ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ

ΣΧΟΛΗ ΗΛΕΚΤΡΟΛΟΓΩΝ ΜΗΧΑΝΙΚΩΝ ΚΑΙ ΜΗΧΑΝΙΚΩΝ ΥΠΟΛΟΓΙΣΤΩΝ

ΗΛΕΚΤΡΟΜΑΓΝΗΤΙΚΑ ΠΕΔΙΑ Β

(2020-2021)

1^{η} Σειρά Ασκήσεων

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Σειρά Ασκήσεων $1^η$

2020 - 2021

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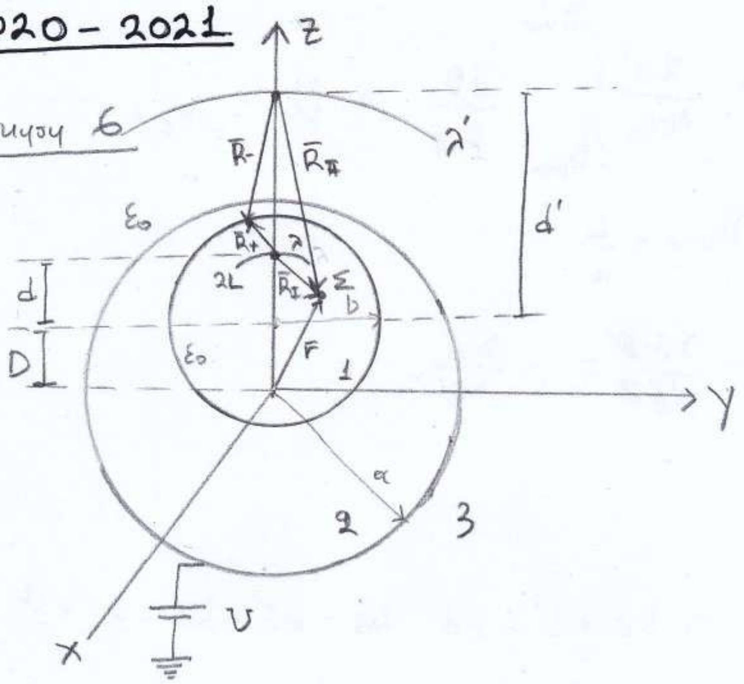
Άσκηση 6

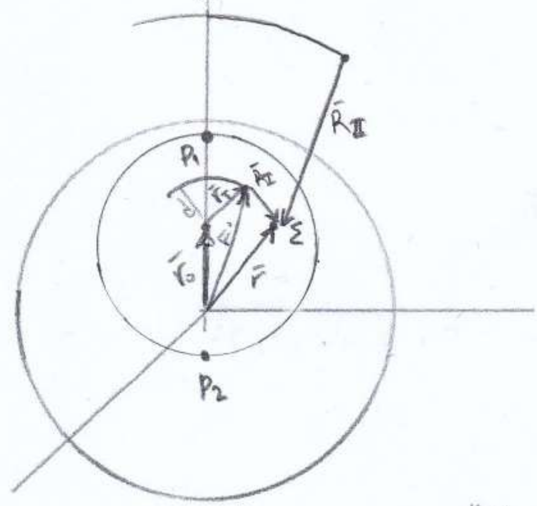


α) Περιοχή 1 : $\Phi=0 \Rightarrow \Phi = \frac{1}{4\pi\varepsilon_0}\int dq'\left(\frac{1}{R^+} + \frac{1}{R^-}\right) = 0 \Rightarrow$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R^+} + \frac{q'}{R^-}\right) = 0 \Rightarrow \frac{q}{R^+} + \frac{q'}{R^-} = 0 \Rightarrow$$

$$\Rightarrow \frac{R^-}{R^+} = -\frac{q'}{q} > 0 \Rightarrow q'\cdot q < 0$$

όπου $q = \lambda\cdot d\cdot d\theta$, $q' = \lambda'\cdot d'\cdot d\theta$



$$\left.\begin{array}{l} \bar{R}_I = \bar{r} - \bar{r}' \\ \bar{r} = x\hat{x} + y\hat{y} + z\hat{z} \\ \bar{r}' = \bar{r}_T + \bar{r}_0 \\ \left.\begin{array}{l} \bar{r}_T = d\cos\varphi\,\hat{y} + d\sin\varphi\,\hat{z} \\ \bar{r}_0 = D\cdot\hat{z} \end{array}\right\} \Rightarrow \bar{r}' = d\cos\varphi\,\hat{y} + [d\sin\varphi + D]\,\hat{z} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \bar{R}_I = x\hat{x} + (y - d\cos\varphi)\hat{y} + (z - d\sin\varphi - D)\hat{z}$$

και όμοια, $\bar{R}_{II} = x\hat{x} + (y - d'\sin\theta)\hat{y} + (z - d'\cos\theta - D)\hat{z}$

Και έχουν μέτρο, $|\bar{R}_I| = R_I = \sqrt{x^2 + (y - d\cos\varphi)^2 + (z - d\sin\varphi - D)^2}$

$$|\bar{R}_{II}| = R_{II} = \sqrt{x^2 + (y - d'\sin\theta)^2 + (z - d'\cos\theta - D)^2}$$

Αφού στην διαχωριστική επιφάνεια της σφαιρικής κοιλότητας δεν υπάρχουν επιφ. ρεύματα, και το δυναμικό είναι συνεχές, θα πρέπει και μέσα στην κοιλότητα να αθροιστεί το δυναμικό U.

Συνεπώς, $$\Phi_I(x,y,z) = \frac{\lambda \cdot d}{4\pi\varepsilon_0}\int_{-\theta_{max}}^{\theta_{max}} \frac{d\theta}{R_I} + \frac{\lambda' \cdot d'}{4\pi\varepsilon_0}\int_{-\theta_{max}}^{\theta_{max}} \frac{d\theta}{R_{II}} + U \quad , r_\Sigma \le b$$

όπου $r_\Sigma = \sqrt{x^2+y^2+(z-D)^2}$, $\theta_{max} = \frac{L}{d}$

Επίσης, $P_1: \frac{b-d}{d'-b} = -\frac{|R+|}{|R-|} = -\frac{q'}{q} = -\frac{\lambda \cdot d \cdot \cancel{d\theta}}{\lambda' d' \cancel{d\theta}} = -\frac{\lambda d}{\lambda' d'}$

