CHRISTOPHER LUEY

315 THEORY OF MACHINES – DESIGN OF ELEMENTS

Fall 2023

HW No. 1 (**10 point each**)

(Use the given setups, add sheets as needed)

Assigned: 9/21

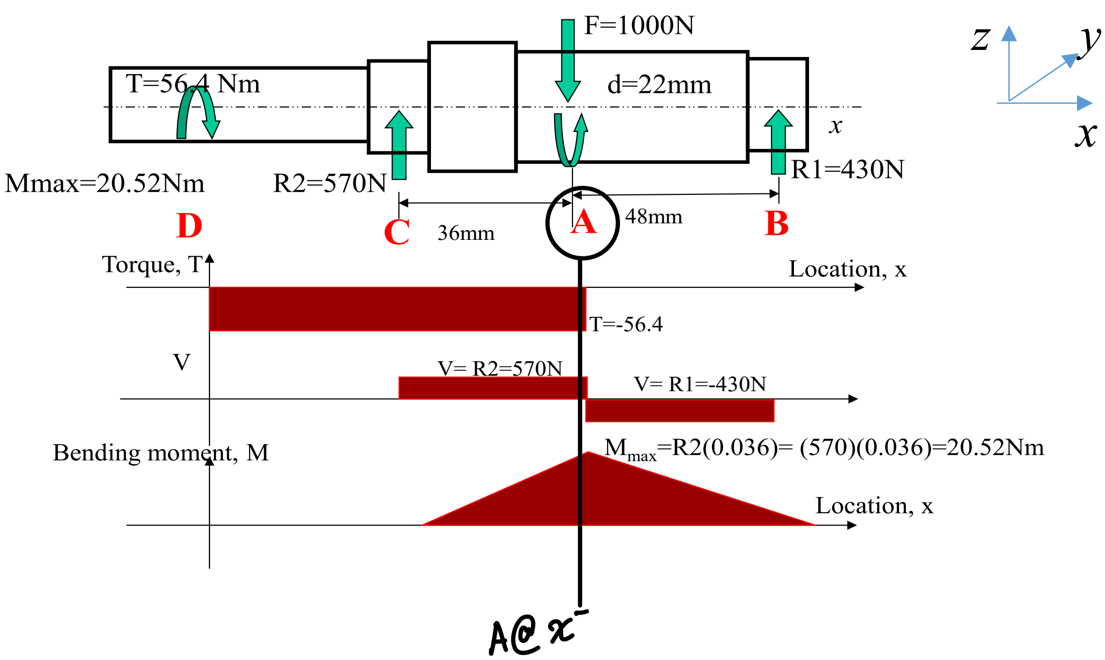
Due: 9/28, online in **ONE** pdf file, before 11:59pm

Review of mechanics is very important. Please follow the list at the end of Chapter 1 and review these topics. Please also review fits and fundamentals of engineering drawing.

Note: please make sure your handwriting is clear and contents are well organized.

1. Re-do the class example calculations for normal force F_y = 1000N at A, torque T=56.4Nm at A and D. Show all details. T is inward at A. The bearings at C and B are the same for radial load supporting, and these bearings offer reaction forces R1 and R2 normal to the shaft. The shaft has no axial loading (i.e., no force is along the x axis). Now you need to calculate for the exact numbers for forces and stresses (for force/moment/torque: xxx.y N, or Nm, for stress: xxx.y MPa)

a. Calculate the reaction forces at C and B, which are R1 and R2.
b. Check the shear force and bending moment diagrams shown below; check the peak values & their locations (no need to submit this portion). Calculate the peak bending moment value (Do show your calculation steps).
c. Indicate the location of the possible critical cross section based on the bending moment diagram. Make a circle to show this cross section; and on this circle, mark the locations of the possible most critical points, 1,2,3,4. Calculate the principal stresses at each point, arrange them in the sequence as $\sigma_1 \geq \sigma_2 \geq \sigma_3$, and calculate the maximum shear stress, τ_{max}, at each of these points as well.



1a. $F = 1000\,N$

$R_1 + R_2 = F$

$-R_2(84\,mm) + F(48\,mm) = 0$

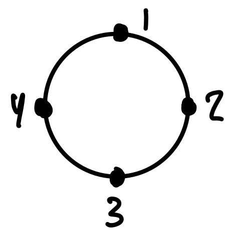
$R_1 = \frac{3000}{7}\,N \;;\; R_2 = \frac{4000}{7}\,N$

$R_1 = 428.6\,N \;;\; R_2 = 571.4\,N$

1b. $M(x) = \int_c^B V\,dx$

$M_{max} = R_2(36\,mm) = 20.6\,Nm$

1c. AT A^- ($T = 56.4\,Nm$, $V = 571.4\,N$, $M = 20.57\,Nm$, $d = 22\,mm$):



$\sigma_{avg} = \frac{(\sigma_x + \sigma_y)}{2}$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_{max} = \tau_{max} + \sigma_{avg}$

$\sigma_{min} = \sigma_{avg} - \tau_{max}$

1: $\sigma_x = -\frac{32M}{\pi D^3} = -19.7\,MPa$;

$\sigma_y = 0.0\,MPa$

$\tau = \frac{16T}{\pi D^3} = 27.0\,MPa$;

