Chapter 7 Sliding Bearings (Fluid-Film Bearings) and Hydrodynamic Lubrication

7-1 Basic concept of lubrication

In a rolling-element bearing, balls or rollers are utilized to connect and separate parts in different motions, such as a shaft in rotation and a bearing bore (or a bearing seat) of a housing that is stationary, or those under a relative motion like what the thighbone and shinbone in a knee joint experiences. In a sliding bearing, a lubricant, which is usually a fluid, a liquid or a gas, is utilized to achieve the same goals without using rolling elements. Actually, sliding bearings are older than rolling-element bearings in the history of technology developments.

Typical sliding (fluid-film) bearings. Similar to rolling-element bearings, sliding bearings can also be classified into radial (journal) and thrust (slider) bearings. **Figure 7-1** shows the basic mechanisms of a journal bearing and a slider bearing. The former consists of a journal (shaft) and a bearing (sleeve), while the latter is simply composed of two non-parallel surfaces. Please take a look of the direction of part motion, why this and not that?

Figure 7-1. Journal bearing (left, for a radial load) and slider bearing (right, for a thrust load).

Because of the simplicity of their structures, only needing wedged surfaces, to be discussed later, sliding bearings can be designed to satisfy special engineering requirements. **Figure 7-2** shows the powertrain of an internal combustion engine (ICE). Can we identify the sliding bearings used in this structure? They are the main bearings (journal bearings), the connecting-rod bearings (journal bearings), the wrist-pin bearings (also journal bearings), the piston ring/cylinder liner bearings (slider bearings), the piston skirt/cylinder liner bearings (slider bearings), and the valve-train and cam/follower bearings (also slider bearings); so many are there.

Why can't we use rolling bearings at these places? Think about the structural features and the needs for engine operation and maintenance. Also, think about the advantages of sliding bearings.

Types of lubrication and types of sliding bearings. Several terms are often used to describe lubrication: hydrodynamic lubrication, hydrostatic lubrication, and Elasto-hydrodynamic lubrication, etc.

Hydrodynamic (HD) lubrication means that a thin fluid film withstanding a sufficiently high fluid pressure is generated under the relative motion of bearing element surfaces, and this film is there to support the applied load and separate the surfaces under different motions. Bearings working with such a hydrodynamic fluid film are called hydrodynamic bearings, or fluid-film bearings. Most the journal bearings in **Figure 7-2** are hydrodynamic bearings.

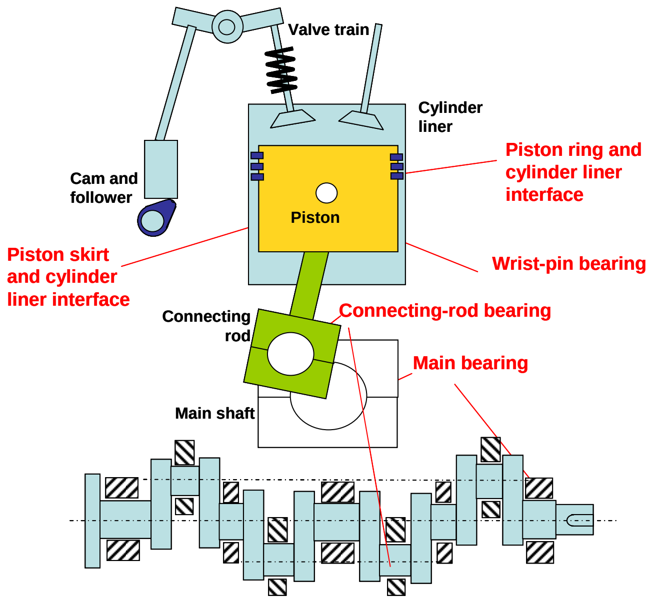

Figure 7-2 Schematic of an internal combustion engine's powertrain; several sliding bearings (journal bearings or slider bearings) are shown. Modified from Fig. 1 in the paper by Goenka et al. [1992].

Hydrostatic (HS) lubrication means that the fluid pressure for load supporting is provided by an external source. Bearings working with such a hydrostatic pressure are hydrostatic bearings. Hydrostatic bearings are usually used to satisfy a high requirement on rotation precision; they are often found in the spindles of high-precision machine tools, such as grinders.

Elasto-hydrodynamic lubrication (EHL) is for the case where elastic deformation contributes to the film thickness. Remember we talked about Martin's experiment in last chapter. EHL is often found in the lubrication of gears, rolling bearings, and cam-follower interfaces where the contacting surfaces are usually non-conformal and the contacts are highly concentrated. The high contact pressures at these interfaces result in significant elastic deformations of the interacting surfaces. Note that the contacts between balls/rollers and the inner race of the outer ring of a rolling element bearing are subjected to EHL, and so are the interfaces between the teeth of an external gear and ring (internal) gear; these contacts are still concentrate contacts although one of the principal curvatures of a race surface, or a ring-gear tooth surface, is concave.

Solid-film lubrication means that either a soft coating of a lubricious solid, such as zinc, graphite or other proper carbon products, silver, etc., or a layer of fine powders, such as MoS_2, graphene, etc., is used as the lubricant. Sliding bearings with a solid lubricant are often used in structures where the fluid lubrication is not possible, such as those working in a high temperature environment, or in vacuum.

Regions of Lubrication. The existence of a thin fluid lubricant layer between two solid surfaces can prevent the direct contact of the surfaces. Theoretically, no failure should occur if this layer could be maintained all the time. There are certain conditions for this film to be built and maintained; however, it is nearly impossible for the lubricant film to cover the entire contacting surfaces all the time except for the cases of hydrostatic bearings. We may see different degrees of lubricant film coverage under different operating conditions, which classifies lubrication into three regimes.

