Ασκήσεις ΣΗΕ - Τμήμα 2

2019 - 2020

Β' ΟΜΑΔΑ

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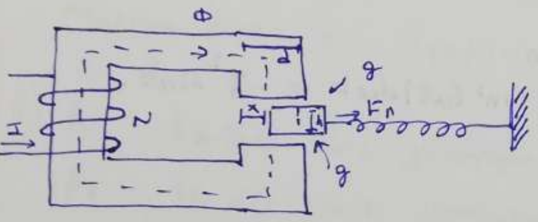
Κεφάλαιο 6

Άσκηση 1



$N = 2000$
$g = 2\ mm$
$d = 10\ cm$
$b = 5\ cm$

α) $I = 5A$

$$R(x) = 2R_g(x) = 2 \cdot \frac{1}{\mu_0} \cdot \frac{g}{A_g(x)} = \frac{2}{\mu_0} \cdot \frac{g}{b(d-x)}$$

$$\varphi(x) = \frac{NI}{R(x)} = \frac{NI \cdot b(d-x) \cdot \mu_0}{2g}$$

$$L(x) = \frac{N^2}{R(x)} = \frac{N^2 \mu_0 b(d-x)}{2g}$$

$$W_m = \frac{1}{2} R(x) \varphi^2(x) = \frac{g}{\mu_0 \cdot b(d-x)} \cdot \frac{N^2 I^2 (d-x)^2 \mu_0^2}{4g^2} = \frac{N^2 I^2 (d-x) b \mu_0}{4g}$$

$$F_m = -\frac{1}{2}\varphi^2(x)\frac{dR}{dx} = -\frac{1}{2} \cdot \frac{N^2 I^2 b^2 (d-x)^2 \mu_0^2}{4g^2} \cdot \frac{2}{\mu_0} \cdot \frac{g}{b} \cdot \frac{1}{(d-x)^2} = -\frac{N^2 I^2 b \mu_0}{4g}$$

$x = 0$: $\varphi(0) = 0{,}0158\ Wb$, $L(0) = 6{,}283\ H$, $W_m(0) = 78{,}53\ J$
$F_m(0) = -785{,}4\ N$

$x = \frac{d}{2}$: $\varphi(0{,}05) = 0{,}00785\ Wb$, $L(0{,}05) = 3{,}141\ H$, $W_m(0{,}05) = 39{,}27\ J$
$F_m(0{,}05) = -785{,}4\ N$

β) $V = 230\sqrt{2}\cos(100\pi t)$
$R = 0{,}5$

$$i = \frac{V}{R} = 460\sqrt{2}\cos(100\pi t)$$

$$F_m = \frac{-4 \cdot 10^6 \cdot 460^2 \cdot 2 \cdot \cos^2(100\pi t) \cdot 5 \cdot 10^{-2}\, 4\pi\, 10^{-7}}{4 \cdot 2 \cdot 10^{-3}} = -460^2 \cdot 20\pi \cdot 10^{9} \cdot 10^{-9} \cdot \cos^2(100\pi t)$$
$$\simeq -13{,}3\cos^2(100\pi t)\ MN$$

$\bar{F}_n = \frac{1}{0{,}01}\int_0^{0{,}01} F_n(t)\,dt = 100\cdot 13{,}3\int_0^{0{,}01}\cos^2(100\pi t)\,dt = -6{,}65\ MN$ ("−": αντίθετη φορά από την σχεδιασμένη)

Άσκηση 6.2 (βιβλίου)

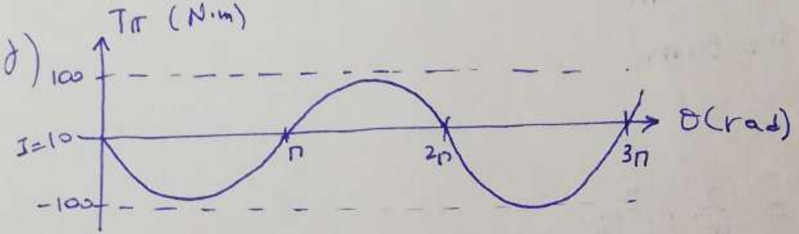
$L_{11} = 0{,}2mH,\ L_{22} = 0{,}1mH,\ L_{12} = 0{,}5mH\cdot\cos\theta,\ i = \sqrt{2}\,I\sin(\omega t)$

α) $T_\pi = \frac{1}{2}\cdot i^2\cdot 0{,}5\cdot 10^{-3}(-\sin\theta) = -2I^2\sin^2(\omega t)\sin\theta \Rightarrow$

$\Rightarrow T_\pi(t) = -2I^2\sin\theta\cdot\sin^2(\omega t)$

β) $\bar{T}_\pi = \frac{1}{2\pi}\int_0^{2\pi} -2I^2\sin^2(\omega t)\sin\theta\, d\omega t = -\frac{I^2\sin\theta}{\pi}\int_0^{2\pi}\sin^2(\omega t)\,d\omega t = -I^2\sin\theta.$

γ)



δ) Μια πιθανή λειτουργία θα ήταν σαν ανεμιστήρας με ένα πτερύγιο το οποίο θα στρέφεται περιοδικά.

Άσκηση 6.8 (βιβλίου)

α) $e_a = U_a - i_a r_a = \frac{d}{dt}(L_{aa} i_a + L_{af}\cdot i_f)$
$e_b = U_b - i_b r_a = \frac{d}{dt}(L_{bb} i_b + L_{bf}\cdot i_f)$ $\Rightarrow$ $i_a = \sqrt{2}I\cos(\omega t)$, $i_f = I_f$, $\theta = \omega t + \delta_{sr}$, $i_b = \sqrt{2}I\sin(\omega t)$

$e_a = -\sqrt{2}I\omega L\sin(\omega t) - I_f\omega M\sin(\omega t + \delta_{sr})$

$e_b = \sqrt{2}I\omega L\cos(\omega t) + I_f\omega M\cos(\omega t + \delta_{sr})$

$P_e = e_a i_a + e_b\cdot i_b =$

$= -2I^2\omega L\cos(\omega t)\sin(\omega t) - \sqrt{2}I I_f\omega M\sin(\omega t + \delta_{sr})\cos(\omega t)$
$+ 2I^2\omega L\cos(\omega t)\sin(\omega t) + \sqrt{2}I I_f\omega M\cos(\omega t + \delta_{sr})\sin(\omega t) \Rightarrow$

$\Rightarrow P_e = \sqrt{2}I I_f\omega M[\cos(\omega t + \delta_{sr})\sin(\omega t) - \sin(\omega t + \delta_{sr})\cos(\omega t)]$

