315 THEORY OF MACHINES – DESIGN OF ELEMENTS
Fall, 2023
HW No. 3

Assigned: 10/5 Due: one week, 10/12, On-line, pdf in **one single file (with CAD 2 and CAD 3)**.

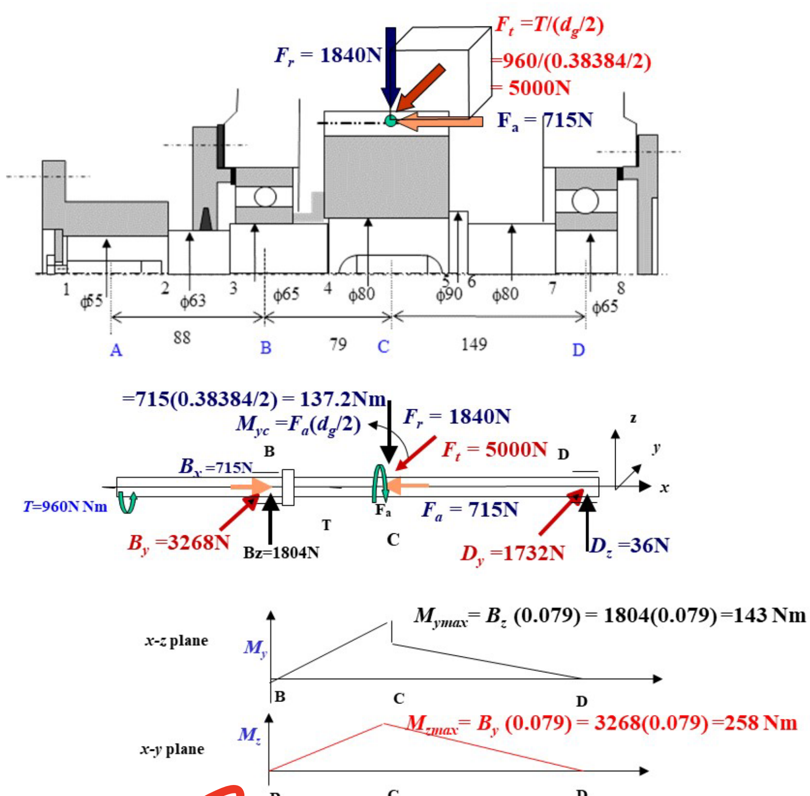
10 points each except otherwise marked.

1. Design the output shaft of a gear reducer with a F. S. $n_s \geq 1.3$ for infinite life. This is the class example, one half of the shar assembly is shown. Forces are in the middle of the corresponding components. The shaft diameters are given in the drawing below:
Given: T= 960 Nm F_r = 1840 N F_a= 715 N
Fatigue stress-concentration factors: $K_f = 3$, $K_{fs} = 2.8$
AISI 1020 steel (S_y = 295 MPa, $S_{ut} = S_u$ =395 MPa)
Ignoring the keyway effect on stress for now.

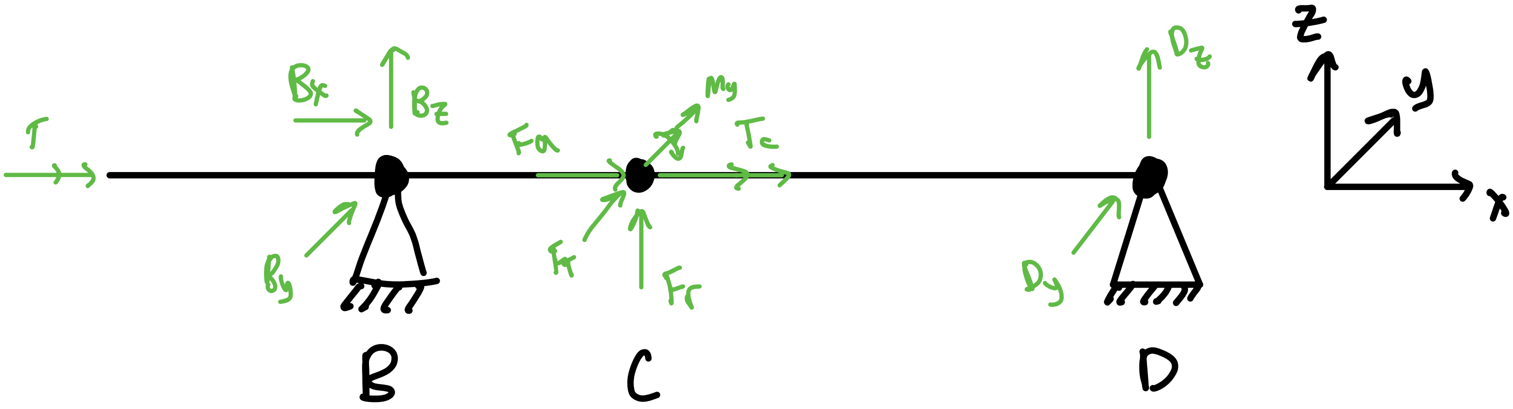
Please show your work on

a) Re-solve (check) the reaction forces at B, C, and D, and the values of the maximum bend moments in the x-z and x-y planes.
b) At C-. re-solve the factor of safety for fatigue (the Goodman's line)
c) the factor of safety for yield, using

$$\frac{\sigma_{VM-a}(no\ \ K's)}{S_y} + \frac{\sigma_{VM-m}}{S_y} = \frac{1}{n_s}$$



