STA6703 SML HW8

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Question 3

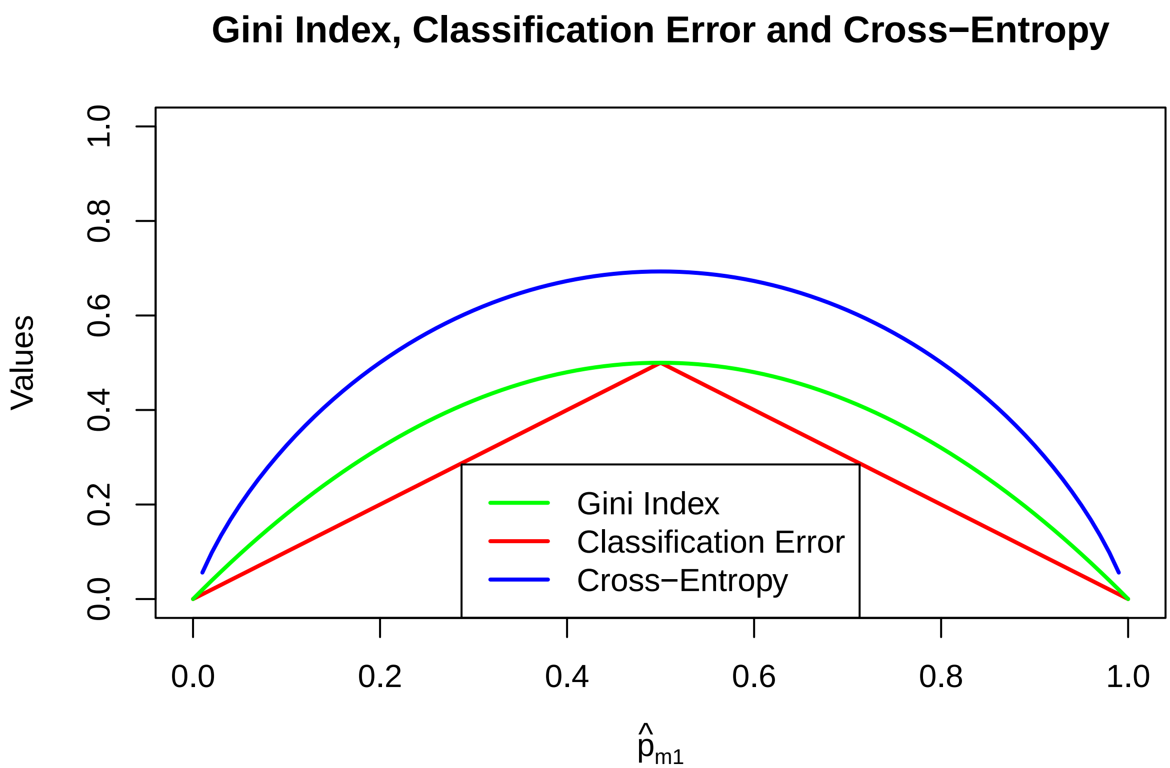
$$E = 1 - max_k(\hat{p}_{mk})$$

$$G = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

$$D = -\sum_{k=1}^{K} \hat{p}_{mk} log(\hat{p}_{mk})$$

Figure 1: Classification Error (E), Gini Index (G), and Entropy (D)

```
# plot Gini Index, Classification Error and Cross-Entropy
p <- seq(0, 1, 0.01)
gini.index <- 2 * p * (1 - p)
class.error <- 1 - pmax(p, 1 - p)
cross.entropy <- - (p * log(p) + (1 - p) * log(1 - p))
df = data.frame(cbind(p, gini.index, class.error, cross.entropy))
{plot(1, type="n", main="Gini Index, Classification Error and Cross-Entropy",
     xlab=expression(hat(p)[m1]), ylab="Values", xlim=seq(0,1), ylim=c(0, 1))
lines(df$p,df$cross.entropy,col="blue",lwd=2)
lines(df$p,df$class.error,col="red",lwd=2)
lines(df$p,df$gini.index,col="green",lwd=2)
legend("bottom",
       legend=c("Gini Index", "Classification Error", "Cross-Entropy"),
       col=c("green", "red", "blue"),
       lty=c(1,1,1),
       lwd=c(2,2,2))}
```



Question 8

a.)

```
# import data
library(ISLR)
library(caTools)
df_data = Carseats

# split data into training and testing sets
set.seed(0)
train_test_filter = sample.split(df_data$Sales, SplitRatio = 0.75)
df_train = subset(df_data, train_test_filter==TRUE)
df_test = subset(df_data, train_test_filter==FALSE)
```

b.)

