# ΦΥΣΙΚΗ Ι

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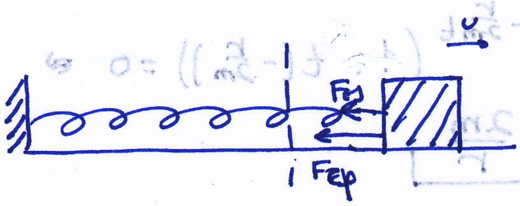
## Δ' ΣΕΙΡΑ ΑΣΚΗΣΕΩΝ

### Ασκ 1

$m, s, r$

$\vec{F}_{\tau\rho} = -r\vec{u}$




α) $m\ddot{x} = -\vec{F}_{\tau\rho} - \vec{F}_{\varepsilon\lambda}$

$m\ddot{x} = -r\dot{x} - sx$

$m\ddot{x} + r\dot{x} + sx = 0$

Αναζητούμε λύση της μορφής $x(t) = Ae^{Bt}$

$\dot{x}(t) = A \cdot B e^{Bt}$

$\ddot{x}(t) = A \cdot B^2 e^{Bt}$

Άρα, $m \cdot B^2 A e^{Bt} + r B \cdot A e^{Bt} + s A e^{Bt} = 0$

$\Leftrightarrow Ae^{Bt}(mB^2 + rB + s) = 0 \Leftrightarrow mB^2 + rB + s = 0$ (Χαρακτηριστικό πολυώνυμο της Δ.Ε.)

λύσεις: $\Delta = r^2 - 4 \cdot m \cdot s$

$B_{1,2} = \dfrac{-r \pm \sqrt{r^2 - 4ms}}{2m}$

και προκύπτει $x(t) = A_1 e^{B_1 t} + A_2 e^{B_2 t}$

με $B_1 = B_2 = -\dfrac{r}{2m} = B < 0 \Rightarrow \boxed{B = -\dfrac{r}{2m}}$

Άρα $x(t) = Ae^{Bt} + Cte^{Bt} = (A + Ct)e^{Bt}$

→ Για $t=0$, $x(0) = (A + C \cdot 0)e^{B \cdot 0} \Leftrightarrow 0 = A \cdot 1 \Leftrightarrow \boxed{A = 0}$

Δηλ. $x(t) = Cte^{Bt} \Rightarrow x(t) = Cte^{-\frac{r}{2m}t}$

και $\dot{x}(t) = C\left[\left(-\dfrac{r}{2m}\right)t + 1\right]e^{-\frac{r}{2m}t}$

→ Για $t=0$, $\dot{x}(0) = C\left[\left(-\dfrac{r}{2m}\right) \cdot 0 + 1\right]e^{-\frac{r}{2m} \cdot 0} \Leftrightarrow U_0 = C \cdot 1 \Leftrightarrow \boxed{C = U_0}$

Δηλ. $x(t) = U_0 t e^{-\frac{r}{2m}t}$

β) Στη μέγιστη απομάκρυνση, έχουμε $\dot{x}(t)=0$.

Άρα, $\dot{x}(t) = U_0 e^{-\frac{r}{2m}t}\left(1 + t\left(-\frac{r}{2m}\right)\right)$

οπότε $\dot{x}(t)=0 \Leftrightarrow U_0 e^{-\frac{r}{2m}t}\left(1+t\left(-\frac{r}{2m}\right)\right)=0 \Leftrightarrow 1+t\left(-\frac{r}{2m}\right)=0 \Leftrightarrow$

$\Leftrightarrow \boxed{t=\frac{2m}{r}}$

και $x(t) = U_0 t\, e^{-\frac{r}{2m}t}$

έχουμε $x\left(\frac{2m}{r}\right) = U_0 \cdot \frac{2m}{r} e^{-\frac{r}{2m}\cdot\frac{2m}{r}} = \frac{2U_0 m}{r} e^{-1} = \frac{2mU_0}{re} \Rightarrow \boxed{x(t) = \frac{2mU_0}{re}}$

γ) Θα έχουμε $x(t) = (A+ct)\, e^{-\frac{r}{2m}t}$

→ Για $t=0$, $x(0) = (A+c\cdot 0)\, e^{-\frac{r}{2m}0} \Leftrightarrow X_0 = A e^0 \Leftrightarrow \boxed{A = X_0}$