$P_m = T_\pi\cdot\omega = M I_f[\sqrt{2}I\sin(\omega t)\cos(\omega t + \delta_{sr}) - \sqrt{2}I\cos(\omega t)\sin(\omega t + \delta_{sr})]\omega$

$= \sqrt{2}I I_f\omega M[\cos(\omega t + \delta_{sr})\sin(\omega t) - \sin(\omega t + \delta_{sr})\cos(\omega t)] = P_e$

Δηλ. $P_m = P_e = -M\sqrt{2}I I_f\omega\sin(\delta_{sr})$

β) $e_r = \frac{d}{dt}\left[L_{ff} \cdot i_f + L_{af} \cdot i_a + L_{bf} \cdot i_b\right] =$

$= -M\omega \sin(\omega t + \delta_{sr})\sqrt{2}I\cos(\omega t) - \sqrt{2}\,\omega \sin(\omega t) M\cos(\omega t + \delta_{sr})$
$+ M\omega\cos(\omega t + \delta_{sr})\sqrt{2}I\sin(\omega t) + \sqrt{2}I\omega\cos(\omega t) M\sin(\omega t + \delta_{sr})$

$\Rightarrow e_r = 0$

Δεν συμμετέχει στην ηλεκτρομηχανική μετατροπή ενέργειας γιατί δεν θα ασκείται ροπή στον δρομέα.

γ) Αν $\delta_{sr} > 0$ τότε γεννήτρια.
Αν $\delta_{sr} < 0$ τότε κινητήρας.
(σελ. 188 βιβλίο)

Κεφάλαιο 8

Άσκηση 1

$S_B = 10\,kVA$, $X_s = 14{,}44\,\Omega$ ανά φάση, $P = 2$ πόλοι
$V_B = 380V$, όταν $I_f = 5A$, $E_f = 380V$
$f = 50Hz$

α) $I_a = 0{,}5\,\alpha\mu$ $\quad \cos\varphi = 0{,}8 \overset{\varepsilon\pi\alpha\gamma.}{\rightsquigarrow} \varphi = 36{,}86° \Rightarrow I_a = 0{,}5 \cdot (0{,}8 - j0{,}6)\,\alpha\mu.$

$I_f' = 5{,}5 \rightsquigarrow I_f = \frac{5{,}5}{5} = 1{,}1\,\alpha\mu.$

Άρα και $E_f = 1{,}1\,\alpha\mu \rightsquigarrow E_f = 1{,}1(\cos\delta + j\sin\delta)$

Έχουμε $X_s = 14{,}44$ με $V_B, S_B \Rightarrow Z_B = \frac{V_B^2}{S_B} = 14{,}44\,\Omega$

$X_s = 1\,\alpha\mu$ $\qquad \omega_s = \frac{2\pi f}{P/2} = 100\pi$

$V_t = 1{,}1(\cos\delta + j\sin\delta) - j0{,}5(0{,}8 - j0{,}6) = 1{,}1\cos\delta - 0{,}3 + j(1{,}1\sin\delta - 0{,}4)$

Άρα, πρέπει $1{,}1\sin\delta - 0{,}4 = 0 \Rightarrow \sin\delta = \frac{0{,}4}{1{,}1} \Rightarrow \delta = 21{,}3°$

και έχουμε $\cos\delta = 0{,}93$

Άρα, $V_t = 0{,}723\,\alpha\mu \Rightarrow V_t = 274{,}7\,V$ (πολική) $\Rightarrow V_{t\varphi} = 158{,}6\,V$ (φασική)

$T = \frac{10 \cdot 10^3}{10^2\pi} \cdot \frac{0{,}723 \cdot 1{,}1}{1} \cdot 0{,}36 = \frac{100}{\pi} \cdot 0{,}723 \cdot 1{,}1 \cdot 0{,}36 = 9{,}11\,Nm$

β) $V_t = 1,1(\cos\delta + j\sin\delta) - j0,5(\cos\theta + j\sin\theta) =$

$= 1,1\cos\delta + 0,5\sin\theta + j(1,1\sin\delta - 0,5\cos\theta)$

Άρα, $\cos\theta = 2,2\sin\delta \sim \cos^2\theta = 2,2^2\sin^2\delta \Rightarrow \cos^2\theta = 2,2^2(1-\cos^2\delta) \Rightarrow$

$\Rightarrow \cos^2\delta = 1 - \dfrac{\cos^2\theta}{2,2^2}$ (*)

και θέλουμε: $V_t = \sqrt{(1,1\cos\delta + 0,5\sin\theta)^2}$ να είναι ελάχιστη

$\cos\delta = \dfrac{V_t}{1,1} - \dfrac{0,5\sin\theta}{1,1} \Rightarrow \cos^2\delta = \left(\dfrac{V_t}{1,1} - 0,45\sin\theta\right)^2$

$\overset{(*)}{\Rightarrow} 1 - \dfrac{\cos^2\theta}{2,2^2} = \left(\dfrac{V_t}{1,1} - 0,45\sin\theta\right)^2 \Rightarrow$

$\Rightarrow \dfrac{V_t^2}{1,1^2} - \dfrac{0,9}{1,1}\sin\theta V_t + 0,45^2\sin^2\theta + \dfrac{\cos^2\theta}{2,2^2} - 1 = 0$

$\Rightarrow V_t^2 - (0,9\cdot 1,1\cdot\sin\theta)V_t + (1,1\cdot 0,45)^2\sin^2\theta + \dfrac{\cos^2\theta}{4} - \dfrac{1}{1,1^2} = 0$

$\Rightarrow V_t^2 - 0,99\sin\theta V_t + 0,245\sin^2\theta + 0,25\cos^2\theta - 0,83 = 0$

$\Rightarrow V_t^2 - 0,99\sin\theta V_t - 0,05\sin^2\theta - 0,58 = 0$

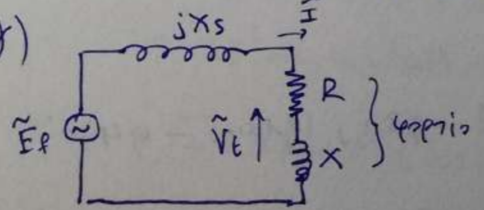
$\Delta = 1,18\sin^2\theta + 2,32$

$V_{t1} = \dfrac{0,99\sin\theta + 1,18\sin^2\theta + 2,32}{2} \xrightarrow[\sin\theta=-1]{} V_{t1\,min} = 0,58$

$V_{t2} = \dfrac{0,99\sin\theta - 1,18\sin^2\theta - 2,32}{2} < 0$, άτοπο.

