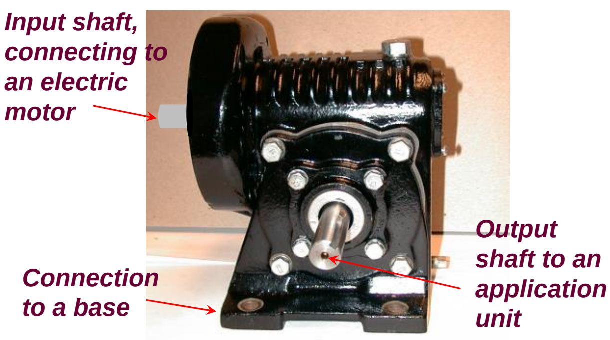
Chapter 1 Getting Ready for Design of Mechanical Elements

It is fair to say that mechanical engineering is the industrial foundation of the country and the world. All engineering equipment, tools, and devices for transportation, communication, navigation, agriculture, construction, public health services, and many others, have to be made and maintained, and the means to support most engineering activities root in mechanical design and manufacturing. Machine elements are building blocks for machines. A qualified mechanical engineer should have a solid knowledge base of machine elements.

1-1 A simple machine

Shown below (**Figure 1-1**) is a simple machine, a worm-gear reducer. It reduces the speed from a power source, for example, an electric motor, to a required application level. It has an input shaft (to be connected to the motor via a coupling or a belt drive) and an output shaft (to be connected to an application unit, such as a machine tool, a pump, etc). It has a base flange, to be attached to a base plate. Everything is assembled inside the housing and enclosed by two sets of end caps.



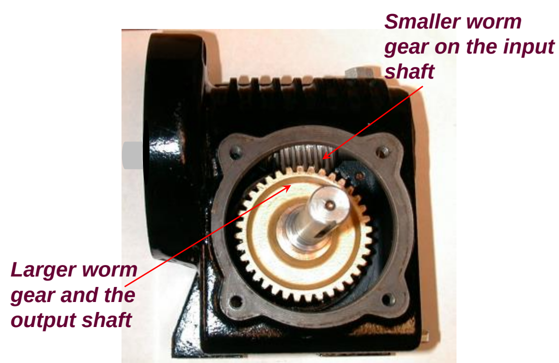


Figure 1-1. Worm-gear reducer. **Figure 1-2**. Worm-gear reducer, end cap removed.

We need to remove the end caps and see the internal structure of this machine. **Figure 1-2** shows the worm-gear reducer after removal of one end cap. There are two gears - a large worm gear, called the gear, or the gear wheel, and a small worm gear, call the worm, like a thread; each is supported by its own shaft system.

The sub-assembly of the shaft system of the large worm gear, or the gear, as shown in **Figure 1-3**, consists of the following elements: the gear and its shaft, a pin, a key, a spacer, two rolling-element bearings, and some shim rings. The sub-assembly of the shaft system for the small worm gear, or the worm, as shown in **Figure 1-4**, consists of the following elements: the worm gear and its shaft, two spacers, and a nut.

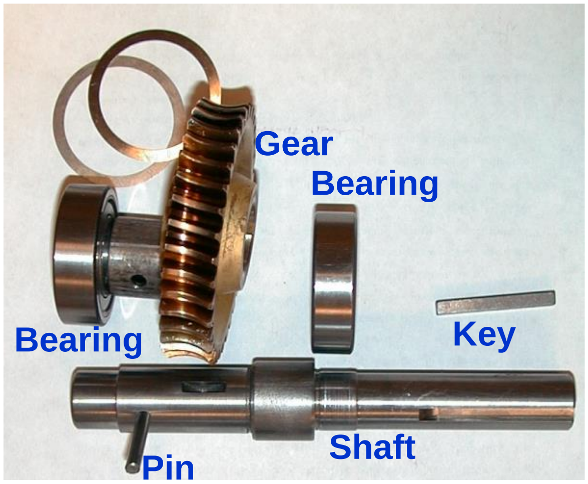


Figure 1-3. Parts of the gear shaft system.

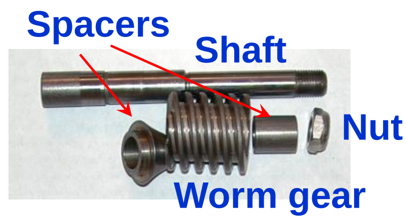


Figure 1-4. Parts of the worm shaft system.

Other parts, such as the end caps, housing, bolts and nuts, etc., are shown in **Figure 1-5**.

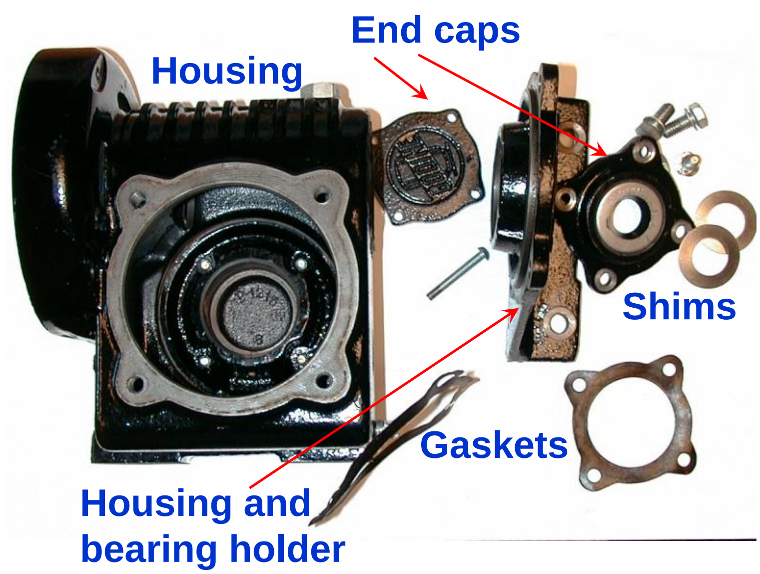


Figure 1-5. Housing and end caps.