$\dot{x}(t) = c\cdot e^{-\frac{r}{2m}t} + (A+ct)\, e^{-\frac{r}{2m}t}\left(-\frac{r}{2m}\right) \Rightarrow$

$\Rightarrow \dot{x}(t) = e^{-\frac{r}{2m}t}\left[(A+ct)\left(-\frac{r}{2m}\right)+c\right]$

→ Για $t=0$, $\dot{x}(0) = e^{-\frac{r}{2m}\cdot 0}\left[(A+c\cdot 0)\left(-\frac{r}{2m}\right)+c\right] \Leftrightarrow$

$\Leftrightarrow -2\gamma X_0 = A\left(-\frac{r}{2m}\right)+c \Leftrightarrow -2\gamma X_0 = -\frac{r}{2m}X_0 + c \Leftrightarrow$

$\Leftrightarrow c = \frac{r}{2m}X_0 - 2\gamma X_0 \Leftrightarrow \boxed{c = X_0\left(\frac{r}{2m}-2\gamma\right)}$ και για $\gamma = \frac{r}{2m}$ έχουμε $\boxed{c = -\gamma X_0}$

οπότε $x(t) = \left(X_0 + X_0\left(\frac{r}{2m}-2\gamma\right)t\right)e^{-\frac{r}{2m}t} = X_0\left(1+\left(\frac{r}{2m}-2\gamma\right)t\right)e^{-\frac{r}{2m}t}$

• Θα διέρχεται από Θ.Ι. όταν $x(t)=0$

$x(t)=0 \Leftrightarrow X_0\left(1+\left(\frac{r}{2m}-2\gamma\right)t\right)e^{-\frac{r}{2m}t}=0 \Leftrightarrow \left(1+\left(\frac{r}{2m}-2\gamma\right)t\right)e^{-\frac{r}{2m}t}=0$

$\Leftrightarrow 1+\left(\frac{r}{2m}-2\gamma\right)t=0 \Leftrightarrow \left(\frac{r}{2m}-2\gamma\right)t=-1 \Leftrightarrow$

$\Leftrightarrow t = \frac{-1}{\frac{r}{2m}-2\gamma} \Rightarrow \boxed{t=\frac{1}{2\gamma-\frac{r}{2m}}}$ θα πρέπει $2\gamma - \frac{r}{2m} > 0$

και για $\gamma = \frac{r}{2m}$ έχουμε $t = \frac{1}{\gamma} = \frac{2m}{r}$

$$\dot{x}(t) = x_0\left(\frac{r}{2m}-2\gamma\right)e^{-\frac{r}{2m}t} + x_0\left(1+\left(\frac{r}{2m}-2\gamma\right)t\right)e^{-\frac{r}{2m}t}\left(-\frac{r}{2m}\right)$$

$$= x_0 \cdot e^{-\frac{r}{2m}t}\left[\left(\frac{r}{2m}-2\gamma\right)+1+\left(\frac{r}{2m}-2\gamma\right)t\left(-\frac{r}{2m}\right)\right]$$

$$= x_0 e^{-\frac{r}{2m}t}\left[\left(\frac{r}{2m}-2\gamma\right)\left(1+t\left(-\frac{r}{2m}\right)\right)+1\right]$$

Οπότε για $t=\dfrac{1}{2\gamma-\frac{r}{2m}}$ έχουμε:

$$\dot{x}(t) = x_0 e^{-\frac{r}{2m}\cdot\frac{1}{2\gamma-\frac{r}{2m}}}\left[\left(\frac{r}{2m}-2\gamma\right)\left(1+\frac{1}{2\gamma-\frac{r}{2m}}\cdot\left(-\frac{r}{2m}\right)\right)+1\right] =$$

$$= x_0 e^{-\frac{r}{2m}\cdot\frac{2m}{4m\gamma-r}}\left[\left(\frac{r}{2m}-2\gamma\right)+1+\overset{-1}{\left(\frac{r}{2m}-\cancel{2\gamma}\right)\cdot\left(\frac{1}{\cancel{2\gamma-\frac{r}{2m}}}\right)}\cdot\left(-\frac{r}{2m}\right)\right]$$

$$= x_0 \cdot e^{-\frac{r}{4m\gamma-r}}\left[\frac{r}{2m}-2\gamma+1+\frac{r}{2m}\right] =$$

