315 THEORY OF MACHINES – DESIGN OF ELEMENTS
Fall, 2023
HW No. 4
Assigned: 10/12 Due: one week, 10/19, On-line, pdf in **one single file**.

Total 70, 10 points each

1. This is from a class example: A helical gear has a normal pressure angle of 20^0, a helical angle of 12^0, and a normal diametral pitch of 6 teeth/in and has 18 teeth. Find: (1) transverse diametral pitch, (2) the pitch circle diameter, (3) the transverse, normal, and axial pitches, (4) the transverse pressure angle, (5) outside circle diameter, (6) radius of the equivalent pitch circle (Reading), and (7) contact ratio if the mating gear has 41 teeth, and the face width is $12/P_n$.

$\phi = 20^\circ$; $\psi = 12^\circ$; $P_n = 6$ TEETH/IN ; $N = 18$ TEETH

(1) $P_t = P_n \cos\psi = \boxed{5.869 \text{ IN}}$ [TRANSVERSE DIAMETRAL PITCH]

(2) $d = N P_t^{-1} = \boxed{3.067 \text{ IN}}$

(3) $p_t = \pi / P_t = \boxed{0.535 \text{ IN}}$

$p_n = p_t \cos\psi = \boxed{0.524 \text{ IN}}$

$p_x = p_t \tan\psi = \boxed{2.518 \text{ IN}}$

(4) $\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \boxed{20.41^\circ}$

(5) $d_{ot} = d_t + 2a = 3 + 2/P_n = \boxed{3.400 \text{ IN}}$

(6) $r_e = r/\cos^2\psi = d_t/2\cos^2\psi = \boxed{1.603 \text{ IN}}$

(7) $N_1 = 18;\ N_2 = 41$

$P_n = 6 \cdot IN^{-1}$

$P_t = 5.869 \cdot IN^{-1}$

$p_t = 0.535\ IN$

$p_n = 0.524\ IN$

$b_w = 2\ IN$

$\phi_t = 20.41^\circ$

$$d_{t1} = \frac{N_1}{P_t}$$

$$d_{t2} = \frac{N_2}{P_t}$$

$$d_{bt1} = d_{t1} \cos\phi_t$$

$$d_{bt2} = d_{t2} \cos\phi_t$$

$$m_1 = \frac{d_{t1}}{N_1}$$

$$m_2 = \frac{d_{t2}}{N_2}$$

$$d_{ot1} = d_{t1} + 2m_1$$

$$d_{ot2} = d_{t2} + 2m_2$$

$$C_{dt} = (d_{t1} + d_{t2})/2$$

```
n1 = 18;
n2 = 41;
Pn = Quantity[6, "Inches^-1"];
Pt = Pn * Cos[Quantity[12, "Degrees"]];
pt = Quantity[0.535, "Inches"];
pn = Quantity[0.524, "Inches"];
bw = Quantity[2, "Inches"];
phit = Quantity[20.41, "Degrees"];