1-2 Classification of commonly used mechanical elements.

Such a simple machine has so many parts, each playing a particular function. A larger machine should have more. The mechanical components to be covered in this study are mainly the common parts in most machines, which are among the following:

Transmission elements that transmit power and motion, such as

Gears (rigid elements): spur, bevel, helical, worm, etc.
Belts and chains (flexible elements)
Clutches, breaks, and couplings (**Figure 1-6**); these are assemblies, for motion connection and separation.

Fasteners and connecting elements that connect parts together:
Screws, bolts and nuts
Keys and keyways
Pins

Supporting elements that hold parts in place:
Housing
Shafts: crank shafts, straight shafts, hollow shafts
Bearings: rolling element bearings, sliding bearings
Frames: bars, columns, plates, shells, etc.
Elastic supports: Springs.

Figure 1-6. A simple coupling set (image from internet).

There are many other elements that we may talk about when we see them in the class.

1-3 Mechanical design, goals of design, and design methods

Mechanical design is an innovation process that transforms a concept, or an idea, into a machine that can perform a certain function. Let's use the machine shown in section 1-1 as an example to see what are involved in the design and how such a machine is designed. We should be given some basic requirements, such as the power to be transmitted, the input and output speeds, and the overall size of the machine. Designing such a machine is an open-ended problem that may have many possible solutions. The designer has the freedom to choose many things - mechanisms, materials, and sizes of parts, etc. The best design should result in the most cost effective product that performs the best and enjoys the longest service life.

In order to change the speed from one shaft to the other, one may have several choices of the mechanisms that can perform this function; for example, gears, worm gears, belts, chains, or even contacting rollers (like those in a continuous variable transmission, CVT). Let's assume that we have the mechanism chosen already: the worm gear system. We then need to decide the materials to be used and the sizes of parts, and most importantly, the locations of the parts in the machine and the relations among them. The parts cannot be assembled arbitrarily. Mechanics, especially mechanics of materials, is heavily involved in the process of part size determination once the materials are properly chosen. We also need to indicate how the part should be made, and what hardness and surface finish should be achieved. In many case, we also need to specify the lubrication method for a machine operation. Performance, strength, life, reliability, cost, energy consumption, machinability, disassembly possibility, and convenience in application, as well as requirements for maintenance, all should be properly considered in the design process and listed in the design report. **The final products of a design are drawings, both part drawings and assembly drawings**. Design is a process from art to part. Everything should be done **exactly** within the tolerances designated, and everything should be shown on drawings **exactly** with the support of dimensions and tolerances.

The design process is also an iteration process. No one can obtain a perfect design from one simple calculation. We can start from an initial design plan, do some calculations, and then do some modifications, and so on. Traditionally, all this was done by “hand” with a hand calculator using simplified formulas and models. In many places, design still remains largely empirical. In this course, however, we will use computer-aided design (CAD) and analyses.

This course consists of about eighteen to twenty lectures, five required CAD sessions, nine to ten homework sets, two exams (a mid-term exam and a final quiz), and one design project where students should work in groups (three people in one group). There are also several short movies before classes. In this course, we will study principles of element design, applications of mechanics, structures of mechanical components and systems. We will analyze the strength and life of several elements and practice structural designs. Integrating mechanical theories with CAD and structural design is the goal of teaching this course. After this course, students are expected to be able to design a simple machines on the basis of mechanics-supported design principles, work with "blue prints," or CAD drawings. This are the goals of student learning.

This book is only a brief introduction to the subjects to be covered in the course. It is **not a complete text**. Students are required to **complete a set of notes in each class**. The book, short movies, handouts, and class notes form a complete set of the class document. With this method, no time should be wasted on drawing structures during the lecture time. Students may use any Machine Design books as references. However, everyone should have two books handy, engineering materials (the one for Materials science 201) and Mechanics of Materials (CE 216).

1-4. Review of mechanics

Take a look at the drawing below, **Figure 1-7**. It is the drawing for a shaft (actually the shaft for the worm gear that we learned in Section 1-1). An immediate question one may ask is, why is it so complicated with so many shoulders? Why is it not just a bar of the same cross section?