German Professor Richard Stribeck is credited for conducting the first systematic experiments that revealed a clear view of the characteristic s of journal-bearing coefficient of friction (COF) versus speed [Stribeck 1902]. In recognition of his contribution, this type of curves is universally referred to as “the Stribeck curve.” COF is defined as friction/normal force.

Note that the original work by Richard Stribeck was for journal bearings, but now this concept has been extended to the lubrication of other interfaces as well. **Figure 7-3** presents a schematic of such a curve as an overall view of friction variation in the entire range of lubrication as a function of a bearing parameter, called the Hersey number, $\eta N/P$ (where η is the dynamic viscosity of the lubricant under investigation, $\boldsymbol{N}$ the rotational speed, in revolutions per second, and P the bearing load in terms of the average pressure). Three lubrication regimes can be clearly observed from the Stribeck curve: namely, the full-film, the mixed, and the boundary lubrication regimes.

The full-film lubrication regime is characterized as the state where the fluid film completely separates the surfaces in contact and relative motion. The load is fully supported by the lubricant. The lubrication may appear in three forms: hydrostatic lubrication when an external static pressure is applied, hydrodynamic lubrication in which the relative motion generates the hydrodynamic lift force, and elastohydrodynamic lubrication where surface elastic deformations contribute to the lubrication film thickness.

On the other hand, if a fluid film cannot separate the surfaces in a relative motion, significant contact of surfaces should occur. Because engineering surfaces are not ideally smooth, they are rough and have asperities (peaks of surface topographic profiles), surface solid-solid contact occurs by pressing asperities. Severe surface asperity contact in the presence of a lubricant indicates the status of boundary lubrication. In this regime, the applied load is mainly supported by contacting asperities although valleys of asperities still retain some lubricant. The boundary lubrication regime is undesirable due to the nature of strong rubbing with high friction and a high energy loss. At the same time, heat generated by rubbing may quickly break off some asperities. Adhesive and abrasive wear can happen, and surface seizure may follow if adhesion accumulates. However, at certain conditions and using proper lubricant additives, a boundary layer can be formed on the surfaces through absorption of additive molecules and/or chemical reactions between lubricant additives and the interacting surfaces, which functions like a buffer layer to prevent the direct contact of bearing-shaft materials. Friction modifiers, anti-wear additives, and extreme-pressure additives are examples of the lubricant additives, and wear of this boundary layer is repaired by replenished additives through lubricant circulation.

Mixed lubrication is the intermediate regime between the boundary and hydrodynamic lubrication regimes. The film thickness in this regime may be of the same order of magnitude as surface roughness. Important features of mixed lubrication are 1) occurrence of a certain degree of solid-to-solid asperity contacts, 2) interrupted lubricant flows by contacting asperities, and 3) joined load supporting by the fluid film and contacting asperities. Asperities may experience contact plastic deformation and wear. The asperity variation due to deformation and wear may in turn lead to the change in the lubricant film thickness. In mixed lubrication, overloading can easily cause excessive wear and failure.

The most desirable lubrication regime for the working state of a journal bearing and other types of elements is the full-film lubrication. However, many components or devices operate in multiple regimes from hydrodynamic to boundary lubrication. Engine bearings run through all three lubrication regimes due to starts and stops. Another example is the wet clutch used in automotive transmission device. More information about Stribeck curves can be found in [Jang and Khonsari, 1999, Wang et al., 2006, and Khonsari and Booser, 2010].

Lubricant developments are challenged by new materials and new engineering systems. Electrical vehicles, for example, may use a lubricant as coolant as well; therefore, a lubricant has to be multifunctional. Certain continuously variable transmissions (CVTs) are traction drives and use friction as the driving force. Therefore, the lubricant there should produce enough friction (traction). Each engineering system has its only characteristics, and there are many lubricants developed to meet the needs of each.

Load, speed, and lubricant, and material properties affect the lubrication conditions. Machine element geometry also plays an important role in lubrication. For example, journal bearings under the same average pressure but having different bearing structures may experience different surface deformation, which in turn affects the lubricant film thickness and friction. In some cases, the difference can be large enough to cause the shift of their lubrication regime [Wang et al., 2006].

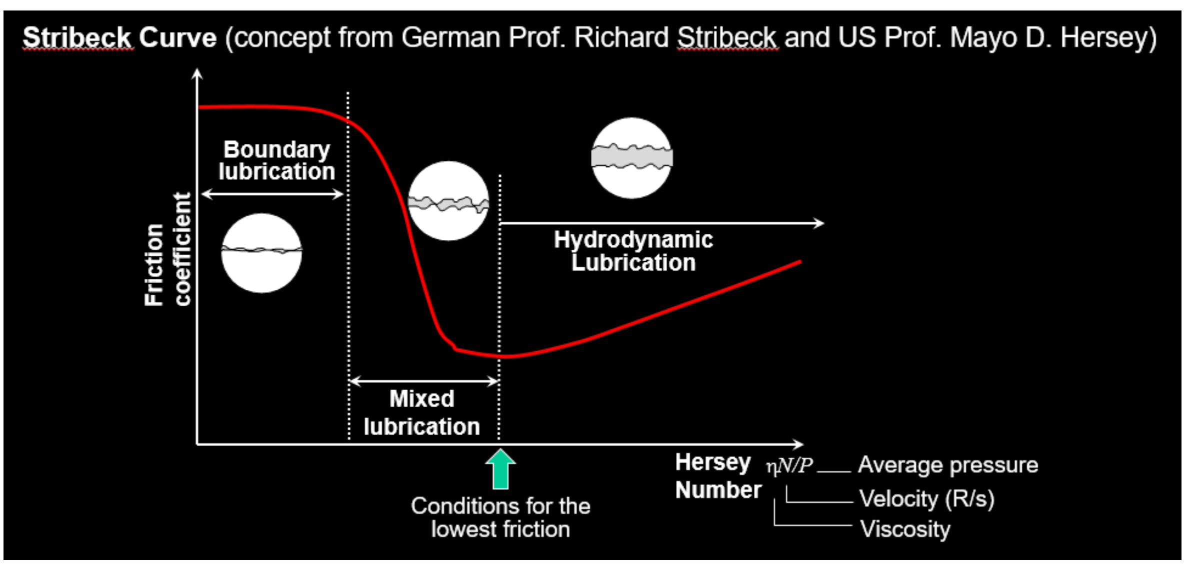

Figure 7-3 Regimes of lubrication in terms of the Stribeck curve, schematic. Note that the shape of this curve may be different for different cases,