dt1 = n1 / Pt;
dt2 = n2 / Pt;
dbt1 = dt1 * Cos[phit];
dbt2 = dt2 * Cos[phit];
m1 = dt1 / n1;
m2 = dt2 / n2;
dot1 = dt1 + 2 * m1;
dot2 = dt2 + 2 * m2;
cdt = (dt1 + dt2) / 2;
crt =
 ((1) / (pt * Cos[phit])) *
   (Sqrt[(dot1 / 2) ^ 2 - (dbt1 / 2) ^ 2] + Sqrt[(dot2 / 2) ^ 2 - (dbt2 / 2) ^ 2]) -
  (cdt * Tan[phit] / pt)
crh = crt + bw * Sin[Quantity[12, "Degrees"]] / pn
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= 1.60834

= 2.4019
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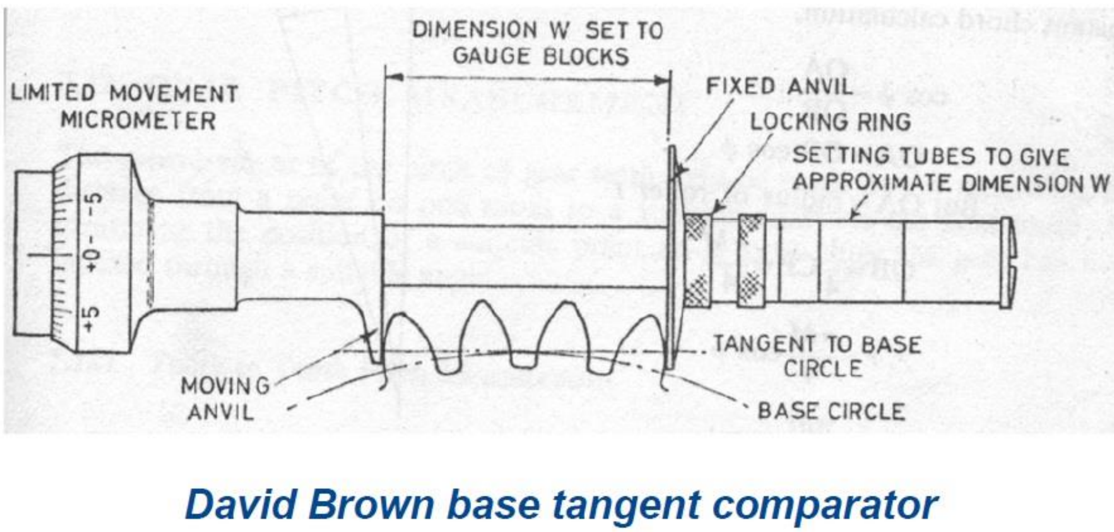
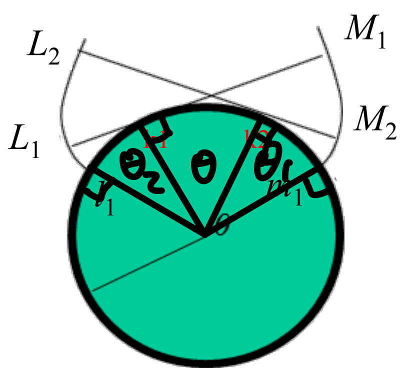
$$\boxed{\begin{array}{l} C_{rt} = 1.608 \\ C_{rh} = 2.37 \end{array}}$$

2. Tooth dimension accuracy can be measured with the David Brown tangent comparator, shown in (a), which is a special caliper measuring the length of common normals, simply illustrated in (b). Prove, the lengths of the common normals between any two opposite involutes are equal, or $L_1M_1 = L_2M_2$
(You may mark the points at which the normals are tangent to the base circle as k_1 and k_2.)



(a) Tooth accuracy measurement (Courtesy of Imgur)



r: RADIUS

(b) Common normals

$$\overline{L_2M_2} = \overline{M_2K_2} + \overline{K_2L_2}$$ [SPLIT LINE SEGMENT INTO TWO PARTS]

$$\overset{\frown}{m_1k_2} + \overset{\frown}{l_1k_2}$$ [PROPERTY OF INVOLUTE]

$$= \theta_1 r + (\theta_2 + \theta) r$$ [ARC ANGLE, ARC LENGTH RELATIONSHIP FOR CIRCLE]

$$= \theta_1 r + \theta_2 r + \theta r$$

$$= (\theta_1 + \theta) r + \theta_2 r$$ [COMMUTATIVE AND DISTRIBUTIVE PROPERTIES]

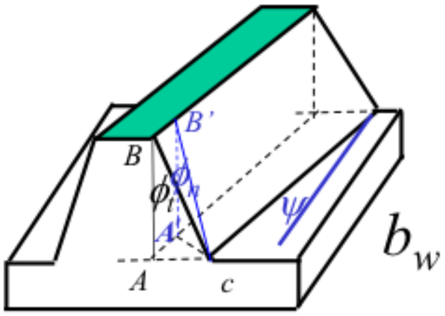
$$= \overset{\frown}{k_1m_1} + \overset{\frown}{l_1k_1}$$ [ARC ANGLE, ARC LENGTH RELATIONSHIP FOR CIRCLE]

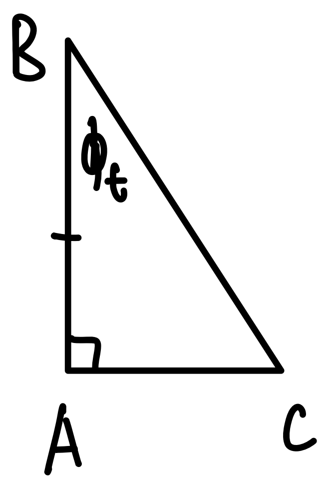
$$= \overline{K_1M_1} + \overline{L_1K_1}$$ [PROPERTY OF INVOLUTE]

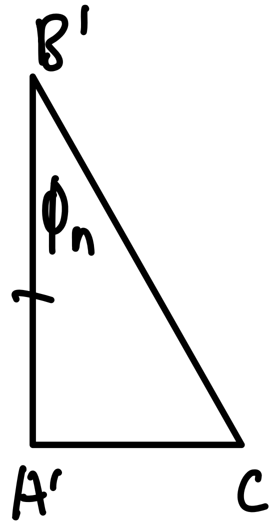