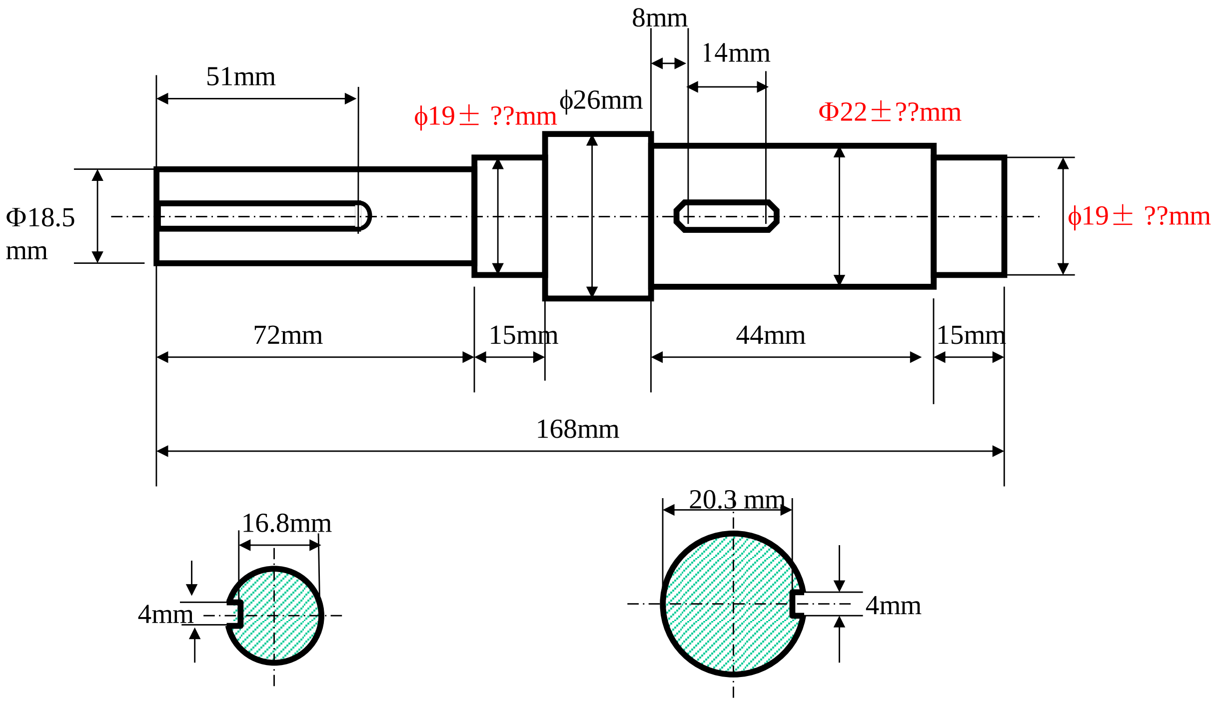


Figure 1-7. A typical shaft drawing. Note that this is a convenient drawing; in reality the two keyways should not be on the same plane; here one of them have been rotated about 90^0. The tolerances are not determined yet.

The shaft is used to support the gear and should be assembled in a housing structure**. Figure 1-8** shows all the necessary parts on the shaft. The shaft has to have shoulders to position parts. It holds a gear, a pair of bearings, and an application element (such as a pulley, not shown); it connects these elements, it rotates with these elements; and it transmits forces from the gear to the housing. The shaft has to have different diameters in different segments, as shown by the shoulders, for strength and assembly considerations. Note that in the assembly drawing shown in **Figure 1-8**, only one half of the structure is drawn in the sectional view.

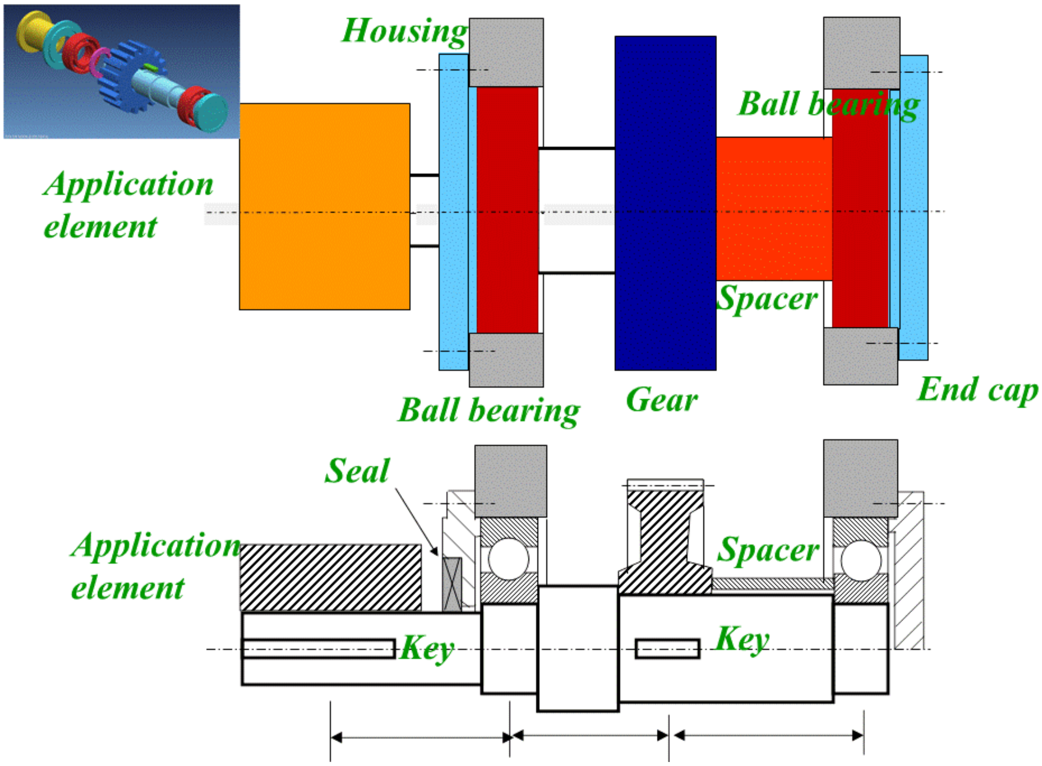


Figure 1-8. Shaft and the elements on the shaft (one half is shown in the cross-sectional view due to symmetry). Note that some clearances are exaggerated for clarity. The short centerlines through the flanges of the end caps indicate connecting bolts, not shown.

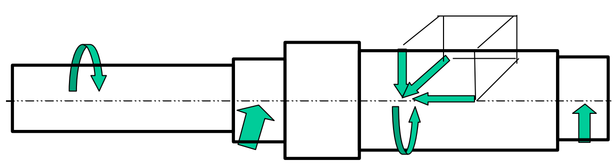


Figure 1-9. A mechanical model for the shaft.

This course is the very first course that brings mechanics to the real world. **Figure 1-9** is a mechanical model of the shaft in **Figure 1-8**. Here, parts have been replaced by forces and moments (note that moments are generalized forces, we may call them all forces for convenience). This figure shows something we are familiar with. We often see bars with several forces in the homework for Mechanics of Materials. In this course, we will utilize the knowledge from mechanics to design real structures. That is something really challenging. However, let's start with what we did in mechanics: stress and strength analyses, because once we construct a design, we need to know if the structure designed is strong enough. The mechanics analysis is the design analysis, which should be done right, and as close to the real-world situation as possible. Suppose now we have such a structure and would like to conduct the mechanics analysis. What should we do in order to generate a mechanical model form the real structure?

Important structural dimensions should be kept, such as diameters, cross-section size, distance between forces, etc. **Forces/moments** should be determined, including their magnitudes, directions, and points of applications. **Material properties** should be recorded because we need to use such properties in our analyses. **Operating conditions** are also important. Static loading and variable loading may result in different failure modes. We may also need to pay attention to possible **stress concentration** because it can accelerate failure of components. A **free-body diagram** is essential for a mechanical model, on which all forces and their locations of applicaitons should be determined.

