315 THEORY OF MACHINES – DESIGN OF ELEMENTS

Fall 2023

HW No. 7, **Total 40 points**

Assigned: 11/2 Due: one week, 11/9, On-line, pdf in **one single file**.

20 point each. In this practice, select the smallest possible bearings whose rating C (C_1) is the closest to the calculated one. Use proper table in the eBook, Chapter 6. Note that in a real engineering work, bearing sizes must meet certain structural requirements.

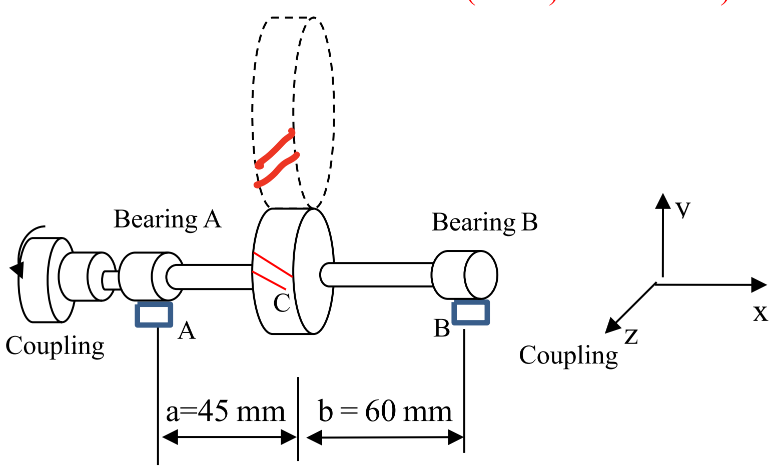
1. A helical-gear shaft transmits a power of H = 52KW at n = 1300 rpm. The coupling is the input element. The gears have teeth N_p = 20, N_G = 51, their normal module is m_n = 5mm, the pressure angle is ϕ_n = 20 degree, and the helical angle is ψ = 17^0. The pinion and its shaft assembly are shown below. The gear torque is balanced by the coupling torque. Two bearings should be used and their L10 life is 11,000 hours. Application factor *Ap*=1.

a) Are the force/moment directions all correct? If not, make the changes. Mark the helical direction of the tooth of the mating gear.

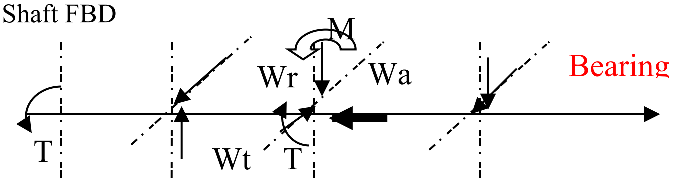
b) Calculate the magnitudes of the radial reaction forces, ***A*** and ***B***, at bearings A and B.

c) Estimate the shaft diameter at the coupling, d_{min}, with the torque on the shaft. The allowable stress is τ_{all} = 200 MPa. Increase the calculated d_{min} by 5% to consider the keyway effect. Use a preferred number ending with 0 or 5 for the final result.

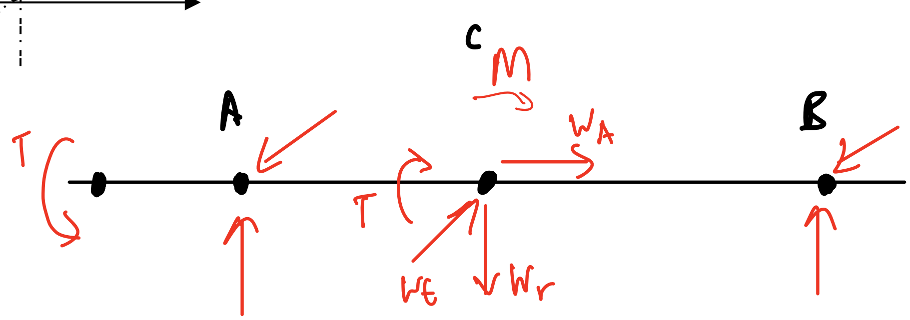
d) Select a pair of SKF single-row deep-groove ball bearings and use the same bearings for both supports. Make sure the bearing bore diameter is at least 5 mm larger than d_{min}. Show your work. List the bearing catalog number, 6xxx, the shaft diameter, *d*, the outside diameter, *D*, and the bearing width. (Assume that the bearing under the larger radial load takes the axial (thrust) load as well)



Shaft FBD



Bearing axial load is not determined here.