$\Rightarrow$

$\sigma_1 = 16.9\,MPa$

$\sigma_2 = 0.0\,MPa$

$\sigma_3 = -36.6\,MPa$

$\tau_{max} = 26.7\,MPa$

2: $\sigma_x = 0.0\,MPa$

$\sigma_y = 0.0\,MPa$

$\tau = \frac{16T}{\pi D^3} + \frac{16V}{3\pi D^2} = 29.0\,MPa$

$\Rightarrow$

$\sigma_1 = 29.0\,MPa$

$\sigma_2 = 0.0\,MPa$

$\sigma_3 = -29.0\,MPa$

$\tau_{max} = 29.0\,MPa$

3: $\sigma_x = \frac{32M}{\pi D^3} = 19.7\,MPa$

$\sigma_y = 0.0\,MPa$

$\tau = \frac{16T}{\pi D^3} = 27.0\,MPa$

$\Rightarrow$

$\sigma_1 = 36.6\,MPa$

$\sigma_2 = 0.0\,MPa$

$\sigma_3 = -16.9\,MPa$

$\tau_{max} = 26.7\,MPa$

4: $\sigma_x = 0.0\,MPa$

$\sigma_y = 0.0\,MPa$

$\tau = \frac{16T}{\pi D^3} - \frac{16V}{3\pi D^2} = 25.0\,MPa$

$\Rightarrow$

$\sigma_1 = 25.0\,MPa$

$\sigma_2 = 0.0\,MPa$

$\sigma_3 = -25.0\,MPa$

$\tau_{max} = 25.0\,MPa$

2. From Problem No. 1, how important is the transverse shear stress?

a. Cross section A-. Determine the ratio of the torsional shear stress over the transverse shear stress, $R_{sA} = \tau_{tsA}/\tau_{vA}$

b. cross section A-. If the ratio of toque T over shear force V is $T/V = 1/10$ (m), how large should diameter d is, in mm, to make the transverse shear stress equivalent to the torsional shear stress, or $R_{sA} = \tau_{tsA}/\tau_{vA} = 1$?

c. Cross section C+, the shaft diameter is d= 18mm. Determine the ratio of the torsional shear stress over the transverse shear stress, $R_s = \tau_{ts}/\tau_v$

d. Cross sections C+ and D+, the shaft diameter at C is d= 18mm and that at D is d=15mm. Determine the ratio of the two torsional shear stresses, τ_{tsD}/τ_{tsC}

e. Based on shear stress comparison, which of the D and C cross section is more critical?

a) FOR LONG SLENDER BEAMS (L>>d), TRANSVERSE SHEAR CONTRIBUTES MINIMALLY TO TOTAL SHEAR.

$\tau_V = \frac{16V}{3\pi D^2} = 2.0$ MPa vs. $\tau_T = \frac{16T}{\pi D^3} = 27.0$ MPa ; $\frac{\tau_T}{\tau_V} = 13.5$ [13.5 ORDER OF MAG. LARGER]

b) $\frac{T}{V} = \frac{1}{10}$ m ; $\tau_T = \tau_V$; $\frac{16T}{\pi D^3} = \frac{16V}{3\pi D^2}$; $\frac{T}{V} = \frac{D}{3}$ => $D = \frac{3}{10}$ m

c) $d = 18$ mm ; $\tau_T \tau_V^{-1} = R_s$; $R_z = 571.4$ N ; $T = 56.4$ Nm

$\tau_V = \frac{16V}{3\pi D^2} = 3.0$ MPa ; $\tau_T = \frac{16T}{\pi D^3} = 49.3$ MPa => $\tau_T \tau_V^{-1} = 16.5$

d) $d_D = 15$ mm ; $d_C = 18$ mm

$\tau_{TD} = \frac{16T}{\pi d_D^3} = 85.1$ MPa => $\tau_{TD} \tau_{TC}^{-1} = 1.73$

$\tau_{TC} = \frac{16T}{\pi d_C^3} = 49.3$ MPa

e) D^+ IS MORE CRITICAL BECAUSE THE TOTAL SHEAR STRESS IS HIGHER.

$\tau_{TD} = 85.1$ MPa

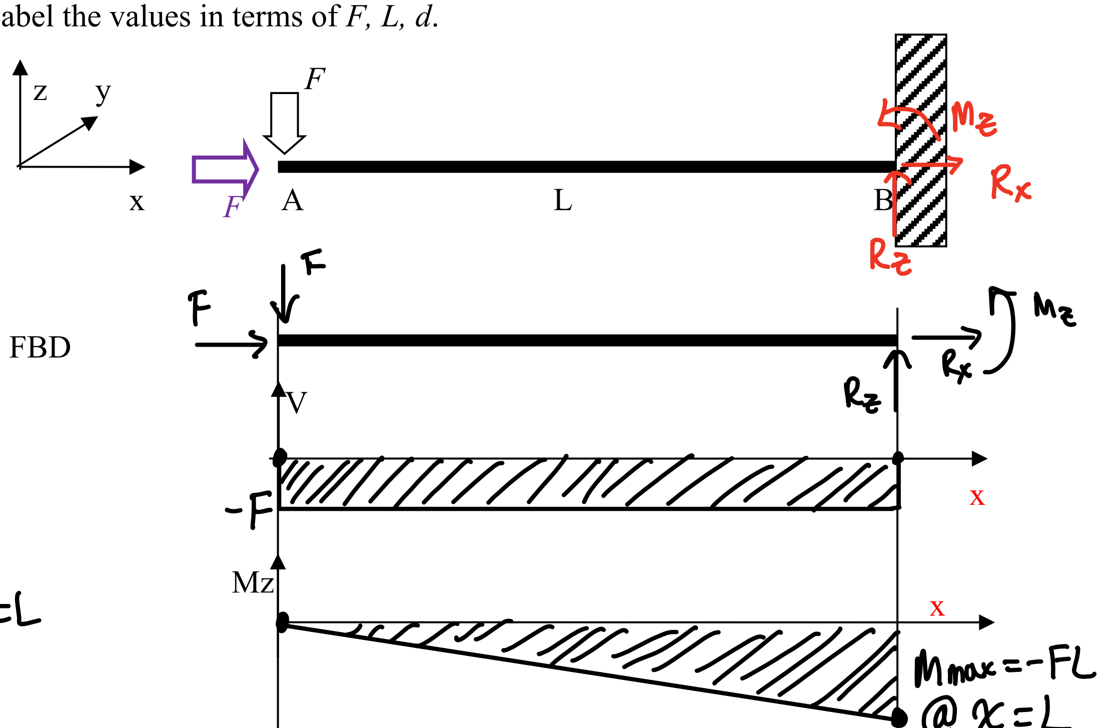
$\tau_C = \tau_{TC} + \tau_{VC} = 52.3$ MPa

