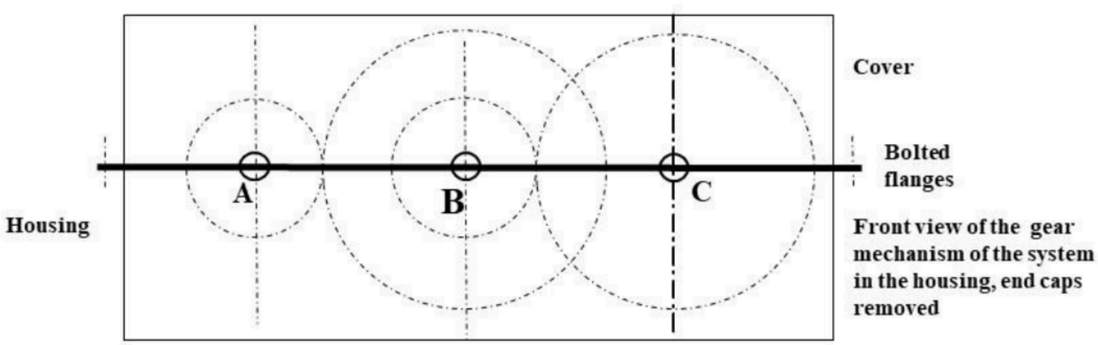
ME 315

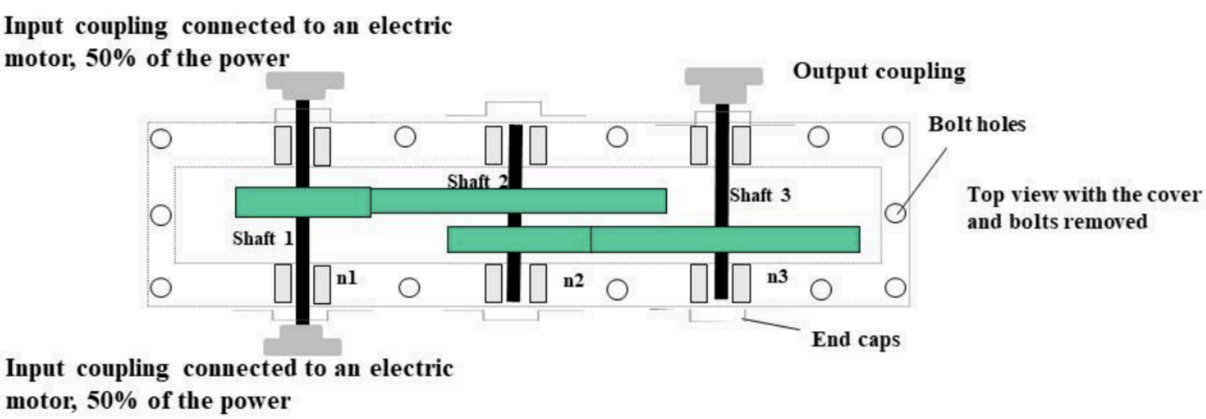
THEORY OF MACHINES - DESIGN OF ELEMENTS

Fall 2023

Design Project

Group Submission





Group members

Chirag Bachani	Shaft system 1
Winston Zhao	Shaft system 2
Christopher Luey	Shaft system 3

Grade

On-time step dues	/5
Structural design, drawings	/40
Design analysis	/35
Report writing	/20

Total __________

Final Project

Christopher Luey, Winston Zhao, Chirag Bachani

Introduction

In our team's engineering project, we designed a robust two-stage gear transmission system depicted in the accompanying figure, aiming to efficiently transmit power from input shaft A to output shaft C with a speed reduction. The system, comprising input shaft A driven by two electric motors, center shaft B with corresponding gears, and output shaft C with a coupling set, was carefully arranged, considering the spatial relationship between A and C. Operating at 2600 rpm, the input motors delivered 70KW to shaft A, with the goal of achieving an output speed (n3) of around 400 ±3 rpm, assuming 100% overall efficiency. Our design incorporated fatigue stress concentration factors to address bending and torsional stresses, and determined geometry using a factor of safety of n=1.1. With a commitment to durability, the transmission system is expected to function continuously for at least 5 years, 7 days a week, 52 weeks a year, and 8 hours a day, with a reliability rate of 99%.

Gear Train

Part	Gear 1	Gear 2	Gear 3	Gear 4
N [teeth]	19	47	27	71
RPM [rev/min]	2600.000	1051.064	1051.064	399.700
Power [kW]	70.0	70.0	70.0	70.0

Gear Train Result	
Ratio	6.5049
Output Speed (RPM)	399.7003
% Error	-0.0749%

Gears

Gear Properties	
Material	AISI 1045 Steel
Module [mm]	5.00
Pitch Angle [Deg]	20.00
Addendum [mm]	5.00
Dedendum [mm]	6.25
Clearance [mm]	1.25
Tooth Depth [mm]	11.25

	Gear 1	**Gear 2**	**Gear 3**	**Gear 4**
Material Grade	2	1	2	1
Hardness [HB]	500	400	500	400
Pitch Diameter [mm]	95.0	235.0	135.0	355.0
Outer Circle [mm]	105.0	245.0	145.0	365.0
Root Circle [mm]	83.0	223.0	123.0	343.0
Face Width [mm]	65.0	60.0	65.0	60.0
Centerline Distance [mm]	165.0		245.0	
Bending Factor of Safety	4.1069	3.0405	2.7465	2.0373
Contact Stress Factor of Safety	1.4576	1.1201	1.4859	1.1462

Shafts

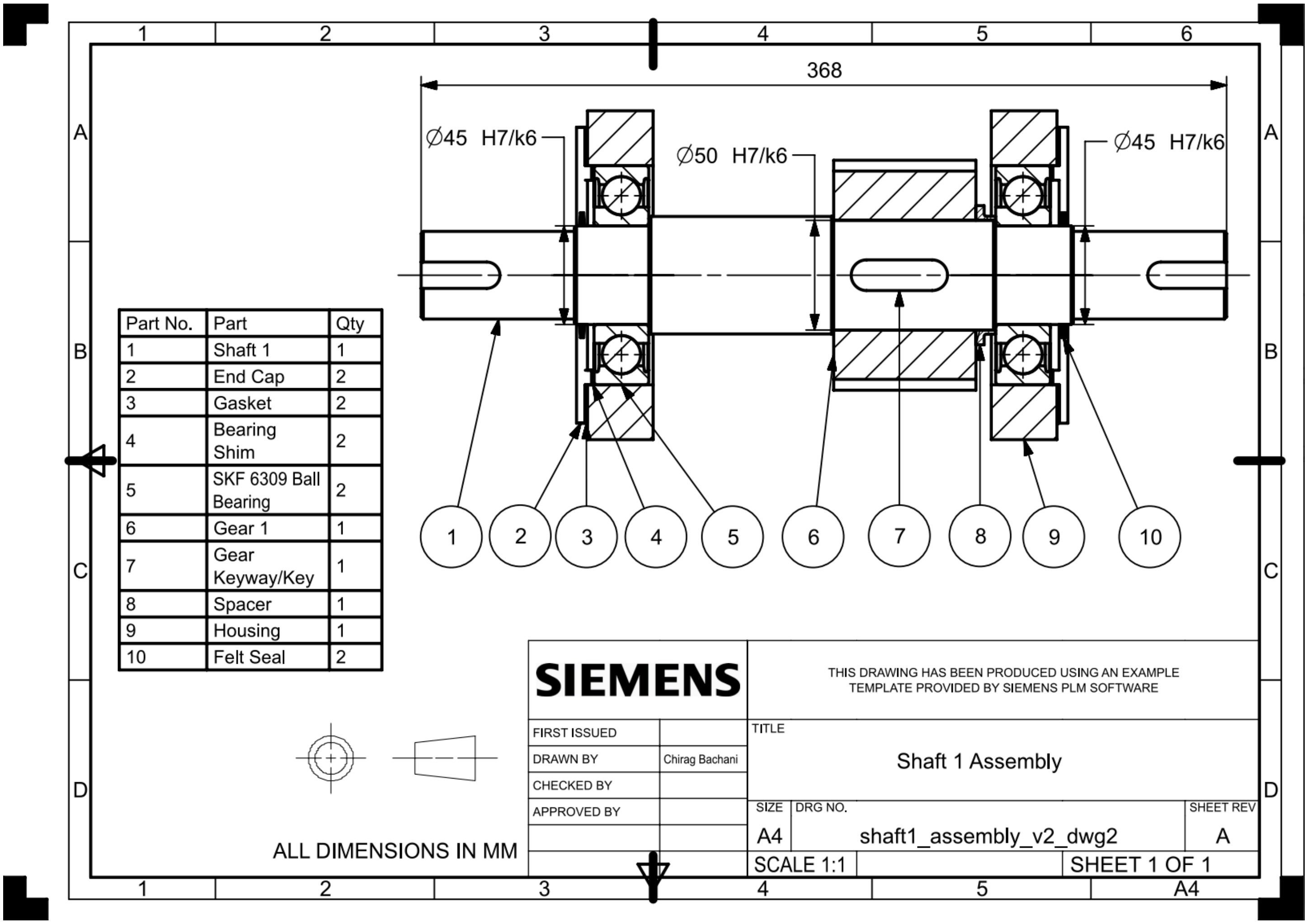
	Shaft 1	Shaft 2	Shaft 3
Material	AISI 1045 Carbon Steel		
RPM [rev/min]	2600.0000	1051.0638	399.7003
Max Bending Magnitude [Nm]	205.7115	451.2470	343.7668
Max Torque [Nm]	257.096	635.975	1672.380
Minimum Shaft Diameter [mm]	17.000	No pure torque	40.000
Critical Cross Section Location	Gear 1 Center	Gear 3 Center	Gear 4 Center
Shaft Diameter at Critical Cross Section [mm]	45.000	60.000	55.000
Goodman Factor of Safety	1.8467	1.8236	1.6866
Yielding Factor of Safety	7.3976	7.3851	6.3999
Lifetime	Infinite (> 10^6 Cycles)		

Bearings

	Shaft 1		Shaft 2		Shaft 3	
As shown in Assembly Drawing	Left	Right	Left	Right	Left	Right
Radial Load [N]	1645.692	4114.230	8204.490	6835.208	6875.336	3151.196
Thrust Load [N]	0		0		0	
Equivalent Load Used in Calculation [N]	4114.230		8204.490		6875.336	
C [N]	54081.734		79744.149		48414.597	
Bearing (SKF)	6309		6312		6311	
Bore [mm]	45		65		55	
OD [mm]	100		140		120	
Width [mm]	25		33		29	
C, bearing [N]	55300		97500		74100	
C0, bearing [N]	31500		60000		45000	
Expected Life [Mil Cycles]	2428.3404		1678.2594		1251.9091	
Expected Life [Yrs]	5.3455		9.1288		17.9265	