Parameter	Units	Pinion	Gear
Torque, T	NM	381.970	974.024
Normal Module, mn	mm	5.000	
Pressure Angle, phi	degree	20.000	
Helix Angle, psi	degree	17.000	
RPM, n	rpm	1300.000	509.804
Number of Teeth, N	no.	20.000	51.000
Gear Ratio		0.392	
Pitch Diameter, d	mm	104.569	266.651
Root Diameter, dr	mm	92.069	254.151
Outer Diameter, do	mm	114.569	276.651
Addendum, a	mm	5.000	
Dedendum, b	mm	6.250	
Transverse Module	mm	5.228	
Transverse Pressure Angle	degree	20.837	
Tangential Force, Wt	N	7305.595	
Raidal Force, Wr	N	2780.514	
Axial Force, Wa	N	2233.544	
Resultant Force, W	N	8129.680	

$$T = \frac{60H}{2\pi n} = 381.97 \text{ Nm}$$

$$n_g = n_p GR$$

$$GR = N_p / N_g$$

$$d = N m_t$$

$$d_r = d - 2b$$

$$d_o = d + 2a$$

$$a = m_n$$

$$b = 1.25 m_n$$

$$m_t = m_n / \cos\psi$$

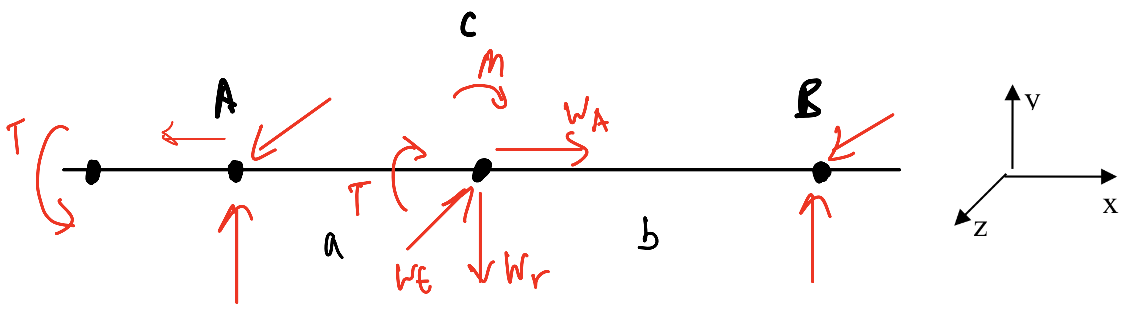
$$\tan\phi_t = \tan\phi_n / \cos\psi$$

$$W^t = \frac{T}{d/2}$$

$$W^r = W \sin\phi_n$$

$$W^a = W \cos\phi_n \sin\psi$$

$$W = W^t / (\cos\phi_n \cos\psi)$$



a = 45mm ; b = 60mm

$$\Sigma F_z = A_z + B_z - W^t = 0$$

$$\Sigma M_{A,z} = 0 = W^t a - (a+b) B_z$$

$$\boxed{\begin{aligned} A_z &= 4174.63\,N \\ B_z &= 3130.97\,N \end{aligned}}$$

$$\Sigma F_y = A_y + B_y - W^r = 0$$

$$\Sigma M_{A,y} = 0 = B_y (a+b) - W^r a - M \; ; \; M = \frac{W^a d}{2}$$

$$\boxed{\begin{aligned} A_y &= 476.68\,N \\ B_y &= 2303.84\,N \end{aligned}}$$

WORST CASE:

BEARING A TAKES W^A BECAUSE $P_{rA} > P_{rB}$

$A_x = -2237.544\,N$

c) $d_{min} = \sqrt[3]{\dfrac{16T}{\pi \tau}} = 21.346\,mm$

$\tau = 200\,MPa$

$T = 381.970\,Nm$

$21.346 \cdot 1.05 = 22.4136$

$$\boxed{25mm}$$

d) $L_{10} = h(60)n \cdot 10^{-6} = 858\text{ MC}$

INTERPOLATION FORMULA:

SLOPE: $\frac{y_2 - y_1}{x_2 - x_1}$

$y = \text{SLOPE}(x - x_1) + y_1$

FOR BEARING A:

$P_a = 2.233\text{ kN}$

$P_r = \text{NORM}(A_z, A_y) = 4.202\text{ kN}$

$P = A_P(XP_r + YP_a)$; $C = PL^{1/m}$

	Ap	1.000	
	m	3.000	
	Pa	2.233	kN
	Pr	4.202	kN
	pa/pr	0.531	
	X	1.000	
	Y	0.000	
	P	4.202	kN
	C	39.929	kN
Bearing 6404	C0	23.600	kN
	C	43.6	kN
	Pa/C0	0.095	
	lower point pa/c0	0.070	
	upper point pa/c0	0.130	
	lower point e	0.270	
	upper point e	0.310	
	slope	0.667	
	e (interpolated)	0.286	
	lower point y	1.600	
	upper point y	1.400	
	slope	-5.000	
	Y (interpolated)	1.518	