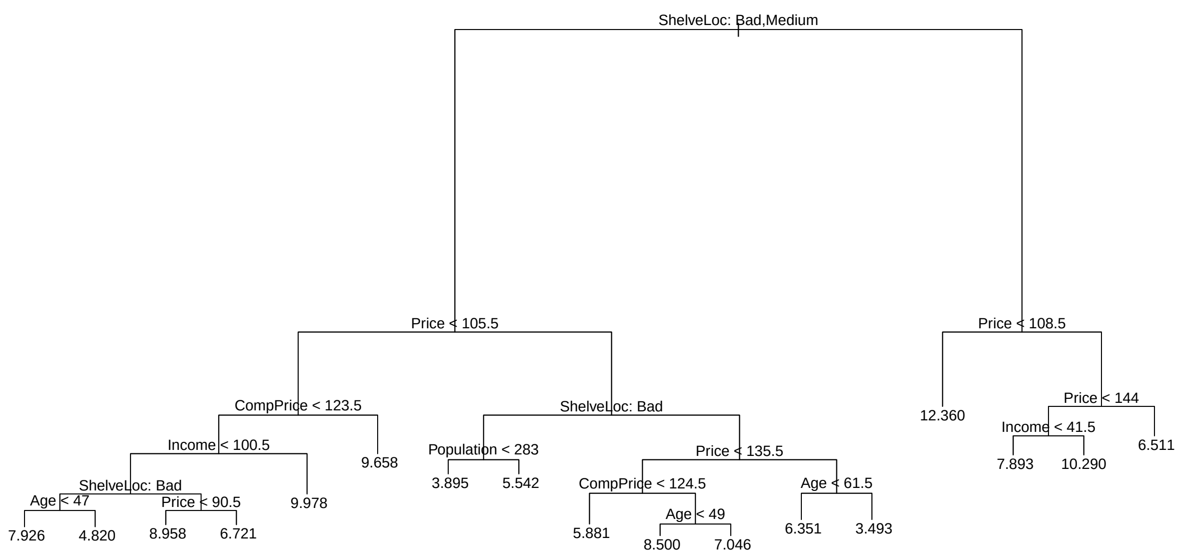
```
# import tree library
library(tree)

# create regression tree model
```

```r
reg_tree_model = tree(Sales~.,data=df_train)
summary(reg_tree_model)
```

```
##
## Regression tree:
## tree(formula = Sales ~ ., data = df_train)
## Variables actually used in tree construction:
## [1] "ShelveLoc"  "Price"      "CompPrice"  "Income"     "Age"
## [6] "Population"
## Number of terminal nodes:  17
## Residual mean deviance:  2.351 = 665.5 / 283
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -4.12200 -1.13100 -0.06688  0.00000  1.04400  4.59800
```

```r
# plot tree
{plot(reg_tree_model)
text(reg_tree_model,pretty=0)}
```



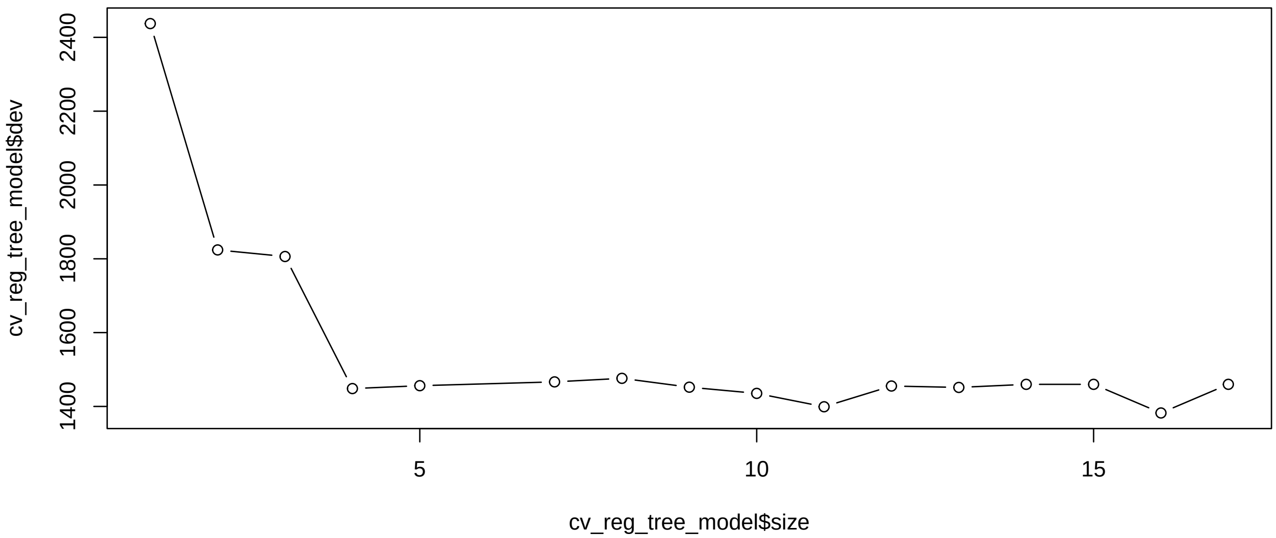

```r
# calculate test MSE
yhat= predict(reg_tree_model ,newdata=df_test)
mse = mean((yhat-df_test$Sales)^2)

print(paste("The test MSE is: ", mse))
```