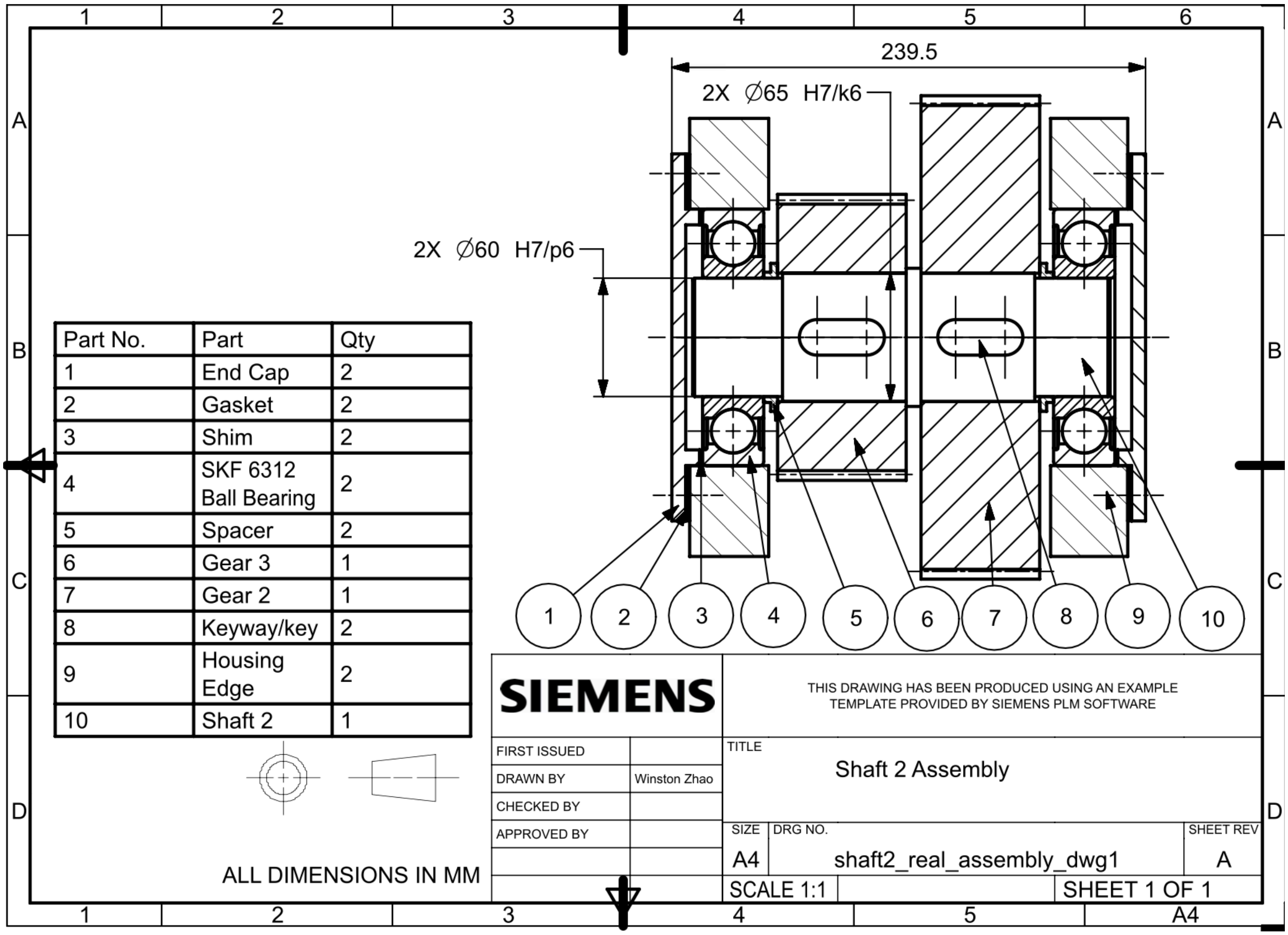
Assembly Drawings

Shaft 1



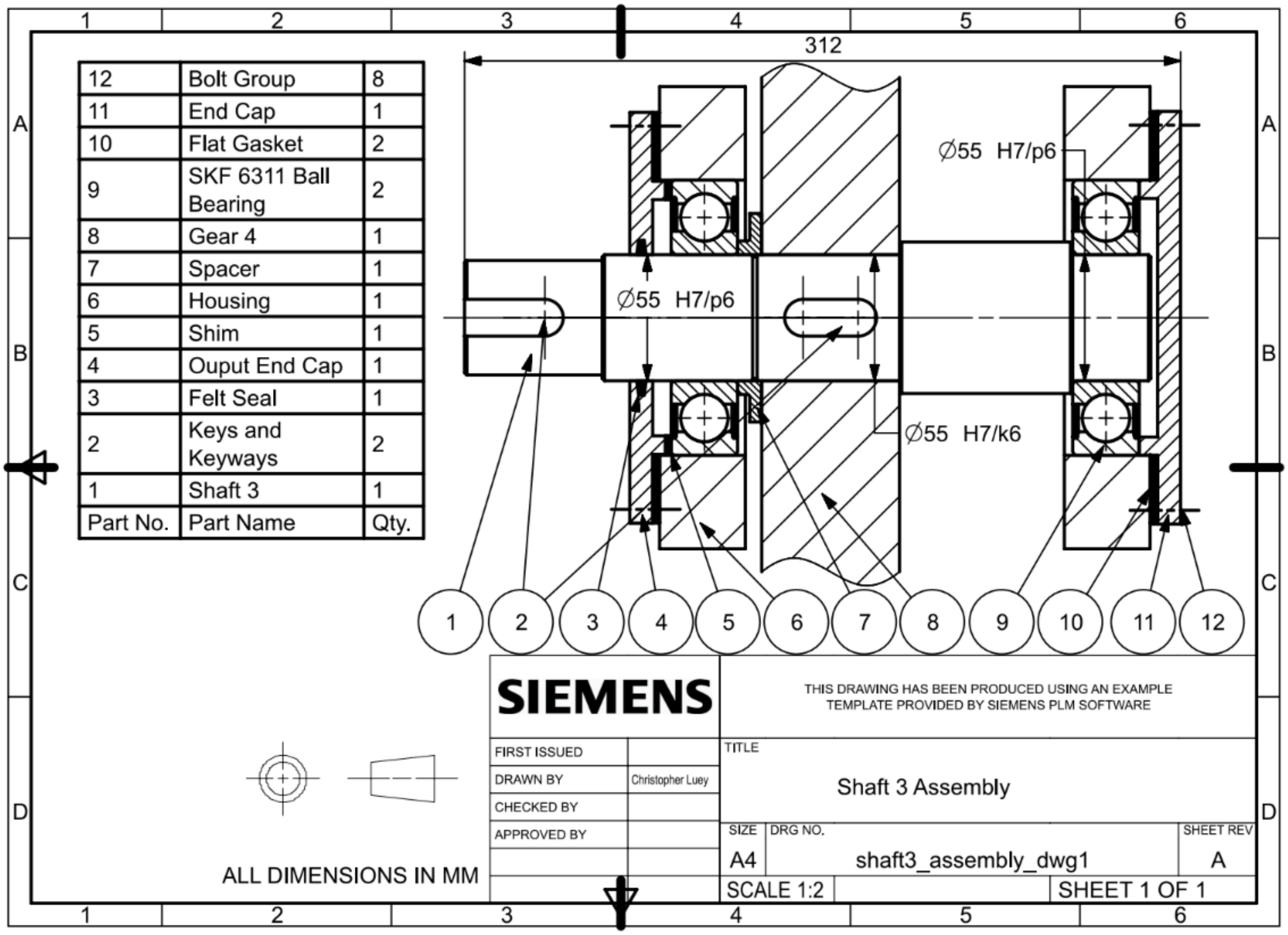
Part No.	Part	Qty
1	Shaft 1	1
2	End Cap	2
3	Gasket	2
4	Bearing Shim	2
5	SKF 6309 Ball Bearing	2
6	Gear 1	1
7	Gear Keyway/Key	1
8	Spacer	1
9	Housing	1
10	Felt Seal	2

Shaft 2



Part No.	Part	Qty
1	End Cap	2
2	Gasket	2
3	Shim	2
4	SKF 6312 Ball Bearing	2
5	Spacer	2
6	Gear 3	1
7	Gear 2	1
8	Keyway/key	2
9	Housing Edge	2
10	Shaft 2	1

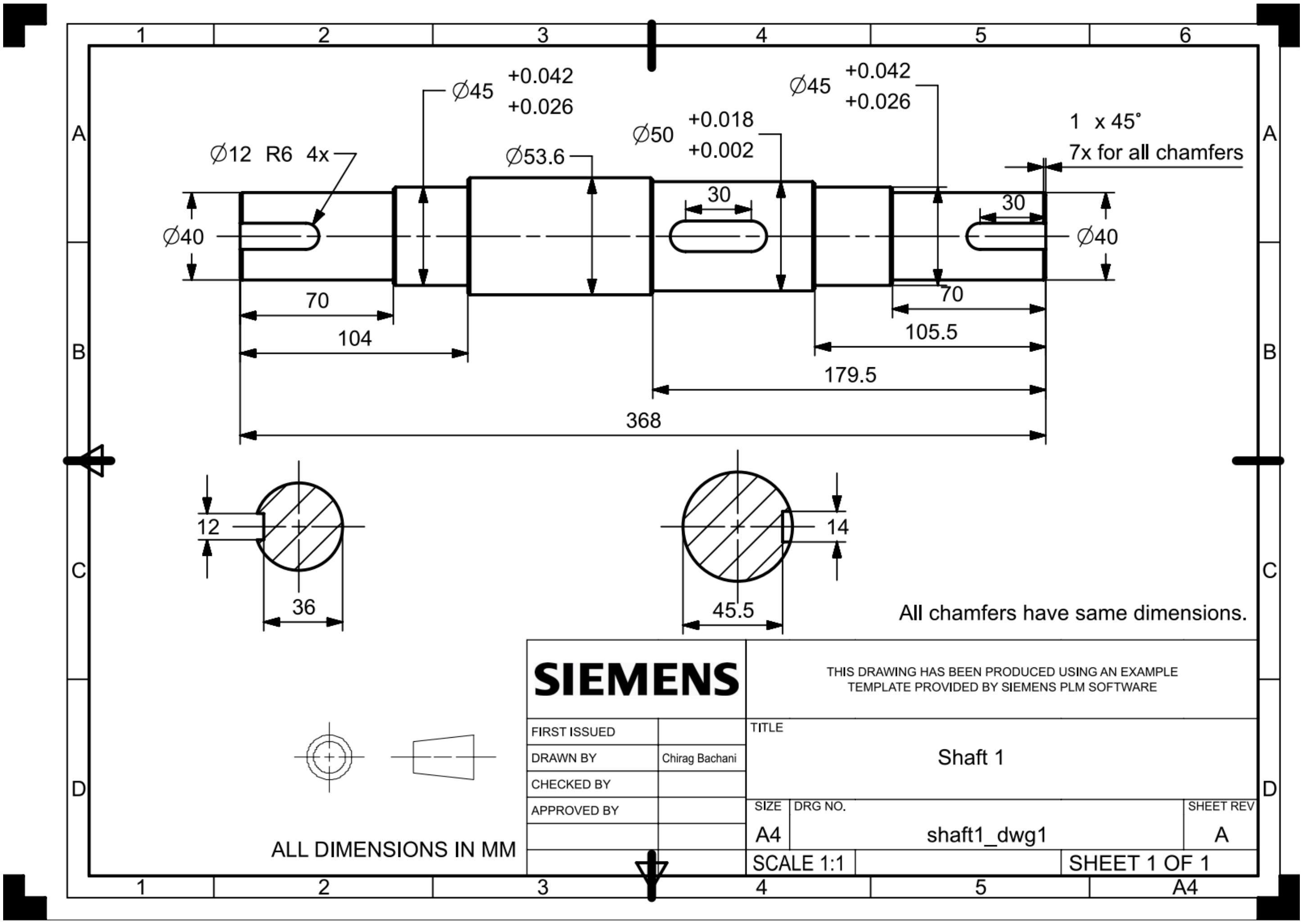
Shaft 3



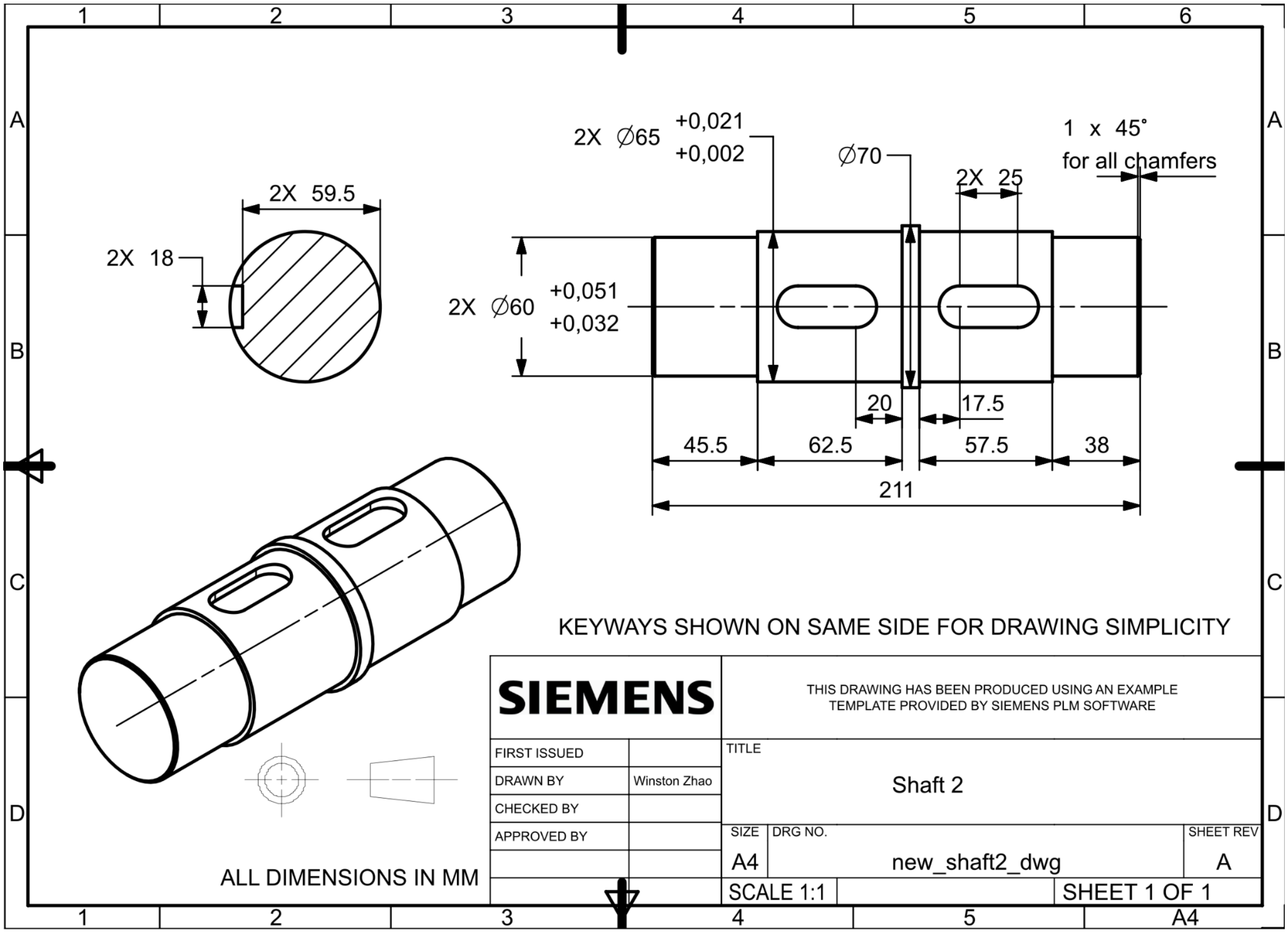
12 Bolt Group 8
11 End Cap 1
10 Flat Gasket 2
9 SKF 6311 Ball Bearing 2
8 Gear 4 1
7 Spacer 1
6 Housing 1
5 Shim 1
4 Ouput End Cap 1
3 Felt Seal 1
2 Keys and Keyways 2
1 Shaft 3 1
Part No. Part Name Qty.
312
Ø55 H7/p6
Ø55 H7/p6
Ø55 H7/k6
SIEMENS
THIS DRAWING HAS BEEN PRODUCED USING AN EXAMPLE TEMPLATE PROVIDED BY SIEMENS PLM SOFTWARE
FIRST ISSUED
DRAWN BY Christopher Luey
CHECKED BY
APPROVED BY
TITLE
Shaft 3 Assembly
SIZE A4
DRG NO. shaft3_assembly_dwg1
SHEET REV A
SCALE 1:2
SHEET 1 OF 1
ALL DIMENSIONS IN MM