CHRISTOPHER LUEY



$CD = 0.149\ m$
$BC = 0.079\ m$
$d_g = 0.3834\ m$

$T = -960\ Nm$
$F_a = -715\ N$
$F_T = -5000\ N$
$F_r = -1840\ N$

SOLVE INDUCED MOMENTS:

$$M_{y,C} = F_a d_g / 2 = 137.22\ Nm$$

STATIC EQUILIBRIUM:

@x:

$$0 = F_a + B_x \Rightarrow \boxed{B_x = 715\ N}$$

@y:

$$\begin{cases} 0 = F_T + B_y + D_y \\ 0 = BC\,F_T + (BC + CD) D_y \end{cases} \Rightarrow \boxed{\begin{matrix} B_y = 3267.54\ N \\ D_y = 1732.46\ N \end{matrix}}$$

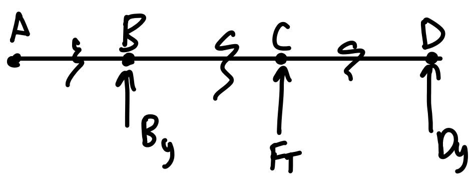
@z:

$$\begin{cases} 0 = F_r + B_z + D_z \\ 0 = BC\,F_r + (BC + CD) D_z + M_{y,C} \end{cases} \Rightarrow \boxed{\begin{matrix} B_z = 1804.31\ N \\ D_z = 35.69\ N \end{matrix}}$$

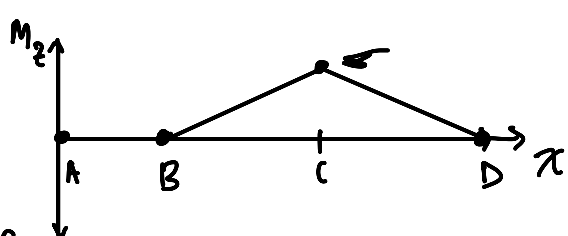
INTERNAL BENDING MOMENTS:

$AC = 0.167\ m$; $AB = 0.088\ m$

@ x-y:

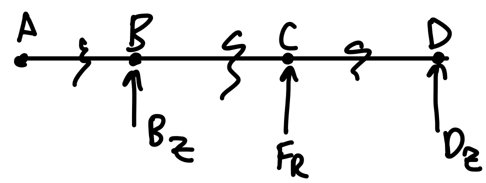


$A \geq x > B$: $M_z = 0$
$B \geq x > C$: $M_z = B_y(x - AB)$
$C \geq x \geq D$: $M_z = B_y(x - AB) + F_T(x - AC)$

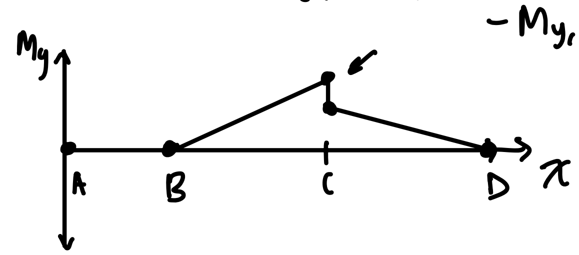


$x = AC$
$\rightarrow M_{z\,max} = B_y(AC - AB) = \boxed{258.14\ Nm}$

@ x-z:



$A \geq x > B$: $M_y = 0$
$B \geq x > C$: $M_y = B_z(x - AB)$
$C \geq x \geq D$: $M_y = B_z(x - AB) + F_R(x - AC) - M_{y,C}$



$x = AC$
$\rightarrow M_{y\,max} = B_z(AC - AB) = \boxed{142.54\ Nm}$

b) @ C^- ($M_z = 258.14\,\text{Nm}$; $M_y = 142.54\,\text{Nm}$; $V_z = 1804.31$; $V_y = 3267.54\,\text{N}$;
$F_x = -715\,\text{N}$; $T = -960\,\text{Nm}$; $d = 0.080\,\text{m}$)

$$|M| = (M_z^2 + M_y^2)^{1/2} = 294.88\,\text{Nm} \qquad |V| = (B_z^2 + B_y^2)^{1/2} = 3732.61\,\text{N}$$

$$\hat{M} = \langle 0, 0.483, 0.875 \rangle$$

$$\sigma_a = \frac{32M}{\pi d^3} = 5.87\,\text{MPa} \qquad \tau_a = \frac{16V}{3\pi d^2} = 0.99\,\text{MPa}$$

$$\sigma_m = -\frac{B_x}{\pi d^2} = -0.142\,\text{MPa} \qquad \tau_m = \frac{16T}{\pi d^3} = 9.55\,\text{MPa}$$

$$\frac{((K_f \sigma_{xa})^2 + 3(K_{fs}\tau_{xya})^2)^{1/2}}{k_f k_s k_r k_t k_m S_e'} + \frac{(\sigma_{xm}^2 + 3\tau_{xym}^2)^{1/2}}{S_{ut}} = \frac{1}{n}$$

