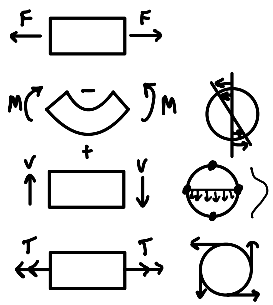
FORCES, STRESS



$\sigma_x = \frac{F}{A} = \boxed{\frac{4F}{\pi D^2}}$

$\sigma_b = \frac{MC}{I} = \boxed{\frac{32M}{\pi D^3}}$; $C = D/2$; $I = \frac{\pi D^4}{64}$

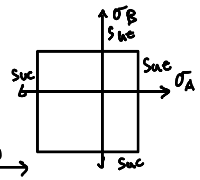
$\tau_v = \frac{4V}{3A} = \boxed{\frac{16V}{3\pi D^2}}$

$\tau_T = \frac{TC}{J} = \boxed{\frac{16T}{\pi D^3}}$; $J = \frac{\pi D^4}{32}$

FAILURE THEORY

MNST

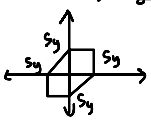
$\sigma_1 \le S_{ut}/n_s$ AND $\sigma_3 \ge -S_{uc}/n_s$



MSST

$(\sigma_1 - \sigma_3) \le S_y/n_s$

$\tau_{max} = (\sigma_1 - \sigma_3)/2$

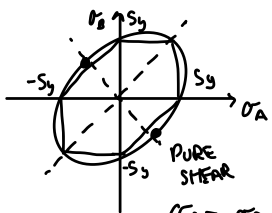


LARGER FoS MORE CONSERVATIVE

VON MISES

$\sigma_{vm} \le S_y/n_s$

$\sigma_{vm} = \sqrt{(\sigma_x)^2 + 3(\tau_{xy})^2} = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2}{2}}$



$\sigma_A = -\sigma_B$

$S_{sy} = 0.5 S_y$ MSST

$S_{sy} = 0.577 S_y$ DET

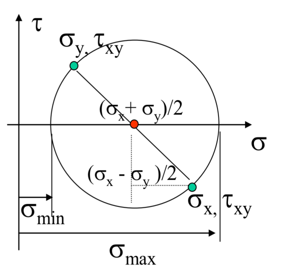
PRINCIPLE STRESSES

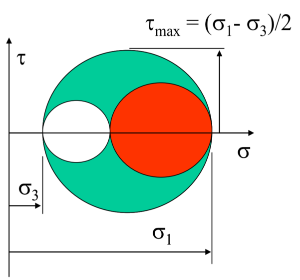
$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$; $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

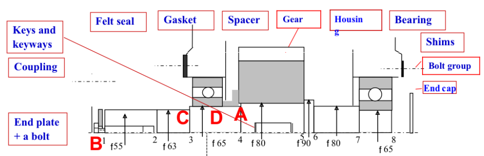
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_1, \sigma_3 = \sigma_{avg} \pm R$; $\sigma_z = 0$

MOHR CIRCLE







FATIGUE

FATIGUE STRENGHT (MATERIAL)

INFINITE:

$S_e' = 0.5 S_{ut}$

$S_{ae}' = 0.45 S_{ut}$

$S_{se}' = 0.29 S_{ut}$

LOW CYCLE FINITE $(0 - 10^3)$

$S_L' = 0.9 S_{ut}$ [VON MISES/BENDING]

$S_{aL}' = 0.75 S_{ut}$ [AXIAL]

$S_{sL}' = 0.72 S_{ut}$ [TORSION]

FINITE $(10^3 - 10^6)$

$b = -\frac{1}{3}\log\left(\frac{S_L'}{S_e'}\right)$ log MPa / log N

$c = \log\left(\frac{S_L'^2}{S_e'}\right)$ log MPa

$S_f' = (10)^c (N)^b$ MPa

MODIFIED FATIGUE LIMIT

$S_e = k_o k_f k_s k_r k_t k_m S_e'$

k_o: STRESS CONCENTRATION REDUCTION

$K_f = 1 + (K_t - 1)q$ $\quad K_{fs} = 1 + (K_{ts} - 1)q$

K_t: THEORETICAL STRESS COEF.