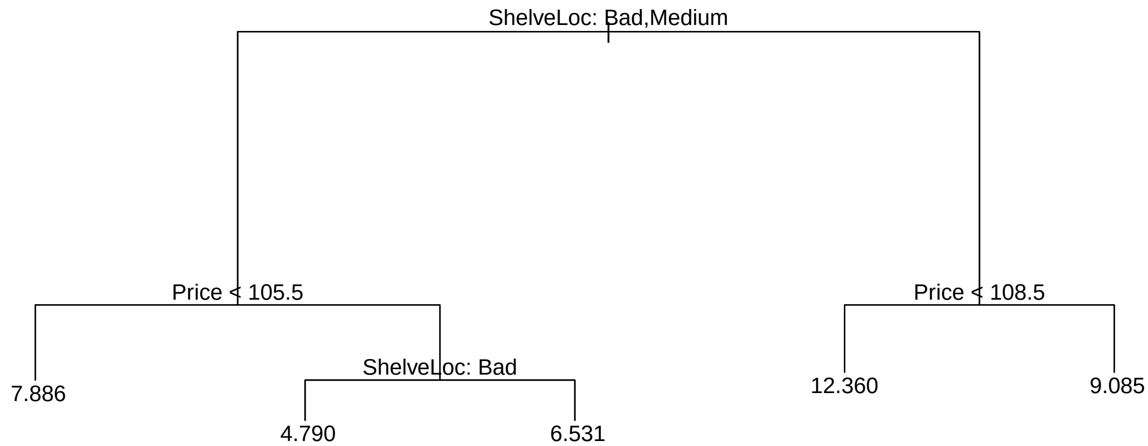
```
## [1] "The test MSE is:  4.70515261090458"
```

c.)

```
# apply cross validation on tree complexity
set.seed(0)
cv_reg_tree_model = cv.tree(reg_tree_model)
plot(cv_reg_tree_model$size ,cv_reg_tree_model$dev ,type="b")
```




```
# prune tree to best size from CV
set.seed(0)
prune_reg_tree_model=prune.tree(reg_tree_model ,best=5)
plot(prune_reg_tree_model )
text(prune_reg_tree_model ,pretty =0)
```




```
# calculate test MSE on pruned tree
yhat= predict(prune_reg_tree_model ,newdata=df_test)
mse = mean((yhat-df_test$Sales)^2)

print(paste("The test MSE after pruning is: ", mse))
```

```
## [1] "The test MSE after pruning is:  4.62268983079291"
```

Pruning the decision tree only slightly improves the MSE.

d.)

```
# import random forest package
library(randomForest)
```

```
## randomForest 4.7-1.1
```

```
## Type rfNews() to see new features/changes/bug fixes.
```

```
# create bagging model
set.seed(0)
bag_tree_model = randomForest(Sales~.,
                         data=df_train,
                         mtry=ncol(df_train)-1,
                         importance =TRUE)

# calculate test MSE on pruned tree
```

```r
yhat = predict(bag_tree_model ,newdata=df_test)
mse = mean((yhat-df_test$Sales)^2)

print(paste("The test MSE for the bagging model is: ", mse))
```

```
## [1] "The test MSE for the bagging model is:  2.5585149484648"
```

```r
importance(bag_tree_model)
```

```
##                %IncMSE IncNodePurity
## CompPrice   30.2792313     229.93406
## Income      12.3444510     149.82951
## Advertising 19.5025809     155.85323
## Population  -1.1909030      81.69699
## Price       68.1588497     630.89837
## ShelveLoc   74.1374927     824.04818
## Age         19.4934903     203.09335
## Education    3.2020555      57.17521
## Urban       -0.3537144      13.31629
## US           3.7656699      14.11128
```

The bagging approach substantially decreases the MSE. The most important predictors are `ShelveLoc` and `Price`.

e.)

```r
# create random forest model
set.seed(0)
rf_tree_model = randomForest(Sales~.,
                         data=df_train,
                         importance =TRUE)

# calculate test MSE on pruned tree
yhat = predict(rf_tree_model ,newdata=df_test)
mse = mean((yhat-df_test$Sales)^2)

print(paste("The test MSE for the RF model is: ", mse))
```

```
## [1] "The test MSE for the RF model is:  2.91624482305896"
```

```r
importance(rf_tree_model)
```

```
##                %IncMSE IncNodePurity
## CompPrice   16.9065928     213.73663
## Income       6.0447745     182.26679
```

```
## Advertising 13.8972554     190.11656
## Population  -1.8545867     137.77206
## Price       42.1641730     525.36243
## ShelveLoc   50.7665440     590.08970
## Age         15.8340355     265.95503
## Education    0.8776803      99.64608
## Urban       -1.8020997      23.94132
## US           5.7207788      35.92824
```

The MSE is reduced substantially for the random forest model. The most important predictors are `ShelveLoc` and `Price`. By reducing `m` the size increases the error rate slightly.