Άρα $V_{t\,min} = 0,58$ αμ για $\sin\theta = -1$, $\cos\theta = 0$

$V_{t\,min} = 220,4\,V$ πολική.

γ) 

$\tan\varphi = \dfrac{X}{R} =$ σταθ.

$I = \dfrac{E_f}{\sqrt{R^2 + (X_L + X_l)^2}} = \dfrac{E_f}{\sqrt{R^2 + (X_s + R\tan\varphi)^2}}$

$P = 3I^2R = \dfrac{3RE_f^2}{R^2 + X_s^2 + 2RX_s\tan\varphi + R^2\tan^2\varphi} \longrightarrow$ $P = P_{max}$ όταν η συνθ. αντίσταση του φορτίου είναι ίση με την εσωτερ.

δηλ. $\sqrt{R^2+X^2} = X_s \Rightarrow X = X_s\cdot\sin\varphi$

$R = X_s\cdot\cos\varphi$

Άρα $I = \frac{E_f}{\sqrt{2} X_s \sqrt{1+\sin^2\varphi}} = \frac{418}{\sqrt{2} \cdot 14,44 \cdot 1,166} = 17,55 A$

$P_{max} = 3RI^2 = \frac{2E_f^2}{2X_2} \cdot \frac{\cos\varphi}{1+\sin\varphi} = 0,050 W$, $V_t = \sqrt{R^2+X^2} \cdot I = 253,42 V$

δ) $V_t = 1 \, a\mu$, $E_f = 1,1 \, a\mu$, $X_s = 1 \, a\mu$, $P_{a\mu} = \frac{5 \cdot 10^3}{10 \cdot 10^3} = 0,5$

$\sin\delta = \frac{P X_s}{V_t E_f} = \frac{0,5 \cdot 1}{1,1} = 0,45 \Rightarrow \delta = 26,7°$

$Q = \frac{1 \cdot 1,1 \cdot 0,893 - 1}{1} = -0,0177 \, a\mu \Rightarrow Q = -177 \, Var$

Άσκηση 4

$P = 12$ $\quad X_s = 0,866 \, a\mu$

$V_B = 380V$ $\quad r_a = 0$

$S_B = 25 kVA$ $\quad$ Αν $I_f = 3A$, $E_f = 380V$

$f = 50Hz$

α) $I_f' = 3,45A$ $\rightarrow I_f = \frac{3,45}{3} = 1,15 \, a\mu \rightarrow E_f = 1,15 \, a\mu$

$T = 300 Nm$, $\omega_s = \frac{2\pi f}{P/2} = \frac{2\pi f}{6} = \frac{50\pi}{3} \, r/s$

$300 = \frac{25 \cdot 10^3}{\frac{50\pi}{3}} \cdot \frac{1 \cdot 1,15}{0,866} \sin\delta \Rightarrow \sin\delta = \frac{3 \cdot 10^2 \cdot 0,866 \cdot 20\pi/3}{10^3 \cdot 1,15} \Rightarrow \sin\delta = 0,47 \Rightarrow \delta = 28,2°$

$\hat{I}_a = \frac{1,15 \, (0,88 + j0,47) - 1}{j0,866} = \frac{0,012 + j0,54}{j0,866} = 0,62 - j0,014 \Rightarrow$

$\Rightarrow \hat{I}_a = 0,64 \angle -1,27°$

$I_a = 0,64 I_B = 0,64 \frac{25 \cdot 10^3}{\sqrt{3} \cdot 380} = 24,3 A$

$\cos\varphi = \cos(1,27°) = 0,99$

β) $T_{max} = \frac{S_B \cdot V_t \cdot E_f}{\omega_s X_s} = \frac{25 \cdot 10^3}{\frac{50\pi}{3}} \cdot \frac{1,1}{0,866} = 551,34 \, Nm$

$\hat{E}_f = j1$

$\hat{I}_a = \frac{j1 - 1}{jX_s} = \frac{j1-1}{j0,866} = 1,63 \, a\mu \angle 45° = 1,15 + j1,15 \, a\mu \rightarrow I_a = 61,7 \angle 45° A$

Άσκηση 8

$P = 6 \quad X_s = 1 \text{af}$

$S_B = 100 \text{ kVA} \quad$ Αν $I_f = 2 \text{ A}$ τότε $E_f = 380 \text{ V}$

$V_B = 380 \text{ V}$

$f = 50 \text{ Hz}$

α) $P = 60 \text{ kW}$

$V_t = 1 \text{ af} \angle 0^\circ \qquad \omega_s = \dfrac{2\pi f}{P/2} = \dfrac{100\pi}{3} \text{ r/s}$

$S = \dfrac{60}{100} = 0{,}6 + j0 \text{ af}$

$\hat{I} = \dfrac{S^*}{\hat{V}^*} = 0{,}6 \text{ af} \angle 0^\circ$

$\hat{E}_f = 1 + j0{,}6 = 1{,}17 \text{ af} \angle 31^\circ \Rightarrow I_f = 1{,}17 \cdot 2 = 2{,}34 \text{ A}$

$\sin\delta = \dfrac{0{,}6 \cdot 1}{1 \cdot 1{,}17} = 0{,}513 \Rightarrow \delta = 30{,}9^\circ$

$T = \dfrac{10^2 \cdot 10^3}{100\pi/3} \cdot \dfrac{1{,}17}{1} \cdot 0{,}513 = \dfrac{3000 \cdot 1{,}17 \cdot 0{,}513}{\pi} = 573{,}2 \text{ Nm}$

β) $T = 600 \text{ Nm}, \ \cos\varphi = 0{,}9, \ V_t = 1 \text{ af} \angle 0^\circ$

$\hat{I}_a = \dfrac{0{,}9 - j0{,}44}{1} = 0{,}9 - j0{,}44$

$\hat{E}_f = 1 + j \cdot 1 \cdot (0{,}9 - j0{,}44) = 1{,}44 + j0{,}9 = 1{,}7 \text{ af} \angle 32^\circ$

$I_f = 1{,}7 \cdot 2 = 3{,}4 \text{ A}$

$T = \dfrac{10^2 \cdot 10^3}{100\pi/3} \cdot \dfrac{1{,}7}{1} \sin\delta \Rightarrow 600 = \dfrac{3 \cdot 10^3 \cdot 1{,}7}{\pi} \sin\delta \Rightarrow \sin\delta = \dfrac{600\pi}{5100} = 0{,}37$

