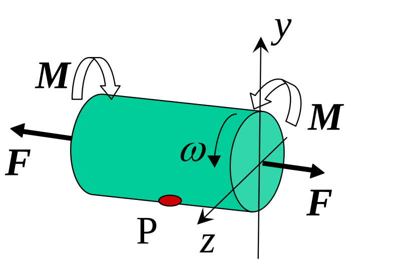
Chapter 3 Variable Loading and Fatigue Failure

3-1 Variable loading

Strictly speaking, there is no static loading whose magnitude, direction, and location of application do not change at all with time. What we see in the real world are loads that more or less vary. The reason why we have to study static loading is due to its simplicity for the analysis. Whether or not a load is variable has to be decided by how stresses behave as a function of time.

Example 3-1. The diagram shown in **Figure 3-1** is a piece of a long shaft that is loaded by a pair of constant tensile loads, *F*, and a pair of constant bending moment, *M* (in the *x*-*y* plane). The shaft rotates at a constant angular speed, ω. The balance of the torque is not shown here in this piece of the shaft, and the effect of torque is not included. It seems that we would have a constant loading case in terms of the loads. Here, are we talking about static or variable loading? Think about the problem carefully and write down the normal stress equation for point P marked by the red dot.



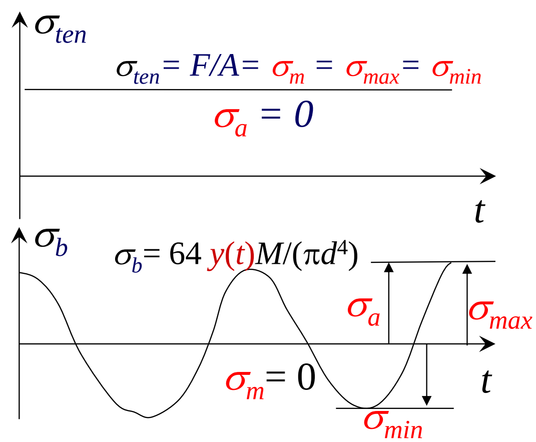


Figure 3-1. A simple shaft loaded with a pair of constant tensile loads, *F*, and a pair of constant bending moments, *M*. Note the stress notations are different for describing different load-stress relations; σ_{ten} is for the normal stress due to tension by *F*, σ_b for the normal stress due to bending by *M*.

Let's take a look at point P at the shaft surface. There should be two components of the normal stresses, one (σ_{ten}) due to the tensile force, F, and the other (σ_b) due to the bending moment M. For this case, the one due to tension should not change with time; however, the one due to bending does change with time. When point P is at the lowest position, the bending stress is tensile and in its maximum value. When point P is at the highest position, the bending stress is compressive and in its minimum value (but still maximum in magnitude). The bending stress is zero when point P passes through the neutral plane. Both stresses are plotted in **Figure 3-1**, and the maximum stress, σ_{max}, the minimum, σ_{min}, the stress variation amplitude, σ_a, and the midrange stress (or average stress, or the mean stress, the same for this case), σ_m, are shown for each stress. Clearly, the case shown in **Figure 3-1** is a variable loading case.

The total stress at point P is plotted in **Figure 3-2** as a function of time and expressed by Equation (3-1).

$$\sigma_p = 4F/(\pi d^2) + 64y(t)M/(\pi d^4) \tag{3-1}$$

Figure 3-2 also plots the maximum stress, σ_{max}, the minimum, σ_{min}, the stress amplitude, σ_a, and the midrange stress, σ_m, for this example, which is also a general case of stress variation. Note that here maximum stress σ_{max} and minimum stress σ_{min} are for the range of stress variations, different from those we used for analyzing principal stresses.

If the stress sequence repeats itself as a function of time, we say that the stress is cyclic. Cyclic stresses may have different behaviors: completely reversed if σ_m=0, released tension if σ_{min}=0, or released compression if σ_{max}=0.

$$\begin{aligned}
\sigma_m &= (\sigma_{max} + \sigma_{min})/2 = F/A \\
\sigma_a &= (\sigma_{max} - \sigma_{min})/2 = 32M/(\pi d^3) \\
\sigma_{max} &= \sigma_m + \sigma_a = F/A + 32M/(\pi d^3) \\
\sigma_{min} &= \sigma_m - \sigma_a = F/A - 32M/(\pi d^3)
\end{aligned} \tag{3-2}$$

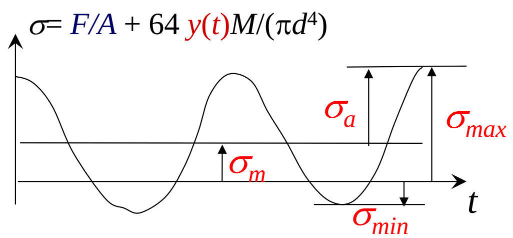


Figure 3-2. A variable stress, and definitions of the maximum stress, σ_{max}, the minimum stress, σ_{min}, the stress amplitude, σ_a, and the midrange stress (or the average stress), σ_m.

