Project 1: The Paper Helicopter Experiment

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November 22, 2018

1 Introduction

A well structured experimental design, makes it easier to analyze the data and estimate different factors by not running a huge number of experimental runs. The aim of this project is to find the optimal design of a paper helicopter. And in this case, we are tasked to plan, run and analyze this experiment. We are only allowed to run 25 experimental runs since our contractor CHC (Cellulosa Helicopter Company) has a budget to withhold. The experiment is also limited to the study of eight factors at two different levels of the paper helicopter. These factors are Material, Wing Length, Body Length, Body Width, Paper Clips, Folded Wings, and whether Taped Body and Taped Wings.

2 Method

2.1 Study-Design and overall method

The assignment, called the paper helicopter experiment, was provided by the website:
www.paperhelicopterexperiment.com. All the information about the experiment and how to construct the helicopters where given there.
Once we decided upon which sort of experiments to run and created the design matrix, a printable document was provided. This document contained the templates of the different helicopters. These templates all had a unique ID. Since we where three people in our group, the workload was divided in order to make the experiment more efficient and minimize the noise. One person clipped the paper, one constructed the helicopters and dropped them, and one measured the flying time and wrote notes. The helicopters were then dropped from the height 230cm, and the time (sec) until the helicopters hit the floor was recorded. This was the response variable y_i for the experiment.

To calculate the confidence interval of each effect, the 2-tail t-distribution table was used, with 5% significance. Standard error *SE(effect)* of each effect is the same for all, as the variance is also the same for all of them, and calculated by summing up the squares of the higher order interactions. A Lenth's plot is also used for finding significance of factors. The factors are considered significant if above the SME(Simultaneous margin of Error)-line which is calculated by $\pm t_{y,d} *_0$ where $y = (1 + 0.95^{1/k})/2$. If the factors are above the ME(Margin of error)-line then they may also be considered significant, but not as strongly. In the first experiment, the fact that this design has *Projectivity* $P = 3$ can be used to re-analyze the data. Three factors can be projected onto a 2^3 full-factorial design. Since three factors from the first experiment was considered active, these calculations can be done.A more fair estimate of the variance was made since we now had a replication. Also new confidence intervals with this variance were calculated.

2.2 Experimental run 1

We used the recommended levels for the eight different factors, see table 1 for further details. It is worth pointing out that the difference in material was small in experimental run 1. Material X was an ordinary paper, and material Y was an ordinary paper painted with chalk pen. This made material Y a little bit heavier and changed the friction of the paper somewhat.

A 16-run 2^{8-4}_{IV} design was constructed. We decided to confound the variables Paper clips(K), Folded Wings(L), Taped Body(M) and Taped Wings(N) with the three-order-interactions abc, acd, abd and bcd.This gave the following generators: $K = abc$, $L = abd$, $M = acd$ and $N = bcd$. By calculation, the shortest defining relation involved four factors, which meant that the resolution for this experiment was IV. This design also have *Projectivity* $P = 3$. The design matrix for this first experimental run is shown in table 2. The 16-runs where randomized by the provider of the experiment.Each helicopter has a unique ID with number 46-61 in a randomized order, which we followed for taking the measurements.

	Factors	Levels	
A	Material	X	Y
B	Wing Length	70mm	120mm
C	Body Length	70mm	120mm
D	Body Width	35mm	50mm
K	Paper Clips	1	3
L	Folded Wings	NO	YES
M	Taped Body	NO	YES
N	Taped Wings	NO	YES

A	B	C	D	K	L	M	N
-	-	-	-	-	-	-	-
-	-	-	+	-	+	+	+
-	-	+	-	+	-	+	+
-	-	+	+	+	+	-	-
-	+	-	-	+	+	-	+
-	+	-	+	+	-	+	-
-	+	+	-	-	+	+	-
-	+	+	+	-	-	-	+
+	-	-	-	+	+	+	-
+	-	-	+	+	-	-	+
+	-	+	-	-	+	-	+
+	-	+	+	-	-	+	-
+	+	-	-	-	-	+	+
+	+	-	+	-	+	-	-
+	+	+	-	+	-	-	-
+	+	+	+	+	+	+	+

Table 1: In the table to the left we see the recommended levels for the eight different factors

Table 2: In the table to the right we see the design matrix for the first experimental run

2.3 Experimental run 2

Se table 3 for details about experimental run 2. It is important to point out that the difference in material was bigger in experimental design 2. Material X was an ordinary paper as before (that weights about 80 gram) but material Y was a heavier paper that weighted 130 gram.