Shaft Drawings

Shaft 1



Shaft 2



2X ∅65 +0,021 +0,002
∅70
1 x 45˚
for all chamfers
2X 25
2X 59.5
2X 18
2X ∅60 +0,051 +0,032
20
17.5
45.5
62.5
57.5
38
211
KEYWAYS SHOWN ON SAME SIDE FOR DRAWING SIMPLICITY
SIEMENS
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Winston Zhao
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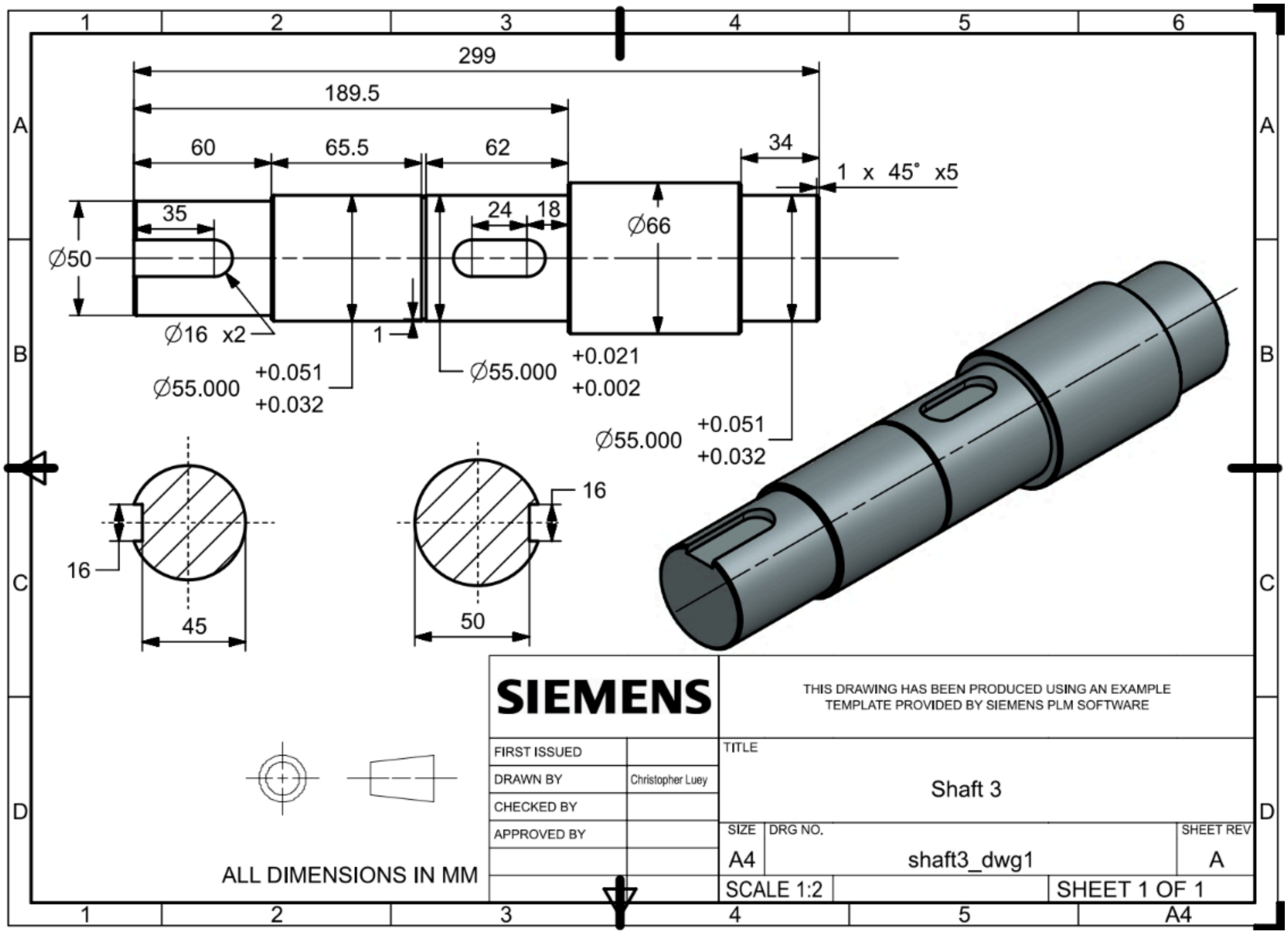
APPROVED BY
TITLE
Shaft 2
SIZE
A4
DRG NO.
new_shaft2_dwg
SHEET REV
A
SCALE 1:1
SHEET 1 OF 1
ALL DIMENSIONS IN MM

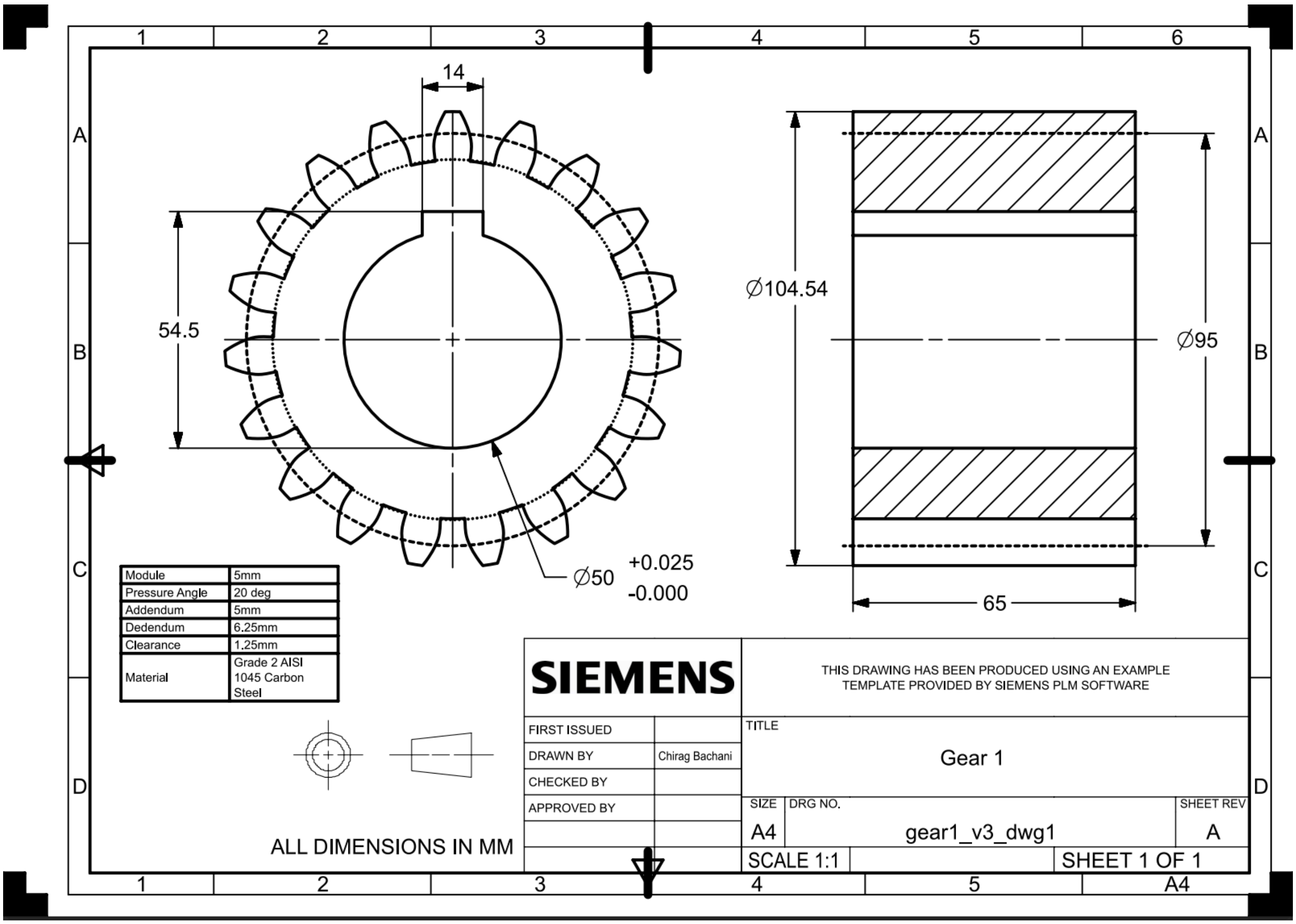
Shaft 3



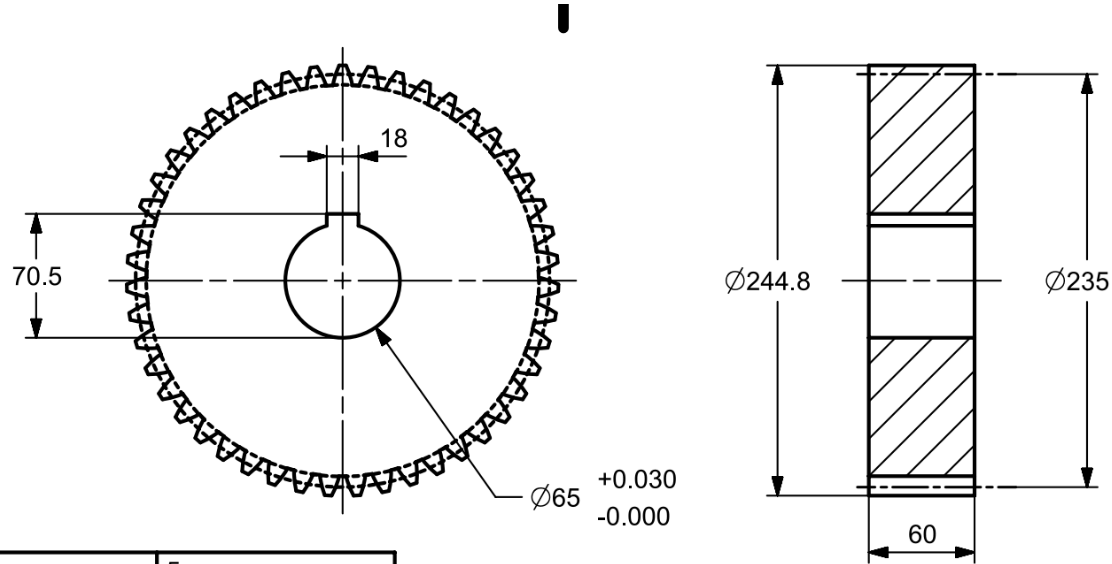
299
189.5
60
65.5
62
34
1 x 45° x5
35
24
18
Ø66
Ø50
Ø16 x2
1
Ø55.000 +0.051 +0.032
Ø55.000 +0.021 +0.002
Ø55.000 +0.051 +0.032
16
16
45
50
SIEMENS
THIS DRAWING HAS BEEN PRODUCED USING AN EXAMPLE TEMPLATE PROVIDED BY SIEMENS PLM SOFTWARE
FIRST ISSUED
DRAWN BY Christopher Luey
CHECKED BY
APPROVED BY
TITLE
Shaft 3
SIZE A4
DRG NO. shaft3_dwg1
SHEET REV A
SCALE 1:2
SHEET 1 OF 1
ALL DIMENSIONS IN MM

Gear Drawings

Gear 1



Gear 2



Module	5mm
Pressure Angle	20 degree
Addendum	5 mm
Dedendum	6.25 mm
Clearance	1.25 mm
Material	AISI 1045 carbon steel grade 1 steel

ALL DIMENSIONS IN MM

SIEMENS

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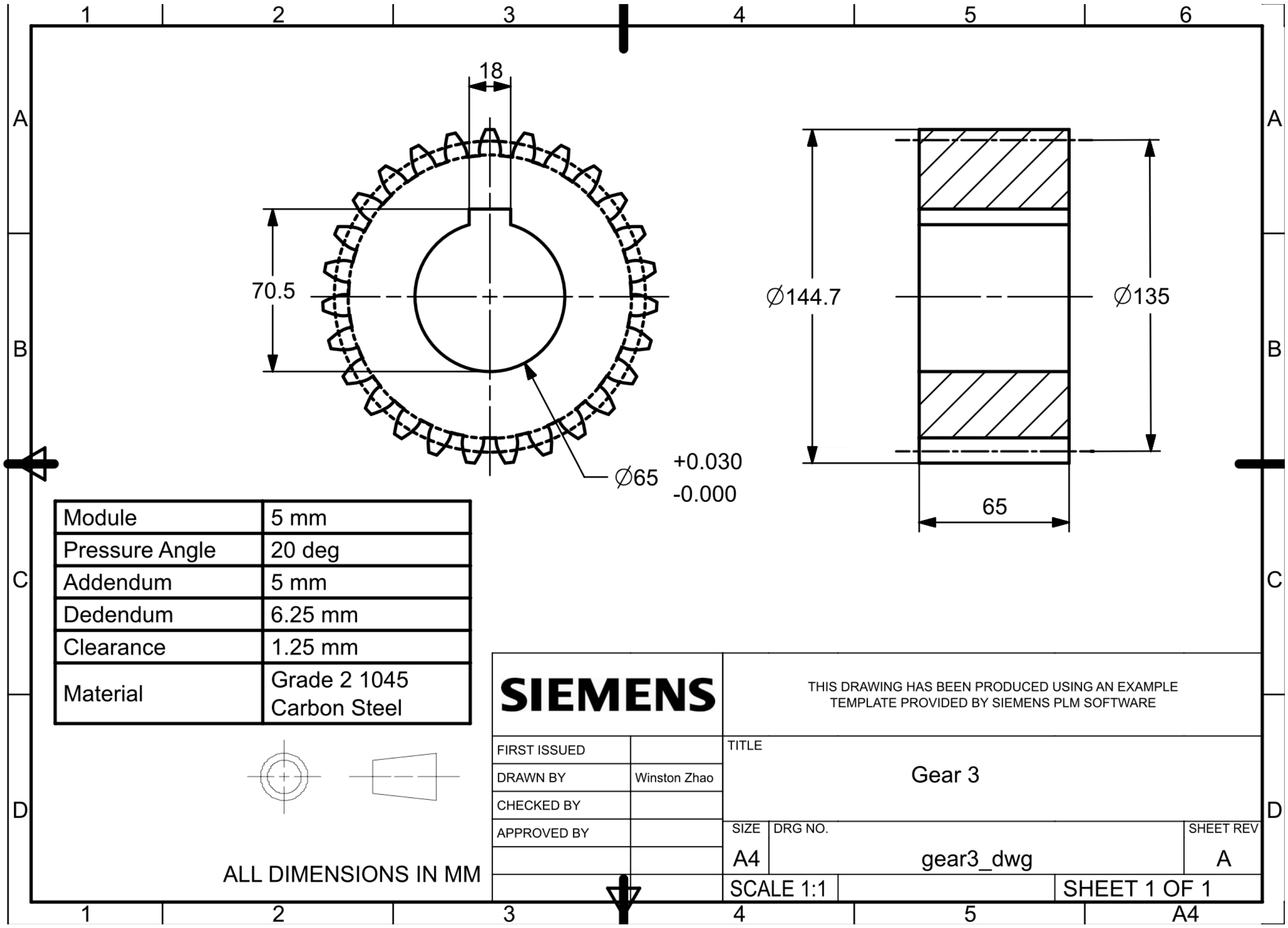
TITLE
Gear 2

