Prompt jump

Start: PRKE. Assume: $\frac{\partial C(0)}{\partial t} = 0$ and a single precursor group.

Start with the delayed neutron precursor concentration equation at $t = 0$.

$$\frac{\partial}{\partial t}C(0) = \frac{\beta}{\Lambda}n(0) - \lambda C(0)$$

Assume $\frac{\partial C(0)}{\partial t} = 0$

$$0 = \frac{\beta}{\Lambda}n(0) - \lambda C(0)$$

$$C(0) = \frac{\beta}{\lambda\Lambda}n(0)$$

Consider the neutron concentration equation and insert $C(0) = \frac{\beta}{\lambda\Lambda}n(0)$

$$\frac{\partial}{\partial t}n(t) = \frac{\rho - \beta}{\Lambda}n(t) + \lambda C(0)$$

$$\frac{\partial}{\partial t}n(t) - \frac{\rho - \beta}{\Lambda}n(t) = \frac{\beta}{\Lambda}n(0)$$

Multiply by an integrating factor $e^{-\alpha t}$ where $\alpha = \frac{\rho-\beta}{\Lambda}$

$$\frac{\partial n(t)}{\partial t}e^{-\alpha t} - \alpha n(t)e^{-\alpha t} = \frac{\beta}{\Lambda}n(0)e^{-\alpha t}$$

$$\frac{\partial}{\partial t}n(t)e^{-\alpha t} = \frac{\beta}{\Lambda}n(0)e^{-\alpha t}$$

Integrate with respect to t from 0 to t.

$$\int_0^t \frac{\partial}{\partial t}n(t)e^{-\alpha t} = \int_0^t \frac{\beta}{\Lambda}n(0)e^{-\alpha t}$$

$$n(t)e^{-\alpha t} - n(0) = -\frac{\beta}{\alpha\Lambda}n(0)e^{-\alpha t} + \frac{\beta}{\alpha\Lambda}n(0)$$

Divide by $n(0)$ and $e^{-\alpha t}$

$$\frac{n(t)}{n(0)}e^{-\alpha t} = \frac{\beta}{\alpha\Lambda}(1 - e^{-\alpha t}) + 1$$

$$\frac{n(t)}{n(0)} = \frac{\beta}{\alpha\Lambda}(e^{\alpha t} - 1) + e^{\alpha t}$$

$$\frac{n(t)}{n(0)} = -\frac{\beta}{\alpha\Lambda} + e^{\alpha t}(1 + \frac{\beta}{\alpha\Lambda})$$

Recall $\alpha = \frac{\rho-\beta}{\Lambda}$

$$\frac{n(t)}{n(0)} = -\frac{\beta}{\Lambda}\frac{\Lambda}{\rho - \beta} + e^{\alpha t}(1 + \frac{\beta}{\Lambda}\frac{\Lambda}{\rho - \beta})$$

$$\frac{n(t)}{n(0)} = \frac{\beta}{\beta - \rho} + e^{\alpha t}(1 - \frac{\beta}{\beta - \rho})$$

Assume $\beta >> \rho$

$$\frac{n(t)}{n(0)} = \frac{\beta}{\beta - \rho} = \frac{1}{1 - \$}$$

Above is the in-hour equation. If the system is not initially critical, the following equation applies.

$$\frac{n_2}{n_1} = \frac{\beta - \rho_1}{\beta - \rho_2} = \frac{1 - \$_1}{1 - \$_2}$$