Question 9

a.)

```r
# import data
set.seed(0)
df_data = OJ

# split data
train_test_filter = sample.split(df_data$Purchase,
                                 SplitRatio = 800/nrow(df_data))
df_train = subset(df_data, train_test_filter==TRUE)
df_test = subset(df_data, train_test_filter==FALSE)
```

b.)

```r
# create tree classifier model
set.seed(0)
class_tree_model = tree(Purchase~.,data=df_train)
summary(class_tree_model)
```

```
##
## Classification tree:
## tree(formula = Purchase ~ ., data = df_train)
## Variables actually used in tree construction:
## [1] "LoyalCH"       "PriceDiff"     "SpecialCH"     "ListPriceDiff"
## [5] "PctDiscMM"
## Number of terminal nodes:  9
## Residual mean deviance:  0.7189 = 568.7 / 791
## Misclassification error rate: 0.1575 = 126 / 800
```

The training error rate of the model is: 0.1575 The number of terminal nodes on the tree is 9

c.)

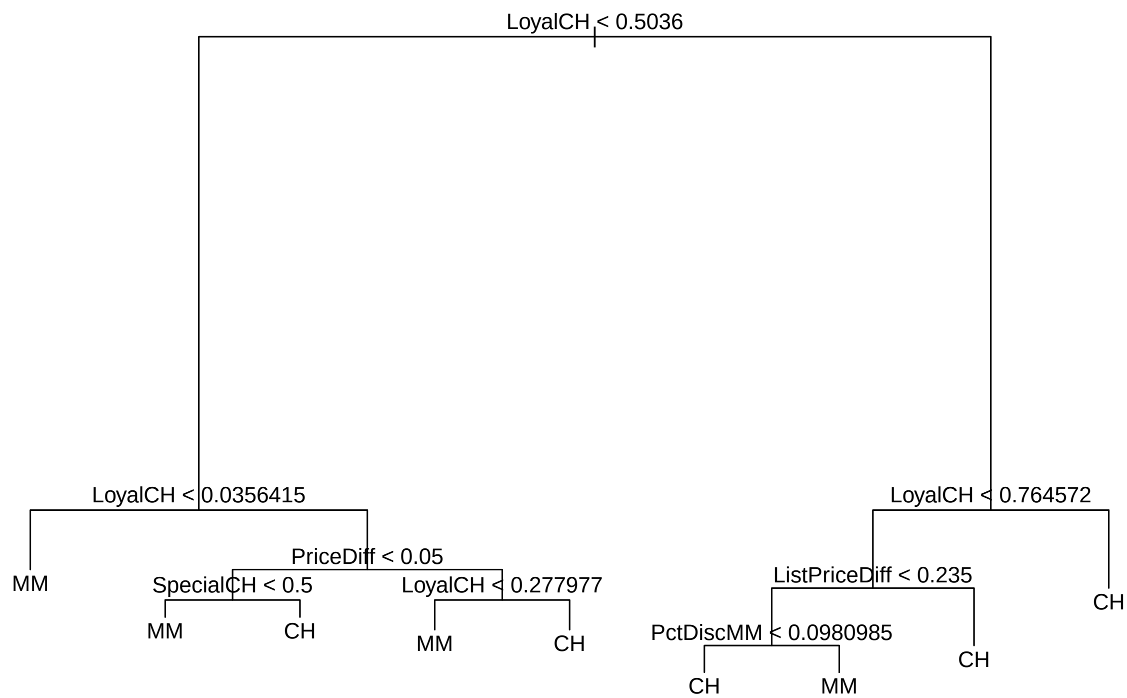
```r
class_tree_model
```

```
## node), split, n, deviance, yval, (yprob)
##       * denotes terminal node
##
##  1) root 800 1070.00 CH ( 0.61000 0.39000 )
##    2) LoyalCH < 0.5036 355  422.10 MM ( 0.28169 0.71831 )
##      4) LoyalCH < 0.0356415 53    0.00 MM ( 0.00000 1.00000 ) *
##      5) LoyalCH > 0.0356415 302  383.50 MM ( 0.33113 0.66887 )
##       10) PriceDiff < 0.05 132  131.00 MM ( 0.19697 0.80303 )
##         20) SpecialCH < 0.5 115   96.35 MM ( 0.14783 0.85217 ) *
##         21) SpecialCH > 0.5 17   23.51 CH ( 0.52941 0.47059 ) *
##       11) PriceDiff > 0.05 170  232.80 MM ( 0.43529 0.56471 )
##         22) LoyalCH < 0.277977 63   66.74 MM ( 0.22222 0.77778 ) *
##         23) LoyalCH > 0.277977 107  146.80 CH ( 0.56075 0.43925 ) *
##    3) LoyalCH > 0.5036 445  340.60 CH ( 0.87191 0.12809 )
##      6) LoyalCH < 0.764572 185  211.80 CH ( 0.74054 0.25946 )
##       12) ListPriceDiff < 0.235 78  108.10 CH ( 0.51282 0.48718 )
##         24) PctDiscMM < 0.0980985 45   54.10 CH ( 0.71111 0.28889 ) *
##         25) PctDiscMM > 0.0980985 33   36.55 MM ( 0.24242 0.75758 ) *
##       13) ListPriceDiff > 0.235 107   66.44 CH ( 0.90654 0.09346 ) *
##      7) LoyalCH > 0.764572 260   78.23 CH ( 0.96538 0.03462 ) *
```

