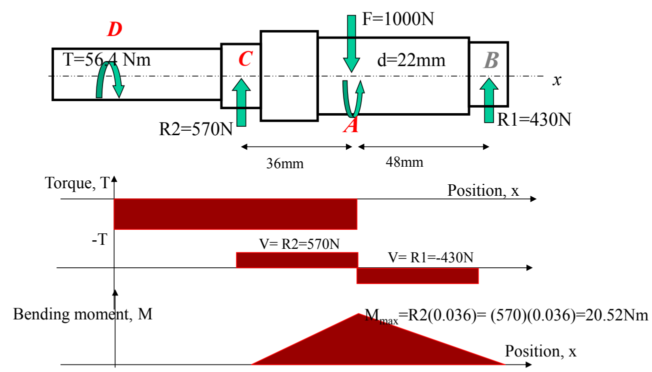
315 THEORY OF MACHINES – DESIGN OF ELEMENTS
Fall, 2023
HW No. 2
Assigned: 9/28 Due: one week, 10/5 Online, pdf in **one single file**.
All diagrams must be well aligned and properly drawn for receiving grades.

1. We have examined the A^- cross section of the shaft below (HW1, No. 1, and the class example). Now, we need to compare the D+ cross section with the A^- cross section and determine which one is truly the most critical cross section, and which point the most critical point. Consider shaft rotation with frequent stops daily (i.e. the torque changes from 0 to 56.4Nm). The diameter is 16 mm at D; the material has S_{ut} = 750 MPa, S_y = 550 MPa; the shaft is processed through a ground finish, and the design reliability is 99%. The stress concentration factors for the bending and shear stresses are K_f=2.2 and K_{fs}= 2.5 at both cross sections D and A due to the keyways there (not shown in the simplified diagram).

a). We have to re-examine A^- because the torque is now fluctuating from 0 to 56.4Nm. Determine the factors of safety based on Goodman' line only (checking yield is important but it is ignored here). Remember there is a keyway at A but ignore the effect of the keyway on section modulus in the stress calculation for simplicity for now.

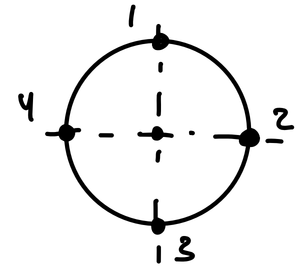
b). At D+, determine the factors of safety based on Goodman' line only. Again, ignore the effect of the keyway on section modulus in the stress calculation for simplicity for now. Even this is a pure shear section, we still handle it via the DET.



CHRISTOPHER LUEY

a)

A^-



@ SURFACE ($d = 22\,mm$, $R_z = 570N$, $M = 20.52\,Nm$)

$$T_m = -28.2\ Nm \quad \Rightarrow \quad \tau_{xy,T,m} = \frac{16T_m}{\pi d^3}$$

$$T_a = 28.2\ Nm \quad \quad \tau_{xy,T,a} = \frac{16T_a}{\pi d^3}$$

$$V_m = 0N \quad \Rightarrow \quad \tau_{xy,V,m} = 0\ MPa$$

$$V_a = R_z \quad \quad \tau_{xy,V,a} = \frac{16V_a}{3\pi d^2}$$

$$\Rightarrow \tau_{xy,m} = \tau_{xy,T,m} + \tau_{xy,V,m} = -13.5\ MPa$$

$$\tau_{xy,a} = \tau_{xy,T,a} + \tau_{xy,V,a} = 15.5\ MPa$$

$$\sigma_{x,m} = 0\ MPa$$

$$\sigma_{x,a} = \frac{Md}{2I} = \frac{32M}{\pi d^3} = 19.6\ MPa$$

$$\frac{\left((K_f\sigma_{xa})^2 + 3(K_{fs}\tau_{xya})^2\right)^{1/2}}{k_f k_s k_r k_t k_m S_e'} + \frac{(\sigma_{xm}^2 + 3\tau_{xym}^2)^{1/2}}{S_{ut}} = \frac{1}{n}$$

$k_f = e(S_{ut})^f$; $e = 1.58\ MPa$; $f = -0.085$; $S_{ut} = 750\,MPa$
$= 0.90$

$\underline{K_f} = 2.2$; $\underline{K_{fs}} = 2.5$

$k_s = 1.189(d)^{-0.112} = 0.84$

$k_r = 0.82$

$k_t = 1$

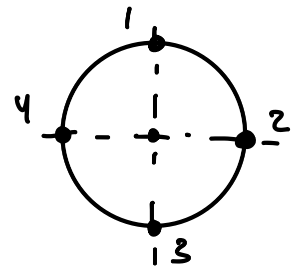
$k_m = 1$

$S_e' = 0.5\ S_{ut}$ [VON MISES]

$$\boxed{n = 2.67}$$

b)

