7η Σειρά Ασκήσεων

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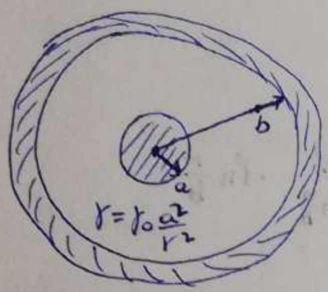
Άσκηση 1



1ος Τρόπος

$$\bar{J} = \frac{I}{2\pi r^2}\hat{r}$$

$$\bar{E} = \frac{\bar{J}}{\gamma} = \frac{I}{2\pi\gamma r^2}\hat{r} = \frac{I}{4\pi\gamma_0 a^2}\hat{r}$$

Άρα, $V = \int_a^b E\,dr = \frac{I}{4\pi\gamma_0 a^2}(b-a)$ οπότε $R = \frac{V}{I} = \frac{b-a}{4\pi\gamma_0 a^2}$

2ος Τρόπος

$$\nabla \bar{J} = 0 \Rightarrow \nabla(\gamma \nabla \Phi) = 0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(\gamma_0 a^2 \frac{\partial \Phi}{\partial r}\right) = 0 \Rightarrow \frac{\partial^2 \Phi}{\partial r^2} = 0 \Rightarrow \Phi = Ar + B$$

- για $r=b$: $\Phi(b) = 0 \Rightarrow B = -Ab$
- για $r=a$: $\Phi(a) = V \Rightarrow Aa + B = V$

οπότε $\Phi = \frac{V}{b-a}(b-r)$

$\bar{E} = -\nabla\Phi = \frac{V}{(b-a)}\hat{r}$ Άρα $\bar{J} = \frac{\gamma V}{b-a}\hat{r}$

$I = \int J\,ds = \int \frac{\gamma_0 a^2 V r^2}{(b-a) r^2}\sin\theta\, d\varphi\, d\theta = \frac{4\pi\gamma_0 a^2 V}{b-a}$ οπότε $R = \frac{V}{I} = \frac{b-a}{4\pi\gamma_0 a^2}$

3ος Τρόπος

$$dR = \frac{1}{\gamma}\cdot\frac{\ell}{S} = \frac{1}{\gamma}\cdot\frac{dr}{4\pi r^2} = \frac{dr}{4\pi\gamma_0 a^2}$$

$$R = \int_a^b \frac{dr}{4\pi\gamma_0 a^2} = \frac{b-a}{4\pi\gamma_0 a^2}$$

4ος Τρόπος

Έστω $\varepsilon(r) = \frac{\varepsilon_0 a^2}{r^2}$, οπότε $\frac{\varepsilon(r)}{\gamma(r)} = \frac{\varepsilon_0}{\gamma_0} =$ σταθ.

Άρα, $RC = \frac{\varepsilon_0}{\gamma_0}$. Όμως $C = \frac{1}{\int_a^b \frac{dr}{\frac{\varepsilon_0 a^2}{r^2}4\pi r^2}} = \frac{4\pi\varepsilon_0 a^2}{b-a}$ Άρα $R = \frac{\varepsilon_0}{\gamma_0 \cdot C} = \frac{\varepsilon_0(b-a)}{\gamma_0 4\pi\varepsilon_0 a^2} = \frac{b-a}{4\pi\gamma_0 a^2}$

Ασκηση 2 (10.20)

α) · $\nabla^2\Phi = 0$ σε όλο τον αγωγό

$$\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) = 0 \Rightarrow r\frac{\partial \Phi}{\partial r} = c_1 \Rightarrow \Phi(r) = c_1 \ln r + c_2$$

· $\Phi(a) = V \Rightarrow c_1 \ln a + c_2 = V \Rightarrow c_1 = -\dfrac{V}{\ln\frac{b}{a}}$

· $\Phi(b) = 0 \Rightarrow c_1 \ln b + c_2 = 0 \Rightarrow c_2 = \dfrac{\ln b \cdot V}{\ln\frac{b}{a}}$

$$\Rightarrow \Phi(r) = -\frac{V}{\ln\frac{a}{b}} \cdot \ln\frac{r}{b}$$

· $\bar{E} = -\nabla\Phi = \dfrac{V}{\ln\frac{b}{a}} \cdot \dfrac{1}{r} \cdot \hat{r}$