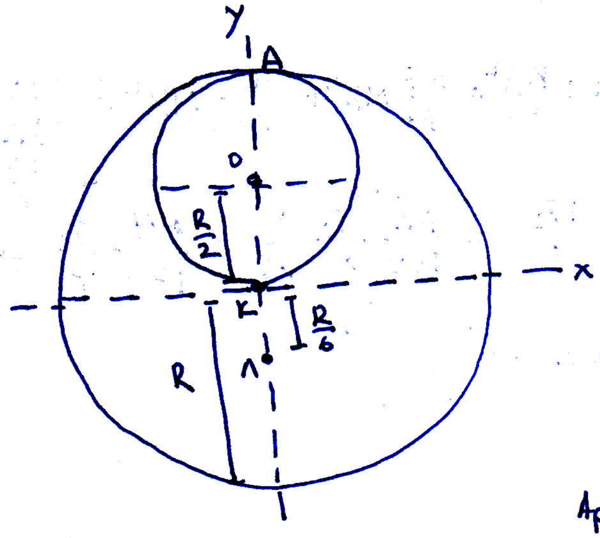
$$= x_0 e^{-\frac{r}{4m\gamma-r}}\left[\frac{r}{m}-2\gamma+1\right]$$

και για $\gamma=\frac{r}{2m}$ έχουμε $\dot{x}(t) = x_0 e^{-\frac{r}{\cancel{4m}\frac{r}{\cancel{2m}}-r}}\left[\frac{r}{m}-\cancel{2}\frac{r}{\cancel{2}m}+1\right] =$

$$= x_0 e^{-\frac{r}{2r-r}}\left[\cancel{\frac{r}{m}}-\cancel{\frac{r}{m}}+1\right] = x_0 e^{-1} = \frac{x_0}{e}$$

ασκ2

R, R/2, M




$$M_{μεγ} - M_{μικ} = M$$

όπου $M_{μεγ} = ρ \cdot S = ρπR^2$

και $M_{μικ} = ρ \cdot S' = ρπ\left(\frac{R}{2}\right)^2 = ρ\frac{πR^2}{4}$

Άρα $πρR^2 - \frac{1}{4}πρR^2 = M$

$\frac{3}{4}πρR^2 = M \Leftrightarrow πρR^2 = \frac{4}{3}M$

Άρα $M_{μεγ} = \frac{4}{3}M$ και $M_{μικ} = \frac{1}{3}M$

α) $$\vec{R}_{cm} = \frac{M_{μεγ} \cdot \vec{R} - M_{μικ} \cdot \vec{R}_{/2}}{M_{μεγ} - M_{μικ}} = \frac{\frac{4}{3}\cancel{M} \cdot 0 - \frac{1}{3}\cancel{M} \cdot \frac{\vec{R}}{2}}{\cancel{M}} = 0 - \frac{\vec{R}}{6} = -\frac{\vec{R}}{6}$$

και $$I_{μεγ} = \frac{1}{2}M_{μεγ} \cdot R^2 = \frac{1}{2} \cdot \frac{4}{3}MR^2 = \frac{2}{3}MR^2$$

$$I_{Λ,μεγ} = I_{μεγ} + M_{μεγ}(KΛ)^2 = \frac{2}{3}MR^2 + \frac{4}{3}M \cdot \left(\frac{R}{6}\right)^2 =$$

$$= \frac{2}{3}MR^2 + \frac{\cancel{4}}{3}M\frac{R^2}{\cancel{36}_9} = \frac{2}{3}MR^2 + \frac{1}{27}MR^2 = \frac{19}{27}MR^2$$

$$I_{μικ} = \frac{1}{2}M_{μικ}R^2 = \frac{1}{2} \cdot \frac{1}{3}M \cdot \left(\frac{R}{2}\right)^2 = \frac{1}{6}M\frac{R^2}{4} = \frac{1}{24}MR^2$$

$$I_{Λ,μικ} = I_{μικ} + M_{μικ}(OΛ)^2 = \frac{1}{24}MR^2 + \frac{1}{3}M \cdot \left(\frac{R}{2} + \frac{R}{6}\right)^2 =$$

$$= \frac{1}{24}MR^2 + \frac{1}{3}M\left(\frac{2R}{3}\right)^2 = \frac{1}{24}MR^2 + \frac{1}{3}M\frac{4}{9}R^2 = \frac{41}{216}MR^2$$