SIZE	DRG NO.	SHEET REV
A4	gear2_dwg	A

SCALE 1:1

SHEET 1 OF 1

Gear 3


18
70.5
∅65 +0.030 -0.000
∅144.7
∅135
65
Module | 5 mm
Pressure Angle | 20 deg
Addendum | 5 mm
Dedendum | 6.25 mm
Clearance | 1.25 mm
Material | Grade 2 1045 Carbon Steel
ALL DIMENSIONS IN MM
SIEMENS
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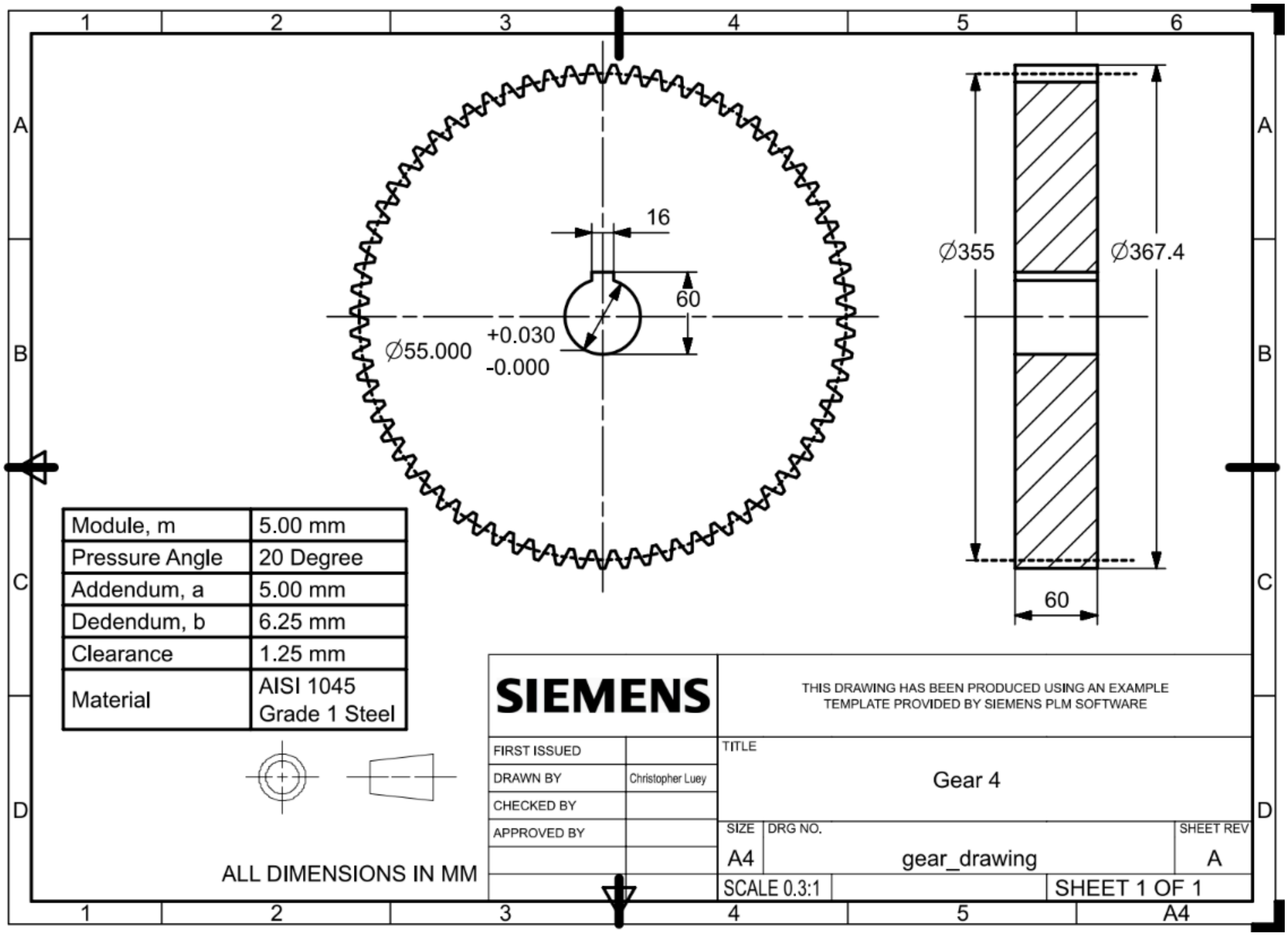
FIRST ISSUED
DRAWN BY Winston Zhao
CHECKED BY
APPROVED BY
TITLE
Gear 3
SIZE A4
DRG NO. gear3_dwg
SHEET REV A
SCALE 1:1
SHEET 1 OF 1
A4

Gear 4


16
60
∅55.000
+0.030
-0.000
∅355
∅367.4
60
Module, m
5.00 mm
Pressure Angle
20 Degree
Addendum, a
5.00 mm
Dedendum, b
6.25 mm
Clearance
1.25 mm
Material
AISI 1045 Grade 1 Steel
SIEMENS
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Christopher Luey
CHECKED BY
APPROVED BY
TITLE
Gear 4
SIZE
A4
DRG NO.
gear_drawing
SHEET REV
A
SCALE 0.3:1
SHEET 1 OF 1
ALL DIMENSIONS IN MM

Shaft One Analysis

Gearing

Given: n1 = 2600 rpm, H = 70000 W, n3 (ideal) = 400, n1/n3 (ideal) = 6.5

<u>Choose gear teeth (odd number, indivisible):</u>

N1 = 19

N2 = 47

N3 = 27

N4 = 71

<u>Choose standard module and pitch angle for off the shelf ordering:</u>

m = a = 0.005 [m]

b = 1.25m = 0.00625 [m]

ϕ = 20 degree

<u>Calculate transmission ratio:</u>

n1/n3 = N4*N2/(N1*N3) = 6.5049

n3 = 2600/(n1/n3) = 399.7003 rpm

% error = |(n3-400)/400| = -0.0749%

Gear 1:

<u>Geometry</u>

N1 = 19

Pitch Diameter: $d_p = N_1 m = 0.0950\ [m]$

Pitch Radius: $r_p = d_p/2 = 0.0480\ [m]$

Face Width: $b_w = 12m = 0.0650\ [m]$

<u>Forces</u>

Note: Forces acting from Gear 1 to Gear 2 are the same as from Gear 2 to Gear 1

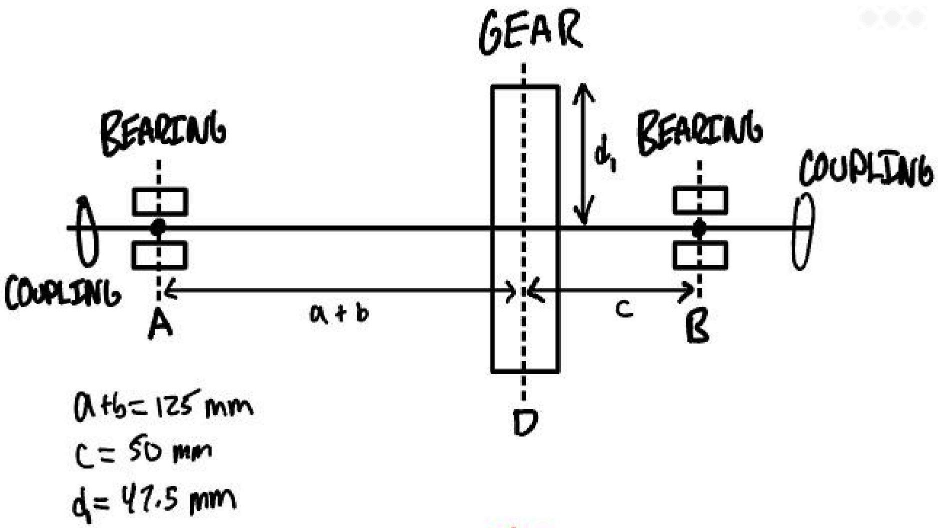
Torque: $T = \frac{60H}{2\pi n_1} = 257.096\ Nm$

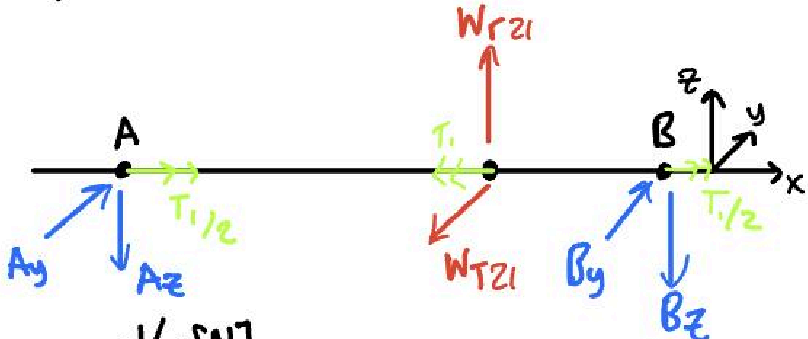
Gear Forces:

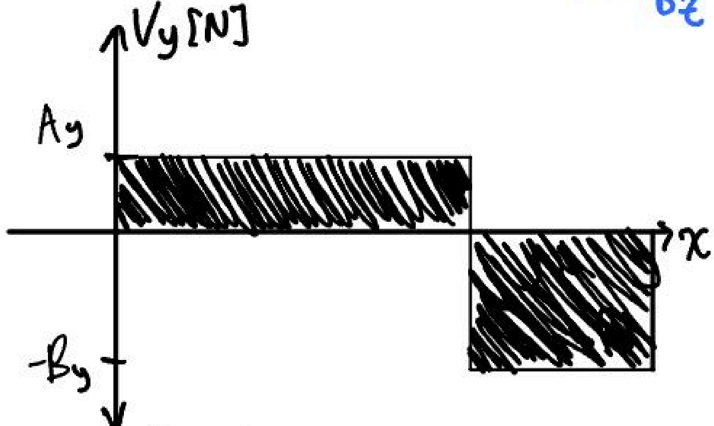
$W_{T12} = T/r_p = 5412.557\ N$

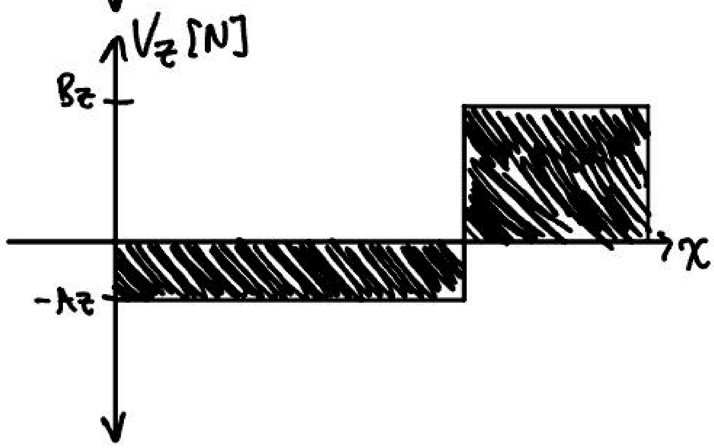
$W_{12} = W_{T12}/cos(\phi) = 5759.923\ N$

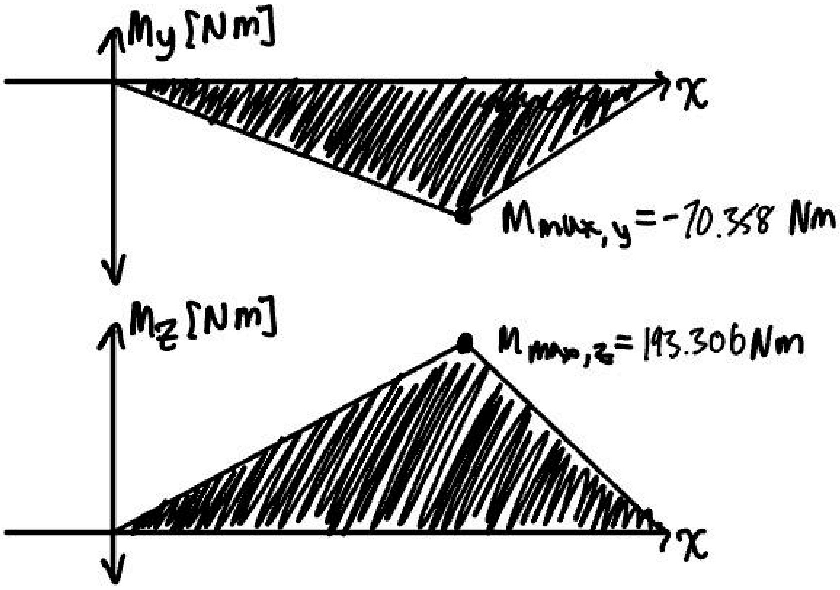
$W_{r12} = W_{12}sin(\phi) = 1970.010\ N$











Note: Torque occurs in the same direction on both sides of the shaft, coming from the coupling.