$\Rightarrow \delta = 21{,}7^\circ \Rightarrow \cos\delta = 0{,}93$

$Q = \dfrac{1{,}17 \cdot 0{,}93 - 1}{1} = 0{,}58 \text{ af} \Rightarrow Q = 0{,}58 \cdot 44 \cdot 10^3 = 25{,}52 \text{ kVar}$

γ) $T_{max} = \dfrac{10^2 \cdot 10^3}{\frac{100\pi}{3}} \cdot \dfrac{1 \cdot E_f}{1} \Rightarrow 600 = \dfrac{3 \cdot 10^3}{\pi} \cdot E_f \Rightarrow E_f = \dfrac{600\pi}{3000} = \dfrac{\pi}{5} = 0{,}63 \text{ af}$

$I_{f\,min} = 2 \cdot 0{,}63 \Rightarrow I_{f\,min} = 1{,}26 \text{ A}$

Κεφάλαιο 9

Άσκηση 1

$P = 6$, αστέρας, $r_1 = r_2 = 0{,}5\,\Omega$, $x_1 = x_2 = 2\,\Omega$, $x_\varphi \to \infty$

$380V$, $f = 50Hz$, $n(\tau\alpha\chi) = 950$ ΣΑΛ, $P_{\alpha\pi\omega\lambda.}$ $400W$

α). $f_r = s \cdot f$

όπου $s = \dfrac{n_s - n}{n_s}$

και $n_s = \dfrac{120 \cdot 50}{6} = 1000\,\text{spm}$ $\Rightarrow s = \dfrac{50}{1000} = 0{,}05 \Rightarrow f_r = 2{,}5\,Hz$

· $T_m = \dfrac{P_m}{\omega_m}$

· $\omega_m = (1-s)\,\omega_s$

$\omega_s = \dfrac{2\pi f}{\frac{P}{2}} = \dfrac{2\pi \cdot 50}{\frac{6}{2}} = \dfrac{100\pi}{3} \Rightarrow \omega_m = \dfrac{19}{20} \cdot \dfrac{100\pi}{3} = \dfrac{95\pi}{3}\ r/s$

· $P_m = P_e - 400 = (1-s) \cdot 3 I_2^2 \cdot \dfrac{r_2}{s} - 400 = \dfrac{1-s}{s} \cdot 3 I_2^2 \cdot r_2 - 400 = \dfrac{19 \cdot 3}{2} \cdot I_2^2 - 400$

· $x_\varphi \to +\infty \Rightarrow \hat{I}_1 = \hat{I}_2$ άρα $\hat{I}_2 = \dfrac{\hat{V}_1}{Z_{o\lambda}} = \dfrac{\frac{380}{\sqrt{3}}}{(r_1 + \frac{r_2}{s}) + j(x_1 + x_2)} = 19{,}5\,\angle -21^\circ$

$P_{em} = 0{,}95 P_g$, $P_g = 3 I_1^2 \dfrac{r_2}{s} = 3 \cdot 19{,}52^2 \cdot 10 = 11.430{,}9\,W$, $P_{em} = 10.859{,}3\,W$

Οπότε, $P_m = 10{,}4\,KW$ $(= P_{em} - P_{\alpha\pi})$

Συνεπώς, $T_m = 105\,N \cdot m$

β) $T_{max} = \dfrac{1}{\omega_s} \cdot \dfrac{\frac{3}{2} \cdot V_{1a}^2}{R_1 + \sqrt{R_1^2 + (x_1+x_2)^2}} \underset{R_1 = r_1,\ X_1 = x_1}{\overset{V_1 = V_{1a}}{=}} \dfrac{3 \cdot 3}{2 \cdot 100\pi} \cdot \dfrac{\left(\frac{380}{\sqrt{3}}\right)^2}{0{,}5 + \sqrt{0{,}5^2 + 16}} = 145{,}15\,Nm$

$S_{max} = \dfrac{r_2}{\sqrt{R_1^2 + (x_1+x_2)^2}} = 0{,}124$

γ) Το μόνο που αλλάζει: $U_Y = \dfrac{U_\Delta}{\sqrt{3}} = \dfrac{380}{\sqrt{3}}$, $U_1 = \dfrac{U_Y}{\sqrt{3}} = 127V$

Ίδια ταχύτητα → ίδια n, ίδια s, ίδια ω_s, ω_m

Θα πρέπει να είναι ίδιο το T_m άρα και το P_m οπότε και το P_e.

$\cdot\ P_e = (1-s)\,P_{g_1} = (1-s)\cdot 3 I_2'^2 \cdot \frac{r_2'}{s} = 10{,}8\cdot 10^3 \Rightarrow 19{,}3 \cdot I_2'^2 \cdot r_2' = 10{,}8\cdot 10^3 \Rightarrow$

$\Rightarrow r_2' = \frac{10{,}8}{19{,}3}\cdot 10^3 \cdot \frac{1}{I_2'^2}$

$\cdot\ \hat{I}_2' = \frac{\hat{V}_1}{Z_{on}} = \frac{380}{(0{,}5 + 20 r_2') + 4j} \Rightarrow |I_2'|^2 = \frac{380^2}{(0{,}5 + 20 r_2')^2 + 16}$

Άρα, $r_2' = \frac{189{,}5}{380^2}\left[(0{,}5 + 20 r_2')^2 + 16\right] \Rightarrow r_2' = A\left[400 r_2'^2 + 20 r_2' + 16{,}25\right] \Rightarrow$

$\Rightarrow r_2' = 0{,}525 r_2'^2 + 0{,}026 r_2' + 0{,}021 \Rightarrow 0{,}525 r_2'^2 - 0{,}974 r_2' + 0{,}021 = 0$

$\Rightarrow r_2' = 1{,}83$ ή $r_2' = 0{,}02$, απορρ. γιατί $< r_2$

Οπότε $\Delta r_2 = r_2' - r_2 = 1{,}33$

Άρα $R_{εξ} = 37{,}5 - 0{,}5 = 37\,\Omega$ από $I_1 = \frac{V_1}{\sqrt{(r_1 + (r_2 + \frac{R_{εξ}}{s}))^2 + (X_1 + X_2)^2}}$

Άσκηση 2

$P = 4$, 10 HP, 380V, 50Hz, αστέρας, $r_1 = 0$, $r_2 = 0{,}4\,\Omega$, $X_1 = X_2 = 0{,}6\,\Omega$, $X_μ = 20\,\Omega$

$P_{απωλ.}$ 900W.

