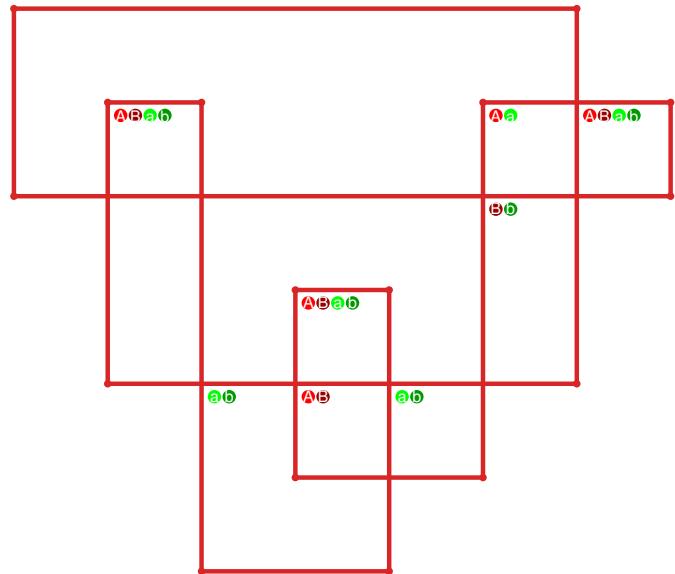


Figure 559: `SnapPy` multiloop plot.

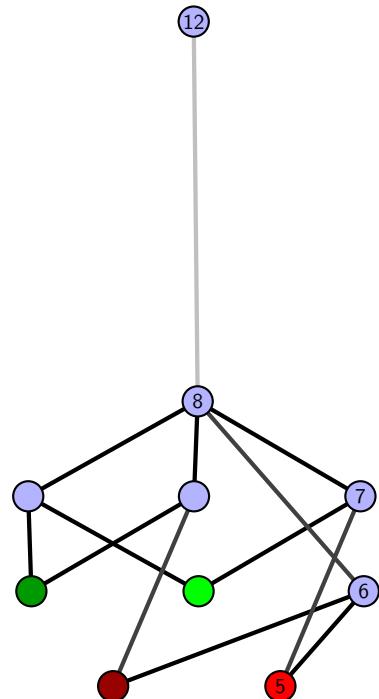


Figure 560: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.132** [[20, 7, 1, 8], [8, 18, 9, 17], [6, 19, 7, 20], [1, 19, 2, 18], [9, 13, 10, 12], [16, 11, 17, 12], [5, 2, 6, 3], [13, 5, 14, 4], [10, 15, 11, 16], [3, 15, 4, 14]]

PD code drawn by `SnapPy`: [(1, 10, 2, 11), (17, 4, 18, 5), (13, 8, 14, 9), (9, 6, 10, 7), (11, 20, 12, 1), (7, 12, 8, 13), (14, 5, 15, 6), (15, 18, 16, 19), (3, 16, 4, 17), (2, 19, 3, 20)]

Planar representation generated by `plantri`: [[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 3, 0], [0, 2, 6, 1], [1, 7, 8, 5], [1, 4, 8, 8], [2, 9, 7, 3], [4, 6, 9, 9], [4, 9, 5, 5], [6, 8, 7, 7]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 4  
 Total pinning sets: 240  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.45  
 Average overall degree: 3.03

Table 279: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	13	45	71	64	33	9	1	236
Average degree	2.4	2.66	2.86	3.02	3.14	3.23	3.29	3.33	

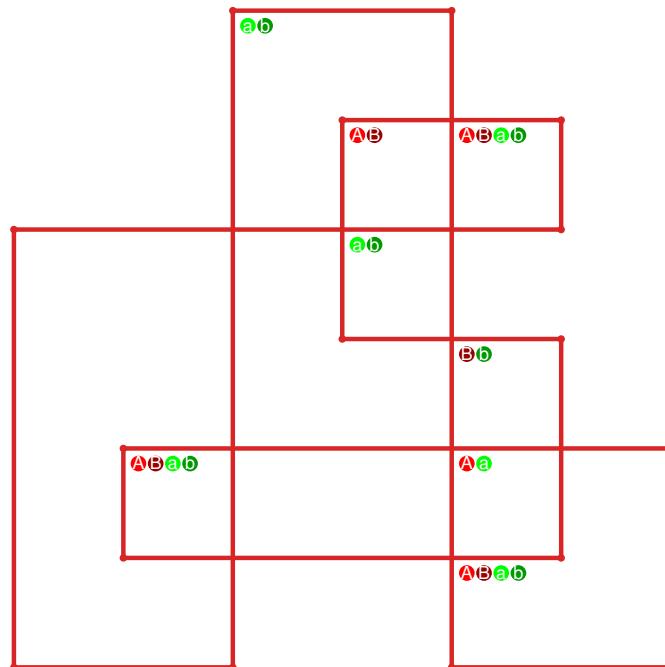


Figure 561: `SnapPy` multiloop plot.

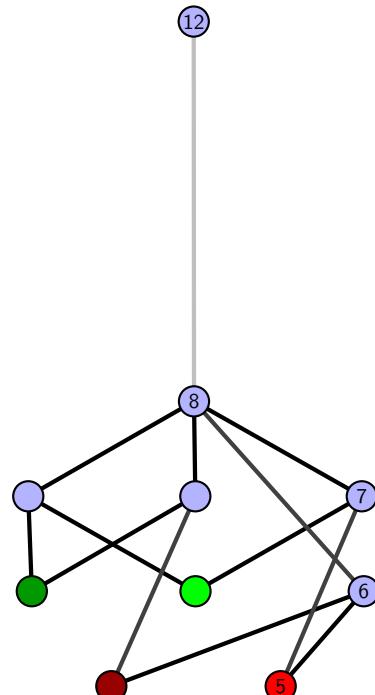


Figure 562: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.133  $[[7, 20, 8, 1], [3, 6, 4, 7], [19, 8, 20, 9], [1, 19, 2, 18], [2, 17, 3, 18], [5, 12, 6, 13], [4, 12, 5, 11], [9, 15, 10, 14], [16, 13, 17, 14], [10, 15, 11, 16]]$

PD code drawn by `SnapPy`:  $[(19, 2, 20, 3), (15, 4, 16, 5), (7, 20, 8, 1), (1, 8, 2, 9), (12, 9, 13, 10), (10, 5, 11, 6), (6, 11, 7, 12), (18, 13, 19, 14), (14, 17, 15, 18), (3, 16, 4, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 7, 3, 0], [0, 2, 4, 4], [1, 3, 3, 8], [1, 8, 6, 6], [1, 5, 5, 9], [2, 9, 9, 8], [4, 7, 9, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 280: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

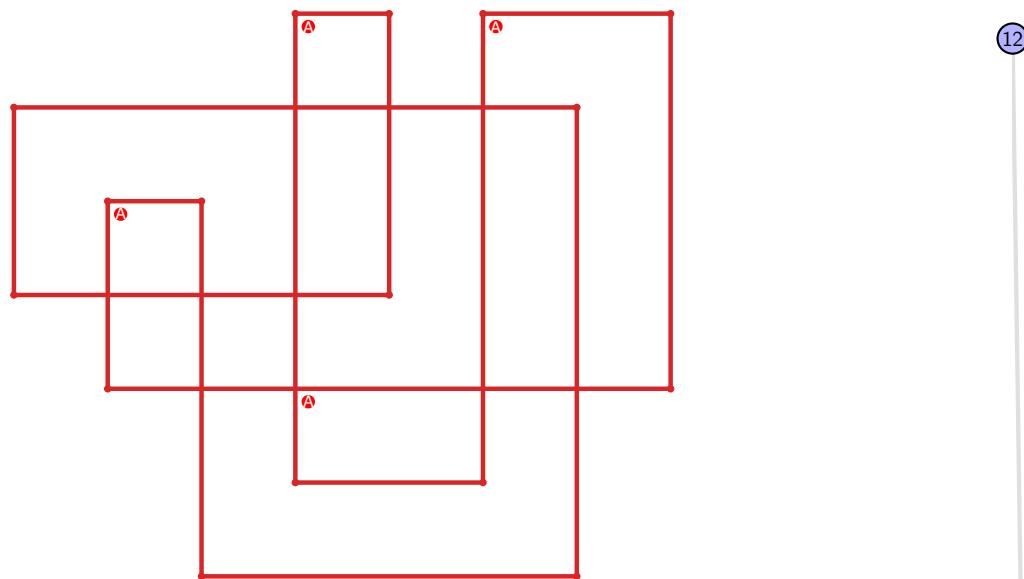


Figure 563: `SnapPy` multiloop plot.

4

Figure 564: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.134**  $[[9, 20, 10, 1], [3, 8, 4, 9], [14, 19, 15, 20], [10, 2, 11, 1], [11, 2, 12, 3], [7, 4, 8, 5], [18, 13, 19, 14], [15, 13, 16, 12], [5, 16, 6, 17], [17, 6, 18, 7]]$

PD code drawn by `SnapPy`:  $[(3, 20, 4, 1), (13, 2, 14, 3), (19, 4, 20, 5), (8, 11, 9, 12), (14, 9, 15, 10), (1, 12, 2, 13), (10, 15, 11, 16), (5, 16, 6, 17), (17, 6, 18, 7), (7, 18, 8, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 7], [0, 4, 4, 0], [1, 3, 3, 7], [1, 8, 9, 1], [2, 9, 7, 2], [2, 6, 8, 4], [5, 7, 9, 9], [5, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 281: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

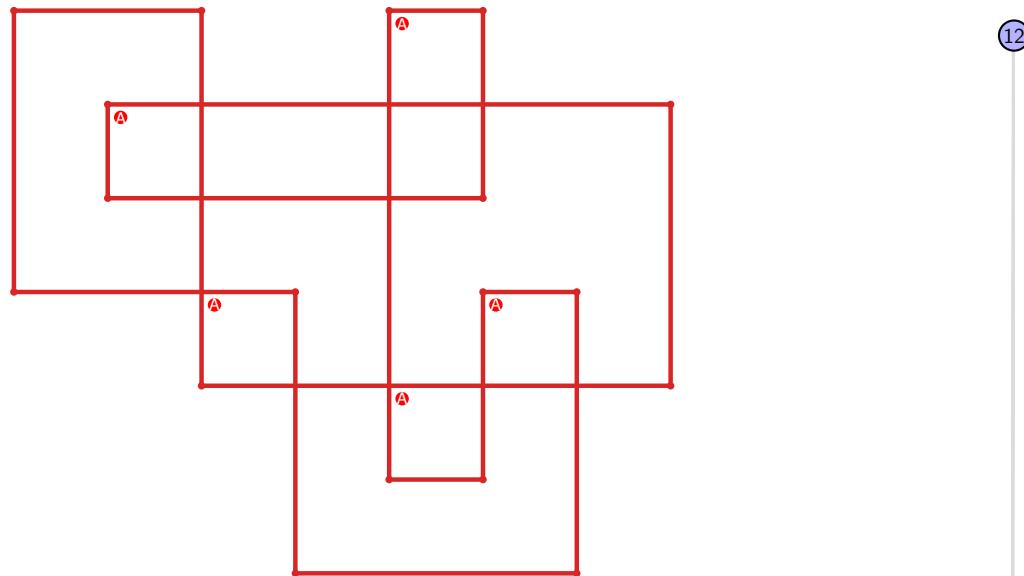


Figure 565: `SnapPy` multiloop plot.

12

5

Figure 566: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.135 [[20, 13, 1, 14], [14, 3, 15, 4], [19, 8, 20, 9], [12, 1, 13, 2], [2, 11, 3, 12], [15, 7, 16, 6], [4, 17, 5, 18], [9, 18, 10, 19], [10, 7, 11, 8], [16, 5, 17, 6]]

PD code drawn by SnapPy: [(12, 1, 13, 2), (14, 3, 15, 4), (4, 13, 5, 14), (8, 5, 9, 6), (17, 6, 18, 7), (20, 9, 1, 10), (18, 11, 19, 12), (2, 15, 3, 16), (7, 16, 8, 17), (10, 19, 11, 20)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 5, 6], [0, 7, 7, 8], [0, 4, 4, 0], [1, 3, 3, 8], [1, 8, 9, 9], [1, 9, 9, 7], [2, 6, 8, 2], [2, 7, 5, 4], [5, 6, 6, 5]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 282: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

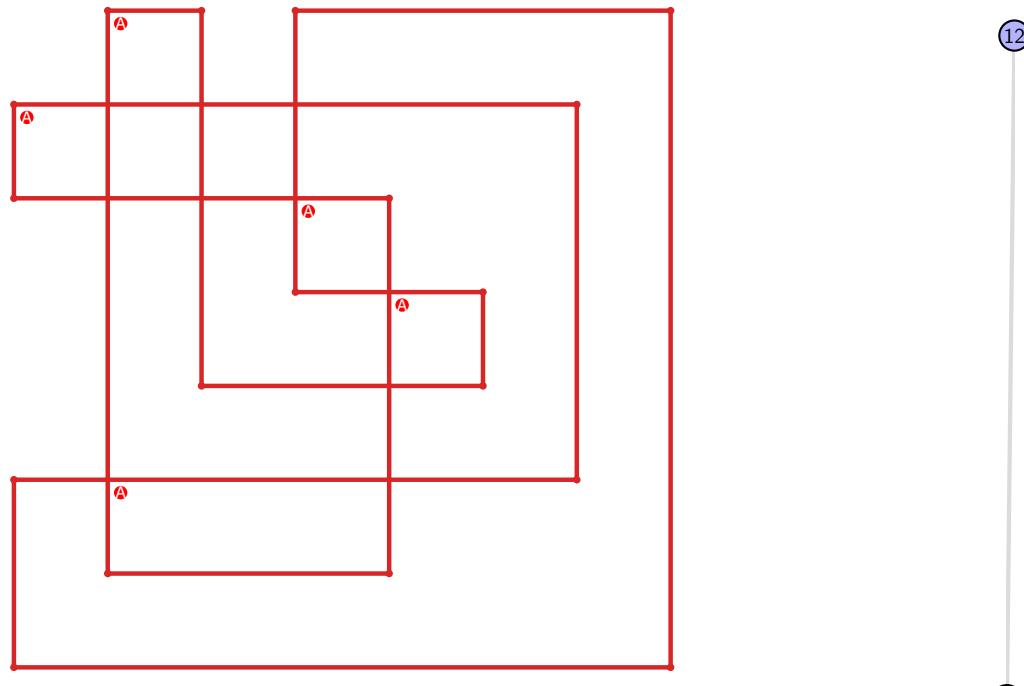


Figure 567: SnapPy multiloop plot.

Figure 568: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.136**  $[[7, 20, 8, 1], [3, 6, 4, 7], [19, 12, 20, 13], [8, 2, 9, 1], [9, 2, 10, 3], [16, 5, 17, 6], [4, 17, 5, 18], [13, 18, 14, 19], [14, 11, 15, 12], [10, 15, 11, 16]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (14, 3, 15, 4), (1, 4, 2, 5), (12, 9, 13, 10), (17, 10, 18, 11), (8, 13, 9, 14), (2, 15, 3, 16), (11, 16, 12, 17), (5, 18, 6, 19), (19, 6, 20, 7)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 6], [0, 7, 7, 8], [0, 4, 4, 0], [1, 3, 3, 9], [1, 9, 6, 6], [1, 5, 5, 7], [2, 6, 8, 2], [2, 7, 9, 9], [4, 8, 8, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 283: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

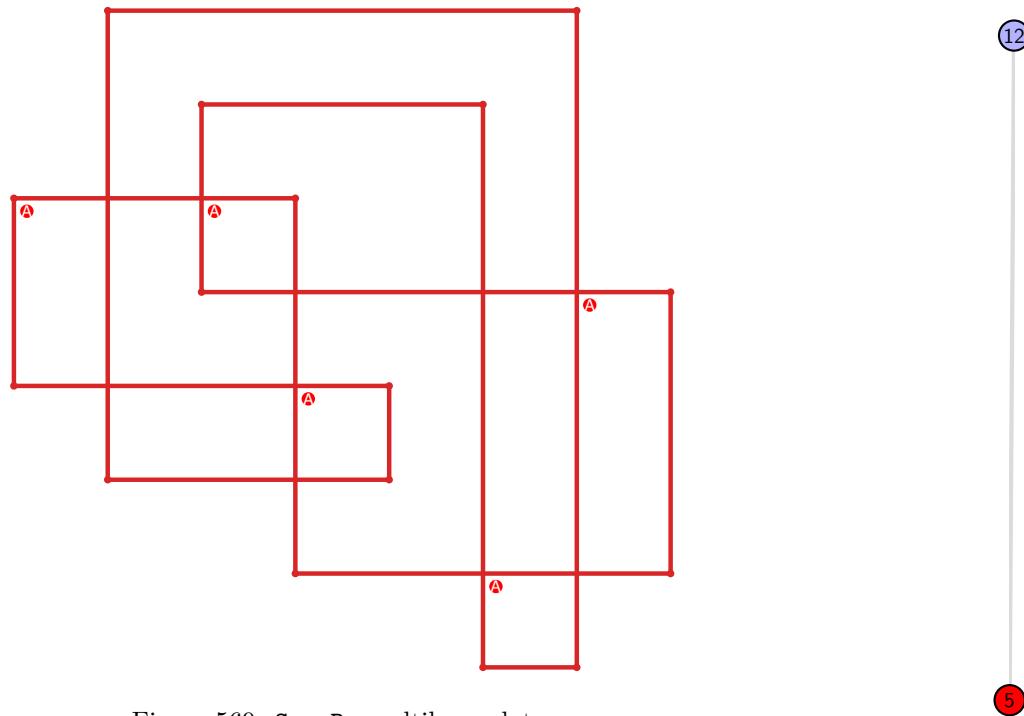


Figure 569: `SnapPy` multiloop plot.

Figure 570: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.137** `[[11, 20, 12, 1], [10, 17, 11, 18], [19, 12, 20, 13], [1, 19, 2, 18], [6, 9, 7, 10], [7, 16, 8, 17], [13, 4, 14, 5], [2, 5, 3, 6], [15, 8, 16, 9], [3, 14, 4, 15]]`

PD code drawn by `SnapPy`: `[(10, 1, 11, 2), (19, 2, 20, 3), (12, 5, 13, 6), (3, 6, 4, 7), (16, 9, 17, 10), (20, 11, 1, 12), (4, 13, 5, 14), (7, 14, 8, 15), (15, 18, 16, 19), (8, 17, 9, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 3, 0], [0, 2, 7, 1], [1, 7, 8, 5], [1, 4, 8, 8], [2, 9, 9, 7], [3, 6, 9, 4], [4, 9, 5, 5], [6, 8, 7, 6]]`

Total optimal pinning sets: 1  
 Total minimal pinning sets: 7  
 Total pinning sets: 244  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.58  
 Average overall degree: 3.05

Table 284: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	1	0	0	0	0	0	6
Nonminimal pinning sets	0	7	43	75	68	34	9	1	237
Average degree	2.4	2.63	2.85	3.02	3.15	3.24	3.29	3.33	

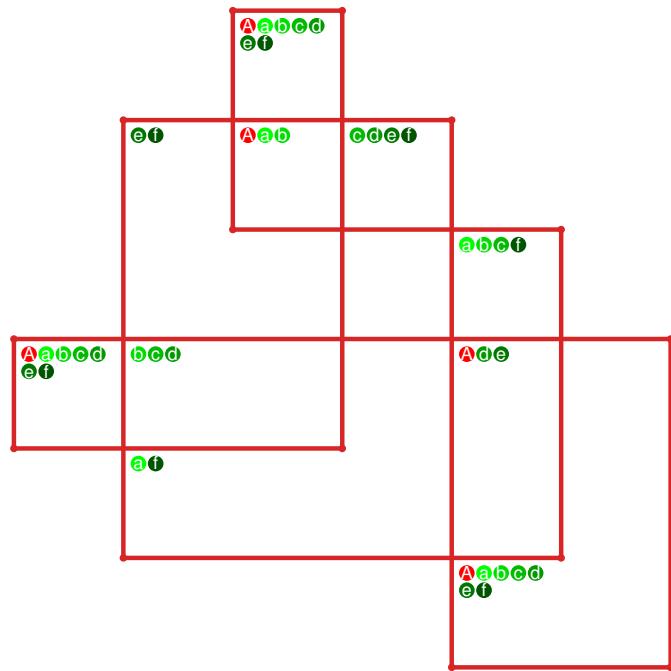


Figure 571: `SnapPy` multiloop plot.

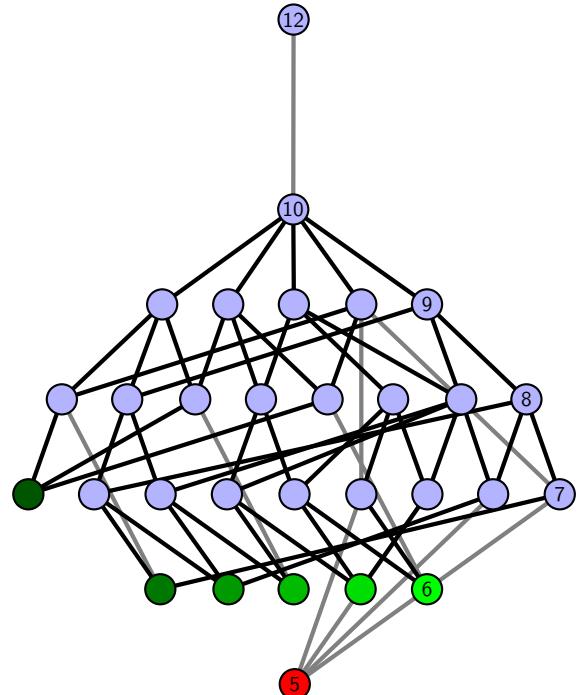


Figure 572: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.138 [[20, 5, 1, 6], [6, 18, 7, 17], [4, 19, 5, 20], [1, 19, 2, 18], [7, 16, 8, 17], [10, 3, 11, 4], [2, 11, 3, 12], [12, 15, 13, 16], [8, 13, 9, 14], [14, 9, 15, 10]]

PD code drawn by SnapPy: [(7, 20, 8, 1), (17, 2, 18, 3), (11, 6, 12, 7), (19, 8, 20, 9), (5, 10, 6, 11), (9, 12, 10, 13), (4, 13, 5, 14), (14, 3, 15, 4), (15, 18, 16, 19), (1, 16, 2, 17)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 3, 4, 4], [0, 5, 3, 0], [0, 2, 6, 1], [1, 7, 8, 1], [2, 9, 6, 6], [3, 5, 5, 7], [4, 6, 9, 8], [4, 7, 9, 9], [5, 8, 8, 7]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4

**Total pinning sets:** 144

**Pinning number:** 6

**Average optimal degree:** 2.33

**Average minimal degree:** 2.33

**Average overall degree:** 2.97

Table 285: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.33	2.66	2.89	3.06	3.18	3.27	3.33	

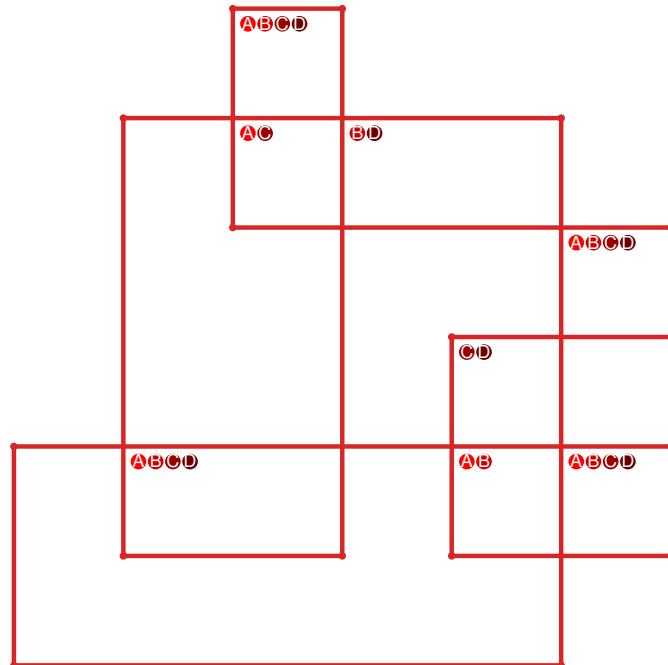


Figure 573: SnapPy multiloop plot.

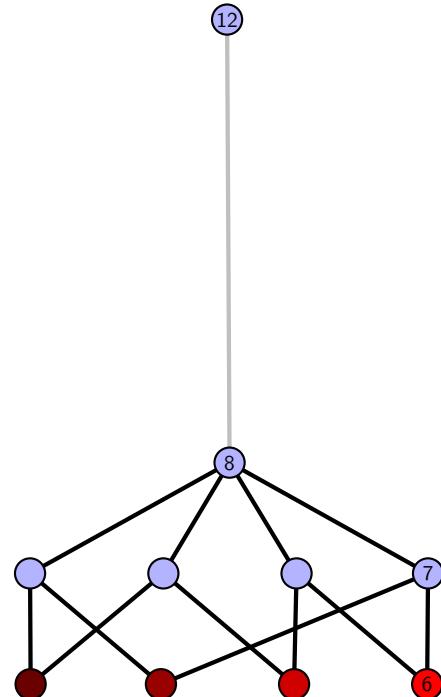


Figure 574: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.139**  $[[7, 20, 8, 1], [6, 9, 7, 10], [19, 8, 20, 9], [1, 11, 2, 10], [5, 16, 6, 17], [18, 15, 19, 16], [11, 15, 12, 14], [2, 14, 3, 13], [17, 4, 18, 5], [12, 4, 13, 3]]$

PD code drawn by `SnapPy`:  $[(1, 10, 2, 11), (15, 2, 16, 3), (17, 6, 18, 7), (4, 7, 5, 8), (8, 19, 9, 20), (11, 20, 12, 1), (12, 9, 13, 10), (16, 13, 17, 14), (3, 14, 4, 15), (5, 18, 6, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 2], [0, 1, 5, 0], [0, 6, 7, 1], [1, 8, 8, 5], [2, 4, 8, 6], [3, 5, 9, 7], [3, 6, 9, 9], [4, 9, 5, 4], [6, 8, 7, 7]]$

Total optimal pinning sets: 1  
Total minimal pinning sets: 7

Total pinning sets: 244

Pinning number: 5

Average optimal degree: 2.4

Average minimal degree: 2.58

Average overall degree: 3.05

Table 286: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	1	0	0	0	0	0	6
Nonminimal pinning sets	0	7	43	75	68	34	9	1	237
Average degree	2.4	2.63	2.85	3.02	3.15	3.24	3.29	3.33	

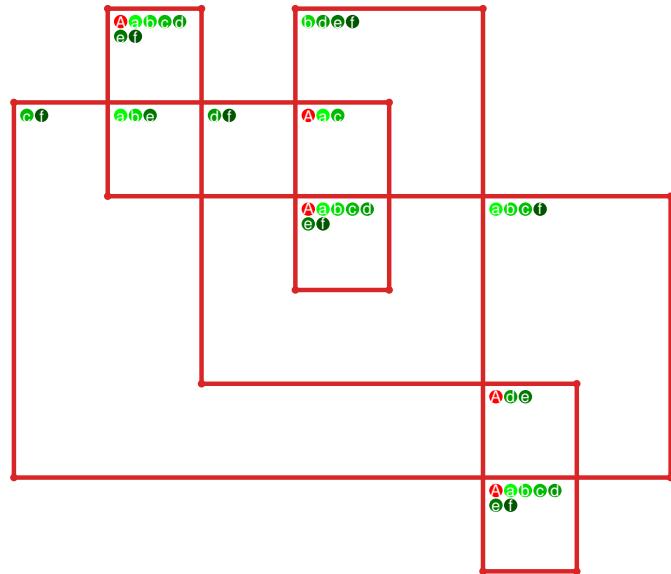


Figure 575: `SnapPy` multiloop plot.

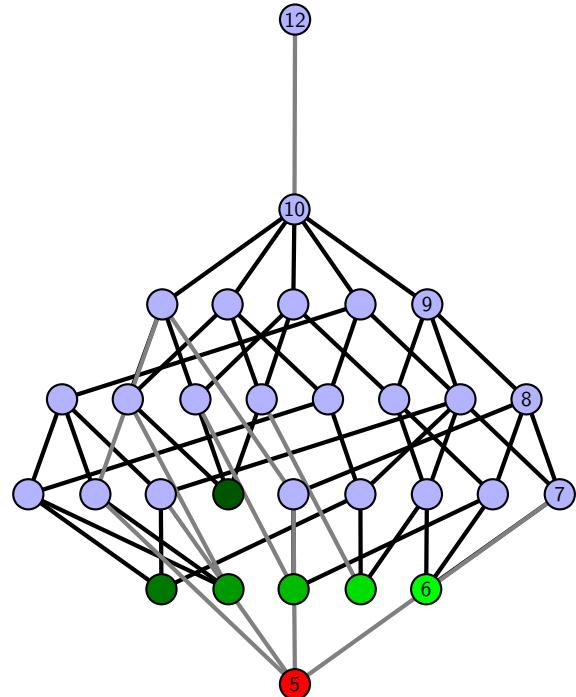


Figure 576: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.140** `[[20, 15, 1, 16], [16, 13, 17, 14], [14, 19, 15, 20], [1, 12, 2, 13], [17, 9, 18, 8], [18, 7, 19, 8], [11, 2, 12, 3], [9, 5, 10, 4], [6, 3, 7, 4], [10, 5, 11, 6]]`

PD code drawn by `SnapPy`: `[(9, 20, 10, 1), (4, 1, 5, 2), (2, 7, 3, 8), (8, 3, 9, 4), (16, 5, 17, 6), (6, 15, 7, 16), (19, 10, 20, 11), (14, 11, 15, 12), (18, 13, 19, 14), (12, 17, 13, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 3, 4, 2], [0, 1, 5, 0], [0, 6, 6, 1], [1, 7, 5, 5], [2, 4, 4, 8], [3, 8, 9, 3], [4, 9, 9, 8], [5, 7, 9, 6], [6, 8, 7, 7]]`

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.33

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.33

**Total pinning sets:** 144

**Average overall degree:** 2.97

**Pinning number:** 6

Table 287: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.33	2.66	2.89	3.06	3.18	3.27	3.33	

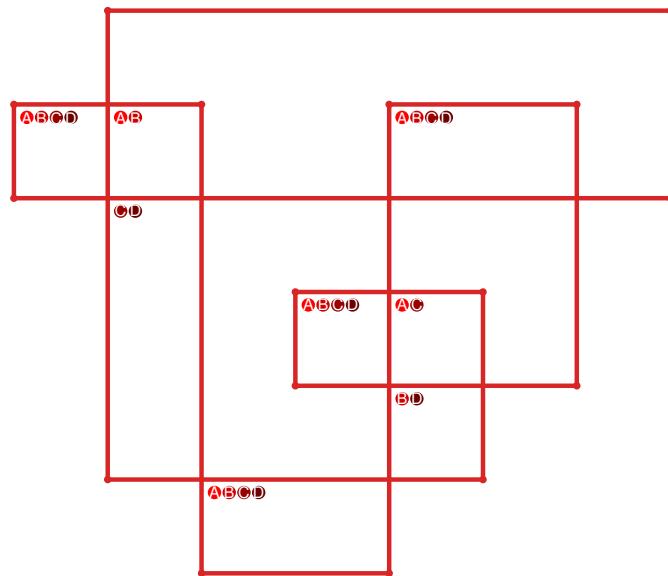


Figure 577: `SnapPy` multiloop plot.

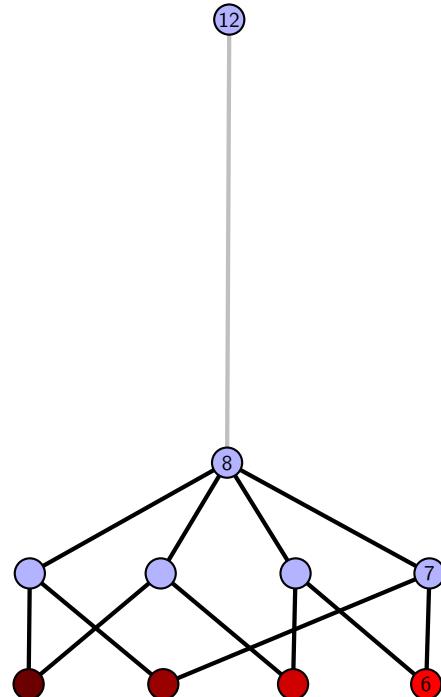


Figure 578: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.141** [[20, 9, 1, 10], [10, 19, 11, 20], [8, 13, 9, 14], [1, 18, 2, 19], [11, 15, 12, 14], [12, 7, 13, 8], [17, 2, 18, 3], [15, 5, 16, 4], [6, 3, 7, 4], [16, 5, 17, 6]]

PD code drawn by `SnapPy`: [(15, 20, 16, 1), (4, 1, 5, 2), (2, 13, 3, 14), (14, 3, 15, 4), (10, 5, 11, 6), (18, 7, 19, 8), (6, 11, 7, 12), (12, 9, 13, 10), (19, 16, 20, 17), (8, 17, 9, 18)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 3, 4, 0], [0, 4, 5, 5], [0, 6, 6, 1], [1, 7, 5, 2], [2, 4, 8, 2], [3, 8, 9, 3], [4, 9, 9, 8], [5, 7, 9, 6], [6, 8, 7, 7]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 3  
 Total pinning sets: 176  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.34  
 Average overall degree: 2.98

Table 288: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.2	2.5	2.75	2.94	3.08	3.19	3.27	3.33	

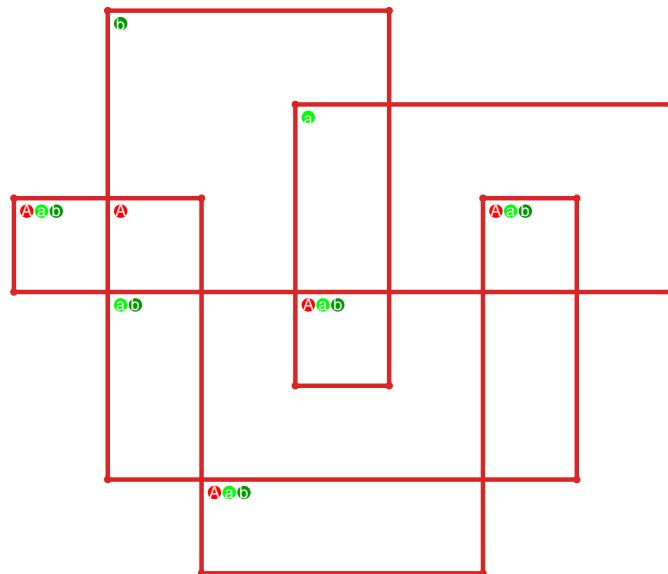


Figure 579: `SnapPy` multiloop plot.

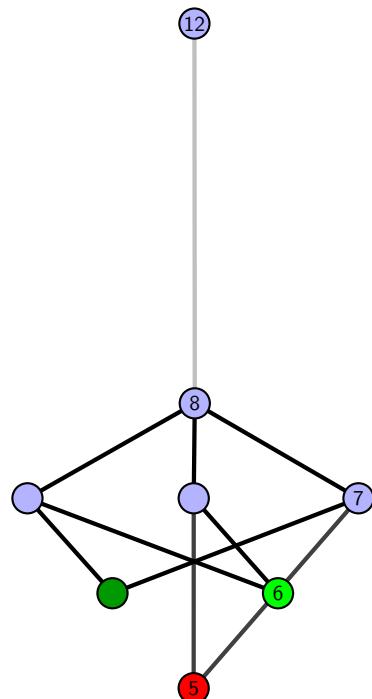


Figure 580: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.142** [[20, 17, 1, 18], [18, 10, 19, 9], [19, 8, 20, 9], [5, 16, 6, 17], [1, 11, 2, 10], [4, 7, 5, 8], [15, 6, 16, 7], [11, 15, 12, 14], [2, 14, 3, 13], [3, 12, 4, 13]]

PD code drawn by `SnapPy`: [(13, 4, 14, 5), (2, 5, 3, 6), (17, 6, 18, 7), (7, 16, 8, 17), (8, 19, 9, 20), (1, 10, 2, 11), (11, 20, 12, 1), (12, 9, 13, 10), (3, 14, 4, 15), (18, 15, 19, 16)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 2, 2], [0, 1, 1, 5], [0, 5, 6, 6], [0, 7, 8, 1], [2, 9, 6, 3], [3, 5, 7, 3], [4, 6, 9, 8], [4, 7, 9, 9], [5, 8, 8, 7]]

Total optimal pinning sets: 1  
Total minimal pinning sets: 7

Total pinning sets: 244

Pinning number: 5

Average optimal degree: 2.4

Average minimal degree: 2.58

Average overall degree: 3.05

Table 289: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	1	0	0	0	0	0	6
Nonminimal pinning sets	0	7	43	75	68	34	9	1	237
Average degree	2.4	2.62	2.85	3.02	3.15	3.24	3.29	3.33	

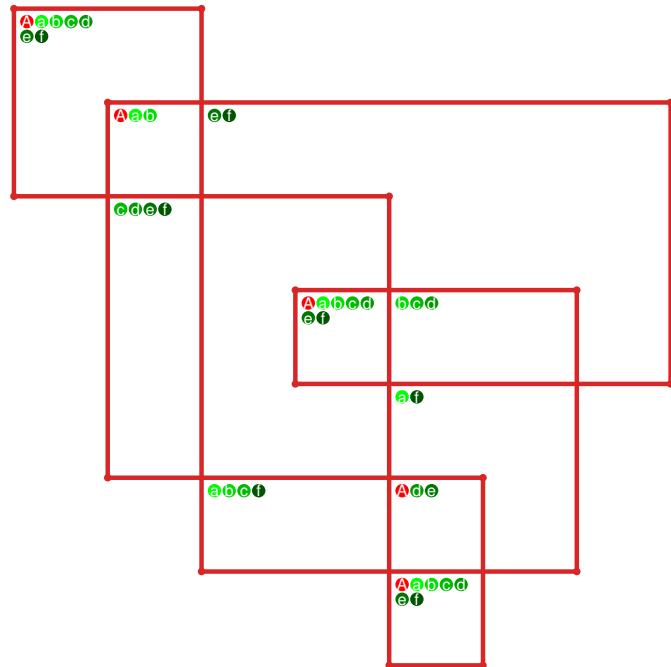


Figure 581: `SnapPy` multiloop plot.

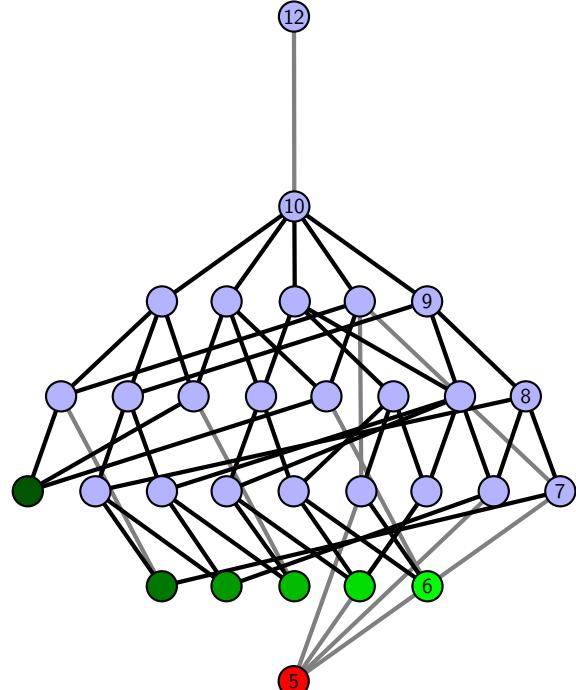


Figure 582: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.143**  $[[9, 20, 10, 1], [11, 8, 12, 9], [14, 19, 15, 20], [10, 2, 11, 1], [4, 7, 5, 8], [12, 5, 13, 6], [6, 13, 7, 14], [18, 15, 19, 16], [2, 18, 3, 17], [3, 16, 4, 17]]$

PD code drawn by `SnapPy`:  $[(10, 3, 11, 4), (2, 5, 3, 6), (9, 6, 10, 7), (4, 11, 5, 12), (16, 13, 17, 14), (1, 14, 2, 15), (15, 20, 16, 1), (12, 17, 13, 18), (7, 18, 8, 19), (19, 8, 20, 9)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 7, 7], [0, 8, 1, 0], [1, 9, 6, 5], [1, 4, 6, 6], [2, 5, 5, 4], [2, 9, 8, 2], [3, 7, 9, 9], [4, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 290: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

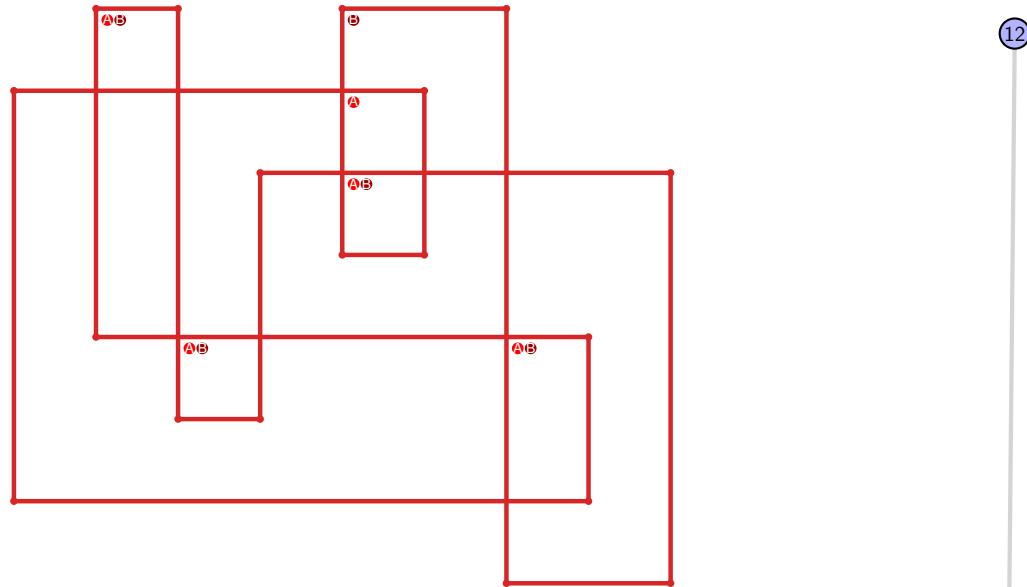


Figure 583: `SnapPy` multiloop plot.

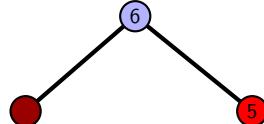


Figure 584: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.144 [[20, 11, 1, 12], [12, 10, 13, 9], [19, 6, 20, 7], [10, 1, 11, 2], [13, 17, 14, 16], [8, 15, 9, 16], [7, 15, 8, 14], [18, 3, 19, 4], [5, 2, 6, 3], [17, 5, 18, 4]]

PD code drawn by SnapPy: [(3, 20, 4, 1), (16, 1, 17, 2), (2, 15, 3, 16), (17, 4, 18, 5), (7, 10, 8, 11), (18, 9, 19, 10), (12, 5, 13, 6), (6, 13, 7, 14), (14, 11, 15, 12), (8, 19, 9, 20)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 7, 8], [0, 8, 1, 0], [1, 9, 6, 5], [1, 4, 6, 6], [2, 5, 5, 4], [2, 9, 9, 8], [2, 7, 9, 3], [4, 8, 7, 7]]

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.4

**Total pinning sets:** 288

**Average overall degree:** 3.03

**Pinning number:** 5

Table 291: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

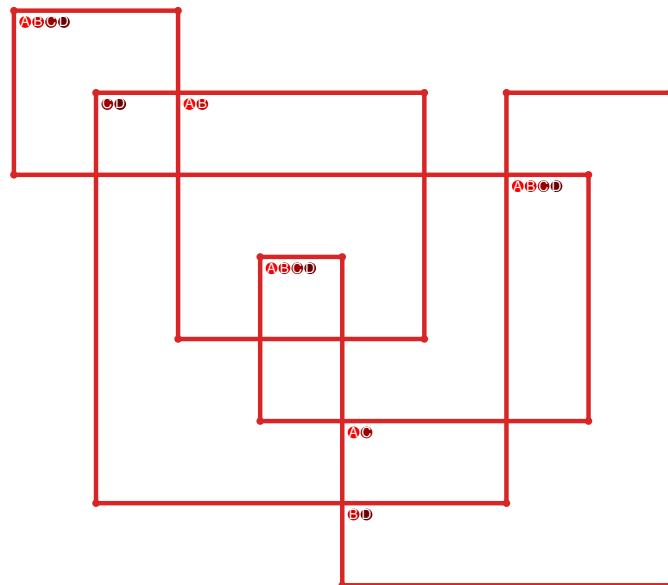


Figure 585: SnapPy multiloop plot.

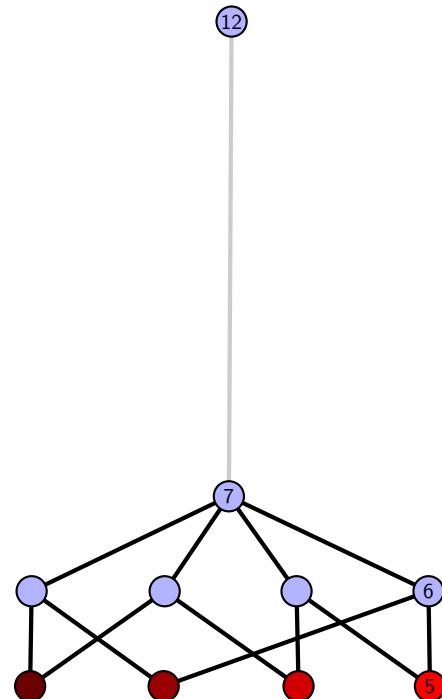


Figure 586: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.145** `[[11, 20, 12, 1], [13, 10, 14, 11], [16, 19, 17, 20], [12, 2, 13, 1], [4, 9, 5, 10], [14, 8, 15, 7], [15, 6, 16, 7], [18, 3, 19, 4], [17, 3, 18, 2], [8, 5, 9, 6]]`

PD code drawn by `SnapPy`: `[(4, 1, 5, 2), (11, 2, 12, 3), (3, 10, 4, 11), (20, 5, 1, 6), (9, 6, 10, 7), (12, 15, 13, 16), (7, 16, 8, 17), (17, 8, 18, 9), (18, 13, 19, 14), (14, 19, 15, 20)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 7, 8], [0, 8, 1, 0], [1, 7, 9, 9], [1, 9, 6, 6], [2, 5, 5, 9], [2, 4, 8, 8], [2, 7, 7, 3], [4, 6, 5, 4]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 292: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

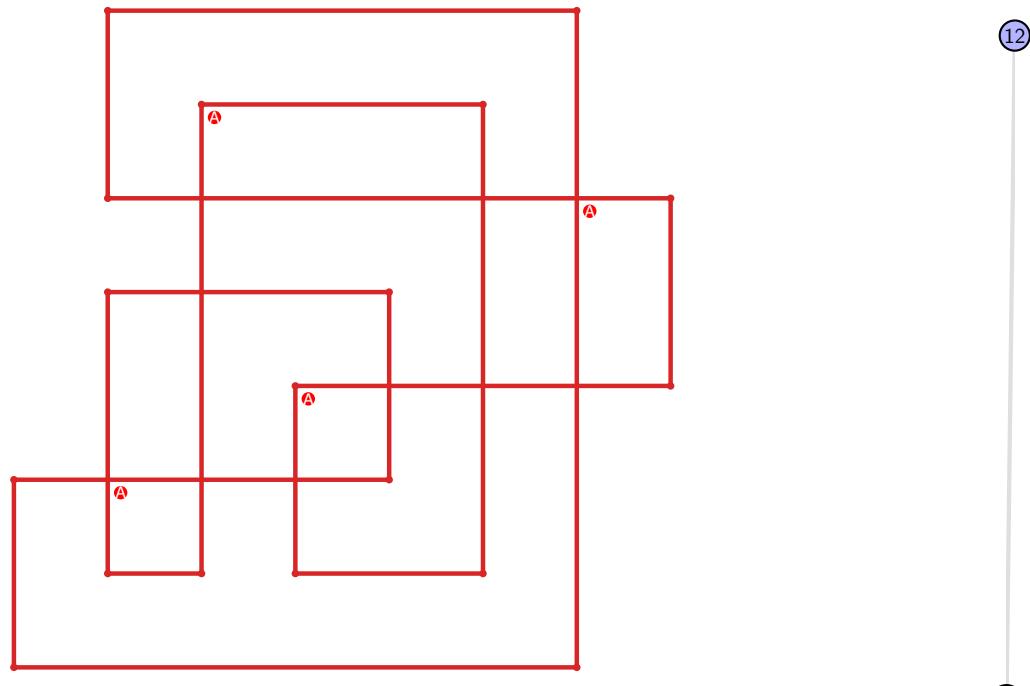


Figure 587: `SnapPy` multiloop plot.

Figure 588: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.146** [[20, 13, 1, 14], [14, 12, 15, 11], [4, 19, 5, 20], [12, 1, 13, 2], [15, 10, 16, 11], [18, 3, 19, 4], [5, 3, 6, 2], [6, 9, 7, 10], [16, 7, 17, 8], [8, 17, 9, 18]]

PD code drawn by `SnapPy`: [(15, 20, 16, 1), (13, 4, 14, 5), (3, 6, 4, 7), (12, 7, 13, 8), (8, 11, 9, 12), (17, 10, 18, 11), (5, 14, 6, 15), (1, 16, 2, 17), (9, 18, 10, 19), (19, 2, 20, 3)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 3, 4, 4], [0, 5, 5, 6], [0, 6, 1, 0], [1, 7, 8, 1], [2, 9, 6, 2], [2, 5, 7, 3], [4, 6, 9, 8], [4, 7, 9, 9], [5, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.34

**Total pinning sets:** 176

**Average overall degree:** 2.98

**Pinning number:** 5

Table 293: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.2	2.5	2.75	2.94	3.08	3.19	3.27	3.33	

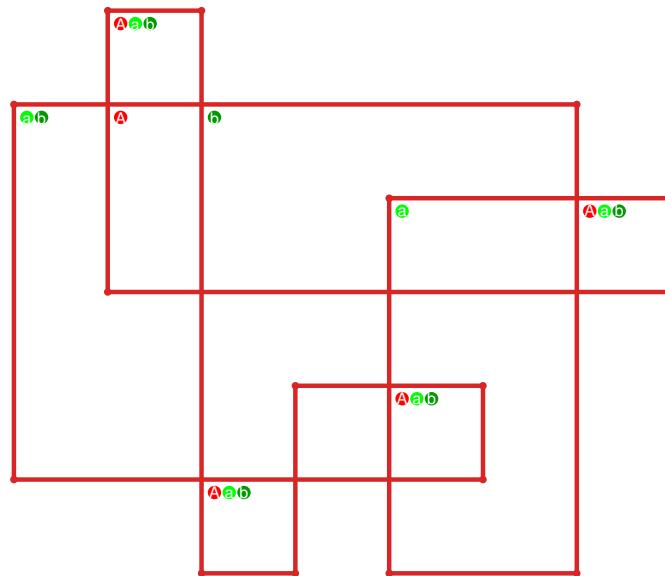


Figure 589: `SnapPy` multiloop plot.

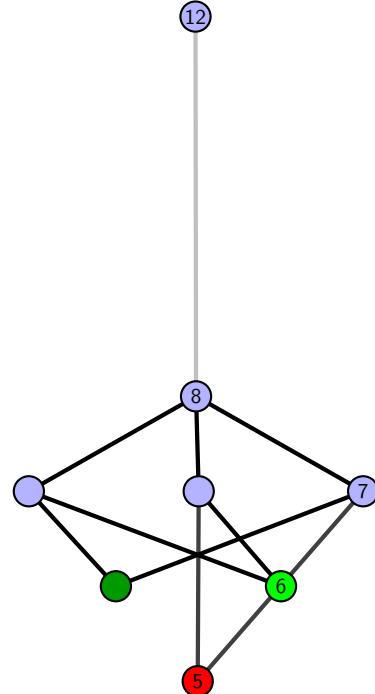


Figure 590: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.147**  $[[9, 20, 10, 1], [11, 8, 12, 9], [19, 4, 20, 5], [10, 2, 11, 1], [7, 12, 8, 13], [5, 15, 6, 14], [3, 18, 4, 19], [2, 18, 3, 17], [13, 17, 14, 16], [6, 15, 7, 16]]$

PD code drawn by `SnapPy`:  $[(16, 1, 17, 2), (8, 5, 9, 6), (4, 9, 5, 10), (11, 2, 12, 3), (3, 12, 4, 13), (13, 10, 14, 11), (14, 19, 15, 20), (20, 15, 1, 16), (6, 17, 7, 18), (18, 7, 19, 8)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 3, 4, 4], [0, 5, 6, 6], [0, 7, 1, 0], [1, 8, 9, 1], [2, 9, 9, 8], [2, 7, 7, 2], [3, 6, 6, 8], [4, 7, 5, 9], [4, 8, 5, 5]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 294: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

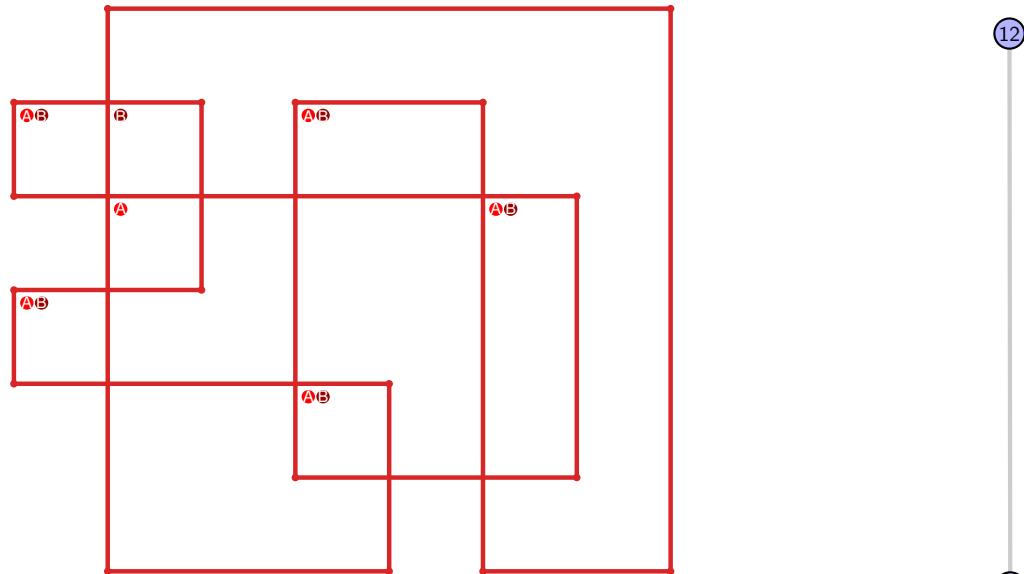


Figure 591: `SnapPy` multiloop plot.

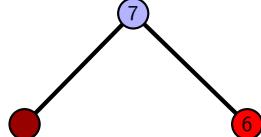


Figure 592: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.148 [[13, 20, 14, 1], [15, 12, 16, 13], [19, 4, 20, 5], [14, 2, 15, 1], [8, 11, 9, 12], [16, 9, 17, 10], [5, 18, 6, 19], [6, 3, 7, 4], [2, 7, 3, 8], [10, 17, 11, 18]]

PD code drawn by `SnapPy`: [(3, 20, 4, 1), (16, 1, 17, 2), (19, 4, 20, 5), (13, 6, 14, 7), (5, 8, 6, 9), (12, 9, 13, 10), (7, 14, 8, 15), (2, 15, 3, 16), (10, 17, 11, 18), (18, 11, 19, 12)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 3, 4, 5], [0, 6, 6, 7], [0, 8, 1, 0], [1, 8, 9, 5], [1, 4, 9, 9], [2, 9, 7, 2], [2, 6, 8, 8], [3, 7, 7, 4], [4, 6, 5, 5]]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 295: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

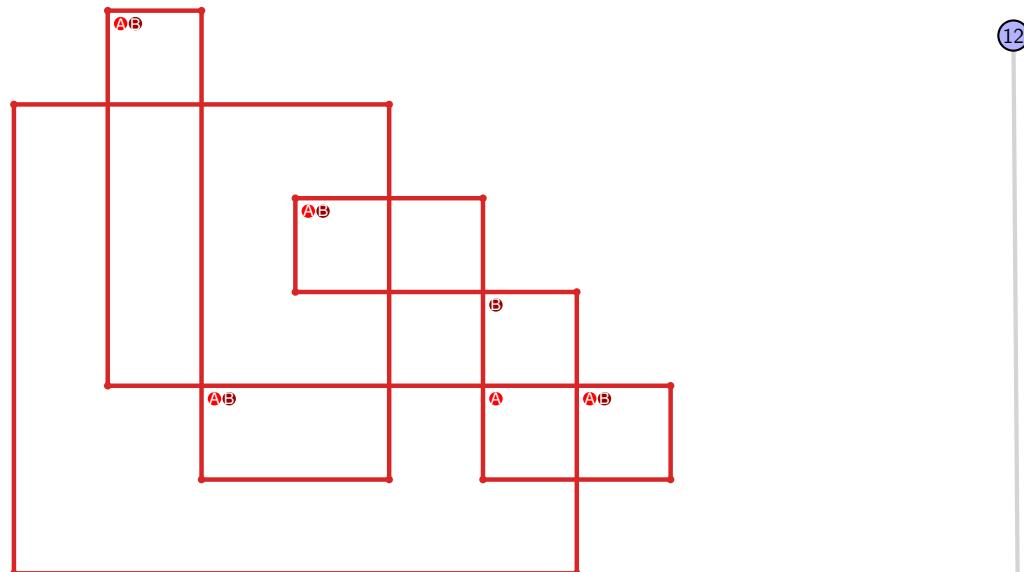


Figure 593: `SnapPy` multiloop plot.

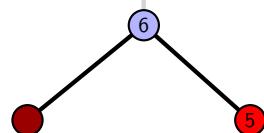


Figure 594: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.149**  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 2, 11, 1], [7, 18, 8, 19], [2, 12, 3, 11], [17, 6, 18, 7], [12, 16, 13, 15], [3, 15, 4, 14], [5, 16, 6, 17], [13, 5, 14, 4]]$

PD code drawn by `SnapPy`:  $[(1, 10, 2, 11), (14, 3, 15, 4), (16, 5, 17, 6), (8, 19, 9, 20), (11, 20, 12, 1), (12, 9, 13, 10), (2, 13, 3, 14), (4, 15, 5, 16), (6, 17, 7, 18), (18, 7, 19, 8)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 5, 1], [2, 6, 7, 2], [3, 8, 8, 3], [4, 8, 9, 7], [4, 6, 9, 9], [5, 9, 6, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.12

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 20

**Average overall degree:** 2.82

**Pinning number:** 8

Table 296: Pinning sets/average degree by cardinal

Cardinal	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	1
Nonminimal pinning sets	0	4	8	5	1	18
Average degree	2.12	2.47	2.85	3.16	3.33	

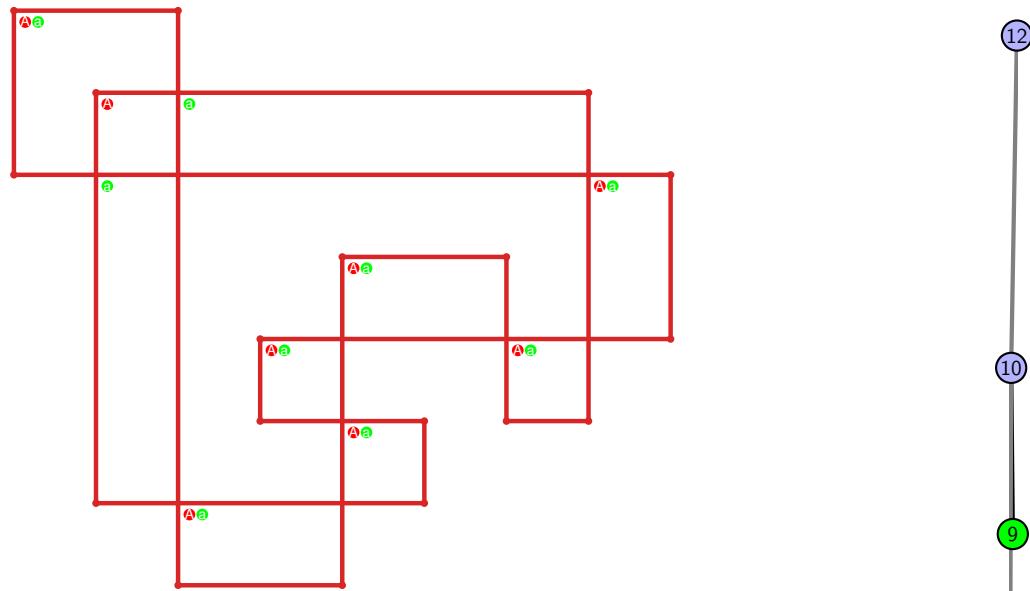


Figure 595: `SnapPy` multiloop plot.

Figure 596: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.150** [[20, 9, 1, 10], [10, 19, 11, 20], [8, 1, 9, 2], [18, 11, 19, 12], [2, 7, 3, 8], [12, 17, 13, 18], [13, 6, 14, 7], [3, 14, 4, 15], [5, 16, 6, 17], [4, 16, 5, 15]]

PD code drawn by `SnapPy`: [(8, 1, 9, 2), (17, 2, 18, 3), (15, 4, 16, 5), (13, 6, 14, 7), (18, 9, 19, 10), (10, 19, 11, 20), (20, 11, 1, 12), (7, 12, 8, 13), (5, 14, 6, 15), (3, 16, 4, 17)]

Planar representation generated by `plantri`: [[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 5, 1], [2, 6, 7, 2], [3, 8, 6, 3], [4, 5, 8, 7], [4, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.81

**Pinning number:** 7

Table 297: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

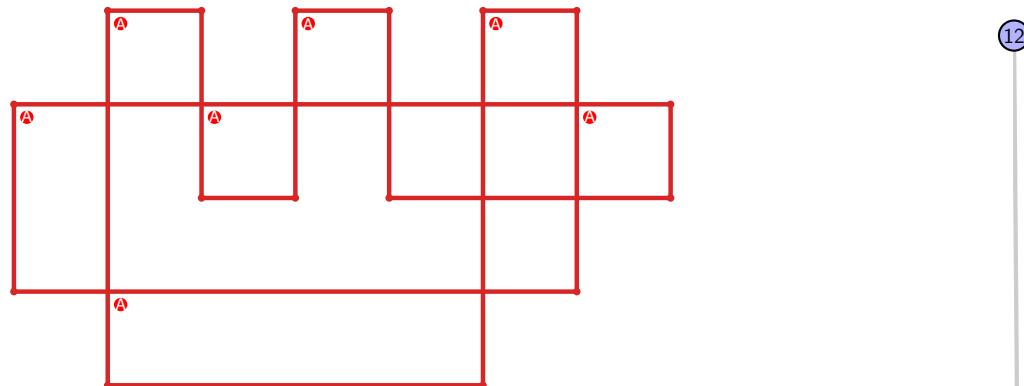


Figure 597: `SnapPy` multiloop plot.



Figure 598: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.151**  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 2, 11, 1], [7, 18, 8, 19], [2, 12, 3, 11], [6, 13, 7, 14], [17, 12, 18, 13], [3, 17, 4, 16], [14, 5, 15, 6], [4, 15, 5, 16]]$

PD code drawn by `SnapPy`:  $[(11, 2, 12, 3), (16, 5, 17, 6), (8, 19, 9, 20), (14, 9, 15, 10), (3, 10, 4, 11), (1, 12, 2, 13), (13, 20, 14, 1), (4, 15, 5, 16), (6, 17, 7, 18), (18, 7, 19, 8)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 6, 7, 2], [3, 8, 8, 6], [3, 5, 7, 4], [4, 6, 9, 9], [5, 9, 9, 5], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.81

**Pinning number:** 7

Table 298: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

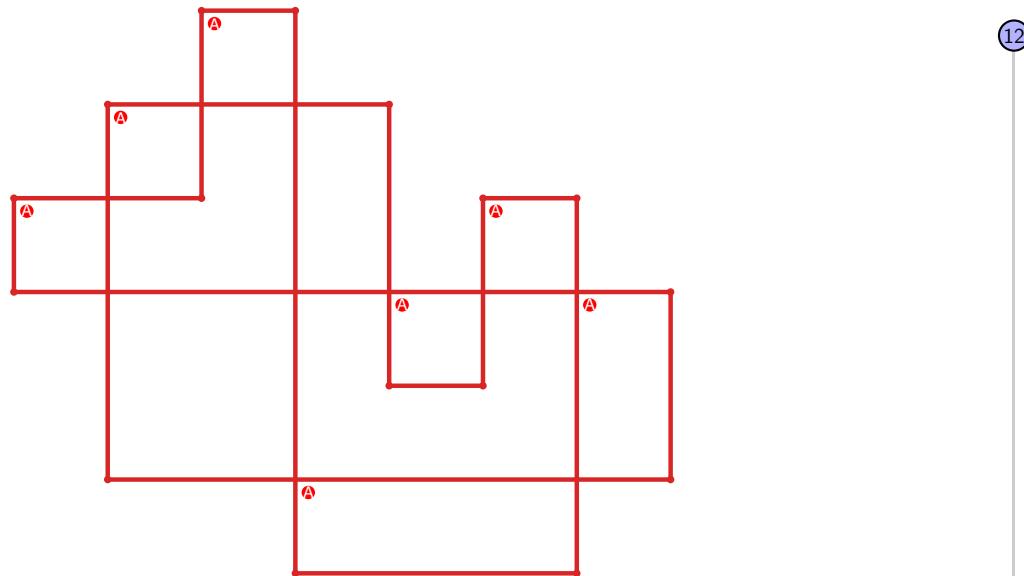


Figure 599: `SnapPy` multiloop plot.

Figure 600: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.152 [[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [6, 15, 7, 16], [1, 11, 2, 10], [18, 10, 19, 9], [14, 5, 15, 6], [11, 3, 12, 2], [4, 13, 5, 14], [3, 13, 4, 12]]

PD code drawn by `SnapPy`: [(20, 9, 1, 10), (12, 3, 13, 4), (18, 5, 19, 6), (7, 16, 8, 17), (8, 19, 9, 20), (10, 1, 11, 2), (2, 11, 3, 12), (4, 13, 5, 14), (17, 14, 18, 15), (15, 6, 16, 7)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 7, 7, 5], [1, 4, 2, 1], [3, 8, 8, 3], [4, 9, 9, 4], [6, 9, 9, 6], [7, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.14

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 40

**Average overall degree:** 2.86

**Pinning number:** 7

Table 299: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	1
Nonminimal pinning sets	0	5	13	13	6	1	38
Average degree	2.14	2.46	2.76	3.02	3.21	3.33	

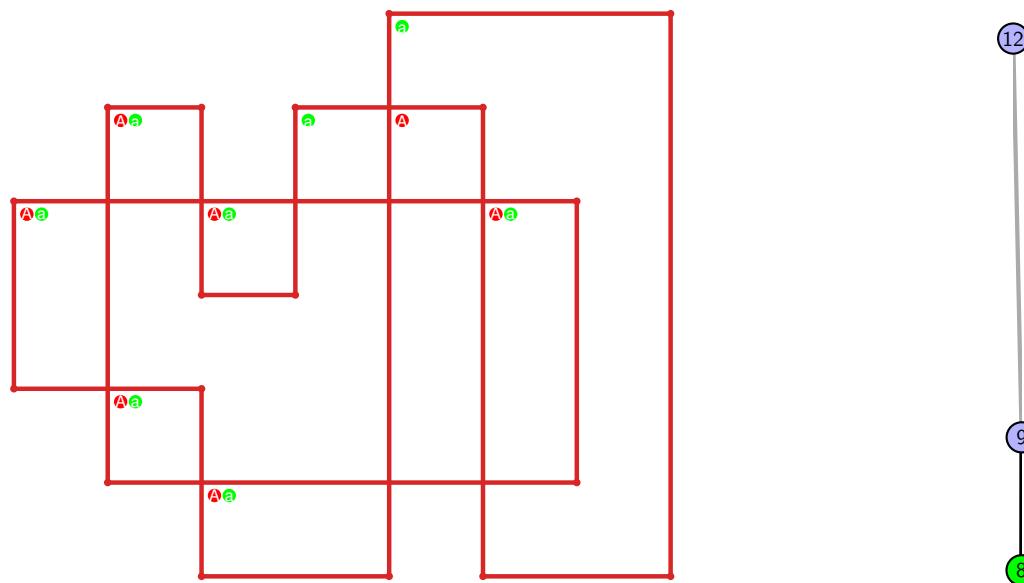


Figure 601: `SnapPy` multiloop plot.



Figure 602: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.153 [[20, 9, 1, 10], [10, 5, 11, 6], [8, 19, 9, 20], [1, 12, 2, 13], [15, 4, 16, 5], [11, 16, 12, 17], [6, 17, 7, 18], [18, 7, 19, 8], [2, 14, 3, 13], [3, 14, 4, 15]]

PD code drawn by SnapPy: [(13, 2, 14, 3), (8, 3, 9, 4), (16, 5, 17, 6), (18, 7, 19, 8), (19, 10, 20, 11), (11, 20, 12, 1), (1, 12, 2, 13), (9, 14, 10, 15), (4, 15, 5, 16), (6, 17, 7, 18)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 5, 6], [0, 7, 7, 0], [0, 5, 8, 8], [1, 9, 9, 5], [1, 4, 3, 6], [1, 5, 7, 7], [2, 6, 6, 2], [3, 9, 9, 3], [4, 8, 8, 4]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.14

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.14

**Total pinning sets:** 48

**Average overall degree:** 2.86

**Pinning number:** 7

Table 300: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.14	2.53	2.82	3.04	3.21	3.33	

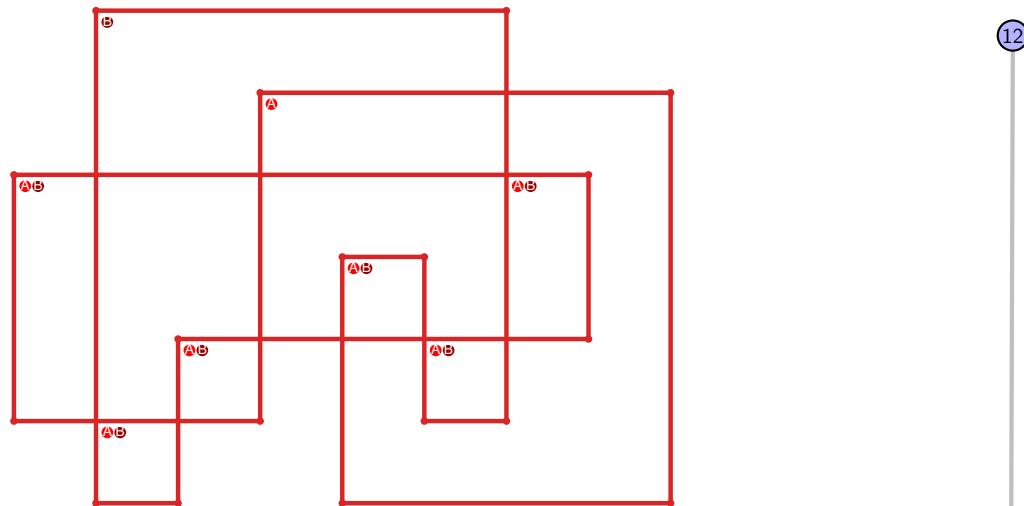


Figure 603: SnapPy multiloop plot.

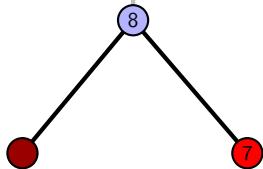


Figure 604: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.154  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 16, 11, 15], [1, 15, 2, 14], [7, 18, 8, 19], [16, 6, 17, 5], [11, 3, 12, 2], [4, 13, 5, 14], [17, 6, 18, 7], [3, 13, 4, 12]]$

PD code drawn by SnapPy:  $[(20, 9, 1, 10), (3, 14, 4, 15), (15, 4, 16, 5), (17, 6, 18, 7), (10, 1, 11, 2), (2, 11, 3, 12), (12, 19, 13, 20), (13, 8, 14, 9), (5, 16, 6, 17), (7, 18, 8, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 3], [0, 2, 6, 7], [1, 8, 8, 1], [2, 8, 8, 7], [2, 9, 9, 3], [3, 9, 9, 5], [4, 5, 5, 4], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 48  
**Pinning number:** 7

**Average optimal degree:** 2.14

**Average minimal degree:** 2.14

**Average overall degree:** 2.86

Table 301: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.14	2.53	2.82	3.04	3.21	3.33	

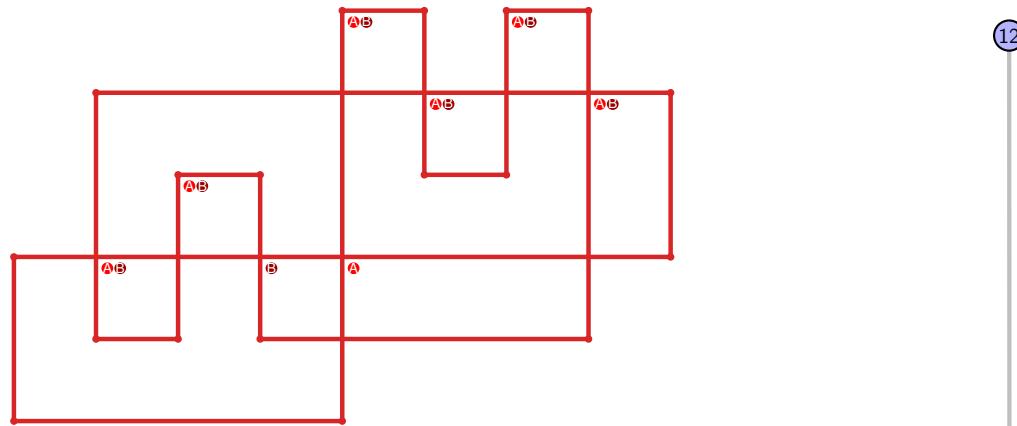


Figure 605: SnapPy multiloop plot.

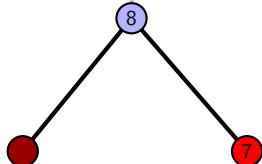


Figure 606: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.155  $[[20, 5, 1, 6], [6, 16, 7, 15], [19, 14, 20, 15], [4, 11, 5, 12], [1, 11, 2, 10], [16, 8, 17, 7], [9, 18, 10, 19], [2, 13, 3, 14], [12, 3, 13, 4], [8, 18, 9, 17]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (18, 3, 19, 4), (5, 14, 6, 15), (8, 19, 9, 20), (2, 9, 3, 10), (20, 11, 1, 12), (17, 12, 18, 13), (13, 4, 14, 5), (15, 6, 16, 7), (7, 16, 8, 17)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 8, 8, 4], [0, 3, 7, 6], [1, 9, 9, 1], [2, 9, 9, 4], [2, 4, 8, 8], [3, 7, 7, 3], [5, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 302: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

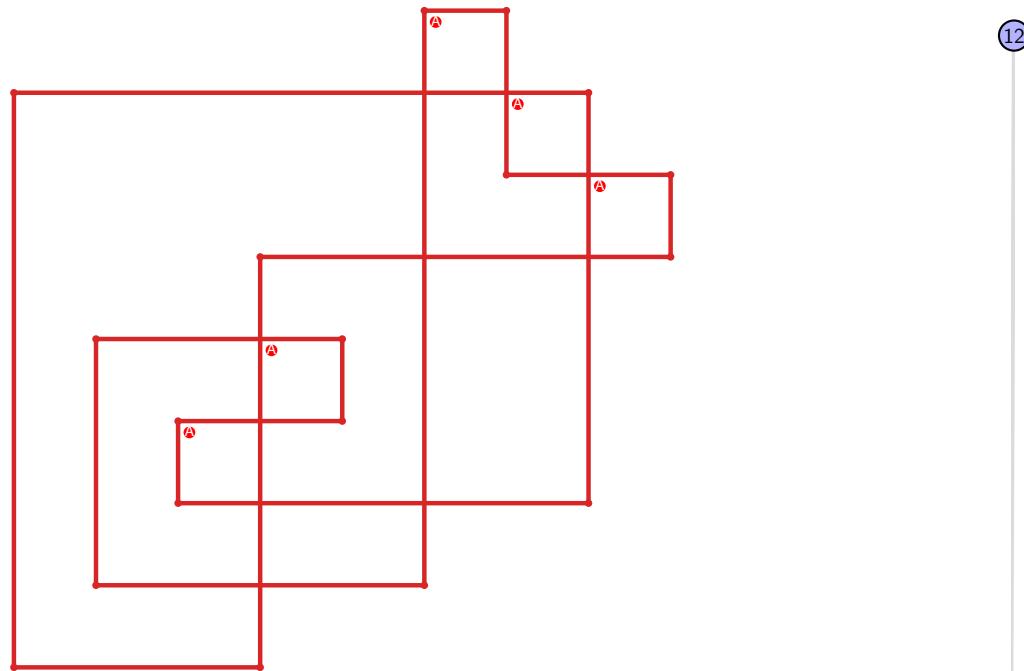


Figure 607: SnapPy multiloop plot.

5

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Figure 608: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.156** [[11, 20, 12, 1], [19, 10, 20, 11], [12, 2, 13, 1], [18, 7, 19, 8], [9, 2, 10, 3], [13, 9, 14, 8], [6, 17, 7, 18], [3, 17, 4, 16], [14, 5, 15, 6], [4, 15, 5, 16]]

PD code drawn by `SnapPy`: [(11, 2, 12, 3), (15, 4, 16, 5), (5, 10, 6, 11), (16, 7, 17, 8), (1, 12, 2, 13), (13, 20, 14, 1), (3, 14, 4, 15), (8, 17, 9, 18), (18, 9, 19, 10), (6, 19, 7, 20)]

Planar representation generated by `plantri`: [[1, 1, 2, 2], [0, 3, 4, 0], [0, 4, 5, 0], [1, 5, 6, 6], [1, 7, 5, 2], [2, 4, 8, 3], [3, 8, 7, 3], [4, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 303: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

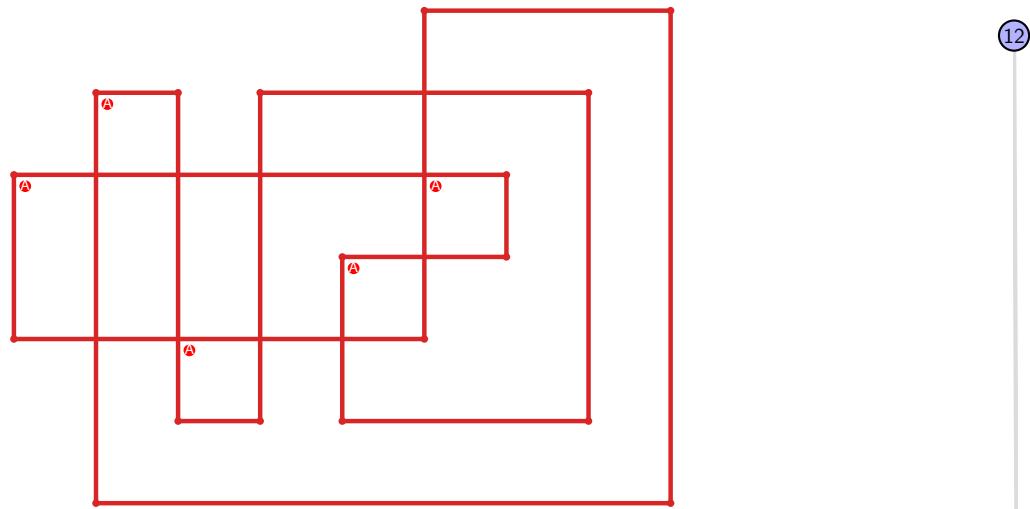


Figure 609: `SnapPy` multiloop plot.

5

Figure 610: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.157** `[[20, 13, 1, 14], [14, 19, 15, 20], [12, 1, 13, 2], [7, 18, 8, 19], [15, 3, 16, 2], [16, 11, 17, 12], [17, 6, 18, 7], [8, 4, 9, 3], [5, 10, 6, 11], [4, 10, 5, 9]]`

PD code drawn by `SnapPy`: `[(13, 2, 14, 3), (5, 20, 6, 1), (1, 6, 2, 7), (12, 7, 13, 8), (16, 9, 17, 10), (3, 14, 4, 15), (10, 15, 11, 16), (8, 17, 9, 18), (18, 11, 19, 12), (19, 4, 20, 5)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 2], [0, 3, 4, 0], [0, 4, 5, 0], [1, 6, 6, 7], [1, 7, 5, 2], [2, 4, 8, 6], [3, 5, 8, 3], [3, 9, 9, 4], [5, 9, 9, 6], [7, 8, 8, 7]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 304: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

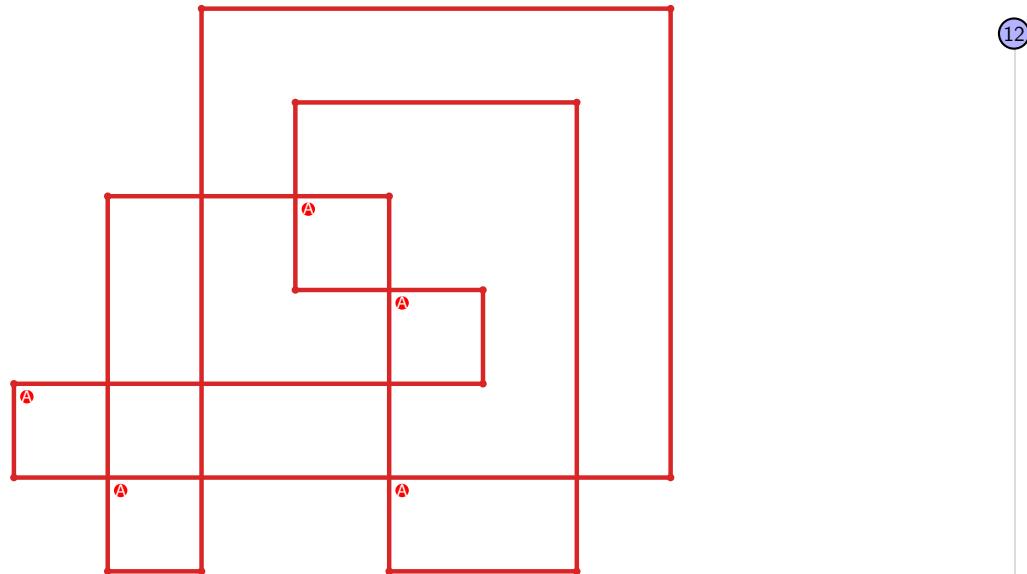


Figure 611: `SnapPy` multiloop plot.

5

Figure 612: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.158 [[13, 20, 14, 1], [19, 12, 20, 13], [14, 12, 15, 11], [1, 16, 2, 17], [18, 5, 19, 6], [15, 10, 16, 11], [2, 8, 3, 7], [17, 7, 18, 6], [9, 4, 10, 5], [8, 4, 9, 3]]

PD code drawn by SnapPy: [(20, 5, 1, 6), (6, 1, 7, 2), (15, 2, 16, 3), (3, 14, 4, 15), (4, 19, 5, 20), (12, 7, 13, 8), (8, 11, 9, 12), (16, 9, 17, 10), (18, 13, 19, 14), (10, 17, 11, 18)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 7], [1, 7, 7, 8], [2, 8, 3, 2], [3, 9, 9, 7], [3, 6, 4, 4], [4, 9, 9, 5], [6, 8, 8, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 305: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

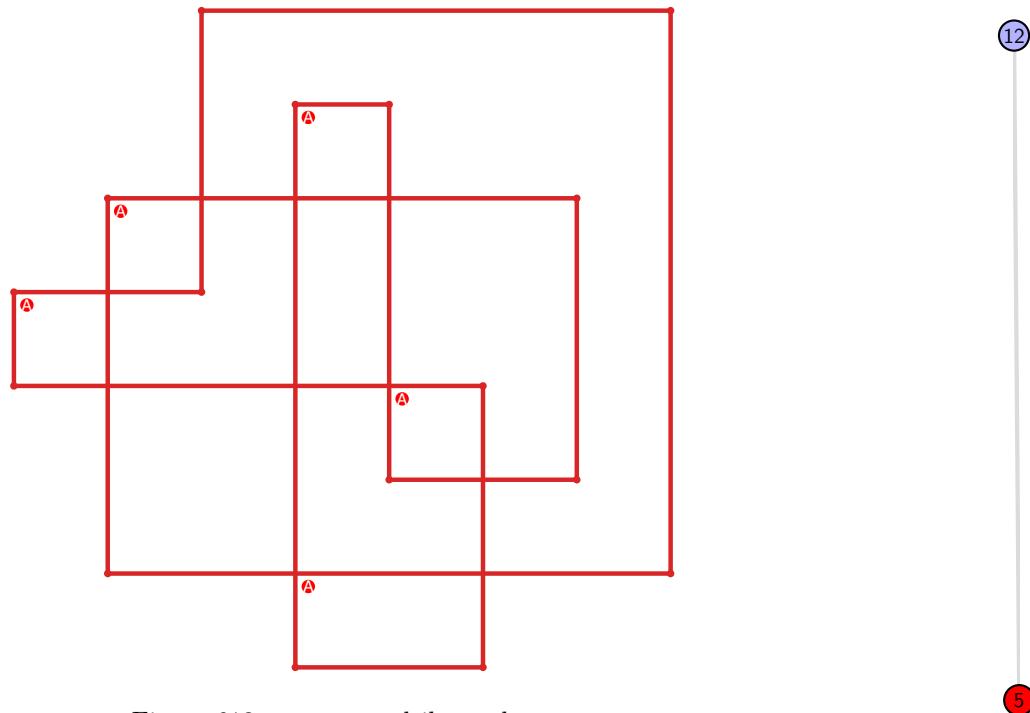


Figure 613: SnapPy multiloop plot.

12

5

Figure 614: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.159  $[[15, 20, 16, 1], [3, 14, 4, 15], [4, 19, 5, 20], [16, 7, 17, 8], [1, 10, 2, 11], [11, 2, 12, 3], [13, 8, 14, 9], [18, 5, 19, 6], [6, 17, 7, 18], [9, 12, 10, 13]]$

PD code drawn by SnapPy:  $[(15, 2, 16, 3), (13, 4, 14, 5), (10, 5, 11, 6), (18, 7, 19, 8), (6, 9, 7, 10), (20, 11, 1, 12), (3, 14, 4, 15), (1, 16, 2, 17), (12, 17, 13, 18), (8, 19, 9, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 7, 7], [0, 8, 8, 6], [0, 9, 5, 5], [1, 4, 4, 9], [1, 9, 9, 3], [2, 8, 8, 2], [3, 7, 7, 3], [4, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 306: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

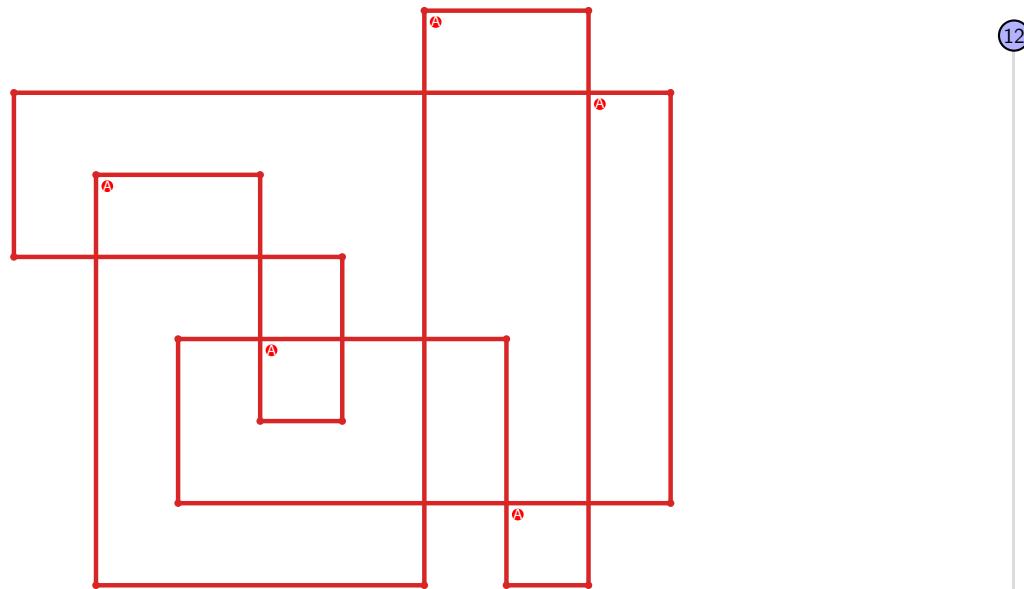


Figure 615: SnapPy multiloop plot.

12

5

Figure 616: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.160**  $[[5, 20, 6, 1], [4, 9, 5, 10], [19, 8, 20, 9], [6, 18, 7, 17], [1, 15, 2, 14], [10, 3, 11, 4], [7, 18, 8, 19], [11, 16, 12, 17], [15, 12, 16, 13], [2, 13, 3, 14]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (17, 6, 18, 7), (2, 7, 3, 8), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (9, 14, 10, 15), (15, 4, 16, 5), (5, 16, 6, 17), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 6], [0, 6, 6, 7], [0, 8, 9, 9], [1, 9, 7, 1], [2, 3, 3, 2], [3, 5, 8, 8], [4, 7, 7, 9], [4, 8, 5, 4]]$

Total optimal pinning sets: 1  
Total minimal pinning sets: 1

Total pinning sets: 128

Pinning number: 5

Average optimal degree: 2.0

Average minimal degree: 2.0

Average overall degree: 2.91

Table 307: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

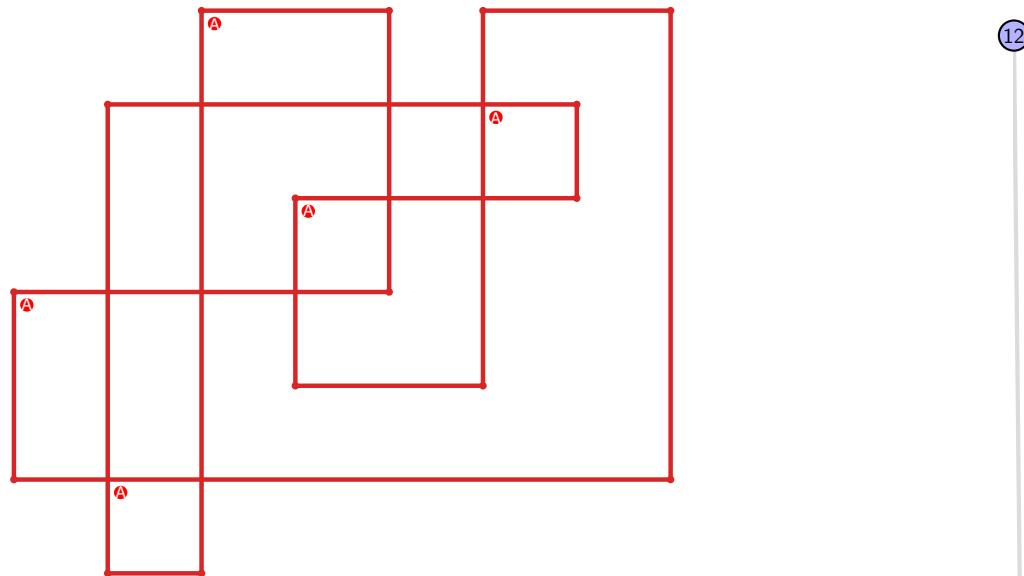


Figure 617: `SnapPy` multiloop plot.

Figure 618: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.161  $[[20, 5, 1, 6], [6, 16, 7, 15], [19, 12, 20, 13], [4, 9, 5, 10], [1, 9, 2, 8], [16, 8, 17, 7], [17, 14, 18, 15], [13, 18, 14, 19], [2, 11, 3, 12], [10, 3, 11, 4]]$

PD code drawn by SnapPy:  $[(7, 20, 8, 1), (17, 2, 18, 3), (4, 13, 5, 14), (5, 18, 6, 19), (1, 6, 2, 7), (19, 8, 20, 9), (16, 9, 17, 10), (14, 11, 15, 12), (12, 3, 13, 4), (10, 15, 11, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 7, 7, 8], [0, 9, 9, 4], [0, 3, 8, 5], [1, 4, 6, 1], [1, 5, 7, 7], [2, 6, 6, 2], [2, 4, 9, 9], [3, 8, 8, 3]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1

**Total pinning sets:** 128

**Pinning number:** 5

**Average optimal degree:** 2.0

**Average minimal degree:** 2.0

**Average overall degree:** 2.91

Table 308: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

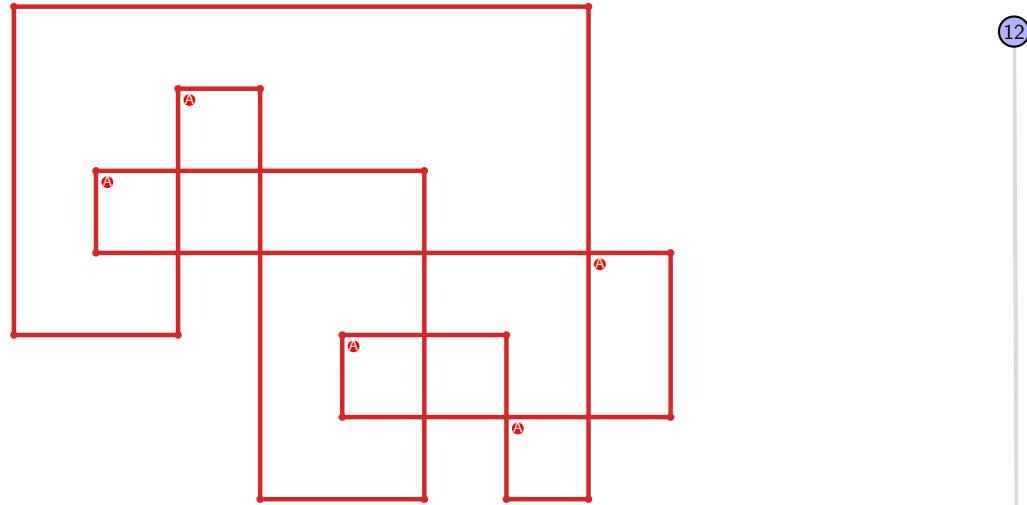


Figure 619: SnapPy multiloop plot.

5

Figure 620: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.162 [[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [6, 11, 7, 12], [1, 11, 2, 10], [18, 10, 19, 9], [2, 15, 3, 16], [12, 5, 13, 6], [14, 3, 15, 4], [4, 13, 5, 14]]

PD code drawn by SnapPy: [(11, 20, 12, 1), (9, 2, 10, 3), (17, 4, 18, 5), (6, 15, 7, 16), (7, 18, 8, 19), (3, 8, 4, 9), (1, 10, 2, 11), (19, 12, 20, 13), (16, 13, 17, 14), (14, 5, 15, 6)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 4], [0, 3, 6, 5], [1, 4, 2, 1], [2, 4, 8, 8], [3, 9, 9, 3], [6, 9, 9, 6], [7, 8, 8, 7]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 309: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

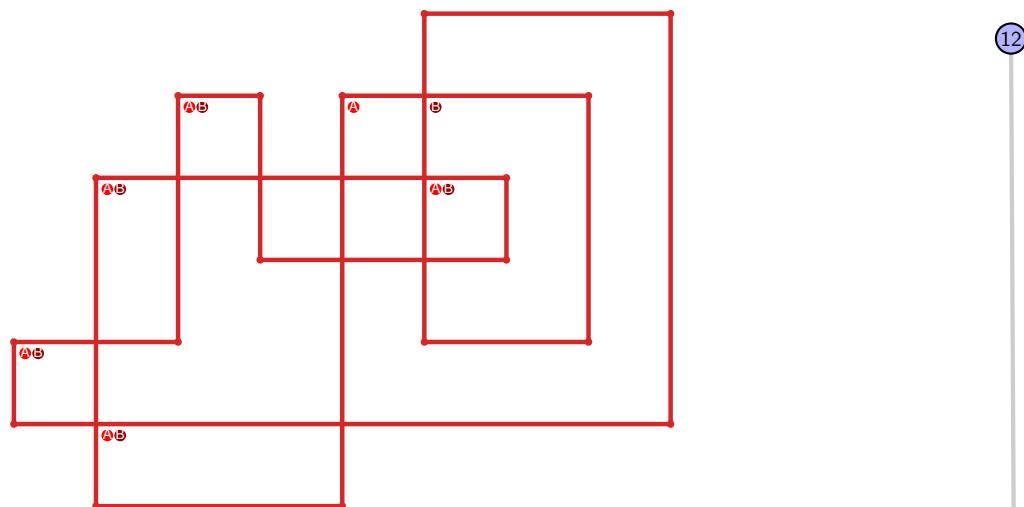


Figure 621: SnapPy multiloop plot.