In the following, *L>>d* All diagrams should be aligned with the forces (moments, torques) in the free-body diagram (FBD). **No grade will be give if they are not aligned**.

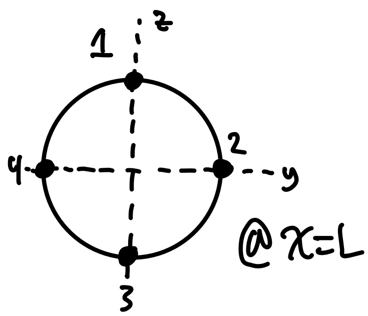
3. Shaft AB is simply shown below as a cantilever beam; its diameter is *d* and its length is *L*. Forces are applied at one end of the shaft, one in the *x* direction (the axial direction) and the other in the *z* direction (a radial direction). Assume uniform shaft cross section (for now), please

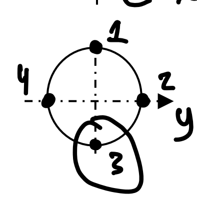
a. Draw the free-body diagram (FBD), label all forces. You need to solve all reactions first.
b. Plot the shear force diagram and indicate the value.
c. Plot the bending moment diagram, mark the peak value and its location.
d. Indicate the location of the critical cross section. The circle below is for a cross section; please drag the circle to the location of this critical cross section, and mark, on this cross section, the location of the most critical point that has the absolute maximum principal stress value. Label the values in terms of *F, L, d*.

$R_z = F$
$R_x = -F$
$M_z = -FL$



3d.





$$1: \sigma_x = \frac{-4F}{\pi D^2} + \frac{32FL}{\pi D^3}$$

$$2: \tau = \frac{16F}{3\pi D^2}$$

$$\sigma_x = \frac{-4F}{\pi D^2}$$

$$3: \sigma_x = \frac{-4F}{\pi D^2} + \frac{-32FL}{\pi D^3}$$

$$4: \tau = \frac{16F}{3\pi D^2}$$

$$\sigma_x = \frac{-4F}{\pi D^2}$$

L>>D, THEREFORE, POINT 3 HAS HIGHEST PRINCIPLE STRESSES. SINCE L CONTRIBUTES MORE TO NORMAL FORCE.

$$\Rightarrow \sigma_{ABS} = \frac{-4F}{\pi D^2} + \frac{-32FL}{\pi D^3}$$ [ABSOLUTE PRINCIPAL STRESS]

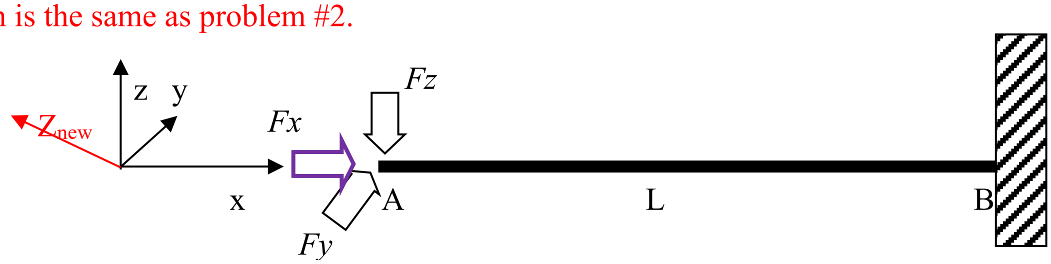
COMPRESSION

4. The same shaft AB is now loaded with three forces at one location, the axial force $Fx = F$ along the x axis, and radial forces $Fy = F$ along the y axis and $Fz = F$ opposite to the z axis, respectively.

You can combine these two radial forces first as Fr and get a new y aligned the combined force and label the new y as Z_{new}.

a. Draw the free-body diagram (FED) with Fx and the combined radial force Fr, label all forces. You need to solve all reactions.
b. Plot the shear force diagram and indicate the shear force value following the above.
c. Plot the bending moment diagram, mark the peak resultant moment and its location.
d. Indicate the location of the critical cross section. The circle below is for a cross section; please drag the circle to the location of this critical cross section, and mark, on this cross section, the location of the most critical point that has the absolute maximum principal stress value. Label the values in terms of F, L, d. Also determine the precise location, by angle (degree) to z, of this critical point and mark it on the circle.

