Deviance in the case of Gaussian MLR model.

$Y_i = x_i^T \beta + \varepsilon_i$, $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$, $i = 1, \dots, n$.

The pdf of Y_i is $f_i(y_i \mid x_i, \beta, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}(y_i - x_i^T\beta)^2\right)$

$$Lik(\beta, \sigma^2) = \prod_{i=1}^{n} f_i(y_i \mid x_i, \beta, \sigma^2) = \dots =$$

$$= (2\pi)^{-n/2} \cdot (\sigma^2)^{-n/2} \cdot \exp\left(-\frac{1}{2\sigma^2}\|\underset{\sim}{y} - X\beta\|_2^2\right). \quad \underset{\sim}{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}.$$

$$Loglik(\beta, \sigma^2) = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\|\underset{\sim}{y} - X\beta\|_2^2$$

$$-2LL(\beta, \sigma^2) = +n \cdot \ln 2\pi + n \ln \sigma^2 + \|\underset{\sim}{y} - X\beta\|_2^2 / \sigma^2$$

joint MLE for (β, σ^2) is $\hat\beta = (X^T X)^{-1} X^T \underset{\sim}{y}$; $\hat\sigma^2 = \frac{\|\underset{\sim}{y} - X\hat\beta\|_2^2}{n}$

If σ^2 is known, Deviance $= n \cdot \ln 2\pi + n \cdot \ln \sigma^2 + \frac{\|\underset{\sim}{y} - X\hat\beta\|_2^2}{\sigma^2}$

$\|\underset{\sim}{y} - X\hat\beta\|_2^2 \equiv RSS$ - resid sum of squares.

If σ^2 is unknown, Deviance $= n \cdot \ln 2\pi + n \cdot \ln(RSS/n) + \underbrace{\frac{RSS}{RSS/n}}_{=n}$.

Deviance $\equiv -2 \cdot Loglik(\hat\theta)$, $\hat\theta$ is the MLE for θ.

$\theta \equiv$ param. vector. Here, $\theta = (\beta, \sigma^2)$.