$$= \overline{L_1M_1}$$ [COMBINE INTO SINGLE LINE SEGMENT]

$$\therefore \boxed{\overline{L_2M_2} = \overline{L_1M_1}}$$

3. Derive the relationship for ψ, ϕ_n, ϕ_t based on the following diagram. Note that A, B, and c are in the transverse plane and A', B', and c are in the normal plane, and that the length of AB equals that of A'B' (the same heights).





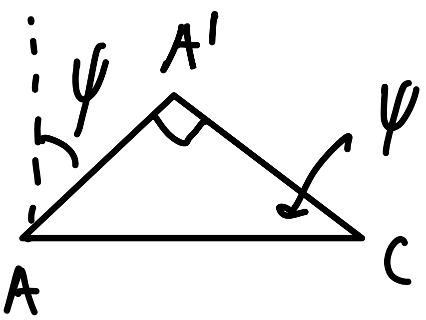


$$\tan\phi_t = \frac{AC}{AB}$$

$$\Rightarrow AC = \tan\phi_t \, AB$$

$$\tan\phi_n = \frac{A'C}{A'B'}$$

$$\Rightarrow A'C = \tan\phi_n \, A'B'$$



$$\cos\psi = \frac{A'C}{AC}$$

$$= \frac{\tan\phi_n \, A'B'}{\tan\phi_t \, AB} \quad ; \quad A'B' = AB$$

$$\Rightarrow \boxed{\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}}$$

4. A pair of gears (a pinion and a gear) has $N_p = 23$, $N_g = 48$, the normal module is $m = 6$ mm, and the pressure angle is $\phi = 20$ degree. Calculate

a) Pitch circle diameters; centerline distance
b) Outside diameters
c) Base circle diameters
d) Contact ratio.
e) If the assembly makes the gear centerline distance, Cd, 0.015 mm shorter than the theoretic centerline distance, what are also changed: diameters of the base circle, root circle, pitch circle, and outside circle, pressure angle, contact ratio? Calculate the new values of the changed ones.

Note that the pressure angles for the two meshing gears are equal.

a) $d_p = mN_p = \boxed{138\text{ mm}}$

$d_g = mN_g = \boxed{288\text{ mm}}$

$C_d = \text{AVG}(d_p, d_g) = \boxed{213\text{ mm}}$

b) $a = m = 6\text{ mm}$

$d_{op} = 2a + d_p = \boxed{150\text{ mm}}$

$d_{og} = 2a + d_g = \boxed{300\text{ mm}}$

c) $d_{bp} = d_p \cos\phi = \boxed{129.68\text{ mm}}$

$d_{bg} = d_g \cos\phi = \boxed{270.63\text{ mm}}$

d) $$C_r = \frac{1}{P_c \cos\phi}\left(\sqrt{r_{op}^2 - r_{bp}^2} + \sqrt{r_{og}^2 - r_{bg}^2}\right) - \frac{C_d \tan\phi}{P_c}$$

$P_c = \pi m$

$\boxed{C_r = 1.6696}$