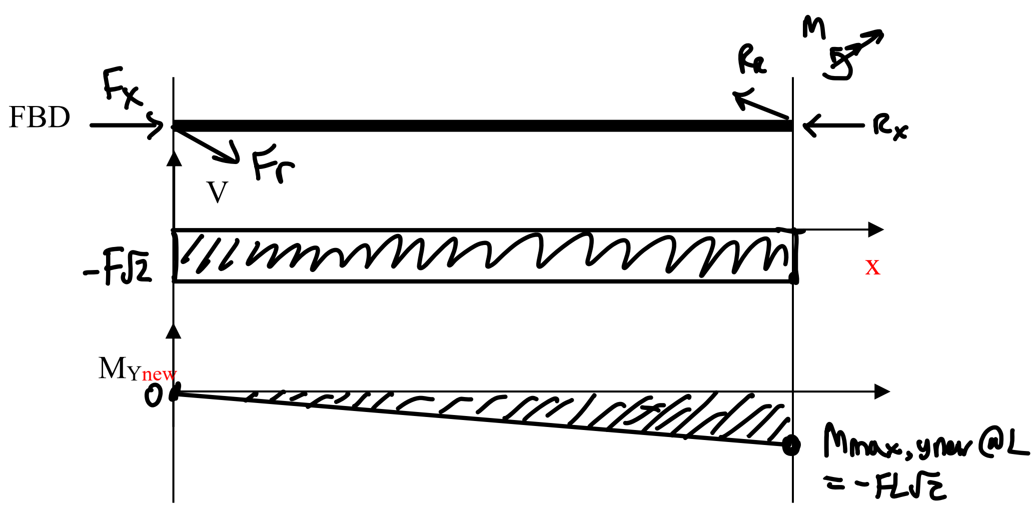
For this problem, you can combine the two radial forces first and get a new y aligned the combined force Fr and label the new y as Z_{new}; then plot the shear force and bending moment diagrams about Z_{new}. The peak bending moment is now the peak resultant bending moment. Note that when Z_{new} is defined, y is also rotated to a new direction Ynew. However, in nature, this problem is the same as problem #2.

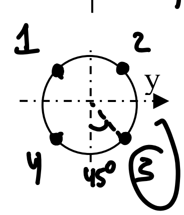


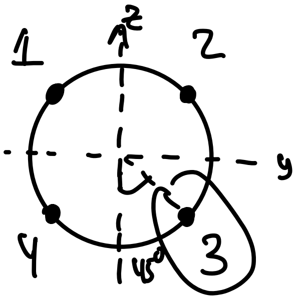
$R_x = F$

$R_e = F\sqrt{2}$

$M = -LF\sqrt{2}$







1 : $\sigma_x = \frac{-4F}{\pi D^2} + \frac{32 FL\sqrt{2}}{\pi D^2}$

2 : $\tau = \frac{16F\sqrt{2}}{3\pi D^2}$

$\sigma_x = \frac{-4F}{\pi D^2}$

3 : $\sigma_x = \frac{-4F}{\pi D^2} + \frac{-32 FL\sqrt{2}}{\pi D^2}$ $\Rightarrow$

4 : $\tau = \frac{16F\sqrt{2}}{3\pi D^2}$

$\sigma_x = \frac{-4F}{\pi D^2}$

L>>D, THEREFORE, POINT 3 HAS HIGHEST PRINCIPLE STRESSES.

$\sigma_{ABS} = \frac{-4F}{\pi D^2} + \frac{-32 FL\sqrt{2}}{\pi D^2}$

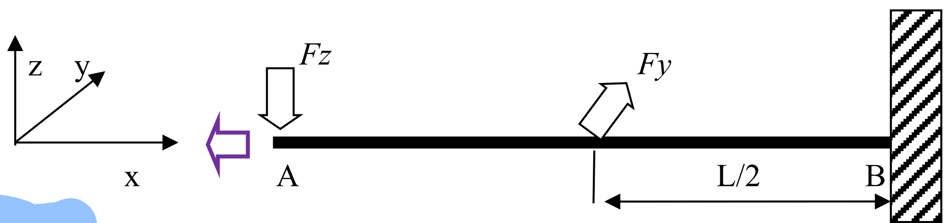
@ POINT 3

3 IS ROTATED 135° FROM +Z AXIS

5. Still the same shaft AB of length L and three forces whose values are $Fx = F$, $Fy = F$, $Fz = F$, but now they are applied at different locations. Fy is at the L/2 location, and Fx is tension.

a. Draw the free-body diagram (FED), label all forces. You need to solve all reactions first.
b. Plot the shear force diagrams and mark the shear force value.
c. Plot the bending moment diagrams, mark the peak values and locations, and calculate the resultant bending moment.
d. Indicate the location of the critical cross section. The circle below is for a cross section; please drag the circle to the location of this critical cross section, and mark, in this cross section, the location of the most critical point that has the absolute maximum principal stress. Label the values in terms of *F, L, d*. Also determine the precise location of this critical point and mark it on the circle by the angle to z.

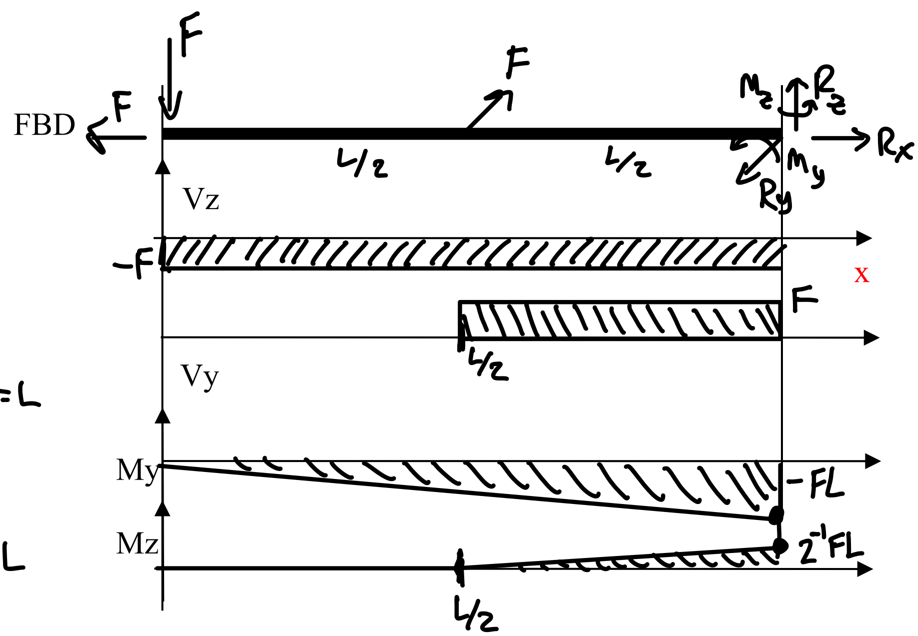
Can you combine these two radial forces first? May be not. You need to work on them separately in different planes and then combine the peak bending moments into the resultant peak moment.