Άρα, $$I_Λ = I_{Λ,μεγ} - I_{Λ,μικ} = \left(\frac{19}{27} - \frac{41}{216}\right)MR^2 = \frac{111}{216}MR^2 = \frac{37}{72}MR^2$$

β) Αν ο δίσκος αναρτηθεί από το Α, τότε:

$$I_A = I_Λ + M(AΛ)^2 = \frac{37}{72}MR^2 + M\left(R+\frac{R}{6}\right)^2 = \frac{37}{72}MR^2 + \frac{49}{36}MR^2 = \frac{135}{72}MR^2 = \frac{15}{8}MR^2$$

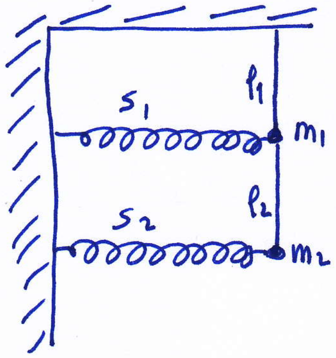
Οπότε, $$\omega_0^2 = \frac{Mg}{I_A}\cdot(AΛ) = \frac{\cancel{M}g}{\frac{135}{72}\cancel{M}R^2}\left(\frac{7\cancel{R}}{6}\right) = \frac{\cancel{72}^{12}}{135}\cdot\frac{7}{\cancel{6}}\cdot\frac{g}{R} = \frac{84}{135}\frac{g}{R} \Rightarrow$$

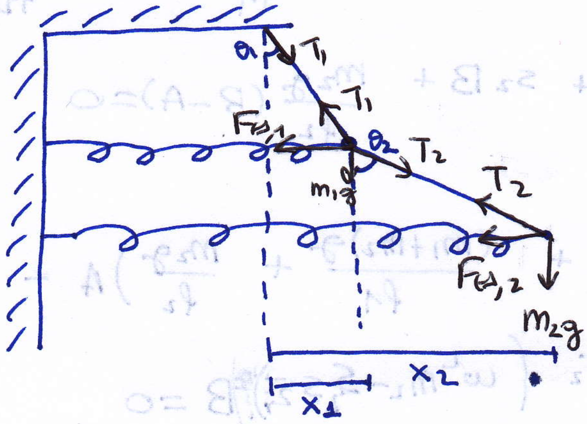
$$\Rightarrow \omega_0^2 = \frac{28}{45}\cdot\frac{g}{R} \Rightarrow \omega_0 = \sqrt{\frac{28}{45}\cdot\frac{g}{R}}$$

## Ασκ 3

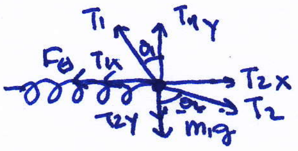
↳ Λυμένη στις σημειώσεις.

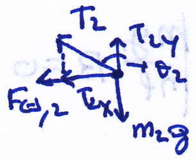
ασκ 4




α)




$$:\begin{cases} T_{1y} = T_{2y} + m_1 g \\ T_{2y} = m_2 g \end{cases} \Rightarrow \begin{cases} T_{1y} = (m_1+m_2) g \\ T_{2y} = m_2 g \end{cases}$$