The structural effect on friction further manifests the dependence of friction on film thickness and its distribution at the interface between a bearing and a journal. For a journal bearing under heavy loading, if any of the factors that influence the film thickness (such as asperity contact, heating, and structural deformation or distortion) is more significant than others, friction may notably become a function of that factor.

The study of this chapter is focused on full-film lubrication. More on lubrication science and methods are in the book by Wang and Zhu [2019].

7-2 Fundamentals of hydrodynamic lubrication, the Reynolds equation

Three conditions have to be satisfied in order to have hydrodynamic pressure generation: 1) there must be a convergent wedge, 2) the lubricant must have a certain viscosity (i. e., cannot be vacuum; note that air has viscosity!), and 3) the two surfaces must be in a relative motion. We see examples of hydrodynamic lubrication everyday. When we drive on an icy road, we try not to brake suddenly because we do not want the car to slip or spin on the road. When we are on a small boat, we tend to sit in the back portion of the boat (**Figure 7-4**) because in this way the boat can go faster. Yes, but why?

Why should these three conditions be satisfied for a hydrodynamic lubrication? And how?

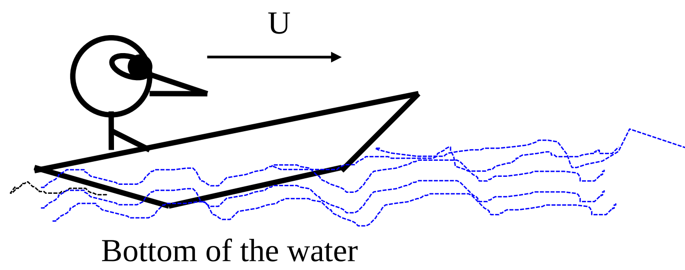

Figure 7-4 A macroscopic "fluid-film" lubrication problem.

The Reynolds equation can help understand all these questions. It is the momentum equation that we will use to develop the basic relationship between hydrodynamic terms, which are the film thickness and the hydrodynamic pressure. The Reynolds equation describes the characteristics of the fluid flow through a thin clearance bounded by two solid surfaces. In the derivation of the Reynolds equation, we assume that

- The fluid is Newtonian,
- Flows are laminar
- There is no boundary slip,
- The body force is very small, ignored
- The fluid film (lubricant film) is very thin, and the derivatives with respect to this direction is more important,
- The curvature effect is very small, ignored
- The hydrodynamic pressure is constant across the fluid film, and
- Bounding solids are rigid and ideally smooth.

Figure 7-5 shows a typical laminar flow situation. The viscosity of a Newtonian fluid obeys Newton's law of viscosity. Here, the dynamic viscosity (or viscosity for short), η, is a measurement of the internal frictional resistance of a fluid and relates fluid share rate to share stress in Equation (7-1). The unit of dynamic viscosity is Pa.S. Kinematic viscosity is defined as $v = \eta/\rho$, which is also used in engineering, and its unit is Stoke (St, or m^2/s), or more often, centistokes, or cSt. We will mainly use dynamic viscosity η in this chapter.

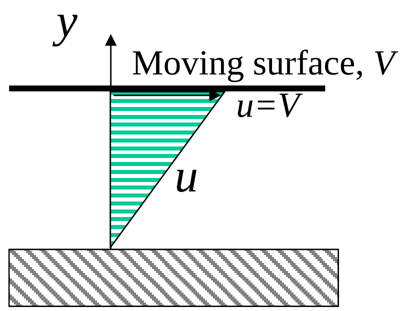

Figure 7-5 Laminar flow in a confined channel.

$$\tau = \eta \frac{\partial u}{\partial y} \qquad (7\text{-}1)$$

Figure 7-6 shows a case of lubrication, where the top surface is stationary but the bottom surface is moving at velocity *V*. A flow element is shown in in this figure, and forces on the element are defined. Note that the width of the element is dz.

The total force balance in *x* is

$$\sum F = -(p + \frac{dp}{dx}dx)dydz + pdydz - (\tau + \frac{d\tau}{dy}dy)dxdz + \tau dxdz = 0$$

which is actually

$$\frac{dp}{dx} = -\frac{d\tau}{dy} \qquad \text{or,} \qquad \frac{dp}{dx} = \frac{d(-\eta \frac{du}{dy})}{dy} = \eta \frac{d^2u}{dy^2}$$

The negative sign of the viscosity term is due to the reduction in *u* with the increase in *y*.

h:	Film thickness
η:	Viscosity $\tau = \eta \frac{du}{dy}$
ρ:	Density
p:	Fluid-film pressure
V:	relative sliding speed
x, y	Coordinates

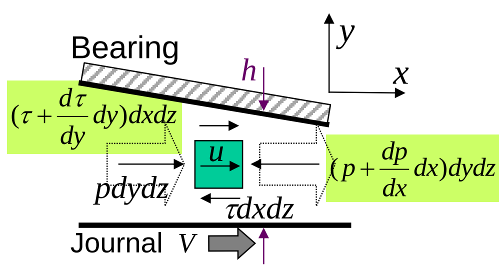

Figure 7-6 Flow and stress definitions.