$R_z = F$ $M_z = FL2^{-1}$

$R_x = F$ $M_y = -FL$

$R_y = F$



$M_{max} = \sqrt{M_z^2 + M_y^2}$

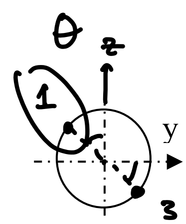
$= \frac{\sqrt{5}}{2} FL$ @ $x = L$

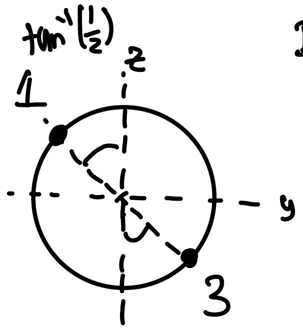
$V_{max} = \sqrt{V_y^2 + V_z^2}$

$= F\sqrt{2}$ @ $x = L$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26.565^\circ$





IGNORING TRANSVERSE SHEAR
BECAUSE MAX SHEAR STRESS DOESN'T ALIGN WITH
NORMAL STRESS MAX.

$$1: \quad \sigma_x = \frac{16FL\sqrt{5}}{\pi D^3} + \frac{4F}{\pi D^2} \quad \Rightarrow \quad \sigma_{ABS} = \frac{16FL\sqrt{5}}{\pi D^3} + \frac{4F}{\pi D^2}$$

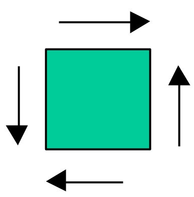
@ POINT 1

$$3: \quad \sigma_x = \frac{-16FL\sqrt{5}}{\pi D^3} + \frac{4F}{\pi D^2}$$

LOCATED $-26.565°$ FROM $+z$ AXIS

6. What is the shear yield limit, S_{sy}, of a steel, determined from a pure shear-stress state, τ, in terms of tensile yield strength S_y based on the

a) Maximum shear stress theory (MSST)? and the
b) Distortion energy theory (DET)? Then
c) Mark your results (values and locations) in the safe-region diagrams, shown below, for σ_A and σ_B, which are two non-zero principal stresses.
d) If the factor of safety based on the MSST is $n_S = 1.1$, what is the factor of safety, n_D, based on the DET for the same stress state and the same material? What is the ratio of the two, n_D/n_S?



τ

a) $S_y = \sigma_1 - \sigma_3$; $\sigma_1 = \tau$; $\sigma_3 = -\tau$

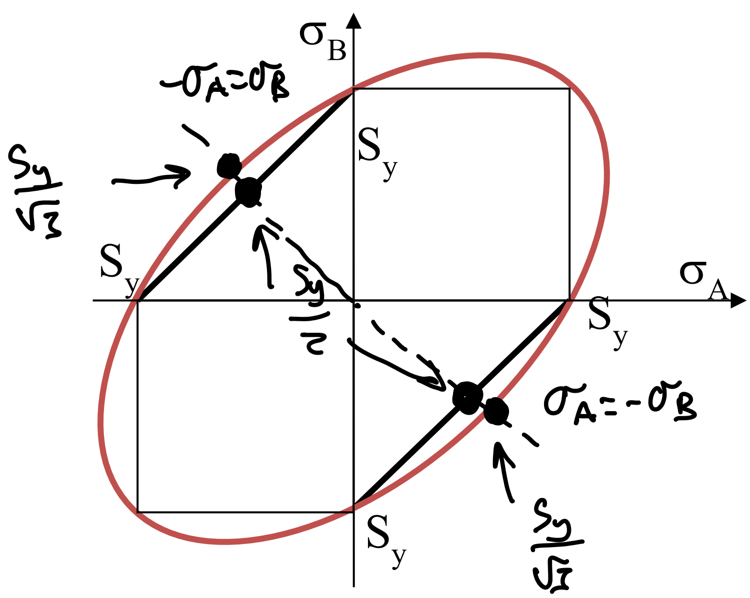
$S_y = 2\tau \Rightarrow S_{sy} = \frac{S_y}{2}$

b) $$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}{2}\right]^{1/2} = S_y$$

