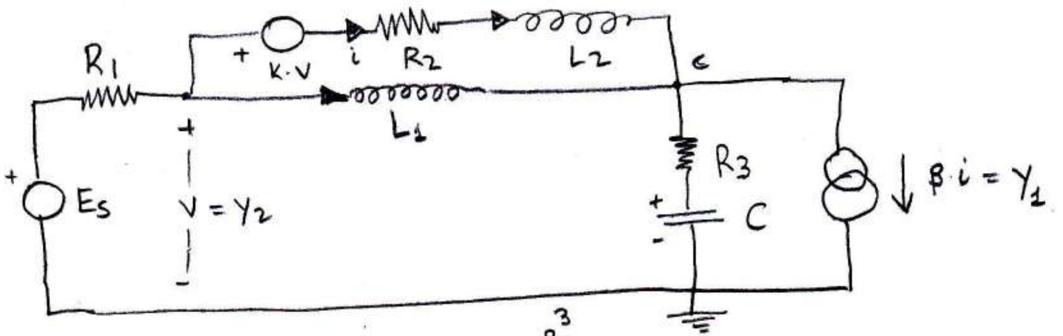
Θεωρία Δικτύων & Κυκλωμάτων

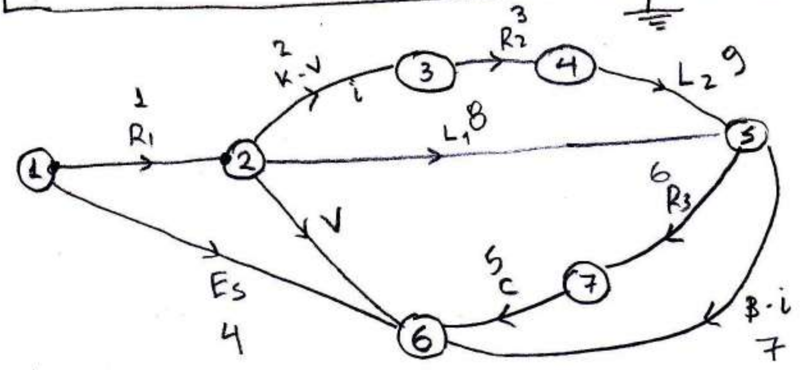
Χρήστος Τσούφης
03117176

2η Σειρά Ασκήσεων

Άσκηση 1



α)



β) • ΝΤΚ στον βρόχο $L_1 - L_2$: $v_{R_1} + v_{L_1} + v_{R_3} + v_C = E_S \Rightarrow$

$$\Rightarrow v_{R_1} + L_1 \cdot \frac{di_{L_1}}{dt} + v_{R_3} + v_C = E_S \quad (1) \quad \Rightarrow$$

$$\Rightarrow v_{R_1} + K \cdot V + v_{R_2} + v_{L_2} + v_{R_3} + v_C = E_S \quad \Rightarrow$$

$$\Rightarrow v_{R_1} + K\left(E_S - v_{R_1}\right) + v_{R_2} + L_2 \cdot \frac{di_{L_2}}{dt} + v_{R_3} + v_C = E_S \Rightarrow$$

$$\Rightarrow (1-K)\, v_{R_1} + v_{R_2} + L_2 \cdot \frac{di_{L_2}}{dt} + v_{R_3} + v_C = (1-K) E_S \quad (2)$$

• ΝΡΚ στο C: $\beta \cdot i + i_C = i_{L_2} + i_{L_1} \Rightarrow \beta \cdot i_{L_2} + C \cdot \frac{dv_C}{dt} = i_{L_1} + i_{L_2} \quad (3)$

• ΝΡΚ στο ②: $i_{R_1} = i_{L_1} + i_{L_2} \Rightarrow v_{R_1} = R_1 (i_{L_1} + i_{L_2}) \quad (4)$

• ΝΡΚ στο ③: $i_{R_2} = i_{L_2} \Rightarrow v_{R_2} = i_{R_2} \cdot R_2 = i_{L_2} \cdot R_2 \quad (5)$

