College of Engineering: School of Nuclear Science and Engineering

PGD IN NUCLEAR REACTOR PHYSICS

An Investigation of A Priori Model Reduction for Nuclear Reactor Physics Calculations

Proper Generalized Decomposition

- Assume separable solution
- Flexibility -> can be physical dimensions, material constants, boundary and initial conditions, etc

$$u(x_1, x_2, \ldots, x_D) = \sum_{i=1}^{n} \prod_{k=1}^{D} X_{i,k}(x_k)$$

- ***"Curse of dimensionality" is overcome by linear relationship!***

Test Problems: Transient Neutron Diffusion

- Assume separated solution in space and time.

$$\phi(x,t) \approx \sum_{j=1}^{n} X_j(x) T_j(t) = \sum_{j=1}^{n-1} X_j(x) T_j(t) + \mathcal{X}(x)\mathcal{T}(t)$$

- Solution is enriched by constructing a nonlinear problem.
- Invoke variational formulation, insert assumed separated solution, and integrate over respective dimensions resulting in a 1D BVP and 1D IVP.

BVP – CFEM

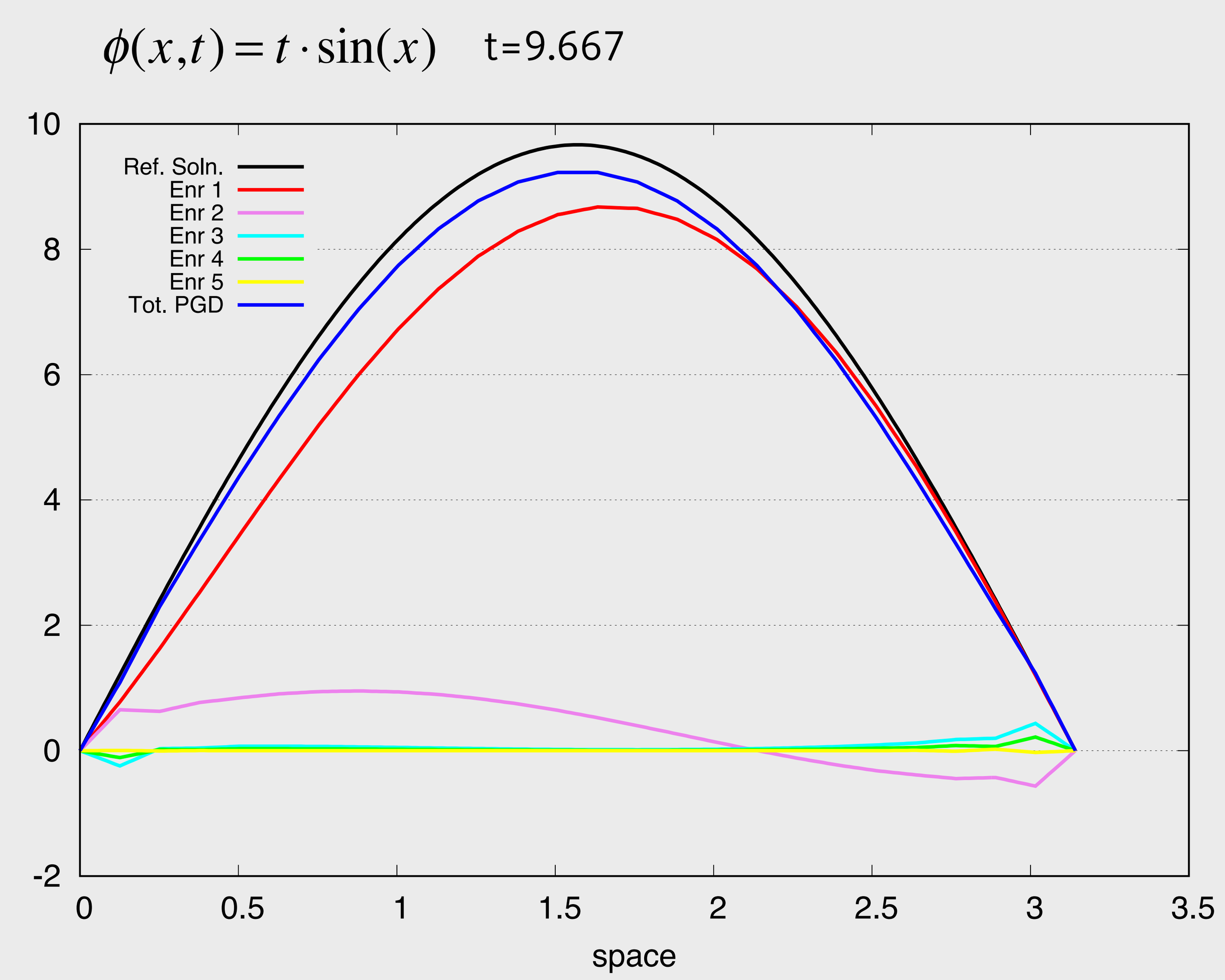
$$\int_{\Omega_x} (t_1 + t_2)\, u\mathcal{X}(x) dx + \int_{\Omega_x} t_1 \frac{du}{dx}\frac{d\mathcal{X}}{dx} dx$$
$$= -\sum_{j=1}^{n}\left[\int_{\Omega_x} uX_i(x)\,(t_{3,i} + t_{4,i})\, dx + \int_{\Omega_x} t_{3,i}\left(\frac{du}{dx}\frac{dX_i}{dx} dx\right)\right] + S_x$$

