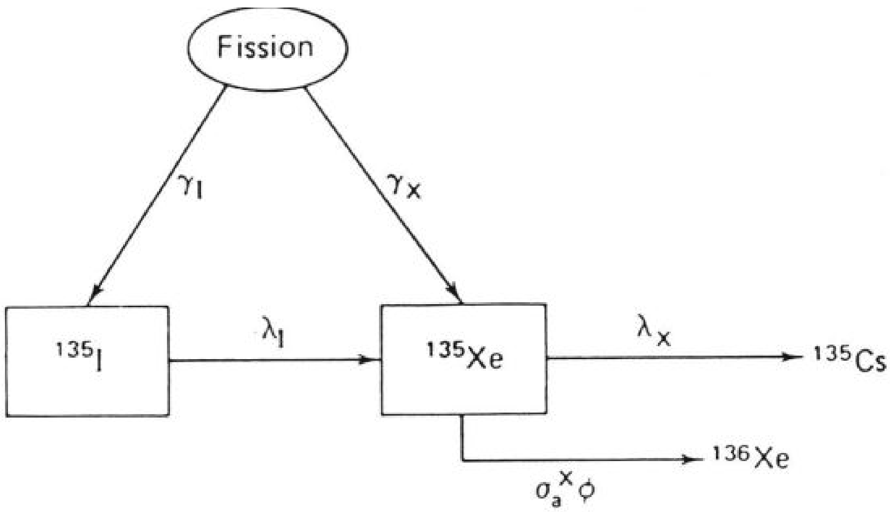
Xenon and Samarium concentration

Xenon

Assume the following decay scheme for Xenon.



A simplified decay scheme for ^{135}Xe.

The following ODEs apply:

$$\frac{\partial I}{\partial t} = \Sigma_f \phi \gamma_1 - \lambda_I I$$

$$\frac{\partial Xe}{\partial t} = \Sigma_f \phi \gamma_2 + \lambda_I I - \sigma_a^X \phi Xe - \lambda_X Xe$$

At equilibrium,$\frac{\partial Xe}{\partial t} = \frac{\partial I}{\partial t} = 0$, the iodine and xenon concentrations would be:

$$0 = \Sigma_f \phi \gamma_1 - \lambda_I I_\infty$$

$$I_\infty = \frac{\Sigma_f \phi \gamma_1}{\lambda_I}$$

$$0 = \Sigma_f \phi \gamma_2 + \lambda_I I_\infty - \sigma_a^X \phi Xe_\infty - \lambda_X Xe_\infty$$

$$\sigma_a^X \phi Xe_\infty + \lambda_X Xe_\infty = \Sigma_f \phi \gamma_2 + \lambda_I I_\infty$$

$$Xe_\infty = \frac{\Sigma_f \phi \gamma_2 + \lambda_I I_\infty}{\sigma_a^X \phi + \lambda_X} = \frac{\Sigma_f \phi \gamma_2 + \Sigma_f \phi \gamma_1}{\sigma_a^X \phi + \lambda_X}$$

$$Xe_\infty = \frac{\Sigma_f \phi (\gamma_1 + \gamma_2)}{\sigma_a^X \phi + \lambda_X}$$

After shutdown, the iodine concentration is modeled with the following ODE.

$$\frac{\partial I}{\partial t} = -\lambda_I I$$
$$\frac{1}{I}\partial I = \lambda_I \partial t$$
$$I = I_0 e^{-\lambda_I t}$$

After shutdown, the xenon concentration is modeled with the following ODE.

$$\frac{\partial Xe}{\partial t} = \lambda_I I - \lambda_X Xe$$
$$\frac{\partial Xe}{\partial t} + \lambda_X Xe = \lambda_I I$$