4) LoyalCH < 0.0356415 53 0.00 MM (0.00000 1.00000) * is a terminal node. This shows us that whenever predictor `LoyalCH` is smaller than `0.0356415` that the classification of Purchase is equal to `MM`, otherwise then it means that the next nested node should be considered.

d.)

```r
{plot(class_tree_model)
text(class_tree_model,pretty=0)}
```



From this tree we can see how nested the predictors are in describing the classifications.

e.)

```r
# predict test data
pred_class_tree_model = predict(class_tree_model,
                                newdata = df_test,
                                type = "class")

# produce confusion matrix
conf_mat = table(pred_class_tree_model,df_test$Purchase)
conf_mat
```

```
##
## pred_class_tree_model  CH  MM
##                    CH 152  37
##                    MM  13  68
```

```r
# calculate error rate
test_error_rate = sum(conf_mat[1,2], conf_mat[2,1])/sum(conf_mat)
print(paste("The test error rate is: ", test_error_rate))
```

```
## [1] "The test error rate is:  0.185185185185185"
```

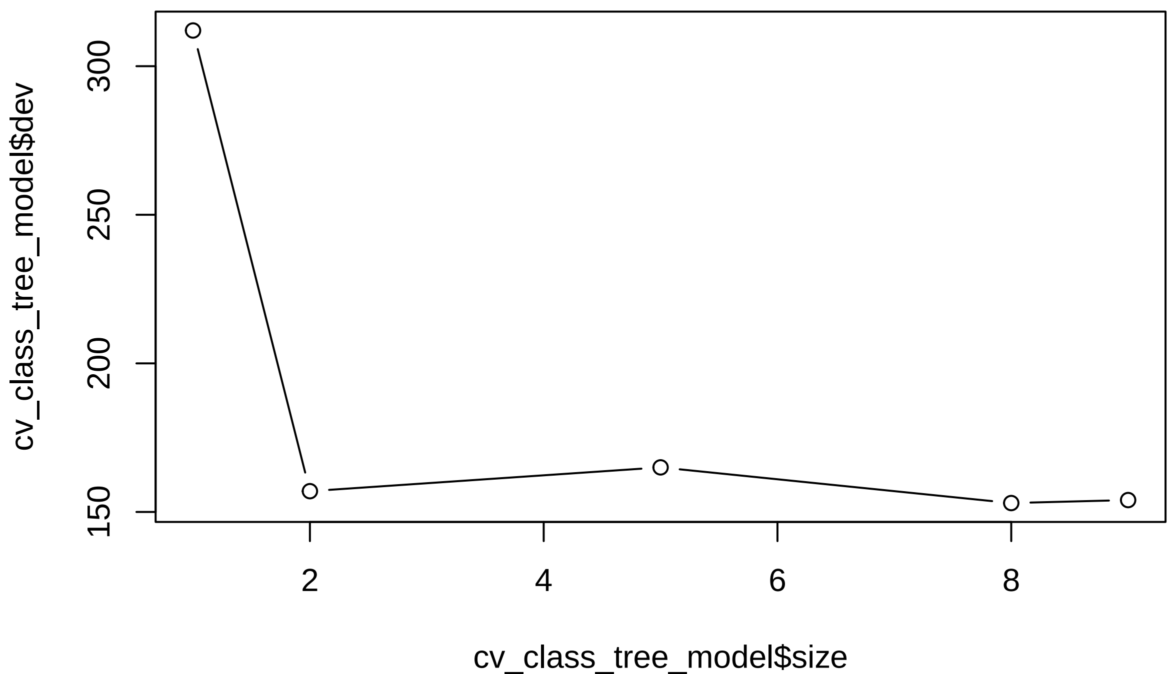
f.)

```r
# cross validation on tree model complexity
set.seed(0)
cv_class_tree_model = cv.tree(class_tree_model, FUN=prune.misclass)
cv_class_tree_model
```

```
## $size
## [1] 9 8 5 2 1
##
## $dev
## [1] 154 153 165 157 312
##
## $k
## [1]       -Inf   1.000000   4.333333   5.666667 155.000000
##
## $method
## [1] "misclass"
##
## attr(,"class")
## [1] "prune"         "tree.sequence"
```

g.)

```r
# plot deviance against tree size
plot(cv_class_tree_model$size ,cv_class_tree_model$dev ,type="b")
```



