9η Σειρά Ασκήσεων

ΧΡΗΣΤΟΣ ΤΣΟΥΦΗΣ
03117176

Άσκηση 1

11.12

a). $\bar{A}_1 = \frac{\mu_0 K_0}{3} r \sin\theta \hat{\varphi}$, $r \leq a$

· $\bar{A}_2 = \frac{\mu_0 K_0 a^3}{3} \cdot \frac{1}{r^2} \sin\theta \hat{\varphi}$, $r \geq a$

· $\bar{B}_1 = \nabla \times \bar{A}_1 = \frac{1}{r\sin\theta} \cdot \frac{\partial}{\partial\theta}(\sin\theta \cdot A_1)\hat{r} - \frac{1}{r}\cdot\frac{\partial}{\partial r}(r A_1)\hat{\theta} = \frac{2\mu_0 K_0}{3}\left[\cos\theta \hat{r} - \sin\theta \hat{\theta}\right]$

· $\bar{B}_2 = \nabla \times \bar{A}_2 = \frac{1}{r\sin\theta} \cdot \frac{\partial}{\partial\theta}(\sin\theta \cdot A_2)\hat{r} - \frac{1}{r}\cdot\frac{\partial}{\partial r}(r A_2)\hat{\theta} = \frac{\mu_0 K_0 a^3}{3}\left[\frac{2\cos\theta}{r^3}\hat{r} + \frac{\sin\theta}{r^3}\hat{\theta}\right]$

· $w_m = \frac{1}{2}\bar{H}\cdot\bar{B} = \frac{1}{2\mu_0}\bar{B}^2 = \begin{cases} w_{m_1} = \frac{2\mu_0 K_0^2}{9} \\ w_{m_2} = \frac{\mu_0 K_0^2 a^6}{18 r^6}(4 - 3\sin^2\theta) \end{cases}$

· $W_m = \underbrace{\int w_{m_1} dV_1}_{I_1} + \underbrace{\int w_{m_2} dV_2}_{I_2}$

· $I_1 = \int_{\varphi=0}^{2\pi}\int_{r=0}^{a}\int_{\theta=0}^{\pi} r^2 \cdot \sin\theta d\theta dr d\varphi \cdot \frac{2\mu_0 K_0^2}{9} = \frac{8\pi\mu_0 K_0^2 a^3}{27}$

· $I_2 = \int_{\varphi=0}^{2\pi}\int_{r=a}^{+\infty}\int_{\theta=0}^{\pi} \frac{r^2}{r^6}(4\sin\theta - 3\sin^3\theta) d\theta dr d\varphi \cdot \frac{\mu_0 K_0^2 a^6}{18} =$

$= \frac{\mu_0 K_0^2 a^6}{18} \cdot 2\pi \int_a^{+\infty} \frac{1}{r^4} dr \cdot \int_0^{\pi} (4\sin\theta - 3\sin^3\theta) d\theta = \frac{4\pi\mu_0 K_0^2 a^3}{27}$

Οπότε $W_m = I_1 + I_2 = \frac{4\pi\mu_0 K_0^2 a^3}{9}$

β) $\cdot\ \bar{H} = \frac{1}{\mu_0}\bar{B} = \begin{cases} \bar{H}_1 = \frac{2K_0}{3}\left(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right) \\ \bar{H}_2 = \frac{K_0 a^3}{3}\left(\frac{2\cos\theta}{r^3}\hat{r} + \frac{\sin\theta}{r^3}\hat{\theta}\right) \end{cases}$

Συνοριακή συνθήκη: $\hat{r}\times\left(\bar{H}_2(r=a) - \bar{H}_1(r=a)\right) = \bar{K} \Rightarrow \bar{K} = \hat{\varphi}\left(\frac{K_0}{3}\sin\theta + \frac{2K_0}{3}\sin\theta\right) \Rightarrow$

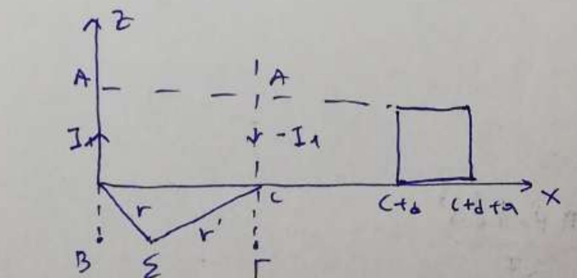
$\Rightarrow \bar{K} = K_0 \sin\theta\,\hat{\varphi}$