D^+



@ SURFACE ($d = 16$ mm, $T = -56.4$ Nm)

$T_m = -28.2$ Nm

$T_a = 28.2$ Nm

$\Rightarrow \tau_{xy,T,m} = \dfrac{16 T_m}{\pi d^3}$

$\tau_{xy,T,a} = \dfrac{16 T_a}{\pi d^3}$

$V_m = 0$ N

$V_a = 0$ N

$\sigma_{x,m} = 0$ MPa

$\sigma_{x,a} = 0$ MPa

$\Rightarrow \tau_{xy,m} = \tau_{xy,T,m}$

$= -35.1$ MPa //

$\tau_{xy,a} = \tau_{xy,T,a}$

$= 35.1$ MPa //

$$\frac{\left((\underline{K_f}\sigma_{xa})^2 + 3(\underline{K_{fs}}\tau_{xya})^2\right)^{1/2}}{k_f k_s k_r k_t k_m S_e'} + \frac{(\sigma_{xm}^2 + 3\tau_{xym}^2)^{1/2}}{S_{ut}} = \frac{1}{n}$$

$k_f = e(S_{ut})^f$; $e = 1.58$ MPa ; $f = -0.085$; $S_{ut} = 750$ MPa

$= 0.90$

$\underline{K_f} = 2.2$; $\underline{K_{fs}} = 2.5$

$k_s = 1.189(d)^{-0.112} = 0.87$

$k_r = 0.82$

$k_t = 1$

$k_m = 1$

$S_e' = 0.5\ S_{ut}$ [VON MISES]

$\boxed{n = 1.41}$

2. We now know the shear yield limits, S_{sy}, of a steel, determined from a pure shear-stress state, τ, in terms of the tensile yield strength, S_y, based on the maximum shear stress theory (MSST) and the distortion energy theory (DET). One step further, knowing that the endurance limit for pure bending stress σ is S_e=0.5S_u, with S_u for the ultimate tensile strength, prove that the endurance limit for pure torsional shear stress τ is S_{se}=0.29S_u by means of the DET.

MSST

$S_{sy} = 0.5 S_y$

DET

$S_{sy} = 0.577 S_y$

$\sigma_{vm} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{3}\,\tau = S_e$

$S_e = 0.5 S_u$

$\tau = 0.289 S_u$

$S_{se} = 0.29 S_u$ //

3. A steel (S_{ut} = 1000MPa) is used for a rotating shaft whose life is expected to be in 650,000 cycles. All forces are static. The shaft diameter is d =30 mm. The keyway (not shown) causes stress concentration, which should be considered, but we ignore the keyway effect on section modulus for now (still consider the shaft cross section to be perfectly circular).

a) If only bending moment M is applied, what should be the maximum possible bending stress, σ, and the corresponding bending moment, M? All fatigue strength modification factors and stress concentration factors are ignored for now. Note that here, the maximum possible stress means ignoring the safety margin, i.e., set the safety factor $SF = n_s = 1$.

b) If the bending stress concentration factor is K_f=2 for the keyway place of the shaft. What should be the maximum possible bending stress, σ, and the corresponding bending moment, M? Ignore all fatigue strength modification factors and set $SF = n_s = 1.0$.

c) Determine the fatigue strength modification factors, $k_f k_s k_r k_t k_m = k_{total}$ if the shaft is grinding finish, and the reliability requirement is 90%. What should be the designed bending stress, σ, and the corresponding bending moment, M? The bending stress concentration factor is K_f=2 and SF= 1.1

d) Apply both bending moment M and torque T, and let torque $T = M$. The stress concentration factors for torsional shear stress and bending normal stress are both K_f=2. The fatigue strength modification factors $k_f k_s k_r k_t k_m$ are the same as those found in c), and SF= 1.1. What should be the torque, T, and the bending moment, M?

a) $S_{ut} = 1000\,\text{MPa}$ $N = 650000$ $d = 30\,\text{mm}$

$S_L' = 0.9\, S_{ut} = 900\,\text{MPa}$; $S_e' = 0.5\, S_{ut} = 500\,\text{MPa}$

$b = -\frac{1}{3}\log\left(\frac{S_L'}{S_e'}\right) = -0.0851\ \frac{\log \text{MPa}}{\log N}$

$C = \log\left(\frac{(S_L')^2}{S_e'}\right) = 3.210\ \log \text{MPa}$

$S_f' = (10)^C (N)^b = \boxed{518.668\ \text{MPa} = \sigma_{max}}$

$\sigma_{max} = \frac{32M}{\pi d^3} \Rightarrow \boxed{M = 1374.84\ \text{Nm}}$

SYMMETRY

b) $\underline{K}_f = 2$

$K_o = 1/\underline{K}_f = 0.5$

$$\boxed{\sigma_{max} = 0.5\,(S_f') = 259.3\ \text{MPa}}$$

$$\boxed{M_{max} = 687.4\ \text{Nm}}$$

c) $n_s = 1.1$; $\underline{K_f} = 2$; $S_{ut} = 1000\ \text{MPa}$; $d = 30\ \text{mm}$

