Εισαγωγή στα Συστήματα Ηλεκτρικής Ενέργειας

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Άσκηση 1

3Φ 6,6 kV / 50 Hz

$Z = R + jX$ ⅄

$S = 65\,kVA$

ΣΙ = 0,8 επαγωγικό ($\cos\theta = 0.8$)

α) $\cos\theta = 0{,}8$ άρα $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \sin\theta = \pm\sqrt{1^2 - 0{,}8^2} = \pm 0{,}6 \Rightarrow \sin\theta = 0{,}6$ (επαγωγ.)

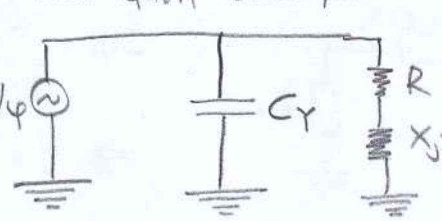
$P = S\cdot\cos\theta = 65k \cdot 0{,}8 = 52\,kW$

$Q = S\cdot\sin\theta = 65k \cdot 0{,}6 = 39\,kVA_r$

$S = 3V_\varphi I_\varphi \Rightarrow I_\varphi = \dfrac{S}{3V_\varphi} = 5.69\,A$

αφού $V_\varphi = \dfrac{V_n}{\sqrt{3}} = \dfrac{6600}{\sqrt{3}} = 3\,810{,}5\,V$

Ανά φάση ισοδύναμο:



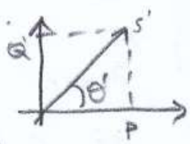
$I_L = I_\varphi \Rightarrow I_L = 5{,}69\,A$

$P = 3RI_L^2 \Rightarrow R = \dfrac{P}{3I_L^2} \Rightarrow R = 535{,}4\,\Omega$

$Q = 3XI_L^2 \Rightarrow X = \dfrac{Q}{3I_L^2} \Rightarrow X = 401{,}5\,\Omega$

β) ΣΙ' = 0,95 επαγ.

Εισάγεται μόνο χωρητική αντίσταση οπότε δεν επηρεάζει την ενεργό ισχύ P.
Όμως, επειδή αλλάζει η S και γίνεται S', αλλάζει και η Q σε Q'.



Άρα, $\Sigma I' = \cos\theta' = 0{,}95 \Rightarrow \sin\theta' = 0{,}31 \Rightarrow \tan\theta' = 0{,}328$

$Q' = \sin\theta'\cdot S' = \dfrac{\sin\theta'\cdot P}{\cos\theta'} = \tan\theta'\cdot P = 0{,}328 \cdot 52K = 17056\,VA_r$

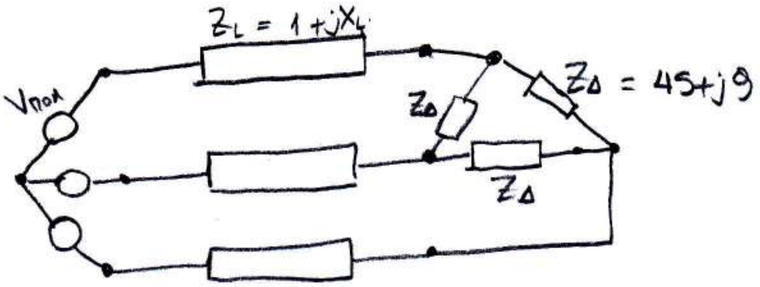
Άρα, άεργος ισχύς του C_Y: $Q_{C_Y} = Q' - Q = 17.056 - 39.000 = -21.944\,VA$

$$Q_{C_Y} = -\frac{3V_\varphi^2}{X_{C_Y}} = \frac{-3\cdot V_\varphi^2}{\frac{1}{\omega C_Y}} = \frac{-3V_\varphi^2}{\frac{1}{2\pi f C_Y}} = -6\pi f C_Y V_\varphi^2 \Rightarrow$$

$$\Rightarrow C_Y = -\frac{Q_{C_Y}}{6\pi f V_\varphi^2} = \frac{21.944}{6\pi 50 (3810{,}5)^2} = 1{,}6\,\mu F$$

Οπότε; $Z_{C_\Delta} = 3Z_{C_Y} \Rightarrow C_\Delta = \dfrac{C_Y}{3} \Rightarrow C_\Delta = 0{,}53\,\mu F$

Άσκηση 2



$f = 50Hz$

$Δ$

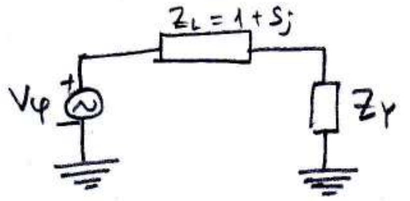
$Z_Δ = (45+j9)\Omega$

$V_n = 380V/50Hz \longrightarrow V_φ = \frac{V_n}{\sqrt{3}} = 219,4V \Rightarrow \hat{V}_φ = 219,4\angle 0$ (τάση αναφοράς)

$r = 1\Omega$, $L = 16mH$ $\Big\}$ $Z_L = 1 + jX_L$

α) $X_L = ωL = 2πfL = 5\Omega \Rightarrow Z_L = (1+5j)\Omega$

$Z_Y = \frac{Z_Δ}{3} = (15+j3)\Omega$



$\hat{I}_φ = \frac{\hat{V}_φ}{Z_L+Z_Y} = \frac{219,4\angle 0}{16+8j} = 12,3\angle -26,57° \ A \Rightarrow$

$\Rightarrow I = 12,3A$ και $I_{max} = \sqrt{2}\cdot I = 17,39 A$

β) $\hat{V}_{Load} = \hat{Z}_Δ \cdot \hat{I}_L = 3\underbrace{(15+3j)}_{Z_Y}(12,3\angle -26,57) \Rightarrow \hat{V}_{Load} = 564,46\angle -15,25° \ V$

γ) $\hat{S} = 3\hat{V}_φ \cdot \hat{I}_φ^* = 3(219,4\angle 0)(12,3\angle 26,57) = 8095,86\angle 26,57 \ VA$

$P = S\cdot\cos θ = 7241,47 \ W$

$Q = S\cdot\sin θ = 8095,86 \sin(26,57) = 3619,94 \ VAr$

$ΣΙ = \cos(26,56) = 0,894$

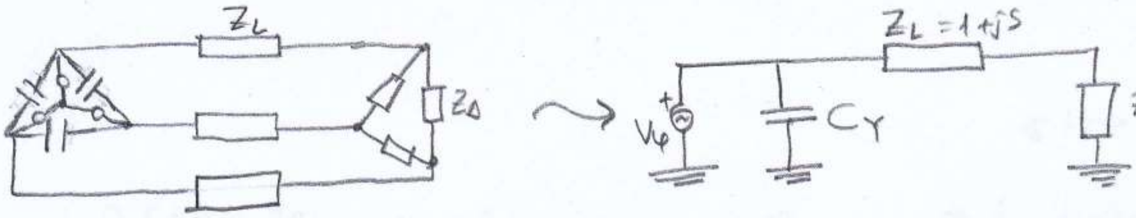
δ) $\hat{S}_{Load} = 3\hat{V}_{Load}\cdot\hat{I}_φ^* = 3\left(\frac{564,46}{\sqrt{3}}\angle -15,25\right)(12,3\angle 26,57) = 6942,75\angle 11,3 \ VA$

$P = S\cos θ = 6808,15W$

$Q = S\sin θ = 1360,4 \ VAr$

$ΣΙ = \cos θ = 0,98$ επαγ.

Άσκηση 3



$Z_L = 1+j5$
$Z_\varphi = 15+j3$
$V_\varphi = \frac{V_n}{\sqrt{3}} = 219{,}4\,V$

α) $\frac{1}{Z_{ολ}} = \frac{1}{Z_{CY}} + \frac{1}{Z_L+Z_Y} = \frac{1}{Z_{CY}} + 0{,}05 - 0{,}025j$

Για να γίνει ελάχιστο το ρεύμα: $\hat{I}_L = \frac{\hat{V}_\varphi}{\hat{Z}_{ολ}} = \hat{V}_\varphi\left[\frac{1}{Z_{CY}} - 0{,}025j + 0{,}05\right]$

Imin: με μηδενισμό πλάτους του phasor → απαλοιφή με το C_Y του μιγαδικού μέρους $Z_{ολ}$ →

$$\to \frac{1}{Z_{ολ,min}} = \cancel{\frac{1}{Z_{CY}}} + (0{,}05 - \cancel{0{,}025j}) = 0{,}05 \longrightarrow \frac{1}{Z_{CY}} = 0{,}025j \longrightarrow$$

$$\to Z_{CY} = -40j$$

Για τρίγωνο: $\hat{Z}_{C\Delta} = 3Z_{CY} = -120j\ \Omega \Rightarrow \hat{Y}_{C\Delta} = \frac{1}{\hat{Z}_{C\Delta}} = 8{,}\bar{3}\cdot 10^{-3} j\ \Omega$

$$\hat{Z}_{C\Delta} = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} \Rightarrow C = \frac{1}{j2\pi f Z_{C\Delta}} = \frac{1}{j\,2\pi 50(-120j)} = 26{,}5\,\mu F$$

β) $\hat{I}_{πηγ} = \frac{\hat{V}_\varphi}{\hat{Z}_{ολ}} = \frac{219{,}4\angle 0}{20} = 10{,}97\,A$

γ) $Q_C = \frac{3V_\varphi^2}{X_{CY}} = \frac{3(219{,}4)^2}{40} = 3610{,}2\ VA_r$

Άσκηση 4

3Φ Μ/Σ

40kVA, 20kV/380V, 50Hz

Συνδέσεις: ΥΤ: τρίγωνο, ΧΤ: αστέρας $\Big\{$ Φορτίο: $P_{Load} = 30kW$, ΣΙ = 0,8 επαγ $\Big\{$ Ανά φάση: $R''_{1\sigma} = 0{,}02\,\Omega$, $X''_{1\sigma} = 0{,}16\,\Omega$ $\Big\}$ ανηγμ. στο ΧΤ.

