Technical Project 2

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1 Task 1

In this section as a mixed binomial model, we can consider a static credit portfolio of m obligers who have loans which is modeled by Merton framework. The maturity is one year. Let's assume the followings such that, the individual default probability is $\bar{p}$, the individual loss is denoted by l and the default correlation is ρ. We are going to use the Large Portfolio Approximation (LPA) approach to derive a formula for the $\mathrm{VaR}_\alpha(L)$.

According to the Large Portfolio Approximation for mixed binomial models, we have the following from (Theorem 2.6.1, Lecture notes, page 14.)

$$\mathbb{P}\left[\frac{N_m}{m} \leq x\right] \rightarrow \mathbb{P}\left[p(Z) \leq x\right]$$

As we consider a Merton mixed binomial model, we get from (Lecture notes, Section 2.6.2, Example 3) the following

$$p(Z) = N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$$

According to the approximation of loss distribution and what we get from (2.7.4) we have that

$$F_{L_m}(x) = \mathbb{P}\left[L_m \leq x\right] = \mathbb{P}\left[\ell N_m \leq x\right] = \mathbb{P}\left[\frac{N_m}{m} \leq \frac{x}{\ell m}\right]$$

by using the LPA (Theorem 2.6.1) and as stated above we get that

$$F_{L_m}(x) \rightarrow \mathbb{P}\left[p(Z) \leq \frac{x}{\ell m}\right]$$

From the probability of right-hand side and plugging $p(Z)$ as mentioned above we get

$$\mathbb{P}\left[p(Z) \leq \frac{x}{\ell m}\right] = \mathbb{P}\left[N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right) \leq \frac{x}{\ell m}\right]$$

Now using the normal inverse we get

$$\mathbb{P}\left[p(Z) \leq \frac{x}{\ell m}\right] = \mathbb{P}\left[\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}} \leq N^{-1}\left(\frac{x}{\ell m}\right)\right]$$

Now multiplying the both sides by $\sqrt{1-\rho}$ and trying to extract Z we get that

$$\mathbb{P}\left[p(Z) \le \frac{x}{\ell m}\right] = \mathbb{P}\left[Z \le \frac{N^{-1}\left(\frac{x}{\ell m}\right)\sqrt{1-\rho} - N^{-1}(\bar{p})}{\sqrt{\rho}}\right] \tag{1}$$

According to section (2.7.1) in lecture notes and considering the definition of Value at Risk represented in (2.7.14), we get that

$$\mathbb{P}\left[L \le \mathrm{VaR}_\alpha(L)\right] = \alpha$$

Where $L_m = \ell N_m$ so we get that

$$\mathbb{P}\left[\ell N_m \le \mathrm{VaR}_\alpha(L)\right] = \alpha$$

Hence

$$\mathbb{P}\left[\frac{N_m}{m} \le \frac{\mathrm{VaR}_\alpha(L)}{\ell m}\right] = \alpha$$

Which based on LPA it is equivalent with

$$\mathbb{P}\left[p(Z) \le \frac{\mathrm{VaR}_\alpha(L)}{\ell m}\right] = \alpha$$

According to what we get in (1) we will conclude that

$$\mathbb{P}\left[p(Z) \le \frac{\mathrm{VaR}_\alpha(L)}{\ell m}\right] = N\left(\frac{N^{-1}\left(\frac{\mathrm{VaR}_\alpha(L)}{\ell m}\right)\sqrt{1-\rho} - N^{-1}(\bar{p})}{\sqrt{\rho}}\right)$$

So what we get is as follows

$$\alpha = N\left(\frac{N^{-1}\left(\frac{\mathrm{VaR}_\alpha(L)}{\ell m}\right)\sqrt{1-\rho} - N^{-1}(\bar{p})}{\sqrt{\rho}}\right)$$

Now we need to solve the equation to sort out the $\mathrm{VaR}_\alpha(L)$.

First we use the normal inverse to get the following

$$N^{-1}(\alpha) = \left(\frac{N^{-1}\left(\frac{\mathrm{VaR}_\alpha(L)}{\ell m}\right)\sqrt{1-\rho} - N^{-1}(\bar{p})}{\sqrt{\rho}}\right)$$

Multiplying the both side by $\sqrt{\rho}$ we get that

$$N^{-1}(\alpha)\sqrt{\rho} = N^{-1}\left(\frac{\mathrm{VaR}_\alpha(\mathrm{L})}{\ell m}\right)\sqrt{1-\rho} - N^{-1}(\bar{p})$$

Hence

$$N^{-1}\left(\frac{\mathrm{VaR}_\alpha(\mathrm{L})}{\ell m}\right) = \frac{N^{-1}(\alpha)\sqrt{\rho} + N^{-1}(\bar{p})}{\sqrt{1-\rho}}$$

Now by having a norm from the both sides and then multiplying by $\ell * m$ we will get the required formula for $\mathrm{VaR}_\alpha(L)$.

Remark: From now on, MATLAB code and created functions for each task, can be found on the Appendix. The order is the same as the tasks.

2 Task 2

In this section we continue with the Merton model and consider an interval for the portfolio credit loss as to be more than 70 million SEK but less than 80 million SEK. In this task, we are going to compute the probability of having loss in the chosen interval, by using the LPA-formula.