A 8-run 2^3 full factorial design was constructed. The design matrix for the second experimental run is shown in table 4. The 8-runs where in this case also randomized by the provider of the experiment. Each helicopter has a unique ID with number 43-50 in a randomized order, which we followed for taking the measurements.

Factors		Levels	
A	Material	X	Y
B	Wing Length	125mm	125mm
C	Body Length	70mm	120mm
D	Body Width	35mm	50mm
K	Paper Clips	1	1
L	Folded Wings	NO	NO
M	Taped Body	NO	NO
N	Taped Wings	NO	NO

A	B	C	D	K	L	M	N
-	-	-	-	-	-	-	-
-	-	-	+	-	-	-	-
-	-	+	-	-	-	-	-
-	-	+	+	-	-	-	-
+	-	-	-	-	-	-	-
+	-	-	+	-	-	-	-
+	-	+	-	-	-	-	-
+	-	+	+	-	-	-	-

Table 3: In the table to the left we see the recommended levels for the eight different factors

Table 4: In the table to the right we see the design matrix for the second experimental run

2.4 Experimental run 3

This is the optimized helicopter derived from the information obtain through experimental run 1 and 2. Wing Length was set to 125mm, one Paper Clip was used and all other factors where as in table 3. In fact, this is one of the helicopters from experimental run 2.

A	B	C	D	K	L	M	N
-	+	-	-	-	-	-	-

3 Results

3.1 First experimental run

Results from the first experimental run, the 16-run 2^{8-4}_{IV} design is shown in table 5. Figure 1 shows a normal plot for the estimated effects. In the normal plot we can see that the estimated effect Wing Length(B) has a big impact on increasing the flight time. Much more than any other effect. The effects Paper Clips(K) and Taped Body(M) also had an effect. But these decreased the flight time of the helicopters. The effects Body Width(D) and Taped Wings(N) had some minor impact on flight time, both shortened it a bit. But in large these effects did not affect so much. Centered around zero are the two-order-interaction effects and some of the main effects, Body Length(C), Material(A) and Folded Wings(L).

The standard deviation for the effects where calculate to $\sqrt{V\hat{a}r(effect)} = 0.0788$. Where $v\hat{a}r(effect)$ is the sum of the last 7 interaction-effects in table 5. This value can be used to calculate confidence intervals for the effects. We use 5% significance and assume that y_i is independent and normally distributed. This assumption is fairly reasonable as can be seen in figure 1. Most factors are rather close to the red line $y = a + bx$. Only Factor B deviate a lot. We consider normality as a plausible model. Therefore a t-test was used with 5 degrees of freedom. Three effects had an confidence interval that did not include zero.These ones where considered as significant, and were the factors B, K and M. Their confidence interval was as following: B: (0.60 , 0.975) K: (-0.483 , -0.109) M: (-0.4278 ,-0.0645).

With the results from the Lenth's plot, similar estimates can be calculated. As can be seen in figure 2 only one factor, Wing Length(B) exceeds the SME. This factor is declared active. The