Multiply by an integrating factor $e^{\lambda_X t}$

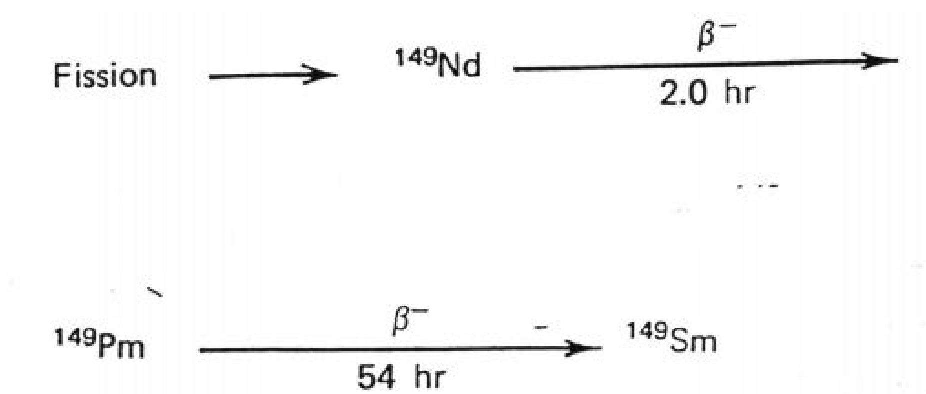
$$\frac{\partial Xe}{\partial t} e^{\lambda_X t} + \lambda_X Xe e^{\lambda_X t} = \lambda_I I e^{\lambda_X t}$$
$$\frac{\partial}{\partial t} Xe e^{\lambda_X t} = \lambda_I I_0 e^{-\lambda_I t} e^{\lambda_X t} = \lambda_I I_0 e^{(\lambda_X - \lambda_I)t}$$

Integrate with respect to t from 0 to t.

$$Xe e^{\lambda_X t} - Xe_0 = \int_0^t \lambda_I I_0 e^{(\lambda_X - \lambda_I)t} dt$$
$$Xe e^{\lambda_X t} - Xe_0 = \frac{\lambda_I I_0}{\lambda_X - \lambda_I}(e^{(\lambda_X - \lambda_I)t} - 1)$$
$$Xe = Xe_0 e^{-\lambda_X t} + \frac{\lambda_I I_0}{\lambda_X - \lambda_I} \frac{1}{e^{\lambda_X t}} (e^{(\lambda_X - \lambda_I)t} - 1)$$
$$Xe = Xe_0 e^{-\lambda_X t} + \frac{\lambda_I I_0}{\lambda_X - \lambda_I}(e^{-\lambda_I t} - e^{-\lambda_X t})$$

Samarium

Assume the following decay scheme for Samarium.



The following ODEs apply:

$$\frac{\partial Pm}{\partial t} = \Sigma_f \phi \gamma_{Pm} - \lambda_{Pm} Pm$$
$$\frac{\partial Sm}{\partial t} = \lambda_{Pm} Pm - \sigma_a^s Sm \phi$$

Assume constant flux and solve for Pm.

$$\frac{\partial Pm}{\partial t} + \lambda_{Pm} Pm = \Sigma_f \phi \gamma_{Pm}$$

Multiply by an integrating factor $e^{\lambda_{Pm} t}$.

$$\frac{\partial Pm}{\partial t} e^{\lambda_{Pm} t} + \lambda_{Pm} Pm e^{\lambda_{Pm} t} = \frac{\partial}{\partial t} Pm e^{\lambda_{Pm} t} = \Sigma_f \phi \gamma_{Pm} e^{\lambda_{Pm} t}$$

Integrate with respect to t from 0 to t.

$$Pm(t) e^{\lambda_{Pm} t} - Pm_0 = \int_0^t \Sigma_f \phi \gamma_{Pm} e^{\lambda_{Pm} t} dt$$
$$Pm(t) e^{\lambda_{Pm} t} - Pm_0 = \frac{\Sigma_f \phi \gamma_{Pm}}{\lambda_{Pm}} (e^{\lambda_{Pm} t} - 1)$$
$$Pm(t) = Pm_0 e^{-\lambda_{Pm} t} + \frac{\Sigma_f \phi \gamma_{Pm}}{\lambda_{Pm}} (1 - e^{-\lambda_{Pm} t})$$