q: NOTCH SENSITIVITY (0.2 CAST IRON)

$k_o = 1/K_f$

k_f: SURFACE FINISH $k_f = e(S_{ut})^f$

Manufacturing process	Factor *e* (MPa)	Factor *e* (Ksi)	Exponent *f*
Grinding	1.58	1.34	-0.085
Machining or cold drawing	4.51	2.70	-0.265
Hot rolling	57.7	14.4	-0.718
As forged	272.0	39.9	-0.995

k_s: SIZE

$k_s = 0.869\ (d)^{-0.112}$ $\quad 0.3" < d < 10"$ $\quad$ (d in inch)

$k_s = 1$ $\quad 0.3" > d$, or $d \le 8$ mm

$k_s = 1.189\ (d)^{-0.112}$ $\quad 8$ mm $< d \le 250$ mm $\quad$ (d in mm)

k_r: RELIABILITY

Percentage probability of survival	Reliability factor, k_r
90	0.90
95	0.87
99	0.82
99.9	0.75

k_t: TEMPERATURE (1) $\quad k_t = S_{ut}/S_{ut,ref}$

k_m: MISC. (1)

A: SLIGHTLY SHORTER THAN GEAR WIDTH

C: END CAP HOLE LARGER THAN SHAFT OD

D: SPACER ID LARGER THAN SHAFT OD

BEARING INNER RING/SHAFT: INTERFERENCE

BEARING OUTER RING/HOUSING: TRANSITION

GEAR AND SHAFT: TRANSITIONAL

VARIABLE STRESS

NORMAL

σ_m = AXIAL

σ_a = BENDING

SHEAR

$\tau_m = 0$

$\tau_a = V$

TORSION

$\tau_m = T$ $\quad \tau_m = T/2$

$\tau_a = 0$ $\quad \tau_a = T/2$

GOODMAN LINE

$\sigma_{vm\text{-}a} = \sqrt{(K_f \sigma_{xa})^2 + 3(K_{fs} \tau_{xya})^2}$

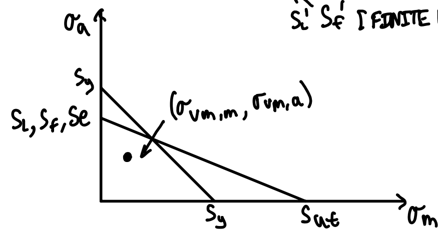
$\sigma_{vm\text{-}m} = \sqrt{(\sigma_{xm})^2 + 3(\tau_{xym})^2}$

$\frac{1}{n_s} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$

[CONSERVATIVE IF σ τ VARY AT DIFF FREQ.]

$\frac{1}{n_s} = \frac{\sigma_{vm\text{-}a}}{k_f k_s k_r k_t k_m S_e'} + \frac{\sigma_{vm\text{-}m}}{S_{ut}}$ [GOODMAN]

S_L' S_f' [FINITE LIFE]



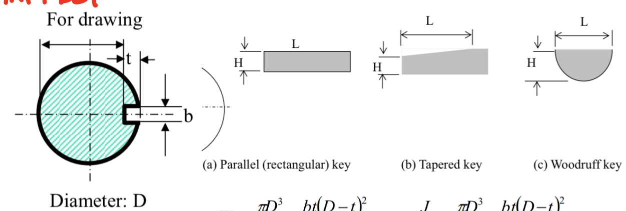
IGNORE τ_{xya}

σ_{xm} BECAUSE SMALL

YIELD LINE

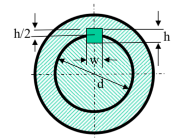
$\frac{1}{n_s} = \frac{\sigma_{vm\text{-}a}(\text{NO } K)}{S_y} + \frac{\sigma_{vm\text{-}m}}{S_y}$

SHAFT KEY



$W_x = \frac{\pi D^3}{32} - \frac{bt(D-t)^2}{2D}$ $\quad \frac{J}{c} \approx \frac{\pi D^3}{16} - \frac{bt(D-t)^2}{2D}$