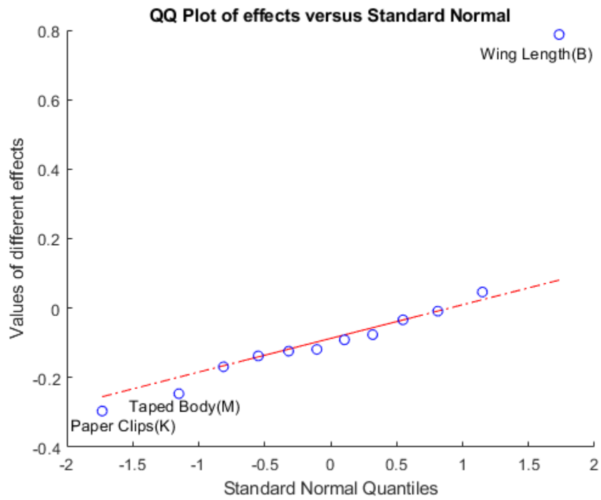

Figure 1: Normal plot for the first experimental run

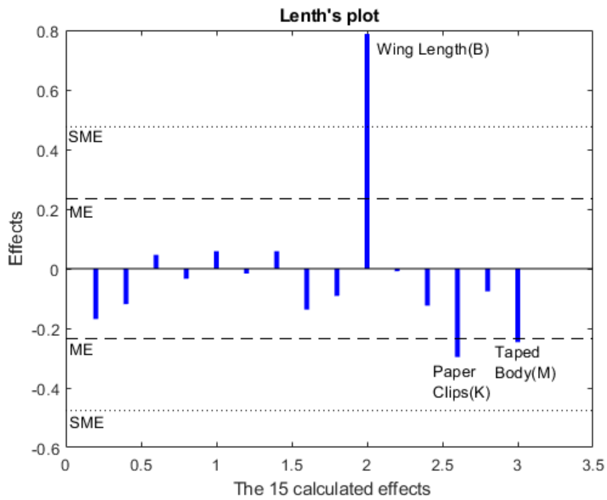

Figure 2: Lenth's plot for first experimental run

factors K and M are above the ME and can also be considered significant, but not as strongly as B.

In the first experimental run there are many two-order-interactions. Two of them are shown in Figure 2 and 3. These shows the interaction effects between effects A and D and B and C. In figure 2 the lines are almost parallel which indicates that there is no interaction-effect between A and D. In figure 3 we can see the lines intersection which points to the existence of some (unclear) interaction-effect between factors B and C.

The results from the first experimental run gave some valuable information about the effects.The experiment acted as a screening design in order to find which effects that affected the flight-time. It seems plausible that if we choose to increase the Wing Length(B), then the flight-time would increase. Also, using 3 paper-clips instead of 1 seemed to decrease the flight-time. There is also some indications toward that taped body lessens the flight-time. These results where later on taken into consideration when constructing the second experimental run.

3.2 Projection of first experimental run

Three factors were by calculations consider active. These factors where projected onto a 2^3 factorial design and new calculations where made. Results from this design can be seen in the Lenth's plot (figure 3). A new variance and standard deviation were calculated. Where $Var(effect) = S^2 \times (1/8+1/8)$ and $SE(effect) = \sqrt{Var(effect)} = 0.1223$. To determine the level of significance the Lenth's plot was not used. We instead use the fact that we now have a genuine replication which makes it possible to calculate a much more accurate variance. This estimation was thereafter used in the t-test to determine significance.

The main effects of Wing Length(B) and Paper Clips(K) were after calculations still considered active. But Taped body(M) was not considered active with the new calculations. New confidence intervals were calculated with the new standard error.
B: (0.498 , 1.077) K: (-0.585 , -0.0676) M: (-0.53 , 0.043). As can be seen, two of these confidence intervals do not include zero. But for factor M it is really close.