Now consider shutdown after steady-state operation long enough to achieve equilibrium. At equilibrium, $\frac{\partial Pm}{\partial t} = \frac{\partial Sm}{\partial t} = 0$, the concentrations would be:

$$0 = \Sigma_f \phi \gamma_{Pm} - \lambda_{Pm} Pm_\infty$$
$$Pm_\infty = \frac{\Sigma_f \phi \gamma_{Pm}}{\lambda_{Pm}}$$
$$0 = \lambda_{Pm} Pm_\infty - \sigma_a^s Sm_\infty \phi$$
$$Sm_\infty = \frac{\lambda_{Pm} Pm_\infty}{\sigma_a^s \phi}$$
$$Sm_\infty = \frac{\lambda_{Pm}}{\sigma_a^s \phi} \frac{\Sigma_f \phi \gamma_{Pm}}{\lambda_{Pm}}$$
$$Sm_\infty = \frac{\Sigma_f \gamma_{Pm}}{\sigma_a^s}$$

After shutdown:

$$\frac{\partial Pm}{\partial t} = -\lambda_{Pm} Pm$$
$$Pm = Pm_\infty e^{-\lambda_{Pm} Pm}$$
$$\frac{\partial Sm}{\partial t} = \lambda_{Pm} Pm$$
$$\frac{\partial Sm}{\partial t} = \lambda_{Pm} Pm_\infty e^{-\lambda_{Pm} Pm}$$

Integrate with respect to t from 0 to t.

$$Sm(t) - Sm_\infty = \int_0^t \lambda_{Pm} Pm_\infty e^{-\lambda_{Pm} Pm} dt = \frac{\lambda_{Pm} Pm_\infty}{-\lambda_{Pm}} (e^{-\lambda_{Pm} Pm} - 1)$$
$$Sm(t) = Sm_\infty + Pm_\infty (1 - e^{-\lambda_{Pm} Pm})$$

Equilibrium Reactivity Worth: Xenon

$$\Delta\rho = \frac{-\Sigma_a}{\nu\Sigma_f} = \frac{-\sigma_a}{\nu\Sigma_f}Xe_\infty = \frac{-\sigma_a}{\nu\Sigma_f}\frac{\Sigma_f\phi(\gamma_1+\gamma_2)}{\lambda_{Xe}+\sigma_a^X e\phi} = \frac{-\sigma_a\phi(\gamma_1+\gamma_2)}{\nu(\lambda_{Xe}+\sigma_a^{Xe}\phi)}$$

$$\Delta\rho = \frac{-\phi(\gamma_1+\gamma_2)}{\nu(\frac{\lambda_{Xe}}{\sigma_a^{Xe}}+\phi)}$$

For large fluxes $\phi >> \frac{\lambda_{Xe}}{\sigma_a^{Xe}}$

$$\Delta\rho = \frac{-\phi(\gamma_1+\gamma_2)}{\nu\phi)} = \frac{-(\gamma_1+\gamma_2)}{\nu)} \approx -0.026$$

Equilibrium Reactivity Worth: Samarium

$$\Delta\rho = \frac{-\Sigma_a}{\nu\Sigma_f} = \frac{-\sigma_a}{\nu\Sigma_f}Sm_\infty$$

From earlier $Sm_\infty = \frac{\Sigma_f\gamma_{Pm}}{\sigma_a}$

$$\Delta\rho = \frac{-\Sigma_a}{\nu\Sigma_f} = \frac{-\sigma_a}{\nu\Sigma_f}\frac{\Sigma_f\gamma_{Pm}}{\sigma_a} = \frac{-\gamma_{Pm}}{\nu} \approx 0.00463$$