Problem Set: Lesson 3.1

CDS 292. Fall, 2023

Instructions

In this document you will find a set of problems intended to be answered **by hand**, scanned and submitted to Blackboard. Make sure your document is well organized and contains readable responses to the questions. **Show your work! Full credit requires this. An answer without procedure is likely to elicit zero points.** There is one addendum to this handwritten rule: any required plot can be either scanned or sent as an additional file to your submission. If you choose to submit separate files with your plots, make sure they are labeled and can be easily identified.

Note: `Python` code questions will occur in problem sets. The answers have to be handwritten and must respect all `Python` syntax rules including proper indentation of code.

Total Points: 13

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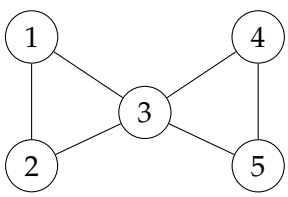
1. *[2 points]* Write the node set, link set, and all the link indicators for this network [2 points]



2. *[2 points]* The set of links of network G is $\{(1,2),(2,3),(3,4),(4,5),\ldots,(n-1,n)\}$. G has no isolated nodes (i.e. there are no nodes with degree 0). How many link indicators do we need to calculate k_2? Note that this calculation requires all *relevant* link indicators regardless of whether their value is 0 or 1. Note that n is a positive integer and your answer should depend on n.

3. *[1 point]* There are circumstances when keeping track of the "missing links" or "vacancies" among node pairs in a network G is important. Say you wanted to define a vacancy indicator called $b_{i,j}$ (as opposed to a link indicator). The vacancy indicator $b_{i,j}$ should be

$$b_{i,j} = \begin{cases} 0 & \text{if } i \text{ and } j \text{ are connected in } G, \\ 1 & \text{otherwise.} \end{cases}$$

   Can you come up with a way to write $b_{i,j}$ as a function of $a_{i,j}$

4. *[1 point]* A network of 5 nodes has the following link indicators given to you:

$$a_{1,2} \quad a_{4,5} \quad a_{5,2} \quad a_{2,4} \quad a_{2,3} \quad a_{5,1} \quad a_{1,3} \quad a_{1,4} \quad a_{3,5} \quad a_{4,3} \quad a_{2,1}.$$

   Can you build the network? Explain. Don't worry that we're not giving you their values 0 or 1, you don't need them

5. Import the Watergate network into your Colaboratory session. Look at the degrees of each node.
   (a) *[1 point]* What is the node with the highest degree? List them if there is more than one.
   (b) *[1 point]* What is the node with the lowest degree? List them if there is more than one.
   (c) *[1 point]* Write `Python` code to list all link indicators (with their values) where Nixon is involved. You can use a dictionary to store the results.
6. *[2 points]* Imagine a management consultant working with Company G. The company has 4 operating units. Communication between units has been an issue. The consultant receives \$1000 for each pair of units that have (or establish) communication, and is obligated to pay back to G \$500 for every pair of units that fail to communicate. Write a formula in terms of link indicators that determines how much the consultant will be paid for any possible combination of communication and non-communication situations. HINT: *A link indicator between units i and j is represented by $a_{i,j}$. You could also use a vacancy indicator (i.e. there is no link between units i and j) as a function of the link indicator. Let b represent such vacancy indicator. It follows that $b_{i,j} = 1 - a_{i,j}$. Also, remember this one formula has to work for any scenario of communications among units.*
7. *[2 points]* Write two different networks, both of which have the same degrees,

$$k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 2.$$

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Problem Set 3.1

(1) Node set, set of all nodes in the network:

$\boxed{V(G), \text{vertices}; \{1,2,3,4,5\}}$

Link set, edges, $\boxed{E(G) = \{(1,2),(1,3),(2,3),(3,4),(3,5),(4,5)\}}$

6 total links, $m=6$. $n=5$

m_{max} = max number of links, $(5)(4)/2 = 10$ max

$(\rho) = r/tt = 6/10, 3/5$

Link indicators for network: a_{ij}, include redundant pairs

	1	2	3	4	5
1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Or

$a_{12}\ a_{13}\ a_{14}\ a_{15}$
$a_{23}\ a_{24}\ a_{25}$
$a_{34}\ a_{35}$
a_{45}

Without Redundancies.

Each "link indicator" (a_{ij}), represents a possible connection between two nodes. Redundant nodes included.

(2) $E(G) = \{(1,2),(2,3),(3,4),(4,5),\ldots,(n-1,n)\}$ 8 (7,8)

No isolated nodes, every node/element of network, called 'G' has at least one degree or at least one link. Need to calculate the degree of node/element "2".

Trying to find out how many connections/links "2" has.
How many link indicators do we need to calculate k_2?

table will be $n \times n$, if 5, we need 5 link indicators to calc, a nodes degree.

