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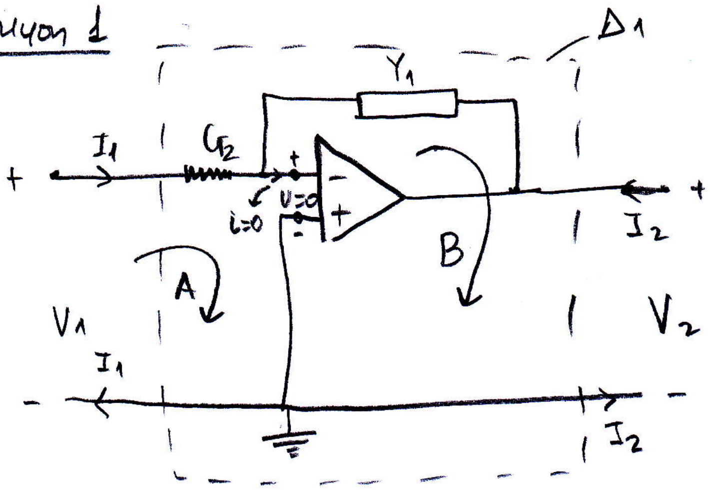
4η Σειρά Ασκήσεων

Άσκηση 1

a)

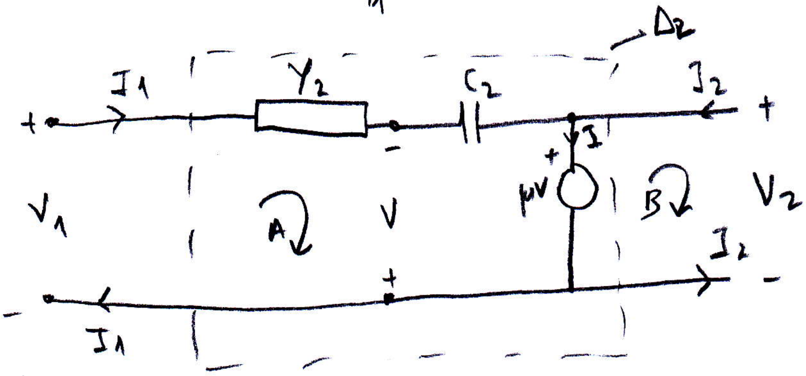


NTK A) : $V_1 = \frac{I_1}{G_2}$

NTK B) : $V_2 = -\frac{I_1}{Y_1}$

$$V_1 = -\frac{Y_1}{G_2} V_2 \Rightarrow T_1 = \begin{bmatrix} -\frac{Y_1}{G_2} & 0 \\ -Y_1 & 0 \end{bmatrix}$$

β)



NTK A) : $V_1 = \frac{I_1}{Y_2} + \frac{I_1}{sC_2} + \mu V$

NTK B) : $V_2 = \mu V$

$$V_1 = \left(\frac{1}{Y_1} + \frac{1}{sC_2}\right) I_1 + V_2$$

NPK : $I_1 + I_2 = I$

NTK : $V_1 = \frac{I_1}{Y_2} - V \Rightarrow V = \frac{I_1}{Y_2} - V_1 \Rightarrow V_1 = \frac{I_1}{Y_2} - \frac{V_2}{\mu}$ $\Rightarrow$

