Demographic Change, House Prices, and the Real Rate[*]

Christopher Schang[†]

December, 2024
Link to Latest Version

Abstract

Aging populations, driven by low fertility rates and increasing longevity, are a defining trend in most advanced economies. This demographic shift has significant implications for asset prices, particularly housing, a primary asset for many households. This paper employs a general equilibrium model with housing and mortgage choice over the life-cycle and a construction sector to study the effects of demographic change on house prices. Calibrated to German microdata, I document the following: In line with past trends (1) demographic factors have contributed significantly to the long-term rise in housing prices. Notably, indirect general equilibrium effects, such as falling real rates, have played a substantial role. (2) Based on projected demographic trends, the model suggests that over the remainder of the 21st century, declining populations and rising old-age dependency ratios place downward pressure on real house prices while (3) the composition of wealth shifts from capital to housing wealth mitigating the drop in real rates.

1 Introduction

Advanced economies, and the world as a whole, are aging as a result of low fertility, rising longevity, and a baby boom generation reaching retirement. The old-age dependency ratio (OADR), has risen by 9pp in the three biggest EU economies between 2000-2020 and this trend is projected to continue.[1] A key concern of this aging process is its effect on rates of return and asset prices, particularly housing which is the single largest asset for households. A number of papers have employed large lifecycle models to study the effects of past demographic change on the rate of return on capital (e.g. Krueger and Ludwig, 2007; Eggertsson et al., 2019; Gagnon et al., 2021; Auclert et al., 2021). Qualitatively they have found aging has led to more capital accumulation and falling real rates. Auclert et al. (2021) make the argument that wealth-to-GDP ratios continue to increase with downward pressure on real rates. Competing hypotheses (e.g. Goodhart and Pradhan, 2020; Poterba, 2001) argue that a decumulation of assets of a large baby boom generation that enters retirement will put downward pressure on asset prices.

The quantitative literature has largely focused on total or financial wealth, however, housing stands apart from other investment assets due to several key characteristics: In addition to being an investment asset it serves the dual role of a consumption good by providing housing services. Households tend to purchase housing early in their lifecycle, often before accumulating significant financial wealth (Fernandez-Villaverde and Krueger, 2011; Pelletier and Tunc, 2019). These early-in-life home purchases are often financed through mortgages, making them sensitive to interest rates. High transaction costs including realtor fees, taxes, and search costs, limit the liquidity of housing, leading to lower levels of dissaving in old age compared to other financial assets (Yang, 2009; De Nardi et al., 2016). The supply of housing tends to be inelastic, and its stock is persistent. One of its factors, land, is naturally limited leading to decreasing returns to scale while the existing housing supply depreciates at a slow rate. Lastly, residential housing is a non-productive asset, meaning its stock does not directly affect the marginal product of capital and, consequently, the

---

[1]The old-age dependency ratio is defined as the ratio of people above 65 to working age population. The largest EU economies by GDP and population size: GER, FRA, ITA. Increase of old-age dependency ratio 2000-2020 for other major economies: SPA 6pp, UK 5pp, CHN 7pp, JPN 23pp, RUS 5pp, CAN 9pp, World 4pp. Source: World Bank https://data.worldbank.org

1

interest rate. At the same time, housing is the most important asset for many households and a predominant asset for retirement savings and intergenerational wealth transfer. Significant increases in real house prices have sparked debates about affordability crises for prospective homeowners (Quigley and Raphael, 2004; Wetzstein, 2017). Moreover, persistent price fluctuations in either direction can have considerable effects on homeowner's consumption through wealth effects (Berger et al., 2018).

The strong connection between housing and the lifecycle makes these features highly relevant when considering the effects of demographic change - a significant shift in the age distribution - on house prices. This paper addresses the key question: What is the effect of demographic change on house prices and through what channels? To answer this question, I construct a general equilibrium heterogeneous agent model with overlapping generations that incorporates the key features distinguishing housing from other assets. The model features a detailed household sector with uninsurable income risk, a consumption-savings choice, housing and mortgage choices, bequests, and retirement transfers. Housing is discrete, can be mortgage financed subject to a downpayment requirement, and households receive utility from housing services which generates early-in-life investments into housing. Non-convex transaction costs limit the adjustments over the lifecycle and lead to a less pronounced dissaving of housing vis-à-vis financial assets in old age. A production sector pins down the real rate and wages, while housing supply is modeled through an explicit construction sector that responds to house and factor prices and is inelastic due to the fixed factor of land. The government runs a pay-as-you-go (PAYG) pension scheme and the demographic age distribution evolves according to past and projected rates of fertility, mortality, and migration.

This setup allows for both direct and indirect general equilibrium effects of demographic change: Direct effects refer to changes in housing demand that come from changes in the age distribution for a given (fixed) housing demand by age.[2] Indirect effects refer to changes in *age-specific* housing demand as a result of equilibrium price changes.

I calibrate the model to the German economy and find that direct effects of population growth and aging over the past four decades increase housing demand and prices. The progression of the baby-boom generation through their home-buying years and their impending retirement generates

---

[2] Also known as shift-share analysis.

age-distributional effects that positively influence *aggregate* housing demand over the past decades. As a result, house prices appreciate. However, a decomposition of house price movements indicates that indirect general equilibrium adjustments play a considerable role, particularly the decline in the real interest rate: As the baby boom generation ages, their demand for financial assets increases, raising the capital-to-GDP ratio which suppresses interest rates and increases wages. The model implied 250-basis-point reduction in the real interest rate over the past four decades induces an increase in housing demand due to a substitution effect towards housing and lower cost of mortgage financing. Increasing wage rates intensify housing demand through an income effect. The increase in housing demand from both direct and indirect channels is juxtaposed with an inelastic increase in supply raising house prices over the past decades.

Throughout the remainder of the 21st century, demographic trends, characterized by low fertility rates below replacement levels, are projected to lead to decreasing populations and further aging of society. As a direct consequence, the resulting fall in housing demand is juxtaposed with an existing stock of housing that depreciates at low rates putting downward pressure on house prices. Indirect general equilibrium effects work in the same direction: The increasing old-age dependency ratio places pressure on the pension system leading to a negative income effect that further reduces housing demand. At the same time, the real interest rate, a primary contributor to house price growth in recent decades, stabilizes, thereby ceasing to exert upward pressure on house prices.

My analysis also contributes to the debate on rates of return on capital by offering a nuanced perspective on the relationship between the wealth-to-GDP ratio and the real interest rate: For single-asset models, a rise in the wealth-to-GDP ratio necessarily implies a drop in the real rate.[3] The presence of housing, however, introduces a second asset which is non-productive. While my model predicts a rise in the wealth-to-GDP ratio over the 21st century as society ages. it predicts a shift in the composition of wealth towards housing wealth, derived from a less pronounced dissaving of housing assets in old age. With housing being nonproductive, the relevant ratio for the real rate becomes the capital-to-GDP ratio which is projected to grow at a lower rate, mitigating the decline of the real rate.

Related Literature. — This paper belongs to the literature that studies the effects of demographic

---

[3]As considered in e.g. Auclert et al. (2021).

change in general equilibrium models with overlapping generations. While previous research has provided valuable insights into the aggregate effects of aging on the real interest rate (Krueger and Ludwig, 2007; Eggertsson et al., 2019; Gagnon et al., 2021; Auclert et al., 2021) quantitative analysis of its impact on house prices remains limited. A key feature of this paper is its explicit focus on housing and its unique characteristics compared to other assets. Existing models incorporating housing lack general equilibrium (Mankiw and Weil, 1989) which, as I demonstrate, is crucial for understanding the transmission of demographic change to house prices.

This work complements a number of empirical studies that have examined the relationship between demographics and house prices. Previous research has documented a positive link between population size and house prices (Hoynes and McFadden, 1996; Takáts, 2012) and investigated the relation between aging and house prices (Takáts, 2012; Hiller and Lerbs, 2016; Kajuth et al., 2016). The main contribution of this paper is to provide a structural framework to explain these patterns, highlight the importance of general equilibrium channels, and enable the analysis of projected demographic change.

While this paper focuses on isolating demographics as a driver for house prices, a complementary literature aims to explain house price growth and the evolution of housing wealth through supply-side factors. E.g. Grossmann et al. (2024) provide a framework that explains rising housing wealth-to-GDP ratios with a construction sector that features lower technological progress and higher land intensity vis-à-vis the rest of the economy.

Lastly, throughout my analysis, I assume demographic developments as exogenous. A large literature is concerned with modeling and explaining the reverse effect of economic variables on fertility choice. I explicitly do not allow for feedback of equilibrium prices on fertility choice. For a review of the literature aiming at explaining fertility see for example Doepke et al. (2023).

The remainder is structured as follows: Section (2) provides empirical evidence on demographic patterns across developed countries, statistical relation between demographics and house prices, and lifecycle patterns of wealth, housing, and income from Germany. Section (3) outlines the general equilibrium lifecycle model. Section (4) discusses the calibration strategy to German data. In section (5), I calculate transition dynamics from 1960 to 2300, feeding in the historical and projected demographic age distribution, and compute endogenous prices along the transition. Section (6)
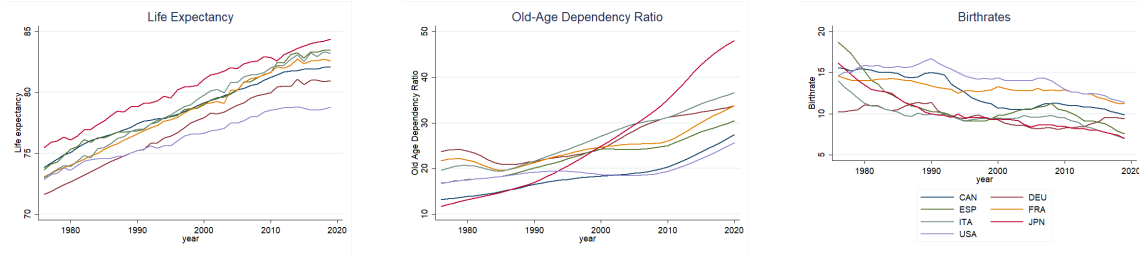
Figure 1: *Demographic trends across selected developed countries. Left: Rising life expectancy of between 4-10 years over the last 45 years. Middle: Upward trend in old-age dependency ratios (ratio of population aged > 65 and working age population). Right: Decline in birthrates. Data source: Worldbank.*

concludes.

2   Empirical Evidence

Demographic Trends.— Virtually all developed countries go through a demographic transition that has seen fertility rates drop below replacement levels while mortality rates have been continuously falling with rising life expectancies. Figure (1) shows the evolution of life expectancy, old age dependency ratios, and birth rates for the 4 biggest EU countries (Germany, France, Italy, Spain), the US, Canada, and Japan. While lower birth rates put downward pressure on total population size, increased longevity as well as migration has kept most countries from declining population size. However, even high net immigration forecasts predict falling population sizes for most advanced countries over the next half-century and although population numbers remain relatively stable for the moment, age distributions have markedly shifted towards older populations across countries. As the most "advanced" country in terms of aging, Japan has reached an old-age dependency ratio of nearly 50% by 2020 (Figure (1) panel (b)).

House Prices and Demographics.— This section provides statistical evidence for the association between demographic factors and house prices. The analysis mostly replicates Takáts (2012) but I use updated data up to 2020 and include birth rates in the regression.[4] Data on house prices comes from the Bank for International Settlements (BIS), while demographic time series and GDP come

---

[4]For a full treatment see Takáts (2012).

