Math 545 - A1 - Due Feb 11

Q1. $\{Z_t\}$. $Z_t \overset{iid}{\sim} (0, \sigma^2)$

a) $X_t = a + bZ_t + cZ_{t-2}$

b) $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$

c) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$

d) $X_t = a + bZ_0$

e) $X_t = Z_0 \cos(ct)$

f) $X_t = Z_t Z_{t-1}$

(a) stationary: $X_t = a + bZ_t + cZ_{t-2}$

$$E(X_t) = E[a + bZ_t + cZ_{t-2}] = a + bE[Z_t] + c \cdot E[Z_{t-2}]$$
$$= a + 0 + 0 = a \quad \forall t$$

$$Cov(X_t, X_{t+h}) = E(X_t \cdot X_{t+h}) = E[(a+bZ_t+cZ_{t-2}) \cdot (a+bZ_{t+h}+cZ_{t+h-2})]$$

for $h=0$:

$$Cov = E[(a+bZ_t+cZ_{t-2})^2]$$
$$= E[a^2 + 2abZ_t + 2acZ_{t-2} + b^2Z_t^2 + 2bcZ_tZ_{t-2} + c^2Z_{t-2}^2]$$
$$= a^2 + 0 + 0 + b^2\sigma^2 + 0 + c^2\sigma^2$$
$$= a^2 + b^2\sigma^2 + c^2\sigma^2$$

for $h=2$:

$$Cov = E[(a+bZ_t+cZ_{t-2})(a+bZ_{t+2}+cZ_t)]$$
$$= E[a^2 + abZ_t + abZ_{t+2} + acZ_{t-2} + b^2Z_tZ_{t+2} + bcZ_{t-2}Z_{t+2} + bcZ_t^2 + c^2Z_{t-2}Z_t]$$
$$= a^2 + 0 + 0 + 0 + 0 + 0 + bc\sigma^2 + 0$$
$$= a^2 + bc\sigma^2$$

for $|h| > 2$:

$$Cov = E[(a+bZ_t+cZ_{t-2}) \cdot (a+bZ_{t+h}+cZ_{t+h-2})]$$
$$= 0 \quad \text{as uncorrelated}, \; t \neq t-2 \neq t+h \neq t+h-2$$

$\therefore \gamma_X(t+h,t) = E(X_{t+h} \cdot X_t)$

$$= \begin{cases} a^2 + b^2\sigma^2 + c^2\sigma^2 & h=0 \\ a^2 + bc\sigma^2 & h=2 \\ 0 & |h|>2 \text{ or } |h|=1 \end{cases}$$

(b) $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$ Stationary

$E(X_t) = E[Z_1 \cos(ct)] + E[Z_2 \cdot \sin(ct)]$

$= E(Z_1)\, E[\cos(ct)] + E[Z_2] \cdot E(\sin(ct)]$ as uncorrelated

$= 0 \quad \forall t, \Rightarrow$ mean is 0.

$\mathrm{COV}(X_t, X_{t+h}) = E(X_t \cdot X_{t+h}) = E[(Z_1 \cos(ct) + Z_2 \sin(ct)) \cdot$

$Z_1 \cos(c(t+h)) + Z_2(\sin(c(t+h))]$

$= E[Z_1^2 \cos(ct) \cdot \cos(c(t+h)) + Z_1 Z_2 \cos(ct) \cdot \sin(c(t+h))$

$+ Z_2^2 \sin(ct)\cos(c(t+h)) + Z_1 Z_2 \sin(ct) \cdot \cos(c(t+h))]$

$= \cos(ct) \cdot \cos(c(t+h))\sigma^2 + \sigma^2 \sin(ct)\sin(c(t+h))$

$= \sigma^2 \cdot \cos(ct + ch - ct) = \sigma^2 \cdot \cos(ch)$

indep of t.

(c) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$. stationary $\Leftrightarrow$ $c = \pm k\pi$.

$E(X_t) = E(Z_t) \cdot \cos(ct) + E(Z_{t-1}) \sin(ct) = 0$

$\mathrm{Cov}(X_{t+h}, X_t) = \cos(c(t+h)) \cos(ct)\, \mathrm{Cov}(Z_{t+h}, Z_t)$

$+ \cos(c(t+h)) \sin(ct)\, \mathrm{COV}(Z_{t+h}, Z_{t-1})$

$+ \sin(c(t+h)) \cos(ct)\, \mathrm{Cov}(Z_{t+h-1}, Z_t)$

$+ \sin(c(t+h)) \sin(ct)\, \mathrm{Cov}(Z_{t+h-1}, Z_{t-1})$

$$= (\cos(c(t+h))\cos(ct)+\sin(c(t+h))\sin(ct))\sigma^2\delta(0)$$
$$+\cos(c(t+h))\sin(ct)\sigma^2\delta(h+1)+\sin(c(t+h))\cos(ct)\sigma^2\delta(h-1)$$

$$=\sigma^2(\cos^2(ct)+\sin^2(ct))\delta(h)$$
$$+\sigma^2\cos(c(t-1))\sin(ct)\delta(h+1)+\sigma^2\sin(c(t+1))\cos(ct)\delta(h-1)$$
$$=\sigma^2\delta(h)+\sigma^2\cos(c(t-1))\sin(ct)\delta(h+1)+\sigma^2\cos(ct)\sin(c(t+1))\delta(h-1)$$

if $c=k\pi,\ k\in\mathbb{Z}$

Then $\gamma_x(t+h,t)=\sigma^2\delta(h)$

which does not depends on t; but for all other values of t, $\gamma_x(t+h,t)$ depends on t.

Hence, X_t is stationary iff $c=\pm k\pi,\ k\in\mathbb{Z}$

(d) $X_t=a+bZ_0$. <u>stationary</u>

$E(X_t)=E(a+bZ_0)=a+b\cdot 0=a$

$Cov(X_t,X_{t+h})=Cov(X_t,X_t)=Var(X_t)=Var(a+bZ_0)$

$$=Var(bZ_0)=b^2\cdot Var(Z_0)$$
$$=b^2\cdot\sigma^2$$

e) $X_t = Z_0 \cos(ct)$

1°: $c = k\pi$ for some $k \in \mathbb{Z}$, then $X_t = (-1)^{kt} Z_0$.

$\gamma_X(t+h, t) = \mathrm{Cov}((-1)^{kt} Z_0, (-1)^{k(t+h)} Z_0)$
$= (-1)^{kt} \cdot (-1)^{k(t+h)} \mathrm{Cov}(Z_0, Z_0)$
$= (-1)^{kh} \cdot \sigma^2$

$E(X_t) = (-1)^{kt} \cdot E[Z_0] = 0$

$\therefore$ Stationary $\{X_t\}$ when $c = k\pi$

2°: $c \neq k\pi$

$E(X_t) = E(Z_0) \cdot \cos(ct)$

$\gamma_X(t+h, t) = \mathrm{Cov}(X_{t+h}, X_t) = \mathrm{Cov}(Z_0 \cos(c(t+h)), Z_0 \cos(ct))$
$= \cos(c(t+h)) \cos(ct) \cdot \mathrm{Cov}(Z_0, Z_0) = \cos(c(t+h)) \cos(ct) \sigma^2$

depends on t.

$\therefore$ $\{X_t\}$ not stationary

Hence $\{X_t\}$ is stationary iff $c = k\pi$ for some $k \in \mathbb{Z}$

f) $X_t = Z_t Z_{t-1}$

$E(X_t) = 0$ since uncorrelated

$|h|>1$: $Cov(X_t, X_{t+h}) = E(Z_t Z_{t-1} \cdot Z_{t+h} \cdot Z_{t+h-1})$

$= 0$ since uncorrelated

$h=0$: $Cov(X_t, X_t) = Var(X_t) = E(Z_t^2 Z_{t-1}^2) = \sigma^4$

$h=1$: $cov(X_t, X_{t+1}) = E(Z_t Z_{t-1} Z_{t+1} Z_t)$

$= E(Z_t^2) E(Z_{t-1}) E(Z_{t+1}) = 0$

$\therefore \ r_x(t, t+h) = \begin{cases} \sigma^4 & \text{if } h=0 \\ 0 & \text{o.w.} \end{cases}$

$\therefore$ X_t is stationary

Q2.

a) $X_t = a + bt + S_t + Y_t \qquad S_t = S_{t-4}$

let $L_t = \nabla_4 X_t = \nabla_4 (a + bt + S_t + Y_t)$

$= \nabla_4 (a) + \nabla_4 (bt) + \nabla_4 (S_t) + \nabla_4 (Y_t)$

$= (a-a) + (bt - b(t-4)) + (S_t - S_{t-4}) + (Y_t - Y_{t-4})$

$= 0 + 4b + 0 + Y_t - Y_{t-4} = 4b + Y_t - Y_{t-4}$

$E[L_t] = E[4b + Y_t - Y_{t-4}] = 4b$ = indep of t

$\gamma_x(t, t+h) = Cov[4b + Y_t - Y_{t-4}, 4b + Y_{t+h} - Y_{t+h-4}]$

$= E[(Y_t - Y_{t-4})(Y_{t+h} - Y_{t+h-4})]$

$= E[Y_{t+h} Y_t - Y_t \cdot Y_{t+h-4} - Y_{t-4} \cdot Y_{t+h} + Y_{t-4} \cdot Y_{t+h-4}]$

$h=0$: $= E[Y_t^2 - Y_t Y_{t-4} - Y_{t-4} Y_t + Y_{t-4} Y_{t-4}]$

$= \sigma^2 - 0 - 0 + \sigma^2 = 2\sigma^2$

$h=4$: $= E[Y_{t+4} Y_t - Y_t \cdot Y_t - Y_{t-4} Y_{t+4} + Y_{t-4} Y_t]$

$= 0 - \sigma^2 - 0 - 0 = -\sigma^2$

o.w.: $= Cov(Y_t, Y_{t+h}) - Cov(Y_{t-4}, Y_{t+h})$

$- Cov(Y_t, Y_{t+h-4}) + Cov(Y_{t-4}, Y_{t+h-4})$

$= \gamma_Y(t, t+h) - \gamma_Y(t-4, t+h) - \gamma_Y(t, t+h-4) + \gamma_Y(t-4, t+h-4)$

indep of t as Y_t is weakly stationary.

$\therefore$ Stationary as $E[\nabla_4 X_t]$ and $\gamma_X(t, t+h)$ are independent of t.

b) $X_t = (a+bt)S_t + Y_t$

Claim: ∇_4^2 is the choice.

1°: $\nabla_4 X_t = X_t - X_{t-4}$

$= \{(a+bt)S_t + Y_t\} - \{(a+b(t-4))S_{t-4} + Y_{t-4}\}$

$= aS_t + btS_t + Y_t - aS_{t-4} - btS_{t-4} + 4bS_{t-4} - Y_{t-4}$

$= a(S_t - S_{t-4}) + bt(S_t - S_{t-4}) + 4bS_{t-4} + (Y_t - Y_{t-4})$

$= 4bS_{t-4} + (Y_t - Y_{t-4})$

2°: $\nabla_4^2 X_t = \nabla_4(\nabla_4 X_t)$

$= \nabla_4 (4bS_{t-4} + Y_t - Y_{t-4})$

$= (4bS_{t-4} + Y_t - Y_{t-4}) - (4bS_{t-8} + Y_{t-4} - Y_{t-8})$

$= 4b(S_{t-4} - S_{t-8}) + Y_t - 2Y_{t-4} + Y_{t-8}$

$= Y_t - 2Y_{t-4} + Y_{t-8}$

$\because Y_t$ is weakly stationary

$\therefore \nabla_4^2 X_t$ is also weakly stationary since it's linear func of Y_t.

i.e. ∇_4^2 can transform X_t to a weakly stationary process.

Q3 MA(m). equal weight $\frac{1}{m+1}$, $X_t = \frac{1}{m+1}\sum_{k=0}^{m} Z_{t-k}$

$$E(X_t) = \frac{1}{m+1}\sum_{k=0}^{m} E(Z_{t-k}) = 0$$

$$\gamma_x(t+h,t) = Cov\left(\frac{1}{m+1}\sum_{k=0}^{m} Z_{t-k}, \frac{1}{m+1}\sum_{k=0}^{m} Z_{t+h-k}\right)$$

$$= \left(\frac{1}{m+1}\right)^2 \cdot Cov\left(\sum_{k=0}^{m} Z_{t-k}, \sum_{k=0}^{m} Z_{t+h-k}\right)$$

$$= \left(\frac{1}{m+1}\right)^2 \cdot E\left[\sum_{k=0}^{m} Z_{t-k} \cdot \sum_{k=0}^{m} Z_{t+h-k}\right]$$

$$= \left(\frac{1}{m+1}\right)^2 \cdot E\left[\sum_{k=0}^{m}\sum_{k=0}^{m} Z_{t-k} \cdot Z_{t+h-k}\right]$$

$h=0$:

$$= \left(\frac{1}{m+1}\right)^2 \cdot \sum_{k=0}^{m} Var(Z_{t-k}) = \frac{1}{m+1}\cdot\sigma^2$$

$m \geq k \geq 1$:

$$= COV\left(\sum_{k=0}^{m} \frac{Z_{t-k}}{m+1}, \sum_{k=0}^{m} \frac{Z_{t-k+h}}{m+1}\right)$$

$$= E\left[\left(\sum_{t=0}^{m-k} \frac{Z_{t-k}}{m+1}\right)\cdot\left(\sum_{h=0}^{m-k} \frac{Z_{t-k+h}}{m+1}\right)\right] = \sigma^2\cdot\left(\sum_{h=0}^{m-k}\frac{1}{(m+1)^2}\right)$$

$$= \sigma^2 \frac{m-k+1}{(m+1)^2}$$

$k \geq m$, $\gamma_x(t+h,t) = 0$

$k \leq 0$, $$\rho_x(k) = \frac{\left(\frac{1}{m+1}\right)^2 (m+k-1)\,\sigma^2}{\left(\frac{1}{m+1}\right)\sigma^2}$$

$$= \frac{1}{m+1}(m+k-1)$$

$$= \frac{1}{m+1}(m-(-k)-1)$$

$$= \rho_x(-k)$$

$$\rho_k = \frac{\gamma(k)}{\gamma(0)} = \begin{cases} \frac{\sigma^2(m-k+1)}{(m+1)^2} \Big/ \frac{\sigma^2(m+1)}{(m+1)^2} & : k=0,1,\cdots,m \\ 0 & : k \geq m \\ \rho(-k) & : k \leq 0 \end{cases}$$

$\longrightarrow$

To simplify

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = \begin{cases} \frac{m-k+1}{m+1} & k = 0, 1, \cdots, m \\ 0 & k > m \\ \rho(-k) & k \leq 0 \end{cases}$$

Q4

(a) $\hat{m}_t = \frac{1}{2q+1} \sum_{i=t-q}^{t+q} X_t \qquad X_t = m_t + Y_t$

$= \frac{1}{2q+1} \sum_{i=t-q}^{t+q} m_i + Y_i$

$= \frac{1}{2q+1} \sum_{i=t-q}^{t+q} m_i + \frac{1}{2q+1} \sum_{i=t-q}^{t+q} Y_i$

$E(\hat{m}_t) = \frac{1}{2q+1} \sum_{i=t-q}^{t+q} E(m_i) + \frac{1}{2q+1} \sum_{i=t-q}^{t+q} E(Y_i)$

$= \frac{1}{2q+1} \sum_{i=t-q}^{t+q} E(m_i) + 0$

$= \frac{1}{2q+1} \sum_{i=t-q}^{t+q} E(a+bi)$

$= a + \frac{b}{2q+1} \sum_{i=t-q}^{t+q} E(i) = a + \frac{b}{2q+1} \sum_{i=t-q}^{t+q} i$

$= a + b \cdot \frac{1}{2q+1} \cdot \frac{2t \cdot 2q+1}{2} = a + bt$

$V(\hat{m}_t) = V\left(\frac{1}{2q+1} \sum_{i=t-q}^{t+q} m_i\right) + V\left(\frac{1}{2q+1} \sum_{i=t-q}^{t+q} Y_i\right)$

$+ 2\mathrm{COV}\left(\frac{1}{2q+1} \sum_{i=t-q}^{t+q} m_i, \frac{1}{2q+1} \sum_{i=t-q}^{t+q} Y_i\right)$

indep

$= V\left(\frac{1}{2q+1} \sum_{i=t-q}^{t+q} m_i\right) + V\left(\frac{1}{2q+1} \sum_{i=t-q}^{t+q} Y_i\right)$

$= \left(\frac{1}{2q+1}\right)^2 \cdot \left(V\left(\sum_{i=t-q}^{t+q} m_i\right) + V\left(\sum_{i=t-q}^{t+q} Y_i\right)\right)$

$= \left(\frac{1}{2q+1}\right)^2 \cdot \left[\sum_{i=t-q}^{t+q} \left(V(m_i) + V(Y_i)\right)\right]$

$$= \left(\frac{1}{2q+1}\right)^2 \cdot \sum_{i=t-q}^{t+q} \left(V(a+bi) + \sigma^2 \right)$$

Y_t W.N.

$$= \left(\frac{1}{2q+1}\right)^2 \cdot \sum_{i=t-q}^{t+q} \left(\underbrace{b^2 V(i)}_{0} + \sigma^2 \right)$$

$$= \frac{1}{2q+1} \cdot \sigma^2$$

for comparison: $V(X_t) = V(a + bt + Y_t)$

$$= b^2 V(t) + V(Y_t) + 2\,\mathrm{Cov}(bt, Y_t)$$

$$= b^2 \cdot V(t) + \sigma^2 > V(\hat{m}_t)$$

unbiasedness:

since $E(\hat{m}_t) = a + bt = m_t$

$\therefore$ $\hat{m}_t$ is an unbiased estimator of m_t.

4(b)

$$E(\hat{m}_t) = E\left[\sum_{j=-\infty}^{\infty} a_j X_{t-j}\right] = E\left[\sum_{j=-\infty}^{\infty} a_j (m_{t-j} + Y_{t-j})\right]$$

$$= \sum_{j=-\infty}^{\infty} a_j m_{t-j} + \sum_{j=-\infty}^{\infty} a_j E[Y_{t-j}]$$

$$= \sum_{j=-\infty}^{\infty} a_j m_{t-j}$$

$\Leftrightarrow$ $\hat{m}_t$ is an unbiased estimator of m_t

$\Leftrightarrow$ $E(\hat{m}_t) = m_t$

$\Leftrightarrow$ $\sum_{j=-\infty}^{\infty} a_j m_{t-j} = m_t$

i.e. $\sum_{j=-\infty}^{\infty} \left\{ a_j (c_0 + c_1(t-j) + c_2(t-j)^2 + \cdots + c_k(t-j)^k) \right\}$

$= c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k, \quad \forall t \in \mathbb{N}$

$c_0 \sum_{j=-\infty}^{\infty} a_j + c_1 \sum_{j=-\infty}^{\infty} a_j (t-j) + \cdots + c_k \sum_{j=-\infty}^{\infty} a_j (t-j)^k$

$= c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k, \quad \forall t \in \mathbb{N}$

$$\Leftrightarrow \begin{cases} \sum_{j=-\infty}^{\infty} a_j = 1 \\ \sum_{j=-\infty}^{\infty} a_j (t-j) = t \\ \sum_{j=-\infty}^{\infty} a_j (t-j)^2 = t^2 \\ \vdots \\ \sum_{j=-\infty}^{\infty} a_j (t-j)^k = t^k \end{cases}$$

$$\Leftrightarrow \begin{cases} \sum_{j=-\infty}^{\infty} a_j = 1 \\ \sum_{j=-\infty}^{\infty} (a_j t - a_j j) = t \\ \sum_{j=-\infty}^{\infty} (a_j t^2 + \binom{2}{1} a_j t(-j) + a_j j^2) = t^2 \\ \vdots \\ \sum_{j=-\infty}^{\infty} \left(a_j t^k + \binom{k}{1} a_j t^{k-1} (-j)^1 + \cdots + a_j (-j)^k \right) = t^k \end{cases}$$

$\Leftrightarrow$ Replace all $\sum_{j=-\infty}^{\infty} a_j$ w/ 1, then we get

$$\begin{cases} \sum_{j=-\infty}^{\infty} a_j = 1 \\ t - \sum_{j=-\infty}^{\infty} a_j j = t \\ t^2 + \sum_{j=-\infty}^{\infty} \left(\binom{2}{1} a_j t(-j) + a_j j^2 \right) = t^2 \\ \vdots \\ t^k + \sum_{j=-\infty}^{\infty} \left(\binom{k}{1} a_j t^{k-1} (-j)^1 + \binom{k}{2} a_j t^{k-2} (-j)^2 \cdots + a_j (-j)^k \right) = t^k \end{cases}$$

$$\Leftrightarrow \begin{cases} \sum_{j=-\infty}^{\infty} a_j = 1 & (1) \\ \sum_{j=-\infty}^{\infty} a_j j = 0 & (2) \\ \sum_{j=-\infty}^{\infty} \left(\binom{2}{1} a_j t(-j)^1 + a_j j^2 \right) = 0 & (3) \\ \vdots \\ \sum_{j=-\infty}^{\infty} \left(\binom{k}{1} a_j t^{k-1} (-j)^1 + \binom{k}{2} a_j t^{k-2} (-j)^2 \cdots + a_j (-j)^k \right) = 0 \end{cases}$$

We then use an iterative method.

$\Longleftarrow$ Starting state:

③ - ②:

$$\binom{2}{1} t \cdot \sum_{j=-\infty}^{\infty} a_j (-j) + \sum_{j=-\infty}^{\infty} a_j j^2 - \sum_{j=-\infty}^{\infty} a_j j = 0 - 0 = 0$$

$$\therefore \sum_{j=-\infty}^{\infty} a_j j^2 = 0$$

Similarly ④ - ③:

$$\binom{3}{1} t^2 \cdot \sum_{t=-\infty}^{\infty} a_j (-j)^1 + \binom{3}{2} t \sum_{j=-\infty}^{\infty} a_j (-j)^2 + \sum_{j=-\infty}^{\infty} a_j j^3 - \binom{2}{1} t \sum_{j=-\infty}^{\infty} a_j (-j)$$

$$- \sum_{j=-\infty}^{\infty} a_j j^2 = 0 - 0 = 0$$

$$\therefore \sum_{j=-\infty}^{\infty} a_j j^3 = 0$$

Iteratively, up to $(k+1)$ - (k), we may have:

$$\sum_{j=-\infty}^{\infty} a_j j^k = 0 \qquad ④$$

Hence, $\hat{m}_t$ is unbiased estimator of m_t

iff $\begin{cases} \sum_j a_j = 1 & (\text{by } ①) \\ \sum_j a_j j^r = 0 & (\text{by } ④) \end{cases}$

$\square$

MATH545-A1-Q5

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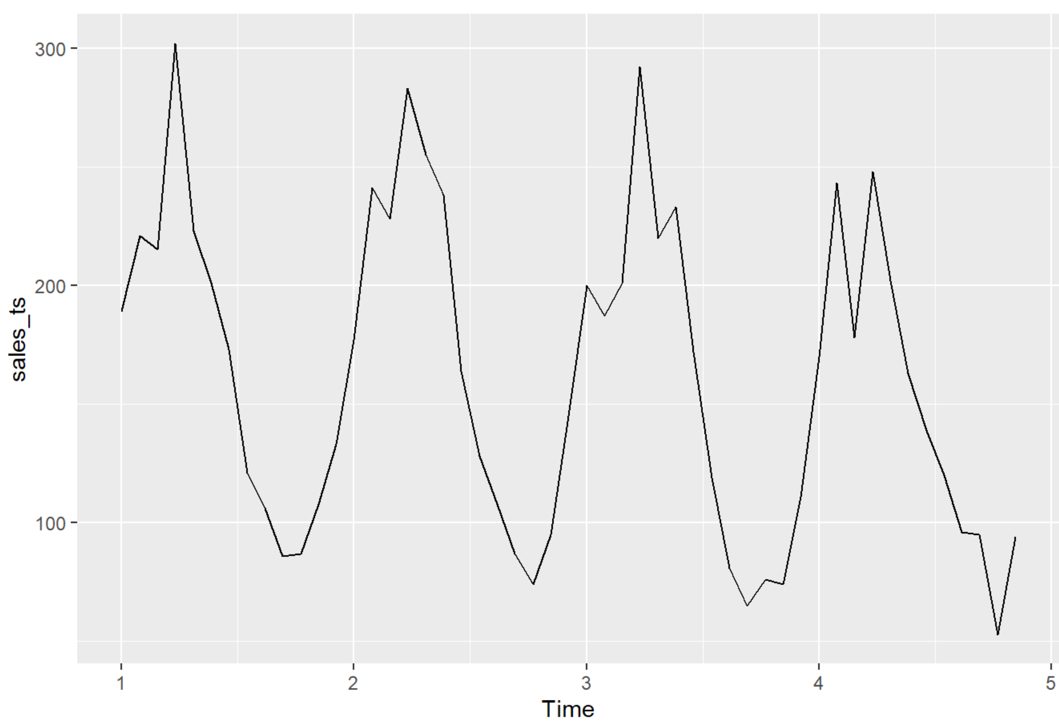
30/01/2020

```
library(readxl) # You may need to install this package first
library(tidyverse)
library(fpp2)
library(knitr)
library(tsbox)
library(gridExtra)
library(tibbletime)
sales_data <- read_excel("Assign1Q5_sales.xlsx")
library(forecast) # You may need to install this package first
sales_ts <- ts(sales_data, frequency=13) # Because there are
# 13 4-week periods per year
#length(sales_ts[1])
```

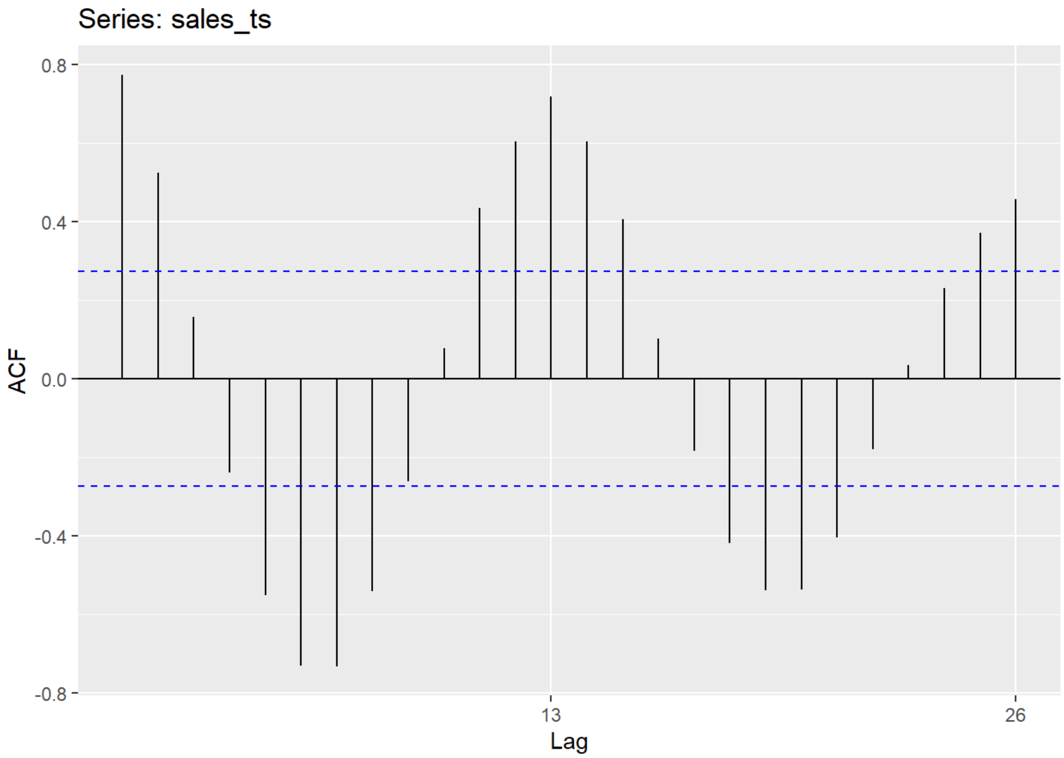
(a) First plot and describe the time series. Note any perceived trend and seasonal components. Do you believe that the sales data series is a stationary series? Explain your answer. Hint: You may want to use an ACF plot.

```
autoplot(sales_ts,facets=TRUE)
```



This time series has a slight decreasing trend, which might be linear, and it also has a fairly strong seasonality whose period is approximately of one unit of the time. In terms of values, this time series ranges from 0 to 300 and is dreasing slowly.

```
ggAcf(sales_ts)
```



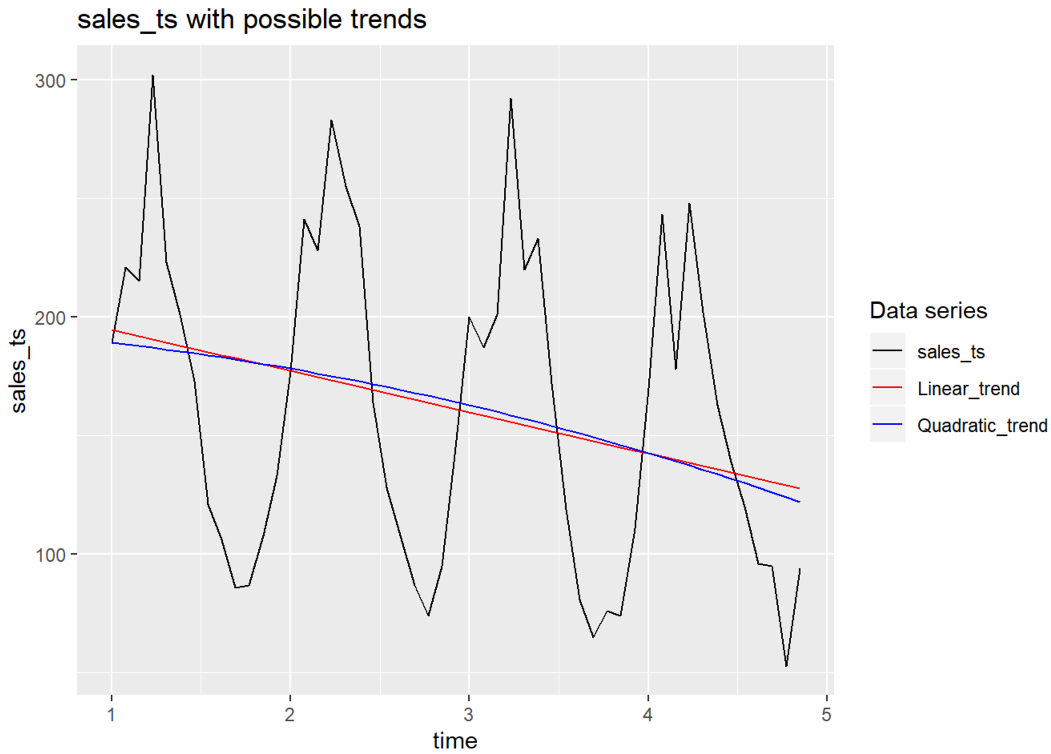
No, it is non-stationary. For a stationary series, we would ultimately expect to see autocorrelations to decay to zero at higher lags (although that is not enough to indicate stationarity). This does not seem to be the case here.

(b) Estimate trend and seasonal components for the time series. Do you find evidence of a trend and seasonal component in the data? Explain. Assess the residuals from your decomposition for evidence that they are resulting from a white noise or iid noise process.

(Compute the trend)

```
sales_linear <- tslm(sales_ts~trend) ## Fit linear trend

sales_quad<- tslm(sales_ts~trend + I(trend^2)) ## Fit linear trend
sales_with_fits<-cbind(sales_ts,
  Linear_trend = fitted(sales_linear),
  Quadratic_trend = fitted(sales_quad))
autoplot(sales_with_fits)+
  ylab("sales_ts") +
  ggtitle("sales_ts with possible trends") + xlab("time") +
  guides(colour=guide_legend(title="Data series"))+
  scale_colour_manual(values=c("black","red","blue"))
```

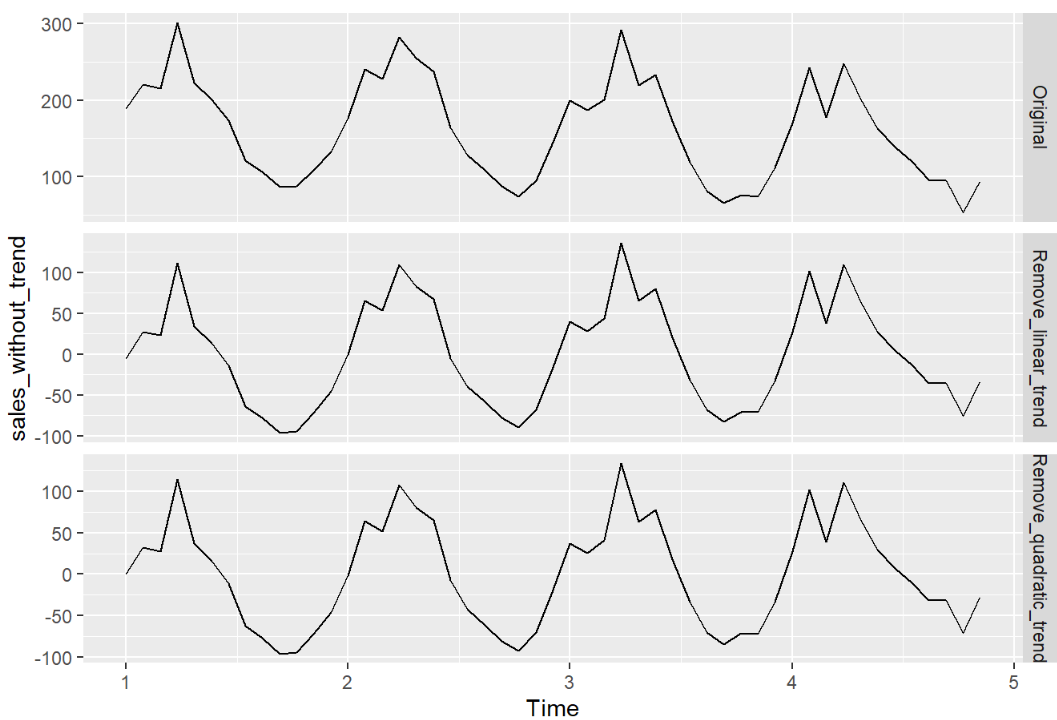


There is a strong indication for a trend component which can be linear or quadratic based on the plot above. The two types of trends have very similar behaviours.

(Remove trend)

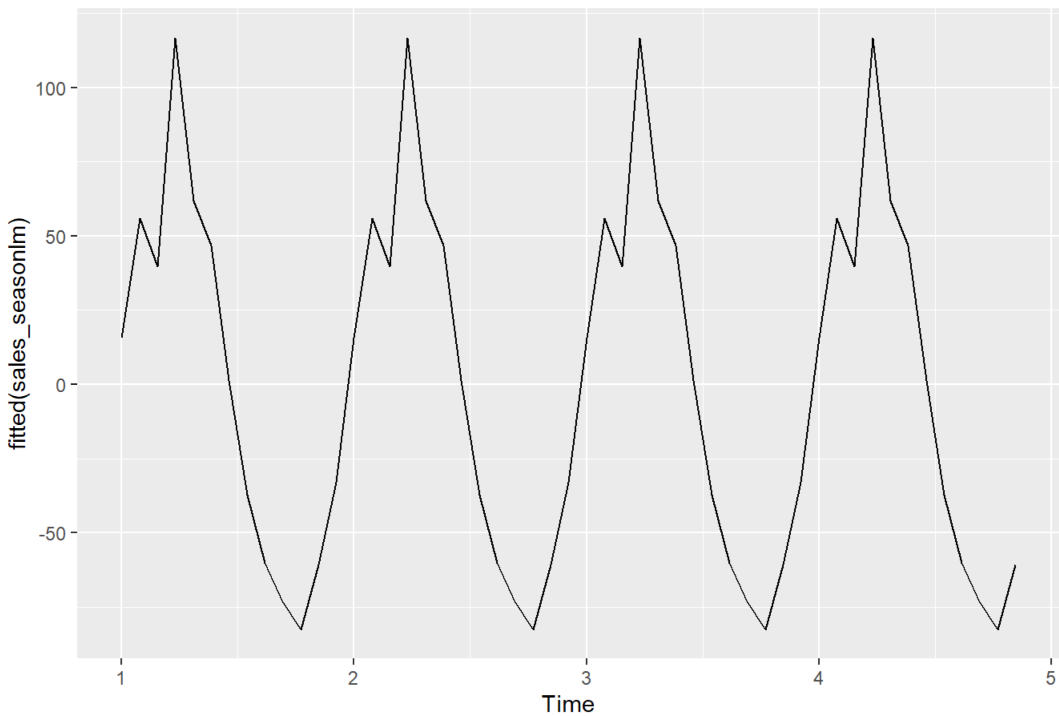
```
sales_without_trend <- cbind(
  Original = sales_with_fits[,"sales_ts"],
  Remove_linear_trend=sales_ts - sales_with_fits[,"Linear_trend"],
  Remove_quadratic_trend=sales_ts - sales_with_fits[,"Quadratic_trend"])

autoplot(sales_without_trend,facet=TRUE)
```



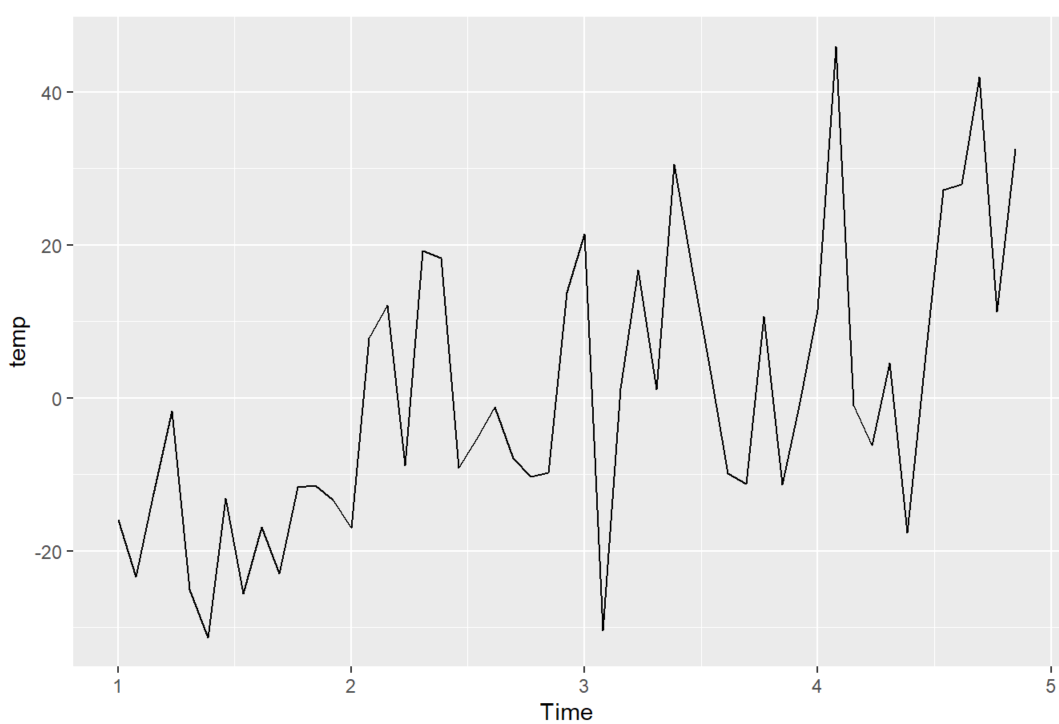
Estimate seasonality

```r
#frequency(sales_without_trend[,"Remove_linear_trend"])
sales_seasonlm <- tslm(Remove_quadratic_trend~season, data = sales_without_trend)
autoplot(fitted(sales_seasonlm))
```



There is also a quite strong seasonality as shown above.

```r
temp <- sales_ts - sales_with_fits[,"Quadratic_trend"] - fitted(sales_seasonlm)
autoplot(temp)
```

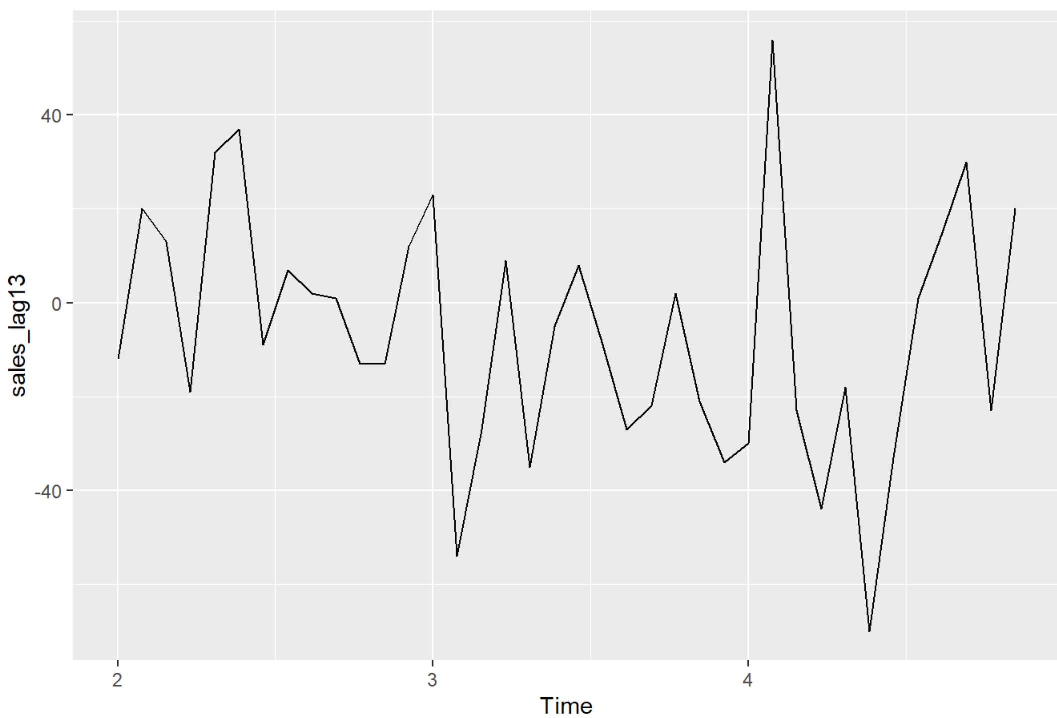


The aforeshown residual plot represents the noise component for the sales time series. As we can see that the major weight centers around 0 with random fluctuations and variations. We can safely assume that this results from a white noise or iid noise process.

(c) Using an appropriate sequence of difference operators, try to eliminate any perceived trend and seasonal components from part (c). Assess the residuals from your decomposition for evidence that they are resulting from a white noise or iid noise process.

(using differences)

```
sales_lag13 <- diff(sales_ts,3)
sales_lag13 <- diff(sales_ts,13)
autoplot(sales_lag13)
```



By first differencing by 3, we remove the linear trend. Then we again difference by 13 to remove the seasonality. From the results above, we can tell that the remaining noise is whitle noise or iid noise.