Integration over *y* yields, $u = \frac{1}{2\eta}\frac{dp}{dx}y^2 + C_1 y + C_2$

We can use the following boundary conditions to solve the constants.

$y = 0 \qquad u = V$
$y = \text{h} \qquad u = 0$

Solve for C_2 first. We now have:

$C_1 = -\frac{h}{2\eta}\frac{dp}{dx} - \frac{V}{h} \qquad C_2 = V$

Then the flow velocity becomes

$$u = \frac{1}{2\eta}\frac{dp}{dx}(y^2 - hy) + \frac{V}{h}(h - y) \tag{7-2}$$

The flow rate is the integration of velocity along y $\quad Q = \int_0^h u dy = -\frac{h^3}{12\eta}\frac{dp}{dx} + \frac{V}{2}h$

The flow conservation means $\frac{dQ}{dx} = -\frac{d}{dx}(\frac{h^3}{12\eta}\frac{dp}{dx}) + \frac{V}{2}\frac{dh}{dx} = 0$

This leads to the following steady-state Reynolds equation for1-D lubrication, with which the bearing is assumed infinitely long in the z direction.

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right) = 6V\frac{\partial(\rho h)}{\partial x} \tag{7-3}$$

The left-hand side of Equation (7-3) is the pressure-driven term, while the right-hand side of the equation is the shear-flow term. We can extend this equation to a two-dimensional one (Equation (7-4)) by adding another pressure driven term. We do not have to add another shear-flow term because there is no relative motion in the z direction.

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial z}\right) = 6V\frac{\partial(\rho h)}{\partial x} \tag{7-4}$$

Now we can easily define the viscous shearing terms, or the frictional shear stresses

$$\tau_{xy} = \eta\frac{\partial u}{\partial y} = \frac{1}{2}\frac{\partial p}{\partial x}(2y - h) - \frac{\eta V}{h} \tag{7-5}$$

$$\tau_{zy} = \eta\frac{\partial w}{\partial y} = \frac{1}{2}\frac{\partial p}{\partial z}(2y - h) \tag{7-6}$$

At surfaces, $y = 0$ and $y = h$

$$\tau_{xy} = \mp\frac{h}{2}\frac{\partial p}{\partial x} - \frac{\eta V}{h} \qquad \text{+ sign is for } y = h \tag{7-7}$$

$$\tau_{zy} = \mp \frac{h}{2}\frac{\partial p}{\partial y} \tag{7-8}$$

Then the total friction is $F_f = \pm \iint_{x,z} (\frac{h}{2}\frac{\partial p}{\partial x} - \frac{\eta V}{h}) dxdz$ (7-9)

Equation (7-3) or (7-4) reveals the basic conditions for hydrodynamic lubrication mentioned at the beginning of this section by ensuring the right-hand side is non-zero:

Geometry:	A convergent wedge is needed. Think why this has to be "convergent."
Lubricant:	The lubricant must have a certain viscosity, η
Motion:	There must be a relative motion, *V*, between the surfaces.

7-3 Hydrodynamic bearings

Journal bearings and film thickness. A journal bearing is simple in structure, having a bushing (or a sleeve, the bearing) and a segment of a shaft (the journal) as mentioned before, with a properly controlled clearance between them. This clearance is the key, as shown in the previous section, it allows a convergent wedge to fill oil, water, or air, or gas, and to form a pressurized thin film for load support and surface separation. **Figure 7-7** shows the simplest case of a long journal bearing formed by a hollow cylinder (sleeve) and a shaft. Note that the journal center is offset due to the nature of hydrodynamic pressure generation that is made possible by the convergent wedge at one side of the offset. This misalignment is intrinsic, and nothing can be done if a plain shaft and plain bearing are used. Equation (7-3) is solved for this bearing and the results are shown in **Figure 7-7** for the pressure distribution along the circumferential direction (middle) in the bearing cross section "cut" at the center of its length and that in the length direction (right) viewed in a length cross section. In set of solutions, we ignore the side leakage because we consider the bearing is infinitely long. We will release this condition later. Note that the clearance shown here is significantly enlarged for clarity. In fact, the journal and the bearing have the same nominal diameter, and the small clearance is guaranteed by the tolerances of the journal outer diameter (OD) and the bearing inner radius (ID).

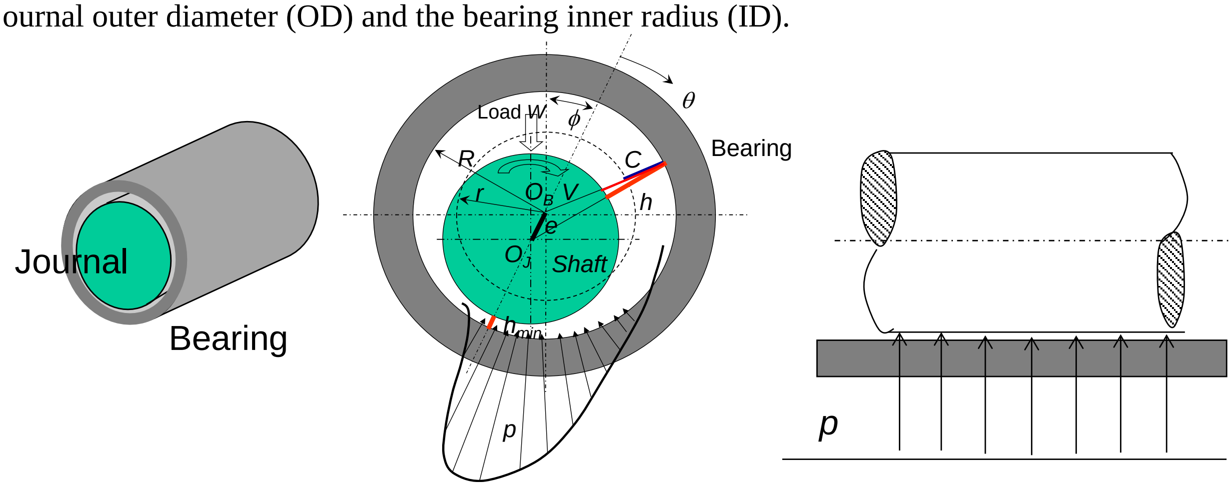

Figure 7-7 Solution to Equation (7-3), long journal bearing (left) formed by a hollow cylinder (sleeve) and a shaft, and a typical pressure distribution viewed in the central cross section (middle) and a length cross section.