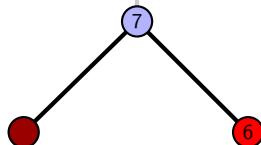


Figure 622: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.163 [[20, 5, 1, 6], [6, 16, 7, 15], [19, 14, 20, 15], [4, 9, 5, 10], [1, 9, 2, 8], [16, 8, 17, 7], [13, 18, 14, 19], [10, 3, 11, 4], [2, 11, 3, 12], [17, 12, 18, 13]]

PD code drawn by SnapPy: [(7, 20, 8, 1), (17, 2, 18, 3), (4, 13, 5, 14), (5, 18, 6, 19), (1, 6, 2, 7), (19, 8, 20, 9), (14, 9, 15, 10), (16, 11, 17, 12), (12, 3, 13, 4), (10, 15, 11, 16)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 6], [0, 7, 7, 4], [0, 3, 8, 5], [1, 4, 9, 1], [2, 9, 9, 2], [3, 8, 8, 3], [4, 7, 7, 9], [5, 8, 6, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 310: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

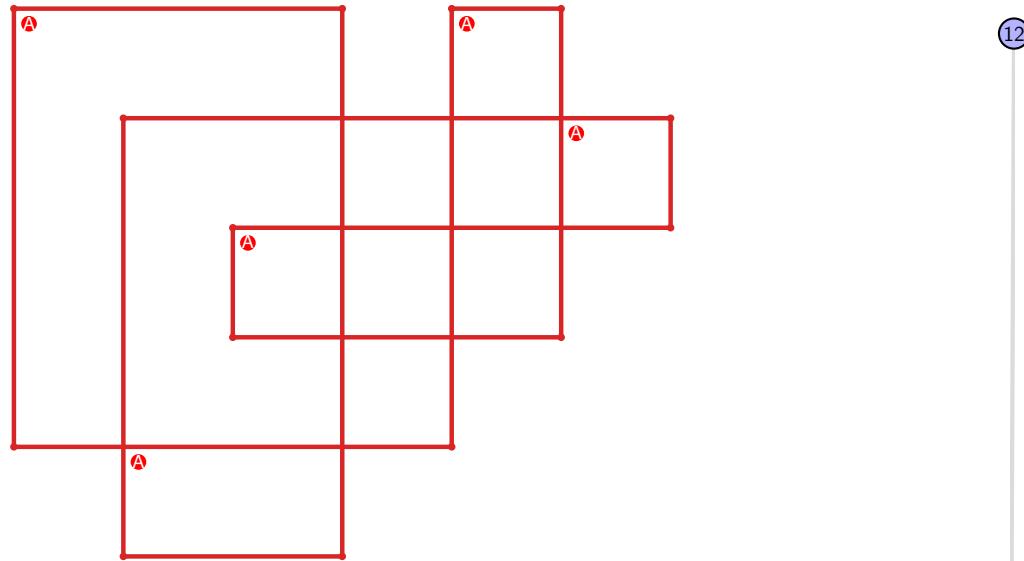


Figure 623: SnapPy multiloop plot.

12

5

Figure 624: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.164 [[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [6, 13, 7, 14], [1, 13, 2, 12], [18, 10, 19, 9], [4, 15, 5, 16], [14, 5, 15, 6], [2, 11, 3, 12], [10, 3, 11, 4]]

PD code drawn by SnapPy: [(8, 1, 9, 2), (12, 3, 13, 4), (18, 5, 19, 6), (7, 16, 8, 17), (20, 9, 1, 10), (10, 19, 11, 20), (4, 11, 5, 12), (2, 13, 3, 14), (17, 14, 18, 15), (15, 6, 16, 7)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 4], [0, 3, 8, 8], [1, 9, 2, 1], [2, 9, 7, 7], [3, 6, 6, 3], [4, 9, 9, 4], [5, 8, 8, 6]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 96  
 Pinning number: 6

Average optimal degree: 2.17  
 Average minimal degree: 2.17  
 Average overall degree: 2.91

Table 311: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

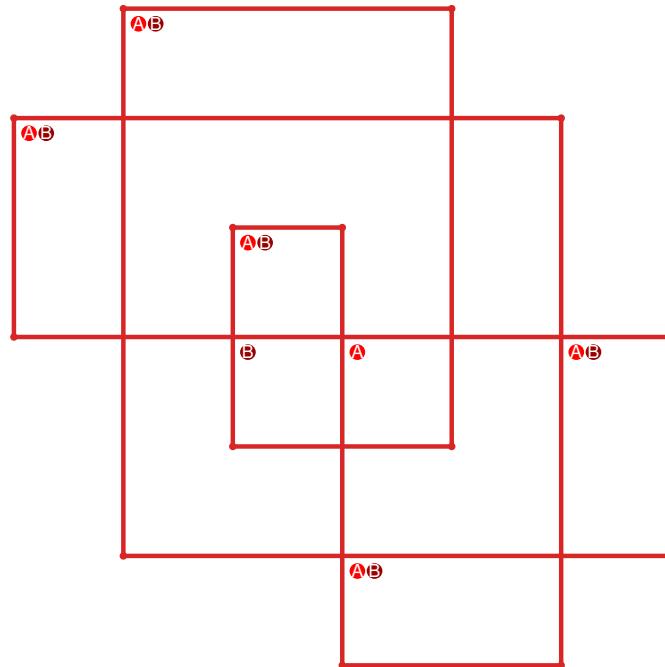


Figure 625: SnapPy multiloop plot.

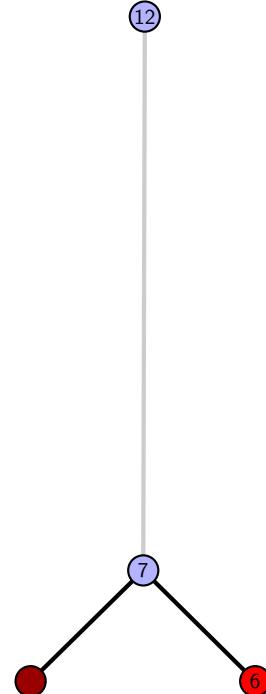


Figure 626: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.165  $[[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 1, 9, 2], [18, 12, 19, 11], [4, 15, 5, 16], [2, 7, 3, 8], [12, 3, 13, 4], [14, 5, 15, 6], [6, 13, 7, 14]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (12, 3, 13, 4), (20, 5, 1, 6), (18, 7, 19, 8), (9, 16, 10, 17), (4, 11, 5, 12), (2, 13, 3, 14), (17, 14, 18, 15), (15, 8, 16, 9), (6, 19, 7, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 6, 0], [1, 7, 2, 1], [2, 7, 8, 8], [3, 9, 7, 3], [4, 6, 9, 5], [5, 9, 9, 5], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 312: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

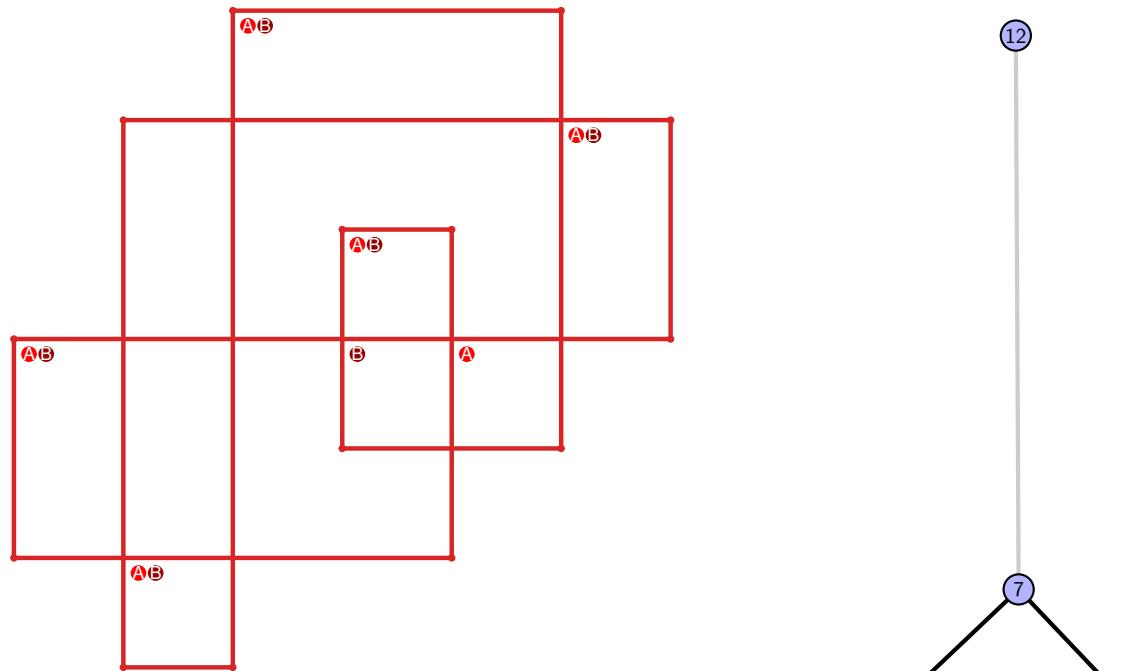


Figure 627: SnapPy multiloop plot.

Figure 628: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.166**  $[[9, 20, 10, 1], [13, 8, 14, 9], [19, 10, 20, 11], [1, 6, 2, 7], [7, 12, 8, 13], [14, 12, 15, 11], [3, 18, 4, 19], [5, 16, 6, 17], [2, 16, 3, 15], [17, 4, 18, 5]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (17, 2, 18, 3), (9, 4, 10, 5), (15, 6, 16, 7), (7, 10, 8, 11), (3, 8, 4, 9), (11, 18, 12, 19), (1, 12, 2, 13), (19, 14, 20, 15), (5, 16, 6, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 5, 6, 0], [0, 7, 8, 4], [1, 3, 5, 1], [1, 4, 8, 2], [2, 8, 9, 9], [3, 9, 9, 8], [3, 7, 6, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3

**Total pinning sets:** 224

**Pinning number:** 5

**Average optimal degree:** 2.27

**Average minimal degree:** 2.27

**Average overall degree:** 2.98

Table 313: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

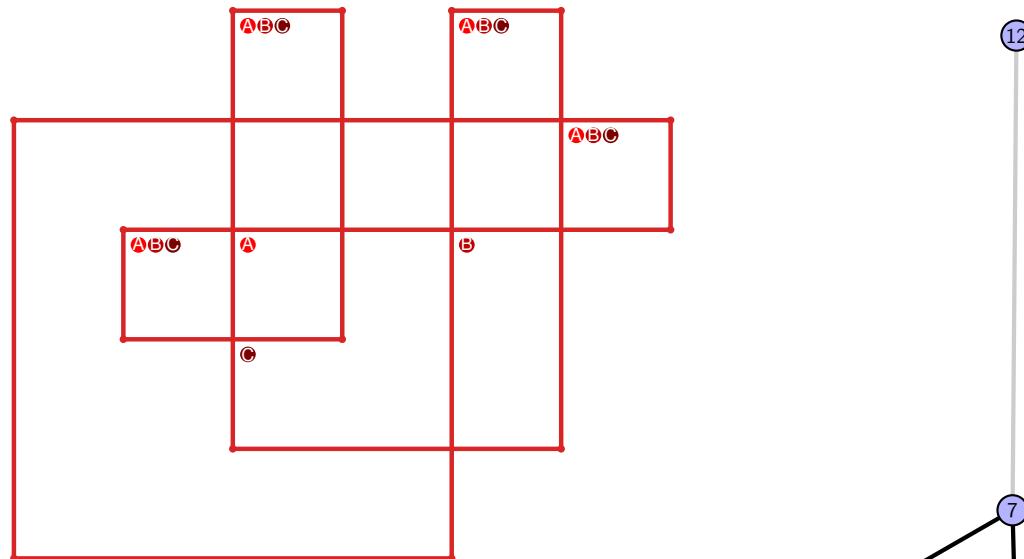


Figure 629: `SnapPy` multiloop plot.

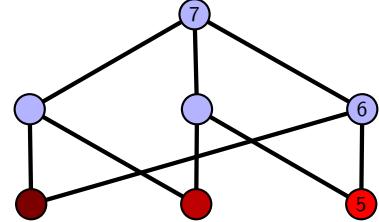


Figure 630: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.167**  $[[11, 20, 12, 1], [10, 17, 11, 18], [19, 16, 20, 17], [12, 6, 13, 5], [1, 8, 2, 9], [18, 9, 19, 10], [15, 6, 16, 7], [13, 4, 14, 5], [7, 2, 8, 3], [3, 14, 4, 15]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (2, 9, 3, 10), (12, 3, 13, 4), (18, 5, 19, 6), (15, 6, 16, 7), (20, 11, 1, 12), (16, 13, 17, 14), (7, 14, 8, 15), (8, 17, 9, 18), (4, 19, 5, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 7], [0, 8, 8, 5], [1, 4, 2, 1], [2, 8, 9, 3], [3, 9, 9, 3], [4, 9, 6, 4], [6, 8, 7, 7]]$

Total optimal pinning sets: 2  
Total minimal pinning sets: 2

Total pinning sets: 192

Pinning number: 5

Average optimal degree: 2.2

Average minimal degree: 2.2

Average overall degree: 2.97

Table 314: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

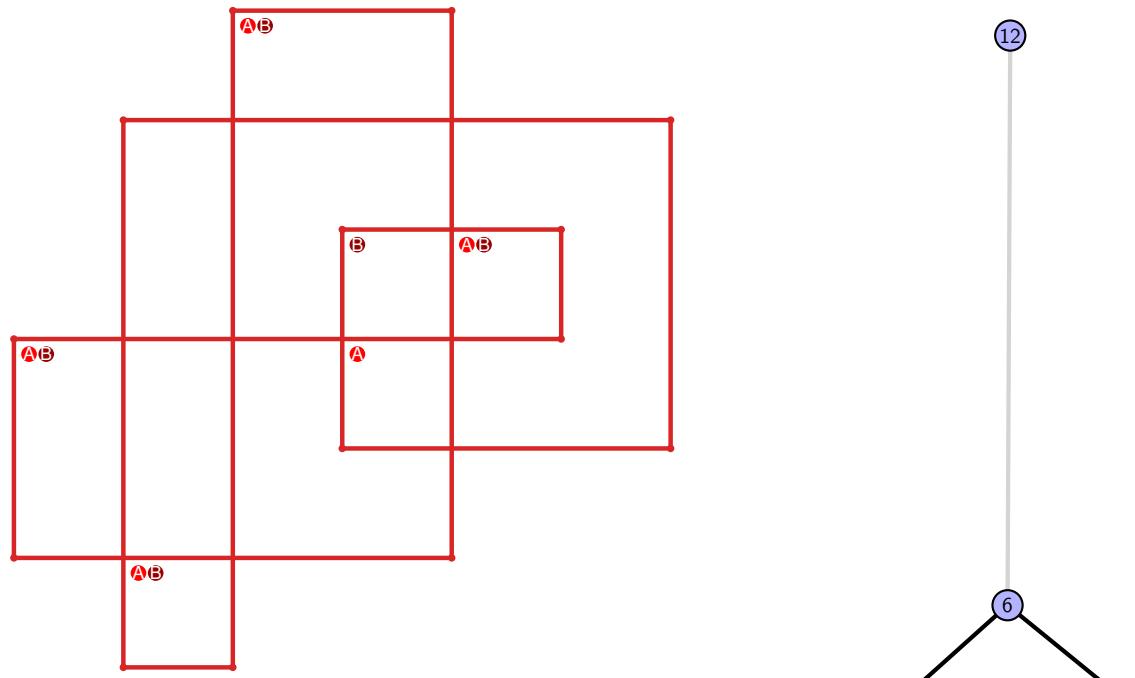


Figure 631: SnapPy multiloop plot.

Figure 632: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.168 [[20, 15, 1, 16], [16, 9, 17, 10], [10, 19, 11, 20], [3, 14, 4, 15], [1, 6, 2, 7], [8, 17, 9, 18], [18, 7, 19, 8], [11, 2, 12, 3], [13, 4, 14, 5], [5, 12, 6, 13]]

PD code drawn by SnapPy: [(9, 20, 10, 1), (17, 2, 18, 3), (14, 3, 15, 4), (12, 5, 13, 6), (1, 8, 2, 9), (19, 10, 20, 11), (16, 11, 17, 12), (4, 13, 5, 14), (6, 15, 7, 16), (7, 18, 8, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 7, 8, 8], [0, 9, 7, 6], [1, 6, 6, 1], [2, 5, 5, 4], [2, 4, 9, 3], [3, 9, 9, 3], [4, 8, 8, 7]]

Total optimal pinning sets: 6  
 Total minimal pinning sets: 6  
 Total pinning sets: 252  
 Pinning number: 5

Average optimal degree: 2.33  
 Average minimal degree: 2.33  
 Average overall degree: 2.97

Table 315: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	27	56	70	56	28	8	1	246
Average degree	2.33	2.65	2.86	3.0	3.11	3.2	3.27	3.33	

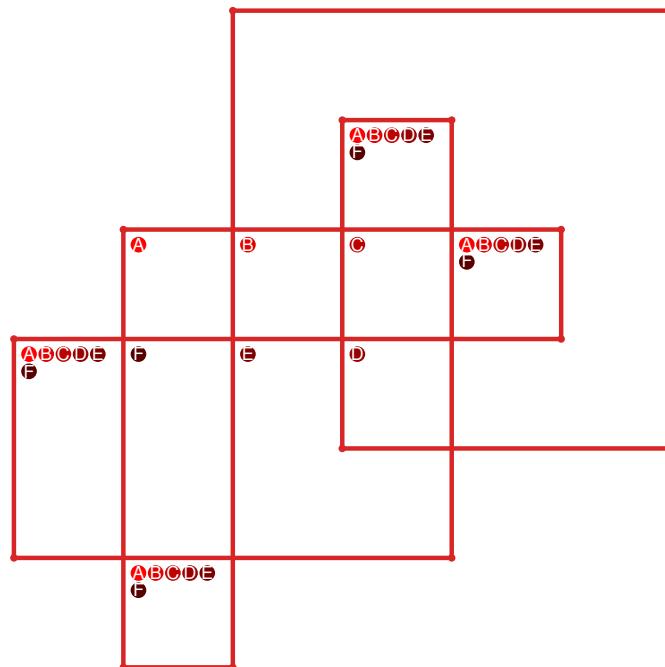


Figure 633: SnapPy multiloop plot.

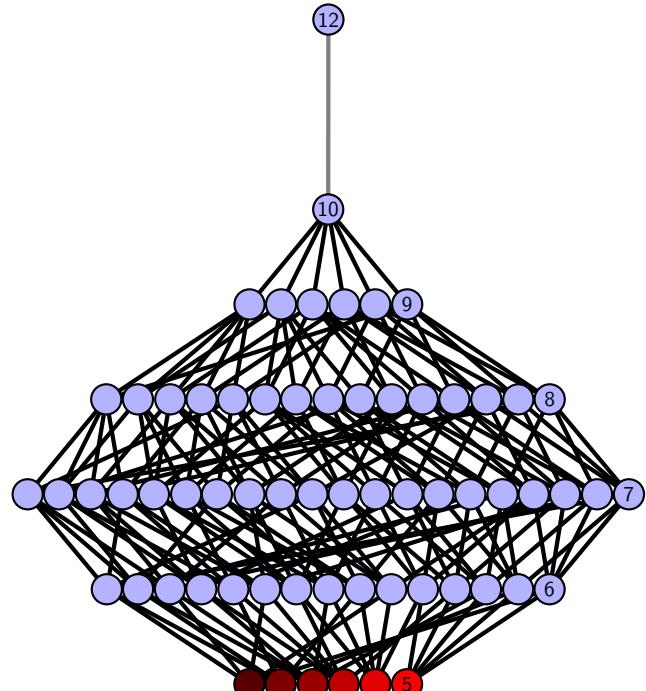


Figure 634: Minimal join sub-semi-lattice of minimal pinning sets.

$$4.8.169 \quad [[9, 20, 10, 1], [19, 8, 20, 9], [10, 8, 11, 7], [1, 12, 2, 13], [13, 18, 14, 19], [11, 6, 12, 7], [2, 15, 3, 16], [4, 17, 5, 18], [14, 5, 15, 6], [3, 17, 4, 16]]$$

PD code drawn by SnapPy: [(12, 3, 13, 4), (9, 4, 10, 5), (5, 8, 6, 9), (17, 6, 18, 7), (19, 10, 20, 11), (20, 13, 1, 14), (14, 1, 15, 2), (2, 15, 3, 16), (11, 16, 12, 17), (7, 18, 8, 19)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 4], [1, 3, 7, 8], [2, 8, 3, 2], [3, 8, 9, 9], [4, 9, 9, 8], [4, 7, 6, 5], [6, 7, 7, 6]]

Total optimal pinning sets: 1  
Total pinning sets: 1

Average optimal degree: 2.0

Total minimal pinning sets: 1

Average minimal degree: 2.0

Total pinning sets: 256

Average minimal degree: 2.0

Pinning number: 4

Average overall degree: 2.97

Table 316: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

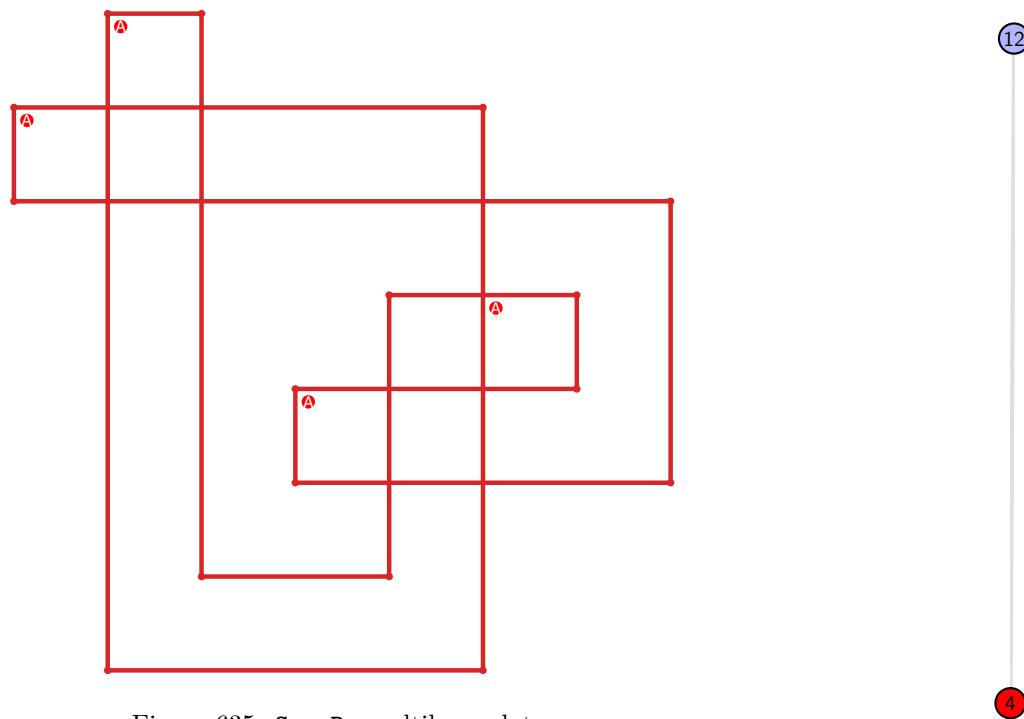


Figure 635: SnapPy multiloop plot.

Figure 636: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.170 [[11, 20, 12, 1], [10, 17, 11, 18], [19, 16, 20, 17], [12, 3, 13, 4], [1, 8, 2, 9], [18, 9, 19, 10], [2, 15, 3, 16], [13, 6, 14, 7], [4, 7, 5, 8], [5, 14, 6, 15]]

PD code drawn by SnapPy: [(10, 1, 11, 2), (11, 4, 12, 5), (2, 5, 3, 6), (19, 6, 20, 7), (16, 7, 17, 8), (3, 12, 4, 13), (20, 13, 1, 14), (17, 14, 18, 15), (8, 15, 9, 16), (9, 18, 10, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 8], [0, 8, 6, 5], [1, 4, 2, 1], [2, 4, 9, 3], [3, 9, 9, 8], [3, 7, 9, 4], [6, 8, 7, 7]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 8  
 Total pinning sets: 556  
 Pinning number: 4

Average optimal degree: 2.5  
 Average minimal degree: 2.73  
 Average overall degree: 3.11

Table 317: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	6	0	0	0	0	0	0	0	6
Nonminimal pinning sets	0	16	78	142	154	104	43	10	1	548
Average degree	2.5	2.76	2.94	3.07	3.16	3.23	3.27	3.31	3.33	

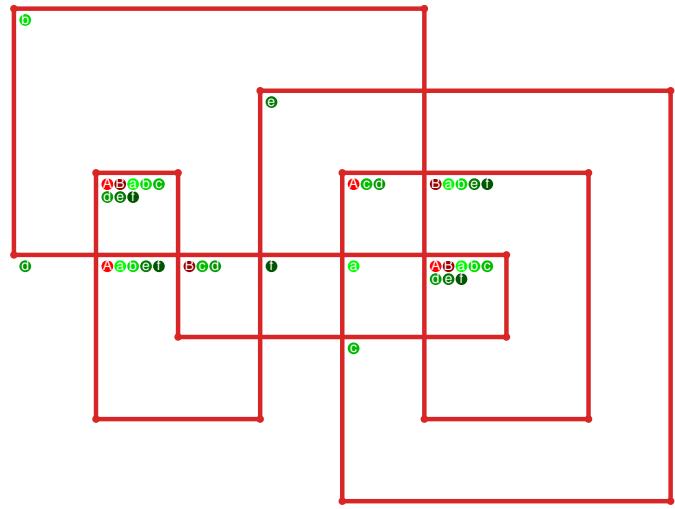


Figure 637: SnapPy multiloop plot.

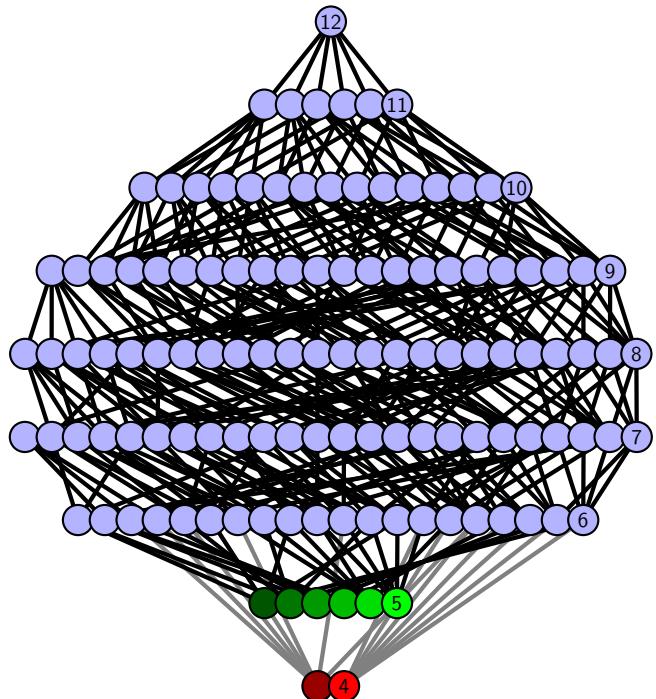


Figure 638: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.171  $[[9, 20, 10, 1], [8, 17, 9, 18], [19, 16, 20, 17], [10, 3, 11, 4], [1, 6, 2, 7], [18, 7, 19, 8], [2, 15, 3, 16], [11, 15, 12, 14], [4, 14, 5, 13], [5, 12, 6, 13]]$

PD code drawn by SnapPy:  $[(8, 1, 9, 2), (19, 4, 20, 5), (16, 5, 17, 6), (3, 10, 4, 11), (11, 2, 12, 3), (12, 9, 13, 10), (20, 13, 1, 14), (17, 14, 18, 15), (6, 15, 7, 16), (7, 18, 8, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 8], [0, 9, 6, 5], [1, 4, 2, 1], [2, 4, 7, 3], [3, 6, 9, 8], [3, 7, 9, 9], [4, 8, 8, 7]]$

Total optimal pinning sets: 8  
 Total minimal pinning sets: 8  
 Total pinning sets: 432  
 Pinning number: 5

Average optimal degree: 2.68  
 Average minimal degree: 2.68  
 Average overall degree: 3.11

Table 318: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	8	0	0	0	0	0	0	0	8
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	44	102	129	96	42	10	1	424
Average degree	2.68	2.89	3.03	3.14	3.22	3.27	3.31	3.33	

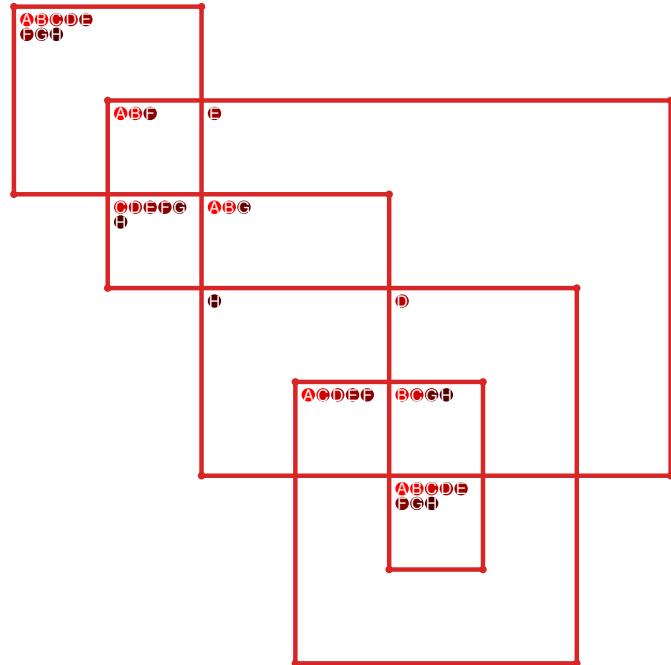


Figure 639: SnapPy multiloop plot.

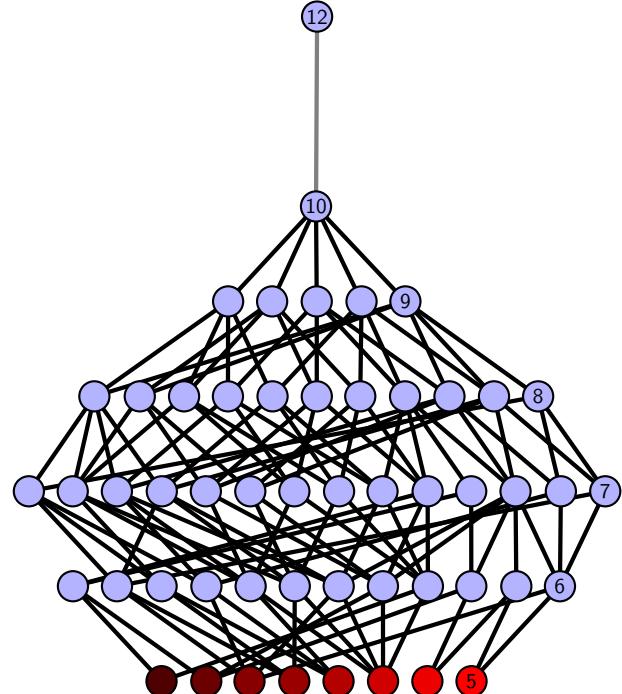


Figure 640: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.172** [[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [3, 8, 4, 9], [1, 13, 2, 12], [18, 12, 19, 11], [2, 15, 3, 16], [7, 4, 8, 5], [13, 7, 14, 6], [14, 5, 15, 6]]

PD code drawn by `SnapPy`: [(3, 20, 4, 1), (10, 1, 11, 2), (2, 9, 3, 10), (19, 4, 20, 5), (16, 5, 17, 6), (7, 14, 8, 15), (8, 17, 9, 18), (18, 11, 19, 12), (15, 12, 16, 13), (13, 6, 14, 7)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 7], [0, 8, 6, 5], [1, 4, 2, 1], [2, 4, 9, 3], [3, 9, 8, 3], [4, 7, 9, 9], [6, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.46

**Total pinning sets:** 368

**Average overall degree:** 3.04

**Pinning number:** 4

Table 319: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	0	3
Nonminimal pinning sets	0	8	43	87	104	77	35	9	1	364
Average degree	2.25	2.56	2.79	2.96	3.08	3.17	3.24	3.29	3.33	

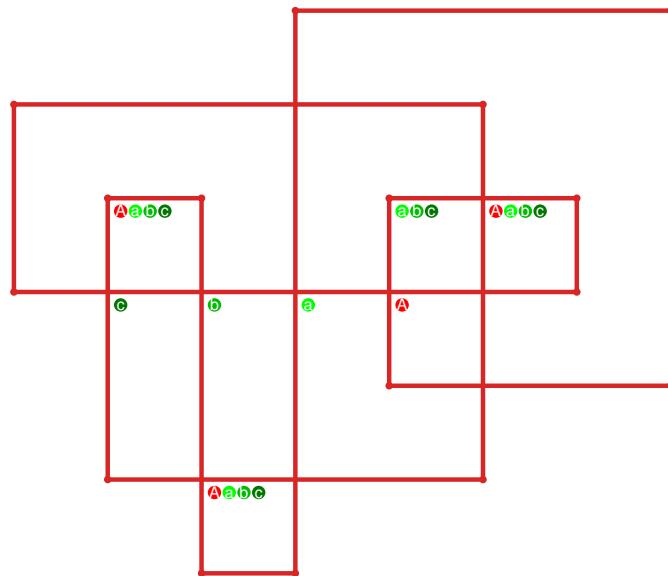


Figure 641: `SnapPy` multiloop plot.

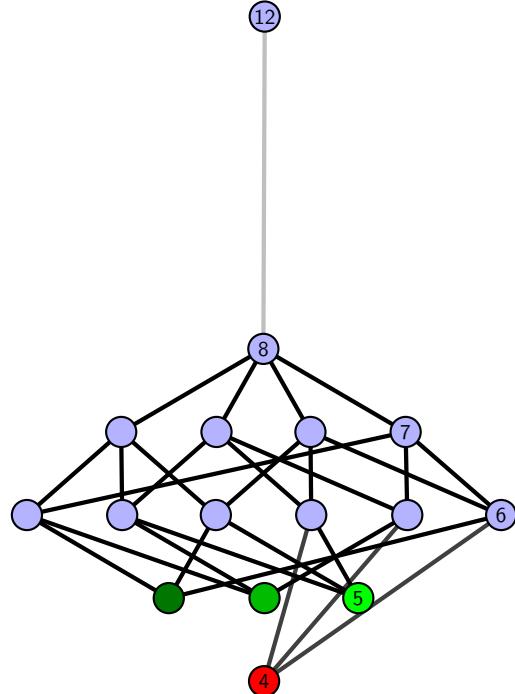


Figure 642: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.173**  $[[5, 20, 6, 1], [4, 17, 5, 18], [19, 16, 20, 17], [6, 10, 7, 9], [1, 13, 2, 12], [18, 3, 19, 4], [15, 10, 16, 11], [7, 15, 8, 14], [8, 13, 9, 14], [2, 11, 3, 12]]$

PD code drawn by `SnapPy`:  $[(16, 1, 17, 2), (7, 4, 8, 5), (13, 8, 14, 9), (20, 9, 1, 10), (10, 19, 11, 20), (6, 11, 7, 12), (12, 5, 13, 6), (17, 14, 18, 15), (2, 15, 3, 16), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 8], [0, 8, 9, 9], [1, 9, 2, 1], [2, 9, 7, 3], [3, 6, 8, 8], [3, 7, 7, 4], [4, 6, 5, 4]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 272  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.47  
 Average overall degree: 3.04

Table 320: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.4	2.68	2.89	3.04	3.15	3.24	3.29	3.33	

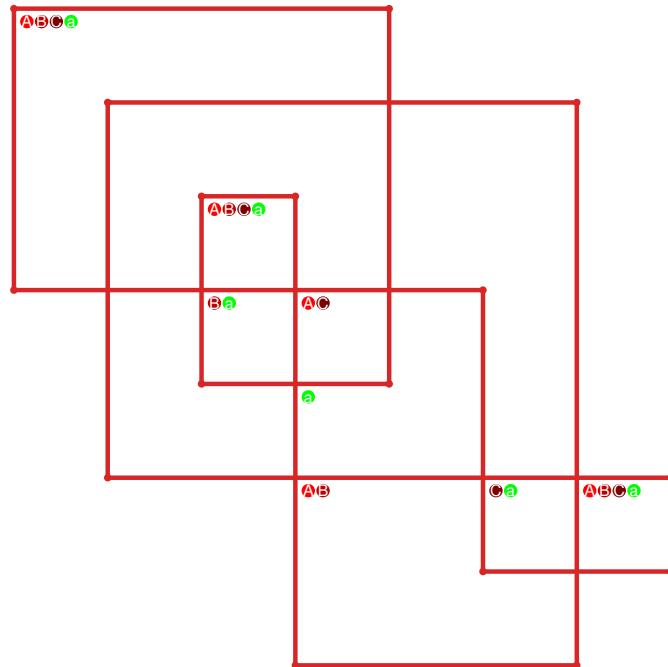


Figure 643: `SnapPy` multiloop plot.

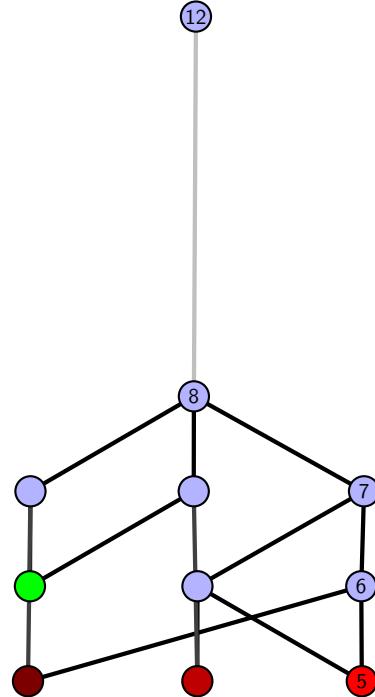


Figure 644: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.174  $[[5, 20, 6, 1], [4, 11, 5, 12], [8, 19, 9, 20], [6, 18, 7, 17], [1, 14, 2, 15], [12, 3, 13, 4], [7, 10, 8, 11], [18, 9, 19, 10], [13, 16, 14, 17], [2, 16, 3, 15]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (15, 6, 16, 7), (4, 7, 5, 8), (1, 8, 2, 9), (18, 11, 19, 12), (12, 19, 13, 20), (10, 13, 11, 14), (14, 3, 15, 4), (5, 16, 6, 17), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 7], [0, 7, 6, 8], [0, 8, 9, 9], [1, 9, 8, 1], [1, 3, 7, 2], [2, 6, 3, 2], [3, 5, 9, 4], [4, 8, 5, 4]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 288  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.4  
 Average overall degree: 3.03

Table 321: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

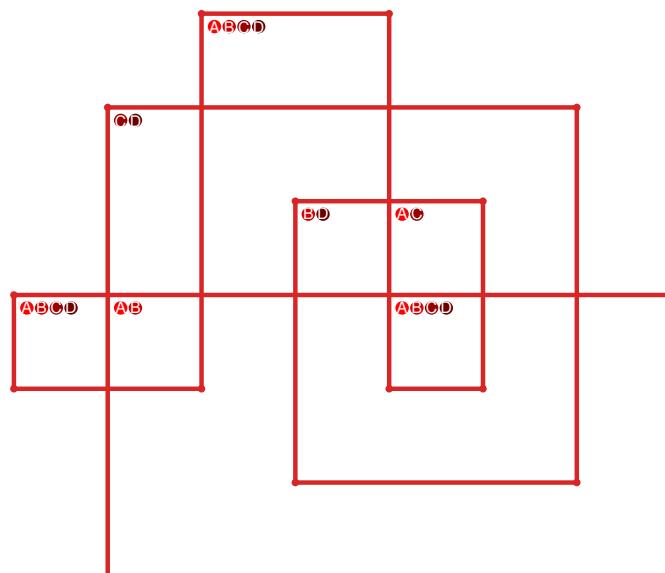


Figure 645: SnapPy multiloop plot.

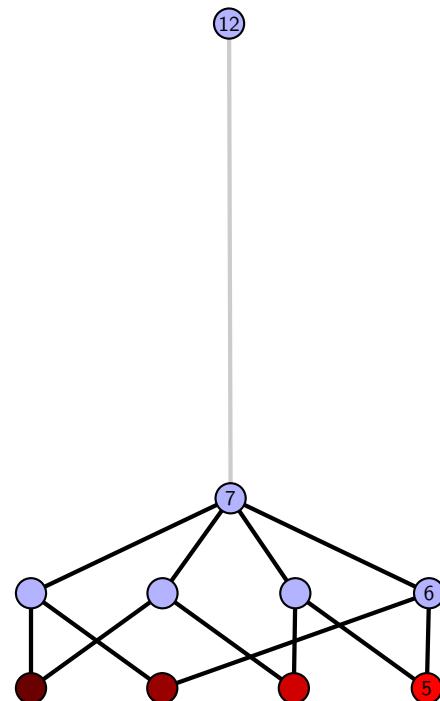


Figure 646: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.175  $[[20, 5, 1, 6], [6, 9, 7, 10], [10, 19, 11, 20], [4, 13, 5, 14], [1, 16, 2, 17], [17, 8, 18, 9], [7, 18, 8, 19], [11, 2, 12, 3], [14, 3, 15, 4], [15, 12, 16, 13]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (12, 1, 13, 2), (18, 3, 19, 4), (15, 6, 16, 7), (4, 7, 5, 8), (13, 10, 14, 11), (2, 11, 3, 12), (19, 14, 20, 15), (5, 16, 6, 17), (8, 17, 9, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 7], [0, 8, 8, 9], [0, 9, 7, 5], [1, 4, 6, 6], [1, 5, 5, 2], [2, 4, 9, 8], [3, 7, 9, 3], [3, 8, 7, 4]]$

**Total optimal pinning sets:** 6

**Average optimal degree:** 2.67

**Total minimal pinning sets:** 7

**Average minimal degree:** 2.67

**Total pinning sets:** 384

**Average overall degree:** 3.11

**Pinning number:** 5

Table 322: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	34	86	115	90	41	10	1	377
Average degree	2.67	2.87	3.01	3.12	3.21	3.27	3.31	3.33	

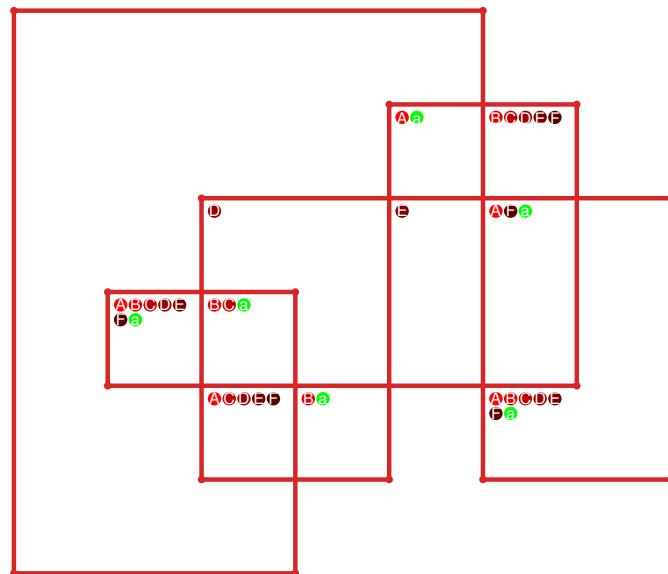


Figure 647: `SnapPy` multiloop plot.

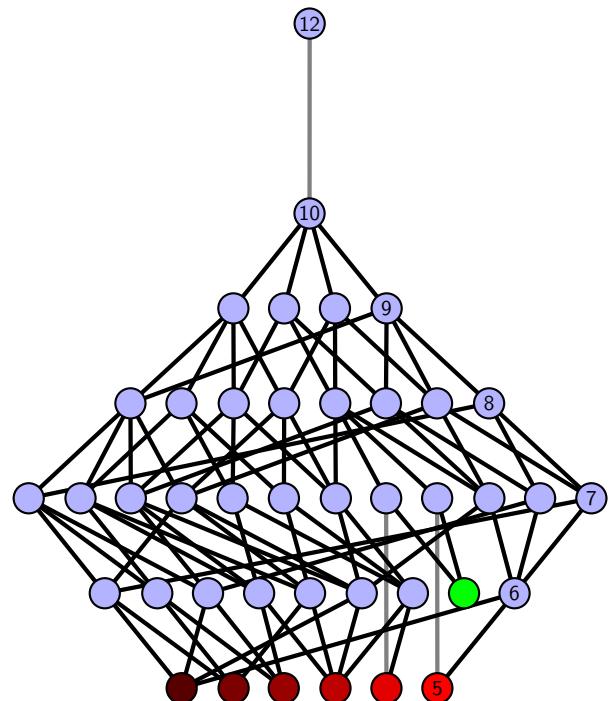


Figure 648: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.176  $[[20, 9, 1, 10], [10, 19, 11, 20], [11, 8, 12, 9], [1, 7, 2, 6], [18, 5, 19, 6], [7, 12, 8, 13], [2, 16, 3, 15], [17, 14, 18, 15], [4, 13, 5, 14], [16, 4, 17, 3]]$

PD code drawn by SnapPy:  $[(20, 13, 1, 14), (15, 2, 16, 3), (3, 18, 4, 19), (8, 5, 9, 6), (17, 6, 18, 7), (4, 9, 5, 10), (1, 10, 2, 11), (14, 11, 15, 12), (12, 19, 13, 20), (7, 16, 8, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 4], [1, 3, 7, 8], [2, 8, 3, 2], [3, 9, 9, 7], [4, 6, 9, 8], [4, 7, 9, 5], [6, 8, 7, 6]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 320  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.33  
 Average overall degree: 3.03

Table 323: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

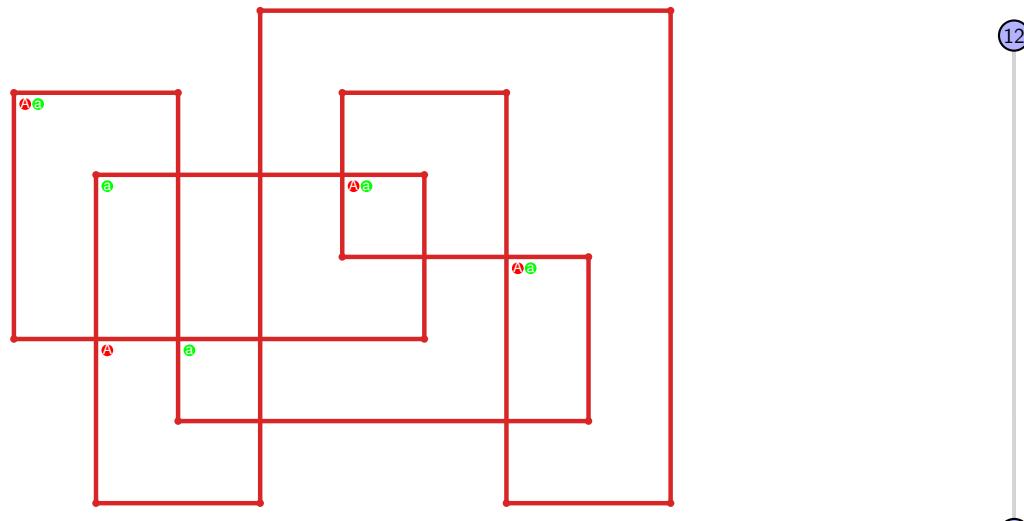


Figure 649: SnapPy multiloop plot.



Figure 650: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.177**  $[[20, 7, 1, 8], [8, 19, 9, 20], [9, 6, 10, 7], [1, 10, 2, 11], [11, 18, 12, 19], [12, 5, 13, 6], [2, 16, 3, 15], [17, 14, 18, 15], [4, 13, 5, 14], [16, 4, 17, 3]]$

PD code drawn by `SnapPy`:  $[(1, 14, 2, 15), (10, 3, 11, 4), (7, 4, 8, 5), (18, 5, 19, 6), (19, 8, 20, 9), (2, 11, 3, 12), (15, 12, 16, 13), (13, 20, 14, 1), (9, 16, 10, 17), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 4], [1, 3, 7, 5], [2, 4, 8, 8], [3, 9, 9, 7], [4, 6, 9, 8], [5, 7, 9, 5], [6, 8, 7, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 5  
 Total pinning sets: 280  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.51  
 Average overall degree: 3.04

Table 324: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	19	58	84	70	34	9	1	275
Average degree	2.4	2.68	2.89	3.04	3.16	3.24	3.29	3.33	

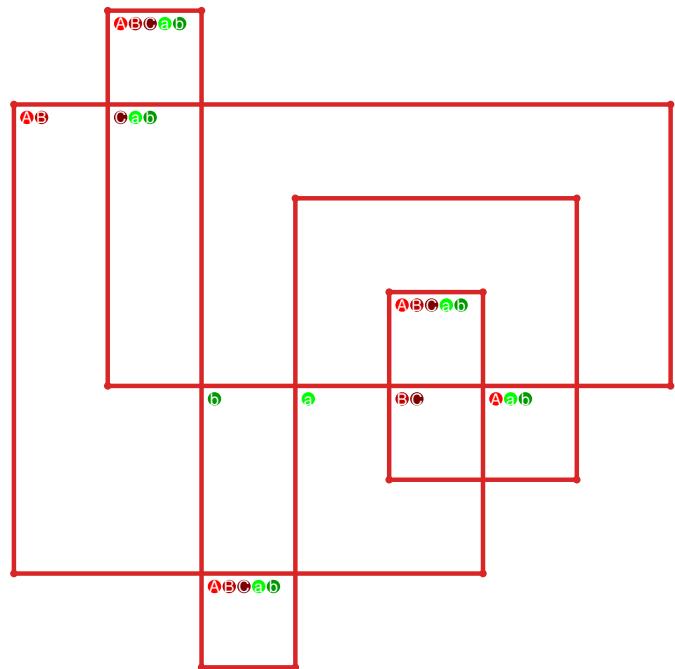


Figure 651: `SnapPy` multiloop plot.

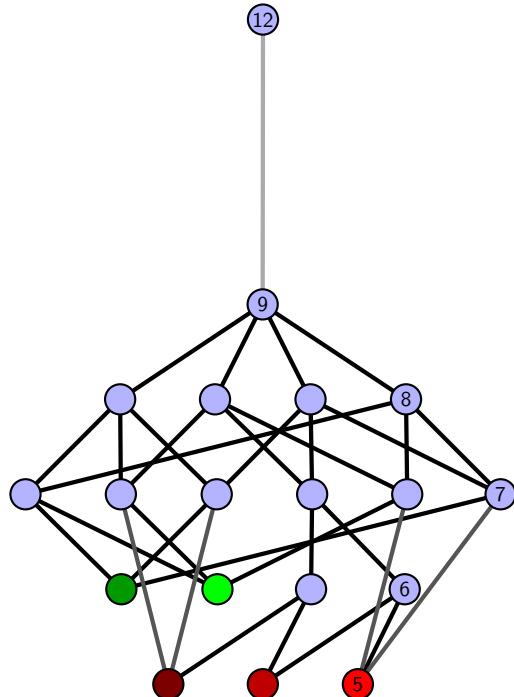


Figure 652: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.178 [[20, 13, 1, 14], [14, 19, 15, 20], [15, 12, 16, 13], [1, 16, 2, 17], [18, 5, 19, 6], [11, 4, 12, 5], [2, 9, 3, 10], [17, 7, 18, 6], [7, 10, 8, 11], [8, 3, 9, 4]]

PD code drawn by `SnapPy`: [(7, 20, 8, 1), (14, 1, 15, 2), (2, 13, 3, 14), (18, 5, 19, 6), (3, 6, 4, 7), (11, 8, 12, 9), (16, 9, 17, 10), (17, 12, 18, 13), (10, 15, 11, 16), (4, 19, 5, 20)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 5], [2, 4, 8, 9], [3, 9, 9, 8], [3, 8, 4, 4], [5, 7, 6, 9], [5, 8, 6, 6]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 288  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.4  
**Average overall degree:** 3.03

Table 325: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

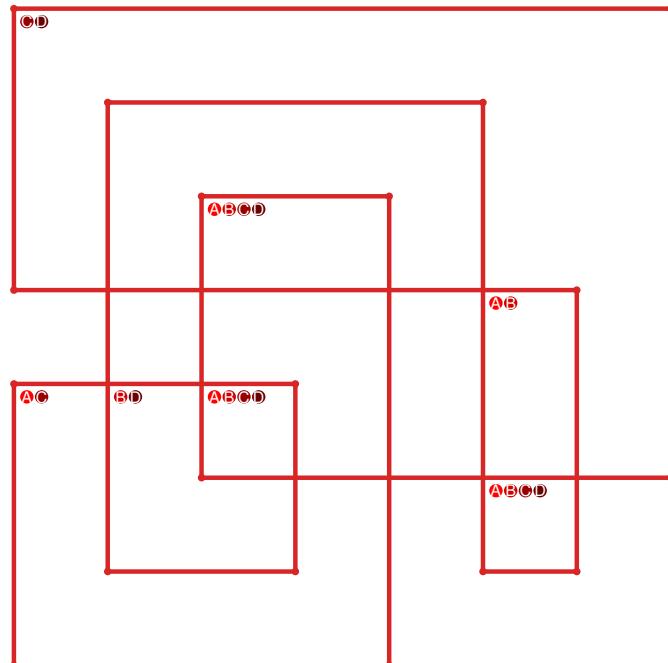


Figure 653: `SnapPy` multiloop plot.

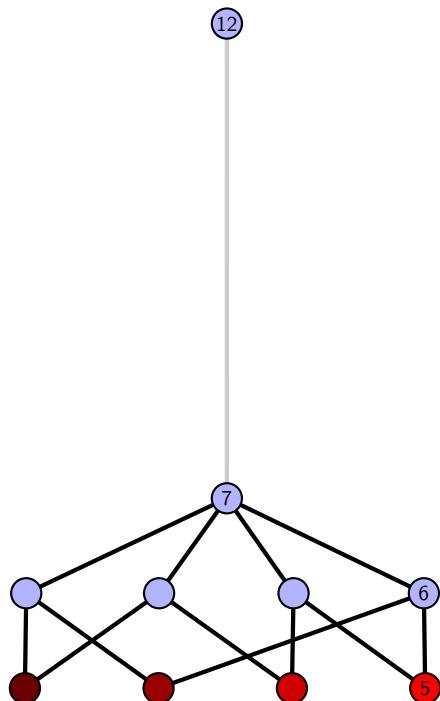


Figure 654: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.179**  $[[5, 20, 6, 1], [4, 9, 5, 10], [19, 16, 20, 17], [6, 16, 7, 15], [1, 12, 2, 13], [10, 3, 11, 4], [8, 17, 9, 18], [18, 7, 19, 8], [11, 14, 12, 15], [2, 14, 3, 13]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (15, 4, 16, 5), (1, 6, 2, 7), (18, 9, 19, 10), (10, 19, 11, 20), (8, 11, 9, 12), (12, 3, 13, 4), (16, 13, 17, 14), (5, 14, 6, 15), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 3], [0, 2, 7, 8], [0, 8, 9, 9], [1, 9, 8, 1], [1, 7, 7, 2], [2, 6, 6, 3], [3, 5, 9, 4], [4, 8, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 5

**Average minimal degree:** 2.51

**Total pinning sets:** 224

**Average overall degree:** 3.04

**Pinning number:** 5

Table 326: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	7	39	67	63	33	9	1	219
Average degree	2.4	2.62	2.84	3.01	3.14	3.23	3.29	3.33	

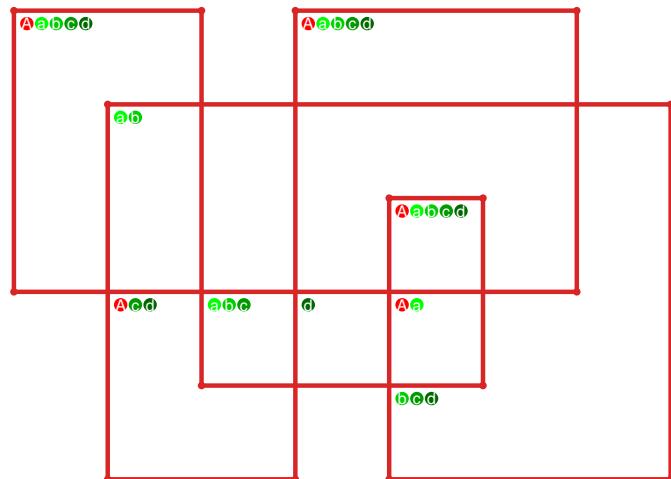


Figure 655: `SnapPy` multiloop plot.

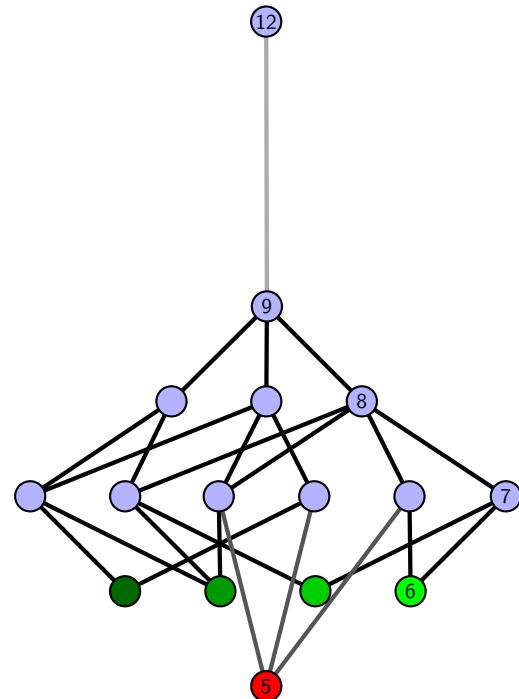


Figure 656: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.180**  $[[9, 20, 10, 1], [8, 17, 9, 18], [19, 16, 20, 17], [10, 13, 11, 14], [1, 6, 2, 7], [18, 7, 19, 8], [2, 15, 3, 16], [3, 12, 4, 13], [11, 4, 12, 5], [14, 5, 15, 6]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (11, 2, 12, 3), (18, 3, 19, 4), (15, 4, 16, 5), (20, 9, 1, 10), (7, 10, 8, 11), (19, 12, 20, 13), (16, 13, 17, 14), (5, 14, 6, 15), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 8, 9], [0, 9, 6, 5], [1, 4, 2, 1], [2, 4, 9, 7], [3, 6, 8, 8], [3, 7, 7, 9], [3, 8, 6, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 9

**Average minimal degree:** 2.74

**Total pinning sets:** 508

**Average overall degree:** 3.12

**Pinning number:** 4

Table 327: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	2	0	0	0	0	0	0	8
Nonminimal pinning sets	0	8	59	127	148	103	43	10	1	499
Average degree	2.5	2.73	2.92	3.06	3.16	3.23	3.27	3.31	3.33	

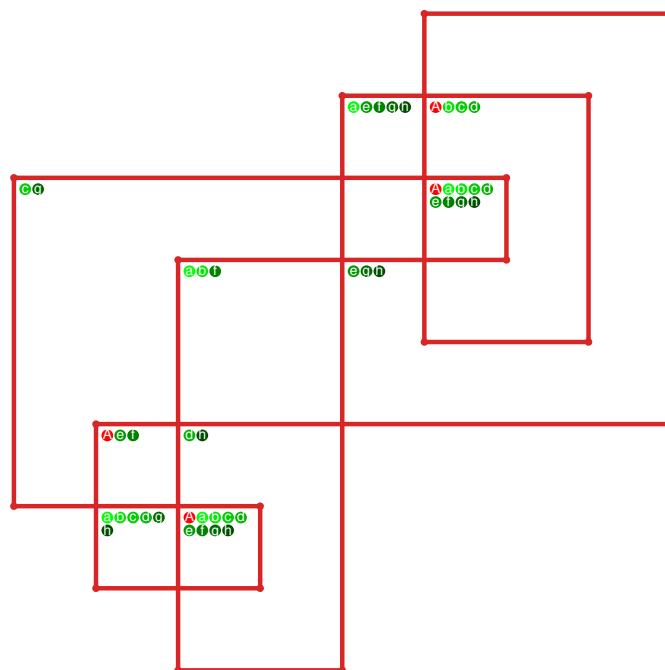


Figure 657: `SnapPy` multiloop plot.

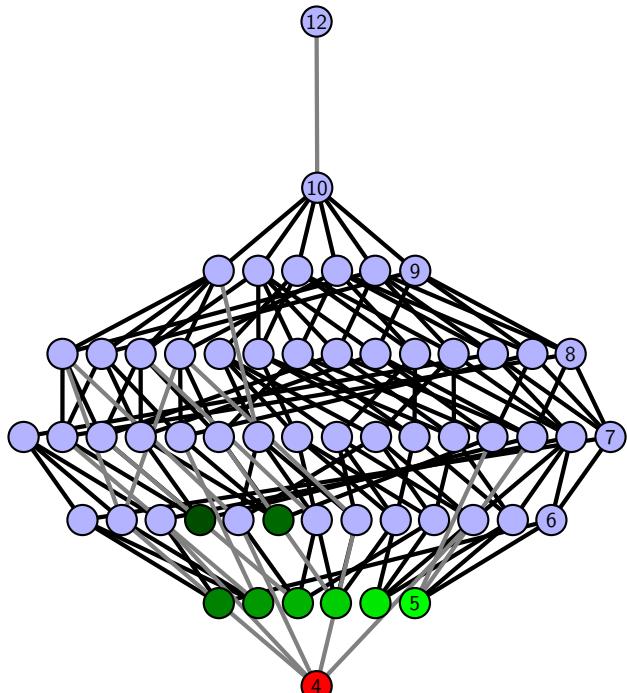


Figure 658: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.181** `[[11, 20, 12, 1], [10, 17, 11, 18], [19, 16, 20, 17], [12, 6, 13, 5], [1, 8, 2, 9], [18, 9, 19, 10], [2, 15, 3, 16], [6, 3, 7, 4], [13, 4, 14, 5], [14, 7, 15, 8]]`

PD code drawn by `SnapPy`: `[(20, 7, 1, 8), (9, 2, 10, 3), (16, 3, 17, 4), (13, 4, 14, 5), (17, 10, 18, 11), (14, 11, 15, 12), (5, 12, 6, 13), (6, 15, 7, 16), (1, 18, 2, 19), (8, 19, 9, 20)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 8, 8], [0, 9, 6, 5], [1, 4, 2, 1], [2, 4, 9, 7], [3, 6, 9, 8], [3, 7, 9, 3], [4, 8, 7, 6]]`

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.6

**Total minimal pinning sets:** 7

**Average minimal degree:** 2.68

**Total pinning sets:** 360

**Average overall degree:** 3.11

**Pinning number:** 5

Table 328: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	25	77	110	89	41	10	1	353
Average degree	2.6	2.82	2.99	3.12	3.21	3.27	3.31	3.33	

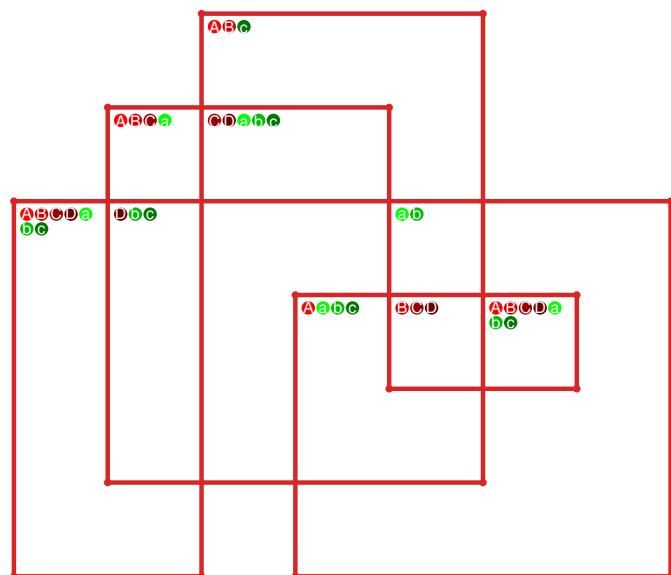


Figure 659: `SnapPy` multiloop plot.

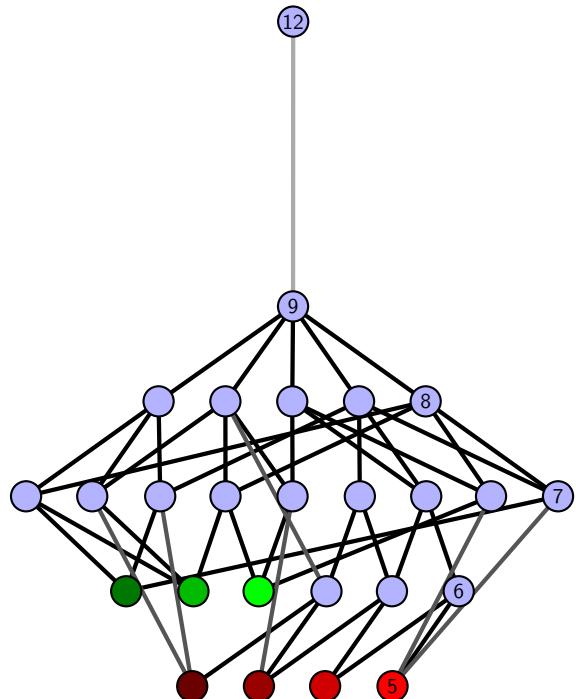


Figure 660: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.182** [[20, 5, 1, 6], [6, 18, 7, 17], [19, 16, 20, 17], [4, 11, 5, 12], [1, 9, 2, 8], [18, 8, 19, 7], [2, 15, 3, 16], [12, 3, 13, 4], [13, 10, 14, 11], [9, 14, 10, 15]]

PD code drawn by `SnapPy`: [(16, 1, 17, 2), (3, 14, 4, 15), (4, 17, 5, 18), (9, 6, 10, 7), (20, 7, 1, 8), (5, 10, 6, 11), (18, 11, 19, 12), (15, 12, 16, 13), (13, 2, 14, 3), (8, 19, 9, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 8], [0, 9, 6, 5], [1, 4, 2, 1], [2, 4, 9, 7], [3, 6, 8, 3], [3, 7, 9, 9], [4, 8, 8, 6]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 312  
**Pinning number:** 5

**Average optimal degree:** 2.45  
**Average minimal degree:** 2.55  
**Average overall degree:** 3.05

Table 329: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	24	68	94	75	35	9	1	306
Average degree	2.45	2.72	2.92	3.07	3.17	3.24	3.29	3.33	

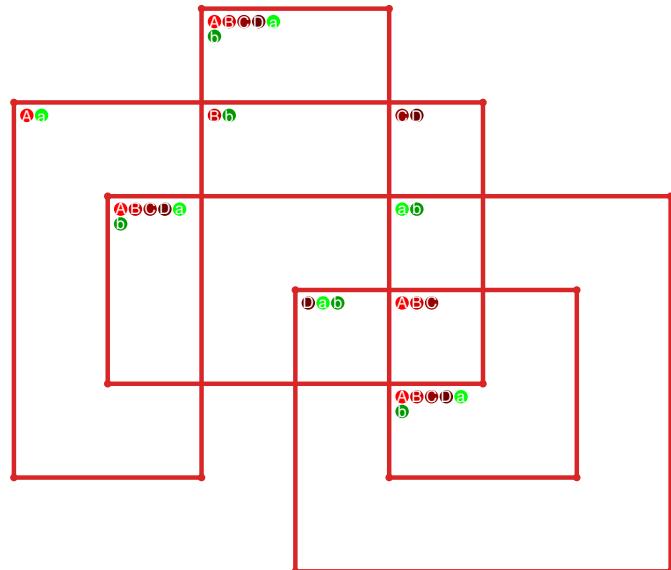


Figure 661: `SnapPy` multiloop plot.

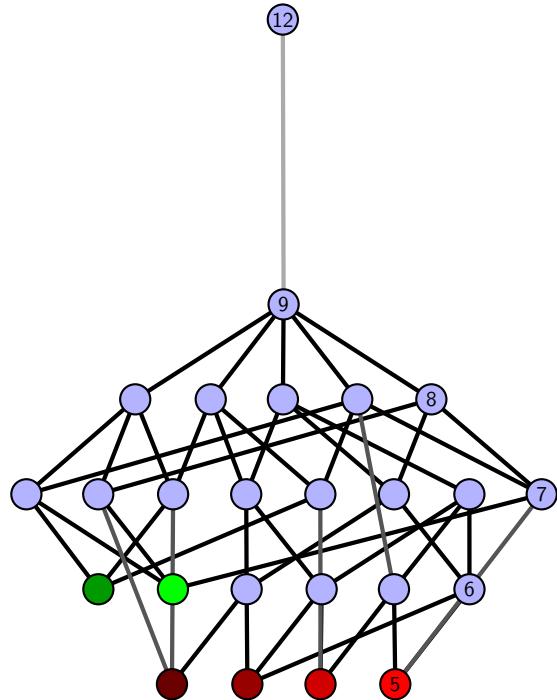


Figure 662: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.183**  $[[20, 3, 1, 4], [4, 18, 5, 17], [19, 16, 20, 17], [9, 2, 10, 3], [1, 10, 2, 11], [18, 6, 19, 5], [12, 15, 13, 16], [13, 8, 14, 9], [11, 7, 12, 6], [7, 14, 8, 15]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (18, 1, 19, 2), (3, 16, 4, 17), (4, 9, 5, 10), (11, 6, 12, 7), (19, 8, 20, 9), (5, 12, 6, 13), (10, 13, 11, 14), (17, 14, 18, 15), (15, 2, 16, 3)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 4, 4], [0, 3, 3, 8], [1, 8, 2, 1], [2, 8, 9, 7], [3, 6, 9, 9], [4, 9, 6, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.47

**Total pinning sets:** 272

**Average overall degree:** 3.04

**Pinning number:** 5

Table 330: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.4	2.68	2.89	3.04	3.15	3.24	3.29	3.33	

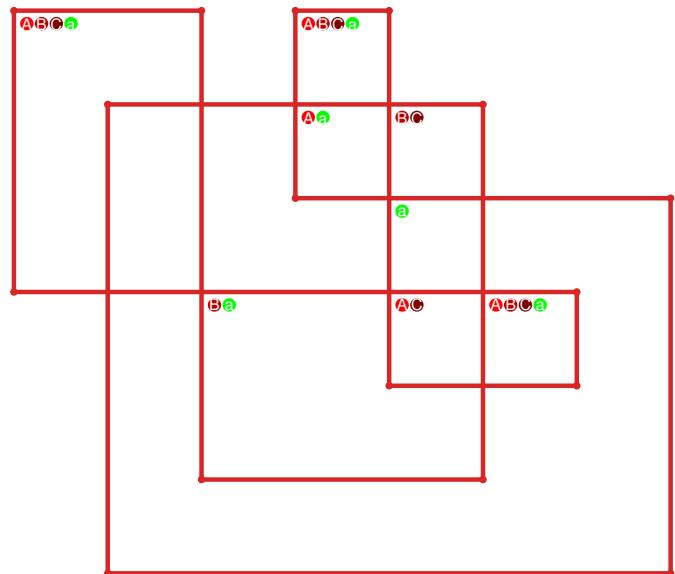


Figure 663: `SnapPy` multiloop plot.

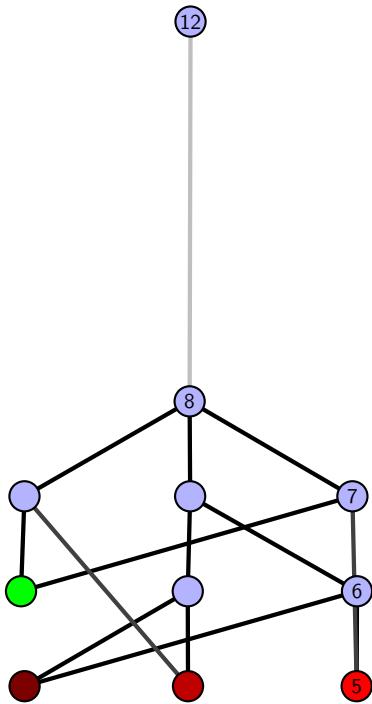


Figure 664: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.184 [[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [6, 1, 7, 2], [18, 10, 19, 9], [12, 15, 13, 16], [2, 13, 3, 14], [5, 10, 6, 11], [11, 4, 12, 5], [14, 3, 15, 4]]

PD code drawn by `SnapPy`: [(12, 1, 13, 2), (9, 2, 10, 3), (18, 5, 19, 6), (7, 16, 8, 17), (3, 8, 4, 9), (20, 11, 1, 12), (10, 13, 11, 14), (17, 14, 18, 15), (15, 6, 16, 7), (4, 19, 5, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 7, 0], [1, 7, 2, 1], [2, 8, 9, 6], [3, 5, 9, 9], [3, 8, 8, 4], [5, 7, 7, 9], [5, 8, 6, 6]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 144  
**Pinning number:** 6

**Average optimal degree:** 2.33  
**Average minimal degree:** 2.33  
**Average overall degree:** 2.97

Table 331: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.33	2.66	2.89	3.06	3.18	3.27	3.33	

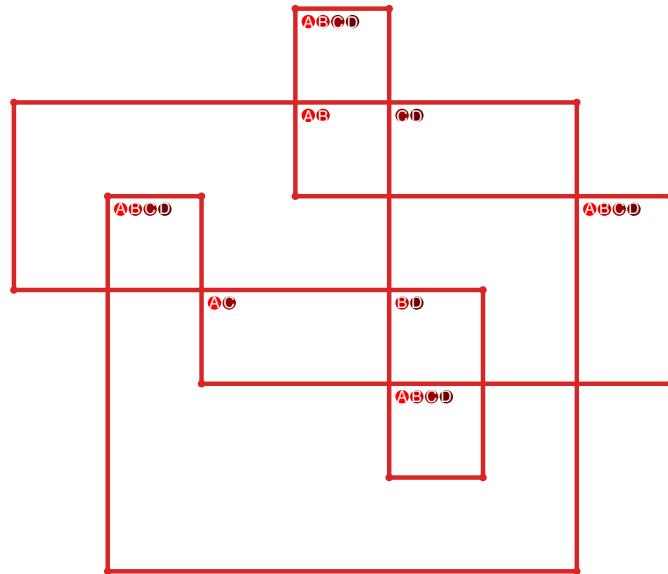


Figure 665: `SnapPy` multiloop plot.

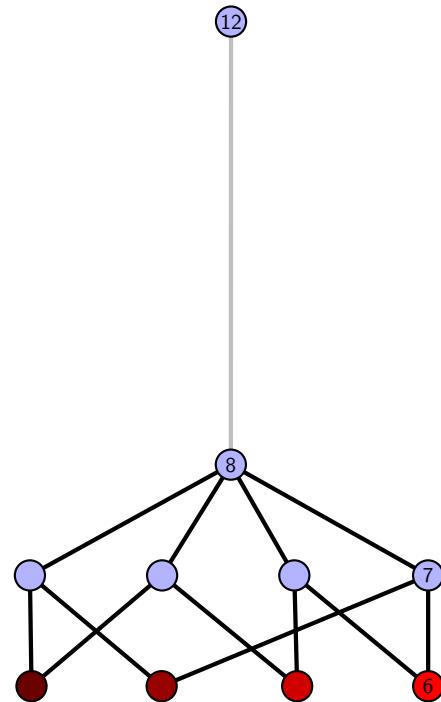


Figure 666: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.185  $[[5, 20, 6, 1], [9, 4, 10, 5], [19, 6, 20, 7], [1, 17, 2, 16], [3, 8, 4, 9], [10, 8, 11, 7], [18, 13, 19, 14], [17, 13, 18, 12], [2, 15, 3, 16], [11, 15, 12, 14]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (16, 3, 17, 4), (4, 7, 5, 8), (20, 5, 1, 6), (9, 14, 10, 15), (15, 10, 16, 11), (11, 8, 12, 9), (19, 12, 20, 13), (13, 18, 14, 19), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 5, 6, 0], [0, 7, 8, 8], [1, 8, 5, 1], [1, 4, 9, 2], [2, 9, 7, 7], [3, 6, 6, 9], [3, 9, 4, 3], [5, 8, 7, 6]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 6  
 Total pinning sets: 252  
 Pinning number: 5

Average optimal degree: 2.33  
 Average minimal degree: 2.33  
 Average overall degree: 2.97

Table 332: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	27	56	70	56	28	8	1	246
Average degree	2.33	2.65	2.86	3.0	3.11	3.2	3.27	3.33	

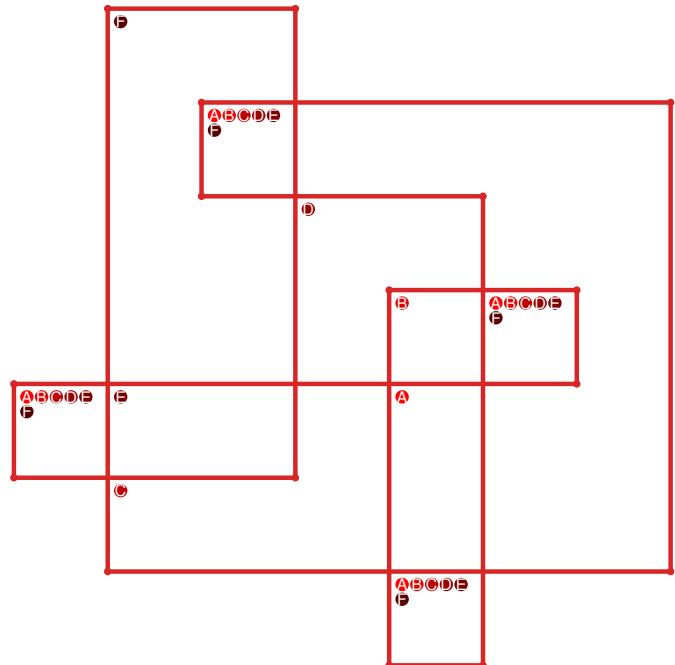


Figure 667: SnapPy multiloop plot.

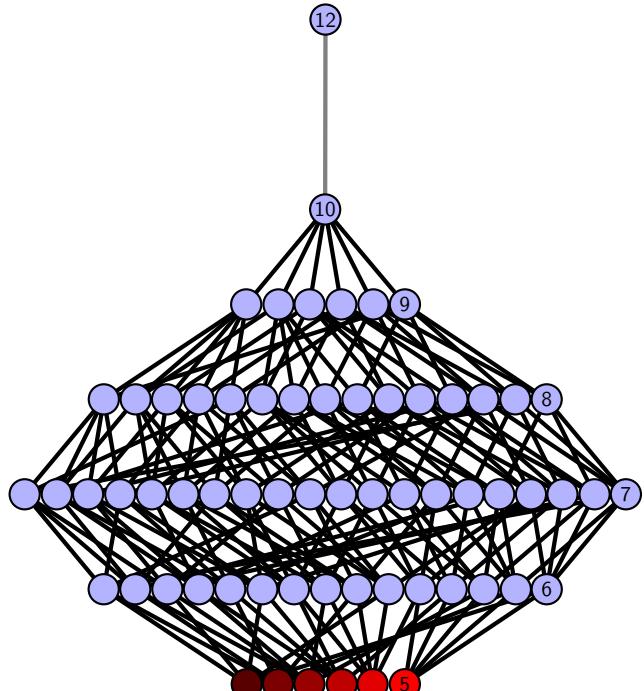


Figure 668: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.186** [[20, 5, 1, 6], [6, 18, 7, 17], [19, 16, 20, 17], [13, 4, 14, 5], [1, 10, 2, 11], [18, 8, 19, 7], [12, 15, 13, 16], [3, 14, 4, 15], [9, 2, 10, 3], [11, 9, 12, 8]]

PD code drawn by `SnapPy`: [(7, 20, 8, 1), (11, 2, 12, 3), (18, 3, 19, 4), (5, 16, 6, 17), (6, 9, 7, 10), (19, 8, 20, 9), (1, 12, 2, 13), (10, 13, 11, 14), (17, 14, 18, 15), (15, 4, 16, 5)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 7], [0, 8, 8, 9], [1, 9, 2, 1], [2, 9, 7, 3], [3, 6, 8, 3], [4, 7, 9, 4], [4, 8, 6, 5]]

Total optimal pinning sets: 4  
 Total minimal pinning sets: 6  
 Total pinning sets: 312  
 Pinning number: 5

Average optimal degree: 2.45  
 Average minimal degree: 2.55  
 Average overall degree: 3.05

Table 333: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	24	68	94	75	35	9	1	306
Average degree	2.45	2.72	2.92	3.07	3.17	3.24	3.29	3.33	

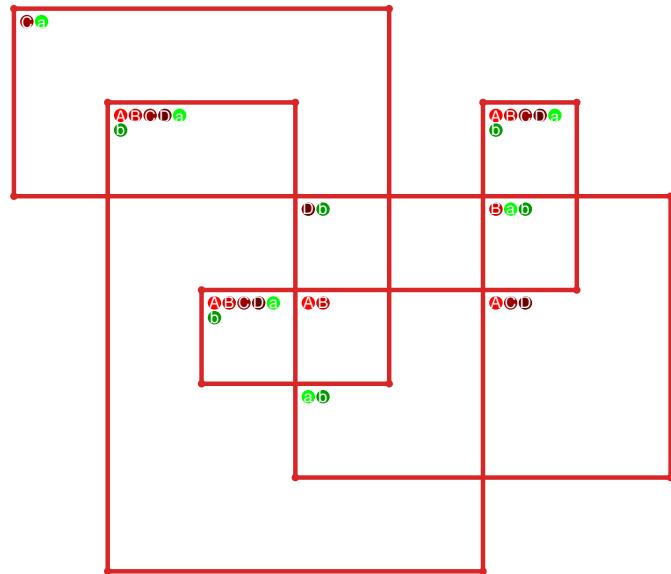


Figure 669: `SnapPy` multiloop plot.

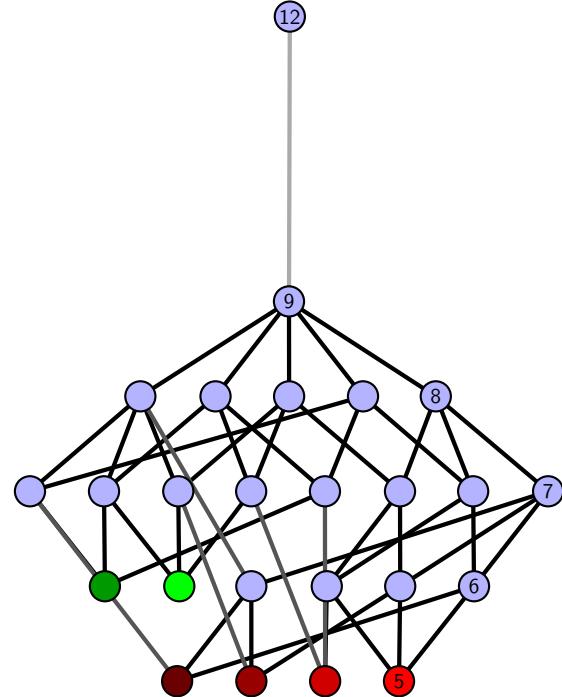


Figure 670: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.187**  $[[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [13, 6, 14, 7], [1, 5, 2, 4], [18, 10, 19, 9], [12, 15, 13, 16], [5, 14, 6, 15], [2, 12, 3, 11], [3, 10, 4, 11]]$

PD code drawn by SnapPy:  $[(1, 18, 2, 19), (10, 3, 11, 4), (17, 4, 18, 5), (6, 15, 7, 16), (20, 7, 1, 8), (8, 19, 9, 20), (2, 11, 3, 12), (9, 12, 10, 13), (16, 13, 17, 14), (14, 5, 15, 6)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 7], [0, 7, 8, 9], [1, 9, 2, 1], [2, 8, 7, 3], [3, 6, 4, 3], [4, 6, 9, 9], [4, 8, 8, 5]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 6  
 Total pinning sets: 312  
 Pinning number: 5

Average optimal degree: 2.45  
 Average minimal degree: 2.55  
 Average overall degree: 3.05

Table 334: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	24	68	94	75	35	9	1	306
Average degree	2.45	2.72	2.92	3.07	3.17	3.24	3.29	3.33	

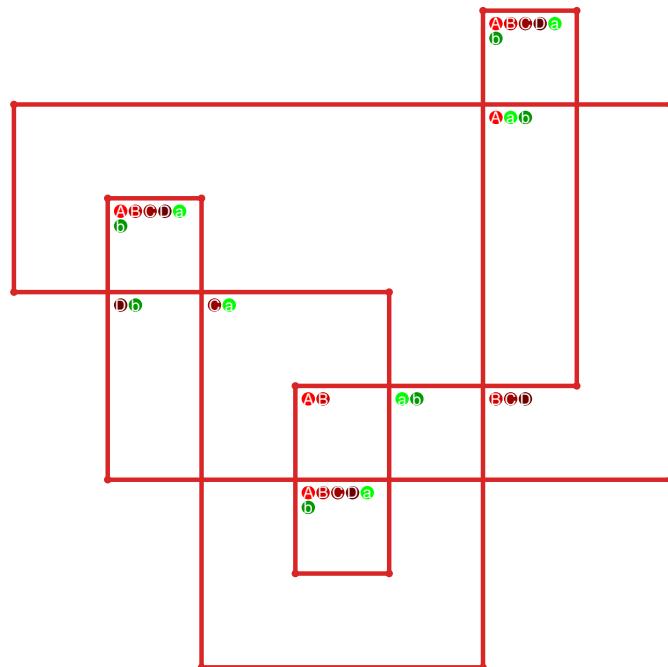


Figure 671: SnapPy multiloop plot.

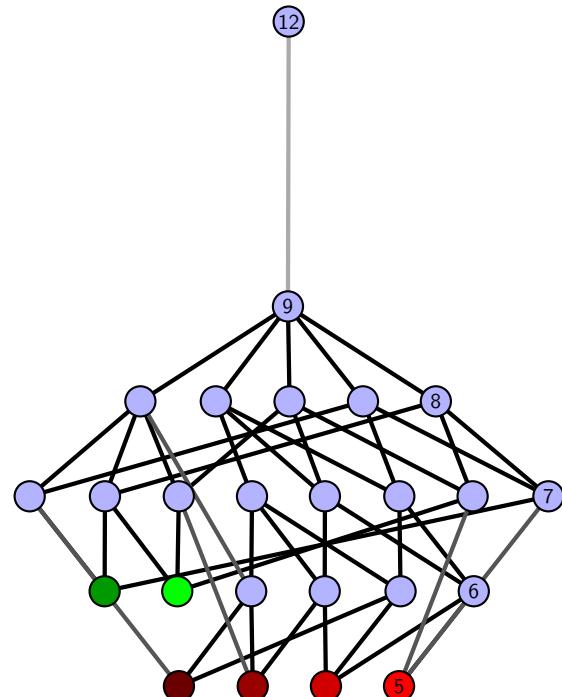


Figure 672: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.188**  $[[20, 7, 1, 8], [8, 3, 9, 4], [6, 19, 7, 20], [1, 10, 2, 11], [11, 2, 12, 3], [9, 12, 10, 13], [4, 15, 5, 16], [16, 5, 17, 6], [18, 13, 19, 14], [14, 17, 15, 18]]$

PD code drawn by SnapPy:  $[(12, 1, 13, 2), (16, 3, 17, 4), (17, 6, 18, 7), (5, 8, 6, 9), (14, 9, 15, 10), (20, 11, 1, 12), (10, 13, 11, 14), (2, 15, 3, 16), (7, 18, 8, 19), (4, 19, 5, 20)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 7, 8, 0], [0, 5, 4, 4], [1, 3, 3, 5], [1, 4, 3, 8], [1, 9, 7, 7], [2, 6, 6, 9], [2, 9, 9, 5], [6, 8, 8, 7]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 192  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.2  
 Average overall degree: 2.97

Table 335: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

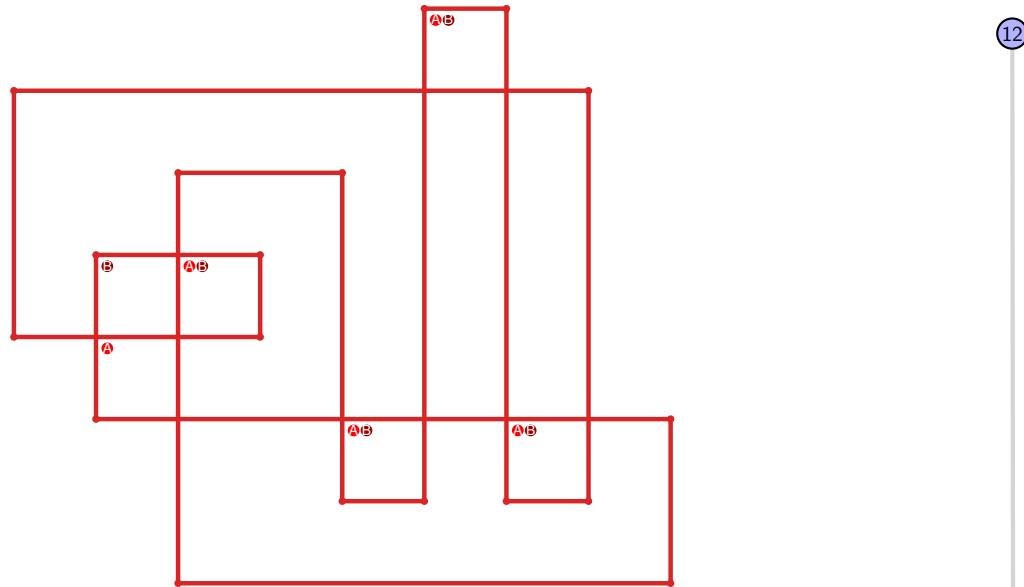


Figure 673: SnapPy multiloop plot.

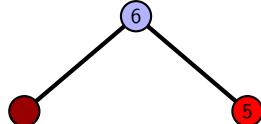


Figure 674: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.189**  $[[20, 15, 1, 16], [16, 3, 17, 4], [12, 19, 13, 20], [14, 9, 15, 10], [1, 6, 2, 7], [7, 2, 8, 3], [17, 5, 18, 4], [18, 11, 19, 12], [13, 11, 14, 10], [5, 8, 6, 9]]$

PD code drawn by `SnapPy`:  $[(5, 20, 6, 1), (14, 1, 15, 2), (2, 15, 3, 16), (19, 4, 20, 5), (3, 6, 4, 7), (12, 7, 13, 8), (8, 11, 9, 12), (16, 9, 17, 10), (18, 13, 19, 14), (10, 17, 11, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 6], [0, 7, 7, 8], [0, 8, 8, 9], [0, 9, 5, 5], [1, 4, 4, 9], [1, 9, 7, 1], [2, 6, 8, 2], [2, 7, 3, 3], [3, 6, 5, 4]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 240  
 Pinning number: 5

Average optimal degree: 2.3  
 Average minimal degree: 2.3  
 Average overall degree: 2.98

Table 336: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	22	52	69	56	28	8	1	236
Average degree	2.3	2.62	2.84	3.0	3.11	3.2	3.27	3.33	

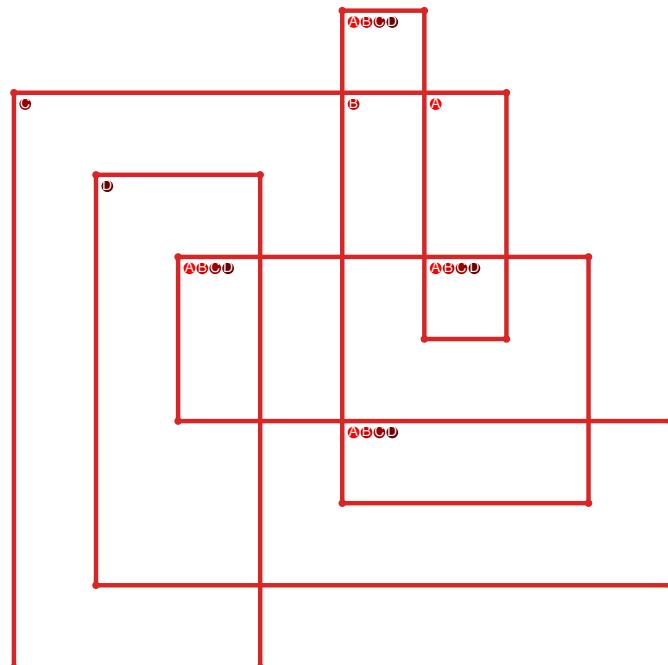


Figure 675: `SnapPy` multiloop plot.

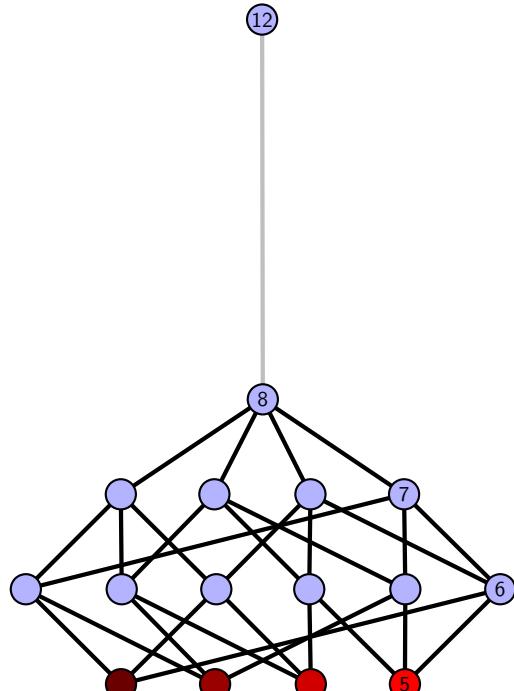


Figure 676: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.190** `[[20, 13, 1, 14], [14, 19, 15, 20], [15, 12, 16, 13], [1, 16, 2, 17], [5, 18, 6, 19], [6, 11, 7, 12], [2, 10, 3, 9], [17, 4, 18, 5], [10, 7, 11, 8], [3, 8, 4, 9]]`

PD code drawn by `SnapPy`: `[(14, 1, 15, 2), (7, 4, 8, 5), (20, 5, 1, 6), (6, 19, 7, 20), (3, 8, 4, 9), (12, 9, 13, 10), (17, 10, 18, 11), (18, 13, 19, 14), (2, 15, 3, 16), (11, 16, 12, 17)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 5], [2, 4, 8, 8], [3, 8, 9, 9], [3, 9, 4, 4], [5, 9, 6, 5], [6, 8, 7, 6]]`

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 337: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

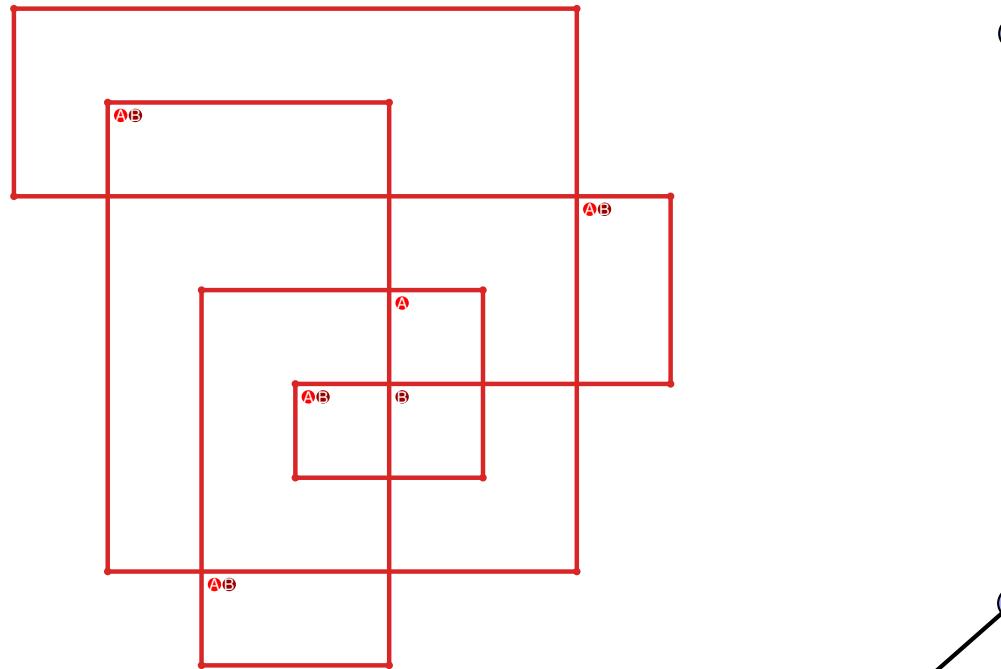


Figure 677: `SnapPy` multiloop plot.

