

A-Priori Reduced Order Modeling for Transient Neutron Diffusion

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1. Introduction

High fidelity simulations in nuclear reactor physics inherently lie in high-parameter spaces and commonly suffer from the “curse of dimensionality”. For example, consider the solution of a 3D transient neutron transport simulation. If a spatial mesh of one million elements is used in conjunction with 28 energy groups, S8 quadrature in angle, and 1000 time steps, the following problem has over 2×10^{12} degrees of freedom. Current methods used to overcome this computational expense include lower-order moments-based angular treatments, fewer-group energy discretizations, and spatial homogenization / upscaling [2]. However, the effect of errors and uncertainties associated with these approaches increase when they are coupled to other physical systems (i.e. fluid dynamics, structural mechanics, material models, etc) [5]. The combination of high parametric space and increasing need for high-fidelity calculations has led to a developing interest in applications of contemporary reduced order modeling methods [1,6].

Several methods have been explored in other fields of science and engineering [3]. However, these methods are primarily data driven and require previous knowledge of a system of interest either through experiments or simulation. As previously illustrated, many problems in reactor physics are of high dimensionality and obtaining high quality experimental or simulation data can be difficult and costly. In an attempt to overcome this difficulty, this work approaches reduced order modeling a-priori, i.e. without any prior data of the system of interest.

2. Methodology

Proper Generalized Decomposition (PGD) is an a priori decomposition based method that seeks a solution that is represented through a series expansion whose components are products of separable functions in each dimension of interest [4]. The exponential nature of the “curse of dimensionality” observed in more classical methods is overcome by a linear relationship.

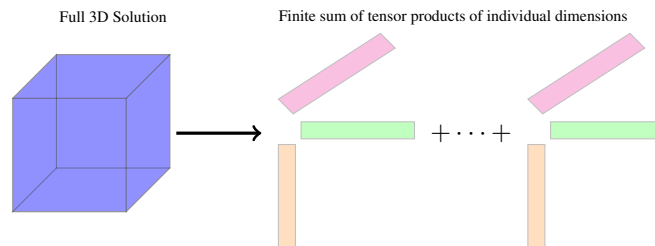


Figure 1: 3D visual example of PGD.

The essential mechanics of PGD are as follows. Consider a one-dimensional transient neutron diffusion problem decomposed in space and time:

$$\phi_n(x, t) = \sum_{i=1}^n X_i(x)T_i(t) \quad (1)$$

Equation 1 is then inserted into a variational formulation of the governing differential equation and a process is developed that results in a progressive construction of the PGD solution. This progression is known as “enrichment” and allows for the accuracy of the PGD ROM to be assessed on-the-fly. This process can be summarized in two steps: 1) Assuming a solution for $T_{n+1}(t)$, solve a 1D boundary value problem for $X_{n+1}(x)$; 2) Using the most recent $X_{n+1}(x)$, solve a 1D initial value problem for $T_{n+1}(t)$. This process is nonlinear in nature and is solved via an alternating direction algorithm.

3. Results

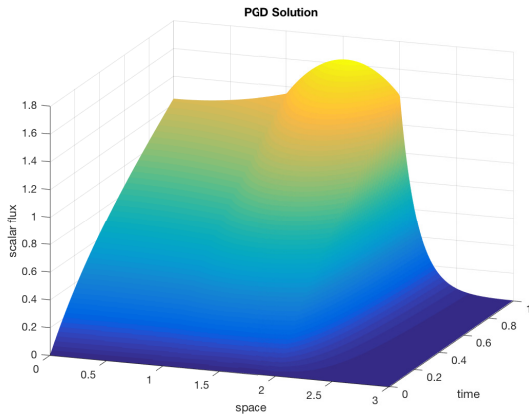
Consider the following heterogeneous, 1D transient neutron diffusion problem defined on $\Omega_x \in [0, 3]$ and $\Omega_t \in [0, 1]$. The BVP in space is solved via continuous Galerkin finite elements (CFEM) while the IVP in time is solved via discontinuous Galerkin finite elements (DFEM). Both discretizations utilize piecewise polynomials.

$$\frac{1}{v} \frac{\partial \phi}{\partial t} - \nabla \cdot D \nabla \phi + \Sigma_a(x) \phi(x, t) = S \quad (2)$$

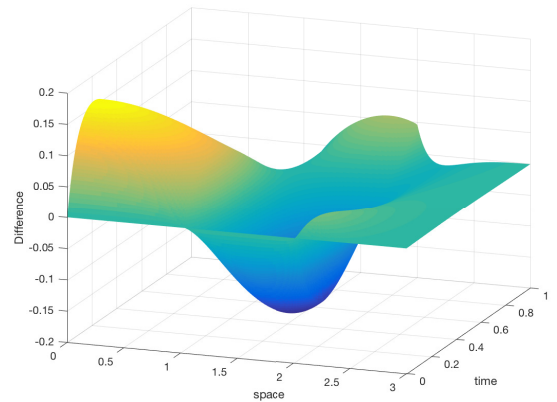
Table I: Cross sections for heterogeneous media problem.

Region	D	Σ_a	Source
$0 \leq x < 1$	5.0	0.001	0.0
$1 \leq x < 2$	1.0	0.5	3.0
$2 \leq x < 3$	0.1	3.5	0.0

Figures 2a and 2b show the solution and deviation of the PGD approximation, respectively. The reference solution for this work was obtained from a discretization of Equation 2 using CFEM in space and Backward Euler in time.



(a) PGD approximated solution.



(b) $\phi_{pgd}(x, t) - \phi_{ref}(x, t)$

From results shown, Proper Generalized Decomposition has the potential to become a viable reduced order modeling technique. The reference solution has 256×256 degrees of freedom while the PGD ROM has $n(256 + 512)$ degrees of freedom.

Figures 2a and 2b are taken when $n = 15$. This produces a $\|\phi_{ref}(x, t) - \phi_{pgd}(x, t)\|_{L_2}$ difference of 0.0913. However, by reducing the number of enrichment steps, one can even further reduce the number of degrees of freedom. For the above simulation, an L_2 difference of 0.1167 can be obtained in one enrichment step, i.e. $n = 1$. This leads to a reduction in degrees of freedom by nearly two orders of magnitudes.

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