Force Analysis

<u>From the gearing section the following are solved:</u>

$W_{T12} = 5412.557\ N$

$W_{r12} = 1970.010\ N$

<u>From geometry we know the following:</u>

$a = 55\ mm;\ b + c = 120\ mm$

$d/2 = 177.5\ mm$

<u>Derived from static equilibrium, solve this system of equations:</u>

$0 = A_y + B_y - W_{t12}$

$0 = A_z + B_z - W_{r12}$

$0 = W_{t12}(a) - B_y(a + b + c)$

$0 = W_{r12}(a) - B_z(a + b + c)$

<u>System of equations yields the following reactions with force directions as drawn:</u>

A_y = 1546.4448 N

A_z = 562.8599 N

B_y = 3866.1120 N
B_z = 1407.1497 N

$$A = \sqrt{A_y^2 + A_z^2} = 1645.6922\ N$$

$$B = \sqrt{B_y^2 + B_z^2} = 4114.2304\ N$$

Maximum Shear, Bending

Maximum bending occurs at cross section C:

$$M_y = A_y a = 70.3575\ Nm$$

$$M_z = A_z a = 193.3056\ Nm$$

$$M = \sqrt{M_y^2 + M_z^2} = 205.7115\ Nm$$

The shear magnitude at cross section C is:

$V_y = A_y$ = 1546.4448 N
$V_z = A_z$ = 562.8599 N

$$V = \sqrt{V_y^2 + V_z^2} = 1645.6922\ N$$

Shaft Stress Analysis

We first choose material:

Material	E (Pa)	UTS (MPa)	Poisson Ratio	Ys (MPa)
AISI 1045 Steel	200000.0000	394.7200	0.2900	294.7400

Minimum Shaft Diameter

Our factor of safety is n=1.1

Calculate the torque magnitude at the coupling:

$T = W_{t12}d/2 = 257.096\ Nm$

Calculate the minimum shaft diameter:

$\tau_{all} = 0.557(Y_s)/n = 156.9727\ MPa$

$d_{min} = \sqrt[3]{\frac{16T/2}{\pi\tau_{all}}} = 16.0967\ mm$

Consider keyway effect, round 5% increase:

$d_{min}' = Round[d_{min}(1.05)] = 17\ mm$

Infinite Life at Critical Cross Section

Choose a shaft diameter of d = 45 mm at cross section C.

Solve for max bending torsion, and shear stresses at cross-section C experienced by point on the surface:

$\sigma_{bs} = \frac{32M}{\pi d^3} = 22.9944\ MPa$

$\tau_T = \frac{16T}{\pi d^3} = 14.3691\ MPa$

$\tau_V = \frac{16V}{3\pi d^2} = 3.4492\ MPa$

Given these stresses, calculate the mean and amplitude stresses for cyclic loading:

$\sigma_a = \sigma_{bs} = 22.9944\ MPa$

$\sigma_m = 0\ MPa$

$\tau_m = \frac{\tau_T}{2} = 7.1845\ MPa$ [The shaft turns on and off]

$\tau_a = \frac{\tau_T}{2} + \tau_V = 10.6337\ MPa$

This is a conservative estimate, since we use the shear and bending stress maximum amplitudes in calculating stress states, although these maximums are out of phase. Furthermore, we do not ignore shear (which is 30% of the mean shear).

Calculate Von-Mises given Kf = 3.000 and Kfs = 2.800

$\sigma_{vm-a} = \sqrt{(K_f\sigma_a)^2 + 3(K_{fs}\tau_a)^2} = 86.1289\ MPa$

$$\sigma_{vm-m} = \sqrt{(\sigma_m)^2 + 3(\tau_a)^2} = 12.4440\ MPa$$

Calculate Infinite Criteria given chosen material:

$k_f = 1.58 * S_{ut}^{-0.085} = 0.9220$ [Grinding]

$k_s = 1.189 * d^{-0.112} = 0.7763$ [Size Factor]

$k_r = 0.82$ [99% Reliability]

$k_t = k_m = 1.000$

$S_e' = 0.5(UTS) = 282.500\ MPa$

$n_s = [\frac{\sigma_{vm-a}}{k_f k_s k_r k_t k_m S_e'} + \frac{\sigma_{vm-m}}{S_{ut}}]^{-1} = 1.8467 > 1.1$ [Chosen shaft diameter at C is safe]

Yielding

$n_y = [\frac{\sigma_{vm-a}}{S_y} + \frac{\sigma_{vm-m}}{S_y}]^{-1} = 7.3976 > 1.1$ [Chosen shaft diameter does not pass yield line]

Gear Stress Analysis

Choose material for gear:

Gear Materia	hardness	E (MPa)	Poisson Ratio
AISI 1045 Carbon Steel	400.0000	200000.0000	0.2850

Bending

Calculate number of shaft cycles:

$$t = 5\ year * 52\ \frac{week}{year} * 7\ \frac{day}{week} * 8\ \frac{hr}{day} * 60\ \frac{min}{hr} = 873600\ min$$

Rotation speed n_1= 2600 RPM.

$$N = t * n_1 = 2271360000\ Cycles$$

Calculate allowable bending stress:

$$Y_n = 1.683(N)^{-0.0323} = 0.8392$$

$S_b = 0.533(HB) + 88.3 = 464.5000 MPa$

$k_t = k_r = 1.000$

$\sigma_{b, all} = \frac{S_b Y_n}{k_t k_r} = 389.8179\ MPa$

Calculate actual bending stress:

$Y_j = 0.350$ [From Class Plot]

$k_a = 1.000$

$k_s = 1.000$ [module = 5 mm]

$k_m = 1.100$ [Gear]

$k_i = 1.000$ [Non Idler]

$k_b = 1.000$

Calculate Kv:

$V = \pi n d/60 = 12.9329\ [m/s]$

$Q_v = 6$ for commercial gears

$B = \frac{(12-Q_v)^{2/3}}{4} = 0.8255$

$A = 50 + 56(1 - B) = 59.7730$

$K_v = (\frac{A+\sqrt{200V}}{A})^B = 1.6623$

$\sigma_{b, actual} = \frac{W^t}{m b_w Y_j} k_a k_s k_m k_v k_i k_b = 94.9170\ MPa$

Comparing the allowable to the actual bending stress:

$n_b = \sigma_{all}/\sigma_b = 4.1069 > 1.1$

The design is sufficient for bending stress since $n_b > n = 1.1$

Contact

Calculate allowable contact stress:

$HB_{pinion} = 500\ (HB)$

$HB_{gear} = 400\ (HB)$

$Z_N = 2.466N^{-0.056} = 0.7380$

$\frac{HB_p}{HB_g} = 1.25$

$A' = (8.89 * 10^{-3})(\frac{HB_p}{HB_g}) - 8.29 * 10^{-3}$ since $1.2 \le \frac{HB_p}{HB_g} = 1.25 \le 1.7$

$\frac{N_g}{N_p} = 2.6296$

$C_H = 1.0 + A'(\frac{N_g}{N_p} - 1.0) = 1.0042$

$K_T = 1$

$K_R = 1$

$S_c = 2.22 * HB_g + 200 = 1437\ MPa$

$\sigma_{c,\,all} = \frac{S_{c\_} Z_N C_{Hg}}{K_T K_R} = 1060.4686$ MPa

Calculate actual contact stress:

$K_a = 1$ [No shock]

$K_s = 1$

$K_E = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g}}} = 466563.1271$ $[\sqrt{Pa}]$

$K_{m,\,gear} = 1.1$

$K_v = 1.6623$ [From bending calculation]

$\phi = 20\ deg;\ d_p = 0.095\ m;\ d_g = 0.235\ m$

$I = \frac{\pi cos(\phi) sin(\phi)}{1 + \frac{d_p}{d_g}} = 0.7190$

$\sigma_{c,actual} = K_E \sqrt{\frac{W^t}{b_w d_p I} K_a K_s K_m K_v} = 727.5603\ MPa$

Factors of Safety for Bending and Contact

$\sigma_{c,actual} = 727.5603\ MPa$

$\sigma_{c,\,all} = 1060.4686$ MPa

$\sigma_{b,\,actual} = 94.9170\ MPa$

$\sigma_{b,\,all} = 389.8179\ MPa$

$n_c = \frac{\sigma_{c,\,all}}{\sigma_{c,actual}} = 1.4576 > 1.1$ [Safe for contact stress]

$n_b = \frac{\sigma_{b,\,all}}{\sigma_{b,actual}} = 4.1069 > 1.1$ [Safe for bending stress]

Bearing Selection

$h = (5\,year)(364\,\frac{day}{year})(8\frac{hr}{day}) = 14560\,hr$ [life hours]

n_3 = 2600 RPM [rotation speed]

$L_{10} = 60(h)(n_3)(10^{-6}) = 2271.3600\ Million\ Revs$

$A_P = 1.000$ [No Shock]

$m = 3$ [Ball Bearing]

$P_r = Max(A, B) = 4114.2304\ N$ [A and B calculated in Force Analysis section]

$X = 1.000$

$P = P_r X = 4114.2304\ N$ [No Axial Force]

$C = PL_{10}^{1/m} = 54081.7335\ N$

Given the calculated C, I specify the following bearing from SKF:

Bearing Spec	C (N)	C0 (N)	d (m)	D (m)	W (m)
6309	55300.0000	31500.0000	0.0450	0.1000	0.0250

Reference: https://www.skf.com/us/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings/productid-6309

This bearing is safe because the bearing specification C is less than our calculated C. Recalculation is not necessary since there is no thrust force and radial factor X is constant, not a function of e or C0.

Expected Lifetime of Bearing

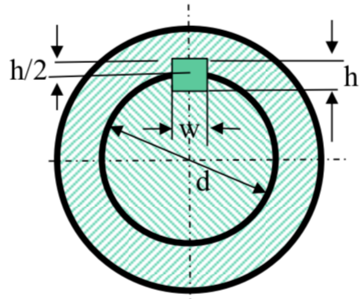
$L = (\frac{C}{P})^m = 2428.3404\ mil\ rev$ [using C=55300 N from SKF]

$Life = L(10^6)(n_1)^{-1}(60\frac{min}{hr})^{-1}(8\frac{hr}{day})^{-1}(7\frac{day}{week})^{-1}(52\frac{week}{yr})^{-1} = 5.3455\ yr$

Bearing does not require maintenance for intended operation of 5 years.

Keyways and Tolerances

<u>Coupling Keyway (40 mm) and Gear Keyway (50 mm):</u>



Shaft diameter d (mm)	Key width W x height H
38 < d <= 44	12 x 8
44 < d <= 50	14 x 9

<u>Fits:</u>

Bearing and Shaft: Interference fit (H7/p6), bearing should be tightly fit with shaft for fluid rotation.

RESULTS		
HOLE		
Parameter	**Value**	**Unit**
Designation	45 H7	---
Hole Upper Deviation	25	µm (0.001mm)
Hole Lower Deviation	0	µm (0.001mm)
Maximum Hole Size	45.025	mm
Minimum Hole Size	45	mm
SHAFT		
Parameter	**Value**	**Unit**
Designation	45 p6	---
Shaft Upper Deviation	42	µm (0.001mm)
Shaft Lower Deviation	26	µm (0.001mm)
Maximum Shaft Size	45.042	mm
Minimum Shaft Size	45.026	mm
FIT		
Parameter	**Value**	**Unit**
Designation	45 H7/p6	---
Fit Type	Interference fit	---
Maximum Interference	42	µm (0.001mm)
Minimum Interference	1	µm (0.001mm)

Gear and Shaft: Transition fit (H7/k6), shaft rotation will be transmitted from gear torque through interlocked key.

RESULTS		
HOLE		
Parameter	**Value**	**Unit**
Designation	45 H7	---
Hole Upper Deviation	25	µm (0.001mm)
Hole Lower Deviation	0	µm (0.001mm)
Maximum Hole Size	45.025	mm
Minimum Hole Size	45	mm
SHAFT		
Parameter	**Value**	**Unit**
Designation	45 k6	---
Shaft Upper Deviation	18	µm (0.001mm)
Shaft Lower Deviation	2	µm (0.001mm)
Maximum Shaft Size	45.018	mm
Minimum Shaft Size	45.002	mm
FIT		
Parameter	**Value**	**Unit**
Designation	45 H7/k6	---
Fit Type	Transition fit	---
Maximum Interference	18	µm (0.001mm)
Maximum Clearance	23	µm (0.001mm)

Source: https://amesweb.info/fits-tolerances/tolerance-calculator.aspx

Shaft Two Analysis

Gearing

Given: n1 = 2600 rpm, n3 = 400 rpm ideal, n1/n3 = 6.5 ideal

N1 = 19

N2 = 47

N3 = 27

N4 = 71

m = 0.005 m

Phi = 20 degree

Power H = 70000 W

n1/n3 = N4*N2/(N1*N3) = 6.5049

n3 = 2600/(n1/n3) = 399.7003 rpm

% error = (n3-400)/400 = -0.0749%

Gear 2:

<u>Geometry</u>

N2 = 47

Pitch Diameter: dp = N*m = 0.235 m

Pitch Radius: rp = dp/2 = 0.1175 m

Outer Diameter: do = dp + 2*m = 0.245 m

Root Diameter: dr = dp - 2*1.25*m = 0.223 m

Face Width: bw = 12*m = 0.06 m

<u>Forces</u>

Torque: T2 = 60*H/(2*pi*n2) = 635.975Nm

Wt23 = T2/rp = 5412.557 N

W23 = Wt23/cos(phi) = 1970.01 N

Wr23 = W23*sin(phi) = 5759.923 N

Gear 3:

<u>Geometry</u>

N3 = 27

Pitch Diameter: dp = N*m = 0.125 m

Pitch Radius: rp = dp/2 = 0.0675 m

Outer Diameter: do = dp + 2*m = 0.145 m
Root Diameter: dr = dp - 2*1.25*m = 0.123 m
Face Width: bw = 13*m = 0.065 m; Pinion has larger bw than gear

<u>Forces</u>

Torque: T3 = T2 = 60*H/(2*pi*n3) = 635.975 Nm
Wt34 = T3/rp = 9421.858 N
W34 = Wt34/cos(phi) = 10026.532 N
Wr34 = W34*sin(phi) = 3429.276 N

