Problem Set 2

Building Blocks of Networks

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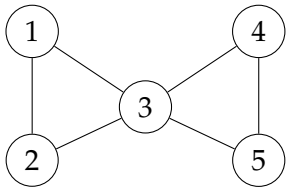
1. Calculate the maximum number of links for a network of
   (a) $n = 5$
   (b) $n = 10$
   (c) $n = 15$
   (d) $n = 20$
2. Network G has a set $V(G)$ with 15 elements and a set $E(G)$ with 35 elements. Cal-culate the density of network G.
3. Using the network below:



   (a) Write the node set
   (b) Write the link set
4. Using the following sets:

$$V(G) = \{1,2,3,4,5,6,7,8\}$$
$$E(G) = \{(1,2),(6,1),(2,3),(3,4),(4,5),(6,5)\}$$

   Draw the network G

5. Follow the steps below to create a network and extract some quantities of it.
   (a) Start by creating 20 nodes.
   (b) Among the nodes created, connect some of them to produce 30 links.
   (c) Calculate the density of your created network using any `Python` function at your disposal (or write your own).
6. A class has 25 students that start the semester not knowing each other. By the end of the semester, 30 friendships were formed. What is the density of this social net-work? How many more friendships would be needed for the social network to be a complete network?

Problem Set 2.

(1a) $n=5$, max number assumes network is a complete network. density, amount of actual / max. Completeness of network?

$m_{complete}$ / m = links, n = nodes / 5 nodes, what's maximum amount of links in the network. links exist between two nodes.

$m_{complete}(5) = \binom{5}{2}$ Binomial looks for combinations, $n(n-1)/2$

$(5)(4)/2 = 20/2$ [10 maximum number of links]

(1b) $n=10$ $V(6)=\{0,0,0,0,0,0,0,0,0,0\}$
Nodes can be anything. $E(6)_{max} = ?$ or $m_{complete}(10)$ or $\binom{10}{2}$ or $10(10-1)/2$

$(10)(9)/2 = 90/2$ [45 links possible]

(1c.) 15 nodes in network, how many combos of 2, links exist between 2 nodes, use binomial for combination checks

$\binom{15}{2}$; $(15)(14)/2$, $210/2$, [105 possible]

(1d) 20 nodes? $(20)(19)/2$ [190 max number]

Every 5 node increase increases gap by +25.

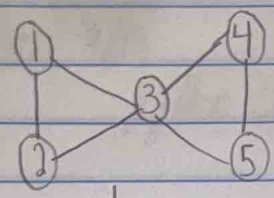
+35
25
+60
25
+85

(2) $V(6)$ = Vertices, or nodes, or n. Set containing all nodes in network
$V(6)$ has 15 nodes / elements.
$E(6)$, edges, links, has 35 elements. / density, P, 0 to 1.

$\binom{15}{2} = (15)(14)/2 = 105$ max number of links, actual = 35 $\frac{1}{10}$

[$p(6) = 35/105 = 1/3$]

Problem Set 2, continued...

(3) 

Node set, V(G), vertices, nodes

$V(G) = \{1,2,3,4,5\}$

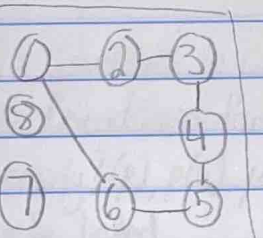
$E(G) = \{(1,2),(1,3),(2,3),(3,4),(3,5),(4,5)\}$

↓ Undirected

$(1,2) = (2,1)$

(4) $V(G) = \{1,2,3,4,5,6,7,8\}$, nodes in network, elements

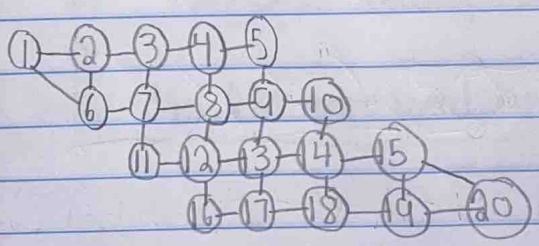
$E(G) = \{(1,2),(6,1),(2,3),(3,4),(4,5),(6,5)\}$, links in network



*Note that not all nodes are connected, not needed to be a network

20 elements in node set, 20 nodes.

(5a)/(5b) $V(G) = \{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}$



(5c) Wants to know density, probability that any two nodes (random) will be connected, or linked

probability = how many links exist / theoretical maximum in network.

m / m_{max}

30 links in our network, need to know what the max number of links we can have in a system of 20 nodes, use binomial.

20 elements or nodes, $(20)(19)/2 = 380/2 = 190$ max links, probability equals $30/190 = 3/19$

Problem Set 2, Continued...

(5c) (p) = 3/19

Python Code:

```
import networkx as nx
#Create network, using nx.graph(),
automatically add nodes by adding links
to network using .add_edges() function

nx.density(G) #Retrieve density
```

(6) 25 students in class
30 friendships formed at end of semester
Density? (p), p(G) probability that any 2 students will have a friendship, or be linked.
p(G) need max number of links in the network, binomial.

n = 25 (25)(24)/2 = 300 maximum friendships/links, 30 real
m = 30 ~~30/300~~ 1/10 Density = 1/10

p(G) = 1/10, 270 more friendships needed for complete network