$P_2: \frac{d+b}{d'+b} = -\frac{\lambda \cdot d}{\lambda' d'}$

Όμως $P_1 = P_2 \Rightarrow \frac{b-d}{d'-b} = \frac{d+b}{d'+b} \Rightarrow bd' + b^2 - dd' - bd = dd' + bd' - bd - b^2 \Rightarrow$

$$\Rightarrow 2b^2 = 2dd' \Rightarrow b^2 = dd' \Rightarrow d' = \frac{b^2}{d}$$

οπότε $\frac{d+b}{d'+b} = \frac{d+b}{\frac{b^2}{d}+b} = \frac{d(d+b)}{b^2+db} = \frac{d(d+b)}{b(d+b)} = \frac{d}{b} \Rightarrow -\frac{\lambda d}{\lambda' d'} = \frac{d}{b} \Rightarrow$

$$\Rightarrow \lambda' = -\lambda\frac{d}{b}$$

Περιοχή 2 : Αφού η σφαίρα είναι αγώγιμη, θα ισχύει $\Phi = U$, $r \le a$, $r_\Sigma \ge b$

Περιοχή 3 : $\nabla^2\Phi_{II} = 0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi_{II}}{\partial r}\right) = 0 \Rightarrow$

$$\Rightarrow \Phi_{II}(r) = -\frac{C_1}{r} + C_2$$

για $\Phi_{II}(r\to\infty) = 0 \Rightarrow C_2 = 0$

για $\Phi_{II}(r=a) = U \Rightarrow C_1 = -aU$

Άρα $\Phi_{II}(r) = U \cdot \frac{a}{r}$, $r \ge a$, όπου $r = \sqrt{x^2+y^2+z^2}$

β) Περιοχή 1:

$$\bar{E}_I(x,y,z) = \frac{1}{4\pi\varepsilon_0}\int_{-\vartheta_{max}}^{\vartheta_{max}} \frac{dq}{R_I^2}\,\hat{\ell}_I + \frac{1}{4\pi\varepsilon_0}\int_{-\vartheta_{max}}^{q_{max}} \frac{dq'}{R_{II}^2}\,\hat{\ell}_{II} \quad , r_\Sigma \leq b$$

όπου $q = \lambda \cdot d \cdot d\theta$, $q' = \lambda' d' d\theta$, $\hat{\ell}_I = \frac{\bar{R}_I}{|\bar{R}_I|}$, $\hat{\ell}_{II} = \frac{\bar{R}_{II}}{|\bar{R}_{II}|}$, θ

Περιοχή 3:

$\cdot\ \bar{E}_{III}(x,y,z) = -\bar{\nabla}\Phi$

$\cdot\ \bar{\nabla}\Phi_I = 0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi_I}{\partial r}\right) = 0 \Rightarrow \Phi_I(r) = -\frac{C_1}{r} + C_2$

για $\Phi_I(r \to \infty) = 0 \Rightarrow C_2 = 0$

για $\Phi_I(r = a) = V \Rightarrow C_1 = -aV$

όμως $\Phi_I(r) = V\frac{a}{r}$, $r \geq a$

Άρα $\bar{E}_{II}(x,y,z) = -\bar{\nabla}\Phi = V\frac{a}{r}\hat{r} = V\frac{a}{r^3}\cdot\bar{r}$, όπου $r = \sqrt{x^2+y^2+z^2}$, $r \leq a$

Περιοχή 2:

$\bar{E}_{II}(x,y,z) = 0$, $r \leq a$, $r_\Sigma \geq b$

6^{η} Άσκηση

α – β) Η επίλυση φαίνεται στις χειρόγραφες σελίδες.

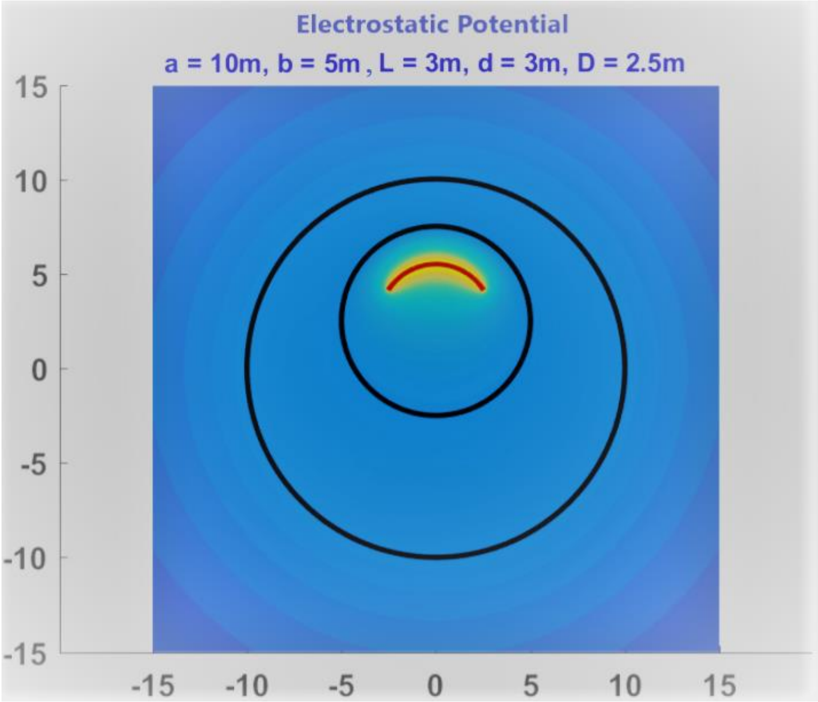
γ) Παρουσιάζεται αποσπασματικά ο κώδικας Matlab:

```
clear;

% initialization
a=10; b=5; d=3; D=2.5; L=3;
theta_max=L/d;
d1=(b^2)/d;

% set Electric Field, Potential & values for y, z
% set surface density in [0, 2*pi]
% set values
val1 = y - d*sin(theta);
val2 = z - d*cos(theta);
val3 = y - d1*sin(theta);
val4 = z - d1*cos(theta);

R = @(theta,y,z) ((val1)^2 + (val2)^2)^0.5;
R1 = @(theta,y,z) ((val3)^2 + (val4)^2)^0.5;

Vy = @(theta,y,z) [(d*(val1))/[R(theta,y,z)^3]] - [(b*(val3))/[R1(theta,y,z)^3]];
Vz = @(theta,y,z) [(d*(val2))/[R(theta,y,z)^3]] - [(b*(val4))/[R1(theta,y,z)^3]];

% set the integral of electrostatic potential of an electric field
potential=@(theta,y,z) (d/R(theta,y,z))-b/R1(theta,y,z);

%calculation of elec. field depending on the location of the point among the circles
for i_y = 1:length(yy)
    for i_z = 1:length(zz)
        y0 = Y(i_y,i_z);
        z0 = Z(i_y,i_z);
        rk = sqrt(y0^2 + (z0-D)^2);
        rs = sqrt(y0^2 + z0^2);

        % inner circle
        if (rk<b)
            z0=z0-D;
            F(i_y,i_z)=integral(@theta)potential(theta, y0, z0),-1, 1, 'RelTol', 0,
'AbsTol', 1e-12) + 1;
            Ey(i_y,i_z)=integral(@theta)Vy(theta,y0,z0),-1,1);
            Ez(i_y,i_z)=integral(@theta)Vz(theta,y0,z0),-1,1);
            if(F(i_y,i_z)>6)
                F(i_y,i_z)=6;
            end
            else
```

```
            % intermediate circle
            if (rs<a)
                F(i_y,i_z)=1;
                Ey(i_y,i_z)=0;
                Ez(i_y,i_z)=0;
            else
                F(i_y,i_z)=a/rs;
                Ey(i_y,i_z)=(a*y0)/(rs^3);
                Ez(i_y,i_z)=(a*z0)/(rs^3);
            end
        end
    end
end

% for the depiction

th = 0:pi/50:2*pi;
x_axis = a*cos(th);
y_axis = a*sin(th);
z_axis = 6 + zeros(size(x_axis));
x1_axis = b*cos(th);
y1_axis = b*sin(th) + D;
z1_axis = 6 + zeros(size(x1_axis));
```