$$\left[\frac{\sigma_1^2+\sigma_3^2+(2\tau)^2}{2}\right]^{1/2} = S_y$$

$$(3S_{sy})^{1/2} = S_y$$

$$S_{sy} = \frac{S_y}{\sqrt{3}}$$



D) $n_s = 1.1$ $\qquad S_{sy} = \frac{S_y}{n_d\sqrt{3}}$

$2\tau = S_y/n_s$

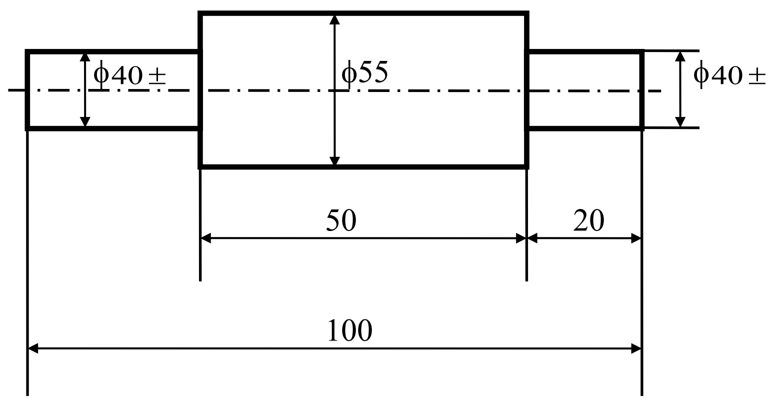
$S_{sy} = S_y/2n_s$

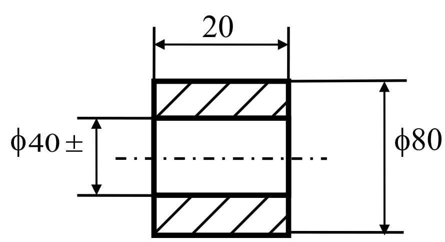
$$1 = \frac{n_d\sqrt{3}}{2n_s}$$

$\frac{n_d}{n_s} = \frac{2}{\sqrt{3}} = 1.15$ $\qquad n_d = \frac{2.2}{\sqrt{3}} = 1.27$

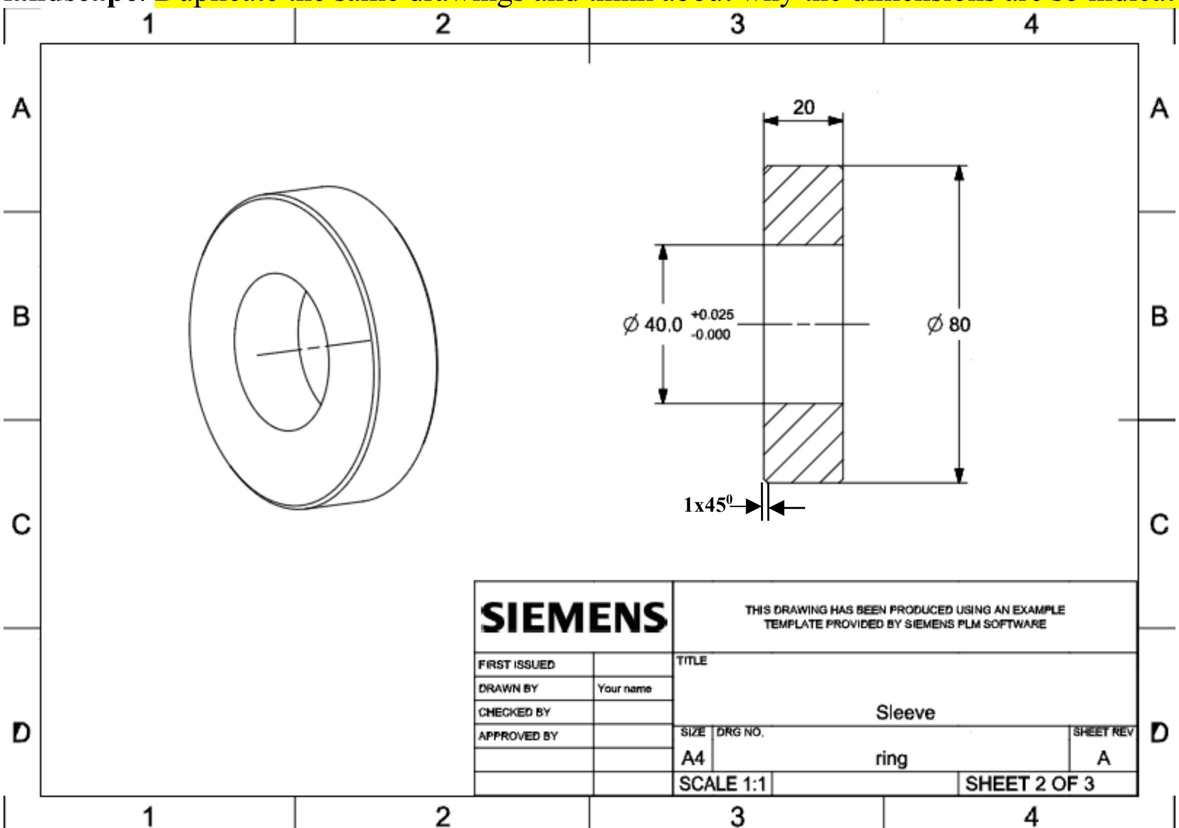
7. CAD 1 warming up. This practice is to draw the shaft and the ring shown below, using Unigraphics, or Solidworks, or AutoCad, or Pro-E, in two separated sheets (scale, 1:1). Sample drawings are shown in the next page. What you need to produce for each of them is a 3-D solid model and one major view of the part; note that one major view is sufficient. Do not draw all front, left, and top views for such simple parts. A front view is sufficient for the shaft but be sure you draw a centerline there and label diameters by ϕ, the common symbol for circular and cylindrical geometry. However, you need to produce a cross-sectional view for the ring to show both inner and outer diameters. Again, one view is enough, provided that you draw a centerline and properly label diameters.