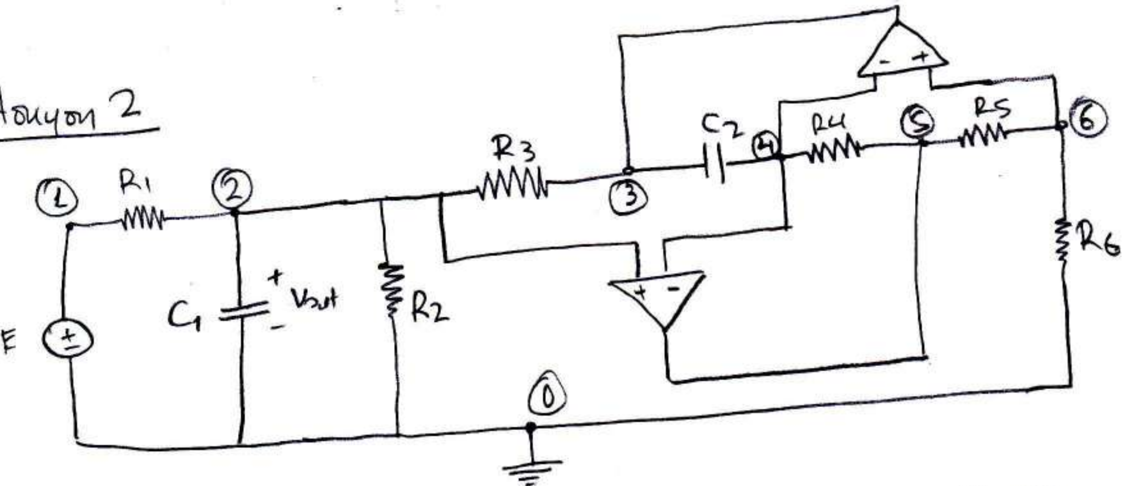
• ΝΡΚ στο ⑤: $i_{R_2} + i_{L_1} = i_{R_3} + \beta \cdot i_{L_2} \overset{(5)}{\Rightarrow} v_{R_3} = i_{L_1} \cdot R_3 + (1-\beta) \cdot i_{L_2} \cdot R_3 \quad (6)$

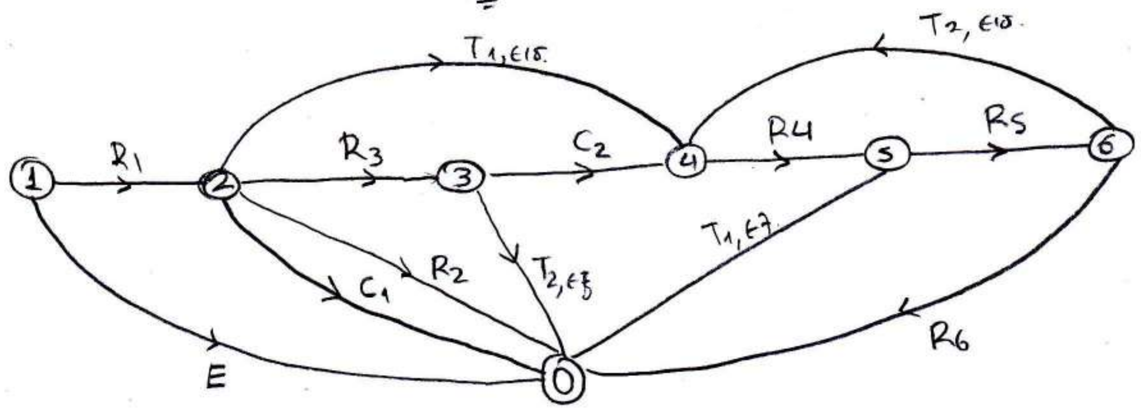
Από (1) − (6):

$$\begin{bmatrix} \dfrac{dV_c}{dt} \\[2ex] \dfrac{di_{L1}}{dt} \\[2ex] \dfrac{di_{L2}}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \dfrac{1}{C} & \dfrac{1-\beta}{C} \\[2ex] -\dfrac{1}{L_1} & \dfrac{R_3 - R_1}{L_1} & \dfrac{-(1-\beta)R_3 - R_1}{L_1} \\[2ex] -\dfrac{1}{L_2} & \dfrac{-(1-K)R_1 + R_3}{L_2} & \dfrac{-(1-K)R_1 + R_2 + (1+\beta)R_3}{L_2} \end{bmatrix}}_{A} \begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\[1ex] \dfrac{1}{L_1} \\[2ex] \dfrac{1-K}{L_2} \end{bmatrix}}_{B} E_s$$

