Ηλεκτρομαγνητικό πεδίο Α

4ο Εξάμηνο

3η Σειρά Ασκήσεων

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Άσκηση 1 (4.13)

- $\vec{E}_1 = \hat{y}\, E_0 \exp[-(t+\frac{z}{c})/T]$, $z<0$ πρέπει $T<0$ ώστε $\lim_{z\to-\infty} \vec{E}_1 = \vec{0}$.

- $\frac{\partial \vec{H}_1}{\partial t} = -\frac{1}{\mu_0} \nabla\times\vec{E}_1 = \hat{x}\frac{1}{\mu_0}\cdot\frac{\partial E_{1y}}{\partial z} = -\hat{x}\cdot\frac{E_0}{\mu_0 cT}\cdot\exp[-(t+\frac{z}{c})/T] \Rightarrow$

  $\Rightarrow \vec{H}_1 = \hat{x}[\frac{E_0}{\mu_0 c}\exp[-(t+\frac{z}{c})/T] + F(z)]$

  Πρέπει $\lim_{t\to-\infty}\vec{H}_1 = \vec{0}$ γιατί κάποια στιγμή φορτίζεται το $\vec{K}(t)$

  Αν $T<0$ τότε $F(z)=0$

  Άρα, $\vec{H}_1 = \hat{x}\cdot\frac{E_0}{\mu_0 c}\exp[-(t+\frac{z}{c})/T]$

- $\vec{H}_2 = -\hat{x} H_0 \exp[-(t-\frac{z}{c})/T]$, $z>0$

- $\frac{\partial \vec{E}_2}{\partial t} = \frac{1}{\varepsilon_0}\nabla\times\vec{H}_2 = \hat{y}\frac{1}{\varepsilon_0}\frac{\partial H_{2x}}{\partial z} = -\hat{y}\frac{H_0}{\varepsilon_0 cT}\exp[-(t-\frac{z}{c})/T] \Rightarrow$

  $\Rightarrow \vec{E}_2 = \hat{y}[\frac{H_0}{\varepsilon_0 c}\exp[-(t-\frac{z}{c})/T] + G(z)]$

  Όμοια, $G(z)=0$

  Άρα $\vec{E}_2 = \hat{y}[\frac{H_0}{\varepsilon_0 c}\exp[-(t-\frac{z}{c})/T]]$

▷ Ορ. Συνθ. στο $z=0$:

i) $\hat{z}\times[\vec{E}_2(z=0) - \vec{E}_1(z=0)] = \vec{0} \Rightarrow \frac{H_0}{\varepsilon_0 c}\exp(-\frac{t}{T}) = E_0\exp(-\frac{t}{T}) \Rightarrow \boxed{H_0 = E_0\cdot\varepsilon_0\cdot c}$

ii) $\hat{z}\times[\vec{H}_2(z=0) - \vec{H}_1(z=0)] = \vec{K}(t) \Rightarrow \boxed{\vec{K}(t) = -\hat{y}[H_0 + \frac{E_0}{\mu_0 c}]\exp(-\frac{t}{T})}$

iii) $\hat{z}\cdot\{\varepsilon_0[\vec{E}_2(z=0) - \vec{E}_1(z=0)]\} = \sigma(t) \Rightarrow \boxed{\sigma(t)=0}$

Άσκηση 2 (5.12)

Από την 4.8 : $\vec{E} = \hat{z} E_0 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \sin\omega t \Rightarrow \vec{E} = \hat{z} E_0 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}$

• $\vec{H} = \hat{x}\left[-\frac{E_0 \pi}{\omega\mu_0 b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} \sin\omega t\right] - \hat{y}\left[-\frac{E_0\pi}{\omega\mu_0 a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} \sin\omega t\right] \Rightarrow$

$\Rightarrow \vec{H} = \hat{x}\left[-\frac{E_0\pi}{j\omega\mu_0 b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b}\right] - \hat{y}\left[-\frac{E_0\pi}{j\omega\mu_0 a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b}\right]$

Άρα, $\vec{H}^* = \hat{x}\left[\frac{E_0\pi}{j\omega\mu_0 b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b}\right] - \hat{y}\left[\frac{E_0\pi}{j\omega\mu_0 a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b}\right]$

• $\langle \vec{N} \rangle \frac{1}{2} Re\{\vec{E} \times \vec{H}^*\}$

• $\vec{E} \times \vec{H}^* = \hat{y}\left[\frac{E_0^2 \pi}{j\omega\mu_0 b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b}\right] + \hat{x}\left[\frac{E_0^2\pi}{j\omega\mu_0 a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b}\right]$

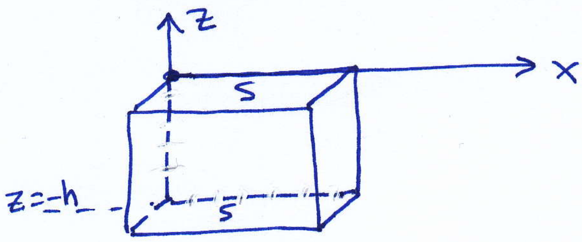
Οπότε $\langle \vec{N} \rangle = \vec{0}$