$\cdot\ W_m = \frac{1}{2}\int_S \bar{K}\cdot\bar{A}\,ds = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\bar{K}\cdot\bar{A}\,a^2\sin\theta\,d\varphi\,d\theta = \frac{\pi\mu_0 K_0^2 a^3}{3}\int_0^{\pi}\sin^3\theta\,d\theta \Rightarrow$

$\Rightarrow W_m = \frac{4\pi\mu_0 K_0^2 a^3}{9}$

Άσκηση 2

11.19



$r = \sqrt{x^2+y^2}$

$r' = \sqrt{(x-c)^2+y^2}$

. Το μαγνητικό δυναμικό λόγω του ρεύματος I_1 στο ΑΒ είναι $\frac{\mu I}{2\pi}\ln\frac{r_{αναφ}}{r}\hat{z}$

. Το μαγνητικό δυναμικό λόγω του ρεύματος $-I_1$ στο ΔΓ είναι $-\frac{\mu I}{2\pi}\ln\frac{r_{αναφ}}{r'}\hat{z}$

Άρα, $\bar{A}_1 = \frac{\mu I}{2\pi}\ln\left(\frac{r'}{r}\right)\hat{z}$

$\cdot\ \Psi_{21} = \oint_{C_2}\bar{A}_1\cdot d\bar{l}_2 = \int_0^b A(x=c+d, y=0)\,dz + \int_b^0 A(x=c+d+a, y=0)\,dz =$

$= b\left(\frac{\mu I}{2\pi}\ln\frac{d}{c+d} - \frac{\mu I}{2\pi}\ln\frac{d+a}{c+d+a}\right) = \frac{\mu I b}{2\pi}\ln\left[\frac{d(c+d+a)}{(c+d)(d+a)}\right]$

Άρα $L_{21} = \frac{\Psi_{21}}{I} = \frac{\mu b}{2\pi}\ln\left[\frac{d(c+d+a)}{(c+d)(d+a)}\right]$

Άσκηση 3

11.16

- $\bar{J} = \hat{z} \frac{I}{\pi(b^2-a^2)}$

- $\bar{K} = -\hat{z} \frac{I}{2\pi c}$

- $r \le a: \oint_{C_1} \bar{H}_1 \cdot d\bar{l}_1 = 0 \Rightarrow \bar{H}_1 = \bar{0}$

- $a < r \le b: \oint_{C_2} \bar{H}_2 \cdot d\bar{l}_2 = \frac{I(r^2-a^2)\pi}{\pi(b^2-a^2)} \Rightarrow \bar{H}_2 = \hat{\varphi} \cdot \frac{I}{2\pi r} \cdot \frac{r^2-a^2}{b^2-a^2}$

- $b \le r \le c: \oint_{C_3} \bar{H}_3 \cdot d\bar{l}_3 = I \Rightarrow \bar{H}_3 = \hat{\varphi} \frac{I}{2\pi r}$

- $c \le r: \bar{H}_4 = 0$, γιατί το συνολικό ρεύμα είναι 0.

- $W_{m,\mu} = \frac{1}{2}\int \bar{H}_2^2 \mu \, dS_2 + \frac{1}{2}\int \bar{H}_3^2 \mu \, dS_3 =$

$$= \frac{1}{2}\int_0^{2\pi} d\varphi \cdot \int_a^b r \left(\frac{I}{2\pi r}\right)^2 \cdot \left(\frac{r^2-a^2}{b^2-a^2}\right) dr + \frac{\mu}{2}\int_b^c \left(\frac{I}{2\pi r}\right)^2 r \, dr \cdot \int_0^{2\pi} d\varphi =$$

$$= \frac{\mu I^2}{4\pi(b^2-a^2)}\left[\frac{b^4-a^4}{4} + a^4 \ln\frac{b}{a} - a^2(b^2-a^2)\right] + \frac{\mu I^2}{4\pi} \ln\frac{c}{b}$$

- $L_\mu = \frac{2 W_{m,\mu}}{I^2} = \frac{\mu}{2\pi}\left[\frac{b^2+a^2}{4} + \frac{a^4}{b^2-a^2} \ln\frac{b}{a} - a^2 + \ln\frac{c}{b}\right]$