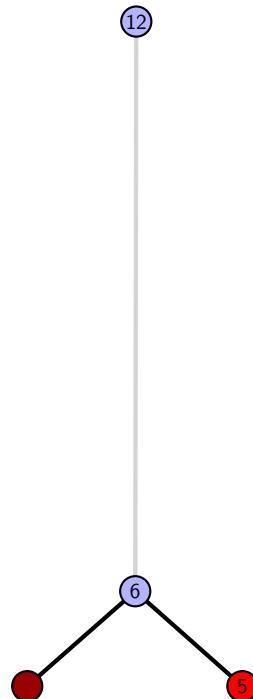


Figure 678: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.191  $[[5, 20, 6, 1], [17, 4, 18, 5], [19, 8, 20, 9], [6, 11, 7, 12], [1, 15, 2, 14], [3, 16, 4, 17], [18, 10, 19, 9], [10, 7, 11, 8], [12, 16, 13, 15], [2, 13, 3, 14]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (6, 3, 7, 4), (15, 4, 16, 5), (5, 14, 6, 15), (2, 7, 3, 8), (18, 9, 19, 10), (10, 17, 11, 18), (11, 8, 12, 9), (19, 12, 20, 13), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 6, 7], [0, 7, 7, 8], [0, 8, 9, 9], [1, 9, 8, 1], [1, 7, 2, 2], [2, 6, 3, 3], [3, 5, 9, 4], [4, 8, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.38

**Total pinning sets:** 184

**Average overall degree:** 2.98

**Pinning number:** 5

Table 338: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	7	33	54	50	27	8	1	180
Average degree	2.2	2.5	2.76	2.95	3.09	3.19	3.27	3.33	

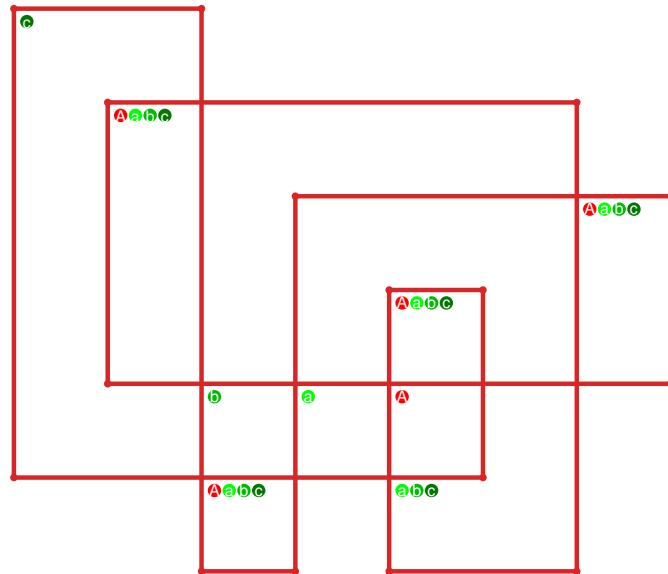


Figure 679: `SnapPy` multiloop plot.

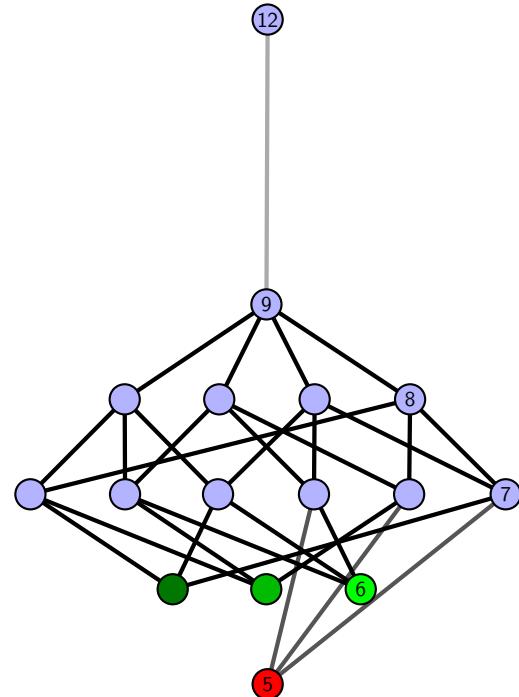


Figure 680: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.192** `[[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [6, 13, 7, 14], [1, 11, 2, 10], [18, 10, 19, 9], [2, 15, 3, 16], [14, 3, 15, 4], [12, 5, 13, 6], [11, 5, 12, 4]]`

PD code drawn by `SnapPy`: `[(7, 20, 8, 1), (15, 2, 16, 3), (4, 13, 5, 14), (5, 16, 6, 17), (1, 8, 2, 9), (17, 10, 18, 11), (14, 11, 15, 12), (12, 3, 13, 4), (9, 18, 10, 19), (19, 6, 20, 7)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 8, 8], [0, 9, 6, 5], [1, 4, 2, 1], [2, 4, 7, 7], [3, 6, 6, 9], [3, 9, 9, 3], [4, 8, 8, 7]]`

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 339: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

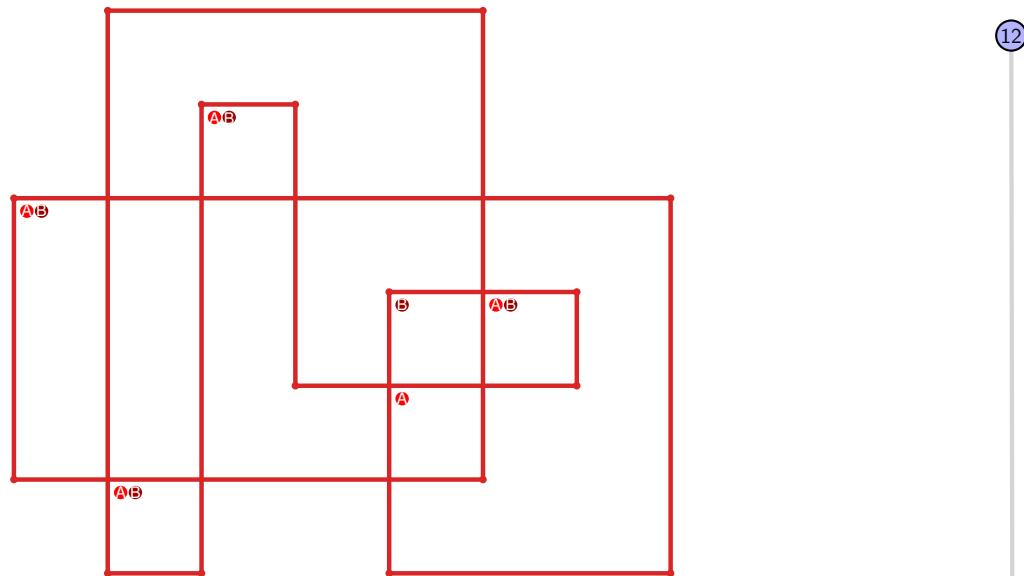


Figure 681: `SnapPy` multiloop plot.

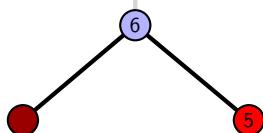


Figure 682: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.193  $[[20, 7, 1, 8], [8, 17, 9, 18], [19, 14, 20, 15], [6, 3, 7, 4], [1, 11, 2, 10], [16, 9, 17, 10], [18, 16, 19, 15], [13, 4, 14, 5], [5, 12, 6, 13], [2, 11, 3, 12]]$

PD code drawn by SnapPy:  $[(4, 1, 5, 2), (11, 2, 12, 3), (18, 5, 19, 6), (14, 7, 15, 8), (8, 19, 9, 20), (20, 9, 1, 10), (3, 10, 4, 11), (17, 12, 18, 13), (13, 16, 14, 17), (6, 15, 7, 16)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 6, 7], [0, 7, 8, 9], [0, 9, 9, 5], [1, 4, 6, 1], [1, 5, 2, 2], [2, 8, 8, 3], [3, 7, 7, 9], [3, 8, 4, 4]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 4  
 Total pinning sets: 184  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.38  
 Average overall degree: 2.98

Table 340: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	7	33	54	50	27	8	1	180
Average degree	2.2	2.5	2.76	2.95	3.09	3.19	3.27	3.33	

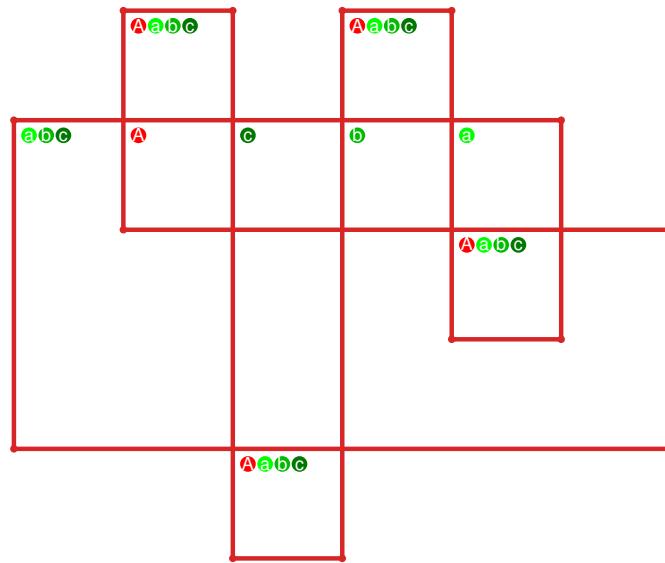


Figure 683: SnapPy multiloop plot.

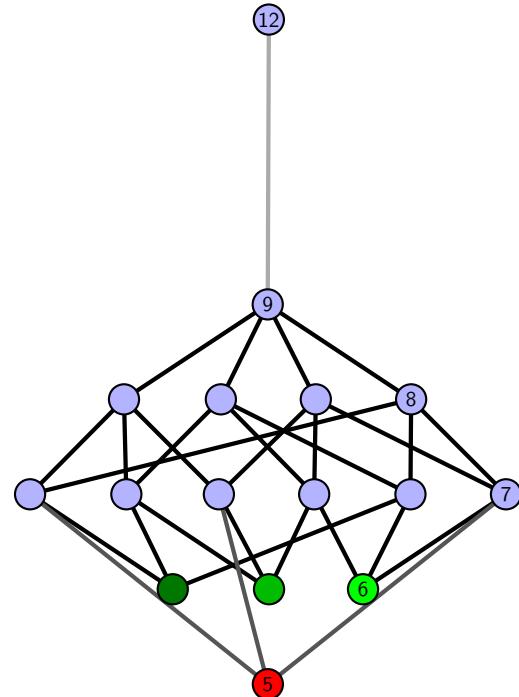


Figure 684: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.194  $[[7, 20, 8, 1], [11, 6, 12, 7], [19, 8, 20, 9], [1, 19, 2, 18], [10, 17, 11, 18], [5, 12, 6, 13], [9, 3, 10, 2], [16, 13, 17, 14], [4, 15, 5, 16], [3, 15, 4, 14]]$

PD code drawn by `SnapPy`:  $[(8, 3, 9, 4), (16, 5, 17, 6), (2, 9, 3, 10), (13, 10, 14, 11), (11, 20, 12, 1), (1, 12, 2, 13), (7, 14, 8, 15), (15, 18, 16, 19), (4, 17, 5, 18), (19, 6, 20, 7)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 5], [0, 6, 3, 0], [0, 2, 6, 4], [1, 3, 6, 7], [1, 7, 8, 1], [2, 9, 4, 3], [4, 9, 8, 5], [5, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 8  
 Total minimal pinning sets: 8  
 Total pinning sets: 216  
 Pinning number: 6

Average optimal degree: 2.56  
 Average minimal degree: 2.56  
 Average overall degree: 3.04

Table 341: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	8	0	0	0	0	0	0	8
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	36	66	63	33	9	1	208
Average degree	2.56	2.82	3.01	3.14	3.23	3.29	3.33	

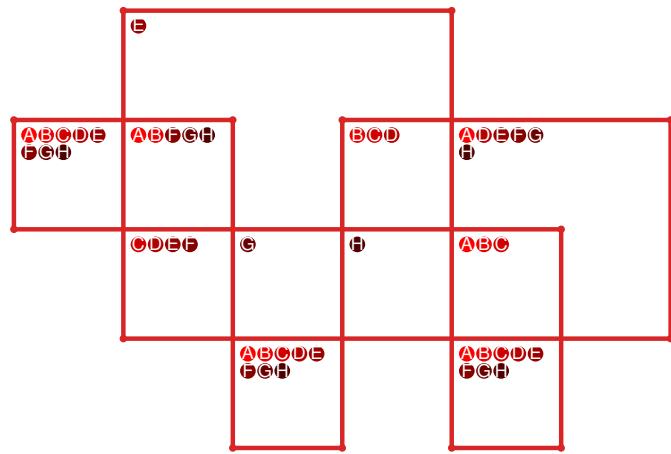


Figure 685: `SnapPy` multiloop plot.

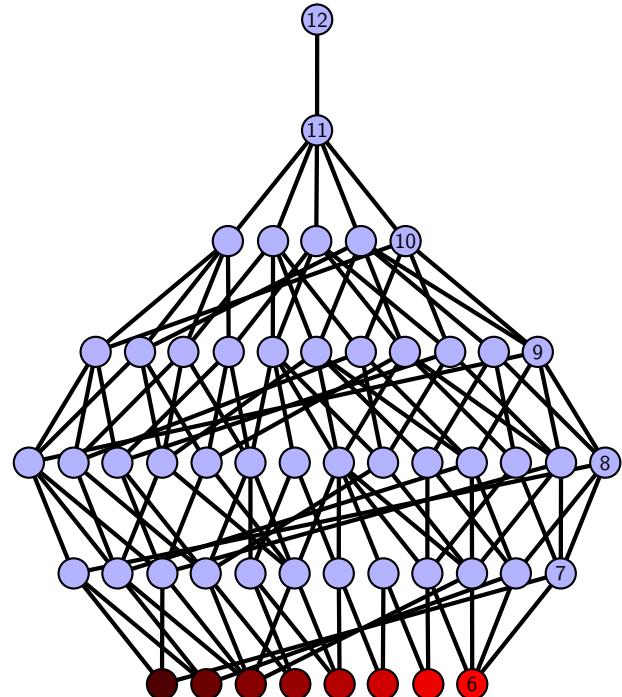


Figure 686: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.195  $[[9, 20, 10, 1], [13, 8, 14, 9], [19, 10, 20, 11], [1, 19, 2, 18], [3, 12, 4, 13], [4, 7, 5, 8], [14, 5, 15, 6], [11, 16, 12, 17], [2, 17, 3, 18], [6, 15, 7, 16]]$

PD code drawn by `SnapPy`:  $[(11, 2, 12, 3), (1, 4, 2, 5), (10, 5, 11, 6), (16, 7, 17, 8), (20, 9, 1, 10), (3, 12, 4, 13), (8, 13, 9, 14), (19, 14, 20, 15), (15, 18, 16, 19), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 7, 3, 0], [0, 2, 8, 8], [1, 8, 7, 5], [1, 4, 9, 6], [1, 5, 9, 9], [2, 9, 4, 8], [3, 7, 4, 3], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 8

**Average minimal degree:** 2.63

**Total pinning sets:** 254

**Average overall degree:** 3.05

**Pinning number:** 5

Table 342: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	1	0	0	0	0	0	7
Nonminimal pinning sets	0	7	47	79	69	34	9	1	246
Average degree	2.4	2.64	2.87	3.04	3.15	3.24	3.29	3.33	

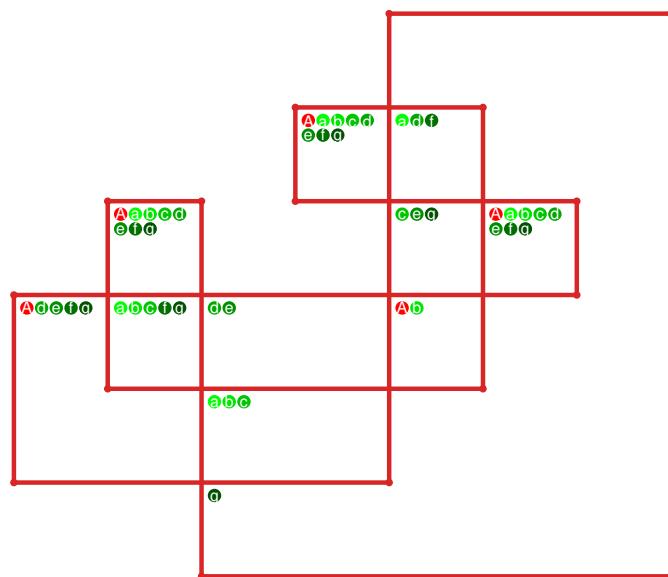


Figure 687: `SnapPy` multiloop plot.

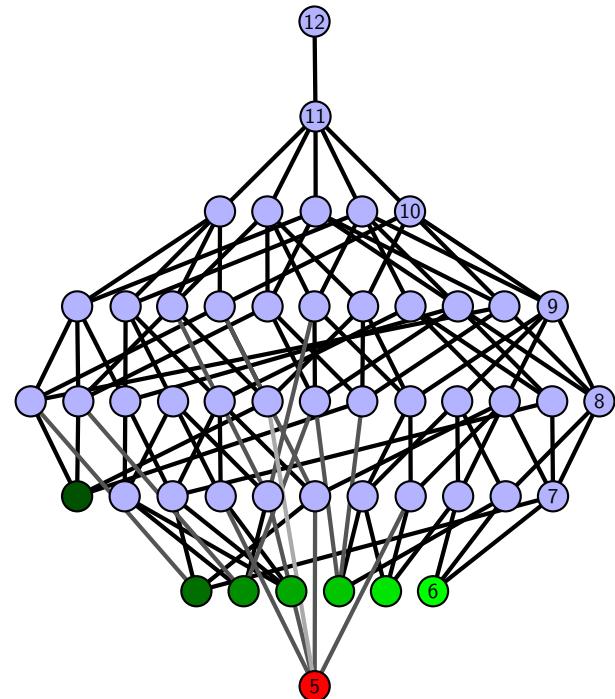


Figure 688: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.196**  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 18, 11, 17], [1, 12, 2, 13], [7, 18, 8, 19], [11, 16, 12, 17], [2, 5, 3, 6], [13, 6, 14, 7], [4, 15, 5, 16], [3, 15, 4, 14]]$

PD code drawn by `SnapPy`:  $[(19, 2, 20, 3), (10, 3, 11, 4), (16, 5, 17, 6), (18, 9, 19, 10), (11, 20, 12, 1), (1, 12, 2, 13), (8, 13, 9, 14), (14, 7, 15, 8), (4, 15, 5, 16), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 5], [0, 5, 6, 7], [1, 7, 2, 1], [2, 8, 3, 2], [3, 8, 9, 7], [3, 6, 9, 4], [5, 9, 9, 6], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 343: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

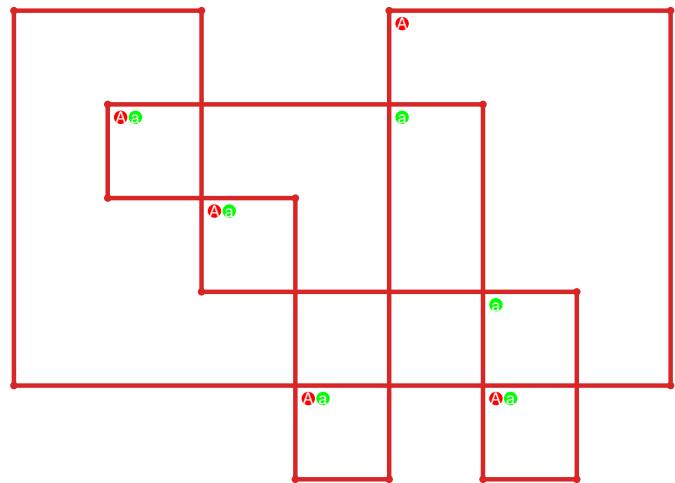


Figure 689: `SnapPy` multiloop plot.



Figure 690: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.197** [[20, 15, 1, 16], [16, 9, 17, 10], [12, 19, 13, 20], [14, 1, 15, 2], [8, 17, 9, 18], [10, 5, 11, 6], [6, 11, 7, 12], [18, 3, 19, 4], [13, 3, 14, 2], [4, 7, 5, 8]]

PD code drawn by `SnapPy`: [(15, 20, 16, 1), (7, 2, 8, 3), (1, 4, 2, 5), (13, 6, 14, 7), (3, 8, 4, 9), (17, 12, 18, 13), (5, 14, 6, 15), (9, 16, 10, 17), (11, 18, 12, 19), (19, 10, 20, 11)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 4, 5], [0, 6, 7, 8], [0, 8, 8, 0], [1, 7, 9, 1], [1, 9, 6, 6], [2, 5, 5, 9], [2, 9, 4, 8], [2, 7, 3, 3], [4, 7, 6, 5]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 4  
 Total pinning sets: 216  
 Pinning number: 5

Average optimal degree: 2.3  
 Average minimal degree: 2.36  
 Average overall degree: 2.98

Table 344: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	13	43	64	55	28	8	1	212
Average degree	2.3	2.58	2.81	2.98	3.11	3.2	3.27	3.33	

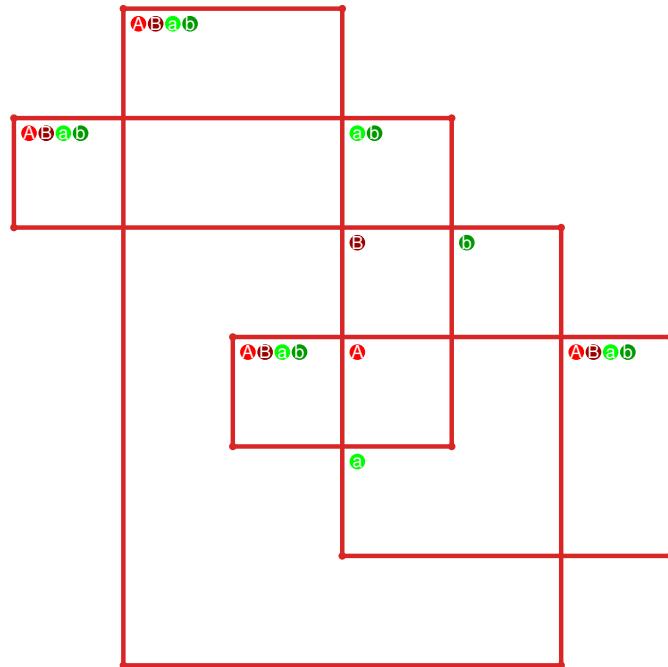


Figure 691: `SnapPy` multiloop plot.

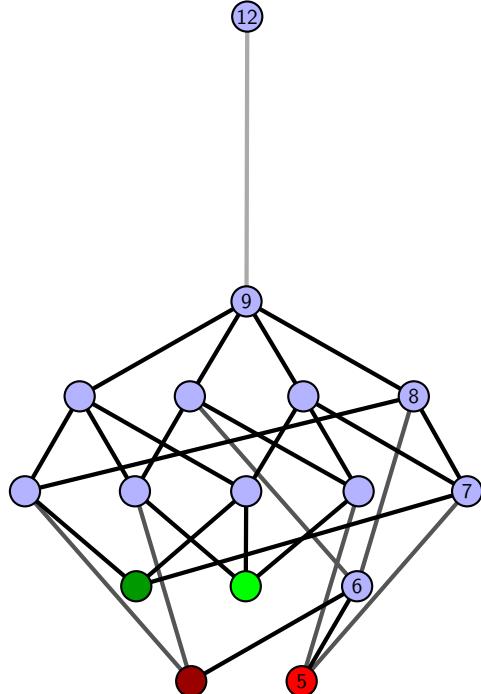


Figure 692: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.198** [[20, 7, 1, 8], [8, 15, 9, 16], [16, 19, 17, 20], [17, 6, 18, 7], [1, 13, 2, 12], [14, 9, 15, 10], [5, 18, 6, 19], [13, 5, 14, 4], [2, 11, 3, 12], [10, 3, 11, 4]]

PD code drawn by `SnapPy`: [(7, 20, 8, 1), (1, 10, 2, 11), (14, 3, 15, 4), (17, 4, 18, 5), (11, 6, 12, 7), (19, 8, 20, 9), (9, 18, 10, 19), (5, 12, 6, 13), (2, 15, 3, 16), (13, 16, 14, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 6, 6], [0, 7, 8, 8], [1, 9, 7, 1], [2, 7, 3, 3], [4, 6, 5, 9], [4, 9, 9, 4], [5, 8, 8, 7]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 160  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.97

Table 345: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

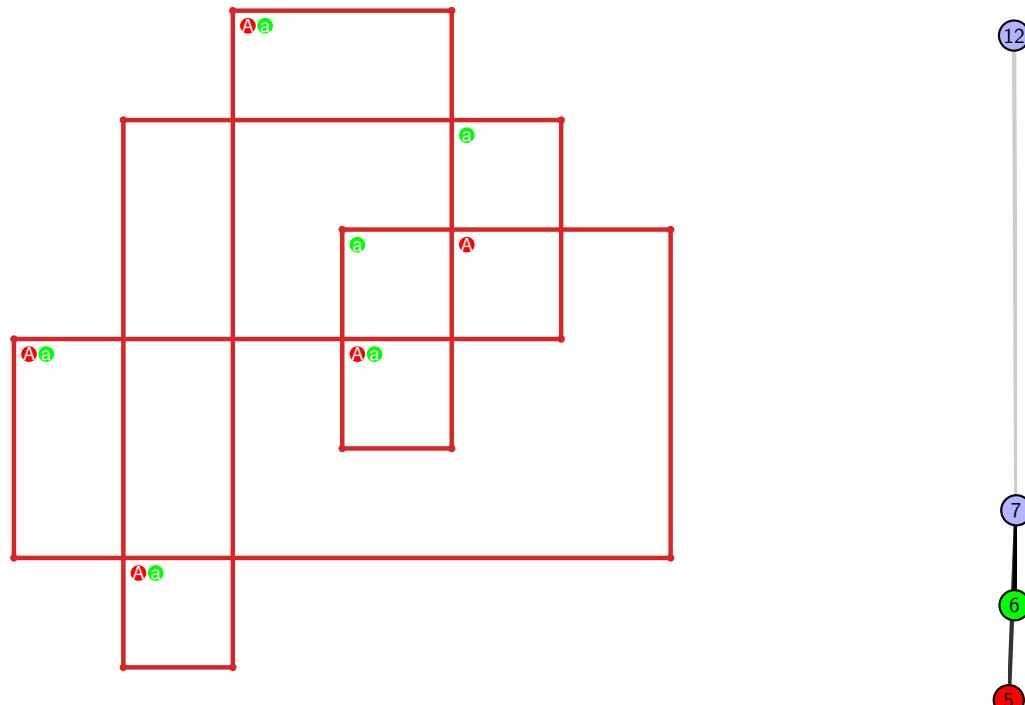


Figure 693: `SnapPy` multiloop plot.



Figure 694: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.199** `[[20, 15, 1, 16], [16, 9, 17, 10], [10, 19, 11, 20], [5, 14, 6, 15], [1, 6, 2, 7], [8, 17, 9, 18], [18, 7, 19, 8], [11, 4, 12, 5], [13, 2, 14, 3], [3, 12, 4, 13]]`

PD code drawn by `SnapPy`: `[(11, 20, 12, 1), (9, 2, 10, 3), (19, 4, 20, 5), (16, 5, 17, 6), (14, 7, 15, 8), (1, 10, 2, 11), (3, 12, 4, 13), (18, 13, 19, 14), (6, 15, 7, 16), (8, 17, 9, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 7, 8, 4], [0, 3, 8, 6], [1, 6, 6, 1], [2, 5, 5, 4], [2, 9, 9, 3], [3, 9, 9, 4], [7, 8, 8, 7]]`

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 346: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

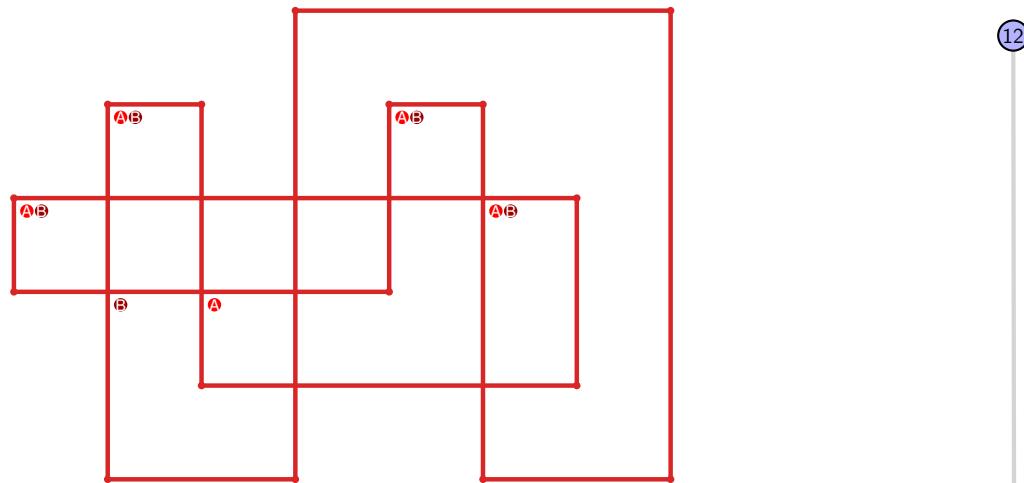


Figure 695: `SnapPy` multiloop plot.

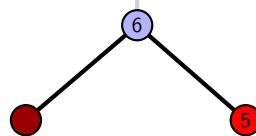


Figure 696: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.200 [[20, 9, 1, 10], [10, 19, 11, 20], [11, 8, 12, 9], [1, 12, 2, 13], [13, 18, 14, 19], [16, 7, 17, 8], [2, 17, 3, 18], [14, 6, 15, 5], [15, 4, 16, 5], [6, 3, 7, 4]]

PD code drawn by `SnapPy`: [(11, 20, 12, 1), (1, 10, 2, 11), (12, 3, 13, 4), (7, 4, 8, 5), (16, 5, 17, 6), (17, 8, 18, 9), (18, 13, 19, 14), (9, 14, 10, 15), (6, 15, 7, 16), (2, 19, 3, 20)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 4], [1, 3, 6, 7], [2, 8, 9, 6], [3, 5, 9, 4], [4, 9, 8, 8], [5, 7, 7, 9], [5, 8, 7, 6]]

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.67

**Total minimal pinning sets:** 12

**Average minimal degree:** 2.79

**Total pinning sets:** 386

**Average overall degree:** 3.12

**Pinning number:** 5

Table 347: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	8	1	0	0	0	0	0	9
Nonminimal pinning sets	0	20	84	122	95	42	10	1	374
Average degree	2.67	2.85	3.01	3.13	3.22	3.27	3.31	3.33	

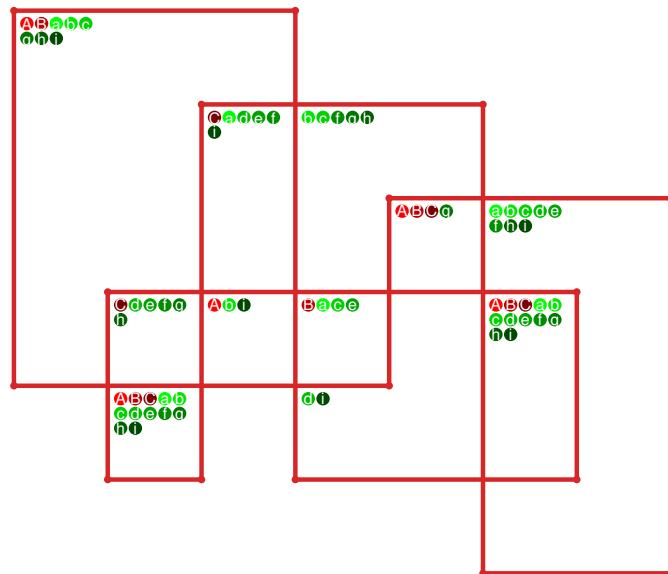


Figure 697: `SnapPy` multiloop plot.

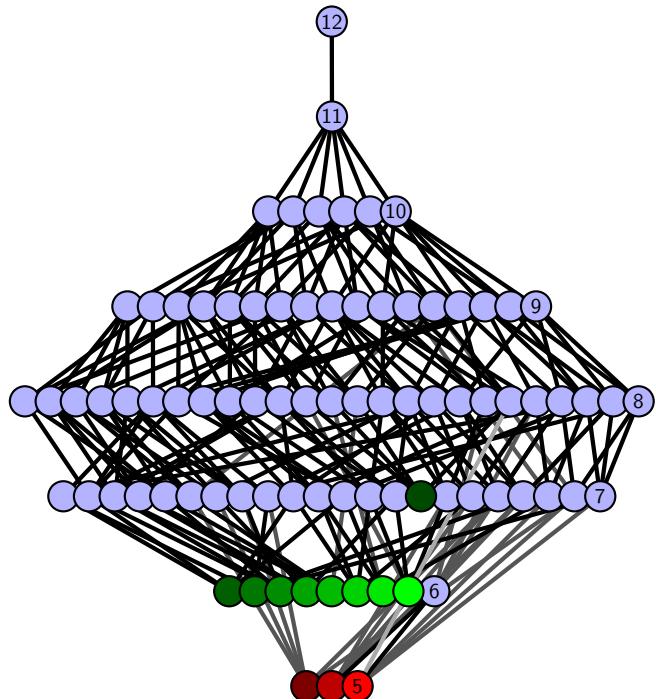


Figure 698: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.201  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 8, 11, 7], [1, 7, 2, 6], [18, 5, 19, 6], [11, 16, 12, 17], [2, 17, 3, 18], [4, 13, 5, 14], [15, 12, 16, 13], [3, 15, 4, 14]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (1, 16, 2, 17), (2, 5, 3, 6), (14, 3, 15, 4), (10, 7, 11, 8), (19, 8, 20, 9), (9, 18, 10, 19), (6, 11, 7, 12), (17, 12, 18, 13), (4, 15, 5, 16)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 4], [1, 3, 6, 7], [2, 8, 8, 6], [3, 5, 9, 4], [4, 9, 9, 8], [5, 7, 9, 5], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 6

**Average minimal degree:** 2.59

**Total pinning sets:** 356

**Average overall degree:** 3.05

**Pinning number:** 4

Table 348: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	4	0	0	0	0	0	0	5
Nonminimal pinning sets	0	8	34	83	103	77	35	9	1	350
Average degree	2.25	2.58	2.79	2.95	3.08	3.17	3.24	3.29	3.33	

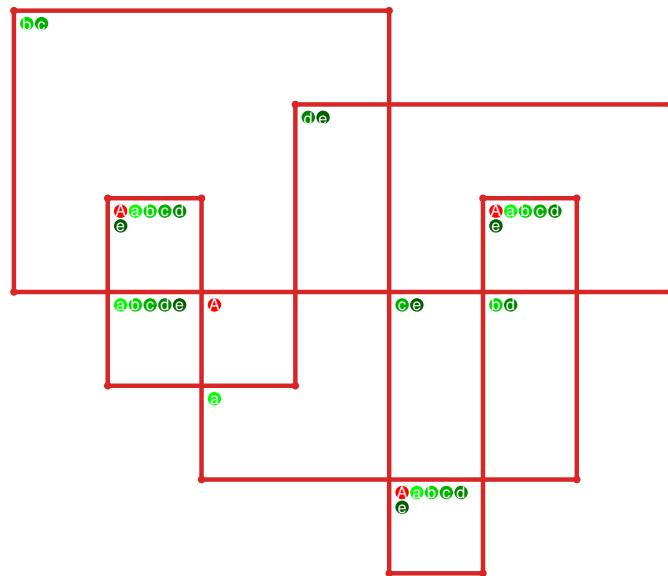


Figure 699: `SnapPy` multiloop plot.

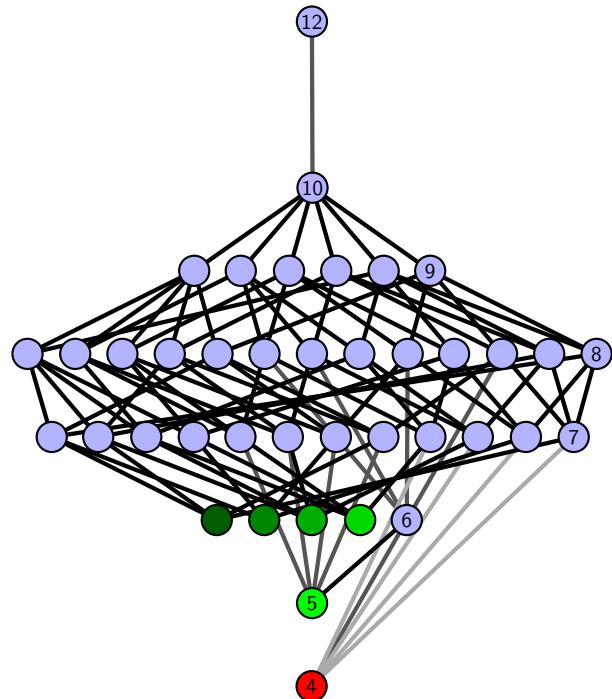


Figure 700: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.202 [[20, 13, 1, 14], [14, 19, 15, 20], [15, 12, 16, 13], [1, 16, 2, 17], [18, 5, 19, 6], [11, 8, 12, 9], [2, 8, 3, 7], [17, 7, 18, 6], [4, 9, 5, 10], [10, 3, 11, 4]]

PD code drawn by `SnapPy`: [(7, 20, 8, 1), (15, 2, 16, 3), (3, 14, 4, 15), (4, 19, 5, 20), (8, 5, 9, 6), (1, 6, 2, 7), (12, 9, 13, 10), (17, 10, 18, 11), (18, 13, 19, 14), (11, 16, 12, 17)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 8], [2, 8, 9, 6], [3, 5, 9, 7], [3, 6, 4, 4], [4, 9, 9, 5], [5, 8, 8, 6]]

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 272  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.47  
**Average overall degree:** 3.04

Table 349: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.4	2.68	2.89	3.04	3.15	3.24	3.29	3.33	

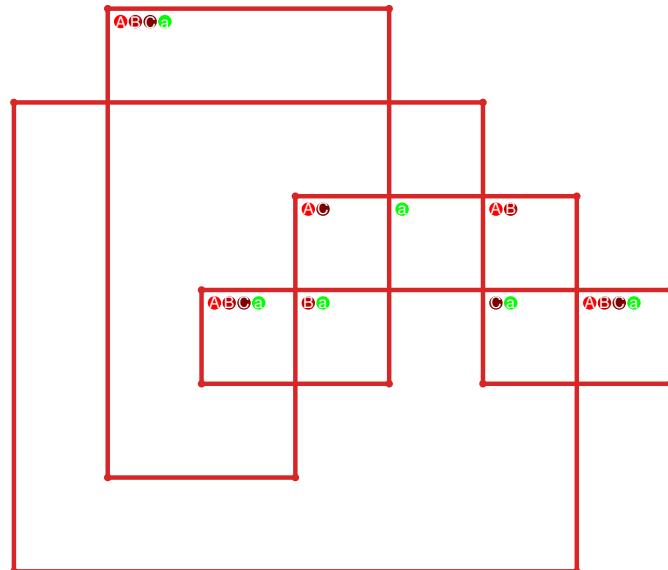


Figure 701: `SnapPy` multiloop plot.

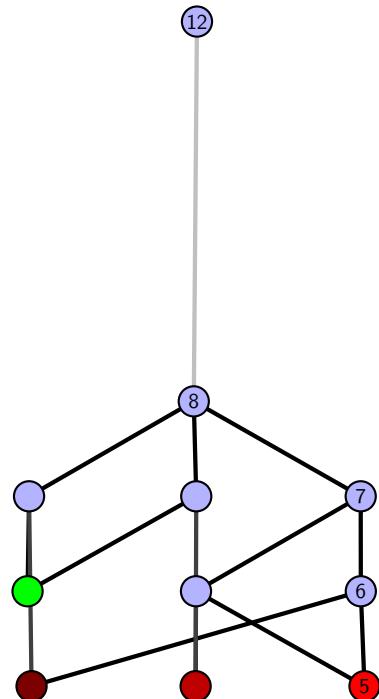


Figure 702: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.203  $[[5, 20, 6, 1], [4, 9, 5, 10], [19, 8, 20, 9], [6, 17, 7, 18], [1, 12, 2, 13], [10, 3, 11, 4], [15, 18, 16, 19], [16, 7, 17, 8], [11, 14, 12, 15], [2, 14, 3, 13]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (13, 4, 14, 5), (16, 5, 17, 6), (1, 6, 2, 7), (18, 9, 19, 10), (10, 19, 11, 20), (8, 11, 9, 12), (3, 14, 4, 15), (12, 15, 13, 16), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 7, 7, 6], [0, 8, 9, 9], [1, 9, 8, 1], [2, 8, 3, 7], [2, 6, 3, 3], [4, 6, 5, 9], [4, 8, 5, 4]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 6  
 Total pinning sets: 284  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.53  
 Average overall degree: 3.04

Table 350: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	19	60	85	70	34	9	1	278
Average degree	2.4	2.68	2.9	3.05	3.16	3.24	3.29	3.33	

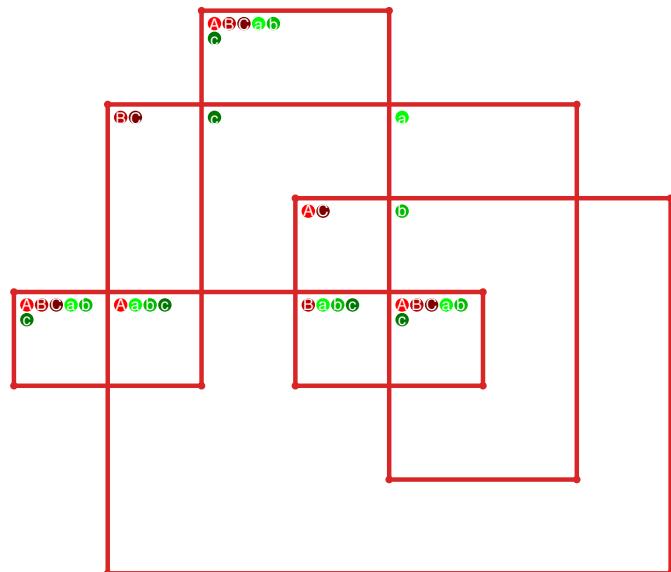


Figure 703: `SnapPy` multiloop plot.

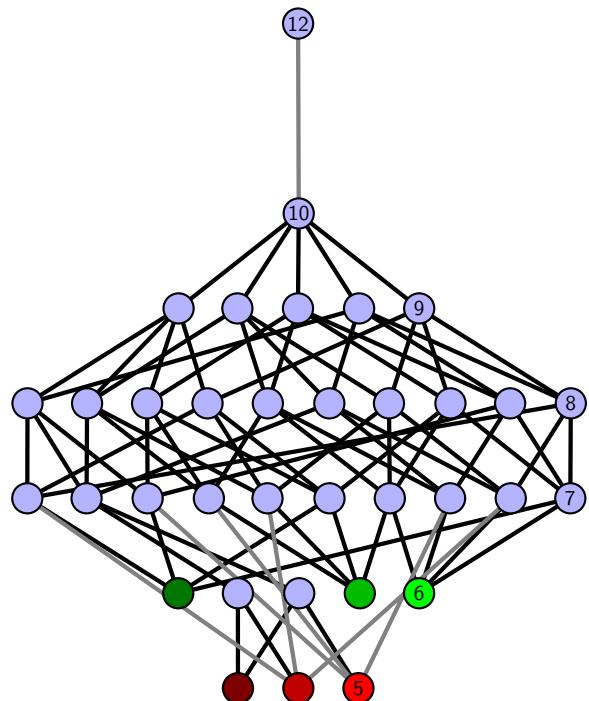


Figure 704: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.204 [[20, 7, 1, 8], [8, 11, 9, 12], [12, 19, 13, 20], [15, 6, 16, 7], [1, 16, 2, 17], [17, 10, 18, 11], [9, 18, 10, 19], [13, 5, 14, 4], [14, 3, 15, 4], [5, 2, 6, 3]]

PD code drawn by `SnapPy`: [(3, 20, 4, 1), (12, 1, 13, 2), (2, 11, 3, 12), (13, 4, 14, 5), (18, 5, 19, 6), (15, 8, 16, 9), (6, 9, 7, 10), (19, 14, 20, 15), (7, 16, 8, 17), (10, 17, 11, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 7], [0, 8, 9, 4], [0, 3, 9, 5], [1, 4, 6, 6], [1, 5, 5, 2], [2, 9, 8, 8], [3, 7, 7, 9], [3, 8, 7, 4]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 9

**Total pinning sets:** 296

**Pinning number:** 5

**Average optimal degree:** 2.6

**Average minimal degree:** 2.7

**Average overall degree:** 3.11

Table 351: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	8	0	0	0	0	0	0	8
Nonminimal pinning sets	0	7	55	92	82	40	10	1	287
Average degree	2.6	2.76	2.95	3.09	3.19	3.27	3.31	3.33	

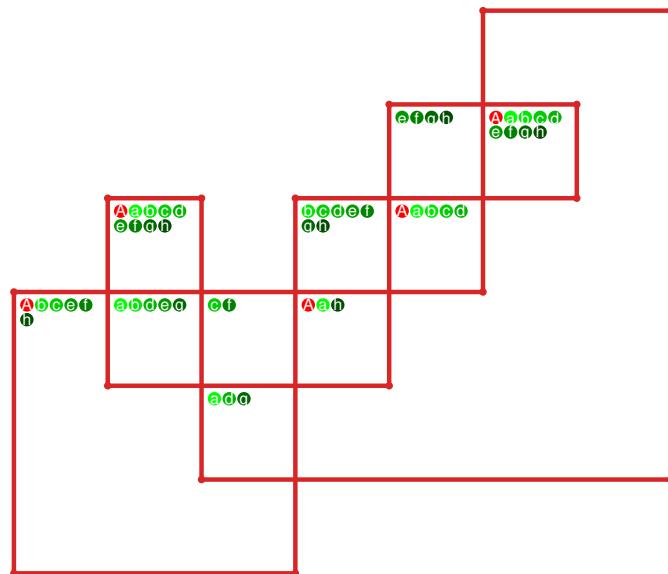


Figure 705: `SnapPy` multiloop plot.

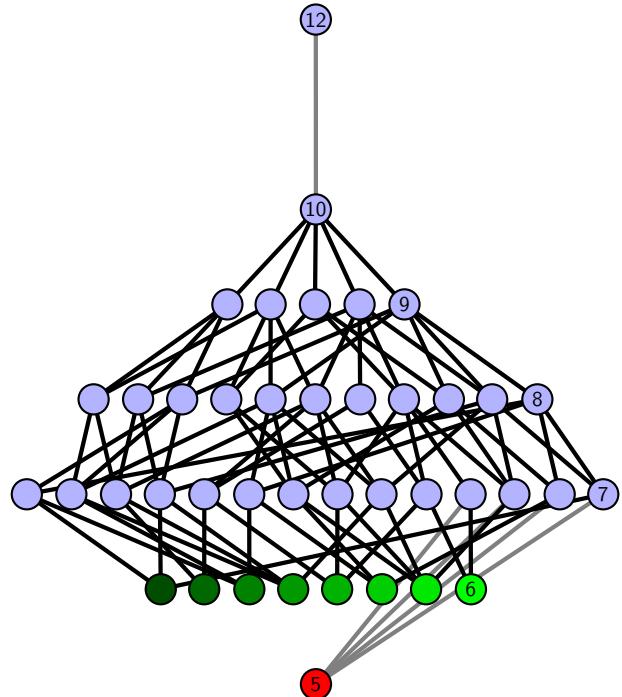


Figure 706: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.205  $[[20, 11, 1, 12], [12, 19, 13, 20], [13, 10, 14, 11], [1, 9, 2, 8], [18, 7, 19, 8], [9, 14, 10, 15], [2, 17, 3, 18], [3, 6, 4, 7], [15, 4, 16, 5], [5, 16, 6, 17]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (19, 2, 20, 3), (14, 3, 15, 4), (4, 17, 5, 18), (9, 6, 10, 7), (16, 7, 17, 8), (5, 10, 6, 11), (1, 12, 2, 13), (18, 13, 19, 14), (8, 15, 9, 16)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 4], [1, 3, 6, 7], [2, 8, 3, 2], [3, 9, 7, 4], [4, 6, 9, 8], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3

**Total pinning sets:** 256

**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.4

**Average overall degree:** 3.03

Table 352: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	19	51	75	65	33	9	1	253
Average degree	2.4	2.68	2.89	3.03	3.15	3.23	3.29	3.33	

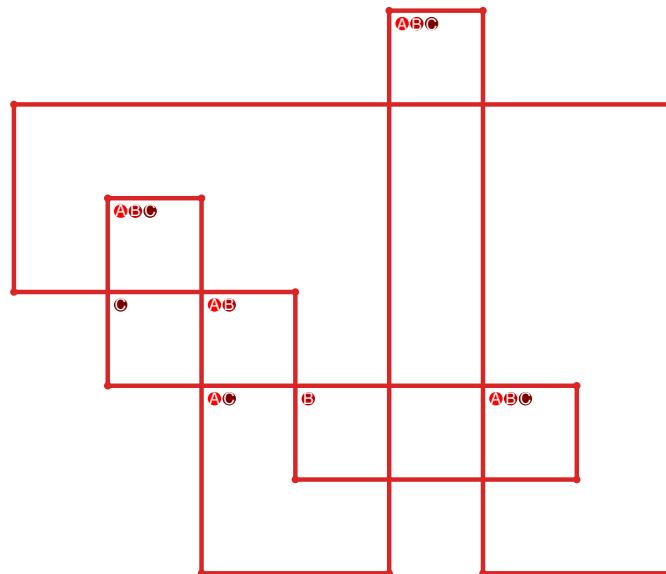


Figure 707: `SnapPy` multiloop plot.

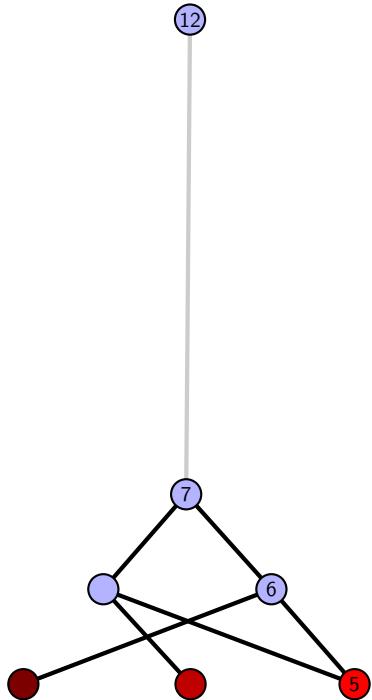


Figure 708: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.206** [[20, 13, 1, 14], [14, 19, 15, 20], [15, 12, 16, 13], [1, 16, 2, 17], [18, 7, 19, 8], [4, 11, 5, 12], [2, 10, 3, 9], [17, 9, 18, 8], [3, 6, 4, 7], [10, 5, 11, 6]]

PD code drawn by `SnapPy`: [(8, 3, 9, 4), (1, 4, 2, 5), (16, 5, 17, 6), (6, 15, 7, 16), (7, 20, 8, 1), (2, 9, 3, 10), (13, 10, 14, 11), (18, 11, 19, 12), (19, 14, 20, 15), (12, 17, 13, 18)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 8], [2, 8, 9, 9], [3, 9, 8, 7], [3, 6, 4, 4], [4, 6, 9, 5], [5, 8, 6, 5]]

Total optimal pinning sets: 4  
Total minimal pinning sets: 4

Total pinning sets: 288

Pinning number: 5

Average optimal degree: 2.4

Average minimal degree: 2.4

Average overall degree: 3.03

Table 353: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

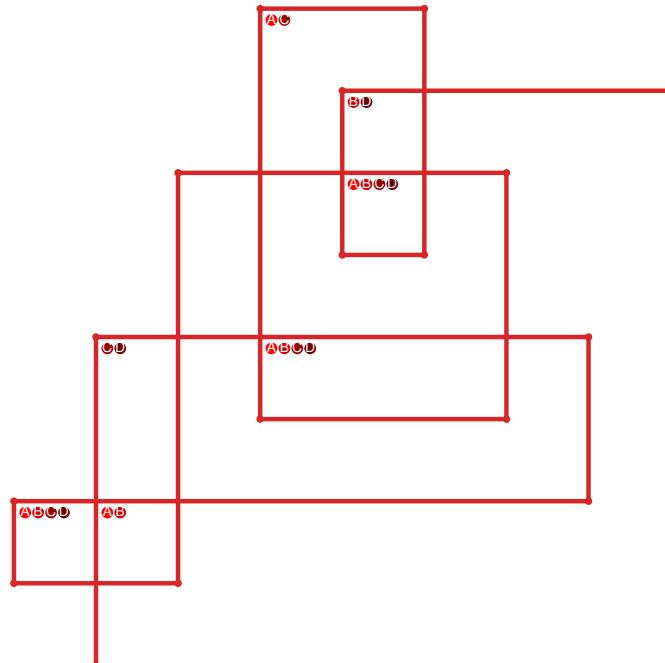


Figure 709: `SnapPy` multiloop plot.

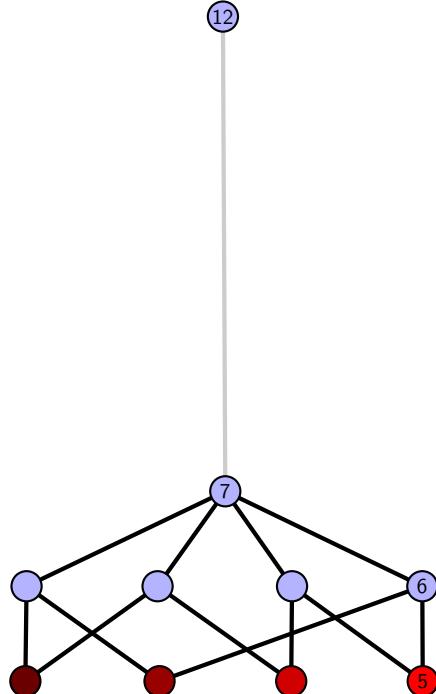


Figure 710: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.207**  $[[5, 20, 6, 1], [4, 11, 5, 12], [19, 10, 20, 11], [6, 10, 7, 9], [1, 14, 2, 15], [12, 3, 13, 4], [18, 7, 19, 8], [8, 17, 9, 18], [13, 16, 14, 17], [2, 16, 3, 15]]$

PD code drawn by `SnapPy`:  $[(14, 3, 15, 4), (5, 2, 6, 3), (15, 6, 16, 7), (20, 7, 1, 8), (17, 10, 18, 11), (11, 18, 12, 19), (9, 12, 10, 13), (4, 13, 5, 14), (1, 16, 2, 17), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 6, 7], [0, 8, 9, 9], [1, 9, 8, 1], [2, 7, 7, 3], [3, 6, 6, 8], [4, 7, 5, 9], [4, 8, 5, 4]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 6  
 Total pinning sets: 192  
 Pinning number: 6

Average optimal degree: 2.5  
 Average minimal degree: 2.5  
 Average overall degree: 3.03

Table 354: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	29	57	58	32	9	1	186
Average degree	2.5	2.78	2.98	3.12	3.23	3.29	3.33	

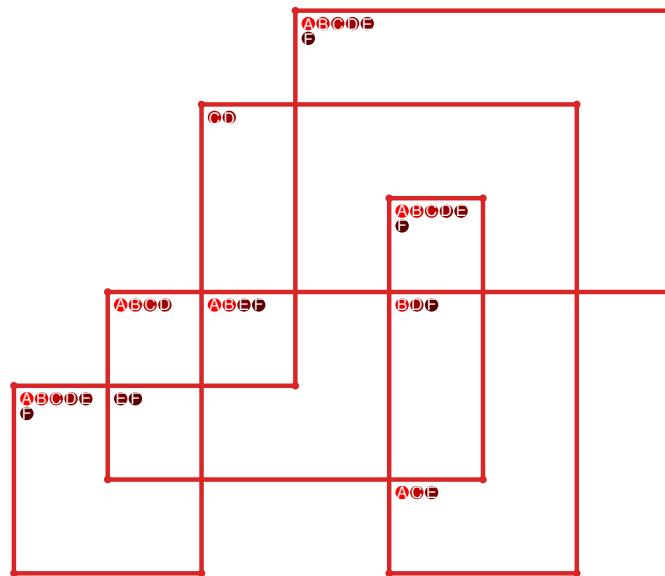


Figure 711: `SnapPy` multiloop plot.

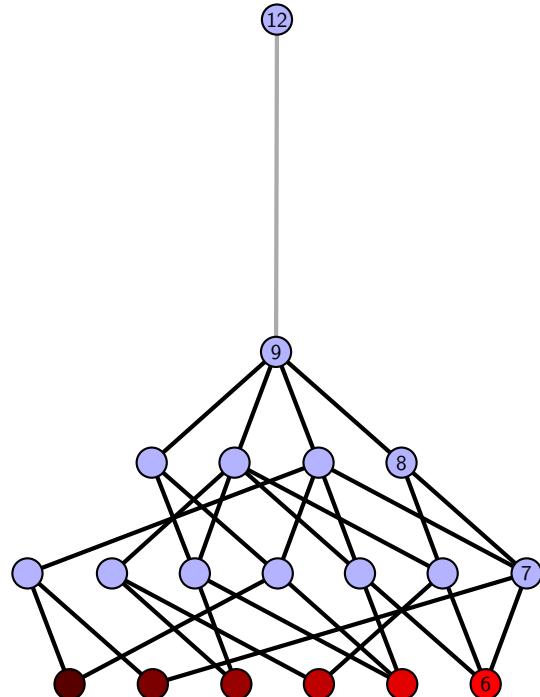


Figure 712: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.208 [[11, 20, 12, 1], [10, 17, 11, 18], [19, 16, 20, 17], [12, 7, 13, 8], [1, 8, 2, 9], [18, 9, 19, 10], [15, 4, 16, 5], [6, 3, 7, 4], [13, 3, 14, 2], [5, 14, 6, 15]]

PD code drawn by SnapPy: [(20, 11, 1, 12), (4, 1, 5, 2), (13, 2, 14, 3), (10, 5, 11, 6), (19, 6, 20, 7), (16, 7, 17, 8), (3, 12, 4, 13), (17, 14, 18, 15), (8, 15, 9, 16), (9, 18, 10, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 8, 4], [0, 3, 8, 5], [1, 4, 2, 1], [2, 9, 9, 7], [3, 6, 9, 8], [3, 7, 9, 4], [6, 8, 7, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.6

**Total minimal pinning sets:** 11

**Average minimal degree:** 2.71

**Total pinning sets:** 308

**Average overall degree:** 3.1

**Pinning number:** 5

Table 355: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	10	0	0	0	0	0	0	10
Nonminimal pinning sets	0	7	60	96	83	40	10	1	297
Average degree	2.6	2.75	2.95	3.09	3.2	3.27	3.31	3.33	

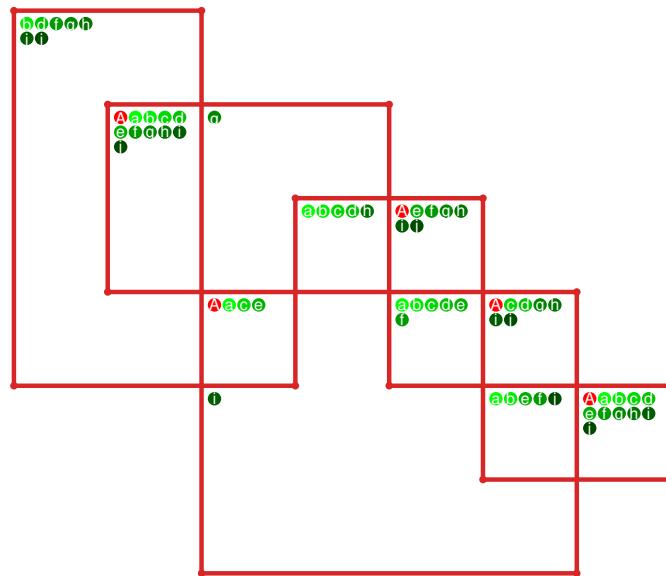


Figure 713: SnapPy multiloop plot.

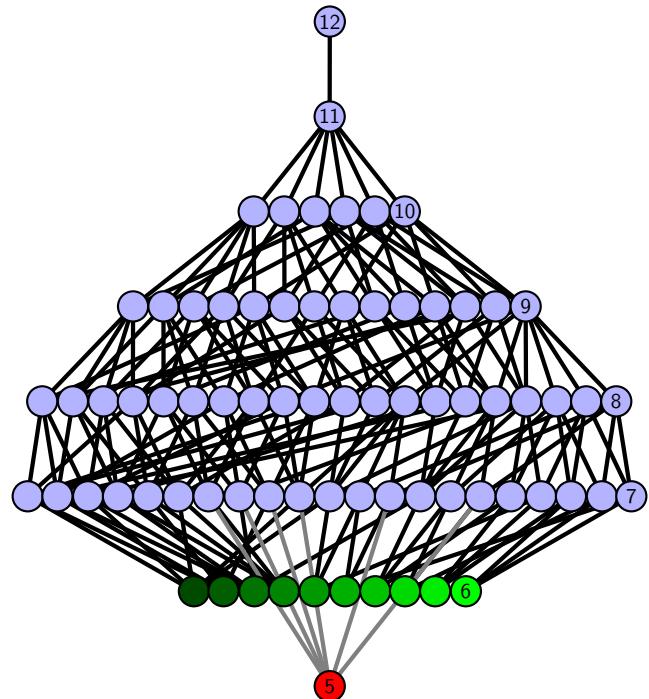


Figure 714: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.209**  $[[9, 20, 10, 1], [8, 17, 9, 18], [19, 16, 20, 17], [10, 5, 11, 6], [1, 6, 2, 7], [18, 7, 19, 8], [12, 15, 13, 16], [13, 4, 14, 5], [11, 3, 12, 2], [3, 14, 4, 15]]$

PD code drawn by `SnapPy`:  $[(20, 9, 1, 10), (11, 2, 12, 3), (8, 3, 9, 4), (19, 4, 20, 5), (16, 5, 17, 6), (1, 12, 2, 13), (10, 13, 11, 14), (17, 14, 18, 15), (6, 15, 7, 16), (7, 18, 8, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 8, 4], [0, 3, 8, 5], [1, 4, 2, 1], [2, 8, 9, 7], [3, 6, 9, 9], [3, 9, 6, 4], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 6

**Average optimal degree:** 2.6

**Total minimal pinning sets:** 10

**Average minimal degree:** 2.73

**Total pinning sets:** 424

**Average overall degree:** 3.11

**Pinning number:** 5

Table 356: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	36	100	129	96	42	10	1	414
Average degree	2.6	2.84	3.01	3.13	3.22	3.27	3.31	3.33	

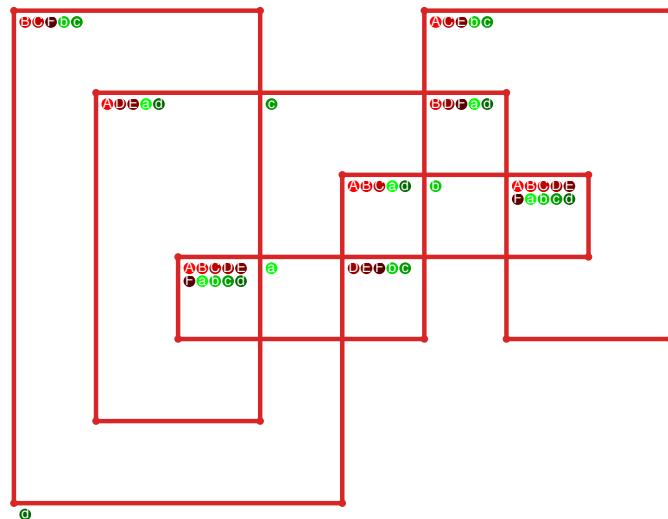


Figure 715: `SnapPy` multiloop plot.

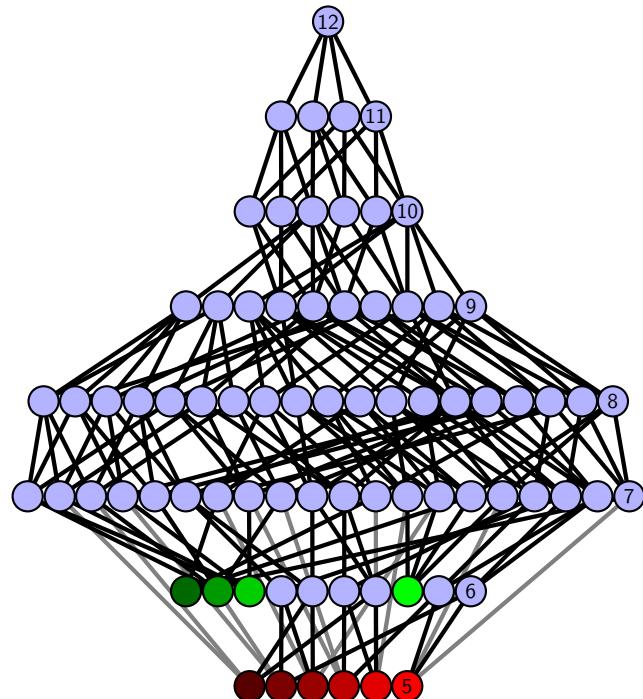


Figure 716: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.210** `[[11, 20, 12, 1], [10, 17, 11, 18], [19, 16, 20, 17], [12, 7, 13, 8], [1, 8, 2, 9], [18, 9, 19, 10], [15, 4, 16, 5], [6, 13, 7, 14], [2, 6, 3, 5], [3, 14, 4, 15]]`

PD code drawn by `SnapPy`: `[(11, 20, 12, 1), (1, 18, 2, 19), (8, 3, 9, 4), (17, 4, 18, 5), (14, 5, 15, 6), (2, 9, 3, 10), (19, 10, 20, 11), (15, 12, 16, 13), (6, 13, 7, 14), (7, 16, 8, 17)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 4], [0, 3, 8, 5], [1, 4, 2, 1], [2, 8, 9, 9], [3, 9, 8, 3], [4, 7, 9, 6], [6, 8, 7, 6]]`

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 264  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.52  
**Average overall degree:** 3.04

Table 357: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	13	52	80	69	34	9	1	258
Average degree	2.4	2.66	2.87	3.04	3.15	3.24	3.29	3.33	

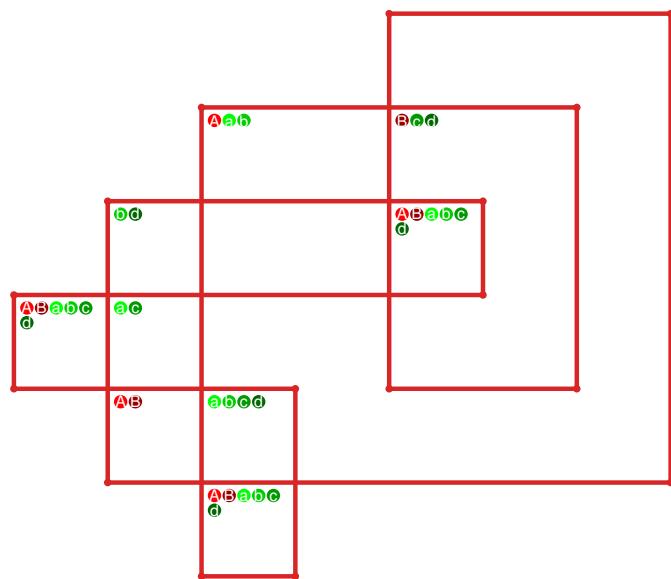


Figure 717: `SnapPy` multiloop plot.

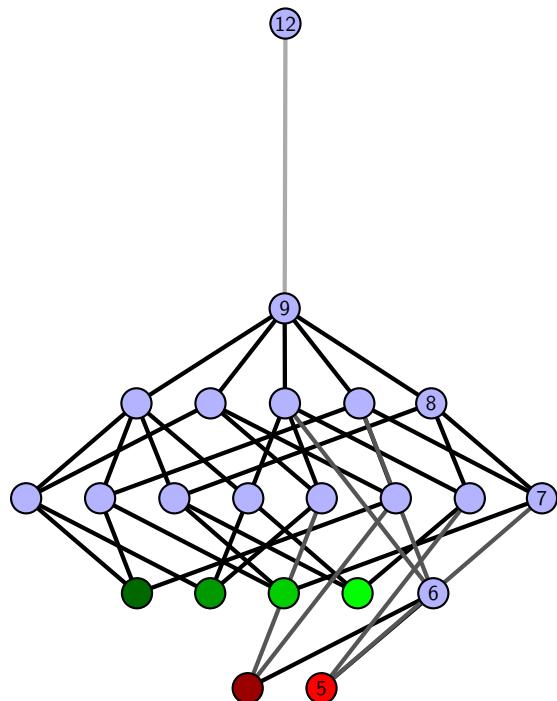


Figure 718: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.211** [[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 13, 9, 14], [1, 13, 2, 12], [18, 12, 19, 11], [4, 15, 5, 16], [14, 5, 15, 6], [7, 2, 8, 3], [3, 6, 4, 7]]

PD code drawn by `SnapPy`: [(5, 20, 6, 1), (12, 3, 13, 4), (1, 4, 2, 5), (18, 7, 19, 8), (9, 16, 10, 17), (10, 19, 11, 20), (6, 11, 7, 12), (2, 13, 3, 14), (17, 14, 18, 15), (15, 8, 16, 9)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 8, 4], [0, 3, 8, 5], [1, 4, 2, 1], [2, 9, 7, 7], [3, 6, 6, 9], [3, 9, 9, 4], [6, 8, 8, 7]]

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 264  
**Pinning number:** 5

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.52  
**Average overall degree:** 3.04

Table 358: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	13	52	80	69	34	9	1	258
Average degree	2.4	2.66	2.87	3.04	3.15	3.24	3.29	3.33	

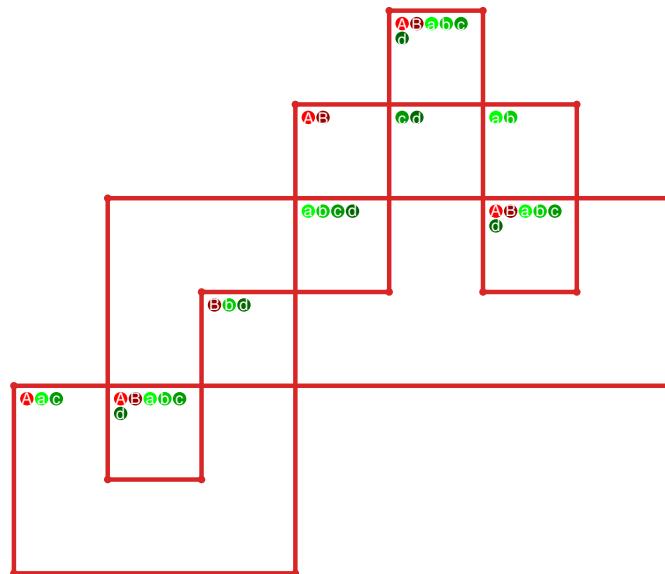


Figure 719: `SnapPy` multiloop plot.

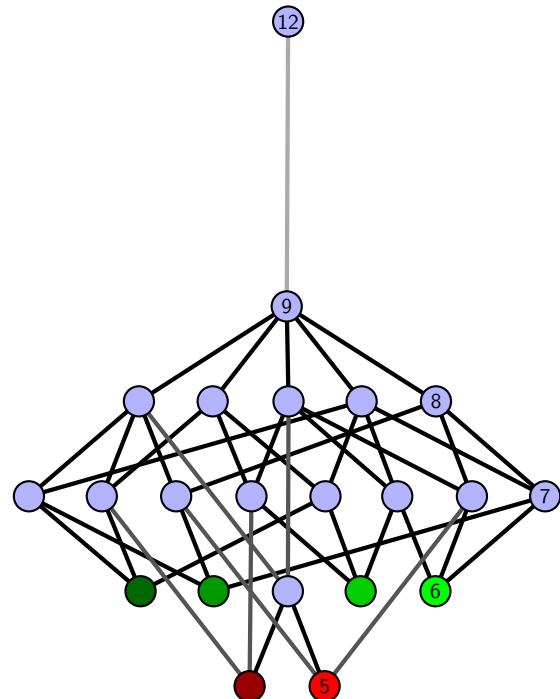


Figure 720: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.212** [[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 5, 9, 6], [1, 5, 2, 4], [18, 12, 19, 11], [15, 6, 16, 7], [7, 14, 8, 15], [2, 14, 3, 13], [3, 12, 4, 13]]

PD code drawn by `SnapPy`: [(1, 18, 2, 19), (5, 2, 6, 3), (12, 3, 13, 4), (17, 6, 18, 7), (8, 15, 9, 16), (20, 9, 1, 10), (10, 19, 11, 20), (4, 11, 5, 12), (16, 13, 17, 14), (14, 7, 15, 8)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 4], [0, 3, 8, 9], [1, 9, 2, 1], [2, 7, 7, 3], [3, 6, 6, 8], [4, 7, 9, 9], [4, 8, 8, 5]]

Total optimal pinning sets: 3  
Total minimal pinning sets: 7

Total pinning sets: 304

Pinning number: 5

Average optimal degree: 2.47

Average minimal degree: 2.58

Average overall degree: 3.05

Table 359: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	19	65	93	75	35	9	1	297
Average degree	2.47	2.72	2.92	3.06	3.17	3.24	3.29	3.33	

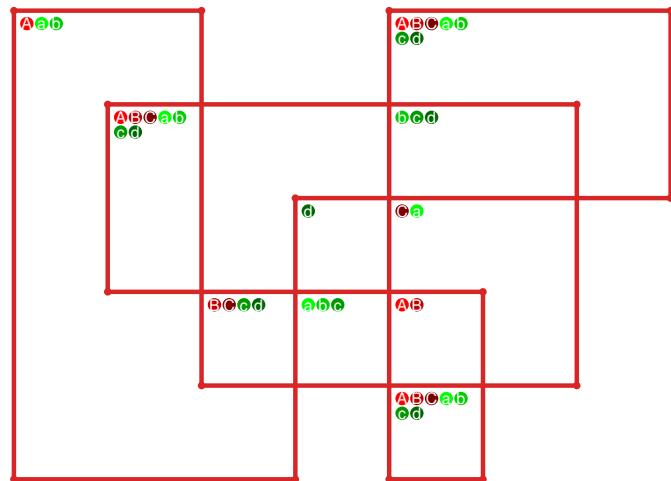


Figure 721: `SnapPy` multiloop plot.

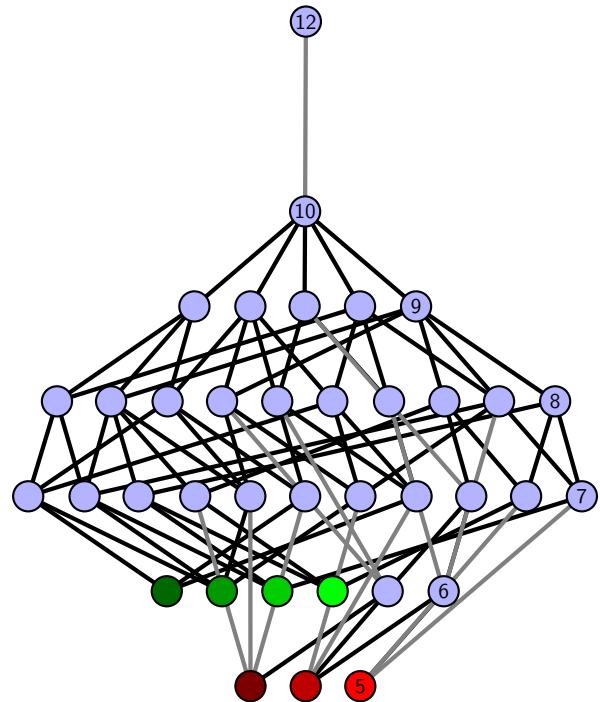


Figure 722: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.213 [[13, 20, 14, 1], [19, 12, 20, 13], [14, 12, 15, 11], [1, 16, 2, 17], [7, 18, 8, 19], [15, 10, 16, 11], [2, 5, 3, 6], [17, 6, 18, 7], [8, 3, 9, 4], [4, 9, 5, 10]]

PD code drawn by `SnapPy`: [(18, 1, 19, 2), (13, 2, 14, 3), (20, 5, 1, 6), (11, 6, 12, 7), (7, 10, 8, 11), (15, 8, 16, 9), (17, 12, 18, 13), (3, 14, 4, 15), (9, 16, 10, 17), (4, 19, 5, 20)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 7], [1, 7, 7, 8], [2, 9, 3, 2], [3, 9, 8, 7], [3, 6, 4, 4], [4, 6, 9, 9], [5, 8, 8, 6]]

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 6

**Total pinning sets:** 228

**Pinning number:** 5

**Average optimal degree:** 2.3

**Average minimal degree:** 2.43

**Average overall degree:** 2.99

Table 360: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	13	48	68	56	28	8	1	222
Average degree	2.3	2.58	2.82	2.99	3.11	3.2	3.27	3.33	

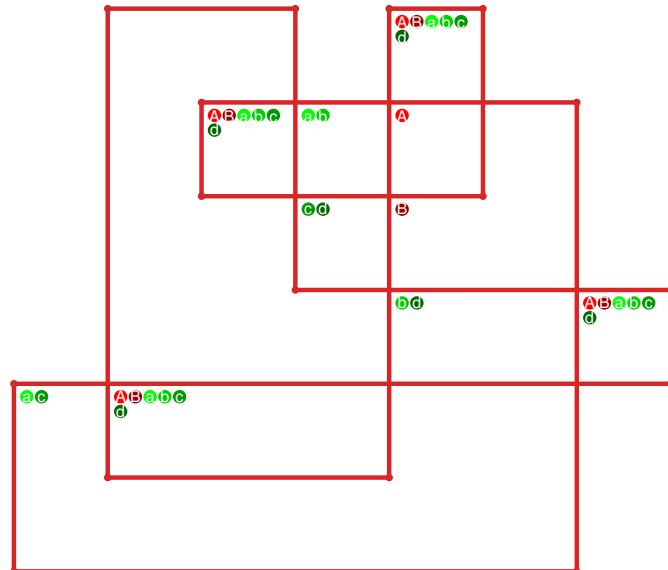


Figure 723: `SnapPy` multiloop plot.

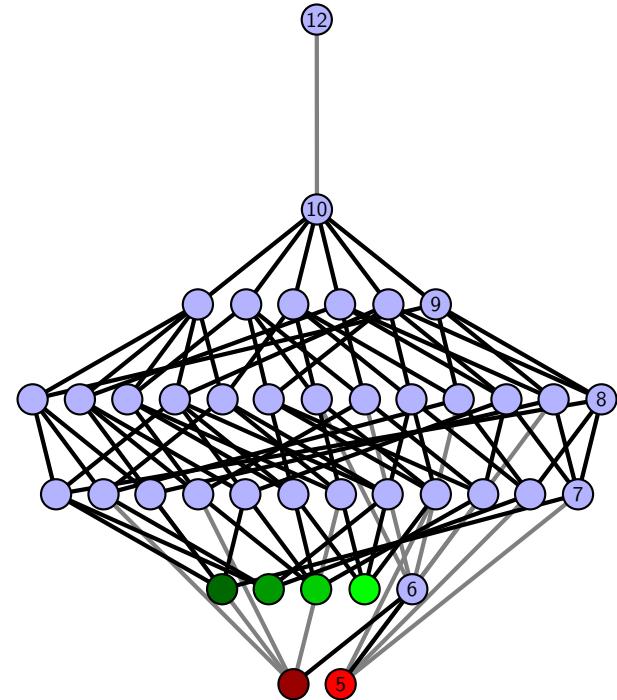


Figure 724: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.214 [[5, 20, 6, 1], [4, 11, 5, 12], [19, 6, 20, 7], [1, 14, 2, 15], [12, 3, 13, 4], [10, 7, 11, 8], [18, 9, 19, 10], [13, 16, 14, 17], [2, 16, 3, 15], [8, 17, 9, 18]]

PD code drawn by SnapPy: [(15, 2, 16, 3), (6, 3, 7, 4), (14, 5, 15, 6), (20, 7, 1, 8), (17, 10, 18, 11), (11, 18, 12, 19), (9, 12, 10, 13), (4, 13, 5, 14), (1, 16, 2, 17), (8, 19, 9, 20)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 4, 5], [0, 5, 6, 0], [0, 7, 8, 8], [1, 8, 7, 1], [1, 9, 6, 2], [2, 5, 9, 9], [3, 9, 4, 8], [3, 7, 4, 3], [5, 7, 6, 6]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 4  
**Total pinning sets:** 144  
**Pinning number:** 6

**Average optimal degree:** 2.33  
**Average minimal degree:** 2.33  
**Average overall degree:** 2.97

Table 361: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.33	2.66	2.89	3.06	3.18	3.27	3.33	

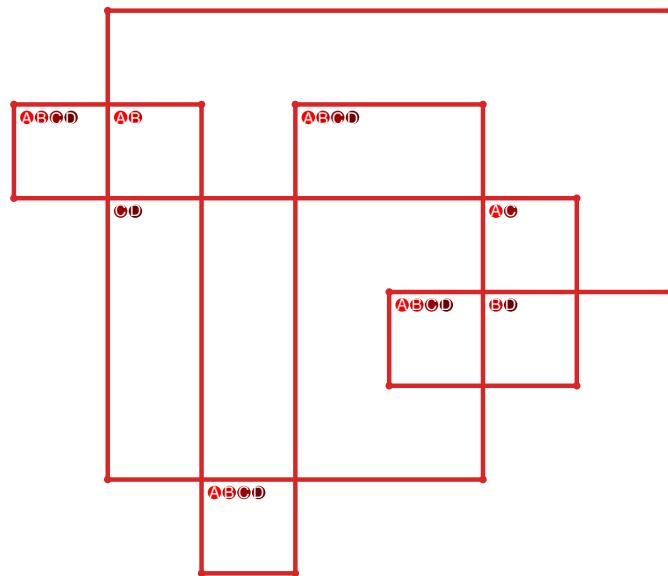


Figure 725: SnapPy multiloop plot.

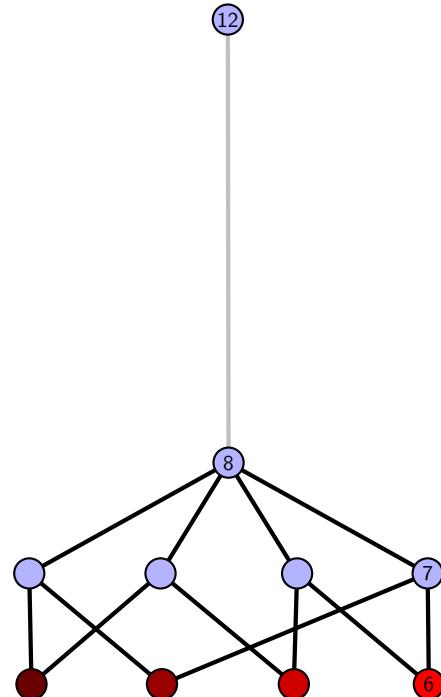


Figure 726: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.215**  $[[5, 20, 6, 1], [15, 4, 16, 5], [16, 19, 17, 20], [6, 17, 7, 18], [1, 10, 2, 11], [3, 14, 4, 15], [18, 7, 19, 8], [9, 12, 10, 13], [2, 12, 3, 11], [13, 8, 14, 9]]$

PD code drawn by `SnapPy`:  $[(13, 2, 14, 3), (16, 5, 17, 6), (11, 6, 12, 7), (19, 8, 20, 9), (9, 20, 10, 1), (7, 10, 8, 11), (1, 12, 2, 13), (4, 15, 5, 16), (14, 17, 15, 18), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 6, 6], [0, 7, 8, 8], [1, 8, 9, 1], [2, 9, 3, 3], [4, 9, 9, 8], [4, 7, 5, 4], [5, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 362: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

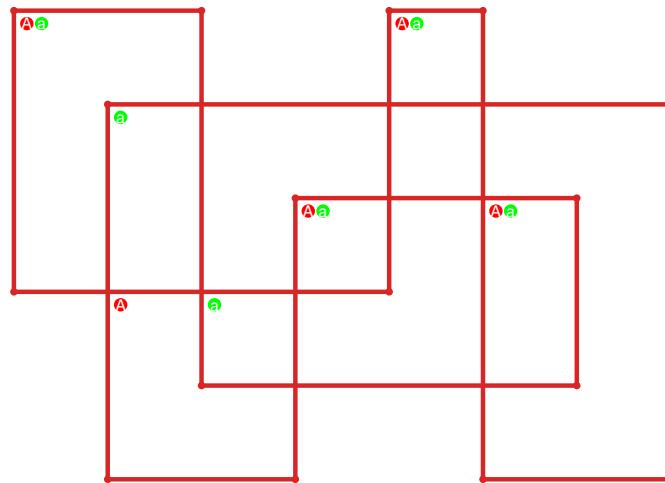


Figure 727: `SnapPy` multiloop plot.



Figure 728: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.216** [[11, 20, 12, 1], [10, 17, 11, 18], [19, 16, 20, 17], [12, 4, 13, 3], [1, 8, 2, 9], [18, 9, 19, 10], [15, 4, 16, 5], [13, 6, 14, 7], [7, 2, 8, 3], [5, 14, 6, 15]]

PD code drawn by `SnapPy`: [(5, 20, 6, 1), (1, 10, 2, 11), (13, 2, 14, 3), (11, 4, 12, 5), (19, 6, 20, 7), (16, 7, 17, 8), (3, 12, 4, 13), (17, 14, 18, 15), (8, 15, 9, 16), (9, 18, 10, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 6, 7, 8], [0, 8, 8, 5], [1, 4, 2, 1], [2, 9, 9, 3], [3, 9, 9, 8], [3, 7, 4, 4], [6, 7, 7, 6]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 6  
 Total pinning sets: 190  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.42  
 Average overall degree: 2.97

Table 363: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	0	0	0	0	0	0	5
Nonminimal pinning sets	0	7	36	55	50	27	8	1	184
Average degree	2.2	2.5	2.78	2.95	3.09	3.19	3.27	3.33	

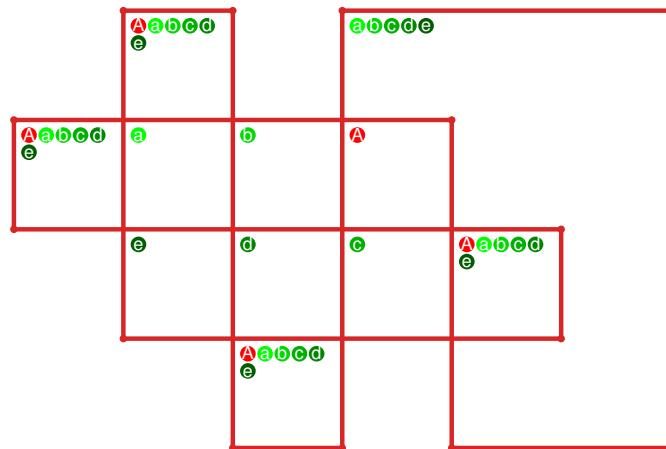


Figure 729: `SnapPy` multiloop plot.

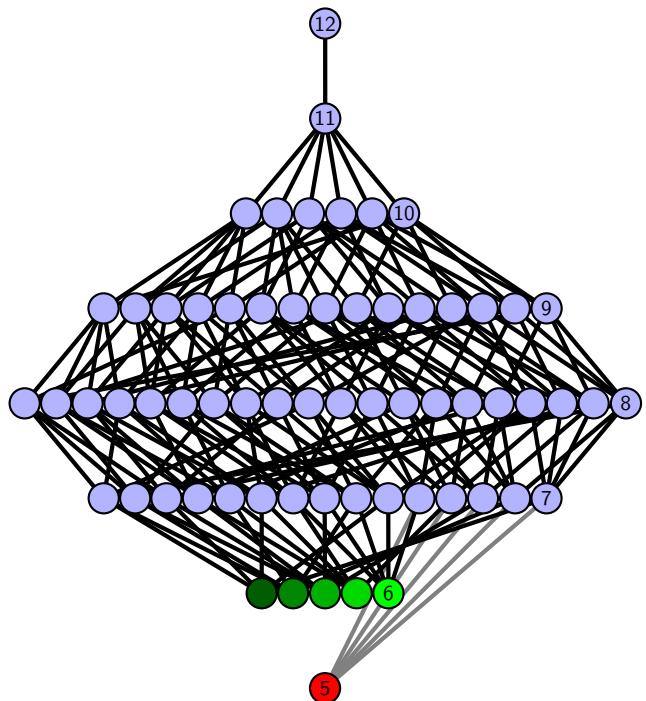


Figure 730: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.217**  $[[9, 20, 10, 1], [17, 8, 18, 9], [19, 14, 20, 15], [10, 5, 11, 6], [1, 6, 2, 7], [7, 16, 8, 17], [18, 16, 19, 15], [4, 13, 5, 14], [11, 3, 12, 2], [12, 3, 13, 4]]$

PD code drawn by `SnapPy`:  $[(20, 9, 1, 10), (10, 1, 11, 2), (8, 3, 9, 4), (19, 4, 20, 5), (15, 6, 16, 7), (2, 11, 3, 12), (17, 12, 18, 13), (13, 16, 14, 17), (5, 14, 6, 15), (7, 18, 8, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 6, 7], [0, 7, 8, 4], [0, 3, 8, 5], [1, 4, 6, 1], [1, 5, 2, 2], [2, 9, 9, 3], [3, 9, 9, 4], [7, 8, 8, 7]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 192  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.2  
 Average overall degree: 2.97

Table 364: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

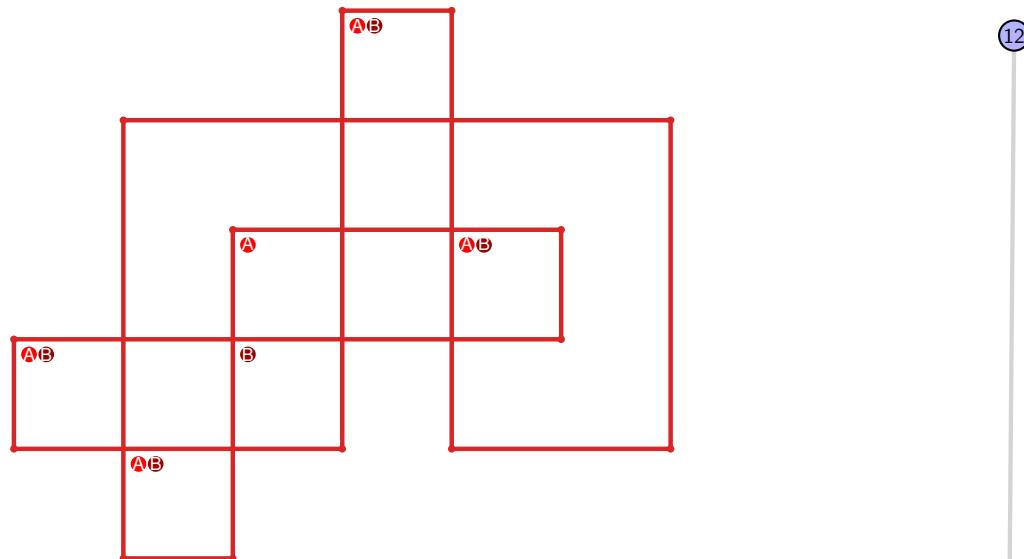


Figure 731: `SnapPy` multiloop plot.

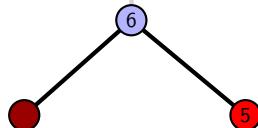


Figure 732: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.218** [[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 3, 9, 4], [1, 13, 2, 12], [18, 12, 19, 11], [6, 15, 7, 16], [4, 7, 5, 8], [2, 13, 3, 14], [14, 5, 15, 6]]

PD code drawn by `SnapPy`: [(6, 1, 7, 2), (12, 3, 13, 4), (2, 5, 3, 6), (18, 7, 19, 8), (9, 16, 10, 17), (10, 19, 11, 20), (20, 11, 1, 12), (4, 13, 5, 14), (17, 14, 18, 15), (15, 8, 16, 9)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 8], [0, 8, 8, 5], [1, 4, 2, 1], [2, 9, 9, 7], [3, 6, 9, 3], [3, 9, 4, 4], [6, 8, 7, 6]]

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 192

**Pinning number:** 5

**Average optimal degree:** 2.2

**Average minimal degree:** 2.2

**Average overall degree:** 2.97

Table 365: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

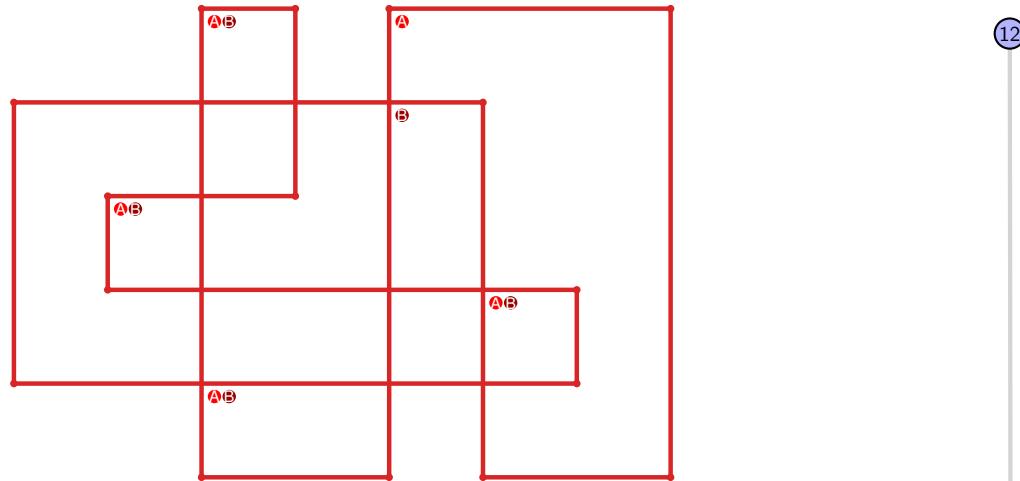


Figure 733: `SnapPy` multiloop plot.

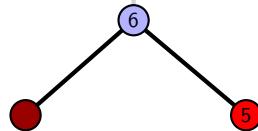


Figure 734: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.219** `[[20, 15, 1, 16], [16, 6, 17, 5], [19, 4, 20, 5], [14, 9, 15, 10], [1, 9, 2, 8], [6, 18, 7, 17], [7, 18, 8, 19], [12, 3, 13, 4], [10, 13, 11, 14], [2, 11, 3, 12]]`

PD code drawn by `SnapPy`: `[(10, 1, 11, 2), (17, 2, 18, 3), (3, 14, 4, 15), (6, 19, 7, 20), (12, 7, 13, 8), (20, 9, 1, 10), (8, 11, 9, 12), (18, 13, 19, 14), (15, 4, 16, 5), (5, 16, 6, 17)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 8, 8, 4], [0, 3, 9, 6], [1, 6, 6, 1], [2, 5, 5, 4], [2, 9, 9, 8], [3, 7, 9, 3], [4, 8, 7, 7]]`

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 224  
 Pinning number: 5

Average optimal degree: 2.27  
 Average minimal degree: 2.27  
 Average overall degree: 2.98

Table 366: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

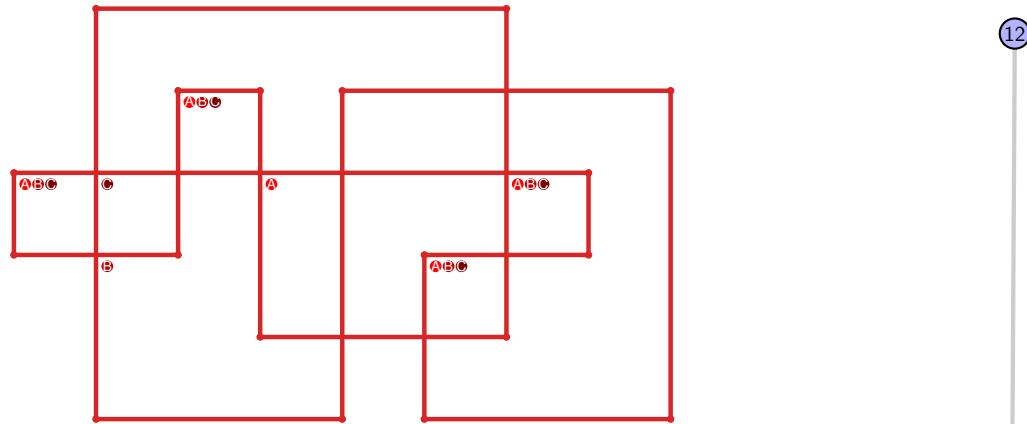


Figure 735: `SnapPy` multiloop plot.