Ανά φάση ισοδ. κυκλ. του Μ/Σ σε α.μ. τιμές:

$$V_{B_1} = 20kV, \quad V_{B_2} = 380V, \quad S_B = 40kVA$$

$$I_{B_1} = \frac{S_B}{\sqrt{3}\,V_{B_1}} = 1{,}15A \quad , \quad I_{B_2} = \frac{S_B}{\sqrt{3}\,V_{B_2}} = 60{,}77A$$

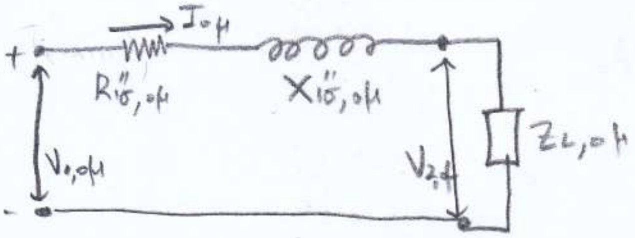
$$Z_{B_1} = \frac{V_{B_1}^2}{S_B} = 10k\Omega \quad , \quad Z_{B_2} = \frac{V_{B_2}^2}{S_B} = \frac{380^2}{40\cdot 10^3} = 3{,}61\Omega$$

Σύστημα α.μ.

$$R''_{1\sigma,\alpha\mu} = \frac{R''_{1\sigma}}{Z_{B_2}} = \frac{0{,}02\,\Omega}{3{,}61\,\Omega} = 5{,}54\cdot 10^{-3}\ \alpha\mu$$

$$X''_{1\sigma,\alpha\mu} = \frac{X''_{1\sigma}}{Z_{B_2}} = \frac{0{,}16\,\Omega}{3{,}61\,\Omega} = 44{,}3\cdot 10^{-3}\ \alpha\mu$$

$$V_{1,\alpha\mu} = \frac{20kV}{V_{B_1}} = 1\ \alpha\mu \quad , \quad V_{2,\alpha\mu} = \frac{380\,kV}{V_{B_2}} = 1\ \alpha\mu$$



$$\hat{I}_{\alpha\mu} = 0{,}75 - 0{,}56j = 0{,}936$$

▷ $\sin\varphi = \sqrt{1-\cos^2\varphi} = 0{,}6$

$$Q_L = S_L \cdot \sin\varphi = 37{,}5\cdot 10^3 \cdot 0{,}6 = 22{,}5\,kVAr$$

Για $\hat{V}_{2,\alpha\mu} = 1\angle 0^\circ$ τάση αναφ.

ΝΤΚ.: $\hat{V}_{1,\alpha\mu} = \hat{V}_{2,\alpha\mu} + (R_{1\sigma,\alpha\mu} + j\,X_{1\sigma,\alpha\mu})\,\hat{I}_{\alpha\mu}$ (1)

$$\hat{I}_{\alpha\mu} = \frac{\hat{S}^*_{2,\alpha\mu}}{\hat{V}_{2,\alpha\mu}} = \frac{0{,}75 - 0{,}56j}{1\angle 0} = 0{,}75 - 0{,}56j$$

$$\hat{S}_{2,\alpha\mu} = \hat{S}_L / S_B = (30 + j22{,}5)\cdot 10^3 / 40\cdot 10^3 = 0{,}75 + j\,0{,}56$$

(1) $\Rightarrow \hat{V}_{1,o\mu} = 1 + (5,54 + 44,3 \cdot j) \cdot 10^{-3} (0,75 - 0,56j) \Rightarrow$

$\Rightarrow \hat{V}_{1,o\mu} = 1,029 + 0,03j = 1,029 \angle 1,68$

$V_{1,\pi o\lambda} = V_{1,o\mu} \cdot V_{B1} = 20,58 \, kV$

$\hat{S}_{1,o\mu} = \hat{V}_{1,o\mu} \cdot \hat{I}^{*}_{o\mu} = (1,029 \angle 1,68)(0,75 - 0,56j) = 0,963 \angle -35$

$\hat{S} = \hat{S}_{1,o\mu} \cdot S_B = 10^3 \cdot 38,53 \angle -35 = 38,53 \cdot 10^3 \, VA$

γ) $\Sigma I' = 0,95 = \cos\theta' \Rightarrow S_L' = \dfrac{P_L}{\Sigma I'}$

$S_L = \dfrac{30k}{0,95} = 31,58 \, kVA$

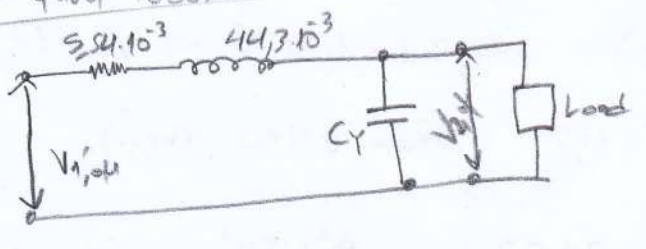
$Q_L' = S\sqrt{1 - \cos^2\theta} = 31,58\sqrt{1 - (0,95)^2} = 9,86 \, kVA_r$

$$C_\Delta = \frac{C_Y}{3} = \frac{Q_{CY}}{3V_\pi^2 \cdot \omega} = \frac{Q_L - Q_L'}{3V_\pi^2 \, 2\pi f} = \frac{(22,5 - 9,86)\,10^3}{2\pi 50 \cdot 3 \cdot 380^2} = 93,4 \, \mu F$$

Επειδή θέλουμε ισοδύναμο κύκλωμα, πρέπει στο C_Δ να γίνει C_Y

$Z_Y = \dfrac{Z_\Delta}{3} \Rightarrow C_Y = 3C_\Delta = 279 \, \mu F$

Ανά φάση ισοδ. κυκλ.



$\cdot \; \hat{V}_{2,o\mu} = \hat{V}_{1,o\mu}'' \dfrac{Z_{CY,o\mu}}{Z_{CY,o\mu} + R_{o\mu}'' + jX_{o\mu}''}$

$\cdot \; Z_{CY,o\mu} = \dfrac{Z_{CY}}{Z_{B2}} = \dfrac{1/_{j2\pi f C_Y}}{3,61} = -3,16j \; o\mu$

$\cdot \; \hat{V}_{1,o\mu}'' = 0,986 \angle 0,1^\circ$

$\cdot \; V_{1,\pi o\lambda}' = V_1 \cdot \hat{V}_{1,o\mu}'' = 20kV \cdot 0,986 = 19,72 \, kV$

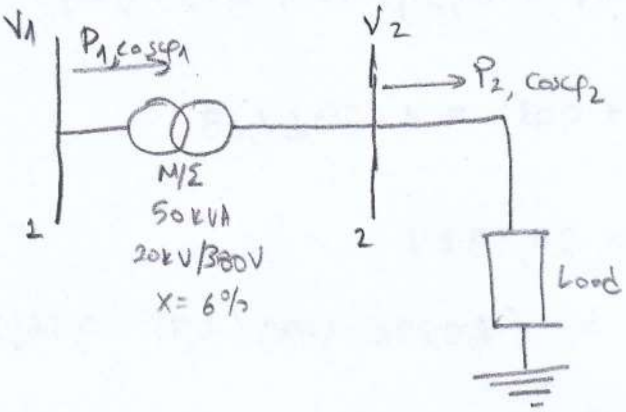
δ) Η συνδεσμολογία του Μ/Σ της άσκησης είναι αυτή που συναντάται σε ΣΗΕ για σύνδεση ΜΤ με ΧΤ. Η Δ-συνδεσμολογία επιτρέπει τα 3Φ ρεύματα να κυκλοφορήσουν στον Μ/Σ και αποτρέπει τα 3Φ ρεύματα από το να κυκλοφορήσουν στην γραμμή παροχής κάτι που θα προκαλούσε παραμόρφωση τάσης. Ακόμη, είναι απαραίτητος ο ουδέτερος του Υ για τα 380V και έτσι, δικαιολογείται ο Υ στο Χ.Τ.

'Άσκηση 5

3Φ Μ/Σ

$V_1 = 20{,}8\,kV$, $f = 50\,Hz$

ΥΤ : άπειρο ΣΗΕ



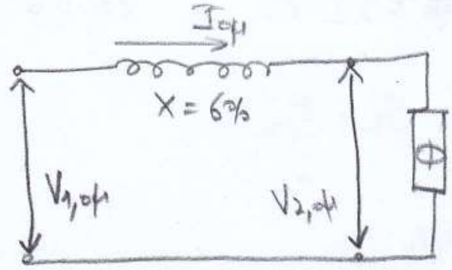
a) $P_1 = 30\,kW$

$ΣΙ = \cos\theta = 0{,}85$ επαγ.