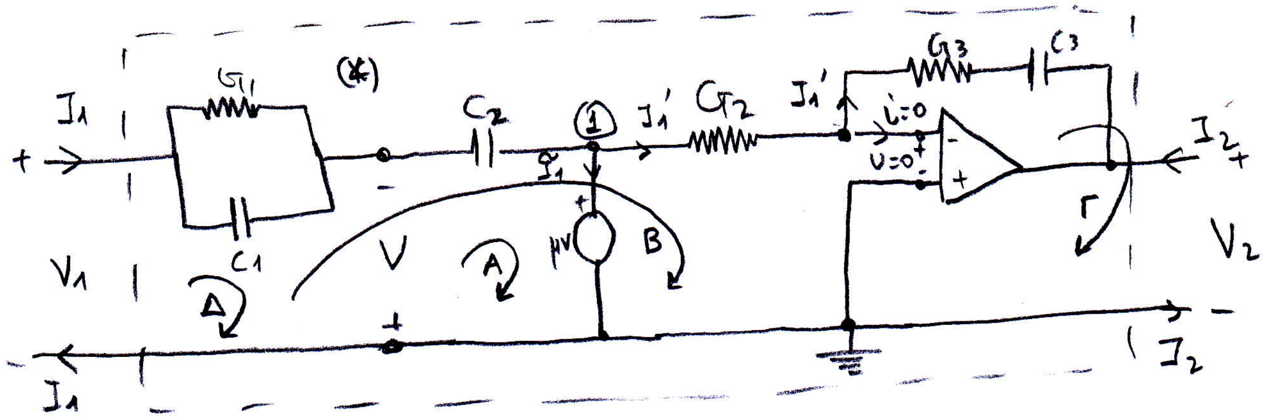
$$\Rightarrow \cancel{\frac{I_1}{Y_2}} - \frac{V_2}{\mu} = \cancel{\frac{I_1}{Y_2}} + \frac{I_1}{sC_2} + V_2 \Rightarrow V_2\left(1 + \frac{1}{\mu}\right) = -\frac{I_1}{sC_2} \Rightarrow$$

$$\Rightarrow I_1 = -sC_2\left(1 + \frac{1}{\mu}\right) V_2 \quad , \quad C' = -sC_2\left(1 + \frac{1}{\mu}\right)$$

$$V_1 = \frac{I_1}{Y_2} - \frac{V_2}{\mu} = -\left(\frac{sC_2(1+\frac{1}{\mu})}{Y_2} + \frac{1}{\mu}\right)V_2 \Rightarrow A' = -\left(\frac{sC_2(1+\frac{1}{\mu})}{Y_2} + \frac{1}{\mu}\right)$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & 0 \\ C' & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Rightarrow T_2 = \begin{bmatrix} A' & 0 \\ C' & 0 \end{bmatrix}$$

γ)



$$(*)\quad Z_1 = \frac{\frac{1}{G_1}\cdot\frac{1}{sC_1}}{\frac{1}{G_1}+\frac{1}{sC_1}} = \frac{1}{sC_1+G_1}$$

NTK A) : $\mu V + V = -I_1 \cdot \frac{1}{sC_2} \Rightarrow V = -\frac{I_1}{sC_2(1+\mu)}$ (1)

NTK B) : $V_1 = \frac{I_1}{sC_1+G_1} + \frac{I_1}{sC_2} + \frac{I_1'}{G_2}$ (2)

NPK (1) : $I_1 = I_1' + \tilde{I}_1$ (3)

NTK Γ) : $V_2 = -I_1'\left(\frac{1}{G_3}+\frac{1}{sC_3}\right) \Rightarrow V_2 = -I_1'\left(\frac{sC_3+G_3}{G_3 sC_3}\right)$ (4)

NTK Δ) : $V = \frac{I_1}{sC_1+G_1} - V_1$ (5)

$(2) \xRightarrow{(4)} V_1 = \left(\frac{1}{sC_1+G_1}+\frac{1}{sC_2}\right)I_1 - \left(\frac{G_3 sC_3}{sC_3+G_3}\right)V_2$ (6)

$(1)(5) \Rightarrow -\frac{I_1}{sC_2(1+\mu)} = \frac{I_1}{sC_1+G_1} - V_1 \Rightarrow V_1 = \left(\frac{1}{sC_2(1+\mu)}+\frac{1}{sC_1+G_1}\right)I_1$ (7)