· $\bar{J} = \begin{cases} \dfrac{\gamma_1 V}{\ln\frac{b}{a}} \cdot \dfrac{1}{r}\hat{r} \\ \dfrac{\gamma_2 V}{\ln\frac{b}{a}} \cdot \dfrac{1}{r}\hat{r} \end{cases}$

· Ισχύει, $\nabla\cdot(\varepsilon_1 \nabla\Phi_1) = \rho_1$ και $\nabla(\varepsilon_2 \nabla\Phi_2) = \rho_2$

$$\underbrace{\begin{matrix} \downarrow & & \downarrow \\ \nabla^2\Phi_1 = \dfrac{\rho_1}{\varepsilon_1} & & \nabla^2\Phi_2 = \dfrac{\rho_2}{\varepsilon_2} \\ 0 = \dfrac{\rho_1}{\varepsilon_1} & & 0 = \dfrac{\rho_2}{\varepsilon_2} \end{matrix}}$$

Άρα $\rho_1 = \rho_2 = 0$

· $\sigma_1 = \left(\bar{0} - \bar{D}_1(b)\right)\cdot\hat{r} = -\dfrac{\varepsilon_1 V}{\ln\frac{b}{a}} \cdot \dfrac{1}{b}$

β) · $R^\mu = \dfrac{V}{I^\mu}$

· $I^\mu = \displaystyle\int_0^\pi J_1 \cdot r \cdot d\varphi + \int_\pi^{2\pi} J_2\, r\, d\varphi = \dfrac{(\gamma_1+\gamma_2) V \pi}{\ln\frac{b}{a}}$

Οπότε $R^\mu = \dfrac{\ln\frac{b}{a}}{(\gamma_1+\gamma_2)\,\pi}$

Άσκηση 3 (10.21)

α) Για σταθερό ισχύει $\nabla^2\Phi = 0$ παντού

Άρα $\cdot \dfrac{\partial^2 \Phi}{\partial z^2} = 0 \Rightarrow \Phi(z) = C_1 z + C_2$

$\cdot\ \Phi(0)=0, \quad \Phi(h)=V$

$\Rightarrow \Phi(z) = \dfrac{V}{h} z$

$\cdot\ \bar{E} = -\nabla\Phi = -\dfrac{V}{h}\hat{z}$

$\cdot\ \bar{J} = \gamma_i \bar{E} = -\dfrac{\gamma_i V}{h}\hat{z}$

Άρα, $I = \oint \bar{J}\cdot d\bar{s} = \iint J_z\, dx\,dy = \dfrac{d\cdot(-V)}{h}\left(a\gamma_1 + (b-a)\gamma_2 + (c-b)\gamma_3\right)$

Οπότε $R = \dfrac{(-V)}{I} = \dfrac{h}{d\left(a\gamma_1 + (b-a)\gamma_2 + (c-b)\gamma_3\right)}$

β) Για $\gamma_2 = \dfrac{\gamma_0 x}{a}$, λόγω μοναδικότητας ισχύει $\Phi(z) = \dfrac{V}{h} z$

Άρα $\bar{E} = -\nabla\Phi = -\dfrac{V}{h}\hat{z}$

$I = \oint \bar{J}\cdot d\bar{s} = \dfrac{d(-V)}{h}\left(\int_0^a \gamma_1 dx + \int_a^b \dfrac{\gamma_0 x}{a} dx + \int_b^c \gamma_3 dx\right) \Rightarrow$

$\Rightarrow I = \dfrac{d(-V)}{h}\left(a\gamma_1 + \dfrac{b^2-a^2}{2a}\cdot\gamma_0 + (c-b)\gamma_3\right)$

Οπότε $R = \dfrac{h}{d\left(a\gamma_1 + \dfrac{b^2-a^2}{2a} + (c-b)\gamma_3\right)}$