$V_{B1} = 20\,kV$

$V_{B2} = 380\,V$

$S_B = 50\,kVA$



$$V_{1,αμ} = \frac{V_1}{V_{B1}} = \frac{20{,}8}{20} = 1{,}04\ αμ$$

$$S_1 = \frac{P}{\cos\varphi} = \frac{30\cdot 10}{0{,}85} = 35{,}29\ kVA \Rightarrow S_{1,αμ} = \frac{S_1}{S_B} = \frac{35{,}29}{50} = 0{,}7\ αμ$$

$$I_{αμ} = \frac{S_{1,αμ}}{V_{1,αμ}} = \frac{0{,}7}{1{,}04} = 0{,}67\ αμ$$

$$NTK: \hat{V}_{2,αμ} = \underset{\substack{\uparrow \\ \text{τάση αναφ.}}}{\hat{V}_{1,αμ}} - jX\cdot I_{αμ} = 1{,}04\angle 0 - 0{,}06j\,(0{,}67\angle -31{,}97) = 1{,}02\angle -1{,}92\ αμ$$

$$\hat{V}_2 = \hat{V}_{2,αμ}\,\hat{V}_{B2} = 380(1{,}02\angle -1{,}92) = 387{,}2\angle -1{,}92\ V \Rightarrow V_2 = 387{,}2\,V$$

$$\cos^{-1}(0{,}85) = 31{,}79 \Rightarrow \hat{I}_{αμ} = 0{,}67\angle -31{,}79 \quad (\text{ρεύμα έπεται τάσης})$$

Χ.Τ.: για ΣΙ: $\theta' = -1{,}92 + 31{,}79 = 29{,}87 \Rightarrow \cos\theta' = ΣΙ' = \cos(29{,}87) = 0{,}87$

β) $R_\Delta = 45\,\Omega$

$$S_{2,αμ} = I_{αμ} V_{αμ} = 0{,}67\cdot 1{,}02 = 0{,}68$$

$$P_{2,αμ} = S_{2,αμ}\cdot\cos\theta' = 0{,}68\cdot 0{,}87 = 0{,}6$$

$$\left.\begin{aligned} Z_{B2} &= \frac{380^2}{50\cdot 10^3} = 2{,}89\,\Omega \\ R_Y &= \frac{R_\Delta}{3} = 15\,\Omega \end{aligned}\right\} Z_{R,αμ} = \frac{15}{2{,}89} = 5{,}19$$

$$P_{R,of} = \frac{V_{2,of}^2}{Z_{R,of}} = \frac{1,02^2}{5,19} = 0,2 \Rightarrow P_R = 0,2 \cdot S_B = 10kW$$

$$P_{κινητ,of} = P_{2,of} - P_{R,of} = 0,4 \Rightarrow P_{κ,κινητ} = 0,4 \cdot S_B = 20kW$$

$$Q_{κ,of} = S_{3of} \cdot \sin\varphi' = 0,68\sqrt{1-0,87^2} = 0,335 \rightarrow Q_{κ,κινητ.} = 16,75\,kVA$$

γ) $P_2 = 25kW$

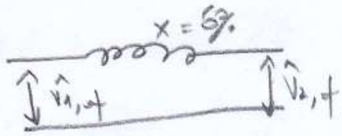
$ΣΙ = \cos\varphi_2 = 0,85$ επαγ. $\rightarrow \sin\varphi = 0,52$

$$S_2 = \frac{P_2}{\cos\varphi_2} = 29,41\,kVA \longrightarrow S_{2,of} = \frac{S_2}{S_B} = 0,59$$

Αν $\hat{V}_2$ τάση αναφ: $\hat{V}_2 = V_2\angle 0$, τότε το ρεύμα θα καθυστερήσει κατά φ_2:

$\varphi_2 = \cos^{-1}(0,85) = 31,78°$

$\hat{I}_{of} = I_{of}\angle -31,78 \quad (1)$ & $I_{of} = \dfrac{S_{2,of}}{V_{2,of}}$



ΝΤΚ: $\hat{V}_{1,of} = \hat{V}_{2,of} + \hat{Z}_{of}\cdot\hat{I}_{of}$

$\hat{Z}_{of} = jX = 0,06\angle 90° \overset{(1)}{\Longrightarrow} \hat{I}_{of}\cdot\hat{Z}_{of} = 0,06\,I_{of}\angle 90 - 31,78° =$

$= 0,06\,I_{of}\angle 68,22$

$\hat{V}_{2,of} = V_{2,of}\angle 0°$

$V_{1,of}^2 = \{V_{2,of} + X I_{of}\cdot\cos(68,22)\}^2 + \{X I_{of}\sin(68,22)\}^2 \Rightarrow$

$\Rightarrow V_{1,of}^4 - 1,0442\,V_{1,of}^2 + 0,00125 = 0 \Rightarrow V_{1,of}^2 = \begin{cases} 1,198\cdot 10^{-3} \ll 1, & \text{απορρ.} \\ 1,043, & \text{δεκτή} \end{cases}$

$\Rightarrow V_{1,of}^2 = 1,043 \longrightarrow V_{2,of} = \sqrt{1,043} = 1,02 \Rightarrow V_{2,of} = 1,02 \Rightarrow$

$\Rightarrow V_2 = V_{2,of}\cdot V_{B_2} = 1,02 \cdot 380 = 387,6V$

Άσκηση 6

$S_N = 100\,kVA$

ονομ. πολ. τάση πρωτεύοντος: $V_N' = 20\,kV$

—ιι— δευτερεύοντος: $V_N'' = 150\,kV$

πλευρά ΧΤ: 20kV

$f = 50\,Hz$

Οι υπολογισμοί έγιναν στο πρωτεύον (πλευρά ΧΤ) έχοντας βραχυκυκλ. & ανοιχτοκυκλ. του δευτερεύοντος.

▷ Βραχυκυκλ. υπό ονομ. ρεύμα $P_1^\beta = 0{,}5\%\, S_N$, $V_1^\beta = 10\%\, V_N'$

▶ ανοιχτοκυκλ. υπό ονομ. τάση $P_1^a = 0{,}3\%\, S_N$, $I_1^a = 2\%\, I_N'$

α) ▷ $P_1^\beta = \dfrac{0{,}5 \cdot 100.000}{100} = 500\,W$

$V_1^\beta = \dfrac{10}{100}\, 20.000 = 2\,kW$

$I_1^\beta = I_N' = \dfrac{S_N}{\sqrt{3}\, V_N'} = 2{,}89\,A$

ανά φάση: $I_{1\varphi}^\beta = I_1^\beta = 2{,}89\,A$

$P_{1\varphi}^\beta = \dfrac{500}{\sqrt{3}} = 288{,}68\,W$

$V_{1\varphi}^\beta = \dfrac{2\,kV}{\sqrt{3}} = 1.154{,}7\,V$

ισοδ. κυκλώματα ανηγμ. στο ΧΤ

$Z_{i\sigma}' = \dfrac{V_{1\varphi}^\beta}{I_{\beta\varphi}} = 399{,}55\,\Omega$

$R_{i\sigma}' = \dfrac{P_{1\varphi}^\beta}{(I_{1\varphi}^\beta)^2} = 34{,}56\,\Omega$

$X_{i\sigma}' = \sqrt{(Z_{i\sigma}')^2 - (R_{i\sigma}')^2} = 398{,}05\,\Omega$

▶ $V_1^a = V_N = 20\,kV$

$P_1^a = 0{,}3\%\, S_N = 300\,W$

$I_1^a = 2\%\, I_N' = 0{,}0576\,A$

ανά φάση: $V_{1\varphi}^a = \dfrac{V_1^a}{\sqrt{3}} = 11{,}55\,kV$

$P_{1\varphi}^a = \dfrac{P_1^a}{\sqrt{3}} = 173{,}21\,W$

$I_{1\varphi}^a = I_1^a = 0{,}0576\,A$

ισοδ. κυκλ. ανηγμ. στο ΧΤ

$Y_\varphi' = \dfrac{I_{1\varphi}^a}{V_{1\varphi}^a} = 4{,}99 \cdot 10^{-6}\,\Omega^{-1}$

$g_c' = \dfrac{P_{1\varphi}^a}{(V_{1\varphi}^a)^2} = 1{,}3 \cdot 10^{-6}\,\Omega^{-1}$

$b_m' = \sqrt{(4{,}99)^2 - (1{,}3)^2} \cdot 10^{-6} = 4{,}82 \cdot 10^{-6}\,\Omega^{-1}$

Ανά μονάδα συστ. με βάσεις τα ονομ. μεγέθη του Μ/Σ :

Επιλογή βάσεων: $S_B = 100kVA$, $V_{B1} = 20kV$, $V_{B2} = 150kV$

$Z_{B1} = \frac{V_{B1}^2}{S_B} = 4k\Omega$ $\qquad I_{B_1} = \frac{S_B}{\sqrt{3} \cdot V_{B_1}} = 2,89A$

$Z_{B2} = \frac{V_{B2}^2}{S_B} = 225k\Omega$ $\qquad I_{B_2} = \frac{S_B}{\sqrt{3} V_{B_2}} = 0,38A$

Παράμετροι ανά μονάδα συστ.:

$Z_{1\sigma,pu} = \frac{Z'_{1\sigma}}{Z_{B1}} = 0,1$ $\qquad Y_{\varphi,pu} = Y'_{\varphi} \cdot Z_{B1} = 0,02$

$R_{1\sigma,pu} = \frac{R'_{1\sigma}}{Z_{B1}} = 8,64 \cdot 10^{-3}$ $\qquad g_{c,pu} = g'_c \cdot Z_{B1} = 0,029$

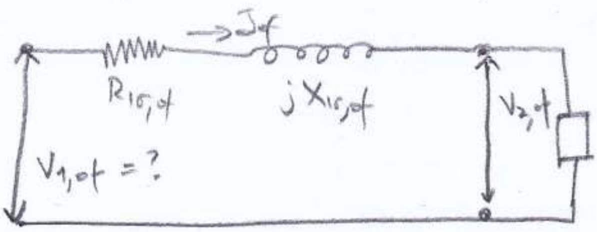
$X_{1\sigma,pu} = \frac{X'_{1\sigma}}{Z_{B1}} = 0,1$ $\qquad b_{m,pu} = b'_m \cdot Z_{B1} = 0,019$

β) $S_2 = 70kVA$

$V_2 = 160kV$

$\cos\varphi_2 = 0,95$ επαγ $\rightarrow \varphi_2 = 18,195°$

Ισοδύν. κυκλ. (αγνοούνται απώλειες πυρήνα & ρεύμ. μαγνητ.)