$A \approx \frac{\pi D^2}{4} - bt$



Shaft diameter d (mm)	Key width W x height h
12<d≤17	5 x 5
17<d≤22	6 x 6
22<d≤30	8 x 7
30<d≤38	10 x 8
38<d≤44	12 x 8
44<d≤50	14 x 9
50<d≤58	16 x 10
58<d≤65	18 x 11

MATERIAL

DUCTILE: HIGH FATIGUE STRENGTH AND ENDURANCE LIMIT

DIAMETER

$\tau_{all} = \frac{S_{sy}}{n_s}$; $S_{sy} = 0.5 S_y$; $S_{sy} = 0.577 S_y$

$d_{min} \ge \sqrt[3]{\frac{16T}{\pi \tau_{all}}}$; KEYWAY: +3-5% ONLY TORQUE

GEAR TRAIN

$i_{pg} = \frac{n_p}{n_g} = \frac{\omega_p}{\omega_g} = \frac{D_g}{D_p} = \frac{N_g}{N_p}$

$d = \frac{N}{P} = mN$

VR>1 : SPEED RED.
$d_2 > d_1$, $N_2 > N_1$

VR<1: SPEED INCR.
$N_2 < N_1$, $d_2 < d_1$

EXTERNAL GEAR: INVERT (-)

INTERNAL GEAR: DON'T (+)

$\frac{\omega_i}{\omega_o} = (-1)^m$

PLANETARY

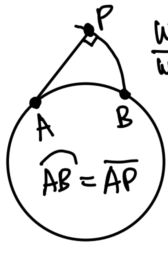
$i_{io} = \frac{\omega_i - \omega_a}{\omega_o - \omega_a}$ $\quad \omega_a n_a = 0 \quad n_i - n_a \Rightarrow n_i$

$i = \frac{\text{PRODUCT OF DRIVEN TEETH}}{\text{PRODUCT OF DRIVING TEETH}} (-1)^m$

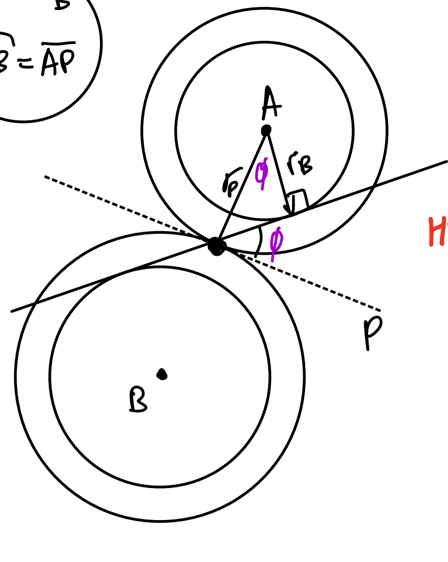
$V = \omega_1 r_1 = \omega_2 r_2$

$\frac{8}{P} < b_w < \frac{16}{P}$
$8m < b_w < 16m$

INVOLUTE



$\frac{W_1}{W_2} = \frac{\overline{AP}}{\overline{BP}} = \frac{r_A}{r_B} = \frac{R_{Ab}}{R_{Bb}}$



SPUR GEAR

$d = \frac{N}{P} = mN$ $\qquad a = P^{-1} = m$

$d_b = d\cos\phi$ $\qquad b = 1.25P^{-1} = 1.25m$

$d_o = d + 2a$ $\qquad h = b + a$ (TOOTH DEPTH)

$d_r = d - 2b$ $\qquad c = b - a$ (CLEARANCE)

$C_g = (d_p + d_g)/2$ $\qquad t = P_c/2$ (TOOTH THICKNESS)

$m = \frac{1}{P}$ $\qquad P_c = \pi d/N = \pi m$ (CIRCULAR PITCH)

$\phi = \cos^{-1}\left(\frac{d_b}{d}\right)$ $\qquad P_b = \pi m \cos\phi$ (BASE PITCH)

$W = J/s = kg \cdot m^2 \cdot s^{-3}$

MESHING

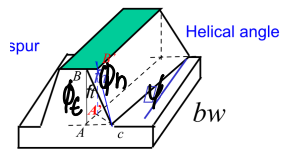
LINE AB: LINE OF ACTION
STARTS AT C: OUTER CIRCLE OF DRIVEN INTERSECTS AB
ENDS AT D: OUTER CIRCLE OF DRIVER INTERSECTS AB
MESHING ONLY BEYOND BASE CIRCLE