$(6)(7) \Rightarrow \frac{I_1}{sC_2(1+\mu)} + \frac{I_1}{sC_1+G_1} = \frac{I_1}{sC_1+G_1} + \frac{I_1}{sC_2} - \left(\frac{G_3 sC_3}{sC_3+G_3}\right)V_2$

$$\Rightarrow I_1\left(\frac{1}{sC_2(1+\mu)}-\frac{1}{sC_2}\right)=-\left(\frac{G_3 s C_3}{G_3+sC_3}\right)V_2 \Rightarrow$$

$$\Rightarrow I_1\left(\frac{1-1+\mu}{sC_2(1+\mu)}\right)=-\left(\frac{G_3 s C_3}{G_3+sC_3}\right)V_2 \Rightarrow$$

$$\Rightarrow I_1 = -\underbrace{\frac{s^2 C_2 C_3 G_3 (1+\mu)}{\mu(sC_3+G_3)}}_{C''} V_2$$

$$(7) \Rightarrow V_1 = \underbrace{\left(\frac{1}{sC_2(1+\mu)}+\frac{1}{sC_1+G_1}\right)\left(-\frac{s^2 C_2 C_3 G_3(1+\mu)}{\mu(sC_3+G_3)}\right)}_{A''} V_2$$

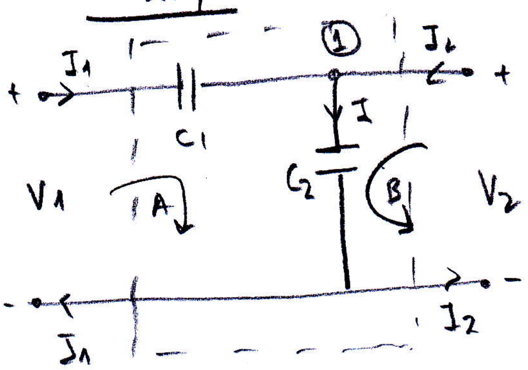
$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Rightarrow T_3 = \begin{bmatrix} A'' & 0 \\ C'' & 0 \end{bmatrix} \quad (T_3 = T_2 \cdot T_1)$$

Άσκηση 2

α) 4.2.α

$$\left.\begin{aligned} V_1 &= -r_2 I_2 \\ V_2 &= r_1 I_1 \Rightarrow I_1 = \frac{1}{r_1} V_2 \end{aligned}\right\} \Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & r_2 \\ 1/r_1 & 0 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Rightarrow T_1 = \begin{bmatrix} 0 & r_2 \\ 1/r_1 & 0 \end{bmatrix}$$

4.2.β



ΝΤΚ A: $V_1 = \frac{I_1}{sC_1} + \frac{I}{sC_2}$ (1)

$$\left.\begin{aligned} &\text{ΝΤΚ B}: V_2 = \frac{I}{sC_2} \\ &\text{ΝΡΚ } 1: I_1 + I_2 = I \end{aligned}\right\} \Rightarrow V_2 = \frac{I_1}{sC_2} + \frac{I_2}{sC_2} \Rightarrow$$

$$\Rightarrow I_1 = sC_2V_2 - I_2 \quad (2)$$

$$(1) \Rightarrow V_1 = \frac{I_1}{sC_1} + V_2 \Rightarrow V_1 = \frac{sC_2V_2}{sC_1} - \frac{I_2}{sC_1} + V_2 \Rightarrow V_1 = \left(\frac{C_2}{C_1}+1\right)V_2 - \left(\frac{1}{sC_1}\right)I_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{C_2}{C_1}+1 & \frac{1}{sC_1} \\ sC_2 & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Rightarrow T_2 = \begin{bmatrix} \frac{C_2}{C_1}+1 & \frac{1}{sC_1} \\ sC_2 & 1 \end{bmatrix}$$

4.2.γ

Ανάμεσα στο Δ1 είναι το Δ2, οπότε 3 δίθυρα σε αλυσιδωτή σύνδεση:

$T_3 = T_1 \cdot T_2 \cdot T_1 \Rightarrow$

$$\Rightarrow T_3 = \begin{bmatrix} 0 & r_2 \\ 1/r_1 & 0 \end{bmatrix} \begin{bmatrix} \frac{C_2}{C_1}+1 & \frac{1}{sC_1} \\ sC_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & r_2 \\ 1/r_1 & 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow T_3 = \begin{bmatrix} \frac{r_2}{r_1} & r_2^2 s C_2 \\ \frac{1}{r_1^2 s C_1} & \frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1} \end{bmatrix}$$