On the other hand, what should we obtain from the mechanical model through mechanics analyses? Bending moment, shear force, and torque distributions are in the first group of the things to be obtained, usually in the form of diagrams, such as the **torque diagram**, **bending moment diagram, and shear-force diagram**, which can help us determine the location of the most critical point where stresses are maxima. When these diagrams are plotted with a good alignment with the free-body diagram that shows all forces (including moments and torques), we can have a clear view of force/moment distributions with respect to our designed component. In this course, similar to what have been done in Mechanics of Materials, torques are referred to the out-of-the–plane twisting actions, while bending moments to the in-plane bending actions, although they are both "moments" in nature, mechanically. The analyses of torque, bending moment, and shear force, along with other types of forces, will lead to the evaluation of **stresses**, normal and shear stresses, which further result in the **principal stresses** that allow us to apply a certain design criterion. With the assistance of design criteria (we call them design theories) we can analyze the **strength** of a mechanical component. In some cases, we will evaluate the **life** of components, such as rolling element bearings. In many situations, we also need to analyze deformation/deflection to ensure proper **stiffness** of a part.

Hope we still remember the sign conventions for bending moment and shear force from Mechanics of Materials. The following diagrams in **Figure 1-10** show the positive cases.

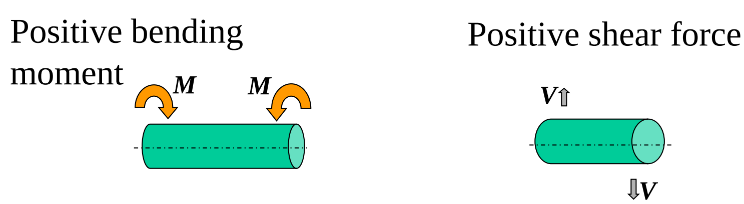


Figure 1-10. Positive bending moment and shear force.

Also hope we remember the differential relationships among the distributed load intensity, $q(x)$, shear force, $V(x)$, and bending moment, $M(x)$, on a beam (or shaft, as for a mechanical component), which are given below, where x is the centerline of the beam/shaft.

$$q(x) = dV(x)/dx \tag{1-1}$$

$$V(x) = dM(x)/dx \tag{1-2}$$

and

$$q(x) = d^2M(x)/dx^2 \tag{1-3}$$

We must be very familiar with the types of stresses and stress expressions. An **axial loading** case is shown below in **Figure 1-11**. Note that the stress is the normal stress, which is σ_y here, simply due to its alignment with the *y* axis for this case. If non-uniformity at the location of the load application is neglected, this normal stress is equal to force *F* divided by cross-sectional area *A*, and it distributes uniformly in each cross section of the bar. The square to the right of the loading figure shows the stress element for this case, and this stress state is the same for every point in the bar far away from the location of the load application. Note that the entire side surface of the bar, or the cylindrical surface, is stress free.

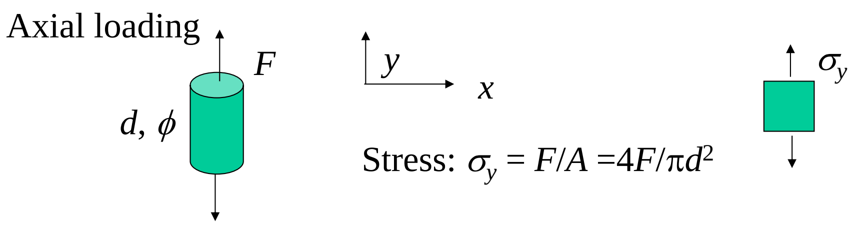


Figure 1-11. Normal stress due to axial loading. The diameter of the bar is *d*, or ϕ.

A **pure bending** case is shown in **Figure 1-12** below. The bending stress is also a normal stress, σ_x, so marked simply because it is a long the *x* direction. However, the bending normal stress does not appear uniformly in a cross section. The expression for the stress at any location of the cross section, σ_x, and the maximum stress on the surface, σ_{bs}, are both given in this figure. The maximum bending stress appears along the surface, the farthest points to the neutral axis at the top and the bottom of the cross section, while along the neutral axis, or in the neutral plane, the bending stress is zero. The stress element for the location marked by the red dot is also illustrated in the figure, which is the red square. Remember that a transverse force also results in a bending. The bending stress expressions are the same as the ones given below as long as the bending moment at an axial location is calculated through a proper moment diagram. Note that the entire bar/shaft surface is stress free.

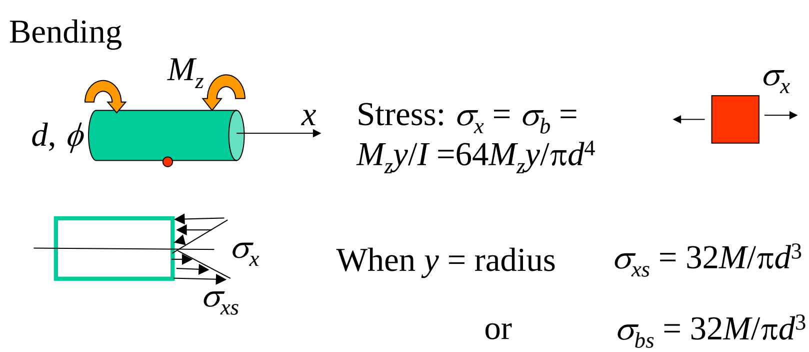


Figure 1-12. Normal stress due to pure bending. Here, x: x-axis direction, z: the z-axis direction (perpendicular to the paper, not shown), s: surface, and b: bending. The red dot indicates a surface point, and σ_{xs} is the bending stress at this point. The diameter of the bar is d, or ϕ.

A **torsional shearing** case is shown in **Figure 1-13** below. The stress is now the shear stress, and it does not appear uniformly in a cross section either. The expression for the stress at any location of the cross section, $\tau = \tau_{xy}$, and the maximum stress, τ_{ts}, are both given in the figure. The maximum torsional shear stress is at every point of the surface, while along the center axis, the shear stress is zero. The stress element for the location marked by the dot is also illustrated in the figure by the red square. Note that the entire surface is stress free.

