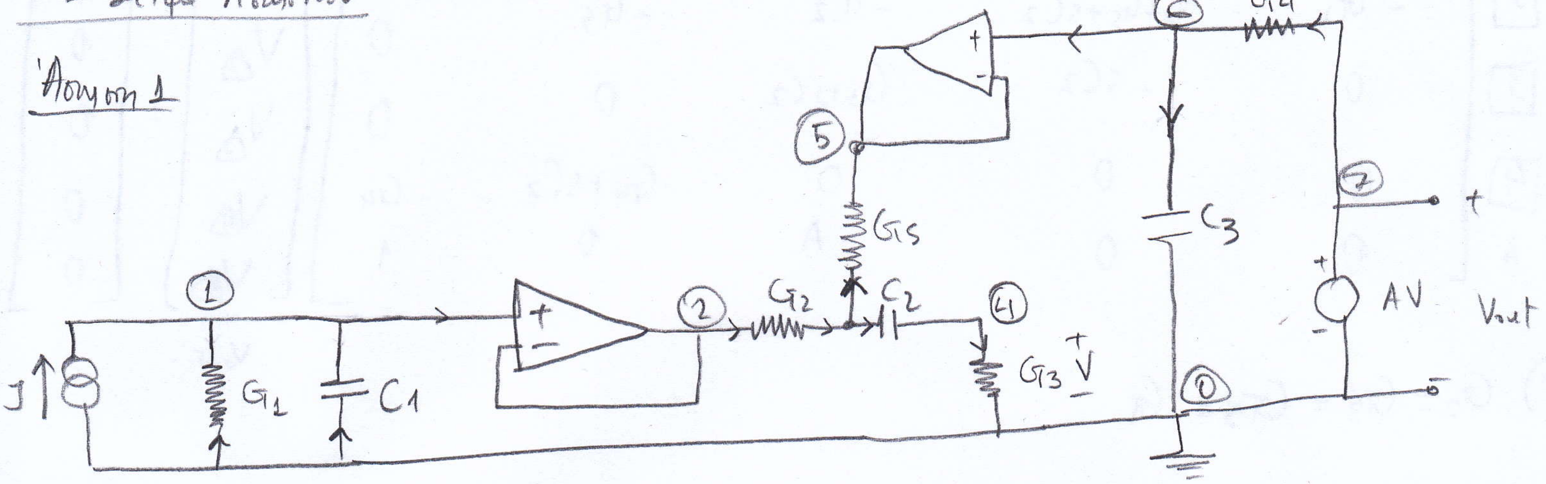
Θεωρία Δικτύων & Κυκλωμάτων

Χρήστος Τσούφης
03117176

3η Σειρά Ασκήσεων

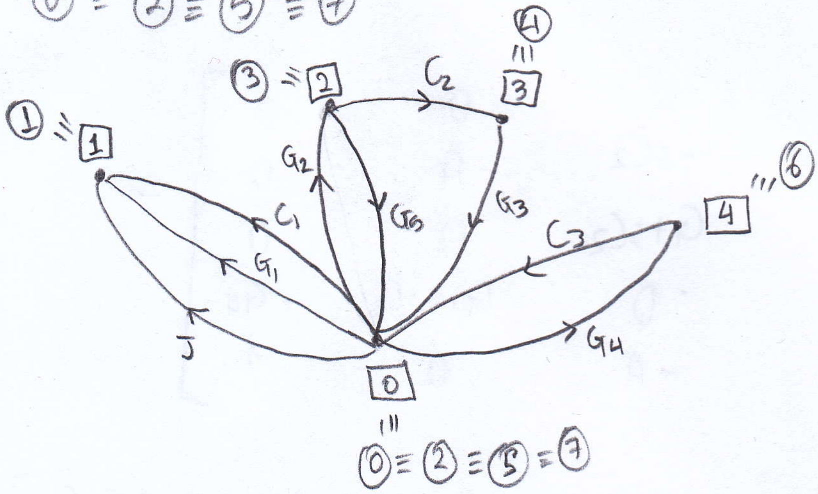
Άσκηση 1



α) Με τα βελάκια φαίνονται οι αυθαίρετες φορές στους παθητικούς κλάδους.

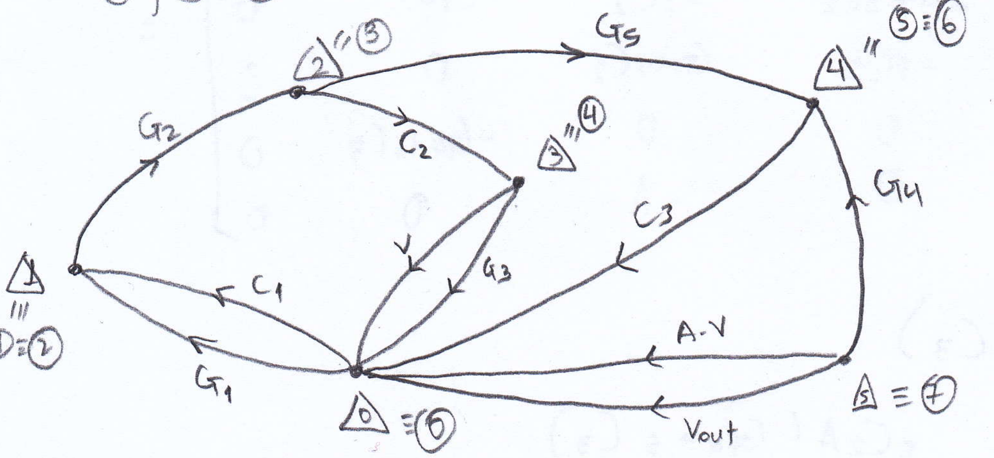
I - γράφος

⓪ ≡ ② ≡ ⑤ ≡ ⑦



V - γράφος

① ≡ ②, ⑤ ≡ ⑥



$$
\begin{array}{c}
\boxed{1}\\ \boxed{2}\\ \boxed{3}\\ \boxed{4}\\ A
\end{array}
\begin{bmatrix}
G_1+sC_1 & 0 & 0 & 0 & 0\\
-G_2 & G_2+G_5+sC_2 & -sC_2 & -G_5 & 0\\
0 & -sC_2 & G_3+sC_2 & 0 & 0\\
0 & 0 & 0 & G_4+sC_3 & -G_4\\
0 & 0 & -A & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V_{1}\\ V_{2}\\ V_{3}\\ V_{4}\\ V_{5}
\end{bmatrix}
=
\begin{bmatrix}
J\\ 0\\ 0\\ 0\\ 0
\end{bmatrix}
$$

$$V_5 = V_{out}$$

β) $G_2 = G_3 = G_5 = G$

$$\left.\begin{array}{l} G(s) = \dfrac{V_{out}(s)}{J} = \dfrac{V_5}{J} \\[2ex] V_5 = \dfrac{\Delta_5(s)}{\Delta(s)} \end{array}\right\} \Rightarrow G(s) = \frac{\Delta_5(s)}{J\cdot\Delta(s)}$$

$$\Delta(s) = \det\begin{bmatrix}
G_1+sC_1 & 0 & 0 & 0 & 0\\
-G & 2G+sC_2 & -sC_2 & -G & 0\\
0 & -sC_2 & G+sC_2 & 0 & 0\\
0 & 0 & 0 & G_4+sC_3 & -G_4\\
0 & 0 & -A & 0 & 1
\end{bmatrix} =$$

$$= \dots = (G_1+sC_1)\left[2G^2C_4 + 2G^2sC_3 + (3-A)G\,G_4\,sC_2 + 3Gs^2C_2C_3\right]$$

$$\Delta_5(s) = \det\begin{bmatrix}
G_1+sC_1 & 0 & 0 & 0 & J\\
-G & 2G+sC_2 & -sC_2 & -G & 0\\
0 & -sC_2 & G+sC_2 & 0 & 0\\
0 & 0 & 0 & -G_4+sC_3 & 0\\
0 & 0 & -A & 0 & 0
\end{bmatrix} =$$

$$= J\,G\cdot sC_2\,A(G_4+sC_3)$$

Onde, $G(s) = \dfrac{\Delta_5(s)}{J\Delta(s)} = \dfrac{sC_2A(G_4+sC_3)}{G(G_1+sC_1)\left[2G(G_4+sC_3)+(3-A)G_4\cdot sC_2+3s^2C_2C_3\right]}$