	(1)	(2)	(3)
	House price	House price	House price
GDP/capita	1.157***	1.238***	0.938***
	(0.0913)	(0.0942)	(0.108)
Old Age Dependency Ratio	-0.894***	-0.744***	-0.725***
	(0.180)	(0.186)	(0.180)
Population	1.152***	1.248***	1.201***
	(0.332)	(0.334)	(0.326)
Birthrate		0.257***	0.247**
		(0.0768)	(0.0747)
L.GDP/capita			0.538***
			(0.106)
Observations	809	791	772
R^2	0.396	0.412	0.442
Time FE	Yes	Yes	Yes
Countries	19	19	19

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1: *Output from fixed effects regression of (1). Data on house prices from BIS, demographic data, and GDP from the World Bank.*

from the World Bank.[5] In particular, I estimate

$$\Delta \ln P_{i,t} = \beta_1 \Delta \ln GDPPC_{i,t} + \beta_2 \Delta \ln OADR_{i,t} + \beta_3 \Delta \ln POP_{i,t} + \beta_4 \Delta \ln BR_{i,t} + \psi_t + \epsilon_{i,t} \quad (1)$$

where $\Delta x_{i,t} = x_{i,t} - x_{i,t-1}$ denotes the the first difference of variable x for country i at time t. I regress the first difference in log house prices on the first difference in log GDP per capita, old-age dependency ratio, population size, and birth rates and include time fixed effects ψ_t. All variables are difference stationary. Results of the regression (1) are shown in table (1). Notably, all demographic variables are significantly associated with house prices and show a positive correlation between population size and birth rates and a negative correlation between the old-age dependency ratio.
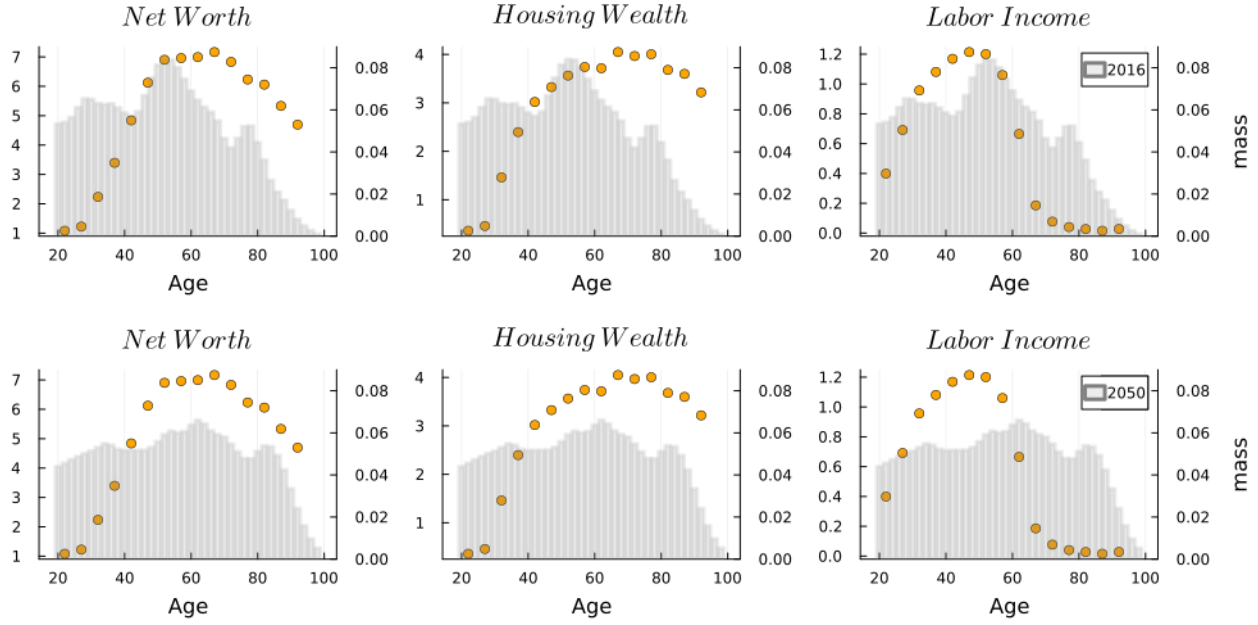
---

[5]https://data.worldbank.org

https://data.bis.org

Figure 2: *Lifecycle profiles for net worth, housing wealth, and labor income (orange dots) overlaid by the population age distribution (grey bars) for 2016 and 2050. Microdata on income and wealth (left axis) from the LWS (German SOEP) panel from 2002, 2007, 2012, and 2017. Wealth profiles are scaled to be consistent with WID (World Inequality Database) aggregates and expressed as ratios to national income. Population shares by age (right axis) are from the UN Population Prospect for 2016 (top row) and 2050 (bottom row). See text for details.*

Lifecycle Profiles and Age Distribution in Germany.— The empirical micro survey evidence in this section focuses on age-specific wealth and income for Germany. The data comes from the Luxembourg Wealth and Income Study Database (LIS/LWS) based on the socioeconomic panel (SOEP) for Germany.[6] Figure (2) shows lifecycle profiles for (net) wealth, housing wealth, and labor income. The overlaid gray bars show the population age distribution in a given age bracket. The top row shows the 2016 population and the bottom row the 2050 population. Wealth profiles are identical for both rows and com from the waves from 2002, 2007, 2012, and 2017. What is visually striking is that the baby boomer generation is in the pre- or early retirement stage and therefore moving into the dissaving of assets. However, dissaving of housing assets is not very

---

[6]https://www.lisdatacenter.org/.

pronounced, indicating that many elderly hold on to their properties. More acute will be the decline in the labor force as average labor income drops sharply at the official retirement age. A resulting increase in the capital-labor ratio, which I will explore in more depth in the model section, has the potential for significant effects on the rate of returns extension house prices.

3   Structural Framework

The quantitative model features a heterogeneous agent household sector, a production sector that makes consumption and investment goods, a construction sector that adds to the stock of housing, a government that runs a pay-as-you-go (PAYG) pension scheme, and a foreign competitive rental sector that supplies rental units. The household side features a realistic lifecycle structure with consumption, savings as well as discrete tenure and housing decisions that include a mortgage in the case of owner-occupied housing similar to Kaplan et al. (2020). On the supply side, there is a standard neo-classical production firm taking capital and labor to produce goods for consumption and capital investment. The construction sector uses capital, labor, and fixed factor of land permits to produce housing units at decreasing returns to scale. The demographic transition is taken as exogenous using the historical evolution of the age distribution paired with projected fertility, mortality, and migration trends from the UN population prospect.[7] Time is indexed by t, age is indexed by j, and households by i. Throughout, if not specified otherwise, lowercase variables $x_{i,j,t}$ denote age-specific household level variables, capital variables with age subscript denote totals by age $X_{j,t} = \int_i x_{i,j,t} di$ and variables without age subscript refer to economy-wide aggregates $X_t = \sum_{j=1}^{J} X_{j,t}$. In what follows individual household indices i are suppressed for readability such that $x_{j,t} := x_{i,j,t}$.

---

[7] United Nations, Department of Economic and Social Affairs, Population Division (2022)

3.1 Households

Preferences.— Households enter the economy at $j = 1$, retire at J_r, live at most up to age J and face lifetime utility

$$\max_{c_{j,t},d_{j,t}\in\mathcal{H},m_{j+1,t+1}} \mathbb{E}\sum_{j=1}^{J}\beta^{t-1}\tilde{\Phi}_j\left\{U(c_{j,t},c_{j,t}^h) + (1-\tilde{\phi}_j)\mathcal{B}(a_{j+1,t+1})\right\}$$

where $c_{j,t} > 0$ is non-durable consumption, $c_{j,t}^h$ are housing services. Discrete choices $d \in \mathcal{H}$ entail tenure status (owning vs renting), different house qualities that come from separate scales for rental $\mathcal{H}_r = \{h^r(1), h^r(2)...\}$ and owner-occupied housing $\mathcal{H}_h = \{h^o(1), h^o(2)...\}$.[8] If households are or become owners they can take out a mortgage $m_{j+1,t+1}$ up to a fraction λ_j of the value of their house. β denotes the discount factor and $\tilde{\phi}_j$ is the perceived survival rate with $\tilde{\Phi}_j = \prod_{i=0}^{j-1}\tilde{\phi}_i$. Utility is a standard CRRA function that takes as input a Cobb-Douglas aggregator for the consumption and housing services

$$U\left(c, c^h\right) = \frac{\left((c)^\gamma(c^h)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$$

with coefficient for relative risk aversion σ and non-durable consumption weight γ. The bequest motive $\mathcal{B}$ considers the value of total assets in the next period which is the sum of housing and the liquid asset net of mortgage debt $a_{j+1,t+1} = (1+r_{t+1})b_{j+1,t+1} + p_{t+1}h_{j+1,t+1} - m_{j+1,t+1}$.[9] The functional form is analogous to contemporaneous utility

$$\mathcal{B}(a_{t+1}) = \kappa\frac{a_{j+1,t+1}^{1-\sigma}}{1-\sigma}$$

with shifter κ regulating the strength of bequests. More generally, the presence of bequests regulates the degree of dissaving in old age and is a key feature to fit realistic lifecycle profiles.

Income.— Households earn labor income over their working age, pay labor taxes τ to fund a government-run pay-as-you-go pension scheme (PAYG), and receive pension payments during retirement. Labor income y is a function of current individual productivity $z_{j,t}$, individual-specific type θ and the economy-wide wage rate w_t

$$y_{j,t,\theta}(z_{j,t}) = z_{j,t}\theta w_t$$

---

[8]Where $\mathcal{H} = \mathcal{H}_r \cup \mathcal{H}_o$.

[9]See Kaplan et al. (2020) for a similar formulation with non-homotheticity.

where productivity z follows a standard formulation with a deterministic χ_j age-specific component and persistent stochastic component $\nu_{j,t}$ that follows an AR(1) process

$$\ln z_{j,t} = \chi_j + \nu_{j,t}$$

$$\nu_{j,t} = \rho_z \nu_{j-1,t-1} + \varepsilon_{j,t}$$

with persistence ρ_z and a Gaussian idiosyncratic shock $\varepsilon_{j,t} \sim \mathcal{N}(0, \sigma_\varepsilon)$. In retirement, labor earnings are zero and households receive a retirement transfer $tr_{j,t}$ which is a fraction ρ of their last pre-retirement income,

$$tr_{j,t} = 0 \ \forall j < J_r$$

$$tr_{j,t} = \rho z_{J_r-1} \theta w_t \ \forall j \geq J_r$$

Lastly, each period households receive bequests $bq_{j,t,\theta}$ that are type- and age-dependent. Total bequests are distributed in two ways: a constant share is distributed among young households as an initial condition over financial assets and housing according to a distribution $F_\theta^1(b, h, m)$, while the remainder is distributed as liquid assets to the working-age population according to a distribution $F_\theta^2(j)$ over age.