$K_f = 3$; $K_{fs} = 2.8$

$S_y = 295\,\text{MPa}$; $S_u = 395\,\text{MPa}$

$n_s \geq 1.3$

$k_f = e(S_u)^f = 0.95$; $e = 1.58$; $f = -0.085$

$k_r = 0.82$ [99% RELIABILITY]

$k_s = 1.189\,d^{-0.112} = 0.728$; $d = 80\,\text{mm}$

$S_e' = 0.5 S_u$

$k_m = k_t = 1$

$\Rightarrow \boxed{n_s = 4.88}$

$S_y = 295\,\text{MPa}$

c)
$$\frac{\sigma_{vm,a}}{S_y} + \frac{\sigma_{vm,m}}{S_y} = \frac{1}{n_s}$$

$$\sigma_{vm,a} = (\sigma_{ax}^2 + 3\tau_{xya}^2)^{1/2} = 6.12\,\text{MPa}$$

$$\sigma_{vm,m} = (\sigma_{mx}^2 + 3\tau_{xym}^2)^{1/2} = 16.54\,\text{MPa}$$

$\Rightarrow \boxed{n_s = 13.02}$

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Christopher's Fatigue Life Calculator

```
In[370]:= sigmaa = Quantity[5.87, "MPa"];
sigmam = Quantity[-0.142, "MPa"];
taua = Quantity[0.99, "MPa"];
taum = Quantity[9.55, "MPa"];
Kf = 3;
Kfs = 2.8;
Sy = Quantity[295, "MPa"];
Su = 395;
e = 1.58;
f = -0.085;
kf = e * (Su) ^ f;
Su = Quantity[Su, "MPa"];
kr = 0.82;
d = 80;
ks = 1.189 * d ^ (-0.112);
se ' = 0.5 * Su;
km = 1;
kt = 1;
Solve[((Kf * sigmaa) ^ 2 + 3 * (Kfs * taua) ^ 2) ^ (1 / 2) / (kf * ks * kr * kt * km * se ') + (sigmam ^ 2 + 3 * (taum ^ 2)) ^ (1 / 2) / (Su) == 1 / n, n]
Out[388]= {{n → 4.88294}}
```

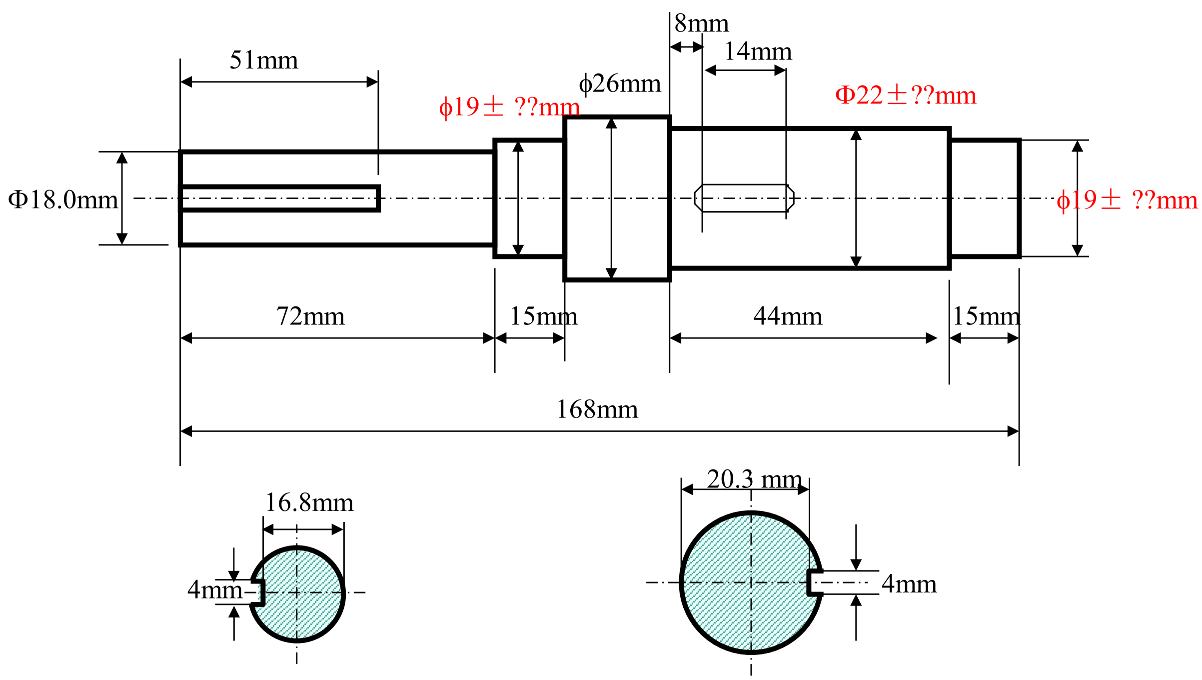
2. (20 points) The shaft below is for a helical gear shaft. Please keep your solution as a template for project calculations. The two keyways are plot at the same view for simplicity. The force, ***F***, on the gear has three components: the radial, axial and the tangential loads, $F_r = 2500$ N, $F_a = 700$ N, and $F_t = 2000$ N. The assembly with major elements is shown in the next page (only the top halves of the elements are shown for simplicity). Use the section modulus equations in Appendix 3 for the cross sections containing a keyway. No need to consider the tolerances indicated by the question marks in the analysis. Please:

a) Given that the width of the coupling is 48 mm, the width of each bearing is 16 mm (although the effective shaft length is 15 mm), the width of the gear is 26 mm, determine the lengths of a, b, and c, which will be used in the force and stress analyses. **Assume that each force is applied at the center of the width of each element**.

b) Determine the bearing reactions at B and D, assuming bearing D takes the axial load. Draw the free-body diagram (FBD) of the shaft (which is now treated as a bar with all diameters neglected) to show all forces (including torques/moments).

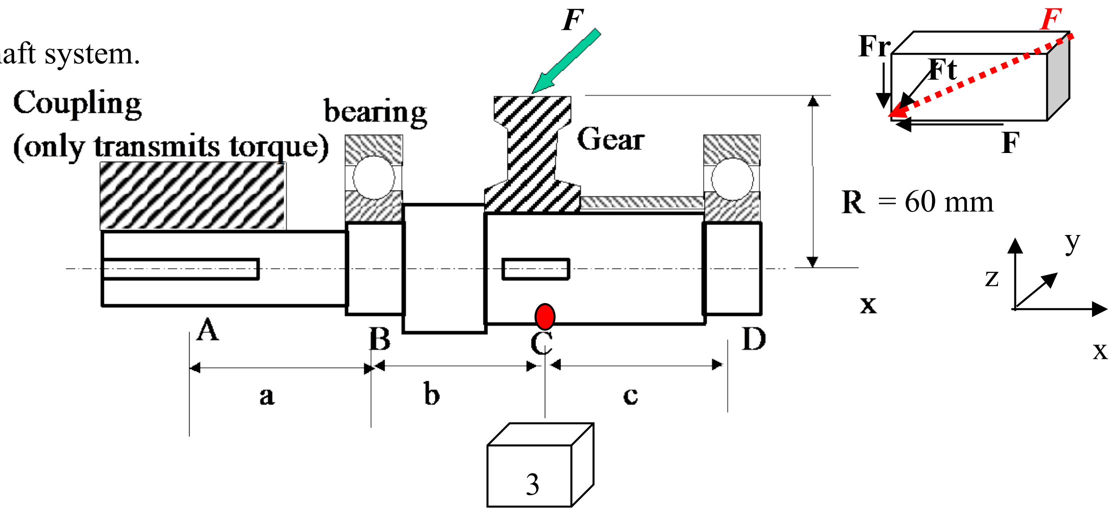
c) Plot the shear force and bending moment diagrams in the xz and xy planes. Properly label the diagrams and indicate peak values. Note, the torque produced by the tangential component of the force is balanced by the torque from the coupling. The resultant bending moment should be obtained from the vector summation of the two moments in the xy and xz planes.

d) The shaft is rotating but the forces are constant. Please calculate the midrange and amplitude of von Mises stresses at most critical point in cross sections C-, consider the stresses due to bending and torsion, and note that the keyway effects on section moduli must be taken into account. K_f=2.0 and K_{fs}= 2.0. You can analyze cross section A yourselves.



For simplicity, you may ignore the transverse shear stress.

The shaft system.