For any positive a we have the following (Lecture notes, 2.7.1)

$$\mathbb{P}\left[L_m \leq a\right] = \mathbb{P}\left[\ell N_m \leq a\right] = \mathbb{P}\left[\frac{N_m}{m} \leq \frac{a}{\ell m}\right]$$

in the case m is "large" we have the following approximation (Lecture notes, 2.7.5)

$$\mathbb{P}\left[L_m \leq a\right] \approx F\left(\frac{a}{\ell m}\right)$$

Let's recall that

$$\mathbb{P}[a < X \leq b] = F_X(b) - F_X(a)$$

and for a chosen interval we get the following approximation

$$\mathbb{P}\left[a < L_m \leq b\right] \approx F\left(\frac{b}{\ell m}\right) - F\left(\frac{a}{\ell m}\right)$$

In our case we assumed that $a = 70m$ SEK and $b = 80m$ SEK.

To compute the results in MATLAB, we are changing one the following parameters each time: $\bar{p}, \rho, m, \ell$ which are the unconditional default probability, the default correlation, the number of obligors and the percentage of individual loss, respectively.

When we have one of them changed, the others remain constant. The constant values are inspired by our lecture notes, exercise sessions and personal intuition. For this task (and basically for the whole project), each loan has notional 1 million SEK.

We will start with our first results: The 2 figures above display the results, from the probability of having a loss from 70 million SEK to 80 million SEK, for different values of $\bar{p}$ and ρ respectively, while the other parameters are kept constant.

It is vital to say that plotting losses against $\bar{p}$ was achieved by keeping $\ell = 0.6 * 10^6, m = 1000, \rho = 0.12$. On the other hand, plotting them for different correlations, achieved by letting $\bar{p} = 0.04$. Using all these constants in MATLAB to compute the loss probability in this interval, the result was very low:

$$\mathbb{P}[70m < L_m \leq 80m] = 0.56\%$$

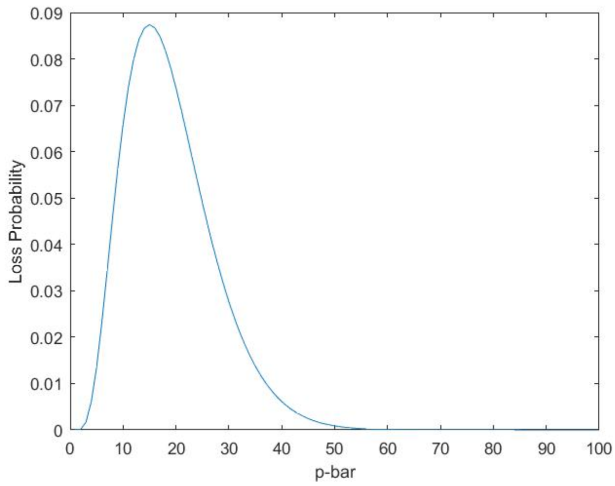

Figure 1: Interval loss Probability for different $\bar{p}$ values.

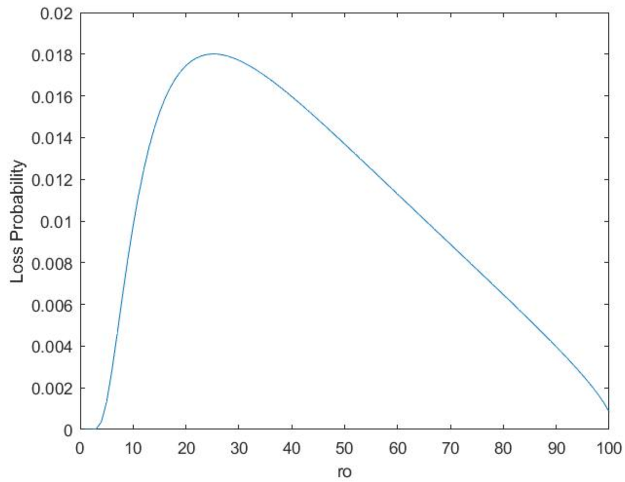

Figure 2: And for different Correlations

To begin with, for Figure 1, it is natural to get values close to zero in the beginning, as the unconditional default probability is very low. As a result, a portfolio loss in our interval is not so likely to happen. Of course, for bigger $\bar{p}$ values, this probability is getting larger. To be more specific, the highest probability of having losses in our interval, is 8.74% and it is achieved when $\bar{p} = 15\%$. Further on the x-axis, a lower interval probability is achieved and that is not strange, as having bigger unconditional default probability probably means losses outside our interval (probably more than 80 million SEK).

With a similar way of thinking we can interpret Figure 2. For this graph, as stated above, we chose $\bar{p} = 0.04$ in order to get results. When the obligors are perfectly correlated, meaning that default the correlation in getting closer to 1, we can see from (2.11.24) on the lecture notes, that the distribution of losses now, depends almost only on the $\bar{p}$, as the term $\sqrt{1-\rho}N^{-1}(x)$ is almost zero for ρ almost 1. For our interval that means that the difference of the distributions will be almost zero. It is more likely to have losses in our interval when the default correlation is aproximately 24%, but that probability is still low ($\approx 1.8\%$).

In this part we will see the results for different portfolio sizes and different individual loss values. The values of the other variables remain constant.