α) $T_{εκ} = \frac{1}{\omega_s}\cdot \frac{3 V_{1α}^2 \cdot r_2}{(R_1 + r_2)^2 + (X_1 + X_2)^2}$

$\hat{V}_{1α} = \frac{20j}{20{,}6j}\cdot \frac{380}{\sqrt{3}} = 213\,V$

$R_1 + jX_1 = \frac{20j\,(0{,}6j)}{j20{,}6} = j\,\frac{20\cdot 0{,}6}{20{,}6} = j\,0{,}583 \Rightarrow R_1 = 0,\ X_1 = 0{,}583$

$\omega_s = \frac{100\pi}{2} = 50\pi$

Άρα $T_{εκ} = 222{,}2$ N·m

β) $n = ;$, $T_e = T_{εκ}$

$T_e = 222{,}2 \Rightarrow \frac{1}{\omega_s}\cdot \frac{3 V_{1α}^2 \cdot \frac{r_2}{s}}{(R_1 + \frac{r_2}{s})^2 + (X_1 + X_2)^2} = 222{,}2 \xrightarrow{z = \frac{r_2}{s}} 222{,}2 = 866{,}5\cdot \frac{z}{z^2 + 1{,}183^2}$

$\Rightarrow 0{,}256 = \frac{z}{z^2 + 1{,}4} \Rightarrow 0{,}256 z^2 - z + 0{,}358 = 0 \Rightarrow z_1 = 3{,}5$, $z_2 = 0{,}399$

$\Rightarrow s_1 = 1{,}002$ ή $s_2 = 0{,}114$

απορρ.

Άρα $n = (1-s) n_s = 1329$ ΣΑΛ

γ) $n = \frac{P_m}{P_1} = \frac{P_e - 900}{P_1} = \frac{T_e \cdot \omega_m - 900}{P_1} = \frac{222,2 \cdot (1-s)\omega_s - 900}{P_1} =$

$= \frac{30,024 \cdot 10^3}{3 V_1 I_1 \cos\theta}$

· $\hat{I}_2 = \frac{\hat{V}_{1a}}{\frac{r_2}{s} + (x_1 + x_2)j} = 57,5 \angle -18,6$

· $\hat{I}_2 = \hat{I}_1 - \hat{I}_\varphi \Rightarrow \hat{I}_\varphi = \hat{I}_1 - \hat{I}_2$

· NTK στο 1: $\hat{V}_1 = \hat{I}_1 \cdot (\cancel{r_1}^{0} + j X_1) + \hat{I}_\varphi \cdot j X_\varphi \Rightarrow$

$\Rightarrow \frac{380}{\sqrt{3}} = \hat{I}_1 \cdot (\cancel{r_1}^{0} + j(X_1 + X_\varphi)) - \hat{I}_2 j X_\varphi$

$\Rightarrow \frac{380}{\sqrt{3}} = \hat{I}_1 \cdot (0 + 20,6j) - 57,5 \angle -18,6 \cdot 20j$

$\Rightarrow \hat{I}_1 = \frac{\frac{380}{\sqrt{3}} + 57,5 \angle -18,6 \cdot 20j}{20,6j} = 60 \angle -28$

Οπότε $n = \frac{30.024}{\sqrt{3} \cdot 380 \cdot 60 \cdot \cos(28)} = 0,861$

Άσκηση 4

$p=4$, 60HP, 380V, 50Hz, 800 W, $r_1 = 0,5\Omega$, $r_2 = 0,2\Omega$, $x_1 = x_2 = 1,5\Omega$, $x_\varphi \to \infty$

α) · $\hat{V}_1 = \hat{I}\left(0,5 + \frac{0,2}{s} + 3j\right) \overset{s=1}{\Rightarrow} \hat{I} = \frac{\frac{380}{\sqrt{3}}}{0,7 + 3j} = 71,2 \angle -77^\circ$, $I = 71,2\ A$

· $P_{\delta 1} = P_1 = 746 \cdot 60 = 44760 W$

· ΣΙ : $\cos(77) = 0,225$

· $T_{EK} = \frac{1}{\omega_s} \cdot \frac{380^2 \cdot 0,2}{0,7^2 + 3^2} = \frac{1}{50\pi} \cdot \frac{0,2 \cdot 380^2}{9 + 0,7^2} = 19,4\ N \cdot m$

β) · $s = 0,02$, τείγωνο

· $P_m = P_e - P_{an} = P_e - 800 = (1-s) \cdot 3 I_2^2 \cdot \frac{r_2}{s} - 800 = 49 \cdot 3 \cdot 0,2 \cdot I^2 - 800 =$
$= 29,4 I^2 - 800$

· $I = \frac{380}{0,5 + \frac{0,2}{0,02} + 3j} = 34,8 \angle -16^\circ$

Οπότε $P_m = 34,804\ W$, $f_s = 0,02 \cdot 50 = 1 Hz$, $\omega_m = 0,98 \cdot 50\pi = 153,9\ r/s$

$n = (1-s) n_s = 0,98 \cdot \frac{120 \cdot 50}{4} = 1740$ ΣΑΛ γ) $\cdot T_{max} = \frac{1,5}{50\pi} \cdot \frac{1}{r_1 + \sqrt{r_1^2 + (x_1+x_2)^2}} \cdot V_1^2 = \begin{cases} \text{αστέρας} = 130\, N\cdot m \\ \\ \text{τρίγωνο} = 389\ Nm \end{cases}$

$S_{max} = \frac{r_2}{\sqrt{R_1^2 + (x_1+x_2)^2}} = 0,066$

$n = (1-s) \cdot \frac{120 \cdot 50}{4} = 1401$ ΣΑΛ

$\omega_m = 146,7\ r/s$

Άσκηση 6

380V, 50 kVA, 50Hz, $X = 1,5$ αμ, $P_1 = 4$

380V, 35HP, 50 Hz, $P_2 = 6$, $r_1 = 0,15\Omega$, $r_2 = 0,06\Omega$, $X_1 = X_2$, $X_\varphi \to \infty$

α) $V_1 = \frac{380}{\sqrt{3}}$ V, $P_1 = 25$ kW

$I = 50A \Rightarrow I_{αμ} = 0,66$

Κινητήρας: $P_1 = 3 V_1 I \cos\theta \Rightarrow 25.000 = \sqrt{3} \cdot 380 \cdot 50 \cos\theta \Rightarrow \cos\theta = 0,76 \Rightarrow \theta = 41°$

Γεννήτρια: $\omega_s = \frac{100\pi}{P_1/2} = 50\pi\ r/s$

$\hat{V}_2 = \hat{E}_f - j X_s \cdot \hat{I}$

$1 = E_{f_{αμ}} \angle\delta - 1,5 j \cdot I_{αμ} \angle -41°$

$E_{f_{αμ}} \angle\delta = 1 + 1,5 j \cdot 0,66 \angle -41° = 1,8 \angle 24° \Rightarrow$

$E_{f_{αμ}} = 1,8$ αμ, $\delta = 24°$

$E_f = 1,8 \cdot 380 \Rightarrow E_f = 684$ V πολική.