γ) Χαρακτηριστικό πολυώνυμο:

$$\Psi(s) = s^3\, 3\, G C_1 C_2 C_3 + s^2 C_1 [2G^2 C_3 + (3-A) G G_4 C_2] + s\, 2 G^2 G_4 C_1 \;+$$

$$+\; s^2\, 3 G C_1 C_2 C_3 + s\, G_1 [2 G^2 C_3 + (3-A) G G_4 C_2] + 2 G^2 G_1 G_4 \;=$$

$$= s^3\, 3 G C_1 C_2 C_3 + s^2 [C_1 2 G^2 C_3 + (3-A) G G_4 C_2 + 3 G G_1 C_2 C_3] \;+$$

$$+\; s [G_1 (2 G^2 C_3 + (3-A) G G_4 C_2) + 2 G^2 G_4 C_1] + 2 G^2 G_1 \cdot G_4 \;=$$

$$= s^3\, 3 G C_1 C_2 C_3 + s^2 [\underbrace{2 G^2 C_1 C_3 + (3-A) G G_4 C_2 + 3 G G_1 C_2 C_3}_{(I)}] \;+$$

$$+\; s [\underbrace{2 G^2 G_1 C_3 + (3-A) G G_1 G_4 C_2 + 2 G^2 G_4 C_1}_{(II)}] + 2 G^2 G_1 G_4$$

$$\begin{array}{c|cc}
s^3 & 3 G_1 C_1 C_2 C_3 & II \\
s^2 & I & 2 G^2 G_1 G_4 \\
s^1 & \dfrac{I \cdot II - 6 G^3 G_1 G_4 C_1 C_2 C_3}{I} & 0 \\
s^0 & 2 G^2 C_1 G_4 &
\end{array}$$

- για <u>ασυμπτωτική ευστάθεια</u> πρέπει $\begin{cases} I > 0 & (1) \\ I \cdot II - 6 G^3 G_1 G_4 C_1 C_2 C_3 > 0 & (2) \end{cases}$

- για να εκτελεί <u>αμείωτες ταλαντώσεις</u> πρέπει μία ολόκληρη γραμμή να μηδενίζεται, δηλ. $I \cdot II - 6 G^3 G_1 G_4 C_1 C_2 C_3 = 0$ και ισχύει η (1).

$$B_{II}(s) = 0 \Rightarrow s^2 A + 2 G^2 G_1 G_4 = 0 \Rightarrow s = \sqrt{\frac{2 G^2 G_1 G_4}{A}} \Rightarrow \omega = \ldots$$

δ) $G = G_1 = G_2 = G_3 = G_4 = G_5 = 1 \frac{1}{\Omega}$

$C_1 = C_2 = 1F$

$C_3 = 2F$

$A = 3$

$J(t) = 5 + 3\sin(2t)$

$$G(s) = \frac{s C_2 A (G_4 + s C_3)}{G (G_1 + s C_1) \left[2G (G_4 + s C_3) + (3-A) G_4 s C_2 + 3 s^2 G_2 C_3 \right]} \Rightarrow$$

$$\Rightarrow G(s) = \frac{s \cdot 3(1+2s)}{(1+s)\left[2(1+2s) + 0 + 6s^2 \right]} = \frac{3s(2s+1)}{\underbrace{2(s+1)(3s^2+2s+1)}} \longrightarrow \begin{array}{l} p_1 = -1 \\ p_2 = \dfrac{-1+j\sqrt{2}}{3} \\ p_3 = \dfrac{-1-j\sqrt{2}}{3} \end{array}$$

$$J(t) = 5 + 3\sin(2t) = 5 + 6 \cdot \tfrac{1}{2} \sin 2t \Rightarrow J(s) = \frac{5}{s} + \frac{6}{s^2+4}$$

$$V_{out} = \mathcal{L}^{-1}\{G(s) \cdot J(s)\} = \frac{3s(2s+1)}{2(s+1)(3s^2+2s+1)} \cdot \left(\frac{5}{s} + \frac{6}{s^2+4} \right) =$$

$$= \frac{7{,}5(2s+1)}{(1+s)(3s^2+2s+1)} + \frac{9s(2s+1)}{(s+1)(s^2+4)(3s^2+2s+1)} =$$

$$= \sum_{i=1}^{3} \frac{k_i}{s-p_i} + \frac{k_0}{s-2j} + \frac{\bar{k}_0}{s+2j}$$

$$\xRightarrow{\mathcal{L}^{-1}} \gamma(t) = k_0 e^{j2t} + \bar{k}_0 e^{-j2t} + \sum_{i=1}^{n} k_i e^{p_i t}$$

για ημιτονική μόνιμη απόκριση ($t \to \infty$): $\gamma_{ss}(t) = k_0 e^{j2t} + \bar{k}_0 e^{-j2t} + \sum_{i=1}^{n} \cancel{k_i e^{p_i t}}^{\,0}$

$$k_0 = G(s) \cdot J(s)(s-2j) \Big|_{s=2j} = \frac{9s(2s+1)}{(s+1)(3s^2+2s+1)(s+2j)} \Bigg|_{s=2j} =$$

$$= \frac{18j(4j+1)}{(1+2j)(-12+4j+1)(4j)} = \frac{-72+j18}{(-8+4j)(-11+4j)} = \frac{-72+j18}{88-32j-44j-16} = \frac{-72+j18}{72-j76} =$$

$$= 0{,}60 - j0{,}38 \Rightarrow \bar{k}_0 = -0{,}60 + j0{,}38$$

Οπότε, $y_{ss}(t) = (-0,60 - j0,38)e^{j2t} + (-0,60 + j0,38)e^{-j2t} =$

$= (-0,60 - j0,38)(\cos 2t + j\sin 2t) + (-0,60 + j0,38)(\cos 2t - j\sin 2t) =$

$= -0,6\cos 2t - 0,6j\,\cancel{\sin 2t} - 0,38\,\cancel{j\cos 2t} + 0,38\sin 2t - 0,6\cos 2t +$

$+ j0,6\,\cancel{\sin 2t} + j0,38\,\cancel{\cos 2t} + 0,38\sin 2t \Rightarrow$

$\Rightarrow y_{ss}(t) = -1,2\cos 2t + 0,76\cos 2t$

ε) $G(s) = \dfrac{3s(2s+1)}{2(s+1)(3s^2+2s+1)}$

(Γενική μορφή: $G(s) = K \dfrac{\prod_{i=1}^{m_1}(1+s\tilde{T}_i) \cdot \prod_{k=1}^{m_2}\left[1+2\frac{\tilde{\zeta}_n}{\tilde{\omega}_k}s+\left(\frac{s}{\tilde{\omega}_k}\right)^2\right]}{\prod_{i=1}^{n_1}(1+sT_i) \cdot \prod_{k=1}^{n_2}\left[1+2\frac{\zeta_n}{\omega_k}s+\left(\frac{s}{\omega_k}\right)^2\right]}$)