Note that the forces on FBD must be at the true direction to ensure correct T, V and M diagrams

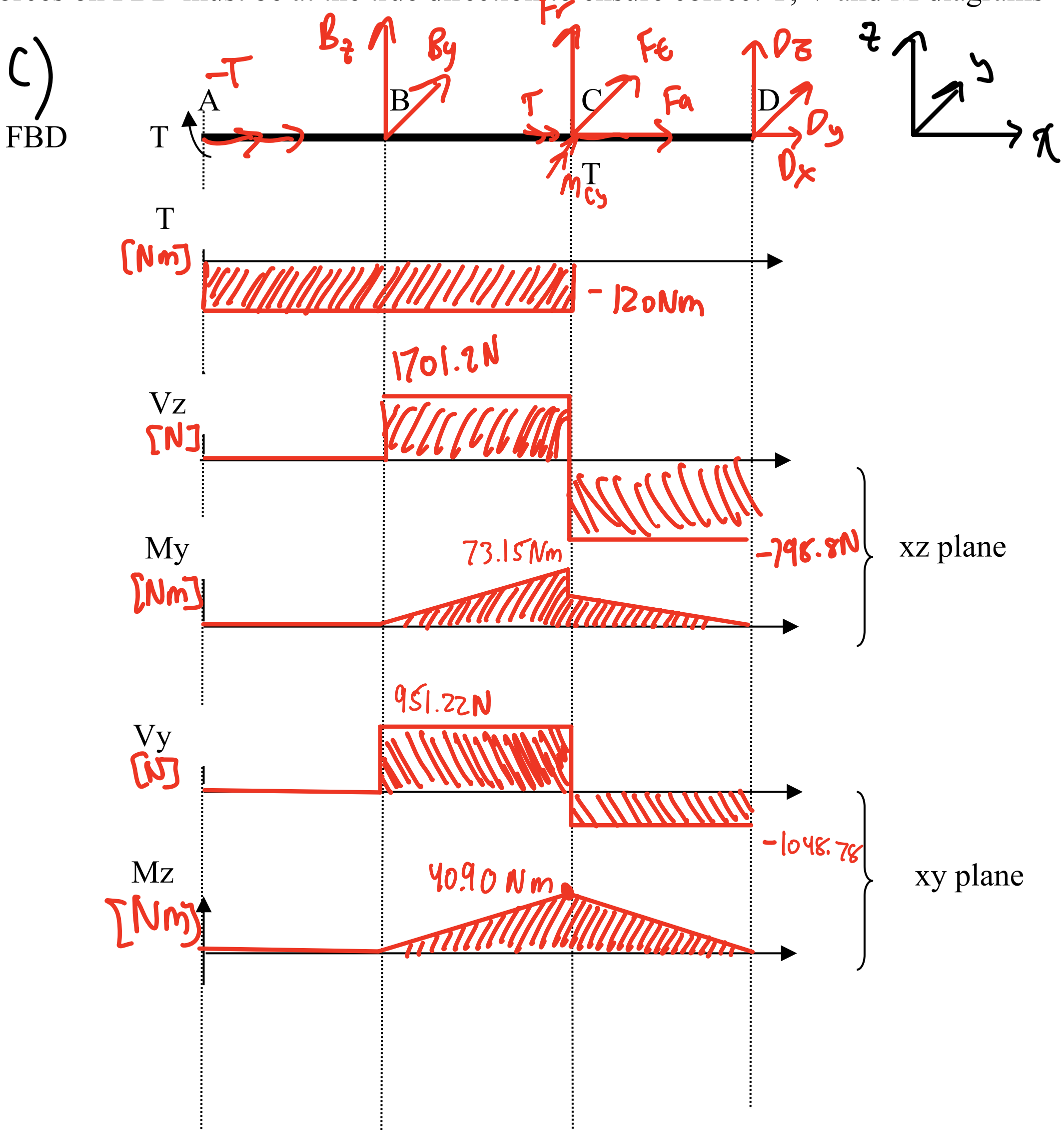
Note that the forces on FBD must be at the true direction to ensure correct T, V and M diagrams

$F_r = -2500\ N$

$F_t = -2000N$

$F_a = -700N$

(c)



a)

$$\boxed{\begin{aligned} A &= 0.055\,m \\ c &= 0.039\,m \\ b &= 0.043\,m \\ d_g &= 0.120\,m \end{aligned}}$$

SOLVE INDUCED MOMENTS:

$$M_{y,c} = F_a d_g / 2 = 42\ Nm$$

$$T = F_t d_g / 2 = -120N$$

$T = -120\,Nm$

$F_r = -2500\ N$

$F_t = -2000N$

$F_a = -700N$

$M_{y,c} = -42Nm$

STATIC EQUILIBRIUM:

@x:

$$0 = F_a + D_x \Rightarrow$$

@y:

$$\left\{\begin{aligned} 0 &= F_T + B_y + D_y \\ 0 &= BC\,F_T + (b+c)D_y \end{aligned}\right\} \Rightarrow$$

@z:

$$\left\{\begin{aligned} 0 &= F_r + B_z + D_z \\ 0 &= BC\,F_r + (b+c)D_z - M_{y,c} \end{aligned}\right\} \Rightarrow$$

b)

$$\boxed{\begin{aligned} D_x &= 700\,N \\ B_y &= 951.22\ N \\ D_y &= 1048.78\,N \\ B_z &= 1701.22\,N \\ D_z &= 798.78\ N \end{aligned}}$$

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```
In[177]:= ab = 0.055
          cd = 0.039
          bc = 0.043
          dg = 0.120
          fa = -700
          ft = -2000
          fr = -2500
          t = ft * dg / 2
          my = fa * dg / 2