β) $\hat{V}_1 = Z_{ολ} \cdot \hat{I} \Rightarrow \frac{380}{\sqrt{3}} = Z_{ολ} \cdot 50 \angle -41° \Rightarrow Z_{ολ} = \frac{\frac{380}{\sqrt{3}}}{50 \angle -41°} = 3,3 + j2,9 \Rightarrow$

$\Rightarrow r_1 + \frac{r_2}{s} + j2X_1 = 3,3 + j2,9 \Rightarrow 0,15 + \frac{0,06}{s} = 3,3 \Rightarrow s = 0,02$, $f_s = 1$ Hz

και $X_1 = \frac{2,9}{2} = 1,45\Omega$

Άρα $n = (1-s) n_s = 0,98 \cdot \frac{120 \cdot 50}{6} = 981$ ΣΑΛ γ) Έστω $-jX_c$ παράλληλα σε όλα

$$Z_{\pi} = \frac{\left(r_1 + \frac{r_2}{s} + j2x_1\right)\cdot(-jX_c)}{r_1 + \frac{r_2}{s} + j(2X_1 - X_c)} = \frac{-(3{,}15 + j2{,}9)jX_c}{3{,}15 + j(2{,}9 - X_c)} = \frac{2{,}9X_c - j3{,}15X_c}{3{,}15 + j(2{,}9 - X_c)} =$$

$$= \frac{(2{,}9X_c - j3{,}15X_c)(3{,}15 - j(-X_c + 2{,}9))}{3{,}15^2 - (X_c - 2{,}9)^2}$$

Πρέπει να είναι πραγματικό, άρα:

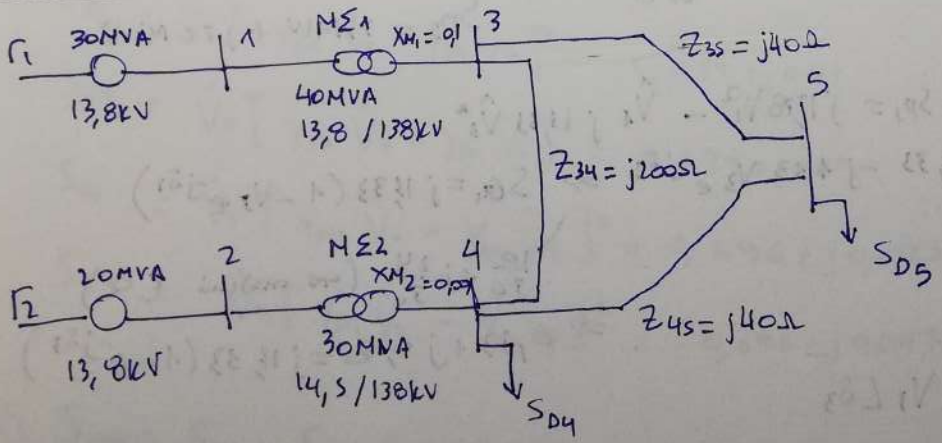
$$-3{,}15^2X_c - 2{,}9X_c(2{,}9 - X_c) = 0$$

$$3{,}15^2 + 2{,}9^2 - 2{,}9X_c = 0 \Rightarrow X_c = \frac{3{,}15^2 + 2{,}9^2}{2{,}9} = 6{,}32\,\Omega$$

και $X_\Delta = 3X_Y = 19 \Rightarrow \frac{1}{\omega C} = 19 \Rightarrow C = \frac{1}{19 \cdot 100\pi} = 168\,\mu F$

Κεφάλαιο 10

Άσκηση 1

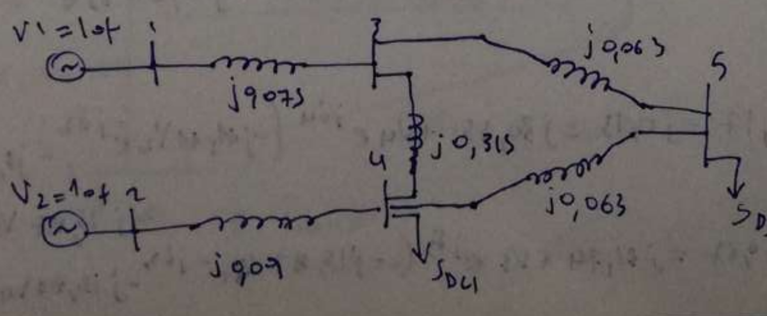


α) $S_B = 30MVA$ $\quad Z_B = V_B^2/S_B = \frac{(138\cdot10^3)^2}{30\cdot10^6} = 634{,}8\,\Omega$

$V_B = 138\,kV$

$$X_{M_1 new} = X_{M_1 old}\left(\frac{138}{138}\right)^2\cdot\left(\frac{30}{40}\right)$$

$$X_{M_2 new} = X_{M_2 old}\cdot 1\cdot\frac{3}{3} = 0{,}09$$



$$Z_{45} = Z_{35} = j\frac{40}{634,8} = j0,063 \text{ of}$$

$$Z_{34} = j\frac{200}{634,8} = j0,315$$

$$V_1 = \frac{13,8}{1,38} = V_2 = 1\text{of}$$

β) $$[Y] = j\begin{bmatrix} -13,33 & 0 & 13,33 & 0 & 0 \\ 0 & -11,11 & 0 & 11,11 & 0 \\ 13,83 & 0 & -32,4 & 3,17 & 15,87 \\ 0 & 11,11 & 3,17 & -30,15 & 15,87 \\ 0 & 0 & 15,87 & 15,87 & -31,74 \end{bmatrix}$$

$Y_{13} = -j13,33$

$Y_{24} = -j11,11$

$Y_{34} = -j3,17$

$Y_{35} = Y_{45} = -j15,87$

γ) $\Gamma_2 \rightarrow P_2 = 20MW$, $Q_2 = 0\,MVAR$, $\Gamma_1 \rightarrow$ ζυγός ταλάντ. $\delta = 0$, $S_{D4} = 5MW + j4MVAR$, $S_{D5} = 25MW + j20MVAR$