Οπότε, $G(s) = \dfrac{1,5 \cdot s \cdot (2s+1)}{(s+1)\left(1+2\frac{\frac{\sqrt{3}}{3}}{\sqrt{\frac{1}{3}}}s+\left(\frac{s}{\sqrt{1/3}}\right)^2\right)} = \dfrac{1,5\,s\,(1+sT_1)}{\left(1+2\frac{\zeta_k}{\omega_k}s+\left(\frac{s}{\omega_k}\right)^2\right)(1+sT_2)}$

Αριθμητής:

$K(\omega) = 20\log 1,5 = 3,5$, $\varphi(\omega) = 0°$ (για όρο 1,5)

$K(\omega) = 20\ dB/dec$, $\varphi(\omega) = 90°$ (για όρο s)

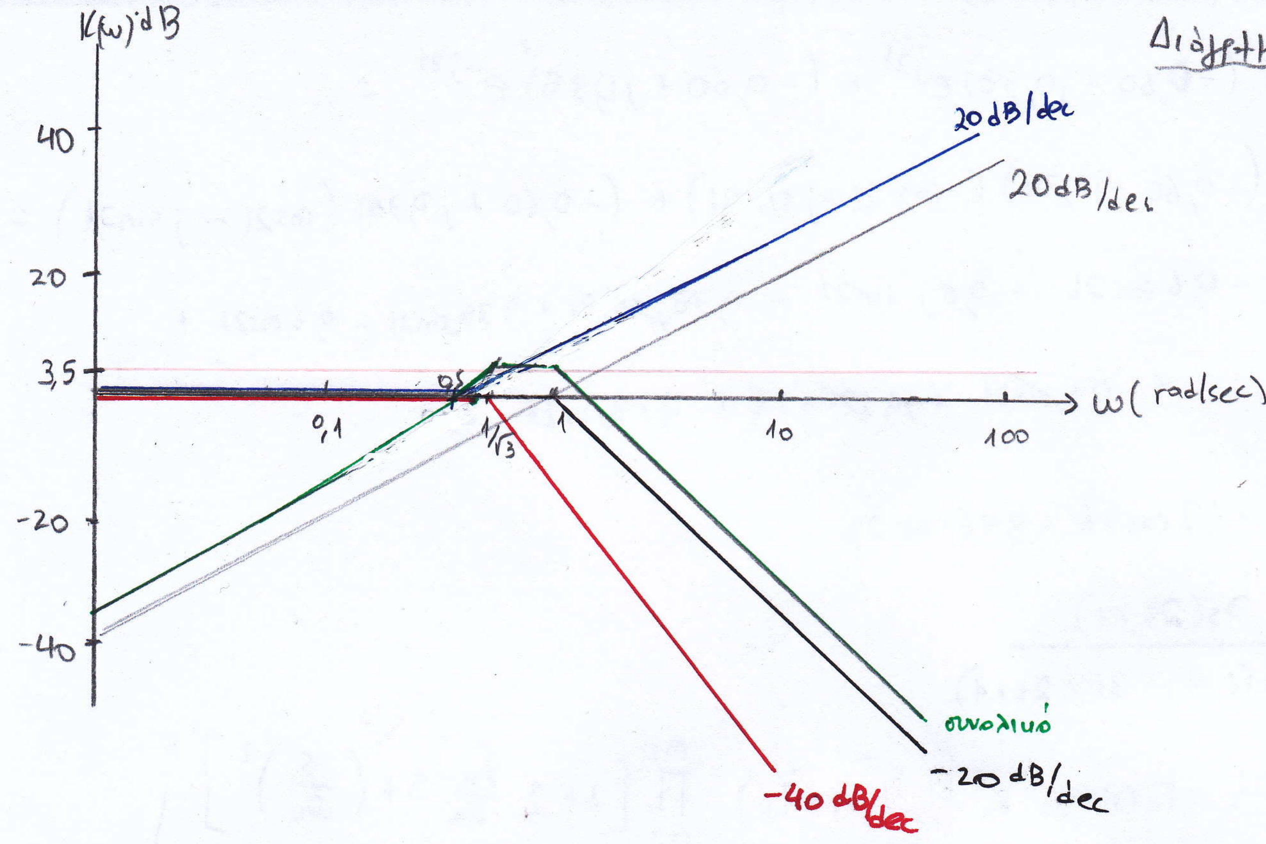
$T_1 = 2$, $\omega_{01} = \dfrac{1}{|T_1|} = 0,5\ rad/sec$ (για όρο 1+2s)

Παρονομαστής:

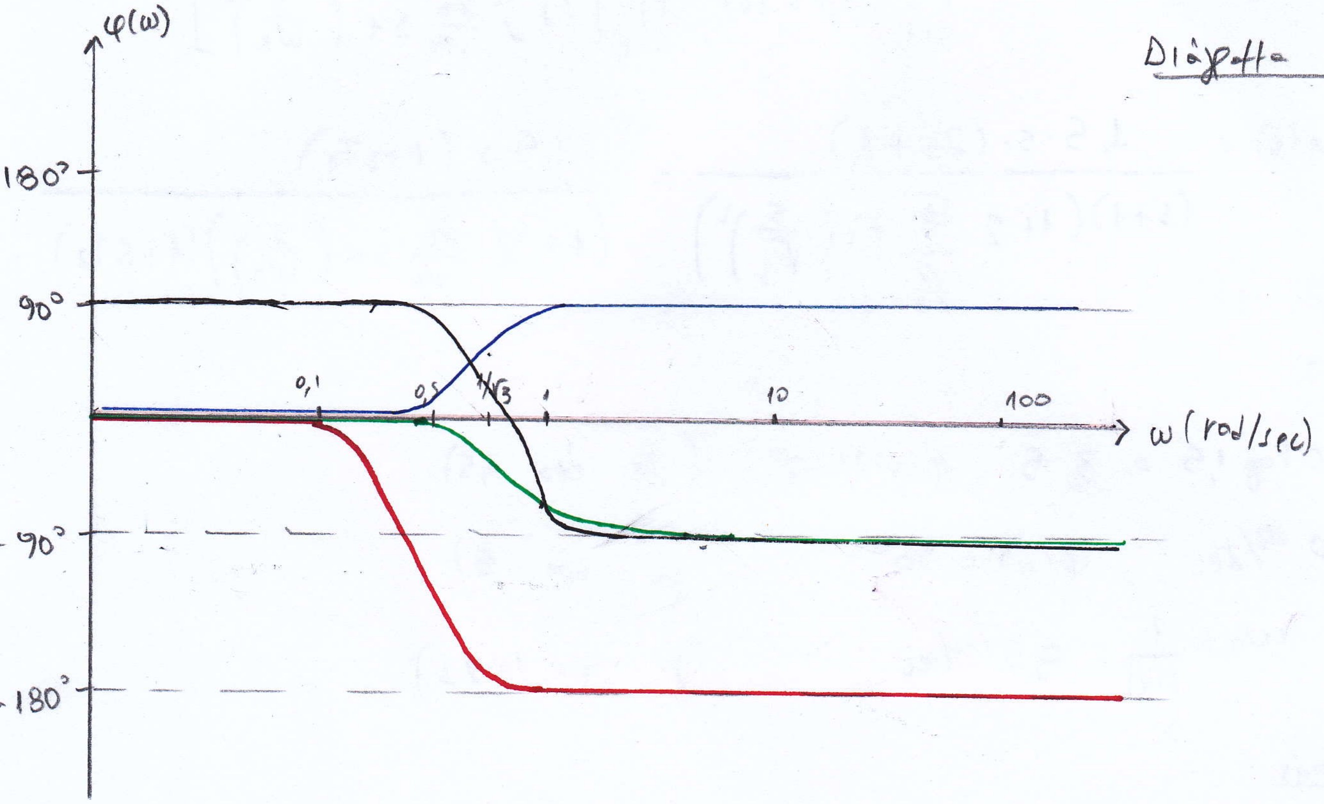
$T_2 = 1$, $\omega_{02} = \dfrac{1}{|T_2|} = 1\ rad/sec$ (για όρο 1+s)