Stress variation accelerates failure of the material of a component. The amplitude of the stress variation and the number of cycles of stress variation matter a great deal.

3-2 Rotating beam tests and *S-N* diagrams

For a mechanical component operated under a variable load, the mode of failure is likely fatigue, which is a damage accumulation phenomenon. Materials and parts may have many flaws at the micro scale, some of which may grow into large cracks and cause the materials to fail. The number of cycles of stress variation means the life of a part subjected to a certain load. Generally, the load that a part can support decreases with the increase in stress cycles. Predicting the life of an actual engineering part is a difficult issue because of complicated part geometry, stress concentration, and operating conditions, etc. Let's first see the fatigue behavior of a material and take a look at its fatigue strength by "conducting" rotating-beam tests with a group of standard specimens shown in **Figure 3-3**. The specimen surfaces should be well polished.

A rotating beam fatigue tester, e.g. the R. R. Moore rotating-beam machine, is virtually used for the "experiments" here. The beam is subjected to pure bending without a transverse shear force. The fatigue strength of the material is measured by the stress at which the specimen fractures into two equal pieces. With the rotating beam tests, we can plot a diagram for the fatigue strength as a function of fatigue life, or the *S-N* diagram (the Wohler diagram) for a material. Here, *N* is the number of cycles of rotation (or stress variation) right before failure.

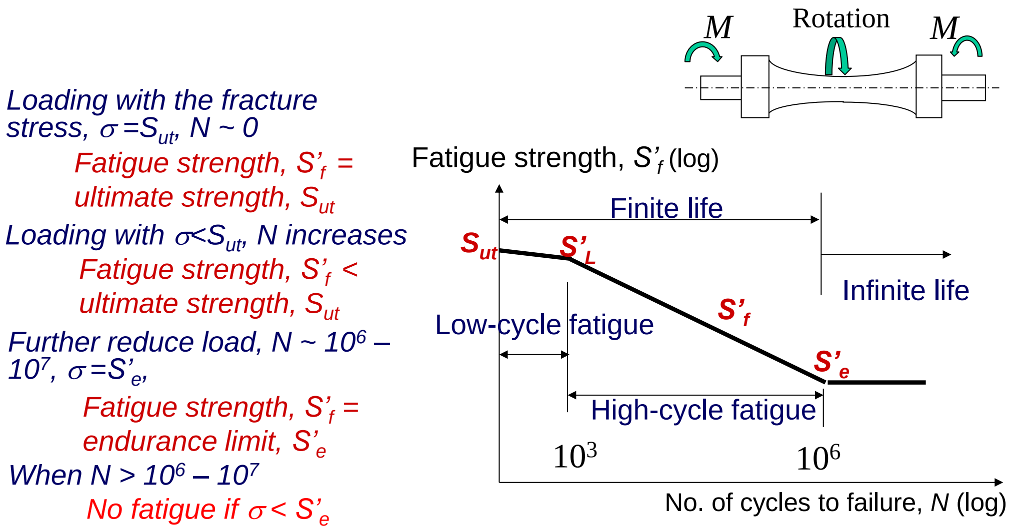


Figure 3-3. Rotating-beam tests, an *S-N* (log-log) diagram, and a standard specimen.

Let's "do" one *S-N* diagram for a ferrous alloy. When a static load is applied, the fatigue (fracture) strength has to be the ultimate limit, S_{ut}, or simply, S_u. The fatigue strength reduces as the number of cycles increases. There are two turning points observed. The first one defines the intersection between low-cycle fatigue and high-cycle fatigue, while the second defines a very important property of the material, the endurance limit. For a specimen whose working stress is lower than the endurance limit, its fatigue life may be infinite, i.e., no fatigue would occur.

Let's summarize our observations in important regimes.

When $N \sim 0$	The fatigue strength, S'_f = ultimate strength, $S_{ut}=S_u$
When N increases	The fatigue strength, S'_f < the ultimate strength, $S_{ut}=S_u$
When $N \sim 10^6$ - 10^7	The fatigue strength, S'_f = endurance limit, S'_e
When $N > 10^6$ - 10^7:	No fatigue, or the life is considered to be infinite.

