Numerical solution to the PRKEs

Start: PRKEs. Assume: $\sum_{i=1}^{6} \lambda_i C_i = \text{constant}$ and $\int_0^t n(t) = \frac{n(t)+n(0)}{2}$

Start with the neutron concentration equation.

$$\frac{\partial}{\partial t} n(t) = \frac{\rho - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{\partial}{\partial t} n(t) - \frac{\rho - \beta}{\Lambda} n(t) = \sum_{i=1}^{6} \lambda_i C_i(t)$$

Define $A = \frac{\rho-\beta}{\Lambda}$ and multiply by an integrating factor e^{-At}

$$\frac{\partial n(t)}{\partial t} e^{-At} - An(t)e^{-At} = \sum_{i=1}^{6} \lambda_i C_i(t) e^{-At}$$

$$\frac{\partial}{\partial t} \left[n(t) e^{-At} \right] = \sum_{i=1}^{6} \lambda_i C_i(t) e^{-At}$$

Integrate with respect to t from 0 to t.

$$n(t)e^{-At} - n(0) = \int_0^t \sum_{i=1}^{6} \lambda_i C_i(t) e^{-At}$$

Assume $\sum_{i=1}^{6} \lambda_i C_i = \text{constant}$

$$n(t)e^{-At} - n(0) = \sum_{i=1}^{6} \lambda_i C_i(t) \int_0^t e^{-At} dt$$

$$n(t)e^{-At} - n(0) = \sum_{i=1}^{6} \lambda_i C_i(t) \frac{1}{-A} (e^{-At} - 1)$$

$$n(t) = n(0)e^{At} + \sum_{i=1}^{6} \lambda_i C_i(t) \frac{1}{-A} (1 - e^{At})$$

$$n(t) = n(0)e^{At} + \sum_{i=1}^{6} \lambda_i C_i(t) \frac{1}{A} (e^{At} - 1)$$

Apply same treatment to delayed neutron precursor equation.

$$\frac{\partial}{\partial t} C_i(t) = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad i = 1...6$$

$$\frac{\partial}{\partial t} C_i(t) + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda} n(t)$$

Multiply by an integrating factor of $e^{\lambda t}$

$$\frac{\partial C_i(t)}{\partial t}e^{\lambda_i t} + \lambda_i C_i(t)e^{\lambda_i t} = \frac{\beta_i}{\Lambda}n(t)e^{\lambda_i t}$$

$$\frac{\partial}{\partial t}\left[C_i(t)e^{\lambda_i t}\right] = \frac{\beta_i}{\Lambda}n(t)e^{\lambda_i t}$$

Integrate with respect to t from 0 to t.

$$C_i(t)e^{\lambda_i t} - C_i(0) = \int_0^t \frac{\beta_i}{\Lambda}n(t)e^{\lambda_i t}dt$$

Assume $\int_0^t n(t) = \frac{n(t)+n(0)}{2}$

$$C_i(t)e^{-\lambda_i t} - C_i(0) = \frac{\beta_i}{\Lambda}\frac{n(t)+n(0)}{2}\int_0^t e^{\lambda_i t}dt$$

$$C_i(t)e^{-\lambda_i t} = C_i(0) + \frac{\beta_i}{\Lambda}\frac{n(t)+n(0)}{2}\frac{1}{\lambda_i}(e^{\lambda_i t} - 1)$$

$$C_i(t) = C_i(0)e^{-\lambda_i t} + \frac{\beta_i}{\Lambda}\frac{n(t)+n(0)}{2}\frac{1}{\lambda_i}(1 - e^{-\lambda_i t}) \quad i = 1...6$$

For the numerical solution, replace t with a time step, and n(t) and n(0) are adjacent time steps.