Free Body Diagram

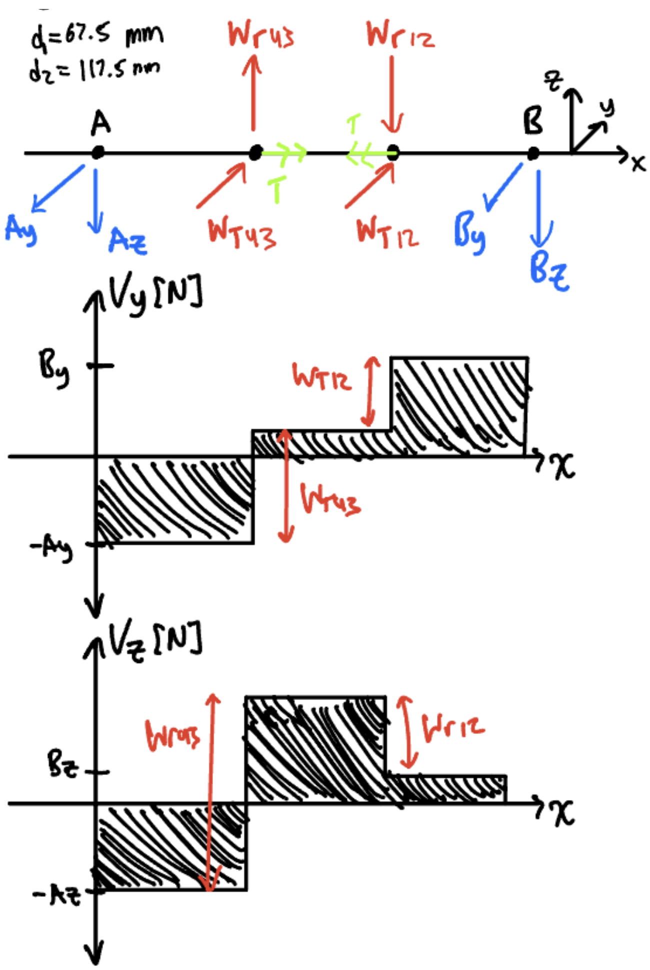
Rotation Speed: n2 = n1*N1/N2 = 1051.0638

Set shaft diameter: d2 = 0.06 m

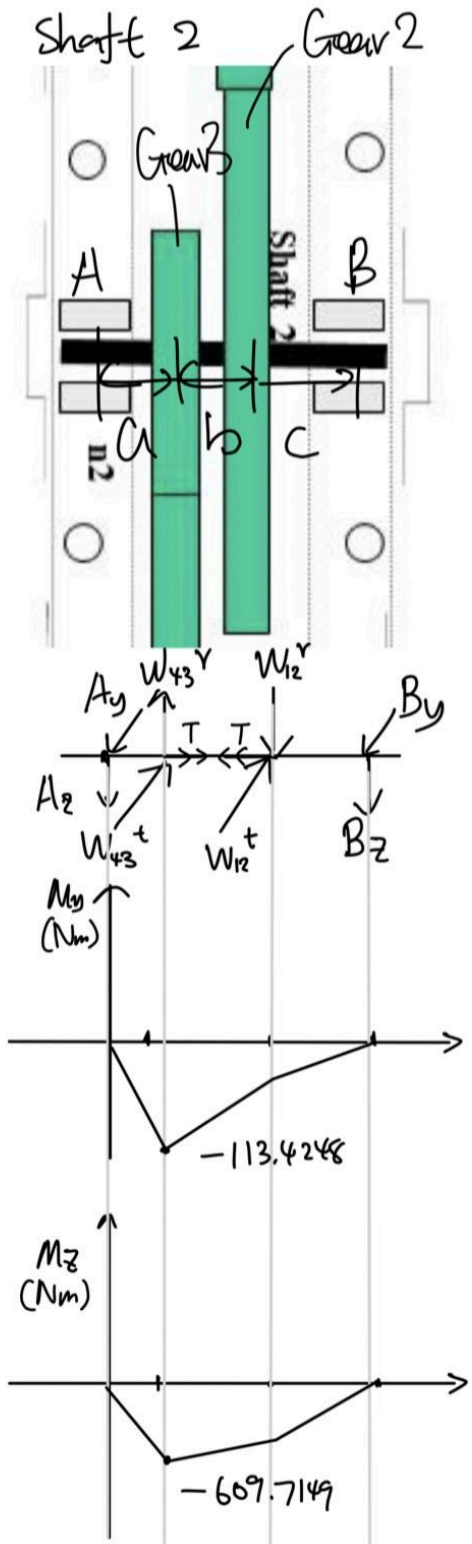
a = 0.0550 m [bw_3/2 + bearing thickness/2 + spacing]

b = 0.0700 m [bw_3/2 +bw_2/2 + spacing]

c = 0.0500 m [bw_2/2 + bearing thickness/2 + spacing]



Shaft 2



Gear 2
Gear 3
Shaft 2
A
B
a
b
c
n2
Ay
W43 r
W12 r
By
T
T
Az
W43 t
W12 t
Bz
My (Nm)
-113.4248
Mz (Nm)
-609.7149

Force Analysis

Shaft 2 Reaction Forces

$$B_y = \frac{W_{t,43}*a+W_{t,12}*(a+b)}{a+b+c} = 6827.2674\ N$$

$$A_y = W_{t,43} + W_{t,12} - B_y = 8007.1475\ N$$

$$B_z = \frac{W_{r,43}*a+W_{r,12}*(a+b)}{a+b+c} = 1788.6436\ N$$

$$A_z = W_{r,43} - W_{r,12} + B_z = 329.3773\ N$$

$$R_A = \sqrt{A_y^2 + A_y^2} = 8204.4900\ N$$

$$R_B = \sqrt{B_y^2 + B_y^2 =} 6835.2081\ N$$

Shear and Bending

Maximum bending occurs at C:

Mz = Ay * a = 440.3931 Nm

My = Az * a = 98.3754 Nm

$$M = \sqrt{M_y^2 + M_z^2} = 451.2470 Nm$$

The shear at point C is:

Vy = Ay = 8007.1475 N

Vz = Az = 1788.6436 N

V = Sqrt(Vy^2 + Vz^2) = 6875.3362 N

Shaft Fatigue Analysis

Solve for bending torsion, and shear stresses at Point 1 at cross-section C:

$$\sigma_{bs} = \frac{32M}{\pi d^3} = 21.2795\ MPa$$

$$\tau_T = \frac{16T}{\pi d^3} = 14.9954\ MPa$$

$$\tau_V = \frac{16V}{3\pi d^2} = 3.8690\ MPa$$

Given these stresses, calculate the mean and amplitude stresses for cyclic loading:

$$\sigma_a = \sigma_{bs} = 21.2795\ MPa$$

$$\sigma_m = 0\ MPa$$ [ignore axial normal stress due to low magnitude]

$$\tau_m = \frac{\tau_t}{2} = 7.4977\ MPa$$ [on/off]

$$\tau_a = \frac{\tau_T}{2} + \tau_v = 11.3667\ MPa$$

Calculate Von-Mises given Kf = 3.000 and Kfs = 2.800.

Note: although max σ_{bs} and max τ_{ts} won't coexist at the same point mechanically, we take both magnitudes into account for a conservative design on calculating the factor of safety for our shafts.

$$\sigma_{vm-a} = \sqrt{(K_f\sigma_a)^2 + 3(K_{fs}\tau_a)^2} = 91.2798\ MPa$$

$$\sigma_{vm-m} = \sqrt{(\sigma_m)^2 + 3(\tau_m)^2} = 16.8598\ MPa$$

Calculate Infinite Criteria given chosen shaft material:

Material	E	UTS	Poisson Ratio	Ys
AISI 1045 Steel	200000.0000	394.7200	0.2900	294.7400

$$k_f = 1.58 * S_{ut}^{-0.085} = 0.9220$$ [Grinding]

$$k_s = 1.189 * d^{-0.112} = 0.7517$$ [Size Factor]

$$k_r = 0.82$$ [99% Reliability]

$$k_t = k_m = 1.000$$ [ignore temperature and miscellaneous factors]

$$S_e' = 0.5(UTS) = 282.500\ MPa$$

Goodman

$$n_s = \left[\frac{\sigma_{vm-a}}{k_f k_s k_r k_t k_m S_e'} + \frac{\sigma_{vm-m}}{S_{ut}}\right]^{-1} = 1.8236 > 1.1$$ [The shaft design is safe]

Yield

$$n_y = \frac{\sigma_{vm-max}}{Sy} = 7.3851$$

Gear Stress Analysis

Material

Gear Material		hardness	E (MPa)	Poisson Ratio	Type
Gear 3	AISI 1045 Carbon Steel	500.0000	200000.0000	0.2850	pinion
Gear 2	AISI 1045 Carbon Steel	400.0000	200000.0000	0.2850	gear

Allowable Contact Stress

$$HB_{gear} = 400\ (HB)$$

$$HB_{pinion} = 500\ (HB)$$

$$Z_N = 2.466N^{-0.056} = 0.7764$$

$$\frac{HB_p}{HB_g} = 1.25$$

$A' = (8.89 * 10^{-3})(\frac{HB_p}{HB_g}) - 8,29 * 10^{-3}$ since $1.2 \le \frac{HB_p}{HB_g} = 1.25 \le 1.7$

$$C_H = 1.0 + A'(\frac{N_g}{N_p} - 1.0) = 1.0042$$

$$K_T = 1$$

$$K_R = 1$$

Gear 2:

$$S_{c2} = 2.22 * HB_g + 200 = 1088\ MPa$$

$$\sigma_{all,2} = \frac{S_{c\_}Z_N C_{Hg}}{K_T K_R} = 848.2032\ \text{MPa}$$

Gear 3:

$$S_{c3} = 2.4 * HB_p + 237 = 1437\ MPa$$

$$\sigma_{all,3} = \frac{S_{c\_}Z_N}{K_T K_R} = 1115.6426\ \text{MPa}$$

<u>Design Contact Stress</u>

Gear 2:

$K_a = 1$ for no shock

$$K_s = 1$$

$$K_E = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p}+\frac{1-v_g^2}{E_g}}} = 466563.1271 \quad [\sqrt{Pa}]$$

$$K_{m,\,gear} = 1.1$$

Calculate Kv:

$Q_v = 6$ for commercial gears

$$B = \frac{(12-Q_v)^{2/3}}{4} = 0.8255$$

$$A = 50 + 56(1 - B) = 59.7730$$

$$V_2 = \frac{\pi * n_2 * d_{p2}}{60} = 12.9329 \text{ m/s}$$

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B = 1.6623$$

Calculate Design Contact Stress

$$\sigma_{c,actual} = K_E\left(\sqrt{\frac{W^t}{b_w d_p I} K_a K_s K_m K_v}\right) = 725.0297 MPa$$

Gear 3:

$K_a = 1$ for no shock

$$K_s = 1$$

$$K_E = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p}+\frac{1-v_g^2}{E_g}}} = 466563.1271 \quad [\sqrt{Pa}]$$

$$K_{m,\,pinion} = 1.2$$

Calculate Kv:

$Q_v = 6$ for commercial gears

$$B = \frac{(12-Q_v)^{2/3}}{4} = 0.8255$$

$$A = 50 + 56(1 - B) = 59.7730$$

$$V_3 = \frac{\pi * n_2 * d_{p3}}{60} = 7.4295 \text{ m/s}$$

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B = 1.5081$$

Calculate Design Contact Stress

$$\sigma_{c,actual} = K_E\left(\sqrt{\frac{W^t}{b_w d_p I} K_a K_s K_m K_v}\right) = 760.4055\ MPa$$

Allowable Bending Stress

Calculate number of shaft cycles:

$hrs = 5\,year * 52\,weeks * 7\,days * 8\,hrs * 60\,minutes = 873600\,mins$

$N = hrs * shaft\,speed(rpm) = 918209362\,Cycles$

Gear 2:

Calculate allowable bending stress

$Y_n = 1.683(N)^{-0.0323} = 0.8915$

$S_b = 0.533(HB_{gear}) + 88.3 = 301.5000\,MPa$

$k_t = k_r = 1.000$

$\sigma_{all,b} = \frac{S_b Y_n}{k_t k_r} = 260.5364\,MPa$

Gear 3:

Calculate allowable bending stress

$Y_n = 1.683(N)^{-0.0323} = 0.8915$

$S_b = 0.533(HB_{pinion}) + 88.3 = 464.5000\,MPa$

$k_t = k_r = 1.000$

$\sigma_{all,b} = \frac{S_b Y_n}{k_t k_r} = 401.3902\,MPa$

Design Bending Stress

Gear 2:

$Y_j = 0.420$

$k_a = 1.000$

$k_s = 1.000$ [module == 5mm]

$k_m = 1.100$ [gear]

$k_i = 1.000$ [not idler]

$k_b = 1.000$

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B = 1.6623$$

$$\sigma_b = \frac{W^t}{mb_w Y_j} k_a k_s k_m k_v k_i k_b = 78.5482\ MPa$$

Gear 3:

$Y_j = 0.350$

$k_a = 1.000$

$k_s = 1.000$ [module == 5mm]

$k_m = 1.200$ [pinion]

$k_i = 1.000$ [not idler]

$k_b = 1.000$

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B = 1.5081$$

$$\sigma_b = \frac{W^t}{mb_w Y_j} k_a k_s k_m k_v k_i k_b = 149.8943\ MPa$$

Factors of Safety

$$n_{c,2} = \frac{\sigma_{all,c2}}{\sigma_{c2}} = 1.1699$$

$$n_{c,3} = \frac{\sigma_{all,c3}}{\sigma_{c3}} = 1.4672$$

$$n_{b,2} = \frac{\sigma_{all,b2}}{\sigma_{b2}} = 3.3169$$

$$n_{b,3} = \frac{\sigma_{all,b3}}{\sigma_{b3}} = 2.6778$$

The design is good

Bearing Selection

$h = (5\, year)(364\, \frac{day}{year})(8 \frac{hr}{day}) = 14560\, hr$ [life hours]

$n_2 = n1*N1/N2 = 1051.0638$ rpm [rotation speed]

$L_{10} = 60(h)(n_3)(10^{-6}) = 918.2094\, million\ cycles$

$A_P = 1.000$ [no shock]

$m = 3$ [Deep Groove Ball Bearing]

$P_r = Max(A, B) = 8204.4900\, N$ [A and B calculated in Force Analysis section]

$X = 1.000$

<u>Initial Selection</u>

$P = P_r X = 8204.4900\, N$ [No Axial Force]

$C = PL_{10}^{1/m} = 79744.1494\, N$

Choose SKF 6312 with $C = 97500\, N, C_0 = 60000\, N$

<u>No reselection necessary since no thrust load</u>

$e = 0.22$ because $\frac{P_a}{C_o} = 0$

$\frac{P_a}{P_r} = 0 < e$

- $X = 1$
- $Y = 0$

Pick SKF 6312 for Final Bearing Selection with following specifications

Bearing Spec	C (N)	C0 (N)	d (m)	D (m)	W (m)
SKF 6312	97500.0000	60000.0000	0.0650	0.1400	0.0330

Reference: https://www.skf.com/group/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings/productid-6312

Expected Lifetime of Bearing

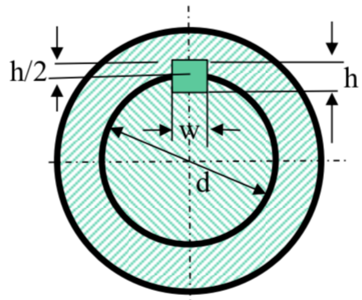
$L = (\frac{C}{P})^m = 1678.2594\, mil\ rev$ [using C=97500 N from SKF]

$Life = L(10^6)(n_2)^{-1}(60 \frac{min}{hr})^{-1}(8 \frac{hr}{day})^{-1}(7 \frac{day}{week})^{-1}(52 \frac{week}{yr})^{-1} = 9.1288\, yr$

Bearing does not require maintenance for intended operation of 5 years.