$k_f k_s k_r k_t k_m = k_{TOT}$

$k_f = e\,(S_{ut})^{f}$; $e = 1.58$; $f = -0.085$

$k_f = 0.878$

$k_s = 1.189\,(d)^{-0.112}$

$k_s = 0.812$

$k_r = 0.90$

$k_t = 1$

$k_m = 1$

$k_{TOT} = 0.642$

$k_o = 1/\underline{K_f} = 0.5$; $S_f' = 518.668\ \text{MPa}$

$\sigma_{max} = k_o\, k_{TOT}\, S_f' = 166.4\ \text{MPa}$

$\sigma_{DESIGN} = \sigma_{max}\, n_s^{-1} = 151.272\ \text{MPa}$

$$\boxed{M = 400.98\ \text{Nm}}$$

d) $\tau_{DESIGN} = \frac{16T}{\pi d^3} = \tau_{xym} = \frac{16M}{\pi d^3}$

$\sigma_{DESIGN} = \frac{32M}{\pi d^3} = \sigma_{xa}$; $d = 30$ mm

$$\frac{\left((K_f \sigma_{xa})^2 + 3(K_{fs}\tau_{xya})^2\right)^{1/2}}{k_f k_s k_r k_t k_m S_f'} + \frac{(\sigma_{xm}^2 + 3\tau_{xym}^2)^{1/2}}{S_{ut}} = \frac{1}{n}$$

$K_f = 2$

$n = 1.1$

$k_f = 0.878$

$k_s = 1.189\,(d)^{-0.112}$

$k_s = 0.812$

$k_r = 0.90$

$k_t = 1$

$k_m = 1$

$S_f' = 518.668$ MPa

$S_{ut} = 1000$ MPa

$$\frac{(K_f \sigma_{xa})}{k_f k_s k_r k_t k_m S_f'} + \frac{\sqrt{3}\,\tau_{xym}}{S_{ut}} = \frac{1}{n}$$

$\Rightarrow \boxed{M = T = 350.6 \text{ Nm}}$

4. The BC segment of the shaft shown below is a cantilever beam “fixed” at B; and its diameter is *d*. The shaft rotates counterclockwise viewed at the right end, but the force, *F,* and the torque, *T,* at C, are stationary. The torque is counterclockwise viewed at the right end.

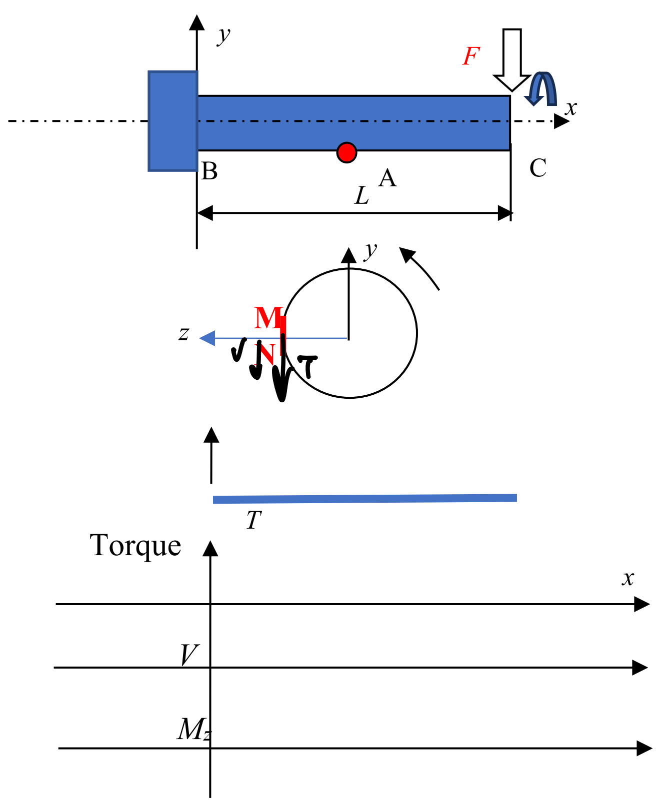
a) Please do the FBD, torque, shear force and bending moment diagrams, and mark characteristic values.

