Problem Set: Lesson 3.2

CDS 292. Fall, 2023

Instructions

In this document you will find a set of problems intended to be answered **by hand**, scanned and submitted to Blackboard. Make sure your document is well organized and contains readable responses to the questions. **Show your work! Full credit requires this. An answer without procedure is likely to elicit zero points.** There is one addendum to this handwritten rule: any required plot can be either scanned or sent as an additional file to your submission. If you choose to submit separate files with your plots, make sure they are labeled and can be easily identified.

Note: `Python` code questions will occur in problem sets. The answers have to be handwritten and must respect all `Python` syntax rules including proper indentation of code.

Total Points: 11

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1. *[1 point]* Construct the network associated with the following adjacency matrix. Label the nodes so that node `a` is given in row/column 1, node `b` in row/column 2, etc.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2. Construct the networks associated with the following adjacency matrices. Label the nodes so that node `a` is given in row/column 1, node `b` in row/column 2, etc.

   (a) *[1 point]*

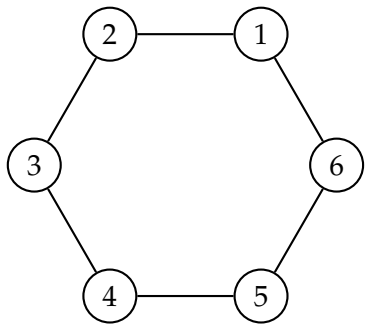
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

   (b) *[1 point]*

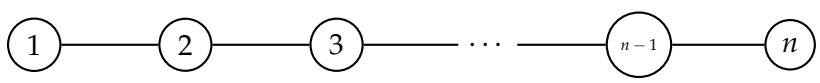
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

3. *[1 point]* How many unique **link indicators** can you find in the line network G, if you only know the node set $V(G) = \{2, 4, 8, \ldots, 2^n\}$?

4. *[1 point]* Construct the adjacency matrix for the network below. Make the rows/columns of the matrix correspond to the labels of the nodes, i.e., node 1 in row/column 1, node 2 in row/column 2, etc.



5. Consider the line network below:



It shows a **line network** of n nodes. Write `Python` code to:

(a) *[1 point]* create a line network of 200 nodes,

(b) *[1 point]* save its adjacency matrix as `A`, and

(c) *[1 point]* use the adjacency matrix to obtain the number of nodes. HINT: *how do the size of the network and the size of the matrix relate?*

6. The sum of elements of a properly constructed adjacency matrix $\mathbf{A}$ of the network G is equal to $1,200$. If the adjacency matrix has dimensions 50×50, determine

(a) *[2 points]* $\langle k \rangle$ and ρ of network G.

(b) *[1 point]* How many 0s does $\mathbf{A}$ have?

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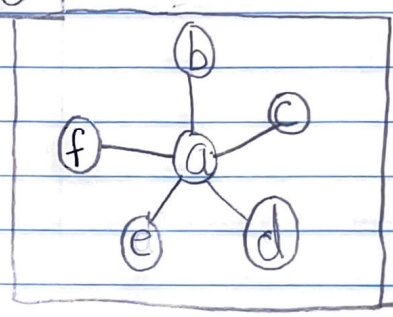
(1) Construct network

$A =$

	a	b	c	d	e	f
a	Ø	1	1	1	1	1
b	1	Ø	Ø	Ø	Ø	Ø
c	1	Ø	Ø	Ø	Ø	Ø
d	1	Ø	Ø	Ø	Ø	Ø
e	1	Ø	Ø	Ø	Ø	Ø
f	1	Ø	Ø	Ø	Ø	Ø

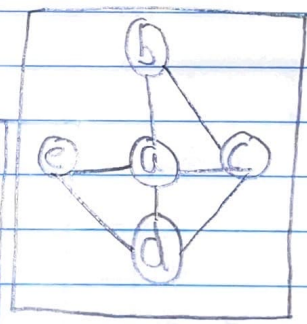
link indicators



(2a.) More construct networks

$A =$

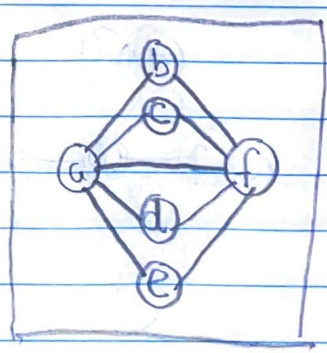
	a	b	c	d	e
a	Ø	1	1	1	1
b	1	Ø	1	Ø	Ø
c	1	1	Ø	1	Ø
d	1	Ø	1	Ø	1
e	1	Ø	Ø	1	Ø



