Point reactor kinetics equations

Start: Diffusion and precursor concentration equations. Main assumption: $\phi(r,t) = vn(t)\psi(r)$

Start with diffusion equation

$$\frac{1}{v}\frac{\partial \phi}{\partial t} - \nabla \cdot D\nabla\phi + \Sigma_a\phi = \nu\Sigma_f(1-\beta)\phi + \sum_{i=1}^{6}\lambda_i C_i$$

Assume D is constant in space and $\nabla^2\phi + B_g^2\phi = 0$

$$\frac{1}{v}\frac{\partial \phi}{\partial t} + DB_g^2\phi + \Sigma_a\phi = \nu\Sigma_f(1-\beta)\phi + \sum_{i=1}^{6}\lambda_i C_i$$

Assume separability $\phi(r,t) = vn(t)\psi(r)$ and $C_i(r,t) = C_i(t)\psi(r)$

$$\frac{1}{v}\frac{\partial}{\partial t}vn(t)\psi(r) + DB_g^2 vn(t)\psi(r) + \Sigma_a vn(t)\psi(r) = \nu\Sigma_f(1-\beta)vn(t)\psi(r) + \sum_{i=1}^{6}\lambda_i C_i(t)\psi(r)$$

Divide through by spatial dependence and simplify.

$$\frac{\partial}{\partial t}n(t) = v[-DB_g^2 - \Sigma_a + \nu\Sigma_f(1-\beta)]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

Pull out Σ_a. $\frac{D}{\Sigma_a} = L^2$.

$$\frac{\partial}{\partial t}n(t) = -v\Sigma_a[L^2B_g^2 + 1 - \frac{\nu\Sigma_f}{\Sigma_a}(1-\beta)]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

The probability of non-leakage is $P_{NL} = \frac{1}{L^2B_g^2+1}$

$$\frac{\partial}{\partial t}n(t) = -v\Sigma_a[\frac{1}{P_{NL}} - \frac{\nu\Sigma_f}{\Sigma_a}(1-\beta)]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

$$\frac{\partial}{\partial t}n(t) = -\frac{v\Sigma_a}{P_{NL}}[1 - P_{NL}\frac{\nu\Sigma_f}{\Sigma_a}(1-\beta)]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

The prompt neutron lifetime is $\ell = \frac{P_{NL}}{v\Sigma_a}$

$$\frac{\partial}{\partial t}n(t) = -\frac{1}{\ell}[1 - P_{NL}\frac{\nu\Sigma_f}{\Sigma_a}(1-\beta)]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

$$\frac{\partial}{\partial t}n(t) = \frac{1}{\ell}[P_{NL}\frac{\nu\Sigma_f}{\Sigma_a}(1-\beta) - 1]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

The multiplication factor is $k = P_{NL}\frac{\nu\Sigma_f}{\Sigma_a}$

$$\frac{\partial}{\partial t}n(t) = \frac{1}{\ell}[k(1-\beta)-1]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

Prompt neutron lifetime is $\ell = \frac{1}{k\Lambda}$

$$\frac{\partial}{\partial t}n(t) = \frac{1}{k\Lambda}[k-k\beta)-1]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

$$\frac{\partial}{\partial t}n(t) = \frac{1}{\Lambda}[\frac{k-1}{k} - \frac{k\beta}{k}]n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

Reactivity is $\rho = \frac{k-1}{k}$. The following is the first equation of the PRKEs.

$$\frac{\partial}{\partial t}n(t) = \frac{\rho-\beta}{\Lambda}n(t) + \sum_{i=1}^{6}\lambda_i C_i(t)$$

Now, consider the delayed neutron precursor equation.

$$\frac{\partial C_i}{\partial t} = \nu\Sigma_f\beta_i\phi - \lambda_i C_i \quad i = 1...6$$

Assume separability $\phi(r,t) = vn(t)\psi(r)$ and $C_i(r,t) = C_i(t)\psi(r)$

$$\frac{\partial}{\partial t}C_i(t)\psi(r) = \nu\Sigma_f\beta_i vn(t)\psi(r) - \lambda_i C_i(t)\psi(r)$$

$$\frac{\partial}{\partial t}C_i(t) = \nu\Sigma_f\beta_i vn(t) - \lambda_i C_i(t)$$

Consider $\ell = \frac{P_{NL}}{v\Sigma_a}$ from before. Neutron velocity is $v = \frac{P_{NL}}{\ell\Sigma_a}$

$$\frac{\partial}{\partial t}C_i(t) = \frac{\beta_i}{\ell}P_{NL}\frac{\nu\Sigma_f}{\Sigma_a}n(t) - \lambda_i C_i(t)$$

$$\frac{\partial}{\partial t}C_i(t) = \frac{\beta_i}{\ell}kn(t) - \lambda_i C_i(t)$$

Again, $\ell = \frac{1}{k\Lambda}$.

$$\frac{\partial}{\partial t}C_i(t) = \frac{\beta_i}{k\Lambda}kn(t) - \lambda_i C_i(t)$$

$$\frac{\partial}{\partial t}C_i(t) = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i(t) \quad i = 1...6$$

Above is the second equation of the PRKEs.