Άρα,

$$\begin{cases} m_1 \ddot{x}_1 = -F_{\varepsilon\lambda,1} - T_{1x} + T_{2x} \\ m_2 \ddot{x}_2 = -F_{\varepsilon\lambda,2} - T_{2x} \end{cases} \Rightarrow \begin{cases} m_1 \ddot{x}_1 = -s_1 x_1 - T_1 \sin\theta_1 + T_2 \sin\theta_2 \\ m_2 \ddot{x}_2 = -s_2 x_2 - T_2 \sin\theta_2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} m_1 \ddot{x}_1 = -s_1 x_1 - T_1 \dfrac{x_1}{\ell_1} + T_2 \dfrac{x_2 - x_1}{\ell_2} \\ m_2 \ddot{x}_2 = -s_2 x_2 - T_2 \dfrac{x_2 - x_1}{\ell_2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} m_1 \ddot{x}_1 = -s_1 x_1 - \dfrac{(m_1+m_2) g \, x_1}{\ell_1} + \dfrac{m_2 g (x_2 - x_1)}{\ell_2} \\ m_2 \ddot{x}_2 = -s_2 x_2 - \dfrac{m_2 g (x_2 - x_1)}{\ell_2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} m_1 \ddot{x}_1 + s_1 x_1 + \dfrac{(m_1+m_2) g}{\ell_1} \cdot x_1 - \dfrac{m_2 g}{\ell_2} (x_2 - x_1) = 0 \\ m_2 \ddot{x}_2 + s_2 x_2 + \dfrac{m_2 g}{\ell_2} (x_2 - x_1) = 0 \end{cases}$$

β) $s_1 = s_2 = s$, $L_1 = L_2 = L$, $m_1 = m_2 = m$, $\dfrac{s}{m} = \dfrac{g}{L} = \omega_0^2$

Υπόθεση εξίσωσης :

$$x_1(t) = A \cos(\omega t + \varphi)$$

$$x_2(t) = B \cos(\omega t + \varphi)$$

όποτε:

$$\begin{cases} -\omega^2 A m_1 \cancel{\cos(\omega t+\varphi)} + s_1 A \cancel{\cos(\omega t+\varphi)} + \dfrac{(m_1+m_2) g}{\ell_1} A \cancel{\cos(\omega t+\varphi)} - \dfrac{m_2 g}{\ell_2} (B-A) \cancel{\cos(\omega t+\varphi)} = 0 \\ -\omega^2 B m_2 \cancel{\cos(\omega t+\varphi)} + s_2 B \cancel{\cos(\omega t+\varphi)} + \dfrac{m_2 g}{\ell_2} (B - A) \cancel{\cos(\omega t+\varphi)} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -\omega^2 A m_1 + s_1 A + \dfrac{(m_1+m_2)g}{\ell_1} A - \dfrac{m_2 g}{\ell_2}(B-A) = 0 \\ -\omega^2 B m_2 + s_2 B + \dfrac{m_2 g}{\ell_2}(B-A) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \left(-\omega^2 m_1 + s_1 + \dfrac{(m_1+m_2)g}{\ell_1} + \dfrac{m_2 g}{\ell_2}\right) A - \dfrac{m_2 g}{\ell_2} B = 0 \\ \dfrac{m_2 g}{\ell_2} A + \left(\omega^2 m_2 - s_2 \; \cancel{\tfrac{g}{\ell}}\right) B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \left(-\dfrac{\omega^2 m}{m} + \dfrac{s}{m} + \dfrac{(m+m)g}{m\ell} + \dfrac{\cancel{m} g}{\cancel{m}\ell}\right) A - \dfrac{mg}{m\ell} B = 0 \\ \dfrac{\cancel{m} g}{\cancel{m}\ell} A + \left(\dfrac{\omega^2 \cancel{m}}{\cancel{m}} - \dfrac{s}{m} - \cancel{\tfrac{g}{\ell}}\right) B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (-\omega^2 + \omega_0^2 + 2\omega_0^2 + \omega_0^2) A - \omega_0^2 B = 0 \\ \omega_0^2 A + (\omega^2 - 2\omega_0^2) B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (-\omega^2 + 4\omega_0^2) A - \omega_0^2 B = 0 & (1) \\ \omega_0^2 A + (\omega^2 - 2\omega_0^2) B = 0 & (2) \end{cases}$$

$$\Rightarrow \begin{vmatrix} \omega^2 - 4\omega_0^2 & \omega_0^2 \\ \omega_0^2 & \omega^2 - 2\omega_0^2 \end{vmatrix} = 0$$

$$\Rightarrow (\omega^2 - 4\omega_0^2)(\omega^2 - 2\omega_0^2) - \omega_0^4 = 0 \Rightarrow$$

$$\Rightarrow \omega^4 - 2\omega^2\omega_0^2 - 4\omega^2\omega_0^2 + 8\omega_0^4 - \omega_0^4 = 0 \Rightarrow$$

$$\Rightarrow \omega^4 - 6\omega^2\omega_0^2 + 7\omega_0^4 = 0$$

$$\Delta = (-6\omega_0^2)^2 - 4 \cdot 1 \cdot 7\omega_0^4 = 36\omega_0^4 - 28\omega_0^4 = 8\omega_0^4$$

$$\omega_{1,2}^2 = \frac{6\omega_0^2 \pm 2\sqrt{2}\,\omega_0^2}{2} = 3\omega_0^2 \pm \sqrt{2}\,\omega_0^2 = (3 \pm \sqrt{2})\,\omega_0^2$$