για $k=1$: $S_1 = S_{G_1} - S_{D_1} = j13,78V_1^2 - \hat{V}_1\, j13,33\, \hat{V}_3^*$

$= j13,33 - j133\, V_3 e^{-j\delta_3} \Rightarrow S_{G_1} = j13,33(1 - V_3 e^{-j\delta_3})$

$\frac{10}{30} + j\frac{24}{30}$ (από ανάλυση P, Q)

$\Rightarrow 0,33 + j0,8 = j13,33(1 - e^{-j\delta_3})$

$\hat{V}_1 = 1\angle 0$ af , $\hat{V}_3 = V_3\angle\delta_3$

για $k=2$: $S_2 = S_{G_2} = 1 = j11,11 + \hat{V}_2(-j11,11)\hat{V}_4^*$

$\hat{V}_4 = V_4\angle\delta_4$

$\hat{V}_2 = V_2\angle\delta_2$

Άρα $1 = j11,11(1 - V_4V_2 e^{-j\delta_4} e^{j\delta_2})$

για $k=3$: $0 = j32,4 + V_3 e^{j\delta_3}(-j13,33 - j3,17V_4e^{-j\delta_4} - j15,87\,V_5e^{-j\delta_5})$

$\hat{V}_5 = V_5\angle\delta_5$

για $k=4$: $-S_{D4} = -0,17 - j0,13 = j30,15 + V_4e^{j\delta_4}(-j11,11V_2e^{-j\delta_2} - j3,17e^{-j\delta_3} - j15,87V_5e^{-j\delta_5})$

για $k=5$: $-S_{D5} = -0,83 - j0,67 = j31,74 + V_5e^{j\delta_5}(-j15,87V_3e^{-j\delta_3} - j15,87V_4e^{-j\delta_4})$

δ) $\hat{V}_1^{(1)} = -\frac{1}{j13,33}\left[\frac{0,33-j0,8}{1} - j13,33\right\} = 1,06 + j0,024$

$\hat{V}_2^{(1)} = -\frac{1}{j11,11}\left\{\frac{1}{1} - j11,11\right\} \doteq 1 + j0,09$

$\hat{V}_3^{(1)} = \frac{1}{j32,4}\left\{0 - j13,33 - j3,17 - j15,87\right\} = 1$

$\hat{V}_4^{(1)} = \frac{1}{-j30,15}\left\{\frac{0,17-j0,13}{1} - j11,11 - j3,17 - j15,87 - j11,11\hat{V}_2^{(1)} - j3,17\right\}$

$\hat{V}_5^{(1)} = -\frac{1}{j31,74}\left\{\frac{0,83-j0,167}{1} - j15,87 - j15,87 - j15,87\hat{V}_3^{(1)} - j15,87\hat{V}_4^{(1)}\right\}$

ε) $S_{13} = \hat{V}_1\left[Y_{13}\hat{V}_1 - Y_{13}\hat{V}_3\right]^* = 0,16 - j0,012$

$S_{24} = \hat{V}_2\left[Y_{24}\hat{V}_2 - Y_{24}\hat{V}_4\right]^* = -0,189 + j0,033$

$S_{34} = \hat{V}_3\left[Y_{34}\hat{V}_3 - Y_{34}\hat{V}_4\right]^* = -0,054 + j0,01$

$S_{35} = \hat{V}_3\left[Y_{35}\hat{V}_3 - Y_{35}\hat{V}_5\right]^* = 0,239 + j0,09$

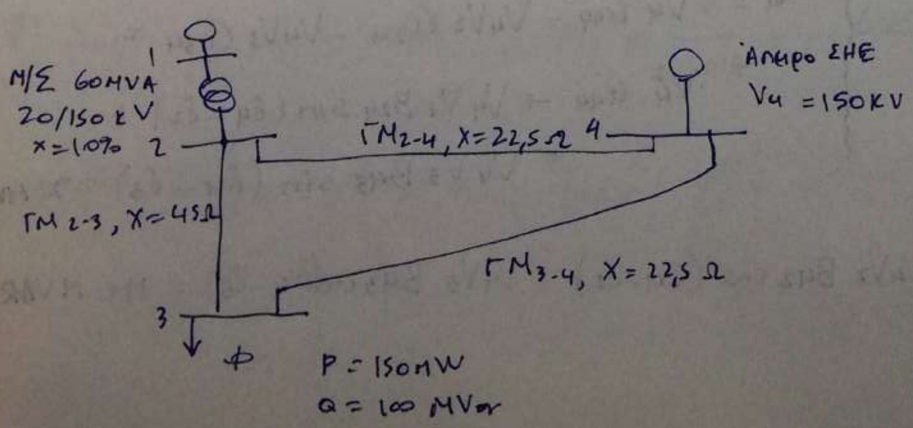
$S_{54} = \hat{V}_5\left[Y_{54}\hat{V}_5 - Y_{54}\hat{V}_4\right]^* = 0,696 + j0,025$

$S_{45} = \hat{V}_4\left[Y_{54}\hat{V}_4 - Y_{54}\hat{V}_5\right]^* = -0,696 - j0,047$

στ) $S_{\alpha\pi} = S_{45} + S_{54} = -j0,022$

$Q_{\alpha\pi} = 660\ MVar$

<u>Άσκηση 6</u>



<u>Γεννήτρια</u>

$P_G = 60MW$

$V_1 = 20kV$

α) $S_B = 100MVA$

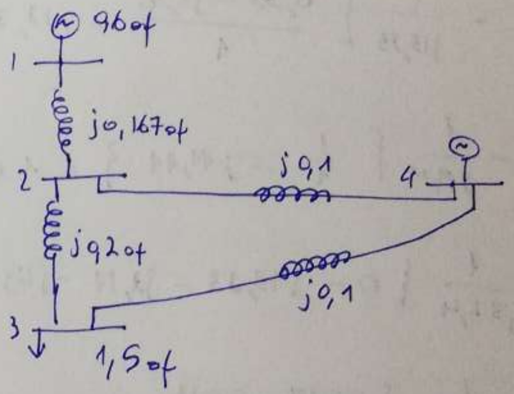
$V_S = 150KV$

$Z_B = \frac{V_B^2}{S_B} = 225\Omega$

$X_{23} = \frac{45}{225} = 0,2$ αμ

$X_{24} = 0,1$ αμ

$X_{34} = 0,1$ αμ



$X_{new} = X_{old}\left(\frac{V_{old}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{old}}\right) = 0,167$ αμ

β) $Y_{12} = \frac{1}{j0,167} = -j5,98$, $Y_{23} = \frac{1}{j0,2} = -j5$, $Y_{24} = \frac{1}{j0,1} = -j10$,