β) $G(s) = \frac{V_2(s)}{E(s)}$

από T_3: $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{r_2}{r_1} & r_2^2 s C_2 \\ \frac{1}{r_1^2 s C_1} & \frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Rightarrow \begin{cases} V_1 = \frac{r_2}{r_1} V_2 - r_2^2 s C_2 I_2 \quad (1) \\ I_1 = \left(\frac{1}{r_1^2 s C_1} \right) V_2 + \\ \quad - \left(\frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1} \right) I_2 \quad (2) \end{cases}$

Από επ. τερματισμού: $\begin{cases} E = I_1 R_s + V_1 \quad (3) \\ V_2 = -I_2 R_\rho \quad (4) \end{cases}$

$$\begin{matrix} (1) \\ (2) \end{matrix} \xRightarrow[(4)]{(3)} \begin{cases} E - I_1 R_s = \frac{r_2}{r_1}(-I_2 R_\rho) - r_2^2 s C_2 I_2 \\ I_1 = \frac{1}{r_1^2 s C_1} \cdot (-I_2 R_\rho) - \left(\frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1} \right) I_2 \end{cases} \Rightarrow$$

$$\Rightarrow E = R_s I_1 - \left(\frac{r_2}{r_1} R_\rho + r_2^2 s C_2 \right) I_2$$

$$0 = I_1 + \left(\frac{R_\rho}{r_1^2 s C_1} + \frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1} \right) I_2$$

$$\begin{bmatrix} E \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & -\left(\frac{r_2}{r_1} R_l + r_2^2 s C_2\right) \\ 1 & \left(\frac{R_l}{r_1^2 s C_1} + \frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$) = \frac{V_2(s)}{E(s)} = -\frac{R_l I_2}{E(s)} = -\frac{R_l}{E(s)} \cdot \frac{\Delta_2}{\Delta} = \cancel{-}\frac{R_l}{\cancel{E(s)}} \cdot \frac{\cancel{-E(s)}}{\Delta} \Rightarrow$$

$$G(s) = \frac{R_l}{R_s\left(\frac{R_l}{r_1^2 s C_1} + \frac{r_2 C_2}{r_1 C_1} + \frac{r_2}{r_1}\right) + \left(\frac{r_2}{r_1} R_l + r_2^2 s C_2\right)}$$

Για $r_1 = r_2 = R_s = R_l = 1\,\Omega$, $C_1 = C_2 = 1F$

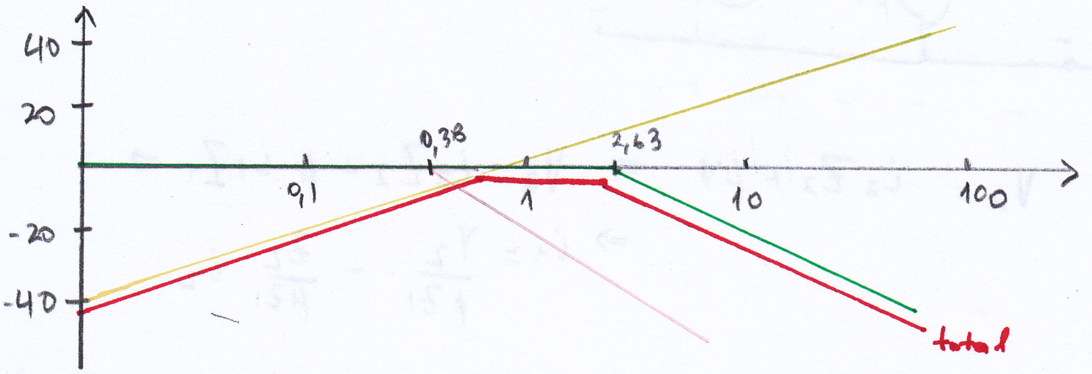
$$(s) = \frac{1}{\left(\frac{1}{s} + 1 + 1\right) + (1+s)} = \frac{1}{s + 3 + \frac{1}{s}} = \frac{s}{s^2 + 3s + 1} =$$

$$= \frac{s}{\left[s - \left(\frac{-3-\sqrt{5}}{2}\right)\right]\left[s - \left(\frac{-3+\sqrt{5}}{2}\right)\right]} = \frac{s}{(1+2{,}63s)(1+0{,}38s)}$$