Για τον υπολογισμό των κανονικών μεταβλητών, προσδιορίζουμε πρώτα το λόγο των πλατών $A/B$ για κάθε κανονικό τρόπο ταλάντωσης, από τις εξισώσεις (1) ή (2), παραπάνω. Συγκεκριμένα:

για $\omega = \omega_1$ : $(1) \Rightarrow \left((3+\sqrt{2})\omega_0^2 - 4\omega_0^2\right)A_1 - \omega_0^2 B_1 = 0 \Leftrightarrow$

$$\Leftrightarrow \left((\sqrt{2}-1)\cancel{\omega_0^2}\right)A_1 = \cancel{\omega_0^2} B_1 \Leftrightarrow \frac{A_1}{B_1} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$$

για $\omega = \omega_2$ : $(1) \Rightarrow \left((3-\sqrt{2})\omega_0^2 - 4\omega_0^2\right)A_2 - \omega_0^2 B_2 = 0 \Leftrightarrow$

$$\Leftrightarrow -(\sqrt{2}+1)\cancel{\omega_0^2} A_2 = \cancel{\omega_0^2} B_2 \Leftrightarrow \frac{A_2}{B_2} = -\frac{1}{\sqrt{2}+1} = -\frac{\sqrt{2}-1}{1} = -\sqrt{2}+1$$

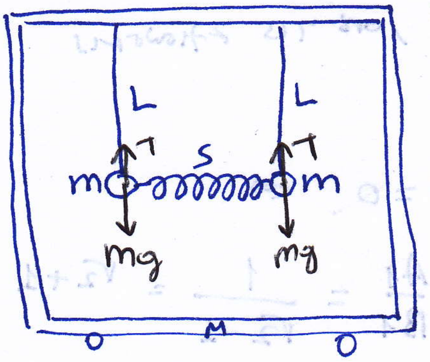
Οπότε, η γενική λύση γράφεται ως εξής:

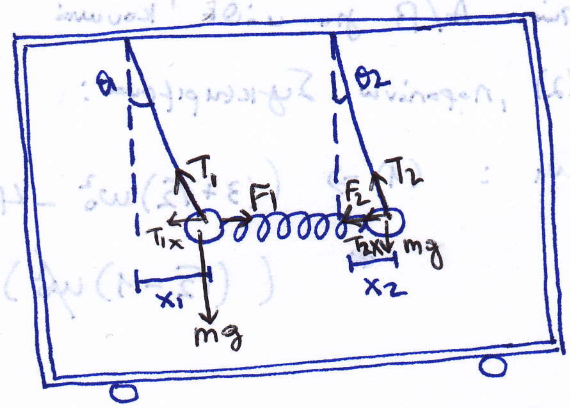
$x_1(t) = A_1 \cos(\omega_1 t + \theta_1) + A_2 \cos(\omega_2 t + \theta_2) \Rightarrow$

και $x_2(t) = B_1 \cos(\omega_1 t + \theta_1) + B_2 \cos(\omega_2 t + \theta_2) \Rightarrow$

$$\Rightarrow x_2(t) = \frac{1}{\sqrt{2}+1} A_1 \cos(\omega_1 t + \theta_1) + \frac{1}{-\sqrt{2}+1} \cos(\omega_2 t + \theta_2)$$