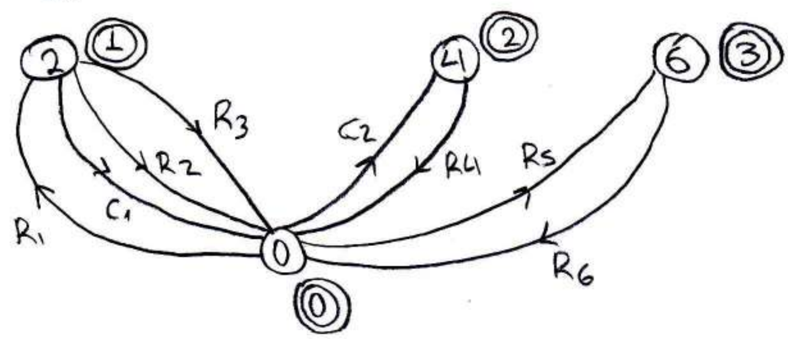
$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & \beta \\ 0 & -R_1 & -R_1 \end{bmatrix}}_{C} \begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{D} E_s$$

Άσκηση 2

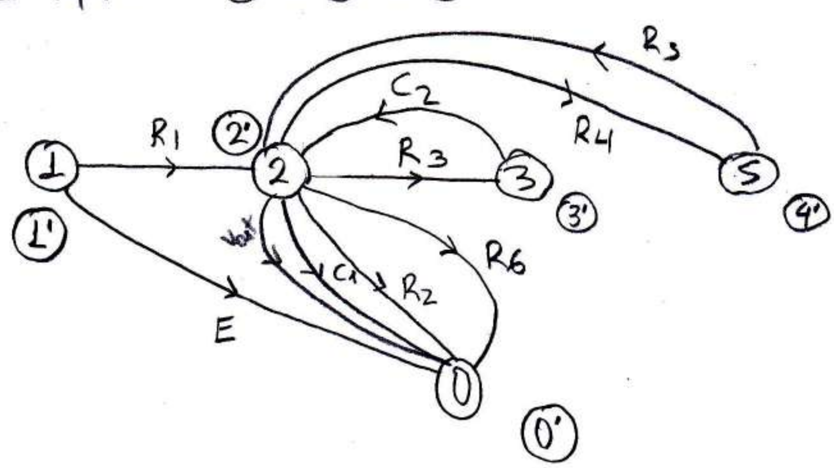




I- Γράφος: πρέπει ⓪ ≡ ① ≡ ③ ≡ ⑤

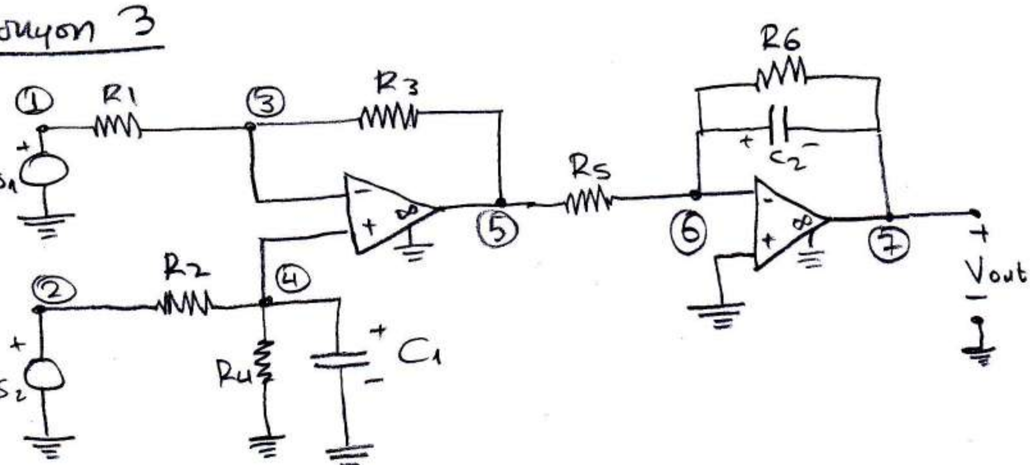


V - Γράφος : πρέπει 2 ≡ 4 ≡ 6

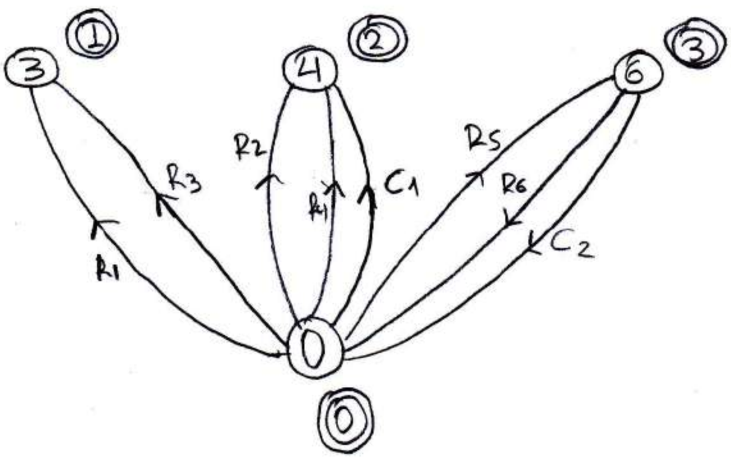


$$
\begin{array}{c|c}
 & \begin{array}{cccc} 1' & 2' & 3' & 4' \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ E \end{array} &
\begin{bmatrix}
-G_1 & sC_1 + G_2 + G_3 & -G_3 & 0 \\
0 & sC_2 + G_4 & -sC_2 & -G_4 \\
0 & G_5 + G_6 & 0 & -G_5 \\