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Επιλογή βάσεων: $S_B = 100kVA$ $\quad V_{B1} = 20kV$ $\quad V_{B2} = 150kV$

(οι υπόλοιπες όπως πριν)

Ανά μονάδα συστ.: $V_{2,pu} = \frac{160kV}{150kV} = 1,067 \rightarrow \hat{V}_{2,pu} = 1,067\angle 0°$

$S_{2,pu} = \frac{70kVA}{100kVA} = 0,7 \rightarrow \hat{S}_{2,pu} = 0,7\angle 18,195°$

$$\hat{S}_{2,\alpha\phi} = \hat{V}_{2,\alpha\phi}\,\hat{I}^{*}_{\alpha\phi} \Rightarrow \hat{I}_{\alpha\phi} = \frac{\hat{S}^{*}_{2,\alpha\phi}}{\hat{V}^{*}_{2,\alpha\phi}} = \frac{0,7}{1,067} \angle -18,195 + 0 \Rightarrow \hat{I}_{\alpha\phi} = 0,656 \angle -18,195^{\circ}$$

NTK: $\hat{V}_{1,\alpha\phi} = \hat{I}_{\alpha\phi}\,\hat{Z}_{1\sigma,\alpha\phi} + \hat{V}_{2,\alpha\phi}$

$$\hat{Z}_{1\sigma,\alpha\phi} = R_{1\sigma,\alpha\phi} + j X_{1\sigma,\alpha\phi} = 8,64 \cdot 10^{-3} + j\,0,1 = 0,1 \angle 85,06^{\circ} \Rightarrow$$

Άρα, $\hat{V}_{1,\alpha\phi} = (0,656 \angle -18,195) \cdot (0,1 \angle 85,06) + 1,067 \angle 0 = \ldots = 1,095 \angle 3,142 \Rightarrow$

$$\Rightarrow V_1 = V_{1,\alpha\phi} \cdot V_{B_1} = 1,095 \cdot 20\,kV = 21,9\,kV$$

Βαθμός απόδοσης Μ/Σ: $P_{Load,3\Phi} = P_2 = S_2 \cos\varphi_2 = 70\,kVA \cdot 0,95 = 66,5\,kW$

$$P_{R,3\Phi} = \underbrace{R_{1\sigma,\alpha\phi} \cdot I_{\alpha\phi}^2}_{P_{R,\alpha\phi}} \cdot S_B = 8,64 \cdot 10^{-3} (0,656)^2 \cdot 100000 = 371,81\ W$$

$$n = \frac{P_2}{P_R + P_2} \cdot 100\% = \frac{66,5\,kW}{(66,5 + 0,37181)\,kW} \cdot 100\% = 99,44\%$$

Άσκηση 7

$$R_g = \frac{l_g}{\mu_0 A_g} = \frac{5 \cdot 10^{-4}}{4\pi 10^{-7} \cdot 8 \cdot 10^{-4}} = 497,36 \cdot 10^{3}\ A\text{-}t/Wb$$

$A = 8\,cm^2$,

$l_g = 0,5\,mm$

N: 1000 σπείρες, $I = 3A$

α) $L = \dfrac{N^2}{R_g} = \dfrac{10^6}{497,36 \cdot 10^3} = 2,01\,H$

β) $W = \frac{1}{2} L I^2 = \frac{1}{2}\, 2,01 \cdot 3^2 = 9,045\,J$

γ) $$F_n = \left.\frac{dW(x)}{dx}\right|_{x=l_g} = \left.\frac{d}{dx}\left(\frac{1}{2} L I^2\right)\right|_{x=l_g} = \left.\frac{d}{dx}\left(\frac{1}{2}\frac{N^2}{R_g} \cdot I^2\right)\right|_{x=l_g} = \left.\frac{d}{dx}\left(\frac{1}{2} N^2 I^2 \frac{\mu_0 A_g}{x}\right)\right|_{x=l_g}$$

$$= \frac{1}{2}\frac{N^2 I^2 \mu_0 A}{l_g^2} = -\frac{1}{2}\frac{10^6 \cdot 9 \cdot 4\pi 10^{-7} \cdot 8 \cdot 10^{-4}}{25 \cdot 10^{-8}} = -18095,57\ N \quad (\text{προς τα αριστερά})$$

Άσκηση 8

$A = 8 \cdot 10^{-4} m^2$ τετρ. διατομή $\longrightarrow a^2 = A \rightarrow a = \sqrt{A} = 2\sqrt{2} \cdot 10^{-2} m \longrightarrow A' = (a+l_g)(a+l_g) =$

$= (a+l_g)^2$

$= 8.285 \cdot 10^{-4} m^2$

$l_g = 5 \cdot 10^{-4} m$

$N = 10^3$

$\mu_0 = 4\pi 10^{-7}$ H/m

α) $L = \dfrac{N^2}{R_g} = \dfrac{N^2 \mu A'}{l_g} = 2{,}08\ H$

β) $W = \frac{1}{2} L I^2 = \frac{1}{2}\, 2{,}08 \cdot 9 = 9{,}36\ J$

γ) $W = \frac{1}{2} N^2 I^2 \mu_0 \dfrac{(a+x)^2}{x} = \frac{1}{2} N^2 I^2 \mu_0 \left(x + 2a + \dfrac{a^2}{x} \right)$

$F = \dfrac{dW}{dx}\Big|_{x=l_g} = \frac{1}{2} N^2 I^2 \mu_0 - \frac{1}{2} N^2 I^2 \mu_0 \dfrac{a^2}{l_g^2} = \frac{1}{2} N^2 I^2 \mu_0 \left(1 - \dfrac{a^2}{l_g^2}\right) = -18089{,}9N$

(προς τα οριακά)

Άσκηση 9

$L_1 = 10 + 4\cos 2\theta$ H

$L_2 = 5 + 2\cos 2\theta$ H

$M = 9{,}9 \cos\theta$ H

α) $N_1 = 1000$

$\theta = 0$

$L_1(0) = 10 + \cos\Theta = 14$ H

$L_1(\Theta) = \dfrac{N_1^2}{R_{ol}(\theta)} \Longrightarrow R_{ol}(0) = \dfrac{N_1^2}{L_1(0)} = 71428{,}57 \ \dfrac{A\cdot\varepsilon}{Wb}$

$\theta = 90$

$L_1(90) = 10 + 4\cos(180) = 6H$

$L_1(\Theta) = \dfrac{N_1^2}{R_{ol}(\theta)} \Rightarrow R_{ol}(90) = \dfrac{N_1^2}{L_1(90)} = 166.666{,}67 \ \dfrac{A\cdot\varepsilon}{Wb}$

β) $I_1 = 0{,}5A$

$I_2 = 2A$

$$T_m = \frac{1}{2} I_1^2 \frac{dL_1}{d\theta} + I_1 I_2 \frac{dM}{d\theta} + \frac{1}{2} I_2^2 \frac{dL_2}{d\theta}$$

$$= -4I_1^2 \sin 2\theta - 9{,}9\, I_1 I_2 \sin\theta - 2I_2^2 \sin 2\theta$$

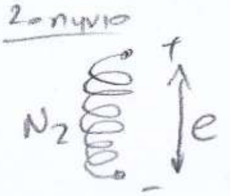
$$= \sin 2\theta\,(-4I_1^2 - 2I_2^2) - 9{,}9\, I_1 I_2 \sin\theta$$

$$= -9 \sin 2\theta - 9{,}9 \sin\theta$$

γ) $I_2 = 0$

$I_1 = 0{,}5\,A$

$\omega = 50\,r/s$



στιγμιαία ΗΕΔ: $e_2 = \frac{d\lambda_2}{dt} = \frac{d(M(\theta) I_1 + \cancel{L_2 I_2})}{dt} = I_1 \frac{dM(\theta(t))}{dt} = I_1 \frac{dM}{d\theta}\frac{d\theta}{dt} \Rightarrow$

$\Rightarrow e_2 = -I_1\, \omega\, 9{,}9 \sin\theta = -247{,}55 \sin\theta$ V

rms ΗΕΔ: $e_{2,rms} = \frac{247{,}55}{\sqrt{2}} = 175$ V

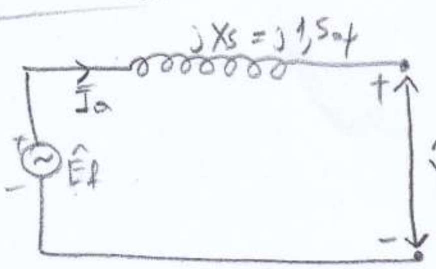
Άσκηση 10

3Φ Σύγχρ. Γενν.: $S_N = 150kVA$, $V_N = 380V$, $f = 50Hz$, Y, $X_{s,f} = 1{,}2$ αμ

ΣΙ: $\cos\theta = 1$ & 100kW ⟶ $I_f = 10A$ (DC)

$P = 100kW$, όμως $\cos\theta = 1 \Rightarrow S = P + \cancel{jQ} = P \Rightarrow P = 100kW$, $S = 100kVA$

Ισοδύν. Κυκλ.