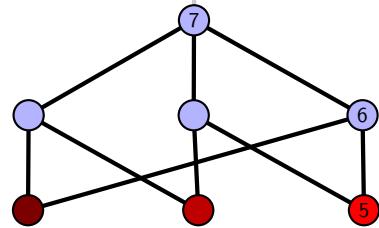


Figure 736: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.220  $[[20, 13, 1, 14], [14, 19, 15, 20], [15, 12, 16, 13], [1, 16, 2, 17], [9, 18, 10, 19], [11, 4, 12, 5], [2, 7, 3, 8], [17, 8, 18, 9], [10, 6, 11, 5], [6, 3, 7, 4]]$

PD code drawn by SnapPy:  $[(4, 1, 5, 2), (9, 2, 10, 3), (3, 8, 4, 9), (20, 5, 1, 6), (15, 6, 16, 7), (13, 10, 14, 11), (18, 11, 19, 12), (19, 14, 20, 15), (7, 16, 8, 17), (12, 17, 13, 18)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 7], [1, 7, 7, 8], [2, 8, 8, 9], [3, 9, 9, 7], [3, 6, 4, 4], [4, 9, 5, 5], [5, 8, 6, 6]]$

Total optimal pinning sets: 2  
Total minimal pinning sets: 2

Total pinning sets: 192

Pinning number: 5

Average optimal degree: 2.2

Average minimal degree: 2.2

Average overall degree: 2.97

Table 367: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

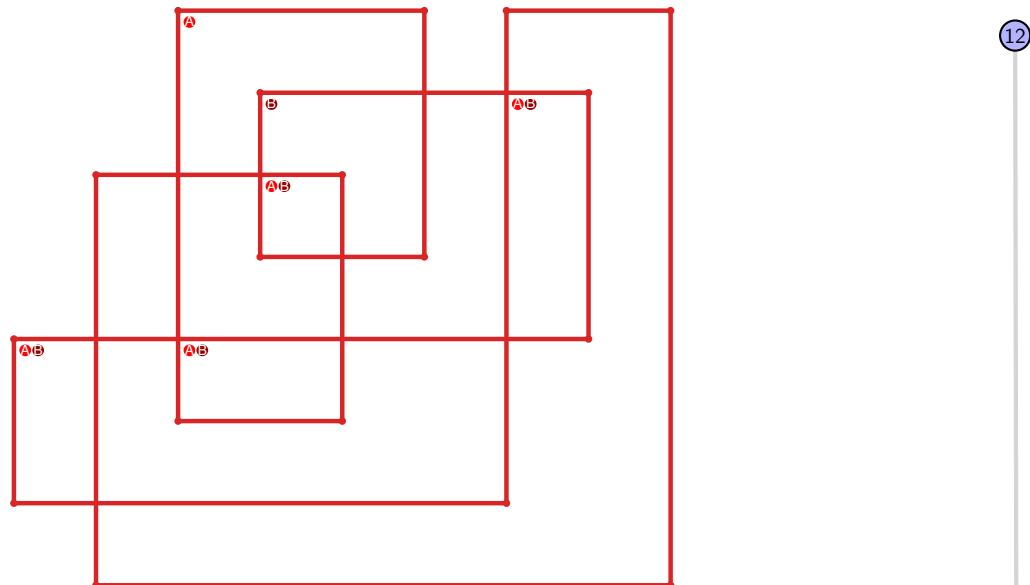


Figure 737: SnapPy multiloop plot.

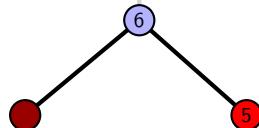


Figure 738: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.221**  $[[5, 20, 6, 1], [13, 4, 14, 5], [14, 19, 15, 20], [6, 18, 7, 17], [1, 11, 2, 10], [3, 12, 4, 13], [18, 15, 19, 16], [7, 16, 8, 17], [11, 8, 12, 9], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (7, 4, 8, 5), (18, 5, 19, 6), (6, 17, 7, 18), (19, 8, 20, 9), (14, 11, 15, 12), (3, 12, 4, 13), (13, 2, 14, 3), (10, 15, 11, 16), (1, 16, 2, 17)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 6], [0, 6, 7, 7], [0, 8, 9, 9], [1, 9, 8, 1], [2, 7, 3, 2], [3, 6, 8, 3], [4, 7, 5, 9], [4, 8, 5, 4]]$

**Total optimal pinning sets:** 6  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 168  
**Pinning number:** 6

**Average optimal degree:** 2.39  
**Average minimal degree:** 2.39  
**Average overall degree:** 2.98

Table 368: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	27	50	49	27	8	1	162
Average degree	2.39	2.71	2.93	3.08	3.19	3.27	3.33	

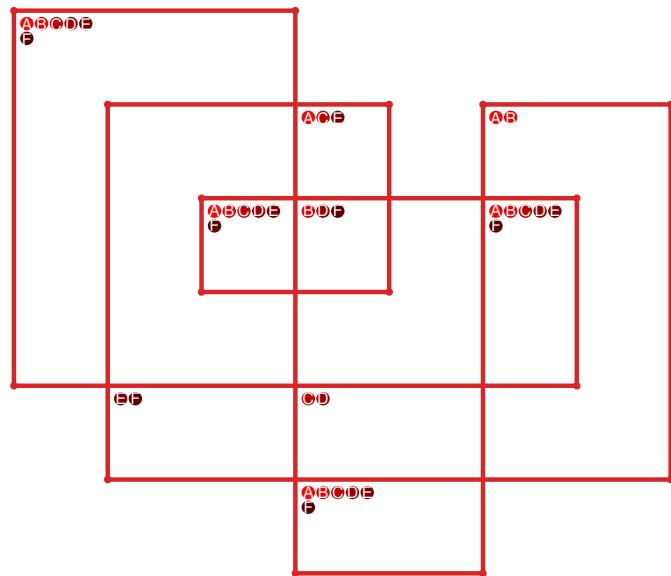


Figure 739: `SnapPy` multiloop plot.

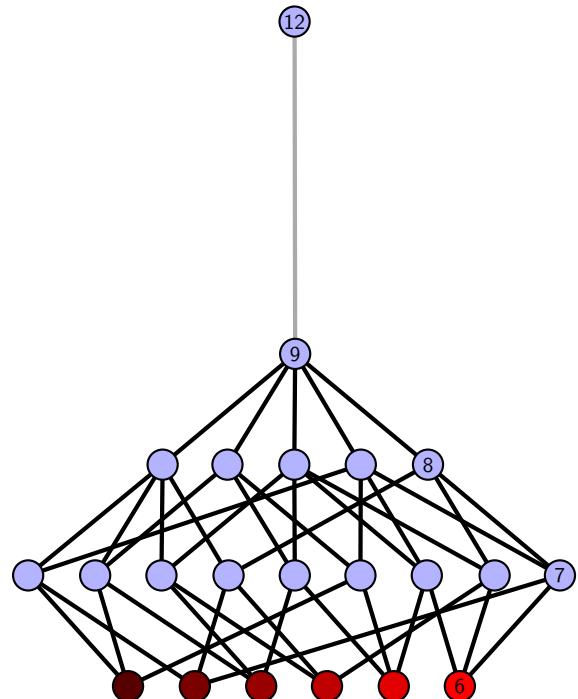


Figure 740: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.222 [[20, 7, 1, 8], [8, 18, 9, 17], [19, 16, 20, 17], [6, 11, 7, 12], [1, 11, 2, 10], [18, 10, 19, 9], [4, 15, 5, 16], [12, 5, 13, 6], [2, 13, 3, 14], [14, 3, 15, 4]]

PD code drawn by `SnapPy`: [(12, 1, 13, 2), (10, 3, 11, 4), (18, 5, 19, 6), (7, 16, 8, 17), (8, 19, 9, 20), (4, 9, 5, 10), (20, 11, 1, 12), (2, 13, 3, 14), (17, 14, 18, 15), (15, 6, 16, 7)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 6], [0, 7, 7, 4], [0, 3, 8, 5], [1, 4, 2, 1], [2, 9, 9, 7], [3, 6, 8, 3], [4, 7, 9, 9], [6, 8, 8, 6]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 369: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

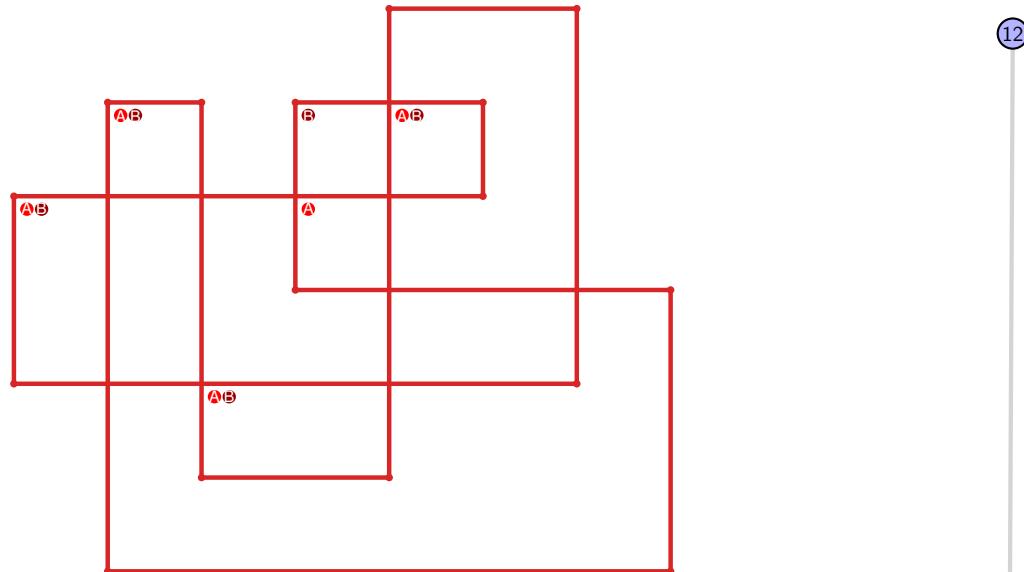


Figure 741: `SnapPy` multiloop plot.

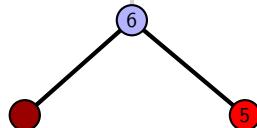


Figure 742: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.223** [[11, 20, 12, 1], [17, 10, 18, 11], [19, 14, 20, 15], [12, 5, 13, 6], [1, 8, 2, 9], [9, 16, 10, 17], [18, 16, 19, 15], [4, 13, 5, 14], [6, 4, 7, 3], [7, 2, 8, 3]]

PD code drawn by `SnapPy`: [(5, 20, 6, 1), (1, 4, 2, 5), (10, 3, 11, 4), (19, 6, 20, 7), (15, 8, 16, 9), (2, 11, 3, 12), (17, 12, 18, 13), (13, 16, 14, 17), (7, 14, 8, 15), (9, 18, 10, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 6, 7], [0, 7, 7, 8], [0, 9, 9, 5], [1, 4, 6, 1], [1, 5, 2, 2], [2, 8, 3, 3], [3, 7, 9, 9], [4, 8, 8, 4]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1

**Total pinning sets:** 128

**Pinning number:** 5

**Average optimal degree:** 2.0

**Average minimal degree:** 2.0

**Average overall degree:** 2.91

Table 370: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

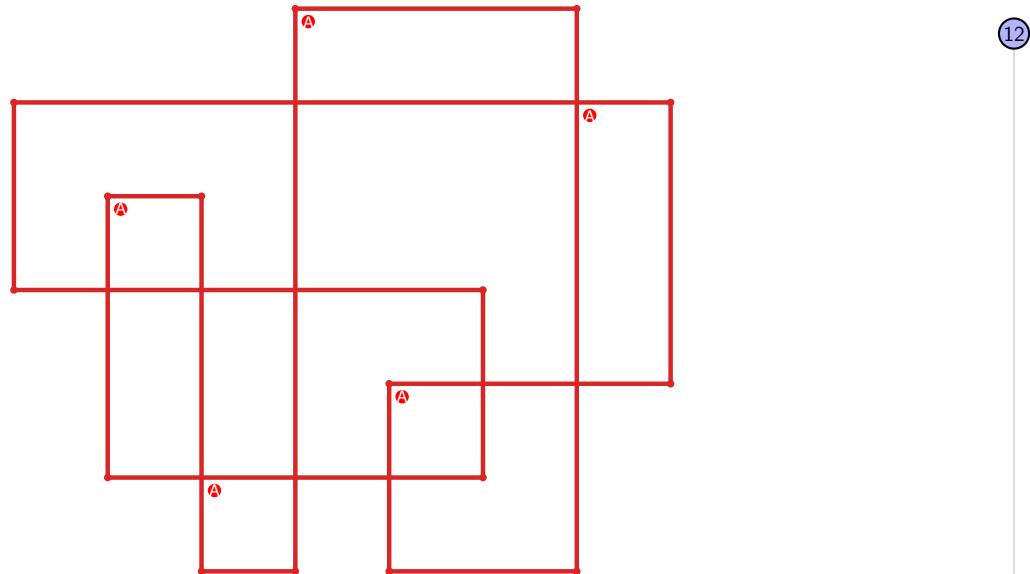


Figure 743: `SnapPy` multiloop plot.



Figure 744: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.224 [[17, 20, 18, 1], [5, 16, 6, 17], [6, 19, 7, 20], [18, 7, 19, 8], [1, 10, 2, 11], [13, 4, 14, 5], [15, 8, 16, 9], [9, 14, 10, 15], [2, 12, 3, 11], [3, 12, 4, 13]]

PD code drawn by SnapPy: [(11, 2, 12, 3), (16, 5, 17, 6), (13, 6, 14, 7), (19, 8, 20, 9), (9, 20, 10, 1), (1, 10, 2, 11), (7, 12, 8, 13), (3, 14, 4, 15), (4, 17, 5, 18), (15, 18, 16, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 3, 3], [0, 2, 2, 6], [0, 7, 8, 8], [1, 9, 9, 7], [1, 7, 7, 3], [4, 6, 6, 5], [4, 9, 9, 4], [5, 8, 8, 5]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 371: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

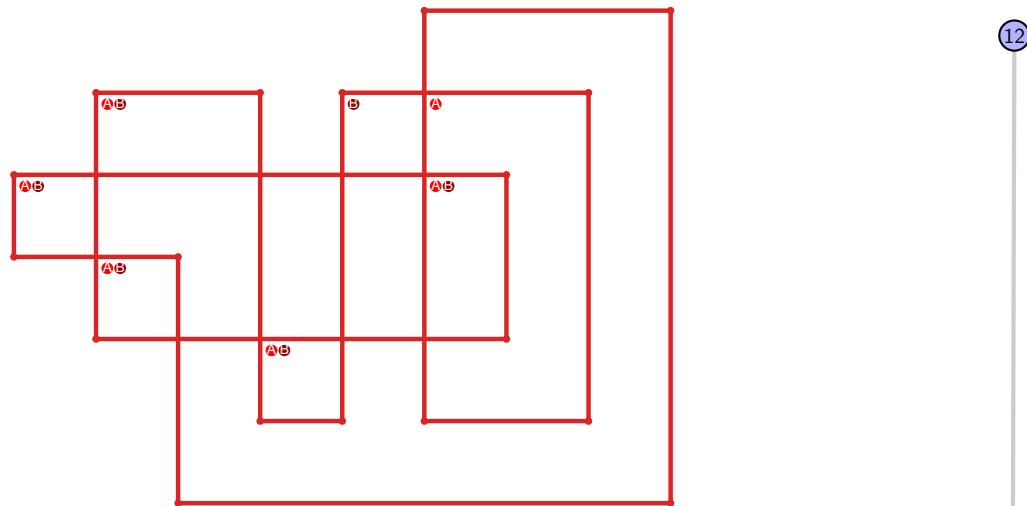


Figure 745: SnapPy multiloop plot.

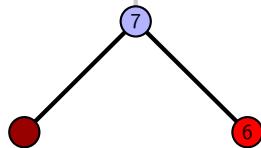


Figure 746: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.225  $[[9, 20, 10, 1], [5, 8, 6, 9], [19, 12, 20, 13], [10, 2, 11, 1], [15, 4, 16, 5], [16, 7, 17, 8], [6, 17, 7, 18], [13, 18, 14, 19], [11, 2, 12, 3], [3, 14, 4, 15]]$

PD code drawn by SnapPy:  $[(12, 1, 13, 2), (13, 4, 14, 5), (2, 5, 3, 6), (16, 9, 17, 10), (20, 11, 1, 12), (3, 14, 4, 15), (10, 15, 11, 16), (6, 17, 7, 18), (18, 7, 19, 8), (8, 19, 9, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 5, 6], [0, 7, 7, 8], [0, 8, 8, 0], [1, 9, 9, 5], [1, 4, 6, 6], [1, 5, 5, 7], [2, 6, 9, 2], [2, 9, 3, 3], [4, 8, 7, 4]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 96  
 Pinning number: 6

Average optimal degree: 2.17  
 Average minimal degree: 2.17  
 Average overall degree: 2.91

Table 372: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

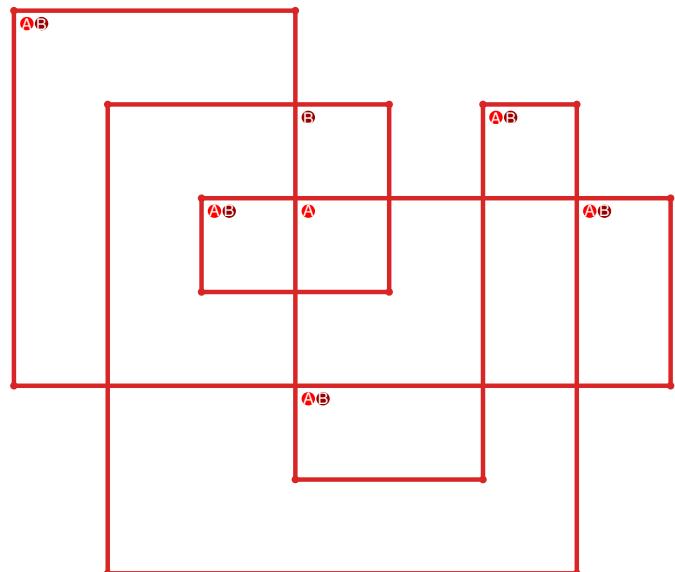


Figure 747: SnapPy multiloop plot.



Figure 748: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.226** [[20, 13, 1, 14], [14, 7, 15, 8], [10, 19, 11, 20], [3, 12, 4, 13], [1, 4, 2, 5], [6, 15, 7, 16], [8, 17, 9, 18], [18, 9, 19, 10], [11, 2, 12, 3], [5, 17, 6, 16]]

PD code drawn by `SnapPy`: [(9, 20, 10, 1), (19, 2, 20, 3), (12, 5, 13, 6), (16, 7, 17, 8), (1, 10, 2, 11), (18, 11, 19, 12), (4, 13, 5, 14), (14, 3, 15, 4), (6, 15, 7, 16), (8, 17, 9, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 6], [0, 7, 7, 8], [0, 8, 8, 4], [0, 3, 8, 9], [1, 9, 9, 1], [1, 9, 7, 7], [2, 6, 6, 2], [2, 4, 3, 3], [4, 6, 5, 5]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 96  
 Pinning number: 6

Average optimal degree: 2.17  
 Average minimal degree: 2.17  
 Average overall degree: 2.91

Table 373: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

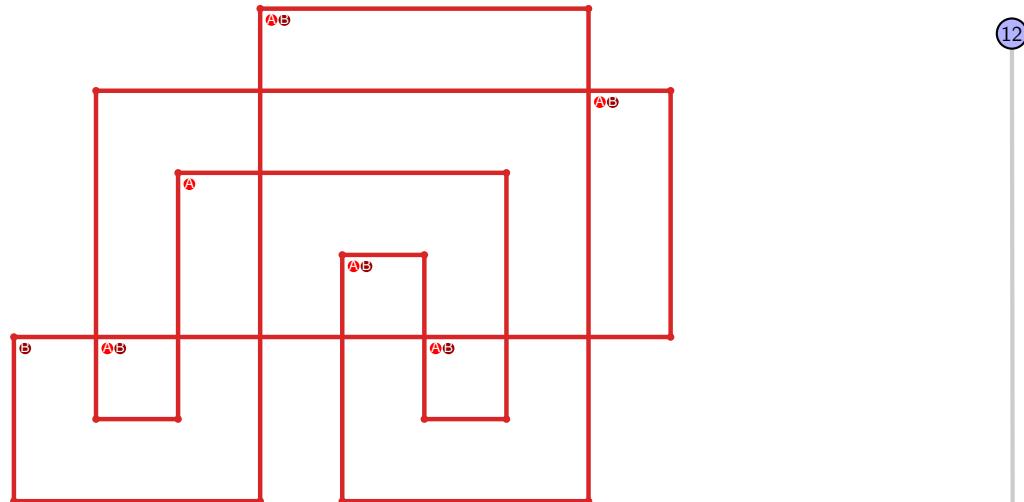


Figure 749: `SnapPy` multiloop plot.

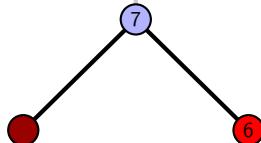


Figure 750: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.227** [[20, 17, 1, 18], [18, 11, 19, 12], [12, 19, 13, 20], [13, 16, 14, 17], [1, 6, 2, 7], [7, 10, 8, 11], [4, 15, 5, 16], [14, 5, 15, 6], [2, 9, 3, 10], [8, 3, 9, 4]]

PD code drawn by `SnapPy`: [(6, 1, 7, 2), (12, 3, 13, 4), (19, 4, 20, 5), (20, 7, 1, 8), (5, 8, 6, 9), (16, 9, 17, 10), (2, 13, 3, 14), (11, 14, 12, 15), (18, 15, 19, 16), (10, 17, 11, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 2, 2], [0, 1, 1, 3], [0, 2, 6, 7], [0, 7, 8, 5], [1, 4, 8, 9], [3, 9, 7, 7], [3, 6, 6, 4], [4, 9, 9, 5], [5, 8, 8, 6]]

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 272  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.47  
 Average overall degree: 3.04

Table 374: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	19	55	81	69	34	9	1	268
Average degree	2.4	2.68	2.89	3.04	3.15	3.24	3.29	3.33	

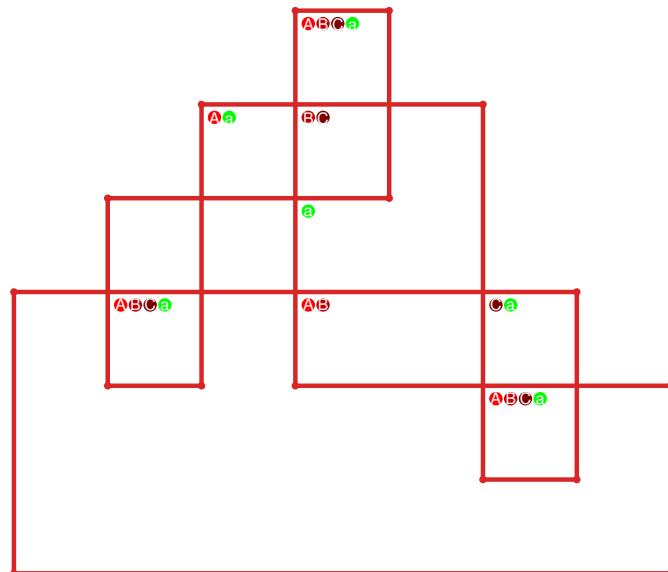


Figure 751: `SnapPy` multiloop plot.

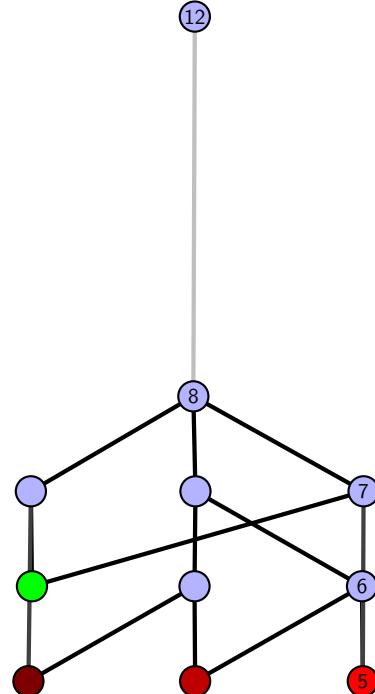


Figure 752: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.228 [[13, 20, 14, 1], [19, 12, 20, 13], [14, 18, 15, 17], [1, 6, 2, 7], [11, 18, 12, 19], [15, 4, 16, 5], [5, 16, 6, 17], [2, 9, 3, 10], [7, 10, 8, 11], [8, 3, 9, 4]]

PD code drawn by SnapPy: [(6, 1, 7, 2), (13, 2, 14, 3), (19, 4, 20, 5), (20, 7, 1, 8), (5, 8, 6, 9), (17, 10, 18, 11), (3, 14, 4, 15), (15, 12, 16, 13), (9, 16, 10, 17), (11, 18, 12, 19)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 6], [0, 6, 7, 8], [1, 8, 2, 1], [2, 9, 6, 6], [2, 5, 5, 3], [3, 9, 9, 8], [3, 7, 9, 4], [5, 8, 7, 7]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 375: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

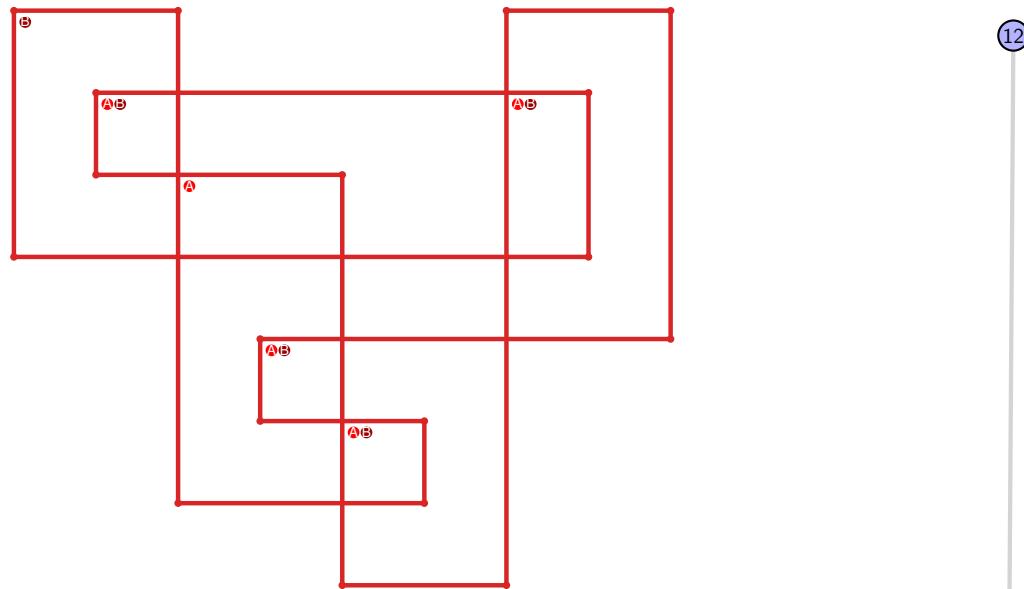


Figure 753: SnapPy multiloop plot.

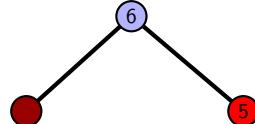


Figure 754: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.229**  $[[17, 20, 18, 1], [3, 16, 4, 17], [4, 19, 5, 20], [18, 5, 19, 6], [1, 10, 2, 11], [11, 2, 12, 3], [15, 8, 16, 9], [6, 14, 7, 13], [9, 12, 10, 13], [7, 14, 8, 15]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (16, 3, 17, 4), (13, 6, 14, 7), (19, 8, 20, 9), (7, 10, 8, 11), (11, 4, 12, 5), (5, 12, 6, 13), (1, 14, 2, 15), (2, 17, 3, 18), (15, 18, 16, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 3, 3], [0, 2, 2, 7], [0, 8, 5, 5], [1, 4, 4, 8], [1, 8, 9, 9], [3, 9, 9, 8], [4, 7, 6, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 376: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

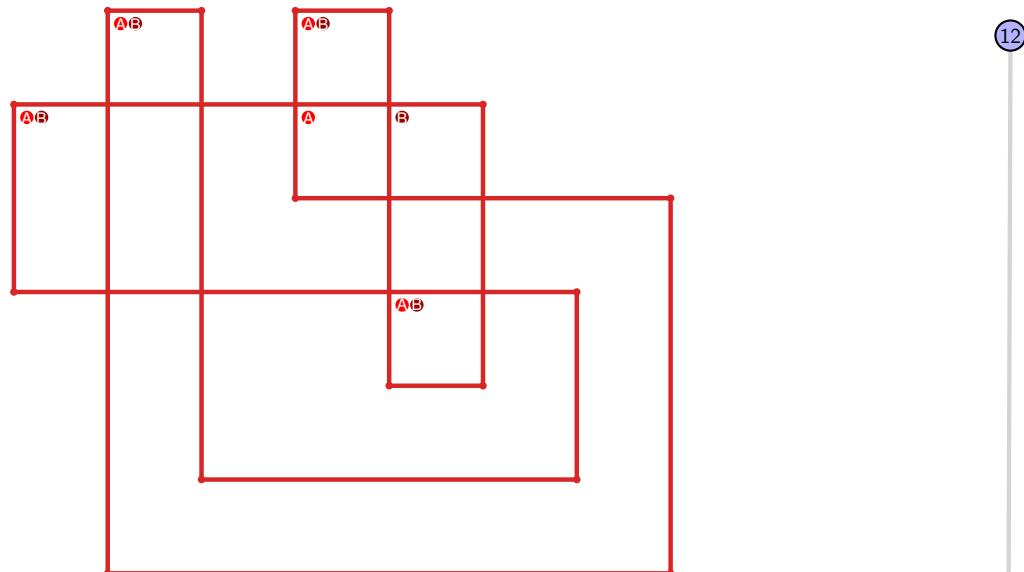


Figure 755: `SnapPy` multiloop plot.

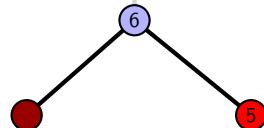


Figure 756: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.230**  $[[7, 20, 8, 1], [6, 13, 7, 14], [16, 19, 17, 20], [8, 3, 9, 4], [1, 4, 2, 5], [14, 5, 15, 6], [15, 12, 16, 13], [18, 11, 19, 12], [17, 11, 18, 10], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (19, 2, 20, 3), (12, 3, 13, 4), (20, 7, 1, 8), (8, 15, 9, 16), (16, 9, 17, 10), (13, 10, 14, 11), (4, 11, 5, 12), (14, 17, 15, 18), (5, 18, 6, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 7, 8], [0, 9, 9, 4], [0, 3, 9, 5], [1, 4, 6, 1], [1, 5, 7, 2], [2, 6, 8, 8], [2, 7, 7, 9], [3, 8, 4, 3]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 288  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.4  
 Average overall degree: 3.03

Table 377: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

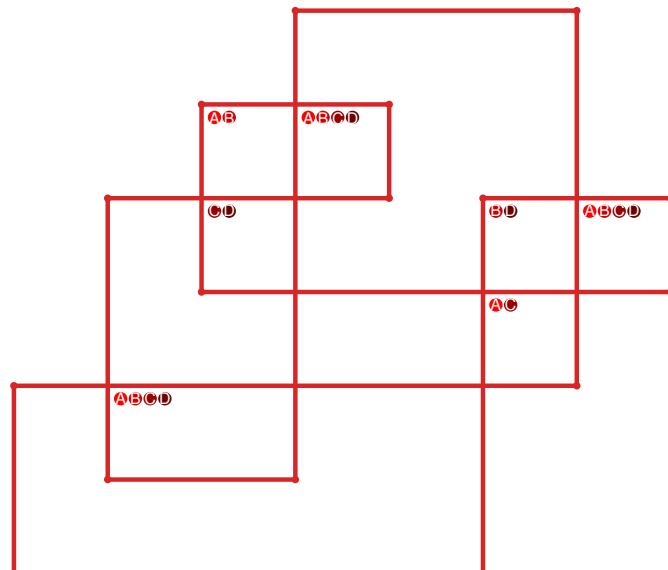


Figure 757: `SnapPy` multiloop plot.

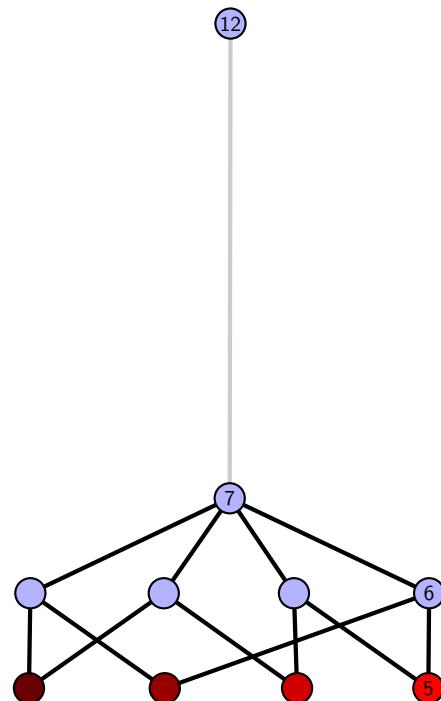


Figure 758: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.231  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 6, 9, 5], [1, 10, 2, 11], [13, 18, 14, 19], [9, 4, 10, 5], [2, 15, 3, 16], [11, 16, 12, 17], [17, 12, 18, 13], [14, 3, 15, 4]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (6, 1, 7, 2), (2, 5, 3, 6), (16, 3, 17, 4), (18, 7, 19, 8), (14, 9, 15, 10), (19, 12, 20, 13), (8, 13, 9, 14), (10, 15, 11, 16), (4, 17, 5, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 5, 6, 7], [1, 8, 8, 9], [2, 9, 3, 2], [3, 9, 9, 7], [3, 6, 8, 8], [4, 7, 7, 4], [4, 6, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 378: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

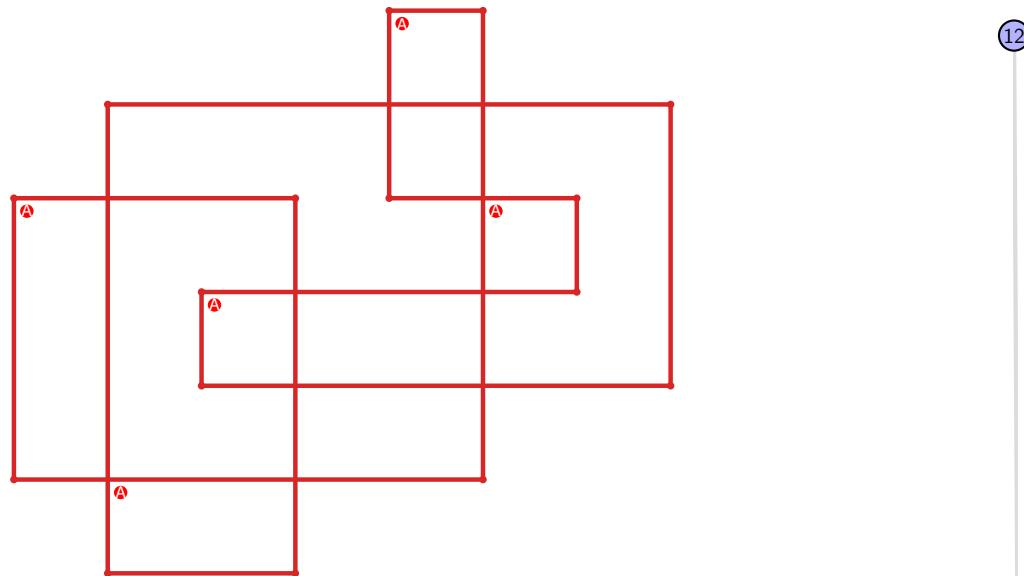


Figure 759: `SnapPy` multiloop plot.



Figure 760: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.232 [[17, 20, 18, 1], [5, 16, 6, 17], [6, 19, 7, 20], [18, 7, 19, 8], [1, 10, 2, 11], [15, 4, 16, 5], [8, 14, 9, 13], [9, 12, 10, 13], [2, 12, 3, 11], [3, 14, 4, 15]]

PD code drawn by `SnapPy`: [(13, 2, 14, 3), (16, 5, 17, 6), (6, 17, 7, 18), (4, 7, 5, 8), (8, 3, 9, 4), (18, 9, 19, 10), (20, 11, 1, 12), (1, 14, 2, 15), (12, 15, 13, 16), (10, 19, 11, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 3, 3], [0, 2, 2, 6], [0, 7, 8, 8], [1, 9, 9, 1], [3, 9, 7, 7], [4, 6, 6, 8], [4, 7, 9, 4], [5, 8, 6, 5]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 379: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

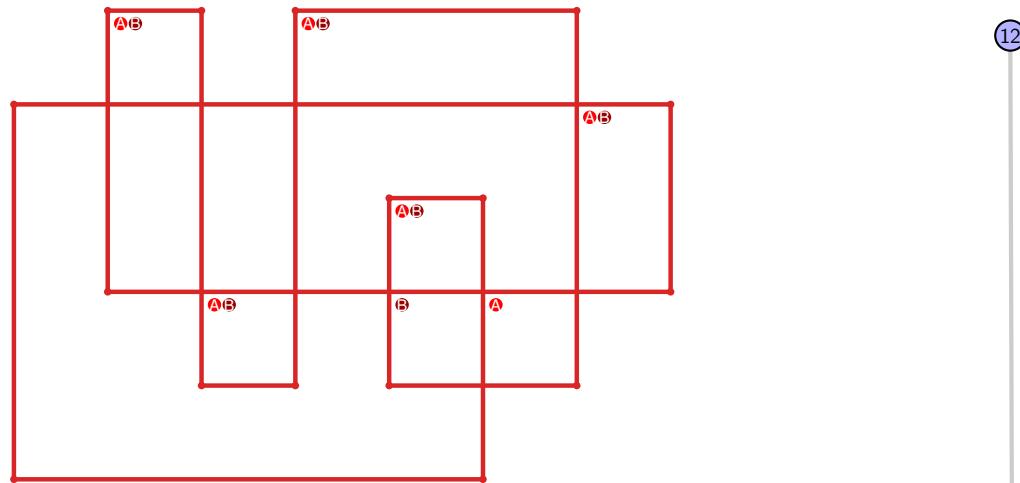


Figure 761: `SnapPy` multiloop plot.

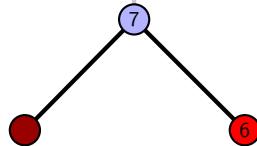


Figure 762: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.233  $[[7, 20, 8, 1], [15, 6, 16, 7], [19, 10, 20, 11], [8, 3, 9, 4], [1, 4, 2, 5], [5, 14, 6, 15], [16, 14, 17, 13], [11, 18, 12, 19], [2, 9, 3, 10], [17, 12, 18, 13]]$

PD code drawn by SnapPy:  $[(6, 1, 7, 2), (19, 2, 20, 3), (13, 4, 14, 5), (20, 7, 1, 8), (17, 8, 18, 9), (15, 10, 16, 11), (11, 14, 12, 15), (3, 12, 4, 13), (9, 16, 10, 17), (5, 18, 6, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 7, 7, 8], [0, 8, 8, 4], [0, 3, 8, 5], [1, 4, 6, 1], [1, 5, 9, 9], [2, 9, 9, 2], [2, 4, 3, 3], [6, 7, 7, 6]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 96  
 Pinning number: 6

Average optimal degree: 2.17  
 Average minimal degree: 2.17  
 Average overall degree: 2.91

Table 380: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

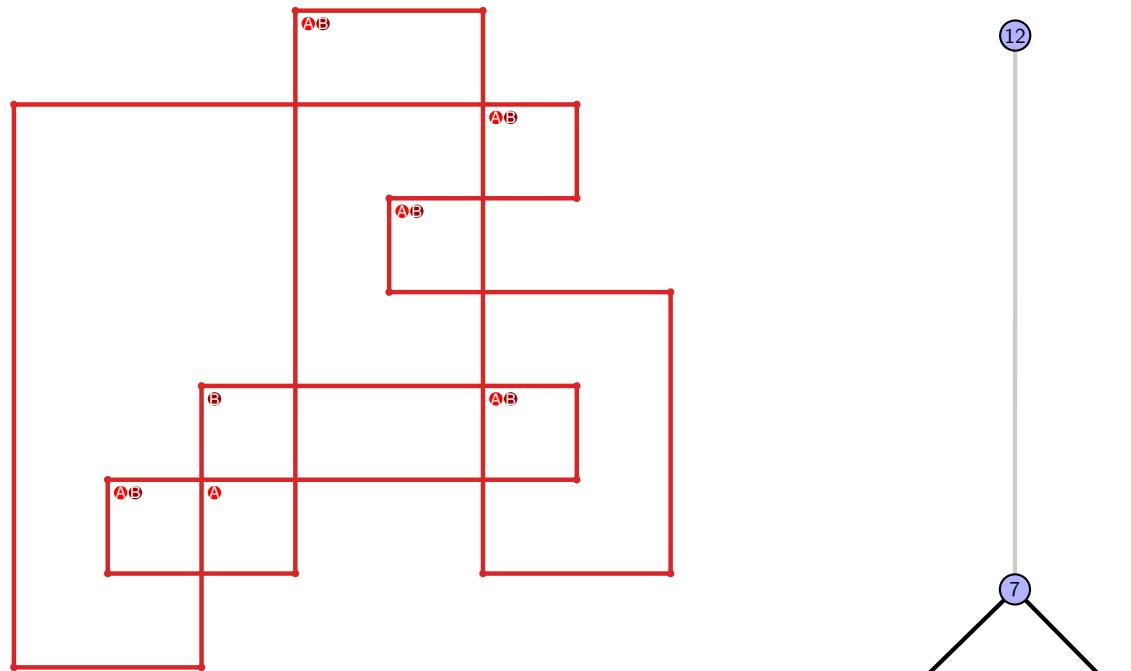


Figure 763: SnapPy multiloop plot.

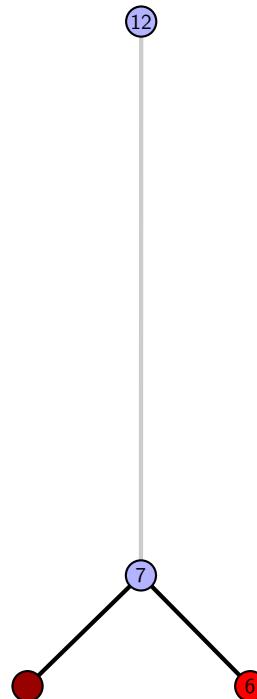


Figure 764: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.234  $[[7, 20, 8, 1], [13, 6, 14, 7], [19, 8, 20, 9], [1, 4, 2, 5], [5, 12, 6, 13], [14, 12, 15, 11], [9, 18, 10, 19], [3, 16, 4, 17], [2, 16, 3, 15], [17, 10, 18, 11]]$

PD code drawn by SnapPy:  $[(18, 1, 19, 2), (10, 3, 11, 4), (16, 5, 17, 6), (14, 7, 15, 8), (8, 11, 9, 12), (2, 9, 3, 10), (12, 19, 13, 20), (20, 13, 1, 14), (6, 15, 7, 16), (4, 17, 5, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 6, 0], [0, 7, 8, 4], [1, 3, 5, 1], [1, 4, 8, 9], [2, 9, 9, 2], [3, 9, 8, 8], [3, 7, 7, 5], [5, 7, 6, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 112  
 Pinning number: 6

Average optimal degree: 2.22  
 Average minimal degree: 2.22  
 Average overall degree: 2.92

Table 381: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	31	34	21	7	1	109
Average degree	2.22	2.57	2.82	3.01	3.14	3.25	3.33	

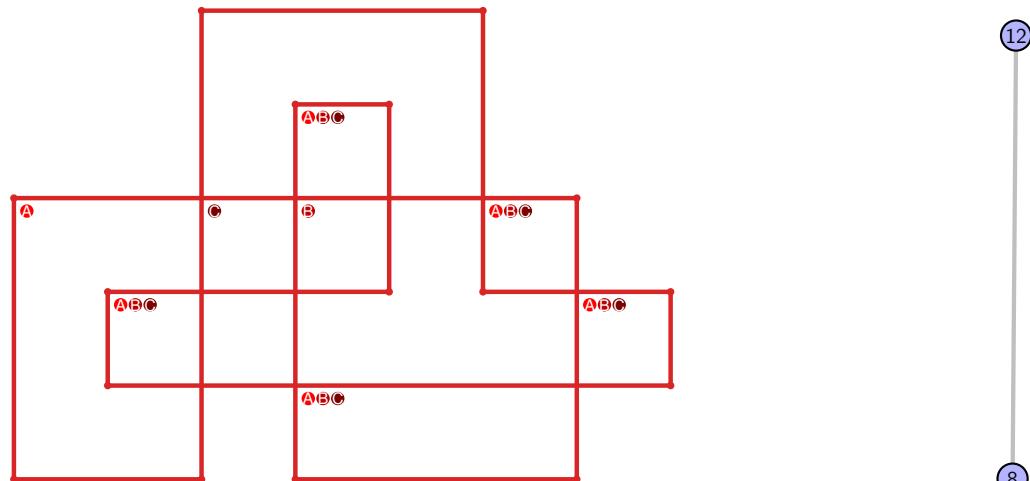


Figure 765: SnapPy multiloop plot.

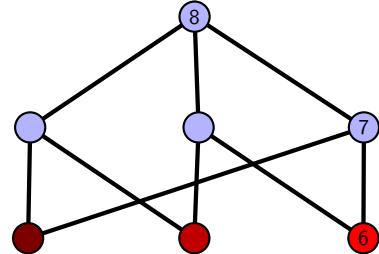


Figure 766: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.235**  $[[9, 20, 10, 1], [8, 15, 9, 16], [19, 10, 20, 11], [1, 6, 2, 7], [16, 7, 17, 8], [17, 14, 18, 15], [11, 18, 12, 19], [5, 2, 6, 3], [13, 4, 14, 5], [12, 4, 13, 3]]$

PD code drawn by `SnapPy`:  $[(20, 9, 1, 10), (10, 1, 11, 2), (18, 3, 19, 4), (13, 4, 14, 5), (16, 7, 17, 8), (14, 11, 15, 12), (5, 12, 6, 13), (8, 15, 9, 16), (6, 17, 7, 18), (2, 19, 3, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 6, 0], [0, 7, 7, 4], [1, 3, 5, 1], [1, 4, 8, 6], [2, 5, 9, 2], [3, 9, 8, 3], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 382: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

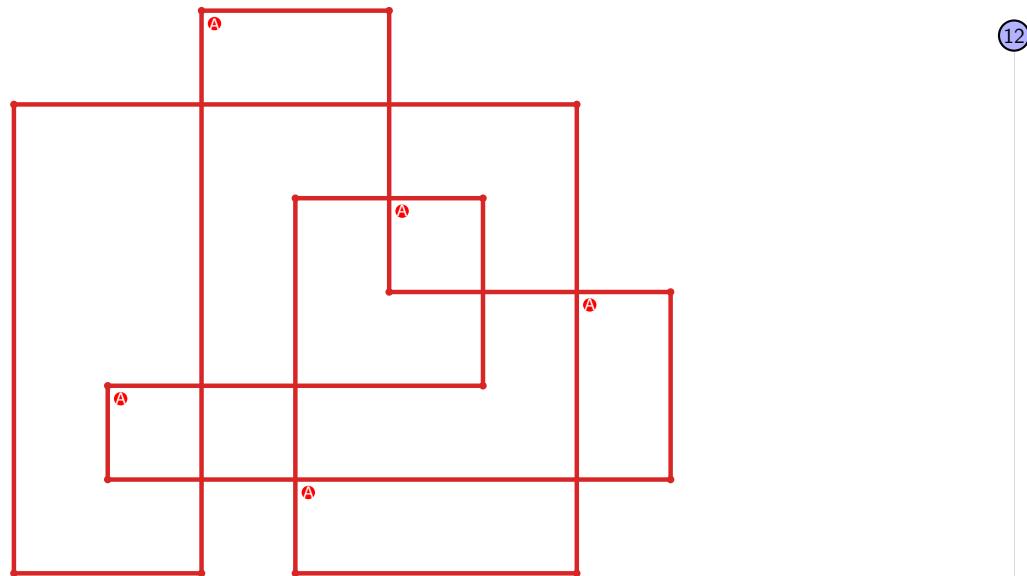


Figure 767: `SnapPy` multiloop plot.



Figure 768: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.236** `[[17, 20, 18, 1], [9, 16, 10, 17], [19, 6, 20, 7], [18, 6, 19, 5], [1, 14, 2, 15], [15, 8, 16, 9], [10, 8, 11, 7], [13, 4, 14, 5], [2, 12, 3, 11], [3, 12, 4, 13]]`

PD code drawn by `SnapPy`: `[(10, 1, 11, 2), (2, 15, 3, 16), (16, 3, 17, 4), (4, 7, 5, 8), (12, 5, 13, 6), (8, 19, 9, 20), (20, 9, 1, 10), (18, 11, 19, 12), (6, 13, 7, 14), (14, 17, 15, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 5, 6], [0, 6, 3, 3], [0, 2, 2, 7], [0, 7, 8, 5], [1, 4, 6, 1], [1, 5, 8, 2], [3, 9, 9, 4], [4, 9, 9, 6], [7, 8, 8, 7]]`

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 224  
 Pinning number: 5

Average optimal degree: 2.27  
 Average minimal degree: 2.27  
 Average overall degree: 2.98

Table 383: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

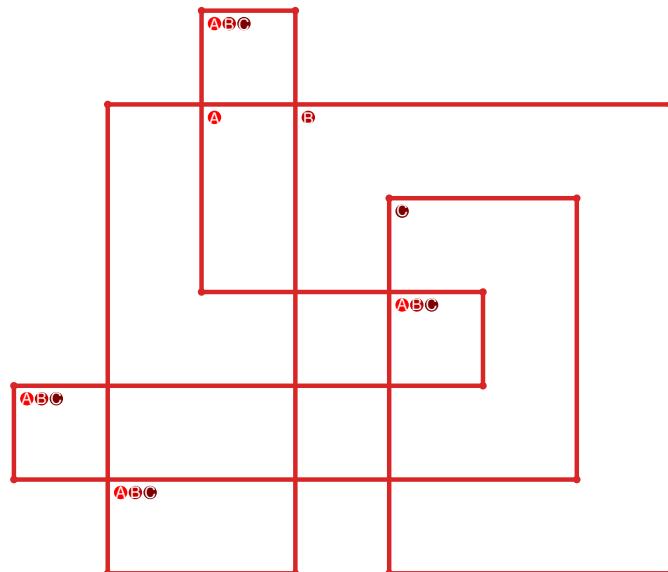


Figure 769: `SnapPy` multiloop plot.

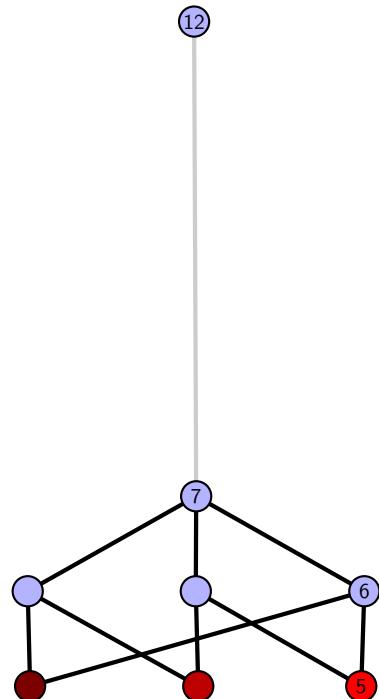


Figure 770: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.237**  $[[7, 20, 8, 1], [6, 15, 7, 16], [10, 19, 11, 20], [8, 11, 9, 12], [1, 4, 2, 5], [16, 5, 17, 6], [17, 14, 18, 15], [18, 9, 19, 10], [12, 3, 13, 4], [2, 13, 3, 14]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (19, 2, 20, 3), (14, 3, 15, 4), (16, 7, 17, 8), (6, 9, 7, 10), (20, 11, 1, 12), (15, 12, 16, 13), (4, 13, 5, 14), (8, 17, 9, 18), (5, 18, 6, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 5, 6], [0, 7, 7, 3], [0, 2, 7, 8], [0, 8, 9, 5], [1, 4, 6, 1], [1, 5, 9, 7], [2, 6, 3, 2], [3, 9, 9, 4], [4, 8, 8, 6]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 6  
 Total pinning sets: 312  
 Pinning number: 5

Average optimal degree: 2.45  
 Average minimal degree: 2.55  
 Average overall degree: 3.05

Table 384: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	24	68	94	75	35	9	1	306
Average degree	2.45	2.72	2.92	3.07	3.17	3.24	3.29	3.33	

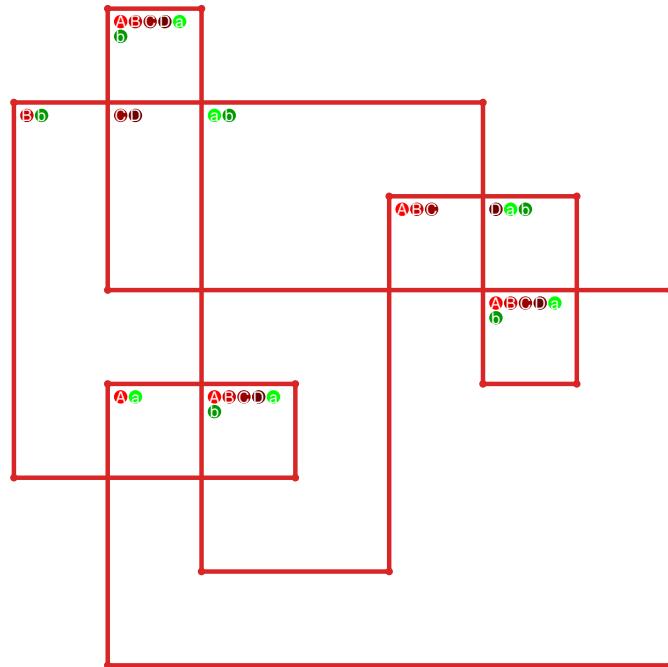


Figure 771: SnapPy multiloop plot.

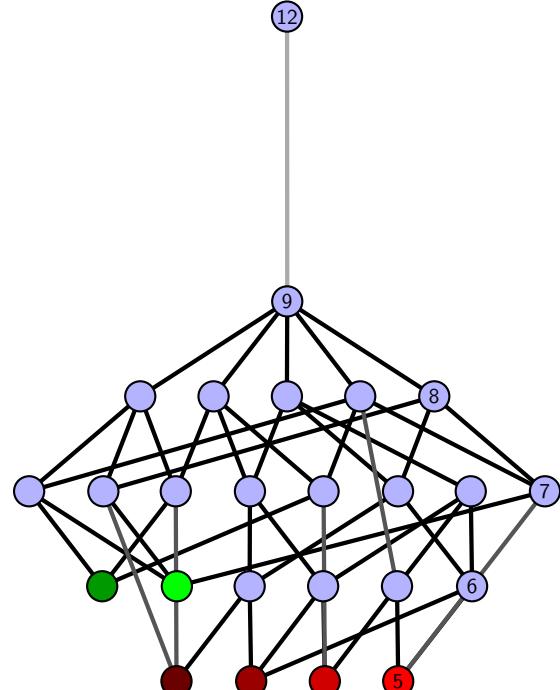


Figure 772: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.238 [[17, 20, 18, 1], [13, 16, 14, 17], [19, 6, 20, 7], [18, 6, 19, 5], [1, 10, 2, 11], [3, 12, 4, 13], [15, 8, 16, 9], [14, 8, 15, 7], [9, 4, 10, 5], [2, 12, 3, 11]]

PD code drawn by `SnapPy`: [(6, 1, 7, 2), (2, 15, 3, 16), (16, 3, 17, 4), (4, 11, 5, 12), (12, 5, 13, 6), (18, 7, 19, 8), (8, 19, 9, 20), (20, 9, 1, 10), (10, 13, 11, 14), (14, 17, 15, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 3, 3], [0, 2, 2, 8], [0, 8, 9, 9], [1, 9, 9, 8], [1, 8, 7, 7], [1, 6, 6, 2], [3, 6, 5, 4], [4, 5, 5, 4]]

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3

**Total pinning sets:** 224

**Pinning number:** 5

**Average optimal degree:** 2.27

**Average minimal degree:** 2.27

**Average overall degree:** 2.98

Table 385: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

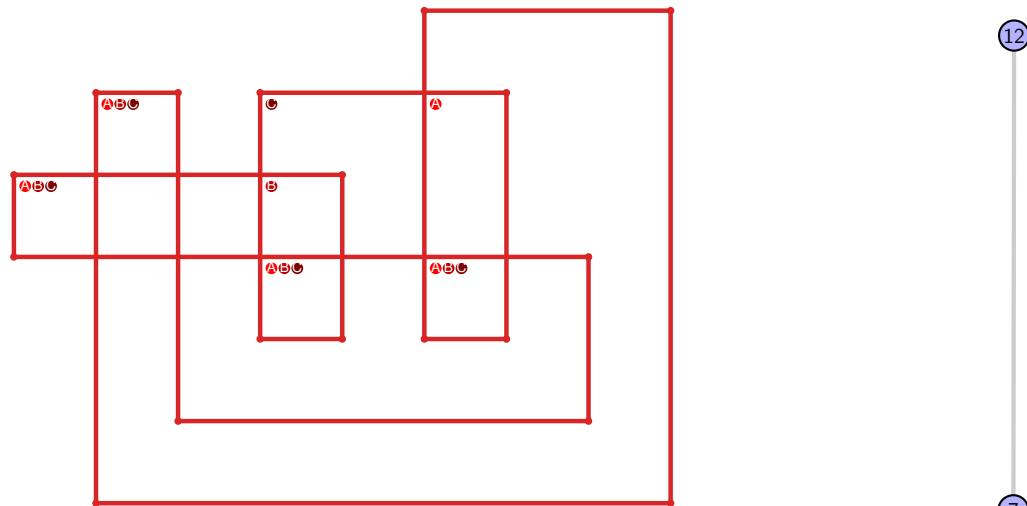


Figure 773: `SnapPy` multiloop plot.

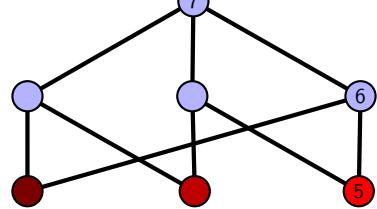


Figure 774: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.239**  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 4, 7, 3], [1, 10, 2, 11], [18, 13, 19, 14], [7, 17, 8, 16], [9, 2, 10, 3], [11, 15, 12, 14], [12, 17, 13, 18], [8, 15, 9, 16]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (1, 4, 2, 5), (17, 2, 18, 3), (19, 8, 20, 9), (16, 11, 17, 12), (12, 9, 13, 10), (6, 13, 7, 14), (14, 5, 15, 6), (10, 15, 11, 16), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 6], [0, 6, 6, 7], [1, 7, 8, 8], [2, 8, 9, 9], [2, 9, 3, 3], [3, 9, 8, 4], [4, 7, 5, 4], [5, 7, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 386: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

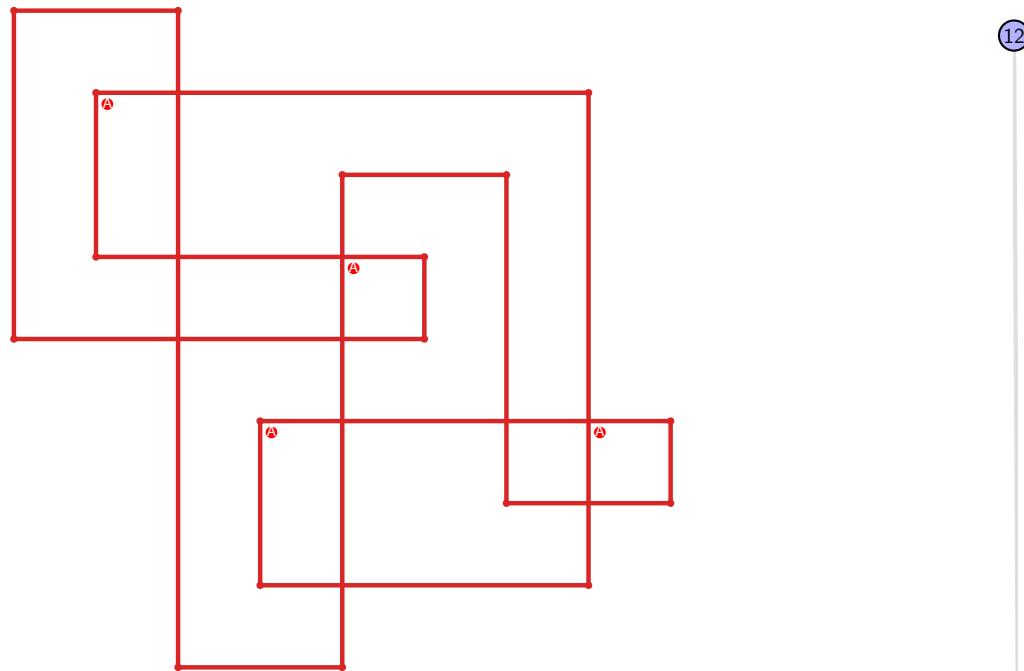


Figure 775: `SnapPy` multiloop plot.

Figure 776: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.240**  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 4, 7, 3], [1, 10, 2, 11], [13, 18, 14, 19], [7, 14, 8, 15], [9, 2, 10, 3], [11, 16, 12, 17], [17, 12, 18, 13], [8, 16, 9, 15]]$

PD code drawn by `SnapPy`:  $[(7, 20, 8, 1), (1, 4, 2, 5), (17, 2, 18, 3), (19, 8, 20, 9), (15, 10, 16, 11), (5, 12, 6, 13), (13, 6, 14, 7), (9, 14, 10, 15), (11, 16, 12, 17), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 6], [0, 6, 6, 7], [1, 8, 8, 5], [2, 4, 9, 9], [2, 9, 3, 3], [3, 9, 8, 8], [4, 7, 7, 4], [5, 7, 6, 5]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 387: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

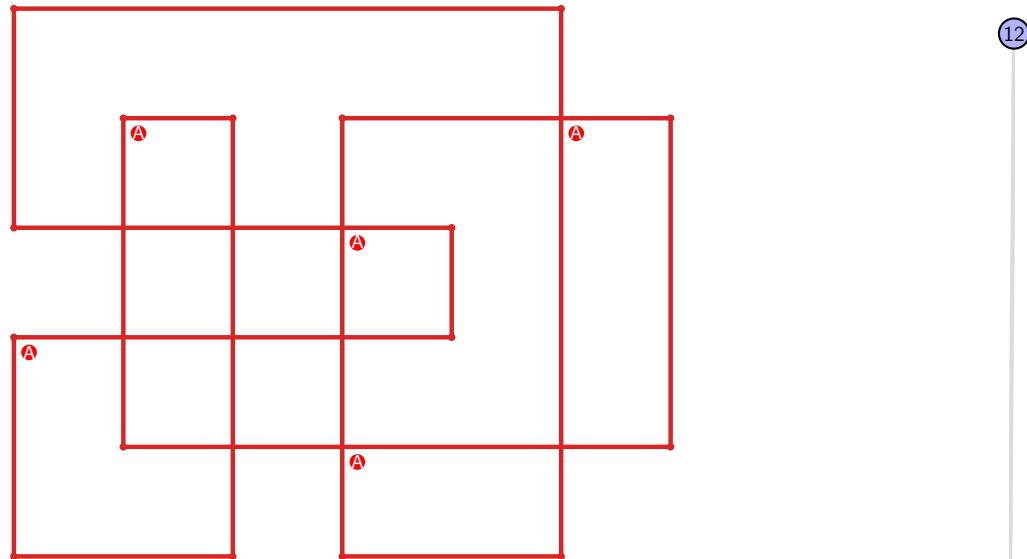


Figure 777: `SnapPy` multiloop plot.



Figure 778: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.241** `[[20, 13, 1, 14], [14, 3, 15, 4], [12, 19, 13, 20], [1, 6, 2, 7], [7, 2, 8, 3], [15, 5, 16, 4], [16, 11, 17, 12], [18, 9, 19, 10], [5, 8, 6, 9], [10, 17, 11, 18]]`

PD code drawn by `SnapPy`: `[(5, 20, 6, 1), (12, 1, 13, 2), (2, 13, 3, 14), (19, 4, 20, 5), (3, 6, 4, 7), (16, 7, 17, 8), (14, 9, 15, 10), (18, 11, 19, 12), (8, 15, 9, 16), (10, 17, 11, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 4, 5, 5], [0, 6, 7, 0], [0, 8, 4, 4], [1, 3, 3, 8], [1, 8, 6, 1], [2, 5, 9, 9], [2, 9, 9, 8], [3, 7, 5, 4], [6, 7, 7, 6]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 388: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

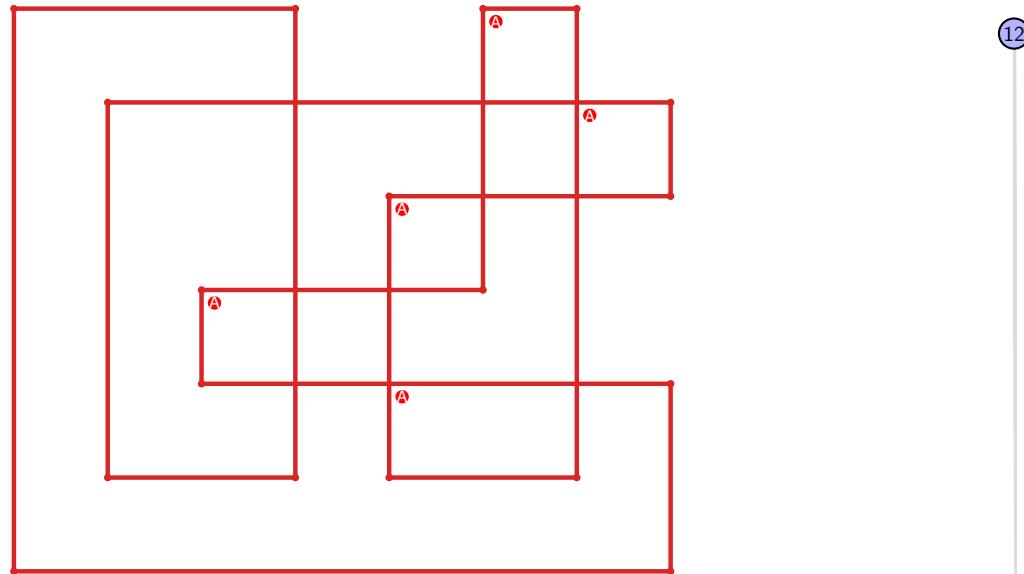


Figure 779: `SnapPy` multiloop plot.

5

Figure 780: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.242**  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 4, 7, 3], [1, 16, 2, 17], [11, 18, 12, 19], [7, 12, 8, 13], [15, 2, 16, 3], [17, 10, 18, 11], [8, 14, 9, 13], [9, 14, 10, 15]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (1, 4, 2, 5), (17, 2, 18, 3), (15, 8, 16, 9), (5, 10, 6, 11), (11, 6, 12, 7), (7, 12, 8, 13), (19, 14, 20, 15), (9, 16, 10, 17), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 6], [0, 6, 6, 7], [1, 7, 7, 5], [2, 4, 8, 8], [2, 9, 3, 3], [3, 9, 4, 4], [5, 9, 9, 5], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 389: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

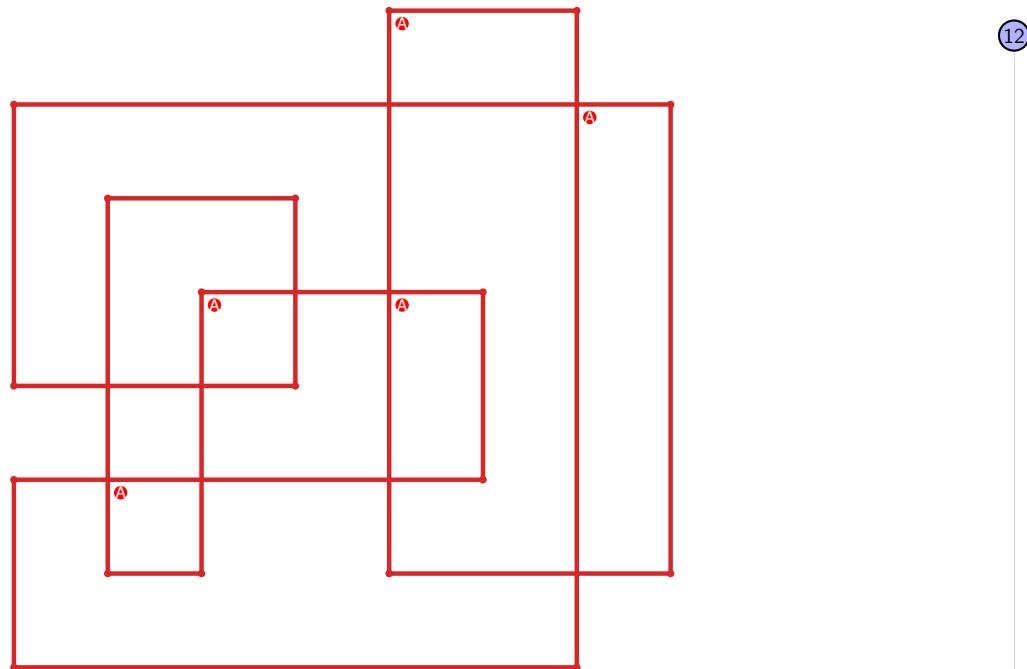


Figure 781: `SnapPy` multiloop plot.

Figure 782: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.243**  $[[20, 9, 1, 10], [10, 3, 11, 4], [6, 19, 7, 20], [8, 17, 9, 18], [1, 14, 2, 15], [15, 2, 16, 3], [11, 5, 12, 4], [12, 5, 13, 6], [18, 7, 19, 8], [13, 16, 14, 17]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (8, 1, 9, 2), (2, 9, 3, 10), (10, 3, 11, 4), (16, 5, 17, 6), (18, 7, 19, 8), (19, 12, 20, 13), (11, 14, 12, 15), (6, 15, 7, 16), (4, 17, 5, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 6], [0, 7, 8, 8], [0, 8, 8, 9], [0, 9, 5, 5], [1, 4, 4, 9], [1, 7, 7, 1], [2, 6, 6, 9], [2, 3, 3, 2], [3, 7, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 390: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

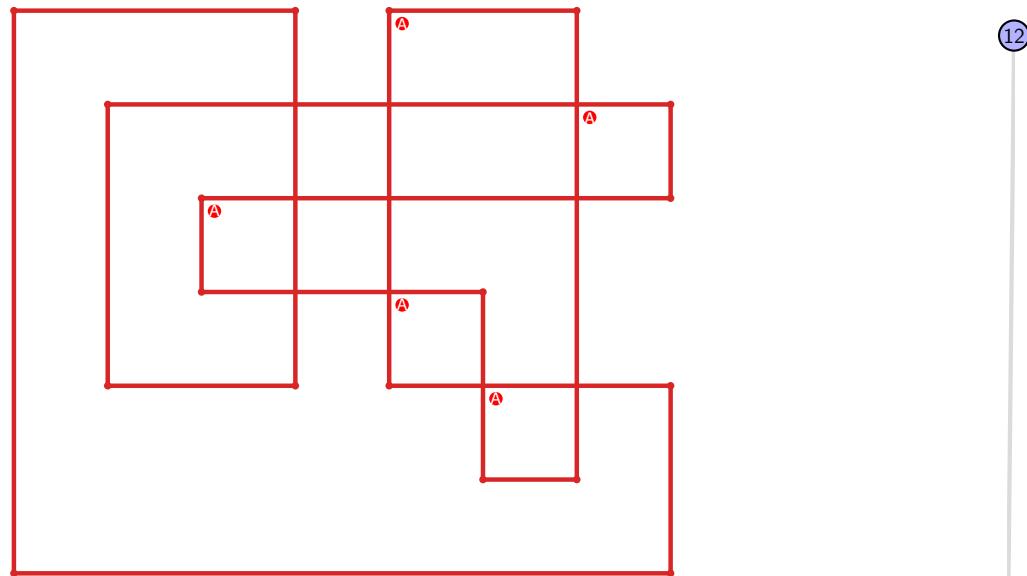


Figure 783: `SnapPy` multiloop plot.

5

Figure 784: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.244 [[15, 20, 16, 1], [3, 14, 4, 15], [19, 12, 20, 13], [16, 10, 17, 9], [1, 6, 2, 7], [7, 2, 8, 3], [13, 4, 14, 5], [11, 18, 12, 19], [10, 18, 11, 17], [5, 8, 6, 9]]

PD code drawn by SnapPy: [(5, 20, 6, 1), (11, 2, 12, 3), (19, 4, 20, 5), (3, 6, 4, 7), (7, 14, 8, 15), (17, 10, 18, 11), (1, 12, 2, 13), (15, 8, 16, 9), (9, 16, 10, 17), (13, 18, 14, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 6], [0, 6, 7, 7], [0, 8, 8, 9], [0, 9, 5, 5], [1, 4, 4, 9], [1, 9, 2, 1], [2, 8, 8, 2], [3, 7, 7, 3], [3, 6, 5, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 391: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

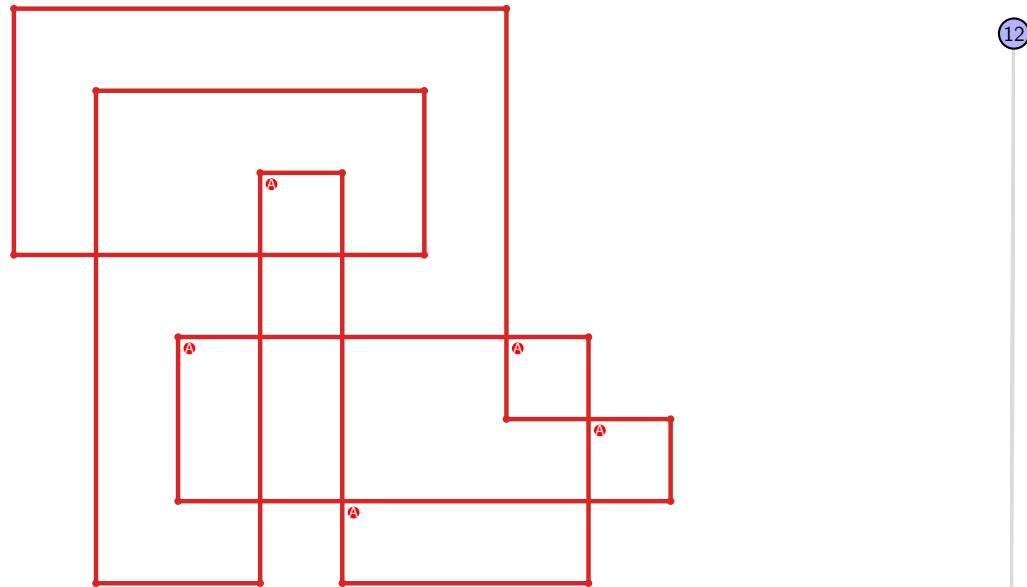


Figure 785: SnapPy multiloop plot.



Figure 786: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.245** `[[13, 20, 14, 1], [3, 12, 4, 13], [19, 14, 20, 15], [1, 6, 2, 7], [7, 2, 8, 3], [11, 4, 12, 5], [15, 18, 16, 19], [5, 8, 6, 9], [17, 10, 18, 11], [16, 10, 17, 9]]`

PD code drawn by `SnapPy`: `[(5, 20, 6, 1), (9, 2, 10, 3), (19, 4, 20, 5), (3, 6, 4, 7), (7, 14, 8, 15), (15, 8, 16, 9), (1, 10, 2, 11), (17, 12, 18, 13), (13, 16, 14, 17), (11, 18, 12, 19)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 4, 5, 5], [0, 6, 6, 0], [0, 7, 4, 4], [1, 3, 3, 7], [1, 7, 8, 1], [2, 8, 9, 2], [3, 9, 5, 4], [5, 9, 9, 6], [6, 8, 8, 7]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 392: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

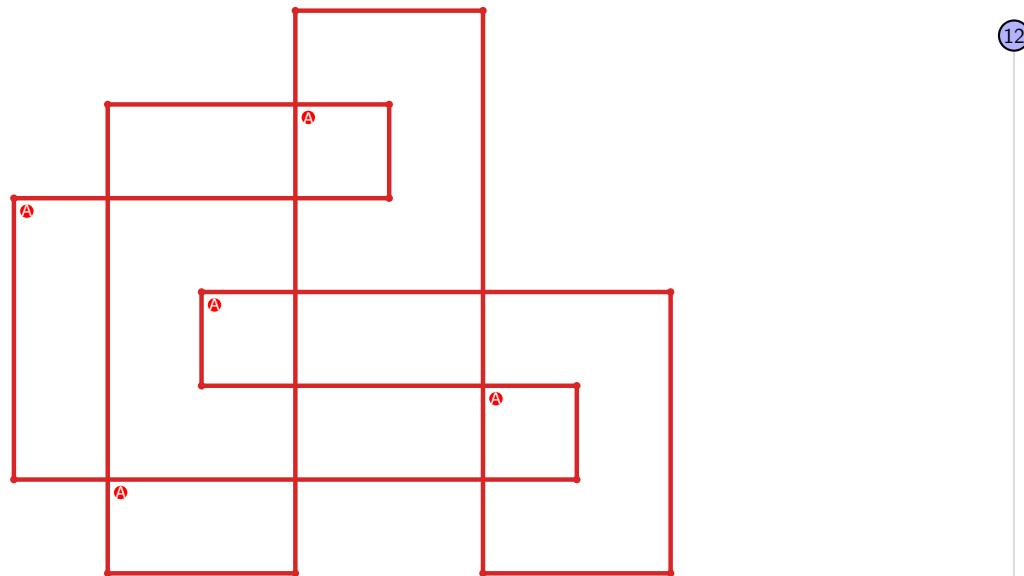


Figure 787: `SnapPy` multiloop plot.

5

Figure 788: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.246** [[15, 20, 16, 1], [3, 14, 4, 15], [19, 16, 20, 17], [1, 9, 2, 8], [2, 7, 3, 8], [13, 4, 14, 5], [17, 13, 18, 12], [18, 11, 19, 12], [9, 6, 10, 7], [5, 10, 6, 11]]

PD code drawn by `SnapPy`: [(11, 2, 12, 3), (7, 4, 8, 5), (20, 5, 1, 6), (6, 19, 7, 20), (3, 8, 4, 9), (16, 9, 17, 10), (10, 15, 11, 16), (1, 12, 2, 13), (17, 14, 18, 15), (13, 18, 14, 19)]

Planar representation generated by `plantri`: [[1, 2, 2, 3], [0, 4, 5, 5], [0, 6, 7, 0], [0, 8, 4, 4], [1, 3, 3, 8], [1, 9, 6, 1], [2, 5, 7, 7], [2, 6, 6, 9], [3, 9, 9, 4], [5, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 393: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

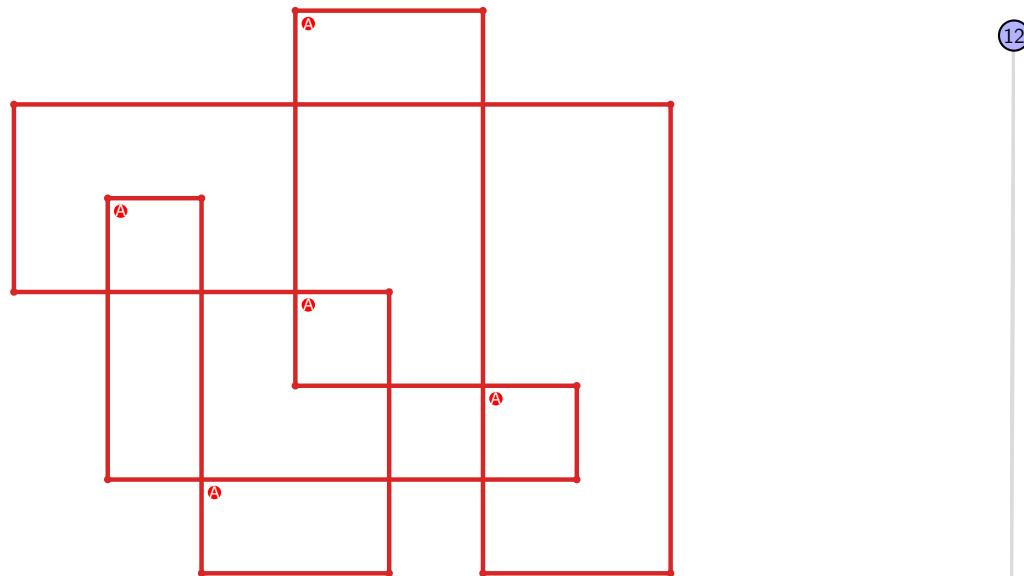


Figure 789: `SnapPy` multiloop plot.



Figure 790: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.247** `[[15, 20, 16, 1], [14, 5, 15, 6], [19, 16, 20, 17], [1, 8, 2, 9], [6, 9, 7, 10], [4, 13, 5, 14], [17, 13, 18, 12], [18, 11, 19, 12], [7, 2, 8, 3], [10, 3, 11, 4]]`

PD code drawn by `SnapPy`: `[(7, 20, 8, 1), (11, 2, 12, 3), (18, 5, 19, 6), (3, 6, 4, 7), (15, 8, 16, 9), (9, 14, 10, 15), (1, 10, 2, 11), (16, 13, 17, 14), (12, 17, 13, 18), (4, 19, 5, 20)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 4, 5, 5], [0, 6, 7, 0], [0, 8, 8, 4], [1, 3, 8, 9], [1, 9, 6, 1], [2, 5, 7, 7], [2, 6, 6, 9], [3, 9, 4, 3], [4, 8, 7, 5]]`

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 394: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

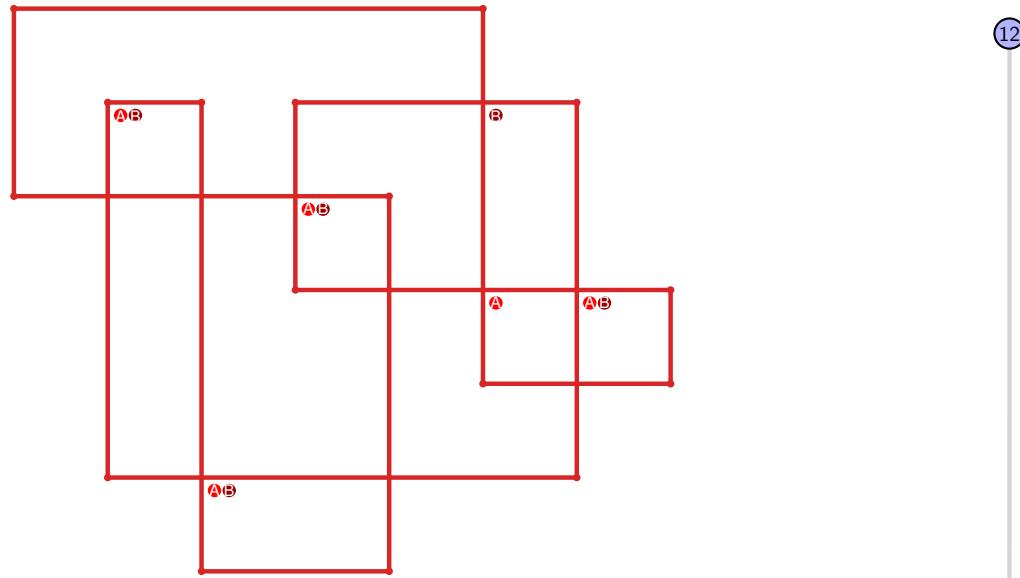


Figure 791: `SnapPy` multiloop plot.

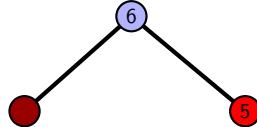


Figure 792: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.248 [[17, 20, 18, 1], [5, 16, 6, 17], [19, 14, 20, 15], [18, 14, 19, 13], [1, 10, 2, 11], [11, 4, 12, 5], [15, 6, 16, 7], [7, 12, 8, 13], [9, 2, 10, 3], [3, 8, 4, 9]]

PD code drawn by `SnapPy`: [(8, 1, 9, 2), (12, 3, 13, 4), (20, 5, 1, 6), (18, 7, 19, 8), (4, 9, 5, 10), (10, 15, 11, 16), (16, 11, 17, 12), (2, 13, 3, 14), (14, 17, 15, 18), (6, 19, 7, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 6], [0, 6, 3, 3], [0, 2, 2, 7], [0, 8, 8, 5], [1, 4, 9, 7], [1, 7, 2, 1], [3, 6, 5, 9], [4, 9, 9, 4], [5, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 395: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

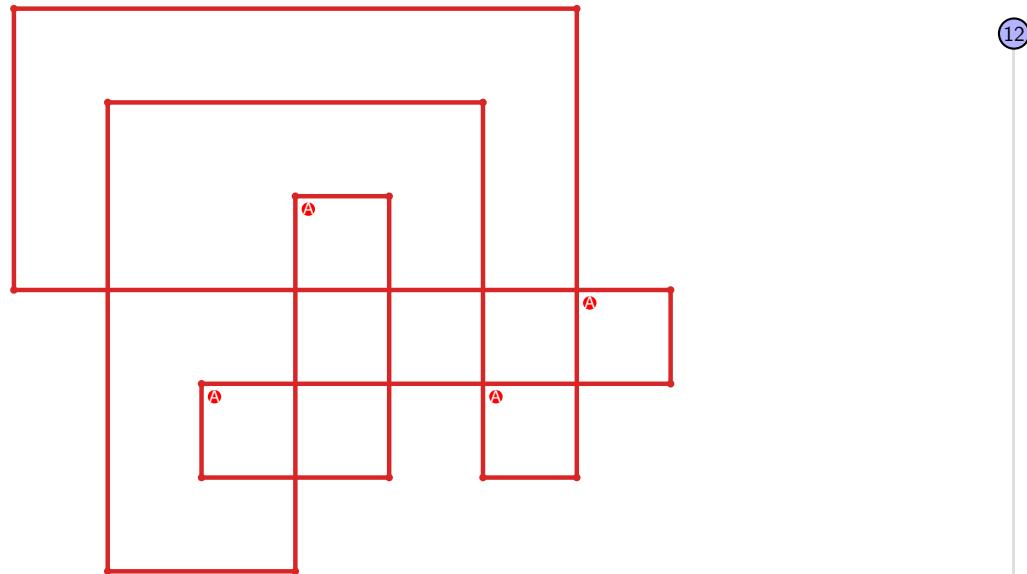


Figure 793: `SnapPy` multiloop plot.

4

Figure 794: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.249**  $[[20, 7, 1, 8], [8, 12, 9, 11], [6, 19, 7, 20], [1, 19, 2, 18], [12, 3, 13, 4], [9, 14, 10, 15], [15, 10, 16, 11], [16, 5, 17, 6], [2, 17, 3, 18], [13, 5, 14, 4]]$

PD code drawn by `SnapPy`:  $[(20, 9, 1, 10), (12, 1, 13, 2), (16, 3, 17, 4), (11, 6, 12, 7), (7, 4, 8, 5), (5, 10, 6, 11), (18, 13, 19, 14), (14, 17, 15, 18), (2, 15, 3, 16), (8, 19, 9, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 7, 3, 0], [0, 2, 8, 8], [1, 8, 9, 9], [1, 9, 6, 6], [1, 5, 5, 7], [2, 6, 9, 8], [3, 7, 4, 3], [4, 7, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 396: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

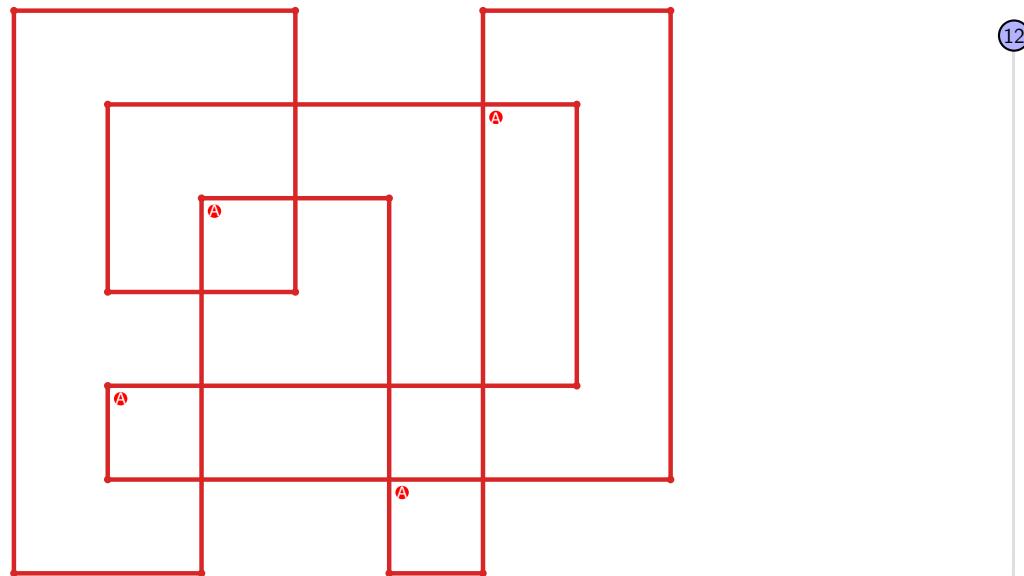


Figure 795: `SnapPy` multiloop plot.

Figure 796: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.250** `[[17, 20, 18, 1], [9, 16, 10, 17], [19, 14, 20, 15], [18, 14, 19, 13], [1, 7, 2, 6], [8, 5, 9, 6], [15, 10, 16, 11], [12, 3, 13, 4], [7, 3, 8, 2], [4, 11, 5, 12]]`

PD code drawn by `SnapPy`: `[(9, 20, 10, 1), (5, 2, 6, 3), (3, 18, 4, 19), (12, 7, 13, 8), (1, 8, 2, 9), (10, 15, 11, 16), (16, 11, 17, 12), (6, 13, 7, 14), (14, 17, 15, 18), (19, 4, 20, 5)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 6, 6], [0, 6, 3, 3], [0, 2, 2, 7], [0, 8, 8, 5], [1, 4, 8, 9], [1, 9, 2, 1], [3, 9, 9, 8], [4, 7, 5, 4], [5, 7, 7, 6]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 397: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

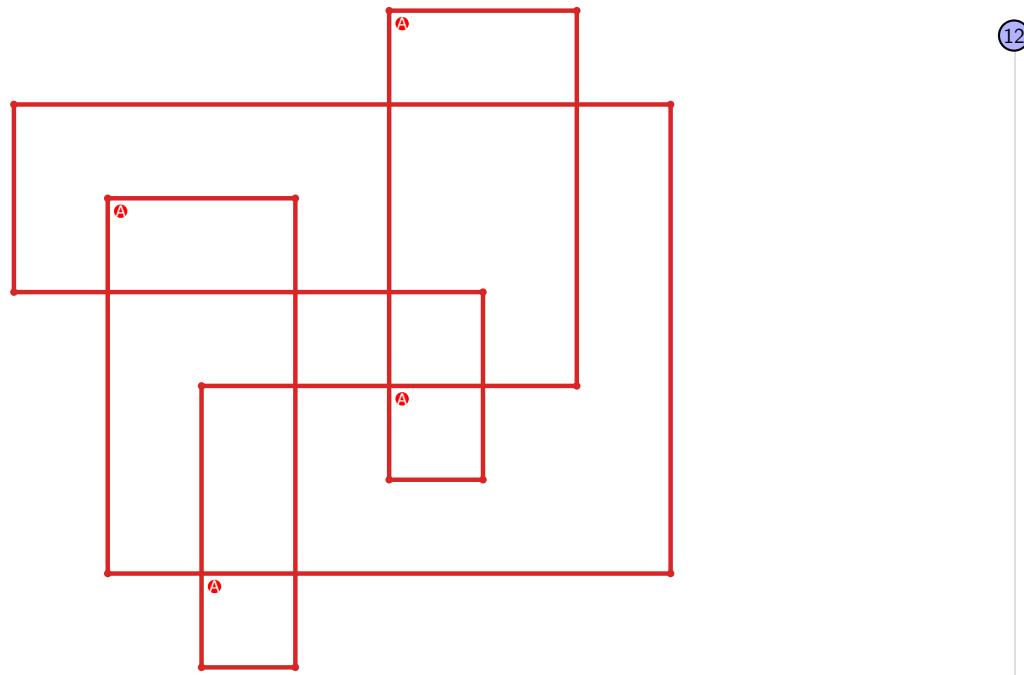


Figure 797: `SnapPy` multiloop plot.

Figure 798: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.251  $[[9, 20, 10, 1], [8, 5, 9, 6], [19, 10, 20, 11], [1, 15, 2, 14], [6, 14, 7, 13], [7, 12, 8, 13], [4, 11, 5, 12], [18, 15, 19, 16], [2, 18, 3, 17], [3, 16, 4, 17]]$

PD code drawn by SnapPy:  $[(6, 3, 7, 4), (17, 8, 18, 9), (2, 9, 3, 10), (5, 10, 6, 11), (11, 4, 12, 5), (12, 19, 13, 20), (16, 13, 17, 14), (1, 14, 2, 15), (15, 20, 16, 1), (7, 18, 8, 19)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 6, 7, 0], [0, 7, 8, 4], [1, 3, 5, 5], [1, 4, 4, 6], [1, 5, 9, 2], [2, 9, 8, 3], [3, 7, 9, 9], [6, 8, 8, 7]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 288  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.4  
 Average overall degree: 3.03

Table 398: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

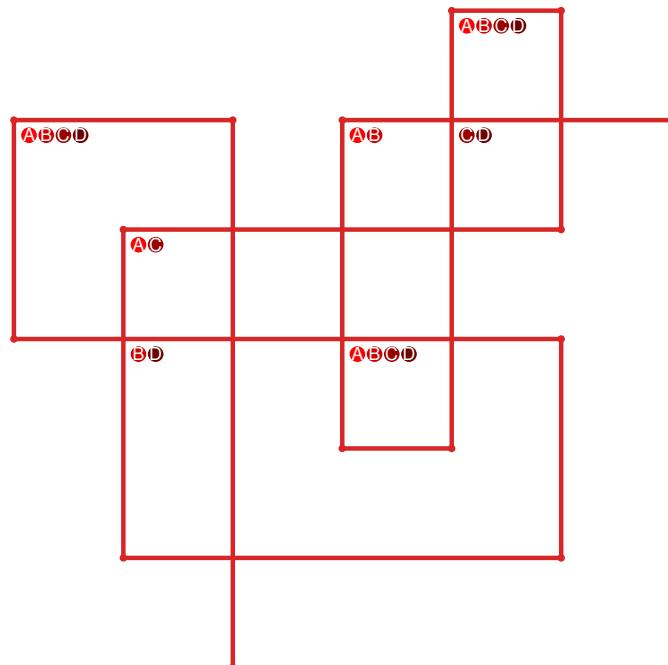


Figure 799: SnapPy multiloop plot.