Παρουσιάζεται η γραφική παράσταση:

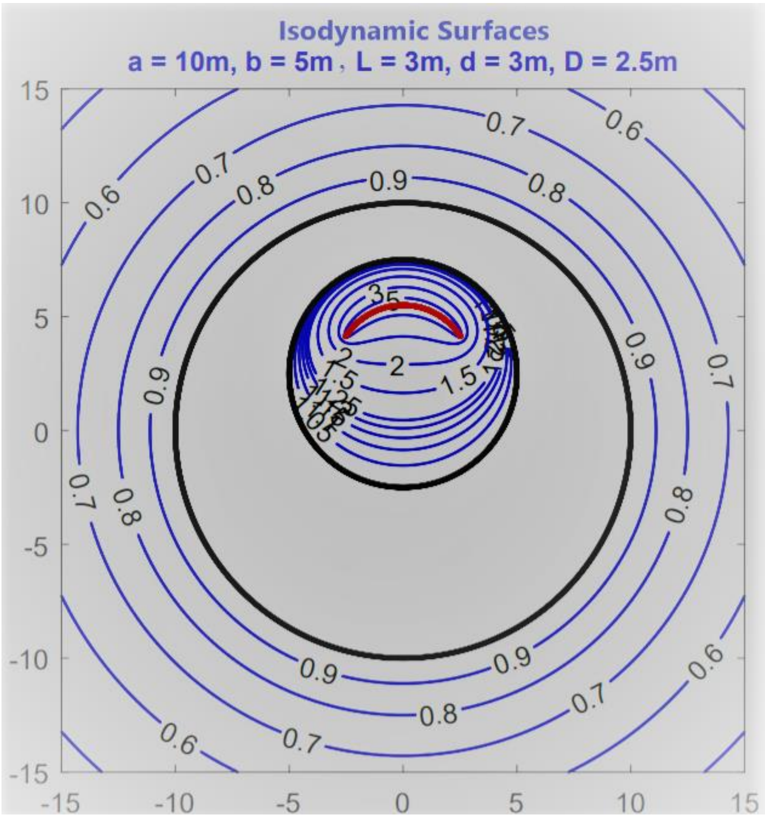
Για το Ηλεκτροστατικό Δυναμικό:



Παρουσιάζεται αποσπασματικά ο κώδικας Matlab:

```
% set Isodynamic Surfaces

cont = [0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1 1.05 1.1 1.25 2 3 5];
fig = fig +1;
figure(fig);

[CS, H] = contour(Y, Z, F, cont, 'Linewidth', 2, 'Color', 'b');
clabel(CS, H, cont);
hold on
set(gca, 'Fontsize', 14)
xlabel('y', 'Fontsize', 14)
ylabel('z', 'Fontsize', 14)
title('Isodynamic Surfaces')
axis equal
grid on
hold off
```

Για Ισοδυναμικές Επιφάνειες:

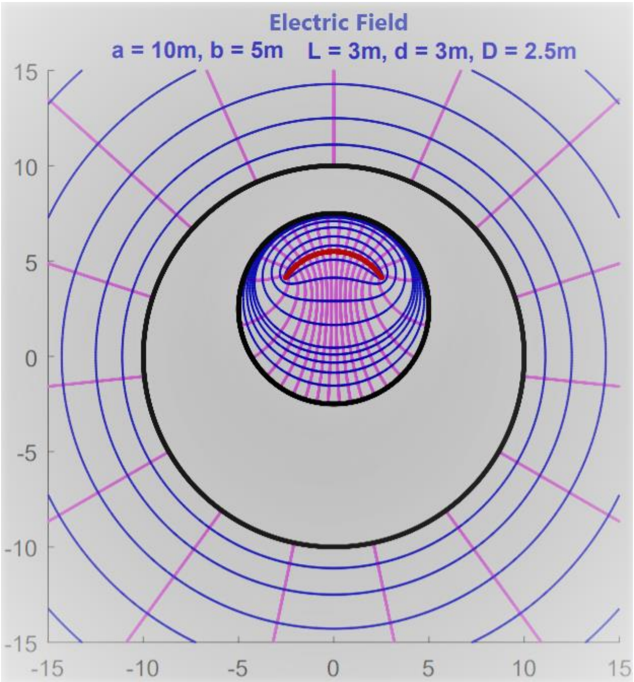


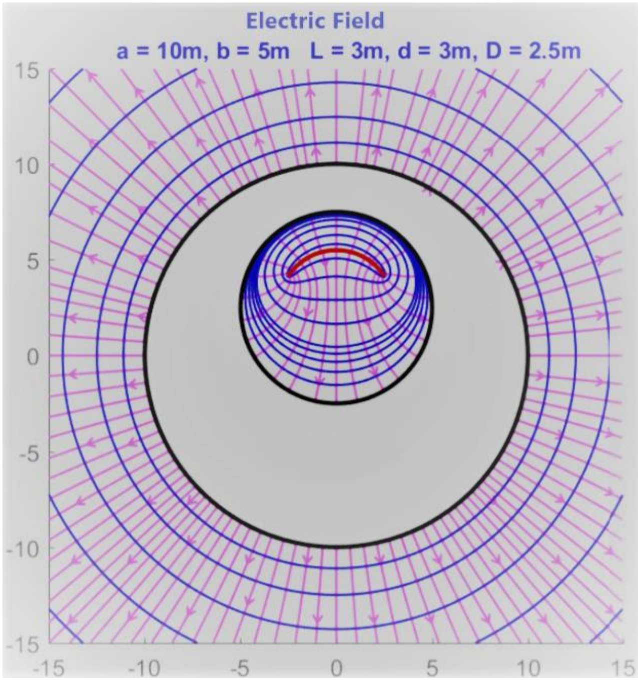
δ) Παρουσιάζεται αποσπασματικά ο κώδικας Matlab:

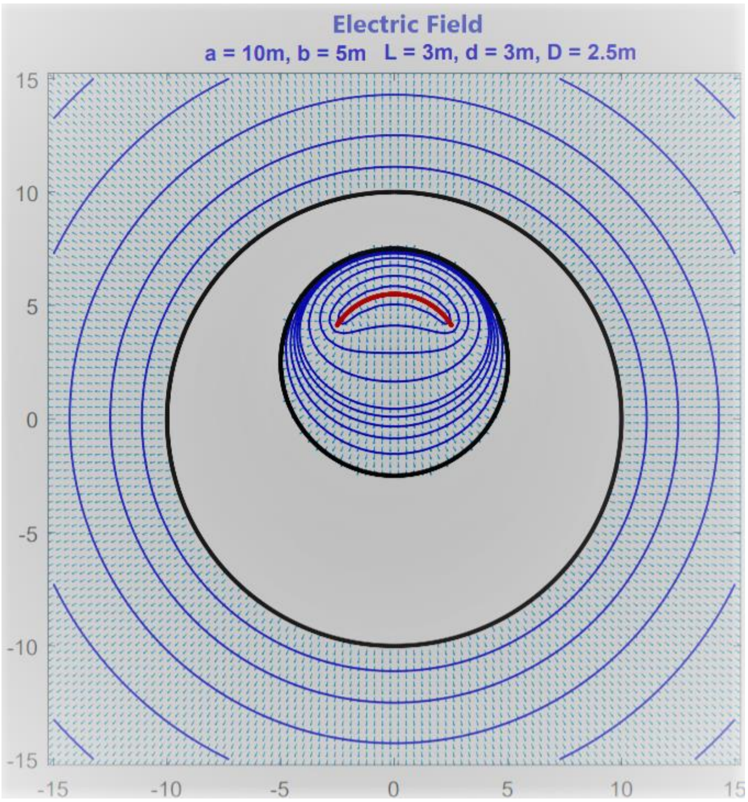
```
% plot with streamslice / quiver
figure(fig);
streamslice(Y, Z, Ey, Ez);
hold on
[CS, H] = contour(Y, Z, F, cont, 'Linewidth', 2, 'Color', 'b');
plot3(x_axis, y_axis, z_axis, 'Linewidth', 2, 'Color', 'b');
plot3(x1_axis, y1_axis, z1_axis, 'Linewidth', 2, 'Color', 'b');
set(gca, 'Fontsize', 14);
xlabel('y', 'Fontsize', 14)
ylabel('z', 'Fontsize', 14)
title('Electric Field')
axis equal
grid on
hold off

% Normalized Potential
fig=fig+1;
Lnorm = ((Ey)^2 + (Ez)^2)^0.5;
figure(fig);
quiver(Y, Z, Ey/Lnorm, Ez/Lnorm, 1/2)
hold on
[CS, H] = contour(Y, Z, F, cont, 'Linewidth', 2, 'Color', 'b');
plot3(x_axis, y_axis, z_axis, 'Linewidth', 2, 'Color', 'b');
plot3(x1_axis, y1_axis, z1_axis, 'Linewidth', 2, 'Color', 'b');
set(gca, 'Fontsize', 14);
xlabel('y', 'Fontsize', 14)
ylabel('z', 'Fontsize', 14)
title('Electric Field')
axis equal
grid on
hold off
fig=fig+1;
```

Παρουσιάζονται οι γραφικές παραστάσεις:

Για το Ηλεκτρικό Πεδίο (εντός & εκτός):







ε) Η επαγόμενη επιφανειακή πυκνότητα φορτίου είναι:

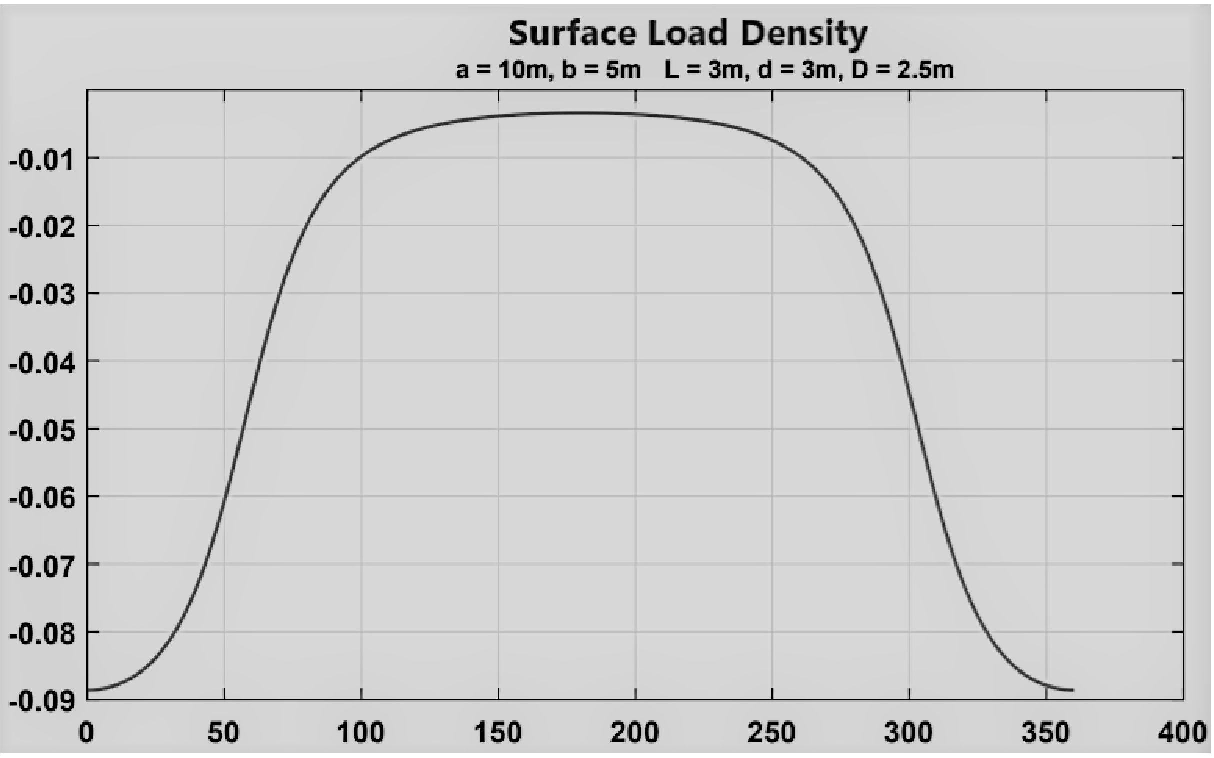
$\sigma_b(\theta, \varphi = \pi/2) = -\, \varepsilon_0 * [\frac{y}{r_b} E_y(y, z) + \frac{z-D}{r_b} E_z(y, z)]$

Όπου $r_b = [y^2 + (z - D)^2]^{1/2}$, $y = b*\sin(\theta)$, $z = b*\cos\theta + D$