(σύμβαση: ρεύμα I_a εξέρχεται από τους ακροδέκτες)

Βάσεις: $S_B = S_N = 150kVA$, $V_B = V_N = 380V$

$$Z_B = \frac{V_B^2}{S_B} = \frac{380^2}{150.000} = 0{,}963$$

$$I_B = \frac{S_B}{\sqrt{3}\,V_B} = \frac{150 \cdot 10^3}{\sqrt{3} \cdot 380} = 227{,}9\ A$$

• $S_{o\mu} = \frac{S}{S_B} = \frac{100.000}{150.000} = 0{,}667 \longrightarrow S_{o\mu} = V_{o\mu} \cdot I_{o\mu} \Rightarrow I_{o\mu} = \frac{0{,}667}{1} = 0{,}667$

Θεωρώ $\hat{V}_{o\mu} = 1$ σφ $\angle 0 \longrightarrow V_{o\mu} = 1$

$I_A = I_B \cdot I_{o\mu} = 227{,}9 \cdot 0{,}667 = 152{,}01 \text{ A}$

• $S = P + j\cancel{Q} = P \Rightarrow \hat{S} = P\angle 0^\circ \Rightarrow \bar{I}_{o\mu} = \frac{\hat{S}^*}{\hat{V}^*} = \frac{0{,}667\angle 0}{1\angle 0} = 0{,}667\angle 0^\circ$

NTK :(στο ισοδ. ανά μονάδα συστ.) : $\hat{E}_{f,o\mu} = I_{o\mu} \cdot jX_{s,o\mu} + \hat{V}_{o\mu} \Rightarrow$

$\Rightarrow \hat{E}_{f,o\mu} = 0{,}667 \cdot j \cdot 1{,}2 + 1\angle 0^\circ = 1{,}28\angle 38{,}67 \longrightarrow \delta = 38{,}67^\circ$

β) $P_{Load} = 125 \text{ kW}$

$I_f' = I_f = 10 \text{ A}$

$P_{o\mu}' = \frac{125\text{k}}{150\text{k}} = 0{,}833$

$\hat{V}_{o\mu} = 1\angle 0, \quad X_{s,o\mu} = 1{,}2, \quad I_f: \text{σταθ} \Rightarrow E_f: \text{σταθ} \Rightarrow E_{f,o\mu} = 1{,}28$

$P_{o\mu} = \frac{V_{o\mu} \cdot E_{f,o\mu} \cdot \sin\delta'}{X_{s,o\mu}} \Rightarrow 0{,}833 = \frac{1 \cdot 1{,}28 \cdot \sin\delta'}{1{,}2} \Rightarrow \sin\delta' = 0{,}781 \Rightarrow$

$\Rightarrow \delta' = 51{,}346$

$Q_{o\mu}' = \frac{V_{o\mu} \cdot E_{f,o\mu} \cdot \cos\delta' - V_{o\mu}^2}{X_{s,o\mu}} = \frac{1 \cdot 1{,}28 \cdot \cos(51{,}346) - 1}{1{,}2} = -0{,}167$

$Q' = Q_{o\mu}' \cdot S_B = -0{,}167 \cdot 150.000 = -25050 \text{ VAr}$

$\cos\varphi' = \frac{P'}{S'} = \frac{125\text{k}}{\sqrt{(25{,}05\text{k})^2 + (125\text{k})^2}} = 0{,}98$

γ) Εν κενώ: $I_A'' = 0 \Rightarrow E_f'' = V \Rightarrow E_{f,o\mu}'' = V_{o\mu} = 1$

$\frac{E_{f,o\mu}}{I_f} = \frac{1{,}28}{10} = 0{,}128 = \frac{E_{f,o\mu}'}{I_f'} \Rightarrow I_f = \frac{1}{0{,}128} = 7{,}8125 \text{ A}$

Άσκηση 11

4πολικός Κινητ. : $P=4$ | $X_s = 3\,\Omega$

$V=380V$ | $P_m = 25kW$

$f=50Hz$ | Τροφοδοσία: άπειρου ζυγ.

Y | $I_f = 4A \Rightarrow \cos\varphi = 0,8$ επαγ.

a) $\omega_s = \dfrac{2\pi f}{P/2} = 50\ r/s$

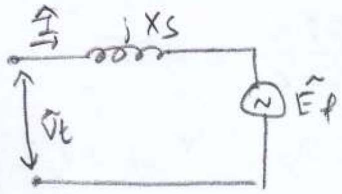
$P_m = P_e = 25kW$

$S = \dfrac{P_m}{\cos\varphi} = \dfrac{25}{0,8} = 31,25kVA$

$\cos\varphi = 0,8 \Rightarrow \sin\varphi = \sqrt{1-0,8^2} = 0,6$

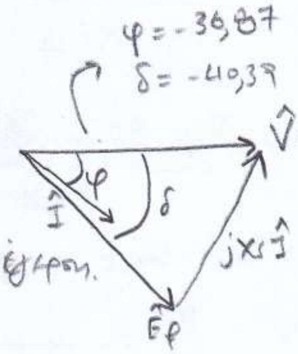
$Q = S\cdot\sin\varphi = 18,75\ kVAr$

β) $I = \dfrac{P_e}{\sqrt{3}\cdot V_t\cdot\cos\varphi} = \dfrac{25\cdot 10^3}{\sqrt{3}\cdot 380\cdot 0,8} = 47,48A$



$\left.\begin{array}{l}\cos\varphi = 0,8\\ \varphi = -36,87\end{array}\right\} \Rightarrow \hat{I} = (47,48\angle -36,87^\circ)\ A$

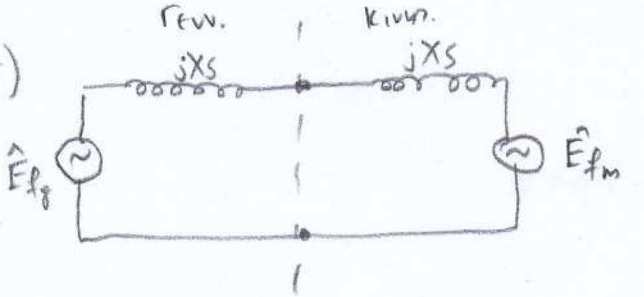
$V_t = \dfrac{380}{\sqrt{3}}\angle 0$ Τάση αναφ.



NTK: $\hat{E}_f = \hat{V}_t - jX_s\hat{I} \Rightarrow \hat{E}_f = 175,85\angle -40,39^\circ$

γεωμ. τόπος: P = σταθ., καθορίζεται από το φορτίο, όχι από διέγερση.

γ)



NTK: $\hat{E}_{fg} = V_t + jX_s\hat{I} \Rightarrow \hat{E}_{fg} = (325,46\angle 20,49)V$

$\dfrac{E_{fg}}{|E_{fm}|} = \dfrac{I_{fg}}{I_{fm}} \Rightarrow I_{fg} = \dfrac{325,46}{175,85}\,4 = 7,4\ A$

δ) άπειρο σύστημα: $P_{max,κινητ} = \dfrac{3E_fV_t}{X_s} = \dfrac{3\cdot 175,85\cdot\frac{380}{\sqrt{3}}}{3} \Rightarrow P_{max} = 38,58kW$

γεννήτρια: $P_{max,γενν.} = \dfrac{3E_{fg}\cdot V_t}{X_s} = 71,4kW$

'Άσκηση 12

4πολικός ασύγχρ. δακτυλιοφόρος: $P=4$

$231/400V$, $50Hz$

$r_1 = 0{,}015\,\Omega$, $X_1 = 0{,}15\,\Omega$

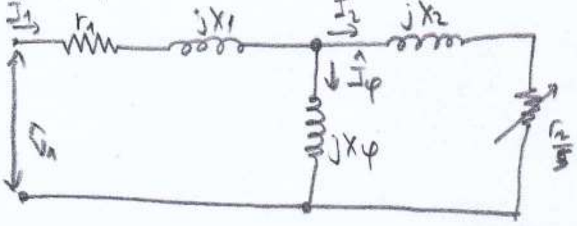
$X_\varphi = 5{,}5\,\Omega$ $r_2 = 0{,}01\,\Omega$

$X_2 = 0{,}15\,\Omega$

$P_{απωλ} = 6kW$

ολίσθηση 1%

a) $n_s = \frac{120f}{P} = 1500$ ΣΑΛ $\Rightarrow$ $n = (1-s)n_s = 0{,}99 \cdot 1500 = 1485$ ΣΑΛ



$$\hat{I}_1 = \frac{\hat{V}_1}{Z_{ολ}} = \frac{\hat{V}_1}{r_1 + jx_1 + (jX_\varphi)\,||\,(\frac{r_2}{s} + jX_2)} \Rightarrow$$

$$\Rightarrow \hat{I}_1 = 222{,}03\angle -26{,}16\ A$$

$$Z_{TH} = jX_\varphi\,||\,(r_1 + jX_1) = 0{,}15\angle 84{,}44\ \Omega$$

$$\hat{V}_{TH} = \frac{jX_\varphi}{r_1 + j(X_1 + X_\varphi)}\hat{V}_1 = 224{,}87\angle 0{,}15\ V$$

$$\omega_s = \frac{2\pi f}{P/2} = 50\pi\ r/s$$

$$T_{em} = \frac{1}{\omega_s} 3 V_{TH}^2 \frac{r_2}{s} \frac{1}{(R_{TH} + \frac{r_2}{s})^2 + (X_{TH} + X_2)^2} = 866{,}43\ Nm$$

$$P_{em} = T_{em} \cdot \omega_m = T_{em}(1-s)\omega_s = 134{,}74\ kW = P_m + P_{απ} = 128{,}74\,kW$$

$$P_g = T_{em} \cdot \omega_s = 135{,}45\ kW$$

$$\eta = \frac{P_m}{P_1} = \frac{P_m}{P_g} = 0{,}947$$

$$T_m = \frac{P_m}{(1-s)\omega_s} = 827{,}85\ Nm$$

β) $$T_{max} = \frac{3V_{TH}^2}{2\omega s\left(R_{TH} + \sqrt{R_{TH}^2 + (X_{th}+X_2)^2}\right)} = \frac{3(224{,}87)^2}{100\pi\left(0{,}014 + \sqrt{0{,}014^2 + 0{,}296^2}\right)} = 1556\ Nm$$