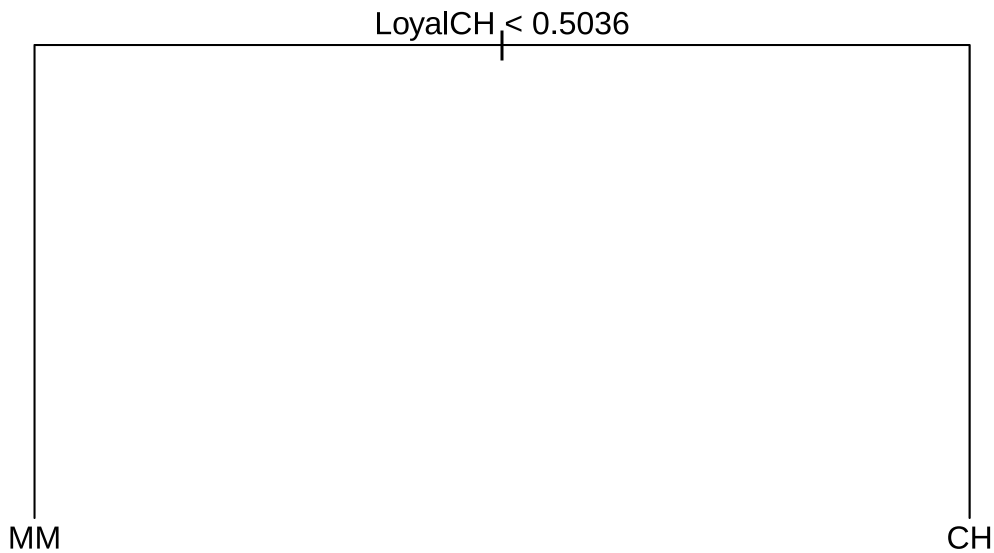
It seems that the ideal tree size might be 2.

h.)

Even though a size of 8 (153) gives the lowest cross-validated classification error rate, it is not too much different from that given by a size of 2 (157). Therefore 2 might be the better option when considering Occam's razor.

i.)

```
# prune tree
prune_class_tree_model <- prune.misclass(class_tree_model, best = 2)
plot(prune_class_tree_model)
text(prune_class_tree_model, pretty = 0)
```



j.)

```r
# compare pruned tree to original unpruned tree
summary(class_tree_model)
```

```
##
## Classification tree:
## tree(formula = Purchase ~ ., data = df_train)
## Variables actually used in tree construction:
## [1] "LoyalCH"       "PriceDiff"     "SpecialCH"     "ListPriceDiff"
## [5] "PctDiscMM"
## Number of terminal nodes:  9
## Residual mean deviance:  0.7189 = 568.7 / 791
## Misclassification error rate: 0.1575 = 126 / 800
```

```r
summary(prune_class_tree_model)
```

```
##
## Classification tree:
## snip.tree(tree = class_tree_model, nodes = 2:3)
## Variables actually used in tree construction:
## [1] "LoyalCH"
```

```
## Number of terminal nodes:  2
## Residual mean deviance:  0.9558 = 762.8 / 798
## Misclassification error rate: 0.1962 = 157 / 800
```

The unpruned tree has a slightly lower training error rate. However, the pruned tree is much less complex.

k.)

```
# predict test data
pred_class_tree_model = predict(prune_class_tree_model,
                                newdata = df_test,
                                type = "class")

# produce confusion matrix
conf_mat = table(pred_class_tree_model,df_test$Purchase)
conf_mat
```

```
##
## pred_class_tree_model  CH  MM
##                    CH 132  24
##                    MM  33  81
```

```
# calculate error rate
test_error_rate = sum(conf_mat[1,2], conf_mat[2,1])/sum(conf_mat)
print(paste("The test error rate is: ", test_error_rate))
```

```
## [1] "The test error rate is:  0.211111111111111"
```

The testing error rate is also slightly higher for the pruned tree.

Question 10

a.)

```
# remove NA for salaries
df_data = Hitters
df_data = df_data[-which(is.na(df_data$Salary)), ]
sum(is.na(df_data$Salary))
```

```
## [1] 0
```

```
# log transform salary data
df_data$Salary = log(df_data$Salary)
```

b.)

```r
# split data
df_train = df_data[1:200, ]
df_test = df_data[-(1:200), ]
```

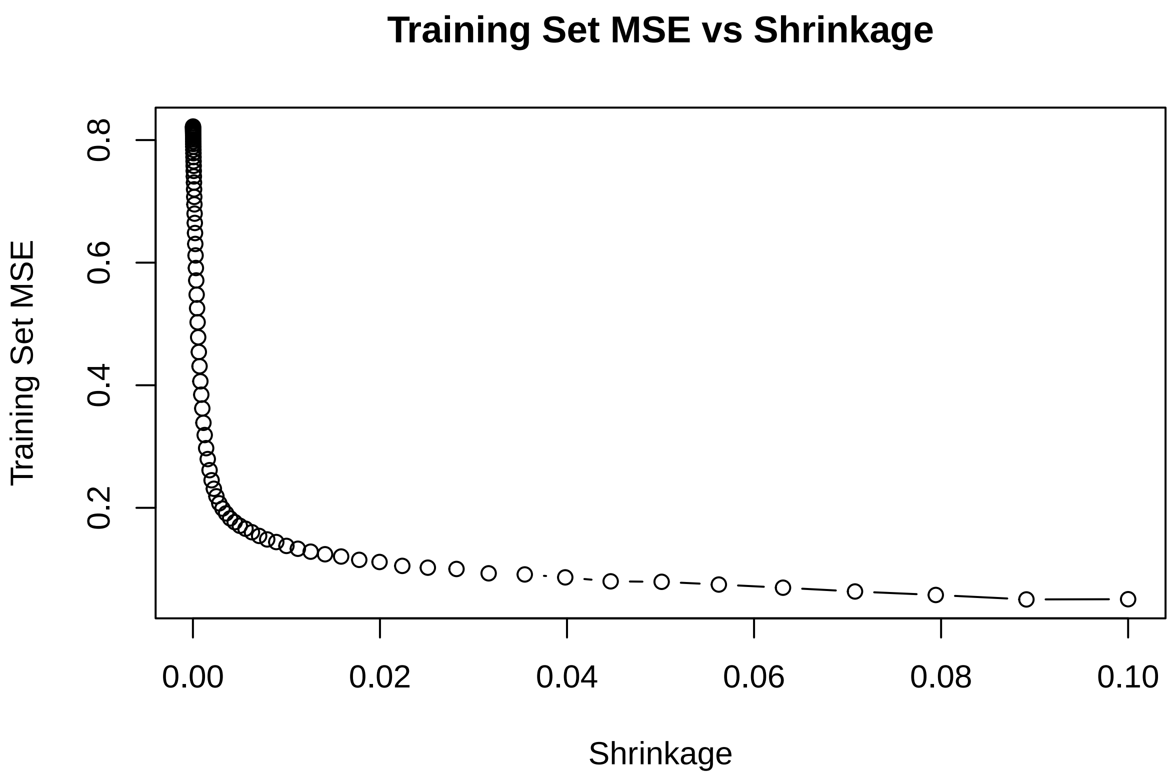
c.)

```r
# import libraries
library(gbm)
```

```
## Loaded gbm 2.1.8.1
```