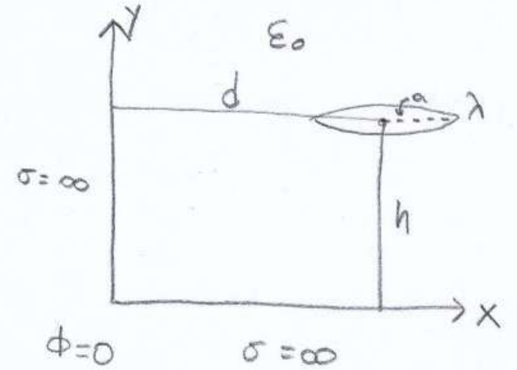
Παρατίθεται ενδεικτικά ο κώδικας Python:

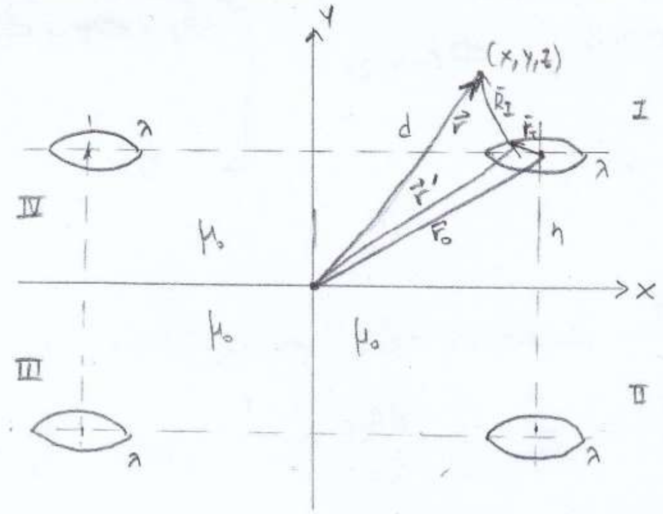
```
import numpy as np
import scipy as sp
from scipy import integrate
import matplotlib
import matplotlib.pyplot as plt

def f(t):
    def x(theta): return (-2.5*8.85)/((5-4*np.cos(t - theta))**1.5)
    a, error = integrate.quad(x, -0.75, 0.75)
    return a

f2 = np.vectorize(f)
t = np.arange(0.0, 2*np.pi, 0.001)
fig, ax = plt.subplots()
ax.plot(t, f2(t), color='blue')
ax.set(xlabel='θ', ylabel='σ', title='Surface Load Desnity')
plt.show()
```

Παρουσιάζεται η γραφική παράσταση:



Άσκηση 7





α) Με χρήση ειδώλων το δυναμικό υπολογίζεται ως εξής:

$$\Phi(x,y,z) = \Phi_I(x,y,z) + \Phi_{II}(x,y,z) + \Phi_{III}(x,y,z) + \Phi_{IV}(x,y,z)$$

▷ Για Φ_I: $\bar{R}_I = \bar{r} - \bar{r}'$

όπου:

$$\left.\begin{array}{l} \bar{r} = x\hat{x} + y\hat{y} + z\hat{z} \\ \left.\begin{array}{l} \bar{r}' = \bar{r}_0 + \bar{r}_T \\ \bar{r}_0 = d\hat{x} + h\hat{y} \\ \bar{r}_T = a\cos\varphi\,\hat{x} + a\sin\varphi\,\hat{z} \end{array}\right\} \Rightarrow \bar{r}' = [d + a\cos\varphi]\hat{x} + h\hat{y} + a\sin\varphi\,\hat{z} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \bar{R}_I = \bar{r} - \bar{r}' = x\hat{x} + y\hat{y} + z\hat{z} - \left[[d + a\cos\varphi]\hat{x} + h\hat{y} + a\sin\varphi\,\hat{z}\right] =$$

$$= (x - d - a\cos\varphi)\hat{x} + (y - h)\hat{y} + (z - a\sin\varphi)\hat{z}$$

Οπότε $R_I = |\bar{R}_I| = \sqrt{(x-d-a\cos\varphi)^2 + (y-h)^2 + (z-a\sin\varphi)^2}$

Άρα, $$\Phi_I(x,y,z) = \frac{1}{4\pi\varepsilon_0}\int \frac{dq_I}{R_I} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\lambda\cdot a}{R_I}\,d\varphi =$$

$$= \frac{\lambda\cdot a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{1}{R_I}\,d\varphi$$

▷ Ομοίως, • $R_{II} = |\bar{R}_{II}| = (x-d-a\cos\varphi)\hat{x} + (y+h)\hat{y} + (z-a\sin\varphi)\hat{z}$

$$\Phi_{II}(x,y,z) = \frac{\lambda\cdot a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{1}{R_{II}}\,d\varphi$$

• $R_{III} = |\bar{R}_{III}| = (x+d-a\cos\varphi)\hat{x} + (y+h)\hat{y} + (z-a\sin\varphi)\hat{z}$

$$\Phi_{III}(x,y,z) = \frac{\lambda\cdot a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{1}{R_{III}}\,d\varphi$$

• $R_{IV} = |\bar{R}_{IV}| = (x+d-a\cos\varphi)\hat{x} + (y-h)\hat{y} + (z-a\sin\varphi)\hat{z}$

$$\Phi_{IV}(x,y,z) = \frac{\lambda a}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{1}{R_{IV}}\,d\varphi$$

Συνεπώς, $\Phi(x,y,z) = \begin{cases} \Phi_I + \Phi_{II} + \Phi_{III} + \Phi_{IV} & , x,y \geq 0 \\ 0 & , \text{αλλού} \end{cases}$

β) Το ηλεκτρικό πεδίο υπολογίζεται ως εξής:

▷ $\bar{E}_I = \frac{1}{4\pi\varepsilon_0} \int \frac{dq_I}{R_I^2} \hat{l}_I$, όπου $\hat{l}_I = \frac{\bar{R}_I}{|\bar{R}_I|}$

Οπότε, $\bar{E}_I = \frac{\lambda \cdot a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\bar{R}_I}{R_I^3} d\varphi = \frac{\lambda \cdot a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{(x-d-a\cos\varphi)\hat{x} + (y-h)\hat{y} + (z-a\sin\varphi)\hat{z}}{R_I^3} d\varphi$

▷ Ομοίως,

• $\bar{E}_{II} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq_{II}}{R_{II}^2} \hat{l}_{II}$, όπου $\hat{l}_{II} = \frac{\bar{R}_{II}}{|\bar{R}_{II}|}$

οπότε, $\bar{E}_{II} = -\frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\bar{R}_{II}}{R_{II}^3} d\varphi = -\frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{(x-d-a\cos\varphi)\hat{x} + (y+h)\hat{y} + (z-a\sin\varphi)\hat{z}}{R_{II}^3} d\varphi$

• $\bar{E}_{III} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq_{III}}{R_{III}^2} \hat{l}_{III}$, όπου $\hat{l}_{III} = \frac{\bar{R}_{III}}{|\bar{R}_{III}|}$