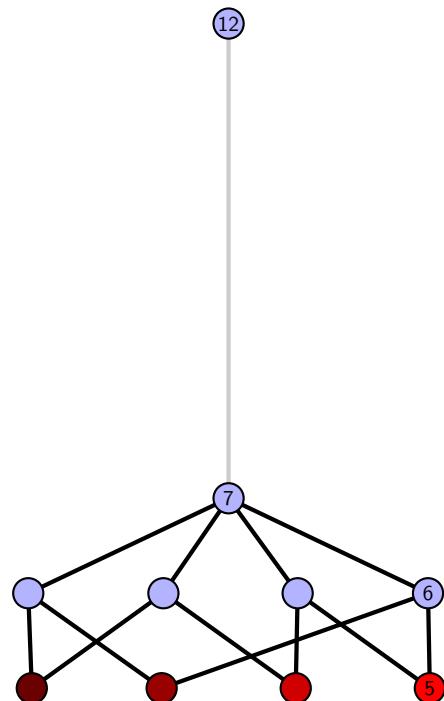


Figure 800: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.252  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 11, 9, 12], [1, 12, 2, 13], [13, 18, 14, 19], [10, 5, 11, 6], [9, 5, 10, 4], [2, 16, 3, 15], [17, 14, 18, 15], [3, 16, 4, 17]]$

PD code drawn by SnapPy:  $[(20, 5, 1, 6), (6, 1, 7, 2), (15, 2, 16, 3), (4, 7, 5, 8), (17, 8, 18, 9), (13, 10, 14, 11), (11, 18, 12, 19), (19, 12, 20, 13), (9, 14, 10, 15), (3, 16, 4, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 5, 0], [0, 5, 6, 3], [0, 2, 7, 4], [1, 3, 8, 8], [1, 6, 6, 2], [2, 5, 5, 9], [3, 9, 9, 8], [4, 7, 9, 4], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1  
**Total pinning sets:** 256  
**Pinning number:** 4

**Average optimal degree:** 2.0  
**Average minimal degree:** 2.0  
**Average overall degree:** 2.97

Table 399: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

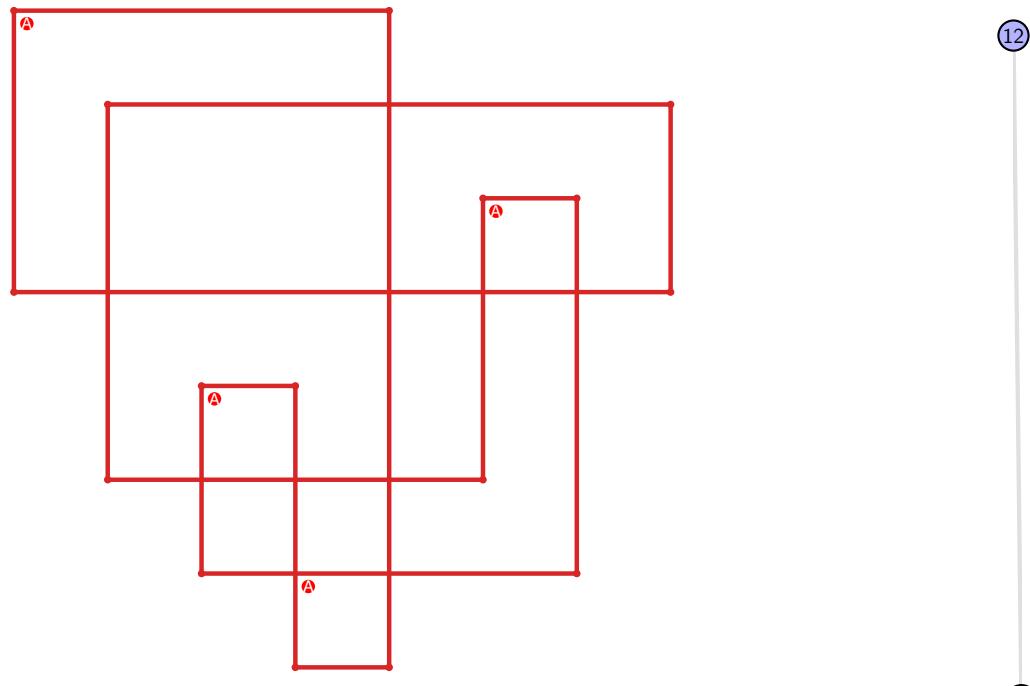


Figure 801: SnapPy multiloop plot.

4

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Figure 802: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.253** `[[20, 7, 1, 8], [8, 3, 9, 4], [4, 19, 5, 20], [6, 17, 7, 18], [1, 15, 2, 14], [2, 13, 3, 14], [9, 13, 10, 12], [18, 5, 19, 6], [16, 11, 17, 12], [15, 11, 16, 10]]`

PD code drawn by `SnapPy`: `[(6, 1, 7, 2), (16, 3, 17, 4), (18, 5, 19, 6), (8, 13, 9, 14), (14, 9, 15, 10), (10, 7, 11, 8), (20, 11, 1, 12), (12, 19, 13, 20), (4, 15, 5, 16), (2, 17, 3, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 7, 7], [0, 7, 7, 8], [0, 9, 5, 5], [1, 4, 4, 6], [1, 5, 9, 8], [2, 3, 3, 2], [3, 6, 9, 9], [4, 8, 8, 6]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 400: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

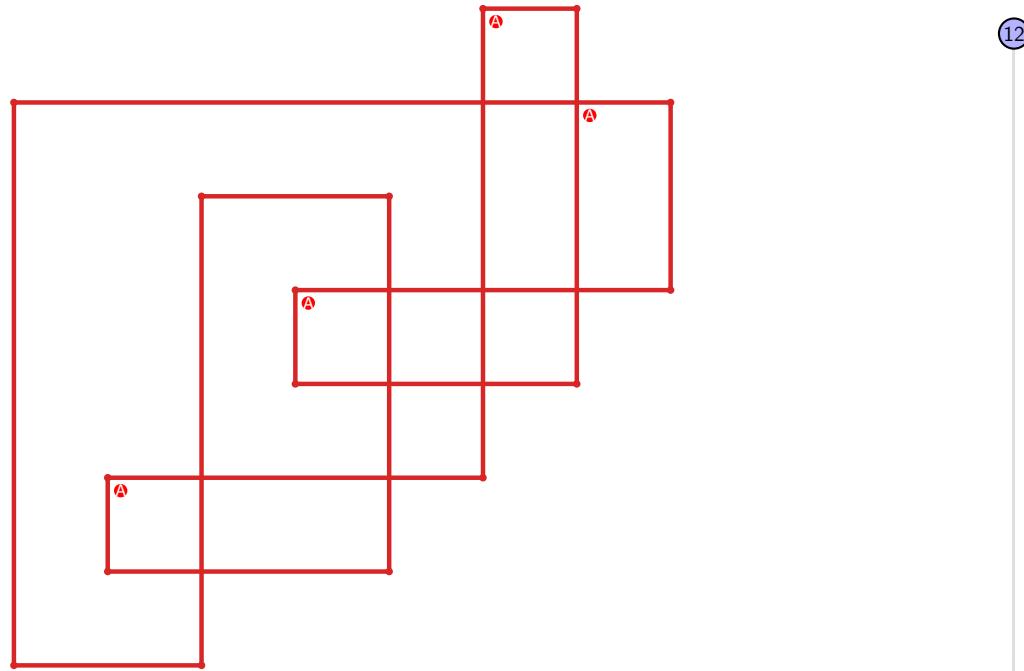


Figure 803: `SnapPy` multiloop plot.

Figure 804: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.254  $[[17, 20, 18, 1], [3, 16, 4, 17], [4, 19, 5, 20], [18, 5, 19, 6], [1, 11, 2, 10], [2, 9, 3, 10], [15, 8, 16, 9], [6, 13, 7, 14], [11, 14, 12, 15], [12, 7, 13, 8]]$

PD code drawn by `SnapPy`:  $[(16, 3, 17, 4), (10, 5, 11, 6), (13, 6, 14, 7), (20, 7, 1, 8), (8, 19, 9, 20), (4, 11, 5, 12), (9, 12, 10, 13), (1, 14, 2, 15), (2, 17, 3, 18), (15, 18, 16, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 3, 3], [0, 2, 2, 7], [0, 8, 5, 5], [1, 4, 4, 6], [1, 5, 8, 9], [3, 9, 9, 8], [4, 7, 9, 6], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.4

**Total pinning sets:** 288

**Average overall degree:** 3.03

**Pinning number:** 5

Table 401: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	24	61	85	70	34	9	1	284
Average degree	2.4	2.69	2.9	3.05	3.16	3.24	3.29	3.33	

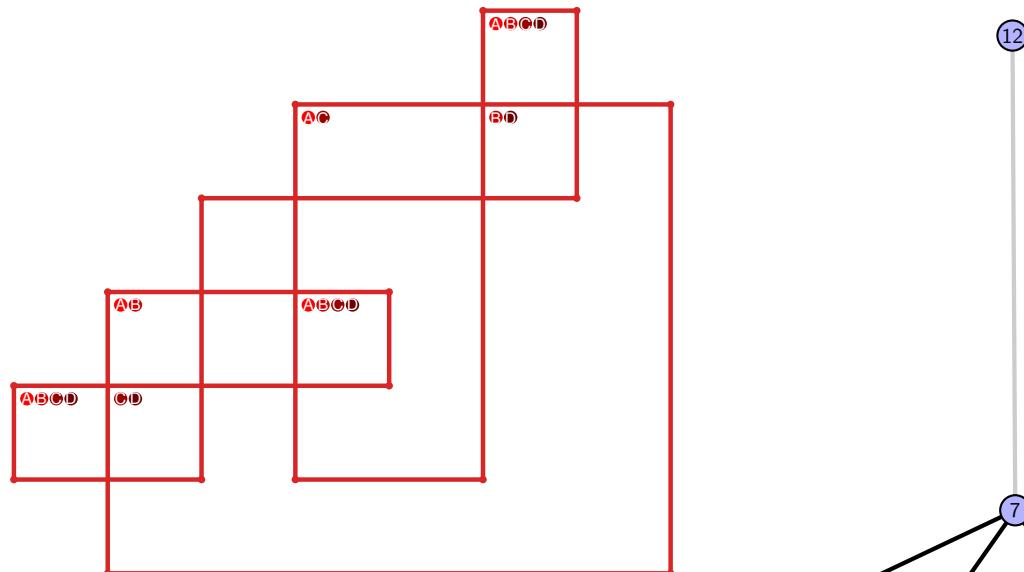


Figure 805: `SnapPy` multiloop plot.

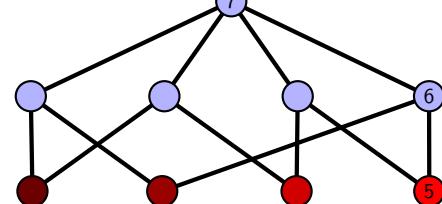


Figure 806: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.255  $[[7, 20, 8, 1], [13, 6, 14, 7], [14, 19, 15, 20], [8, 3, 9, 4], [1, 4, 2, 5], [5, 12, 6, 13], [18, 15, 19, 16], [2, 9, 3, 10], [11, 16, 12, 17], [17, 10, 18, 11]]$

PD code drawn by `SnapPy`:  $[(5, 20, 6, 1), (18, 1, 19, 2), (10, 3, 11, 4), (19, 6, 20, 7), (14, 7, 15, 8), (2, 9, 3, 10), (15, 12, 16, 13), (8, 13, 9, 14), (11, 16, 12, 17), (4, 17, 5, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 6], [0, 7, 7, 4], [0, 3, 7, 5], [1, 4, 8, 1], [2, 8, 9, 2], [3, 9, 4, 3], [5, 9, 9, 6], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 402: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

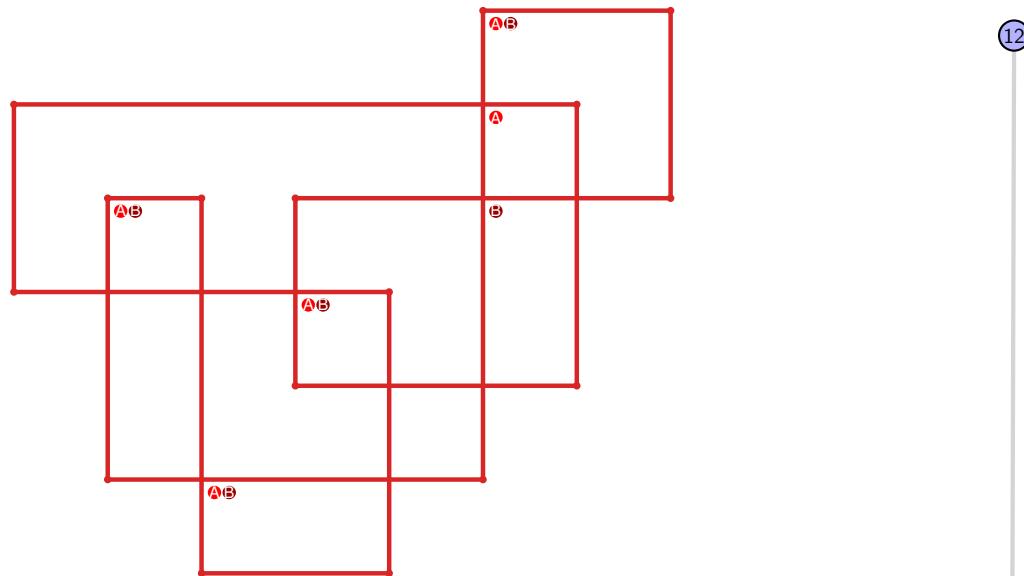


Figure 807: `SnapPy` multiloop plot.

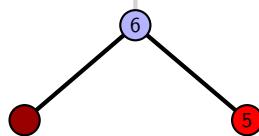


Figure 808: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.256 [[9, 20, 10, 1], [19, 8, 20, 9], [10, 2, 11, 1], [18, 13, 19, 14], [7, 12, 8, 13], [2, 12, 3, 11], [14, 5, 15, 6], [6, 17, 7, 18], [3, 17, 4, 16], [4, 15, 5, 16]]

PD code drawn by `SnapPy`: [(11, 2, 12, 3), (16, 5, 17, 6), (8, 19, 9, 20), (4, 9, 5, 10), (15, 10, 16, 11), (1, 12, 2, 13), (13, 20, 14, 1), (3, 14, 4, 15), (6, 17, 7, 18), (18, 7, 19, 8)]

Planar representation generated by `plantri`: [[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 5, 0], [1, 6, 7, 4], [1, 3, 7, 5], [2, 4, 8, 2], [3, 9, 9, 7], [3, 6, 8, 4], [5, 7, 9, 9], [6, 8, 8, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 403: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

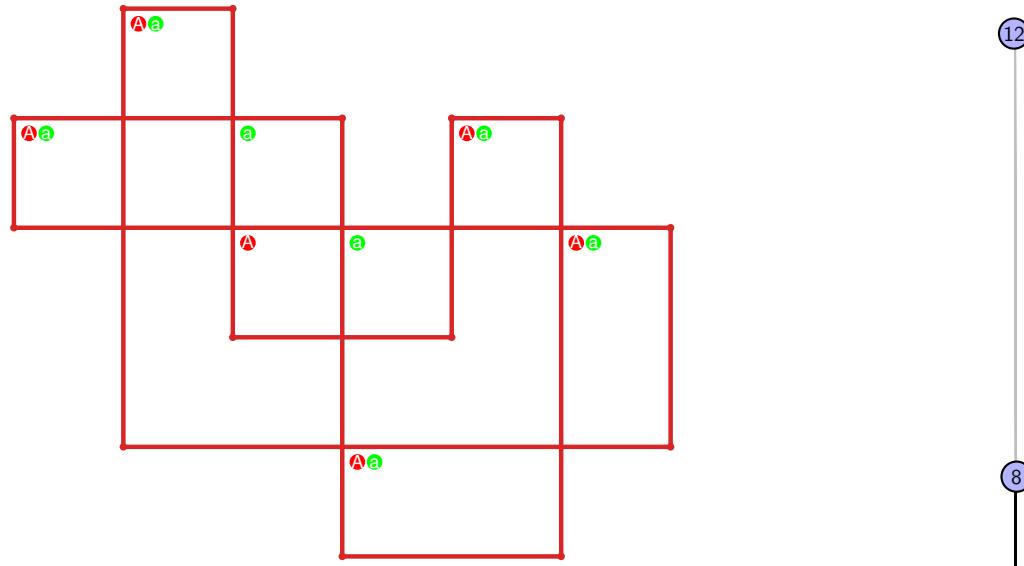


Figure 809: `SnapPy` multiloop plot.



Figure 810: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.257** `[[11, 20, 12, 1], [19, 10, 20, 11], [12, 10, 13, 9], [1, 9, 2, 8], [18, 3, 19, 4], [13, 3, 14, 2], [14, 7, 15, 8], [4, 17, 5, 18], [6, 15, 7, 16], [16, 5, 17, 6]]`

PD code drawn by `SnapPy`: `[(20, 11, 1, 12), (16, 1, 17, 2), (14, 3, 15, 4), (12, 5, 13, 6), (6, 19, 7, 20), (7, 10, 8, 11), (17, 8, 18, 9), (4, 13, 5, 14), (2, 15, 3, 16), (9, 18, 10, 19)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 5, 6], [1, 7, 7, 5], [2, 4, 6, 3], [3, 5, 8, 8], [4, 9, 9, 4], [6, 9, 9, 6], [7, 8, 8, 7]]`

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 64  
 Pinning number: 7

Average optimal degree: 2.29  
 Average minimal degree: 2.29  
 Average overall degree: 2.92

Table 404: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	22	18	7	1	61
Average degree	2.29	2.63	2.9	3.1	3.25	3.33	

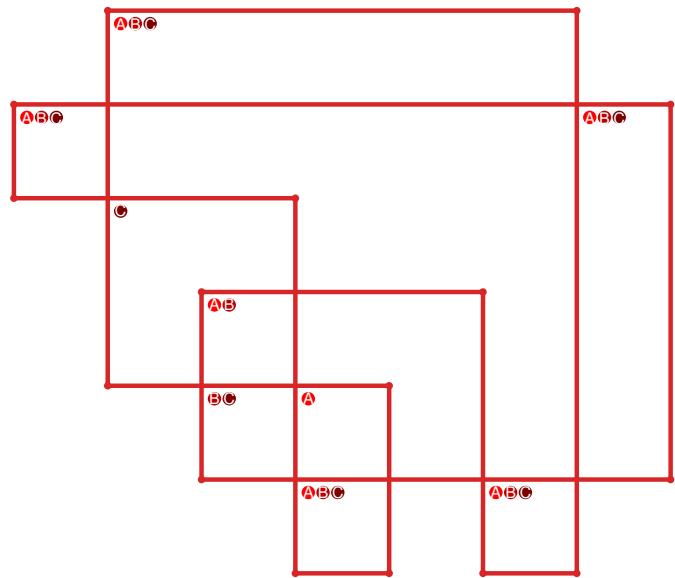


Figure 811: `SnapPy` multiloop plot.

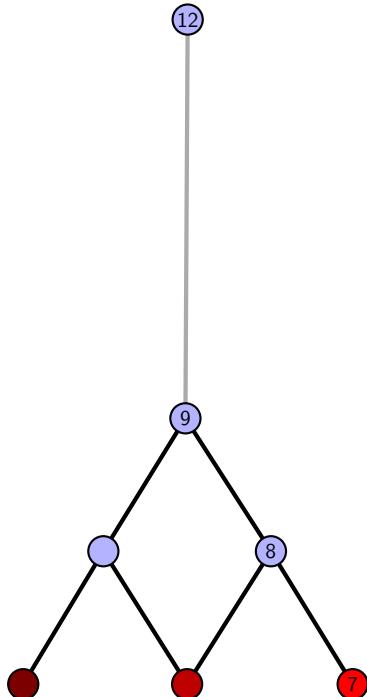


Figure 812: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.258  $[[7, 20, 8, 1], [17, 6, 18, 7], [19, 8, 20, 9], [1, 15, 2, 14], [16, 11, 17, 12], [5, 10, 6, 11], [18, 10, 19, 9], [15, 5, 16, 4], [2, 13, 3, 14], [12, 3, 13, 4]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (1, 12, 2, 13), (14, 3, 15, 4), (16, 5, 17, 6), (6, 15, 7, 16), (2, 7, 3, 8), (13, 8, 14, 9), (19, 10, 20, 11), (11, 18, 12, 19), (4, 17, 5, 18)]$

Planar representation generated by plantri:  $[[1, 2, 2, 3], [0, 4, 5, 6], [0, 6, 6, 0], [0, 7, 8, 8], [1, 9, 7, 5], [1, 4, 7, 6], [1, 5, 2, 2], [3, 5, 4, 9], [3, 9, 9, 3], [4, 8, 8, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 128  
 Pinning number: 6

Average optimal degree: 2.33  
 Average minimal degree: 2.33  
 Average overall degree: 2.97

Table 405: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	16	35	40	25	8	1	125
Average degree	2.33	2.64	2.87	3.04	3.18	3.27	3.33	

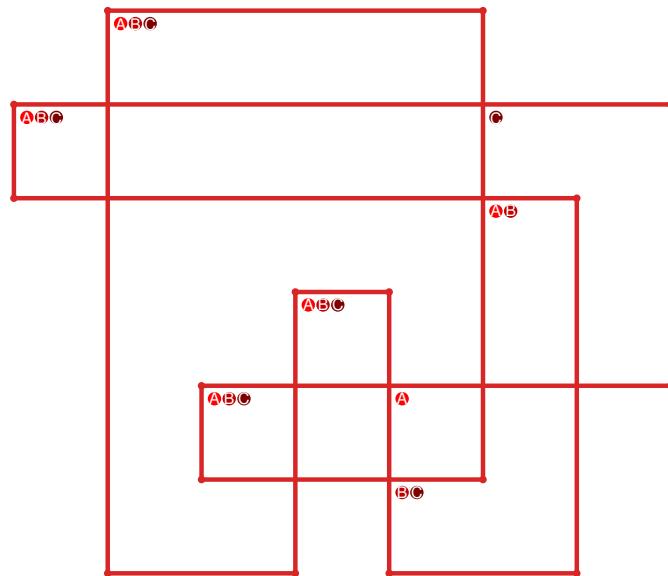


Figure 813: SnapPy multiloop plot.

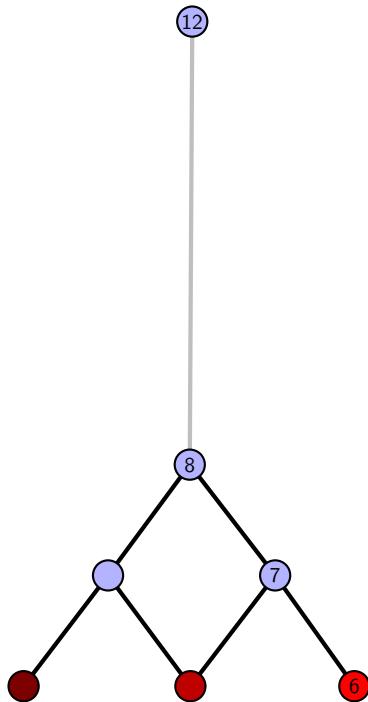


Figure 814: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.259**  $[[7, 20, 8, 1], [6, 9, 7, 10], [19, 8, 20, 9], [1, 15, 2, 14], [10, 17, 11, 18], [18, 5, 19, 6], [15, 12, 16, 13], [2, 13, 3, 14], [3, 16, 4, 17], [11, 4, 12, 5]]$

PD code drawn by `SnapPy`:  $[(9, 2, 10, 3), (16, 3, 17, 4), (17, 6, 18, 7), (4, 7, 5, 8), (13, 10, 14, 11), (20, 11, 1, 12), (12, 19, 13, 20), (1, 14, 2, 15), (8, 15, 9, 16), (5, 18, 6, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 4, 5, 2], [0, 1, 5, 0], [0, 6, 7, 7], [1, 8, 9, 5], [1, 4, 9, 2], [3, 9, 8, 7], [3, 6, 8, 3], [4, 7, 6, 9], [4, 8, 6, 5]]$

Total optimal pinning sets: 5  
 Total minimal pinning sets: 9  
 Total pinning sets: 220  
 Pinning number: 6

Average optimal degree: 2.67  
 Average minimal degree: 2.69  
 Average overall degree: 3.1

Table 406: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	4
Nonminimal pinning sets	0	26	66	70	38	10	1	211
Average degree	2.67	2.86	3.03	3.17	3.26	3.31	3.33	

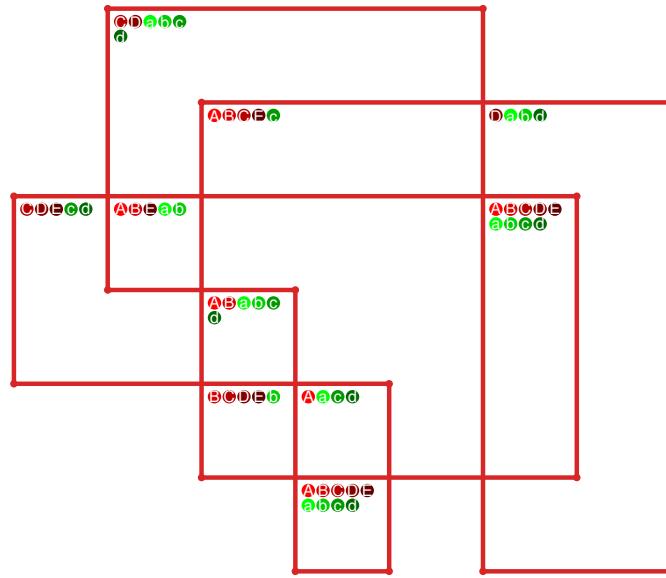


Figure 815: `SnapPy` multiloop plot.

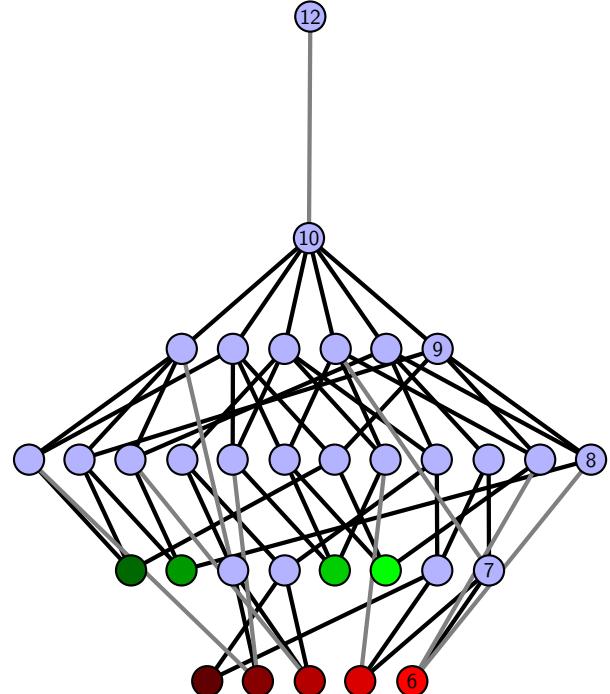


Figure 816: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.260  $[[20, 7, 1, 8], [8, 19, 9, 20], [9, 6, 10, 7], [1, 14, 2, 15], [18, 11, 19, 12], [5, 10, 6, 11], [13, 16, 14, 17], [2, 16, 3, 15], [12, 3, 13, 4], [4, 17, 5, 18]]$

PD code drawn by SnapPy:  $[(4, 1, 5, 2), (15, 2, 16, 3), (20, 5, 1, 6), (13, 6, 14, 7), (16, 9, 17, 10), (10, 17, 11, 18), (8, 11, 9, 12), (19, 12, 20, 13), (3, 14, 4, 15), (7, 18, 8, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 6, 7, 7], [1, 8, 9, 5], [2, 4, 9, 2], [3, 9, 8, 7], [3, 6, 8, 3], [4, 7, 6, 9], [4, 8, 6, 5]]$

Total optimal pinning sets: 5  
 Total minimal pinning sets: 7  
 Total pinning sets: 188  
 Pinning number: 6

Average optimal degree: 2.53  
 Average minimal degree: 2.56  
 Average overall degree: 3.04

Table 407: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	25	56	58	32	9	1	181
Average degree	2.53	2.78	2.98	3.12	3.22	3.29	3.33	

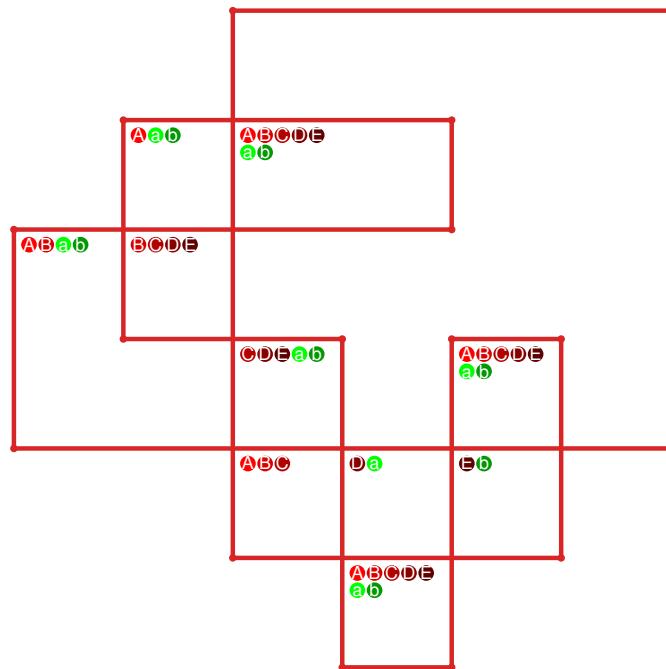


Figure 817: SnapPy multiloop plot.

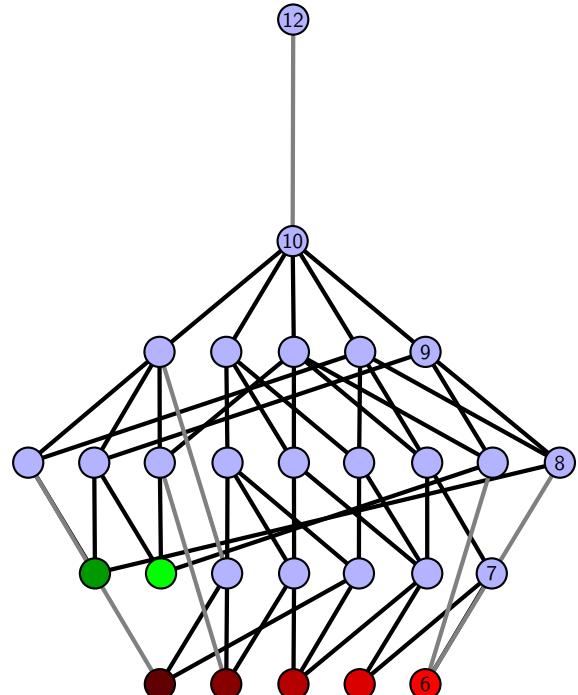


Figure 818: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.261  $[[20, 7, 1, 8], [8, 19, 9, 20], [9, 6, 10, 7], [1, 14, 2, 15], [5, 18, 6, 19], [10, 18, 11, 17], [13, 16, 14, 17], [2, 16, 3, 15], [4, 11, 5, 12], [12, 3, 13, 4]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (6, 3, 7, 4), (17, 4, 18, 5), (1, 8, 2, 9), (18, 11, 19, 12), (12, 19, 13, 20), (10, 13, 11, 14), (14, 7, 15, 8), (2, 15, 3, 16), (5, 16, 6, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 4, 5], [0, 6, 7, 7], [1, 8, 5, 2], [2, 4, 8, 6], [3, 5, 9, 7], [3, 6, 9, 3], [4, 9, 9, 5], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 7

**Average minimal degree:** 2.57

**Total pinning sets:** 180

**Average overall degree:** 3.04

**Pinning number:** 6

Table 408: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	3
Nonminimal pinning sets	0	21	53	57	32	9	1	173
Average degree	2.5	2.75	2.96	3.12	3.23	3.29	3.33	

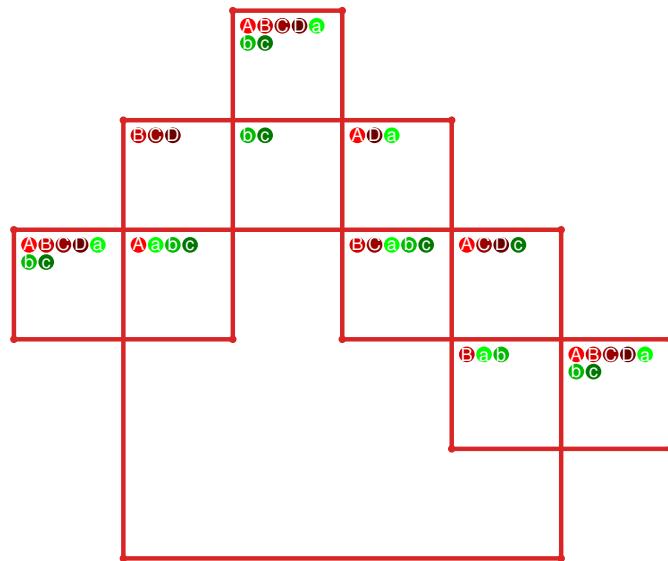


Figure 819: SnapPy multiloop plot.

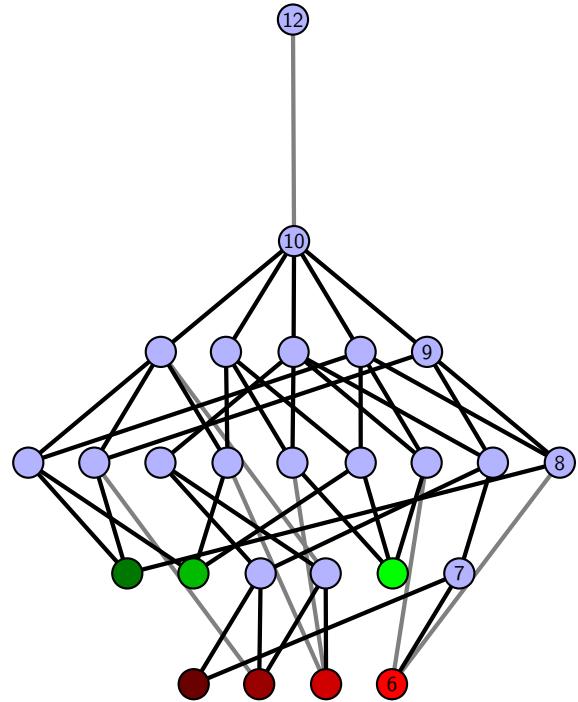


Figure 820: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.262  $[[13, 20, 14, 1], [5, 12, 6, 13], [6, 19, 7, 20], [14, 7, 15, 8], [1, 8, 2, 9], [17, 4, 18, 5], [18, 11, 19, 12], [15, 3, 16, 2], [9, 16, 10, 17], [10, 3, 11, 4]]$

PD code drawn by SnapPy:  $[(7, 20, 8, 1), (14, 1, 15, 2), (11, 4, 12, 5), (18, 5, 19, 6), (15, 8, 16, 9), (2, 9, 3, 10), (6, 13, 7, 14), (3, 16, 4, 17), (10, 17, 11, 18), (19, 12, 20, 13)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 3], [0, 2, 7, 4], [0, 3, 7, 8], [1, 8, 9, 6], [1, 5, 9, 2], [3, 9, 8, 4], [4, 7, 9, 5], [5, 8, 7, 6]]$

Total optimal pinning sets: 2

Average optimal degree: 3.0

Total minimal pinning sets: 17

Average minimal degree: 3.0

Total pinning sets: 492

Average overall degree: 3.23

Pinning number: 5

Table 409: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	15	0	0	0	0	0	0	15
Nonminimal pinning sets	0	14	102	160	130	56	12	1	475
Average degree	3.0	3.05	3.15	3.23	3.29	3.32	3.33	3.33	

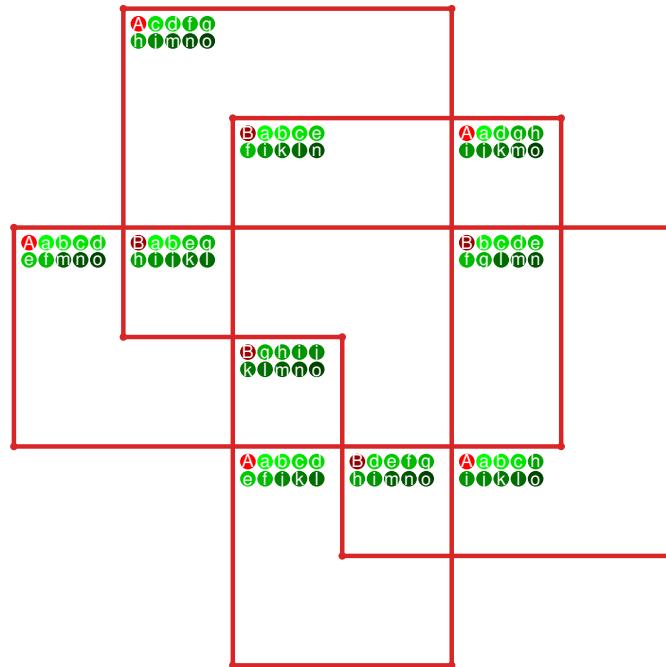


Figure 821: SnapPy multiloop plot.

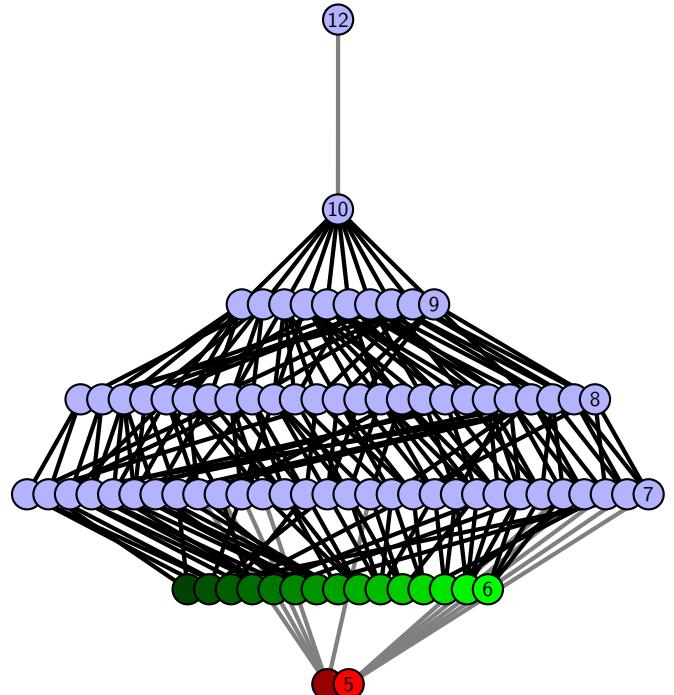


Figure 822: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.263 [[20, 7, 1, 8], [8, 17, 9, 18], [6, 19, 7, 20], [1, 12, 2, 13], [16, 9, 17, 10], [18, 5, 19, 6], [11, 14, 12, 15], [2, 14, 3, 13], [10, 3, 11, 4], [4, 15, 5, 16]]

PD code drawn by SnapPy: [(8, 1, 9, 2), (16, 3, 17, 4), (18, 5, 19, 6), (14, 7, 15, 8), (19, 10, 20, 11), (11, 20, 12, 1), (9, 12, 10, 13), (2, 13, 3, 14), (6, 15, 7, 16), (4, 17, 5, 18)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 4, 5], [0, 5, 5, 0], [0, 6, 7, 7], [1, 8, 9, 1], [1, 9, 2, 2], [3, 9, 8, 7], [3, 6, 8, 3], [4, 7, 6, 9], [4, 8, 6, 5]]

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 128  
 Pinning number: 6

Average optimal degree: 2.33  
 Average minimal degree: 2.33  
 Average overall degree: 2.97

Table 410: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	16	35	40	25	8	1	125
Average degree	2.33	2.64	2.87	3.04	3.18	3.27	3.33	

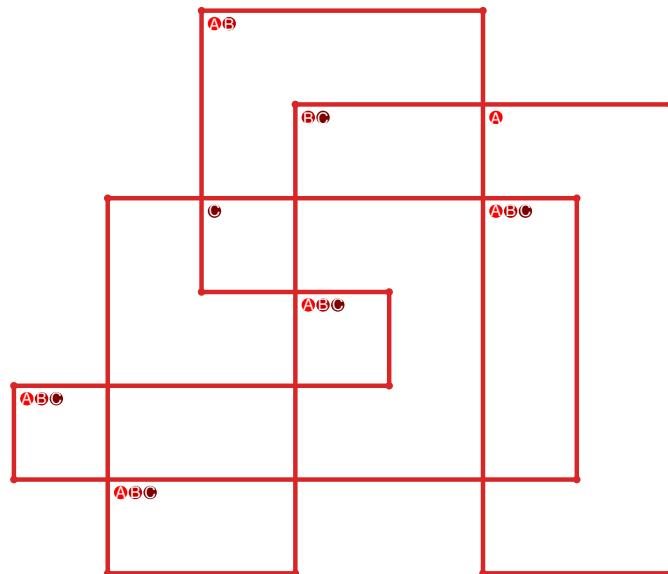


Figure 823: SnapPy multiloop plot.

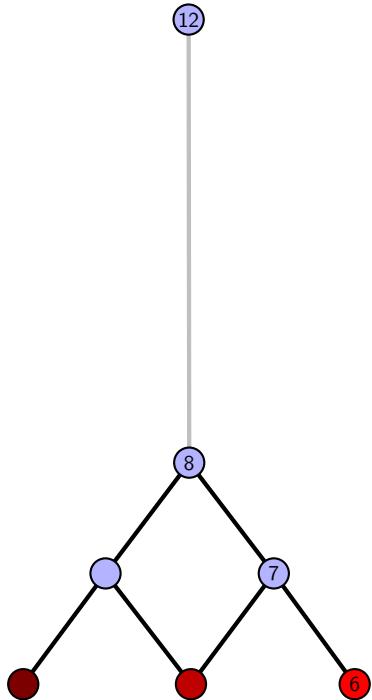


Figure 824: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.264  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 18, 9, 17], [1, 17, 2, 16], [5, 18, 6, 19], [9, 3, 10, 2], [10, 15, 11, 16], [4, 13, 5, 14], [3, 13, 4, 12], [14, 11, 15, 12]]$

PD code drawn by SnapPy:  $[(20, 7, 1, 8), (17, 4, 18, 5), (12, 9, 13, 10), (10, 1, 11, 2), (2, 11, 3, 12), (8, 13, 9, 14), (14, 19, 15, 20), (15, 6, 16, 7), (3, 16, 4, 17), (5, 18, 6, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 3], [0, 2, 5, 6], [1, 7, 2, 1], [2, 8, 6, 3], [3, 5, 9, 9], [4, 9, 8, 8], [5, 7, 7, 9], [6, 8, 7, 6]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 144  
 Pinning number: 6

Average optimal degree: 2.38  
 Average minimal degree: 2.38  
 Average overall degree: 2.98

Table 411: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.38	2.68	2.9	3.06	3.18	3.27	3.33	

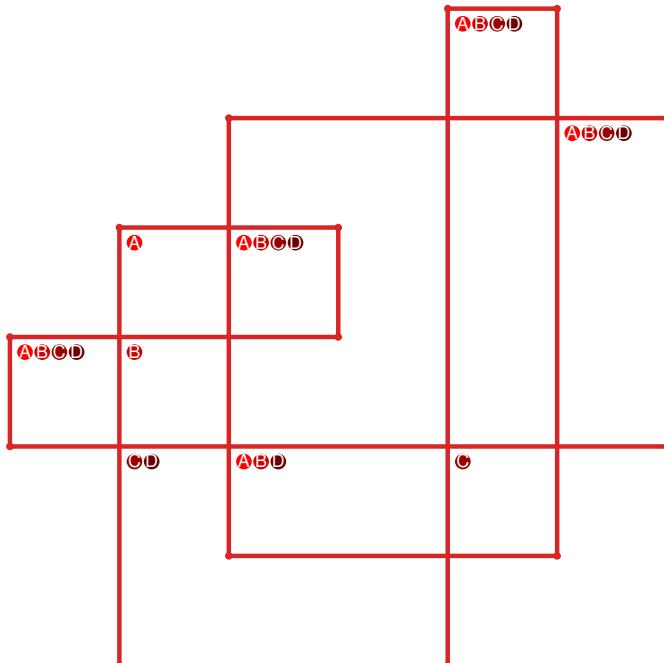


Figure 825: SnapPy multiloop plot.

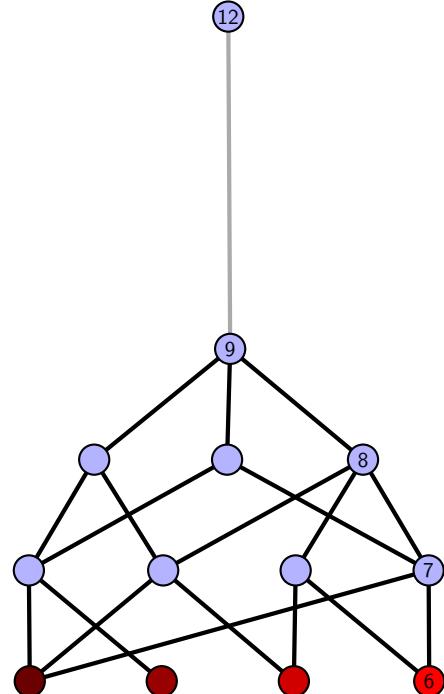


Figure 826: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.265  $[[11, 20, 12, 1], [17, 10, 18, 11], [19, 4, 20, 5], [12, 4, 13, 3], [1, 14, 2, 15], [16, 7, 17, 8], [9, 6, 10, 7], [18, 6, 19, 5], [13, 2, 14, 3], [15, 9, 16, 8]]$

PD code drawn by SnapPy:  $[(11, 20, 12, 1), (1, 10, 2, 11), (17, 2, 18, 3), (3, 16, 4, 17), (7, 4, 8, 5), (14, 5, 15, 6), (6, 13, 7, 14), (15, 8, 16, 9), (19, 12, 20, 13), (9, 18, 10, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 7, 3], [0, 2, 8, 8], [0, 8, 8, 9], [1, 9, 9, 6], [1, 5, 9, 7], [1, 6, 2, 2], [3, 4, 4, 3], [4, 6, 5, 5]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 144  
 Pinning number: 6

Average optimal degree: 2.38  
 Average minimal degree: 2.38  
 Average overall degree: 2.98

Table 412: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	20	41	44	26	8	1	140
Average degree	2.38	2.68	2.9	3.06	3.18	3.27	3.33	

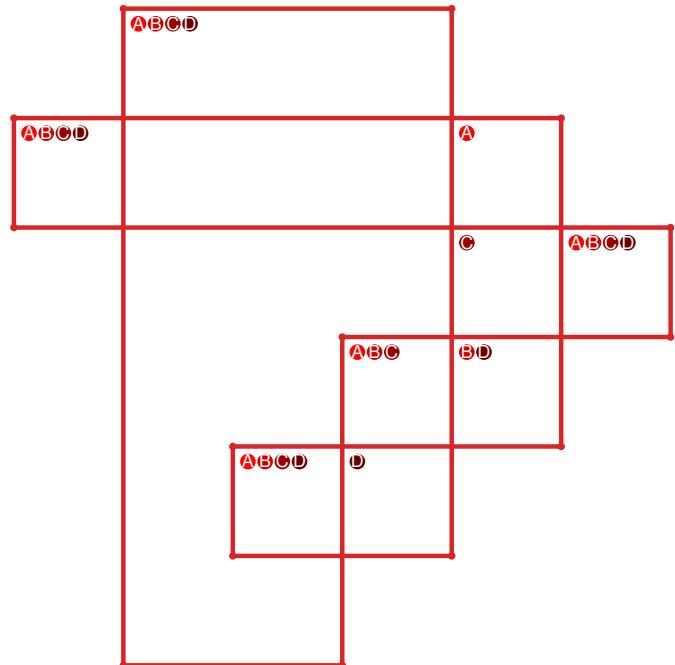


Figure 827: SnapPy multiloop plot.

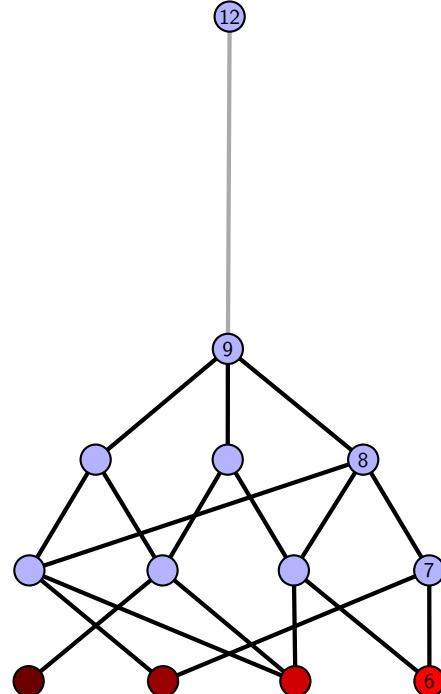


Figure 828: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.266 [[20, 13, 1, 14], [14, 8, 15, 7], [19, 2, 20, 3], [12, 1, 13, 2], [8, 5, 9, 6], [15, 6, 16, 7], [3, 18, 4, 19], [4, 11, 5, 12], [9, 17, 10, 16], [10, 17, 11, 18]]

PD code drawn by `SnapPy`: [(5, 2, 6, 3), (12, 3, 13, 4), (4, 11, 5, 12), (1, 8, 2, 9), (17, 10, 18, 11), (13, 6, 14, 7), (7, 14, 8, 15), (20, 15, 1, 16), (9, 18, 10, 19), (16, 19, 17, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 3], [0, 2, 7, 0], [1, 7, 8, 5], [1, 4, 8, 1], [2, 9, 7, 2], [3, 6, 9, 4], [4, 9, 9, 5], [6, 8, 8, 7]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 156  
**Pinning number:** 6

**Average optimal degree:** 2.38  
**Average minimal degree:** 2.46  
**Average overall degree:** 2.99

Table 413: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	20	46	48	27	8	1	150
Average degree	2.38	2.68	2.91	3.08	3.19	3.27	3.33	

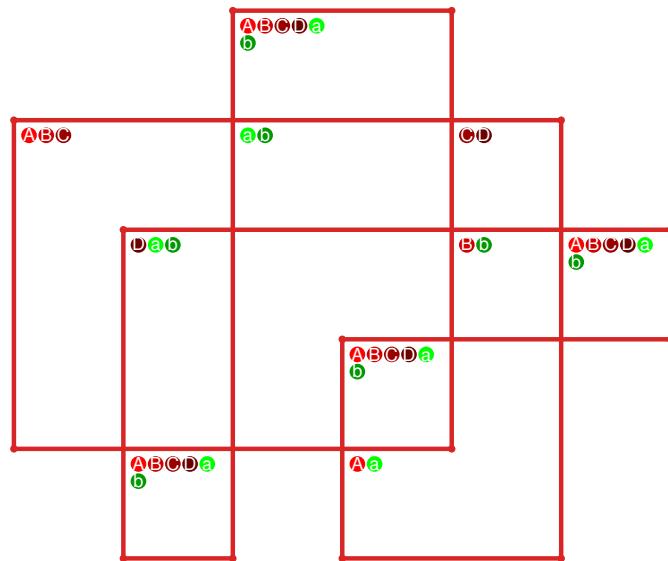


Figure 829: `SnapPy` multiloop plot.

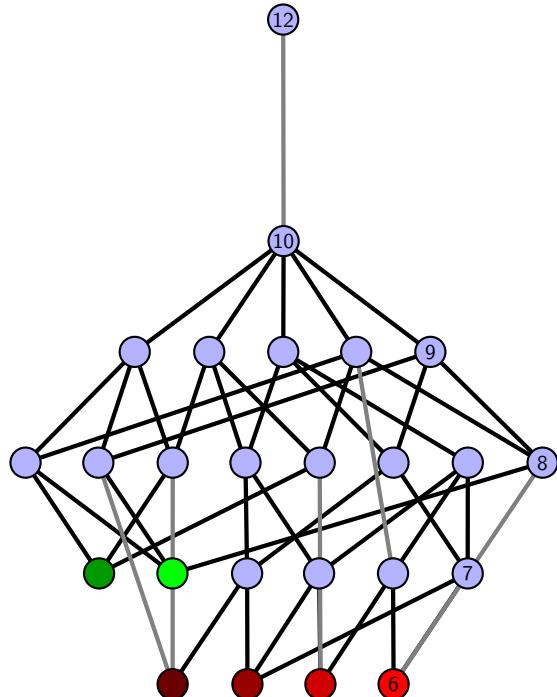


Figure 830: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.267** `[[7, 20, 8, 1], [6, 13, 7, 14], [19, 2, 20, 3], [8, 2, 9, 1], [14, 11, 15, 12], [12, 5, 13, 6], [3, 16, 4, 17], [18, 9, 19, 10], [10, 17, 11, 18], [15, 4, 16, 5]]`

PD code drawn by `SnapPy`: `[(12, 1, 13, 2), (9, 2, 10, 3), (15, 6, 16, 7), (11, 8, 12, 9), (3, 10, 4, 11), (20, 13, 1, 14), (7, 14, 8, 15), (4, 17, 5, 18), (18, 5, 19, 6), (16, 19, 17, 20)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 7, 0], [1, 8, 9, 5], [1, 4, 9, 1], [2, 9, 9, 8], [2, 8, 8, 3], [4, 7, 7, 6], [4, 6, 6, 5]]`

Total optimal pinning sets: 3  
 Total minimal pinning sets: 5  
 Total pinning sets: 140  
 Pinning number: 6

Average optimal degree: 2.33  
 Average minimal degree: 2.43  
 Average overall degree: 2.98

Table 414: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	16	40	44	26	8	1	135
Average degree	2.33	2.63	2.88	3.06	3.18	3.27	3.33	

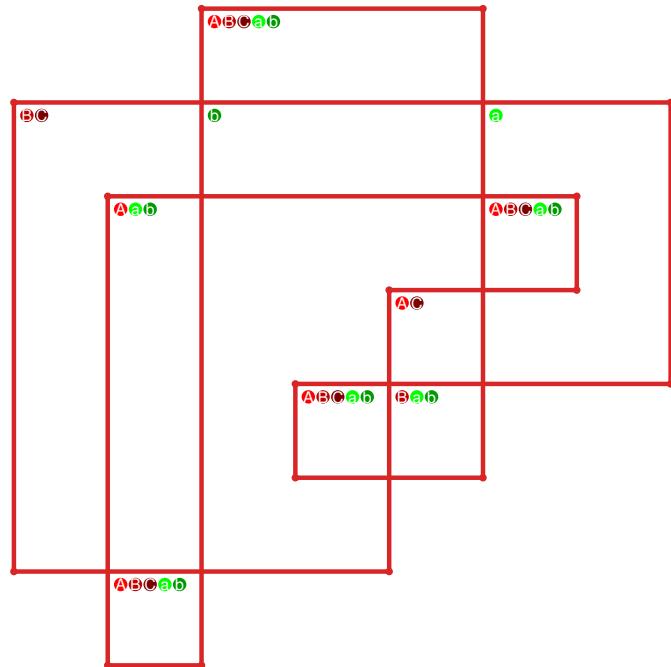


Figure 831: `SnapPy` multiloop plot.

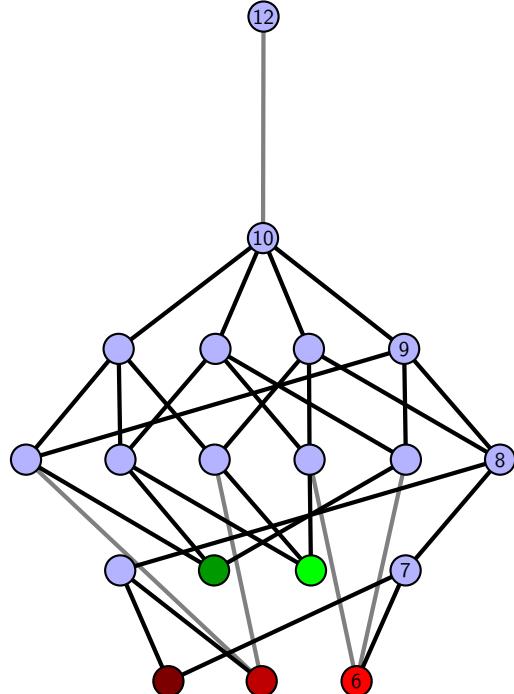


Figure 832: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.268**  $[[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 15, 9, 16], [1, 7, 2, 6], [18, 12, 19, 11], [14, 7, 15, 8], [2, 5, 3, 6], [12, 3, 13, 4], [4, 13, 5, 14]]$

PD code drawn by `SnapPy`:  $[(9, 20, 10, 1), (19, 2, 20, 3), (3, 18, 4, 19), (11, 4, 12, 5), (17, 6, 18, 7), (8, 15, 9, 16), (1, 10, 2, 11), (5, 12, 6, 13), (16, 13, 17, 14), (14, 7, 15, 8)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 6, 7, 7], [1, 8, 2, 1], [3, 9, 4, 3], [4, 9, 8, 4], [5, 7, 9, 9], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 5  
**Total minimal pinning sets:** 5  
**Total pinning sets:** 152  
**Pinning number:** 6

**Average optimal degree:** 2.4  
**Average minimal degree:** 2.4  
**Average overall degree:** 2.98

Table 415: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	5	0	0	0	0	0	0	5
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	23	44	45	26	8	1	147
Average degree	2.4	2.7	2.91	3.07	3.18	3.27	3.33	

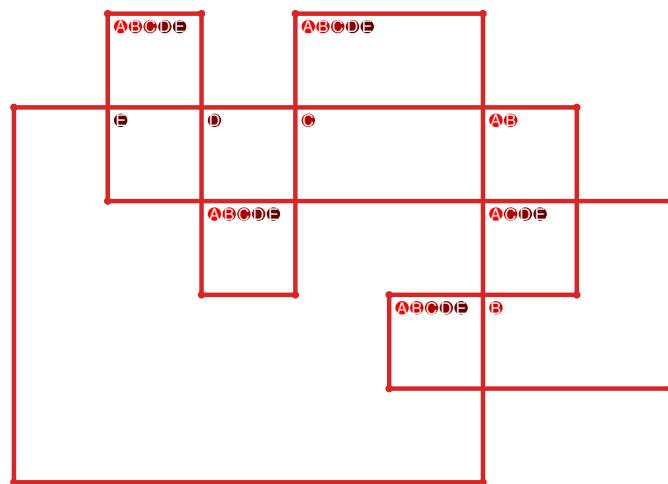


Figure 833: `SnapPy` multiloop plot.

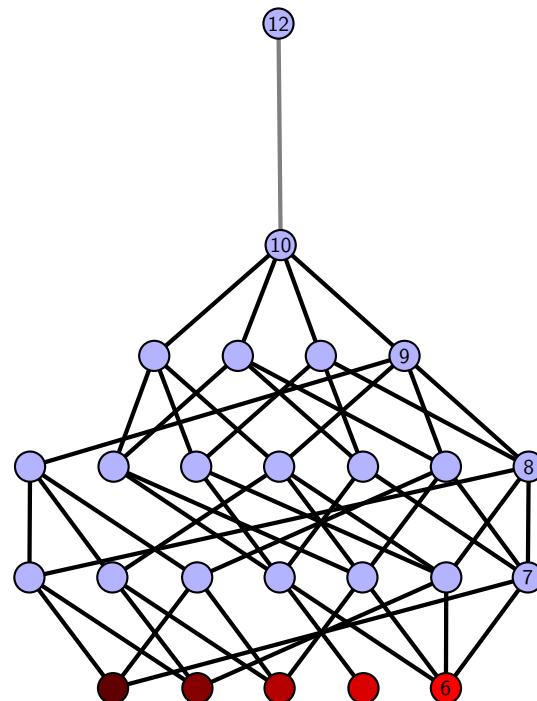


Figure 834: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.269 [[11, 20, 12, 1], [19, 10, 20, 11], [12, 2, 13, 1], [18, 5, 19, 6], [9, 2, 10, 3], [13, 9, 14, 8], [6, 17, 7, 18], [4, 15, 5, 16], [3, 15, 4, 14], [16, 7, 17, 8]]

PD code drawn by SnapPy: [(15, 2, 16, 3), (13, 4, 14, 5), (5, 10, 6, 11), (16, 7, 17, 8), (11, 20, 12, 1), (1, 12, 2, 13), (3, 14, 4, 15), (8, 17, 9, 18), (18, 9, 19, 10), (6, 19, 7, 20)]

Planar representation generated by plantri: [[1, 1, 2, 2], [0, 3, 4, 0], [0, 4, 5, 0], [1, 6, 6, 7], [1, 8, 5, 2], [2, 4, 8, 9], [3, 9, 9, 3], [3, 9, 8, 8], [4, 7, 7, 5], [5, 7, 6, 6]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 416: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

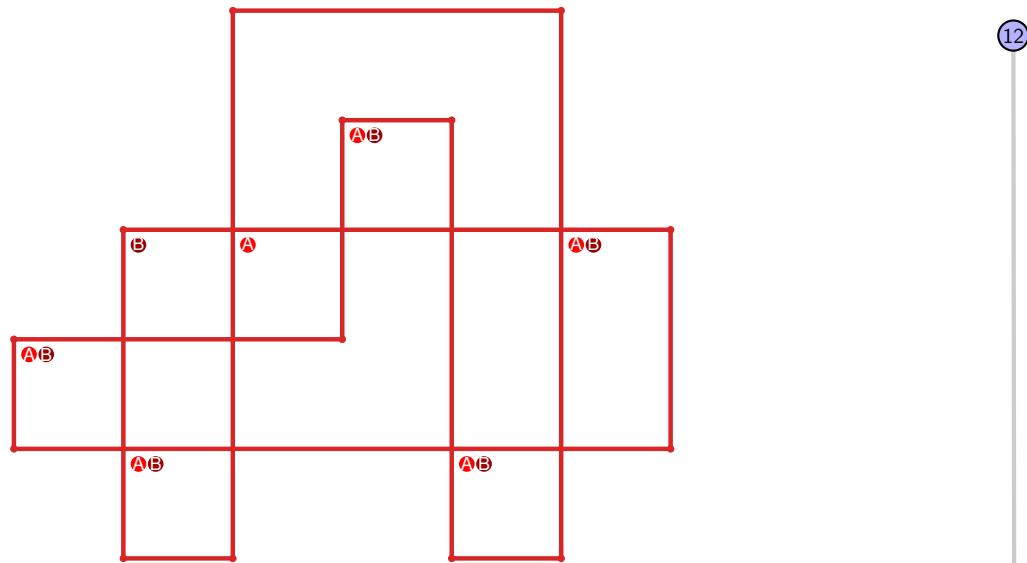


Figure 835: SnapPy multiloop plot.

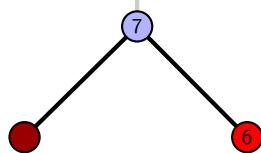


Figure 836: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.270  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 8, 11, 7], [1, 17, 2, 16], [5, 18, 6, 19], [11, 6, 12, 7], [17, 12, 18, 13], [2, 15, 3, 16], [4, 13, 5, 14], [14, 3, 15, 4]]$

PD code drawn by `SnapPy`:  $[(11, 20, 12, 1), (8, 3, 9, 4), (4, 7, 5, 8), (16, 5, 17, 6), (14, 9, 15, 10), (1, 10, 2, 11), (19, 12, 20, 13), (13, 18, 14, 19), (2, 15, 3, 16), (6, 17, 7, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 6, 7, 7], [1, 8, 6, 5], [2, 4, 6, 2], [3, 5, 4, 8], [3, 9, 9, 3], [4, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.29

**Total pinning sets:** 88

**Average overall degree:** 2.92

**Pinning number:** 6

Table 417: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	2
Nonminimal pinning sets	0	6	22	29	20	7	1	85
Average degree	2.17	2.46	2.74	2.97	3.13	3.25	3.33	

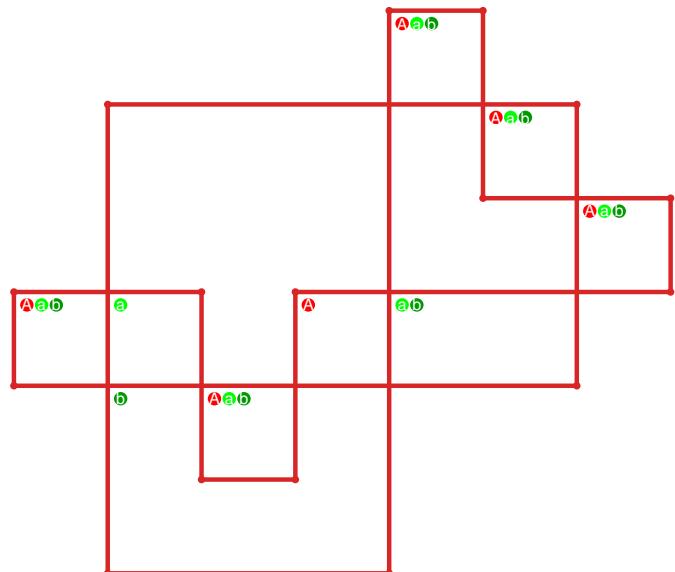


Figure 837: `SnapPy` multiloop plot.

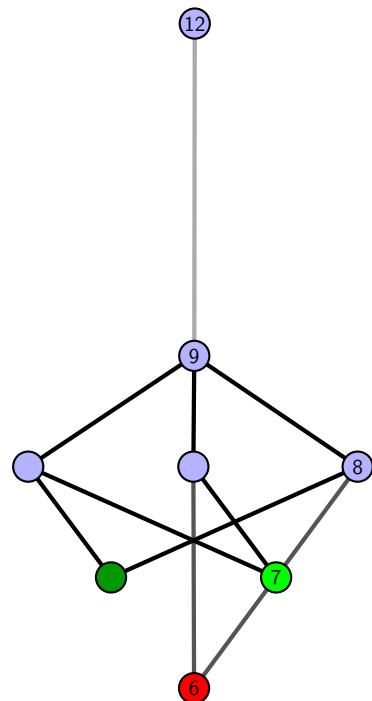


Figure 838: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.271 [[20, 15, 1, 16], [16, 5, 17, 6], [6, 19, 7, 20], [7, 14, 8, 15], [1, 10, 2, 11], [4, 17, 5, 18], [18, 3, 19, 4], [13, 8, 14, 9], [9, 12, 10, 13], [2, 12, 3, 11]]

PD code drawn by `SnapPy`: [(15, 20, 16, 1), (13, 2, 14, 3), (10, 5, 11, 6), (18, 7, 19, 8), (8, 19, 9, 20), (6, 9, 7, 10), (4, 11, 5, 12), (17, 12, 18, 13), (1, 14, 2, 15), (3, 16, 4, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 7, 7], [0, 8, 9, 9], [1, 6, 6, 1], [2, 5, 5, 9], [3, 8, 8, 3], [4, 7, 7, 9], [4, 8, 6, 4]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 418: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

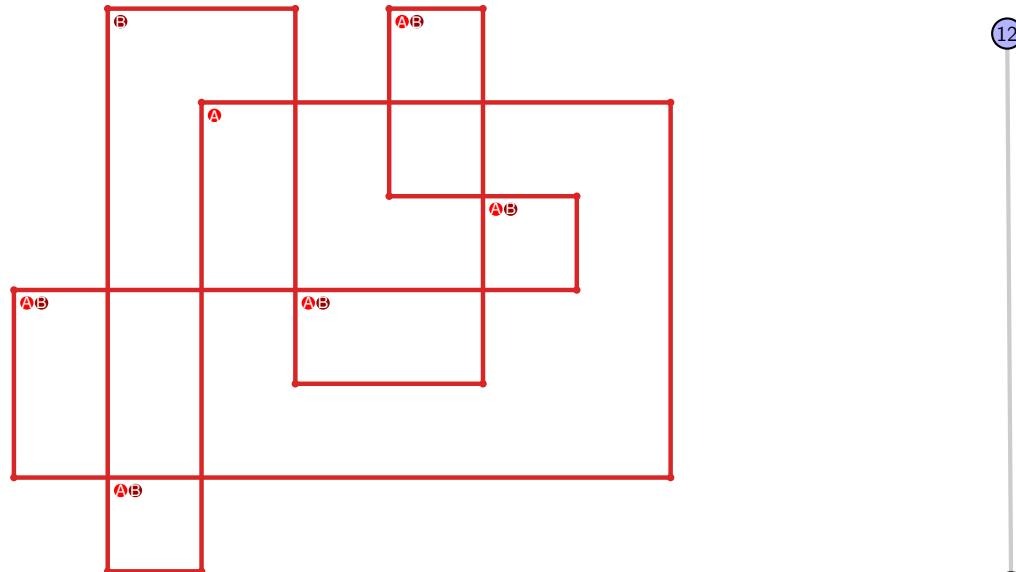


Figure 839: `SnapPy` multiloop plot.

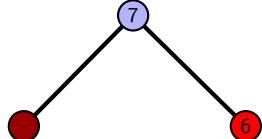


Figure 840: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.272  $[[7, 20, 8, 1], [6, 13, 7, 14], [19, 2, 20, 3], [8, 2, 9, 1], [14, 5, 15, 6], [15, 12, 16, 13], [3, 18, 4, 19], [9, 4, 10, 5], [11, 16, 12, 17], [17, 10, 18, 11]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (11, 2, 12, 3), (14, 7, 15, 8), (12, 9, 13, 10), (3, 10, 4, 11), (8, 13, 9, 14), (20, 15, 1, 16), (4, 17, 5, 18), (18, 5, 19, 6), (16, 19, 17, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 4, 5], [0, 6, 6, 3], [0, 2, 7, 0], [1, 7, 5, 1], [1, 4, 8, 8], [2, 9, 7, 2], [3, 6, 9, 4], [5, 9, 9, 5], [6, 8, 8, 7]]$

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3  
**Total pinning sets:** 112  
**Pinning number:** 6

**Average optimal degree:** 2.22  
**Average minimal degree:** 2.22  
**Average overall degree:** 2.92

Table 419: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	31	34	21	7	1	109
Average degree	2.22	2.57	2.82	3.01	3.14	3.25	3.33	

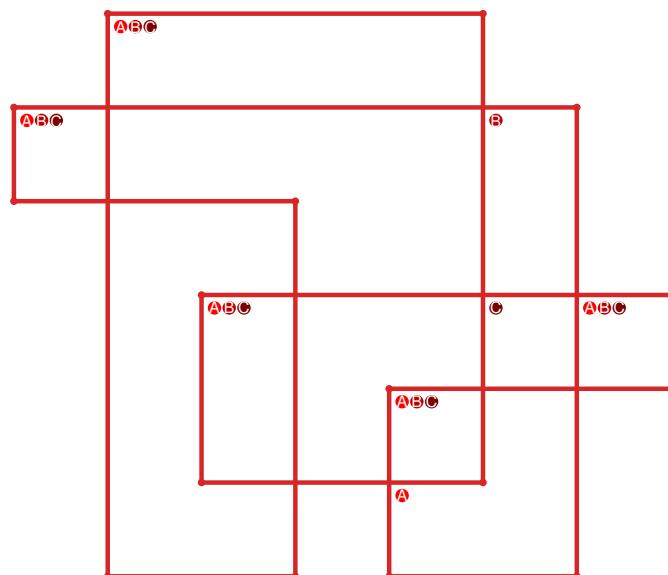


Figure 841: `SnapPy` multiloop plot.

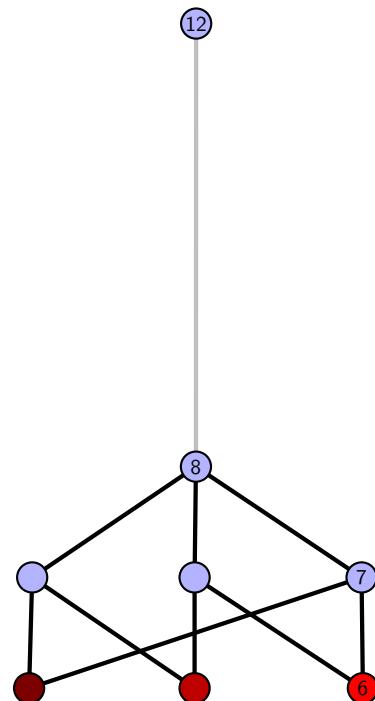


Figure 842: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.273 [[20, 11, 1, 12], [12, 19, 13, 20], [13, 10, 14, 11], [1, 14, 2, 15], [7, 18, 8, 19], [9, 2, 10, 3], [15, 4, 16, 5], [17, 6, 18, 7], [8, 4, 9, 3], [16, 6, 17, 5]]

PD code drawn by `SnapPy`: [(20, 5, 1, 6), (12, 1, 13, 2), (2, 13, 3, 14), (14, 3, 15, 4), (10, 7, 11, 8), (17, 8, 18, 9), (18, 11, 19, 12), (4, 15, 5, 16), (9, 16, 10, 17), (6, 19, 7, 20)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 5, 6], [1, 7, 7, 8], [2, 8, 8, 3], [3, 8, 9, 9], [4, 9, 9, 4], [4, 6, 5, 5], [6, 7, 7, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 420: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

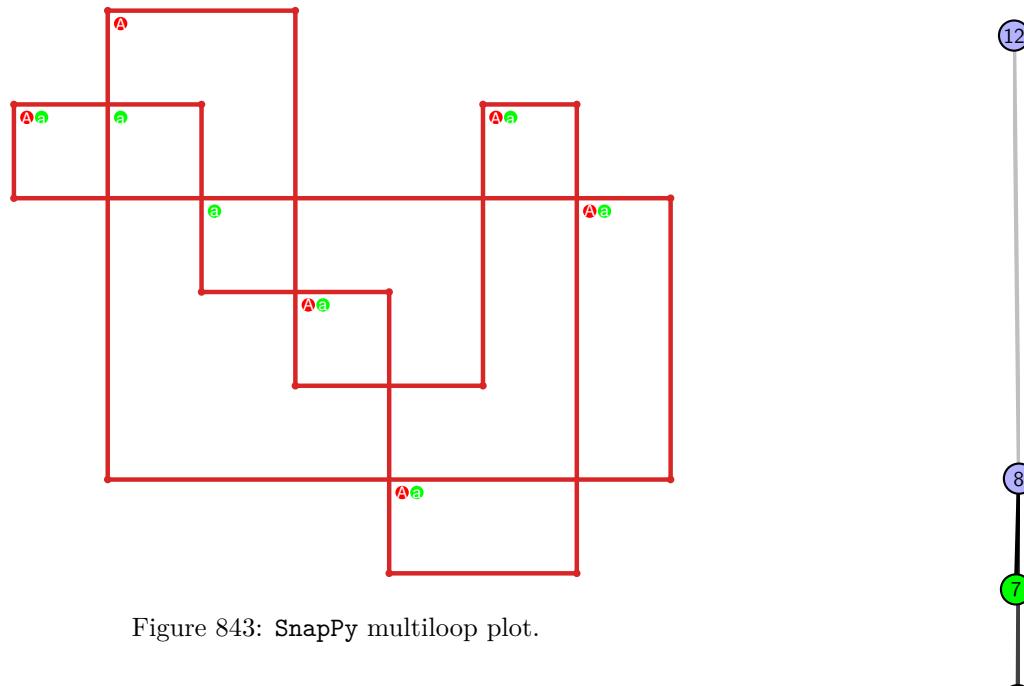


Figure 843: `SnapPy` multiloop plot.

Figure 844: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.274 [[20, 11, 1, 12], [12, 5, 13, 6], [8, 19, 9, 20], [10, 1, 11, 2], [15, 4, 16, 5], [13, 16, 14, 17], [6, 17, 7, 18], [18, 7, 19, 8], [9, 3, 10, 2], [3, 14, 4, 15]]

PD code drawn by `SnapPy`: [(7, 20, 8, 1), (13, 2, 14, 3), (15, 6, 16, 7), (18, 9, 19, 10), (8, 11, 9, 12), (1, 12, 2, 13), (3, 14, 4, 15), (5, 16, 6, 17), (17, 4, 18, 5), (10, 19, 11, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 6], [0, 7, 7, 8], [0, 8, 8, 0], [1, 9, 9, 5], [1, 4, 9, 6], [1, 5, 7, 7], [2, 6, 6, 2], [2, 9, 3, 3], [4, 8, 5, 4]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 2  
**Total pinning sets:** 80  
**Pinning number:** 6

**Average optimal degree:** 2.17  
**Average minimal degree:** 2.23  
**Average overall degree:** 2.91

Table 421: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

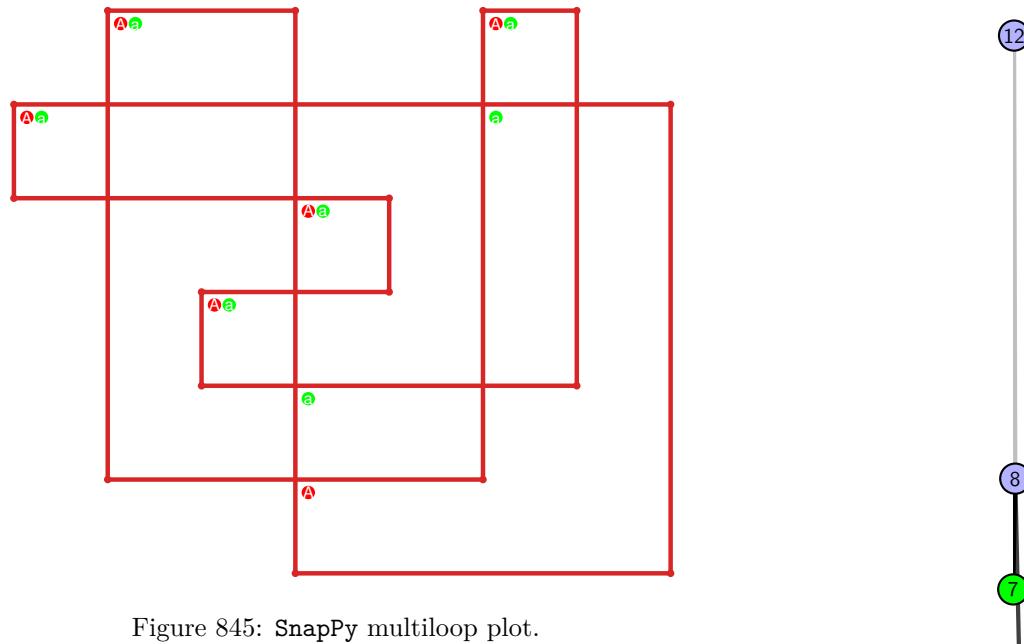


Figure 845: `SnapPy` multiloop plot.

Figure 846: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.275  $[[20, 7, 1, 8], [8, 19, 9, 20], [9, 6, 10, 7], [1, 14, 2, 15], [18, 11, 19, 12], [5, 10, 6, 11], [13, 4, 14, 5], [2, 16, 3, 15], [12, 17, 13, 18], [3, 16, 4, 17]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (6, 3, 7, 4), (17, 4, 18, 5), (2, 7, 3, 8), (15, 8, 16, 9), (18, 11, 19, 12), (12, 19, 13, 20), (20, 13, 1, 14), (9, 14, 10, 15), (5, 16, 6, 17)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 6, 7, 7], [1, 8, 8, 5], [2, 4, 6, 2], [3, 5, 8, 9], [3, 9, 9, 3], [4, 9, 6, 4], [6, 8, 7, 7]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 120  
 Pinning number: 6

Average optimal degree: 2.25  
 Average minimal degree: 2.25  
 Average overall degree: 2.92

Table 422: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	34	35	21	7	1	116
Average degree	2.25	2.6	2.85	3.02	3.14	3.25	3.33	

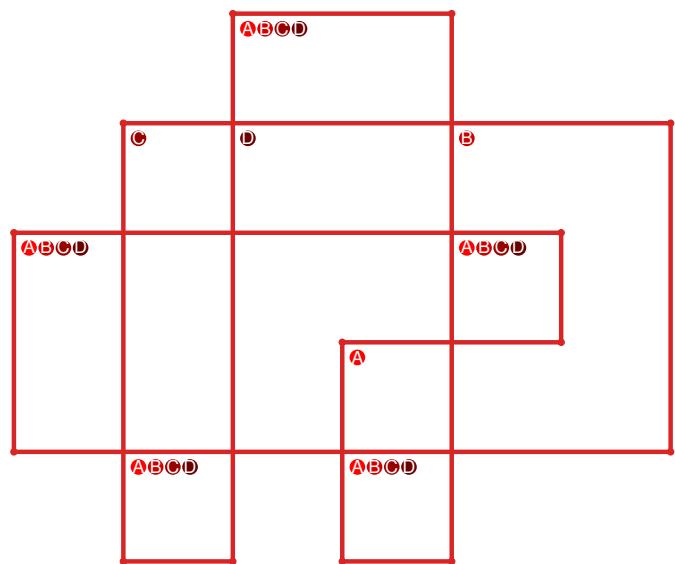


Figure 847: SnapPy multiloop plot.

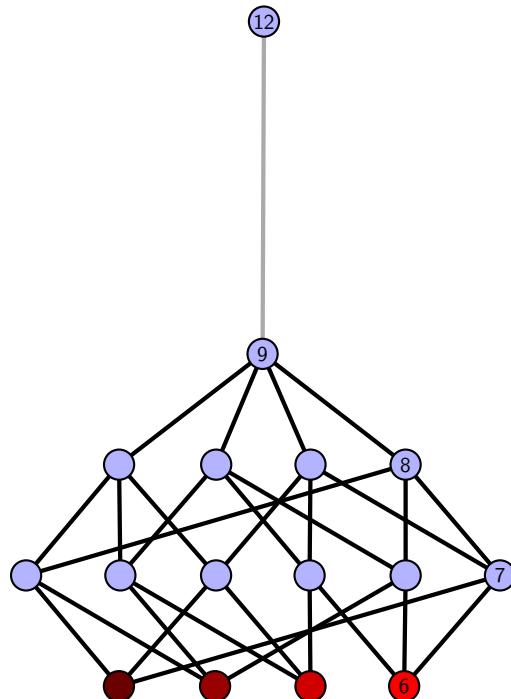


Figure 848: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.276 [[20, 9, 1, 10], [10, 19, 11, 20], [11, 8, 12, 9], [1, 12, 2, 13], [5, 18, 6, 19], [7, 14, 8, 15], [2, 14, 3, 13], [17, 4, 18, 5], [6, 16, 7, 15], [3, 16, 4, 17]]

PD code drawn by SnapPy: [(6, 3, 7, 4), (15, 4, 16, 5), (16, 7, 17, 8), (8, 17, 9, 18), (2, 9, 3, 10), (10, 1, 11, 2), (18, 11, 19, 12), (20, 13, 1, 14), (5, 14, 6, 15), (12, 19, 13, 20)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 6], [1, 7, 7, 8], [2, 8, 8, 6], [3, 5, 9, 3], [4, 9, 9, 4], [4, 9, 5, 5], [6, 8, 7, 7]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 423: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

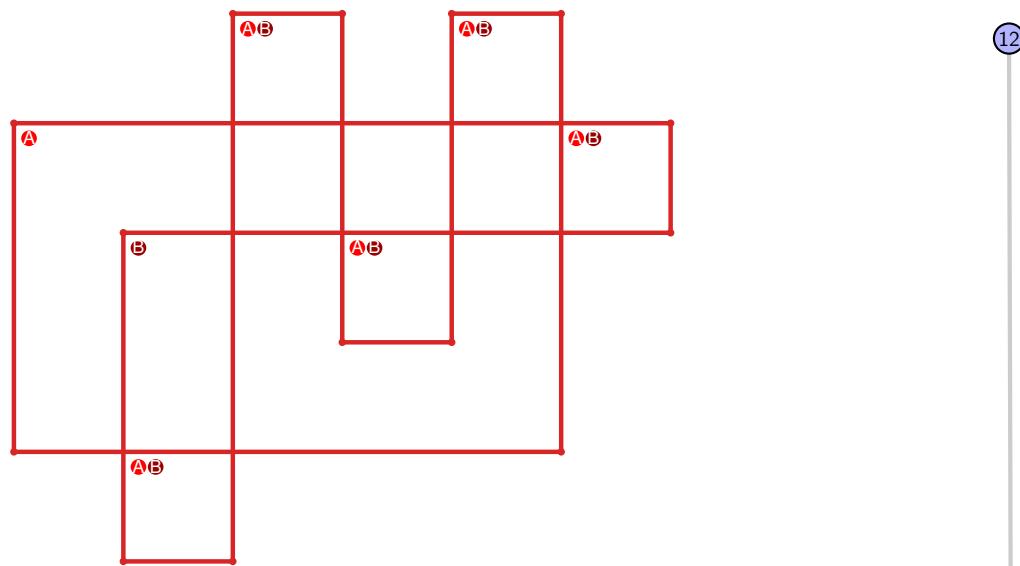


Figure 849: SnapPy multiloop plot.

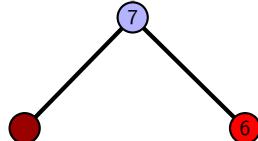


Figure 850: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.277  $[[11, 20, 12, 1], [17, 10, 18, 11], [19, 2, 20, 3], [12, 2, 13, 1], [9, 16, 10, 17], [18, 4, 19, 3], [13, 7, 14, 6], [15, 8, 16, 9], [4, 8, 5, 7], [14, 5, 15, 6]]$

PD code drawn by SnapPy:  $[(20, 7, 1, 8), (1, 18, 2, 19), (9, 2, 10, 3), (11, 4, 12, 5), (3, 10, 4, 11), (5, 14, 6, 15), (15, 6, 16, 7), (13, 16, 14, 17), (17, 12, 18, 13), (8, 19, 9, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 5], [0, 5, 5, 3], [0, 2, 6, 0], [1, 7, 7, 1], [1, 8, 2, 2], [3, 8, 9, 9], [4, 9, 8, 4], [5, 7, 9, 6], [6, 8, 7, 6]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 424: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

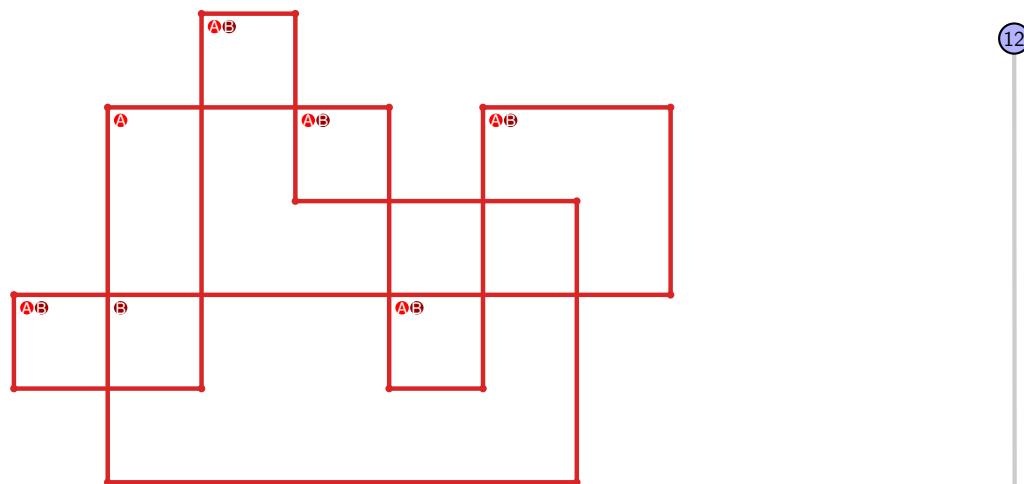


Figure 851: SnapPy multiloop plot.

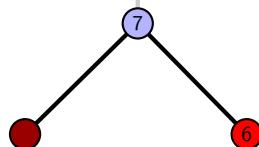


Figure 852: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.278  $[[5, 20, 6, 1], [4, 17, 5, 18], [19, 16, 20, 17], [6, 16, 7, 15], [1, 10, 2, 11], [18, 3, 19, 4], [7, 14, 8, 15], [9, 12, 10, 13], [2, 12, 3, 11], [13, 8, 14, 9]]$

PD code drawn by `SnapPy`:  $[(15, 20, 16, 1), (10, 5, 11, 6), (18, 7, 19, 8), (8, 19, 9, 20), (6, 9, 7, 10), (4, 11, 5, 12), (12, 3, 13, 4), (16, 13, 17, 14), (1, 14, 2, 15), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 7, 8, 8], [1, 8, 2, 1], [3, 9, 9, 3], [4, 9, 9, 8], [4, 7, 5, 4], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.23

**Total pinning sets:** 80

**Average overall degree:** 2.91

**Pinning number:** 6

Table 425: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	6	19	26	19	7	1	78
Average degree	2.17	2.47	2.73	2.94	3.12	3.25	3.33	

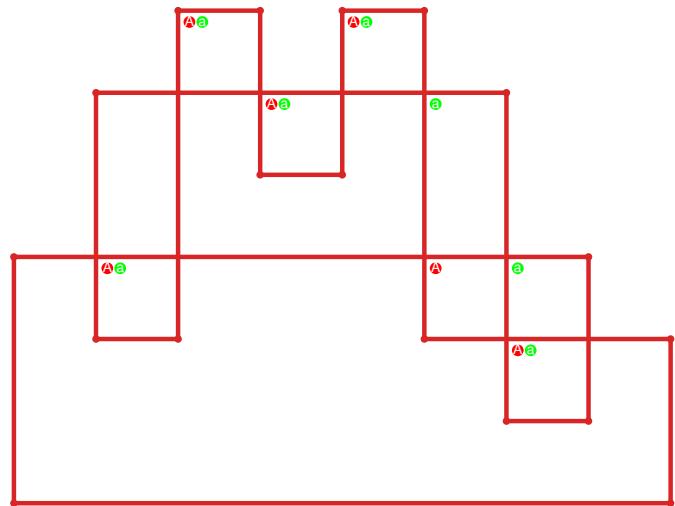


Figure 853: `SnapPy` multiloop plot.



Figure 854: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.279  $[[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 1, 9, 2], [18, 12, 19, 11], [6, 15, 7, 16], [2, 7, 3, 8], [12, 5, 13, 6], [14, 3, 15, 4], [4, 13, 5, 14]]$

PD code drawn by SnapPy:  $[(7, 20, 8, 1), (17, 2, 18, 3), (15, 4, 16, 5), (6, 13, 7, 14), (19, 8, 20, 9), (1, 10, 2, 11), (14, 11, 15, 12), (12, 5, 13, 6), (3, 16, 4, 17), (9, 18, 10, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 6, 0], [1, 7, 2, 1], [2, 7, 8, 6], [3, 5, 8, 3], [4, 9, 9, 5], [5, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.17

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.17

**Total pinning sets:** 96

**Average overall degree:** 2.91

**Pinning number:** 6

Table 426: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	11	25	30	20	7	1	94
Average degree	2.17	2.52	2.78	2.98	3.13	3.25	3.33	

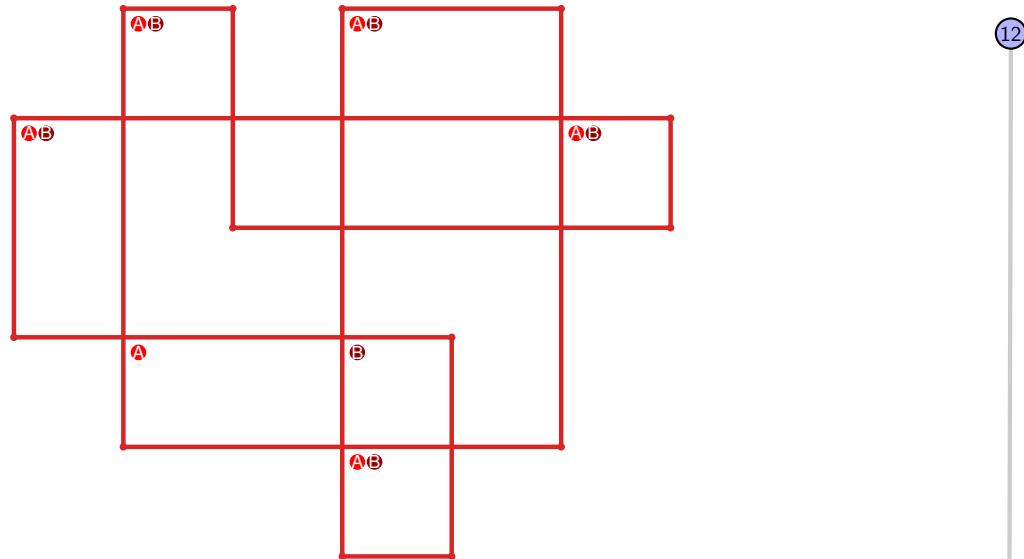


Figure 855: SnapPy multiloop plot.

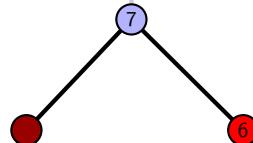


Figure 856: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.280**  $[[5, 20, 6, 1], [11, 4, 12, 5], [12, 19, 13, 20], [6, 13, 7, 14], [1, 16, 2, 17], [17, 10, 18, 11], [18, 3, 19, 4], [7, 15, 8, 14], [8, 15, 9, 16], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(15, 20, 16, 1), (8, 1, 9, 2), (19, 6, 20, 7), (14, 7, 15, 8), (3, 10, 4, 11), (11, 4, 12, 5), (5, 12, 6, 13), (18, 13, 19, 14), (9, 16, 10, 17), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 3], [0, 2, 7, 7], [0, 8, 9, 5], [1, 4, 9, 6], [1, 5, 9, 2], [3, 8, 8, 3], [4, 7, 7, 9], [4, 8, 6, 5]]$

Total optimal pinning sets: 1  
Total minimal pinning sets: 7

Total pinning sets: 264

Pinning number: 5

Average optimal degree: 2.6

Average minimal degree: 2.68

Average overall degree: 3.1

Table 427: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	1	0	0	0	0	0	6
Nonminimal pinning sets	0	7	44	80	76	39	10	1	257
Average degree	2.6	2.75	2.93	3.07	3.18	3.26	3.31	3.33	

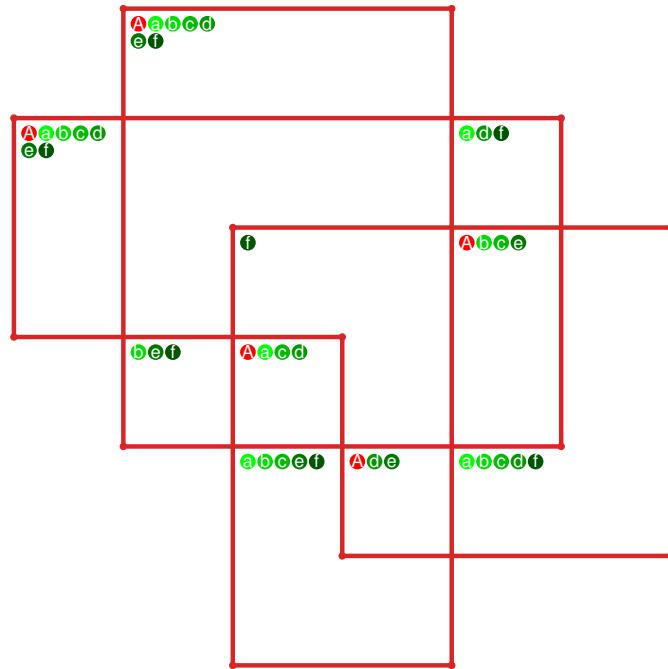


Figure 857: `SnapPy` multiloop plot.

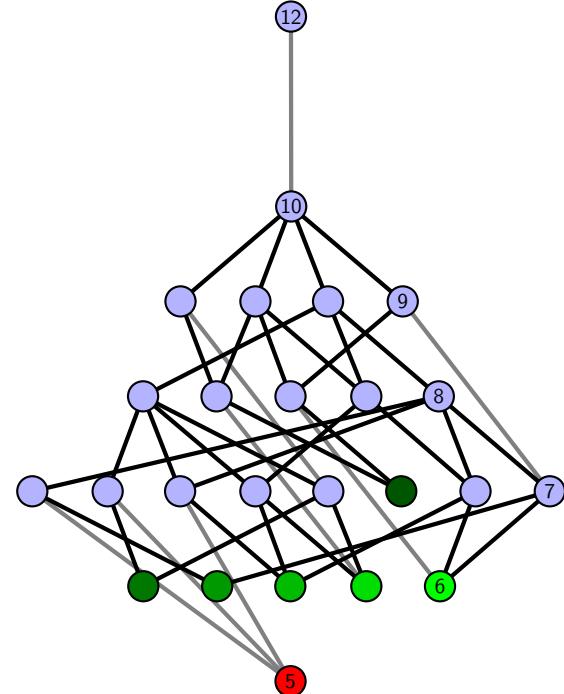


Figure 858: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.281  $[[5, 20, 6, 1], [4, 9, 5, 10], [19, 8, 20, 9], [6, 14, 7, 13], [1, 16, 2, 17], [10, 17, 11, 18], [18, 3, 19, 4], [7, 14, 8, 15], [15, 12, 16, 13], [2, 12, 3, 11]]$

PD code drawn by SnapPy:  $[(15, 20, 16, 1), (16, 5, 17, 6), (1, 6, 2, 7), (8, 19, 9, 20), (12, 9, 13, 10), (10, 3, 11, 4), (4, 11, 5, 12), (18, 13, 19, 14), (7, 14, 8, 15), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 7], [0, 7, 7, 8], [0, 8, 9, 5], [1, 4, 9, 6], [1, 5, 9, 2], [2, 8, 3, 3], [3, 7, 9, 4], [4, 8, 6, 5]]$

Total optimal pinning sets: 1

Average optimal degree: 2.75

Total minimal pinning sets: 12

Average minimal degree: 2.92

Total pinning sets: 612

Average overall degree: 3.19

Pinning number: 4

Table 428: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	9	2	0	0	0	0	0	0	11
Nonminimal pinning sets	0	8	73	155	179	123	50	11	1	600
Average degree	2.75	2.93	3.05	3.14	3.21	3.26	3.3	3.32	3.33	

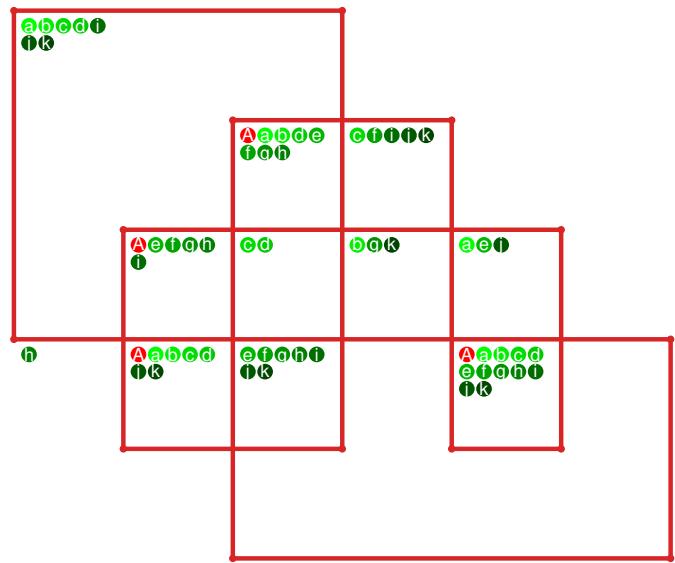


Figure 859: SnapPy multiloop plot.