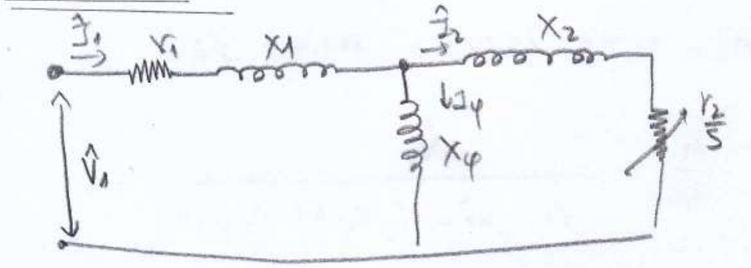
Θέλουμε $S_{εκκ} = S_{Tmax}$:

$$S_{Tmax} = \frac{r_2'}{\sqrt{R_{TH}^2 - (X_2+X_{TH})^2}} \Rightarrow r_2' = 0{,}296\ \Omega$$

$$S_{εκκ} = \frac{n_s - \cancel{n}}{n_s} = 1$$

οπότε σε κάθε φάση $r_2' - r_2 = 0{,}286\ \Omega$

Άσκηση 13



α) $$V_{TH} = \frac{jX_\varphi}{r_1 + j(X_1+X_\varphi)}\hat{V}_1 \Rightarrow \hat{V}_{TH} = 372{,}55\angle 0{,}11\ V$$

$$Z_{TH} = \frac{jX_\varphi(r_1+jX_1)}{r_1 + j(X_1+X_\varphi)} \Rightarrow Z_{TH} = 0{,}01 + 0{,}0981\ \Omega$$

$$T = \frac{3}{\omega_s}\cdot\frac{V_{TH}\cdot r_2/s}{(R_{TH}+\frac{r_2}{s})^2 + (X_{TH}+X_2)^2} = 4000$$

$$\omega_s = \frac{2\pi f}{P/2} = \frac{100\pi}{3}\ r/s\ ,\quad z' = \frac{r_2}{s}$$

$$\Rightarrow \frac{9\ V_{TH}^2\cdot z}{100\pi\left[(R_{TH}+z')^2 + (X_{TH}+X_2)^2\right]} = 4000 \Rightarrow$$

$$\Rightarrow 4000z^2 - 3896{,}14z + 157{,}22 = 0$$

$z_1 = 0{,}932 \Rightarrow \frac{r_2}{s_1} = 0{,}932 \Rightarrow s_1 = 1\%$, ✓

$z_2 = 0{,}042 \Rightarrow \frac{r_2}{s_2} = 0{,}042 \Rightarrow s_2 = 20\%$, ✗

β) $s = 0{,}5\%$, $P = 6$, $f = 50Hz$

$$n = (1-s)\cdot\frac{120f}{P} \Rightarrow n = 995\ ΣΑΛ$$

$$\omega = (1-s)\omega_s,\quad \omega_s = \frac{2\pi f}{P/2} \Rightarrow \omega = \frac{11\pi}{3}\ r/s$$

γ) $$T_{εκκ} = \frac{3}{\omega_s}\cdot\frac{3V_{TH}^2\cdot r_2}{(r_2+R_{TH})^2 + (X_L+X_{TH})^2} = 1003{,}974\ Nm$$

Για εκκ., πρέπει: $T_m < T_{εκκ}$, όμως $T_m = 4000\ Nm$, $T_{εκ} = 1003{,}974\ Nm$

άρα δεν θα εκκινήσει.

Άσκηση 14

ΣΗΕ : $f = 50 Hz$, ΓΜ: 150 kV , άπειρο ΣΗΕ , $V_1 = 400 kV$

Δ ωμικός : $P_\Phi = 100 MW$, 0,95 επαγ

$Γ_3$: $P_3 = 60 MW$, 165 kV

$Γ_4$: $P_4 = 80 MW$, Q . τ.ω. 0,9 επαγ

α) Ζ4 : $P_4 = P_{Γ_4} - P_\Phi = -20 MW$

φορτίο : $ΣΙ\varphi = 0{,}95 \Rightarrow \varphi = 18{,}19^o$

$Q_\varphi = P_\varphi \tan\varphi = 100 \cdot \tan(18{,}19) = 32{,}87\ MVA_r$

Ζ4 : $\cos\varphi_4 = 0{,}9$, επαγ

$Q_\varphi = S_\varphi \cdot \sin(\cos^{-1}(\cos\varphi_4)) = -9{,}69\ MVA_r$

Γενν. $Γ_4$: $Q_{Γ_4} = Q_\varphi + Q_4 = 23{,}18\ MVA_r$

β) Τάσεις ζυγών : $V_{B_1} = 400 kV$, $V_{B_2} = 150 kV$, $V_{B_3} = 150 kV$, $V_{B4} = 150 kV$

Αντιστ. ΓΜ : $Z_B = \frac{V_B^2}{S_B} = \frac{(150 \cdot 10^3)^2}{100 \cdot 10^6} = 225\ \Omega$

$$\hat{Z}_{2,5,\,of} = \hat{Z}_{3,4,\,of} = \hat{Z}_{2,4,\,of} = \frac{jX}{Z_B} = 0{,}12j\ of$$

ΜΣ : $S_{old} = 150 MVA$, $S_{new} = S_B = 100 MVA$

$$\hat{Z}_{1,2\,new,of} = \hat{Z}_{1,2\,old,of}\left(\frac{V_{2,old}}{V_{2,new}}\right)^2\left(\frac{S_{new}}{S_{old}}\right) = 0{,}067j\ of$$

$$P_{Γ_3,of} = \frac{P_{Γ_3}}{S_B} = 0{,}6$$

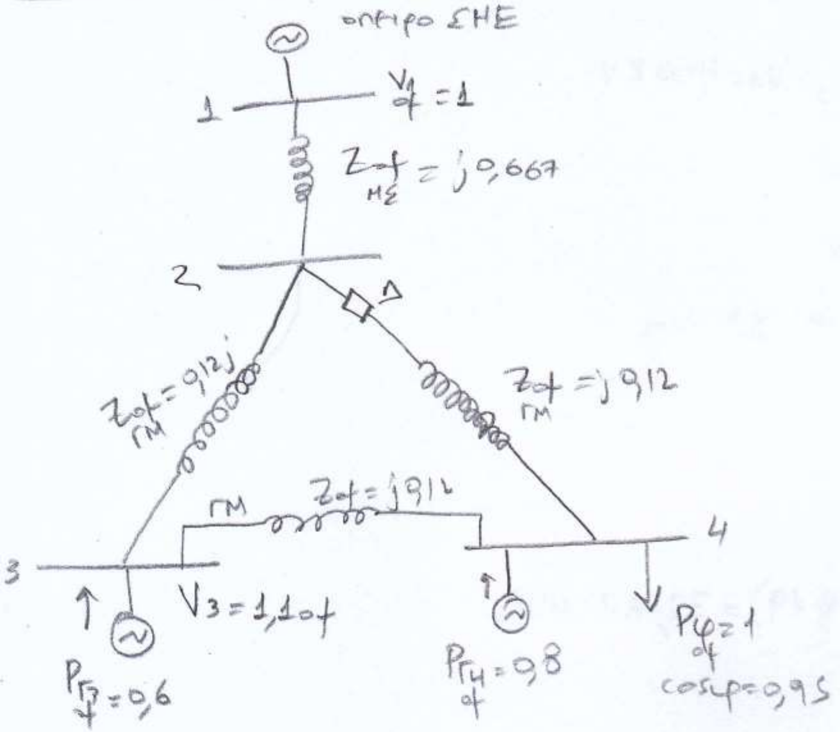
$$P_{Γ_4,of} = \frac{P_{Γ_4}}{S_B} = 0{,}8$$

$$P_{φ,of} = \frac{P_\Phi}{S} = 1$$

$$V_{1,of} = \frac{V_1}{V_{1,B}} = 1$$

$$V_{3,of} = \frac{V_3}{V_{3,B}} = 1{,}1$$

AM ισοδ. 1Φ κυκλ.



γ) Έστω Z1 ως ζυγός αναφ:

Z_2, Z_4: φορτίου

Z_3: παραγωγής

$$Μ/Σ: \quad Y_{1,2\,of} = \frac{1}{Z_{1,2\,of}} = -14{,}92j$$

$$Γ.Μ.: \quad Y_{2,3\,of} = Y_{2,4\,of} = Y_{3,4\,of} = \frac{1}{Z_{2,3\,of}} = -8{,}33j$$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} Y_{12} & -Y_{12} & 0 & 0 \\ -Y_{12} & Y_{12}+Y_{23}+Y_{24} & -Y_{23} & -Y_{24} \\ 0 & -Y_{23} & Y_{23}+Y_{34} & -Y_{34} \\ 0 & -Y_{24} & -Y_{34} & Y_{24}+Y_{34} \end{bmatrix}_{of} =$$

$$= \begin{bmatrix} -14{,}92j & 14{,}92j & 0 & 0 \\ 14{,}92j & -31{,}58j & 8{,}33j & 8{,}33j \\ 0 & 8{,}33j & -16{,}66j & 8{,}33j \\ 0 & 8{,}33j & 8{,}33j & -16{,}66j \end{bmatrix}_{of}$$

δ) Δ ανοιχτός:

ΓΜ 2-4 εκτός : $P_3 = \frac{V_3 V_1}{X_{1,3}} \sin(\delta_3 - \delta_1)$

$\left.\begin{array}{l}\hat{V}_1 = 1\angle 0^\circ \Rightarrow \delta_1 = 0 \quad \text{(ταση αναφ.)} \\ X_{1,3\,\alpha f} = X_{12\,\alpha f} + X_{23\,\alpha f} = 0{,}187\end{array}\right\} \Rightarrow \sin\delta_3 = 0{,}102 \Rightarrow \delta = 5{,}85^\circ$

$V_{3\,\alpha f} = 1{,}1\angle 5{,}85$

$Q_{3\,\alpha f} = S_{3\,\alpha f}\sin\delta_3 = P_{3\,\alpha f}\tan\delta_3 = 0{,}06 \Rightarrow S_{3\,\alpha f} = P_{\alpha f} + jQ_{\alpha f} = 0{,}6 + j0{,}06$

$S_{\varphi\,\alpha f} = 0{,}8 + j0{,}02318 - 1 - j0{,}05287 = -0{,}2 - j0{,}00989$

Z1 : παραγωγή : $S_{\alpha f} = S_{3,\alpha f} + S_{4,\alpha f} = 0{,}4 + j0{,}05031 \Rightarrow$

$\Rightarrow P = 40MW$ & $Q = 5{,}031$ MVAr

παράγεται από το ΣΗΕ

Άσκηση 15

$V_{B_1} = 15kV \quad V_{B_2} = 150kV \quad V_{B_3} = 150kV \quad V_{B_4} = 20kV$

ΓΜ: $Z_B = \frac{V_B^2}{S_B} = 225\,\Omega$, $S_B = 100$ MVA

AM Συσ.