1 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\begin{bmatrix} e_{1'} \\ e_{2'} \\ e_{3'} \\ e_{4'} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E \end{bmatrix}
$$

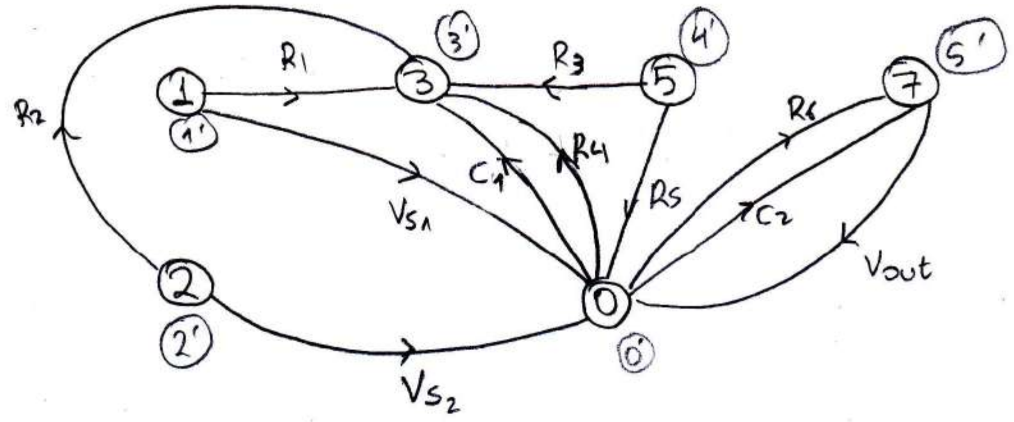
Άσκηση 3



α) I - Γράφος : πρέπει : 0 ≡ 5 ≡ 7 ≡ 1 ≡ 2



V - Γράφος: πρέπει ③ ≡ ④ , ⑥ ≡ ⓪



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 1' & 2' & 3' & 4' & 5' \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ V_{S_1} \\ V_{S_2} \end{array} &
\begin{bmatrix}
-G_1 & 0 & G_1+G_3 & -G_3 & 0 \\
0 & -G_2 & G_2+G_4+sC_1 & 0 & 0 \\
0 & 0 & 0 & -G_5 & -G_6-sC_2 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\begin{bmatrix} e_{1'} \\ e_{2'} \\ e_{3'} \\ e_{4'} \\ e_{5'} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_{S_1} \\ V_{S_2} \end{bmatrix}
$$