Financial Assets.— Households can save in a liquid asset b which earns a return r_t. There is no unsecured borrowing in the liquid asset such that $b_{j+1,t+1} \geq 0$. Households can borrow against their owned house by originating a mortgage m^o up to the loan-to-value (LTV) collateral constraint λ_j which binds only at origination

$$m_{j+1,t+1}^o \leq \lambda_j h_{j+1,t+1} p_t$$

A household that does not originate or refinance is not subject to the LTV constraint such that $m_{j,t} = m_{j,t}^o$ the period after origination. The unconstrained mortgage balance amortizes at rate ξ of the house value or until it is paid in full such that the amortization payment is

$$amort_{j,t} = \min \left[\xi p_{j,t} h_{j,t}, \ m_{j,t} \right] \tag{2}$$

implying a mortgage payment of

$$m_{j,t}^{payment} = r_t m_{j,t} + amort_{j,t} \tag{3}$$

10

per period that combines interest and amortization payments. The unconstrained mortgage evolves according to $m_{j+1,t+1} = m_{j,t} - amort_{j,t}$. Refinancing an existing mortgage incurs a fixed refinancing cost υ.

Housing.— Besides liquid savings, households choose their housing needs. A household that decides to live in an owner-occupied house, i.e. chooses $h^o \in \{h^o(1), h^o(2)..\}$ and receives housing services $h^c = h^o$. For the transaction, they face house price p_t and a non-convex transaction cost $\zeta(h_t + h_{t+1})p_t$ whenever purchasing or selling a house, which includes up- or downsizing. Transaction costs capture costs associated with for example realtor fees and property acquisition tax that are sizable in Germany. Every period the housing stock depreciates at rate δ_H and households are required to pay maintenance costs to counteract depreciation. If households decide to rent they chooses $h^r \in \{h^r(1), h^r(2)..\}$ and receives housing services $h^c = h^r$. Renters pay a per-period rate f_t and can adjust the rental size costlessly.

Recursive form.— Recasting the problem in recursive form, I can write the household value function in a nested fashion with households first making the discrete choice d and subsequently the consumption choice c and if applicable mortgages m. The outer value function for renters then comprises the discrete choice between staying a renter and choosing size h^r or becoming an owner with size h^o

$$V_{j,t,\theta}^r (b, z) = \max \left\{ v_{j,t}^{r,r} (b, z; \mathbf{h^r}), v_{j,t}^{r,o} (b, z; \mathbf{h^{o\prime}}) \right\} \tag{4}$$

The problem for owners consists of becoming a renter with size h^r, adjusting the house to size h^o refinancing or not adjusting at all

$$V_{j,t,\theta}^o (b, h, m, z) = \max \left\{ v_{j,t,\theta}^{o,r} (b, h, m, z; \mathbf{h^r}), v_{j,t,\theta}^{o,a} (b, h, m, z; \mathbf{h^{o\prime}}), v_{j,t,\theta}^{o,re} (b, h, m, z;), v_{j,t,\theta}^{o,na} (b, h, m, z;) \right\} \tag{5}$$

where unprimed variables are today's values and primed variables denote future values. For a full description of the discrete choice-specific value functions including their associated budget constraints see appendix (B.1). Accordingly there are policies for liquid assets $b' = b_{j,t,\theta}(b, h, m, z)$, consumption $c = c_{j,t,\theta}(b, h, m, z)$, rental housing $h^r = h^r{}_{j,t,\theta}(b, h, m, z)$, owner-occupied housing $h^{o\prime} = h^o{}_{j,t,\theta}(b, h, m, z)$ and mortgages $m' = m_{j,t,\theta}(b, h, m, z)$.

3.2 Demographics

Total population N_t is the sum over the population of all age groups $N_t = \sum_{j=1}^{J} N_{j,t}$ with a demographic law of motion by age

$$N_{j+1,t+1} = \phi_{j,t} N_{j,t} + \mathcal{M}_{j+1,t+1} \ \forall j \geq 1$$
$$N_{1,t} = \psi_t N_t$$

where ψ_t denotes the birthrate, $\phi_{j,t}$ the age-specific survival rate and $\mathcal{M}_{j,t}$ is net migration by age. The demographic distribution is thus determined by three driving forces: fertility, mortality (modeled directly as a birthrate), and net migration.

3.3 Government

The sole purpose of the government is to run a pay-as-you-go (PAYG) pension scheme with a balanced budget

$$\sum_{j=1}^{J_r-1} N_{j,t} \int_{z_j} \tau_t y_{j,t}(z_j) dF(z_j) = \sum_{j=J_r}^{J} N_{j,t} \int_{z_{J_r}} tr_{j,t}(z_{J_r}) dF(z_{J_r}) \tag{6}$$

where $F(z)$ is the CDF over z. The condition (6) ensures that total tax revenue from the labor income of the working-age population equals total transfers to the retired population.

3.4 Supply Side

Production Sector.— There is a representative firm that chooses capital and labor to maximize profits. Output is given by the Cobb-Douglas production function $Y_t = K_{Y,t}^{\alpha} \tilde{N}_{Y,t}^{1-\alpha}$ where α is capital's share in output and $\tilde{N}_{Y,t} = \sum_{j=1}^{J_r-1} \sum_{\theta} \int_z N_{Y,j,t} z_j \theta dF(z_j)$ is aggregate labor in the production sector in efficiency units. Capital depreciates at rate δ. Factor prices can be derived from the first-order condition of the firm as

$$r_t = \alpha \left(\frac{K_{Y,t}}{\tilde{N}_{Y,t}} \right)^{\alpha-1} - \delta \tag{7}$$

$$w_t = (1-\alpha) \left(\frac{K_{Y,t}}{\tilde{N}_{Y,t}} \right)^{\alpha} \tag{8}$$

where r_t is the interest rate on capital and w_t is the wage rate for labor per efficiency unit.

Housing Construction Sector.— Housing investments with fixed factor land permits Q features diminishing returns to scale in capital and labor $I_{H,t} = K_{H,t}^{\alpha_1} \tilde{N}_{H,t}^{\alpha_2} Q^{1-\alpha_1-\alpha_2}$. Construction firms choose housing investment by maximizing profits

$$\Pi_{H,t} = \max_{K_{H,t} \tilde{N}_{H,t}} p_t I_{H,t} - r_t K_{H,t} - w_t N_{H,t} \tag{9}$$

yielding a housing construction equation of

$$I_{H,t} = p_t^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}} \left(\frac{r_t + \delta}{\alpha_1} \right)^{\frac{-\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{w_t}{\alpha_2} \right)^{\frac{-\alpha_2}{1-\alpha_1-\alpha_2}} Q \tag{10}$$

where $\varepsilon_{H,p} = \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}$ is the price elasticity of housing supply. The aggregate stock of housing H_t depreciates at rate δ_H and follows the law of motion

$$H_{t+1} = (1 - \delta_H)H_t + I_{H,t}$$

Rental Sector.— Rental supply comes from foreign investors that buy properties H_t^r with borrowed funds, face operating cost ι, and let at price f to renters. In the following period, they can resell the property at tomorrow's price net of depreciation. The investor problem then reads

$$\max_{H_t^r} f_t H_t^r - (1 + r_t + \iota)p_t H_t^r + p_{t+1} H_t^r (1 - \delta_H)$$

yielding a rent-price ratio

$$\frac{f_t}{p_t} = (1 + r_t + \iota) - \frac{p_{t+1}}{p_t}(1 - \delta_H) \tag{11}$$

which is an increasing function of the interest rate and a decreasing function of house price growth.

3.5 Aggregates

Aggregates.— My measure of GDP Y^{tot} is the sum of construction and the production of the consumption and investment good

$$Y_t^{tot} = Y_t + p_t I_{H,t} \tag{12}$$

and total wealth W is defined as the sum of financial assets and housing assets net of mortgage debt

$$W_t = B_t + p_t H_t^o - M_t \tag{13}$$

Total capital in the economy is the sum of capital in production and construction $K_t = K_{Y,t} + K_{H,t}$ with the law of motion

$$K_{t+1} = I_t + (1 - \delta)K_t \tag{14}$$

Market clearing.— To close the model, I need to clear the asset and housing market and equalize bequests received with bequests given. For the asset market, the total supply of financial assets equals the demand for mortgages and total capital $B_t = M_t + K_t$, housing supply equals the demand for owner-occupied and rental housing units. $H_t = H_t^o + H_t^r$. The total sum of bequests given equals all bequests received[10]

$$\sum_{j=1}^{J} BQ_{j,t,\theta} = \sum_{j=2}^{J} (1 - \phi_{j-1,t-1})\hat{A}_{j-1,t-1} \tag{15}$$

For a formal definition of the equilibrium see appendix (B.2).

4   Model Solution and Calibration

To solve the model, I discretize the continuous state variables into a log-spaced grid for the liquid asset $\bar{b}$, an equidistant mortgage grid $\bar{m}$, and I discretize the idiosyncratic income process using Tauchen (1986)-method. I solve for the policy functions by backward induction, starting from the terminal age using the discrete-choice endogenous gridpoint method following Iskhakov et al. (2017). With the policy function in hand, I simulate the economy using Young (2010)-method with a continuum of agents and a joint transition matrix. Finally, I aggregate the economy by using the age-specific demographic distribution.

---

[10]where $\hat{A}_{j-1,t-1}$ correspond to last period's end-of-period assets of households aged $j - 1$.

4.1 Calibration

The model is calibrated to a time-invariant steady-state with constant prices. I match a number of key moments of household portfolios over the lifecycle and aggregate ratios. For the latter, I use the demographic age distribution of the target year (2017) to arrive at aggregate moments.[11] Throughout I normalize house prices to 1 and set the interest rate to $r = 0.024$ which corresponds to the average return on capital relative to housing.[12]

Supply.— On the supply side, I calibrate several parameters to target economic aggregates. Namely, deprecation of capital δ and housing δ_H, the capital share in production α and the land permits Q are pinned down by the investment ratio I/Y^{tot}, capital ratio K/Y^{tot}, housing ratio pH/Y^{tot} and the construction share pI_H/Y^{tot}. This yields annual depreciation of total capital of $\delta = 0.10$ and the production capital share $\alpha = 0.28$ imply an investment/output ratio of 0.23 and capital/output ratio of 2.21 respectively. The depreciation of the housing stock is set to $\delta_H = 0.014$. On the construction side, I set the land permit share to $(1 - \alpha_1 - \alpha_2) = 0.56$ which implies an elasticity of residential housing construction of 0.8. Caldera and Johansson (2013) study house price elasticities across 21 OECD countries and find a wide variety from 0.15 (Switzerland) and 2.01 (USA) for long-run price-elasticities of residential investment with a point estimate for Germany of 0.43. Lerbs (2014) estimate house price elasticities for Germany and discuss large variations in the degree of urbanization with higher elasticities for major cities (0.80) and urbanized counties (0.46). I use the most the most conservative value of 0.8. Capital and labor shares in construction are set such that the capital-labor ratio will be equal across sectors. This implies $\alpha_1 = 0.12$ and $\alpha_2 = 0.32$.[13] Operating cost of the rental firm ι is chosen to match an annualized price-to-rent ratio of $p/f = 26$.

Households.— Households enter the economy at age 20 and die with certainty at age 100. One model period corresponds to two years such that the terminal age in the model is $J = 40$. Retirement is fixed at age 65 which corresponds to the cut-off for the old-age dependency ratio (OADR)

---

[11]Some papers calibrate the model along the transition path (e.g. Krueger and Ludwig, 2007), however, in a model with heterogeneous agents and discrete housing choices this becomes computationally prohibitively costly.

[12]Jordà et al. (2019) and www.macrohistory.net for the period between 2002 and 2020.