          Solve[0 == d_x + fa, d_x]
          Solve[{0 == ft + b_y + d_y, 0 == bc * ft + (bc + cd) * d_y}, {b_y, d_y}]
          Solve[{0 == fr + b_z + d_z, 0 == bc * fr + (bc + cd) * d_z + my}, {b_z, d_z}]

Out[177]= 0.055
Out[178]= 0.039
Out[179]= 0.043
Out[180]= 0.12
Out[181]= -700
Out[182]= -2000
Out[183]= -2500
Out[184]= -120.
Out[185]= -42.
Out[186]= {{d_x → 700}}
Out[187]= {{b_y → 951.22, d_y → 1048.78}}
Out[188]= {{b_z → 676.829, d_z → 1823.17}}
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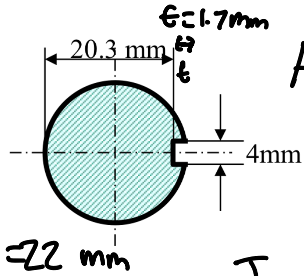
$$\boxed{\begin{aligned} M_{z,max} &= B_y b = 40.90\ Nm \\ M_{y,max} &= B_z b = 73.15\ Nm \end{aligned}}$$

d)



$$A = \frac{\pi D^2}{4} - bt$$

$$= 373.533\ mm^2$$

$$\frac{J}{c} = \frac{\pi D^3}{16} - \frac{bt(D-t)^2}{2D}$$

$$= 2027.04\ mm^3$$

$$W = \frac{\pi D^3}{32} - \frac{bt(D-t)^2}{2D} = 981.678\,mm^3$$

$$V = (B_z^2 + B_y^2)^{1/2} = 1949.09\ N$$

$$M = (M_{y\,max}^2 + M_{z\,max}^2)^{1/2} = 83.808\ Nm$$

$$\sigma_a = \frac{M}{W} = 85.372\ MPa$$

$$\sigma_m = \frac{F}{A} = 1.875\ MPa$$

$$\tau_a = 4V(3A)^{-1} = 6.96\ MPa$$

$$\tau_m = \frac{Tc}{J} = 59.20\ MPa$$

$$\sigma_{vm,a} = \left((K_f \sigma_a)^2 + 3(K_{fs}\tau_a)^2\right)^{1/2}$$

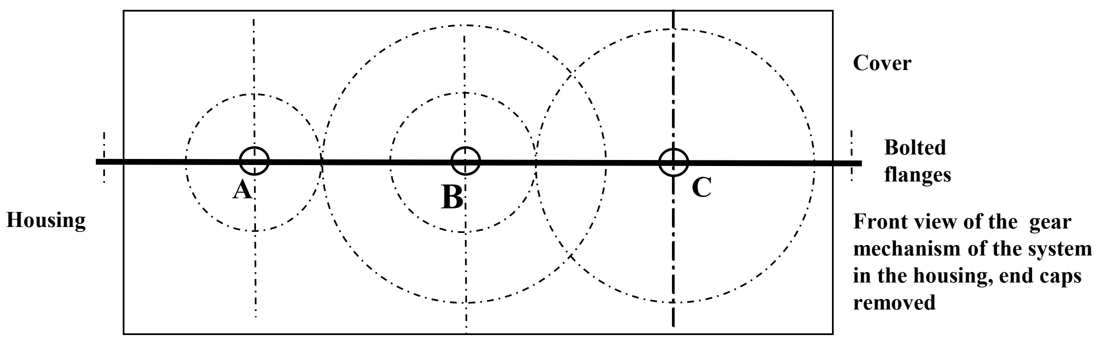
$$\boxed{= 172.44\ MPa}$$

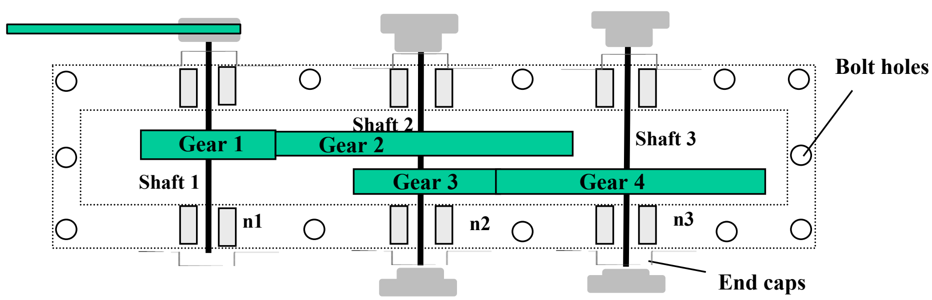
$$\sigma_{vm,m} = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2}$$

$$\boxed{= 102.555\ MPa}$$

3. This is a two-stage gear reducer with four gears, Gear 1-Gear 4 (or G_1, G_2, G_3, G_4), in two sets. The input to the first shaft and G_1 is 3800 rpm and the expected train ratio is $i_{ideal}=8$. G_2 and G_3 are mounted on the middle shaft, and therefore, they have the same speed. The numbers of teeth of G_2-G_4 are $N_2 = 60$, $N_3= 23$, $N_4 = 72$.