β) Μεταβλητές Κατάστασης:

$$X_1 = v_{c_1} = -e_3 \Rightarrow e_3 = -x_1$$

$$X_2 = v_{c_2} = -e_5 \Rightarrow e_5 = -x_2$$

$$
\left\{
\begin{array}{l}
-G_1 \cdot e_1 + (G_1+G_3)\, e_3 - G_3 \cdot e_4 = 0 \\
-G_2 \cdot e_2 + (G_2+G_4+sC_1)\cdot e_3 = 0 \\
-G_5 \cdot e_4 - (sC_2+G_6)\cdot e_5 = 0
\end{array}
\right\} \Rightarrow
$$

$$
\Rightarrow \left\{
\begin{array}{l}
-G_1 \cdot V_{S_1} - x_1 (G_1+G_3) = G_3 \cdot e_4 \\
sC_1 \cdot e_3 = G_2 \cdot V_{S_2} - (G_2+G_4)\cdot e_3 \\
-sC_2 \cdot e_5 = G_3 \cdot e_4 + G_5 \cdot e_5
\end{array}
\right\} \Rightarrow
$$

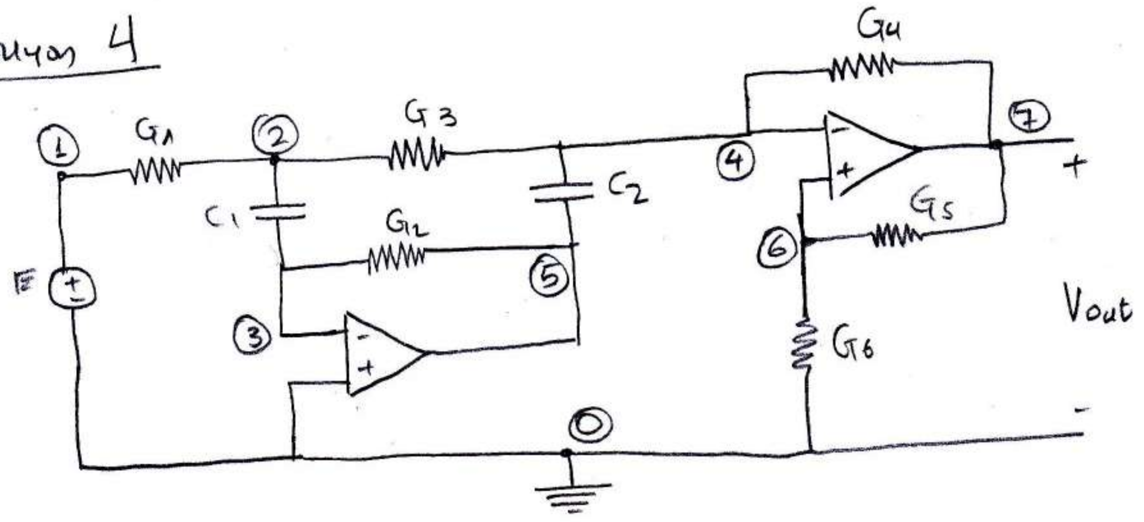
$$\Rightarrow \begin{cases} -G_1 \cdot U_{S_2} - X_1(G_1+G_3) = G_3 \cdot e_4 \\ -sX_1 C_1 = G_2 \cdot U_{S_2} + (G_2+G_4) \cdot X_1 \\ sX_2 C_2 = -G_1 U_{S_2} - X_1(G_1+G_3) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} sX_1 = -\dfrac{G_2}{C_1} U_{S_2} - \dfrac{(G_2+G_4)}{C_1} \cdot X_1 \\ sX_2 = -\dfrac{G_1}{C_2} \cdot U_{S_2} - \dfrac{G_1+G_3}{C_2} \cdot X_1 \end{cases}$$