(2b)

$A =$

	a	b	c	d	e	f
a	Ø	1	1	1	1	1
b	1	Ø	Ø	Ø	Ø	1
c	1	Ø	Ø	Ø	Ø	1
d	1	Ø	Ø	Ø	Ø	1
e	1	Ø	Ø	Ø	Ø	1
f	1	1	1	1	1	Ø



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(3) Unique link indicators in line, only knowing node set

$V(G) = \{2, 4, 8 \ldots, 2^n\}$

n, number of nodes $= 4 / \{2, 4, 8, 16\}$ $2^4 = 16$? $2^5 = 32$.
$n = 5 / \{2, 4, 8, 16, 32\}$

$\underset{1}{2} - \underset{2}{4} - \underset{3}{8} - \underset{4}{16} - 32$ 10 possible links.
Not in line network though

link indicators for 2 $\boxed{\text{\# of unique} = n-1}$

a_{24} a_{28} a_{216} a_{232}

(4) Adjacency matrix 6 x 6

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \cancel{a_{11}} & a_{12} & \cancel{a_{13}} & \cancel{a_{14}} & \cancel{a_{15}} & a_{16} = \\ 2 & a_{21} & \cancel{a_{22}} & a_{23} & \cancel{a_{24}} & \cancel{a_{25}} & \cancel{a_{26}} = \\ 3 & \cancel{a_{31}} & a_{32} & \cancel{a_{33}} & a_{34} & \cancel{a_{35}} & \cancel{a_{36}} = \\ 4 & \cancel{a_{41}} & \cancel{a_{42}} & a_{43} & \cancel{a_{44}} & a_{45} & \cancel{a_{46}} = \\ 5 & \cancel{a_{51}} & \cancel{a_{52}} & \cancel{a_{53}} & a_{54} & \cancel{a_{55}} & a_{56} = \\ 6 & a_{61} & \cancel{a_{62}} & \cancel{a_{63}} & \cancel{a_{64}} & a_{65} & \cancel{a_{66}} = \end{array}$$

$$A = \begin{pmatrix} \emptyset & 1 & \emptyset & \emptyset & \emptyset & 1 \\ 1 & \emptyset & 1 & \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset & 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 & \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 & \emptyset & 1 \\ 1 & \emptyset & \emptyset & \emptyset & 1 & \emptyset \end{pmatrix}$$

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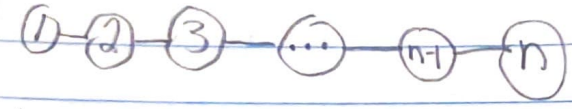
5a



5b

Write python code to:
Create line network of 200 nodes

5c

```
import networkx as nx

G = nx.Graph() # Create empty network

for e in range(1, 201):
    G.add_node(e)
    if e < 200:
        G.add_edge(e, e+1)

A = nx.adjacency_matrix(G)

#Obtain number of nodes from matrix, matrix is n x n
# So 200 x 200. Therefore, number of nodes is just 200

num_nodes = A.size / 2
```

6

Sum of link indicators in matrix = 1,200
50 x 50 matrix

a) Determine average degree, $\langle k \rangle$ and p, density

$\langle k \rangle = 2m/n$

$1{,}200/2 = 600$ $m = 600$

$n = 50$

$600/1225$

$\langle k \rangle = 1200/50 = 24$

$p = 0.49$

$600/1225$

$50(49)/2 = 1{,}225$

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(6) b.) Number of Ø's?

2,500 link indicators in matrix

$$\begin{array}{r} 2{,}500 \\ -\ 1{,}200 \\ \hline 1{,}300 \end{array}$$ 1's

$\boxed{1{,}300 \text{ Ø's}}$