a) Determine N_1=? You may have two choices, which one to choose?
b) What is designed train value i_{14}, or speed reduction ratio?
c) Determine the real speed (rpm) of the middle shaft and output shaft determined by the teeth of the gears, considering the rotation direction with respect to that of shaft 1 as well. Now the output speed, n_4, is different from what expected. What is the absolute percentage error of the designed output speed, n_4, with respect to the expected output speed?





$i_{IDEAL} = 8$

$n_1 = 3600$ RPM ; $N_2 = 60$; $N_3 = 23$; $N_4 = 72$

a) $i_{IDEAL} = \frac{N_2 N_4}{N_1 N_3}$

$\Rightarrow \boxed{N_1 = \frac{540}{23} = 23.48 \quad N_1 = 23 \text{ (CHOOSE CLOSER INTERGER)}}$

FAVOR LOWER FOR FITMENT

b) $i_{14} = \frac{N_2 N_4}{N_1 N_3} = \boxed{8.166}$

c) $n_1 \frac{N_1}{N_2} = n_2 = n_3 = \boxed{1456.67 \text{ RPM CCW}}$

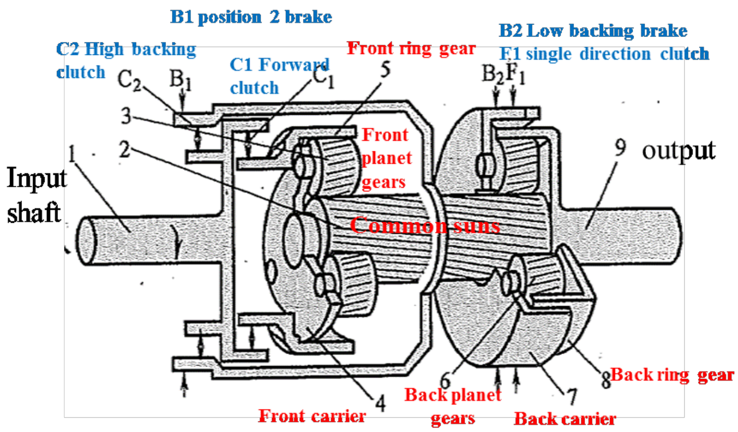
$n_4 = n_3 \frac{N_3}{N_4} = \boxed{465.32 \text{ RPM CW}}$

$n_{4,IDEAL} = n_1 \, i_{IDEAL}^{-1} = 475$ RPM CW

$\text{ERROR} = \frac{|n_{4,IDEAL} - n_4|}{n_{4,IDEAL}} = 0.02037$

$\boxed{2.037\%}$

4. This is the class reading example. Please solve this one by yourself and show details. Prove the input and output are in the same direction and solve for the transmission ratio from the input shaft to the output shaft.



Forward 1: C1 encaged, the front ring gear inputs

F1 engaged and (that makes) the back system a fixed axis system

The back carrier is fixed

Front carrier 4 and back ring 8 are connected (designed)

Knowing N_{sf}, N_{pf}, N_{rf}, N_{sb}, N_{pb}, N_{rb},

The link between the two systems are $n_{sf} = n_{sb}$, $n_{armf} = n_{rb}$

MAKE RB STATIONARY REFERENCE FRAME:

$$\frac{n_{SF} - n_{RB}}{n_{RF} - n_{RB}} = -\frac{N_{RF}}{N_{SF}}$$

FLIPS ONCE

$$n_{SF} - n_{RB} = -\frac{N_{RF}}{N_{SF}}(n_{RF} - n_{RB}) \quad (1)$$

RELATE SB TO SF AND RB:

$$\frac{n_{SB}}{n_{RB}} = -\frac{N_{RB}}{N_{SB}}$$

FLIPS ONCE

$$n_{SB} = n_{SF} = -n_{RB}\left(\frac{N_{RB}}{N_{SB}}\right) \quad (2)$$

USE EQ 1 AND 2:

$$-n_{RB}\left(\frac{N_{RB}}{N_{SB}}\right)-n_{RB}=-\frac{N_{RF}}{N_{SF}}\left(n_{RF}-n_{RB}\right)$$