[13]Targeting the fraction of labor employed in construction (see for example Kaplan et al. (2020)) yields a slightly higher labor intensity in construction.

Description	Parameter	Value	Comment
Households			
Discounting	β	0.957	Internal
Risk Aversion	σ	2	Standard
Consumption share	γ	0.76	Internal
Bequest strength	κ	14.5	Internal
Transaction cost	ζ	0.052	Internal
Refinancing cost	υ	0.01	
Amortization rate	ξ	0.032	25 year mortgage
Perceived survival	$\tilde{\phi}_j$		Lifetables
Deterministic part of income	χ_j		LIS
Income process persistence	ρ_z	0.93	LIS
Income process variance	σ_ε	0.19	LIS
Housing			
House sizes	$\mathcal{H}_h$	$\{4.7, 5.8, 11.7\}$	
Rental sizes	$\mathcal{H}_r$	$\{2.6\}$	
Minimum downpayment	$1 - \lambda$	0.3	for $(j \leq J_r)$
Production			
Interest rate	r	0.024	Return on capital vs housing
Capital Depreciation	δ	0.1	Investment-to-GDP
Capital Share	α	0.28	Capital-to-GDP
Construction			
Housing depreciation	δ_H	0.014	Housing-to-GDP
Relative capital share	α_1/α_2	0.39	Matching relative share in production
Land share	$1 - \alpha_1 - \alpha_2$	0.56	Residential investment elasticity of 1.0
Land/Permits	Q	0.19	Construction-to-GDP
Government			
Tax rate	τ	0.141	Matching replacement rate

Table 2: *Model parameters. All values are annualized where applicable. For details see text.*

as defined by the UN. The loan-to-value ratio λ is set to 0.7 to reflect the conventional balance on newly originated residential mortgages in Germany.[14] The amortization rate ξ is set to correspond to a maturity of 25 years for an 80% LTV mortgage in line with values for the German housing market discussed in Geiger et al. (2016). Following Kaas et al. (2017), I set the coefficient for risk aversion to 2. The perceived survival probabilities are set according to official lifetables from the

---

[14]Lang et al. (2020) discuss loan-to-value ratios in Europe for the 2016-2018 period. I set the LTV to 0.2 for retired households reflecting the difficulty of receiving mortgage financing in old age.

German statistical office for the year 2016.[15] The replacement rate for the pension scheme is set to target a 48% replacement rate over average working age income[16] which implies a labor tax rate of $\tau = 14.1\%$. The initial distribution of assets for the young households $F^1(b, h, m)$ is estimated from the empirical distribution in the data for 20-25 year-olds and I assume the remainder of bequests $F_\theta^2(j)$ is distributed uniformly over working-age households. The number of available sizes for owner-occupied houses is set to three, with sizes equal to the average size as well as a small and large size at 80% and 200% of the average. There is one rental size reflecting the average rental size. All house sizes are relative to average GDP per capita. I use the discount factor β, housing preferences $(1 - \gamma_j)$, the bequest parameters κ and transaction cost ζ to jointly target the (net) wealth-to-GDP ratio W/Y^{tot}, housing wealth-to-GDP ratio pH^o/Y^{tot}, the total home ownership rate and the wealth-to-GDP at age 80.[17] Net wealth and housing wealth-to-GDP are sensitive to β and γ and are well identified by these parameters. Kaas et al. (2017) discuss the importance of transaction costs in explaining the low homeownership rate in Germany. Lastly, κ, which scales the strength of bequests, regulates the amount of dissaving in old age. See Appendix (D.2) for the sensitivity of moments to parameters. Refinancing costs are set to $\upsilon = 0.01$. The existence of refinancing costs in the model is important to avoid excessive refinancing in old age which would be counterfactual. In the baseline version, I only consider one type such that $\theta \in \{1.0\}$. The deterministic part of the income process χ_j is estimated with a fixed effect regression on the log of labor income and a $4th-$order age polynomial with coefficient vector $[\beta_0, \beta_1, \beta_2, \beta_3, \beta_4]$, which pins down the lifecycle profile of labor income. For the stochastic components I choose a standard value of $\rho_z = 0.93$ and calculate $\sigma_\varepsilon = 0.19$ which is consistent with the variance of the residual of the fixed effects regression. Table (2) gives a summary of the parameter values.

---

[15]https://www.destatis.de/

[16]This is calculated as the ratio of the average transfers divided by the income of an average worker over the lifecycle.

[17]I take the average between 75-85.

Moment	Model	Data	Target	Comment
Wealth Ratios				
Wealth/GDP	5.13	5.13	(Y)	WID (2016)
Capital/GDP	2.21	2.21	Y	WID (2016)
Housing assets/GDP	2.92	2.92	Y	WID (2016)
Wealth/GDP at 80	6.15	6.15	Y	LWS / WID (2016)
Mortgage debt/GDP	0.65	0.61	N	WID (2016)
Production				
Investment/GDP	0.23	0.23	Y	Destatis
Construction/GDP	0.06	0.06	Y	Destatis
Labor in Construction	0.03	0.06	N	Destatis
Housing				
Ownership	0.49	0.49	Y	LWS
Price/Rent	0.26	0.26	Y	Bundesbank (2017)
Households				
Pension Replacement rate	0.48	0.48	Y	OECD
Gini (labor income)	0.33	0.32	N	World Bank (2017)
Labor Income Owner/Renter	1.55	1.6	N	LWS
Fraction of labor income for rent	0.21	0.23	N	LWS (conditional on renting)

Table 3: *Model and data moments. Y signifies a targeted moment in the calibration. Wealth/GDP is implicitly targeted as the sum of housing assets and capital. All moments are annualized where applicable.*

4.2 Aggregate and Lifecycle Moments

Lifecycle moments come from the LIS version of the German SOEP pooled for the years 2002, 2007, 2012, and 2017.[18] Although the transition starts in 1960, it is both infeasible and impractical to calibrate the initial steady-state to the data: Firstly, data availability for 1960 is scarce in general and non-existent for the detailed household statistics required for fitting lifecycle profiles. Secondly, as I care about the dynamics in the recent past and near future, fitting the microdata in the 2000s will yield more accurate results for this time span.

To make lifecycles consistent with aggregate national wealth moments, I scale the lifecycle profiles such that when aggregated with the demographic distribution they are consistent with aggregate moments. Aggregate moments for (net) wealth-to-GDP W/Y^{tot}, housing assets-to-GDP

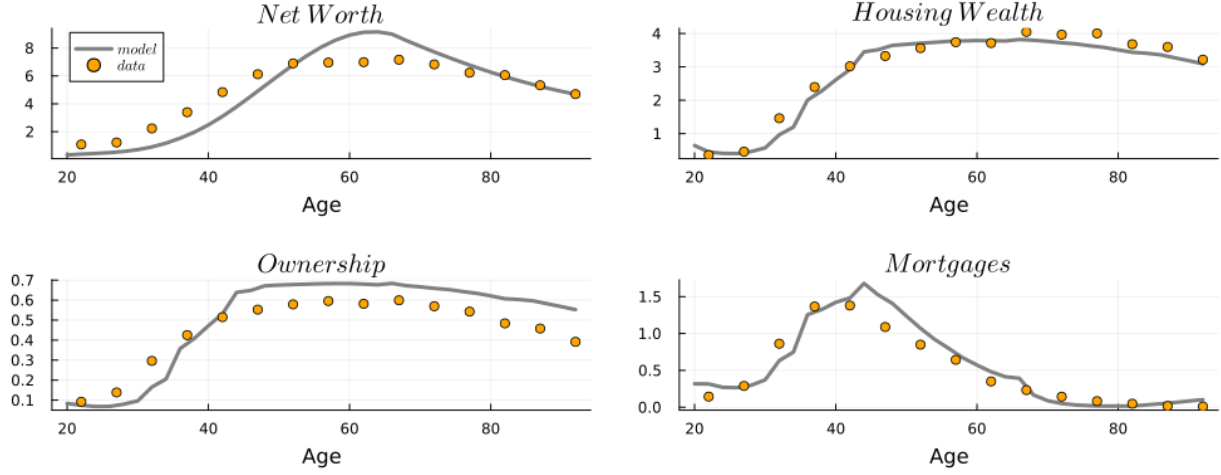[18]Which are the years with detailed wealth data in the LIS.

Figure 3: *Lifecycle profiles empirical vs model. Grey lines are model moments, and orange dots are corresponding data moments. Net worth, housing wealth, and mortgages are relative to GDP.*

pH^o/Y^{tot}, debt-to-GDP M/Y^{tot} come from the World Inequality Database (WID).[19] All WID values are for 2016 and I use net private wealth from WID as my wealth variable, private housing assets as the sum of private dwellings and land underlying dwellings, I map all private debt to mortgages in the model and use the national income variable to map into model GDP. With these ratios in hand, I calculate productive capital as the residual of $\frac{K}{Y^{tot}} = \frac{W}{Y^{tot}} - \frac{pH^o}{Y^{tot}} = 2.21$.

Table (3) summarizes targeted and untargeted aggregate model and data moments and figure (3) shows the empirical and model implied lifecycle moments for wealth-to-GDP, housing wealth-to-GDP, home ownership and mortgages.

---

[19]https://wid.world

5   Demographic Transition

To calculate the transitional dynamics, I need to fix an initial and terminal period. I choose as the starting point the year 1960 and impose the age distribution of that year as my initial condition $t = 1$. Up until 2021, I use the historical evolution of demographics after which I switch to demographic projections from the UN Population Prospects.[20] Current projections are available up until 2100. From this date on, I assume constant mortality and fertility at replacement, such that the age distribution reaches a stationary steady state by 2200 after one full generation has passed (100 years). I allow for another 100 years for all equilibrium variables to converge to their terminal steady-state values. The full transition is therefore over 340 years or $T = 170$ model periods. I solve the transition fully non-linearly by making use of the insights in Auclert et al. (2021) and calculating the sequence-space Jacobians to update the guesses for the prices using Newton's method. I need to solve for the price sequences $\{p_t, r_t, bq_{j,t,\theta}, tr_{j,t}\}_{t=1}^T$ such that the housing market and capital market clear each period, bequests received equals bequests given and transfers are equal to total tax revenues.[21] I report the resulting transition dynamics for the period 1980-2100.

Figure (4) shows the exogenous evolution of three demographic indicators over the considered time horizon: the adult population has increased by around 8% over the past three decades and is projected to decline over the remainder of the 21st century as a consequence of persistently low fertility and the baby boom generation that is reaching lower survival rates. The aging of the baby boom generation into retirement has already led to a persistent decline in the working population which is projected to continue over the remainder of the century. These forces lead to a continuous

---

[20]One of the driving forces of the age distribution is net migration which forces me to take a stance on the effect on changes in income and wealth distribution due to net migration. Mathä et al. (2011) for example report that the average income (net wealth) for immigrants in Germany is at around 90% (50%) of that of non-immigrant natives. One issue that arises is that only net migration numbers are available by age, which does not allow me to separate immigration from emigration. Here I take a parsimonious approach and assume that changes in net migration only alter between-age and not within-age income and asset distributions. This makes net migration by age a sufficient statistic without the need for separate immigration/emigration accounts.

[21]These sequences pin down the paths for wages and rents $\{w_t, f_t\}_{t=1}^T$. For the PAYG pension scheme, I take the approach that tax rates stay constant while transfers per capita adjust endogenously.
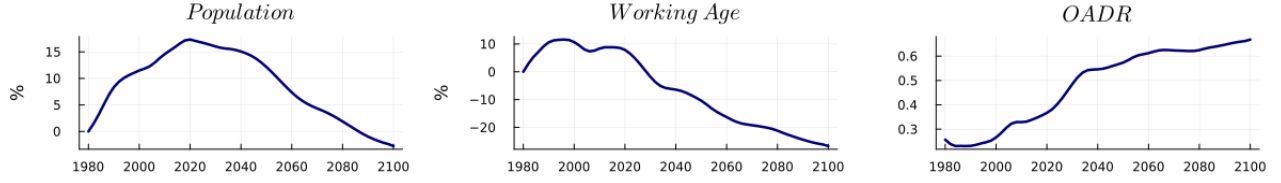
Figure 4: *Demographic change between 1980 and 2100. Population includes all model ages from 20 to 100. Working age population includes ages 20 to 65 which is the OECD cutoff for the old age dependency ratio (OADR). OADR is the ratio of retired (>65) over working age population.*

increase in the old age dependency ratio which is projected to rise to over 60% by the end of the century.

5.1 The Aggregate Effect on Equilibrium Prices

Figure (5) shows the equilibrium paths of prices for the closed economy transition. The house price path continuously increases over the first four decades of the transition as total population levels rise and the baby boom generation moves into peak housing demand. As population levels drop and the baby boom enters retirement in the 2020s house prices start to decline over the remainder of the 21st century. The aging society puts downward pressure on interest rates until around 2030 at which time it stabilizes at around 250bp lower than at the beginning of the century.[22] Due to the Cobb-Douglas production function, the real rate and wages are inversely related, resulting in an approximately 8 percentage point increase in the real wage over the same period.

These evolutions are a consequence of the drop in the relative size of the working-age population which reduces labor supply while the dissaving of the old does not reduce the capital stock to the same extent. The increasing capital-labor ratio then puts downward (upward) pressure on the real rate (wage). This dynamic has non-negligible effects on house price growth as I will discuss in more detail in the next section: The increased wages result in a positive income effect making households richer and falling interest rates increase housing demand through a substitution effect as the relative return of the housing asset increases while mortgage financing costs decrease. On the other hand, lower interest rates imply an income effect and make agents poorer, particularly

---

[22]Compare with 86 basis points calculated by Krueger and Ludwig (2007) for the period 2005-2080 for a world interest rate.
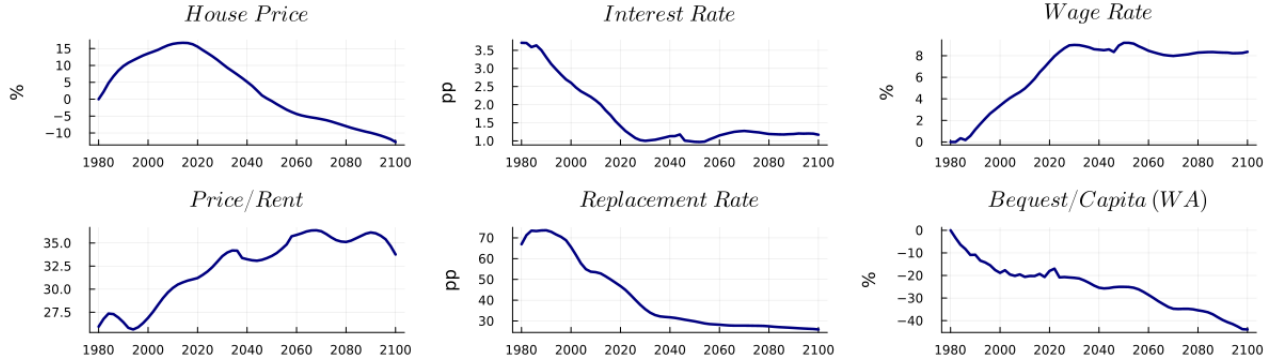
Figure 5: *Time series of the transition between 1980 and 2100. All variables are annualized. House prices, wage rate, and bequests/capita in % changes from 1980. Interest rate in percentage points. Bequest/Capita are bequests received per working-age household.*

the asset-rich.

The price-rent ratio, which is a function of the interest rate and house price growth, increases throughout the first half of the transition: As real rates fall rental developers find it cheaper to acquire property, while positive house price growth yields capital gains on housing investments. During the second half, these forces are traded off when developers make losses on their property while benefiting from low financing costs.

The aggregate replacement rate of the economy drops sharply between 2000 and 2030, which marks the strongest increase in the old-age dependency ratio. Bequests per working-age household decline over the entire transition as falling pension benefits reduce the ability of households to leave bequests while falling house prices reduce the value of bequests. This mechanism is partially mitigated between 2020 and 2040 during which a large baby-boom generation leaves more bequests than other cohorts.

5.2   Decomposition of House Price Growth

The demographic transition puts into motion a host of endogenous mechanisms that translate into shifts in housing demand and prices. This section aims at decomposing the demographic effect on house prices into its channels. It is constructive to consider the household problem in isolation for a moment: The endogenous variables the household cares about are the sequences for house prices,

rents, interest rates, wages, transfers, and bequests. $\{p_t, f_t, r_t, w_t, tr_{j,t,\theta}, br_{j,t,\theta}\}$ are thus a sufficient statistic to determine household policies over time. Together with the aggregating evolution of the age distribution, housing demand can be pinned down. In this exercise I will compute partial equilibrium solutions, fixing (non-housing) endogenous variables at their initial steady state while only allowing one sequence to follow its general equilibrium path calculated above. Because I am still interested in a (partial) equilibrium in the housing market, I will continue to clear the housing (and rental) market along this transition. This will isolate the contribution of that particular variable on the effect on the housing market. Due to the strong interaction effects of prices with the demographic evolution, I always allow for demographic change and then take the difference between the partial equilibrium solution where I allow one price and the age distribution to change, and the baseline where only the age distribution changes. Figure (6) shows the decomposition by the direct effect of demographic change, interest rates, wages, transfers, and bequests. The blue bars correspond to the direct contribution of demographic change to house prices in an economy where all other endogenous variables are fixed. This corresponds to a small open economy solution in which transfers and bequests are held constant as well and isolates the compositional effect of the changing age distribution on housing demand.

The demographic contribution comes from two changes: (1) changes in the age distribution as the larger baby boom generation moves along the lifecycle. Over the lifecycle housing demand differs such that demand adjusts depending on the relative weight of the demographic distribution at each age. The (2) change comes from population size. Even if the age distribution were to remain constant, an increase in the total population would increase housing demand. Because housing supply is modeled with a fixed factor and therefore exhibits decreasing returns to scale, larger populations imply higher house prices. In the demographic evolution, the period between 1980 and around 2025 is marked by a baby boom generation that is moving towards the peak of the housing demand of their lifecycle while at the same time total population increases as shown in figure (4). This implies that both demographic forces work in the same direction. It is worth noting that this direct demographic contribution makes up less than 1/3 of the increase over the past four decades. The projected demographic evolution sees a reversal of these trends. Total population levels are projected to decline, which is a consequence of continued low levels of fertility

and the baby boom generation facing progressively lower survival rates as they age. At the same time, as the baby boom moves into retirement, their housing demand does not increase any further but experiences a slight drop towards the end of their lifecycle. Both forces again work in the same direction but this time putting downward pressure on house prices towards the 2nd half and end of the 21st century.

As anticipated, the interesting and non-trivial interaction lies in the evolution of the capital-GDP ratio, its effect on the real rate (and wage rate), and their contribution to the equilibrium of house prices. Figure (7) shows an increase in the capital-GDP ratio for the past decades. This effect is well studied[23] and derives from the evolution of the baby boom cohorts through their lifecycle into age brackets that feature the highest stock of wealth and capital. Because the interest rate is a monotone function of the capital-labor (or capital-output) ratio, we observe a decline in the interest rate. Falling interest rates make financial assets less profitable and therefore housing a relatively more attractive investment asset, at the same time, financing of a house becomes cheaper through lower mortgage rates.[24] This substitution effect has been the dominant contributor to the growth in house prices over the past four decades.

The increase in the wage rate is the flip side of falling real rates. Note that the increase in the wage rate w is not representative of the average level of labor income - which directly depends on the age distribution - but gives the within-age change in labor income. It has an unambiguously positive effect on house prices along the entire transition while fading out towards the end of the century. This is driven by a pure income effect: households are richer and demand more of everything.

Due to the assumption of constant taxes, transfers drop as a consequence of an increasing old-age-dependency ratio resulting in lower disposable income for retired households. This creates both an income effect due to lower lifetime income as well as a substitution effect: Because households need to privately save more for retirement during their working age they substitute away from housing and towards financial assets which they can liquidate cost-efficiently in old age.

Lastly, falling average bequests work through a pure income effect but are overall small in magnitude.

---

[23]See for example Auclert et al. (2021) and Krueger and Ludwig (2007).

[24]In terms of interest rate exposure the household only cares about its net financial position i.e. financial assets net of mortgage debt.
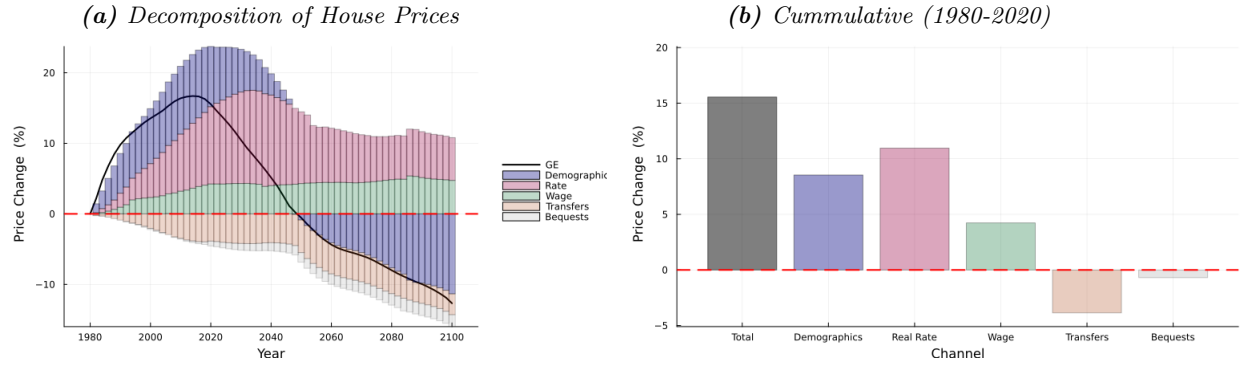
Figure 6: *Decomposition of house price dynamics into contributions of the direct effect of demographic change (blue), interest rates (purple), wages (green), transfers (orange) and bequests (grey). The solid line (black) gives total general equilibrium house prices. Panel (a) shows the dynamics over time, and panel (b) shows cumulative effects from 1980 to 2020. The sum of partial effects does not necessarily equal the total effect due to interaction effects. Decompositions are calculated by holding equilibrium sequences at their initial steady-state value and only allowing the channel of interest to follow its general equilibrium path. I then only clear the housing market which yields a partial equilibrium house price (and rent) path.*