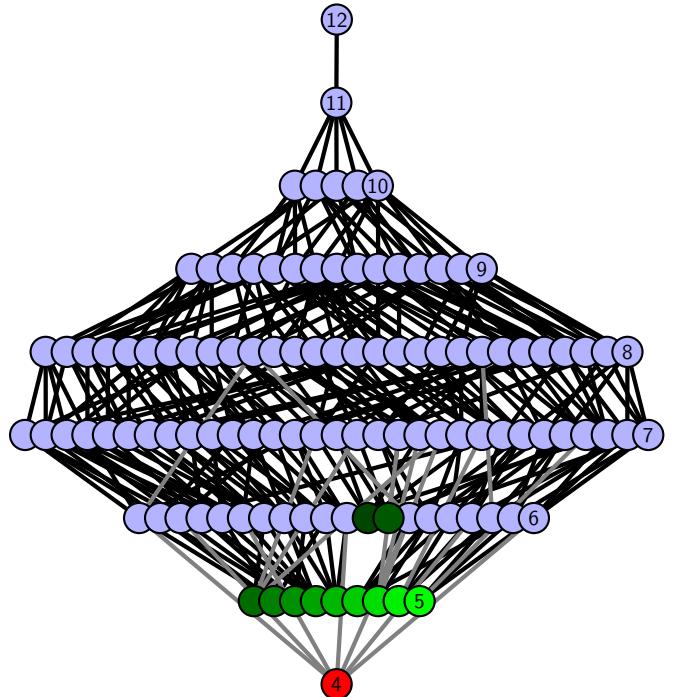


Figure 860: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.282 [[13, 20, 14, 1], [17, 12, 18, 13], [6, 19, 7, 20], [14, 7, 15, 8], [1, 8, 2, 9], [9, 16, 10, 17], [4, 11, 5, 12], [18, 5, 19, 6], [15, 3, 16, 2], [10, 3, 11, 4]]

PD code drawn by SnapPy: [(7, 20, 8, 1), (15, 2, 16, 3), (11, 4, 12, 5), (18, 5, 19, 6), (1, 8, 2, 9), (14, 9, 15, 10), (6, 13, 7, 14), (3, 16, 4, 17), (10, 17, 11, 18), (19, 12, 20, 13)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 7, 3], [0, 2, 8, 4], [0, 3, 8, 5], [1, 4, 8, 9], [1, 9, 9, 7], [1, 6, 2, 2], [3, 9, 5, 4], [5, 8, 6, 6]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 10  
 Total pinning sets: 344  
 Pinning number: 5

Average optimal degree: 2.6  
 Average minimal degree: 2.75  
 Average overall degree: 3.11

Table 429: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	8	0	0	0	0	0	0	8
Nonminimal pinning sets	0	14	71	108	89	41	10	1	334
Average degree	2.6	2.8	2.98	3.12	3.21	3.27	3.31	3.33	

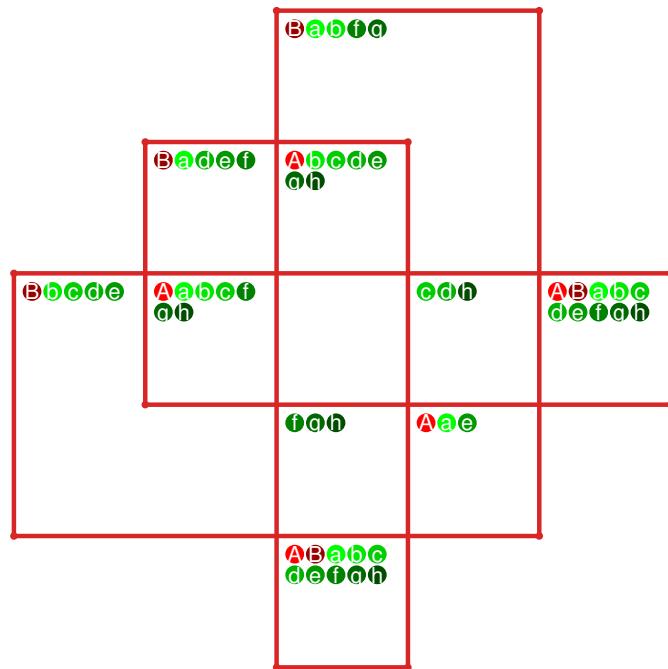


Figure 861: SnapPy multiloop plot.

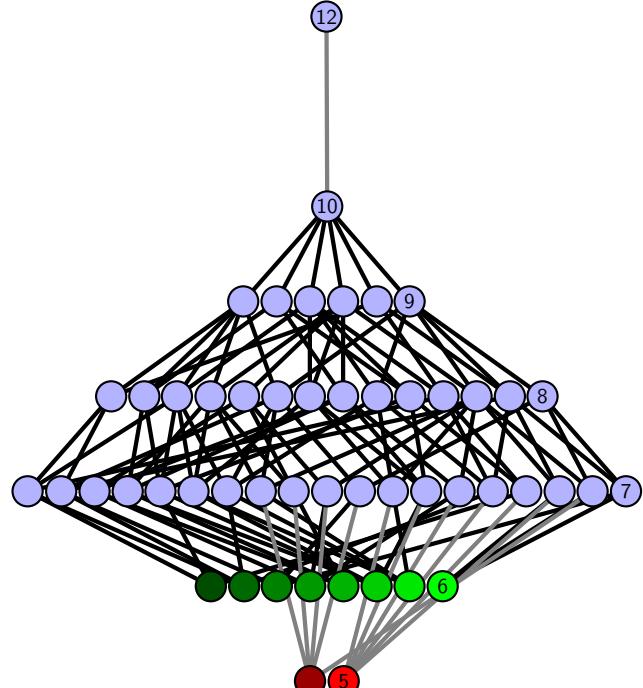


Figure 862: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.283  $[[5, 20, 6, 1], [17, 4, 18, 5], [12, 19, 13, 20], [6, 13, 7, 14], [1, 14, 2, 15], [9, 16, 10, 17], [10, 3, 11, 4], [18, 11, 19, 12], [7, 3, 8, 2], [15, 8, 16, 9]]$

PD code drawn by SnapPy:  $[(20, 5, 1, 6), (8, 1, 9, 2), (15, 2, 16, 3), (13, 6, 14, 7), (16, 9, 17, 10), (4, 11, 5, 12), (19, 12, 20, 13), (7, 14, 8, 15), (10, 17, 11, 18), (3, 18, 4, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 7, 3], [0, 2, 8, 4], [0, 3, 8, 9], [1, 9, 9, 6], [1, 5, 8, 7], [1, 6, 2, 2], [3, 6, 9, 4], [4, 8, 5, 5]]$

Total optimal pinning sets: 2  
 Total minimal pinning sets: 9  
 Total pinning sets: 336  
 Pinning number: 5

Average optimal degree: 2.6  
 Average minimal degree: 2.73  
 Average overall degree: 3.11

Table 430: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	7	0	0	0	0	0	0	7
Nonminimal pinning sets	0	14	68	105	88	41	10	1	327
Average degree	2.6	2.79	2.97	3.11	3.2	3.27	3.31	3.33	

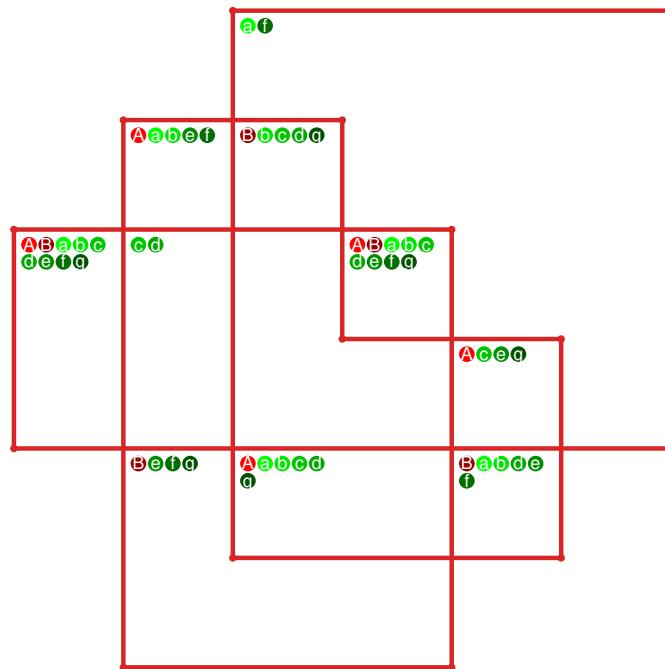


Figure 863: SnapPy multiloop plot.

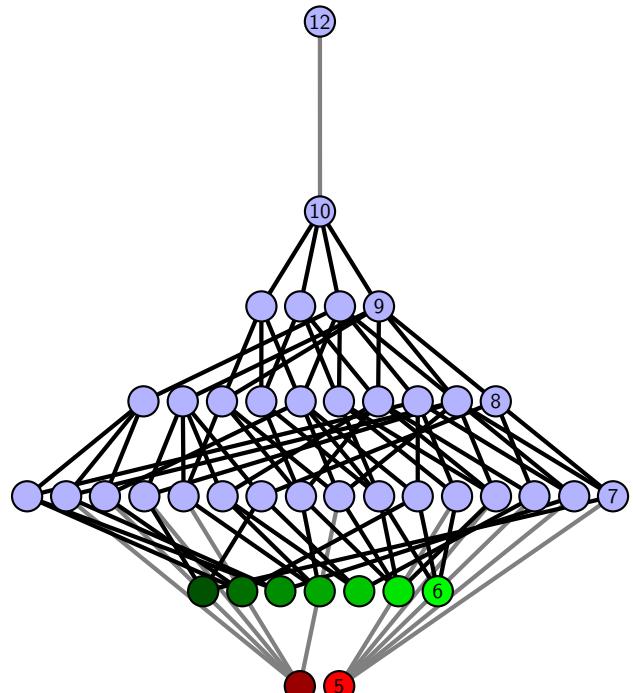


Figure 864: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.284 [[5, 20, 6, 1], [11, 4, 12, 5], [19, 6, 20, 7], [1, 14, 2, 15], [15, 10, 16, 11], [16, 3, 17, 4], [12, 17, 13, 18], [7, 18, 8, 19], [8, 13, 9, 14], [2, 9, 3, 10]]

PD code drawn by `SnapPy`: [(13, 20, 14, 1), (16, 3, 17, 4), (9, 4, 10, 5), (2, 7, 3, 8), (15, 8, 16, 9), (6, 11, 7, 12), (1, 12, 2, 13), (19, 14, 20, 15), (10, 17, 11, 18), (5, 18, 6, 19)]

Planar representation generated by `plantri`: [[1, 2, 2, 3], [0, 4, 5, 6], [0, 7, 7, 0], [0, 8, 9, 4], [1, 3, 9, 5], [1, 4, 9, 6], [1, 5, 8, 7], [2, 6, 8, 2], [3, 7, 6, 9], [3, 8, 5, 4]]

**Total optimal pinning sets:** 4  
**Total minimal pinning sets:** 6  
**Total pinning sets:** 360  
**Pinning number:** 5

**Average optimal degree:** 2.65  
**Average minimal degree:** 2.66  
**Average overall degree:** 3.11

Table 431: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	26	77	110	89	41	10	1	354
Average degree	2.65	2.85	3.0	3.12	3.21	3.27	3.31	3.33	

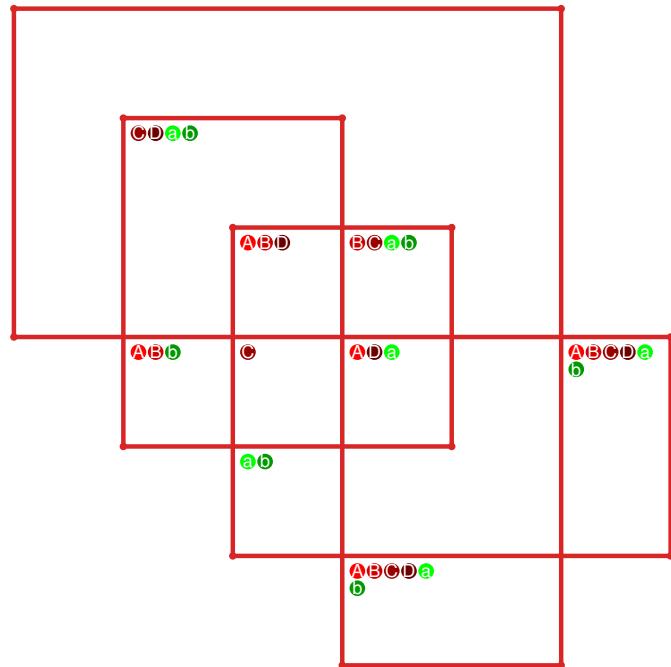


Figure 865: `SnapPy` multiloop plot.

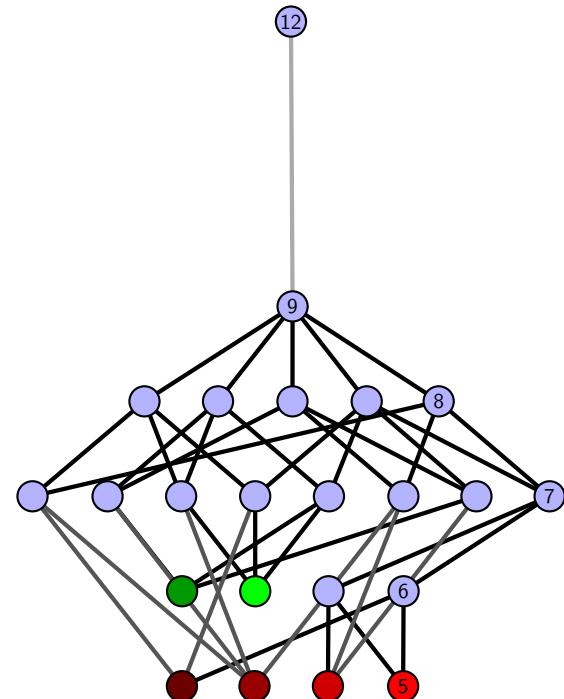


Figure 866: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.285 [[20, 13, 1, 14], [14, 5, 15, 6], [12, 19, 13, 20], [1, 8, 2, 9], [4, 15, 5, 16], [6, 11, 7, 12], [7, 18, 8, 19], [2, 18, 3, 17], [9, 17, 10, 16], [10, 3, 11, 4]]

PD code drawn by SnapPy: [(9, 20, 10, 1), (16, 3, 17, 4), (1, 6, 2, 7), (15, 8, 16, 9), (19, 10, 20, 11), (11, 4, 12, 5), (12, 17, 13, 18), (2, 13, 3, 14), (7, 14, 8, 15), (5, 18, 6, 19)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 4, 4, 5], [0, 5, 6, 0], [0, 6, 7, 8], [1, 8, 9, 1], [1, 9, 6, 2], [2, 5, 7, 3], [3, 6, 9, 8], [3, 7, 9, 4], [4, 8, 7, 5]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 6

**Average minimal degree:** 2.68

**Total pinning sets:** 464

**Average overall degree:** 3.11

**Pinning number:** 4

Table 432: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	0	0	0	0	0	0	0	5
Nonminimal pinning sets	0	8	54	112	134	97	42	10	1	458
Average degree	2.5	2.74	2.92	3.05	3.15	3.22	3.27	3.31	3.33	

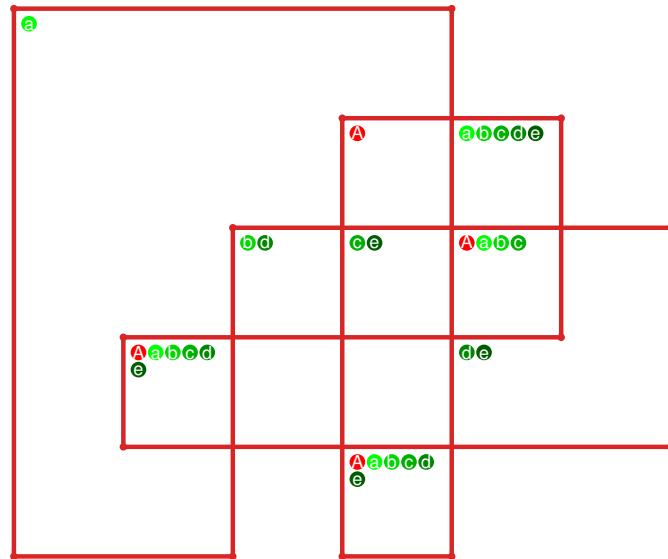


Figure 867: SnapPy multiloop plot.

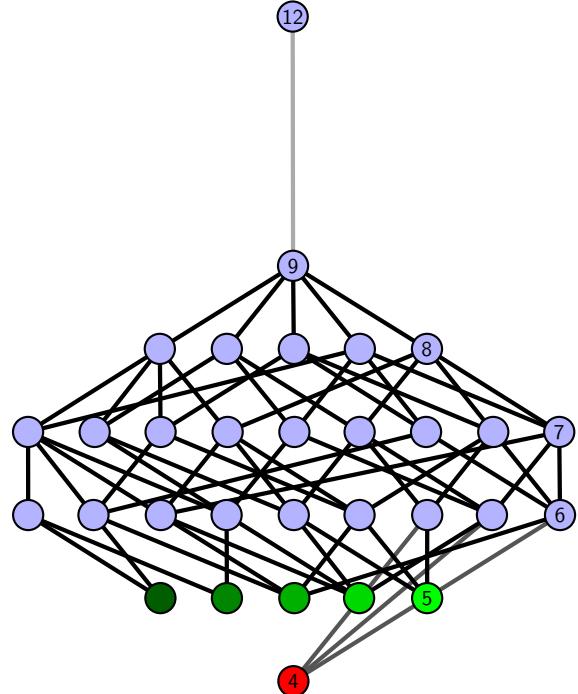


Figure 868: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.286** [[20, 11, 1, 12], [12, 7, 13, 8], [8, 19, 9, 20], [10, 3, 11, 4], [1, 16, 2, 17], [17, 6, 18, 7], [13, 18, 14, 19], [9, 5, 10, 4], [15, 2, 16, 3], [5, 14, 6, 15]]

PD code drawn by `SnapPy`: [(13, 20, 14, 1), (8, 1, 9, 2), (2, 7, 3, 8), (15, 4, 16, 5), (10, 5, 11, 6), (18, 9, 19, 10), (3, 12, 4, 13), (19, 14, 20, 15), (11, 16, 12, 17), (6, 17, 7, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 7], [0, 7, 7, 8], [0, 8, 8, 5], [1, 4, 9, 6], [1, 5, 9, 2], [2, 9, 3, 3], [3, 9, 4, 4], [5, 8, 7, 6]]

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3

**Total pinning sets:** 512

**Pinning number:** 4

**Average optimal degree:** 2.5

**Average minimal degree:** 2.5

**Average overall degree:** 3.11

Table 433: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	22	70	126	140	98	42	10	1	509
Average degree	2.5	2.76	2.94	3.06	3.15	3.22	3.27	3.31	3.33	

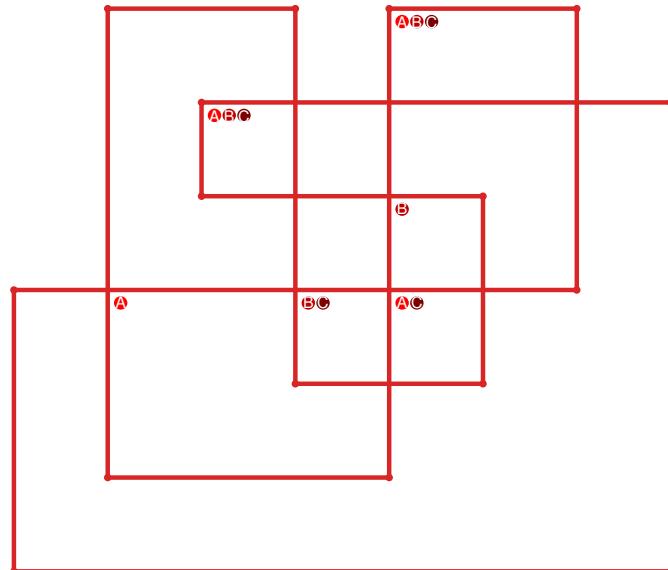


Figure 869: `SnapPy` multiloop plot.

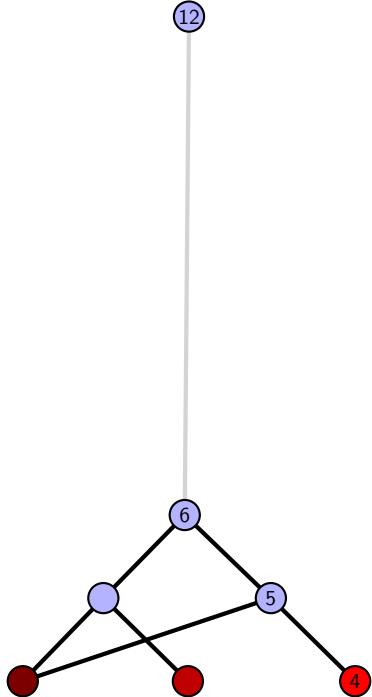


Figure 870: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.287** `[[7, 20, 8, 1], [11, 6, 12, 7], [19, 4, 20, 5], [8, 4, 9, 3], [1, 16, 2, 17], [17, 10, 18, 11], [5, 12, 6, 13], [13, 18, 14, 19], [9, 14, 10, 15], [15, 2, 16, 3]]`

PD code drawn by `SnapPy`: `[(20, 5, 1, 6), (16, 1, 17, 2), (8, 3, 9, 4), (14, 7, 15, 8), (2, 9, 3, 10), (6, 11, 7, 12), (18, 13, 19, 14), (10, 15, 11, 16), (4, 17, 5, 18), (12, 19, 13, 20)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 6, 6], [0, 6, 7, 3], [0, 2, 8, 9], [0, 9, 9, 5], [1, 4, 8, 7], [1, 7, 2, 1], [2, 6, 5, 8], [3, 7, 5, 9], [3, 8, 4, 4]]`

**Total optimal pinning sets:** 3

**Average optimal degree:** 2.83

**Total minimal pinning sets:** 15

**Average minimal degree:** 2.86

**Total pinning sets:** 708

**Average overall degree:** 3.15

**Pinning number:** 4

Table 434: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	12	0	0	0	0	0	0	0	12
Nonminimal pinning sets	0	21	113	193	192	118	45	10	1	693
Average degree	2.83	2.95	3.04	3.13	3.19	3.24	3.28	3.31	3.33	

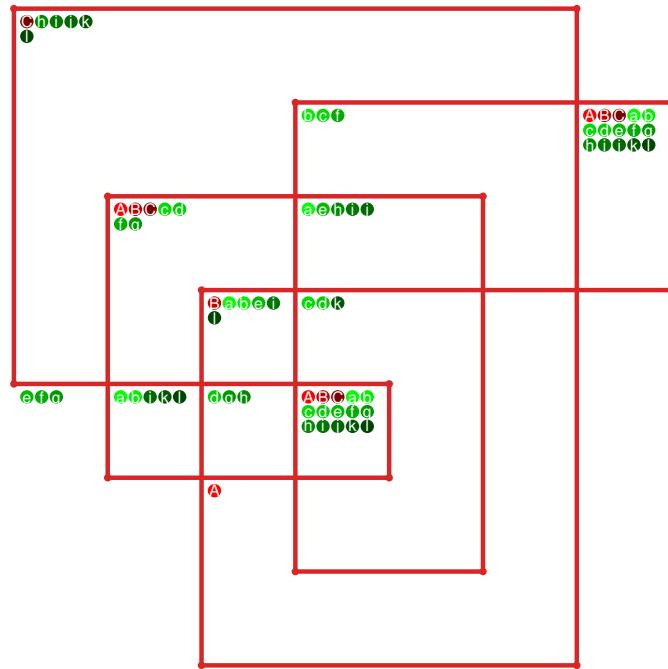


Figure 871: `SnapPy` multiloop plot.

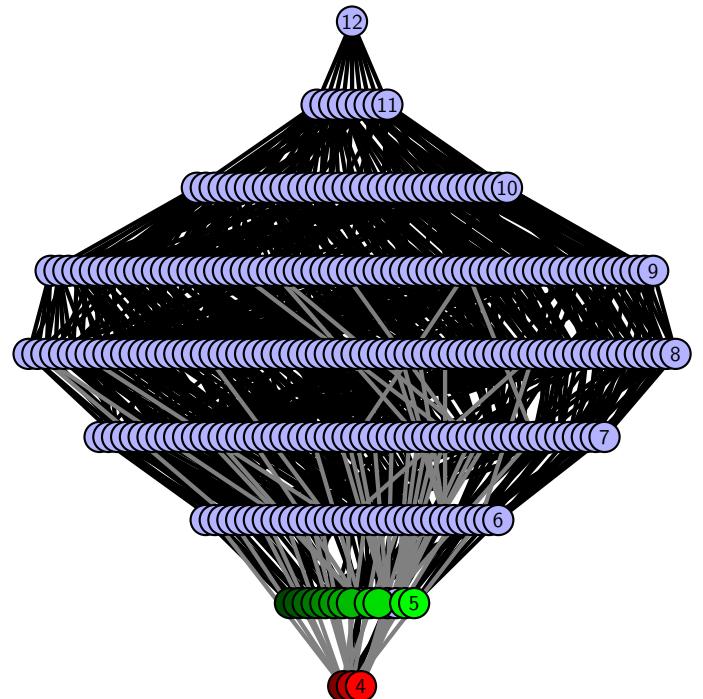


Figure 872: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.288 [[13, 20, 14, 1], [17, 12, 18, 13], [19, 4, 20, 5], [14, 4, 15, 3], [1, 9, 2, 8], [16, 7, 17, 8], [11, 6, 12, 7], [18, 6, 19, 5], [15, 11, 16, 10], [2, 9, 3, 10]]

PD code drawn by SnapPy: [(1, 12, 2, 13), (17, 2, 18, 3), (3, 16, 4, 17), (9, 4, 10, 5), (14, 5, 15, 6), (6, 19, 7, 20), (20, 7, 1, 8), (8, 13, 9, 14), (15, 10, 16, 11), (11, 18, 12, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 7, 3], [0, 2, 8, 9], [0, 9, 9, 5], [1, 4, 8, 6], [1, 5, 8, 7], [1, 6, 2, 2], [3, 6, 5, 9], [3, 8, 4, 4]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 8  
 Total pinning sets: 552  
 Pinning number: 4

Average optimal degree: 2.5  
 Average minimal degree: 2.78  
 Average overall degree: 3.12

Table 435: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	6	0	0	0	0	0	0	0	6
Nonminimal pinning sets	0	15	76	141	154	104	43	10	1	544
Average degree	2.5	2.79	2.96	3.08	3.16	3.23	3.27	3.31	3.33	

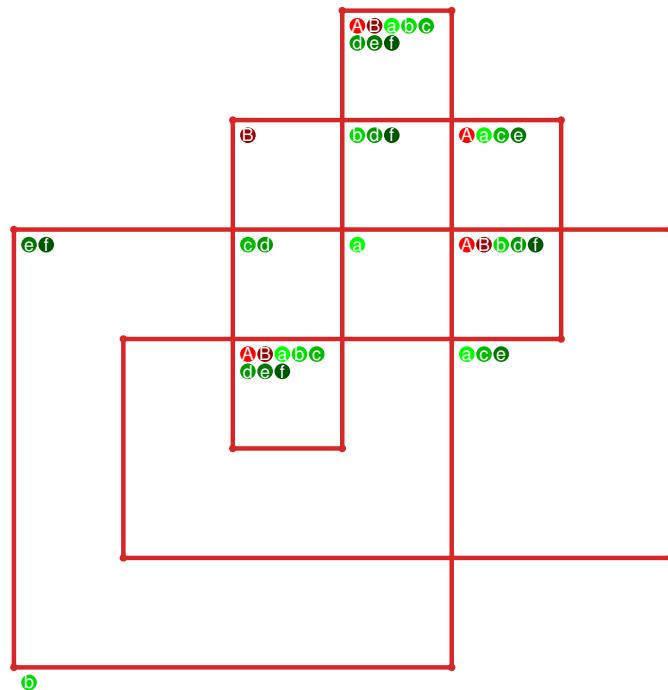


Figure 873: SnapPy multiloop plot.

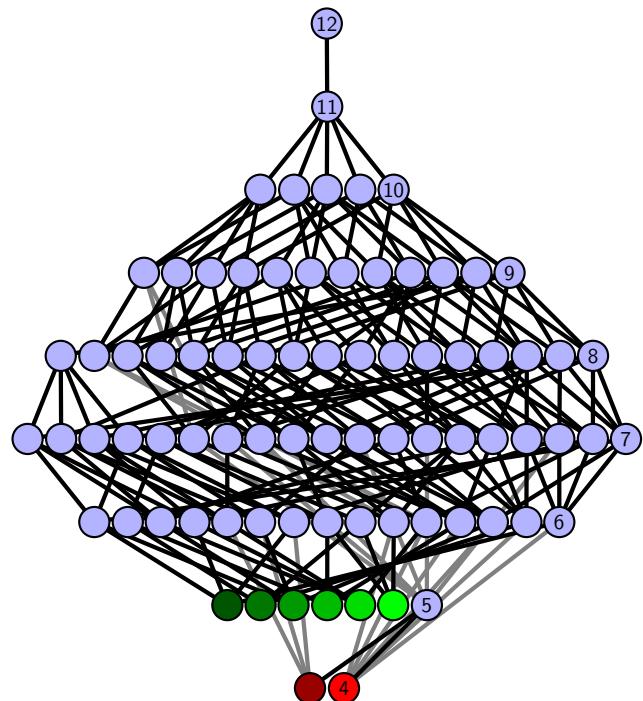


Figure 874: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.289**  $[[9, 20, 10, 1], [19, 8, 20, 9], [10, 2, 11, 1], [7, 18, 8, 19], [2, 16, 3, 15], [11, 15, 12, 14], [6, 13, 7, 14], [17, 4, 18, 5], [16, 4, 17, 3], [12, 5, 13, 6]]$

PD code drawn by `SnapPy`:  $[(1, 14, 2, 15), (15, 2, 16, 3), (10, 3, 11, 4), (17, 6, 18, 7), (4, 11, 5, 12), (12, 9, 13, 10), (13, 20, 14, 1), (5, 16, 6, 17), (7, 18, 8, 19), (19, 8, 20, 9)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 5, 0], [1, 6, 7, 1], [2, 8, 8, 5], [2, 4, 9, 6], [3, 5, 9, 9], [3, 9, 8, 8], [4, 7, 7, 4], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 436: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

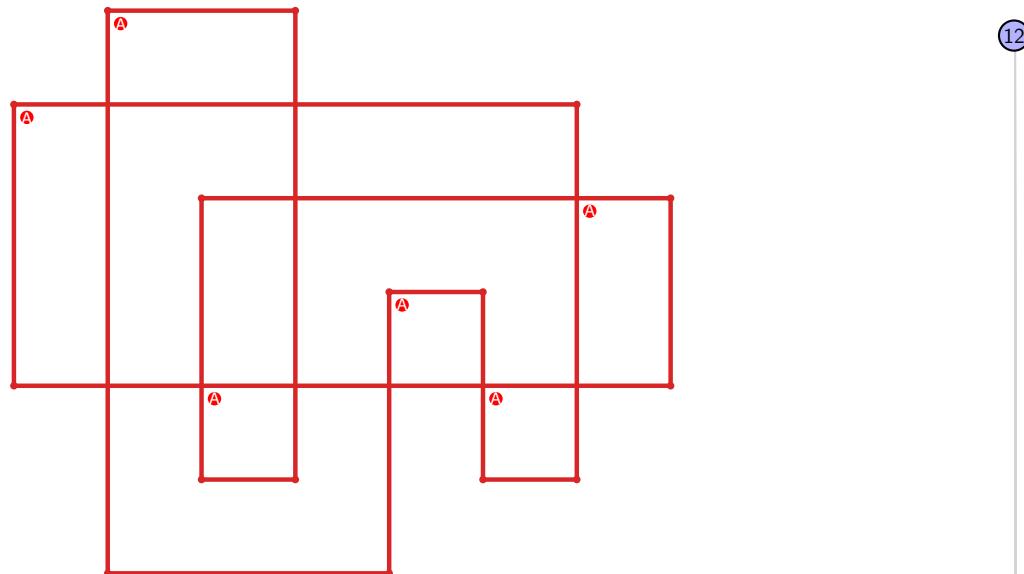


Figure 875: `SnapPy` multiloop plot.

6

Figure 876: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.290**  $[[9, 20, 10, 1], [8, 13, 9, 14], [19, 6, 20, 7], [10, 2, 11, 1], [14, 11, 15, 12], [12, 7, 13, 8], [5, 18, 6, 19], [2, 16, 3, 15], [17, 4, 18, 5], [16, 4, 17, 3]]$

PD code drawn by `SnapPy`:  $[(20, 13, 1, 14), (14, 1, 15, 2), (16, 3, 17, 4), (9, 4, 10, 5), (11, 8, 12, 9), (5, 10, 6, 11), (12, 19, 13, 20), (2, 15, 3, 16), (6, 17, 7, 18), (18, 7, 19, 8)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 5], [0, 5, 6, 6], [0, 7, 4, 0], [1, 3, 7, 5], [1, 4, 2, 1], [2, 8, 8, 2], [3, 9, 9, 4], [6, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 437: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

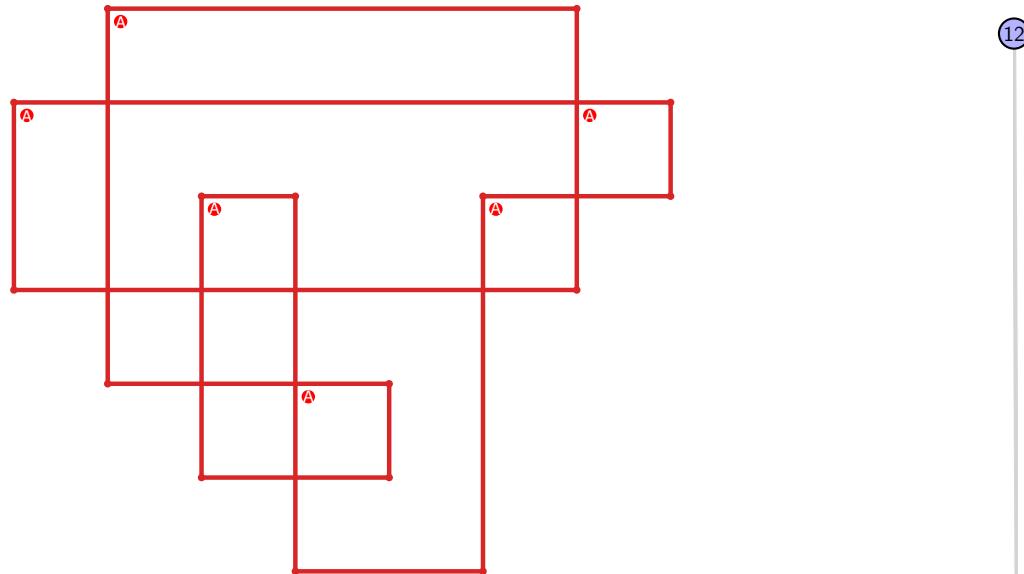


Figure 877: `SnapPy` multiloop plot.



Figure 878: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.291  $[[20, 5, 1, 6], [6, 15, 7, 16], [16, 19, 17, 20], [4, 11, 5, 12], [1, 9, 2, 8], [14, 7, 15, 8], [18, 13, 19, 14], [17, 13, 18, 12], [10, 3, 11, 4], [9, 3, 10, 2]]$

PD code drawn by `SnapPy`:  $[(20, 7, 1, 8), (8, 1, 9, 2), (18, 3, 19, 4), (12, 5, 13, 6), (6, 19, 7, 20), (2, 9, 3, 10), (10, 15, 11, 16), (16, 11, 17, 12), (4, 13, 5, 14), (14, 17, 15, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 7, 8, 8], [0, 9, 9, 5], [1, 4, 6, 1], [2, 5, 7, 7], [2, 6, 6, 3], [3, 9, 9, 3], [4, 8, 8, 4]]$

Total optimal pinning sets: 4  
 Total minimal pinning sets: 4  
 Total pinning sets: 120  
 Pinning number: 6

Average optimal degree: 2.25  
 Average minimal degree: 2.25  
 Average overall degree: 2.92

Table 438: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	34	35	21	7	1	116
Average degree	2.25	2.6	2.85	3.02	3.14	3.25	3.33	

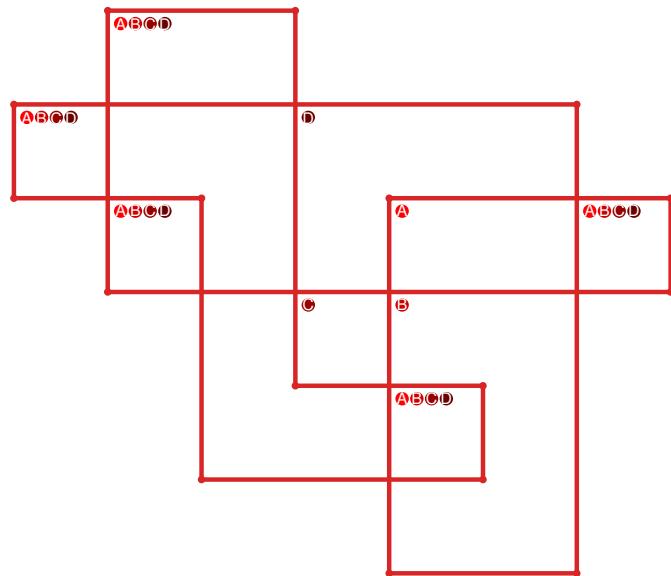


Figure 879: `SnapPy` multiloop plot.

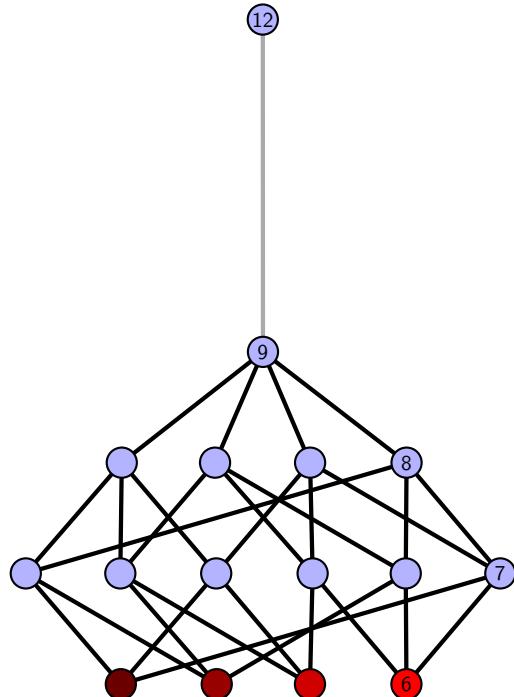


Figure 880: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.292** `[[20, 11, 1, 12], [12, 19, 13, 20], [10, 1, 11, 2], [18, 13, 19, 14], [2, 5, 3, 6], [6, 9, 7, 10], [14, 7, 15, 8], [4, 17, 5, 18], [3, 17, 4, 16], [8, 15, 9, 16]]`

PD code drawn by `SnapPy`: `[(20, 3, 1, 4), (12, 5, 13, 6), (4, 7, 5, 8), (18, 9, 19, 10), (16, 11, 17, 12), (6, 13, 7, 14), (14, 1, 15, 2), (2, 15, 3, 16), (10, 17, 11, 18), (8, 19, 9, 20)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 5, 0], [1, 6, 7, 1], [2, 7, 8, 5], [2, 4, 9, 6], [3, 5, 9, 9], [3, 8, 8, 4], [4, 7, 7, 9], [5, 8, 6, 6]]`

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 112  
 Pinning number: 6

Average optimal degree: 2.22  
 Average minimal degree: 2.22  
 Average overall degree: 2.92

Table 439: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	31	34	21	7	1	109
Average degree	2.22	2.57	2.82	3.01	3.14	3.25	3.33	

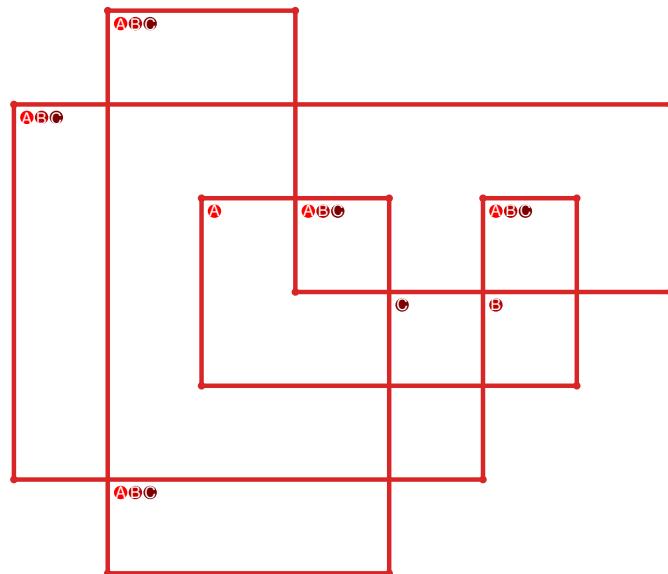


Figure 881: `SnapPy` multiloop plot.

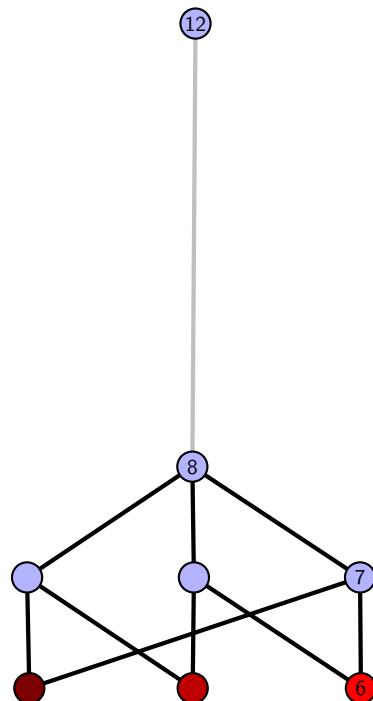


Figure 882: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.293  $[[13, 20, 14, 1], [12, 5, 13, 6], [19, 8, 20, 9], [14, 2, 15, 1], [6, 3, 7, 4], [4, 11, 5, 12], [9, 18, 10, 19], [7, 16, 8, 17], [2, 16, 3, 15], [17, 10, 18, 11]]$

PD code drawn by `SnapPy`:  $[(4, 1, 5, 2), (10, 3, 11, 4), (17, 6, 18, 7), (15, 8, 16, 9), (2, 9, 3, 10), (13, 20, 14, 1), (5, 14, 6, 15), (7, 16, 8, 17), (11, 18, 12, 19), (19, 12, 20, 13)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 7], [0, 8, 8, 0], [1, 8, 7, 5], [1, 4, 9, 1], [2, 9, 9, 2], [2, 9, 4, 8], [3, 7, 4, 3], [5, 7, 6, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 440: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

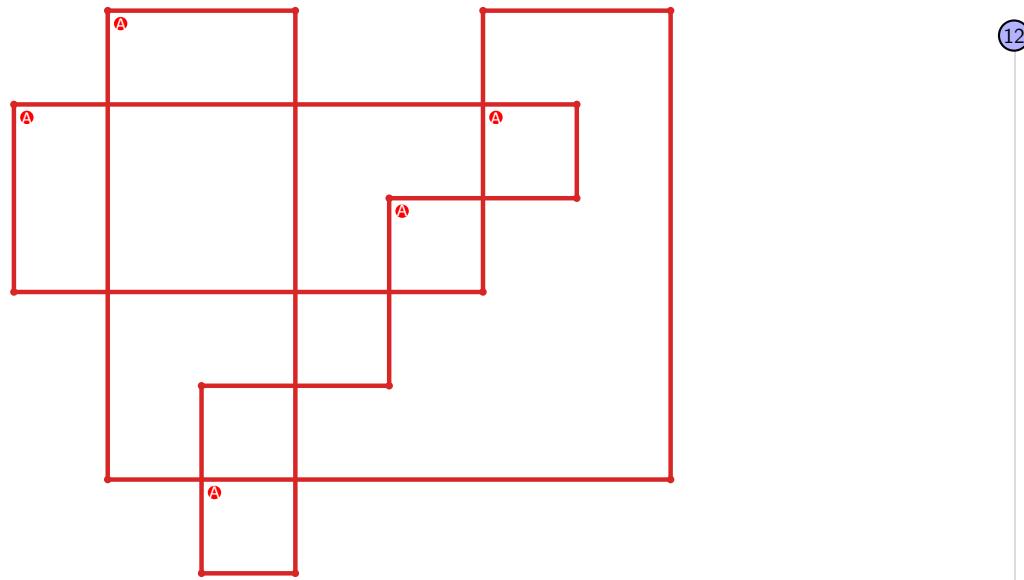


Figure 883: `SnapPy` multiloop plot.

5

Figure 884: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.294 [[20, 15, 1, 16], [16, 5, 17, 6], [19, 10, 20, 11], [14, 1, 15, 2], [4, 7, 5, 8], [17, 7, 18, 6], [11, 18, 12, 19], [12, 9, 13, 10], [2, 13, 3, 14], [8, 3, 9, 4]]

PD code drawn by `SnapPy`: [(5, 20, 6, 1), (13, 2, 14, 3), (3, 14, 4, 15), (1, 4, 2, 5), (9, 6, 10, 7), (16, 7, 17, 8), (19, 10, 20, 11), (17, 12, 18, 13), (8, 15, 9, 16), (11, 18, 12, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 7], [0, 8, 8, 0], [1, 9, 9, 5], [1, 4, 6, 1], [2, 5, 7, 2], [2, 6, 9, 8], [3, 7, 9, 3], [4, 8, 7, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 441: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

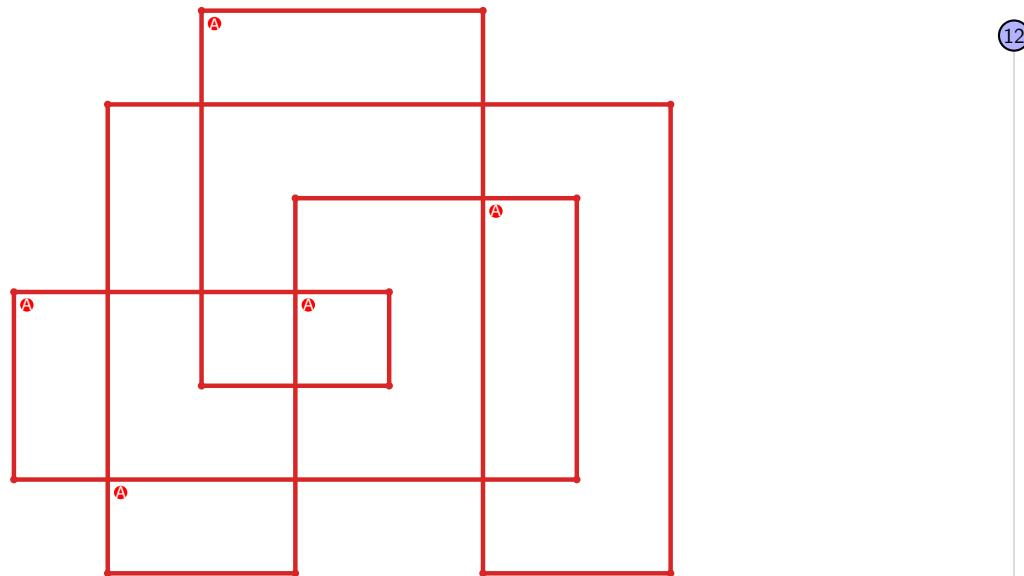


Figure 885: `SnapPy` multiloop plot.



Figure 886: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.295** `[[20, 13, 1, 14], [14, 19, 15, 20], [12, 9, 13, 10], [1, 9, 2, 8], [18, 5, 19, 6], [15, 11, 16, 10], [16, 11, 17, 12], [2, 7, 3, 8], [6, 3, 7, 4], [4, 17, 5, 18]]`

PD code drawn by `SnapPy`: `[(4, 1, 5, 2), (16, 3, 17, 4), (20, 5, 1, 6), (6, 19, 7, 20), (10, 7, 11, 8), (8, 13, 9, 14), (14, 9, 15, 10), (18, 11, 19, 12), (2, 15, 3, 16), (12, 17, 13, 18)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 5, 0], [0, 5, 6, 3], [0, 2, 7, 7], [1, 8, 9, 9], [1, 6, 6, 2], [2, 5, 5, 9], [3, 8, 8, 3], [4, 7, 7, 9], [4, 8, 6, 4]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 442: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

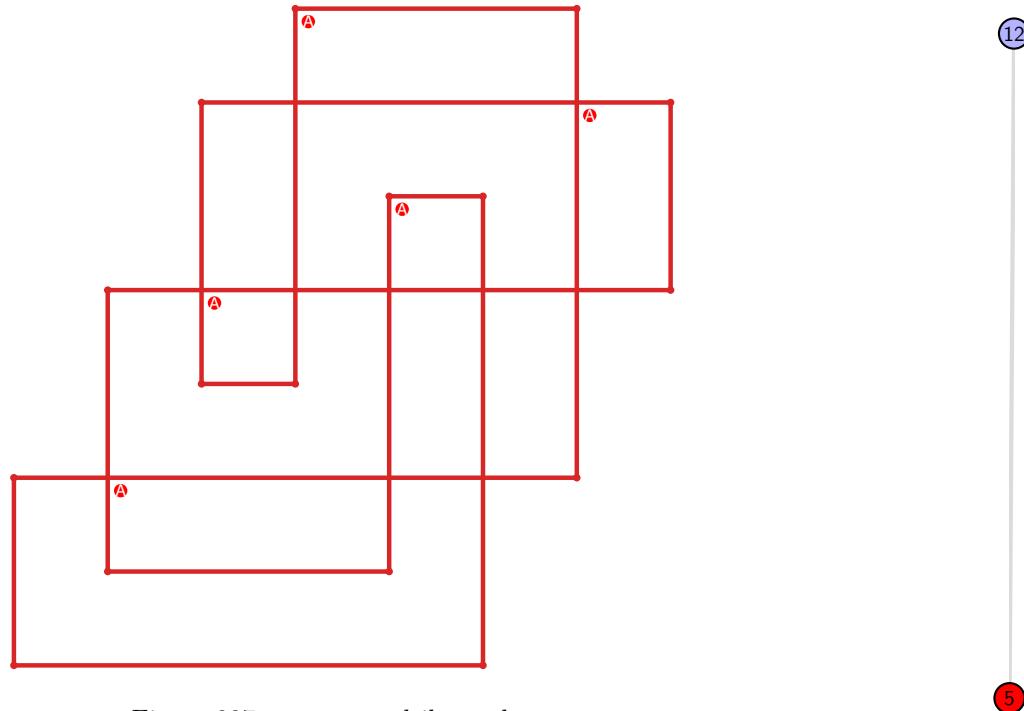


Figure 887: `SnapPy` multiloop plot.

Figure 888: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.296** [[20, 5, 1, 6], [6, 13, 7, 14], [14, 19, 15, 20], [4, 1, 5, 2], [12, 7, 13, 8], [18, 15, 19, 16], [2, 10, 3, 9], [3, 8, 4, 9], [11, 16, 12, 17], [17, 10, 18, 11]]

PD code drawn by `SnapPy`: [(18, 3, 19, 4), (10, 5, 11, 6), (1, 6, 2, 7), (7, 20, 8, 1), (15, 8, 16, 9), (4, 11, 5, 12), (16, 13, 17, 14), (9, 14, 10, 15), (12, 17, 13, 18), (2, 19, 3, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 5], [0, 6, 7, 0], [1, 7, 8, 1], [2, 8, 9, 2], [3, 9, 7, 7], [3, 6, 6, 4], [4, 9, 9, 5], [5, 8, 8, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 443: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

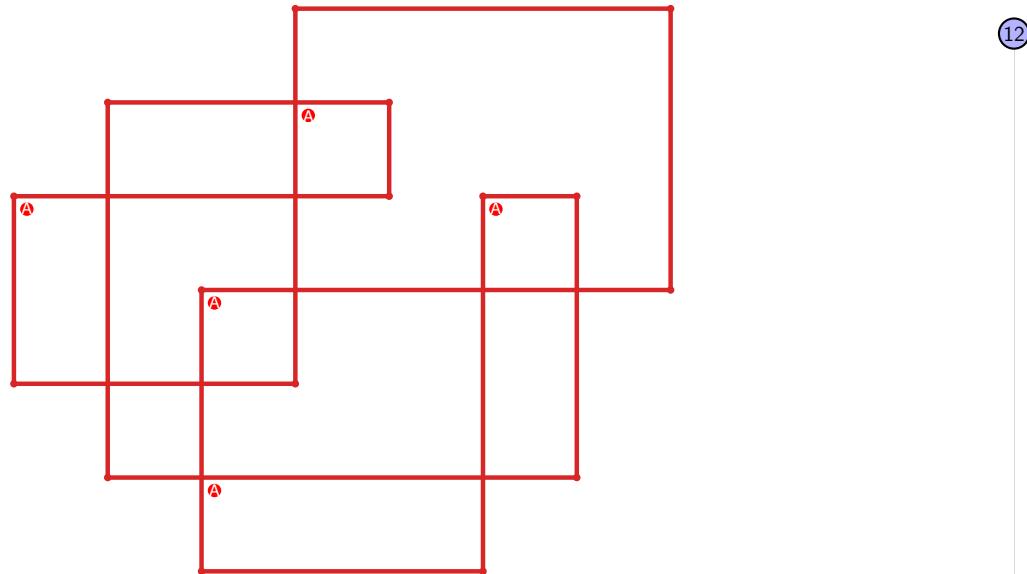


Figure 889: `SnapPy` multiloop plot.



Figure 890: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.297**  $[[7, 20, 8, 1], [15, 6, 16, 7], [16, 19, 17, 20], [8, 2, 9, 1], [5, 14, 6, 15], [18, 13, 19, 14], [17, 13, 18, 12], [2, 12, 3, 11], [9, 4, 10, 5], [3, 10, 4, 11]]$

PD code drawn by `SnapPy`:  $[(6, 1, 7, 2), (12, 3, 13, 4), (20, 7, 1, 8), (8, 19, 9, 20), (9, 14, 10, 15), (15, 10, 16, 11), (2, 11, 3, 12), (13, 16, 14, 17), (4, 17, 5, 18), (18, 5, 19, 6)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 6], [0, 7, 8, 0], [1, 8, 5, 1], [2, 4, 6, 6], [2, 5, 5, 7], [3, 6, 9, 9], [3, 9, 9, 4], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 128

**Average overall degree:** 2.91

**Pinning number:** 5

Table 444: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	7	21	35	35	21	7	1	127
Average degree	2.0	2.38	2.65	2.86	3.02	3.14	3.25	3.33	

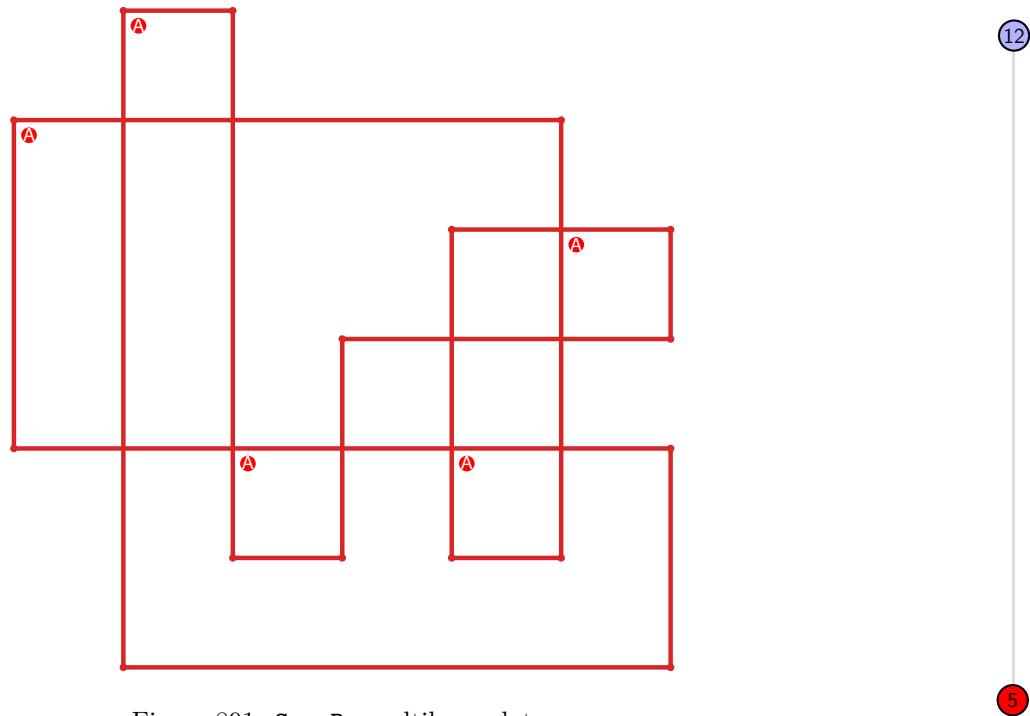


Figure 891: `SnapPy` multiloop plot.

Figure 892: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.298 [[20, 3, 1, 4], [4, 15, 5, 16], [16, 19, 17, 20], [2, 9, 3, 10], [1, 9, 2, 8], [14, 7, 15, 8], [5, 13, 6, 12], [18, 11, 19, 12], [17, 11, 18, 10], [6, 13, 7, 14]]

PD code drawn by SnapPy: [(18, 1, 19, 2), (12, 5, 13, 6), (6, 19, 7, 20), (20, 7, 1, 8), (8, 15, 9, 16), (16, 9, 17, 10), (10, 3, 11, 4), (4, 11, 5, 12), (2, 13, 3, 14), (14, 17, 15, 18)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 7, 8], [0, 8, 4, 4], [0, 3, 3, 5], [1, 4, 9, 9], [1, 9, 9, 7], [2, 6, 8, 8], [2, 7, 7, 3], [5, 6, 6, 5]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.4

**Total minimal pinning sets:** 6

**Average minimal degree:** 2.4

**Total pinning sets:** 208

**Average overall degree:** 2.99

**Pinning number:** 5

Table 445: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	5	0	0	0	0	0	0	5
Nonminimal pinning sets	0	7	40	63	55	28	8	1	202
Average degree	2.4	2.56	2.79	2.98	3.11	3.2	3.27	3.33	

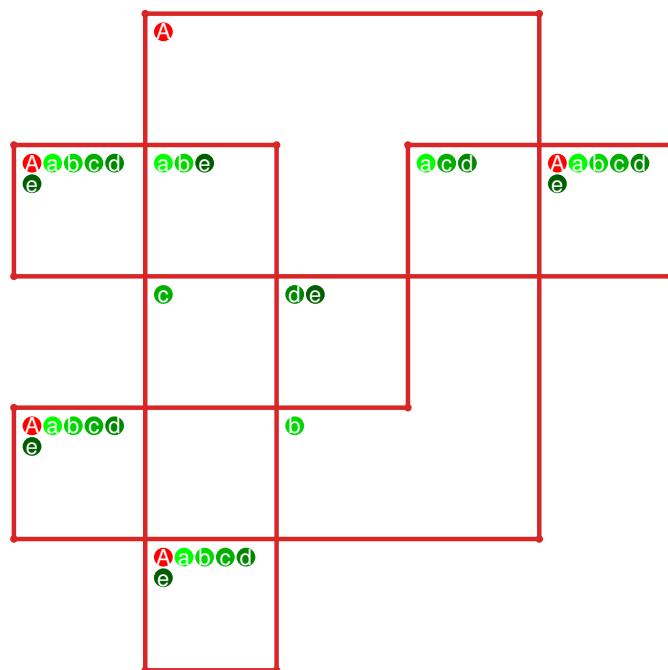


Figure 893: SnapPy multiloop plot.

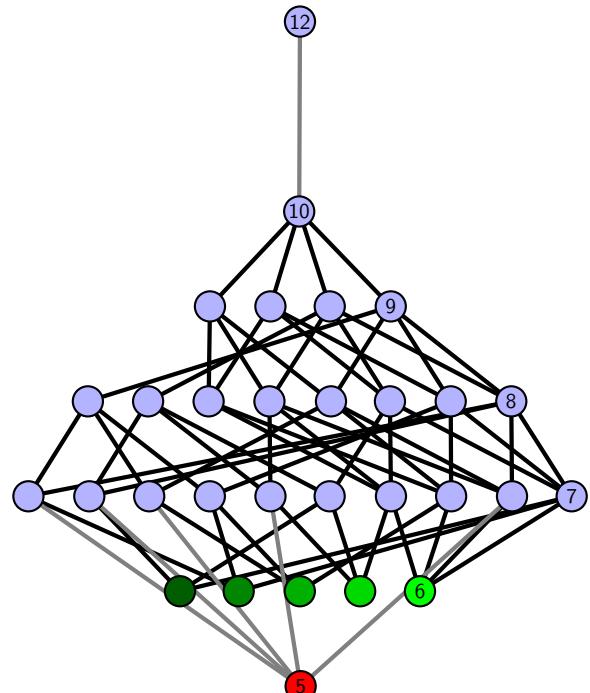


Figure 894: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.299**  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 16, 9, 15], [1, 15, 2, 14], [5, 18, 6, 19], [16, 10, 17, 9], [2, 11, 3, 12], [4, 13, 5, 14], [17, 10, 18, 11], [3, 13, 4, 12]]$

PD code drawn by SnapPy:  $[(17, 4, 18, 5), (7, 14, 8, 15), (15, 8, 16, 9), (20, 9, 1, 10), (10, 1, 11, 2), (2, 11, 3, 12), (12, 19, 13, 20), (13, 6, 14, 7), (3, 16, 4, 17), (5, 18, 6, 19)]$

Planar representation generated by plantri:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 5, 3], [0, 2, 6, 7], [1, 7, 8, 1], [2, 8, 8, 2], [3, 8, 9, 9], [3, 9, 9, 4], [4, 6, 5, 5], [6, 7, 7, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 446: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

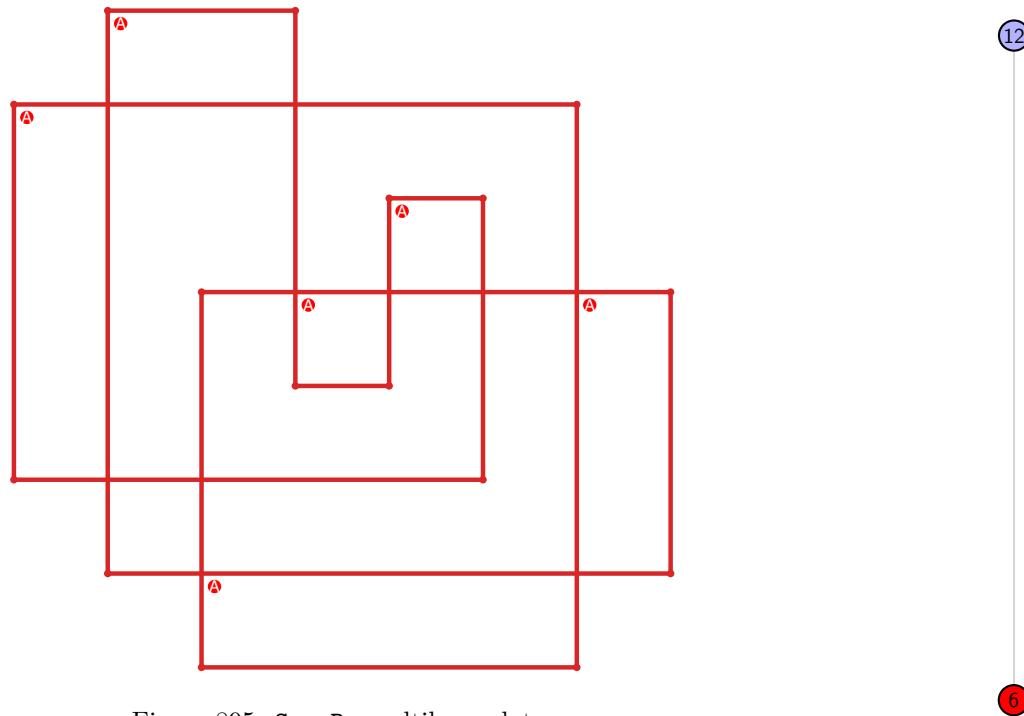


Figure 895: SnapPy multiloop plot.

Figure 896: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.300** [[20, 5, 1, 6], [6, 16, 7, 15], [19, 14, 20, 15], [4, 11, 5, 12], [1, 9, 2, 8], [16, 8, 17, 7], [13, 18, 14, 19], [12, 18, 13, 17], [10, 3, 11, 4], [9, 3, 10, 2]]

PD code drawn by `SnapPy`: [(20, 7, 1, 8), (8, 1, 9, 2), (18, 3, 19, 4), (5, 14, 6, 15), (6, 19, 7, 20), (2, 9, 3, 10), (17, 12, 18, 13), (13, 4, 14, 5), (15, 10, 16, 11), (11, 16, 12, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 6], [0, 7, 8, 8], [0, 9, 9, 5], [1, 4, 7, 1], [2, 7, 7, 2], [3, 6, 6, 5], [3, 9, 9, 3], [4, 8, 8, 4]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 447: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

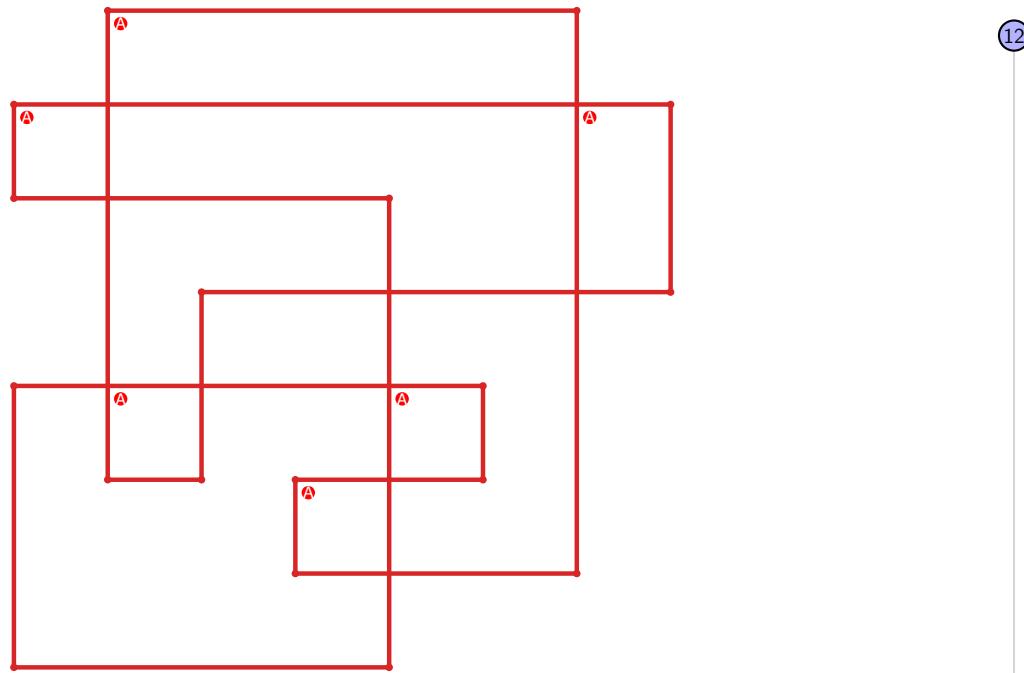


Figure 897: `SnapPy` multiloop plot.

Figure 898: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.301 [[13, 20, 14, 1], [19, 12, 20, 13], [14, 4, 15, 3], [1, 16, 2, 17], [18, 7, 19, 8], [11, 6, 12, 7], [4, 10, 5, 9], [15, 2, 16, 3], [17, 9, 18, 8], [5, 10, 6, 11]]

PD code drawn by `SnapPy`: [(13, 20, 14, 1), (1, 12, 2, 13), (9, 4, 10, 5), (16, 5, 17, 6), (6, 15, 7, 16), (7, 2, 8, 3), (3, 8, 4, 9), (17, 10, 18, 11), (19, 14, 20, 15), (11, 18, 12, 19)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 5, 0], [0, 6, 7, 7], [0, 7, 7, 8], [1, 8, 8, 5], [1, 4, 9, 9], [2, 9, 9, 8], [2, 3, 3, 2], [3, 6, 4, 4], [5, 6, 6, 5]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 448: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

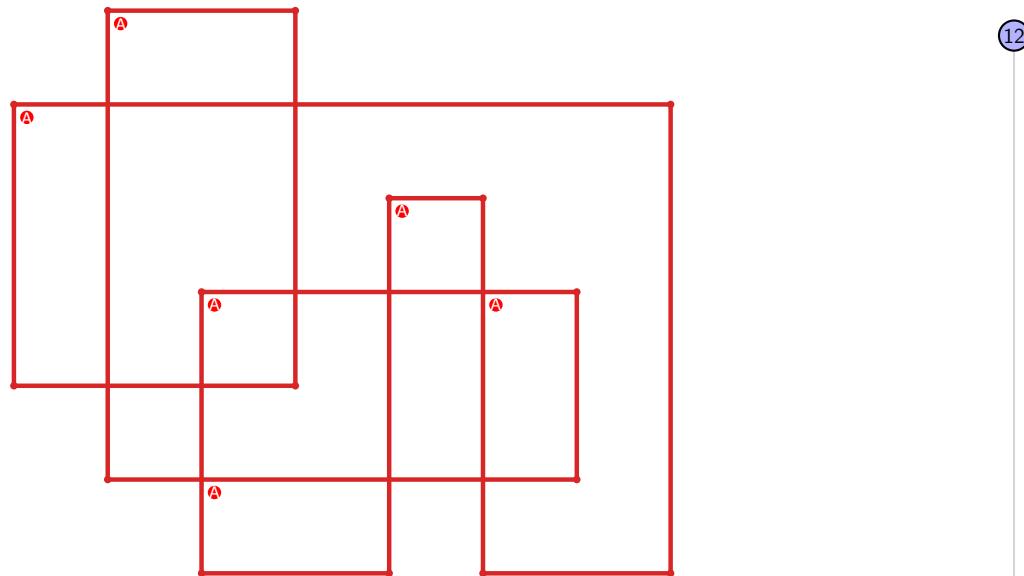


Figure 899: `SnapPy` multiloop plot.



Figure 900: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.302**  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 2, 7, 1], [18, 11, 19, 12], [3, 16, 4, 17], [2, 16, 3, 15], [7, 13, 8, 12], [10, 17, 11, 18], [14, 9, 15, 10], [13, 9, 14, 8]]$

PD code drawn by `SnapPy`:  $[(16, 1, 17, 2), (5, 10, 6, 11), (11, 6, 12, 7), (15, 8, 16, 9), (9, 4, 10, 5), (12, 19, 13, 20), (20, 13, 1, 14), (7, 14, 8, 15), (2, 17, 3, 18), (18, 3, 19, 4)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 6, 0], [1, 6, 7, 7], [1, 7, 5, 5], [2, 4, 4, 8], [2, 9, 9, 3], [3, 8, 4, 3], [5, 7, 9, 9], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 449: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

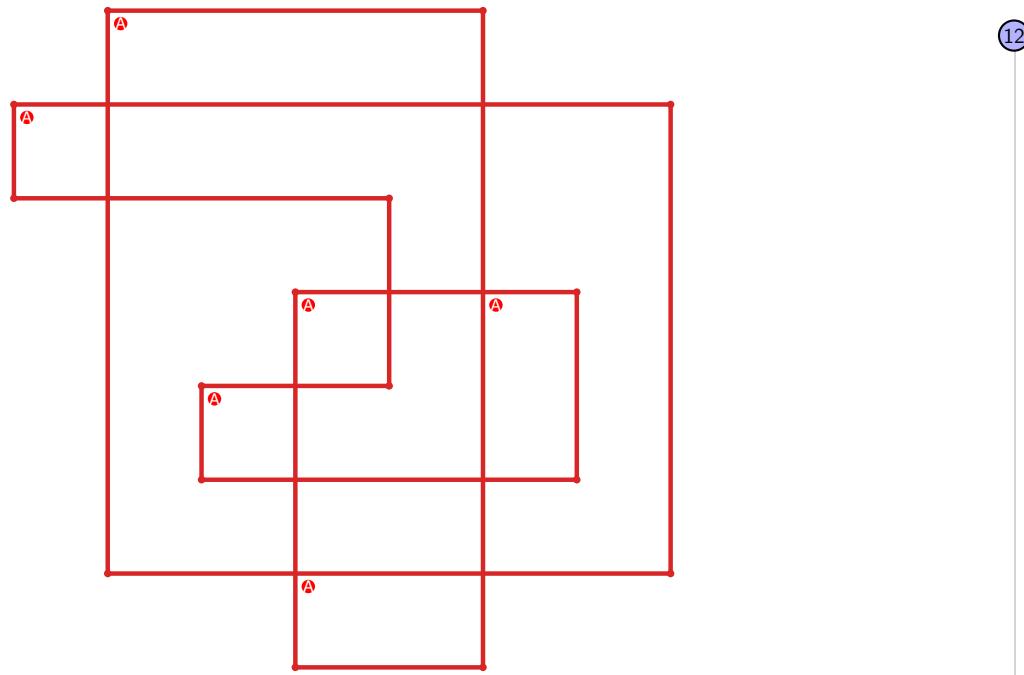


Figure 901: `SnapPy` multiloop plot.

6

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Figure 902: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.303** `[[13, 20, 14, 1], [19, 12, 20, 13], [14, 10, 15, 9], [1, 9, 2, 8], [18, 5, 19, 6], [11, 16, 12, 17], [10, 16, 11, 15], [2, 7, 3, 8], [6, 3, 7, 4], [4, 17, 5, 18]]`

PD code drawn by `SnapPy`: `[(4, 1, 5, 2), (16, 3, 17, 4), (20, 5, 1, 6), (6, 19, 7, 20), (7, 12, 8, 13), (17, 10, 18, 11), (13, 8, 14, 9), (9, 14, 10, 15), (2, 15, 3, 16), (11, 18, 12, 19)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 5, 0], [0, 6, 6, 3], [0, 2, 7, 7], [1, 8, 9, 9], [1, 9, 6, 6], [2, 5, 5, 2], [3, 8, 8, 3], [4, 7, 7, 9], [4, 8, 5, 4]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 450: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

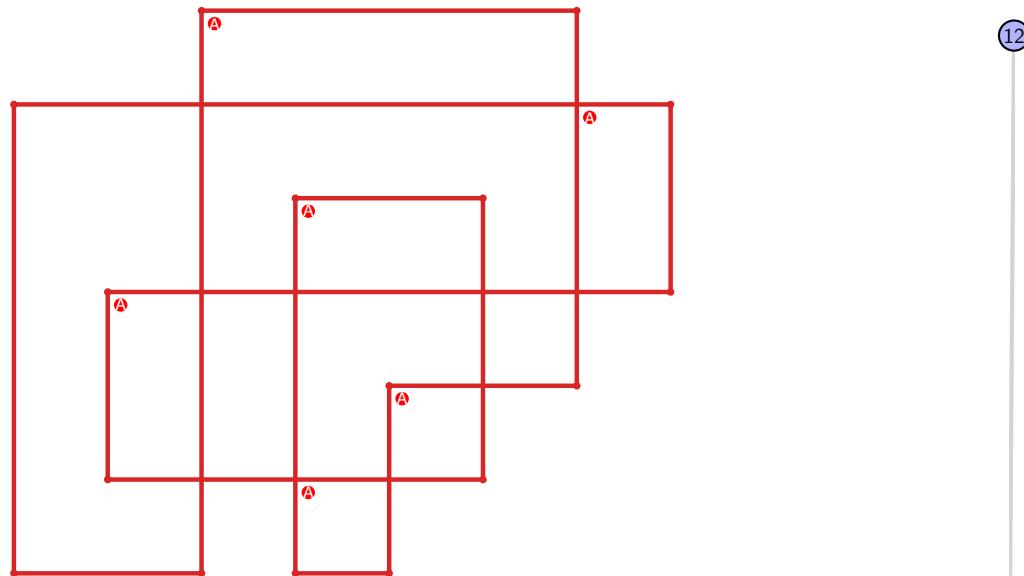


Figure 903: `SnapPy` multiloop plot.



Figure 904: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.304  $[[9, 20, 10, 1], [8, 17, 9, 18], [19, 16, 20, 17], [10, 2, 11, 1], [18, 7, 19, 8], [15, 2, 16, 3], [11, 6, 12, 7], [3, 14, 4, 15], [5, 12, 6, 13], [13, 4, 14, 5]]$

PD code drawn by SnapPy:  $[(10, 1, 11, 2), (8, 3, 9, 4), (15, 4, 16, 5), (2, 9, 3, 10), (20, 11, 1, 12), (16, 13, 17, 14), (5, 14, 6, 15), (6, 17, 7, 18), (18, 7, 19, 8), (12, 19, 13, 20)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 5, 6, 0], [1, 6, 2, 1], [2, 7, 7, 3], [3, 8, 8, 4], [5, 9, 9, 5], [6, 9, 9, 6], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 48

**Pinning number:** 7

**Average optimal degree:** 2.14

**Average minimal degree:** 2.14

**Average overall degree:** 2.86

Table 451: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.14	2.53	2.82	3.04	3.21	3.33	

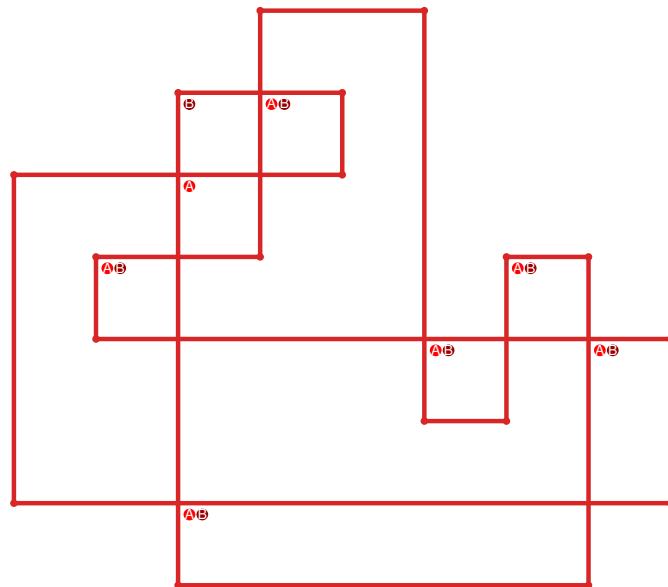


Figure 905: SnapPy multiloop plot.

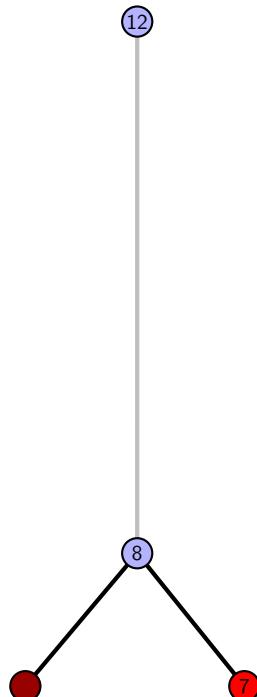


Figure 906: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.305 [[20, 7, 1, 8], [8, 19, 9, 20], [11, 6, 12, 7], [1, 12, 2, 13], [18, 9, 19, 10], [10, 17, 11, 18], [5, 14, 6, 15], [2, 14, 3, 13], [3, 16, 4, 17], [15, 4, 16, 5]]

PD code drawn by `SnapPy`: [(11, 20, 12, 1), (7, 2, 8, 3), (16, 3, 17, 4), (14, 5, 15, 6), (17, 8, 18, 9), (9, 18, 10, 19), (1, 10, 2, 11), (19, 12, 20, 13), (6, 13, 7, 14), (4, 15, 5, 16)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 3], [0, 2, 7, 7], [1, 5, 5, 1], [2, 4, 4, 8], [2, 9, 9, 7], [3, 6, 8, 3], [5, 7, 9, 9], [6, 8, 8, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 452: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

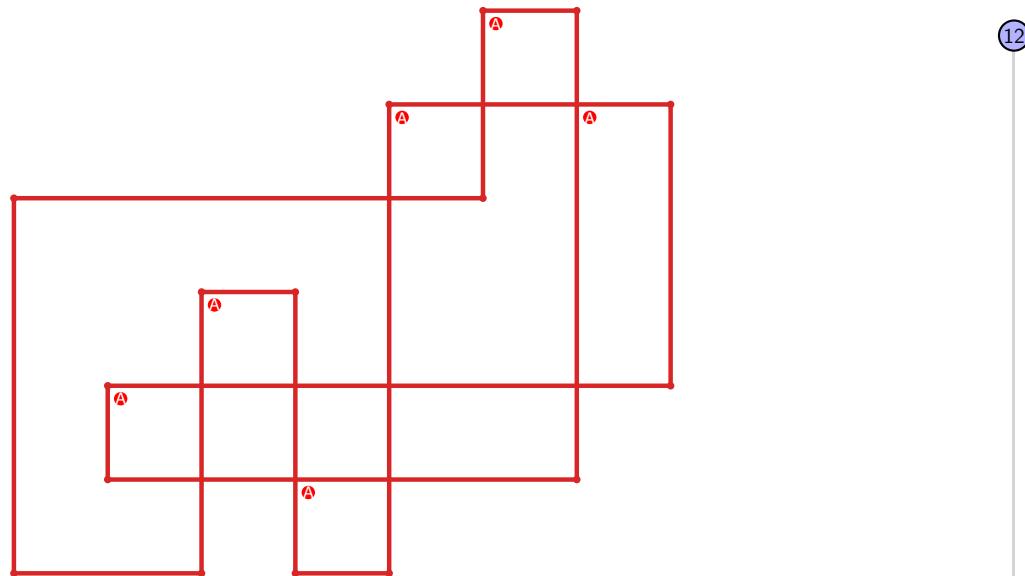


Figure 907: `SnapPy` multiloop plot.

6

Figure 908: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.306 [[20, 15, 1, 16], [16, 8, 17, 7], [19, 6, 20, 7], [14, 1, 15, 2], [8, 18, 9, 17], [9, 18, 10, 19], [12, 5, 13, 6], [2, 13, 3, 14], [10, 3, 11, 4], [4, 11, 5, 12]]

PD code drawn by SnapPy: [(8, 1, 9, 2), (15, 2, 16, 3), (3, 12, 4, 13), (20, 7, 1, 8), (18, 9, 19, 10), (16, 11, 17, 12), (13, 4, 14, 5), (5, 14, 6, 15), (10, 17, 11, 18), (6, 19, 7, 20)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 6], [0, 7, 7, 0], [1, 5, 5, 1], [2, 4, 4, 8], [2, 9, 9, 7], [3, 6, 8, 3], [5, 7, 9, 9], [6, 8, 8, 6]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 1  
 Total pinning sets: 64  
 Pinning number: 6

Average optimal degree: 2.0  
 Average minimal degree: 2.0  
 Average overall degree: 2.85

Table 453: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

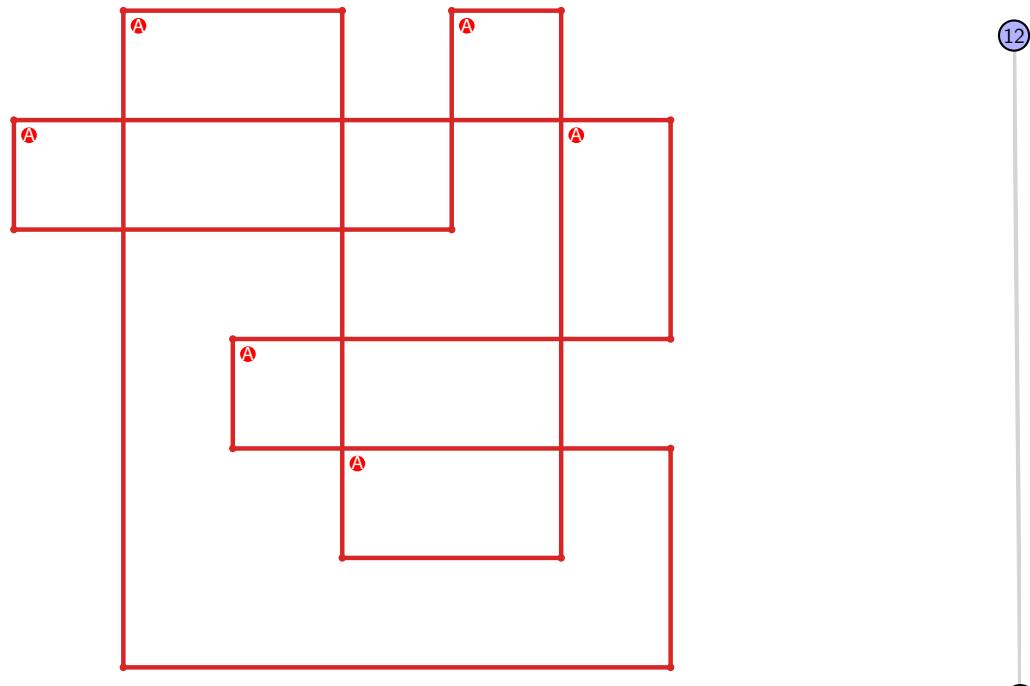


Figure 909: SnapPy multiloop plot.

Figure 910: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.307  $[[20, 9, 1, 10], [10, 18, 11, 17], [19, 16, 20, 17], [8, 1, 9, 2], [18, 12, 19, 11], [6, 15, 7, 16], [2, 7, 3, 8], [12, 3, 13, 4], [14, 5, 15, 6], [13, 5, 14, 4]]$

PD code drawn by SnapPy:  $[(9, 20, 10, 1), (1, 10, 2, 11), (11, 2, 12, 3), (19, 4, 20, 5), (17, 6, 18, 7), (8, 15, 9, 16), (3, 12, 4, 13), (16, 13, 17, 14), (14, 7, 15, 8), (5, 18, 6, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 6, 0], [1, 7, 2, 1], [2, 8, 8, 6], [3, 5, 7, 3], [4, 6, 9, 9], [5, 9, 9, 5], [7, 8, 8, 7]]$

**Total optimal pinning sets:** 2  
**Total minimal pinning sets:** 2

**Total pinning sets:** 48

**Pinning number:** 7

**Average optimal degree:** 2.14

**Average minimal degree:** 2.14

**Average overall degree:** 2.86

Table 454: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.14	2.53	2.82	3.04	3.21	3.33	

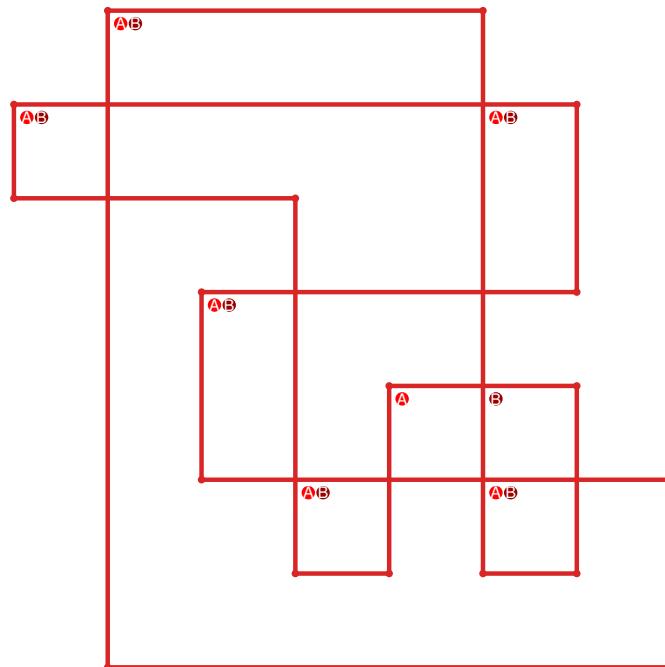


Figure 911: SnapPy multiloop plot.

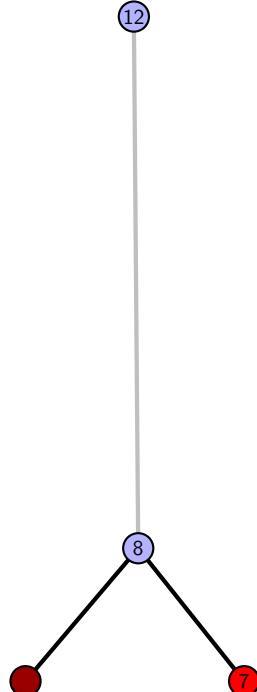


Figure 912: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.308 [[9, 20, 10, 1], [8, 17, 9, 18], [19, 16, 20, 17], [10, 2, 11, 1], [18, 7, 19, 8], [15, 4, 16, 5], [2, 12, 3, 11], [13, 6, 14, 7], [5, 14, 6, 15], [3, 12, 4, 13]]

PD code drawn by SnapPy: [(12, 1, 13, 2), (10, 3, 11, 4), (15, 4, 16, 5), (8, 19, 9, 20), (20, 9, 1, 10), (2, 11, 3, 12), (16, 13, 17, 14), (5, 14, 6, 15), (6, 17, 7, 18), (18, 7, 19, 8)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 4, 5], [0, 6, 6, 0], [1, 7, 2, 1], [2, 8, 8, 9], [3, 9, 9, 3], [4, 9, 8, 8], [5, 7, 7, 5], [5, 7, 6, 6]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 48  
 Pinning number: 7

Average optimal degree: 2.14  
 Average minimal degree: 2.14  
 Average overall degree: 2.86

Table 455: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.14	2.53	2.82	3.04	3.21	3.33	

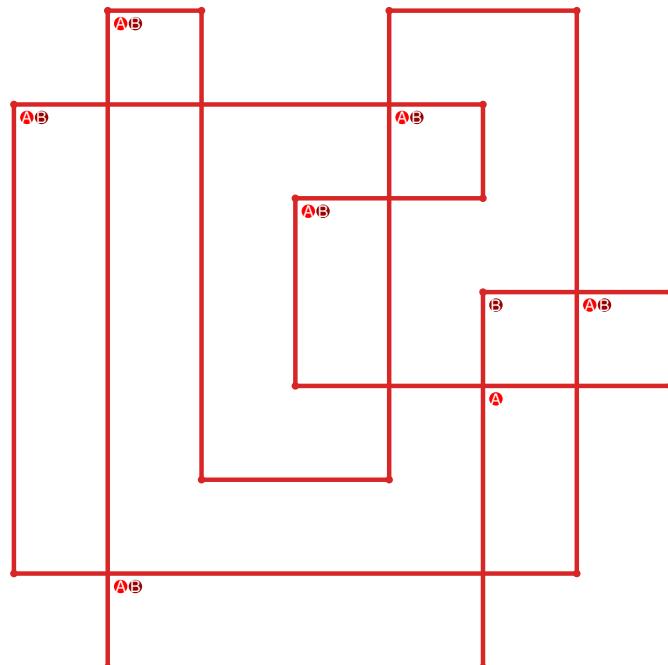


Figure 913: SnapPy multiloop plot.

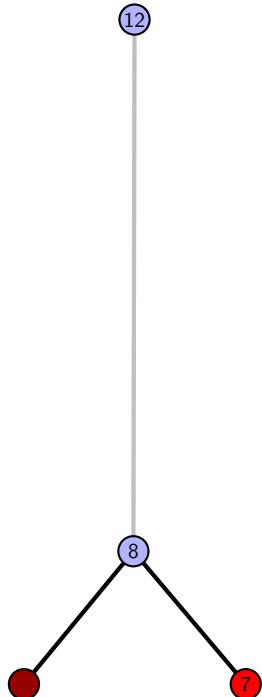


Figure 914: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.309**  $[[13, 20, 14, 1], [19, 12, 20, 13], [14, 10, 15, 9], [1, 9, 2, 8], [11, 18, 12, 19], [10, 18, 11, 17], [15, 5, 16, 4], [2, 7, 3, 8], [16, 5, 17, 6], [6, 3, 7, 4]]$

PD code drawn by `SnapPy`:  $[(3, 20, 4, 1), (1, 12, 2, 13), (13, 2, 14, 3), (19, 4, 20, 5), (5, 18, 6, 19), (6, 11, 7, 12), (16, 9, 17, 10), (14, 7, 15, 8), (8, 15, 9, 16), (10, 17, 11, 18)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 3], [0, 2, 7, 7], [1, 5, 5, 1], [2, 4, 4, 8], [2, 8, 8, 9], [3, 9, 9, 3], [5, 9, 6, 6], [6, 8, 7, 7]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 1  
 Total pinning sets: 64  
 Pinning number: 6

Average optimal degree: 2.0  
 Average minimal degree: 2.0  
 Average overall degree: 2.85

Table 456: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

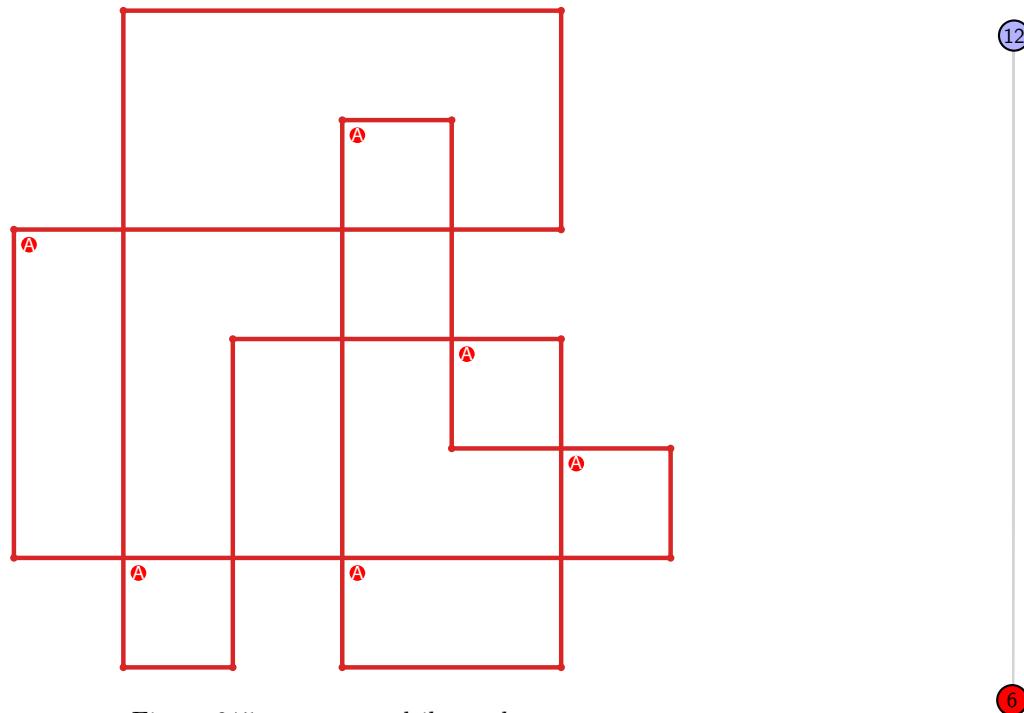


Figure 915: `SnapPy` multiloop plot.