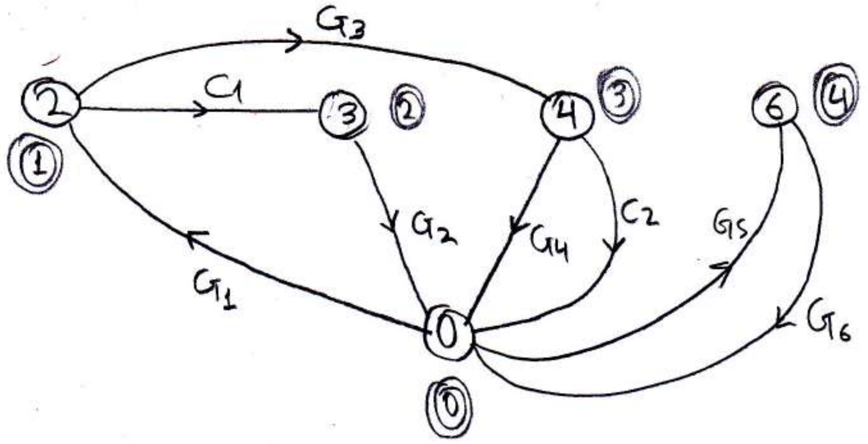
$$\begin{bmatrix} sX_1 \\ sX_2 \end{bmatrix} = \overbrace{\begin{bmatrix} -\left(\dfrac{G_2+G_4}{C_1}\right) & 0 \\ 0 & -\left(\dfrac{G_1+G_3}{C_2}\right) \end{bmatrix}}^{A} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \overbrace{\begin{bmatrix} 0 & -\dfrac{G_2}{C_1} \\ 0 & -\dfrac{G_1}{C_1} \end{bmatrix}}^{B} \begin{bmatrix} U_{S_1} \\ U_{S_2} \end{bmatrix}$$

$$y = \underbrace{[0 \quad 1]}_{C} \begin{bmatrix} U_{C_1} \\ U_{C_2} \end{bmatrix} + \underbrace{[0 \quad 0]}_{D} \begin{bmatrix} U_{S_1} \\ U_{S_2} \end{bmatrix}$$

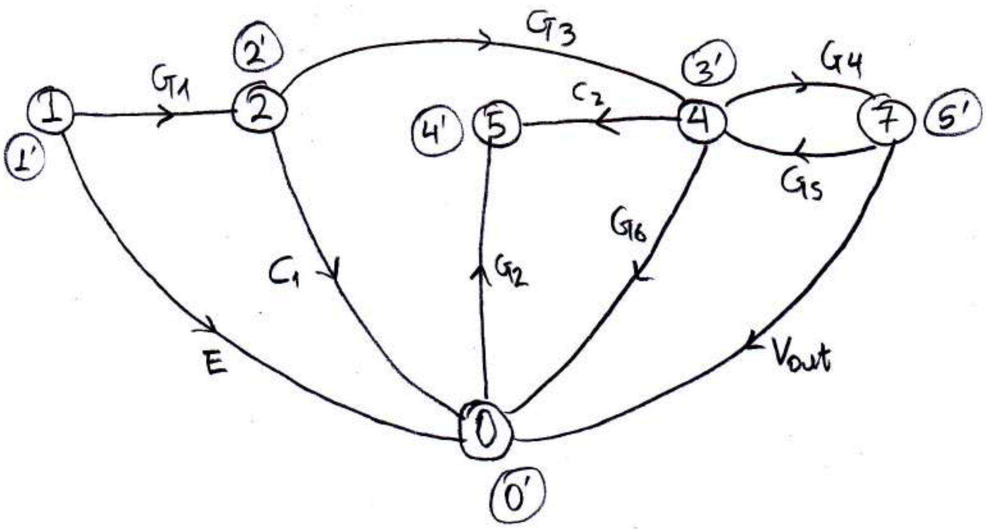
'Ασκηση 4



ράφος : πρέπει $0 \equiv 1 \equiv 5 \equiv 7$



άφος : πρέπει $3 \equiv 0$, $4 \equiv 6$



$$
\begin{array}{ccccc}
1' & 2' & 3' & 4' & 5' \\
\end{array}
$$

$$
\begin{bmatrix}
-G_1 & G_1+G_3+sC_1 & -G_3 & 0 & 0 \\
0 & -sC_1 & 0 & -G_2 & 0 \\
0 & G_3 & G_3+sC_2+G_4 & -sC_2 & -G_4 \\
0 & 0 & G_5+G_6 & 0 & -G_5 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} e_1' \\ e_2' \\ e_3' \\ e_4' \\ e_5' \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix}
$$