Το s: $K(\omega) = 20$ dB/dec

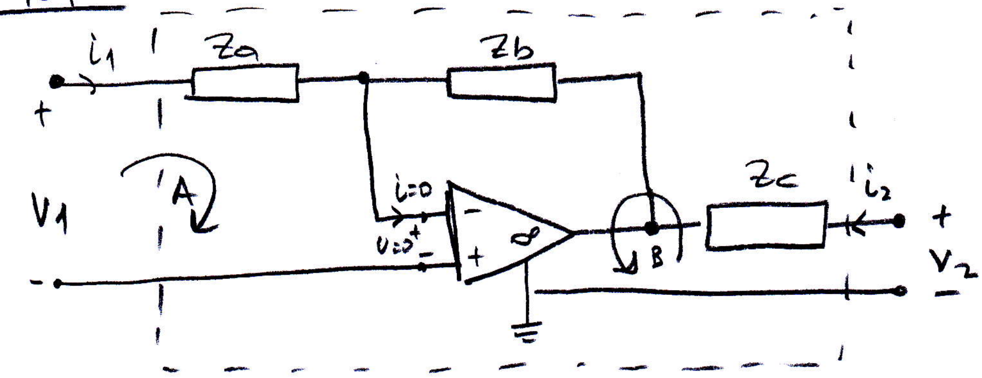
$$\frac{1}{1+2{,}63s} = \frac{1}{1+sT_1}, \quad T_1 = 2{,}63 \Rightarrow \omega_1 = \frac{1}{T_1} = 0{,}38 \text{ rad}$$

$$\frac{1}{1+0{,}38s} = \frac{1}{1+sT_2}, \quad T_2 = 0{,}38 \Rightarrow \omega_2 = \frac{1}{T_2} = 2{,}63 \text{ rad}$$



Άσκηση 3

α)



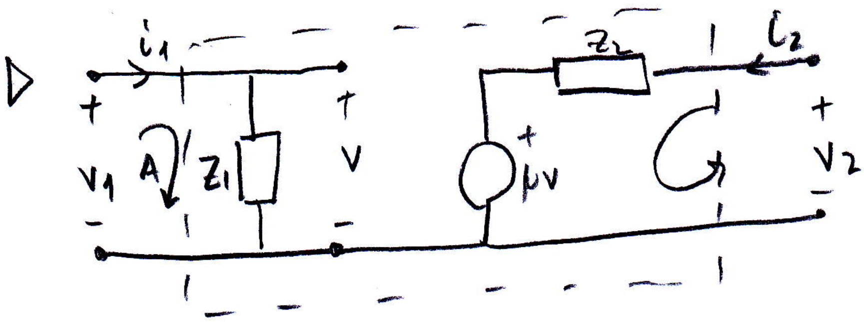
NTK (B): $V_2 = i_2 Z_c - i_1 Z_b \Rightarrow i_1 = \dfrac{Z_c i_2 - V_2}{Z_b} = \left(-\dfrac{1}{Z_b}\right)V_2 + \left(\dfrac{Z_c}{Z_b}\right) i_2$

NTK (A): $V_1 = i_1 Z_a \Rightarrow V_1 = \left(-\dfrac{Z_a}{Z_b}\right) V_2 + \left(\dfrac{Z_a Z_c}{Z_b}\right) i_2$

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \underbrace{\begin{bmatrix} -\dfrac{Z_a}{Z_b} & -\dfrac{Z_a Z_c}{Z_b} \\ -\dfrac{1}{Z_b} & -\dfrac{Z_c}{Z_b} \end{bmatrix}}_{T_{\Delta_1}} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

$$Y_{\Delta_1} = \frac{1}{D}\begin{bmatrix} B & \Delta_{T_{\Delta_1}} \\ -1 & C \end{bmatrix} = -\frac{Z_b}{Z_c}\begin{bmatrix} -\dfrac{Z_a Z_c}{Z_b} & \dfrac{Z_a \cancel{Z_c}}{Z_b^2} - \dfrac{Z_a \cancel{Z_c}}{Z_b^2} \\ -1 & -\dfrac{1}{Z_b} \end{bmatrix} =$$

$$= \begin{bmatrix} Z_a & 0 \\ \dfrac{Z_b}{Z_c} & \dfrac{1}{Z_c} \end{bmatrix}$$



NTK (B): $V_2 = i_2 Z_2 + \mu V \Rightarrow V_2 = i_2 Z_2 + \mu i_1 Z_1 \Rightarrow$