$\hat{Z}_{\alpha f} = \frac{r + jx}{Z_B} = 0{,}01 + j0{,}067$ αf

Μεταξύ Z2-Z3 υπάρχουν παράλληλες γραμμές μεταφοράς: $\hat{Z}_{2,3} = \hat{Z} \,//\, \hat{Z} \Rightarrow$

$\Rightarrow \hat{Z}_{2,3} = 0{,}005 + j0{,}0335$ αf

M/Σ : $\hat{Z}_{1,2\,\alpha f}\,new = \hat{Z}_{1,2\,\alpha f},old \left(\frac{V_{2,old}}{V_{2,new}}\right)^2 \frac{S_{new}}{S_{old}} = 0{,}067j$ αf

$\hat{Z}_{3,4\,\alpha f}\,new = \hat{Z}_{1,2\,\alpha f}\,old \left(\frac{V_{3,old}}{V_{3,new}}\right)^2 \frac{S_{new}}{S_{old}} = 0{,}09j$ αf

Z1 : $V_{1\,\alpha f} = \frac{V_1}{V_{1B}} = 0{,}97\,\alpha f$

Γεvv.: $S_{\alpha f} = \frac{120\text{ MVA}}{100\text{ MVA}} = 1{,}2\,\alpha f$

Z_3 (φορτίο): $S_{3\,pu} = \dfrac{40\,MVA}{100\,MVA} = 0{,}4$ pu $\Big\}$ $\Rightarrow P_3 = S_3 \cos\varphi = 0{,}36$ pu

$ΣΙ = \cos\varphi = 0{,}9$

$Q_3 = S_3 \cdot \sin\varphi = 0{,}17$ pu

$\cos\varphi = 0{,}9 \rightarrow \sin\varphi = 0{,}436$

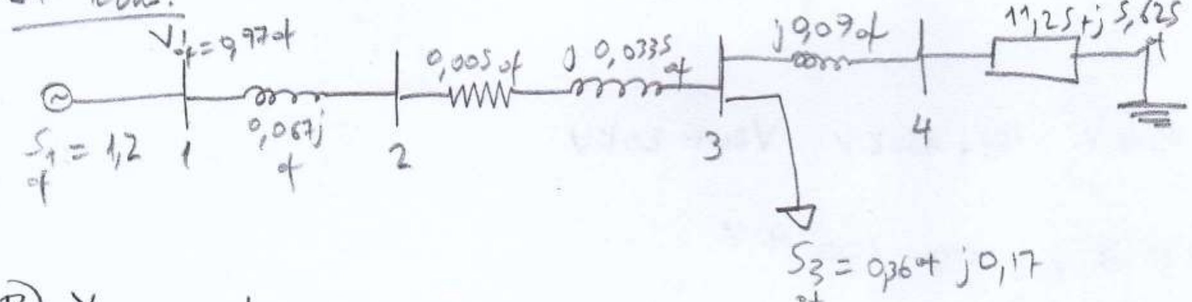
$\hat{S}_3 = 0{,}36 + j\,0{,}17$ pu

Z_4 (φορτίο): (τρίγωνο) $Z_\varphi = Z + \dfrac{Z}{2} = 45 + 22{,}5j\ \Omega$

$Z_{B\varphi} = \dfrac{V_{B\varphi}^2}{S_B} = 4\,\Omega$

$\hat{Z}_{4\,pu} = \dfrac{Z_4}{Z_{B\varphi}} = 11{,}25 + j5{,}625$ pu

1Φ Κυκλ.



β) $Y_{1,2\,pu} = \dfrac{1}{Z_{1,2\,pu}} = -14{,}92j$

$Y_{2,3\,pu} = \dfrac{1}{Z_{2,3\,pu}} = 4{,}36 - 24{,}2j$

$Y_{3,4\,pu} = \dfrac{1}{Z_{34\,pu}} = -11{,}11$

$Y_{4\,pu} = \dfrac{1}{Z_{4\,pu}} = 0{,}07 - 0{,}036j$

$$[Y]_{pu} = \begin{bmatrix} Y_{12} & -Y_{12} & 0 & 0 \\ -Y_{12} & Y_{12}+Y_{23} & -Y_{23} & 0 \\ 0 & -Y_{23} & Y_{23}+Y_{34} & -Y_{34} \\ 0 & 0 & -Y_{34} & Y_{34}+Y_{44} \end{bmatrix} =$$

$$\Rightarrow [Y]_{\text{αμ}} = \begin{bmatrix} -14{,}92j & 14{,}92j & 0 & 0 \\ 14{,}92j & 4{,}36-49{,}12j & -4{,}36+29{,}2j & 0 \\ 0 & -4{,}36+29{,}2j & 4{,}36-40{,}3j & 11{,}1j \\ 0 & 0 & 11{,}1j & 0{,}07-11{,}136j \end{bmatrix}$$

γ) $\underline{Z3}$: $P_{g3} - P_{D3} = G_{33}V_3^2 + V_3V_2 + G_{32}\cos(\delta_3-\delta_2) + V_3V_4 \cancel{G_{34}} \cos(\delta_3-\delta_4) +$

$+ V_3V_2B_{32}\sin(\delta_3-\delta_2) + V_3V_4B_{34}\sin(\delta_3-\delta_4) =$

$= 4{,}36V_3^2 - 4{,}36V_3V_2\cos(\delta_3-\delta_2) + 29{,}2V_3V_2\sin(\delta_3-\delta_2) +$

$+ 11{,}1V_3V_4\sin(\delta_3-\delta_4)$

$Q_{g3} - Q_{D3} = -B_{33}V_3^2 + V_3V_2G_{32}\sin(\delta_3-\delta_2) + V_3\cancel{V_4 G_{34}}\sin(\delta_3-\delta_4) -$

$- V_3V_2B_{32}\cos(\delta_3-\delta_2) - V_3V_4B_{34}\cos(\delta_3-\delta_4) =$

$= 40{,}3V_3^2 - 4{,}36V_3V_2\sin(\delta_3-\delta_2) - 29{,}2V_3V_2\cos(\delta_3-\delta_2) -$

$- 11{,}1V_3V_4\cos(\delta_3-\delta_4)$

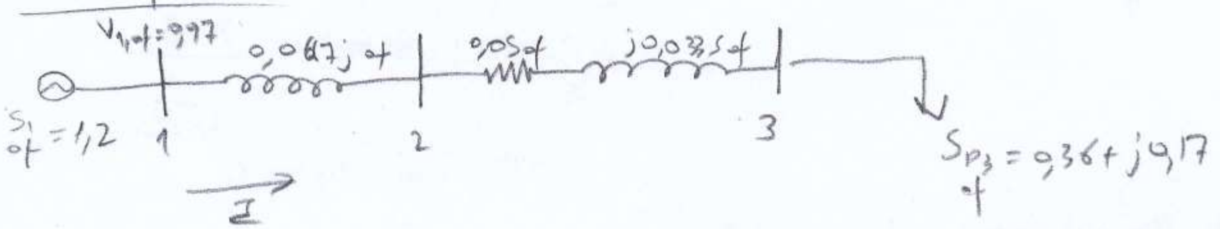
$\underline{\underline{Z4}}$: $P_{g4} - P_{D4} = G_{44}V_4^2 + V_4V_3G_{43}\cancel{\cos(\delta_4-\delta_3)}^{0} + V_4V_3B_{43}\sin(\delta_4-\delta_3)$

$= 0{,}07V_4^2 + 11{,}1V_4V_3\sin(\delta_4-\delta_3)$

$Q_{g4} - Q_{D4} = -B_{44}V_4^2 + V_4V_3G_{43}\cancel{\sin(\delta_4-\delta_3)} - V_4V_3B_{43}\cos(\delta_4-\delta_3) =$

$= 11{,}136V_4^2 - 11{,}1V_4V_3\cos(\delta_4-\delta_3)$

δ) Ισοδ. αμ κυκλ.



