315 THEORY OF MACHINES – DESIGN OF ELEMENTS

Fall 2023

HW No. 6

Assigned: 10/26 Due: one week, 11/2, On-line, pdf in **one single file**.

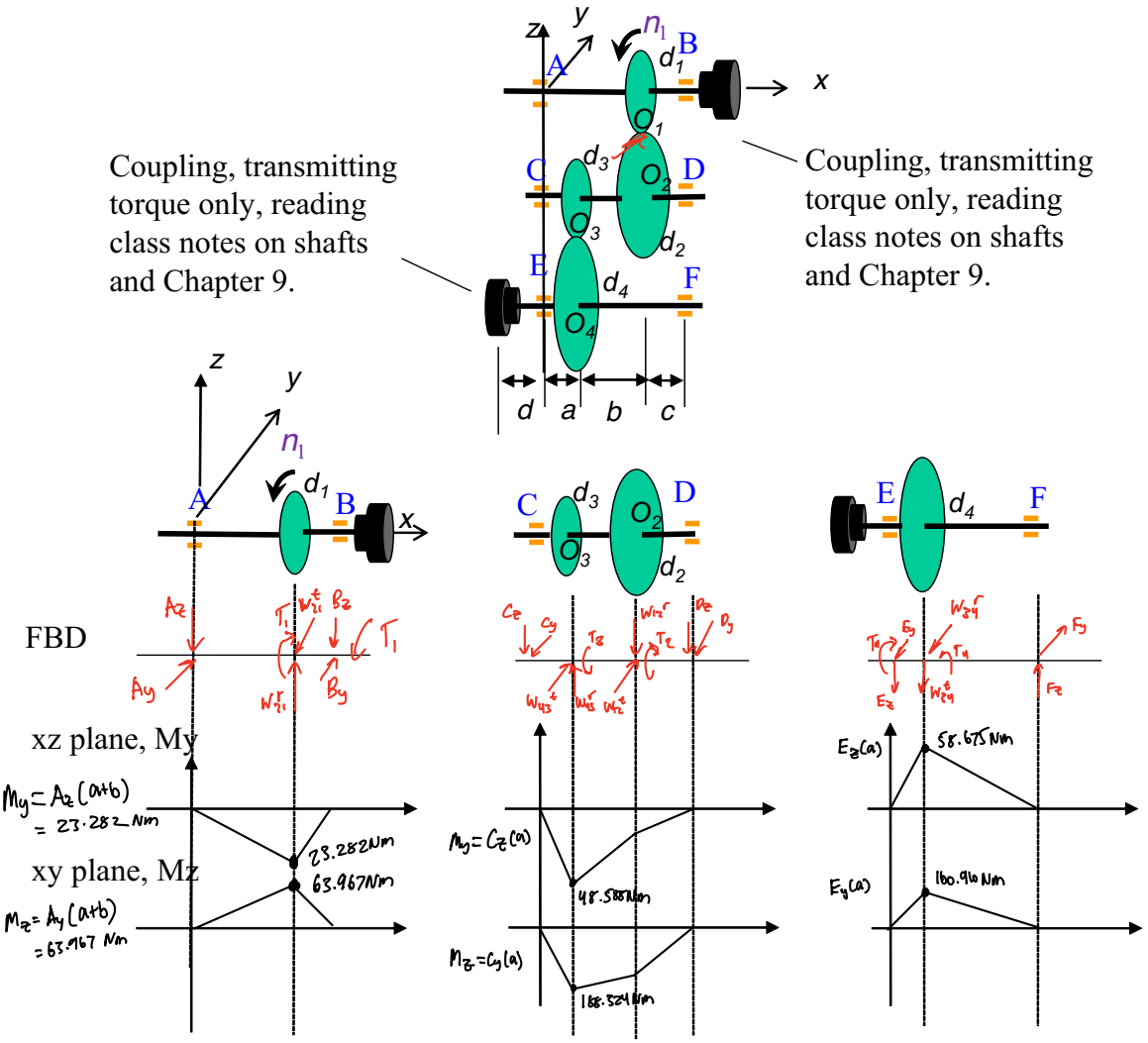
50 points

1. (30 points) This gear train is similar to that in the assigned project; it has four gears, 1, 2, 3 and 4. The input is from the pulley (belt transmission) to the shaft Gear 1, n_1= 2400rpm and the input power is H = 30KW (no power loss). The rotation direction of gear 1 (also shaft 1) is shown. Gear parameters are: m_1 = 5, m_3 = 6, N_1= 19, N_2= 81, N_3= 23, N_4 = 97, and pressure angles are all 20 degrees. The gear centers are O_1, O_2, O_3, and O_4. For a = 30 mm, b = 40 mm, c = 40 mm, d = 35 mm, calculate and determine