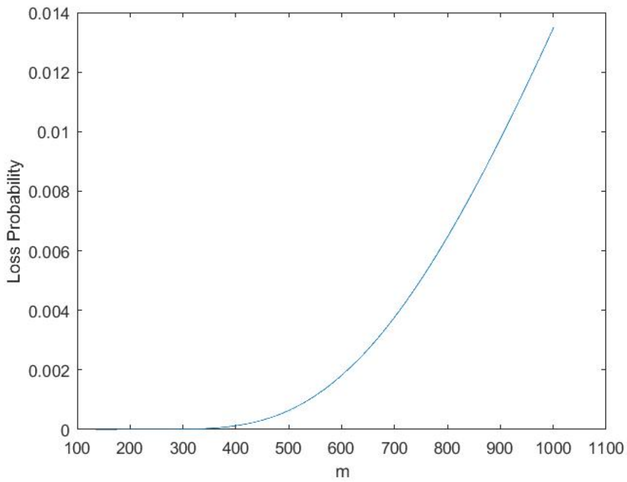

Figure 3: Interval loss Probability for different portfolio size.

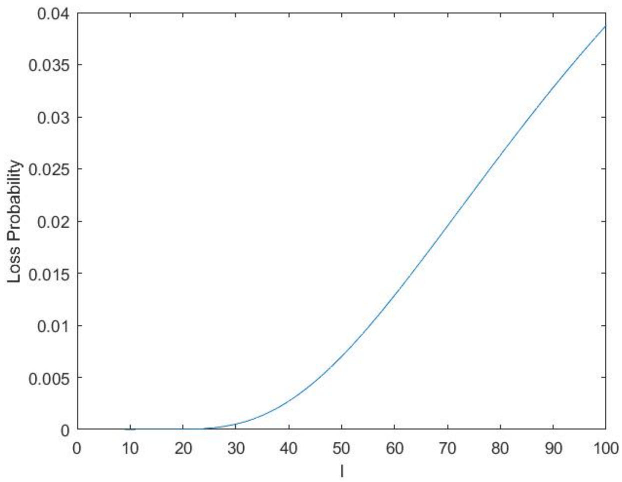

Figure 4: And for different individual loss

On Figure 3 the probability of having the loss in our interval seems to go higher as the portfolio size is growing. But again, this probability is still very low, with the maximum being only 1.4%. We tried another loss interval that gave a higher probability than this. For more obligors, it is more probable to have losses from 20 million SEK up to 30 million SEK with almost 20% which is way higher than our interval's result. The portfolio can do of course worse than that, but just with lower probability. Probabilities are getting extremely low after some point so there is an 'upper limit' on the losses.

As those 2 plots look kind of the same, we also tried different intervals to see what is happening when individual loss is growing, as seen in Figure 4. For the same test interval as before, we got higher probability of lower losses as seen in Figure 5. In other words, again there is almost 19.7% probability of our total portfolio credit loss

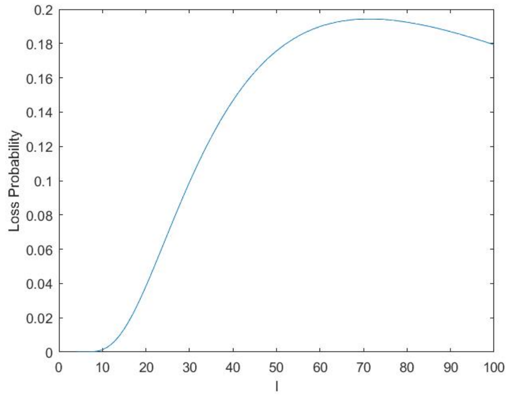

Figure 5: Probability of loosing 20-30million SEK for different l values

to lay in the 20-30million SEK interval and not in ours.

We guess that this is happening due to our selection of values for the constants. 70million-80 million credit loss, may be considered an extreme case for our chosen model and constants, especially when it comes to the selection of $\bar{p}$. Also, our interval may be considered small.

3 Task 3

In this section we are going to use the VaR formula we derived in Task 1 to compute the results for when $\alpha = 0.95$, $\alpha = 0.99$ and $\alpha = 0.999$.

According to the derived formula for Value at Risk in Task 1, we have that

$$\mathrm{VaR}_\alpha(L) = \ell \cdot m \cdot N\left(\frac{N^{-1}(\alpha)\sqrt{\rho} + N^{-1}(\bar{p})}{\sqrt{1-\rho}}\right)$$

and according to the lecture notes (Section 2.8), we have the following formula for the expected shortfall

$$ES_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(L)du$$

3.1 LPA-Value at Risk

To derive results from MATLAB, we again, use the same values for the constants as in Task 2. For every new figure, we are plotting the LPA-Value at Risk by changing one of the 4 variables and keep the others constant.

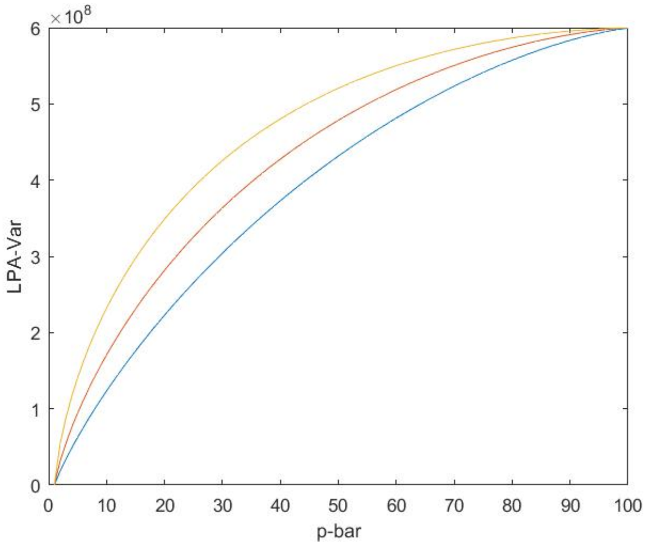

Figure 6: LPA-VaR for different $\bar{p}$ values for 3 different confidence levels

Displayed in Figure 6, one can find the LPA-Value at Risk for three different confidence levels α: 95% (blue), 99% (red) and 99.9%(yellow), taking all the possible values of the individual default probability $\bar{p}$.