Here, we use S'_f for the fatigue strength of a material, not for a mechanical component, and likely for others, superscript " ' " indicates the property of a material, not of a component. For the low-cycle fatigue ($N < 10^3$), we can simply reduce the level of the ultimate strength to obtain the low-cycle fatigue strength, S'_L for a material:

Bending:	$S'_L = 0.9\, S_{ut}$	(3-3-1)
Axial loading:	$S'_L = 0.75 S_{ut}$	(3-3-2)
Torsional loading:	$S'_L = 0.72 S_{ut}$	(3-3-3)

For the high-cycle fatigue ($10^6 \geq N \geq 10^3$), if we know S'_L and S'_e (the endurance limit of the material), we can find the fatigue strength, S'_f, on the log-scale straight line (note the units):

$$\log S_f' = b \log N + c \tag{3-4-1}$$

$$\text{Slope, } b = -\frac{1}{3}\log\left(\frac{S_L^{'}}{S_e^{'}}\right) = -\frac{1}{3}\left(\log\left(S_L^{'}\right) - \log\left(S_e^{'}\right)\right) \tag{3-4-2}$$

$$\text{Intercept, } \quad c = \log\left(\frac{\left(S_L^{'}\right)^2}{S_e^{'}}\right) \tag{3-4-3}$$

Thus, the fatigue strength can be found as $S_f^{'} = (10)^c (N)^b$ in MPa (3-2-4)

The unit for b is logMPa/logN and that for c is logMPa.

Endurance limit. The endurance limit, S'_e , may be identified for ferrous materials, as that clearly shown by the knee when N is about 10^6~10^7 in **Figure 3-3**. This is the case for most ferrous materials. However, most non-ferrous materials, such as aluminum alloys, copper alloys, and polymer materials, do not have an apparent endurance limit. The knee does not appear in their *S-N* diagrams. These materials will fatigue regardless of the stress amplitude. Instead, the fatigue strength at $50(10)^7$ cycles is given in many handbooks as the so-called endurance limit.

The endurance limits of steels for different types of loading may be approximated as follows.

Bending:	$S'_e = 0.5\, S_{ut}$	(3-5-1)
Axial loading:	$S'_e = 0.45 S_{ut}$	(3-5-2)
Torsional loading:	$S'_e = 0.29 S_{ut}$	(3-5-3)

The endurance limits for other materials may be found from reference books listed at the end of this chapter, or from materials handbooks (such as Metals Handbook). Note that these reductions are for pure loading modes. For a combined loading mode, we need to use either the distortion energy theory or the maximum shear stress theory to combine the stresses, and then use Eq. (3-5-1) or (3-5-3). Think, why more reduction is given to torsional loading? Is there any relationship between Eqs (3-5-1) and (3-5-3)?

Example 3-2. A steel (S_u = 1080MPa) is used for a shaft whose life should be 10,000 cycles. What is the maximum possible stress that this component can sustain?

Solution

This is a high-cycle fatigue problem, as marked in **Figure 3-4**. We need to use the power-law straight-line equation (3-4) to solve it.

Step 1: Find S'_e and S'_L

From Equations (3-5-1) and (3-3-1)

$S'_e = 0.5\, S_{ut} = 0.5\, S_u \quad S'_L = 0.9\, S_{ut}$

Step 2: Solve the power-law straight-line equation

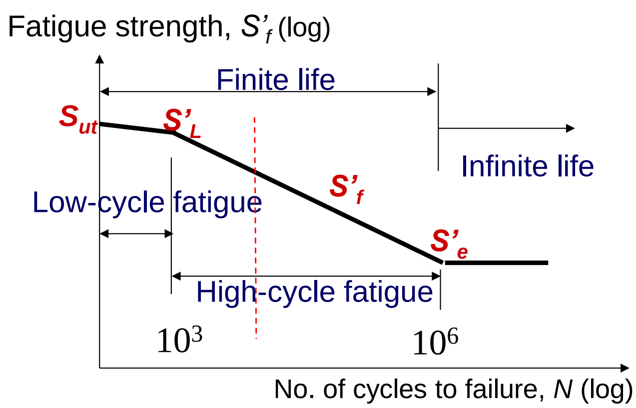


Figure 3-4 Cycle number and fatigue strength for the steel in Example 3-2.

The slope, $b = -\frac{1}{3}\log\left(\frac{S'_L}{S'_e}\right)$ =(1/3)log(0.9 S_{ut} /0.5 S_{ut})= -0.0851 (logMPa/logN)

The intercept, c = $c = \log\left(\frac{\left(S'_L\right)^2}{S'_e}\right)$ = log([0.9*1080]^2/[0.5*1080])= 3.2439 (logMPa)

Thus, the fatigue strength is $S_f' = (10)^c (N)^b = (10)^{3.2439}(10000)^{-0.0851}$ = 800 MPa, and this is the maximum possible stress that the shaft can sustain.