```
m = Quantity[6., "mm"];
dp = m * 23;
dg = m * 48;
cd = (dp + dg) / 2;
phi = Quantity[20, "Degree"];
rop = (2 * m + dp) / 2;
rbp = (dp * Cos[phi]) / 2;
rog = (2 * m + dg) / 2;
rbg = (dg * Cos[phi]) / 2;
pc = Pi * m;
cr = (Sqrt[rop ^ 2 - rbp ^ 2] + Sqrt[rog ^ 2 - rbg ^ 2]) / (pc * Cos[phi]) - cd * Tan[phi] / pc

1.66955
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e) $C_{new} = C_d - 0.015\,mm = 212.985\ mm$

BASE CIRCLE CONSTANT; ROOT CIRCLE CONSTANT; OUTSIDE CIRCLE CONSTANT; PITCH CIRCLE DIAMETERS CHANGE PROPORTIONAL TO BASE CIRCLE LENGTHS:

$$d_p' d_g'^{-1} = d_{bp} d_{bg}^{-1} \Rightarrow \frac{2C_{new}}{d_g'} - 1 = \frac{d_{bp}}{d_{bg}} \Rightarrow d_g' = \frac{2C_{new}}{\frac{d_{bp}}{d_{bg}} + 1}$$

$$d_p' + d_g' = 2C_{new}$$

[PITCH CIRCLE CENTERLINE RELATIONSHIP]

$$= \boxed{287.977\,mm}$$

$$d_p' = d_{bp} d_{bg}^{-1} d_g' = \boxed{137.993\ mm}$$

NEW PITCH CIRCLE DIAMETERS

```
Cnew = Quantity[213 - 0.015, "mm"]
dbp = Quantity[129.68, "mm"];
dbg = Quantity[270.63, "mm"];
dg ' = 2 * Cnew / (dbp / dbg + 1)
dp ' = dbp * dg ' / dbg
ArcCos[dbg / dg '] / Degree // FullSimplify

212.985 mm

287.977 mm

137.993 mm

19.9885
```

SINCE PITCH CIRCLES CHANGE RELATIVE TO BASE CIRCLES, ϕ CHANGES:

NEW PRESSURE ANGLE:

$$\phi = \cos^{-1}\left(\frac{d_{bg}}{d_g'}\right) = \boxed{19.989^\circ}$$

CONTACT RATIO CHANGES MINIMALLY BECAUSE ϕ AND C_d CHANGE:

$$C_r = \frac{1}{P_c \cos\phi}\left(\sqrt{r_{op}^2 - r_{bp}^2} + \sqrt{r_{og}^2 - r_{bg}^2}\right) - \frac{C_d \tan\phi}{P_c}$$