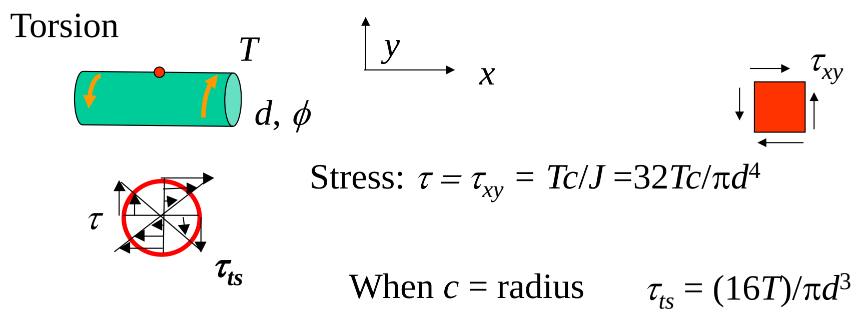


Figure 1-13. Shear stress due to a pair of torque. Here, *t*: torsional shear, and *s*: surface, *c*: distance to the center of the shaft measured in the cross section. The red dot indicates a surface point, and τ_{ts} is the torsional shear stress at this point. The diameter of the bar is *d*, or ϕ.

The shear stress due to a **transverse force** is shown below in **Figure 1-14**, where both the cross-sectional distribution and the maximum, τ_v, are presented. Note that again, the entire surface is stress free. The maximum of this shear stress is at the neutral axis, or the neutral plane.

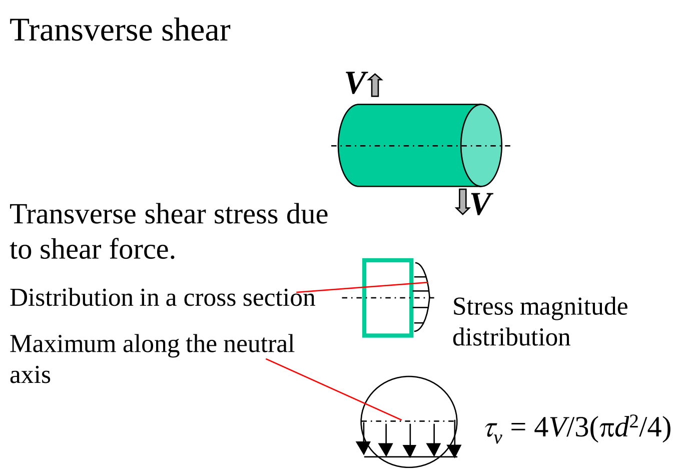


Figure 1-14. Shear stress due to transverse loading.

A combined **torsional shearing and bending** case, **Figure 1-15**, is common in mechanical components. Both non-uniform normal and shear stresses appear in a cross section. The maximum torsional shear stress, τ_{ts}, and the maximum bending normal stress, σ_{xs}, are on the surface. The stress element for the location marked by the red dot is illustrated by the red square in the figure. However, the entire cylindrical surface away from the location of load application is stress free.

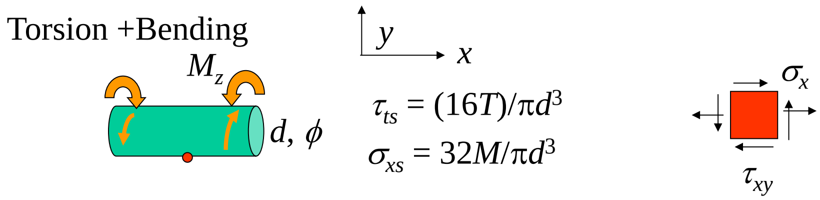


Figure 1-15. Normal and shear stresses due to combined torsion and bending. The red dot indicates a surface point. Here, σ_{xs} is the bending stress and τ_{ts} is the torsional shear stress at this surface point. For simplicity, we can use σ_x, or even σ, for σ_{xs}, and τ_{xy}, or even τ, for τ_{ts}.

We only need to pay attention to the maximum stresses. As shown in **Figure 1-15**, σ_{xs} is the largest bending stress and τ_{ts} is the largest torsional shear stress at a surface point. They are the ones to be analyzed. We will later use σ_x, or even σ, for σ_{xs}, and τ_{xy}, or even τ, for τ_{ts}.

Example 1-1. Let's try to do the bending moment diagram and stresses of a simply supported shaft.

The simplest shaft as a straight bar is shown below in **Figure 1-16**. Plot the shear force and bending moment diagrams, label the peak values, calculate σ and τ at point **A,** and show these stresses on the stress element marked by **A** (which is the top view of point **A**). The length of the shaft is L and the shaft diameter is d.

Solution. The bearing (or whatever the support) reactions at C and D are both $F/2$. The resultant shear force diagram and bending moment diagram are plotted below the shaft diagram. At location **A** in cross section E, the normal stress is tensile and maximal in value.

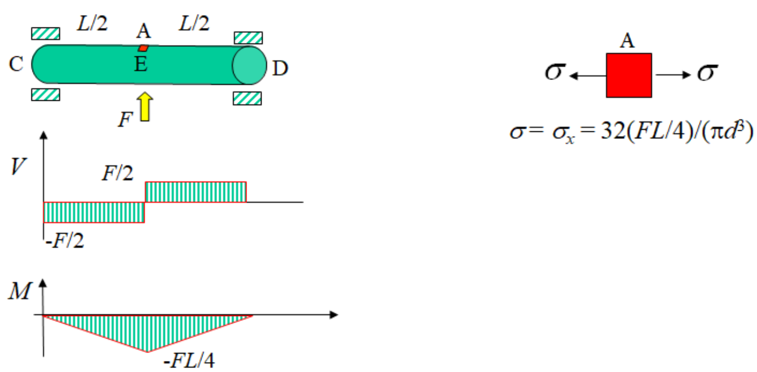


Figure 1-16. Example 1-1 and solution.

