pdf of features for kth class

Quadratic discriminant analysis:

$$\Pr(X = x \mid Y = k) \equiv f_k(x)$$

$$f_k(x) = \text{const} \cdot |\det(\Sigma_k)|^{-1/2} \cdot \exp\left(-\frac{(x-\mu_k)^T \Sigma_k^{-1} (x-\mu_k)}{2}\right).$$

$$\log f_k(x) = \text{const} - \frac{1}{2} \log\det \Sigma_k - \frac{1}{2}(x-\mu_k)^T \Sigma_k^{-1} (x-\mu_k).$$

In the numerator of $\Pr(Y=k \mid X=x)$, we have $\pi_k \cdot f_k(x)$; here we are working with the log of the numerator.

$$\tilde{\delta}_k(x) = \log \text{numerator} = \log f_k(x) + \log \pi_k$$

Case 1: $\Sigma_k = \Sigma$ for all $k = 1, \dots, K$. $\equiv$ LDA

$$(x-\mu_k)^T \Sigma^{-1} (x-\mu_k) = x^T \Sigma^{-1} x - 2\mu_k^T \Sigma^{-1} \cdot x + \mu_k^T \Sigma^{-1} \mu_k$$

Hence
$$\tilde{\delta}_k(x) = \underline{-\frac{1}{2}\log\det \Sigma} \, \underline{-\frac{1}{2} x^T \Sigma^{-1} x} + \mu_k^T \Sigma^{-1} x - \frac{1}{2}\mu_k^T \Sigma^{-1} \mu_k + \log \pi_k.$$

=> underlined expressions do not depend on k and do not influence classifications.

Case 2: QDA, $\Sigma_j \neq \Sigma_k$ for $j \neq k$. =>

$\tilde{\delta}_k(x) = \log f_k(x) + \log \pi_k$; terms $-\frac{1}{2}\log\det \Sigma_k$ and $-\frac{1}{2} x^T \Sigma_k^{-1} x$ now influence the classification.