For surface point A, in the middle of shaft BC, in terms of rotation angle counting (or time) from the current position, please

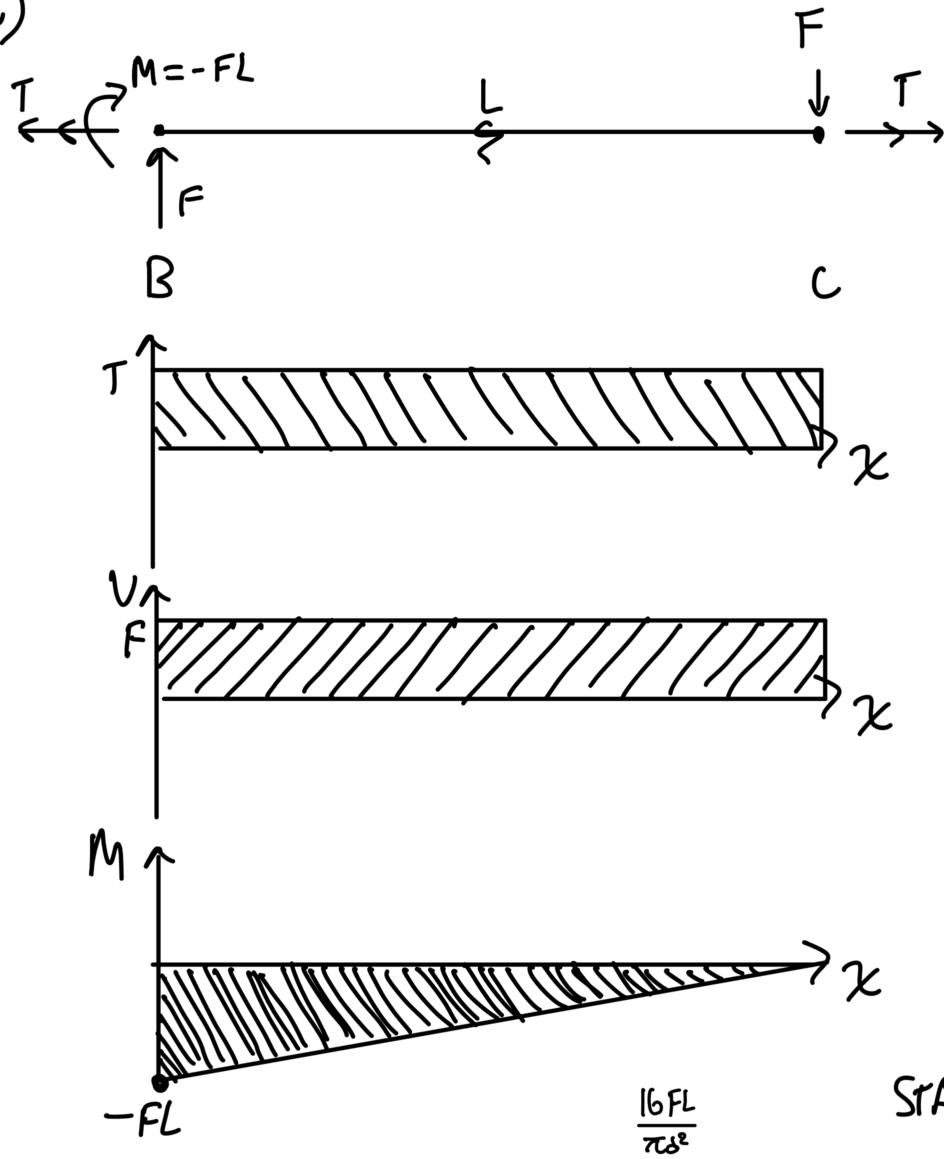
b) Sketch the variation of the bending stress (normal), σ_b.
c) Sketch the variation of the torsional shear stress, τ_{ts}.
d) Sketch the variation of the *F*-induced transverse shear stress, τ_v.
For all, label the peak values in terms of *L*, *d*, *F*, *T*. You may start from our usual points of 1,2,3,4 at A, and figure out the variation of each of σ_b, τ_{ts}, and τ_v in one cycle of rotation.

Note: 1) the shear force does not change its direction, but the shaft rotates, we can follow the motion of one edge of a shear-stress element, e.g. MN, to see the shear-stress variation; 2) we assume that the torsional shear stress is larger than the transverse shear stress in value.

“Cut” at A and look at AB from A.