Μμοδ. Ισχ. Z2: $S_3 = G_{g3} - S_{D3} = -0{,}36 - 0{,}17j$ αμ

$$\left.\begin{aligned} \hat{S}_1 &= 0{,}36 + 0{,}005I^2 + j(0{,}1005I^2 + 0{,}17) \\ S_1 &= 1{,}2 \end{aligned}\right\} \Rightarrow$$

$\Rightarrow (1{,}2)^2 = (0{,}36 + 0{,}005I^2) + j(0{,}1005I^2 + 0{,}17) \Rightarrow I = 3{,}07$ αμ

$S_{D_2} = (0{,}005 + j\,0{,}0335) I^2 + S_{D_3} = 0{,}408 + j\,0{,}49$ of

$S_2 = S_{g_2} - S_{D_2} = -0{,}408 - 0{,}49 j$ of

Gauss - Siedel

$V_2 = 1\angle 0$ of

$V_3 = 1\angle 0$ of

$$V_2^{(1)} = \frac{1}{Y_{22}} \left\{ \frac{P_2 - jQ_2}{[V_2^{(0)}]^*} - Y_{21} V_1 - Y_{23} V_3^{(0)} \right\} = 0{,}978 \angle -0{,}415 \text{ of}$$

$$V_3^{(1)} = \frac{1}{Y_{33}} \left\{ \frac{P_3 - jQ_3}{[V_3^{(0)}]^*} - Y_{23} V_2^{(1)} \right\} = 0{,}97 \angle -1{,}08 \text{ of}$$

$Z3$: $V_3^{(1)} = 0{,}97 \cdot 150 = 145{,}5\,kV$

Άσκηση 16

$V_{B_1} = 100\,kV$

a) $\Sigma I = 1 \Rightarrow Q_{o\lambda} = 0 \Rightarrow Q_{\varphi_1} + Q_C = 0$

$Q_{\varphi_1} = P_{\varphi_1} \tan\varphi \Rightarrow Q_{\varphi_1} = 34$ MVAr

$$\Rightarrow Q_C = \frac{3 V_{B_1}^{y^2}}{X_C} \Rightarrow$$

$$\Rightarrow -34 \cdot 10^6 = \frac{3 V_{B_1}^{y^2}}{\frac{-1}{2\pi f C_Y}} \Rightarrow$$

$$\Rightarrow C_Y = 10{,}8\,\mu F$$

β) $\underbrace{V_{B_1} = 100\,kV,\ V_{B_2} = 100\,kV,\ V_{B_3} = 100\,kV}_{V_B = 100\,kV}$

ΓΜ: $Z_B = \frac{V_B^2}{S_B} = 200\,\Omega$

$$\hat{Z}_{1,2,pu} = \frac{\hat{Z}_{1,2}}{Z_B} = \frac{(0{,}2 + j0{,}4)\,50}{200} = 0{,}05 + 0{,}1 j$$

$$\hat{Z}_{2,3,pu} = \frac{\hat{Z}_{2,3}}{Z_B} = \frac{j0{,}4 \cdot 150}{200} = j\,0{,}3$$

$$\hat{Z}_{1,3,\alpha\mu} = \frac{\hat{Z}_{1,3}}{Z_B} = \frac{j\,0{,}4\cdot 100}{200} = j\,0{,}2$$

$$\hat{Z}_{C,\alpha\mu} = \frac{\hat{Z}_C}{Z_B} = \frac{-\frac{j}{2\pi f C_Y}}{Z_B} = -j\,1{,}474$$

<u>Z3</u> (χωρ. φορτίο)

$P_{\varphi_3} = 40MW$

$Q_{\varphi_3} = -10MVAr$

$$\hat{S}_{\varphi_3} = P_{\varphi_3} + j\,Q_{\varphi_3} = 40 - j\,10 \ MVA$$

$$\hat{S}_{\varphi_3,\alpha\mu} = \frac{\hat{S}_{\varphi_3}}{S_B} = 0{,}20 - j\,0{,}2$$

<u>Z1</u> (επαγ. φορτίο)

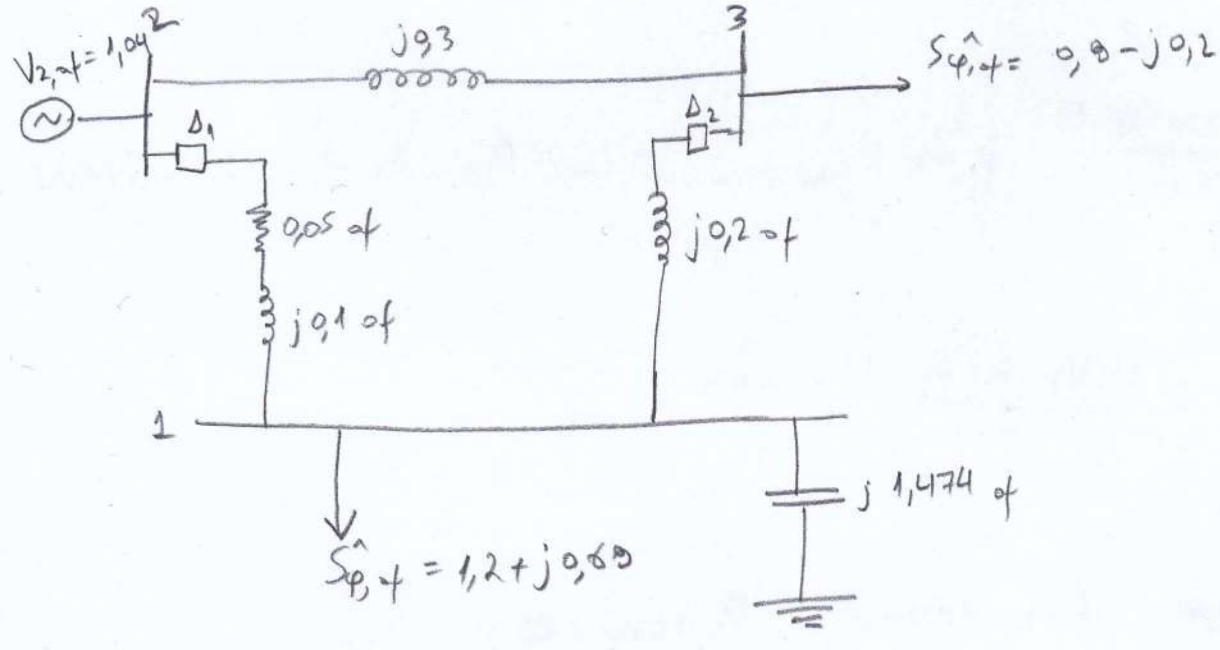
$P_{\varphi_1} = 60\ MW$

$Q_{\varphi_1} = 34\ MVAr$

$$\hat{S}_{\varphi_1} = 60 + j\,34 \ MVA$$

$$\hat{S}_{\varphi_1,\alpha\mu} = 1{,}2 + j\,0{,}68$$

$$V_{2,\alpha\mu} = \frac{V_2}{V_B} = 1{,}04$$



γ) $$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{1,2,\alpha\mu} = \frac{1}{Z_{1,2}} = 4 - 8j$$

$$Y_{2,3,\alpha\mu} = \frac{1}{Z_{2,3}} = -3{,}33j$$

$$Y_{1,3,\alpha\mu} = \frac{1}{Z_{1,3}} = -5j$$

$$Y_{11,\alpha\mu} = Y_1 + Y_{12} + Y_{13} = 4 - 12{,}32j$$

$$Y_{12,\alpha\mu} = Y_{21} = -Y_{1,2} = -4 + 8j$$

$$Y_{22,\alpha\mu} = Y_{1,2} + Y_{2,3} = 4 - 11{,}33j$$

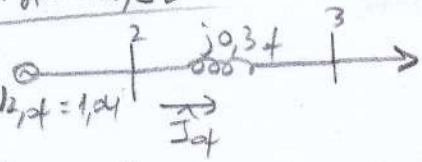
$Y_{23,of} = Y_{32} = -y_{2,3} = 3,53j$

$Y_{33,of} = y_{1,3} + y_{2,3} = -8,333j$

$Y_{13} = Y_{31} = -y_{1,3} = 5j$

$$[Y]_{of} = \begin{bmatrix} 4-12,33j & -4+8j & 5j \\ -4+8j & 4-11,33j & 3,33j \\ 5j & 3,33j & -8,333j \end{bmatrix}$$

β) Ανάγκη Δ_1, Δ_2



$\hat{S}_{\varphi,of} = 0,8 - j0,2$

$\hat{S}_{3,of} = \hat{S}_{G3} - \hat{S}_{B3} = -0,8 + j0,2$

$$\left.\begin{aligned} \hat{V}_{2,of} &= \hat{I}_{of} \cdot j0,3 + \hat{V}_{3,of} \\ \hat{S}_{3,of} &= \hat{V}_{3,of}\, \hat{I}_{of}^{*} \end{aligned}\right\} \Rightarrow \hat{S}_{3,of} = \frac{\hat{V}_{2,of}^{*} \cdot \hat{V}_{3,of} - V_{3,of}^2}{-j0,3} \Rightarrow$$

$$\Rightarrow (V_{3,of}^2 + 0,06) + j0,24 = 1,04\, V_{3,of} \angle \delta_3 \Rightarrow$$

$$\Rightarrow (V_{3,of}^2 + 0,06)^2 + 0,24^2 = (1,04 V_{3,of})^2 \Rightarrow V_3 = 94,5\,kV$$

$$I_{of} = \frac{S_{3,of}}{V_{3,of}} = 0,87, \quad I_B = \frac{S_B}{\sqrt{3} V_B} = 288,7\,A, \quad I = I_{of} \cdot I_B = 251,17\,A$$

Ροή στο 2 της ΓΜ 2,3:

$$\left.P_{2,3\,of} = \frac{V_2 V_3 \sin(\delta_2 - \delta_3)}{X_{23}}\right\} \Rightarrow P_{2,3of} = -0,8 \Rightarrow P_{23} = P_B \cdot P_{2,3of} = -40\,MW$$

$\rightarrow \delta_3 = 14,14$

$$Q_{2,3\,of} = \frac{V_2^2}{X_{23}} = \frac{V_2 V_3 \cos(\delta_2 - \delta_3)}{X_{23}} = 0,43$$

$Q_{2,3} = 21,5\,MVAr$

Το ΓΜ δεν έχει ωμική αντίσταση: $P_{2,3\,loss} = 0$

όμως, $Q_{2,3\,loss,of} = X_{2,3\,of} \cdot I_{of}^2 = 0,23 \Rightarrow Q_{2,3\,loss} = 0,23 \cdot 50 = 11,5\,MVAr$