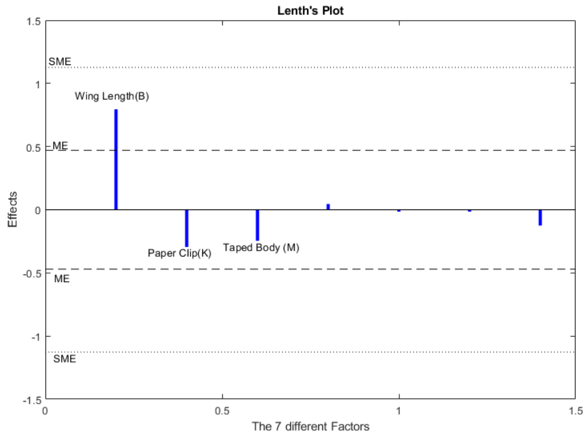

Figure 3: Lenth's plot for the three projected factors

a A	b B	c C	d D	abc K	abd L	acd M	bcd N	y_i	Average	Effect
-	-	-	-	-	-	-	-	2.05	$\bar{y} = 1.94$	
-	-	-	+	-	+	+	+	1.44	$l_A \to A$	-0.09125
-	-	+	-	+	-	+	+	1.35	$l_B \to B$	0.7875
-	-	+	+	+	+	-	-	1.32	$l_C \to C$	- 0.00875
-	+	-	-	+	+	-	+	2.22	$l_D \to D$	- 0.12375
-	+	-	+	+	-	+	-	2.38	$l_K \to K$	-0.29625
-	+	+	-	-	+	+	-	2.54	$l_L \to L$	-0.07625
-	+	+	+	-	-	-	+	2.66	$l_M \to M$	-0.24625
+	-	-	-	+	+	+	-	1.46	$l_N \to N$	-0.16875
+	-	-	+	+	-	-	+	1.40	$l_{AB} \to AB+CK+DL+MN$	-0.11875
+	-	+	-	-	+	-	+	1.57	$l_{AC} \to AC+BK+DM+LN$	0.04625
+	-	+	+	-	-	+	-	1.84	$l_{AD} \to AD+BL+CM+KN$	-0.03375
+	+	-	-	-	-	+	+	2.45	$l_{BC} \to BC+AK+DN+LN$	0.05875
+	+	-	+	-	+	-	-	2.23	$l_{BD} \to BD+AL+CN+KM$	-0.01625
+	+	+	-	+	-	-	-	2.45	$l_{CD} \to CD+AM+BN+KL$	0.05875
+	+	+	+	+	+	+	+	1.83	$l_{AN} \to AN+DK+CL+DM$	-0.1375

Table 5: Results from the first experimental run

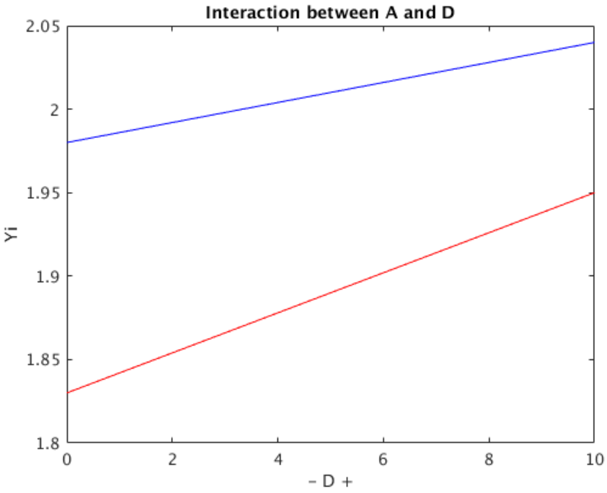

Figure 4: Interaction between effects A and D

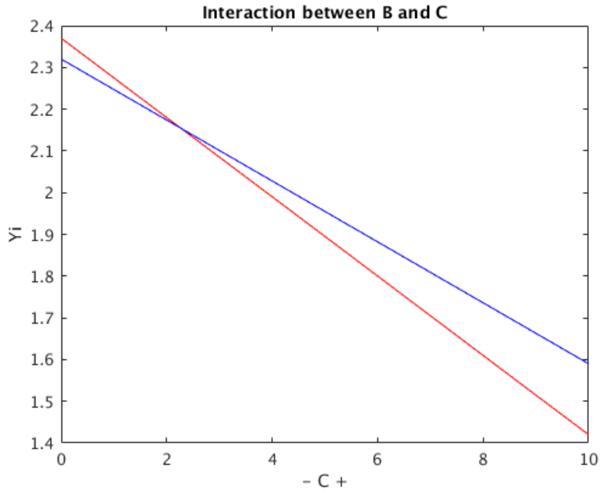

Figure 5: Interaction between effects B and C

3.3 Second experimental run

Results from the second experimental run is shown in table 6. We can see that the heavier paper material, factor A, had the largest negative main effect of -0.5. Meaning that the heavier paper type decreased the flight time. The interaction between effects A and D had the highest positive interaction, 0.235. These can be seen in the normal plot(figure 5.) as the first and last point.

In figure 7. the interaction-effect between Body Length(C) and Body Width(D) is shown. Both lines intercept each other, and it is clear that there exist some interaction between these factors.
The standard deviation for the effects where calculate to $\sqrt{\hat{Var}(effect)} = 0.22$. Because, in this experiment we only have one three-order-interaction. We use 5% significance and assume that y_i is independent and normally distributed so we can use the t-test. None of the effects where significant in this experimental run. All confidence intervals included zero. The 'closest' one was factor(A) with a confidence interval of (-1.327, 0.327).