$\Rightarrow i_1 = \dfrac{V_2}{\mu Z_1} - \dfrac{Z_2}{\mu Z_1} i_2$

$V = i_1 Z_1$

NTK A): $V_1 = Z_1 \cdot i_1 \Rightarrow V_1 = \frac{Z_1}{\mu Z_1} V_2 - \frac{Z_1 Z_2}{\mu Z_1} i_2 \Rightarrow$

$$\Rightarrow V_1 = \frac{1}{\mu} V_2 - \frac{Z_2}{\mu} i_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1/\mu & Z_2/\mu \\ 1/\mu Z_1 & Z_2/\mu Z_1 \end{bmatrix}}_{T_{\Delta_2}} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$Y_{D_2} = \frac{1}{D}\begin{bmatrix} B & \Delta_{T_{\Delta_2}} \\ -1 & C \end{bmatrix} = \frac{\mu Z_1}{Z_2}\begin{bmatrix} \frac{Z_2}{\mu} & \frac{\cancel{Z_2}}{\mu^2 Z_1} - \frac{\cancel{Z_2}}{\mu^2 Z_1} \\ -1 & \frac{1}{\mu Z_1} \end{bmatrix} =$$

$$= \begin{bmatrix} Z_1 & 0 \\ -\frac{\mu Z_1}{Z_2} & \frac{1}{Z_2} \end{bmatrix}$$

B) Ισχύει για Δ: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow$

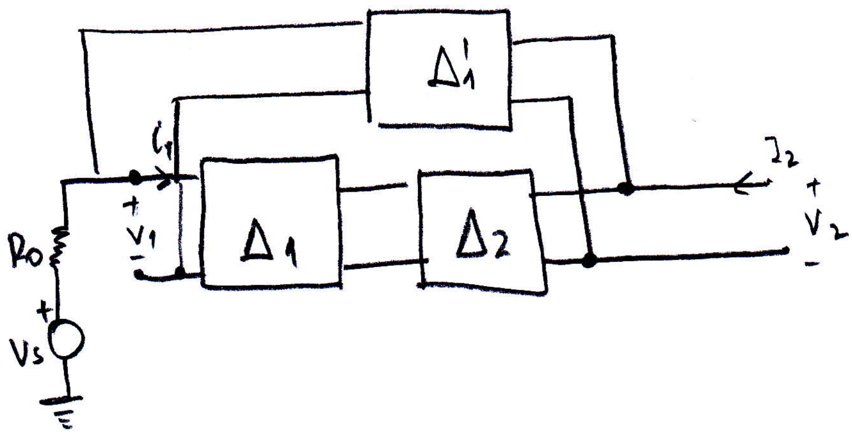
$$\Rightarrow \begin{cases} I_1 = y_{11} V_1 + y_{12} V_2 & (1) \\ I_2 = y_{21} V_1 + y_{22} V_2 & (2) \end{cases}$$

Ισχύει για Δ': $\left.\begin{aligned} I_1' &= I_2 \\ I_2' &= I_1 \end{aligned}\right\} \quad \begin{aligned} V_1' &= V_2 \\ V_2' &= V_1 \end{aligned}$

$$(1),(2) \Rightarrow \begin{cases} I_2' = y_{11} V_2' + y_{12} V_1' \\ I_1' = y_{21} V_2' + y_{22} V_1' \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \underbrace{\begin{bmatrix} y_{22} & y_{21} \\ y_{12} & y_{11} \end{bmatrix}}_{Y_{\Delta'}} \begin{bmatrix} V_1' \\ V_2' \end{bmatrix}$$