Figure 916: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.310**  $[[13, 20, 14, 1], [19, 12, 20, 13], [14, 12, 15, 11], [1, 9, 2, 8], [18, 5, 19, 6], [15, 10, 16, 11], [9, 16, 10, 17], [2, 7, 3, 8], [6, 3, 7, 4], [4, 17, 5, 18]]$

PD code drawn by `SnapPy`:  $[(13, 20, 14, 1), (17, 2, 18, 3), (3, 16, 4, 17), (10, 5, 11, 6), (6, 9, 7, 10), (14, 7, 15, 8), (4, 11, 5, 12), (19, 12, 20, 13), (8, 15, 9, 16), (1, 18, 2, 19)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 5], [0, 6, 7, 7], [1, 8, 9, 9], [2, 6, 6, 2], [3, 5, 5, 9], [3, 8, 8, 3], [4, 7, 7, 9], [4, 8, 6, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 457: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

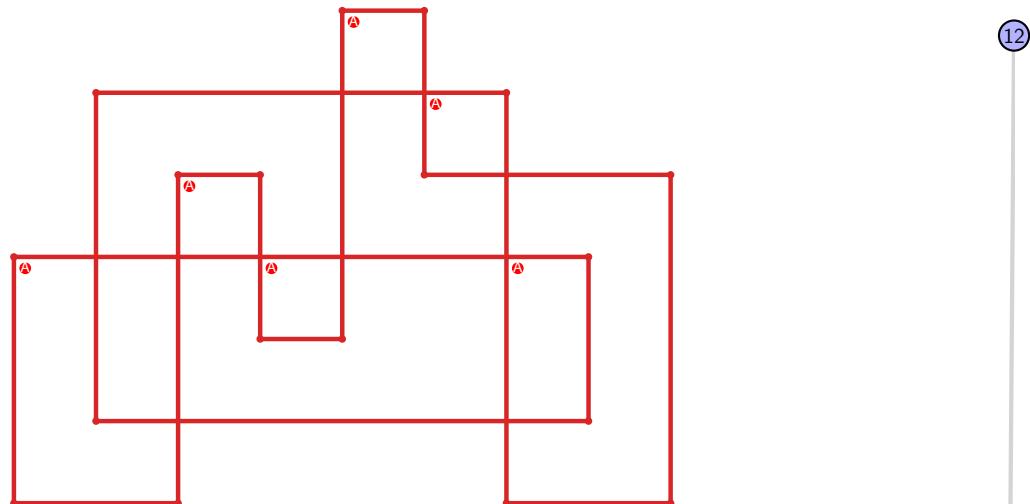


Figure 917: `SnapPy` multiloop plot.



Figure 918: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.311** [[20, 5, 1, 6], [6, 15, 7, 16], [19, 10, 20, 11], [4, 1, 5, 2], [14, 7, 15, 8], [16, 14, 17, 13], [11, 18, 12, 19], [9, 2, 10, 3], [3, 8, 4, 9], [17, 12, 18, 13]]

PD code drawn by `SnapPy`: [(17, 2, 18, 3), (11, 4, 12, 5), (20, 5, 1, 6), (16, 7, 17, 8), (14, 9, 15, 10), (10, 13, 11, 14), (3, 12, 4, 13), (8, 15, 9, 16), (1, 18, 2, 19), (6, 19, 7, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 4, 5], [0, 6, 6, 7], [0, 7, 8, 0], [1, 8, 5, 1], [1, 4, 9, 9], [2, 9, 9, 2], [2, 8, 8, 3], [3, 7, 7, 4], [5, 6, 6, 5]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 1  
 Total pinning sets: 64  
 Pinning number: 6

Average optimal degree: 2.0  
 Average minimal degree: 2.0  
 Average overall degree: 2.85

Table 458: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

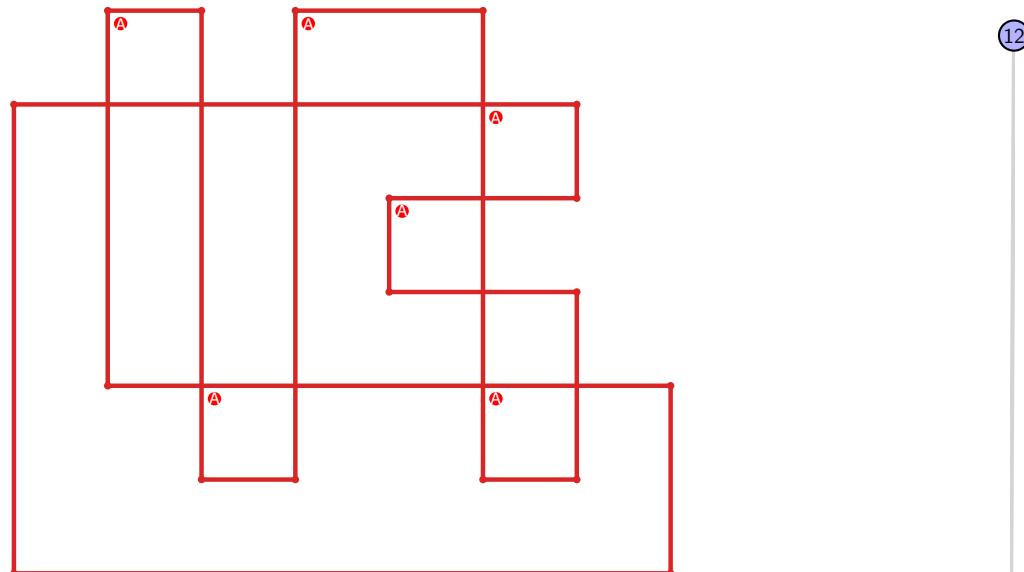


Figure 919: `SnapPy` multiloop plot.



Figure 920: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.312** [[20, 9, 1, 10], [10, 19, 11, 20], [13, 8, 14, 9], [1, 7, 2, 6], [18, 11, 19, 12], [12, 17, 13, 18], [7, 14, 8, 15], [2, 5, 3, 6], [3, 16, 4, 17], [15, 4, 16, 5]]

PD code drawn by `SnapPy`: [(12, 1, 13, 2), (19, 2, 20, 3), (3, 18, 4, 19), (10, 5, 11, 6), (17, 6, 18, 7), (15, 8, 16, 9), (4, 11, 5, 12), (20, 13, 1, 14), (9, 14, 10, 15), (7, 16, 8, 17)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 4, 0], [0, 5, 6, 6], [0, 6, 7, 7], [1, 5, 5, 1], [2, 4, 4, 8], [2, 9, 3, 2], [3, 9, 8, 3], [5, 7, 9, 9], [6, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 459: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

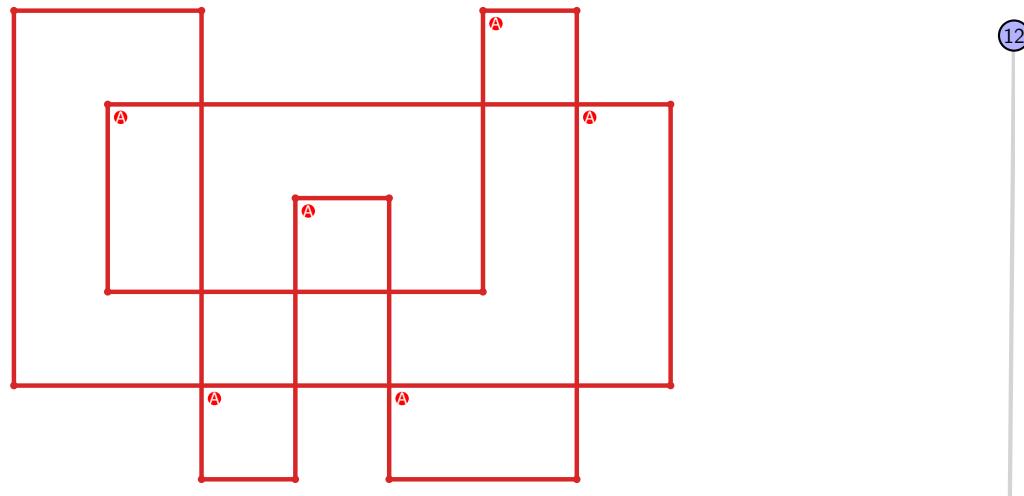


Figure 921: `SnapPy` multiloop plot.

6

Figure 922: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.313** [[20, 15, 1, 16], [16, 8, 17, 7], [12, 19, 13, 20], [14, 1, 15, 2], [8, 5, 9, 6], [17, 6, 18, 7], [18, 11, 19, 12], [13, 3, 14, 2], [4, 9, 5, 10], [10, 3, 11, 4]]

PD code drawn by `SnapPy`: [(3, 20, 4, 1), (12, 1, 13, 2), (2, 11, 3, 12), (19, 4, 20, 5), (13, 6, 14, 7), (15, 10, 16, 11), (7, 14, 8, 15), (9, 16, 10, 17), (17, 8, 18, 9), (5, 18, 6, 19)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 5], [0, 6, 6, 7], [0, 7, 7, 0], [1, 8, 8, 5], [1, 4, 6, 1], [2, 5, 9, 2], [2, 9, 3, 3], [4, 9, 9, 4], [6, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 64

**Average overall degree:** 2.85

**Pinning number:** 6

Table 460: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	6	15	20	15	6	1	63
Average degree	2.0	2.38	2.67	2.89	3.07	3.21	3.33	

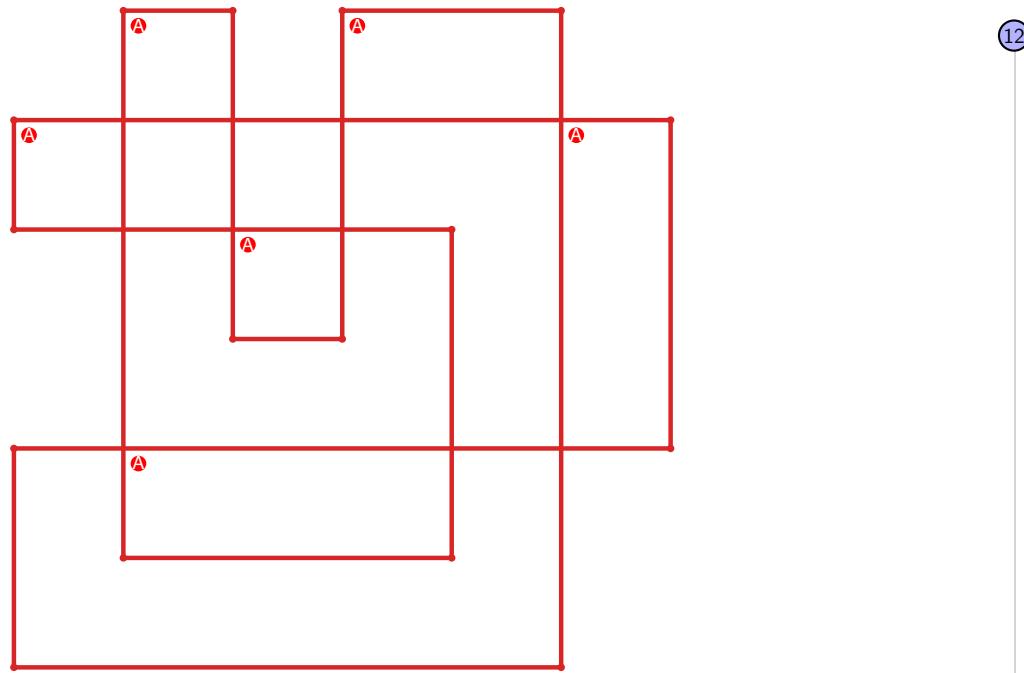


Figure 923: `SnapPy` multiloop plot.

6

12

Figure 924: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.314 [[15, 20, 16, 1], [19, 14, 20, 15], [16, 14, 17, 13], [1, 13, 2, 12], [18, 7, 19, 8], [17, 7, 18, 6], [2, 11, 3, 12], [8, 5, 9, 6], [10, 3, 11, 4], [4, 9, 5, 10]]

PD code drawn by SnapPy: [(7, 20, 8, 1), (5, 2, 6, 3), (3, 14, 4, 15), (15, 4, 16, 5), (1, 6, 2, 7), (19, 8, 20, 9), (9, 18, 10, 19), (10, 13, 11, 14), (16, 11, 17, 12), (12, 17, 13, 18)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 2, 0], [0, 1, 5, 3], [0, 2, 6, 6], [1, 7, 5, 5], [2, 4, 4, 7], [3, 8, 8, 3], [4, 9, 9, 5], [6, 9, 9, 6], [7, 8, 8, 7]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 48  
 Pinning number: 7

Average optimal degree: 2.14  
 Average minimal degree: 2.14  
 Average overall degree: 2.86

Table 461: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	9	16	14	6	1	46
Average degree	2.14	2.53	2.82	3.04	3.21	3.33	

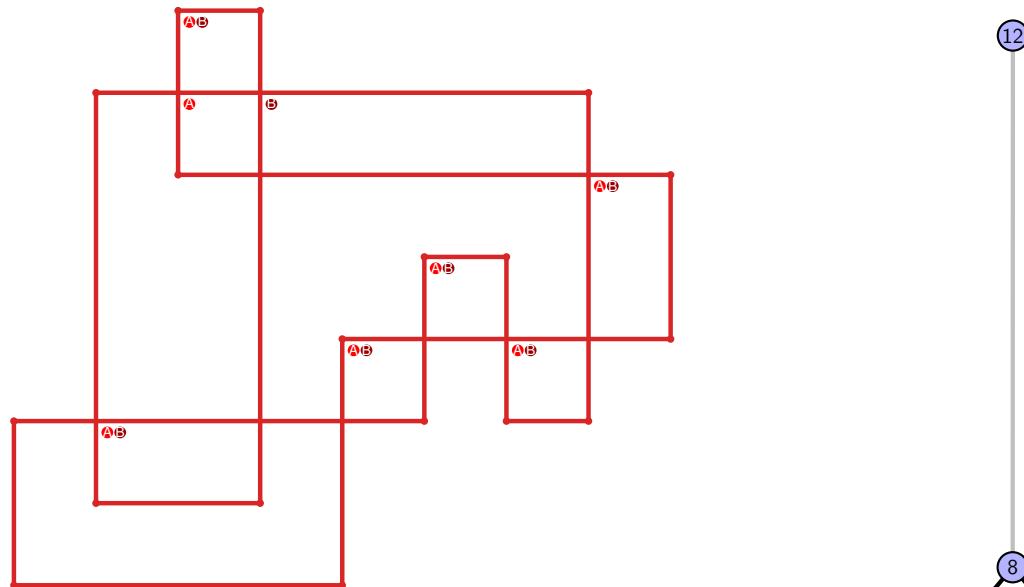


Figure 925: SnapPy multiloop plot.

Figure 926: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.315**  $[[5, 20, 6, 1], [19, 4, 20, 5], [6, 18, 7, 17], [1, 13, 2, 12], [3, 18, 4, 19], [7, 16, 8, 17], [13, 10, 14, 11], [2, 11, 3, 12], [15, 8, 16, 9], [9, 14, 10, 15]]$

PD code drawn by `SnapPy`:  $[(17, 2, 18, 3), (13, 6, 14, 7), (11, 8, 12, 9), (20, 9, 1, 10), (10, 19, 11, 20), (7, 12, 8, 13), (5, 14, 6, 15), (15, 4, 16, 5), (1, 16, 2, 17), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 3], [0, 4, 4, 0], [0, 4, 5, 5], [0, 6, 7, 7], [1, 7, 2, 1], [2, 8, 8, 2], [3, 9, 9, 7], [3, 6, 4, 3], [5, 9, 9, 5], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.81

**Pinning number:** 7

Table 462: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

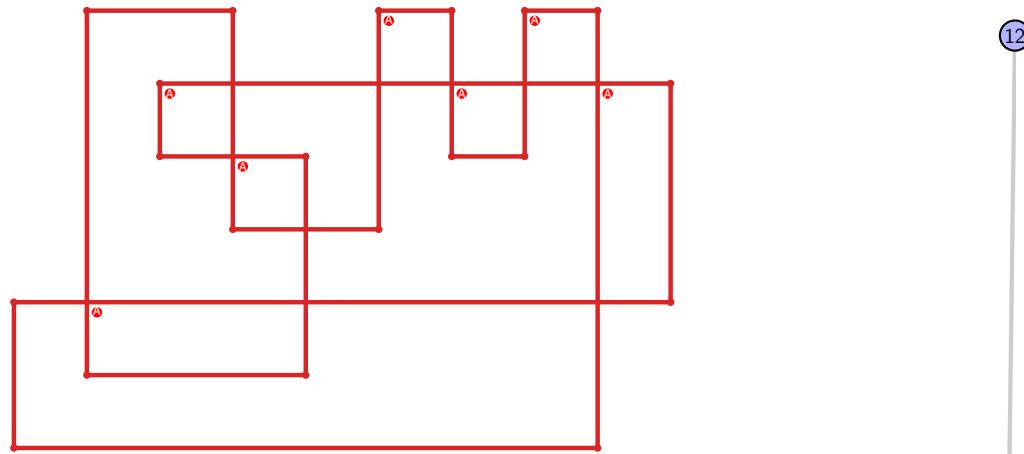


Figure 927: `SnapPy` multiloop plot.



Figure 928: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.316**  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 2, 9, 1], [5, 18, 6, 19], [2, 10, 3, 9], [4, 13, 5, 14], [17, 10, 18, 11], [3, 15, 4, 14], [15, 12, 16, 13], [11, 16, 12, 17]]$

PD code drawn by `SnapPy`:  $[(16, 3, 17, 4), (6, 19, 7, 20), (14, 7, 15, 8), (12, 9, 13, 10), (1, 10, 2, 11), (11, 20, 12, 1), (8, 13, 9, 14), (2, 15, 3, 16), (4, 17, 5, 18), (18, 5, 19, 6)]$

Planar representation generated by `plantri`:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 6, 7, 2], [3, 7, 7, 8], [3, 9, 9, 4], [4, 8, 5, 5], [5, 7, 9, 9], [6, 8, 8, 6]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.81

**Pinning number:** 7

Table 463: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

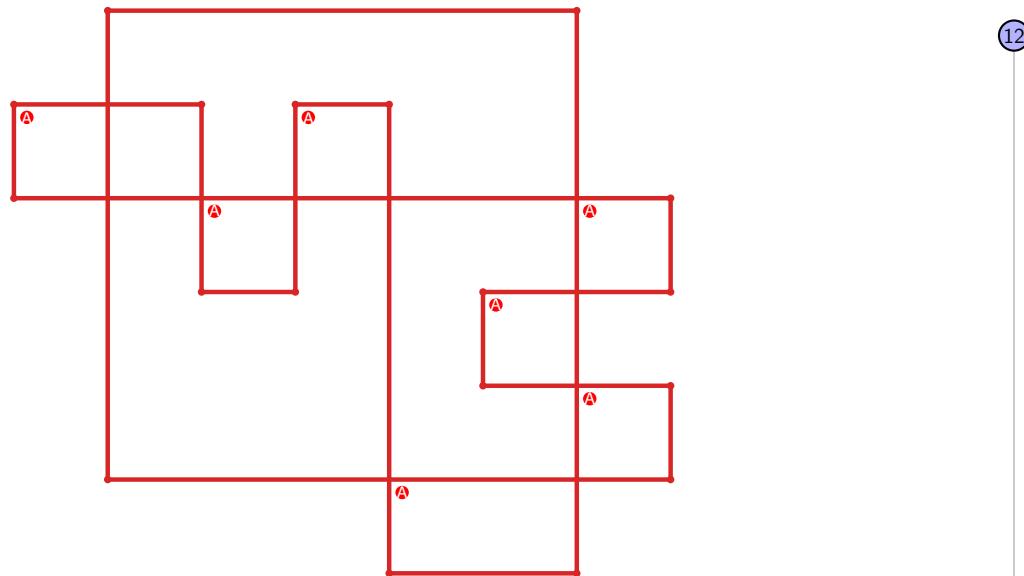


Figure 929: `SnapPy` multiloop plot.

7

Figure 930: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.317** [[20, 11, 1, 12], [12, 19, 13, 20], [10, 1, 11, 2], [5, 18, 6, 19], [13, 8, 14, 9], [2, 9, 3, 10], [17, 4, 18, 5], [6, 16, 7, 15], [7, 14, 8, 15], [3, 16, 4, 17]]

PD code drawn by `SnapPy`: [(14, 1, 15, 2), (10, 5, 11, 6), (19, 6, 20, 7), (17, 8, 18, 9), (4, 11, 5, 12), (12, 3, 13, 4), (20, 13, 1, 14), (2, 15, 3, 16), (9, 16, 10, 17), (7, 18, 8, 19)]

Planar representation generated by `plantri`: [[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 5, 0], [1, 6, 6, 7], [1, 8, 8, 5], [2, 4, 9, 2], [3, 9, 9, 3], [3, 9, 8, 8], [4, 7, 7, 4], [5, 7, 6, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 32

**Average overall degree:** 2.81

**Pinning number:** 7

Table 464: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

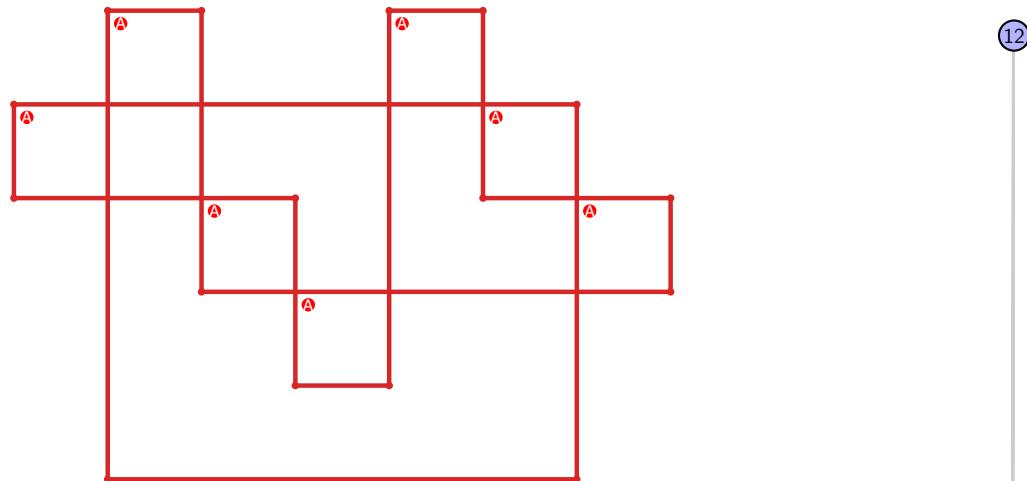


Figure 931: `SnapPy` multiloop plot.

12

7

Figure 932: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.318  $[[7, 20, 8, 1], [19, 6, 20, 7], [8, 2, 9, 1], [5, 18, 6, 19], [2, 10, 3, 9], [4, 13, 5, 14], [17, 12, 18, 13], [10, 16, 11, 15], [3, 15, 4, 14], [11, 16, 12, 17]]$

PD code drawn by SnapPy:  $[(17, 4, 18, 5), (7, 20, 8, 1), (15, 10, 16, 11), (2, 11, 3, 12), (12, 1, 13, 2), (13, 8, 14, 9), (9, 14, 10, 15), (3, 16, 4, 17), (5, 18, 6, 19), (19, 6, 20, 7)]$

Planar representation generated by plantri:  $[[1, 1, 2, 2], [0, 3, 3, 0], [0, 4, 4, 0], [1, 5, 6, 1], [2, 7, 8, 2], [3, 8, 8, 6], [3, 5, 9, 9], [4, 9, 9, 8], [4, 7, 5, 5], [6, 7, 7, 6]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 1  
 Total pinning sets: 32  
 Pinning number: 7

Average optimal degree: 2.0  
 Average minimal degree: 2.0  
 Average overall degree: 2.81

Table 465: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

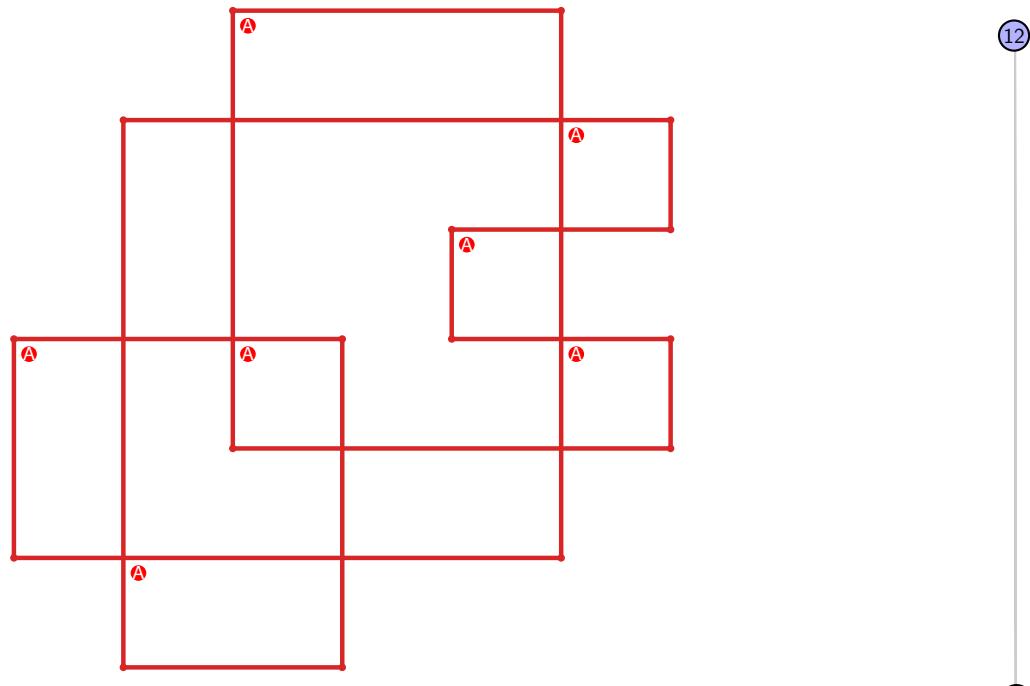


Figure 933: SnapPy multiloop plot.

7

12

Figure 934: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.319** [[20, 11, 1, 12], [12, 19, 13, 20], [10, 1, 11, 2], [5, 18, 6, 19], [13, 6, 14, 7], [2, 9, 3, 10], [17, 4, 18, 5], [14, 8, 15, 7], [15, 8, 16, 9], [3, 16, 4, 17]]

PD code drawn by `SnapPy`: [(14, 1, 15, 2), (10, 3, 11, 4), (4, 11, 5, 12), (12, 5, 13, 6), (19, 6, 20, 7), (17, 8, 18, 9), (20, 13, 1, 14), (2, 15, 3, 16), (9, 16, 10, 17), (7, 18, 8, 19)]

Planar representation generated by `plantri`: [[1, 1, 2, 2], [0, 3, 4, 0], [0, 5, 5, 0], [1, 6, 6, 4], [1, 3, 7, 7], [2, 8, 9, 2], [3, 9, 9, 3], [4, 8, 8, 4], [5, 7, 7, 9], [5, 8, 6, 6]]

**Total optimal pinning sets:** 1  
**Total minimal pinning sets:** 1

**Total pinning sets:** 32

**Pinning number:** 7

**Average optimal degree:** 2.0

**Average minimal degree:** 2.0

**Average overall degree:** 2.81

Table 466: Pinning sets/average degree by cardinal

Cardinal	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0
Nonminimal pinning sets	0	5	10	10	5	1	31
Average degree	2.0	2.4	2.71	2.96	3.16	3.33	

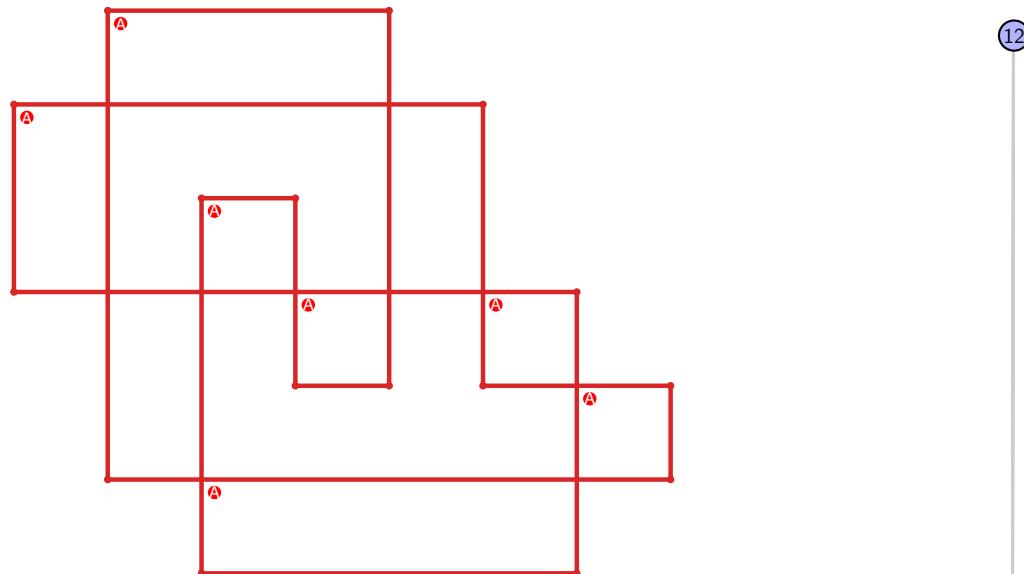


Figure 935: `SnapPy` multiloop plot.

12

7

Figure 936: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.320**  $[[20, 9, 1, 10], [10, 3, 11, 4], [4, 19, 5, 20], [5, 8, 6, 9], [1, 16, 2, 17], [17, 2, 18, 3], [11, 18, 12, 19], [7, 14, 8, 15], [6, 14, 7, 13], [15, 12, 16, 13]]$

PD code drawn by SnapPy:  $[(15, 20, 16, 1), (8, 1, 9, 2), (3, 6, 4, 7), (18, 7, 19, 8), (13, 10, 14, 11), (11, 4, 12, 5), (5, 12, 6, 13), (19, 14, 20, 15), (9, 16, 10, 17), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 3], [0, 2, 7, 8], [0, 9, 5, 5], [1, 4, 4, 6], [1, 5, 9, 2], [3, 9, 8, 8], [3, 7, 7, 9], [4, 8, 7, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 512  
 Pinning number: 4

Average optimal degree: 2.5  
 Average minimal degree: 2.5  
 Average overall degree: 3.11

Table 467: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	22	70	126	140	98	42	10	1	509
Average degree	2.5	2.76	2.94	3.06	3.15	3.22	3.27	3.31	3.33	

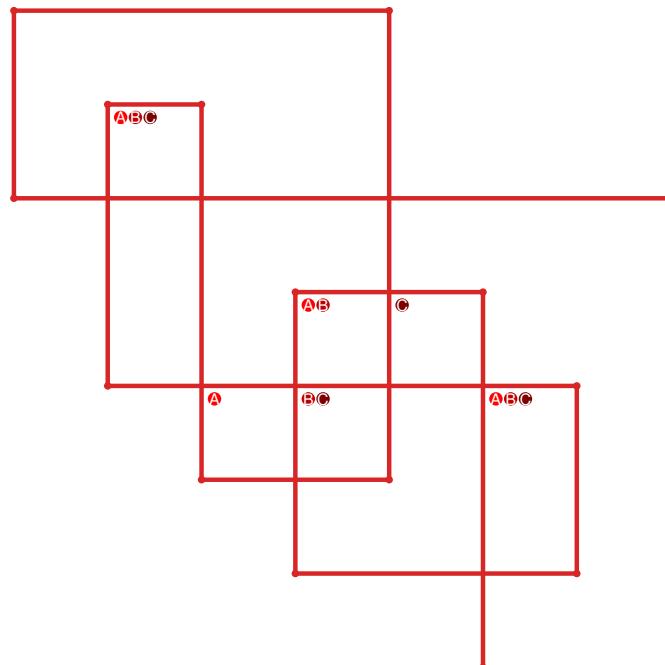


Figure 937: SnapPy multiloop plot.

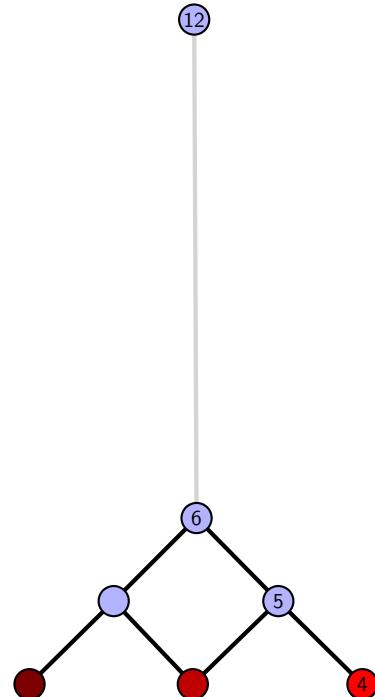


Figure 938: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.321** [[20, 11, 1, 12], [12, 8, 13, 7], [19, 16, 20, 17], [10, 15, 11, 16], [1, 9, 2, 8], [13, 2, 14, 3], [3, 6, 4, 7], [17, 4, 18, 5], [5, 18, 6, 19], [14, 9, 15, 10]]

PD code drawn by `SnapPy`: [(11, 20, 12, 1), (15, 2, 16, 3), (1, 4, 2, 5), (5, 10, 6, 11), (6, 19, 7, 20), (12, 7, 13, 8), (18, 9, 19, 10), (8, 13, 9, 14), (17, 14, 18, 15), (3, 16, 4, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 7, 8, 3], [0, 2, 9, 9], [0, 9, 5, 1], [1, 4, 9, 6], [1, 5, 8, 7], [2, 6, 8, 8], [2, 7, 7, 6], [3, 5, 4, 3]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 6  
 Total pinning sets: 504  
 Pinning number: 4

Average optimal degree: 2.62  
 Average minimal degree: 2.68  
 Average overall degree: 3.12

Table 468: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	0	4
Nonminimal pinning sets	0	15	67	125	140	98	42	10	1	498
Average degree	2.62	2.81	2.96	3.07	3.16	3.22	3.27	3.31	3.33	

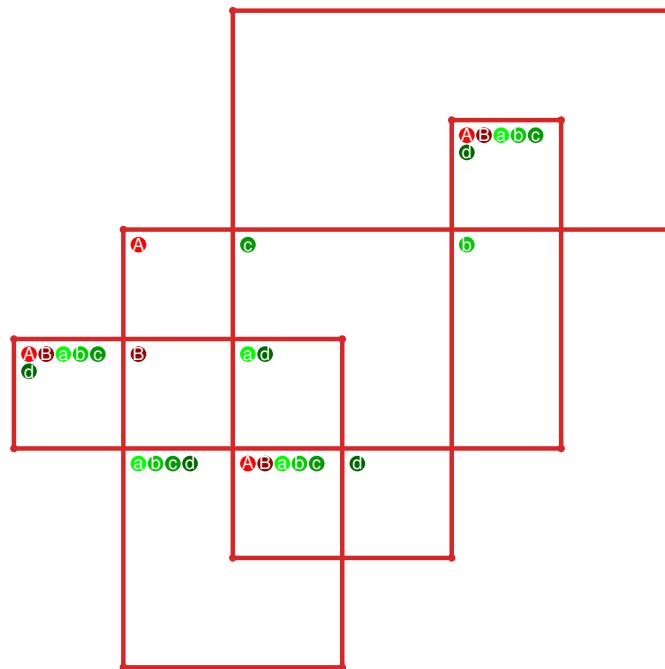


Figure 939: `SnapPy` multiloop plot.

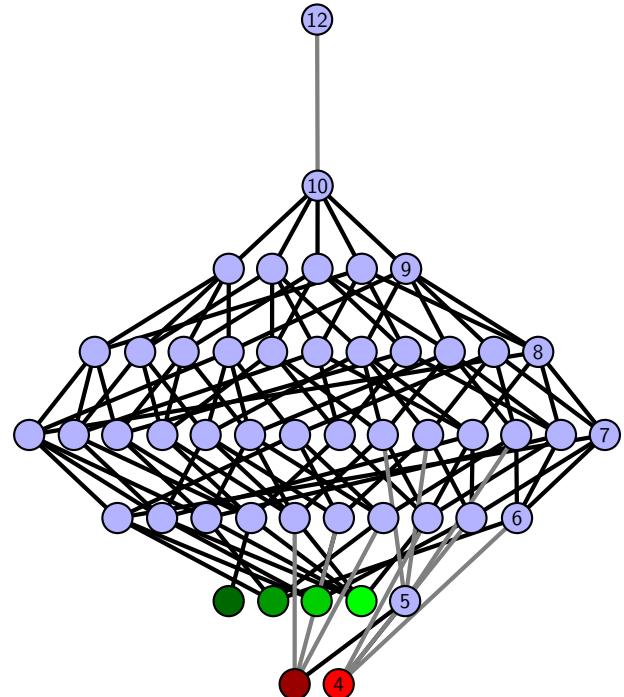


Figure 940: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.322** [[13, 20, 14, 1], [12, 5, 13, 6], [19, 4, 20, 5], [14, 4, 15, 3], [1, 7, 2, 6], [18, 11, 19, 12], [15, 11, 16, 10], [2, 7, 3, 8], [8, 17, 9, 18], [16, 9, 17, 10]]

PD code drawn by `SnapPy`: [(20, 11, 1, 12), (16, 1, 17, 2), (9, 2, 10, 3), (3, 18, 4, 19), (19, 4, 20, 5), (13, 6, 14, 7), (7, 12, 8, 13), (15, 8, 16, 9), (5, 14, 6, 15), (10, 17, 11, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 3], [0, 2, 6, 7], [0, 7, 7, 1], [1, 8, 6, 2], [3, 5, 9, 9], [3, 8, 4, 4], [5, 7, 9, 9], [6, 8, 8, 6]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.38

**Total pinning sets:** 288

**Average overall degree:** 3.03

**Pinning number:** 4

Table 469: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	28	61	80	66	33	9	1	286
Average degree	2.25	2.58	2.78	2.93	3.05	3.15	3.23	3.29	3.33	

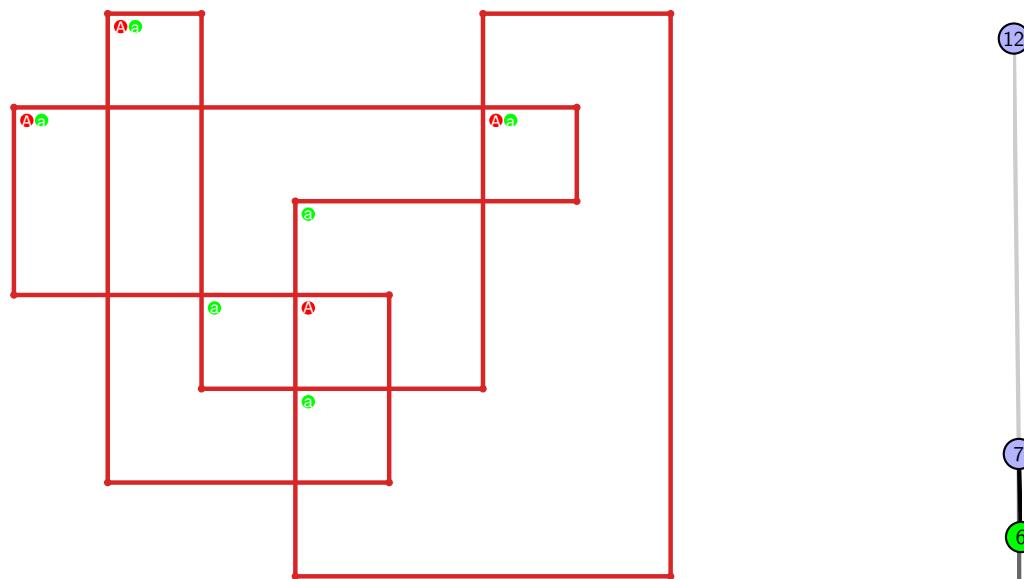


Figure 941: `SnapPy` multiloop plot.

Figure 942: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.323**  $[[13, 20, 14, 1], [3, 12, 4, 13], [4, 19, 5, 20], [14, 5, 15, 6], [1, 6, 2, 7], [7, 2, 8, 3], [18, 11, 19, 12], [15, 11, 16, 10], [8, 17, 9, 18], [16, 9, 17, 10]]$

PD code drawn by `SnapPy`:  $[(5, 20, 6, 1), (12, 3, 13, 4), (19, 4, 20, 5), (14, 7, 15, 8), (8, 13, 9, 14), (16, 9, 17, 10), (1, 10, 2, 11), (6, 15, 7, 16), (2, 17, 3, 18), (11, 18, 12, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 3], [0, 2, 7, 4], [0, 3, 5, 5], [1, 4, 4, 8], [1, 8, 7, 2], [3, 6, 9, 9], [5, 9, 9, 6], [7, 8, 8, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 256  
 Pinning number: 5

Average optimal degree: 2.4  
 Average minimal degree: 2.4  
 Average overall degree: 3.03

Table 470: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	19	51	75	65	33	9	1	253
Average degree	2.4	2.68	2.89	3.03	3.15	3.23	3.29	3.33	

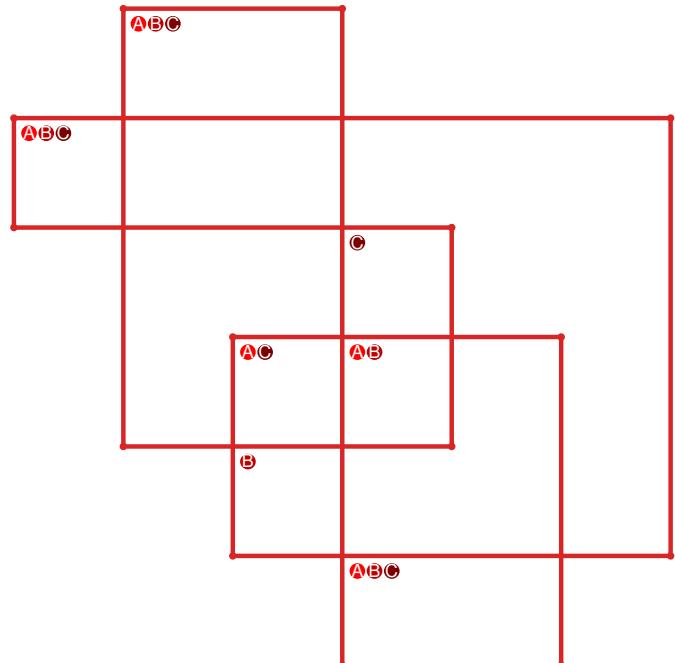


Figure 943: `SnapPy` multiloop plot.

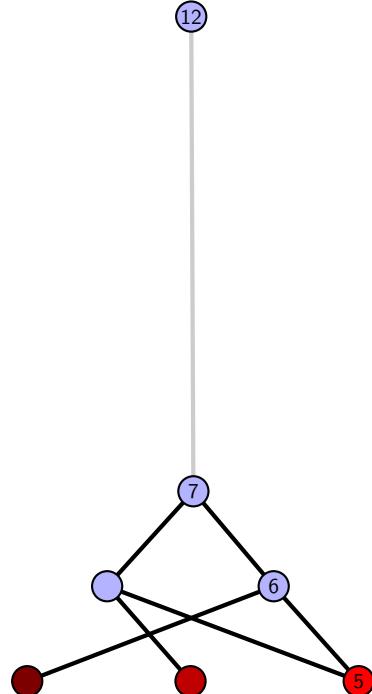


Figure 944: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.324 [[17, 20, 18, 1], [16, 9, 17, 10], [4, 19, 5, 20], [18, 5, 19, 6], [1, 11, 2, 10], [2, 15, 3, 16], [3, 8, 4, 9], [6, 13, 7, 14], [11, 14, 12, 15], [12, 7, 13, 8]]

PD code drawn by SnapPy: [(16, 3, 17, 4), (10, 5, 11, 6), (1, 6, 2, 7), (14, 7, 15, 8), (8, 19, 9, 20), (4, 11, 5, 12), (9, 12, 10, 13), (20, 13, 1, 14), (2, 17, 3, 18), (15, 18, 16, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 3, 3], [0, 2, 2, 7], [0, 8, 5, 1], [1, 4, 8, 6], [1, 5, 9, 2], [3, 9, 9, 8], [4, 7, 9, 5], [6, 8, 7, 7]]

**Total optimal pinning sets:** 8

**Average optimal degree:** 2.75

**Total minimal pinning sets:** 10

**Average minimal degree:** 2.77

**Total pinning sets:** 450

**Average overall degree:** 3.13

**Pinning number:** 5

Table 471: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	8	0	0	0	0	0	0	0	8
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	46	110	134	97	42	10	1	440
Average degree	2.75	2.92	3.06	3.15	3.22	3.27	3.31	3.33	

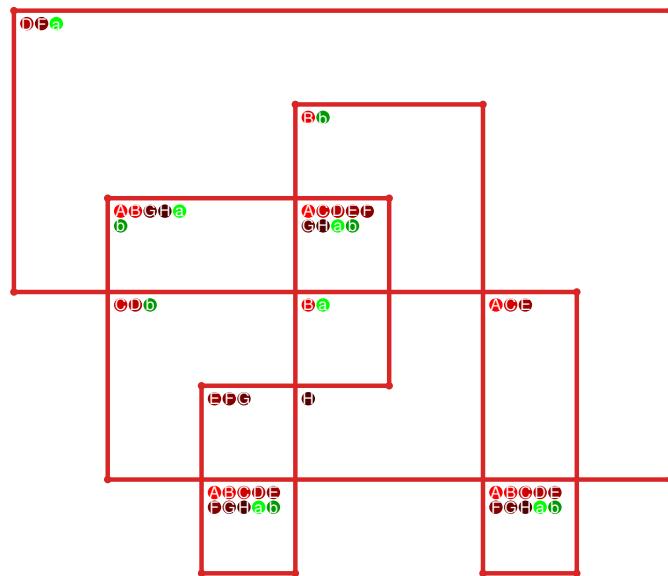


Figure 945: SnapPy multiloop plot.

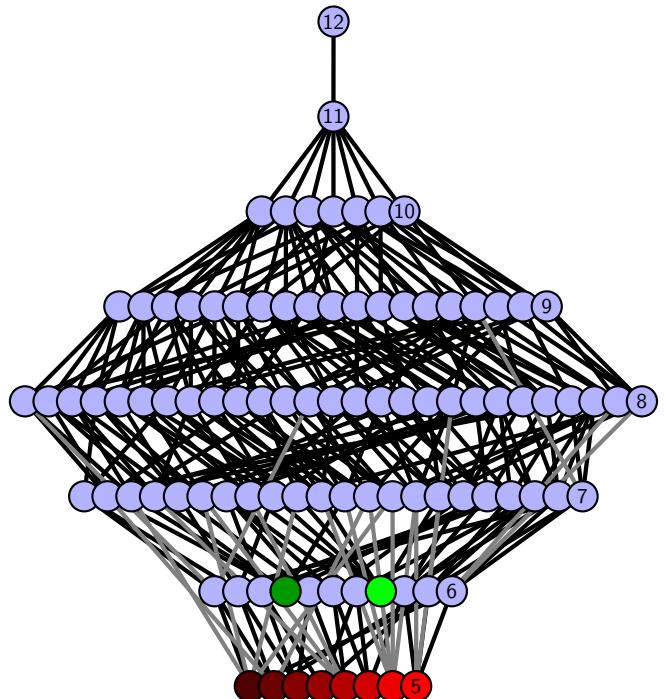


Figure 946: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.325 [[20, 5, 1, 6], [6, 15, 7, 16], [4, 19, 5, 20], [1, 14, 2, 15], [7, 2, 8, 3], [16, 3, 17, 4], [11, 18, 12, 19], [13, 8, 14, 9], [17, 10, 18, 11], [12, 10, 13, 9]]

PD code drawn by SnapPy: [(5, 20, 6, 1), (14, 1, 15, 2), (18, 3, 19, 4), (11, 6, 12, 7), (7, 10, 8, 11), (15, 8, 16, 9), (19, 12, 20, 13), (4, 13, 5, 14), (9, 16, 10, 17), (2, 17, 3, 18)]

Planar representation generated by plantri: [[1, 2, 2, 3], [0, 3, 4, 5], [0, 5, 6, 0], [0, 7, 4, 1], [1, 3, 7, 5], [1, 4, 8, 2], [2, 8, 8, 9], [3, 9, 9, 4], [5, 9, 6, 6], [6, 8, 7, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.38

**Total pinning sets:** 288

**Average overall degree:** 3.03

**Pinning number:** 4

Table 472: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	28	61	80	66	33	9	1	286
Average degree	2.25	2.57	2.78	2.93	3.05	3.15	3.23	3.29	3.33	

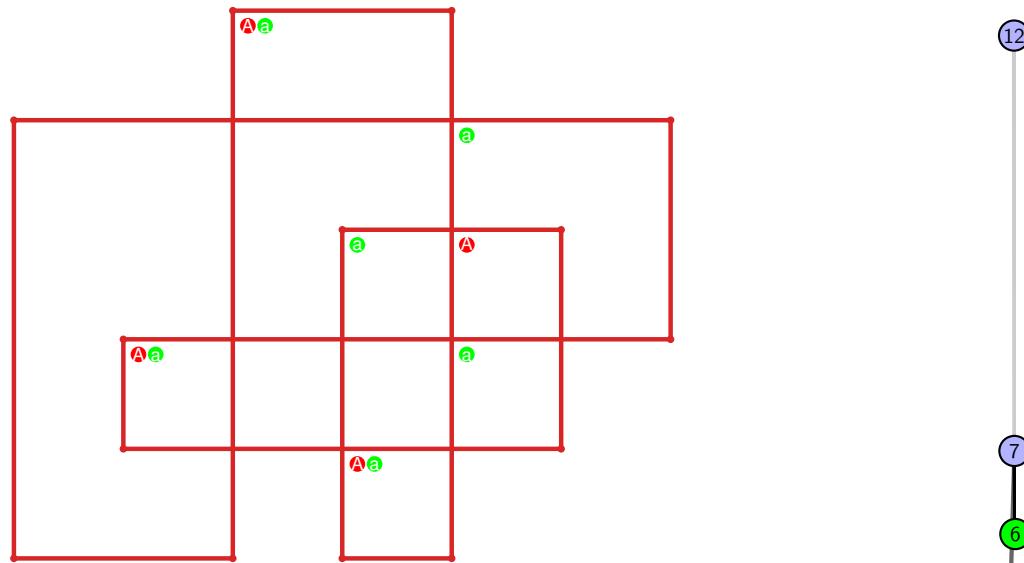


Figure 947: SnapPy multiloop plot.

Figure 948: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.326 [[20, 5, 1, 6], [6, 9, 7, 10], [10, 19, 11, 20], [11, 4, 12, 5], [1, 12, 2, 13], [13, 8, 14, 9], [7, 14, 8, 15], [15, 18, 16, 19], [16, 3, 17, 4], [2, 17, 3, 18]]

PD code drawn by SnapPy: [(14, 1, 15, 2), (9, 2, 10, 3), (18, 3, 19, 4), (11, 6, 12, 7), (4, 7, 5, 8), (19, 10, 20, 11), (5, 12, 6, 13), (20, 15, 1, 16), (13, 16, 14, 17), (8, 17, 9, 18)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 7, 3], [0, 2, 8, 4], [0, 3, 9, 5], [1, 4, 6, 6], [1, 5, 5, 7], [2, 6, 9, 8], [3, 7, 9, 9], [4, 8, 8, 7]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 6  
 Total pinning sets: 528  
 Pinning number: 4

Average optimal degree: 2.62  
 Average minimal degree: 2.67  
 Average overall degree: 3.12

Table 473: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	0	4
Nonminimal pinning sets	0	15	69	132	149	103	43	10	1	522
Average degree	2.62	2.81	2.96	3.08	3.16	3.23	3.27	3.31	3.33	

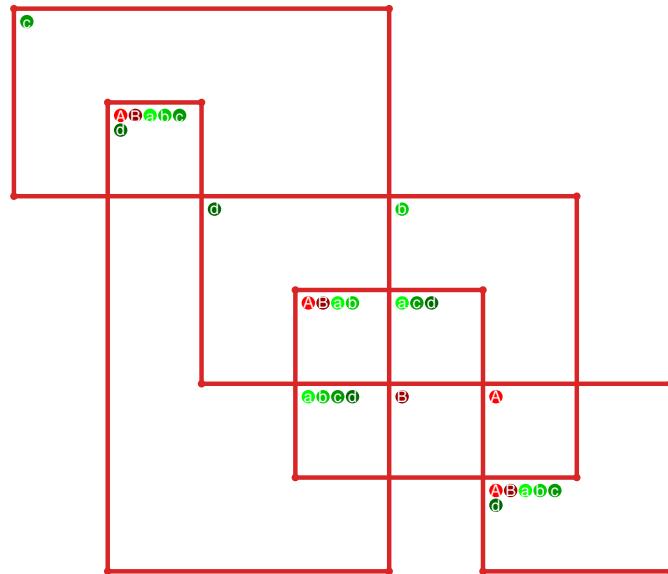


Figure 949: SnapPy multiloop plot.

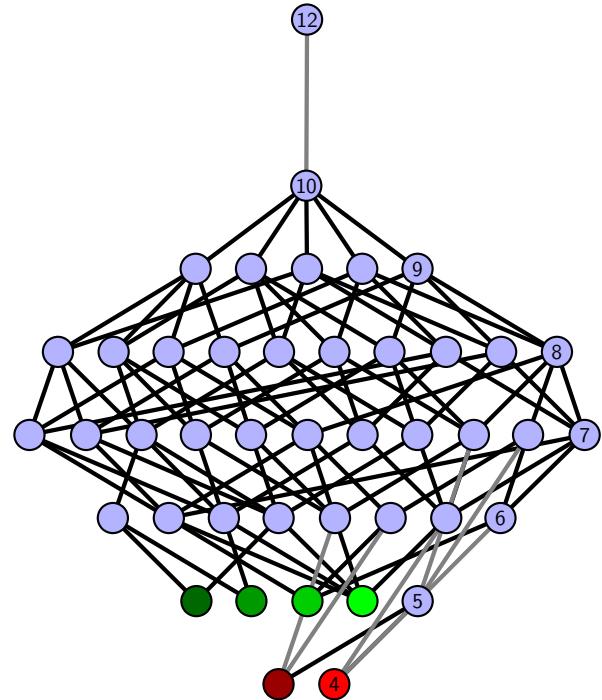


Figure 950: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.327** [[20, 5, 1, 6], [6, 14, 7, 13], [8, 19, 9, 20], [9, 4, 10, 5], [1, 10, 2, 11], [14, 11, 15, 12], [7, 12, 8, 13], [15, 18, 16, 19], [16, 3, 17, 4], [2, 17, 3, 18]]

PD code drawn by `SnapPy`: [(13, 20, 14, 1), (6, 1, 7, 2), (17, 2, 18, 3), (18, 7, 19, 8), (11, 8, 12, 9), (4, 9, 5, 10), (10, 3, 11, 4), (19, 14, 20, 15), (12, 15, 13, 16), (5, 16, 6, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 6], [0, 6, 7, 3], [0, 2, 8, 4], [0, 3, 9, 5], [1, 4, 7, 6], [1, 5, 2, 1], [2, 5, 9, 8], [3, 7, 9, 9], [4, 8, 8, 7]]

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.62

**Total minimal pinning sets:** 6

**Average minimal degree:** 2.68

**Total pinning sets:** 636

**Average overall degree:** 3.13

**Pinning number:** 4

Table 474: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	0	2
Nonminimal pinning sets	0	30	99	166	170	110	44	10	1	630
Average degree	2.62	2.84	2.99	3.1	3.18	3.23	3.28	3.31	3.33	

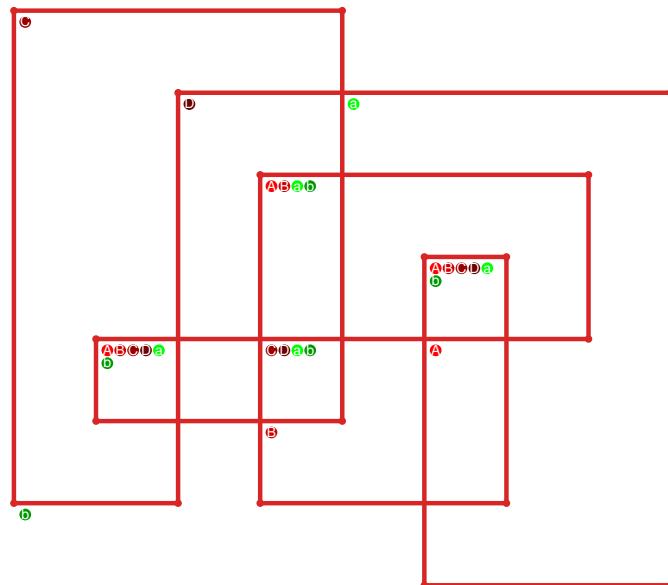


Figure 951: `SnapPy` multiloop plot.

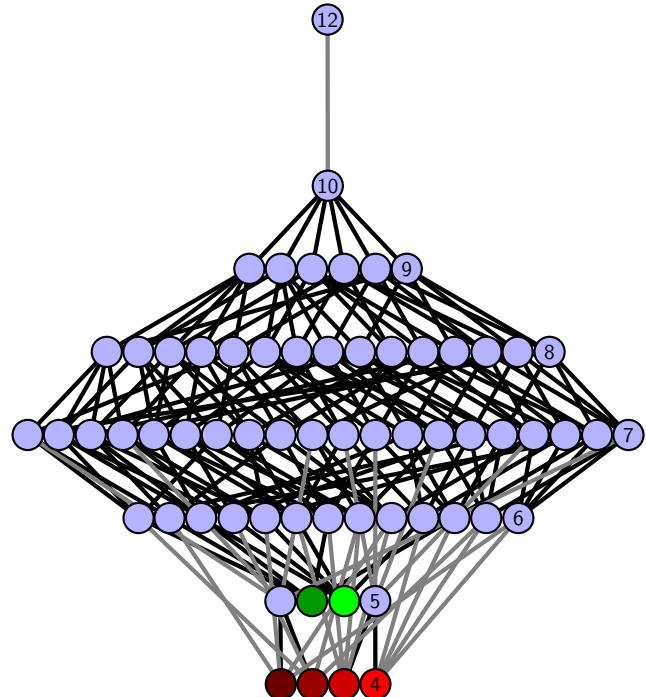


Figure 952: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.328 [[9, 20, 10, 1], [8, 13, 9, 14], [19, 12, 20, 13], [10, 5, 11, 6], [1, 6, 2, 7], [14, 7, 15, 8], [15, 18, 16, 19], [4, 11, 5, 12], [2, 17, 3, 18], [16, 3, 17, 4]]

PD code drawn by SnapPy: [(14, 1, 15, 2), (8, 3, 9, 4), (19, 4, 20, 5), (12, 5, 13, 6), (2, 9, 3, 10), (17, 10, 18, 11), (6, 11, 7, 12), (20, 15, 1, 16), (13, 16, 14, 17), (7, 18, 8, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 7], [0, 7, 7, 4], [0, 3, 8, 5], [1, 4, 6, 1], [2, 5, 8, 9], [2, 9, 3, 3], [4, 9, 9, 6], [6, 8, 8, 7]]

Total optimal pinning sets: 11  
 Total minimal pinning sets: 12  
 Total pinning sets: 430  
 Pinning number: 5

Average optimal degree: 2.6  
 Average minimal degree: 2.62  
 Average overall degree: 3.06

Table 475: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	11	0	0	0	0	0	0	0	11
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	54	111	123	84	36	9	1	418
Average degree	2.6	2.84	3.0	3.11	3.19	3.24	3.29	3.33	

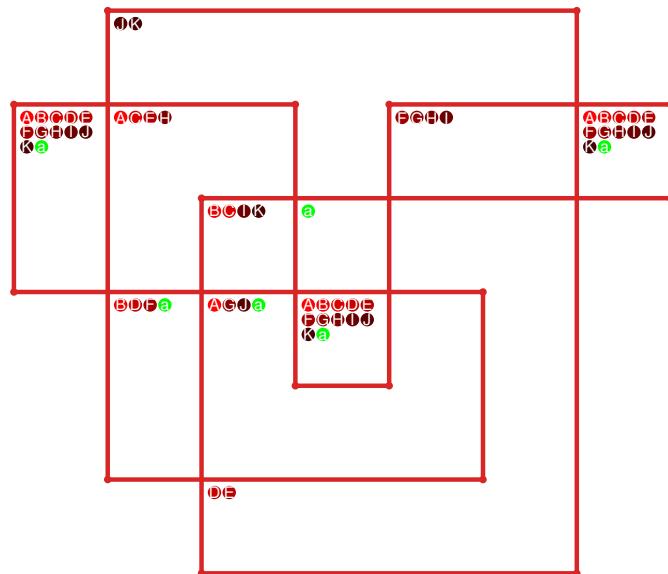


Figure 953: SnapPy multiloop plot.

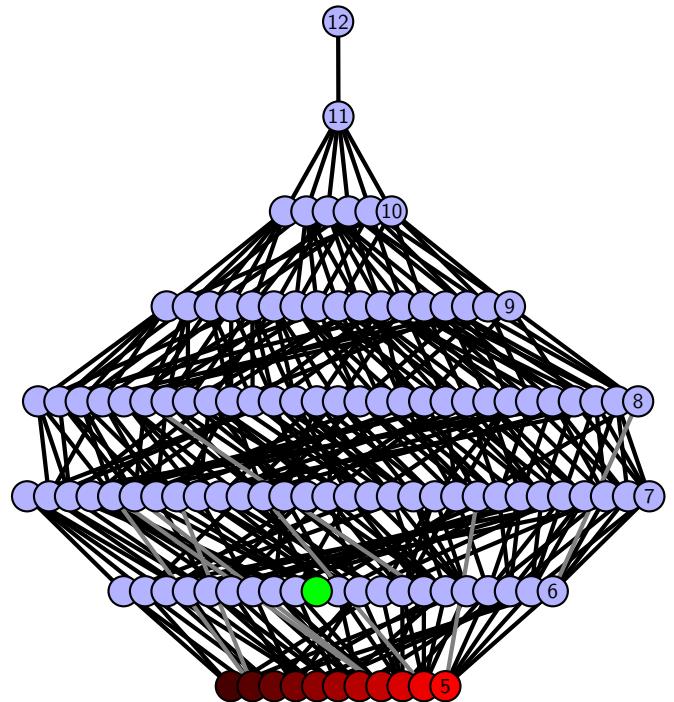


Figure 954: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.329**  $[[5, 20, 6, 1], [4, 13, 5, 14], [19, 12, 20, 13], [6, 12, 7, 11], [1, 11, 2, 10], [14, 3, 15, 4], [15, 18, 16, 19], [7, 16, 8, 17], [2, 9, 3, 10], [17, 8, 18, 9]]$

PD code drawn by `SnapPy`:  $[(12, 1, 13, 2), (15, 6, 16, 7), (20, 7, 1, 8), (8, 19, 9, 20), (9, 4, 10, 5), (17, 10, 18, 11), (2, 11, 3, 12), (5, 14, 6, 15), (13, 16, 14, 17), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 5, 2], [0, 1, 6, 3], [0, 2, 7, 4], [0, 3, 8, 8], [1, 8, 6, 1], [2, 5, 9, 7], [3, 6, 9, 9], [4, 9, 5, 4], [6, 8, 7, 7]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.47

**Total pinning sets:** 304

**Average overall degree:** 3.04

**Pinning number:** 4

Table 476: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	8	28	65	86	70	34	9	1	301
Average degree	2.25	2.58	2.78	2.93	3.06	3.16	3.24	3.29	3.33	

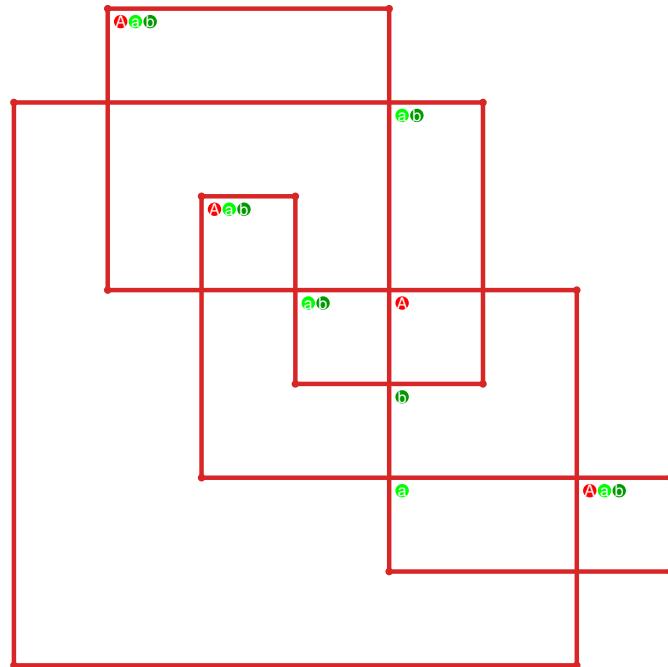


Figure 955: `SnapPy` multiloop plot.

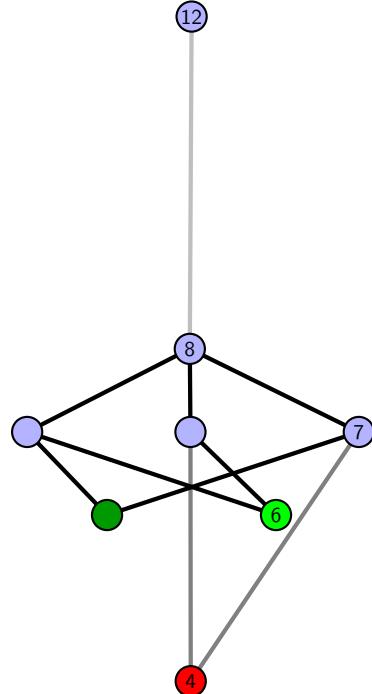


Figure 956: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.330** `[[20, 7, 1, 8], [8, 3, 9, 4], [4, 19, 5, 20], [15, 6, 16, 7], [1, 13, 2, 12], [2, 11, 3, 12], [9, 18, 10, 19], [5, 14, 6, 15], [16, 14, 17, 13], [17, 10, 18, 11]]`

PD code drawn by `SnapPy`: `[(6, 1, 7, 2), (13, 4, 14, 5), (18, 5, 19, 6), (15, 8, 16, 9), (20, 9, 1, 10), (10, 19, 11, 20), (11, 14, 12, 15), (3, 12, 4, 13), (7, 16, 8, 17), (2, 17, 3, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 5, 6, 2], [0, 1, 6, 7], [0, 7, 7, 8], [0, 8, 5, 5], [1, 4, 4, 9], [1, 9, 9, 2], [2, 8, 3, 3], [3, 7, 9, 4], [5, 8, 6, 6]]`

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 384  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.25  
 Average overall degree: 3.03

Table 477: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

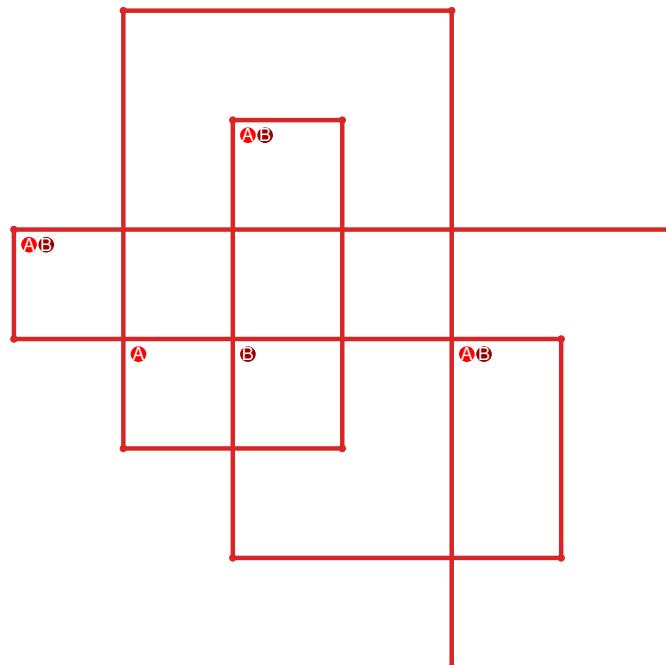


Figure 957: `SnapPy` multiloop plot.

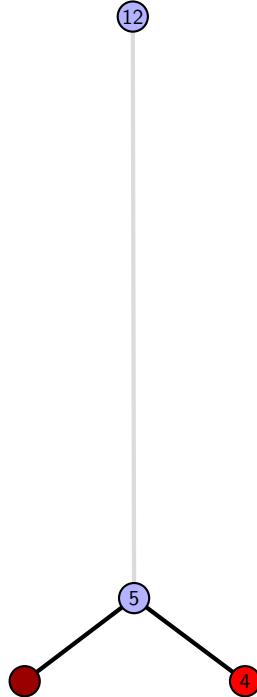


Figure 958: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.331 [[15, 20, 16, 1], [19, 14, 20, 15], [16, 7, 17, 8], [1, 8, 2, 9], [9, 18, 10, 19], [6, 13, 7, 14], [17, 3, 18, 2], [10, 5, 11, 6], [12, 3, 13, 4], [4, 11, 5, 12]]

PD code drawn by SnapPy: [(20, 15, 1, 16), (10, 1, 11, 2), (8, 3, 9, 4), (14, 5, 15, 6), (19, 6, 20, 7), (2, 9, 3, 10), (4, 11, 5, 12), (17, 12, 18, 13), (7, 16, 8, 17), (13, 18, 14, 19)]

Planar representation generated by plantri: [[1, 1, 2, 3], [0, 4, 5, 0], [0, 5, 6, 3], [0, 2, 6, 4], [1, 3, 6, 7], [1, 7, 8, 2], [2, 8, 4, 3], [4, 9, 9, 5], [5, 9, 9, 6], [7, 8, 8, 7]]

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 320

**Average overall degree:** 3.03

**Pinning number:** 4

Table 478: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

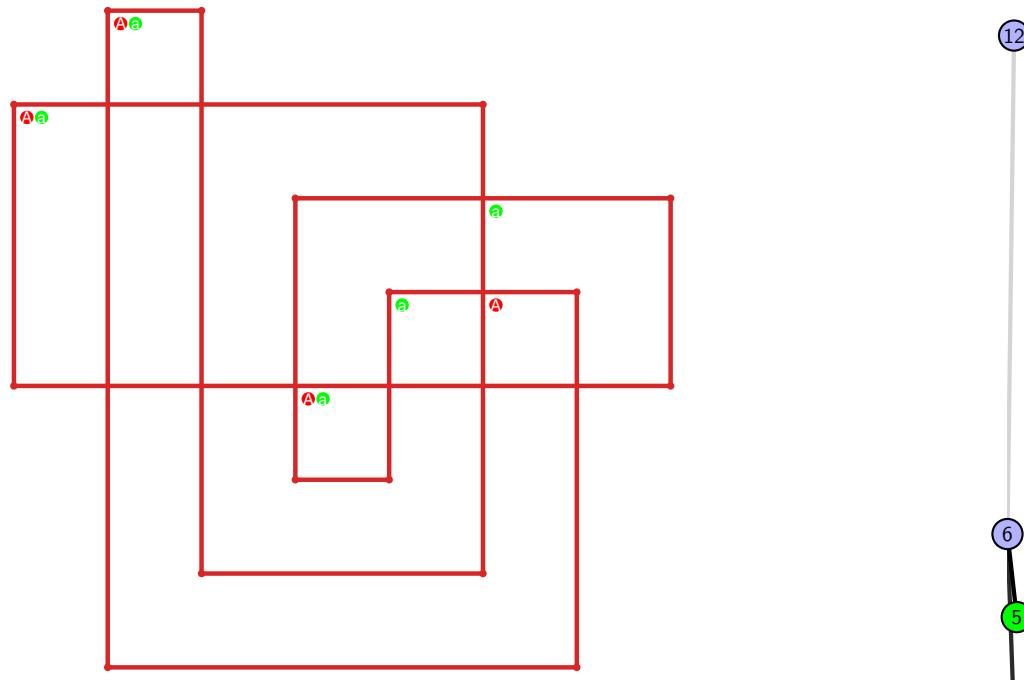


Figure 959: SnapPy multiloop plot.

Figure 960: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.332** `[[15, 20, 16, 1], [19, 14, 20, 15], [16, 8, 17, 7], [1, 7, 2, 6], [18, 5, 19, 6], [13, 8, 14, 9], [17, 3, 18, 2], [11, 4, 12, 5], [9, 12, 10, 13], [3, 10, 4, 11]]`

PD code drawn by `SnapPy`: `[(8, 1, 9, 2), (15, 2, 16, 3), (3, 18, 4, 19), (4, 13, 5, 14), (10, 5, 11, 6), (20, 7, 1, 8), (6, 9, 7, 10), (16, 11, 17, 12), (12, 17, 13, 18), (19, 14, 20, 15)]`

Planar representation generated by `plantri`: `[[1, 1, 2, 3], [0, 4, 5, 0], [0, 5, 6, 3], [0, 2, 6, 4], [1, 3, 6, 7], [1, 8, 8, 2], [2, 9, 4, 3], [4, 9, 9, 8], [5, 7, 9, 5], [6, 8, 7, 7]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.25

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.33

**Total pinning sets:** 320

**Average overall degree:** 3.03

**Pinning number:** 4

Table 479: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

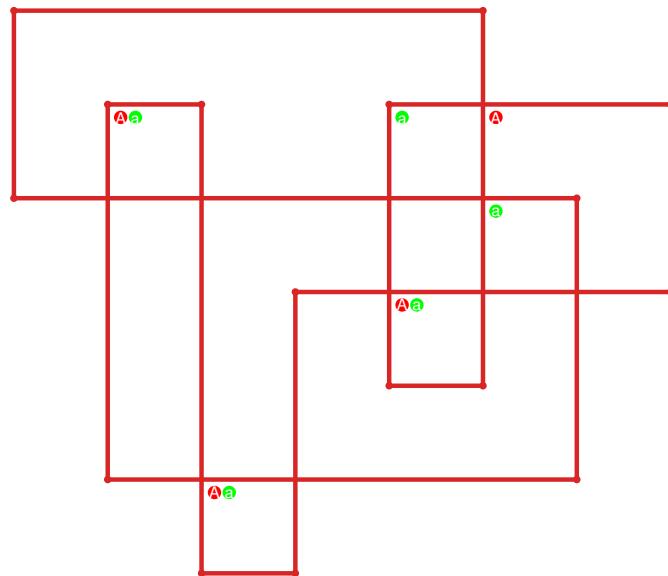


Figure 961: `SnapPy` multiloop plot.



Figure 962: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.333  $[[7, 20, 8, 1], [11, 6, 12, 7], [19, 16, 20, 17], [8, 16, 9, 15], [1, 15, 2, 14], [10, 13, 11, 14], [5, 12, 6, 13], [17, 5, 18, 4], [18, 3, 19, 4], [9, 3, 10, 2]]$

PD code drawn by `SnapPy`:  $[(20, 7, 1, 8), (16, 1, 17, 2), (2, 15, 3, 16), (9, 4, 10, 5), (3, 10, 4, 11), (8, 11, 9, 12), (12, 19, 13, 20), (13, 6, 14, 7), (17, 14, 18, 15), (5, 18, 6, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 5, 6, 6], [0, 7, 8, 3], [0, 2, 9, 4], [0, 3, 9, 5], [1, 4, 9, 6], [1, 5, 7, 1], [2, 6, 8, 8], [2, 7, 7, 9], [3, 8, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.5

**Total minimal pinning sets:** 5

**Average minimal degree:** 2.66

**Total pinning sets:** 448

**Average overall degree:** 3.11

**Pinning number:** 4

Table 480: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	0	4
Nonminimal pinning sets	0	8	50	106	130	96	42	10	1	443
Average degree	2.5	2.73	2.91	3.05	3.14	3.22	3.27	3.31	3.33	

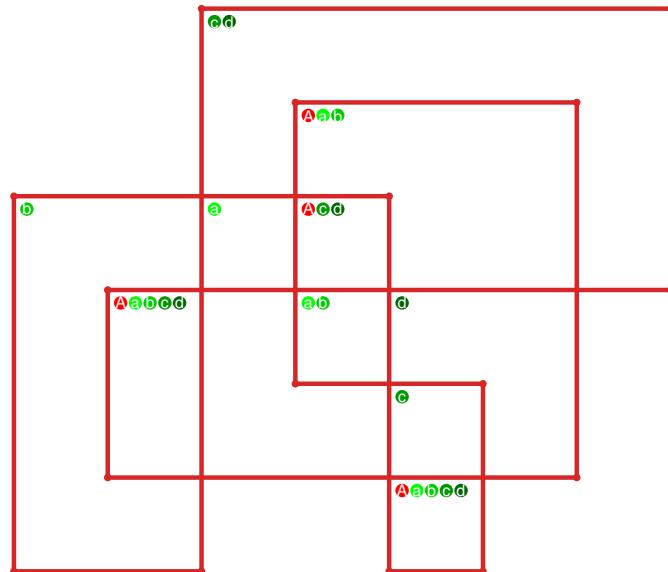


Figure 963: `SnapPy` multiloop plot.

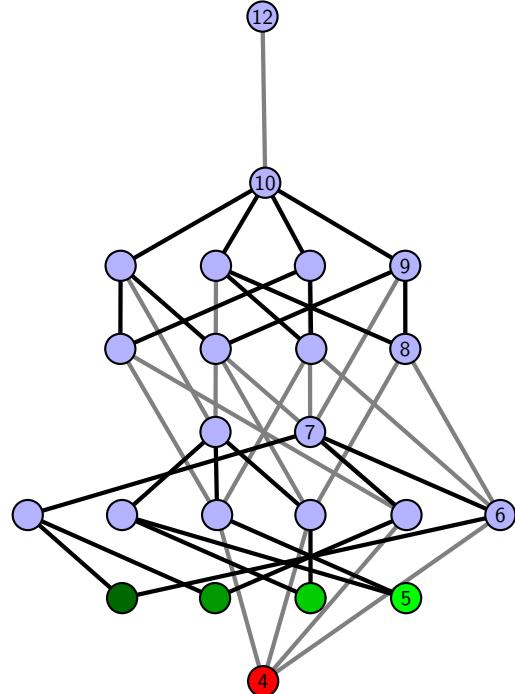


Figure 964: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.334 [[20, 9, 1, 10], [10, 17, 11, 18], [19, 4, 20, 5], [8, 1, 9, 2], [16, 7, 17, 8], [11, 7, 12, 6], [18, 6, 19, 5], [3, 14, 4, 15], [2, 14, 3, 13], [15, 12, 16, 13]]

PD code drawn by SnapPy: [(16, 1, 17, 2), (2, 15, 3, 16), (3, 8, 4, 9), (13, 4, 14, 5), (17, 6, 18, 7), (12, 9, 13, 10), (10, 19, 11, 20), (20, 11, 1, 12), (7, 14, 8, 15), (5, 18, 6, 19)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 5, 6], [0, 6, 6, 7], [0, 8, 4, 0], [1, 3, 9, 5], [1, 4, 9, 6], [1, 5, 2, 2], [2, 9, 8, 8], [3, 7, 7, 9], [4, 8, 7, 5]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 320  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.33  
 Average overall degree: 3.03

Table 481: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	0	1
Nonminimal pinning sets	0	8	34	71	90	71	34	9	1	318
Average degree	2.25	2.56	2.77	2.94	3.06	3.16	3.24	3.29	3.33	

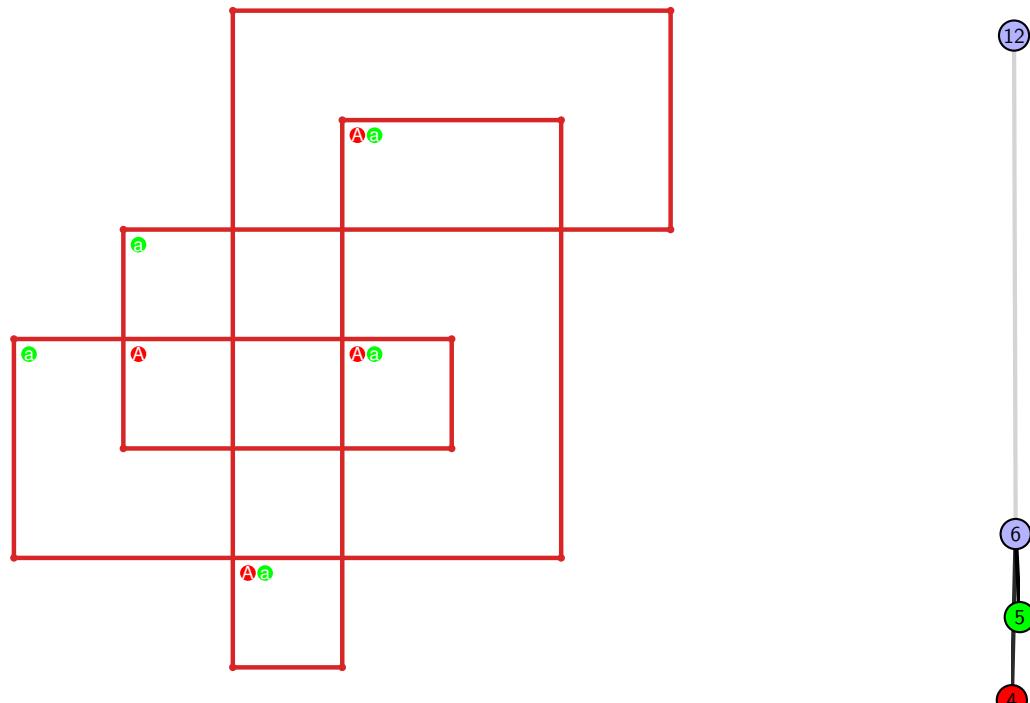


Figure 965: SnapPy multiloop plot.

Figure 966: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.335 [[15, 20, 16, 1], [14, 5, 15, 6], [19, 4, 20, 5], [16, 2, 17, 1], [6, 17, 7, 18], [18, 13, 19, 14], [3, 10, 4, 11], [2, 10, 3, 9], [7, 12, 8, 13], [11, 8, 12, 9]]

PD code drawn by `SnapPy`: [(16, 1, 17, 2), (11, 2, 12, 3), (8, 5, 9, 6), (6, 19, 7, 20), (20, 7, 1, 8), (4, 9, 5, 10), (15, 10, 16, 11), (3, 14, 4, 15), (12, 17, 13, 18), (18, 13, 19, 14)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 2], [0, 1, 5, 6], [0, 7, 4, 0], [1, 3, 8, 5], [1, 4, 8, 2], [2, 9, 7, 7], [3, 6, 6, 9], [4, 9, 9, 5], [6, 8, 8, 7]]

**Total optimal pinning sets:** 3  
**Total minimal pinning sets:** 3

**Total pinning sets:** 256

**Pinning number:** 5

**Average optimal degree:** 2.4

**Average minimal degree:** 2.4

**Average overall degree:** 3.03

Table 482: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	19	51	75	65	33	9	1	253
Average degree	2.4	2.68	2.89	3.03	3.15	3.23	3.29	3.33	

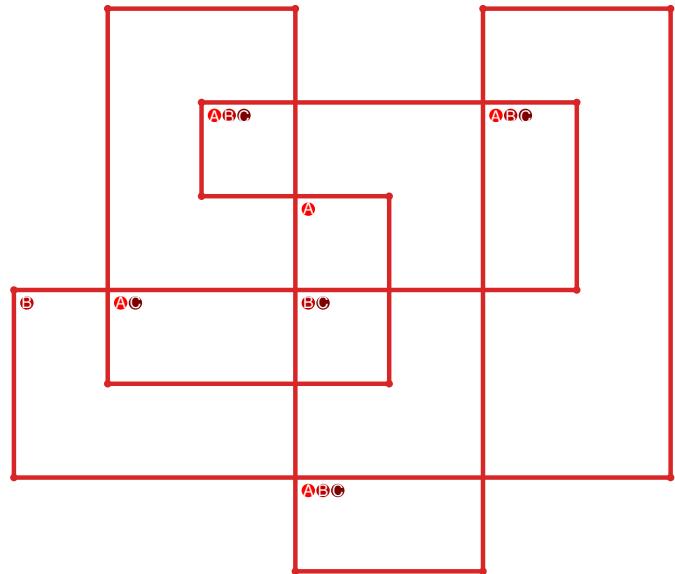


Figure 967: `SnapPy` multiloop plot.

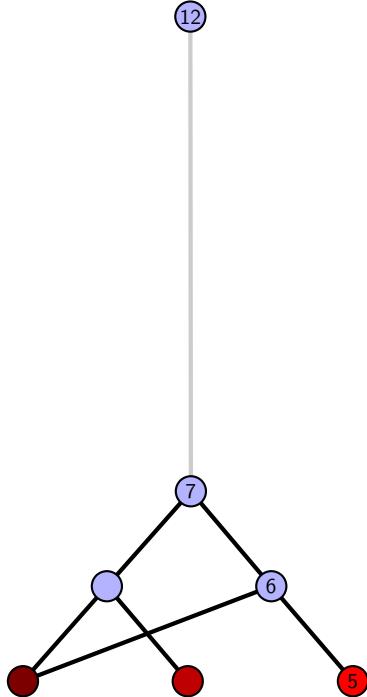


Figure 968: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.336** [[20, 7, 1, 8], [8, 17, 9, 18], [14, 19, 15, 20], [6, 1, 7, 2], [16, 5, 17, 6], [9, 12, 10, 13], [18, 13, 19, 14], [15, 3, 16, 2], [11, 4, 12, 5], [10, 4, 11, 3]]

PD code drawn by `SnapPy`: [(20, 7, 1, 8), (8, 1, 9, 2), (13, 2, 14, 3), (17, 4, 18, 5), (6, 9, 7, 10), (15, 10, 16, 11), (5, 14, 6, 15), (11, 16, 12, 17), (3, 18, 4, 19), (19, 12, 20, 13)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 6], [0, 6, 6, 7], [0, 7, 4, 0], [1, 3, 7, 8], [1, 8, 9, 6], [1, 5, 2, 2], [2, 9, 4, 3], [4, 9, 9, 5], [5, 8, 8, 7]]

Total optimal pinning sets: 3  
 Total minimal pinning sets: 6  
 Total pinning sets: 476  
 Pinning number: 4

Average optimal degree: 2.33  
 Average minimal degree: 2.57  
 Average overall degree: 3.05

Table 483: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	0	3
Nonminimal pinning sets	0	21	73	121	125	84	36	9	1	470
Average degree	2.33	2.67	2.87	3.01	3.11	3.19	3.24	3.29	3.33	

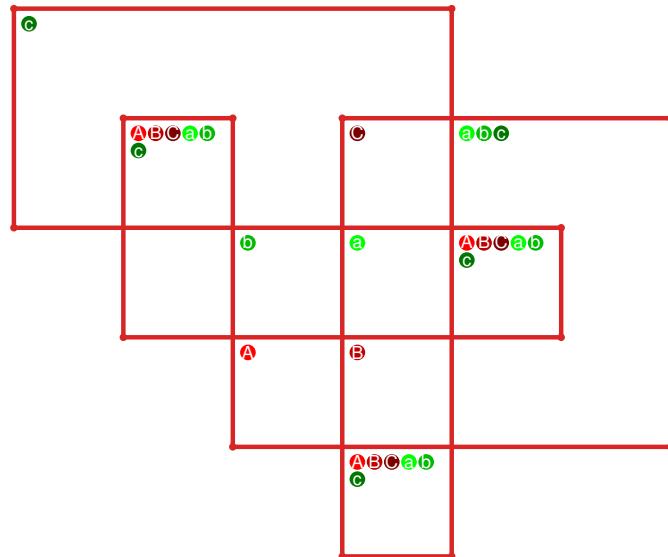


Figure 969: `SnapPy` multiloop plot.

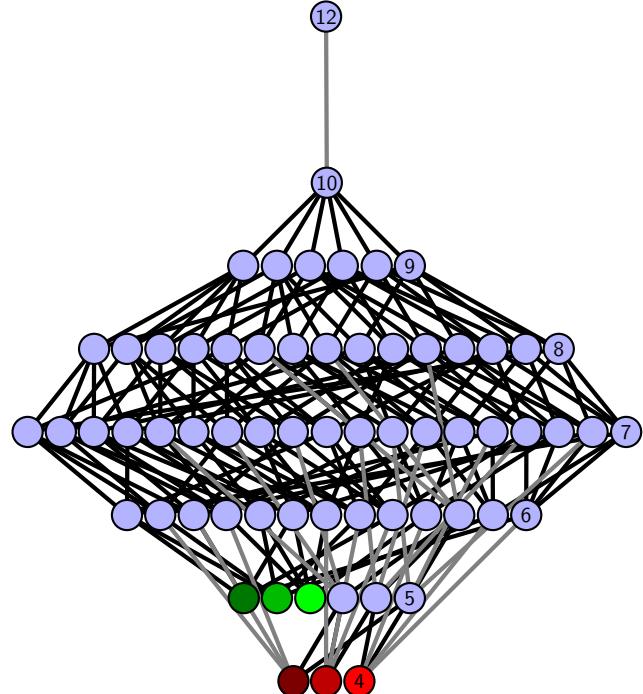


Figure 970: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.337** [[20, 13, 1, 14], [14, 5, 15, 6], [6, 19, 7, 20], [12, 1, 13, 2], [4, 11, 5, 12], [15, 18, 16, 19], [7, 3, 8, 2], [8, 3, 9, 4], [17, 10, 18, 11], [16, 10, 17, 9]]

PD code drawn by `SnapPy`: [(5, 20, 6, 1), (11, 2, 12, 3), (6, 13, 7, 14), (14, 7, 15, 8), (1, 8, 2, 9), (17, 10, 18, 11), (12, 15, 13, 16), (3, 16, 4, 17), (9, 18, 10, 19), (19, 4, 20, 5)]

Planar representation generated by `plantri`: [[1, 2, 3, 3], [0, 4, 5, 2], [0, 1, 5, 6], [0, 6, 4, 0], [1, 3, 7, 8], [1, 8, 9, 2], [2, 7, 7, 3], [4, 6, 6, 9], [4, 9, 9, 5], [5, 8, 8, 7]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 2  
 Total pinning sets: 384  
 Pinning number: 4

Average optimal degree: 2.25  
 Average minimal degree: 2.25  
 Average overall degree: 3.03

Table 484: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	15	49	91	105	77	35	9	1	382
Average degree	2.25	2.59	2.81	2.97	3.08	3.17	3.24	3.29	3.33	

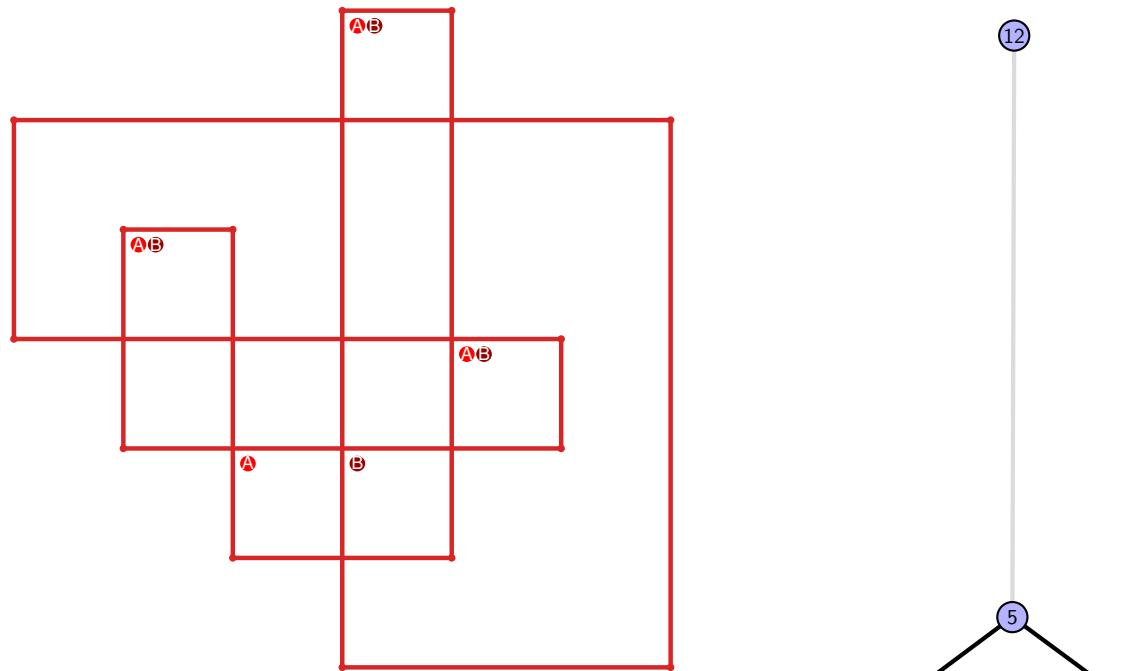


Figure 971: `SnapPy` multiloop plot.

Figure 972: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.338 [[20, 7, 1, 8], [8, 14, 9, 13], [19, 12, 20, 13], [6, 1, 7, 2], [14, 10, 15, 9], [15, 18, 16, 19], [11, 2, 12, 3], [5, 10, 6, 11], [17, 4, 18, 5], [16, 4, 17, 3]]

PD code drawn by SnapPy: [(15, 2, 16, 3), (4, 9, 5, 10), (20, 5, 1, 6), (14, 7, 15, 8), (8, 3, 9, 4), (10, 13, 11, 14), (1, 16, 2, 17), (6, 17, 7, 18), (18, 11, 19, 12), (12, 19, 13, 20)]

Planar representation generated by plantri: [[1, 2, 3, 3], [0, 4, 4, 2], [0, 1, 5, 6], [0, 6, 7, 0], [1, 7, 5, 1], [2, 4, 8, 9], [2, 9, 7, 3], [3, 6, 8, 4], [5, 7, 9, 9], [5, 8, 8, 6]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 6  
 Total pinning sets: 444  
 Pinning number: 4

Average optimal degree: 2.38  
 Average minimal degree: 2.49  
 Average overall degree: 3.05

Table 485: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	0	4
Nonminimal pinning sets	0	15	63	111	120	83	36	9	1	438
Average degree	2.38	2.64	2.85	3.0	3.1	3.18	3.24	3.29	3.33	

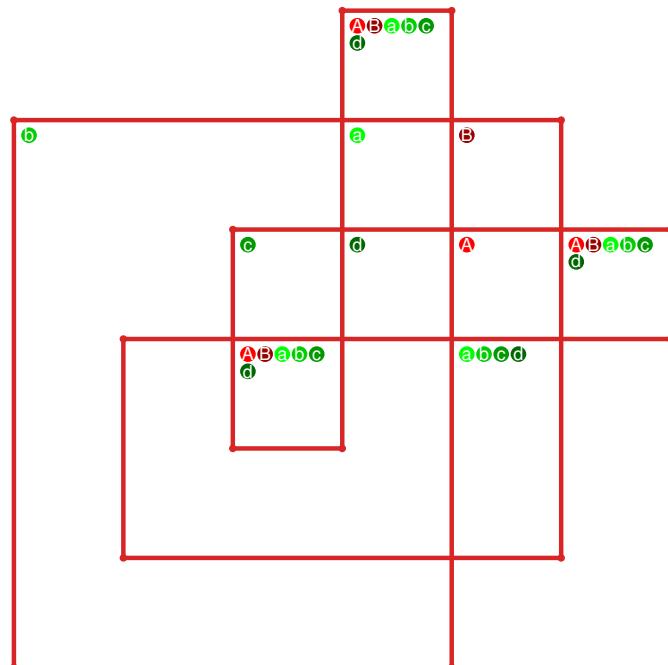


Figure 973: SnapPy multiloop plot.