$\zeta = \frac{\sqrt{3}}{3} < 1 \rightarrow$ δηλαδή $\omega_{03} = \omega_k = \sqrt{1/3}\ rad/sec$ (για όρο $1+2\frac{\sqrt{3}/3}{\sqrt{1/3}}s+\left(\frac{s}{\sqrt{1/3}}\right)^2$)

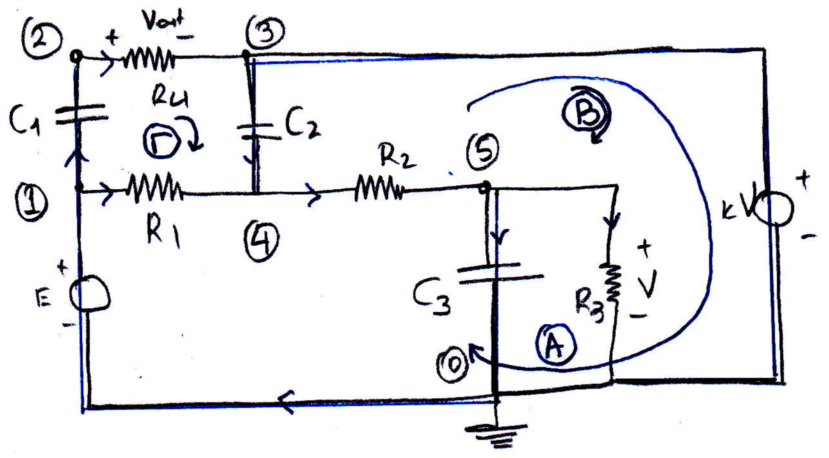
Διάγραμμα Κέρδους



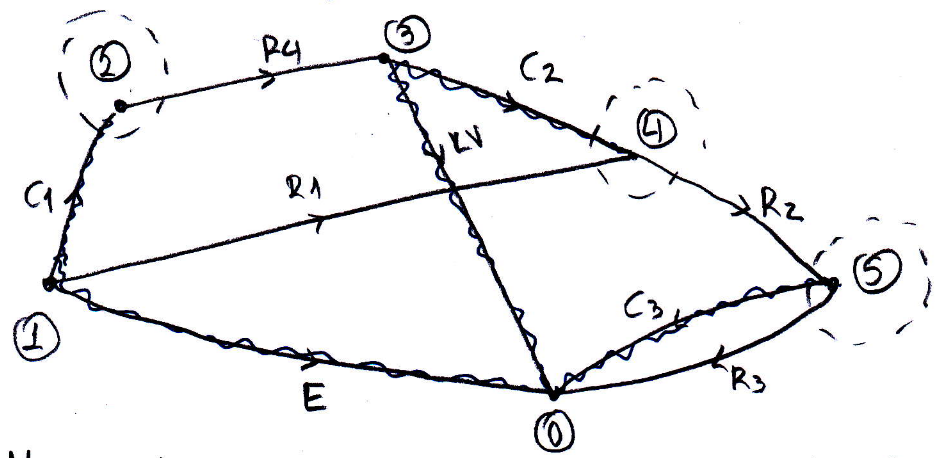
Διάγραμμα συχνότητας



Άσκηση 2



α) Κανονικό δέντρο:



Με διανύσματα κατάστασης: $x(t) = \begin{bmatrix} U_{C\delta}(t) \\ i_{L\ell}(t) \end{bmatrix} = \begin{bmatrix} U_{C1} \\ U_{C2} \\ U_{C3} \end{bmatrix}$, $u(t) = [E]$, $y = [V_{out}] = V_{R4}$

ΝΡΚ στη θ.ο.δ. που αντισ. στον C_1: $i_{C1} = i_{R4} \Rightarrow C_1 \frac{dU_{C1}}{dt} = i_{R4}$ (1)

ΝΡΚ στη θ.ο.δ που αντισ. στο C_2: $i_{C2} + i_{R1} = i_{R2} \Rightarrow C_2 \frac{dU_{C2}}{dt} = i_{R2} - i_{R1}$ (2)

ΝΡΚ στη θ.ο.δ που αντισ. στο C_3: $i_{R3} = i_{C3} + i_{R3} \Rightarrow C_3 \frac{dU_{C2}}{dt} = i_{R2} - i_{R3}$ (3)

$V_{R3} = V_{C3} \Rightarrow i_{R3} \cdot R_3 = V_{C3} \Rightarrow i_{R3} = \frac{V_{C3}}{R_3}$ (4)

$V = V_{R3} \overset{(4)}{=} V_{C3}$ (5)

ΝΤΚ Δ: $E = V_{C1} + V_{R4} + kV \Rightarrow E = V_{C1} + R_4 i_{R4} + kV_{C3} \Rightarrow$

$\Rightarrow i_{R4} = \frac{E - kV_{C3} - V_{C1}}{R_4}$ (6)

NTK B) : $kV = V_{C_2} + V_{R_2} + V \Rightarrow kV_{C_3} = V_{C_2} + R_2 i_{R_2} + V_{C_3} \Rightarrow$

$$\Rightarrow i_{R_2} = \frac{[(k-1)V_{C_3} - V_{C_2}]}{R_2} \qquad (7)$$

NTK D) :

$$V_{C_1} + V_{R_4} + V_{C_2} - V_{R_1} \overset{(6)}{=} 0 \Rightarrow \cancel{V_{C_1}} + V_{C_2} + (E - kV_{C_3} - \cancel{V_{C_1}}) = R_1 \cdot i_{R_1}$$

$$\Rightarrow i_{R_1} = \frac{E + V_{C_2} - kV_{C_3}}{R_1} \qquad (8)$$

$$(1) \Rightarrow \frac{dV_{C_1}}{dt} = \frac{i_{R_4}}{C_1} \overset{(6)}{=} \frac{E - kV_{C_3} - V_{C_1}}{R_4 C_1} = \frac{E - kV_{C_3} - V_{C_1}}{C_1 R_4} = \frac{-V_{C_1} - kV_{C_3} + E}{C_1 R_4}$$

$$(2) \Rightarrow \frac{dV_{C_2}}{dt} = \frac{i_{R_2} - i_{R_1}}{C_2} \overset{(7)}{\underset{(8)}{=}} \frac{(k-1)V_{C_3} - V_{C_2}}{C_2 R_2} - \frac{E + V_{C_2} - kV_{C_3}}{C_2 R_1} =$$

$$= \frac{\left[\left(-\frac{1}{R_1} - \frac{1}{R_2}\right)V_{C_2} + \left(\frac{(k-1)}{R_2} + \frac{k}{R_1}\right)V_{C_3} - \frac{1}{R_1}E\right]}{C_2}$$

$$(3) \Rightarrow \frac{dV_{C_3}}{dt} = \frac{i_{R_2} - i_{R_3}}{C_3} \overset{(4)}{\underset{(7)}{=}} \frac{\left[\frac{(k-1)V_{C_3} - V_{C_2}}{R_2} - \frac{V_{C_3}}{R_3}\right]}{C_3} =$$