• $\langle w \rangle = \langle W_e \rangle + \langle W_m \rangle = \frac{1}{4}\varepsilon_0 E^2 + \frac{1}{4}\mu_0 H^2 =$

$= \varepsilon_0 E_0^2 \sin^2\left(\frac{\pi x}{a}\right) \sin^2\left(\frac{\pi x}{b}\right) \cos^2(\omega t) + \frac{E_0^2 \pi^2}{\omega^2 \mu_0 b^2} \sin^2\left(\frac{\pi x}{a}\right) \cos^2\left(\frac{\pi y}{b}\right) \sin^2(\omega t) +$

$+ \frac{E_0^2\pi^2}{\omega^2\mu_0 a^2} \cos^2\left(\frac{\pi x}{a}\right) \sin^2\left(\frac{\pi y}{b}\right) \sin^2(\omega t)$

Άσκηση 3 (5.13)



- $$\vec{N} = \vec{E}\times\vec{H} = \begin{cases} -\hat{z}\,\dfrac{E_0^2}{\mu_0 c}\exp\left[-2\left(t+\frac{z}{c}\right)/T\right] = \vec{N}_1, & z<0 \\ \hat{z}\,\dfrac{H_0^2}{\varepsilon_0 c}\exp\left[-2\left(t-\frac{z}{c}\right)/T\right] = \vec{N}_2, & z>0 \end{cases}$$

- $P_{μετ} = -\vec{E}\cdot\vec{J} = 0$, για $\vec{J} = \vec{0}$

- $$w = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 = \begin{cases} \frac{1}{2}\varepsilon_0 E_0^2 \exp\left[-2\left(t+\frac{z}{c}\right)\right] + \frac{1}{2}\dfrac{E_0^2}{\mu_0 c^2}\exp\left[-2\left(t+\frac{z}{c}\right)/T\right], & z<0 \\ \frac{1}{2}\dfrac{H_0^2}{\varepsilon_0 c^2}\exp\left[-2\left(t-\frac{z}{c}\right)\right] + \frac{1}{2}H_0^2\mu_0\exp\left[-2\left(t-\frac{z}{c}\right)/T\right], & z>0 \end{cases}$$

- $$\frac{\partial}{\partial t}\int_V w(t,z)\,dV = \frac{1}{2}\frac{\partial}{\partial t}\left[\exp\left(-\frac{2t}{T}\right)\right]\cdot\left[\varepsilon_0 E_0^2 + \frac{E_0^2}{\mu_0 c^2}\right]\int_{z=-h}^{0}\int_{s'=0}^{S}\exp\left(-\frac{2z}{cT}\right)dz\,ds' =$$

όγκος που περικλείεται από το παραλληλεπίπεδο

$$= \frac{S}{2}\left[E_0^2\varepsilon_0 c + \frac{E_0^2}{\mu_0 c}\right]\cdot\left[\exp\left(-\frac{2t}{T}\right) - \exp\left[-2\left(t-\frac{h}{c}\right)/T\right]\right]$$

- Poynting: $\displaystyle\oint_S \vec{N}\,d\vec{s} + \frac{\partial}{\partial t}\int_V w\,dV = \int_V P_{μετ} \Rightarrow$

$$\Rightarrow \iint N_z\,dx\,dy = \frac{S}{2}\left[E_0^2\varepsilon_0 c + \frac{E_0^2}{\mu_0 c}\right]\cdot\left[\exp\left(-\frac{2t}{T}\right) - \exp\left[-2\left(t-\frac{h}{c}\right)/T\right]\right] \Rightarrow$$

$$\Rightarrow -N_2(z=0)\cdot S - N_1(z=-h)\cdot S = -\frac{S}{2}\left[E_0^2\varepsilon_0 c + \frac{E_0^2}{\mu_0 c}\right]\cdot\left[\exp\left(-\frac{2t}{T}\right) - \exp\left(-2\left(t-\frac{h}{c}\right)/T\right)\right]$$

Και ισχύει: • $N_2(z=0)\cdot S = \dfrac{S}{2}\left[E_0^2\varepsilon_0 c + \dfrac{E_0^2}{\mu_0 c}\right]\exp\left(-\dfrac{2t}{T}\right) \Rightarrow$

$$\Rightarrow \frac{2H_0^2}{\varepsilon_0 c} = E_0^2\varepsilon_0 c + \frac{E_0^2}{\mu_0 c} \xRightarrow{H_0 = E_0\varepsilon_0 c} E_0^2\varepsilon_0 c = \frac{E_0^2}{\mu_0 c} \Rightarrow \varepsilon_0\mu_0 = \frac{1}{c^2} = \frac{1}{(\sqrt{\varepsilon_0\mu_0})^2}$$

• $N_1(z=-h)\cdot S = -\dfrac{S}{2}\left[E_0^2\varepsilon_0 c + \dfrac{E_0^2}{\mu_0 c}\right]\exp\left(-2\left(t-\dfrac{h}{c}\right)\right) \Rightarrow$

$$\Rightarrow \frac{2E_0^2}{\mu_0 c} = \frac{E_0^2}{\mu_0 c} + E_0^2\varepsilon_0 \Rightarrow \varepsilon_0\mu_0 = \frac{1}{c^2}$$

(επαληθεύεται)