Problem Set: Lesson 4.1

CDS 292. Fall, 2023

Instructions

In this document you will find a set of problems intended to be answered **by hand**, scanned and submitted to Blackboard. Make sure your document is well organized and contains readable responses to the questions. **Show your work! Full credit requires this. An answer without procedure is likely to elicit zero points.** There is one addendum to this handwritten rule: any required plot can be either scanned or sent as an additional file to your submission. If you choose to submit separate files with your plots, make sure they are labeled and can be easily identified.

Note: `Python` code questions will occur in problem sets. The answers have to be handwritten and must respect all `Python` syntax rules including proper indentation of code.

Total Points: 11

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1. *[2 points]* The following is the list of degrees of all the nodes of a *single* network (don't get confused by the two lines, those are the degrees for one network). The list of degrees of a network has a special name, the *degree sequence of a network*. Construct the respective histogram.

$$\begin{array}{cccccccccccc} 4, & 8, & 6, & 9, & 1, & 12, & 1, & 1, & 1, & 2, & 1, & 2, \\ 1, & 2, & 2, & 1, & 2, & 2, & 8, & 2, & 1, & 1, & 1, & 1 \end{array}$$

   Draw the histogram by hand. You can show the heights (i.e. $H(k)$) either by bars or just simply points (but please make them large enough to be visible).

2. *[2 points]* Use `Python` to create a *random network* with $n = 1000$ and $\rho = 0.25$. Obtain its degree histogram and plot it. *HINT: You can use the* `gnp_random_graph()` *function from* `networkx` *(see the documentation for details) or a nested* `for` *loop with a random number generator (use the* `random.random()` *function from* `numpy` *to obtain a number from a uniform distribution in the interval* $(0, 1)$

3. We run $10,000$ simulations of tomorrow's commute time and find that a specific stretch of road (call it road X) between 8:15 AM and 8:30 AM shows the passing of between $c = 421$ to $c = 450$ cars. In the $10,000$ simulations, every value of c between 421 and 450 shows up.

   (a) *[1 point]* What is the following sum equal to?

$$\sum_{c=421}^{450} H(c) = ?$$

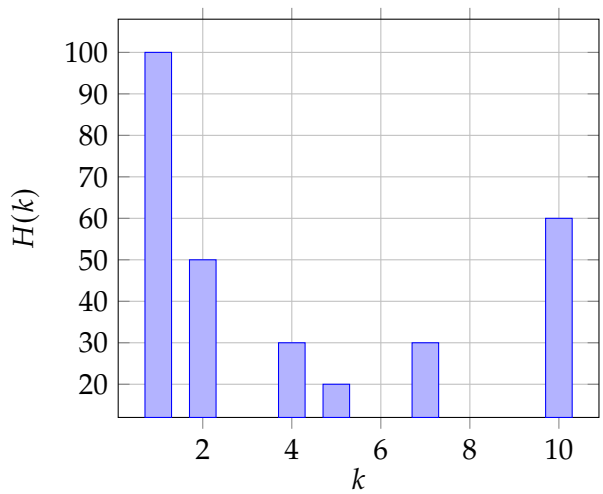
   Assume that $H(421) = H(422) = \cdots = H(435) = M$ and $H(436) = H(437) = \cdots = H(450) = M/3$.

(b) *[1 point]* What is M?

(c) *[1 point]* What is (are) the value(s) of c^*, defined as the c for which $H(c)$ achieves its largest value. Note that sometimes more than one value c can lead to $H(c)$ having a largest value. In this case, indicate all c with this property.

(d) *[1 point]* Calculate $\langle c \rangle$, the average of c. You can do this by hand or use `python` code. In either case, be very explicit about how you have done the calculation.

4. *[3 points]* Consider the $H(k)$ below. Determine the number of nodes n, number of links m, k_{min}, k_{max}, k^*, the average degree $\langle k \rangle$, and the density ρ of the network.



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1.) A list of various degrees associated with every node in a network.

4, 8, 6, 9, 1, 12, 1, 1, 1, 2, 1, 2, 1, 2, 2, 1, 2, 2, 8, 2, 1, 1, 1, 1 → 24 total degrees listed, thus 24 nodes in network

Degree sequence is its name for the list of degrees written out like this. Construct a histogram.

Degrees in order: 1, 2, 4, 6, 8, 9, 12

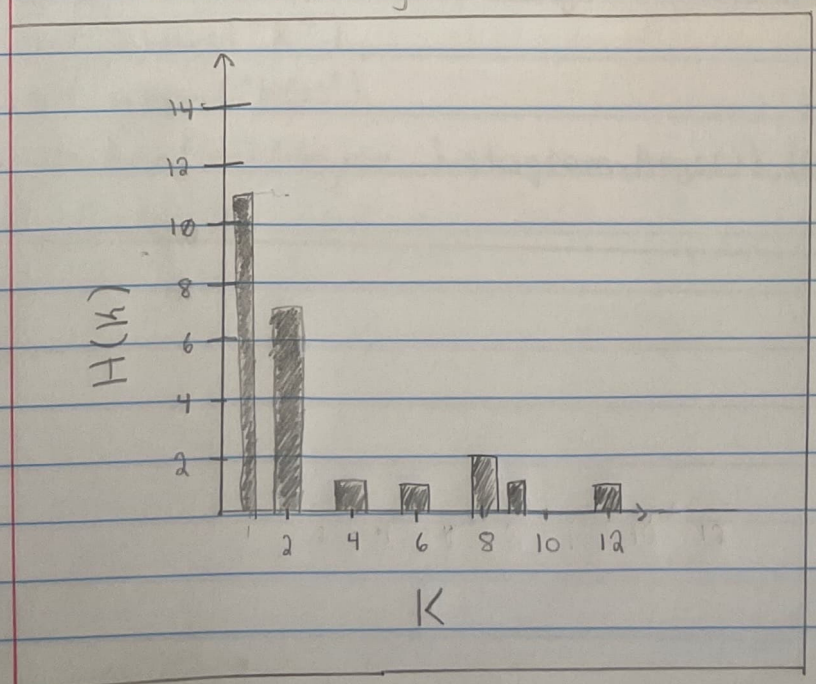
$H(k=1|G)=11$ / $H(k=2|G)=7$ / $H(k=4|G)=1$ / $H(k=6|G)=1$ / $H(K=8|G)=2$ / $H(K=9|G)=1$ / $H(k=12|G)=1$

