Problem Set: Lesson 7.1

CDS 292. Fall, 2023

Instructions

In this document you will find a set of problems intended to be answered **by hand**, scanned and submitted to Blackboard. Make sure your document is well organized and contains readable responses to the questions. **Show your work! Full credit requires this. An answer without procedure is likely to elicit zero points.** There is one addendum to this handwritten rule: any required plot can be either scanned or sent as an additional file to your submission. If you choose to submit separate files with your plots, make sure they are labeled and can be easily identified.

Note: `Python` code questions will occur in problem sets. The answers have to be handwritten and must respect all `Python` syntax rules including proper indentation of code.

Total Points: 11

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1. *[1 point]* How many **triangle indicators** do you need to check for all possible triangles in a network with $n = 9$?

2. *[1 point]* Network G has $\rho = 1$. If it contains 84 triangles (global count, i.e. $T = 84$), how many nodes are in the network?

3. Network G has adjacency matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

   If the node set is $V(G) = \{2, 4, 6, 8, 10\}$ and the rows/columns of $\mathbf{A}$ are in the same order as the nodes in $V(G)$:

   (a) *[1 point]* List all triangles attached to node 4

   (b) *[1 point]* Find all global triangles

4. *[2 points]* In a network G, we are told that $T(G) = 120$ and that $n = 10$. In addition, we know that 8 of the nodes each has 36 local triangles. If we add them together, how many local triangles do the other two nodes have? How many local triangles do each of the two remaining nodes have? *Hint: What is the maximum number of triangles in a network of n nodes?*

5. *[3 points]* A frequent flyer has plenty of miles to spend. For work related reasons, this person can only take short trips, i.e. over the weekend. To maximize the number of visited airports, the optimal way to travel is to start from a destination visit two more airports and then go back

to the first airport. This *path* describes a triangle, and therefore, we can look at the `airline` data to find the optimal starting airports for this traveler. To be explicit, you need to look for the airports that have the most triangles attached to them. Just as an exercise, list the 5 airports with the most triangles attached to them, with their respective t_i

6. *[2 points]* L is a line network with n nodes. Assume the node set as $V(R) = \{1, 2, 3, \ldots, n\}$ with n even. How many triangles (global) are in L? Now, if you add the links $\{(2,4), (4,6), (6,8), \ldots, (n-2, n)\}$ to the network, how many global triangles can you identify? *HINT: A useful strategy to think about this problem is to create a diagram with a network using a small value for n, and then, generalize your result.*

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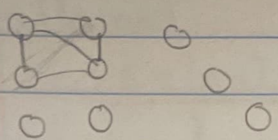
1.) $n=9$, triangle indicator → T_{ihq}, 3 nodes
how many combinations of three nodes in network



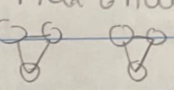
$$\frac{n(n-1)(n-2)}{1 \times 2 \times 3} = \frac{9 \times 8 \times 7}{6} = \frac{504}{6} = 84$$

84 combinations of three nodes possible in $n=9$ network, therefore $t_{ihq} \times 84$ triangle indicators to check if triangle exists.

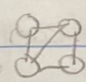
2.) global triangle = # of triangles in the network. triangles are powerful structures. In this problem, we're saying that some network, random, named 'G', contains 84 triangles in it. That is, 3 nodes, 3 links total forming an infinite path between each other

84 triangles, each with 3 nodes. $84 \cdot 3 = 252$ nodes maximum (↗ but $p \neq 1$) in network G.

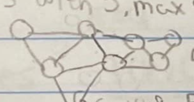
max 6 links
max 6 nodes



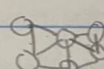
5 links
2 with 4 nodes, max 6



7 links 2 7 links
3 with 5, max 9



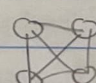
4 with 6, max 12
6 with 8 5 with 7, max 15
max 18



9 links max
4 tri 12

84 triangles, max 252
min 86
↘ but $p \neq 1$

n triangles,
min 'n' = $n+2$ → but $p \neq 1$
max 'n' = $n \cdot 3$



$p=1$

Number of triangles 'nt'
min # of nodes to form 'nt' triangles is = $nt+2$
max # of nodes to form 'nt' triangles is = $nt \cdot 3$

We also are told that the density is = to 1. Probability that any two nodes, chosen at random, have a link, this means that we have a complete network, quite rare. →

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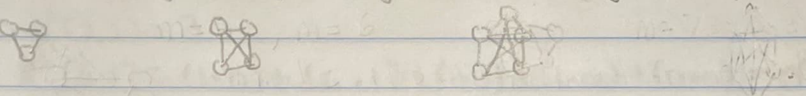
2.) ... Continued..

So we know that this random network 'G' is a complete network, in which the maximum number of links are attained.

$\frac{n \times (n-1)}{2}$. We know that the number of nodes must lie between 86 and 252.

In a network with 84 triangles, where every node is connected to each other. If $p=1$ in a network, then there is an exact number of triangles that are formed.

$n=3$, 1 triangle / $n=4$, 4 triangles / $n=5$, triangles $=8$ / $n=6$



$\frac{n(n-1)(n-2)}{6}$ → gives number of triangles possible triangles when $p=1$. Process of elimination.