5.3   The Evolution of Wealth-to-GDP Ratios

Figure (7) shows the evolution of wealth-to-GDP and its components capital and housing wealth-to-GDP. One of the insights in Auclert et al. (2021) is that the demographic effect on wealth-to-GDP ratios is projected to remain positive. In a model with a consumption-savings choice and a single productive asset, this necessarily implies a continued decline in the real rate. This can easily be seen as wealth equals capital $W/Y = K/Y$ and from the first order condition of the firm we have $r + \delta = \alpha \frac{Y}{W}$ and therefore unambiguously $\frac{dr}{dW/Y} = -\alpha(W/Y)^{-2} < 0$. However, in the present model with housing, we have that total (net) wealth is comprised of the sum of capital and housing wealth net of debt i.e. $W/Y = K/Y + pH^o/Y$. Therefore, while the model still projects a rise in the total wealth-to-GDP ratio, the relative share of housing wealth increases, such that the capital-to-GDP ratio stagnates from around 2030 on. This result is a consequence of the savings behavior of households in retirement. While total wealth begins to decline in retirement, a disproportionate share of productive (and in the model liquid) assets are being dissaved. Housing on the other hand
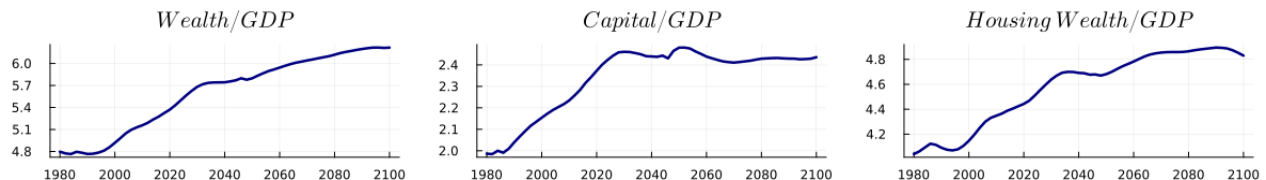
Figure 7: *Evolution of wealth-to-GDP ratios between 1980 and 2100. All variables are annualized.*

is costly to dissave as it incurs a transaction cost. This result highlights an important effect of demographic change on the real rate due to the presence of housing that is potentially neglected in an analysis where (non-productive) housing assets are subsumed as a productive asset.[25]

6   Conclusion

In this paper, I calibrate a quantitative lifecycle model to German microdata and document the effects of demographic change on house prices and their interaction with rates of return. The transition paths suggest that past demographic trends have increased house prices while a rising capital-output ratio suppressed the real rate. A decomposition of the house price path reveals that indirect general equilibrium effects matter, particularly through a substitution effect in response to a falling real rate that has pushed up house prices. As population levels are projected to decline over the 21st century, house prices fall. While wealth-to-GDP ratios are projected to increase as a consequence of an aging society, the composition of wealth shifts towards housing wealth mitigating the drop of the real rate as capital becomes a smaller share of total wealth.

While other forces can have a significant impact on house prices and real rates, this study isolates the effect from exogenous demographic trends in a controlled model environment. With this, I contribute to the literature on the demographic effects on house and asset prices by providing - to the best of my knowledge - the first analysis with a structural lifecycle model and housing in general equilibrium.

Limitations of the analysis open several avenues for future research: A limitation of the current study is the assumption of a closed economy where prices are affected by *domestic* demographics. International mobility of capital provides an argument for incorporating the effect of *global* demo-

---

[25]Auclert et al. (2021) do acknowledge the fact that housing can potentially attenuate some of their results.

graphics on rates of return. An extension of the current setup could incorporate a "rest of the world country" whose demographic evolution affects the supply of capital and interest rates but has no direct effect on the domestic housing market. Different stages of global demographic trends might then have an indirect effect on local housing markets and prices through interest rates and wages. On the other hand, some studies have documented a positive degree of home bias with respect to investment behavior that cast doubt on perfect capital mobility.[26] Endogenously parameterizing a degree of home bias would address this issue and allow for flexible weighting of domestic versus foreign demographic trends.

Another straightforward extension of the present analysis is the calibration to other countries, which differ with regard to both household portfolio characteristics, like homeownership, as well as demographic transitions. Differences in lifecycle profiles and old age dependency ratios might result in lagged or qualitatively different effects on house prices in some countries. A cross-country comparison might give additional insights into characteristics that drive housing market responses.

---

[26]See for example Oehler et al. (2007) for a discussion on German investment home bias.

References

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2021). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica 89*(5), 2375–2408.

Auclert, A., H. Malmberg, F. Martenet, and M. Rognlie (2021). Demographics, wealth, and global imbalances in the twenty-first century. NBER Working Papers 29161.

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra (2018). House prices and consumer spending. *The Review of Economic Studies 85*(3), 1502–1542.

Caldera, A. and Å. Johansson (2013). The price responsiveness of housing supply in oecd countries. *Journal of Housing Economics 22*(3), 231–249.

De Nardi, M., E. French, and J. B. Jones (2016). Savings after retirement: A survey. *Annual Review of Economics 8*(1), 177–204.

Doepke, M., A. Hannusch, F. Kindermann, and M. Tertilt (2023). The economics of fertility: A new era. In *Handbook of the Economics of the Family*, Volume 1, pp. 151–254. Elsevier.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019). A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics 11*(1), 1–48.

Fernandez-Villaverde, J. and D. Krueger (2011). Consumption and saving over the life cycle: How important are consumer durables? *Macroeconomic Dynamics 15*(5), 725–770.

Gagnon, E., B. K. Johannsen, and D. López-Salido (2021). Understanding the new normal: the role of demographics. *IMF Economic Review 69*(2), 357–390.

Geiger, F., J. Muellbauer, and M. Rupprecht (2016). The housing market, household portfolios and the german consumer. Working Paper Series 1904, European Central Bank.

Goodhart, C. A. E. and M. Pradhan (2020). *The Great Demographic Reversal: Ageing Societies, Waning Inequality, and an Inflation Revival.* Springer International Publishing.

Grossmann, V., B. Larin, and T. Steger (2024, 06). Das House Kapital: A Long-Run Theory of House Prices and Housing Wealth. *Journal of the European Economic Association*, jvae038.

Hiller, N. and O. W. Lerbs (2016). Aging and urban house prices. *Regional Science and Urban Economics 60*, 276–291.

Hoynes, H. W. and D. L. McFadden (1996). The impact of demographics on housing and nonhousing wealth in the united states. In M. D. Hurd and N. Yashiro (Eds.), *The Economic Effects of Aging in the United States and Japan*. University of Chicago Press.

Iskhakov, F., T. H. Jørgensen, J. Rust, and B. Schjerning (2017). The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks. *Quantitative Economics 8*(2), 317–365.

Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019). The rate of return on everything, 1870–2015. *The Quarterly Journal of Economics 134*(3), 1225–1298.

Kaas, L., G. Kocharkov, E. Preugschat, and N. Siassi (2017). Low homeownership in germany—a quantitative exploration. *Journal of the European Economic Association*.

Kajuth, F., T. A. Knetsch, and N. Pinkwart (2016). Assessing house prices in germany: Evidence from a regional data set. *Journal of European Real Estate Research*.

Kaplan, G., K. Mitman, and G. L. Violante (2020). The housing boom and bust: Model meets evidence. *Journal of Political Economy 128*(9), 000–000.

Krueger, D. and A. Ludwig (2007). On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics 54*(1), 49–87.

Lang, J. H., M. Pirovano, M. Rusnák, C. Schwarz, et al. (2020). Trends in residential real estate lending standards and implications for financial stability. Financial stability review, European Central Bank.

Lerbs, O. W. (2014). House prices, housing development costs, and the supply of new single-family housing in german counties and cities. *Journal of Property Research 31*(3), 183–210.

Mankiw, N. G. and D. N. Weil (1989). The baby boom, the baby bust, and the housing market. *Regional Science and Urban Economics 19*(2), 235–258.

Mathä, T. Y., A. Porpiglia, and E. Sierminska (2011). The immigrant/native wealth gap in germany, italy and luxembourg. Working Paper Series 1302, European Central Bank.

Oehler, A., M. Rummer, T. Walker, and S. Wendt (2007). *Are Investors Home Biased? Evidence from Germany*, pp. 57–77. London: Palgrave Macmillan UK.

Pelletier, D. and C. Tunc (2019). Endogenous life-cycle housing investment and portfolio allocation. *Journal of Money, Credit and Banking 51*(4), 991–1019.

Poterba, J. M. (2001). Demographic structure and asset returns. *Review of Economics and Statistics 83*(4), 565–584.

Quigley, J. M. and S. Raphael (2004). Is housing unaffordable? why isn't it more affordable? *Journal of Economic Perspectives 18*(1), 191–214.

Takáts, E. (2012). Aging and house prices. *Journal of Housing Economics 21*(2), 131–141.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters 20*(2), 177–181.

United Nations, Department of Economic and Social Affairs, Population Division (2022). World population prospects 2022: Data sources. *(UN DESA/POP/2022/DC/NO. 9)..*

Wetzstein, S. (2017). The global urban housing affordability crisis. *Urban Studies 54*(14), 3159–3177.

Yang, F. (2009). Consumption over the life cycle: How different is housing? *Review of Economic Dynamics 12*(3), 423–443.

Young, E. R. (2010). Solving the incomplete markets model with aggregate uncertainty using the krusell–smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control 34*(1), 36–41.

Appendix

A   Calibration Data

- Aggregate data for net private wealth, private housing assets, private debt, and national income are from the World Inequality Database. Values for the steady-state calibration are taken for the year 2017 in 2023 Euros and are available at https://wid.world. Asset-to-GDP ratios are calculated by dividing by net national income. Here I classify all debt as mortgage debt. Capital is calculated by subtracting private housing assets from net private wealth. Gross financial asset holdings, which would correspond to liquid assets b in the model are the sum of capital and private debt.

- Distributional data for the lifecycle profiles come from the Luxembourg Wealth Study Database https://www.lisdatacenter.org/. I use the 2002, 2007, 2012, and 2017 waves of the German household survey data (based on Germany's Socio-Economic Panel (SOEP)) and merge household level (h) and person level (p) files. To determine the age for the household I use the age of the household head as indicated in the panel. My definition of housing assets comes from the principal residence (hanrp) and mortgage debt from principal residence loans (hlrp). My definition of capital includes financial assets (haf), business equity (hannb), other real estate including commercial real estate (hanro), vehicles (hanncv), and voluntary life insurance and pensions (hasi). Financial assets are comprised of deposit accounts and cash (hafc), bonds and other debt securities, stocks and other equity investment funds and alternative investments (hafi), and other non-pension financial assets (hafo). Net worth is then financial assets plus housing assets net of housing debt. I use household weights when calculating averages and lifecycle profiles. Given that all wealth variables are on the household level I attribute wealth to individuals by dividing by household size where I count the household head and adult spouses. I do not consider children when attributing wealth.

- The demographic data on the past and projected evolution of the age distribution, and survival and fertility rates comes from the UN Population Prospect.

- The price-rent ratio for apartments in Germany for 2017 comes from Deutsche Bundesbank and is based on data from bulwiengesa AG (https://www.bundesbank.de).

- Labor in construction is calculated as the ratio of persons employed in construction divided by all persons employed. Both data come from the German statistical office Destatis https://www.destatis.de.