Keyways and Tolerances

Gear Keyway (65 mm):



Shaft diameter d (mm)	Key width W x height H
58 < d <= 65	18 x 11

Fits:

Bearing inner ring and Shaft:

Interference fit (H7/p6) for hole basis. Interference needed for inner ring to rotate with shaft

Gear and Shaft:

Transition fit (H7/k6) for hole basis. Locational transition since shaft rotation will be transmitted from gear torque through interlocked key.

Table 3. ***(Continued)*** **American National Standard Preferred Hole Basis Metric Transition and Interference Fits** *ANSI B4.2-1978 (R1999)*

Basic Size[a]		Locational Transition			Locational Transition			Locational Interference			Medium Drive			Force		
		Hole H7	Shaft k6	Fit[b]	Hole H7	Shaft n6	Fit[b]	Hole H7	Shaft p6	Fit[b]	Hole H7	Shaft s6	Fit[b]	Hole H7	Shaft u6	Fit[b]
30	Max	30.021	30.015	+0.019	30.021	30.028	+0.006	30.021	30.035	−0.001	30.021	30.048	−0.014	30.021	30.061	−0.027
	Min	30.000	30.002	−0.015	30.000	30.015	−0.028	30.000	30.022	−0.035	30.000	30.035	−0.048	30.000	30.048	−0.061
40	Max	40.025	40.018	+0.023	40.025	40.033	+0.008	40.025	40.042	−0.001	40.025	40.059	−0.018	40.025	40.076	−0.035
	Min	40.000	40.002	−0.018	40.000	40.017	−0.033	40.000	40.026	−0.042	40.000	40.043	−0.059	40.000	40.060	−0.076
50	Max	50.025	50.018	+0.023	50.025	50.033	+0.008	50.025	50.042	−0.001	50.025	50.059	−0.018	50.025	50.086	−0.045
	Min	50.000	50.002	−0.018	50.000	50.017	−0.033	50.000	50.026	−0.042	50.000	50.043	−0.059	50.000	50.070	−0.086
60	Max	60.030	60.021	+0.028	60.030	60.039	+0.010	60.030	60.051	−0.002	60.030	60.072	−0.023	60.030	60.106	−0.057
	Min	60.000	60.002	−0.021	60.000	60.020	−0.039	60.000	60.032	−0.051	60.000	60.053	−0.072	60.000	60.087	−0.106
80	Max	80.030	80.021	+0.028	80.030	80.039	+0.010	80.030	80.051	−0.002	80.030	80.078	−0.029	80.030	80.121	−0.072
	Min	80.000	80.002	−0.021	80.000	80.020	−0.039	80.000	80.032	−0.051	80.000	80.059	−0.078	80.000	80.102	−0.121
100	Max	100.035	100.025	+0.032	100.035	100.045	+0.012	100.035	100.059	−0.002	100.035	100.093	−0.036	100.035	100.146	−0.089
	Min	100.000	100.003	−0.025	100.000	100.023	−0.045	100.000	100.037	−0.059	100.000	100.071	−0.093	100.000	100.124	−0.146
120	Max	120.035	120.025	+0.032	120.035	120.045	+0.012	120.035	120.059	−0.002	120.035	120.101	−0.044	120.035	120.166	−0.109
	Min	120.000	120.003	−0.025	120.000	120.023	−0.045	120.000	120.037	−0.059	120.000	120.079	−0.101	120.000	120.144	−0.166
160	Max	160.040	160.028	+0.037	160.040	160.052	+0.013	160.040	160.068	−0.003	160.040	160.125	−0.060	160.040	160.215	−0.150
	Min	160.000	160.003	−0.028	160.000	160.027	−0.052	160.000	160.043	−0.068	160.000	160.100	−0.125	160.000	160.190	−0.215
200	Max	200.046	200.033	+0.042	200.046	200.060	+0.015	200.046	200.079	−0.004	200.046	200.151	−0.076	200.046	200.265	−0.190
	Min	200.000	200.004	−0.033	200.000	200.031	−0.060	200.000	200.050	−0.079	200.000	200.122	−0.151	200.000	200.236	−0.265
250	Max	250.046	250.033	+0.042	250.046	250.060	+0.015	250.046	250.079	−0.004	250.046	250.169	−0.094	250.046	250.313	−0.238
	Min	250.000	250.004	−0.033	250.000	250.031	−0.060	250.000	250.050	−0.079	250.000	250.140	−0.169	250.000	250.284	−0.313
300	Max	300.052	300.036	+0.048	300.052	300.066	+0.018	300.052	300.088	−0.004	300.052	300.202	−0.118	300.052	300.382	−0.298
	Min	300.000	300.004	−0.036	300.000	300.034	−0.066	300.000	300.056	−0.088	300.000	300.170	−0.202	300.000	300.350	−0.382
400	Max	400.057	400.040	+0.053	400.057	400.073	+0.020	400.057	400.098	−0.005	400.057	400.244	−0.151	400.057	400.471	−0.378
	Min	400.000	400.004	−0.040	400.000	400.037	−0.073	400.000	400.062	−0.098	400.000	400.208	−0.244	400.000	400.435	−0.471
500	Max	500.063	500.045	+0.058	500.063	500.080	+0.023	500.063	500.108	−0.005	500.063	500.292	−0.189	500.063	500.580	−0.477
	Min	500.000	500.005	−0.045	500.000	500.040	−0.080	500.000	500.068	−0.108	500.000	500.252	−0.292	500.000	500.540	−0.580

[a] The sizes shown are first-choice basic sizes (see Table 1). Preferred fits for other sizes can be calculated from data given in ANSI B4.2-1978 (R1999).
[b] A plus sign indicates clearance; a minus sign indicates interference.
All dimensions are in millimeters.

Shaft Three Analysis

Gearing

Given: n1 = 2600 rpm, H = 70000 W

Choose gear teeth (odd number, indivisible):

N1 = 19

N2 = 47

N3 = 27

N4 = 71

Choose standard module m=5mm and pitch angle phi = 20 deg for off the shelf ordering:

m = a = 0.005 [m]

b = 1.25m = 0.00625 [m]

ϕ = 20 degree

Calculate transmission ratio:

$n_1/n_3 = (N4)(N2)(N1)^{-1}(N3)^{-1} = 6.5049$

n3 = 2600/(n1/n3) = 399.7003 rpm

% error = |(n3-400)/400| = 0.0749%

Gear 4:

Geometry

N4 = 71 [Teeth]

Pitch Diameter: $d_p = N_4 m = 0.3550\ [m]$

Pitch Radius: $r_p = d_p/2 = 0.1775\ [m]$

Face Width: $b_w = 12m = 0.0600\ [m]$

Forces

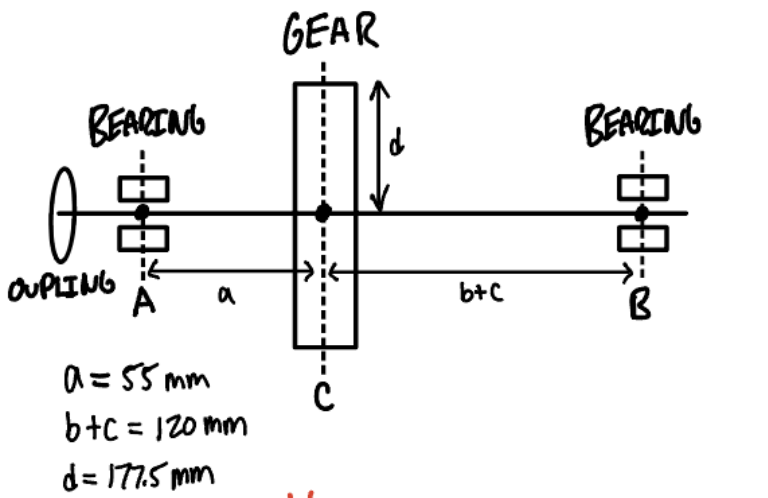
Torque: $T = \frac{60H}{2\pi n_3} = 1672.380\ Nm$

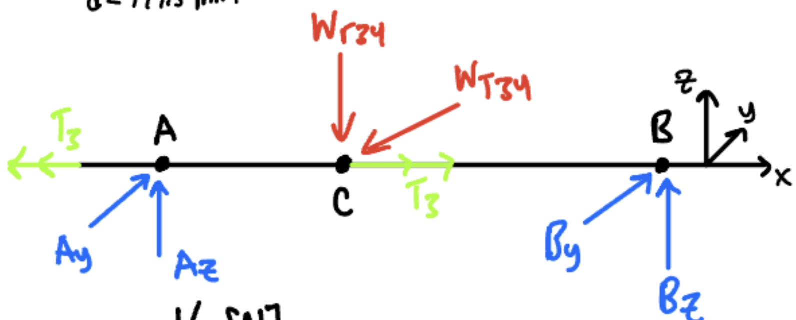
Gear Forces: $W_{T34} = T/r_p = 9421.8581\ N$

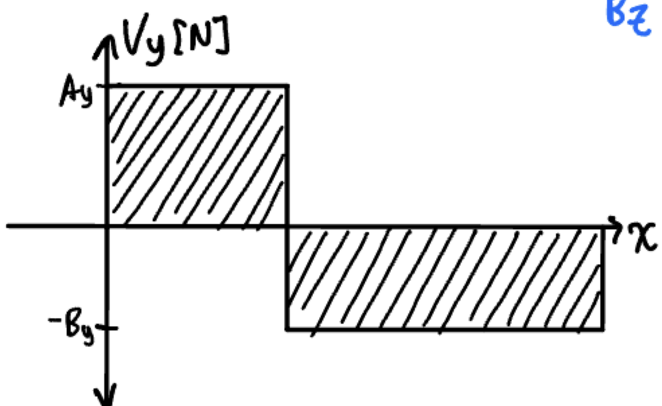
$W_{34} = W_{T34}/cos(\phi) = 10026.532$

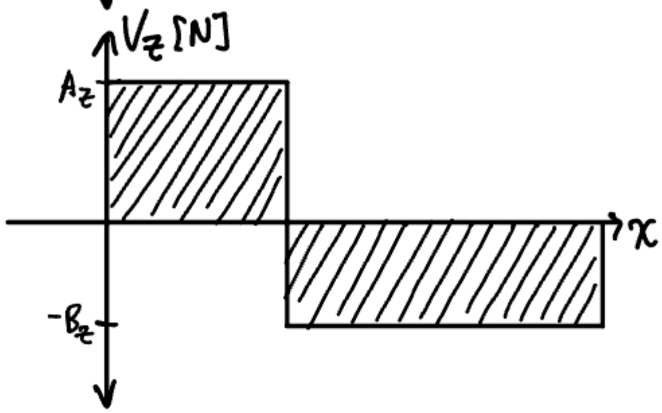
$W_{r34} = W_{34} sin(\phi) = 3429.276\ N$

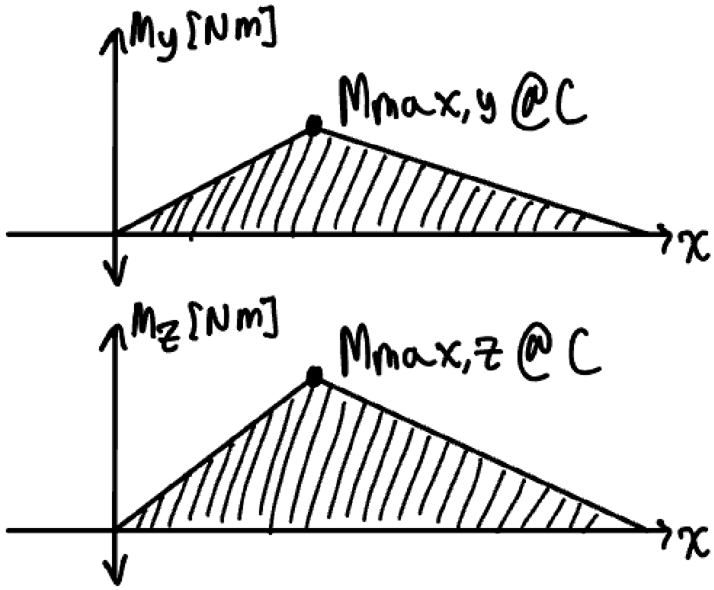
Geometry











Force Analysis

<u>From the gearing section the following are solved:</u>

$W_{r34} = 3429.2759\ N$

$W_{t34} = 9421.8581\ N$

<u>From geometry we know the following:</u>

$a = 55\ mm;\ b + c = 120\ mm$

$d/2 = 177.5\ mm$

<u>Derived from static equilibrium, solve this system of equations:</u>

$0 = A_y + B_y - W_{t34}$

$0 = A_z + B_z - W_{r34}$

$0 = W_{t34}(a) - B_y(a + b + c)$

$0 = W_{r34}(a) - B_z(a + b + c)$

<u>System of equations yields the following reactions with force directions as drawn:</u>