a) The torque transmitted on each gear and forces on each gear, and the forces on each shaft; plot the free-body diagram for each shaft. All forces should be in their correct directions.

b) The moment diagrams of each shaft, and the magnitude and location of the maximum bending moment on each shaft.

You need to plot the forces and bending moment diagrams in the xz and xy planes.



$m_1 = 5\,mm$; $m_2 = 6\,mm$

$d_1 = m_1 N_1 = 95\,mm$

$d_2 = m_1 N_2 = 405\,mm$

$d_3 = m_2 N_3 = 138\,mm$

$d_4 = m_2 N_4 = 582\,mm$

$n_2 = n_1 \frac{N_1}{N_2} = 562.96\,RPM$

$T_1 = \frac{H}{w_1} = \frac{60H}{2\pi n_1} = 119.4\,Nm$

$|W_{12}^t| = |W_{21}^t| = \frac{2T_1}{d_1} = 2512.97\,N$

$T_2 = 2W_{12}^t / d_2 = 508.88\,Nm = T_3$

$|W_{12}^r| = |W_{21}^r| = W_{21}^t \tan\phi = 914.647\,N$

$|W_{34}^t| = |W_{43}^t| = 2T_2 / d_3 = 7375.03\,N$

$|W_{34}^r| = |W_{43}^r| = W_{43}^t \tan\phi = 2684.29\,N$

$T_4 = 2W_{43}^t / d_4 = 2146.13\,Nm$

$|W_{21}| = |W_{12}| = W_{12}^t / \cos\phi = 2674.25\,N$

$|W_{34}| = |W_{43}| = W_{34}^t / \cos\phi = 7848.34\,N$

$a = 0.03$, $b = 0.04$, $c = 0.04$, $d = 0.035$

SHAFT 1:

$(a+b) W_{21}^t - B_y (a+b+c) = 0$

$B_y = 1599.165\,N$

$A_y = W_{21}^r - B_y = 913.808\,N$

$(a+b) W_{21}^r - B_z (a+b+c) = 0$

$B_z = 582.048\,N$

$A_z = W_{21}^r - B_z = 332.599\,N$

$M_{max} = \sqrt{M_{y1}^2 + M_{z1}^2}$

$= 68.072\,Nm$

SHAFT 2:

$D_y (a+b+c) = W_{43}^t (a) + W_{12}^t (a+b)$

$D_y = 3610.536\,N$

$C_y = W_{43}^t + W_{12}^t - D_y = 6277.466\,N$

$D_z (a+b+c) = W_{12}^r (a+b) - W_{43}^r (a)$

$D_z = 150.031\,N$

$C_z = W_{43}^r - W_{12}^r + D_z = 1619.613\,N$

$M_{max} = \sqrt{M_{y2}^2 + M_{z2}^2}$

$= 194.491\,Nm$

SHAFT 3:

$F_z (a+b+c) = a W_{34}^r$

$F_z = 783.443\,N$

$E_z = W_{34}^r - F_z = 1955.847\,N$

$F_y (a+b+c) = a (W_{34}^t)$

$F_y = 2011.372\,N$

$E_y = W_{34}^t - F_y = 5363.658\,N$

$M_{max} = \sqrt{M_{y3}^2 + M_{z3}^2}$

$= 171.274\,Nm$

2. (20 points) A pair of through hardened spur gears has the following parameters: $Np = 20$, $Ng = 59$, $m = 6$ mm, $\phi = 20$ degree. The power to transmit is 30KW, the speed of the pinion is $n_p = 2000$ rpm. The set is driven by an electric motor. Gear materials are the grades 1 and 2 gear steels, with the better one for the pinion; through hardened, and the gear hardness is $HB_G = 290HB$, the pinion hardness is $HB_P = 1.5\ HB_G$. For steel, $E = 210$ GPa, $v = 0.28$. The face width is $b_w = 12m$, life of the pinion is 30 million cycles, please calculate:

a) Pitch circle diameters and face widths of both the pinion and the gear
b) Tangential force
c) Allowable stresses
d) Factors for stresses; assuming 99% reliability.
e) Tooth bending and contact stresses
f) ;
g) Factors of safety

a) $d_P = m N_P = 120$ mm

$d_g = m N_g = 354$ mm

$b_w = 12m = 72$ mm

b) $H = 30$ kW

$$T = \frac{60H}{2\pi n_p} = 143.239 \text{ Nm}$$

$$W_t = \frac{2T}{d_P} = 2387.32 \text{ N}$$

c,d)

$$\sigma_{all,b} = \frac{S_b Y_n}{K_T K_R} \; ; \; \sigma_{all,c} = \frac{S_c Z_n C_H}{K_T K_R}$$

$HB_G = 290$ HB $\qquad HB_P = 435$ HB

$S_{GB} = 0.533 HB_G + 88.3 = 243.45$ $\qquad S_{GC} = 2.22 HB_G + 200 = 843.8$

$S_{PB} = 0.703 HB_P + 113 = 418.805$ $\qquad S_{PC} = 2.41 HB_P + 237 = 1285.35$

$NC_p = 30$ MIL CYCLES ; $NC_g = N_{CP} \cdot \frac{N_p}{N_g} = 10.2$ MIL CYCLE

$N_p = 20$; $N_g = 59$

$Y_{NP} = 1.6831 NC_p^{-0.0323} = 0.965$

$Y_{NG} = 1.6831 NC_g^{-0.0323} = 0.999$

$Z_{NP} = 2.466\ NC_p^{-0.056} = 0.940$

$Z_{NG} = 2.466\ NC_g^{-0.056} = 0.999$

$HB_p / HB_g = 1.5$

$A' = (8.69 \times 10^{-3})(HB_p/HB_g) - 8.29 \times 10^{-3}$

$= 0.005045$

$C_{Hg} = 1 + A'(N_g/N_p - 1) = 1.00984$

$K_T = 1$

$K_R = 1$

$\sigma_{all,b,G} = \frac{S_b Y_n}{K_T K_R} = 243.32$ MPa $\quad \sigma_{all,c,G} = \frac{S_c Z_N C_H}{K_T K_R} = 851.286$ MPa

$\sigma_{all,b,P} = \frac{S_b Y_n}{K_T K_R} = 404.21$ MPa $\quad \sigma_{all,c,P} = \frac{S_c Z_N}{K_T K_R} = 1208.64$ MPa

e) $\sigma_b = \frac{W^t}{b_w Y_j m} K_a K_s K_m K_v K_i K_0$

$W_6 = 2387.32\ N$; $d_p = 120\ mm$

$b_w = 12m = 72\ mm$

$m = 6mm$

$K_a = 1$

$K_s = 1.05$ ($m = 6mm$)

$K_m = 1.2$

$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = 1.653$

$V = \pi n_p d_p / 60 = 12.5664\ m/s$

$A = 50 + 56(1-B) = 59.773$

$B = \frac{(12 - Q_v)^{2/3}}{4} = 0.8255$

$K_i = K_0 = 1$

$Y_{j,p} = 0.32$

$Y_{j,g} = 0.41$

$\sigma_{B,p} = 35.975\ MPa$

$\sigma_{B,g} = 28.078\ MPa$

$\sigma_{c,g} = \sigma_{c,p} = 417.054\ MPa$

$\sigma_c = K_e \left(\frac{W^t}{b_w d_p} \frac{1}{I} K_a K_s K_m K_v\right)^{1/2}$ ⇒

$I = \frac{\pi \cos\phi \sin\phi}{\left(1 + \frac{d_p}{d_g}\right)} = 0.754$

$K_e = \left(2\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g}\right)^{-1}\right)^{1/2} = 477352\ \sqrt{Pa}$