3-3 Endurance limit of a component, modified endurance limit

The endurance limit of a mechanical component, S_e, is related to, but different from the endurance limit of a material. A mechanical component is different from the rotating beam in many aspects. The following equation defines the endurance limit of a mechanical component, which is the modified endurance limit based on the endurance limit obtained from the rotating-beam test (or that listed in a handbook):

$$S_e = k_0 k_f k_s k_r k_t k_m\, S'_e \tag{3-6}$$

- S'_e: *Endurance limit from the rotating-beam tests, S'_e*
- k_m: *Miscellaneous factor*
- k_t: *Temperature factor*
- k_r: *Reliability factor*
- k_s: *Size factor*
- k_f: *Surface finish effect*
- k_0: *Stress-concentration reduction factor*
- S_e: *Endurance limit of a machine element*

3-3-1 Stress-concentration reduction factor, k_0. Stress concentration happens when a geometric discontinuity appears, Section 3.5 at the end of this chapter explains the analysis of stress concentration through notch sensitivity. The stress-concentration reduction factor is defined as

$$k_0 = 1/K_f \tag{3-7-1}$$

Here, K_f is named the fatigue-stress-concentration factor for normal stresses. For a shear stress, the fatigue stress concentration factor is K_{fs}.

$$K_f = \frac{\text{Maximum stress in a notched element}}{\text{Stress in a notch-free element}}$$

$$= 1 + (\underbrace{K_c}_{\text{Theoretical stress concentration factor}} -1)\, \underbrace{q}_{\text{Notch sensitivity}} \tag{3-7-2}$$

The stress concentration effect can be taken into account either by reducing the endurance limit by k_0, or by increasing the work stress by K_f . The latter may be more practical because different stress components (shear and normal, under different loads, corresponding to different notches) may have different fatigue stress concentration factors. See Section 3-5 at the end of this chapter for details.

3-3-2 Surface finish factor, k_f. Surface finish of a mechanical component is different from that of the rotating-beam specimens. A factor has to be determined to take into account the effect of surface finish.

The surface finish factor, k_f**,** is empirically fit into the following equation:

$$k_f = e(S_{ut})^f \tag{3-8}$$

Coefficients *e* and *f* are defined in **Table 3-1**.

Table 3-1 Coefficients *e* and *f* in Equation (3-8)
(Shigley and Mischke 1989)

Manufacturing process	Factor *e* (MPa)	Factor *e* (Ksi)	Exponent *f*
Grinding	1.58	1.34	-0.085
Machining or cold drawing	4.51	2.70	-0.265
Hot rolling	57.7	14.4	-0.718
As forged	272.0	39.9	-0.995

3-3-3 Size factor, k_s. The geometry of a mechanical component is definitely not the same as that of the rotating-beam specimens. A factor has to be defined to take into account the effect of size.

The size factor for a round bar is defined based on the type of loading. For bending and torsion, the following equations are useful:

$k_s = 0.869\ (d)^{-0.112}$	$0.3" < d < 10"$	(d in inch)	(3-9-1)
$k_s = 1$	$0.3" > d$, or $d \leq 8$ mm		(3-9-2)
$k_s = 1.189\ (d)^{-0.112}$	$8\text{ mm} < d \leq 250\text{ mm}$	(d in mm)	(3-9-3)

For axial loading,

$$k_s = 1 \tag{3-9-4}$$

3-3-4 Reliability factor, k_r. Because fatigue is closely related to pre-existing material defects, uncertainty is always a concern. **Table 3-2** lists some of the commonly used reliability factors based on the requirement of probability of survival.

Table 3-2 Reliability factor, k_r (Hamrock et al 1999)

Percentage probability of survival	Reliability factor, k_r
90	0.90
95	0.87
99	0.82
99.9	0.75

3-3-5 Temperature factor, k_t. Temperature affects material properties and stresses, and therefore affects fatigue of a part. A temperature factor may be defined by the following ratio.

$$k_t = S_{ut}/S_{ut,\ ref} \tag{3-10}$$

Here, S_{ut} is the ultimate tensile strength at a desired temperature, $S_{ut,ref}$ is the ultimate tensile strength at a reference temperature, which is usually the room temperature.

3-3-6 Miscellaneous factor, k_m. Many other processes and conditions may also affect fatigue of a component. These influences can be taken into account by this miscellaneous factor.

3-4 Fatigue failure criteria

The specimen in the rotating beam tests is subjected to the bending stress amplitude only, or its normal stress is completely reversed. In reality, loading conditions of mechanical components are much more complicated than that of the rotating beam. Both normal and shear stresses may exist. Both stress midrange and amplitude may be not zero. There are quite a few theories and criteria that consider both non-zero midrange and amplitude of stresses. The Goodman line is one of the widely accepted criteria for designing parts against fatigue failure. **Figure 3-5** shows the Goodman line for the design for an infinite fatigue life (because S_e is used) together with the yield line. The Goodman line is so constructed that the midrange and the amplitude of a stress state, in terms of the von Mises stress or other type of stresses obtained from a failure theory, are used as the horizontal and vertical coordinates. The horizontal axis corresponds to static loading (σ_a =0), while the vertical axis is for completely reversed loading (σ_m =0). When a stress state has a large midrange component, checking yield is of the primary importance. The safe-design region is under both the Goodman line and the yield line.