In figure 6 (Lenth's plot) it is obvious that no factor has an effect that exceeds the ME-line. Since we had less estimated effects in the second run, and also larger standard deviation, the gap for ME and SME is larger. Material (A) is shown in the plot as the factor with largest effect. This method also showed that no effects were significant.

For the optimized helicopter we got a time of 2.69s. Which is the best recorded time out of any helicopter. There is no data to analyze on this run.

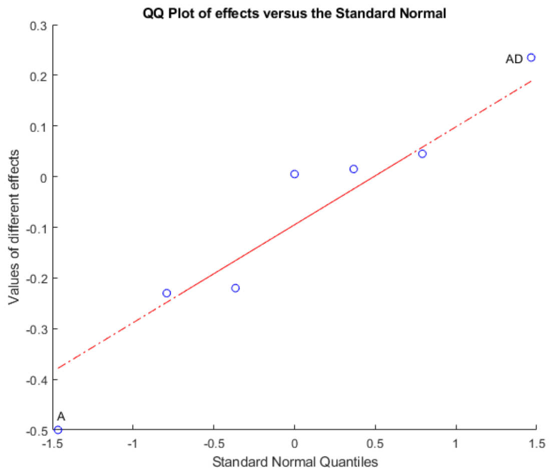

Figure 6: Normal plot for the second experimental run

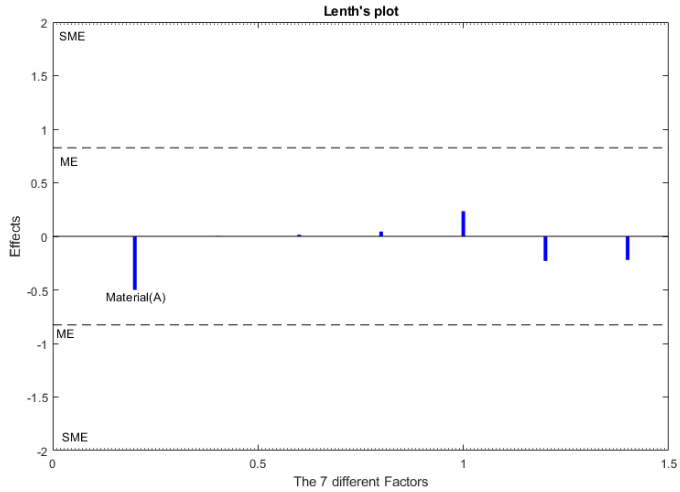

Figure 7: Lenth's plot for second experimental run

A	B	C	D	K	L	M	N	y_i	Effect
-	-	-	-	-	-	-	-	2.50	$\bar{y} = 2.13$
-	-	-	+	-	-	-	-	2.29	A: -0.5
-	-	+	-	-	-	-	-	2.47	C: -0.0075
-	-	+	+	-	-	-	-	2.24	D: 0.0525
+	-	-	-	-	-	-	-	1.5	AC: 0.045
+	-	-	+	-	-	-	-	2.2	AD: 0.235
+	-	+	-	-	-	-	-	2.0	CD: -0.23
+	-	+	+	-	-	-	-	1.8	ACD -0.22

Table 6: Results from the second experimental run

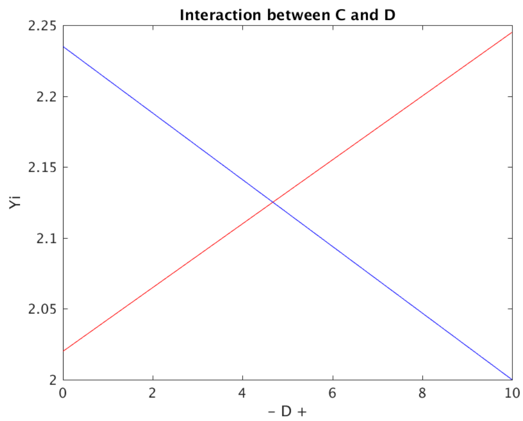

Figure 8: Interaction between effects C and D

3.4 Model

We also tried to construct a model for a better approach to the average flight time of the helicopter. Below is the multiple linear regression model with the effects only from the projected design. So we get a model based on the three significant effects and their interactions. We choose to do it this way because we wanted to have an simple model.