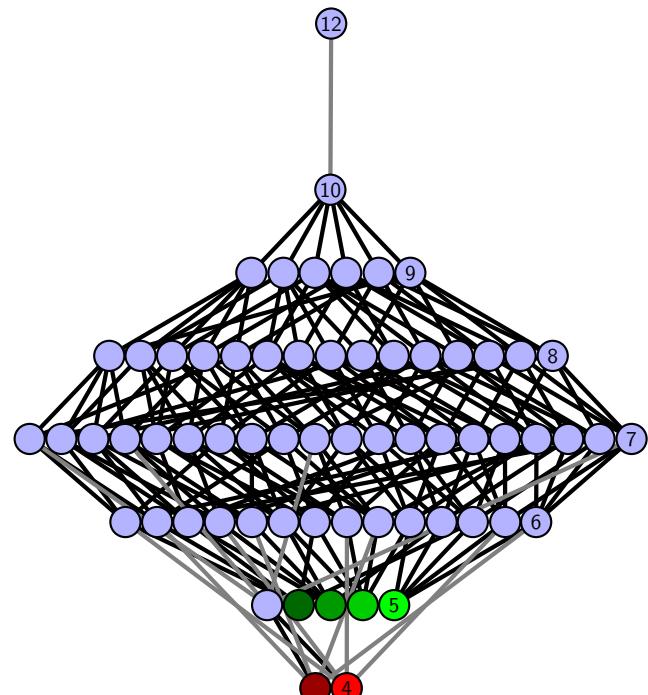


Figure 974: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.339**  $[[9, 20, 10, 1], [3, 8, 4, 9], [19, 16, 20, 17], [10, 16, 11, 15], [1, 15, 2, 14], [2, 13, 3, 14], [7, 12, 8, 13], [4, 18, 5, 17], [5, 18, 6, 19], [11, 6, 12, 7]]$

PD code drawn by SnapPy:  $[(9, 4, 10, 5), (1, 6, 2, 7), (5, 10, 6, 11), (20, 11, 1, 12), (12, 19, 13, 20), (13, 8, 14, 9), (17, 14, 18, 15), (15, 2, 16, 3), (3, 16, 4, 17), (7, 18, 8, 19)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 5, 6, 7], [0, 7, 8, 3], [0, 2, 9, 4], [0, 3, 5, 5], [1, 4, 4, 6], [1, 5, 9, 9], [1, 8, 8, 2], [2, 7, 7, 9], [3, 8, 6, 6]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 6  
 Total pinning sets: 336  
 Pinning number: 5

Average optimal degree: 2.5  
 Average minimal degree: 2.5  
 Average overall degree: 3.05

Table 486: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	33	77	99	76	35	9	1	330
Average degree	2.5	2.76	2.94	3.07	3.17	3.24	3.29	3.33	

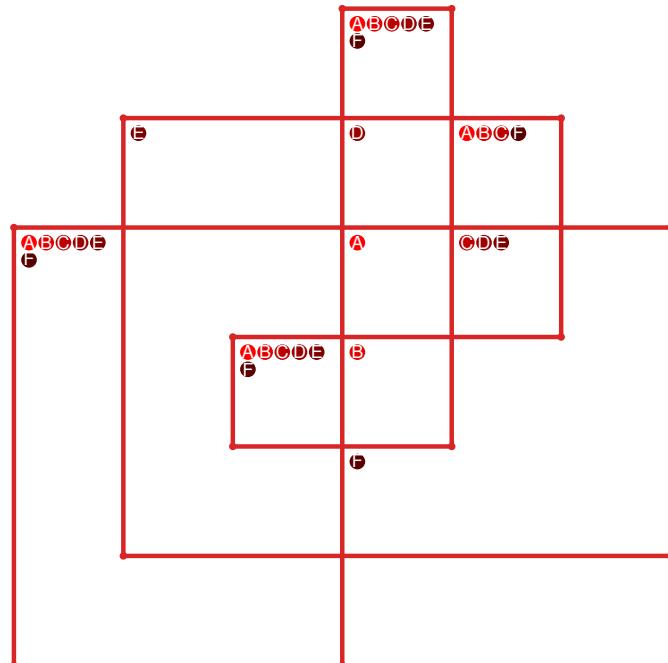


Figure 975: SnapPy multiloop plot.

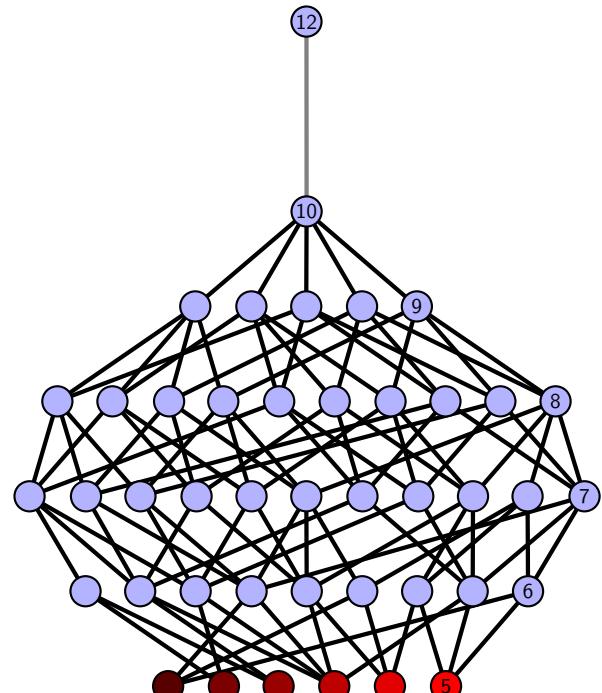


Figure 976: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.340** `[[11, 20, 12, 1], [17, 10, 18, 11], [19, 4, 20, 5], [12, 4, 13, 3], [1, 16, 2, 17], [9, 6, 10, 7], [18, 6, 19, 5], [13, 9, 14, 8], [15, 2, 16, 3], [7, 15, 8, 14]]`

PD code drawn by `SnapPy`: `[(13, 20, 14, 1), (1, 10, 2, 11), (17, 2, 18, 3), (3, 16, 4, 17), (7, 4, 8, 5), (6, 11, 7, 12), (15, 8, 16, 9), (12, 5, 13, 6), (19, 14, 20, 15), (9, 18, 10, 19)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 3], [0, 2, 7, 8], [0, 8, 8, 1], [1, 9, 7, 6], [1, 5, 2, 2], [3, 5, 9, 9], [3, 9, 4, 4], [5, 8, 7, 7]]`

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.38

**Total minimal pinning sets:** 4

**Average minimal degree:** 2.44

**Total pinning sets:** 432

**Average overall degree:** 3.05

**Pinning number:** 4

Table 487: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	0	2
Nonminimal pinning sets	0	15	58	107	119	83	36	9	1	428
Average degree	2.38	2.65	2.85	2.99	3.1	3.18	3.24	3.29	3.33	

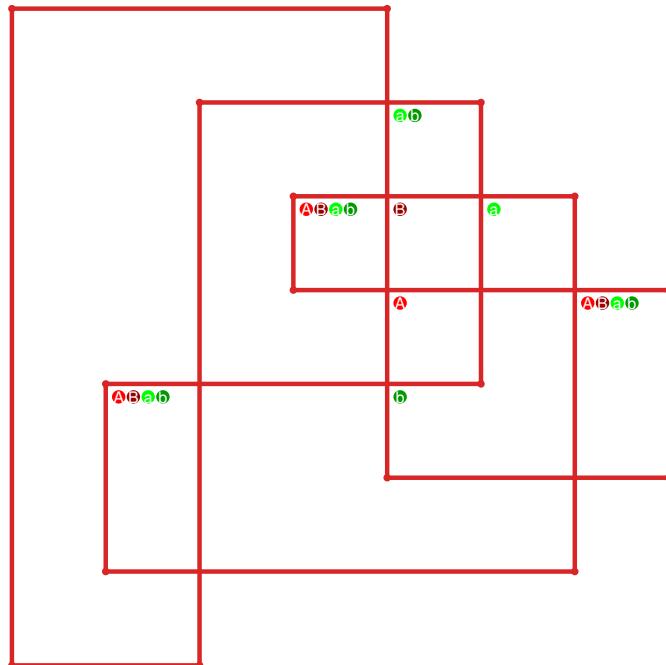


Figure 977: `SnapPy` multiloop plot.

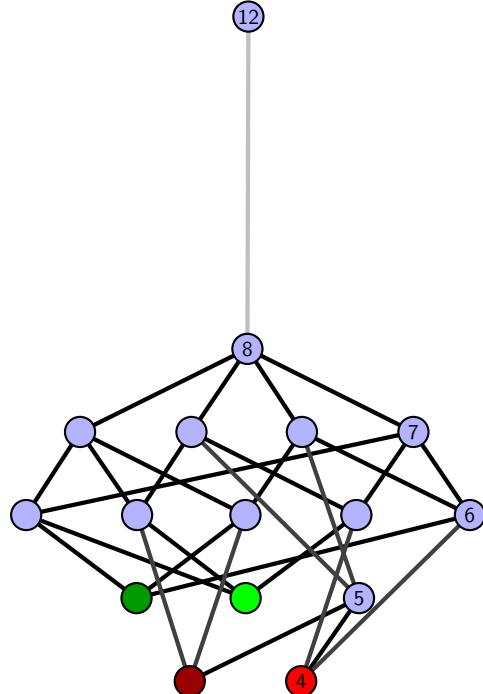


Figure 978: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.341  $[[5, 20, 6, 1], [17, 4, 18, 5], [19, 8, 20, 9], [6, 14, 7, 13], [1, 16, 2, 17], [3, 10, 4, 11], [18, 10, 19, 9], [7, 14, 8, 15], [15, 12, 16, 13], [2, 12, 3, 11]]$

PD code drawn by SnapPy:  $[(14, 1, 15, 2), (16, 5, 17, 6), (6, 15, 7, 16), (20, 7, 1, 8), (8, 19, 9, 20), (12, 9, 13, 10), (10, 3, 11, 4), (4, 11, 5, 12), (18, 13, 19, 14), (2, 17, 3, 18)]$

Planar representation generated by plantri:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 7], [0, 7, 7, 8], [0, 8, 9, 1], [1, 9, 9, 6], [1, 5, 2, 2], [2, 8, 3, 3], [3, 7, 9, 4], [4, 8, 5, 5]]$

Total optimal pinning sets: 6  
 Total minimal pinning sets: 9  
 Total pinning sets: 372  
 Pinning number: 5

Average optimal degree: 2.57  
 Average minimal degree: 2.6  
 Average overall degree: 3.06

Table 488: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	6	0	0	0	0	0	0	0	6
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	34	89	112	82	36	9	1	363
Average degree	2.57	2.79	2.97	3.09	3.18	3.24	3.29	3.33	

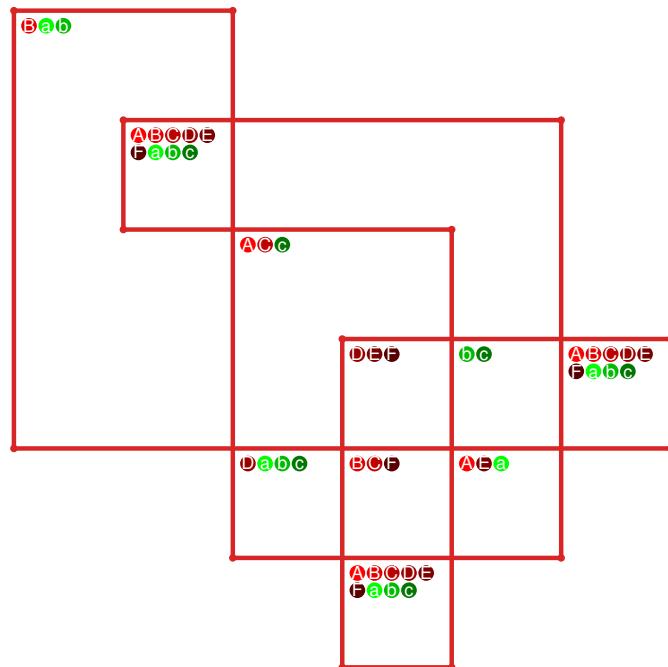


Figure 979: SnapPy multiloop plot.

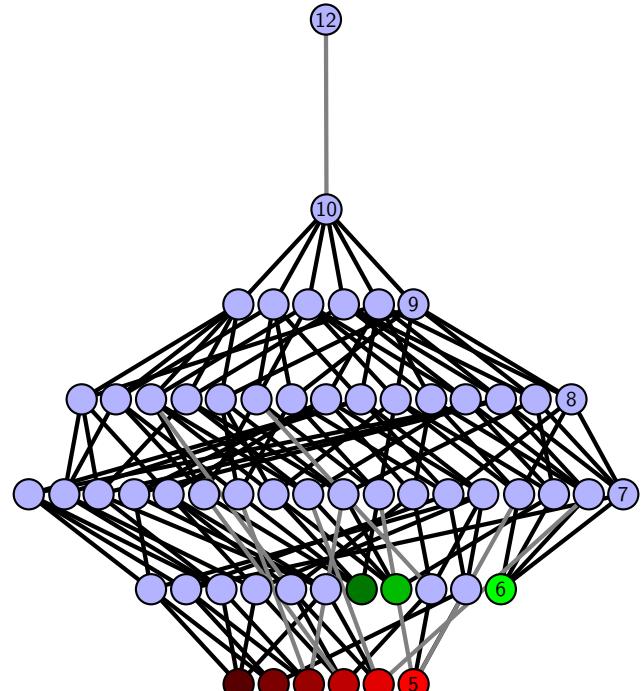


Figure 980: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.342** `[[20, 13, 1, 14], [14, 9, 15, 10], [10, 19, 11, 20], [12, 5, 13, 6], [1, 8, 2, 9], [15, 18, 16, 19], [11, 7, 12, 6], [7, 4, 8, 5], [2, 17, 3, 18], [16, 3, 17, 4]]`

PD code drawn by `SnapPy`: `[(14, 1, 15, 2), (5, 2, 6, 3), (10, 3, 11, 4), (4, 9, 5, 10), (19, 6, 20, 7), (12, 7, 13, 8), (18, 11, 19, 12), (20, 15, 1, 16), (13, 16, 14, 17), (8, 17, 9, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 6], [0, 6, 6, 7], [0, 7, 8, 1], [1, 8, 9, 2], [2, 7, 3, 3], [3, 6, 9, 4], [4, 9, 9, 5], [5, 8, 8, 7]]`

Total optimal pinning sets: 1  
 Total minimal pinning sets: 9  
 Total pinning sets: 520  
 Pinning number: 4

Average optimal degree: 2.5  
 Average minimal degree: 2.74  
 Average overall degree: 3.12

Table 489: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	8	0	0	0	0	0	0	0	8
Nonminimal pinning sets	0	8	66	131	149	103	43	10	1	511
Average degree	2.5	2.76	2.94	3.07	3.16	3.23	3.27	3.31	3.33	

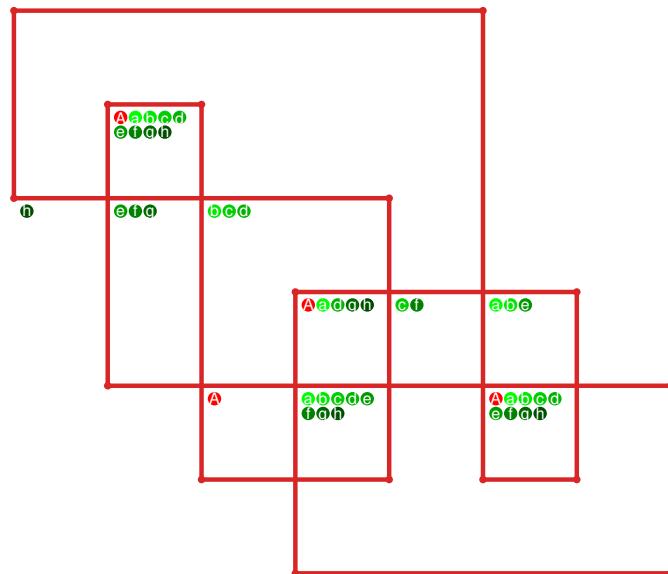


Figure 981: `SnapPy` multiloop plot.

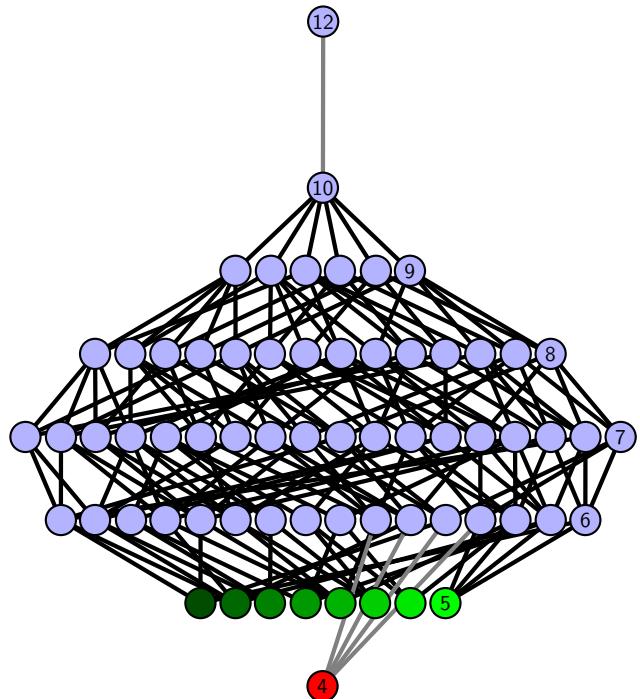


Figure 982: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.343** [[20, 15, 1, 16], [16, 11, 17, 12], [12, 19, 13, 20], [14, 7, 15, 8], [1, 10, 2, 11], [17, 5, 18, 4], [18, 3, 19, 4], [13, 9, 14, 8], [9, 6, 10, 7], [2, 6, 3, 5]]

PD code drawn by `SnapPy`: [(15, 20, 16, 1), (1, 14, 2, 15), (2, 19, 3, 20), (6, 3, 7, 4), (11, 4, 12, 5), (5, 10, 6, 11), (18, 7, 19, 8), (13, 8, 14, 9), (17, 12, 18, 13), (9, 16, 10, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 6, 7], [0, 7, 7, 8], [0, 8, 9, 1], [1, 9, 6, 6], [2, 5, 5, 9], [2, 8, 3, 3], [3, 7, 9, 4], [4, 8, 6, 5]]

**Total optimal pinning sets:** 4

**Average optimal degree:** 2.75

**Total minimal pinning sets:** 12

**Average minimal degree:** 2.79

**Total pinning sets:** 634

**Average overall degree:** 3.13

**Pinning number:** 4

Table 490: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	4	0	0	0	0	0	0	0	0	4
Minimal (suboptimal) pinning sets	0	7	1	0	0	0	0	0	0	8
Nonminimal pinning sets	0	26	98	164	169	110	44	10	1	622
Average degree	2.75	2.89	3.0	3.1	3.18	3.23	3.28	3.31	3.33	

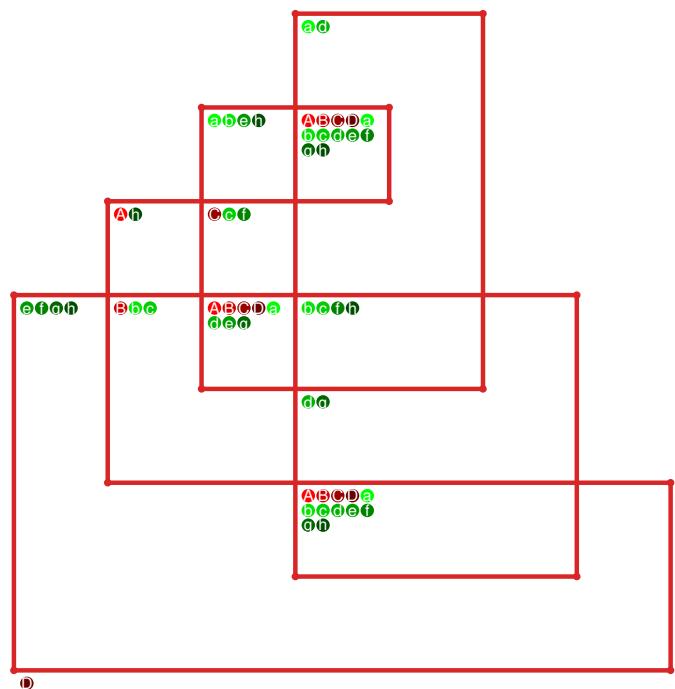


Figure 983: `SnapPy` multiloop plot.

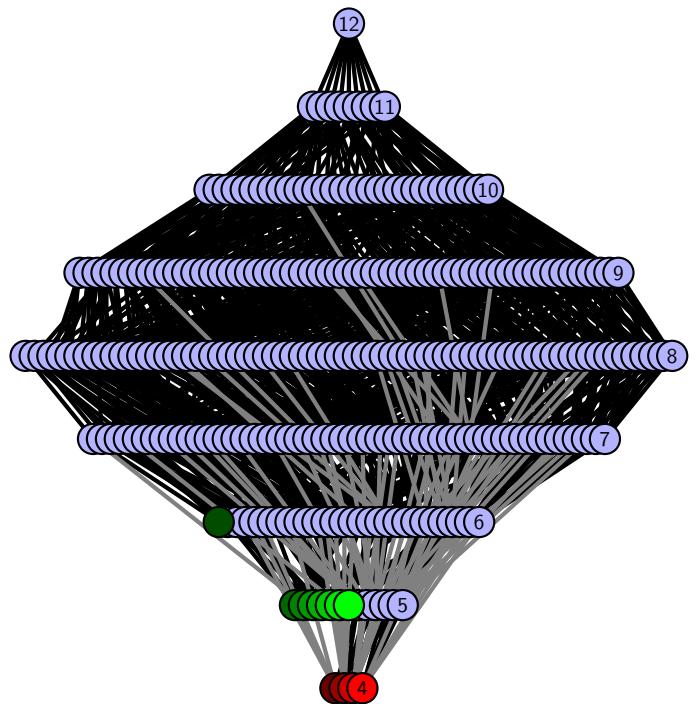


Figure 984: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.344** [[5, 20, 6, 1], [11, 4, 12, 5], [12, 19, 13, 20], [6, 15, 7, 16], [1, 10, 2, 11], [18, 3, 19, 4], [13, 9, 14, 8], [14, 7, 15, 8], [16, 9, 17, 10], [2, 17, 3, 18]]

PD code drawn by `SnapPy`: [(15, 20, 16, 1), (8, 1, 9, 2), (11, 4, 12, 5), (5, 10, 6, 11), (13, 6, 14, 7), (18, 7, 19, 8), (3, 12, 4, 13), (19, 14, 20, 15), (9, 16, 10, 17), (2, 17, 3, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 6], [0, 7, 7, 8], [0, 8, 9, 1], [1, 9, 9, 2], [2, 8, 7, 7], [3, 6, 6, 3], [3, 6, 9, 4], [4, 8, 5, 5]]

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 280  
 Pinning number: 5

Average optimal degree: 2.47  
 Average minimal degree: 2.52  
 Average overall degree: 3.05

Table 491: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	20	58	84	70	34	9	1	276
Average degree	2.47	2.71	2.91	3.05	3.16	3.24	3.29	3.33	

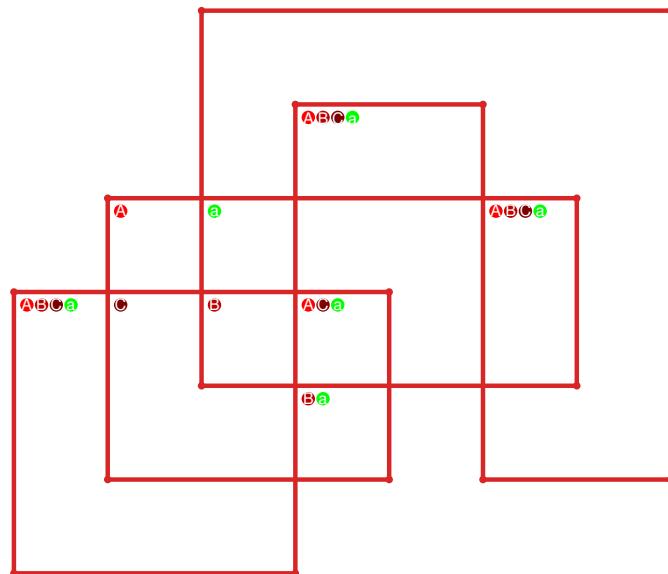


Figure 985: `SnapPy` multiloop plot.

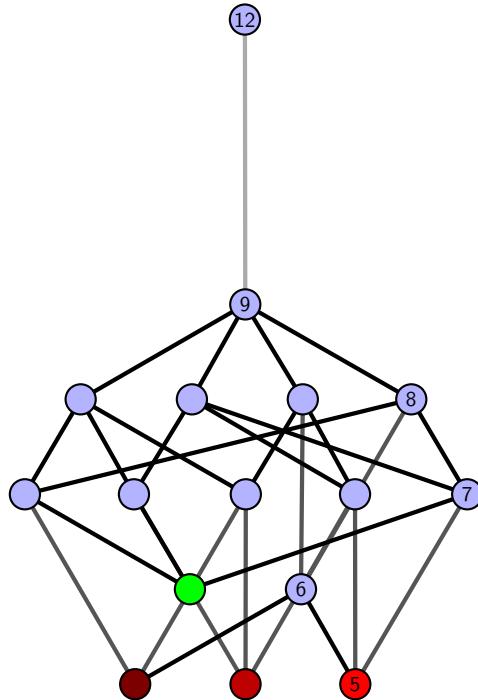


Figure 986: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.345**  $[[5, 20, 6, 1], [4, 9, 5, 10], [19, 8, 20, 9], [6, 14, 7, 13], [1, 11, 2, 10], [18, 3, 19, 4], [7, 14, 8, 15], [15, 12, 16, 13], [11, 16, 12, 17], [2, 17, 3, 18]]$

PD code drawn by `SnapPy`:  $[(15, 20, 16, 1), (16, 5, 17, 6), (1, 6, 2, 7), (7, 18, 8, 19), (12, 9, 13, 10), (10, 3, 11, 4), (4, 11, 5, 12), (8, 13, 9, 14), (19, 14, 20, 15), (2, 17, 3, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 6], [0, 6, 6, 7], [0, 8, 9, 1], [1, 9, 9, 2], [2, 7, 3, 3], [3, 6, 8, 8], [4, 7, 7, 9], [4, 8, 5, 5]]$

Total optimal pinning sets: 3  
Total minimal pinning sets: 5

Total pinning sets: 284

Pinning number: 5

Average optimal degree: 2.47

Average minimal degree: 2.55

Average overall degree: 3.05

Table 492: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	20	60	85	70	34	9	1	279
Average degree	2.47	2.71	2.91	3.05	3.16	3.24	3.29	3.33	

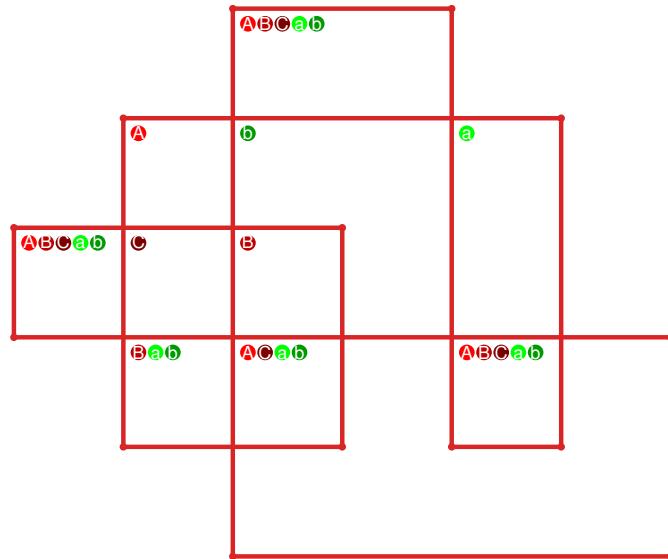


Figure 987: `SnapPy` multiloop plot.

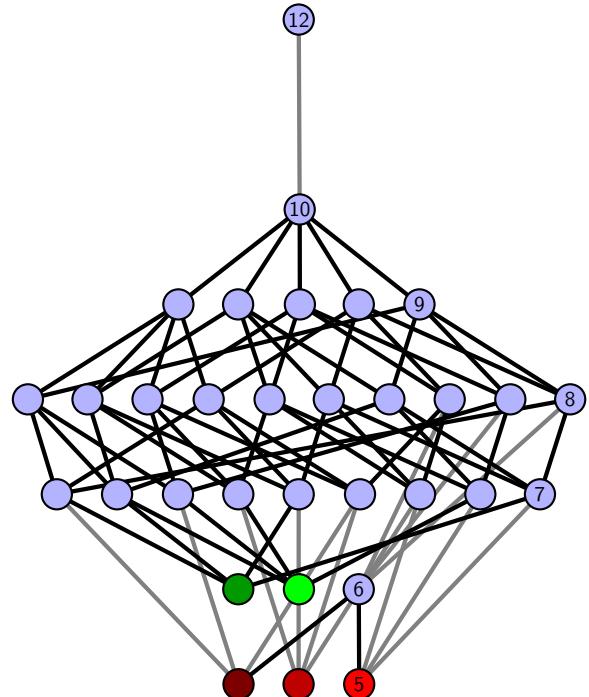


Figure 988: Minimal join sub-semi-lattice of minimal pinning sets.

$$4.8.346 \quad [[20, 15, 1, 16], [16, 9, 17, 10], [6, 19, 7, 20], [7, 14, 8, 15], [1, 8, 2, 9], [17, 4, 18, 5], [10, 5, 11, 6], [11, 18, 12, 19], [13, 2, 14, 3], [3, 12, 4, 13]]$$

PD code drawn by SnapPy: [(11, 20, 12, 1), (15, 2, 16, 3), (8, 3, 9, 4), (19, 6, 20, 7), (14, 7, 15, 8), (1, 10, 2, 11), (5, 12, 6, 13), (18, 13, 19, 14), (9, 16, 10, 17), (4, 17, 5, 18)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 3], [0, 2, 8, 4], [0, 3, 8, 1], [1, 9, 7, 6], [1, 5, 7, 2], [2, 6, 5, 9], [3, 9, 9, 4], [5, 8, 8, 7]]

Total optimal pinning sets: 1  
Total pinning sets: 2

Average optimal degree: 3.0

Total minimal pinning sets: 8

Average minimal degree: 2.83

Total pinning sets: 536

Average minimal degree: 2.3

Pinning number: 4

Table 493: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	6	0	1	0	0	0	0	0	7
Nonminimal pinning sets	0	8	60	127	157	115	49	11	1	528
Average degree	3.0	2.97	3.06	3.14	3.21	3.26	3.3	3.32	3.33	

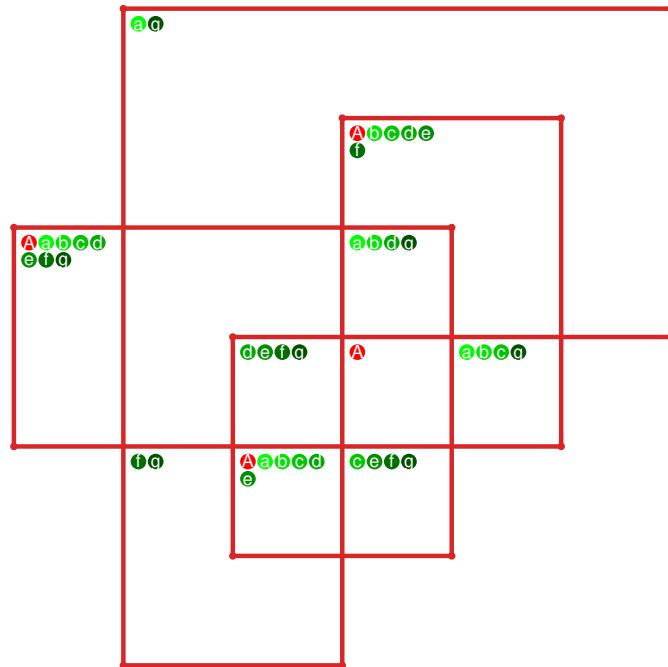


Figure 989: SnapPy multiloop plot.

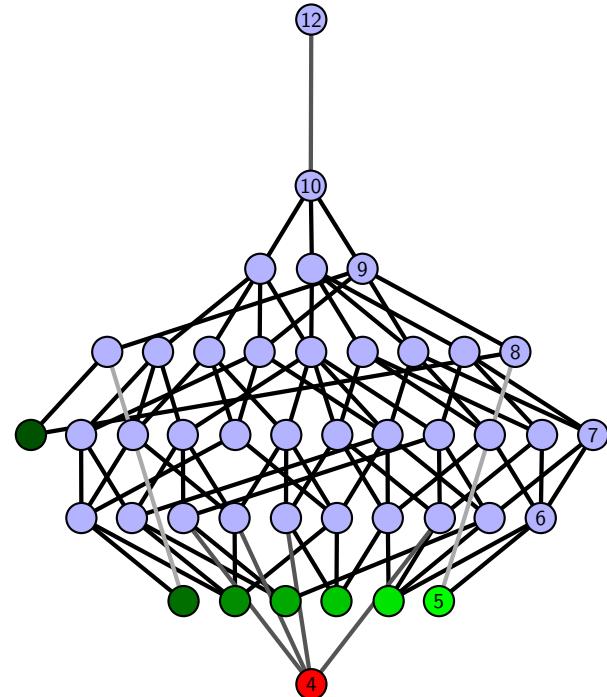


Figure 990: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.347** [[20, 15, 1, 16], [16, 6, 17, 5], [12, 19, 13, 20], [14, 7, 15, 8], [1, 7, 2, 6], [17, 10, 18, 11], [11, 4, 12, 5], [18, 3, 19, 4], [13, 9, 14, 8], [2, 9, 3, 10]]

PD code drawn by SnapPy: [(16, 1, 17, 2), (9, 2, 10, 3), (3, 14, 4, 15), (4, 19, 5, 20), (12, 5, 13, 6), (6, 11, 7, 12), (20, 7, 1, 8), (15, 8, 16, 9), (18, 13, 19, 14), (10, 17, 11, 18)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 7, 8], [0, 8, 8, 4], [0, 3, 9, 1], [1, 9, 7, 6], [1, 5, 7, 2], [2, 6, 5, 9], [2, 9, 3, 3], [4, 8, 7, 5]]

Total optimal pinning sets: 2

Average optimal degree: 2.88

Total minimal pinning sets: 11

Average minimal degree: 2.96

Total pinning sets: 688

Average overall degree: 3.2

Pinning number: 4

Table 494: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	9	0	0	0	0	0	0	0	9
Nonminimal pinning sets	0	15	93	179	197	130	51	11	1	677
Average degree	2.88	3.0	3.1	3.17	3.23	3.27	3.3	3.32	3.33	

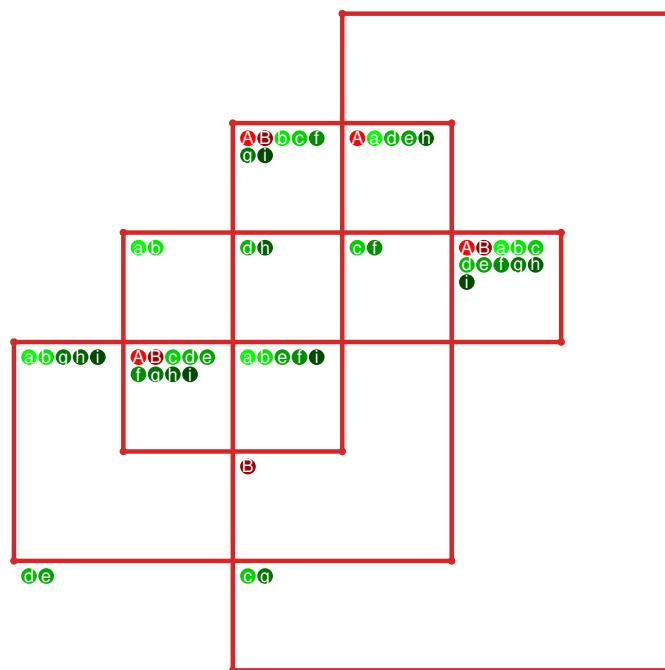


Figure 991: SnapPy multiloop plot.

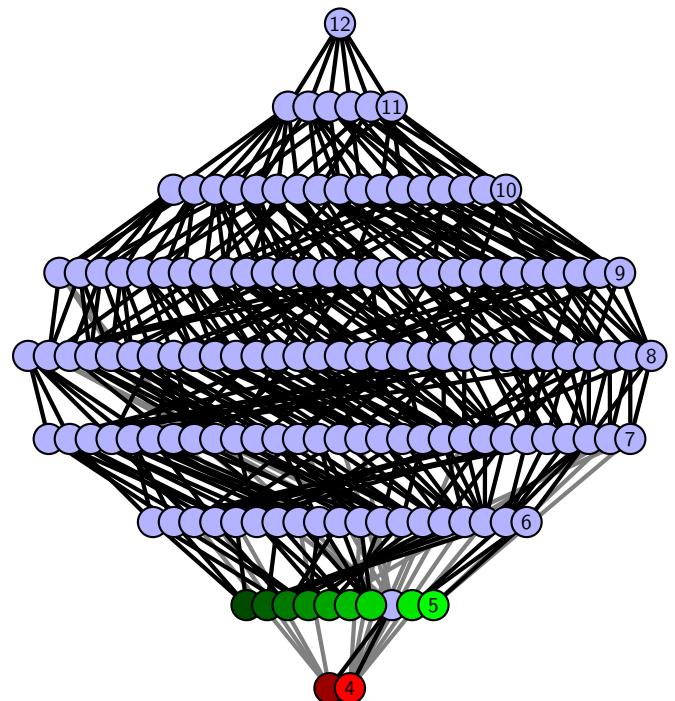


Figure 992: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.348**  $[[20, 7, 1, 8], [8, 17, 9, 18], [14, 19, 15, 20], [15, 6, 16, 7], [1, 16, 2, 17], [9, 2, 10, 3], [18, 13, 19, 14], [5, 12, 6, 13], [10, 4, 11, 3], [11, 4, 12, 5]]$

PD code drawn by `SnapPy`:  $[(20, 7, 1, 8), (8, 1, 9, 2), (12, 3, 13, 4), (17, 4, 18, 5), (2, 9, 3, 10), (15, 10, 16, 11), (18, 13, 19, 14), (5, 14, 6, 15), (11, 16, 12, 17), (6, 19, 7, 20)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 3], [0, 2, 7, 4], [0, 3, 5, 1], [1, 4, 8, 8], [1, 7, 2, 2], [3, 6, 9, 9], [5, 9, 9, 5], [7, 8, 8, 7]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 128  
 Pinning number: 6

Average optimal degree: 2.33  
 Average minimal degree: 2.33  
 Average overall degree: 2.97

Table 495: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	16	35	40	25	8	1	125
Average degree	2.33	2.64	2.87	3.04	3.18	3.27	3.33	

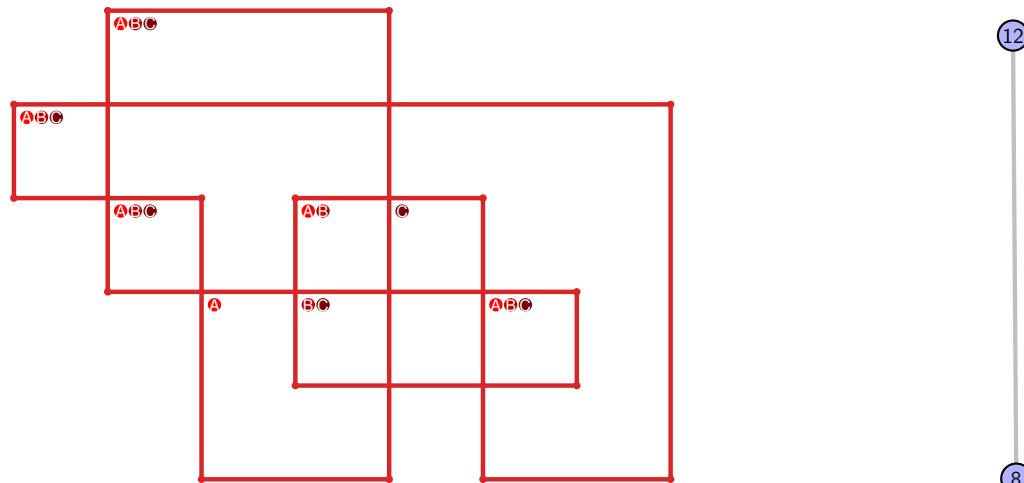


Figure 993: `SnapPy` multiloop plot.

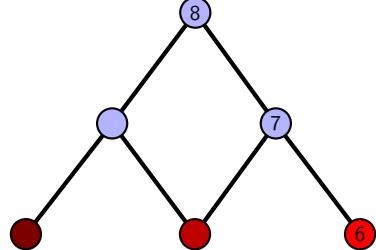


Figure 994: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.349**  $[[20, 5, 1, 6], [6, 15, 7, 16], [12, 19, 13, 20], [13, 4, 14, 5], [1, 14, 2, 15], [7, 17, 8, 16], [18, 11, 19, 12], [3, 10, 4, 11], [2, 10, 3, 9], [17, 9, 18, 8]]$

PD code drawn by `SnapPy`:  $[(11, 2, 12, 3), (18, 3, 19, 4), (6, 15, 7, 16), (7, 20, 8, 1), (1, 8, 2, 9), (16, 9, 17, 10), (19, 12, 20, 13), (4, 13, 5, 14), (14, 5, 15, 6), (10, 17, 11, 18)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 6, 3], [0, 2, 7, 4], [0, 3, 8, 1], [1, 9, 9, 1], [2, 9, 7, 2], [3, 6, 8, 8], [4, 7, 7, 9], [5, 8, 6, 5]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 160  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.27  
 Average overall degree: 2.97

Table 496: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

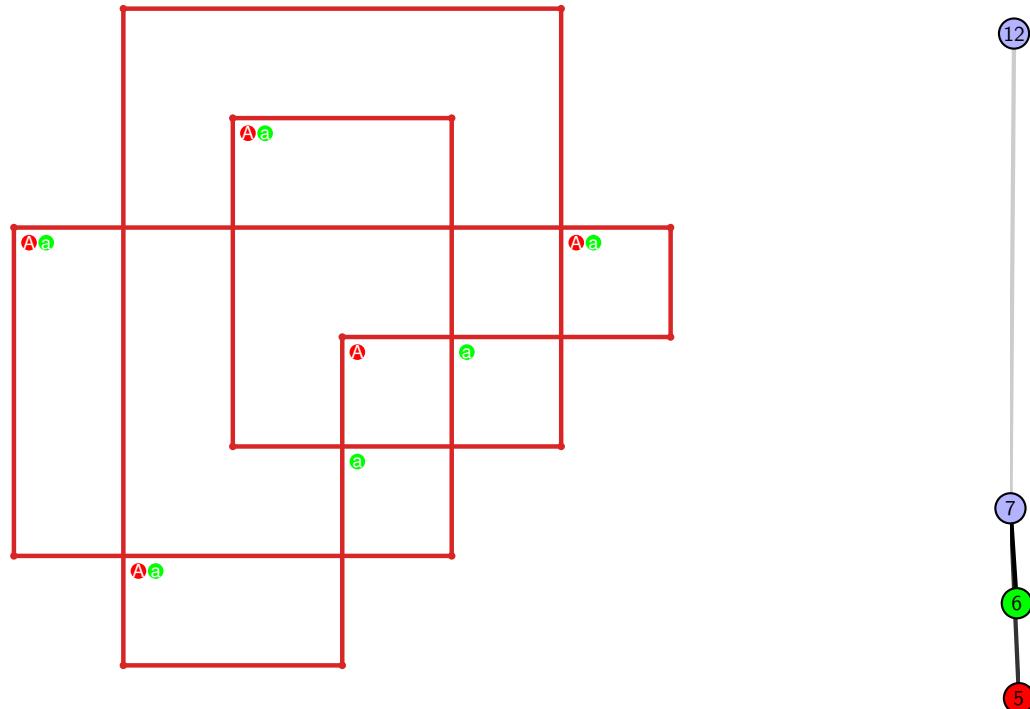


Figure 995: `SnapPy` multiloop plot.

Figure 996: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.350** `[[13, 20, 14, 1], [12, 7, 13, 8], [19, 6, 20, 7], [14, 4, 15, 3], [1, 9, 2, 8], [18, 11, 19, 12], [5, 16, 6, 17], [4, 16, 5, 15], [2, 9, 3, 10], [10, 17, 11, 18]]`

PD code drawn by `SnapPy`: `[(1, 12, 2, 13), (17, 4, 18, 5), (10, 5, 11, 6), (6, 19, 7, 20), (20, 7, 1, 8), (16, 9, 17, 10), (13, 2, 14, 3), (3, 14, 4, 15), (8, 15, 9, 16), (11, 18, 12, 19)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 6], [0, 7, 7, 8], [0, 8, 8, 1], [1, 9, 9, 2], [2, 9, 7, 7], [3, 6, 6, 3], [3, 9, 4, 4], [5, 8, 6, 5]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.27

**Total pinning sets:** 160

**Average overall degree:** 2.97

**Pinning number:** 5

Table 497: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	0	1
Nonminimal pinning sets	0	7	26	45	45	26	8	1	158
Average degree	2.2	2.5	2.74	2.92	3.07	3.18	3.27	3.33	

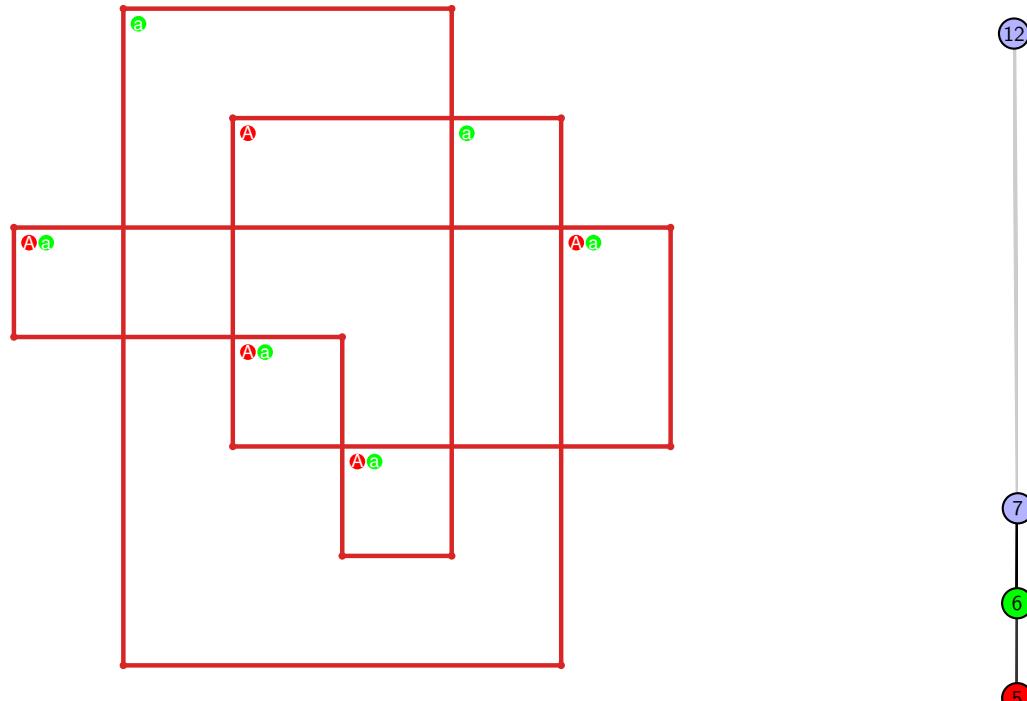


Figure 997: `SnapPy` multiloop plot.

Figure 998: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.351 [[20, 9, 1, 10], [10, 17, 11, 18], [6, 19, 7, 20], [15, 8, 16, 9], [1, 16, 2, 17], [11, 4, 12, 5], [18, 5, 19, 6], [7, 14, 8, 15], [2, 14, 3, 13], [3, 12, 4, 13]]

PD code drawn by `SnapPy`: [(8, 1, 9, 2), (15, 2, 16, 3), (12, 5, 13, 6), (17, 6, 18, 7), (20, 9, 1, 10), (10, 19, 11, 20), (4, 11, 5, 12), (18, 13, 19, 14), (7, 14, 8, 15), (3, 16, 4, 17)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 7], [0, 7, 7, 4], [0, 3, 8, 1], [1, 9, 9, 6], [1, 5, 2, 2], [2, 8, 3, 3], [4, 7, 9, 9], [5, 8, 8, 5]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 498: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

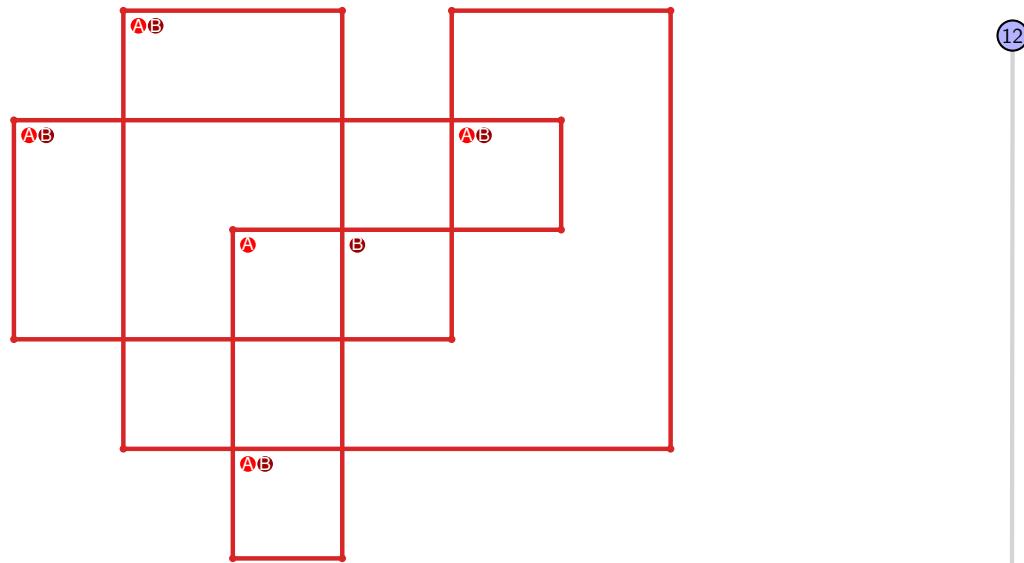


Figure 999: `SnapPy` multiloop plot.

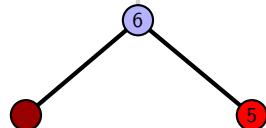


Figure 1000: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.352**  $[[5, 20, 6, 1], [17, 4, 18, 5], [12, 19, 13, 20], [6, 15, 7, 16], [1, 16, 2, 17], [10, 3, 11, 4], [18, 11, 19, 12], [13, 9, 14, 8], [14, 7, 15, 8], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(8, 1, 9, 2), (15, 2, 16, 3), (12, 5, 13, 6), (6, 11, 7, 12), (20, 7, 1, 8), (16, 9, 17, 10), (4, 13, 5, 14), (19, 14, 20, 15), (10, 17, 11, 18), (3, 18, 4, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 6], [0, 6, 6, 7], [0, 8, 8, 4], [0, 3, 9, 1], [1, 9, 9, 6], [1, 5, 2, 2], [2, 9, 8, 8], [3, 7, 7, 3], [4, 7, 5, 5]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 4  
 Total pinning sets: 184  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.38  
 Average overall degree: 2.98

Table 499: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	7	33	54	50	27	8	1	180
Average degree	2.2	2.5	2.76	2.95	3.09	3.19	3.27	3.33	

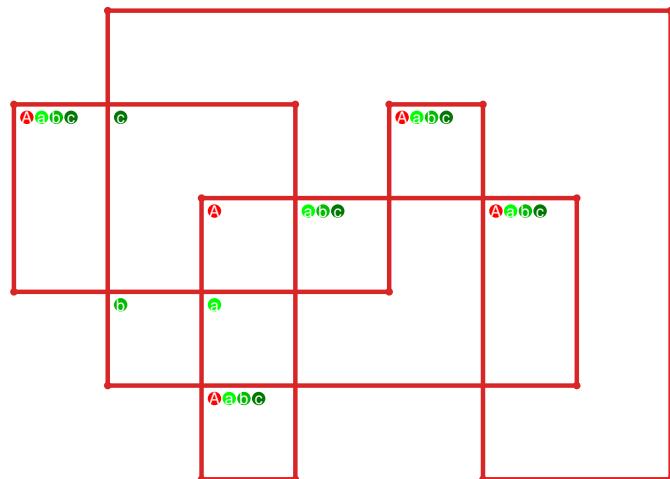


Figure 1001: `SnapPy` multiloop plot.

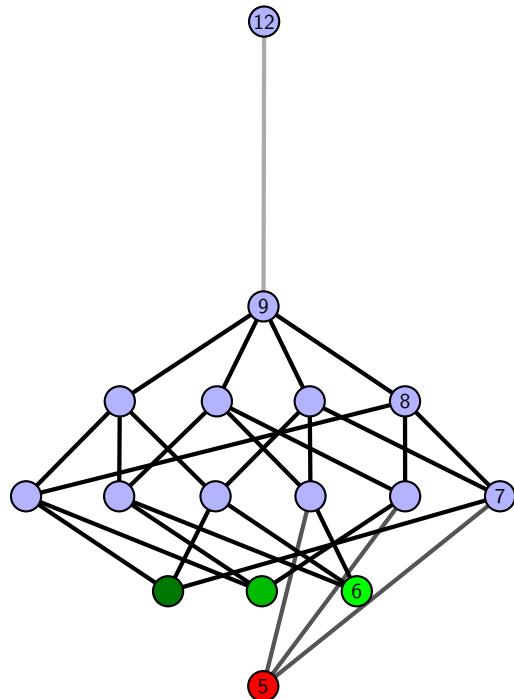


Figure 1002: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.353 [[15, 20, 16, 1], [14, 7, 15, 8], [19, 6, 20, 7], [16, 6, 17, 5], [1, 9, 2, 8], [18, 13, 19, 14], [17, 13, 18, 12], [4, 11, 5, 12], [9, 3, 10, 2], [10, 3, 11, 4]]

PD code drawn by SnapPy: [(20, 7, 1, 8), (8, 1, 9, 2), (3, 14, 4, 15), (17, 4, 18, 5), (12, 5, 13, 6), (6, 19, 7, 20), (2, 9, 3, 10), (10, 15, 11, 16), (16, 11, 17, 12), (13, 18, 14, 19)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 3], [0, 2, 6, 7], [0, 8, 8, 1], [1, 6, 6, 2], [3, 5, 5, 7], [3, 6, 9, 9], [4, 9, 9, 4], [7, 8, 8, 7]]

Total optimal pinning sets: 3  
 Total minimal pinning sets: 4  
 Total pinning sets: 128  
 Pinning number: 6

Average optimal degree: 2.39  
 Average minimal degree: 2.4  
 Average overall degree: 2.98

Table 500: Pinning sets/average degree by cardinal

Cardinal	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	15	35	40	25	8	1	124
Average degree	2.39	2.67	2.88	3.05	3.18	3.27	3.33	

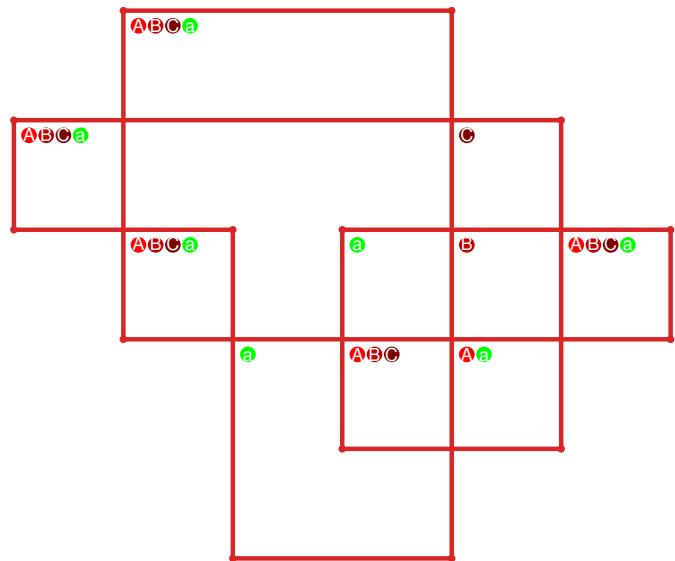


Figure 1003: SnapPy multiloop plot.

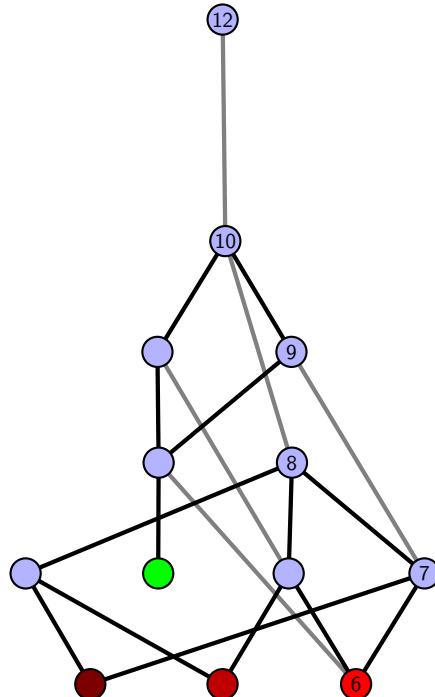


Figure 1004: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.354 [[13, 20, 14, 1], [12, 7, 13, 8], [19, 6, 20, 7], [14, 6, 15, 5], [1, 9, 2, 8], [18, 11, 19, 12], [15, 4, 16, 5], [9, 3, 10, 2], [10, 17, 11, 18], [3, 16, 4, 17]]

PD code drawn by SnapPy: [(11, 20, 12, 1), (1, 10, 2, 11), (15, 2, 16, 3), (8, 3, 9, 4), (4, 17, 5, 18), (14, 7, 15, 8), (19, 12, 20, 13), (6, 13, 7, 14), (9, 16, 10, 17), (18, 5, 19, 6)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 4, 5, 2], [0, 1, 5, 3], [0, 2, 6, 6], [0, 7, 7, 1], [1, 8, 8, 2], [3, 9, 9, 3], [4, 9, 8, 4], [5, 7, 9, 5], [6, 8, 7, 6]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 2  
 Total pinning sets: 144  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.31  
 Average overall degree: 2.97

Table 501: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	1	0	0	0	0	0	1
Nonminimal pinning sets	0	7	21	39	41	25	8	1	142
Average degree	2.2	2.52	2.74	2.91	3.05	3.18	3.27	3.33	

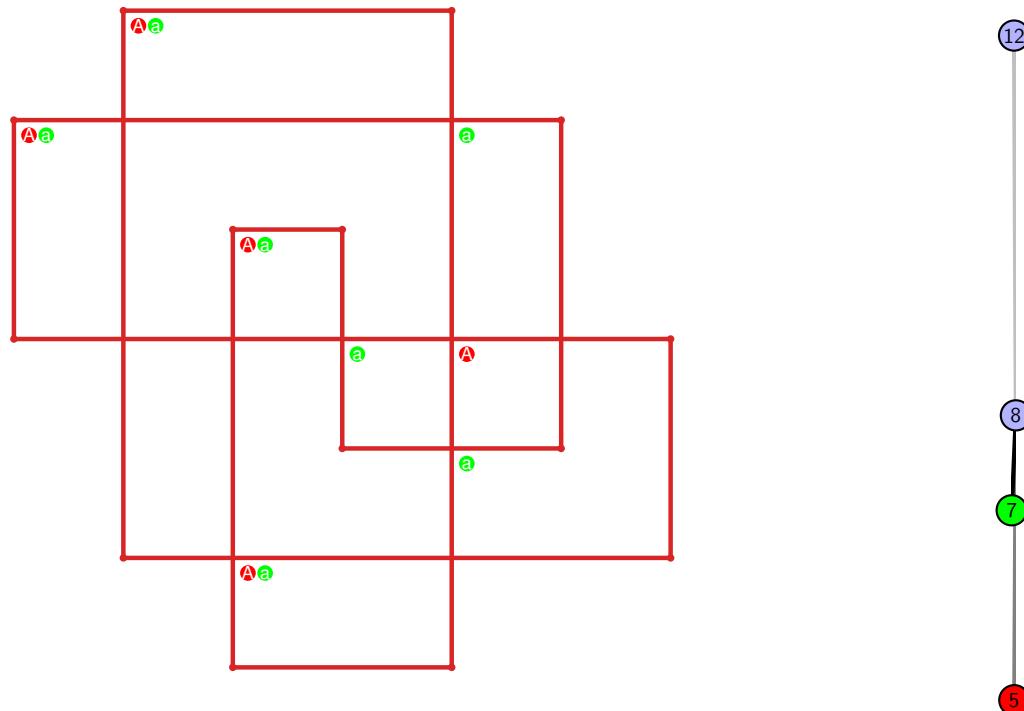


Figure 1005: SnapPy multiloop plot.

Figure 1006: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.355  $[[15, 20, 16, 1], [14, 7, 15, 8], [19, 4, 20, 5], [16, 4, 17, 3], [1, 9, 2, 8], [6, 13, 7, 14], [5, 13, 6, 12], [18, 11, 19, 12], [17, 11, 18, 10], [2, 9, 3, 10]]$

PD code drawn by `SnapPy`:  $[(1, 14, 2, 15), (17, 2, 18, 3), (12, 5, 13, 6), (6, 19, 7, 20), (20, 7, 1, 8), (8, 15, 9, 16), (16, 9, 17, 10), (10, 3, 11, 4), (4, 11, 5, 12), (13, 18, 14, 19)]$

Planar representation generated by `plantri`:  $[[1, 2, 3, 4], [0, 4, 5, 5], [0, 6, 7, 3], [0, 2, 8, 9], [0, 9, 9, 1], [1, 6, 6, 1], [2, 5, 5, 7], [2, 6, 8, 8], [3, 7, 7, 9], [3, 8, 4, 4]]$

Total optimal pinning sets: 1  
 Total minimal pinning sets: 5  
 Total pinning sets: 196  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.46  
 Average overall degree: 2.99

Table 502: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	1	0	0	0	0	0	4
Nonminimal pinning sets	0	7	34	59	54	28	8	1	191
Average degree	2.2	2.53	2.77	2.96	3.1	3.2	3.27	3.33	

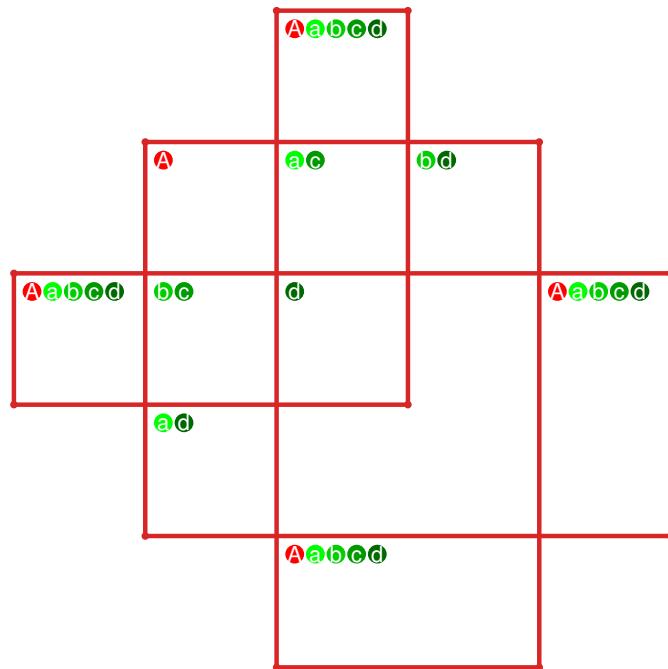


Figure 1007: `SnapPy` multiloop plot.

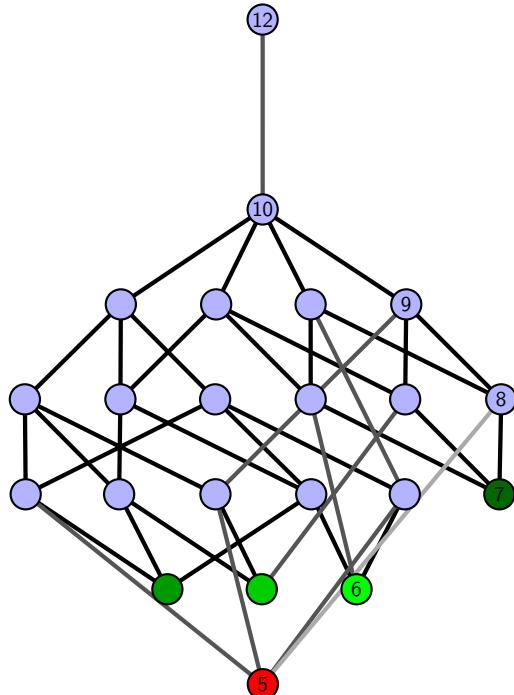


Figure 1008: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.356**  $[[11, 20, 12, 1], [10, 5, 11, 6], [19, 12, 20, 13], [1, 7, 2, 6], [16, 9, 17, 10], [17, 4, 18, 5], [13, 18, 14, 19], [7, 3, 8, 2], [8, 15, 9, 16], [3, 14, 4, 15]]$

PD code drawn by `SnapPy`:  $[(14, 1, 15, 2), (7, 2, 8, 3), (3, 18, 4, 19), (13, 6, 14, 7), (16, 9, 17, 10), (20, 11, 1, 12), (5, 12, 6, 13), (10, 15, 11, 16), (8, 17, 9, 18), (19, 4, 20, 5)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 6, 0], [0, 7, 7, 1], [1, 8, 8, 5], [1, 4, 9, 6], [2, 5, 9, 2], [3, 9, 8, 3], [4, 7, 9, 4], [5, 8, 7, 6]]$

Total optimal pinning sets: 3  
 Total minimal pinning sets: 3  
 Total pinning sets: 224  
 Pinning number: 5

Average optimal degree: 2.27  
 Average minimal degree: 2.27  
 Average overall degree: 2.98

Table 503: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	3	0	0	0	0	0	0	0	3
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	18	46	65	55	28	8	1	221
Average degree	2.27	2.59	2.82	2.98	3.11	3.2	3.27	3.33	

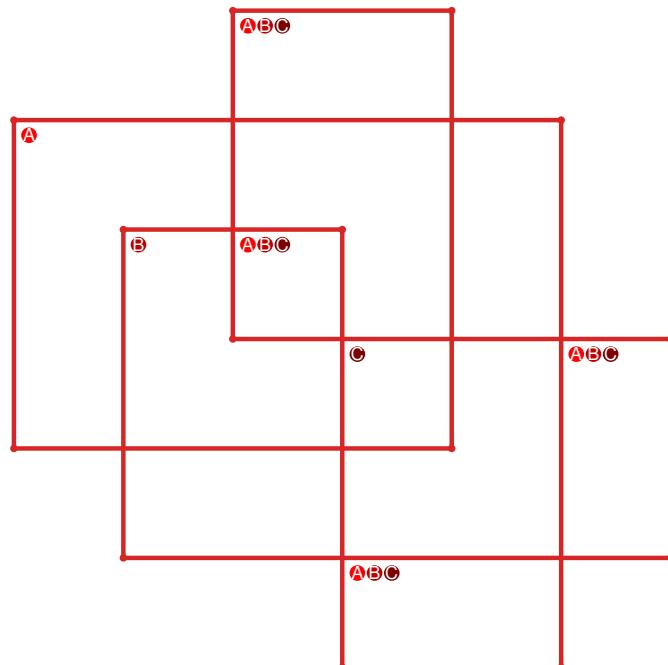


Figure 1009: `SnapPy` multiloop plot.

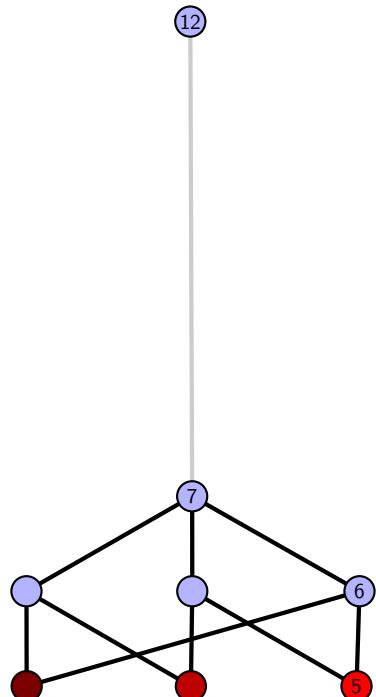


Figure 1010: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.357** `[[5, 20, 6, 1], [4, 9, 5, 10], [19, 6, 20, 7], [1, 11, 2, 10], [14, 3, 15, 4], [15, 8, 16, 9], [7, 16, 8, 17], [18, 11, 19, 12], [2, 13, 3, 14], [17, 13, 18, 12]]`

PD code drawn by `SnapPy`: `[(11, 20, 12, 1), (16, 3, 17, 4), (12, 5, 13, 6), (1, 6, 2, 7), (7, 18, 8, 19), (15, 8, 16, 9), (9, 14, 10, 15), (19, 10, 20, 11), (4, 13, 5, 14), (2, 17, 3, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 7, 0], [0, 7, 8, 1], [1, 8, 8, 5], [1, 4, 6, 6], [2, 5, 5, 9], [2, 9, 9, 3], [3, 9, 4, 4], [6, 8, 7, 7]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 504: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

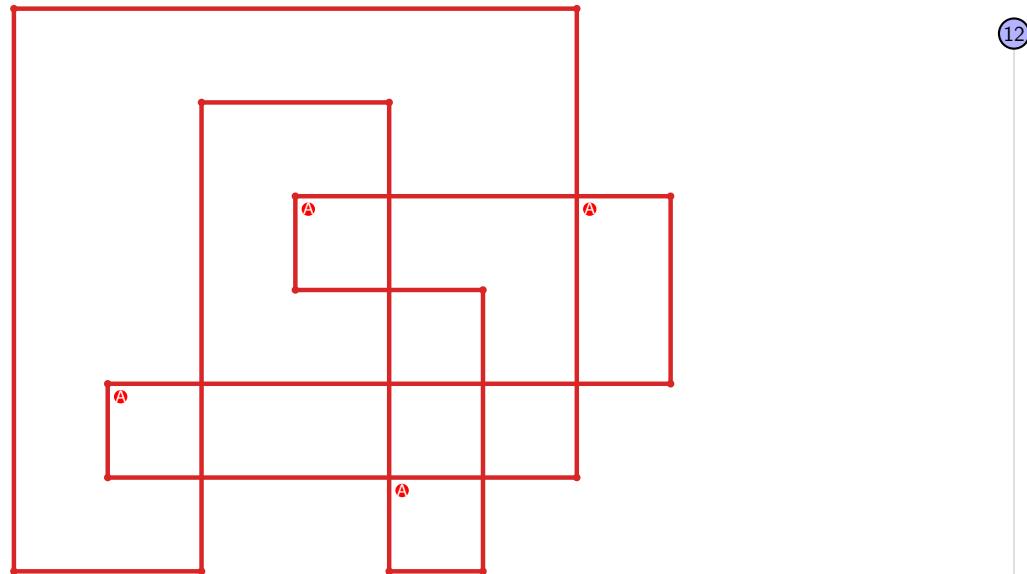


Figure 1011: `SnapPy` multiloop plot.



Figure 1012: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.358** `[[5, 20, 6, 1], [4, 11, 5, 12], [19, 6, 20, 7], [1, 13, 2, 12], [16, 3, 17, 4], [17, 10, 18, 11], [7, 18, 8, 19], [13, 8, 14, 9], [2, 15, 3, 16], [9, 14, 10, 15]]`

PD code drawn by `SnapPy`: `[(13, 20, 14, 1), (16, 3, 17, 4), (10, 5, 11, 6), (14, 7, 15, 8), (1, 8, 2, 9), (9, 18, 10, 19), (6, 11, 7, 12), (19, 12, 20, 13), (4, 15, 5, 16), (2, 17, 3, 18)]`

Planar representation generated by `plantri`: `[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 6, 0], [0, 7, 8, 1], [1, 8, 8, 5], [1, 4, 9, 6], [2, 5, 7, 2], [3, 6, 9, 9], [3, 9, 4, 4], [5, 8, 7, 7]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 505: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

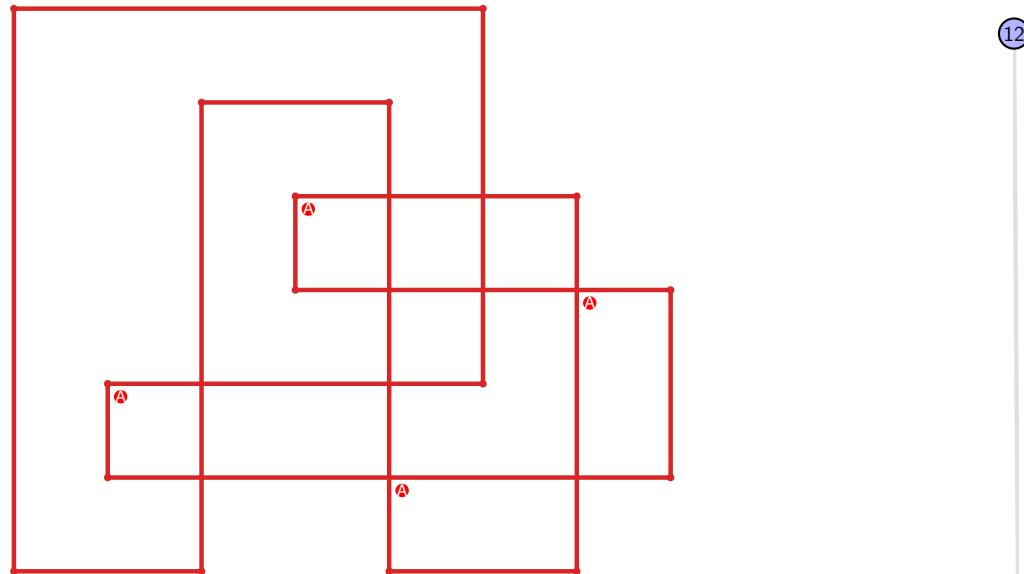


Figure 1013: `SnapPy` multiloop plot.

Figure 1014: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.359**  $[[9, 20, 10, 1], [8, 13, 9, 14], [19, 10, 20, 11], [1, 15, 2, 14], [16, 7, 17, 8], [12, 5, 13, 6], [11, 5, 12, 4], [18, 3, 19, 4], [15, 3, 16, 2], [6, 17, 7, 18]]$

PD code drawn by `SnapPy`:  $[(15, 20, 16, 1), (1, 8, 2, 9), (9, 2, 10, 3), (13, 4, 14, 5), (16, 7, 17, 8), (5, 10, 6, 11), (11, 18, 12, 19), (3, 14, 4, 15), (6, 17, 7, 18), (19, 12, 20, 13)]$

Planar representation generated by `plantri`:  $[[1, 2, 2, 3], [0, 3, 4, 5], [0, 6, 7, 0], [0, 8, 8, 1], [1, 8, 9, 9], [1, 9, 6, 6], [2, 5, 5, 7], [2, 6, 9, 8], [3, 7, 4, 3], [4, 7, 5, 4]]$

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.0

**Total minimal pinning sets:** 1

**Average minimal degree:** 2.0

**Total pinning sets:** 256

**Average overall degree:** 2.97

**Pinning number:** 4

Table 506: Pinning sets/average degree by cardinal

Cardinal	4	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	8	28	56	70	56	28	8	1	255
Average degree	2.0	2.4	2.67	2.86	3.0	3.11	3.2	3.27	3.33	

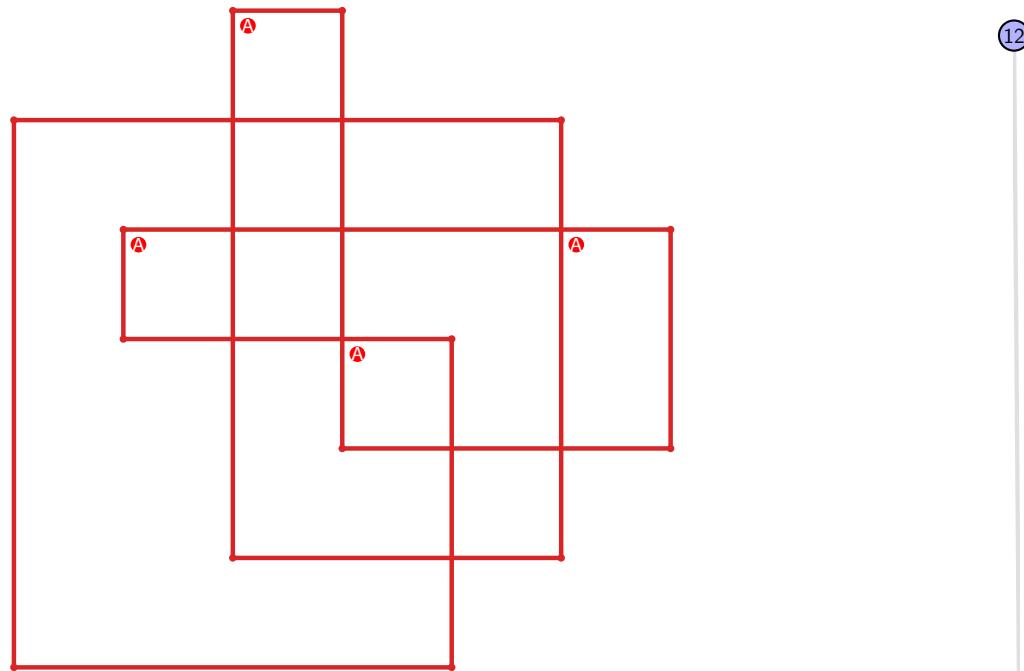


Figure 1015: `SnapPy` multiloop plot.

Figure 1016: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.360** `[[13, 20, 14, 1], [12, 7, 13, 8], [19, 6, 20, 7], [14, 2, 15, 1], [8, 3, 9, 4], [18, 11, 19, 12], [5, 16, 6, 17], [2, 16, 3, 15], [9, 5, 10, 4], [10, 17, 11, 18]]`

PD code drawn by `SnapPy`: `[(6, 1, 7, 2), (2, 7, 3, 8), (17, 4, 18, 5), (10, 5, 11, 6), (16, 9, 17, 10), (13, 20, 14, 1), (3, 14, 4, 15), (8, 15, 9, 16), (11, 18, 12, 19), (19, 12, 20, 13)]`

Planar representation generated by `plantri`: `[[1, 2, 3, 3], [0, 4, 5, 2], [0, 1, 5, 6], [0, 7, 7, 0], [1, 7, 8, 8], [1, 9, 9, 2], [2, 9, 8, 7], [3, 6, 4, 3], [4, 6, 9, 4], [5, 8, 6, 5]]`

**Total optimal pinning sets:** 1

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 3

**Average minimal degree:** 2.34

**Total pinning sets:** 176

**Average overall degree:** 2.98

**Pinning number:** 5

Table 507: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	2	0	0	0	0	0	0	2
Nonminimal pinning sets	0	7	30	51	49	27	8	1	173
Average degree	2.2	2.5	2.75	2.94	3.08	3.19	3.27	3.33	

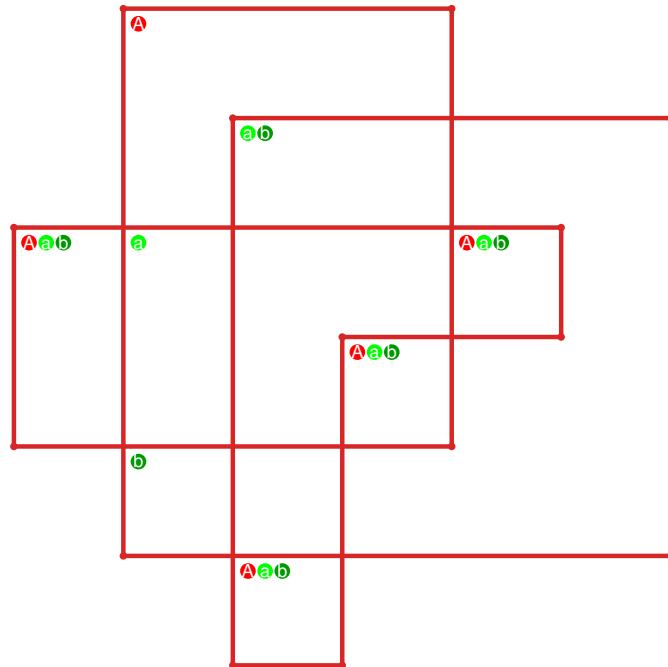


Figure 1017: `SnapPy` multiloop plot.

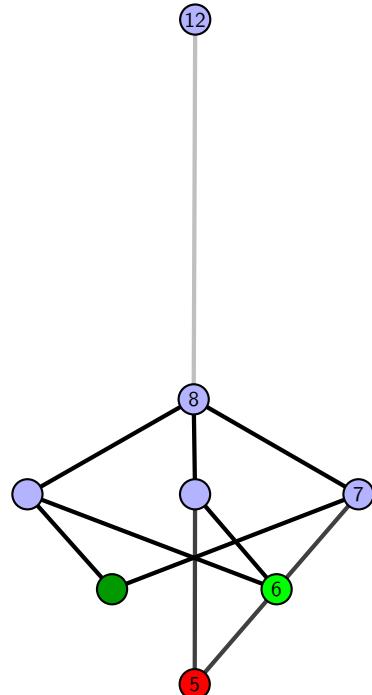


Figure 1018: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.361** [[9, 20, 10, 1], [19, 8, 20, 9], [10, 4, 11, 3], [1, 15, 2, 14], [18, 13, 19, 14], [7, 12, 8, 13], [4, 12, 5, 11], [2, 15, 3, 16], [6, 17, 7, 18], [5, 17, 6, 16]]

PD code drawn by `SnapPy`: [(1, 8, 2, 9), (17, 6, 18, 7), (9, 2, 10, 3), (5, 10, 6, 11), (16, 11, 17, 12), (12, 19, 13, 20), (20, 13, 1, 14), (14, 3, 15, 4), (4, 15, 5, 16), (7, 18, 8, 19)]

Planar representation generated by `plantri`: [[1, 1, 2, 3], [0, 4, 5, 0], [0, 6, 6, 7], [0, 7, 7, 4], [1, 3, 8, 5], [1, 4, 8, 6], [2, 5, 9, 2], [2, 9, 3, 3], [4, 9, 9, 5], [6, 8, 8, 7]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 508: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

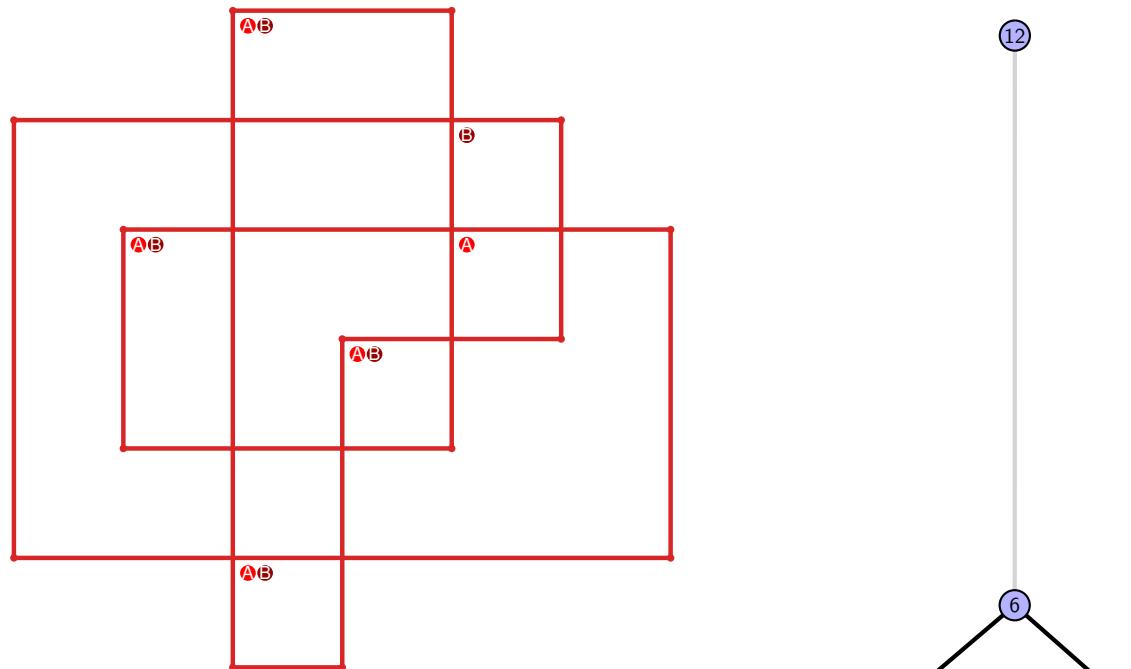


Figure 1019: `SnapPy` multiloop plot.

Figure 1020: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.362** [[20, 13, 1, 14], [14, 9, 15, 10], [12, 19, 13, 20], [1, 17, 2, 16], [8, 15, 9, 16], [10, 3, 11, 4], [4, 11, 5, 12], [5, 18, 6, 19], [17, 6, 18, 7], [2, 7, 3, 8]]

PD code drawn by `SnapPy`: [(8, 1, 9, 2), (15, 4, 16, 5), (20, 5, 1, 6), (12, 7, 13, 8), (2, 9, 3, 10), (18, 11, 19, 12), (6, 13, 7, 14), (14, 19, 15, 20), (3, 16, 4, 17), (10, 17, 11, 18)]

Planar representation generated by `plantri`: [[1, 2, 2, 3], [0, 4, 4, 5], [0, 6, 7, 0], [0, 8, 9, 4], [1, 3, 9, 1], [1, 9, 6, 6], [2, 5, 5, 7], [2, 6, 8, 8], [3, 7, 7, 9], [3, 8, 5, 4]]

**Total optimal pinning sets:** 2

**Average optimal degree:** 2.2

**Total minimal pinning sets:** 2

**Average minimal degree:** 2.2

**Total pinning sets:** 192

**Average overall degree:** 2.97

**Pinning number:** 5

Table 509: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	0	0	0	0	0	0	0	0
Nonminimal pinning sets	0	13	36	55	50	27	8	1	190
Average degree	2.2	2.54	2.78	2.95	3.09	3.19	3.27	3.33	

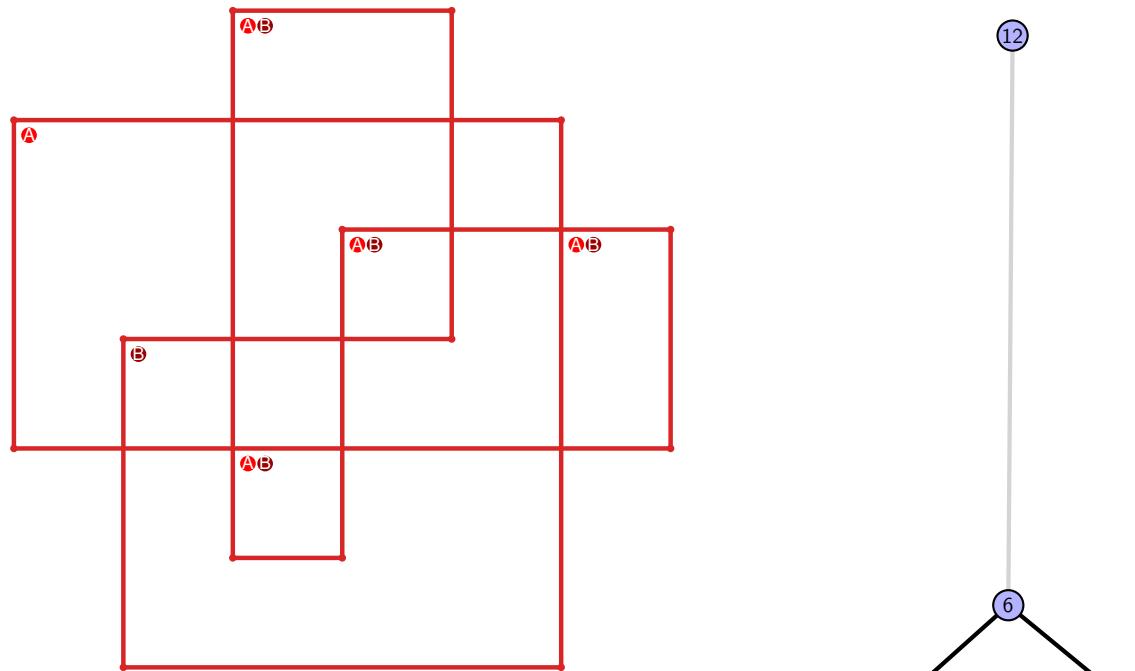


Figure 1021: `SnapPy` multiloop plot.

Figure 1022: Minimal join sub-semi-lattice of minimal pinning sets.

4.8.363 [[20, 15, 1, 16], [16, 3, 17, 4], [12, 19, 13, 20], [7, 14, 8, 15], [1, 8, 2, 9], [9, 2, 10, 3], [17, 5, 18, 4], [18, 11, 19, 12], [13, 6, 14, 7], [10, 6, 11, 5]]

PD code drawn by SnapPy: [(7, 20, 8, 1), (14, 1, 15, 2), (2, 15, 3, 16), (3, 8, 4, 9), (11, 4, 12, 5), (19, 6, 20, 7), (16, 9, 17, 10), (5, 12, 6, 13), (18, 13, 19, 14), (10, 17, 11, 18)]

Planar representation generated by plantri: [[1, 2, 3, 4], [0, 5, 6, 6], [0, 7, 7, 8], [0, 8, 8, 4], [0, 3, 5, 5], [1, 4, 4, 9], [1, 9, 7, 1], [2, 6, 9, 2], [2, 9, 3, 3], [5, 8, 7, 6]]

Total optimal pinning sets: 2  
 Total minimal pinning sets: 6  
 Total pinning sets: 228  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.51  
 Average overall degree: 2.98

Table 510: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	2	0	0	0	0	0	0	0	2
Minimal (suboptimal) pinning sets	0	4	0	0	0	0	0	0	4
Nonminimal pinning sets	0	13	48	68	56	28	8	1	222
Average degree	2.2	2.57	2.82	2.99	3.11	3.2	3.27	3.33	

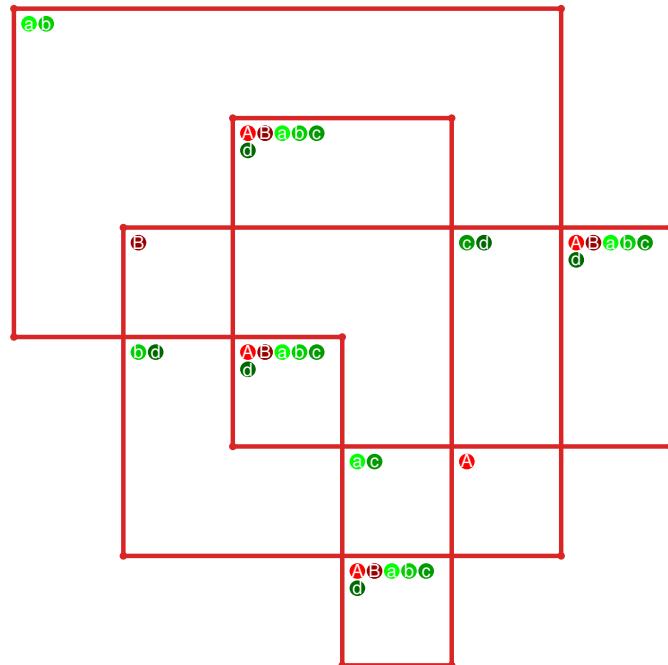


Figure 1023: SnapPy multiloop plot.

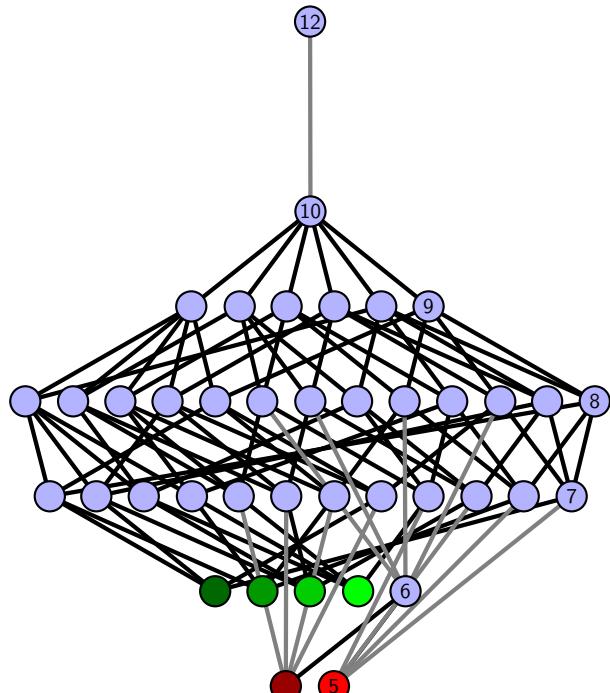


Figure 1024: Minimal join sub-semi-lattice of minimal pinning sets.

**4.8.364** [[20, 7, 1, 8], [8, 3, 9, 4], [12, 19, 13, 20], [13, 6, 14, 7], [1, 14, 2, 15], [15, 2, 16, 3], [9, 5, 10, 4], [18, 11, 19, 12], [5, 16, 6, 17], [10, 17, 11, 18]]

PD code drawn by `SnapPy`: [(20, 5, 1, 6), (9, 2, 10, 3), (16, 3, 17, 4), (14, 7, 15, 8), (17, 10, 18, 11), (1, 12, 2, 13), (6, 13, 7, 14), (8, 15, 9, 16), (11, 18, 12, 19), (4, 19, 5, 20)]

Planar representation generated by `plantri`: [[1, 2, 3, 4], [0, 5, 6, 6], [0, 7, 7, 3], [0, 2, 8, 4], [0, 3, 5, 5], [1, 4, 4, 8], [1, 8, 9, 1], [2, 9, 9, 2], [3, 9, 6, 5], [6, 8, 7, 7]]

Total optimal pinning sets: 1  
 Total minimal pinning sets: 4  
 Total pinning sets: 184  
 Pinning number: 5

Average optimal degree: 2.2  
 Average minimal degree: 2.38  
 Average overall degree: 2.98

Table 511: Pinning sets/average degree by cardinal

Cardinal	5	6	7	8	9	10	11	12	Total
Optimal pinning sets	1	0	0	0	0	0	0	0	1
Minimal (suboptimal) pinning sets	0	3	0	0	0	0	0	0	3
Nonminimal pinning sets	0	7	33	54	50	27	8	1	180
Average degree	2.2	2.5	2.76	2.95	3.09	3.19	3.27	3.33	

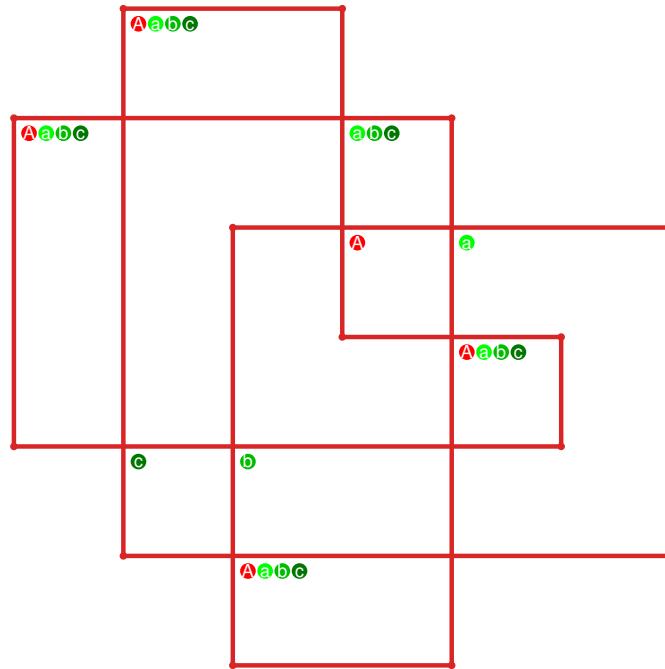


Figure 1025: `SnapPy` multiloop plot.

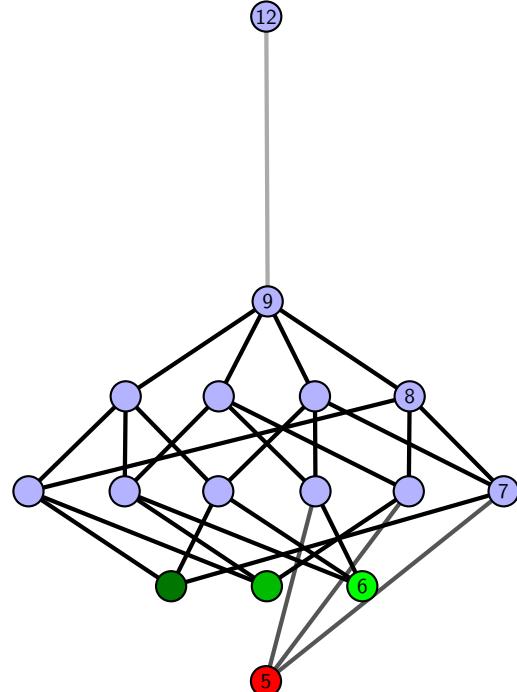


Figure 1026: Minimal join sub-semi-lattice of minimal pinning sets.