In the LPA approach, the Value at Risk does not really depend on the losses and the chosen loss interval, but as it can be seen in (2.11.1.7) on the lecture notes, it depends on confidence level, correlation, $\bar{p}$ and individual loss. A quick test can be seen in Table 1. We used the LPA-Merton formula and kept constant $\ell = 0.6 * 10^6, m = 1000, \rho = 0.12$, $\bar{p} = 0.04$ inspired by the lecture notes and exercise sessions.

α=**95%**	$VaR_{LPA} = 62.4$ m SEK
α=**99%**	$VaR_{LPA} = 94.1$ m SEK
α=**99.9%**	$VaR_{LPA} = 140.5$ m SEK

Table 1: LPA-VaR for different levels by keeping everything else constant

Taking into account the results from Table 1, Figure 6 does not seem so surprising, even if it contains big values for LPA-Value at Risk. Table 1 was derived by $\bar{p} = 4\%$, so imagine letting it grow more and more. For $\bar{p} = 34\%$ there is 5% probability of the loss to exceed 300 million SEK. This amount is growing when we are moving further on the x-axis and demanding larger confidence level. So the higher the $\bar{p}$ and the confidence level, the higher the VaR. There is a 0.1% probability for the losses of the portfolio to exceed 600 million SEK for $\bar{p}$ close to 1!

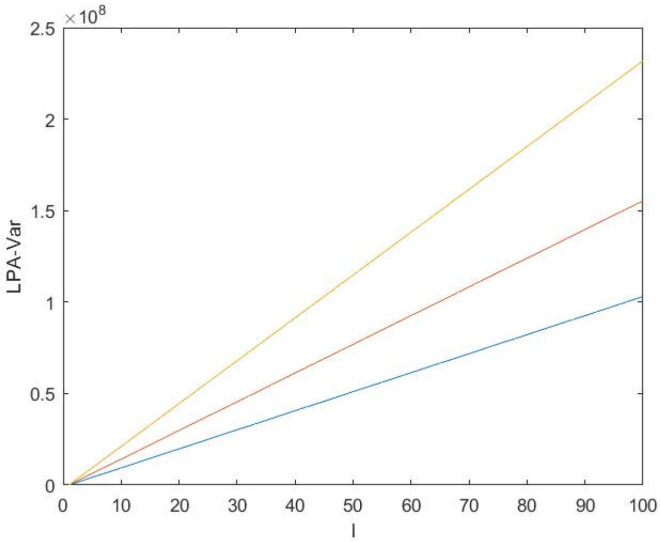

Figure 7: LPA-VaR for different ℓ values for 3 different confidence levels

In Figure 7, the results for different values of the individual loss ℓ are displayed. Keeping everything constant and just changing ℓ, makes the LPA-VaR formula linear in terms of ℓ. A graph like this, was a result we were expecting as we used that formula and linearity was obvious. The difference now is that, we are not obtaining that big values for VaR comparing with Figure 6.

Therefore we can safely say, that higher individual default probability will let more extreme events to happen. Keeping it at 4% as we did to obtain Figure 7, will give $\approx$ 230 million SEK LPA-VaR for 99.9% confidence. It is the most extreme we can get, and comparing it with 600 million from the previous figure, is significantly lower. We believe that $\bar{p}$ plays a very important role in our model.

3.2 LPA-Expected Shortfall

We continue with LPA-Expected Shortfall calculation. Firstly, on the following table we will display some test results, derived by plugging in our chosen constants in the MATLAB function for expected shortfall:

α=**95%**	ES_{LPA} = 82.1 m SEK
α=**99%**	ES_{LPA} = 114.2 m SEK
α=**99.9%**	ES_{LPA} = 160.5 m SEK

Table 2: LPA-ES for different levels by keeping everything else constant

Remember, that expected shortfall, is a measure of 'how bad a situation can get', given that losses exceed Value at Risk. Having that in mind, and taking into account the results from Table 1, expected shortfall values for different confidence intervals are more extreme than VaR values. One can confirm it by just comparing the two tables. This is happening because now we are moving further to the x-axis and obtaining more extreme values, i.e the worst event that can happen.

So, for $\ell = 0.6 * 10^6, m = 1000, \rho = 0.12,\ \bar{p} = 0.04$ we should expect with 5% probability portfolio credit loss of 82.1 million SEK, given that losses exceeded 62.4 million. For more extreme cases, if our losses exceed 140.5 million, we expect 160.5 million credit loss with a very small probability, when the situation is not good for the portfolio. Things could of course, be worse than that, with a different choice of $\bar{p}$, for example.

Figure 8 comes to confirm this, by letting expected shortfall take larger values like $\approx$ 600 million SEK, making it almost the same with the Value at Risk plot for $\bar{p}$, which is an interesting fact. As the confidence level is getting closer to 100% (or 1), the expected shortfall and value at risk coincide. One can think about it as an area and a point, which are almost the same on the limit. That is why we are getting 600 million maximum on the y-axis for Figures 6 and 8.

Now observe that on Figure 9, higher individual loss, allowed access to bigger values of portfolio credit loss. This plot displays the worst case scenarios for the three different confidence levels, so those results were expected.

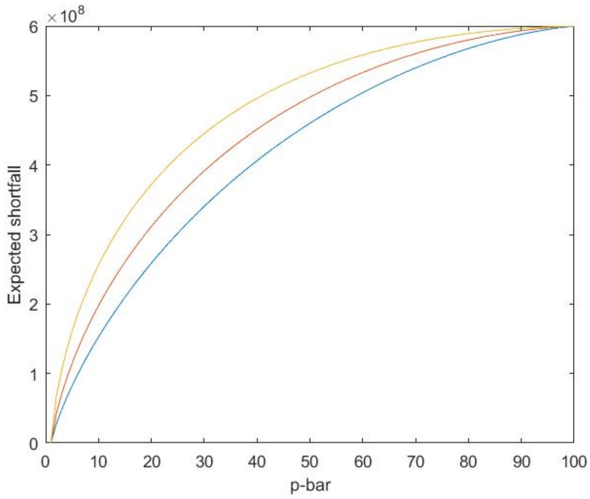

Figure 8: LPA-ES for different $\bar{p}$ values and confidence levels

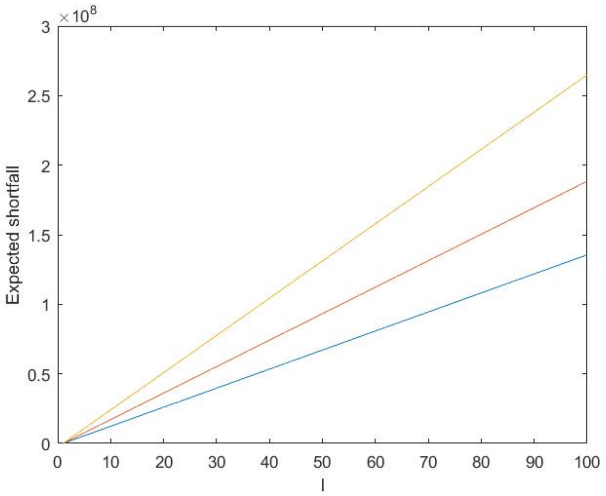

Figure 9: And for different individual loss

4 Task 4

In this task we will continue, by using a mixed binomial model with beta distribution. That means, that the mixing variable $p(Z) = Z$ for example, follows a Beta distribution with parameters a, b which can generate heavy tails.

Our first task, is to compute the LPA-Value at Risk for some suitable parameters. Large Portfolio Approximation comes very handy in this situation as we have to compute:

$$VaR_a(L) = lmF^{-1}(a)$$

,where $F(x)$ is the distribution of $p(Z)$. That is the point of the LPA approach. Value at Risk now depends only on the distribution of the mixing variable. In our case it follows a Beta distribution. The choice of the parameters is up to us. Basically we need to calculate the Value at Risk taking the inverse of the Beta CDF at the point a, which depends of the level of confidence we want.

We will be back at the choice of parameters for the LPA approach in a while. A more demanding part of this task, is calculating the exact Value at Risk, when the mixing variable is Beta distributed. The exact VaR is given by:

$$VaR_a(L) = F_{L_m}^{-1}(a)$$

In other words, the quintile of the Beta distribution evaluated at α. We will use formula (2.9.7) to instead, find the quintile of the 'number of losses distribution':

$$\mathbb{P}[N_m = k] = \binom{m}{k} \frac{\beta(a+k, b+m-k)}{\beta(a, b)}$$

,where k is the number of defaults up to time T, m is the number of obligors in the credit portfolio, a, b are the Beta distribution parameters and $\beta(a, b)$ is given by (2.9.2). For the exact method, it is enough to make our studies for number of defaults being less than or equal to 55, according to the hint. We will do it for k up to 50.

The idea is the following: We are summing up the probabilities of every k from zero to 50, in order to create the CDF. Then, we want an area (CDF value) which is approximately equal with the confidence level $\alpha \in (0, 1)$ of our choice. When this area is achieved, we stop the procedure and take as a result the value of the last k that made our area to be as close as possible to the confidence level. This is a value on the x-axis, so it is basically the quintile of the Beta distribution evaluated at point α.

We used MATLAB for this procedure with the following values:

- $m = 50$ obligors
- $p(Z) \sim \ Beta(2,5)$
- $\alpha = 0.95$

Of course the maximum number of defaults is 50, as $m = 50$. We chose the Beta distribution to have those parameters, because they generate heavier tails. The plot of the specific $Beta(2,5)$ PDF seems to fit better in our analysis.

Remark: We only get integers with this method. As k is the number of defaults.

With those parameters and the method stated above, the exact 95% Value at Risk MATLAB result is:

$$VaR_{0.95}(L) = F_{L_m}^{-1}(0.95) = 30$$

For the LPA-VaR, our intuition was to use $l = 1$ for 50 obligors, because after some tests we were getting results very close to the exact VaR. Therefore:

$$VaR_{0.95_{LPA}}(L) = lmF^{-1}(0.95) = 50F^{-1}(0.95) = 29.0902$$

,where $F^{-1}(0.95)$ is the inverse of the Beta distribution CDF with shape parameters $a = 2, b = 5$ evaluated at $\alpha = 0.95$. We firmly believe that, there has to be a better approximation for bigger number of obligors, as that is basically the idea behind the LPA approach.