άσκ5




a)
$$\begin{cases} m_1\ddot{x}_1 = -T_1\sin\theta_1 - s(x_1-x_2) \\ m_2\ddot{x}_2 = -T_2\sin\theta_2 - s(x_2-x_1) \\ M\ddot{x} = T_1\sin\theta_1 + T_2\sin\theta_2 \end{cases} \Leftrightarrow \begin{cases} m_1\ddot{x}_1 = -\frac{m_1 g}{L}x_1 - s(x_1-x_2) \\ m_2\ddot{x}_2 = -\frac{m_2 g}{L}x_2 - s(x_2-x_1) \\ M\ddot{x} = \frac{m_1 g}{L}x_1 + \frac{m_2 g}{L}x_2 \end{cases} \Leftrightarrow$$

b) Θέτουμε εξισώσεις:
$$\begin{cases} x_1(t) = A\cos(\omega t+\varphi) \\ x_2(t) = B\cos(\omega t+\varphi) \\ x_3(t) = \Gamma\cos(\omega t+\varphi) \end{cases}$$

Άρα
$$\begin{cases} -\omega^2 A m_1 \cancel{\cos(\omega t+\varphi)} = -\frac{m_1 g}{L}A\cancel{\cos(\omega t+\varphi)} - s(A-B)\cancel{\cos(\omega t+\varphi)} \\ -\omega^2 B m_2 \cancel{\cos(\omega t+\varphi)} = -\frac{m_2 g}{L}B\cancel{\cos(\omega t+\varphi)} - s(B-A)\cancel{\cos(\omega t+\varphi)} \\ -\omega^2\Gamma M\cancel{\cos(\omega t+\varphi)} = \frac{m_1 g}{L}A\cancel{\cos(\omega t+\varphi)} + \frac{m_2 g}{L}B\cancel{\cos(\omega t+\varphi)} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -\omega^2 A m_1 + \frac{m_1 g}{L}A + s(A-B) = 0 \\ -\omega^2 B m_2 + \frac{m_2 g}{L}B + s(B-A) = 0 \\ -\omega^2\Gamma M - \frac{m_1 g}{L}A - \frac{m_2 g}{L}B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \left(-\omega^2 m_1 + m_1\frac{g}{L} + s\right)A - sB = 0 \\ \left(-\omega^2 m_2 + m_2\frac{g}{L} + s\right)B - sA = 0 \\ \frac{m_1 g}{L}A + \frac{m_2 g}{L}B + \omega^2 M\Gamma = 0 \end{cases}$$

γ) $$\begin{cases} \left(-\omega^2+\frac{g}{L}+\frac{s}{m}\right)A-\frac{s}{m}B=0 \\ -\frac{s}{m}A+\left(-\omega^2+\frac{g}{L}+\frac{s}{m}\right)B=0 \\ \frac{g}{L}A+\frac{g}{L}B+\omega^2\Gamma=0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} (-\omega^2+2\omega_0^2)A-\omega_0^2B=0 \\ -\omega_0^2A+(-\omega^2+2\omega_0^2)B=0 \\ \omega_0^2A+\omega_0^2B+\omega^2\Gamma=0 \end{cases} \Leftrightarrow$$

$$\Rightarrow \begin{vmatrix} -\omega^2+2\omega_0^{2\,+} & -\omega_0^{2\,-} & 0^+ \\ -\omega_0^{2\,-} & -\omega^2+2\omega_0^{2\,+} & 0^- \\ \omega_0^{2\,+} & \omega_0^{2\,-} & \omega^{2\,+} \end{vmatrix}=0 \Rightarrow$$

$$\Rightarrow \omega^2\begin{vmatrix} -\omega^2+2\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & -\omega^2+2\omega_0^2 \end{vmatrix}=\omega^2\left[(-\omega^2+2\omega_0^2)^2-\omega_0^4\right]=0 \Rightarrow$$

$$\Rightarrow \omega^2\left[(-\omega^2+2\omega_0^2)-\omega_0^2\right]\left[(-\omega^2+2\omega_0^2)+\omega_0^2\right]=0 \Rightarrow$$

$$\Rightarrow \omega^2\left[(-\omega^2+\omega_0^2)(-\omega^2+3\omega_0^2)\right]=0 \Rightarrow$$

$$\Rightarrow \omega^2(\omega^2-\omega_0^2)(\omega^2-3\omega_0^2)=0$$

$\Rightarrow$ $\omega^2=0$ ή $\omega^2-\omega_0^2=0$ ή $\omega^2-3\omega_0^2=0$

$\downarrow$ $\qquad\qquad$ $\omega^2=\omega_0^2$ ή $\omega^2=3\omega_0^2$

προέρχεται από την μεταφορική κίνηση του Κ.Μ.