$$= \frac{\left[-\frac{1}{R_2}V_{C_2} + \left(\frac{(k-1)}{R_2} - \frac{1}{R_3}\right)V_{C_3}\right]}{C_3}$$

$$\dot{x}(t) = Ax(t) + Bu(t) \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \frac{dV_{C_1}}{dt} \\ \frac{dV_{C_2}}{dt} \\ \frac{dV_{C_3}}{dt} \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{1}{C_1R_4} & 0 & \frac{-k}{C_1R_4} \\ 0 & \left(-\frac{1}{R_1C_2} - \frac{1}{R_2C_2}\right) & \left(\frac{k-1}{R_2C_2} + \frac{k}{R_1C_2}\right) \\ 0 & -\frac{1}{R_2C_3} & \left(\frac{k-1}{R_2C_3} - \frac{1}{R_3C_3}\right) \end{bmatrix}}^{A} \begin{bmatrix} V_{C_1} \\ V_{C_2} \\ V_{C_3} \end{bmatrix} + \overbrace{\begin{bmatrix} \frac{1}{C_1R_4} \\ -\frac{1}{R_1C_2} \\ 0 \end{bmatrix}}^{B} E$$

$$y(t) = Cx(t) + Du(t)$$

$$\Rightarrow V_{out} = -V_{C1} - kV_{C3} + E \Rightarrow V_{out} = \underbrace{\begin{bmatrix} -1 & -k & 0 \end{bmatrix}}_{C} \begin{bmatrix} U_{C1} \\ U_{C2} \\ U_{C3} \end{bmatrix} + \underbrace{1}_{D} \cdot E$$

B) $R_1 = R_2 = R_3 = R_4 = 1\Omega$

$C_1 = C_2 = C_3 = 1F$

$$A = \begin{bmatrix} -\frac{1}{C_1R_4} & 0 & -\frac{k}{C_1R_4} \\ 0 & \left(-\frac{1}{R_1C_2} - \frac{1}{R_2C_2}\right) & \left(\frac{k-1}{R_2C_2} + \frac{k}{R_1C_2}\right) \\ 0 & -\frac{1}{R_2C_3} & \left(\frac{k-1}{R_2C_3} - \frac{1}{R_3C_3}\right) \end{bmatrix} = \begin{bmatrix} -1 & 0 & -k \\ 0 & -2 & 2k-1 \\ 0 & -1 & k-2 \end{bmatrix}$$

Χαρακτ. πολυων. $\varphi(s) = \det\{sI - A\}$

$$\varphi(s) = \begin{bmatrix} s+1 & 0 & k \\ 0 & s+2 & s-2k+1 \\ 0 & 1 & s-k+2 \end{bmatrix} = (s+1)\left[(s+2)(s+k-2) - (s-2k+1)\right] =$$

$$= (s+1)(s^2 - ks + 2s + 2s - 2k + 4 - s + 2k - 1) =$$

$$= (s+1)(s^2 + (3-k)s + 3) = s^3 + (3-k)s^2 + 3s + s^2 + (3-k)s + 3 =$$

$$= s^3 + (4-k)s^2 + (6-k)s + 3$$

$$\begin{array}{c|ccc} s^3 & 1 & 6-k & 0 \\ s^2 & 4-k & 3 & 0 \\ s^1 & \frac{(4-k)(6-k)-3}{4-k} & 0 & \\ s^0 & 3 & & \end{array}$$

• Για ασυμπτωτική ευστάθεια πρέπει: $4-k>0 \Rightarrow k<4$

$k^2-10k+21>0 \Rightarrow k\in(-\infty,3)\cup(7,+\infty)$ $\Big\}\ k<4$

• Για να έχουμε οριακώς ευσταθές σύστημα πρέπει να υπάρχει ολόκληρη γραμμή με μηδενικά στοιχεία. Η μόνη γραμμή που μπορεί να μηδενιστεί είναι η s^1.

Για $k=7$, υπάρχει αλλαγή προσήμου στη γραμμή s^2.

Για $k=3$, μηδενίζεται χωρίς αλλαγή προσήμου με $B(s)=s^2+3=0 \Rightarrow s=\pm j\sqrt{3} \Rightarrow$

$\Rightarrow \omega=\sqrt{3}$

δ) για $k=0$, $E(t)=0$, $V_{C1}(0)=2V$, $V_{C2}(0)=V_{C3}(0)=0V$

$$\underline{x}(0)=\begin{pmatrix}2\\0\\0\end{pmatrix}$$

$$Y_{AME}(t)=\mathcal{L}^{-1}\{c(sI-A)^{-1}\cdot\underset{\sim}{x}(0)\}$$

$$A=\begin{pmatrix}-1&0&-k\\0&-2&2k-1\\0&-1&k-2\end{pmatrix}=\begin{pmatrix}-1&0&0\\0&-2&-1\\0&-1&-2\end{pmatrix}$$

$$C^T=\begin{pmatrix}-1\\-k\\0\end{pmatrix}=\begin{pmatrix}-1\\0\\0\end{pmatrix}$$

$$(sI-A)^{-1}=\begin{pmatrix}s+1&0&0\\0&s+2&1\\0&1&s+2\end{pmatrix}^{-1}=\frac{adj(sI-A)}{det(sI-A)}$$

$$det(sI-A)=(s+1)\left[(s+2)^2-1\right]=(s+1)(s^2+4s+3)$$

$$adj(sI-A)=\begin{pmatrix}s^2+4s+3&0&0\\0&s^2+3s+2&s+1\\0&s+1&s^2+3s+2\end{pmatrix}$$

$$C(sI-A)^{-1}x(0) = \frac{1}{(s+1)(s^2+4s+3)} \begin{bmatrix} -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} (s+1)(s+3) & 0 & 0 \\ 0 & (s+1)(s+2) & s+1 \\ 0 & s+1 & (s+1)(s+2) \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s^2+4s+3)} \begin{bmatrix} -(s+1)(s+3) & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \frac{-2\cancel{(s+1)}(s+3)}{(s+1)^2(s+3)} =$$

$$= -\frac{2}{s+1}$$

Άρα $y_{AME}(t) = \mathcal{L}^{-1}\left\{ -\frac{2}{s+1} \right\} = -2e^{-t}$.

Άσκηση 3

α) Από την σχέση 2.2 της $2^{ης}$ Σειράς Ασκήσεων.

$$\begin{array}{c|c} & \begin{array}{cccc} \triangle 1 & \triangle 2 & \triangle 3 & \triangle 4 \end{array} \\ \begin{array}{c} \boxed{1} \\ \boxed{2} \\ \boxed{3} \\ E \end{array} & \begin{bmatrix} -G_1 & G_1+G_2+G_3+sC_1 & -G_3 & 0 \\ 0 & G_4+sC_2 & -sC_2 & -G_4 \\ 0 & G_5+G_6 & 0 & -G_5 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{\triangle 1} \\ V_{\triangle 2}=V_{out} \\ V_{\triangle 3} \\ V_{\triangle 4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E \end{bmatrix} \end{array}$$