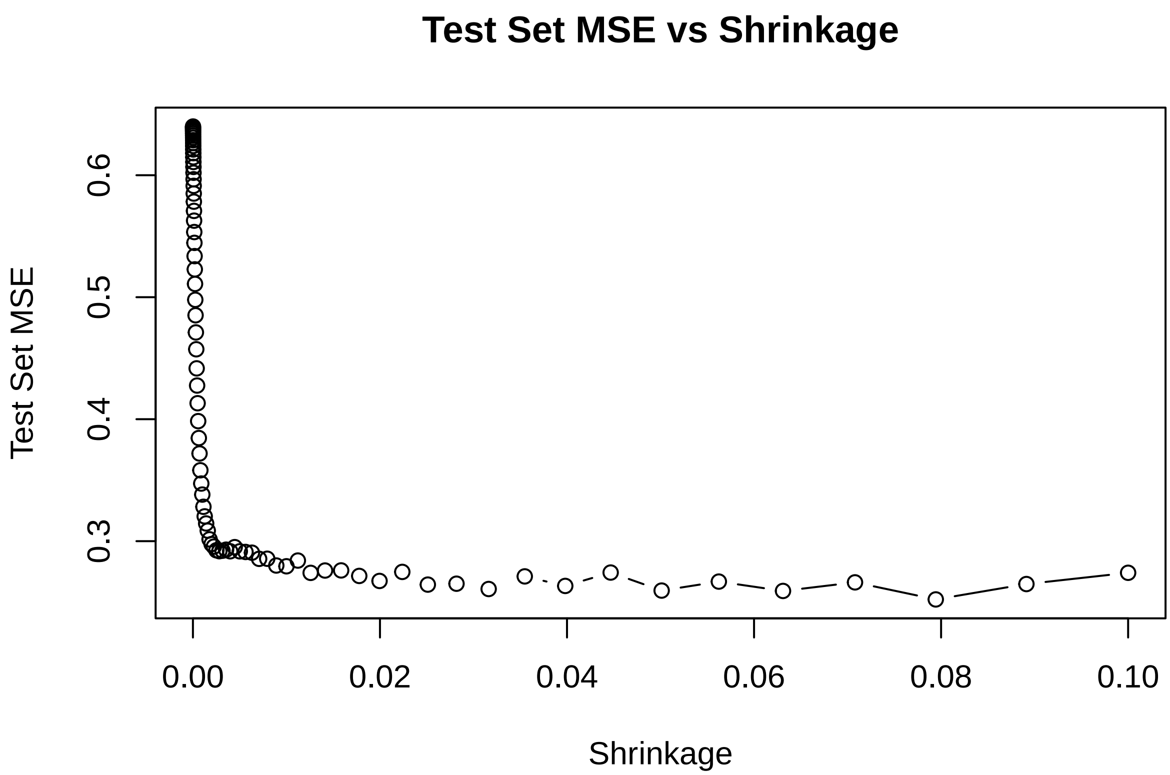
```r
# boosting
set.seed(0)
lambdas = 10^seq(-5, -1, by = 0.05)
train_mse = rep(NA, length(lambdas))
test_mse = rep(NA, length(lambdas))
for (i in 1:length(lambdas)) {
    boost= gbm(Salary ~ ., data = df_train, distribution = "gaussian",
        n.trees = 1000, shrinkage = lambdas[i])
    train_pred = predict(boost, df_train, n.trees = 1000)
    test_pred = predict(boost, df_test, n.trees = 1000)
    train_mse[i] = mean((df_train$Salary - train_pred)^2)
    test_mse[i] = mean((df_test$Salary - test_pred)^2)
}

# plot results
plot(lambdas,
     train_mse,
     type = "b",
     main = 'Training Set MSE vs Shrinkage',
     xlab = "Shrinkage",
     ylab = "Training Set MSE")
```