Complications with Neutron Transport

- Consider transient neutron transport equation:

$$\frac{1}{v(E)}\frac{\partial \psi}{\partial t} + \mathbf{\Omega}\cdot\nabla\psi + \Sigma_t(\mathbf{r},E)\psi(\mathbf{r},E,\mathbf{\Omega},t)$$
$$= \int_{4\pi} d\mathbf{\Omega}' \int_0^\infty \Sigma_s(\mathbf{r}, E' \to E, \mathbf{\Omega}' \to \mathbf{\Omega}, t)\, \psi(\mathbf{r}, E', \mathbf{\Omega}', t) dE' + S$$

- 7 degrees of freedom quickly lead to intractability!
- Simplifying assumptions required



IVP – DFEM

$$\int_{\Omega_t} x_1 \left(w\mathcal{T}(t)\big|_{bndry} - \int_{\Omega_t} \mathcal{T}(t)\frac{dw}{dt} dt\right) + \int_{\Omega_t} (x_2 + x_3)\, w\mathcal{T}(t) dt$$
$$= -\sum_{j=1}^{n}\left[\int_{\Omega_t} \frac{1}{v} x_{4,i} w \frac{dT_i}{dt} dt + \int_{\Omega_t} (x_{5,i} - x_{6,i})\, wT_i(t) dt\right] + S_t$$

Monoenergetic Transient Neutron Diffusion

- Remove spatial dependence in neutron transport by assuming linearly anisotropic scattering – i.e. scattering physics are *weakly* dependent upon angle
- Assume *no energy dependence* in scattering or external sources.

$$\frac{1}{v}\frac{\partial\phi}{\partial t} - \nabla\cdot D(x)\nabla\phi + \Sigma_a\phi(x,t) = S$$