- Data in section (2) for house prices comes from the BIS data on residential property prices https://data.bis.org/topics/RPP/data. Demographic and GDP data comes from the World Bank https://data.worldbank.org/indicator with variables (codes): old-age dependency ratio (SP.POP.DPND.OL), birthrates (SP.DYN.CBRT.IN), and total population (SP.POP.TOTL), GDP per capita in constant 2015 dollars (NY.GDP.PCAP.KD). The following countries are part of the regression: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, South Africa, South Korea, Spain, Sweden, Switzerland, United Kingdom, United States. The data is at annual frequency from 1977-2020.

B Model Details

B.1 Household Problem

In this section, I cast the formal household problem in recursive form. Let $\Omega_{j,t} = (b_{j,t}, h_{j,t}, m_{j,t}, z_{j,t})$ be the individual state vector, where $b_{j,t}$ is the real holdings of liquid assets, $h_{j,t}$ is the size of the home the household owns, $m_{j,t}$ is the outstanding mortgage balance, and $z_{j,t}$ is the idiosyncratic productivity. For renters, we always have that $h_{j,t} = m_{j,t} = 0$.

B.1.1 Renters

They enter the period with their liquid assets $b_{j,t}$ and idiosyncratic productivity $z_{j,t}$ and make the discrete choice between staying a renter and choosing housing quality $h^r_{j,t} \in \mathcal{H}^r$ or becoming a homeowner and choosing house quality $h^o_{j+1,t+1} \in \mathcal{H}^o$. Let $d \in \mathcal{D}$ be the set of discrete choices.

The problem of the renter is given by

$$V_{j,t}^r(\Omega_{j,t}) = \max_d \left[v^{r,r}(\Omega_{j,t}; \mathbf{h^r}_{j,t}), v^{r,o}(\Omega_{j,t}; \mathbf{h^o}_{j+1,t+1}) \right] \tag{B.16}$$

$$h_{j,t}^r \in \mathcal{H}^r \tag{B.17}$$

$$h_{j+1,t+1}^o \in \mathcal{H}^o \tag{B.18}$$

where V_t^r is the value of starting as a renter at time t, $v_t^{r,r}$ be the value of remaining a renter, and $v_t^{r,o}$ is the value of transitioning to home-ownership. I now outline their problem conditional on each discrete choice.

Stay renter.— If the household stays a renter, they must choose consumption $c_{j,t}$ liquid savings $b_{j+1,t+1}$, conditional on rental quality $h_{j,t}^r$ maximizing

$$v_{j,t}^{r,r}(\Omega_{j,t}; h_{j,t}^r) = \max_{c_{j,t}, b_{j+1,t+1}} u(c_{j,t}, h_{j,t}^r) + \tilde{\phi}_j \beta \mathbb{E} V_{j+1,t+1}^r(\Omega_{j+1,t+1}) + (1 - \tilde{\phi}_j) \mathcal{B}(a_{j+1,t+1}) \tag{B.19}$$

subject to the budget constraint

$$b_{j+1,t+1} + c_{j,t} + f_t h_{j,t}^r = (1 + r_t) b_{j,t} + (1 - \tau_t) y_{j,t}(z_{j,t}) + tr_{j,t} + bq_{j,t} \tag{B.20}$$

$$a_{j+1,t+1} = (1 + r_{t+1}) b_{j+1,t+1} \tag{B.21}$$

which requires savings in liquid assets, non-durable consumption, and their rental costs to equal the sum of the return on their liquid assets, their gross labor income, transfers, and bequests.

Purchase home.— If the household becomes a home-owner, it must choose a house size $h_{j,t}^o$ consumption $c_{j,t}$, mortgage debt $m_{j+1,t+1}$ and liquid savings $b_{j+1,t+1}$ to maximize

$$v_{j,t}^{r,o}(\Omega_{j,t}; h_{j+1,t+1}^o) = \max_{c_{j,t}, b_{j+1,t+1}, m_{j+1,t+1}} u(c_{j,t}, h_{j+1,t+1}^o) + \tilde{\phi}_j \beta \mathbb{E} V_{j+1,t+1}^o(\Omega_{j+1,t+1}) + (1 - \tilde{\phi}_j) \mathcal{B}(a_{j+1,t+1}) \tag{B.22}$$

subject to the following constraints

$$b_{j+1,t+1} + c_{j,t} + (1 + \zeta) p_t h_{j+1,t+1} = (1 + r_t) b_{j,t} + (1 - \tau_t) y_{j,t}(z_{j,t}) + m_{j+1,t+1} + tr_{j,t} + bq_{j,t} \tag{B.23}$$

$$m_{j+1,t+1} \leq \lambda_t h_{j+1,t+1} p_t \tag{B.24}$$

$$a_{j+1,t+1} = (1 + r_{t+1}) b_{j+1,t+1} + p_{t+1} h_{j+t,t+1}^o \tag{B.25}$$

Equation (B.23) is the budget constraint. It states that savings in liquid assets, the consumption of non-durable goods, and the costs of purchasing a home must equal the sum of the returns from

liquid assets, the labor income, the new borrowing through the mortgage, transfers, and bequests. Equation (B.24) gives the LTV constraint to be a fraction λ_t of the market value of the home.

B.1.2 Owners

They enter the period with their liquid assets $b_{j,t}$, idiosyncratic productivity $z_{j,t}$, the current house quality $h^o_{j,t}$ and outstanding mortgage debt $m_{j,t}$. They make the discrete choice between 1) not adjusting 2) adjusting into another house quality $\mathbf{h^o}_{j+1,t+1} \in \mathcal{H}^o$ with a new mortgage 3) refinancing their mortgage for the current home 4) becoming a renter and choosing house quality $\mathbf{h^r}_{j,t} \in \mathcal{H}^r$. Formally, their problem is

$$V^o_{j,t}(\Omega_{j,t}) = \max_j \left[v^{o,na}(\Omega_{j,t}), v^{o,a}(\Omega_{j,t}; \mathbf{h^o}_{j+1,t+1}), v^{o,re}(\Omega_{j,t}), v^{o,r}(\Omega_{j,t}; \mathbf{h^r}_{j,t}) \right] \tag{B.26}$$

$$\mathbf{h^r}_{j,t} \in \mathcal{H}^r, \tag{B.27}$$

$$\mathbf{h^o}_{j+1,t+1} \in \mathcal{H}^o \tag{B.28}$$

where V^o_t is the value of starting as an owner at time t, $v^{o,na}_t$ be the value of not adjusting, $v^{o,a}_t$ is the value of purchasing a new home, $v^{o,re}_t$ is the value of refinancing but staying in the same home, and $v^{o,r}_t$ is the value of becoming a renter. I now outline their problem conditional on each discrete choice.

No adjustment.— Owners that do not adjust choose non-durable consumption $c_{j,t}$ and liquid savings $b_{j+1,t+1}$ to maximize

$$v^{o,na}_{j,t}(\Omega_{j,t}) = \max_{c_{j,t}, b_{j+1,t+1}} u(c_{j,t}, h^o_{j,t}) + \tilde{\phi}_j \beta \mathbb{E} V^o_{j+1,t+1}(\Omega_{j+1,t+1}) + (1 - \tilde{\phi}_j)\mathcal{B}(a_{j+1,t+1})$$

subject to the following constraints

$$b_{j+1,t+1} + c_{j,t} + \delta_H h_{j,t} p_t + r_t m_{j,t} + amort_{j,t} = (1 + r_t)b_{j,t} + (1 - \tau_t)y_{j,t}(z_{j,t}) + tr_{j,t} + bq_{j,t} \tag{B.29}$$

$$m_{j+1,t+1} = m_{j,t} - amort_{j,t} \tag{B.30}$$

$$amort_{j,t} = \min \left[\xi h^o_{j,t} p_t, m_{j,t} \right] \tag{B.31}$$

$$a_{j+1,t+1} = (1 + r_{t+1})b_{j+1,t+1} + p_{t+1} h^o_{j+t,t+1} \tag{B.32}$$

Equation (B.29) is the budget constraint. It states that savings in liquid assets, the consumption of non-durable goods, the home maintenance costs, and the mortgage payments, must equal the

sum of the returns from liquid assets, labor income, transfers, and bequests. Equation (B.30) is the evolution of mortgage balances (B.30). Equation (B.31) gives the amortization.

Adjust.— Owners that adjust their home size, choose non-durable consumption $c_{j,t}$ the quality of the new home $h^o_{j+1,t+1}$, their mortgage debt $m_{j+1,t+1}$ and savings in liquid assets $b_{j+1,t+1}$ to solve

$$v^{o,a}_{j,t}(\Omega_{j,t}; h_{j+1,t+1}) = \max_{c_{j,t}, b_{j+1,t+1}, m_{j+1,t+1}} u(c_{j,t}, h^o_{j+1,t+1}) + \tilde{\phi}_j \beta \mathbb{E} V^o_{j+1,t+1}(\Omega_{j+1,t+1}) + (1 - \tilde{\phi}_j)\mathcal{B}(a_{j+1,t+1})$$

(B.33)

subject to

$$b_{j+1,t+1} + c_{j,t} + h_{j+1,t+1}p_t + \zeta p_t(h_{j,t} + h_{j+1,t+1}) + (1 + r_t)m_{j,t}$$
$$= (1 + r_t)b_{j,t} + (1 - \tau_t)y_{j,t}(z_{j,t}) + m_{j+1,t+1} + (1 - \delta_H)h_{it}p_t + tr_{j,t} + bq_{j,t}$$

(B.34)

$$m_{j+1,t+1} \leq \lambda_t h_{j+1,t+1}p_t \tag{B.35}$$

$$a_{j+1,t+1} = (1 + r_{t+1})b_{j+1,t+1} + p_{t+1}h^o_{j+t,t+1} \tag{B.36}$$

as well as the LTV constraint (B.35). The budget constraint (B.34) is similar to those of renters who choose to purchase. The difference is that homeowners must pay the maintenance and transaction costs of the old home and pay back their old mortgage in full.

Refinance.— Owners that want to refinance without moving homes choose non-durable consumption $c_{j,t}$, their mortgage debt $m_{j+1,t+1}$ and savings in liquid assets $b_{j+1,t+1}$ to solve

$$v^{o,re}_{j,t}(\Omega_{j,t}) = \max_{c_{j,t}, b_{j+1,t+1}, m_{j+1,t+1}} u(c_{j,t}, h^r_{j,t}) + \tilde{\phi}_j \beta \mathbb{E} V^o_{j+1,t+1}(\Omega_{j+1,t+1}) + (1 - \tilde{\phi}_j)\mathcal{B}(a_{j+1,t+1})$$

subject to

$$b_{j+1,t+1} + c_{j,t} + \upsilon + (1 + r_t)m_{j,t} - m_{j+1,t+1} = (1 + r_t)b_{j,t} + (1 - \tau_t)y_{j,t}(z_{j,t}) + tr_{j,t} + bq_{j,t} \tag{B.37}$$

$$m_{j+1,t+1} \leq \lambda_t h_{j+1,t+1}p_t \tag{B.38}$$

$$h^0_{j+1,t+1} = h^o_{j,t} \tag{B.39}$$

$$a_{j+1,t+1} = (1 + r_{t+1})b_{j+1,t+1} + p_{t+1}h^o_{j+t,t+1} \tag{B.40}$$

as well as the LTV constraint (B.38) and the restriction that they do not adjust their house. Their budget constraint is similar to that of owners that do not adjust, with the difference that they must pay refinancing costs to be able to issue a new mortgage.[27]

Become a renter.— Owners that choose to become renters choose non-durable consumption $c_{j,t}$ and savings in liquid assets $b_{j,t}$ to maximize

$$V_{o,t}^r(\Omega_{j,t}; h_{j,t}^r) = \max_{c_{j,t}, b_{j+1,t+1}} u(c_{j,t}, h_{j,t}^r) + \tilde{\phi}_j \beta \mathbb{E} V_{j+1,t+1}^r(\Omega_{j+1,t+1}) + (1 - \tilde{\phi}_j) \mathcal{B}(a_{j+1,t+1}) \qquad \text{(B.41)}$$

subject to the budget constraint

$$b_{j+1,t+1} + c_{j,t} + (1 + r_t)m_{j,t} + f_t h_{j,t}^r \qquad \text{(B.42)}$$

$$= (1 + r_t)b_{j,t} + (1 - \tau_t)y_{j,t}(z_{j,t}) + (1 - \delta_H - \zeta)h_{it}p_t + tr_{j,t} + bq_{j,t} \qquad \text{(B.43)}$$

$$a_{j+1,t+1} = (1 + r_{t+1})b_{j+1,t+1} + p_{t+1}h_{j+t,t+1}^o \qquad \text{(B.44)}$$

which states that the savings in liquid assets, consumption of the non-durable good, the rental cost, and the repayment of the mortgage must equal the sum of returns from liquid savings, the labor income, and the proceeds from selling their old home net of maintenance and transaction costs.