d.)

```
# plot MSe vs Shrinkage
plot(lambdas,
     test_mse,
     type = "b",
     main = 'Test Set MSE vs Shrinkage',
     xlab = "Shrinkage",
     ylab = "Test Set MSE")
```




```r
min(test_mse)
```

```
## [1] 0.252252
```

```r
lambdas[which.min(test_mse)]
```

```
## [1] 0.07943282
```

e.)

```r
# import library
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.1-4
```

```r
# Regression model
set.seed(0)
lm = lm(Salary ~ ., data = df_train)
lm_pred = predict(lm, df_test)
mean((df_test$Salary - lm_pred)^2)
```

```
## [1] 0.4917959
```

```r
x_train = model.matrix(Salary ~ ., data = df_train)
x_test = model.matrix(Salary ~ ., data = df_test)
lasso = glmnet(x_train, df_train$Salary, alpha = 1)
lasso_pred = predict(lasso, s = 0.01, newx = x_test)
mean((df_test$Salary - lasso_pred)^2)
```

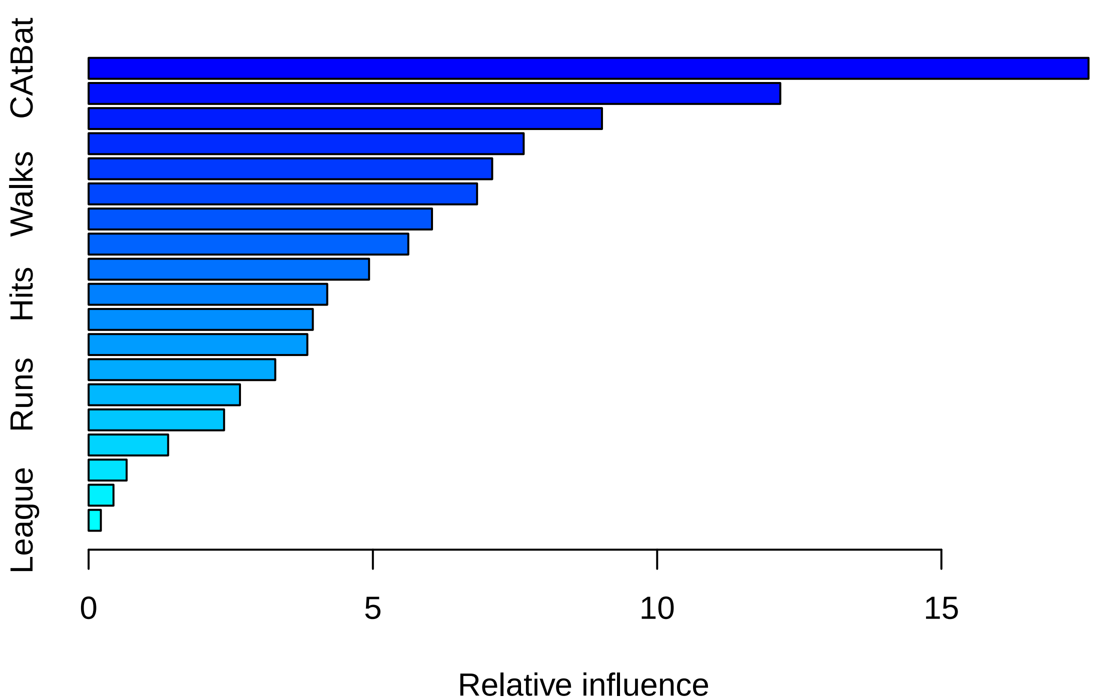
```
## [1] 0.4700537
```

The test MSE of boosting model is lower than linear and Lasso models.

f.)

```r
set.seed(0)
best = gbm(Salary ~ ., data = df_train, distribution = "gaussian",
    n.trees = 1000, shrinkage = lambdas[which.min(test_mse)])

summary(best)
```




```
##                 var    rel.inf
## CAtBat       CAtBat 17.5880810
```

```
## CRuns          CRuns 12.1657282
## PutOuts      PutOuts  9.0297031
## CRBI            CRBI  7.6523867
## CHmRun        CHmRun  7.0986463
## Walks          Walks  6.8327535
## CWalks        CWalks  6.0398111
## Years          Years  5.6228571
## RBI              RBI  4.9330850
## Hits            Hits  4.1969825
## Assists      Assists  3.9439999
## AtBat          AtBat  3.8472490
## HmRun          HmRun  3.2829758
## Runs            Runs  2.6624755
## Errors        Errors  2.3828680
## CHits          CHits  1.3986994
## Division   Division  0.6689345
## NewLeague NewLeague  0.4376038
## League        League  0.2151596
```

CAtBat, CRuns and PutOuts are most important predictors in the boosted model

g.)

```
set.seed(0)
rf = randomForest(Salary ~ ., data = df_train, ntree = 500, mtry = 19)
rf_pred = predict(rf, df_test)
mean((df_test$Salary - rf_pred)^2)
```

```
## [1] 0.2304067
```

The test MSE of bagging is lower than boosting.