hours	11000.000	
rotation speed	1300.000	rpm
L10	858.000	MC

USING $y - y_1 = m(x - x_1)$

	Ap	1.000	
	m	3.000	
	Pa	2.233	kN
	Pr	4.202	kN
	pa/pr	0.531	
	X	0.560	
	Y	1.518	
	P	5.743	kN
	C	54.569	kN
Bearing 6407	C0	31	kN
	C	55.3	kN
	Pa/C0	0.072	
	lower point pa/c0	0.070	
	upper point pa/c0	0.130	
	lower point e	0.270	
	upper point e	0.310	
	slope	0.667	
	e (interpolated)	0.271	
	lower point y	1.600	
	upper point y	1.400	
	slope	-5.000	
	Y (interpolated)	1.593	

hours	11000.000	
rotation speed	1300.000	rpm
L10	858.000	MC

USING $y - y_1 = m(x - x_1)$

USING $y - y_1 = m(x - x_1)$

	Ap	1.000	
	m	3.000	
	Pa	2.233	kN
	Pr	4.202	kN
	pa/pr	0.531	
	X	0.560	
	Y	1.593	
	P	5.911	kN
	C	56.166	kN
Bearing 6408	C0	36.5	kN
	C	63.7	kN
	Pa/C0	0.061	
	lower point pa/c0	0.040	
	upper point pa/c0	0.070	
	lower point e	0.240	
	upper point e	0.270	
	slope	1.000	
	e (interpolated)	0.261	
	lower point y	1.800	
	upper point y	1.600	
	slope	-6.667	
	Y (interpolated)	1.659	

hours	11000.000	
rotation speed	1300.000	rpm
L10	858.000	MC

USING $y - y_1 = m(x - x_1)$

USING $y - y_1 = m(x - x_1)$

Ap	1.000	
m	3.000	
Pa	2.233	kN
Pr	4.202	kN
pa/pr	0.531	
X	0.560	
Y	1.659	
P	6.057	kN
C	57.558	kN

hours	11000.000	
rotation speed	1300.000	rpm
L10	858.000	MC

CHOOSE 6408:
d = 40 mm
D = 110 mm
W = 27

57.558 kN < 63.7 kN

2. For the same problem above, continue to
1) Select a pair of Timken ISO 355 bearings and use the same bearings for both supports in a face-to-face mounting. Also make sure the bearing bore diameter is at least 5 mm larger than d_{min}. Show your work. List the bearing catalog number, 3xxxx, the shaft diameter, *d*, and the bearing outside diameter, *D*, and *T*. Make sure the axial loading direction is correctly determined.
2) Repeat above but select a pair of Timken ISO 355 bearings for the use in a back-to-back mounting.

You may use the calculated C value from the last step of Problem No. 1 for the first trial selection.

1. $\lambda = 0.5$ [TIMKEN ISO] $X = 0.4$

$C_{CALC} = 57.558\,kN$

30208 ; $C = 60.1\,kN$

$$P_a = \max \begin{cases} XF_{rA} + Y_A\left(\frac{\lambda F_{rB}}{Y_B} + W_{ae}\right) \\ F_{rA} \end{cases}$$

$$P_b = \max \begin{cases} XF_{rB} + Y_B\left(\frac{\lambda F_{rA}}{Y_A} - W_{ae}\right) \\ F_{rB} \end{cases}$$

$F_{rA} = 4.202\,kN$; $F_{rB} = 3.887\,kN$; $W_{ae} = -2.233\,kN$

2. $$P_a = \max \begin{cases} XF_{rA} + Y_A\left(\frac{\lambda F_{rB}}{Y_B} - W_{ae}\right) \\ F_{rA} \end{cases}$$

$$P_b = \max \begin{cases} XF_{rB} + Y_B\left(\frac{\lambda F_{rA}}{Y_A} + W_{ae}\right) \\ F_{rB} \end{cases}$$

$F_{rA} = 4.202\,kN$; $F_{rB} = 3.887\,kN$; $W_{ae} = 2.233\,kN$

	hours	11000	
	rotation speed	1300	rpm
	L10	858	MC
	Fra	4.202	N
	Frb	3.887	N
	Wae	-2.233	N
Bearing 30208	**m**	1/2	
c: 60.1 kN	**lambda**	0.500	
	X	0.560	
	Ya	1.600	
	Yb	1.600	
	Pa	4.202	N
	Pb	7.851	N
	Ca	31.879	N
	Cb	59.559	N

	hours	11000	
	rotation speed	1300	rpm
	L10	858	MC
	Fra	4.202	kN
	Frb	3.887	kN
	Wae	-2.233	kN
Bearing 30208	**m**	1/2	
c: 60.1 kN	**lambda**	0.500	
	X	0.560	
	Ya	1.600	
	Yb	1.600	
	Pa	7.869	kN
	Pb	3.887	kN
	Ca	59.702	kN
	Cb	29.489	kN

BEARING 30208:

$d = 40$ mm

$D = 80$ mm

$W = 19.75$ mm