B.2   Equilibrium

Definition 1 (Closed Economy Equilibrium). *An equilibrium is a set of (non-stationary) age- and time-dependent policy function* $b_{j,t,\theta}(b, h, m, z)$, $h_{j,t,\theta}^r(b, h, m, z)$, $h_{j,t,\theta}^o(b, h, m, z)$, $m_{j,t,\theta}(b, h, m, z)$, *value functions* $V_{j,t,\theta}^r(b, z)$, $V_{j,t,\theta}^o(b, h, m, z)$, *discrete choice specific value functions* $v_{j,t,\theta}^{r,r}(b, z)$, $v_{j,t,\theta}^{r,o}(b, z)$, $v_{j,t,\theta}^{o,r}(b, h, m, z)$, $v_{j,t,\theta}^{o,a}(b, h, m, z)$, $v_{j,t,\theta}^{o,re}(b, h, m, z)$, $v_{j,t,\theta}^{o,na}(b, h, m, z)$, *and sequences of prices* $\{p_t, f_t, r_t, w_t\}_{t=1}^{\infty}$, *pension replacement rates* $\{\rho_t\}_{t=1}^{\infty}$ *and bequests* $\{bq_{j,t,\theta}\}_{t=1}^{\infty}$, *a measure* $\mu_{j,t,\theta}(b, h, m, z)$, *given initial conditions for* $N_{j,1} \forall j$ *and sequences* $\{\phi_{j,t}, \psi_t, \mathcal{M}_{j,t}\}_{j=1,t=1}^{J,\infty}$ *that satisfy*

1. *the household optimization problem as outlined in (3.1)*

2. *construction and rents are set according to (10) and (11)*

3. *factor prices are set according to (7) and (8)*

---

[27]Whenever a household adjusts it borrows up to the limit. Therefore, there is no pre-payment in the model.

4. *the government budget (6) is balanced*

5. *bequests given equal bequests received and the market for housing, capital and labor clears as outlined in (3.5)*

C   Household vs Individual Wealth

To calculate individual level wealth by age I follow the approach as in Auclert et al. (2021) and divide household level wealth by household size. I only count the household head or their spouse and do not include children. Figure (8) shows wealth profiles juxtaposing household with individual-level wealth. Household and individual-level profiles are rescaled to integrate to the same number.
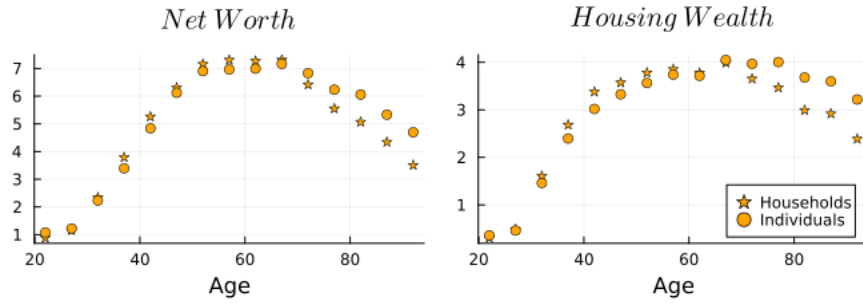

Figure 8: *Lifecycle profiles household vs individual wealth.*

D   Calibration

D.1   Income Process

Table (4) gives the parameters estimated of the 4th-order age polynomial for the income process.

D.2   Sensitivity

This section explores the sensitivity of the four key household moments to the internally calibrated parameters. Although all parameters can and do interact in a complex fashion affecting all moments

Description	Parameter	Value
Income process persistence	ρ_z	0.93
Income process variance	σ_ε	0.19
Age components	β_0	1.83344
	β_1	.7182734
	β_2	$-.0223433$
	β_3	.000323
	β_4	$-1.82e^{-06}$

Table 4: *Parameters of the estimated income process.*

jointly it is useful to inspect whether a particular moment is sensitive to a particular parameter. This can be seen in figure (9): Capital-to-GDP is sensitive to β which regulates the discounting of future periods and is a key parameter in many models to regulate the degree of savings. Housing wealth is sensitive to the consumption/housing weight γ which regulates the preference for non-durable consumption versus housing. The transaction cost parameter ζ helps pin the ownership rate and the bequest strength κ regulates wealth holdings in retirement (age 80). To highlight the joint determination of moments, figure (10) shows the sensitivity of all moments to all parameters. The discount factor has an impact on all wealth and ownership variables as it regulates total savings. While the housing weight γ also affects all moments, it predominantly affects the tradeoff between capital and housing wealth and less the total level of wealth. The transaction cost predominantly affects ownership choice but plays little role in identifying retirement wealth. The flat portions of housing wealth and ownership with respect to the bequest κ suggest that those moments do not identify κ well which is predominantly identified by old age wealth as it regulates the saving in retirement.
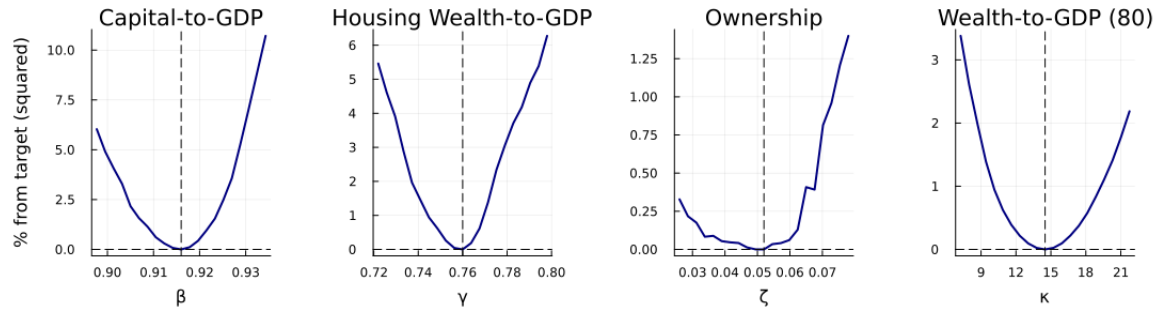
Figure 9: *Sensitivity of moments to parameters that identify them well. Vertical axes give % deviation from target squared. See (10) for all cross sensitivities. β is not annualized.*
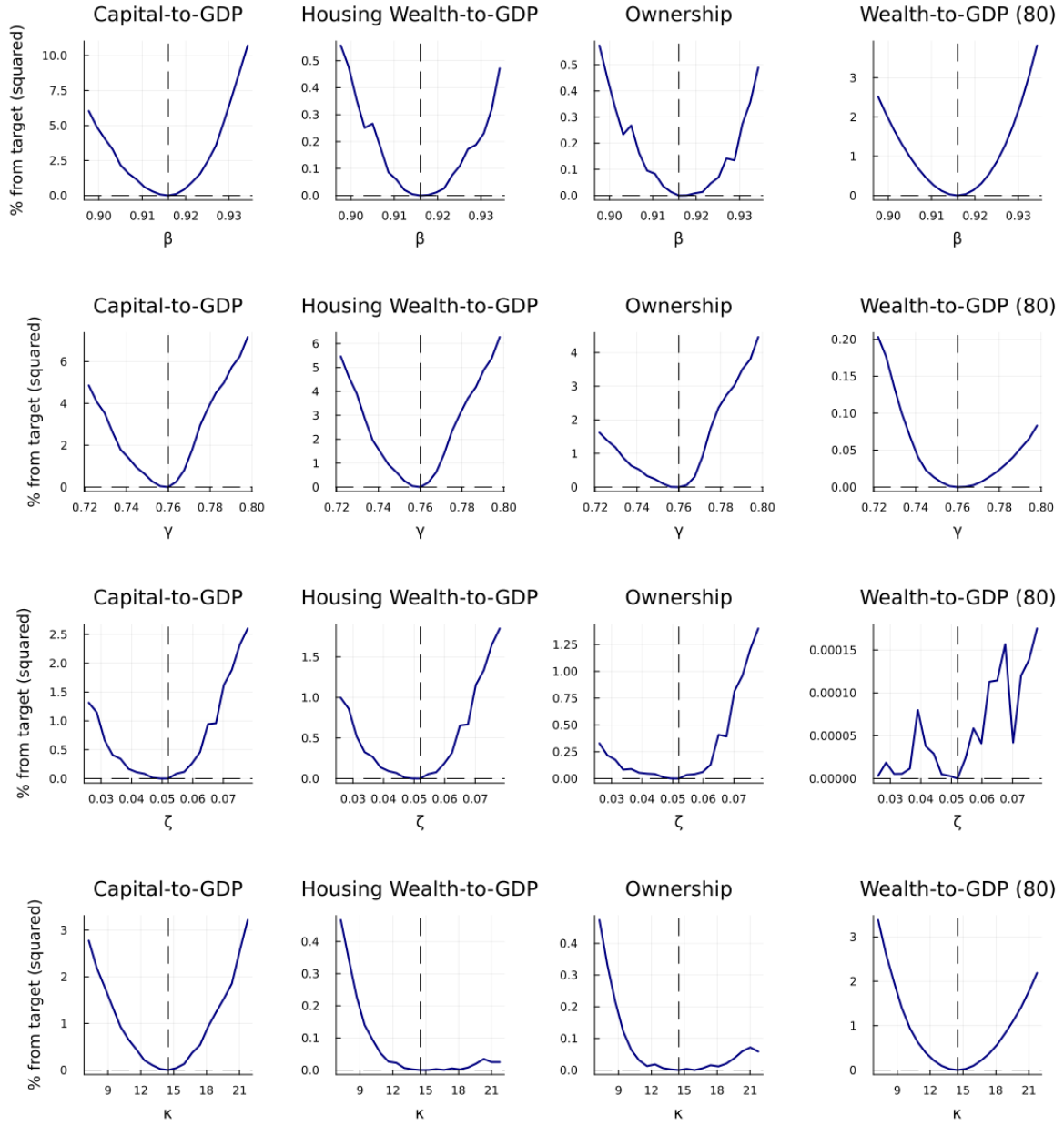
Figure 10: *Sensitivity of moments to parameters, all moments to all parameters. Vertical axes give % deviation from target squared. β is not annualized.*