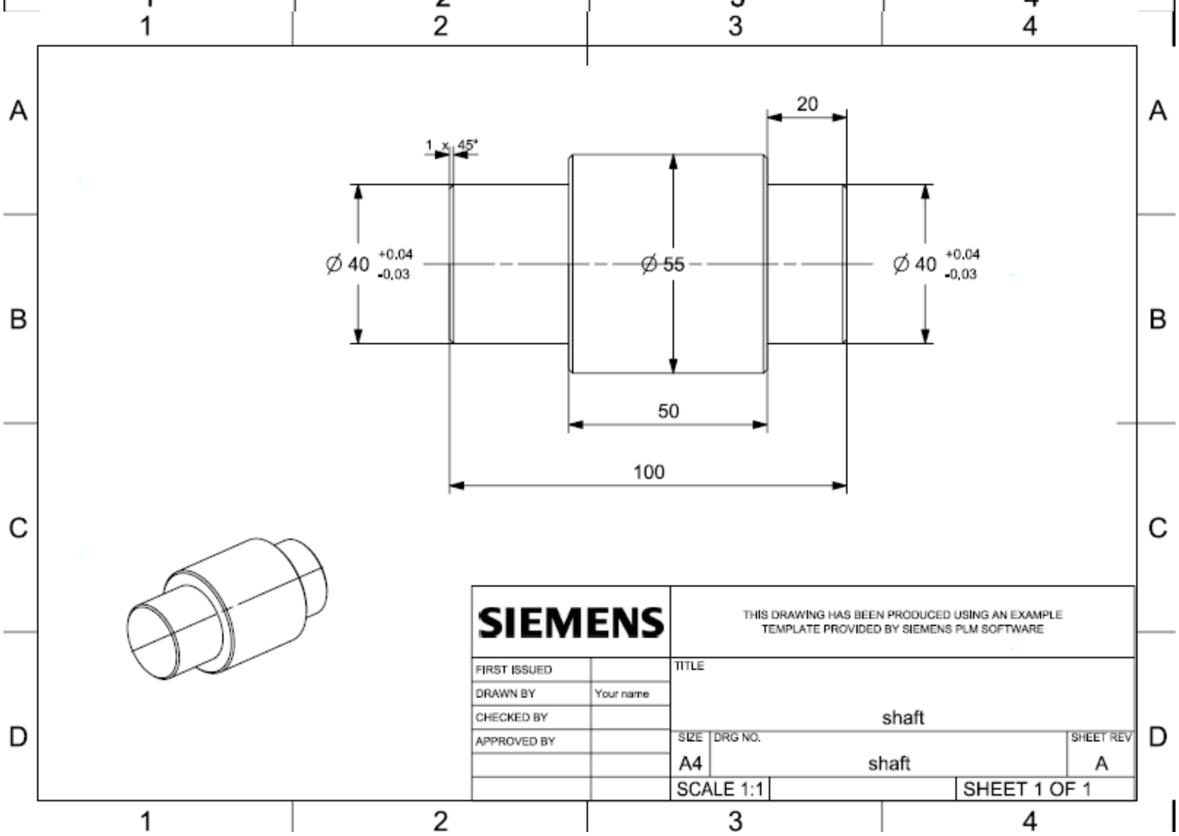
Remember to plot borders and a title box for each. Write your name in the title box. You may fill the title box with the material and quantity of the part, etc., just for a practice. Arrange the overall drawing nicely. You can save this as a template. For the parts below, the paper layout should be **landscape**. Use **one FULL page for one part**.

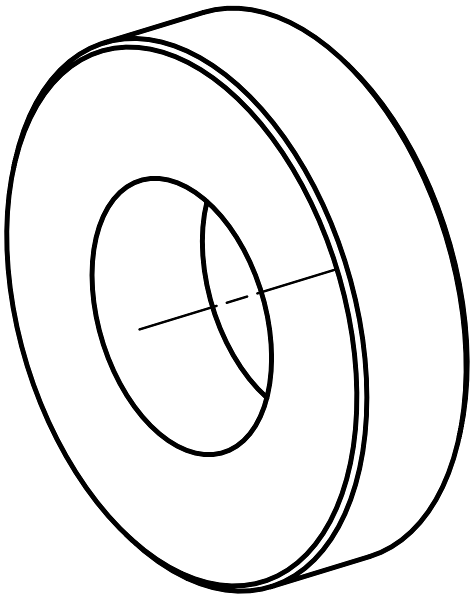


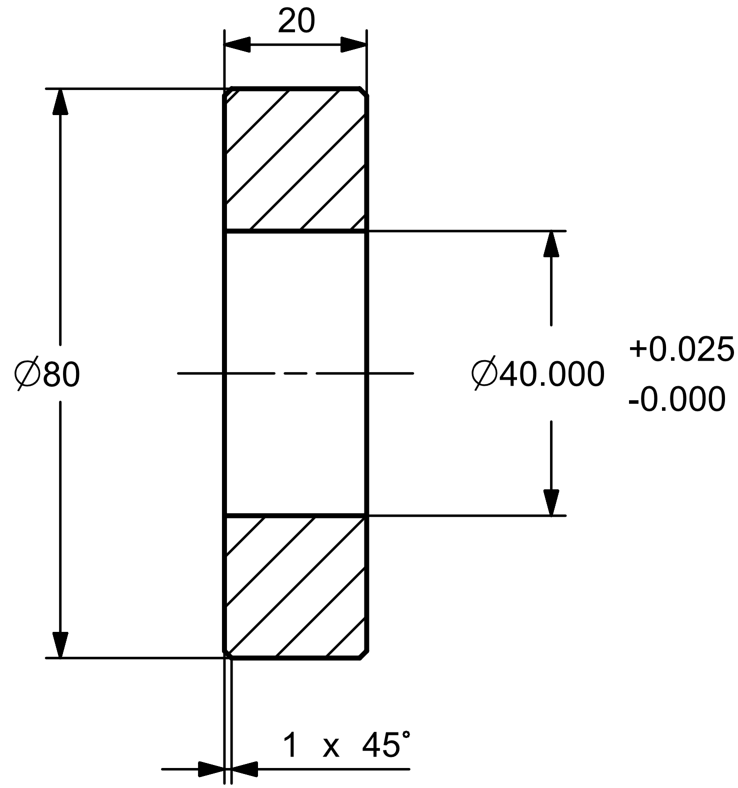


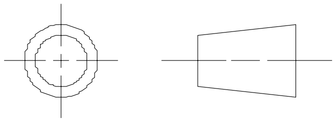
Samples are given below; but you need to submit two separate drawings, **each uses a full page**, **landscape**. Duplicate the same drawings and think about why the dimensions are so indicated.