The Goodman line is created using the fully modified endurance limit and the ultimate tensile strength of the material under consideration. Can we construct a Goodman line for the fatigue strength with respect to a finite life?

Goodman's line for failure: $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1 \qquad \text{(3-11-1)}$$

Goodman's line for design: $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_s} \qquad \text{(3-11-2)}$$

Yield line for failure: $$\frac{\sigma_a}{S_y} + \frac{\sigma_m}{S_y} = 1 \qquad \text{(3-12-1)}$$

Yield line for design: $$\frac{\sigma_a}{S_y} + \frac{\sigma_m}{S_y} = \frac{1}{n_s} \qquad \text{(3-12-1)}$$

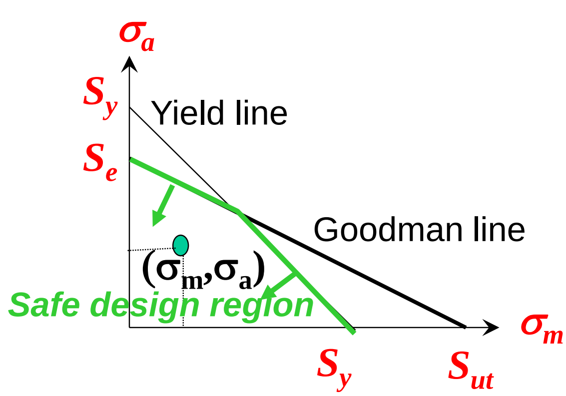


Figure 3-5. Goodman line for the design for infinite life, together with the yield line. The von Mises stress may be used. The green dot indicates the working stress state of a part inside the safe-design region.

In **Figure 3-5**, the failure lines are described by Equations (3-11-1) and (3-12-1). Equations 3-11-2) and (3-12-2) are for the safe design use because a factor of safety, n_s, is involved. The design calculation process may be summarized as follows:

A. Draw a mechanical model from the designed structure, FBD
B. Conduct force analyses.
C. Conduct stress analyses for bending, torsion, tension, compression, and transverse shear; analyze the principal stresses, both amplitude and midrange.
D. Apply a design theory. For ductile materials (shafts are usually made of ductile materials), use the maximum shear stress theory or the distortion energy theory, to determine the amplitude and midrange of stresses.
E. Determine the endurance limit, or the fatigue strength, of the material used, and the modification factors.
F. Consider stress concentration factor K_f or K_{fs} for the stress amplitude (then, do not use k_0).
H. Apply the Goodman line.
I. Check yield.
J. Modify the design and repeat A-H if necessary until n_s is satisfactory.

Example 3-3 The mechanics analysis of the most critical point of a shaft results in the following stress components: σ_{xm} = 30 MPa (midrange), σ_{xa} = 70 MPa (amplitude), and τ_{xym} = 50MPa (midrange only, or static). The material is an AISI 1040 medium-carbon steel (yield strength: S_y = 350 MPa, ultimate strength, S_{ut} = 520 MPa). The result of the correction factors is $k_f k_s k_r k_t k_m = 0.9$. The fatigue stress-concentration factors for the normal and shear stresses are respectively K_f=1.5, K_{fs}=1.3. Let's calculate the factors of safety by using the distortion-energy theory for the failure stress and the Goodman line for fatigue.

Solution

This problem is for the rotating shaft shown below, **Figure 3-6**. Steps A~C have been done.

Figure 3-6. Example 3-3, loading and the stresses in the Goodman diagram with both the Goodman line and the yield line.

Step D: von Mises stress, $\sigma_{vm} = \sqrt{(\sigma_x)^2 + 3(\tau_{xy})^2}$

How can we handle the midrange and amplitude of the von Mises stress? Assuming the shear stress and normal stress vary at the same frequency, **Figure 3-7** shows the ranges of the variations of the normal stress (along the vertical axis) and $\sqrt{3}\tau$, as well as $\sigma_{VM} = \sqrt{(\sigma\ )^2 + 3(\tau\ )^2}$, which is like a vector of σ and $\sqrt{3}\tau$. Therefore,

$\sigma_{VM\_m} = \sqrt{(\sigma_m)^2 + 3(\tau_m)^2}$, and $\sigma_{VM\_a} = \sqrt{(\sigma_a)^2 + 3(\tau_a)^2}$. Make sure that K_{sf} and K_f are applied to the stress amplitudes.

Note that **Figure 3-7** is accurate if the normal and shear stresses have the same frequency; otherwise, this is a conservative approach because usually the bending-related stress frequency is higher, the same as the rotation frequency. The torsional shear stress variation is likely related to the duty cycle of a component, for example, start-stop, reverse of rotation, which is at a much lower frequency.

