(1)

$$TS = \frac{a^T \hat\beta - a^T \beta}{\hat\sigma \sqrt{a^T (X^T X)^{-1} a}} \sim t(n-p) \qquad \hat\beta = \begin{pmatrix} \hat\beta_0 \\ \hat\beta_1 \\ \hat\beta_2 \end{pmatrix} ; \ \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}.$$

CC example:

AA: $Y_i = \beta_0 + \varepsilon_i$;
Asian: $Y_i = \beta_0 + \beta_1 + \varepsilon_i$
Caucasian: $Y_i = \beta_0 + \beta_2 + \varepsilon_i$.

~~Ex~~ Let $E(Y_i) = \mu_j$ if Y_i belongs to group j.

① $H_0: \beta_0 = c_1$; $a^T = (1, 0, 0)$. $\beta_0 = a^T \beta$.
This is estimated by $\hat\beta_0 = a^T \hat\beta$.
Under H_0, $(a^T \hat\beta - \underbrace{a^T \beta}_{= c_1}) / (\hat\sigma \sqrt{a^T (X^T X)^{-1} a}) \sim t(n-p)$.

② $H_0: \mu_2 = \boxed{\beta_0 + \beta_1} = c_2$; $a^T = (1, 1, 0)$; $a^T \beta = c_2$ is known.
What needs to be recomputed? $a^T \hat\beta = \hat\beta_0 + \hat\beta_1$; $\hat\sigma \equiv RSE$
Also, compute $a^T \cdot (X^T X)^{-1} \cdot a$ for $a^T = (1, 1, 0)$.
To finish, compute the obs. value of TS under H_0; call this t_0.
Then, p-value $\Pr(T > |t_0|$ under $H_0)$ i.e., assuming T is Student-t with $(n-p)$ deg. of freedom.

pretest, problem 2.3 : $A \cdot x = b$

pick the value of rhs b such that

$\| A x - b \|_2^2 = 0$ for some $x = x^*$.

Is x^* a unique solution?

Notice: $A \cdot c = 0$, where $c = (1, -1, -1)^T$.

let $x^*(\lambda) = x^* + \lambda \cdot c$, where $\lambda \in \mathbb{R}$

then this is also a solution to $A x = b$.

$A x^*(\lambda) = A(x^* + \lambda \cdot c) = A \cdot x^* + \lambda \cdot \underbrace{A \cdot c}_{=0} = A \cdot x^*$,

for any value of λ.

$\Rightarrow$ infinitely many solutions to $A \cdot x = b$.