β) $G_T(s) = \frac{V_{out}(s)}{E(s)} = \frac{V_{\triangle 2}}{E} = \frac{\Delta_2(s)}{E\,\Delta(s)}$

$$\Delta(s) = -1 \begin{vmatrix} G_1+G_2+G_3+sC_1 & -G_3 & 0 \\ G_4+sC_2 & -sC_2 & -G_4 \\ G_5+G_6 & 0 & -G_5 \end{vmatrix} =$$

$$= -G_3G_4G_6 - G_1G_5sC_2 - G_2G_5sC_2 - G_5s^2C_1C_2$$

$$\Delta_2(s) = \begin{bmatrix} -G_1 & 0 & -G_3 & 0 \\ 0 & 0 & sC_2 & -G_4 \\ 0 & 0 & 0 & G_5 \\ 1 & E & 0 & 0 \end{bmatrix} = E\left[-G_1\left(G_5\, s\, C_2\right)\right] =$$

$$= -E\, G_1\, G_5\, s\, C_2$$

$$G(s) = \frac{V_{out}(s)}{E(s)} = \frac{V_\Delta}{E} = \frac{\Delta_2(s)}{E\,\Delta(s)} \Rightarrow$$

$$\Rightarrow G(s) = \frac{G_1\, G_5\, s\, C_2}{G_3\, G_4\, G_6 + G_1\, G_5\, s\, C_2 + G_2 G_5\, s C_2 + G_5\, s^2 C_1 C_2}$$

γ) $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1\Omega$, $C_1 = 2F$, $C_2 = 1F$

$E(t) = 10u(t)$ V $\xRightarrow{\mathcal{L}}$ $E(s) = \frac{10}{s}$

$$G(s) = \frac{s}{(1+s+s+2s^2)} = \frac{s}{2s^2+2s+1}$$

$$V_{out}(s) = E(s)\cdot G(s) = \frac{10s}{s(2s^2+2s+1)} = \frac{10}{2\left[(s+\frac{1}{2})^2+\frac{1}{4}\right]} = \frac{5\cdot 2\cdot \frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{1}{2})^2} \Rightarrow$$

$$\Rightarrow V_{out}(s) = 10\,\frac{1/2}{(s+\frac{1}{2})^2+(\frac{1}{2})^2} \rightarrow$$

$$\Rightarrow V_{out}(t) = \mathcal{L}^{-1}\{V_{out}(s)\} = 10e^{-\frac{t}{2}}\sin\left(\frac{t}{2}\right)$$

δ) $G(s) = \frac{s}{2s^2 + 2s + 1}$

(Γενική μορφή: $G(s) = K \frac{\prod_{i=1}^{m_1} (1+s\tilde{T}_i) \cdot \prod_{k=1}^{m_2} \left[1 + 2\frac{\tilde{\zeta}_k}{\tilde{\omega}_k} s + \left(\frac{s}{\tilde{\omega}_k}\right)^2\right]}{\prod_{i=1}^{n_1} (1+sT_i) \cdot \prod_{k=1}^{n_2} \left[1 + 2\frac{\zeta_k}{\omega_k} s + \left(\frac{s}{\omega_k}\right)^2\right]}$)

$$G(s) = \frac{s}{1 + 2\frac{\sqrt{1/2}}{\sqrt{1/2}} s + \left(\frac{s}{\sqrt{1/2}}\right)^2} = \frac{s}{1 + 2\frac{\zeta_k}{\omega_k} s + \left(\frac{s}{\omega_k}\right)^2}$$

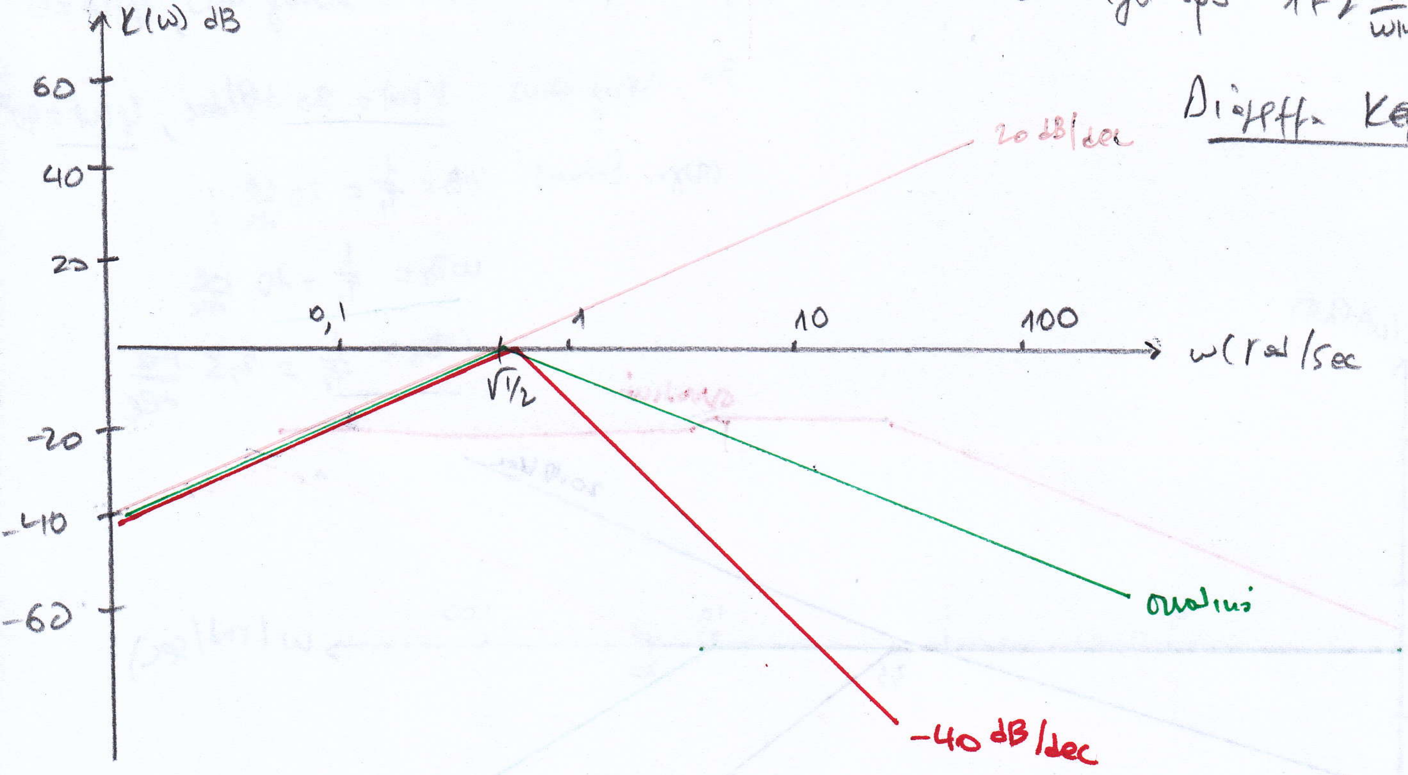
Αριθμητής:

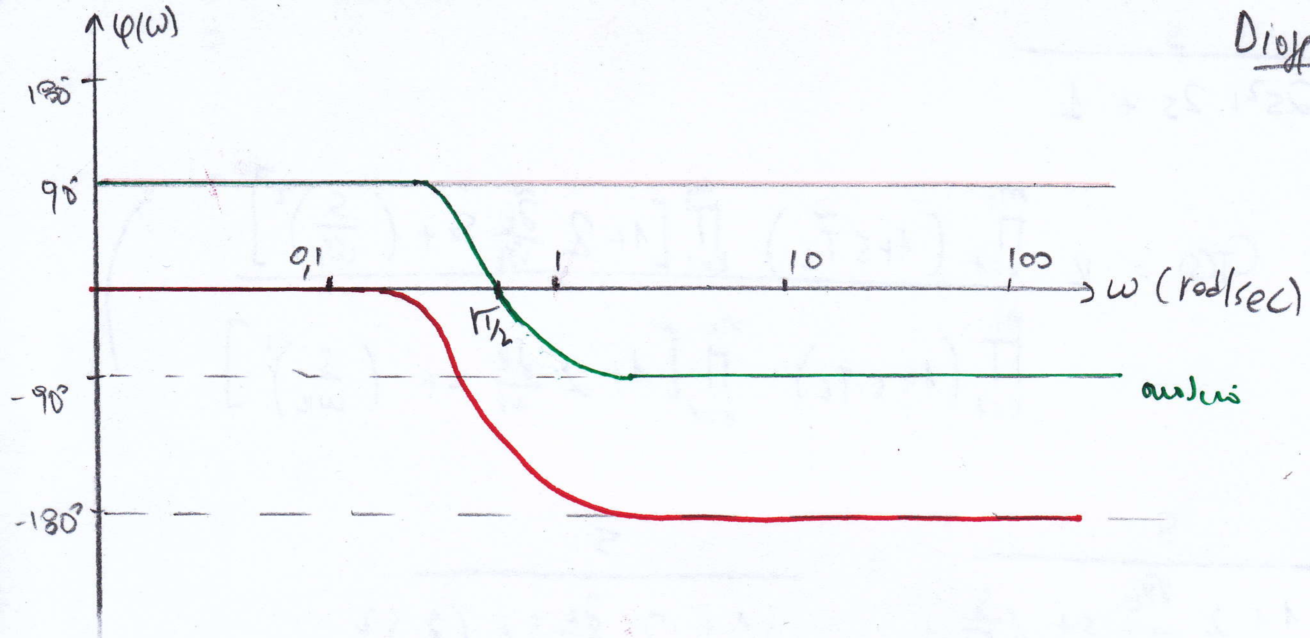
$K(\omega) = 20$ dB/dec, $\varphi(\omega) = 90$ (για όρο απλό s)

Παρονομαστής:

$\zeta = \sqrt{1/2} < 1 \Rightarrow$ [illegible], $\omega_0 = \omega_k = \sqrt{1/2}$ rad/sec (για όρο $1 + 2\frac{\zeta_k}{\omega_k} s + \left(\frac{s}{\omega_k}\right)^2$)

Διάγραμμα Κέρδους





Άσκηση 4

a) $G(s) = \dfrac{10s(40+2s)}{(1+0{,}1s)(1+0{,}4s)} = \dfrac{10s\,40(1+\frac{s}{20})}{(1+0{,}1s)(1+0{,}4s)} = \dfrac{400s(1+sT_1)}{(1+sT_2)(1+sT_3)}$

όπου $T_1 = \frac{1}{20}$, $T_2 = 0{,}1$, $T_3 = 0{,}4$

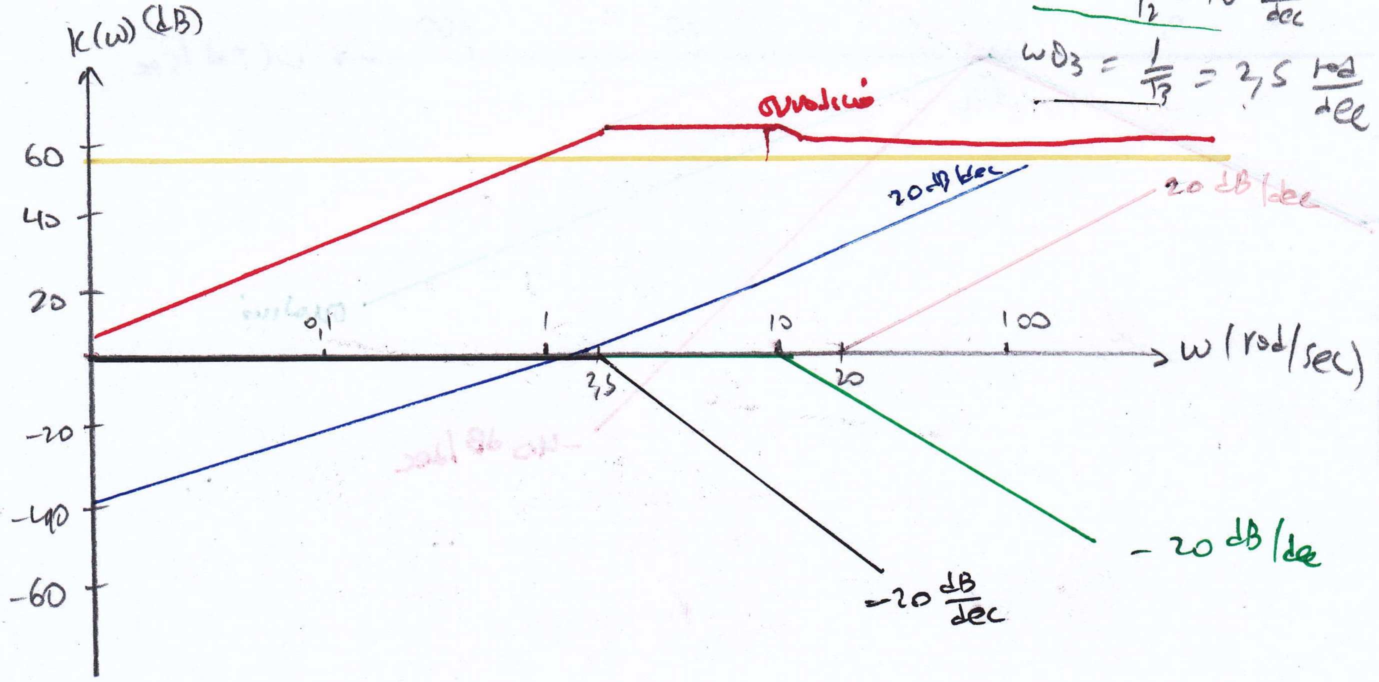
για κέρδος: $K(\omega) = 20\log 400 = 52{,}4$, $\varphi(\omega) = 0°$

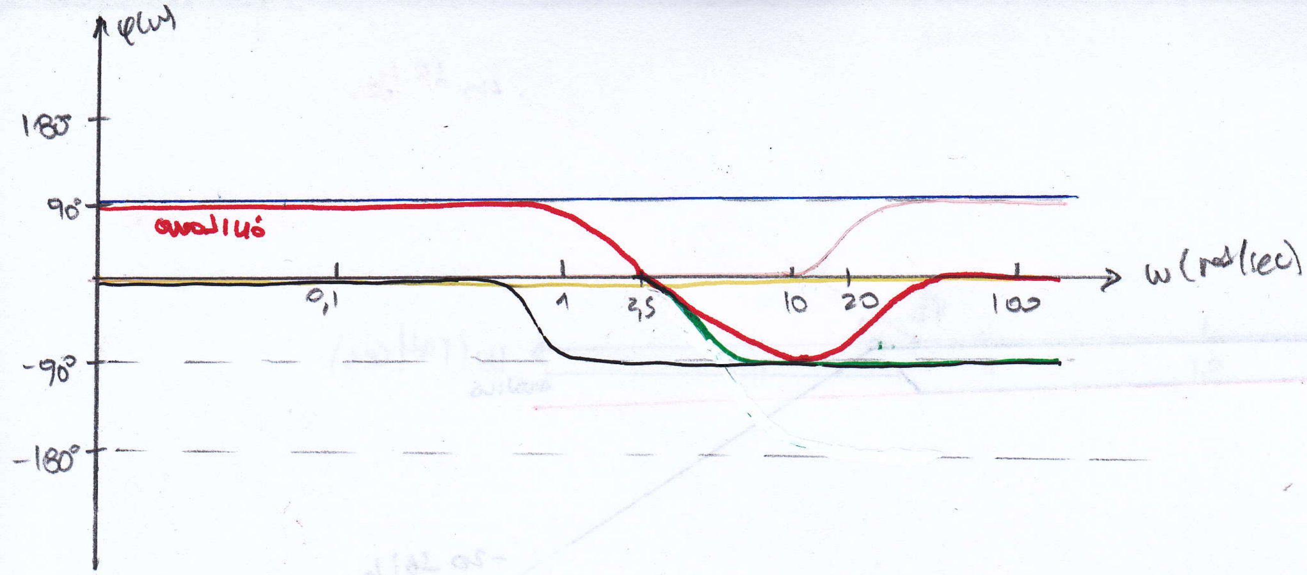
για παραγωγό: $K(\omega) = 20$ dB/dec, $\varphi(\omega) = 90°$

συχν. θλάσης: $\omega_{\theta 1} = \frac{1}{T_1} = 20 \frac{rad}{sec}$,

$\omega_{\theta 2} = \frac{1}{T_2} = 10 \frac{rad}{sec}$

$\omega_{\theta 3} = \frac{1}{T_3} = 2{,}5 \frac{rad}{sec}$





β) $G(s) = \dfrac{s^2+2s+2}{s^2+4s+4} = \dfrac{s^2+2s+2}{(s+2)(s+2)} = \dfrac{2\left(1+2\frac{\left(\frac{\sqrt{2}}{2}\right)s}{\sqrt{2}} + \left(\frac{s}{\sqrt{2}}\right)^2\right)}{4\left(1+\frac{s}{2}\right)\left(1+\frac{s}{2}\right)} =$

$= \dfrac{0,5\left(1+2\zeta\frac{s}{\omega_k}+\left(\frac{s}{\omega_k}\right)^2\right)}{(1+sT_1)(1+sT_2)}$

Αριθμητής:

$\zeta = \frac{\sqrt{2}}{2} < 1 \Rightarrow$ δέκτης

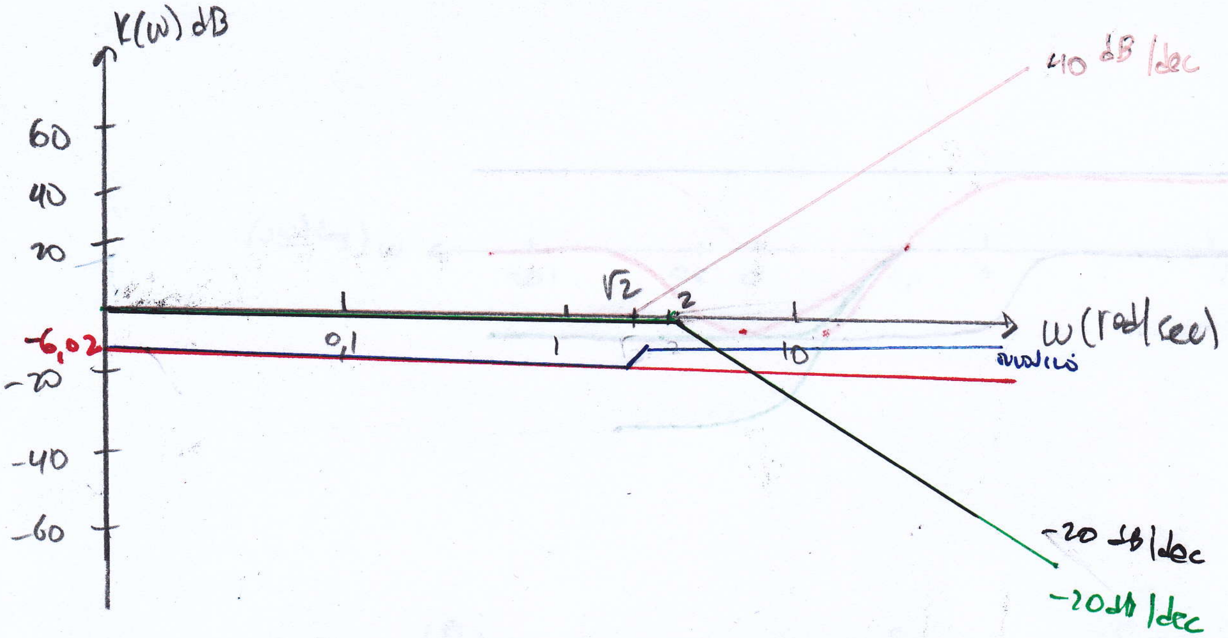
$\omega_{\theta 1} = \omega_k = \sqrt{2}\ \frac{rad}{sec}$

$K(\omega) = 20\log 0{,}5 = -6{,}02$, $\quad \varphi(\omega) = 0°\ (0{,}5>0)$

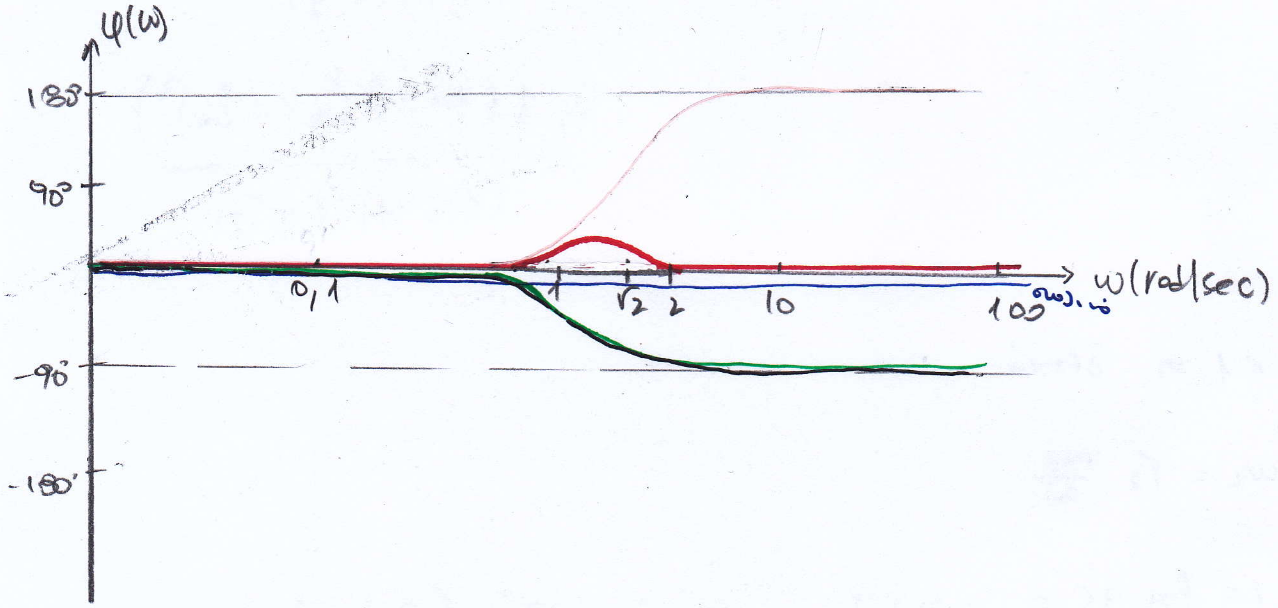
Παρονομαστής:

$T_1 = T_2 = 0{,}5$

συχν. θλάσης: $\omega_{\theta 2} = \omega_{\theta 3} = \frac{1}{T_1} = 2\ \frac{rad}{sec}$



K(ω) dB
60
40
20
-6,02
-20
-40
-60
0,1
1
√2
2
10
ω (rad/sec)
40 dB/dec
-20 dB/dec
-20dB/dec



φ(ω)
180°
90°
-90°
-180°
0,1
1
√2
2
10
100
ω (rad/sec)