Think, how large is the shear stress at point **A**?

Now let's do some analyses of the shaft shown in **Figure 1-8** and use the analysis process to guide our mechanics review further. Here, the mechanics has to be three-dimensional (3-D). Remember, there are no two-dimensional structures, although in some cases we can simplify a complicated structure into a two-dimensional (2-D) one. People may do some simplifications to make a mechanics analysis easier under certain reasonable assumptions. Some simplifications have been made in **Figure 1-17**, e.g. only the vertical components of the forces are considered, as if the gear were a spur gear, just to start our analysis easier. Here, the forces are only in the x-y plane, or the paper plane, but the torques have to be in the *y*-*z* plane (perpendicular to the paper). Note that *x*, *y*, *z* follow the right-hand rule, and only the *x* axis is shown.

A designed mechanical component can be considered safe as long as the material can survive the worst loading in an expected lifetime. Here are some of the important questions. Where is the most critical cross section? Where is the most critical point? Is it in the cross section where the bending moment is the maximum? Or is it in the cross section where the torque is the maximum? Or is it the place where the cross section is the thinnest? Not really for any of the above. The most critical cross section should be determined by the maximum stress. The bending stress at the thinnest cross section may be insignificant if the bending moment there is small. In this example, the location of the most critical cross section and the location of the most critical point in the critical cross section may be straightforward if the torque is not large, which is at the location of the maximum bending moment. However, for this case, we are not sure yet. The most critical cross section could be at A, C, or D. Why not at B?

Let's analyze the mechanics at location A. The most critical cross section is slightly to the left of A, or A-, where both *F* and *T* are applied. All the results for the diagrams of the torque, bending moment, and shear force are given below.

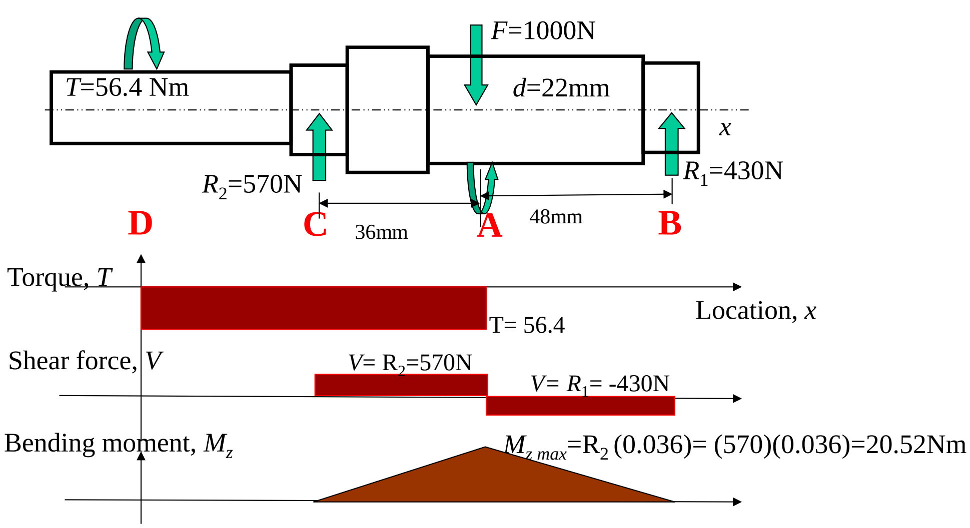


Figure 1-17. The gear shaft with simplified forces, Torque and moment diagrams.

Let's further analyze the shaft we have worked on. What are the stresses we need to analyze, and what are the maximum stress of each stress component? **Figure 1-18** shows the forces, the stress it causes, and the maximum stress of each.