γ) "πάνω" : Δ_1' , "κάτω-αριστερά" : Δ_1 , "κάτω-δεξιά" : Δ_2



▷ Δ_1, Δ_2 : δίθυρα σε αλυσιδωτή σύνδεση : $T_{\Delta_{12}} = T_{\Delta_1} \cdot T_{\Delta_2}$

$$T_{\Delta_{21}} = \begin{bmatrix} -\frac{Z_a}{Z_b} & -\frac{Z_a Z_c}{Z_b} \\ -\frac{1}{Z_b} & -\frac{Z_c}{Z_b} \end{bmatrix} \begin{bmatrix} \frac{1}{\mu} & \frac{Z_2}{\mu} \\ \frac{1}{\mu Z_1} & \frac{Z_2}{\mu Z_1} \end{bmatrix}$$

$Z_a = R_1$, $Z_b = \frac{1}{Y_1}$, $Z_c = \frac{1}{Y_2}$; $Z_1 = R_4$, $Z_2 = R_s$

$$T_{\Delta_{21}} = \begin{bmatrix} -R_1 Y_1 & -\frac{R_1 Y_1}{Y_2} \\ -Y_1 & -\frac{Y_1}{Y_2} \end{bmatrix} \begin{bmatrix} \frac{1}{\mu} & \frac{R_s}{\mu} \\ \frac{1}{\mu R_4} & \frac{R_s}{\mu R_4} \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{R_1 Y_1}{\mu}\left(1+\frac{1}{Y_2 R_4}\right) & -\frac{R_1 R_s Y_1}{\mu}\left(1+\frac{1}{Y_2 R_4}\right) \\ -\frac{Y_1}{\mu}\left(1+\frac{1}{Y_2 R_4}\right) & -\frac{Y_1 R_s}{\mu}\left(1+\frac{1}{Y_2 R_4}\right) \end{bmatrix}$$

$$Y_{\Delta_{21}} = \frac{1}{B}\begin{bmatrix} D & -\Delta_{T_{\Delta_{21}}} \\ -1 & A \end{bmatrix} = \frac{1}{-\frac{R_1 R_s Y_1}{\mu}\left(1+\frac{1}{Y_2 R_4}\right)} \cdot \begin{bmatrix} -\frac{Y_1 R_s}{\mu}\left(1+\frac{1}{Y_2 R_4}\right) & 0 \\ -1 & -\frac{R_1 Y_1}{\mu}\left(1+\frac{1}{Y_2 R_4}\right) \end{bmatrix}$$

$$\Rightarrow Y_{\Delta 21} = \begin{bmatrix} \dfrac{1}{R_1} & 0 \\ \dfrac{R_1 R_5 Y_1}{\mu}\left(1+\dfrac{1}{Y_2 R_4}\right) & \dfrac{1}{R_5} \end{bmatrix}$$

Δ_{12} και Δ_1' είναι δίθυρα σε παράλληλη - παράλληλη σύνδεση $\Rightarrow$

$$\Rightarrow Y_{\Delta o \lambda} = Y_{\Delta 12} + Y_{\Delta'}$$

$$Y_{\Delta 1} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} Z_a & 0 \\ \dfrac{Z_b}{Z_c} & \dfrac{1}{Z_c} \end{bmatrix}$$

$$Z_a = R_3$$
$$Z_b = \frac{1}{Y_3}$$
$$Z_c = R_3$$

$$Y_{\Delta 1'} = \begin{bmatrix} y_{22} & y_{21} \\ y_{12} & y_{11} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{Z_c} & \dfrac{Z_b}{Z_c} \\ 0 & Z_a \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_3} & \dfrac{1}{Y_3 R_3} \\ 0 & R_3 \end{bmatrix}$$

$$Y_{\Delta o \lambda} = \begin{bmatrix} \dfrac{1}{R_1} & 0 \\ \dfrac{R_1 R_5 Y_1}{\mu}\left(1+\dfrac{1}{Y_2 R_4}\right) & \dfrac{1}{R_5} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{R_3} & \dfrac{1}{Y_3 R_3} \\ 0 & R_3 \end{bmatrix} =$$

$$= \begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_3} & \dfrac{1}{Y_3 R_3} \\ \dfrac{R_1 R_5 Y_1}{\mu}\left(1+\dfrac{1}{Y_2 R_4}\right) & \dfrac{1}{R_5} + R_3 \end{bmatrix}$$