οπότε, $\bar{E}_{III} = \frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\bar{R}_{III}}{R_{III}^3} d\varphi = \frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{(x+d-a\cos\varphi)\hat{x} + (y+h)\hat{y} + (z-a\sin\varphi)\hat{z}}{R_{III}^3} d\varphi$

• $\bar{E}_{IV} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq_{IV}}{R_{IV}^2} \hat{l}_{IV}$, όπου $\hat{l}_{IV} = \frac{\bar{R}_{IV}}{|\bar{R}_{IV}|}$

οπότε, $\bar{E}_{IV} = -\frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\bar{R}_{IV}}{R_{IV}^3} d\varphi = -\frac{\lambda a}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{(x+d-a\cos\varphi)\hat{x} + (y-h)\hat{y} + (z-a\sin\varphi)\hat{z}}{R_{IV}^3} d\varphi$

Συνεπώς, $\bar{E}(x,y,z) = \begin{cases} \bar{E}_I + \bar{E}_{II} + \bar{E}_{III} + \bar{E}_{IV} & , x,y \geq 0 \\ 0 & , \text{αλλού} \end{cases}$

γ) $\sigma = (D^+ - D^-)\hat{y} = \varepsilon_0 E_y^+(y=0) = \frac{\lambda \cdot a}{4\pi} \cdot \int_0^{2\pi} \left[\frac{h}{[R_I(y=0)]^3} - \frac{h}{[R_{II}(y=0)]^3} + \frac{h}{[R_{III}(y=0)]^3} + \frac{h}{[R_{IV}(y=0)]^4} \right]$

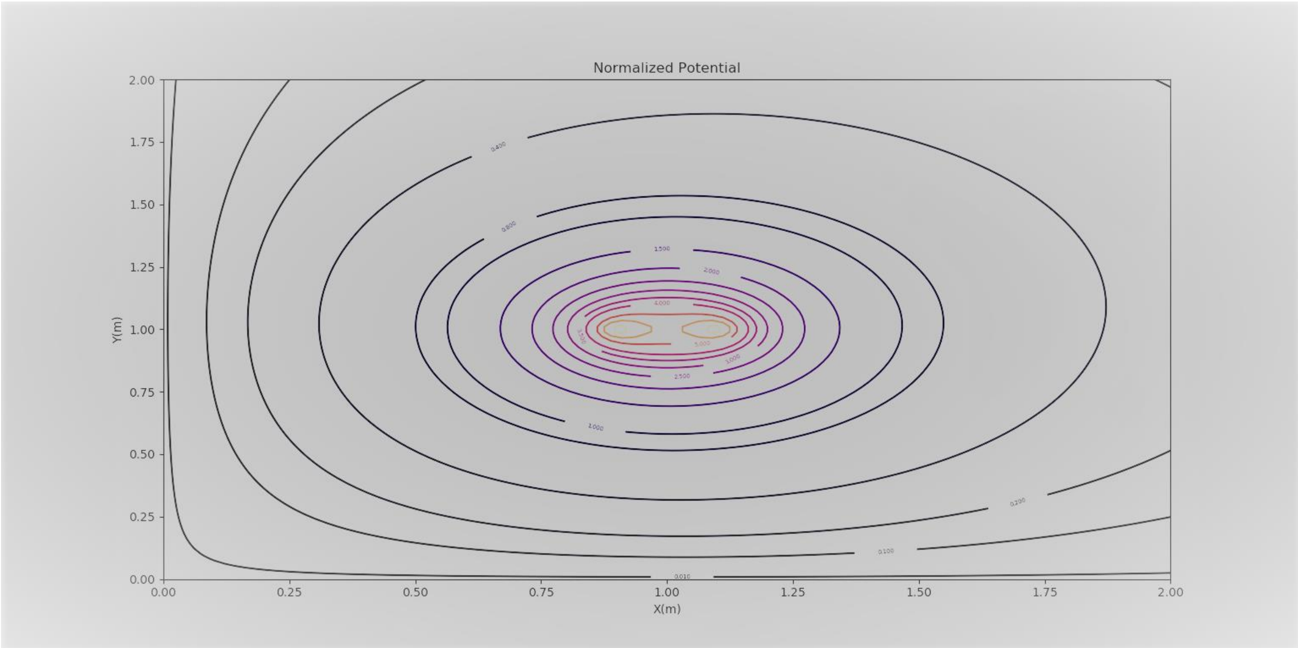
7^{η} Άσκηση

α – γ) Η επίλυση φαίνεται στις χειρόγραφες σελίδες.

δ) 1° σκέλος: Παρατίθεται ενδεικτικά ο κώδικας Python:

```
import numpy as np
import scipy as sp
from scipy import integrate
import matplotlib
import matplotlib.pyplot as plt
from mpl_toolkits import mplot3d

def f(x, y):
    def Fi1(TH): return (0.1)/(np.sqrt((x - 1 - 0.1*np.cos(TH)) ** 2 + (y-
1)**2 + 0.01*((np.sin(TH))**2)))
    fi1, error1 = integrate.quad(Fi1, 0, np.pi*2)

    def Fi2(TH): return (-0.1)/(np.sqrt((x + 1 - 0.1*np.cos(TH)) ** 2 + (y-
1)**2 + 0.01*((np.sin(TH))**2)))
    fi2, error2 = integrate.quad(Fi2, 0, np.pi*2)

    def Fi3(TH): return (0.1)/(np.sqrt((x + 1 - 0.1*np.cos(TH)) ** 2 + (y+1)**2 + 0.
01*((np.sin(TH))**2)))
    fi3, error3 = integrate.quad(Fi3, 0, np.pi*2)

    def Fi4(TH): return (-
0.1)/(np.sqrt((x - 1 - 0.1*np.cos(TH))** 2 + (y+1)**2 + 0.01*((np.sin(TH))**2)))
    fi4, error4 = integrate.quad(Fi4, 0, np.pi*2)
    return fi1 + fi2 + fi3 + fi4

f1 = np.vectorize(f)
x = np.linspace(0, 2, 100)
y = np.linspace(0, 2, 100)
X, Y = np.meshgrid(x, y)
Z = f1(X, Y)

fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z, zdir = 'xy', offset = 11,
                levels = [0.01, 0.1, 0.2, 0.4, 0.8, 1.0, 1.5, 2, 2.5, 3, 3.5, 4, 5,
6, 7.5],
                cmap = matplotlib.cm.magma,
                color = 'blue')
ax.clabel(CS, CS.levels, inline = True, fontsize = 5)
ax.set_title('Normalized Potential')
ax.set_xlabel('X(m)')
ax.set_ylabel('Y(m)')
plt.show()
```

Και η γραφική παράσταση είναι:



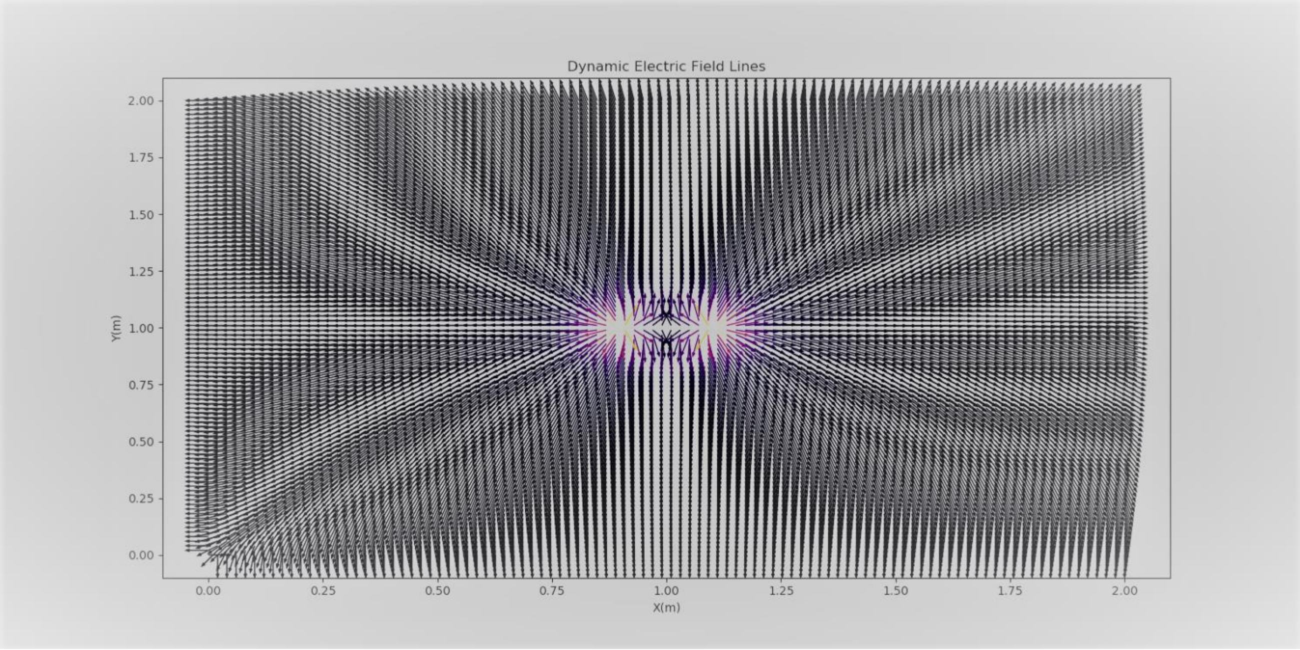
<u>2º Σκέλος</u>: Παρατίθεται ενδεικτικά ο κώδικας Python:

```
import numpy as np
import scipy as sp
from scipy import integrate
import matplotlib
import matplotlib.pyplot as plt
from mpl_toolkits import mplot3d

def f(x, y):
    def Ex1(TH): return (x - 1 - 0.1*np.cos(TH))/(((x - 1 - 0.1 * np.cos(TH))**2 + (
y - 1)**2 + 0.01*(np.sin(TH))**2)**1.5)
    ex1, error1 = integrate.quad(Ex1, 0, np.pi*2)

    def Ey1(TH): return (y-1)/(((x - 1 - 0.1*np.cos(TH)) ** 2 + (y-
1)**2 + 0.01 * (np.sin(TH))**2)**1.5)
    ey1, error1 = integrate.quad(Ey1, 0, np.pi*2)

    def Ex2(TH): return (-
1)*(x + 1 - 0.1*np.cos(TH))/(((x + 1 - 0.1*np.cos(TH))**2 + (y - 1)**2 + 0.01*(np.si
n(TH))**2)**1.5)
    ex2, error2 = integrate.quad(Ex2, 0, np.pi*2)

    def Ey2(TH): return (-1)*(y-1)/(((x + 1 - 0.1*np.cos(TH)) ** 2 + (y-
1)**2 + 0.01 * (np.sin(TH))**2)**1.5)
    ey2, error2 = integrate.quad(Ey2, 0, np.pi*2)
```

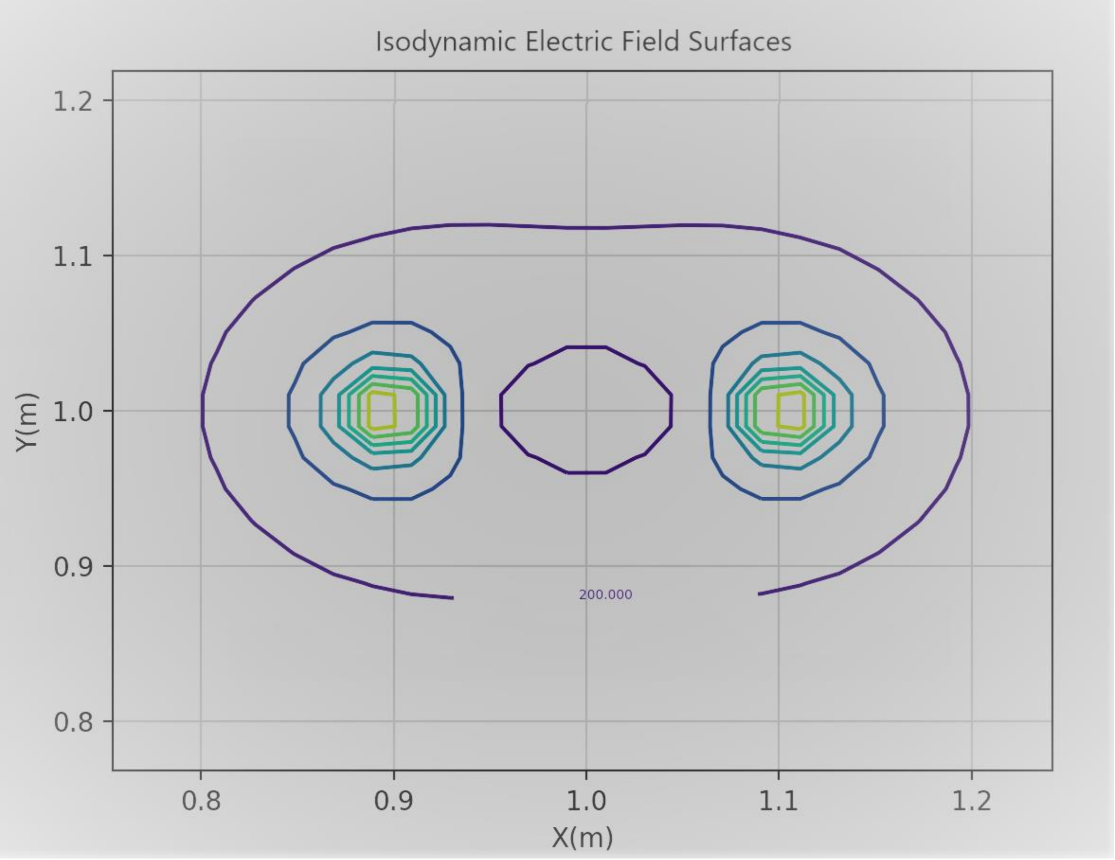
```
    def Ex3(TH): return (x + 1 - 0.1*np.cos(TH))/(((x + 1 - 0.1 * np.cos(TH))**2 + (
y + 1)**2 + 0.01*(np.sin(TH))**2)**1.5)
    ex3, error3 = integrate.quad(Ex3, 0, np.pi*2)

    def Ey3(TH): return (y+1)/(((x + 1 - 0.1*np.cos(TH)) ** 2 + (y+1)**2 + 0.01 * (n
p.sin(TH))**2)**1.5)
    ey3, error3 = integrate.quad(Ey3, 0, np.pi*2)

    def Ex4(TH): return (-
1)*(x - 1 - 0.1*np.cos(TH))/(((x - 1 - 0.1*np.cos(TH))**2 + (y + 1)**2 + 0.01*(np.si
n(TH))**2)**1.5)
    ex4, error4 = integrate.quad(Ex4, 0, np.pi*2)

    def Ey4(TH): return (-
1)*(y+1)/(((x - 1 - 0.1*np.cos(TH)) ** 2 + (y+1)**2 + 0.01 * (np.sin(TH))**2)**1.5)
    ey4, error4 = integrate.quad(Ey4, 0, np.pi*2)

    Ex = ex1 + ex2 + ex3 + ex4
    Ey = ey1 + ey2 + ey3 + ey4
    return Ex, Ey

f1 = np.vectorize(f)
x = np.linspace(0, 2, 100)
y = np.linspace(0, 2, 100)
X, Y = np.meshgrid(x, y)
Ex, Ez = f1(X, Y)

fig, ax = plt.subplots()
ax.quiver(X, Y, Ex/((Ex**2+Ez**2)**0.5), Ez/((Ex**2+Ez**2)**0.5), (Ex**2+Ez**2)**0.5
,cmap=matplotlib.cm.inferno, units='xy', scale=15, zorder=3, width=0.0035, headwidth
=3., headlength=4.)
ax.set_title('Dynamic Electric Field Lines')
ax.set_xlabel('X(m)')
ax.set_ylabel('Y(m)')
plt.show()
```