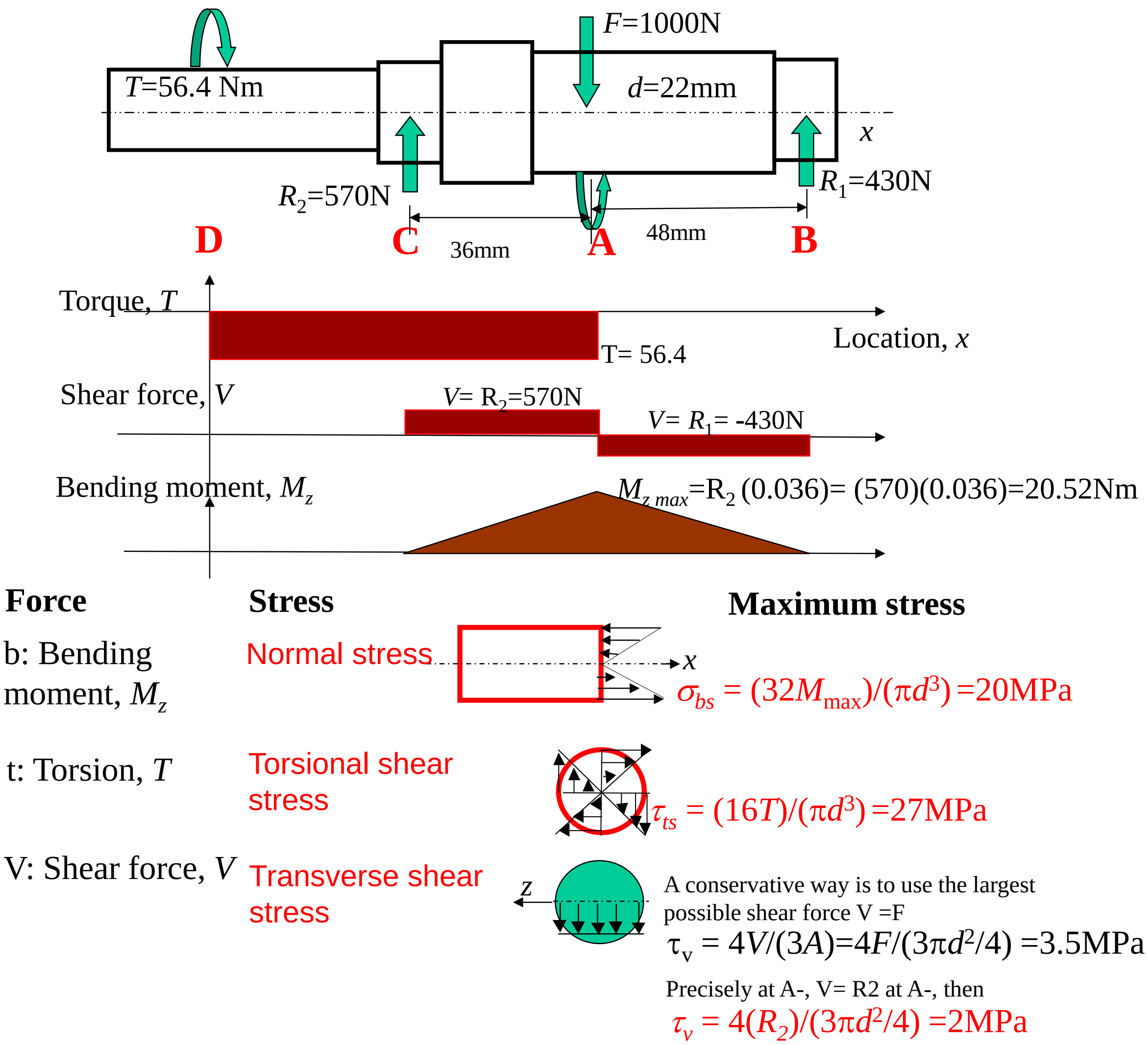


Figure 1-18. Stress analyses at cross section A-.

Here, we can use the largest possible shear force $V = F$ to calculate the transverse shear stress for the worst case. Actually $V = R_2$ should be fine if we use the A- cross section.

Can we identify the most critical point in the A- cross section? We then need the information of the principal stresses, σ_1, σ_2, and σ_3, as well as the maximum shear stress, τ_{max}, because we cannot simply add the normal stress due to bending and the shear stresses due to torsional shear. They are tensors! Note that the maximum shear stress, τ_{max}, is not the torsional shear stress, not the transverse shear stress; it is the maximum shear stress of one point at all possible orientations of the stress state.

We can draw this cross section in **Figure 1-19** below with all stresses shown. The total stresses at points 1, 2, 3, and 4 are also given in the figure. The most critical points should come from pints 1, 3 and 4. Why not point 2?

Even though we are analyzing 3D mechanics for a 3D structure, the resultant stresses at each of these four points are 2D, with σ_x, σ_y=0, τ_{xy}, which is a plane-stress case. At points 2 and 4, we only have shear stress $\tau_{xy} = \tau_{ts} \pm \tau_v$, which is a pure shear stress case.

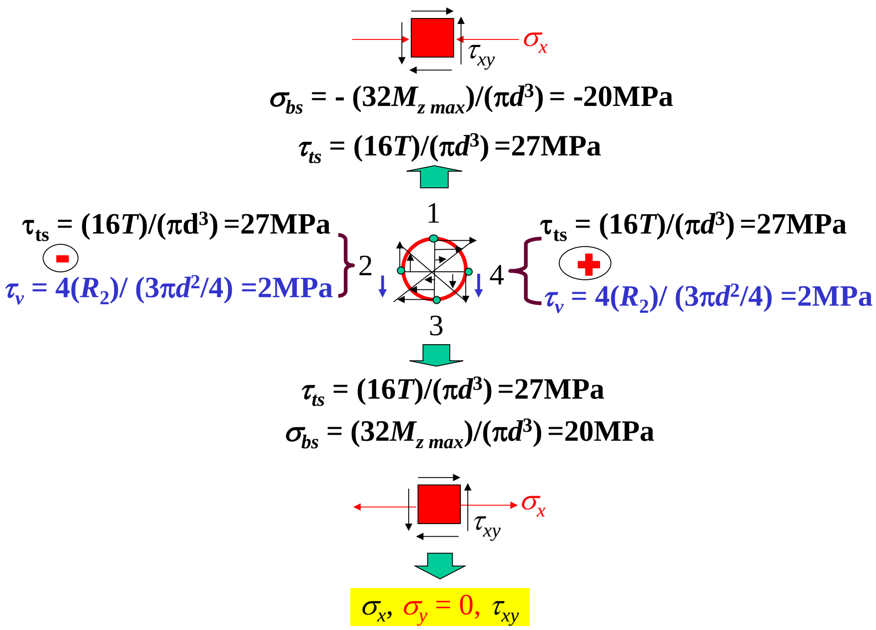


Figure 1-19. Total stresses at points 1,2,3,4 in possible critical cross section A-. The stress states at points 1 and 3 are (σ_x, σ_y=0, τ_{xy})

Principle stresses

Assuming non-zero σ_x, σ_y, τ_{xy}, for a plane-stress case, corresponding to the normal and shear stresses mentioned above, the maximum and minimum normal stresses of one stress element but different orientation can be found as follows.

$$\sigma_{max} = (\sigma_x + \sigma_y)/2 + [((\sigma_x - \sigma_y)/2)^2 + (\tau_{xy})^2]^{1/2} \quad (1\text{-}4)$$
$$\sigma_{min} = (\sigma_x + \sigma_y)/2 - [((\sigma_x - \sigma_y)/2)^2 + (\tau_{xy})^2]^{1/2} \quad (1\text{-}5)$$

Remember, they may or may be not the real maximum or minimum. The true maximum and minimum normal stresses as the principal stresses can be determined from the following sequence. The principal stresses are named $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Here in the plane-stress state, one principle stress is zero.