$Y_{34} = -j10$

$$[Y] = j\begin{bmatrix} -5,98 & 5,98 & 0 & 0 \\ 5,98 & 20,98 & 5 & 10 \\ 0 & 5 & -5 & 10 \\ 0 & 10 & 10 & -20 \end{bmatrix}$$

Ζυγός 1: παραγωγής PV, γνωστά P, V

Ζυγός 2: ζυγός PQ

Ζυγός 3: -||-

Ζυγός 4: ζυγός αναφοράς

γ) $\hat{V}_1 = 1\angle 3,98^\circ$ αμ

$\hat{V}_2 = 0,98\angle 4,76^\circ$ αμ

$\hat{V}_3 = 0,915\angle -6,16^\circ$ αμ

$\hat{V}_4 = 1\angle 0^\circ$ αμ

$\Gamma_1: Q_{G_1} = -V_1^2 B_{11} - V_1V_2B_{12} = 131$ MVAR

$P_{G_4} = V_4^2 G_{44} - V_4V_2 Q_{24} - V_4V_3 Q_{34} =$

$= V_4^2 G_{44} + V_4V_2B_{24}\sin(\delta_4 - \delta_2) +$

$+ V_4V_3B_{43}\sin(\delta_4 - \delta_3) = 90MW$

$Q_{G4} = -V_4^2 B_{44} - V_4V_2 B_{42}\cos(\delta_4 - \delta_2) - V_4V_3 B_{43}\cos(\delta_4 - \delta_3) = 110$ MVAR δ) $X = 8\,\Omega$

$S_{a1} = 0,6 + j0,132$ α.μ.

$S_{a1} = \hat{V}_1 \dfrac{\hat{E}_f^* - \hat{V}_t^*}{-jX_s} = \hat{V}_1 \dfrac{\hat{E}_f^* - \hat{V}_1^*}{-jX_s} \Rightarrow E_f = 1,74 \angle 47,54^\circ$

Άρα $\delta = 47,54^\circ - 3,98^\circ = 43,55^\circ$

Άσκηση 10

α) $\left.\begin{matrix} S_B = 100\,MVA \\ V_B = 150\,kV \end{matrix}\right\} Z_B = 225\,\Omega$

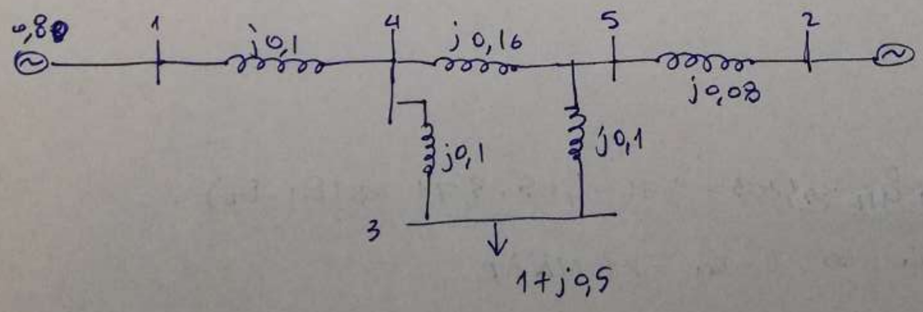
$X_{15} = \dfrac{80 \cdot 0,45}{225} = 0,16$ α.μ.

$X_{34} = X_{35} = \dfrac{50 \cdot 0,45}{225} = 0,1$ α.μ.

$X_{14} = 0,1 \cdot \left(\dfrac{150}{150}\right)^2 \cdot \dfrac{100 \cdot 10^6}{100 \cdot 10^6} = 0,1$ α.μ.

$X_{25} = 0,16 \cdot 1 \cdot \dfrac{1}{2} = 0,08$

$S_{D3} = \dfrac{100 \cdot 10^6 + j50 \cdot 10^6}{100 \cdot 10^6} = 1 + j0,5$ α.μ.



β) $Y_{14} = \dfrac{1}{j0,1} = -j10$ α.μ. , $Y_{45} = -\dfrac{1}{j0,16} = -j6,25$ α.μ.

$Y_{52} = -\dfrac{1}{j0,08} = -j12,5$ α.μ. , $Y_{43} = -\dfrac{1}{j0,1} = -j10$ α.μ. , $Y_{53} = -j10$ α.μ.

$$[Y] = j \begin{bmatrix} -10 & 0 & 0 & 10 & 0 \\ 0 & -12{,}5 & 0 & 0 & 12{,}5 \\ 0 & 0 & -20 & 10 & 10 \\ 10 & 0 & 10 & -26{,}25 & -6{,}25 \\ 0 & 12{,}5 & 10 & -6{,}25 & -28{,}75 \end{bmatrix}$$

γ) Ζυγός 1: ζυγός PV,
Ζυγός 2: ζυγός αναφοράς
Ζυγός 3,4,5: ζυγός PQ

δ) $S_{25} = \hat{V}_2 \left(Y_{25} \hat{V}_2 - Y_{25} \hat{V}_5^{*} \right)^{*} = 1 \angle \delta_2 \left(j12{,}5 \angle \delta_2 - j12{,}5\, V_5 \angle \delta_5 \right)^{*}$

ε) Με το άνοιγμα του διακόπτη $Z_{45} = \dfrac{j0{,}16\,(j0{,}1 + j0{,}1)}{j0{,}16 + j0{,}1 + j0{,}1} = j0{,}089\ \Omega$

Έτσι, έχουμε $Z_{12} = j0{,}1 + j0{,}089 + j0{,}08 \Rightarrow Z_{12} = j0{,}267$, $Y_{12} = -j3{,}71$

$P_{G1} = V_1^2 G_1 - V_1 V_2 Q_{12} \Rightarrow 0{,}8 = -1{,}05 \left(-\dfrac{1}{0{,}267} \right) \sin(\theta_1 - \theta_2) \Rightarrow$

$\Rightarrow \sin(\theta_1 - \theta_2) = 0{,}2 \Rightarrow \theta_1 - \theta_2 = 11{,}87°$

$V_1 = \dfrac{15{,}75}{15} = 1{,}05$

$Q_{G1} = -V_1^2 B_{11} - V_1 V_2 B_{12} = 1{,}103 - 3{,}71 - 1{,}05 \cdot 3{,}71 \cos(\theta_1 - \theta_2)$

$\Rightarrow Q_{G1} = 0{,}28$ α.μ. $\Rightarrow Q_{G1} = 28$ MVAR