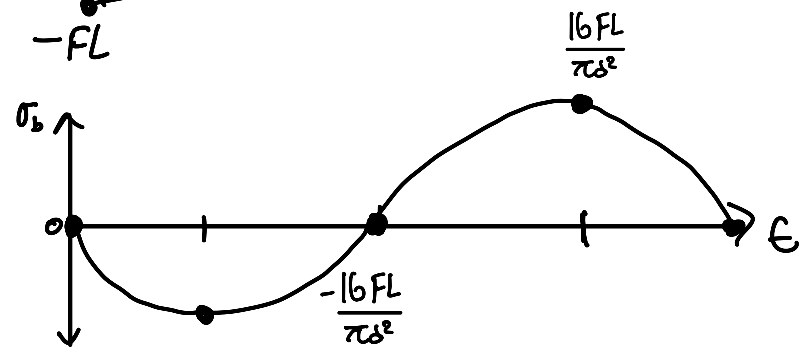


a)

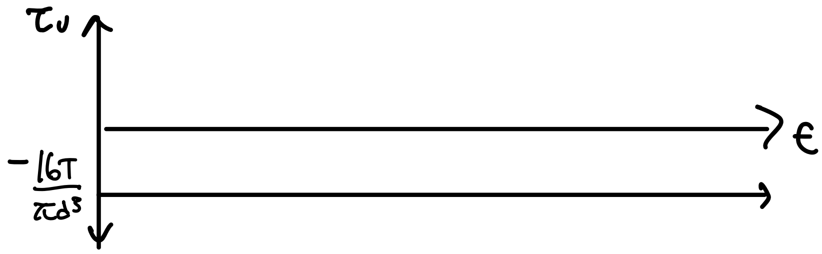


START AT MN

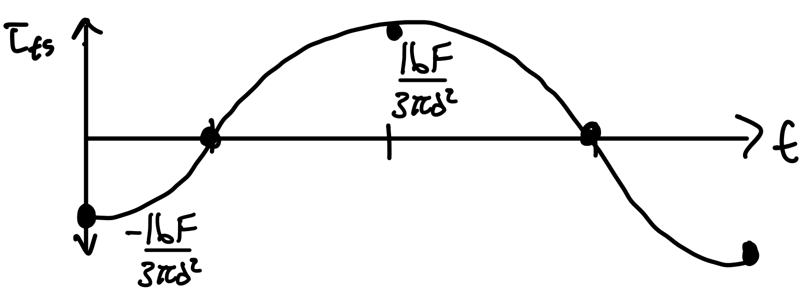
b)



c)



d)

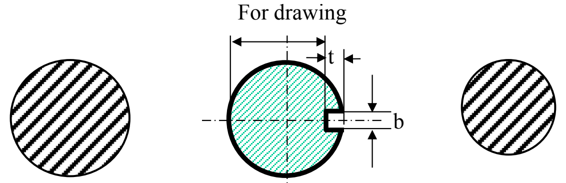


5. When we deal with a keyway, we increase the calculated minimum diameter by 3-5% in order to consider the weakening effect of the keyway on section modulus. Consider a flat bottom keyway of a shaft. Our simplified way to determine the minimum diameter is to increase the diameter d from $d = \frac{T}{J\tau}$ for about 3%. Here, J is the polar second moment of area for a keyway-free cross section of diameter d. We can define J/c as the section modulus for torsional shear stress with $c=d/2$. However, the true section modulus for torsional shear, $J/c\__{\text{Ture}}$, is from Appendix 3. Our simplified method is actually using something like $\frac{J}{c} = \frac{\pi(d-0.03d)^3}{16}$ to approach $J/c\__{\text{Ture}}$. So, the question is "is the diameter increase by 3% sufficient for the d =16mm section of the shaft below?" If yes, we can simply reduce the diameter by 3% when we analyze the shear stress at a keyway-containing cross section. Note that d and ϕ are both symbols for diameter, but ϕ usually appears in drawings.

Read Appendices 1 and 3 first, and think about the following three cases, c is the radius of each cross section. Which two J/c should be equal, if you want to increase d_{min} by x% to get d2?

J1/(d1/2), d1=d2 J2/(d2/2) d2>d3) J3/(d3/2), d3=d_{min} calculated for a design case.

For drawing



1 2 3

a) Determine the keyway depth, t= h/2, and width, w (or b), from Appendix 1 using the diameters of the keyway-containing shaft sections, which is d= 16 mm here.
b) The true section modulus, labeled as $J/c\__{\text{Ture}}$, for torsional shear stress should be calculated with the formulas given in Appendix 3. Please do so for the d= 16 mm section.
c) Determine a new diameter, d_n, from $\frac{J}{c} = \frac{\pi(d_n)^3}{16} = J/c\__{\text{Ture}}$, and compare d_n with d in two ways, $(d- d_n)/d$ (reducing d) and $(d- d_n)/d_n$ (increasing d_n).
d) The keyway dimensions for d= 16 mm work for a range of diameters. Please use the largest and the smallest diameter in this range and repeat b) and c), what can you conclude?

a) $d = 16mm$; $\boxed{W = 5mm \text{ ; } t = 2.5\ mm}$; $h = 5mm$

b) $\frac{J_{true}}{c} \approx \frac{\pi d^3}{16} - \frac{W t (d-t)^2}{2d} = \boxed{7.331 \times 10^{-7}\ m^3}$

c) $\frac{J_{true}}{c} = \frac{\pi (d_n)^3}{16} \Rightarrow \boxed{d_n = 15.5\ mm}$