Now we are going to see how this difference is affected by the credit portfolio size. We plotted the difference of exact VaR and LPA VaR against number of obligors, as one can see on Figure 10:

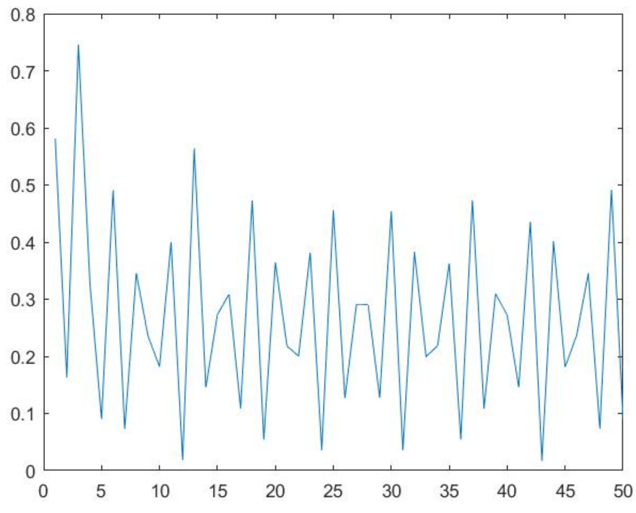

Figure 10: The absolute value of the difference between the 2 methods, against number of obligors

Those fluctuations are the result of our code. Sometimes, the beta distribution quintile is derived by stopping the sum below 0.95 and sometimes above 0.95. It basically selects the value which is closer to 0.95.

Therefore, the difference is also influenced by this fact. Even if this plot seems kind off confusing, we can observe a trend in the picks of the process. It is easy to observe that the difference of the two methods for computing Value at Risk is getting lower for bigger m, i.e there is a trend on the plot. If we had tested it for large m, e.g $m = 1000$ we are pretty sure that the difference would have been be a lot smaller.

Next we are going to find out the impact of the parameters of the Beta distribution for this difference between VaRs. Firstly, we will assume that $p(Z) \sim \ Beta(a, 5)$ and derive results for the difference of LPA and Exact VaR, for different values of α. Next, we are doing the same with $p(Z) \sim \ Beta(2, b)$, changing b this time.

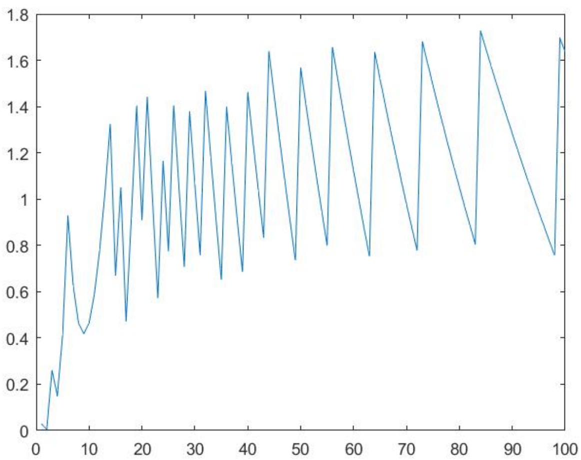

Figure 11: Difference changing a, keeping $b = 5$.

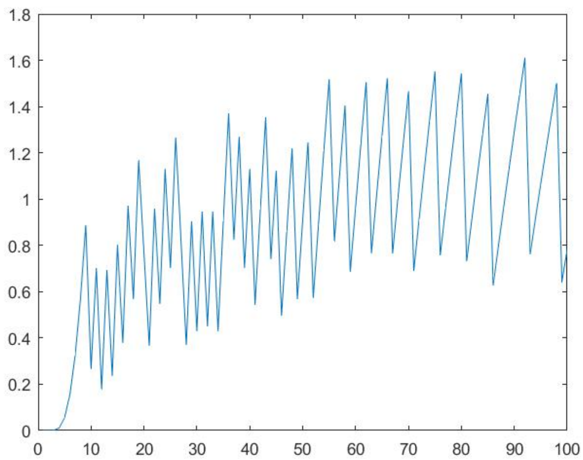

Figure 12: Changing b, keeping $a = 2$.

Figures 11 and 12 look very similar. From the trend of the 2 plots, one can easily see that the difference between LPA and exact VaR is growing. Since LPA is using the inverse of the Beta CDF, our changes in the shape parameters affect of course, our results. Probably, some other combinations between a, b give different tails. Also, having in mind that we are testing it for a credit portfolio of $m = 50$ obligors, so maybe, the LPA approach fails to be close to the exact VaR. With those two points, we cannot skip commenting on the individual loss. We deem that, with different l values, one could get better estimations (also worse) of the exact VaR via the LPA approach, but this can be left for further investigation.

5 Task 5

In this section we are going to briefly explain how to estimate the parameters of the Mixed Binomial Model, when the mixing variable is beta distributed. In other words, by definition: $p(Z) = Z$, where $Z \sim \ Beta(a, b)$.

We are going to use the method of moments, as the first and second moment of $p(Z) = Z$ are enough for all our calculations.

When $Z \sim \ Beta(a, b)$ with $a, b > 0$, it is known to us that:

$$\mathbb{E}(Z) = \frac{a}{a+b} \qquad (1)$$

and

$$\mathbb{E}(Z^2) = \frac{a(a+1)}{(a+b)(a+b+1)} \qquad (2)$$

We can prove this by using the definition of the expectation along with (2.9.1), (2.9.2) and (2.9.4) from the lecture notes. Remember that the definition of the expectation is:

$$\mathbb{E}(Z) = \int_0^1 x f_Z(x) dx$$

and similarly, the definition of the second moment:

$$\mathbb{E}(Z^2) = \int_0^1 x^2 f_Z(x) dx$$

So, using those three properties from the lecture notes we can compute the two integrals. After some calculations, (1) and (2) is our result for the first and second moment respectively.