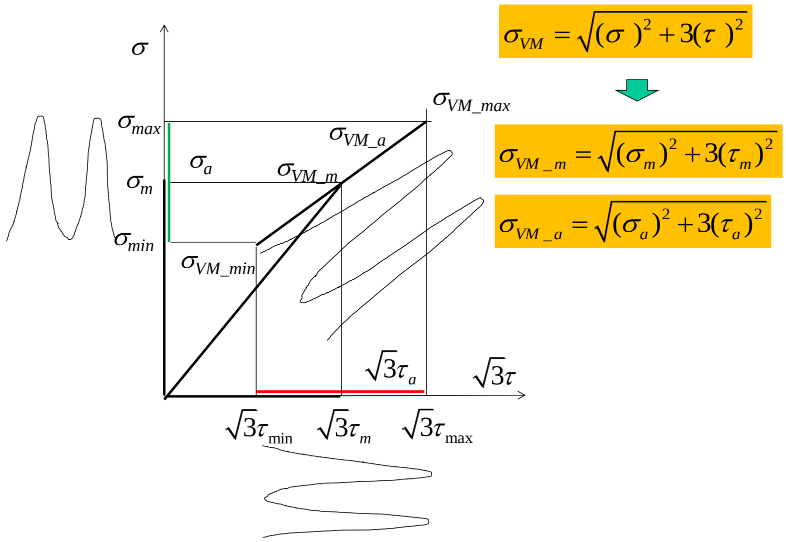


Figure 3-7 Midrange and amplitude of the von Mises stress

With what mentioned above, Equation (3-11-2) becomes the following

$$\frac{\sqrt{(K_f\sigma_{xa})^2 + 3(K_{fs}\tau_{xya})^2}}{k_f k_s k_r k_t k_m S'_e} + \frac{\sigma_{VM-m}}{S_{ut}} = 1/n_s \qquad (3\text{-}11\text{-}2')$$

where

Midrange $\sigma_{vm-m} = \sqrt{(\sigma_{xm})^2 + 3(\tau_{xym})^2} = [(30)^2 + 3(50)^2\ ]^{1/2} = 91.7$ MPa

Amplitude $\sigma_{vm-a} = \sqrt{(K_f\sigma_{xa})^2 + 3(K_s\tau_{xya})^2} = [(1.5*70)^2 + 3(1.3*0)^2\ ]^{1/2} = 105$ MPa

Step E: Endurance limit:

$S_e = k_f k_s k_r k_t k_m S'_e = 0.9(0.5)(520) = 234$ MPa

Step F-G: Goodman line, Equation (3-11-2) $\frac{\sigma_{vm-a}}{S_e} + \frac{\sigma_{vm-m}}{S_{ut}} = \frac{1}{n_s}$, or Equation (3-11-2')

$\frac{\sigma_{vm-a}}{S_e} + \frac{\sigma_{vm-m}}{S_{ut}} = 1/n_s = 105 / 234 + 91.7 / 520$ $n_s = 1.6$

Step H: Yield line $\frac{\sigma^0{}_{vm-a}}{S_y} + \frac{\sigma_m}{S_y} = \frac{1}{n_s}$. Note that the peak stress has no modification by the stress concentration factors.

The peak von Mises stress: $\sigma_{vm} = \sqrt{(\sigma_{xa} + \sigma_{xm})^2 + 3(\tau_{xya} + \tau_{xym})^2}$

$$\sigma_{vm} = \sqrt{(\sigma_{xa} + \sigma_{xm})^2 + 3(\tau_{xya} + \tau_{xym})^2} = \sqrt{(70+30)^2 + 3(50)^2} = 132.3Mpa$$

$\frac{\sigma^0{}_{vm-a}}{S_y} + \frac{\sigma_m}{S_y} = \frac{\sigma_{vm}}{S_y} = \frac{1}{n_s}$, $n_s = S_y / \sigma_{vm} = 350 / 132.3$ $n_S = 2.65$

We only need to check the peak stress for static loading. Most shafts are made of ductile materials, and there is no need to consider stress concentration for static loading.

Step I is not needed for this case.

Other lines for design

There are other criteria. However, for this class, we will only focus on the Goodman line.

Gerber's line for failure: $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1$ (3-13-1)

Gerber's line for design: $\frac{n_s\sigma_a}{S_e} + \left(\frac{n_s\sigma_m}{S_{ut}}\right)^2 = 1$ (3-13-2)

Soderberg's line for failure: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$ (3-14-1)

Soderberg's line for design: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n_s}$ (3-14-2)

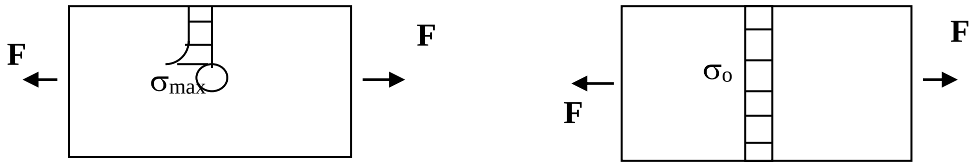
3-5. Determination of stress concentration factors

1 **Theoretical stress-concentration factors for normal and shear stresses: *K_c, K_{cs}*.** Capital *K*, not the lower case, is used for the stress concentration factor. The lower case of *k* is used for the factors that modify the endurance limit.