INCREASE:

$\frac{(d-d_n)}{d_n} = \boxed{0.0314}$

DECREASE:

$\frac{d-d_n}{d} = \boxed{0.0304}$

$\boxed{\text{3\% INCREASE IS CLOSE}}$

d) RANGE: 12 mm-17 mm

FOR 12 mm:

$$\frac{J_{TUBE}}{C} = 2.923 \cdot 10^{-7} m^3$$

$d_n = 11.4$ mm

INCREASE: $\frac{d-d_n}{d_n} = 0.0510$

DECREASE: $\frac{d-d_n}{d} = 0.0485$

3% INCREASE NOT ENOUGH

FOR 17 mm:

$$\frac{J_{TUBE}}{c} = 8.87 \cdot 10^{-7} m^3$$

$d_n = 16.5$ mm

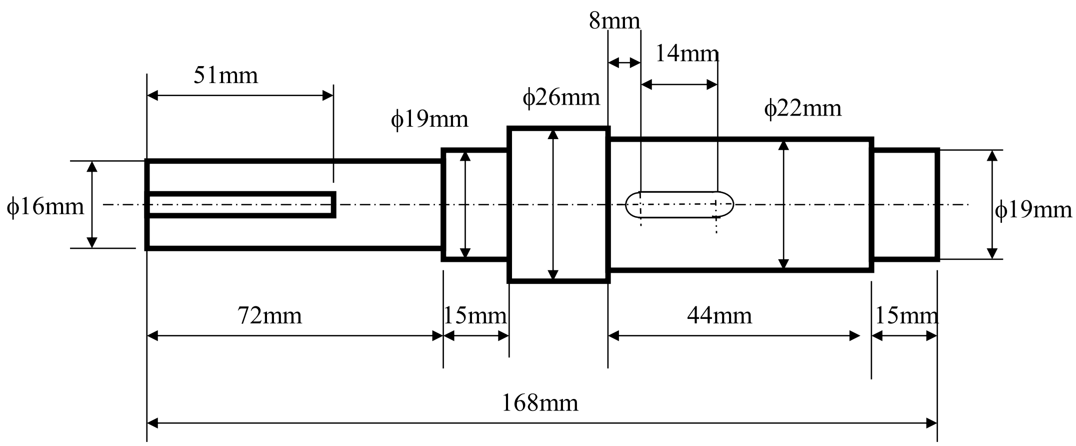
INCREASE:

$\frac{|d-d_n|}{d_n} = 0.0303$

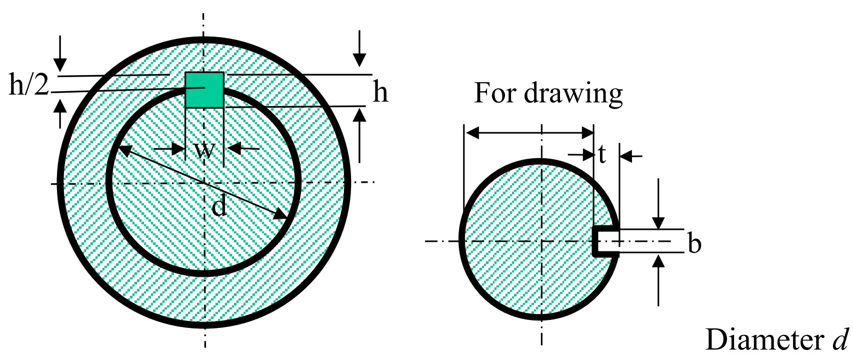
DECREASE:

$\frac{|d-d_n|}{d} = 0.0313$

I CONCLUDE THAT YOU USE 5% FOR SMALLER D AND 3% FOR LARGER D WITHIN THE GIVEN RANGE!



From appendix 1



In the section modulus calculation, b=w =bkey, t =h/2

6. CAD 2. Please draw the shaft shown below, using Unigraphics, Solidworks, or AutoCad, or Pro-E (scale, 1:1). This time, you need to produce a 3-D solid model, one major view of the part, and two cut-away views that show the keyway cross sections. Do not draw all front, left, and top views for such a simple part. Remember to draw centerlines and label diameters by ϕ, the common symbol for the circular and cylindrical geometry. See the attached sample. Please pay a close attention to the way that dimensions are given. Use Appendix A4 on Canvas, or your ME 240 knowledge (or Table A-11 ~ 14 in Shigley and Mischke, or Section10.2 in Hamrock, Jacobson, and Schmid) to determine the tolerance shown with question marks, assuming IT6 for the tolerance grade of the shaft, and the fundamental deviation for the shaft segment of ϕ=19 is p6 (basic hole), and that of ϕ=22 is h6 (basic shaft). The tolerance for the ϕ=18.5 segment is ignored in this work.