Of course, having those, means that we are also able to calculate the Variance. Just plug in (1) and (2) in the definition of Variance to get the result:

$$Var(Z) = \frac{ab}{(a+b)^2(a+b+1)}$$

The next step is to estimate the parameters. This will be achieved by solving the following system of equations for a and b:

$$\bar{X} = \frac{a}{a+b} \qquad (3)$$

$$S^2 = \frac{ab}{(a+b)^2(a+b+1)} \qquad (4)$$

In other words, the estimates are found by letting the sample mean $\bar{X}$ and variance S^2, to be equal with the population mean and variance. After that, we solve equations (3) and (4) for a, b in terms of the sample variance and sample mean.

Equation (3) gives:

$$b = \frac{a}{\bar{X}} - a$$

and equation (4) after some calculations and solving for a, and having b plugged in from the previous step:

$$a = \bar{X}\Big(\frac{\bar{X}(1-\bar{X})}{S^2} - 1\Big)$$

Therefore, our b given this a is:

$$b = (1-\bar{X})\Big(\frac{\bar{X}(1-\bar{X})}{S^2} - 1\Big)$$

Our final answer is that our method of moments estimates for the Beta distribution a and b are:

$$\hat{a} = \bar{X}\Big(\frac{\bar{X}(1-\bar{X})}{S^2} - 1\Big)$$

$$\hat{b} = (1-\bar{X})\Big(\frac{\bar{X}(1-\bar{X})}{S^2} - 1\Big)$$

□

6 Appendix

Matlab Code

6.1 Script

Task 2: Loss Probabilities and plots

```
%% Task 2 Interval Probability

[tryingstuff] = probloss(0.4*10^6,1000,80*10^6,0.12,0.5);  %test

m =1:1:10000;
pibarzero = 0;      %unc default probability
rozero = 0;         %correlation
mzero = 0;          %obligers
lzero = 0;          % ind loss

for i=1:length(m)
    if pibarzero <= 1   %loss probability for different pibar
     [problosspibar_t_80] = probloss(0.6*10^6,1000,80*10^6,0.12,pibarzero);
     [problosspibar_t_70] = probloss(0.6*10^6,1000,70*10^6,0.12,pibarzero);
        problosspibar(i) = problosspibar_t_80-problosspibar_t_70;
        pibarzero = pibarzero + 0.01;
    end
    if rozero <=1 %diffrent correlations
        [problossro_t_80] = probloss(0.6*10^6,1000,80*10^6,rozero,0.04);
        [problossro_t_70] = probloss(0.6*10^6,1000,70*10^6,rozero,0.04);
        problossro(i) = problossro_t_80-problossro_t_70;
        rozero = rozero + 0.01;
    end
    if mzero <= 1000  %diffrent port. size
        [probloss_m_t80] = probloss(0.6*10^6,mzero,80*10^6,0.12,0.04);
        [probloss_m_t70] = probloss(0.6*10^6,mzero,70*10^6,0.12,0.04);
        probloss_m(i) = probloss_m_t80-probloss_m_t70;
        mzero = mzero +1;
    end

    if lzero <= 1       %different 'l'
        [probloss_l_t80] = probloss(lzero*10^6,1000,80*10^6,0.12,0.04);
        [probloss_l_t70] = probloss(lzero*10^6,1000,70*10^6,0.12,0.04);
        probloss_l(i) = probloss_l_t80-probloss_l_t70;
        lzero = lzero +0.01;
    end

end

figure
plot(problosspibar)
xlabel('p-bar')        %loss probability for different pibar
ylabel('Loss Probability')

figure
plot(problossro)
xlabel('ro')                    %diffrent correlations
ylabel('Loss Probability')

figure
plot(probloss_m)        %diffrent port. size
xlabel('m')
ylabel('Loss Probability')

figure
plot(probloss_l)
```

```
xlabel('l')                    %different 'l'
ylabel('Loss Probability')
%%
```

Task 3: LPA-VaR for different parameters

```
%% Task 3 LPA-VaR

level= [0.95,0.99,0.999];
m =1:1:10000;
pibarzero = 0;
lzero = 0;
for i=1:length(m)
    for t=1:length(level)
        if pibarzero <= 1 %lpa var for different pibar
          [varLPA_pibar] = MertVaR(0.6*10^6,1000,level(t),0.12,pibarzero);
                varpibar(i,t) = varLPA_pibar;

        end
            if lzero <= 1 %different 'l'
                [varLPA_l] = MertVaR(lzero*10^6,1000,level(t),0.12,0.04);
                var_l(i,t) = varLPA_l;
            end
    end
   pibarzero = pibarzero + 0.01;
   lzero = lzero +0.01;
end
testing_var = MertVaR(0.6*10^6,1000,0.99,0.12,0.04); %specific param. test

figure              %plots for different values of pibar and l
plot(varpibar)
xlabel('p-bar')
ylabel('LPA-Var')

figure
plot(var_l)
xlabel('l')
ylabel('LPA-Var')
%%
```