$$\eta = 1.94 + 0.394x_1 - 0.148x_2 - 0.123x_3 + 0.022x_1x_2 - 0.03x_1x_3 - 0.0625x_2x_3 - 0.11x_1x_2x_3 + \epsilon$$

3.5 Prediction interval based on optimal configuration

We construct a prediction interval for the optimal configuration. In this calculation we use the variance calculated from the projected design. At 5 % significance we determine the confidence interval to be the following.

$$\eta = 2.69 \pm t_{15}(0.025) * Se_{effect} = [2.43, 2.95]$$

4 Discussion

The estimated effect Wing Length (B) has the highest impact on increasing the flight time. The effect of this factor to the response variable in this case was positive 0,7875. But when it comes to the other factors, their main effect is negative, meaning that their contribution decreases the flight time. Paper Clips (K) factor has the most impact on decreasing flight time with an effect of - 0,29625. Furthermore, Taped Body (M) and Taped Wings (N) had a quite high negative effect too, according to our results, with -0,24625 and -0,16875 respectively. It seems quite reasonable to believe that, having three paper clips decreases the flight time comparing to one paper clip since the helicopter in that case will be more heavy. We deem that the same holds for Taped Body (M) and Taped Wings (N).

Our original thought was that both Body Length and Wing Length would be important factors that play a significant role at the flight time. The effect of Body Length was the lowest of all factors (only -0.00875). This shows that our belief about body length in the beginning was not correct.

But when it comes to wing length our expectation in the beginning was correct. The wing length in general, has a massive impact to the response variable. Having that in mind during our design of the second experimental run, we chose the wing length to be constant. And to be more precise, at it's higher level: 125mm, expecting our paper helicopter to fly longer. And in fact it did. The average flight time during the 16-run 2_{IV}^{8-4} design was ≈ 1.94 sec. and ≈ 2.13 sec. on the second run. Even if we did not manage to beat the record high 2.66 seconds of the first run, we at least managed to increase the average. We highly believe that our failure on obtaining a new record flight time on the second run, is due to systematic differences. This is the case because we did not use the same Paper Clip as in experimental run 1. In fact we used a slightly heavier one, which we think played some role into the result.

As stated on section 2.1, chalk was used to distinguish material X from Y. It was expected that the impact of factor A (material) would be very small and so its interaction with any other factor. This can be easily seen by Figure 2, where the two lines are (almost) parallel, meaning, (almost) no interaction between factors A and D. Furthermore, our second assumption was that a slightly heavier (thicker) paper could bring home longer flight time, since we thought that it could be more stable and spin better. Experimental run 2 managed to refute our assumption, as we obtained a big Main Effect of the Material i.e $L_A = -0.5$. As a result, we were convinced that a thicker paper gives the helicopter shorter flying times. Given that, we chose the plain A4 paper to optimize our helicopter.

The optimized helicopter got the best time. Even tho this is the same helicopter which was in experimental run 2. The difference here was that we used the original paper clip, which was somewhat lighter. This might describe the difference obtained. Or it could simply be due to random variation.

It is obvious from figure 7 that, there is some interaction between Body Length(C) and Body Width(D). We are not sure about this interaction but the graph shows a negative correlation between those two factors. One can guess that there could exist an Optimal Ratio between them, making the helicopter more stable and moreover, fly longer. Another possibility is that the total area is the key thing. The larger the area, the longer the flight time. These new questions can lead to further data analysis or to another experimental design.

5 Conclusion

The experiment concluded that the most important factor for longer flight time was increased Wing Length(B). More Paper Clips(K) seemed to decreased flight time due to additional weight. Taped Body also had an significant effect which decreased flight time, most likely due to more stability. We choose not to investigate this factor further. When heavier material is used, as in experimental run 2, the flight time also decreases. There may exist some Optimal Ratio between Body Width and Body Length. This result came from experimental run 2, so there was no possibility to investigate this more thoroughly. For another experiment there might be a good idea to analyze the factor Taped Body and the interaction-effect between Body Length and Body Width further.