ALL DIMENSIONS IN MM

SIEMENS

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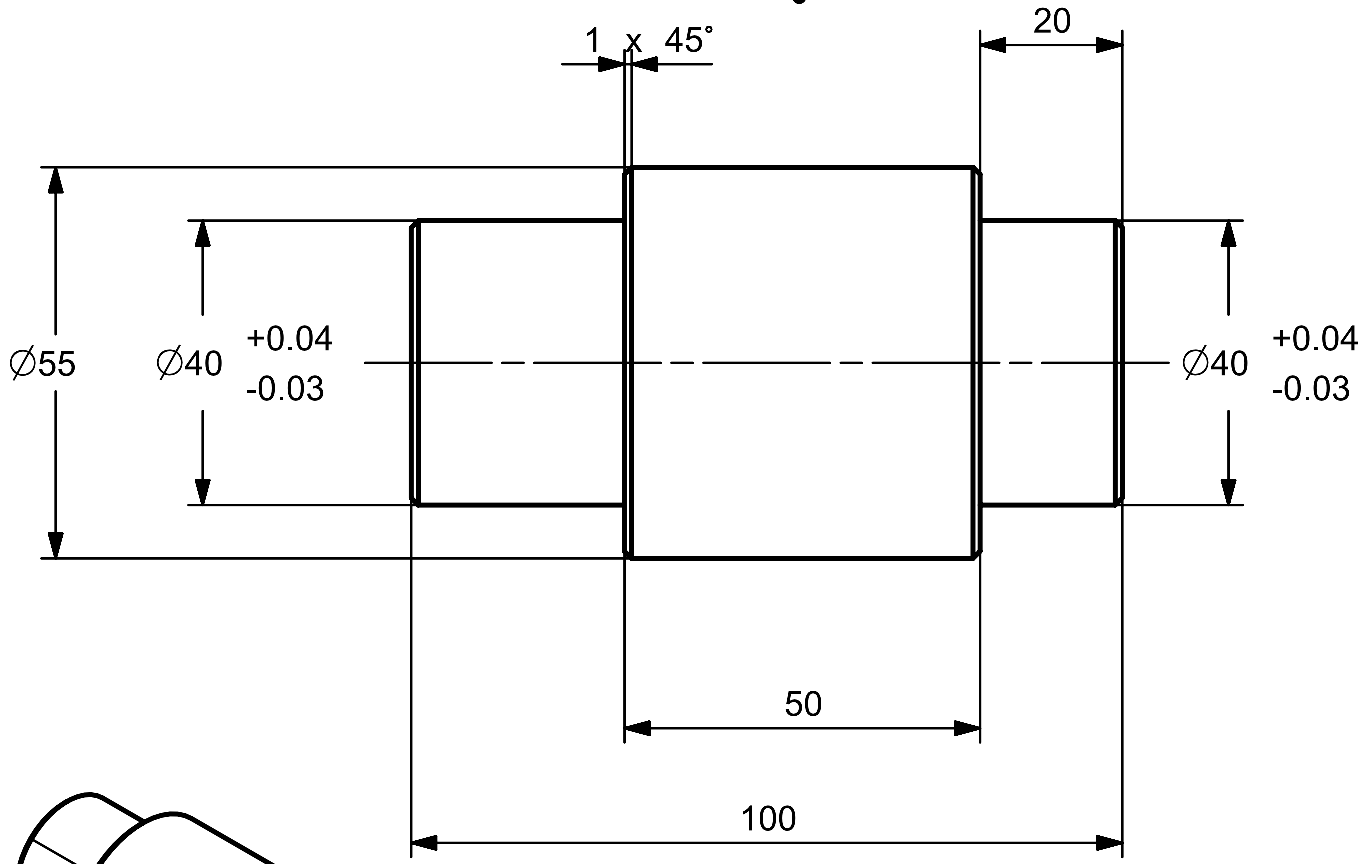
TITLE

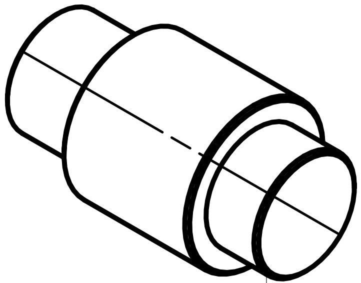
Ring

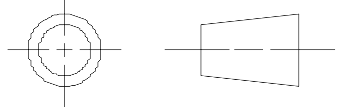
SIZE	DRG NO.	SHEET REV
A4	ring_dwg1	A

SCALE 1:1		SHEET 1 OF 1

A4







ALL DIMENSIONS IN MM

SIEMENS		THIS DRAWING HAS BEEN PRODUCED USING AN EXAMPLE TEMPLATE PROVIDED BY SIEMENS PLM SOFTWARE	
FIRST ISSUED		TITLE shaft	
DRAWN BY	Christopher Luey		
CHECKED BY			
APPROVED BY		SIZE A4	DRG NO. shaft · SHEET REV A
		SCALE 1:1	SHEET 1 OF 1

A4