K_c = <u>Maximum stress in notched specimen</u>
Stress in notch-free specimen (or nominal stress)
$= \sigma_{max}/\sigma_o$

$K_{cs} = \tau_{max}/\tau_o$

A geometric discontinuity is called a "notch." **Figure 3-8** compares the normal stresses in plates with and without a notch, which is a hole there.



Notched Notch free

Figure 3-8 Notch effect.

2 **Fatigue stress-concentration factor for normal and shear stresses K_f, K_{fs}.** K_c and K_{cs} may over estimate the effect of stress concentration and should be reduced by considering the sensitivity of a material to stress concentration. K_f and K_{fs} are defined as:

$$K_f = 1 + q\,(K_c - 1)$$
$$K_{fs} = 1 + q(K_{cs} - 1)$$

3 Notch sensitivity *q*. Notch sensitivity q is experimentally determined to reflect the sensitivity of a material to the stress concentration.

If $q = 0$ then $K_f = 1$, the material is not sensitive to notches.
If $q = 1$then $K_f = K_c$, the material is very sensitive to notches.

Figure 3-9 plot the values of theoretical stress concentration factor, K_c, at a shoulder due to different loading modes.

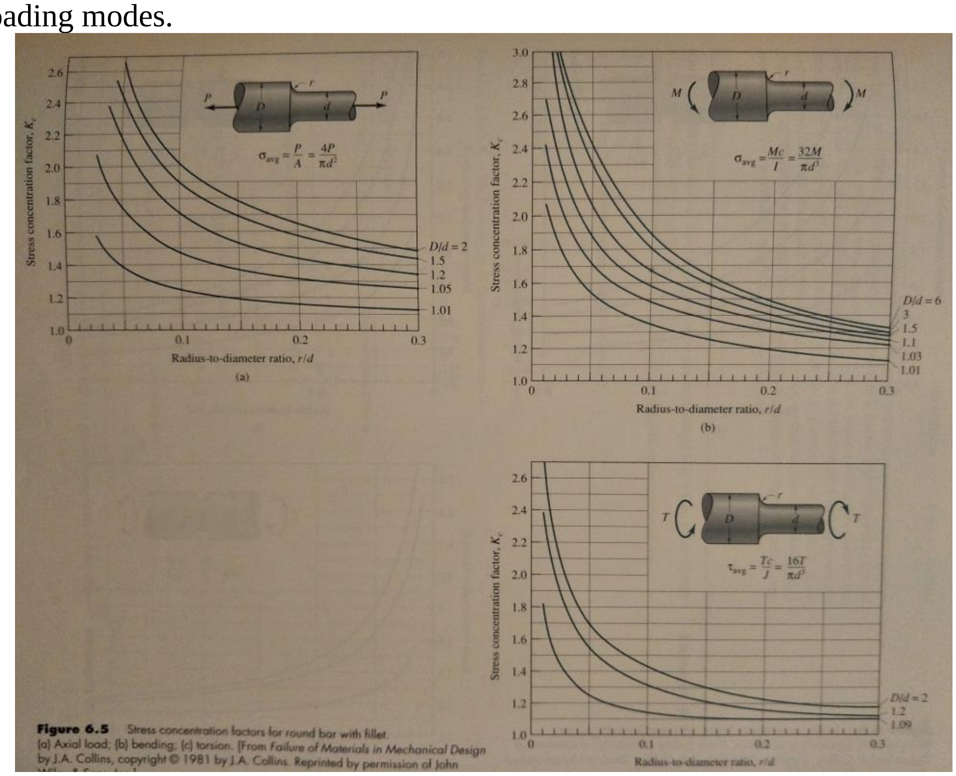


Figure 3-9 Theoretical stress concentration factor, K_c, for different loading modes. From Hamrock, et al., 2005, "Fundamentals of Machine Elements," McGraw Hills.

Figure 3-10 plot the values of q for different materials and loading forms. For cast iron: q = 0.2, which means not very sensitive because too many defects are already in materials, and the effects of these defects are reflected by their strengths. Several curves cited from the references are pasted at the end of this chapter.