SOLVE FOR $i=\frac{n_{RF}}{n_{RB}}$:

$$\frac{N_{RF}}{N_{SF}}n_{RF}=\frac{N_{RF}}{N_{SF}}n_{RB}+n_{RB}\left(\frac{N_{RB}}{N_{SB}}\right)+n_{RB}$$

$$=n_{RB}\left(\frac{N_{RF}}{N_{SF}}+\frac{N_{RB}}{N_{SB}}+1\right)$$

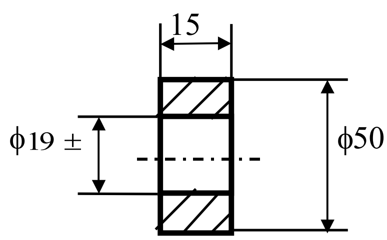
$$\boxed{\Rightarrow i=\frac{n_{RF}}{n_{RB}}=\frac{\left(\frac{N_{RF}}{N_{SF}}+\frac{N_{RB}}{N_{SB}}+1\right)}{\frac{N_{RF}}{N_{SF}}}}$$

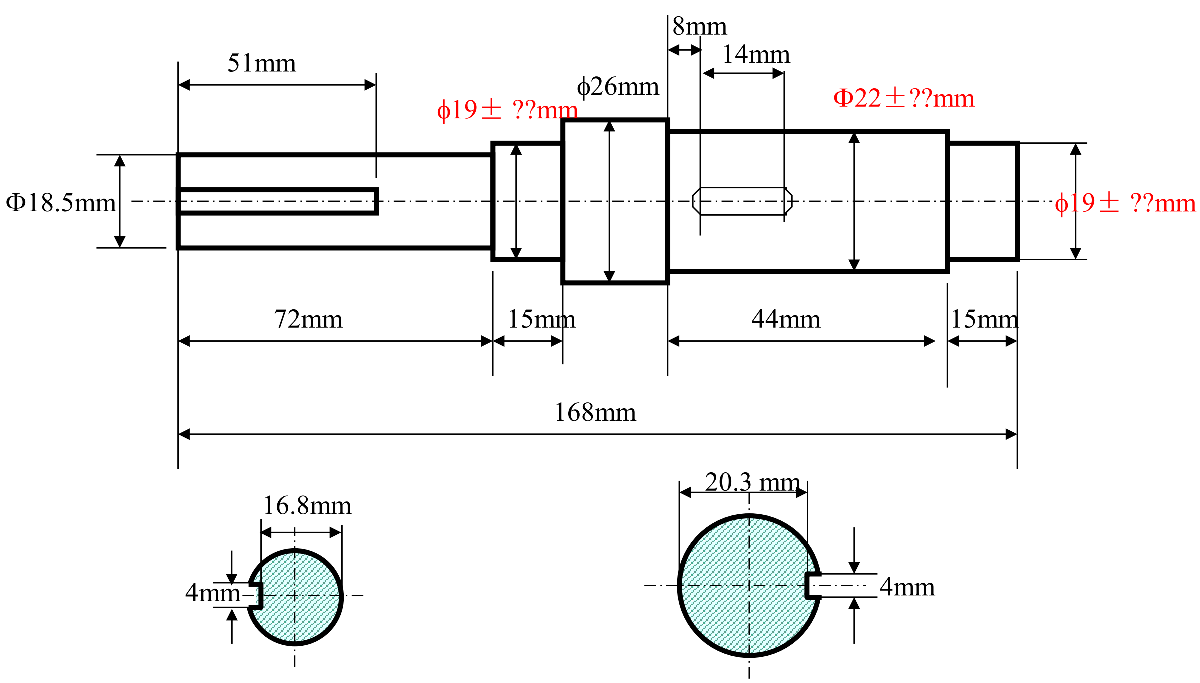
$$\underset{\text{SAME}}{n_{RF}\rightarrow n_{PF}}\ \underset{\text{FLIP}}{\rightarrow n_{SF}}\ \underset{\text{SAME}}{\rightarrow n_{SB}}\ \underset{\text{FLIP}}{\rightarrow n_{PB}}\ \underset{\text{SAME}}{\rightarrow n_{RB}}$$

FLIPS TWICE $\Rightarrow$ n_{RF} IS SAME DIRECTION AS n_{RB}

5. CAD 3. These are the two elements we practiced in CADs 1 & 2. First, make sure your drawings are correct. Then, create an assembly of a shaft-ring system by attaching one ring from one side of the shaft until the rings touch the shoulders of the shaft. So, you will have a system of a shaft with two rings. You only need to create a 3-D model, from which you make a major view of the assembly, which is the cross-sectional view of the assembly with parts properly labeled. Only one such view is sufficient. See the next page for an example of the drawing and part labeling. Note: the part list is bottom up.

Please 1) plot borders and a title block (use your own template); 2) arrange the overall drawing nicely; 3) use the full page, landscape. And 4) most importantly, in a cross-section assembly like this one, the shaft-type elements, such as shafts, pins, and balls, still appear in their regular views as if they were not "cut" in the cross-sectional view, 5) only the assembly dimensions, such as the overall dimensions and fits (and center distances as well if we have more shafts) are needed in the assembly drawing, no detailed dimensions of each element are needed. Use the landscape layout.





CAD DELAYED BECAUSE NX ISSUE

A sample (yours should be full page, landscape)