A_y = 6460.7027 N

A_z = 2351.5035 N

B_y = 2961.1554 N

B_z = 1077.7724 N

$$A = \sqrt{A_y^{\ 2} + A_z^{\ 2}} = 6875.3362\ N$$

$$B = \sqrt{B_y^{\ 2} + B_z^{\ 2}} = 3151.1958\ N$$

Maximum Shear, Bending

Maximum bending occurs at cross section C:

$$M_{max,y} = A_y a = 117.5752\ Nm$$

$$M_{max,z} = A_z a = 323.0351\ Nm$$

$$M = \sqrt{M_y^{\ 2} + M_z^{\ 2}} = 343.7668\ Nm$$

The shear magnitude at cross section C is:

$V_y = A_y = 6460.7027$ N

$V_z = A_z = 2351.5035$ N

$$V = \sqrt{V_y^{\ 2} + V_z^{\ 2}} = 6875.3362\ N$$

Shaft Stress Analysis

We first choose material:

Material	E [Pa]	UTS [MPa]	Poisson Ratio	Ys [MPa]
AISI 1045 Medium Carbon Steel	200000.0000	565.0000	0.2850	310.0000

Minimum Shaft Diameter

Our factor of safety is n=1.1

Calculate the torque magnitude at the coupling:

$$T = W_{t34}d/2 = 1672.380\ Nm$$

Calculate the minimum shaft diameter:

$$\tau_{all} = 0.557(Y_s)/n = 156.9727\ MPa$$

After unit conversion:

$$d_{min} = \sqrt[3]{\frac{16T}{\pi\tau_{all}}} = 37.858\ mm$$

Consider keyway effect, round 5% increase [conservative]:

$$d_{min}' = Round[d_{min}(1.05)] \approx 40\ mm$$

Infinite Life at Critical Cross Section

Choose a shaft diameter of d = 55 mm at cross section C.

Solve for max bending torsion, and shear stresses at cross-section C experienced by point on the shaft surface:

$$\sigma_{bs} = \frac{32M}{\pi d^3} = 21.0463\ MPa$$

$$\tau_T = \frac{16T}{\pi d^3} = 19.4680\ MPa$$

$$\tau_V = \frac{16V}{3\pi d^2} = 3.8585\ MPa$$

Given these stresses, calculate the mean and amplitude stresses for cyclic loading:

$\sigma_a = \sigma_{bs} = 21.0463\ MPa$

$\sigma_m = 0\ MPa$

$\tau_m = \frac{\tau_T}{2} = 9.7340 MPa$ [The shaft turns on and off]

$\tau_a = \frac{\tau_T}{2} + \tau_V = 13.5925\ MPa$

This is a conservative estimate, since we use the shear and bending stress maximum amplitudes in calculating stress states, although these maximums are out of phase. Furthermore, we do not ignore shear (which is 30% of the mean shear).

<u>Calculate Von-Mises given Kf = 3.000 and Kfs = 2.800 [conservative estimate]:</u>

$$\sigma_{vm-a} = \sqrt{(K_f\sigma_a)^2 + 3(K_{fs}\tau_a)^2} = 91.2798\ MPa$$

$$\sigma_{vm-m} = \sqrt{(\sigma_m)^2 + 3(\tau_a)^2} = 16.8598\ MPa$$

<u>Calculate Infinite Criteria given chosen material:</u>

$k_f = 1.58 * UTS^{-0.085} = 0.9220$ [Grinding]

$k_s = 1.189 * d^{-0.112} = 0.7590$ [Size Factor]

$k_r = 0.82$ [99% Reliability]

$k_t = k_m = 1.000$

$S_e' = 0.5(UTS) = 282.500\ MPa$

$n_s = [\frac{\sigma_{vm-a}}{k_f k_s k_r k_t k_m S_e'} + \frac{\sigma_{vm-m}}{UTS}]^{-1} = 1.6866\ > 1.1$ [Chosen shaft diameter at C is safe]

Yielding

$n_y = [\frac{\sigma_{vm-a}}{S_y} + \frac{\sigma_{vm-m}}{S_y}]^{-1} = 6.3999\ >\ 1.1$ [55mm shaft diameter does not pass yield line]

Gear Stress Analysis

Choose material for gear:

Gear Material	Hardness [HB]	E (MPa)	Poisson Ratio	Material Grade
AISI 1045 Carbon Steel	400.0000	200000.0000	0.2850	1

Bending

Calculate number of shaft cycles:

$t = 5\,year * 52\,\frac{week}{year} * 7\,\frac{day}{week} * 8\,\frac{hr}{day} * 60\,\frac{min}{hr} = 873600\,min$

Rotation speed n_3= 399.7003 RPM.

$N = t * n_3 = 349178207.9712\,Cycles$

Calculate allowable bending stress:

$Y_n = 1.683(N)^{-0.0323} = 0.8915$ [Stress cycle factor]

$S_b = 0.533(HB) + 88.3 = 301.5000\,MPa$ [Material strength for gear grade 1]

$k_t = k_r = 1.000$ [Temperature not higher than 125 C, 99% reliability]

$\sigma_{b,all} = \frac{S_b Y_n}{k_t k_r} = 268.8011\,MPa$

Calculate actual bending stress:

$Y_j = 0.420$ [Hamrock et al., 2005, N_3=27, N_4=71]

$k_a = 1.000$ [No shock, Electric motor]

$k_s = 1.000$ [Module = 5 mm]

$k_m = 1.17$ [From Hamrock et al, 1999 table] [$b_w/d_p = 0.4444,\ b_w = 60\,mm$]

$k_i = 1.000$ [Non Idler]

$k_b = 1.000$ [Rim thickness for solid disk]

Calculate Kv dynamic factor:

$V = \pi n_3 d/60 = 7.4295\,[m/s]$

$Q_v = 6$ [Commercial gear]

$B = \frac{(12-Q_v)^{2/3}}{4} = 0.8255$

$A = 50 + 56(1 - B) = 59.7730$

$K_v = (\frac{A+\sqrt{200V}}{A})^B = 1.5081$

$\sigma_{b,\, actual} = \frac{W^t}{mb_w Y_j} k_a k_s k_m k_v k_i k_b = 131.9382\ MPa$

Comparing the allowable to the actual bending stress:

$n_b = \sigma_{all}/\sigma_b = 2.0373 > 1.1$

The design is sufficient for bending stress since $n_b > n = 1.1$

Contact

Calculate allowable contact stress:

$HB_{pinion} = 500\ [HB]$

$HB_{gear} = 400\ [HB]$

$Z_N = 2.466N^{-0.056} = 0.8196$ [Cycle factor]

$\frac{HB_p}{HB_g} = 1.25$ [Hardness ratio]

$A' = (8.89 * 10^{-3})(\frac{HB_p}{HB_g}) - 8.29 * 10^{-3}$ $[1.2 \le \frac{HB_p}{HB_g} = 1.25 \le 1.7]$

$\frac{N_g}{N_p} = 2.6296$

$C_H = 1.0 + A'(\frac{N_g}{N_p} - 1.0) = 1.0046$ [Hardness Factor, for gear only]

$K_T = 1$ [Temperature less than 125 C]

$K_R = 1$ [99% reliability]

$S_c = 2.22 * HB_g + 200 = 1088\ MPa$ [Material strength for contact]

$\sigma_{c,\, all} = \frac{S_{c\_} Z_N C_{Hg}}{K_T K_R} = 895.7861$ MPa

Calculate actual contact stress:

$K_a = 1$ [No shock, electric motor]

$K_s = 1$ [m = 5 mm]

$$K_E = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g}}} = 466563.1271 \quad [\sqrt{Pa}]$$

$K_{m,\,gear} = 1.17$ [Same from bending]

$K_v = 1.5081$ [From bending calculation]

$\phi = 20\,deg;\ d_p = 0.135\,m;\ d_g = 0.355\,m$

$$I = \frac{\pi cos(\phi) sin(\phi)}{1+\frac{d_p}{d_g}} = 0.7315$$ [Geometry factor]

$$\sigma_{c,actual} = K_E\sqrt{\frac{W^t}{b_w d_p I} K_a K_s K_m K_v} = 781.4993\,MPa$$

Factors of Safety for Bending and Contact

$\sigma_{c,actual} = 781.4993\,MPa$

$\sigma_{c,\,all} = 895.7861$ MPa

$\sigma_{b,\,actual} = 131.9382\,MPa$

$\sigma_{b,\,all} = 268.8011\,MPa$

$$n_c = \frac{\sigma_{c,\,all}}{\sigma_{c,actual}} = 1.1462 > 1.1$$ [Safe for contact stress]

$$n_b = \frac{\sigma_{b,\,all}}{\sigma_{b,actual}} = 2.0373 > 1.1$$ [Safe for bending stress]

Bearing Selection

$h = (5\, year)(364\, \frac{day}{year})(8 \frac{hr}{day}) = 14560\, hr$ [lifetime hours]

n_3 = 399.7003 RPM

$L_{10} = 60(h)(n_3)(10^{-6}) = 349.1782\, Million\, Revs$

$A_P = 1.000$ [No Shock, electric motor]

$m = 3$ [Ball Bearing]

$P_r = Max(A, B) = 6875.3362\, N$ [A and B calculated in Force Analysis section]

$X = 1.000$

$P = P_r X = 6875.3362\, N$ [No Axial Force]

$C = PL_{10}^{1/m} = 48414.5965\, N$

Given the calculated C, I specify the following bearing from SKF:

Bearing Spec	C (N)	C0 (N)	d (m)	D (m)	W (m)
6311	74100.0000	45000.0000	0.0550	0.1200	0.0290

Reference: https://www.skf.com/us/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings/productid-6311

This bearing is safe because the bearing specification C is less than our calculated C. Recalculation is not necessary since there is no thrust force and radial factor X is constant, not a function of e or C0.

Expected Lifetime of Bearing

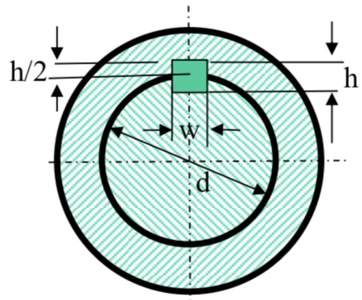
$L = (\frac{C}{P})^m = 1251.9091\, mil\, rev$ [using C=74100 N from SKF]

$Life = L(10^6)(n_3)^{-1}(60 \frac{min}{hr})^{-1}(8 \frac{hr}{day})^{-1}(7 \frac{day}{week})^{-1}(52 \frac{week}{yr})^{-1} = 17.9265\, yr$

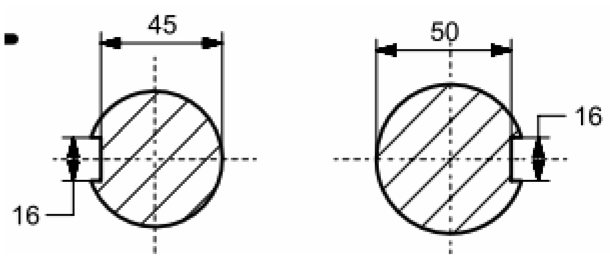
Bearing does not require maintenance for intended operation of 5 years.

Keyways and Tolerances

Coupling Keyway (50 mm) and Gear Keyway (55 mm):



Shaft diameter d (mm)	Key width W x height H
50 < d <= 58	16 x 10



Fits:

Bearing and Shaft: Interference fit (H7/p6), bearing should be tightly fit with shaft for fluid rotation.

RESULTS		
HOLE		
Parameter	**Value**	**Unit**
Designation	55 H7	---
Hole Upper Deviation	30	µm (0.001mm)
Hole Lower Deviation	0	µm (0.001mm)
Maximum Hole Size	55.03	mm
Minimum Hole Size	55	mm
SHAFT		
Parameter	**Value**	**Unit**
Designation	55 p6	---
Shaft Upper Deviation	51	µm (0.001mm)
Shaft Lower Deviation	32	µm (0.001mm)
Maximum Shaft Size	55.051	mm
Minimum Shaft Size	55.032	mm
FIT		
Parameter	**Value**	**Unit**
Designation	55 H7/p6	---
Fit Type	Interference fit	---
Maximum Interference	51	µm (0.001mm)
Minimum Interference	2	µm (0.001mm)

Gear and Shaft: Transition fit (H7/k6), shaft rotation will be transmitted from gear torque through interlocked key.

RESULTS		
HOLE		
Parameter	**Value**	**Unit**
Designation	55 H7	---
Hole Upper Deviation	30	µm (0.001mm)
Hole Lower Deviation	0	µm (0.001mm)
Maximum Hole Size	55.03	mm
Minimum Hole Size	55	mm
SHAFT		
Parameter	**Value**	**Unit**
Designation	55 k6	---
Shaft Upper Deviation	21	µm (0.001mm)
Shaft Lower Deviation	2	µm (0.001mm)
Maximum Shaft Size	55.021	mm
Minimum Shaft Size	55.002	mm
FIT		
Parameter	**Value**	**Unit**
Designation	55 H7/k6	---
Fit Type	Transition fit	---
Maximum Interference	21	µm (0.001mm)
Maximum Clearance	28	µm (0.001mm)

Source: https://amesweb.info/fits-tolerances/tolerance-calculator.aspx