This is a simplified practice; remember that in reality, the two keyways should not be in the same plane. But let's make them at the same view for simplicity. Don't forget chamfers and fillets, but in this practice, let's just do the chamfers at the two ends. For the part below, the paper layout should be landscape. **Use a full page, landscape**.

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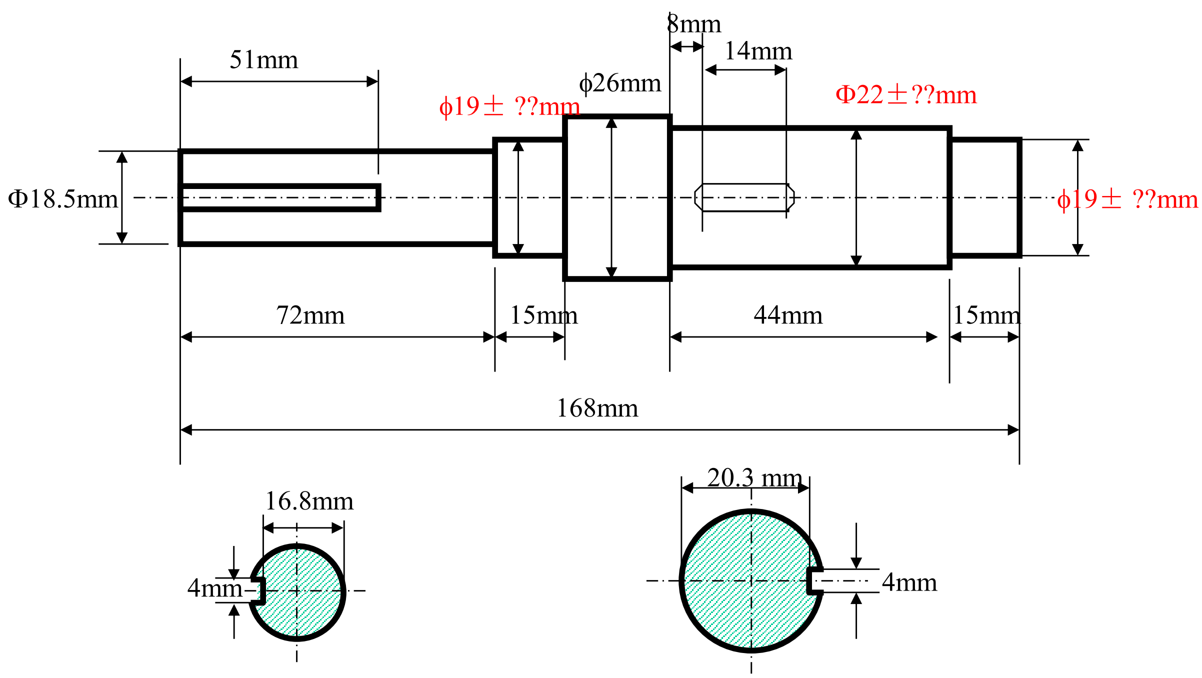
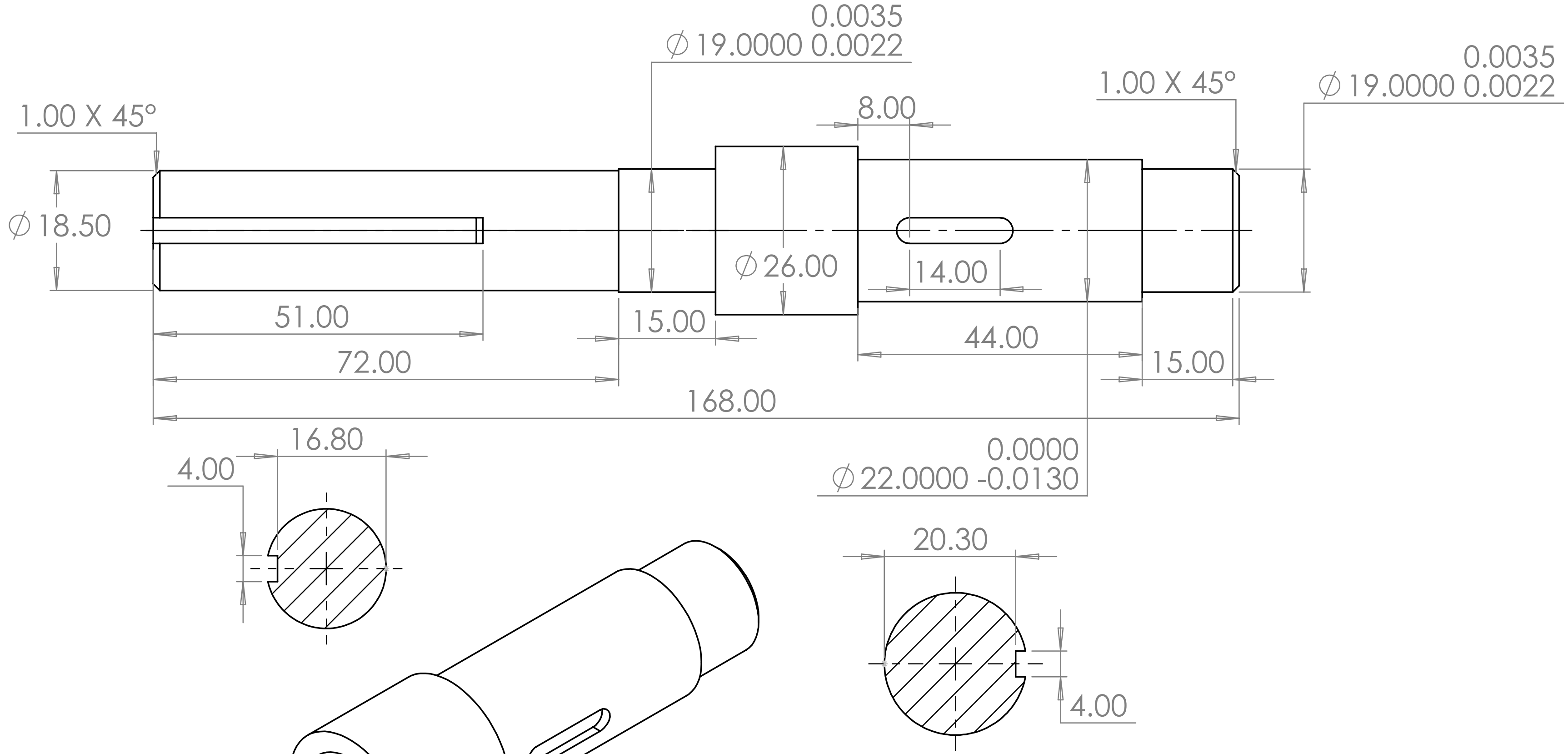
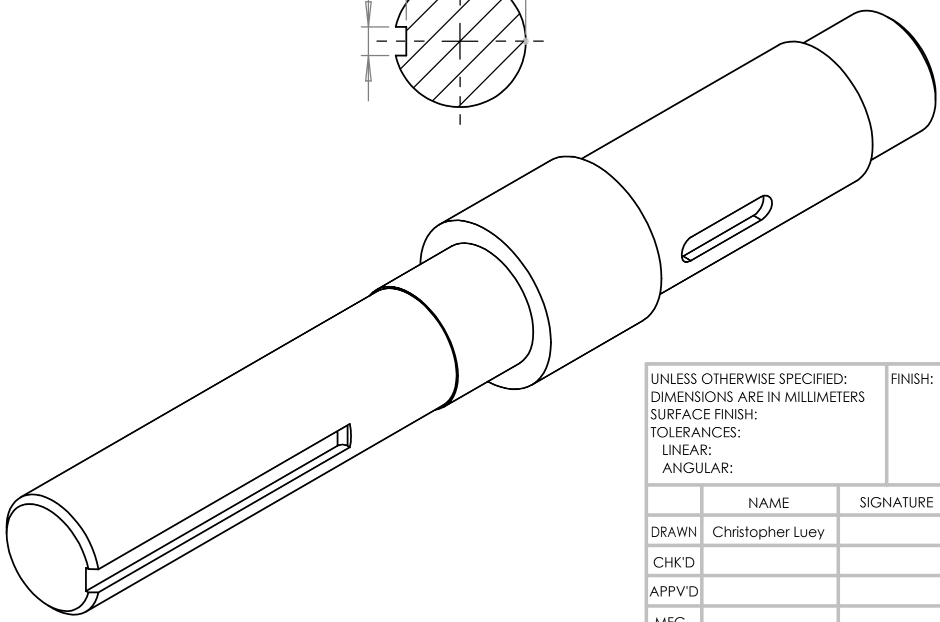


For the cutaway views, no need for arrows if they are right under the places of the cuts, and no need to show the background either.





UNLESS OTHERWISE SPECIFIED:
DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH:
TOLERANCES:
LINEAR:
ANGULAR:

FINISH:

DEBURR AND BREAK SHARP EDGES

DO NOT SCALE DRAWING

REVISION

	NAME	SIGNATURE	DATE
DRAWN	Christopher Luey		
CHK'D			
APPV'D			
MFG			
Q.A			

TITLE:
CAD 2 - Shaft

MATERIAL:

DWG NO.
Shaft

A4

WEIGHT:

SCALE:1:1

SHEET 1 OF 1