Task 3: LPA-ES for different parameters

```
%% Task 3 LPA-ES
level= [0.95,0.99,0.999];
m =1:1:10000;
pibarzero = 0;
lzero = 0;
for i=1:length(m)
    for t=1:length(level)
        if pibarzero <= 1
            [esLPA_pibar] = ESlpa(0.6*10^6,1000,level(t),0.12,pibarzero);
             espibar(i,t) = esLPA_pibar;

        end

           if lzero <= 1
                [esLPA_l] = ESlpa(lzero*10^6,1000,level(t),0.12,0.04);
                es_l(i,t) = esLPA_l;
            end
    end
   pibarzero = pibarzero + 0.01;
   lzero = lzero +0.01;
end


figure
```

```
plot(espibar)
xlabel('p-bar')
ylabel('Expected shortfall')

figure
plot(es_l)
xlabel('l')
ylabel('Expected shortfall')
%%
```

Task 4: Exact VaR Beta

```
%% Task 4 exact value at risk
stop=0;
sum=0;            %cdf
k=-1;

while k <= 50 && stop == 0             %50 defaults
    k=k+1;
    p=betaexact(50,k,2,5);           %probability
    sum=sum+p;                    %sum of the probabilities (cdf)

    if sum >=0.95            %area bigger or equal of 0.95
        x=k;
        stop=1;    %change stop to stop the while loop
    end
end

if x~=0
    if abs(0.95-sum)>abs(0.95-(sum-p))      %closer?

        x=k-1;
    end
end

x
%%
```

Task 4: Difference between VaRs for different m

```
%% Task 4 difference changing m
%fun = @(x)
s=0;
sum = zeros(50,1);    %probability sum


for m=1:50
    p=0;            %p is the exat probability for every k
    for k=0:m
        if sum(m) < 0.95       %create the 0.95 area
        p = betaexact(m,k,2,5);
            if isnan(p) &&  s==0
                NaNerror = m;ans
                NaNerrork = k;
                s=1;
            end
        sum(m) = sum(m)+p;
        betaquintile = k;                %inverse at 0.95 i.e VaR
        end
    end


    testsumsmall(m) = sum(m)-p;


        area(m) = sum(m)-p;          %area gained when stopped
```

```
        VaR_exact(m) = betaquintile-1;


LPA_Beta_VaR(m) = m*betainv(0.95,2,5);          %l=1

diff(m) = abs(LPA_Beta_VaR(m)-VaR_exact(m)); % differenece between exact var
end                                              %and lpa var

figure
plot(diff)
%%
```

Task 4: Difference between VaRs for different a

```
%% Task 4 difference changing parameter a
sum2 = zeros(100,1);

for t=1:100
    a=t/10;
    p2=0;         %exact probability of each k
    for k=0:50
        if sum2(t) < 0.95 %construct the area
        p2 = betaexact(50,k,a,5);
        sum2(t) = sum2(t)+p2;   %summing up the probs
        betaquintile2 = k;     %value that gives the area
        end
    end
    testsumsmall2(t) = sum2(t)-p2;


        area2(t) = sum2(t);
        VaR_exact2(t) = betaquintile2;

    LPA_Beta_VaR2(t) = 50*betainv(0.95,a,5);     %lpa var using l=1

    diff2(t) = abs(LPA_Beta_VaR2(t)-VaR_exact2(t));
end

figure
plot(diff2)
%%
```

Task 4: Difference between VaRs for different b

```
%% Task 4 difference changing parameter b
sum3 = zeros(100,1);

for t=1:100
    b=t/10;
    p3=0;         %exact probability of each k
    for k=0:50
        if sum3(t) < 0.95  %constructing the area
        p3 = betaexact(50,k,2,b);
        sum3(t) = sum3(t)+p3;        %summing up each prob
        betaquintile3 = k;          %value of x axis giving the wanted area
        end
    end
    testsumsmall3(t) = sum3(t)-p3;


        area3(t) = sum3(t);
        VaR_exact3(t) = betaquintile3;       %value at risk is the quintile

    LPA_Beta_VaR3(t) = 50*betainv(0.95,2,b);    %lpa var using l=1

    diff3(t) = abs(LPA_Beta_VaR3(t)-VaR_exact3(t)); %difference of them
```

```
end

figure                                %plot of the difference when
plot(diff3)
%%
```

6.2 Functions

Task 2: Probability of the loss

```
function [lossprob] = probloss(l,m,x,ro,pbar)


lossprob = normcdf((1/sqrt(ro))*(sqrt(1-ro)*norminv(x/(l*m))-norminv(pbar)));




end
```

Task 3: Merton LPA-VaR

```
function [VaR] = MertVaR(l,m,alpha,ro,pbar)


normalex = normcdf((sqrt(ro)*norminv(alpha)+norminv(pbar))/sqrt(1-ro));



VaR = l*m*normalex;

end
```

Task 3: Merton LPA-ES

```
function [ES] = ESlpa(l,m,alpha,ro,pibar)

 for i=1:length(alpha)

        alpha_t = alpha(i);
        constant = (l*m)/(1-alpha_t);

  funct_es = @(x) normcdf((sqrt(ro)*norminv(x)+norminv(pibar))/sqrt(1-ro));

   esintegral = integral(funct_es,alpha_t,1);

   ES(i) = constant*esintegral;
 end

end
```

Task 4: $\mathbb{P}[N_m = k]$

```
function [exactbeta] = betaexact(m,k,a,b)

ena = nchoosek(m,k);

dyo = gamma(a+b)*gamma(a+k)*gamma(b+m-k);

```

```
tria = gamma(a)*gamma(b)*gamma(a+b+m);

exactbeta = ena*(dyo/tria);

end
```

References

[1] Herbertsson A., Static Credit Portfolio Models Lecture Notes, Centre for Finance, Department of Economics and Law, University of Gothenburg, April 7, 2018