$\boxed{\text{We need "n" many link indicators to calculate degree of "2"}}$

Problem Set 3.1

(3) Sometimes, need to keep track of missing links in a network.

link indicator - a_{ij}, vacancy indicator - b_{ij}, can't you just look at Ø's in matrix? If not:

$$a_{ij} = \begin{cases} 1, \text{ connected} \\ \emptyset, \text{ not} \end{cases} \qquad b_{ij} = \begin{cases} 1, \text{ not connected} \\ \emptyset, \text{ connected} \end{cases}$$

Write b_{ij} as a function of a_{ij}?, opposite of a_{ij}

$y = mx + b$ $\quad f(x) = x^2 + 3$ $\quad f(a_{ij}) = a_{ij}$ / returns 1, need to subtract 1

y is a function of x $\quad$ return a 1 or a zero

$f(a_{ij}) = a_{ij}$ / returns Ø, need to add 1.

$b_{ij}(a_{ij}) = \overset{1}{a_{ij}} - 1$ / To return Ø ✓ $\quad \boxed{b_{ij}(a_{ij}) = 1 - a_{ij}}$

$b_{ij}(a_{ij}) = \overset{\emptyset}{a_{ij}} + 1$ / To return 1 ✓

(4) 5 nodes in network, $n = 5$, 25 indicators in matrix, given:

a_{12} a_{45} a_{52} a_{24} a_{23} a_{51} a_{13} a_{14} a_{35} a_{43} a_{21}

Can we build the network? We know there're 5 nodes.

find max links in network. $(20)/2 = 10$

Max links in network is 10, we are given 11 link indicators. a_{12} or a_{21} is redundant, and in undirected networks, $a_{ij} = a_{ji}$, so we can "eliminate" one of the link indicators, leaving us with 10 total. If all indicators return "1", their sum would be 10, allowed by m complete. Nodes in network do not have to be connected/linked to be considered a network, so they could all be Ø too, which is fine. Hence, yes, building a network is possible.

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(5) a.) Node with highest degree; Ehrlichman, $k_{Ehrlichman} = 24$

b.) Node with lowest degree; Howard, O, H, Shipley / $k = 1$

c.)
```
import networkx as nx
from Readlist import readlist

G=readlist("Watergate.dat") #Renamed file

#Need link indicator function
def a(G,i,j):
    if G.has_edge(i,j):
        return 1
    else:
        return 0

#Create dictionary, empty.
Nixon_Links = {}

#Use loops to populate dictionary with keys(nodes) and values
for e in G.nodes():
    Nixon_Links[e] = a(G,"Nixon",e)

print(Nixon_Links)
```

Problem Set 3.1

(6) $\boxed{G}$ ① ② + \$1000 for each pair
④ ③ − \$500 for each pair not connected

6 max links, hence 6 possible pairs $12/2 = 6$

a_{12} a_{13} a_{14}
a_{23} a_{24}
a_{34}

$$\text{money} = 1{,}000(m) - 500(6-m)$$

$5{,}000 - 500 = \$4500$ 5 communications
$2000 - 2000 = \$0$ 2 communications

Need to write in terms of link indicators. Need sum of link indicators for those connected, and those not connected

$$\sum_{j=2}^{4} a_{1j} + \sum_{j=3}^{4} a_{2j} + \sum_{j=4}^{4} \times 1{,}000 - 500\left(6 - \sum_{j=2}^{4} a_{1j} + \sum_{j=3}^{4} a_{2j} + \sum_{j=4}^{4} a_{3j}\right)$$

$$\boxed{\text{Compensation} = 1{,}000\left(\sum_{j=2}^{4} a_{1j} + \sum_{j=3}^{4} a_{2j} + \sum_{j=4}^{4} a_{3j}\right) - 500\left(6 - \sum_{j=2}^{4} a_{1j} + \sum_{j=3}^{4} a_{2j} + \sum_{j=4}^{4} a_{3j}\right)}$$

(7) Write 2 networks, that are different from each other.

$k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 2$

6 total nodes in network, all have same number of links, 2.

max $= (6)(5) = 2 = 30/2 = 15$ links

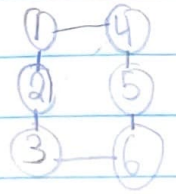
12 links total in network

1, 2, 3, 4, 5, 6
|| || || || || ||

Network 1:

$V(G) = \{1, 2, 3, 4, 5, 6\}$

$E(G) = \{(1,2), (1,4), (2,3), (3,6), (5,6), (4,5)\}$



Problem Set 3.1

(7) Continued...

link indicators for network 1:

a_{12}^1 a_{23}^1 a_{34}^0 a_{45}^1 a_{56}^1

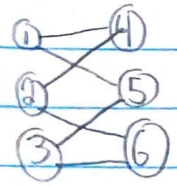
a_{13}^0 a_{24}^0 a_{35}^0 a_{46}^0

a_{14}^1 a_{25}^0 a_{36}^1

a_{15}^0 a_{26}^0

a_{16}^0

Network #2:



$V(G) = \{1,2,3,4,5,6\}$

$E(G) = \{(1,4),(1,5),(2,4),(2,6),(3,5),(3,6)\}$

link indicators:

$a_{12}=0$ $a_{23}=0$ $a_{34}=0$ $a_{45}=0$ $a_{56}=0$

$a_{13}=0$ $a_{24}=1$ $a_{35}=1$ $a_{46}=0$

$a_{14}=1$ $a_{25}=0$ $a_{36}=1$

$a_{15}=1$ $a_{26}=1$

$a_{16}=0$