Recall the three regimes of lubrication we learned at the beginning of this chapter, which are the full film, mixed (partial film), and boundary lubrication regimes. Journal bearings are expected to work in the full-film lubrication regime; however, they may experience all of these regimes due to starts and stops. Therefore, our design should be based on the hydrodynamic lubrication theory to ensure that the bearing's steady-state operation is in the full-film regime. We should also have measures for the bearings to safely pass the boundary and mixed lubrication regimes during their operation transitions, such as starts and stops.

The thinnest gap is called the minimum film thickness. The geometry in **Figure 7-7** shows two wedges, where the convergent wedge is seen along the θ direction before the location of the minimum film thickness, which is followed by the divergent wedge. This figure also defines the bearing geometry and terminologies to be used. The offset of the journal and bearing centers is measured by eccentricity *e*.

More terminologies are defined as follows. The calculation of the film thickness as a function of location is expressed in Equation (7-10) and illustrated in **Figure 7-8**.

Nominal diameter: D (R)
Shaft diameter: d (r)
Bearing width: L
Radial clearance: $C = (D-d)/2$

Minimum film thickness; h_{min}

Eccentricity (max. shaft disp.): $e = C - h_{min}$
Eccentricity ratio: $\varepsilon = e/C = (C - h_{min})/C$

Load: W, the normal load
Load angle, Φ, (the position angle, the attitude angle)
Average pressure: $P = W/(LD)$

Sommerfeld number: $S = \frac{\eta N}{P}(\frac{R}{C})^2$
$R = D/2$, N: rev/s

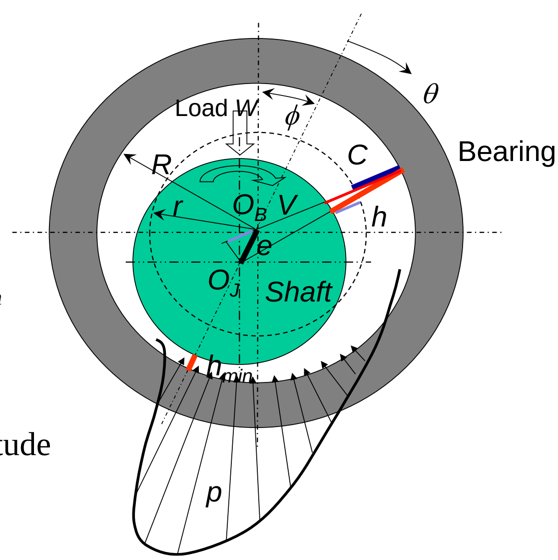

Figure 7-8. Journal bearing geometry

An important geometry item is the film thickness, which is schematically shown in the figure above and expressed as follow. It seems to be more precise to defined *h* from the journal center, but more convenient to express *h* from the bearing center.

$$h \approx C + e\cos\theta = C(1 + \varepsilon\cos\theta) \qquad (7\text{-}10)$$

Infinitely long bearings, Sommerfeld number *S*. It is difficult to solve the two dimensional Reynolds analytically. However, based on the assumption that there is no side leakage, or the bearing is infinitely long (actually for *L/D* >> 1), a closed-form solution was derived by Sommerfeld from the one-dimensional lubrication equation, Equation (7-3), simplified below for an incompressible and isoviscous fluid:

$$\frac{\partial}{\partial x}\left(\frac{h^3 \partial p}{\partial x}\right) = 6V\eta \frac{\partial h}{\partial x} \tag{7-11}$$

Define $x = R\theta$. By applying the periodic boundary condition, $p(0) = p(2\pi)$, and the Sommerfeld transform ($1+\varepsilon\cos\theta = \dfrac{1-\varepsilon^2}{1-\varepsilon\cos\gamma}$, where γ is the Sommerfeld variable), Equation (7-11) yields the solution for p as a function of ε and θ, which is shown in Equation (7-12), and a load capacity expression, which is given in Equation (7-13), in terms of Sommerfeld number S. The equation for load capacity is from the integration of pressure along the load direction over the entire region, 2π.

$$p = \frac{6\eta V R\varepsilon}{C^2}\frac{(2+\varepsilon\cos\theta\ )\sin\theta}{(2+\varepsilon^2)(1+\varepsilon\cos\theta\ )^2} \tag{7-12}$$

$$S = \frac{\eta N}{P}\left(\frac{R}{C}\right)^2 = \frac{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}{12\pi^2\varepsilon} \tag{7-13}$$

The Sommerfeld number, shown in **Figure 7-10**, is thus defined as an important non-dimensional parameter for bearing design.

$$S = \frac{\eta N}{P}\left(\frac{R}{C}\right)^2 \tag{7-14}$$

Look carefully at Equation (7-12). It is symmetric about π with a huge negative pressure side. This negative pressure is not real. Here, the Sommerfeld number is obtained by integrating the pressure over 2π, including the negative pressure region. Equation (7013) is not a realistic load solution; however, the Sommerfeld number is a useful non-dimensional parameter for journal bearing analyses.