$C_r = \frac{1}{P_c\cos\phi}\left(\sqrt{r_{op}^2 - r_{bp}^2} + \sqrt{r_{og}^2 - r_{bg}^2}\right) - \frac{C_d\tan\phi}{P_c} = \frac{CD}{P_b}$

$1 \le C_r < 2$ $\quad$ 0.4 $\quad$ 0.4 $\quad$ DOUBLE TOOTH : 1.4

$1.6 <$ SPUR GEAR

HELICAL GEAR



$\phi_n : \angle A'B'C$ $\quad \phi_t : \angle ABC$

$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$; $AB = A'B'$

$P_n = P_t\cos\psi$; $\tan\psi = \frac{P_t}{P_x}$ $\qquad \frac{b_w}{P_x}$

PARALLEL AXIS:
EXT GEAR: $\psi_p = \psi_g$
INT GEAR: $\psi_p = \psi_g$
GENERALLY:
$\psi_p + \psi_g$ = SHAFT ANGLE (90°)

GEOMETRY:
$P_t = \pi m_t$ $\quad P_n = \pi m_n$
$P_n = P_t\cos\psi$
$a = m_n = \frac{1}{P_n}$; $b = 1.25m_n = \frac{1.25}{P_n}$

CONTACT RATIO

$d \to d_t = N m_t$ $\quad d_b \to d_{bt} = d_t\cos\phi_t$
$d_o \to d_{ot} = d_t + 2m_n$ $\quad d_r \to d_{rt} = d_t - 2.5m_n$

$C_{rH} = C_{rt} + C_{rx}$
$= C_{rt} + \frac{b_w}{P_x} = \left(\frac{1}{P_t\cos\phi_t}\left(\sqrt{r_{opt}^2 - r_{bpt}^2} + \sqrt{r_{ogt}^2 - r_{bgt}^2}\right) - \frac{C_d\tan\phi_t}{P_t}\right) + \frac{b_w\sin\psi}{P_n}$

GEAR FORCES

SPUR GEAR

$\vec{W} = \vec{W}^t + \vec{W}^r$

DRIVER: AGAINST MOTION
DRIVEN: WITH MOTION

$W^t = \frac{T}{d/2} = \frac{60H}{\pi d n}$ [PITCH CIRCLE d] $\quad T = \frac{H}{\omega}$

$W^t = W\cos\phi$; $W^r = W^t\tan\phi$

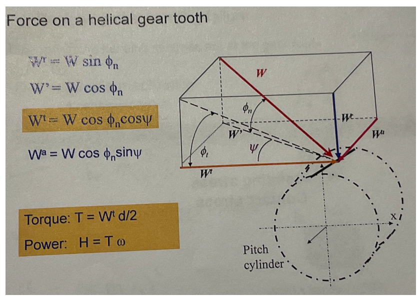
$HP = 550\,ft \cdot lbf/s$
$= 745.6$ Watts

$W = \frac{W^t}{\cos\phi}$ $\qquad H = \frac{W^t V}{33000} = W^t V = T\omega$

$1HP = 33000\,ft \cdot lbf/min$

DRIVER: TORQUE DIR OPPOSITE ROT DIR
DRIVEN: TORQUE DIR SAME ROT DIR

HELICAL GEAR



GEAR STRESSES

$W^T = \frac{T}{d/2} = \frac{60H}{\pi d n}$ [BENDING]

$W = \frac{W^T}{\cos\phi}$ [CONTACT]

GEAR BENDING

$\sigma_B \le \sigma_{all,b}$ $\qquad n_b = \sigma_{all}/\sigma_b$

$\sigma_b = \frac{W^t}{b_w m Y_j} K_a K_s K_m K_v K_i K_b$

$\sigma_b = \frac{W^t P}{b_w Y_j} K_a K_s K_m K_v K_i K_b$

Y_j: GEOMETRY FACTOR
K_a: APPLICATION FACTOR
K_b: RIM THICKNESS (1)
K_i: IDLER FACTOR (NO IDLER: 1)
K_v: DYNAMIC FACTOR
K_m: LOAD DISTRIBUTION FACTOR
K_s: SIZE FACTOR