$$P_c = \pi m$$

$$\boxed{C_r = 1.67034}$$

```
m = Quantity[6., "mm"];
dp = m * 23;
dg = m * 48;
cd = Quantity[213 - 0.015, "mm"];
phi = Quantity[19.989, "Degree"];
rop = (2 * m + dp) / 2;
rbp = (dp * Cos[phi]) / 2;
rog = (2 * m + dg) / 2;
rbg = (dg * Cos[phi]) / 2;
pc = Pi * m;
cr = (Sqrt[rop ^ 2 - rbp ^ 2] + Sqrt[rog ^ 2 - rbg ^ 2]) / (pc * Cos[phi]) - cd * Tan[phi] / pc

1.67034
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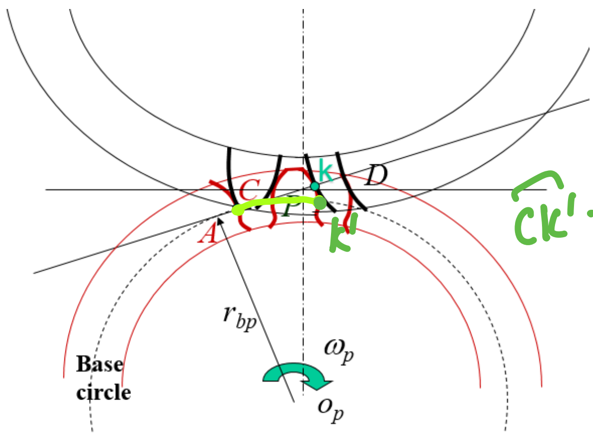
5. Involute profiles

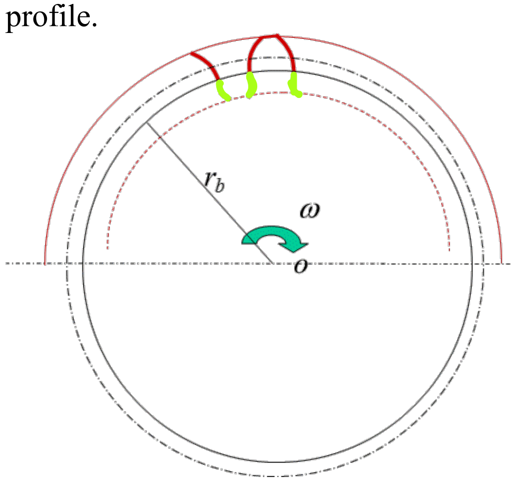
INVOLUTE
ARC LENGHT
IS EQUAL TO
INVOLUTE TANGENT LENGHT

a) Mark a segment that equals Ck in length.



$\overset{\frown}{CK'} = \overline{CK}$

b. Mark, on one of the tooth profiles in the diagram below, the non-involute portion of the tooth profile.



FEATURES BELOW THE BASE CIRCLE AND ABOVE OR AT THE OUTER CIRCLE ARE NOT INVOLUTE.

6. The size of a gear is determined by its number of teeth if the module has been given. Interference in gearing, or undercutting in manufacturing, occurs if the gear is too small. We need to understand and determine the minimum number of teeth to avoid such interference, by analyzing a gear-rack meshing set shown in the figure below.

a) Label, in the diagram below, the pitch circle radius, r, and the base circle radius, r_b, of the gear. Here, the outside circle radius of the gear is r_o.