Short bearing solution (Ocvirk's Solution). On the other hand, bearings may be narrow, and we call them short bearings. An assumption can be made that the side flows in the width directions are the major flows for such short (narrow) bearings. We now can so simplify the Reynolds equation that the pressure-driven flow in the circumferential direction disappears (less important as compared to the side flows out of the two edges), as shown in Equation (7-15). The pressure is now a function of θ and y. Again, $x = R\theta$.

$$\frac{\partial}{\partial z}\left(\frac{h^3 \partial p}{\partial z}\right) = 6V\eta \frac{\partial h}{\partial x} \tag{7-15}$$

The boundary conditions are that the pressures at the bearing edges are the atmospheric pressure, or simply zero (the relative pressure is considered), and that $p(0) = p(2\pi)$. The latter is the periodic pressure boundary condition in the circumference direction used in the previous section for long bearings. The pressure and load solutions are in the following two equations.

$$p = \frac{3\eta V}{RC^2}\left(\frac{L^2}{4} - z^2\right)\frac{\varepsilon \sin\theta}{(1+\varepsilon\cos\theta)^3} \tag{7-16}$$

This pressure is also symmetric about π with a negative pressure region. Integrating the pressure over 0-π, neglecting the negative pressure, result in

$$S\left(\frac{L}{D}\right)^2 = \frac{(1-\varepsilon^2)^2}{\pi\varepsilon\left[\pi^2(1-\varepsilon^2)+16\varepsilon^2\right]^{1/2}} \tag{7-17}$$

Example 7-1

A journal bearing has the following structural and operating parameters:

Radius:	R = 10 mm	Radial clearance:	C = 0.009 mm
Width:	L = 5 mm	Speed:	N = 20Rev/s
Viscosity:	η = 0.01 Pa.S		

Calculate the Sommerfeld number, S, and the corresponding loads using the long and short bearing solutions at ε = 0.6, compare the results with S = 1.07, which is from the accurate numerical solution to Equation (7-4), and draw a short conclusion.

Solution.

This is a bearing with a finite length, L/D = 5/20 =0.25, not too far away from being a short bearing. Let's try to use Equations (7-13) and (7-17) and see which comes closer to the numerical result.

Load: $W = P(LD) = \frac{\eta N}{S}\left(\frac{R}{C}\right)^2 (LD)$ = 24.7/S

Equation 7-13, the infinitely long bearing solution, yields

$$S = \frac{\eta N}{P}\left(\frac{R}{C}\right)^2 = \frac{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}{12\pi^2\varepsilon} = 0.02656 \quad \text{then, } W = 930 \text{ N}$$

Equation 7-17, the short-bearing solution, yields

$$S\left(\frac{L}{D}\right)^2 = \frac{(1-\varepsilon^2)^2}{\pi\varepsilon\left[\pi^2(1-\varepsilon^2)+16\varepsilon^2\right]^{1/2}} = 0.0625 \quad \text{and} \quad S = 1 \quad \text{then, } W = 24.7\text{N}$$

The accurate load calculated by the numerical calculation is W = 23.1 N. We can conclude that for bearings with L/D = 1/4 or smaller, the short-bearing solution is a wise choice.

Example 7-2

Repeat the same problem in **Example 7-1** for a different L/D ratio. A journal bearing has the following structural and operating parameters:

Radius:	R = 10 mm	Radial clearance:	C = 0.009 mm
Width:	L = 20 mm	Speed:	N = 20Rev/s
Viscosity:	η = 0.01 Pa.S		

Calculate the Sommerfeld number, S, and the corresponding loads, with Equations (7-13) and (7-17) at ε = 0.6, compare the solutions with the numerical result of Sommerfeld number of S = 0.12, which is the accurate solution to Equation (7-4),

Solution

This is a bearing with a finite length, L/D = 20/20 = 1, far from being a short bearing, and it is not an infinitely long bearing either. Let's try Equations (7-13) and (7-17) again.

Load: $W = P(LD) = \dfrac{\eta N}{S}\left(\dfrac{R}{C}\right)^2 (LD) = 98.8/\mathrm{S}$

Equation (7-13), the infinitely long bearing solution, yields

$$S = \frac{\eta N}{P}\left(\frac{R}{C}\right)^2 = \frac{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}{12\pi^2\varepsilon} = 0.02656 \text{ Then, } W = 3719 \text{ N}$$

Equation (7-17), the short-bearing solution, yields

$$S\left(\frac{L}{D}\right)^2 = \frac{(1-\varepsilon^2)^2}{\pi\varepsilon\left[\pi^2(1-\varepsilon^2)+16\varepsilon^2\right]^{1/2}} = 0.0625 \quad S = 0.0625 \quad \text{then, } W = 1580\text{N}$$

The real load by the numerical calculation is W = 823 N.

WE can further conclude:
(a) For L/D smaller than 1/4, the short bearing solution is applicable.
(b) For bearings with L/D about or larger than 1, none of the simple solutions may be used. However, the short-bearing solution is closer to the real solution.
(b) Both of Equations (7-13) and (7-17) over estimate the load, but the former over estimates more.

Numerical solutions. Reynolds equation (7-4) works for realistic bearings, its solution relies on the use of numerical methods. Solving this second-order partial differential equation requires four boundary conditions. Some negative pressure may occur, but the magnitude cannot be as high as what is seen in the simple solutions (long or short bearing solutions). Film rupture conditions should be properly defined. We may assume a smooth pressure decay at the film rapture location, θ_0, namely, $\bar{p}(\theta_0)=0$ and $\frac{\partial \bar{p}(\theta_0)}{\partial \bar{x}}=0$. These conditions for film rupture are referred to as the Reynolds boundary conditions. For $\theta > \theta_0$, the pressure should be largely zero, i.e. $p(\theta > \theta_0)=0$. **Figure 7-9** shows the pressure-distribution results from the numerical solutions for a group of bearings of the same diameter but with different widths running at the same speed and lubricated with the same oil, which shows

- Along the circumferential direction, the pressure reaches maximum in the convergent side before the minimum film thickness; it reaches zero in the divergent side at several degrees passing the location of the minimum film thickness;
- The pressure distribution along the axial (width) direction is parabolic for bearings with a finite length;
- The wider the bearing width is, the higher the pressure peak.
- If there were no side leakage (ideally infinitely long), the pressure could be constant along the width direction.