σ_1= ? σ_2 = ? σ_3 = ? ($\sigma_1 \geq \sigma_2 \geq \sigma_3$)

$\sigma_1 = \sigma_{max},\quad \sigma_2 = \sigma_{min} \quad \sigma_3 = 0 \quad$ if $\sigma_{min} > 0$

$\sigma_1 = \sigma_{max},\quad \sigma_2 = 0 \quad \sigma_3 = \sigma_{min} \quad$ if $\sigma_{min} < 0$ but $\sigma_{max} > 0$

$\sigma_1 = 0 \quad \sigma_2 = \sigma_{max},\quad \sigma_3 = \sigma_{min} \quad$ if $\sigma_{max} < 0$

Mohr's circles below in **Figure 1-20** well show these stresses.

We cannot forget the maximum shear stress for one stress element, which is given below.
In one Mohr circle,

$$\tau_{max} = [((\sigma_x - \sigma_y)/2)^2 + (\tau_{xy})^2]^{1/2} \tag{1-6}$$

This is the radius of the Mohr circle.

Overall $\quad \tau_{max} = (\sigma_1 - \sigma_3)/2 \quad$ (1-7)

This is the radius of the largest Mohr circle, see the right plot of **Figure 1-20**, and the overall maximum shear stress at one point, or one candidate critical point, of a mechanical shaft (or whatever component).

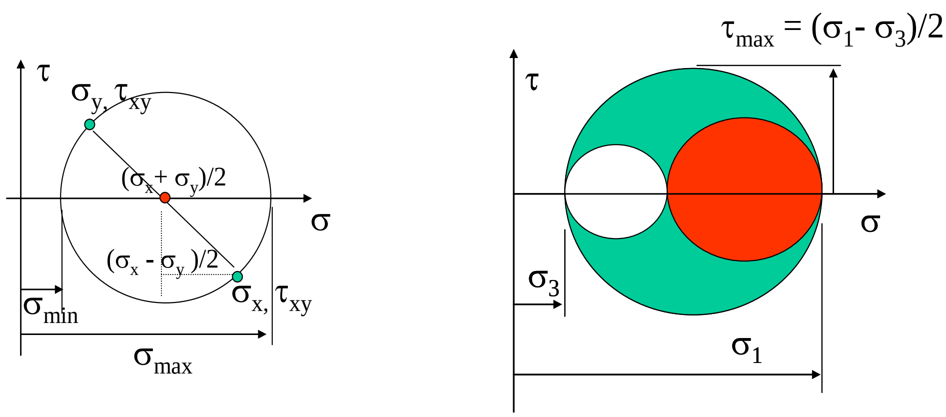


Figure 1-20. Mohr's circles showing σ_{max}, σ_{min}, and τ_{max}. Left: stress state rotation in one plane, right: 3D and the overall maximum shear stress.

Example 1-2. Determine the principal stresses, σ_1, σ_2, and σ_3, for the pure shear stress state shown below.

Solution.

$$\begin{matrix}\sigma_{max} \\ \sigma_{min}\end{matrix} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\tau_{xy} = \pm 80\text{MPa}$$

$\sigma_1 = 80$MPa; $\sigma_2 = 0$; and $\sigma_3 = -80$MPa.

Example 1-3. Principal stresses for the shaft in Figure 1-17 are analyzed. Points 1 and 3 are the most critical points based on the amplitudes of the principal and maximum shear stresses. We still do not know which of point 1, 3, or 4 is more critical. We need to know failure criteria in the next chapter.

Details will be explained in class and practiced in a homework problem.

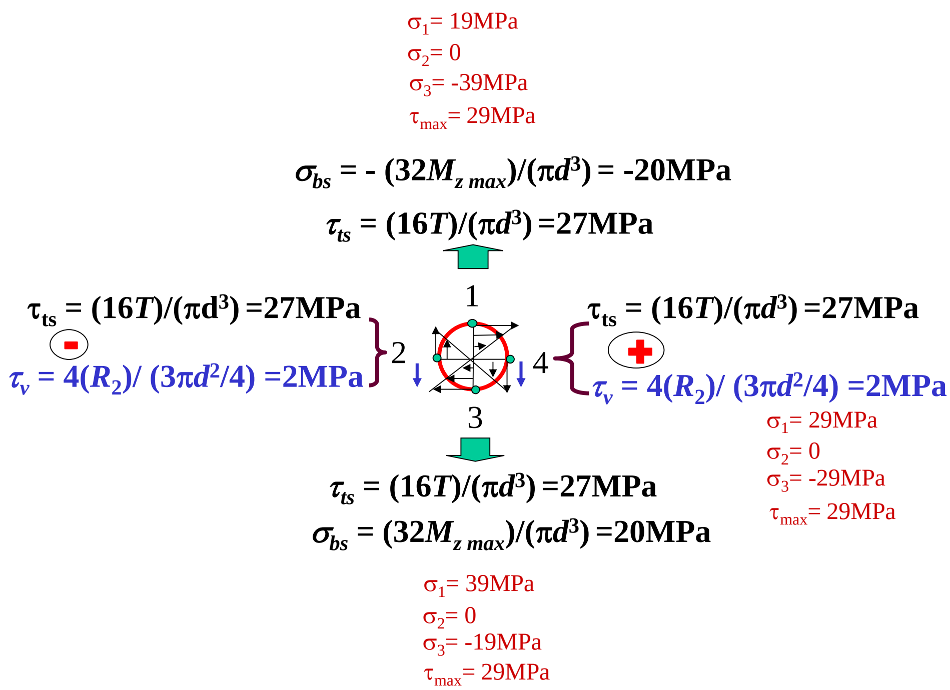


Figure 1-21. Principal stresses at points 1, 3, and 4

The following topics should be well reviewed:

A Reaction force analysis, (Statics, or EA2)
B Shearing force, bending moment and torque diagrams
C Normal stresses: bending, tensile
  Shear stresses: torsional, shearing, and bending
D Deformations (tensile) and deflections (bending)
E most critical point
F Strength of materials (yield strength, ultimate strength, fatigue endurance limit)
G Principle stresses, Mohr's circle
H Generalized Hooke's law
I Stress-strain relationships
J Strain energy.

References to any books of Mechanics of materials.

Media: Review Mechanics 1, 2, 3, 4, 5, and 6 on U-Tube, in the first reading assignment.
315 students should watch these short movies before the first class starts.