$H(k)=\emptyset$ if $k \neq 1, 2, 4, 6, 8, 9,$ or 12

$11+7+1+1+2+1+1=18+6=24$

```
# Python Code, not sure if needed.
    import HK Analysis as HK
    from Readlist import readlist

    G = readlist("Problem1.txt")
    GH = HK.Hdeg(G)
```



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2.) In Python, create "random network" with 1000 nodes & density = 0.25. Then, obtain degree histogram and plot it.

→ gnp_random_graph() / 2 arguments (n, p) :)

```
import networkx as nx
import matplotlib.pyplot as plt.
n= 1000
p = 0.25
G = nx.gnp_random_graph(n,p)
G_degrees = dict(nx.degree(G))
degree_histogram = {}

for node in G.nodes():
    degree_histogram[node] = 0
for node in G_degrees.keys():
    k = G_degrees[node]
    degree_histogram[k] += 1

plt.title("Frequencies of Degrees in Network")
plt.xlabel("K")
plt.ylabel("H(K)")
plt.bar(list(degree_histogram.keys()), list(degree_histogram.values()))
```

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3.) 10,000 Simulations
Road X, 8:15am - 8:30am, 421-450 cars pass
10,000 Simulations of commute time on road X, all values between 421 and 450 show-up.

a.) $\sum_{c=421}^{450} H(c) = \sum_{421}^{435} 15M + \sum_{436}^{450} 15 \cdot M/3 \rightarrow \frac{15m}{3} \rightarrow 5M + 15m = \boxed{20M}$

$H(c=?) = M$
↳ amount of simulations that that amount of cars pass through road X.

$H(421) \rightarrow H(435)$ are all the same. $= M$
$H(436) \rightarrow H(450)$ are all the same $= M/3$

b.) M is the amount of simulations that each amount of cars pass through road X between 8:15-8:30.

c.) Since c from $421 \rightarrow 435 = m$ and c from $436 \rightarrow 450 = m/3$.
The largest freqeuencies occur from $c = 421 \rightarrow c = 435$.

$\boxed{c^* = 421, 422, \ldots, 435}$

d.) $\cancel{20}^{2}M / \cancel{30}_{3} = \boxed{2m/3 = \langle 15 \rangle}$

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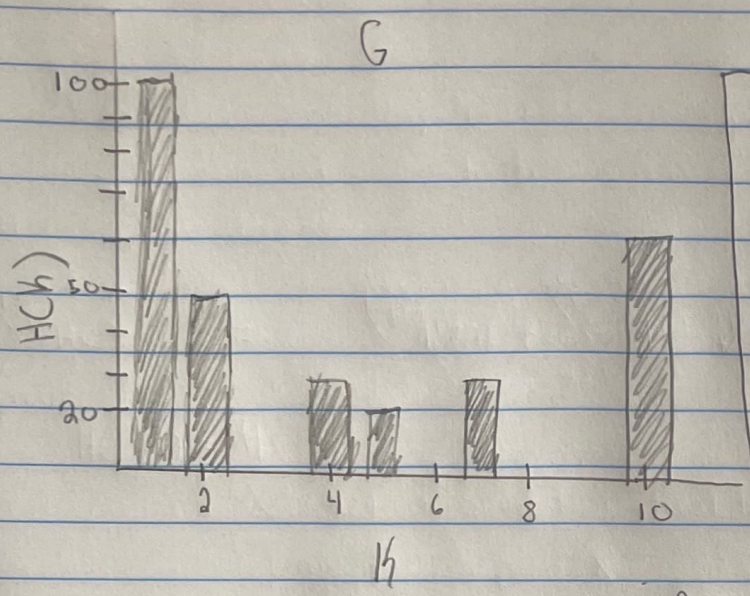
4.)



$n = 290$
$m = 615$
$k_{min} = 1$
$k_{max} = 10$
$k^* = 1$
$\langle k \rangle = 4.2$
$p = 0.01$

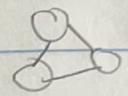
r/ER $615/41905$

$290 \cdot 289 / 2 = 41{,}905$ m complete

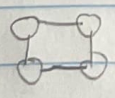
$H(k=1) = 100$
$H(k=2) = 50$ } 150 } 200
$H(k=4) = 30$
$H(k=5) = 20$ } 50 } 290 nodes
$H(k=7) = 30$ } 90 } 90
$H(k=10) = 60$

$\langle k \rangle = 2 \cdot 615 / 290 = \frac{1230}{290} = 4.2$

100 nodes with 1 link 100 } 200
50 nodes with 2 links + 100
30 nodes with 4 links + 120 } 220 + } 420
20 nodes with 5 links + 100
30 nodes with 7 links + 210 } 810 } 1230 / 2 = 615 links
60 nodes with 10 links + 600



3 nodes 2 links 3 links
6 links?



4 nodes 2 links 4 links.
8 links ?