b) If the gear is the driver, and the rack tooth depth is shown by the straight-line tooth profile, find points C and D and label them. Would interference occur or not?

c) a=1.0m, b=1.25m, ϕ =20 degree, what is the minimum number of teeth to avoid interference?

d) a=1.0m, b=1.25m, ϕ =25 degree, what is the minimum number of teeth to avoid interference?

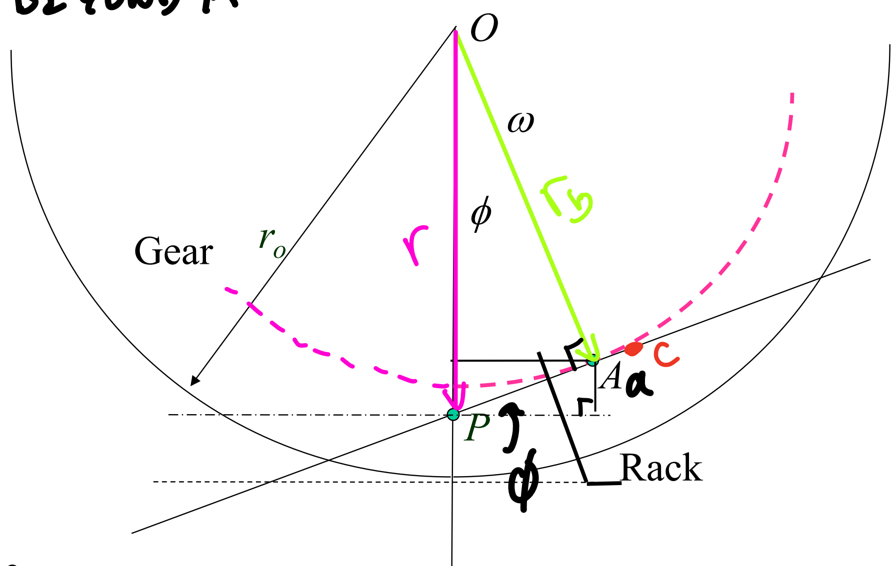
b) YES INTERFERENCE; C IS BEYOND A

c)

$17.097 \Rightarrow \boxed{n=18}$

D)

$11.198 \Rightarrow \boxed{n=12}$



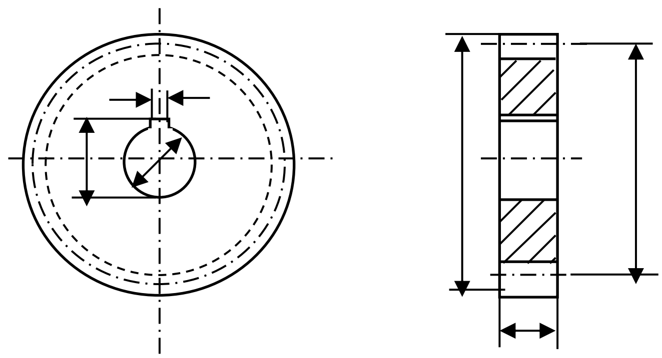
$2r = mN \; ; \; a = m$

$$\sin\phi = \frac{PA}{r} \qquad \sin\phi = \frac{a}{PA} \Rightarrow \sin^2\phi = \frac{PA}{r}\frac{a}{PA} = \frac{2m}{mN}$$

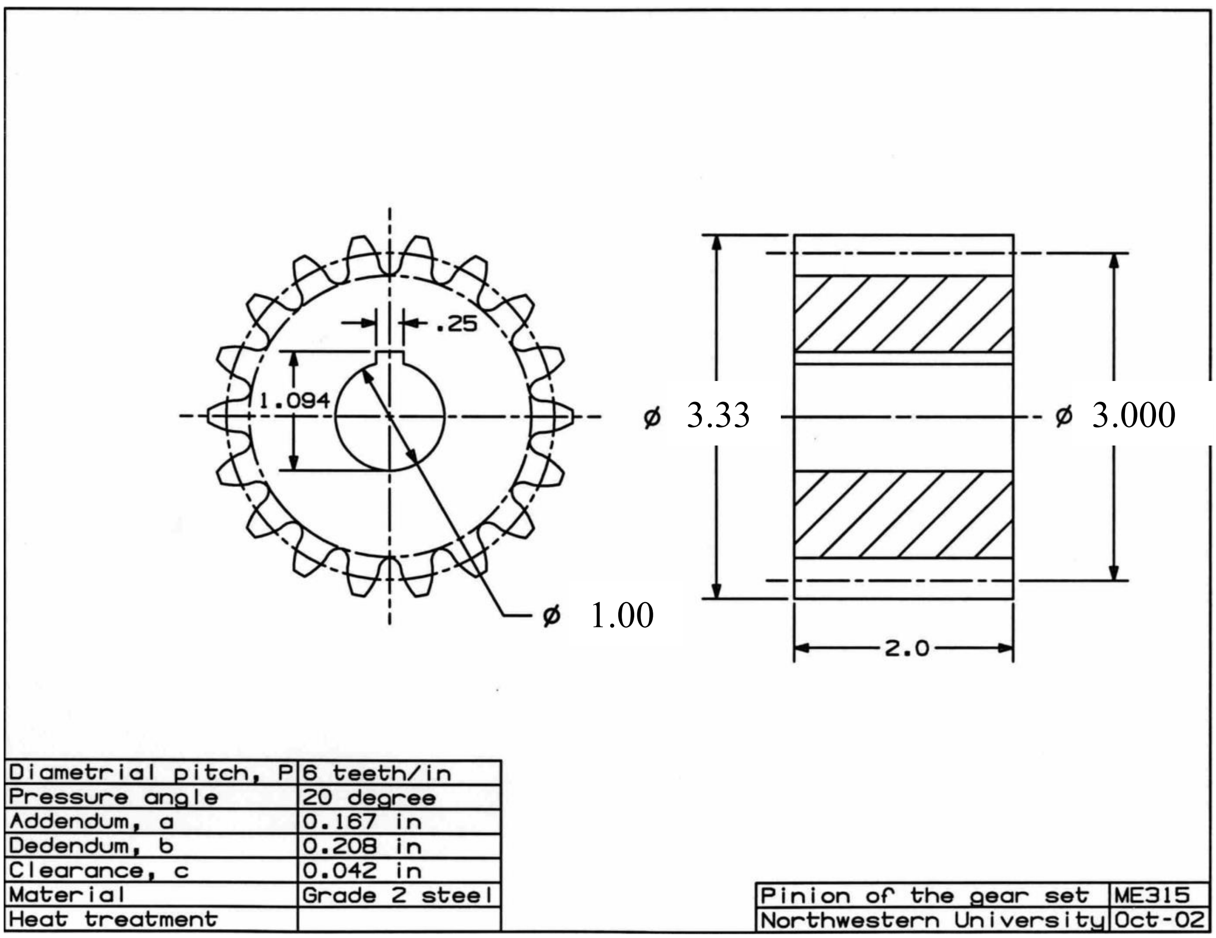
$$\Rightarrow N = \frac{2}{\sin^2\phi}$$

7. CAD Develop a CAD drawing for the pinion shown below. This is a solid-dish gear, N= 18, pressure angle =20 degree, and m = 3mm. The dash-dot circle, solid circle, and the dotted circle are for the pitch circle, the outside circle, and the root circle. Do not show the base circle although it is very important. Again, draw the gear in a landscape setup, and use the full page.

Choose the shaft diameter to be 20mm, then the key is 6x6mm.
Choose the gear face width in the middle of 8m> b_w>16m, which is 12m



Sample drawing (note the length unit hear is inch, tolerances are ignored in this practice)



Diametrial pitch, P	6 teeth/in
Pressure angle	20 degree
Addendum, a	0.167 in
Dedendum, b	0.208 in
Clearance, c	0.042 in
Material	Grade 2 steel
Heat treatment	

DELAYED

Project initiation (No submission now)
This week
Gear train design

The input speed is given.
Select N1 for the input gear and select the numbers of teeth, N_i, for other gears. Then calculate the speed for each gear; and adjust N_i until you get a satisfactory output speed.

Gear geometry

Select m for each pair of gears. You should use the same module for one pair of gears. Later, you will modify the geometry for each gear until the strength requirements are satisfied.

Next week and later
Gear force/stress/strength analysis

The power is given. The input speed is given.
Calculate the tangential force on the input gear and then the other forces, and then the forces on the other gears.
Select materials.
Gear strength, factors of safety.
Knowing the gear forces, you can now calculate the factors of safety for gears.

Shaft design and force analysis, draft of your assembly

The power is given. The input speed is given.
Knowing the gear forces, you can then obtain the forces on each shaft.
Knowing the gear geometry, you can then begin your shaft assembly design, assuming bearings are selected. Draw each element to its size.
Select the material.
Analyze the strength (deflection, if the shaft is thin and long) of your shaft.

You may repeat some of the steps to adjust and modify your design.