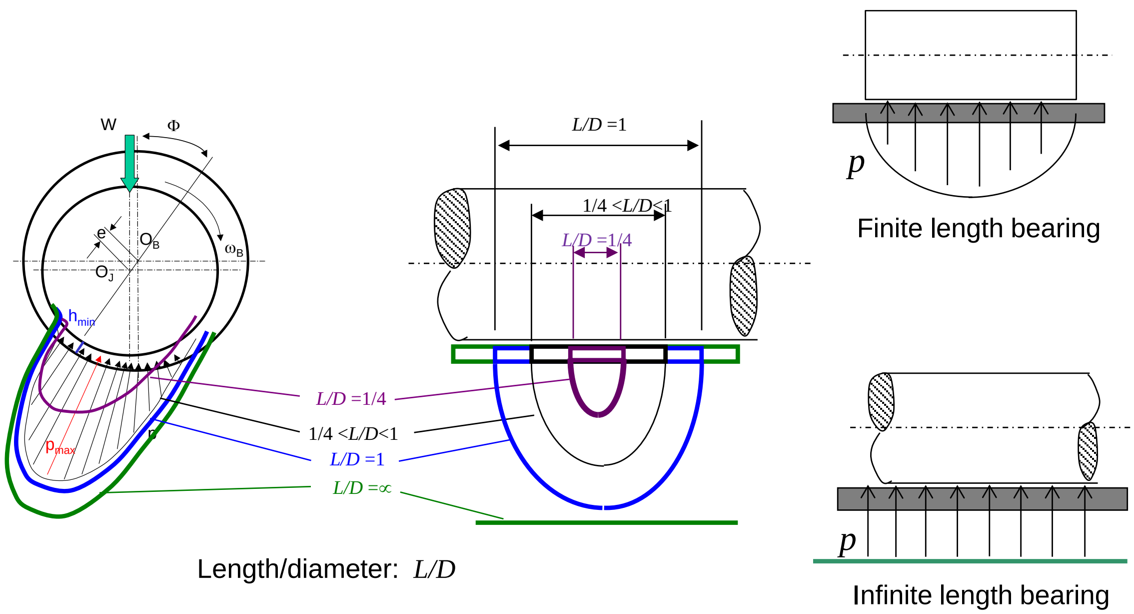

Figure 7-9 Pressure distributions influenced by bearing width. Left: viewed at the central cross section, middle: viewed at a length direction cross section, right: two special cases.

7-4 Slider bearings

Can a pressure be built between the shaft end surface and its mating plate shown below in **Figure 7-10** if the shaft turns and a lubricant is provided?

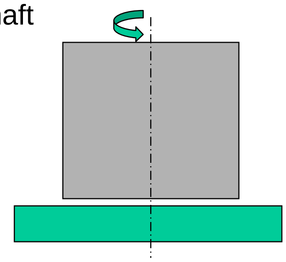

Figure 7-10 Shaft end surface and a plate.

Recall the conditions for generating hydrodynamic pressure. We need to create convergent wedges. Consider the structure below, **Figure 7-11**, where several sloped pads are made for one of the surfaces. When the shaft rotates, its end face drags the fluid into each of the convergent wedges, thus producing hydrodynamic pressures that are capable of supporting a load along the shaft direction. The pads can be made to tilt, and thus to create the tilting-pad thrust bearings.

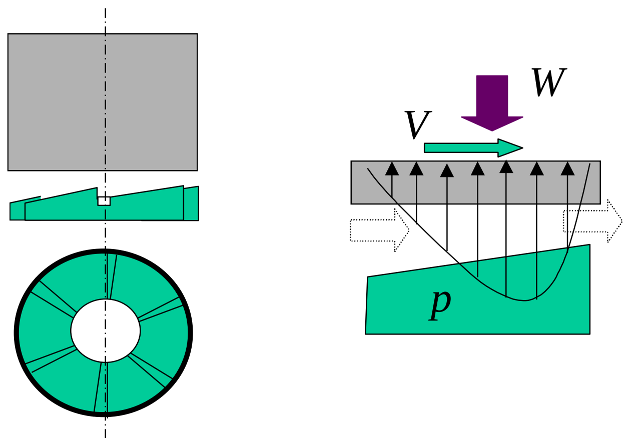

Figure 7-11 Slider bearing. Left: pad configuration, right: performance of each pad.

7-5 Hydrostatic bearings

However, hydrodynamic bearings have some problems. The speed of the relative motion should be sufficiently high, and start-stop processes may involve higher friction. Most importantly, the shaft runs eccentrically, resulting in a low rotation accuracy (the shaft and the journal are not concentric) and low stiffness (the eccentricity changes with speed). These problems are intrinsic for hydrodynamic journal bearings.

Hydrostatic bearings are designed to overcome some of these problems. An externally supplied pressure is used to make the bearing performance independent of speed. No start-stop friction problems exist anymore because the shaft is always suspended. Thus, the shaft and the journal can run concentrically, which means a high rotation accuracy and high stiffness.

Hydrostatic slider bearings. **Figure 7-12** shows a hydrostatic slider bearing consisting of a runner and a pad with a bearing recess. A working fluid is provided through a restrictor, which maintains the pressure in the recess at a certain level. Because the fluid leaks through the runner edge, a constant lubricant supply should be maintained in bearing operation. The pressure distribution looks like a trapezoidal shape; it drops to the environmental pressure at the runner edge.