S_B, S_c:
10m CYCLES
99%
T<125°C

GEAR CONTACT STRESS

$\sigma_c = K_E\left(\frac{W^t}{b_w d_p}\frac{1}{I}K_a K_s K_m K_v\right)^{1/2}$ $\qquad R = \frac{1}{\frac{1}{R_g} + \frac{1}{R_p}} = \frac{1}{\left(\frac{1}{d_g} + \frac{1}{d_p}\right)\frac{2}{\sin\phi}}$

$I = \frac{\pi\cos\phi\sin\phi}{\left(1 \pm \frac{d_p}{d_g}\right)}$; $K_E = \sqrt{E'} = \sqrt{\frac{2}{\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}}}$

$n_c = \sigma_{all,c}/\sigma_c$

$\sigma_{all,b} = \frac{S_b Y_n}{K_T K_R}$ $\quad$ CYCLE FACTOR $\quad$ ← RELIABILITY $\quad$ TEMPERATURE

$\sigma_{all,c} = \frac{S_c Z_n C_H}{K_T K_R}$ $\quad$ CYCLE FACTOR $\quad$ CONTACT HARDNESS

PINION HIGHER GRADE HARDNESS
PINION MAT. BETTER: MORE CYCLES
PINION HARDER: MORE CONTACT STRESS CYCLES
DIFF MAT. BC ADHESION

GEAR FORCES EXAMPLE

$$T_1 = \frac{H}{w_1} = \frac{60H}{2\pi n_1}$$

$$w = \frac{2\pi n}{60}$$

$$W^t = \frac{T_1}{d/2} \quad W = \frac{W^t}{\cos\phi} \quad W^r = W^t \tan\phi$$

$$\sigma_b \le \sigma_{all,b} \; ; \; n_b = \sigma_{all,b}/\sigma_b$$

$$\frac{W^t P}{b w Y_j} K_a K_s K_m K_v K_i K_b \le \frac{S_b Y_n}{K_T K_R}$$

$$\sigma_c \le \sigma_{all,c} \; ; \; n_c = \sigma_{all,c}/\sigma_c$$

$$K_e \left(\frac{W^t}{b w d_p} \frac{1}{I} K_a K_s K_m K_v \right)^{1/2} \le \frac{S_c Z_n C_H}{K_T K_R}$$

Y_j : USE TABLE / CHART, X-AXIS GEAR TALKED ABOUT

HARDNESS RATIO FACTOR (C_H)

PINION 50 ~ 80 HB

$1.2 < HB_p / HB_g \le 1.7$

$C_H = 1.0 + A'(N_g/N_p - 1)$

$A' = 0 \qquad HB_p/HB_g < 1.2$

$A' = (8.89 \times 10^{-3})(HB_p/HB_g) - 8.29 \times 10^{-3} \qquad 1.2 \le HB_p/HB_g \le 1.7$

$A' = 0.00698 \qquad HB_p/HB_g > 1.7$

NO CONTACT STRESS RATIO FOR PINION

SIZE FACTOR (K_s)

DIAMETRAL PITCH (1/in)	MODULE (mm)	K_s
≥ 5	≤ 5	1
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

LOAD DISTRIBUTION (K_m)

USE CHART (FACE WIDTH)

PERCENTAGE OF SURVIVAL	RELIABILITY K_r
50	0.70
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Y_n ; Z_n : CYCLE FACTOR

$$NC_g = NC_p \cdot \frac{n_g}{n_p} = NC_p \cdot \frac{N_p}{N_g}$$

STRESS CYCLE FACTOR CHOOSE LOWEST

APPLICATION (K_a)

NO SHOCK	1 ← ELECTRIC MOTOR
LIGHT SHOCK	1.2
MOD. SHOCK	1.3

DYNAMIC FACTOR (K_v)

$$K_v = \begin{cases} \left(\frac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\frac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases}$$

$$B = \frac{(12 - Q_v)^{2/3}}{4} \qquad A = 50 + 56(1 - B)$$

Q_v (ACCURACY): $Q_v = 3 \sim 7$ [COMMERCIAL]

$Q_v = 8 \sim 12$ [PRECISION]