$n=10$

$\frac{10(9)(8)}{6} = 120$ possible triangles

$n=9$

$\frac{9(8)(7)}{6} = 84$

Wow, ok, so $\binom{n}{3}$ also gives us the possible number of triangles that can be formed, informally meaning that $p=1$. So if $p=1$ and there are 84 triangles in this network, then there are 9 nodes in the network

→ The calculations I was doing in the beginning failed to incorporate p being equal to 1.

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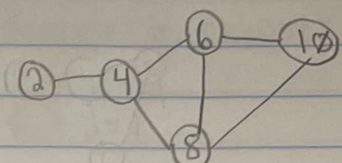
3.)

$$A = \begin{array}{c|c} & \begin{array}{ccccc} 2 & 4 & 6 & 8 & 10 \end{array} \\ \hline \begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array} \rightarrow$$



a.) Visually: t_{468}

Mathematically:

$A \times A$ for $[2,2]$ for matrix $C = (1\cdot1)+(0\cdot0)+(1\cdot1)+(1\cdot1)+(0\cdot0)$

$A^2[2,2] = 3$

$A^2[2,1] = (1\cdot0)+(0\cdot1)+(1\cdot0)+(1\cdot0)+(0\cdot0) = \varnothing$

$[2,3] = (1\cdot0)+(0\cdot1)+(1\cdot0)+(1\cdot1)\ (0\cdot1) = 1$

$[2,4] = 1\cdot0 + 0\cdot1 + 1\cdot1 + 1\cdot0 + 0\cdot1 = 1$

$[2,5] = 1\cdot0 + 0\cdot0 + 1\cdot1 + 1\cdot1 + 0\cdot0 = 2$

0 3 1 1 2

$A^3[2,2] = 0\cdot1 + 3\cdot0 + 1\cdot1 + 1\cdot1 + 2\cdot0 = 2$

$t_4 = \frac{A^3_{22}}{2} = \frac{2}{2} =$ $\boxed{1 \text{ triangle total} \rightarrow t_{468}}$

b.) Visually: $\boxed{t_{468},\ t_{6810},}$

4.) We have some random network, let's call it 'G'. This random network has 10 'nodes' in it. This means that the most amount of triangles that this network, or any network with 10 'nodes', can have is $\binom{10}{3}$ $10(9)(8)/6 = 12\cancel{0}$ triangles. Coincidentally, we also know that there are 120 'global' triangles in this network called 'G', which means that every node in this network is connected to all other 9 nodes. Thus, there are $\binom{10}{2}$ links in the network as well, being 45 links

So, to recap: 'G' has:

- 10 nodes
- 45 links
- 12~~0~~ triangles
- 8 nodes have 36 local triangles, each.

We additionally know that: 8 of the nodes in this network have 36 local triangles each.

$36+36+36+36+36+36+36+36 = 288$ total local triangles from the 8 nodes. This means, that since every node is linked to all other nodes, that the remaining 2 nodes must also have 36 local triangles because:

→ $36 \cdot 1\cancel{0} = 360/3 = 120$

$36 \times 2 = 72$ between the two

36
+36
/2
7

↳ The number of global triangles in the network

5.) We have some traveler, named John, for instance. John is a frequent flier in airports who has accumulated many 'flyer miles' over the years, called having 'miles to spend'. Because of John's proffession, John can only take short-distance trips. →

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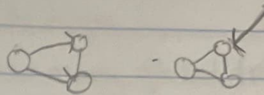
5.) ... Continued ... on the weekends, 2 days.
If John were trying to visit the most number of airports, he should start at some destination, travel to two other airports and then travel back to the first, so: forms a triangle



We can look at airport data to find the best starting airports for John to start at in order to visit the most number of airports. The best starting airports are the ones that have the most local triangles. Find those networks in the file dataset, the top 5, and list how many local triangles they have

- The sum of all local triangles = 3 x global triangles
- 3,425 airports in data.

...

```
import networkx as nx
from readlist import readlist
G = readlist('Airports.txt', 0)  → ti = nx.triangles(G)
all_local_triangles = nx.triangles(G) # Returns dictionary with number of local
                                      # triangles for each airline
ranked = Sorted(ti.items(), Key = lambda x: x[1], reverse = True)
```

↳ Shows Nodes with most local triangles:

- 'AMS' / $t_{AMS} = 4543$
- 'FRA' / $t_{FRA} = 4357$
- 'CDG' / $t_{CDG} = 4136$
- 'MUC' / $t_{MUC} = 3658$
- 'LHR' / $t_{LHR} = 3168$

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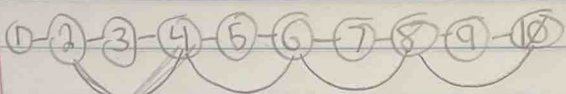
6.) We have some network, let's call it L with the following characteristics:

- line network
- $V(L) = \{1, 2, 3, \dots n\}$
- n is even
- how many global triangles?

∅ global triangles in L since it is a line network



Add links -> (2-4), (4-6), (6-8), (8-10)

↳ Now how many global triangles?

Still ∅ global triangles