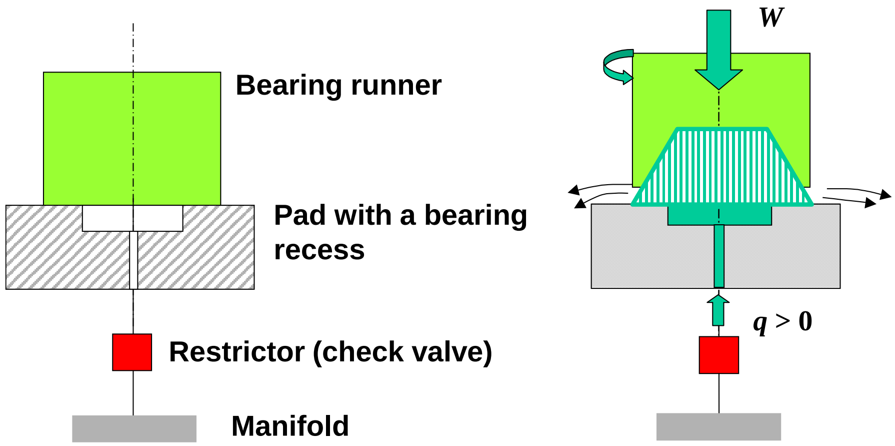

Figure 7-12 Hydrostatic slider bearing, structure (left) and operation (right).

Hydrostatic journal bearings. The structure of a hydrostatic journal bearing, **Figure 7-13**, is similar to that of a hydrostatic thrust bearing. Several recesses should be made in the bearing. The operation of this bearing is exactly the same as that of the thrust bearing. The lubrication should be completely hydrostatic if p_r is sufficiently high. Otherwise, both hydrostatic and hydrodynamic pressures co-exist if p_r is not high enough to support the entire applied load, and the bearing becomes a hybrid bearing, which is also widely used in machine-tool building. The rotation of the shaft may help produce a certain amount of hydrodynamic pressure as a result of the bearing-shaft offset if there is shortage of the hydrostatic pressure.

Keep in mind that hydrostatic lubrication involves an expensive lubricant supply system and consistent lubricant running.

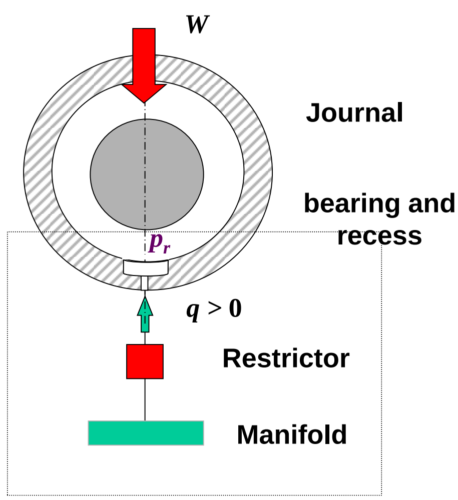

Figure 7-13 Hydrostatic journal bearing.

7-6 Bearing materials

Typically, a journal-bearing lubrication interface involves a long path of fluid flow, a long path of debris transport, and a large area of surface interaction, if any; and solid surface contact occurs at starts and stops. The following basic requirements should be considered for bearing material selection.

- Sufficient strength
- Strong heat conductiveity
- No adhesion
- Compliant to deformations
- Good embedability
- High fatigue resistant
- High corrosion resistant
- Low cost

The following materials have been widely used as journal bearing materials. Unfortunately, lead-based materials are no longer recommended to use due to the health consideration.

- Tin-based babbitt
- Bronze (solid)
- Aluminum alloy (solid)
- Silver overlay
- Cast iron
- Carbon-based materials
- Polymers and composites

Journal bearings mentioned in this chapter are only one group of examples of lubrications and lubrication technologies. Lubrication is one of the basic corner stones of modern industry, and an everlasting area of technology development as long as wheels turn. New materials, new lubricants, and new methods are making our industries and industrial products more capable and reliable in doing many things that were impossible in the past.

7-7 Bearing comparison

Now we are in a stronger position to do bearing comparison because we know all important types of bearings and understand the essential science behind.

Table 7-1 Bearing comparison

Performance	Rolling bearings	Sliding bearings		
		Bearings with mixed-lubrication	Bearings with full-film lubrication	Hydrostatic bearings
Load (W)-speed (N)	N/A	W$\Downarrow$ as N$\Uparrow$	W$\Uparrow$ as N $\Uparrow$	N/A

relation				
Structural flexibility	Not flexible	Very flexible	Very flexible	Can be flexible
Shock buffering ability	low	low	high	high
High-speed operation	fair	poor	good	good
Starting resistance	low	high	May be high	low
Power loss	may be low	high	low	may be low
Life	fair	low	can by long	can be very long
Noise	high	low	can be very low	can be very low
Stiffness	high	fair	fair	can be very high
Rotation accuracy	high	low	can be high	very high
Radial dimension	large	small	small	small
Axial dimension	(0.2-0.5)d	(0.5-4)d	(0.5-0.4)d	fair
Lubricant	oil or grease	oil, grease, solid	oil, gas	oil, gas
Lubricant consumption	very low	low	high	very high
Maintenance	easy	change sleeves, shaft re-polishing	change sleeves, shaft re-polishing	change sleeves, shaft re-polishing
Cost	fair	may be low	high	may be very high

Chapter summary

This chapter introduces the fundamental theory of hydrodynamic lubrication and simplified solutions of the Reynolds equation. The concept and geometry of journal bearings and thrust bearings are explained. The effect of journal bearing eccentricity and width/diameter ratio on bearing load capacity are studied. Hydrostatic bearings are briefly mentioned. Material selection is also introduced, and comparison with rolling element bearings is further explored.

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Media: Several U-Tube movies for the concept of lubrication, lubrication regimes, and film thickness are on line. 315 students should watch the short U-Tube movies before the class.