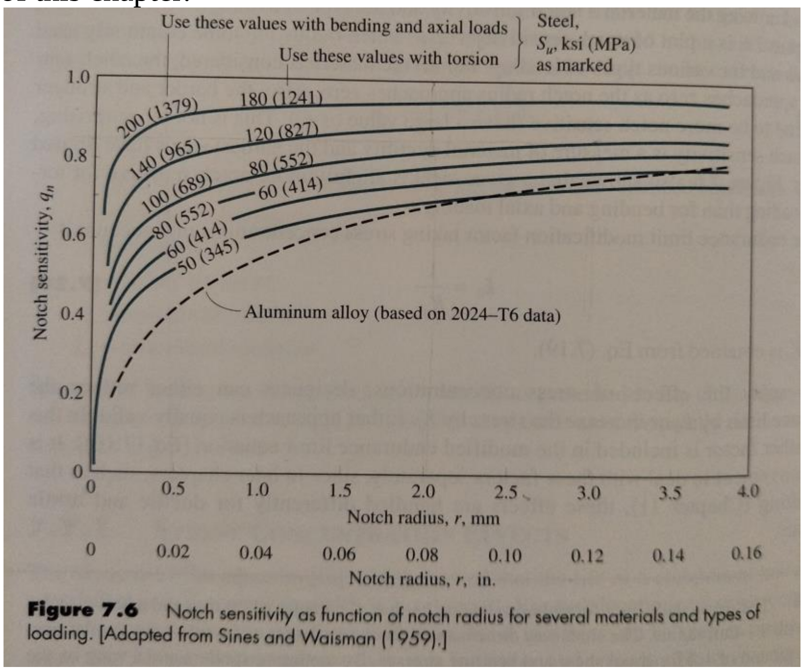


Figure 3-10. Data for notch sensitivity. From Hamrock, et al, 2005.

Applications. Brittle materials require the consideration of stress concentration if a notch exists. For ductile materials, stress concentration is only considered for the amplitude of a variable stress.

Example 3-4. Determine the fatigue stress-concentration factors, K_f and K_{fs}, at the shaft shoulders, shown in **Figure 3-11**, for (a) cast iron and (b) drawn steel, S_{ut} = 700 MPa.

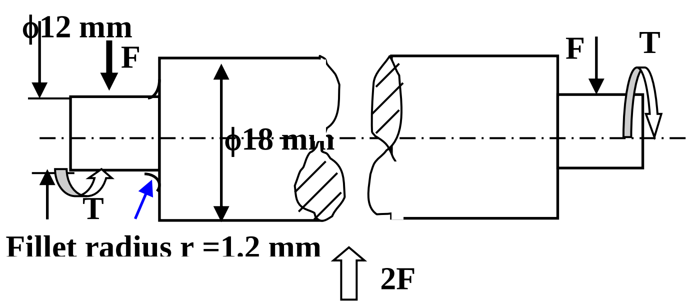


Figure 3-11.   Example 3-4.

Solution:

(a) $K_f = 1 + q(K_c - 1)$ - for normal stress (bending)

$K_{fs} = 1 + q(K_{cs} - 1)$ - for shear stress (torsion)

Find K_c (for bending) and K_{cs} (for torsional shear) from **Figure 3-9**

K_c and K_{cs} are related to geometry and the types of loading

D/d = 18/12 = 1.5 r/d = 1.2/12 = 1/10 = 0.1
Figure 3-9 (b) $K_c \approx 1.67$

Figure 3-9 (c)
$K_{cs} \approx 1.42$ (Data are highly nonlinear, and the curve for D/d=1.5 should be very close to that for D/d = 2)

(b) Notch sensitivity q is related to materials and types of loading

(1) For the cast iron, $q = 0.2$
Therefore, $K_f = 1 + 0.2(1.68 - 1) = 1.14$
and $K_{fs} = 1 + 0.2(1.42 - 1) = 1.08$

(2) For the drawn steel whose S_{ut} = 700 MPa,

For bending, in **Figure 3-10,** r = 1.2 mm, use the curve for S_u = 689 MPa
$q \approx 0.77$
$K_f = 1 + 0.77*(1.68 - 1) = 1.524$

For torsion, in **Figure 3-10**, r = 1.2 mm, do interpolation between the curves for S_u = 552 and 827 MPa
$q \approx 0.82$
$K_{fs} = 1 + 0.82(1.42 - 1) = 1.34$

Chapter summary. This chapter introduces the concepts of variable loading and fatigue failure as a result of variable loading. Stress amplitude and midrange are defined. Fatigue strength vs. fatigue life, or the *S-N* curve, is introduced. The low-cycle fatigue life, high-cycle fatigue life, and infinite life are explained. Several fatigue related items are also defined, which are the fatigue strength and endurance limit of a material, the fatigue strength and endurance limit of an engineering component, between them are modification factors involved in the latter. Goodman's line is defined and used as a portion of the safe design border guarding against fatigue failure, while the yield line is implemented for the other part of the safe design border against yield under a heavy static load peek. The application of the Goodman line for shaft design will be introduced in the next chapter.

References
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Reley, W., Sturges, L., and Morris, D., 1999, "Mechanics of Materials," Willey.

Media: VariableLoading, A-MPlot are on U-Tube. **315 students should watch these short movies before the class.**