Transport to One-Speed Diffusion

Start: Transport equation. Assume: isotropic scattering, one energy group, Fick's law.

Start with the transport equation:

$$\left[\frac{1}{v}\frac{\partial}{\partial t} + \vec{\Omega}\cdot\nabla + \Sigma_t(\vec{r},E,t)\right]\psi(\vec{r},\vec{\Omega},E,t) = \int_{4\pi} d\vec{\Omega}' \int_0^\infty dE'\Sigma_s(\vec{\Omega}' \to \vec{\Omega}, E' \to E)\psi(\vec{r},\vec{\Omega}',E',t) + S$$

Where S is a source. The fission term would be:

$$\frac{X_p(E)}{4\pi}\int_0^E dE'\nu(E')\Sigma_f(E')\phi(\vec{r},E',t)$$

To arrive at the diffusion equation, start by integrating over all angles. The terms, in order, are:

$$\frac{1}{v}\frac{\partial}{\partial t}\int_{4\pi} d\Omega\psi(\vec{r},\vec{\Omega},E,t) = \frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},E,t)$$

$$\int_{4\pi} d\Omega(\vec{\Omega}\cdot\nabla\psi(\vec{r},\vec{\Omega},E,t)) = \nabla\cdot\int_{4\pi} d\Omega(\vec{\Omega}\psi(\vec{r},\vec{\Omega},E,t)) = \nabla\cdot\vec{J}$$

$$\int_{4\pi} d\Omega\Sigma_t(\vec{r},E,t)\psi(\vec{r},\vec{\Omega},E,t) = \Sigma_t(\vec{r},E,t)\phi(\vec{r},E,t)$$

$$\int_{4\pi} d\vec{\Omega}\int_{4\pi} d\vec{\Omega}'\int_0^\infty dE'\Sigma_s(\vec{\Omega}' \to \vec{\Omega}, E' \to E)\psi(\vec{r},\vec{\Omega}',E',t) = \int_0^\infty dE'\Sigma_s(E' \to E)\phi(\vec{r},E',t)$$

Assume one speed, producing the following equation:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) + \nabla\cdot\vec{J} + \Sigma_t(\vec{r},t)\phi(\vec{r},t) = \Sigma_s(\vec{r},t)\phi(\vec{r},E',t) + S$$

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) + \nabla\cdot\vec{J} + \Sigma_a(\vec{r},t)\phi(\vec{r},t) = S$$

Substitute with Fick's Law $\vec{J} = -D\nabla\phi$

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) - \nabla\cdot D\nabla\phi + \Sigma_a(\vec{r},t)\phi(\vec{r},t) = S$$