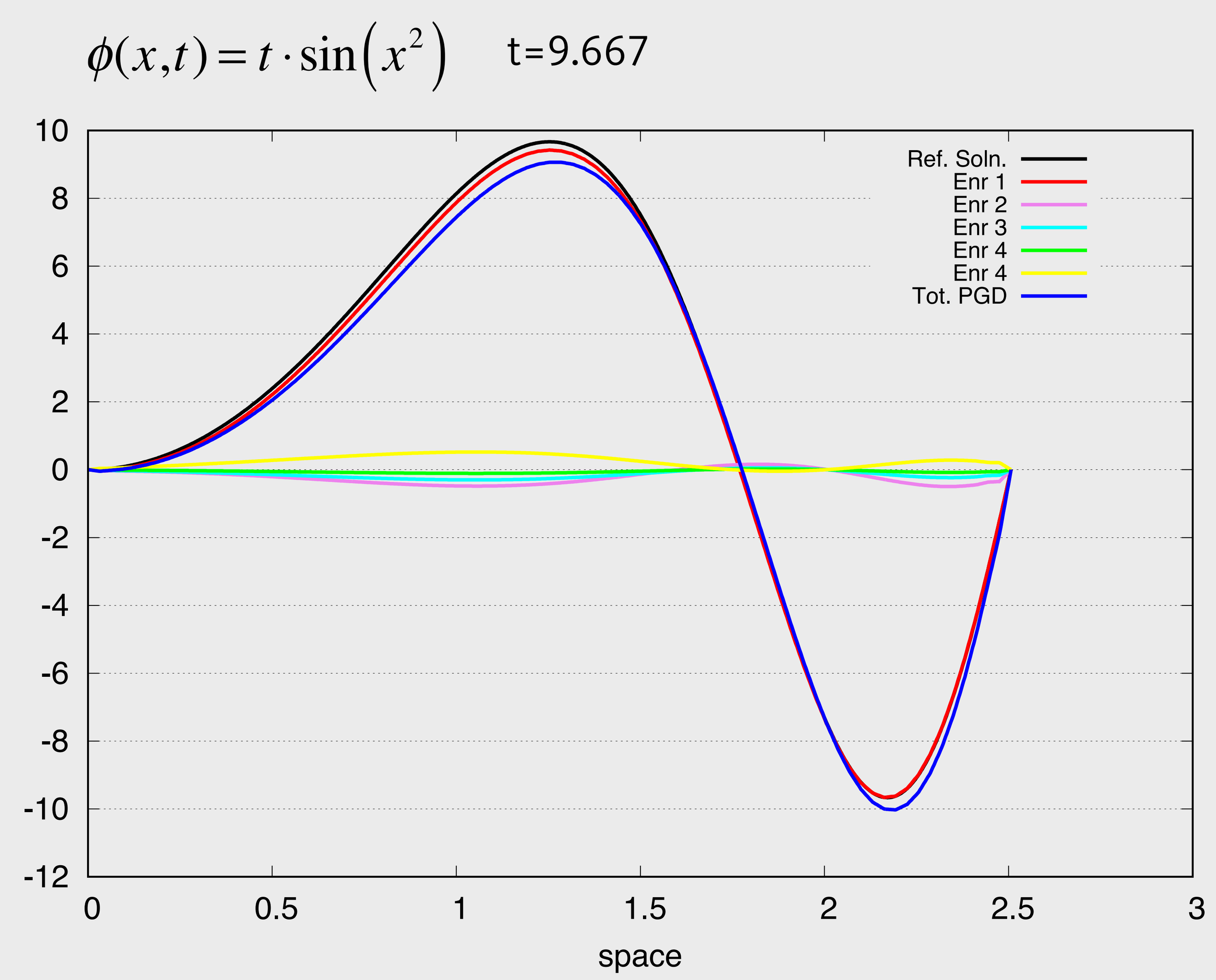


Table 1: L2 norm of difference between reference and approximated solutions.

Enr. Num.	$\phi(x,t) = t\cdot\sin(x)$	$\phi(x,t) = t\cdot\sin(x^2)$
1	3.5022	0.5422
2	2.9955	1.3486
3	2.9460	1.8117
4	2.9410	1.9600
5	2.9405	1.4243
6	2.9401	1.9105

DOE Nuclear Engineering Advanced Modeling and Simulation

- Understanding the operational characteristics and overall safety of the core of a nuclear reactor is arguably one of the most important aspects of any reactor design.
- Goal of program is to develop state-of-the-art computational methods to accelerate the development and deployment of novel nuclear power technologies.

Basic Algorithm

for $m = 1$ to m_{max}
define $X^{(0)}, T^{(0)}$ (initialization)
for $k = 1$ to k_{max}
compute $X^{(k)} = F_m\left(T^{(k-1)}\right)$
compute $T^{(k)} = G_m\left(X^{(k)}\right)$
check convergence of $X^{(k)}T^{(k)}$
end
normalize $X^{(k)} = \frac{X^{(k)}}{\|X^{(k)}\|\|T^{(k)}\|}$ $T^{(k)} = \frac{T^{(k)}}{\|X^{(k)}\|\|T^{(k)}\|}$
set $K^{(k)} = K_m$ and $T^{(k)} = T_m$
set $\phi_m = \phi_{m-1} + K_m T_m$ and check convergence
end

Future Work

- Solution enrichment analysis
  - Why doesn't test #2 improve monotonically?
  - Unrecoverable error in both test cases.
- Nonlinear sources.
- Analysis of higher order polynomials.
- Spatial discontinuities.

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