Και η γραφική παράσταση είναι:

Για τις Δυναμικές Γραμμές Ηλεκτρικού Πεδίου:



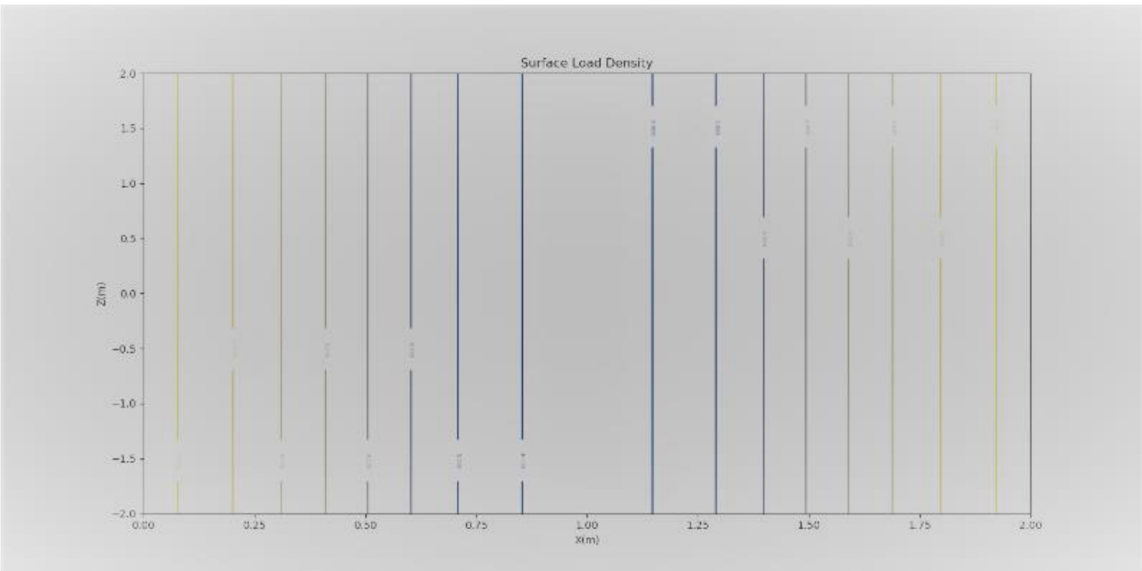
Για τις Ισοδυναμικές Επιφάνειες Ηλεκτρικού Πεδίου (ομοίως):



<u>3ο Σκέλος</u>: Παρατίθεται ενδεικτικά ο κώδικας Python:

```python
import numpy as np
import scipy as sp
from scipy import integrate
import matplotlib
import matplotlib.pyplot as plt
from mpl_toolkits import mplot3d

def f(x, y):
    e0 = 8.854 * (10**(-12))
    def Sx(TH): return (2*e0) / (((x + 1 - 0.1*np.cos(TH))**2 + 1 + 0.01*(np.sin(TH))**2)**1.5)
    sx, error1 = integrate.quad(Sx, 0, np.pi*2)
    def Sy(TH): return (-1)/(((x - 1 - 0.1*np.cos(TH)) ** 2 + 1 + 0.01 * (np.sin(TH))**2)**1.5)
    sy, error1 = integrate.quad(Sy, 0, np.pi*2)
    return sx+sy
f1 = np.vectorize(f)
x = np.linspace(0, 2, 100)
y = np.linspace(-2, 2, 100)
X, Y = np.meshgrid(x, y)
Z = f1(X, Y)
fig, ax = plt.subplots()
CS = ax.contour(X, Y, Z, zdir='xy', offset=11, cmap=matplotlib.cm.cividis, color='blue')
ax.clabel(CS, CS.levels, inline=True, fontsize=5)
ax.set_title('Surface Load Density')
ax.set_xlabel('X(m)')
ax.set_ylabel('Z(m)')
plt.show()
```

Και η γραφική παράσταση είναι:



Εναλλακτικά, σε Matlab, το αντίστοιχο απόσπασμα θα ήταν:

```
fig = fig +1;
figure(fig);

[CS, H] = contour(X, Z, Si, 'Linewidth', 2, 'Color', 'b');
clabel(CS, H, cont);
hold on
set(gca, 'Fontsize', 14)
xlabel('y', 'Fontsize', 14)
ylabel('z', 'Fontsize', 14)
title('Surface Load Density')
axis equal
grid on
hold off
```

Και η γραφική παράσταση είναι: