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Mathematical Modeling of Figure Skating Jumps

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Abstract. In this paper the mathematical model of body movement during a figure skating jump is developed. The goal of this project is to investigate how body and hands movements in the takeoff and flight phases of the jump influences jump parameters (angular velocity, number of revolutions in the air). The research studied three phases of the figure skating jump: the preparation, the entrance with the takeoff and the flight. A motion capture system was used to obtain experimental data on hands and body movement.

1. INTRODUCTION

Jumps are among the most important elements in figure skating. Execution of every jump depends on a great number of different parameters, which an athlete must control. That is why even for experienced coaches it is often difficult to define characteristics that are crucial for the successful jump landing. Nowadays researchers use biomechanics for analyzing jump technique, improving athletic performance and minimizing the probability of injuries [1].

A figure skating jump is usually divided into several phases: preparation for the jump, the entrance phase, which ends with the takeoff, the flight phase, and the landing phase. To prepare for a jump the skater usually slides on an edge of one skate in a fixed position to stabilize the body posture and normalize the speed. During the entrance and takeoff phases the skater pushes off the ice and starts rotating in the air, using coordinated movements of the takeoff leg, the free leg, the torso and arms. In the flight phase the skater moves arms and the free leg close to the body to rotate faster and perform the required number of revolutions in the air (from 1 to 4). The jump ends with the landing phase, which starts when the blade touches the ice and continues with the athlete skating backwards on the outside edge with one leg behind in the air.

The most popular method for biomechanical analysis of figure skating jumps is experimental investigation by using video analysis and different motion capture systems [2, 3]. The researcher who made the most significant contribution to the field is Deborah King [4–6]. She investigates vertical and horizontal takeoff velocity, time in the air, rotational velocity and other parameters for different types of jumps in single and pair skating. Other studies use computer simulation of body movement in the air to calculate and optimize the moment of inertia during the flight [7–9]. Another important question is prevention of injuries caused by leg's joint overload upon landing from multiple revolution jumps. To study joint reactions during the takeoff and landing of a jump multi-segment models of the athlete's body are usually used [10–13].

Although mathematical modeling of figure skating elements can be a powerful tool, this field remains underdeveloped. Famous coach Alexey Mishin was the first to propose simple mathematical models of such figure skating elements as a glide on a curve, an upright spin, a death spiral [14]. In 2013 V. Vinogradova published a study [15], where she described a number of models for a glide on a curve, creating an initial angular momentum and spinning in the air during the jump. Present work builds on the above-mentioned works to create a more detailed model of a figure skating jump.

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2. METHODS

Studying all the phases of the figure skating jump is a very difficult task. That is why researches usually focus on one phase: the flight [7, 8], the takeoff [5, 10] or the landing [12, 13]. The aim of the present research is to create a model that includes several phases of the jump (the preparation, entrance, takeoff and flight) and explains how movements of the torso and arms affect execution of the jump.

To describe skater's motion during execution of a jump a model of skate's body was created. This model consists of several geometrical objects: a sphere for the head, truncated cones for the torso and legs, two rods for arms and a cone for skates with feet (Figure 1a).

Now let us consider the first phase of the jump – preparation. The basic motion in figure skating is a glide on a curve on one foot. Leaning into the circle makes the skate glide on the outside or inside edge of the blade. If the skater does not change body position, his skate draws on the ice a circular arc, which radius depends on skater's body lean and speed. The glade is used as a basic motion for steps and as the preparation for jumps and spins.

To create a model it is assumed that the skater moves in a circle around the fixed axis *Oz* (Fig. 1b). The axis of symmetry of the skate's body is located in the *yOz* plane, and there is an angle α between the body axis and vertical axis *Oz* (Fig. 1b). Application of D'Alembert's principle of inertial forces for this motion gives the following system:

$$\begin{cases} N_x + m x_C \omega^2 + m y_C \varepsilon = 0, \\ N_y + m y_C \omega^2 - m x_C \varepsilon = 0, \\ -mg + N_z = 0, \\ -mg y_C + N_z \rho - J_{yz}\omega^2 + J_{zx}\varepsilon = 0, \\ J_{zx}\omega^2 + J_{yz}\varepsilon = 0, \\ N_x \rho - J_z \varepsilon = 0, \end{cases} \tag{1}$$

where N_x, N_y and N_z are components of the ground reaction force; *m* is the body mass; x_C and y_C are coordinates of the center of mass; ω and ε are the angular velocity and acceleration; J_z, J_{yz} and J_{zx} are moments of inertia about different axes; $\rho = OA$ is the radius of the circle and g is the gravitational acceleration.

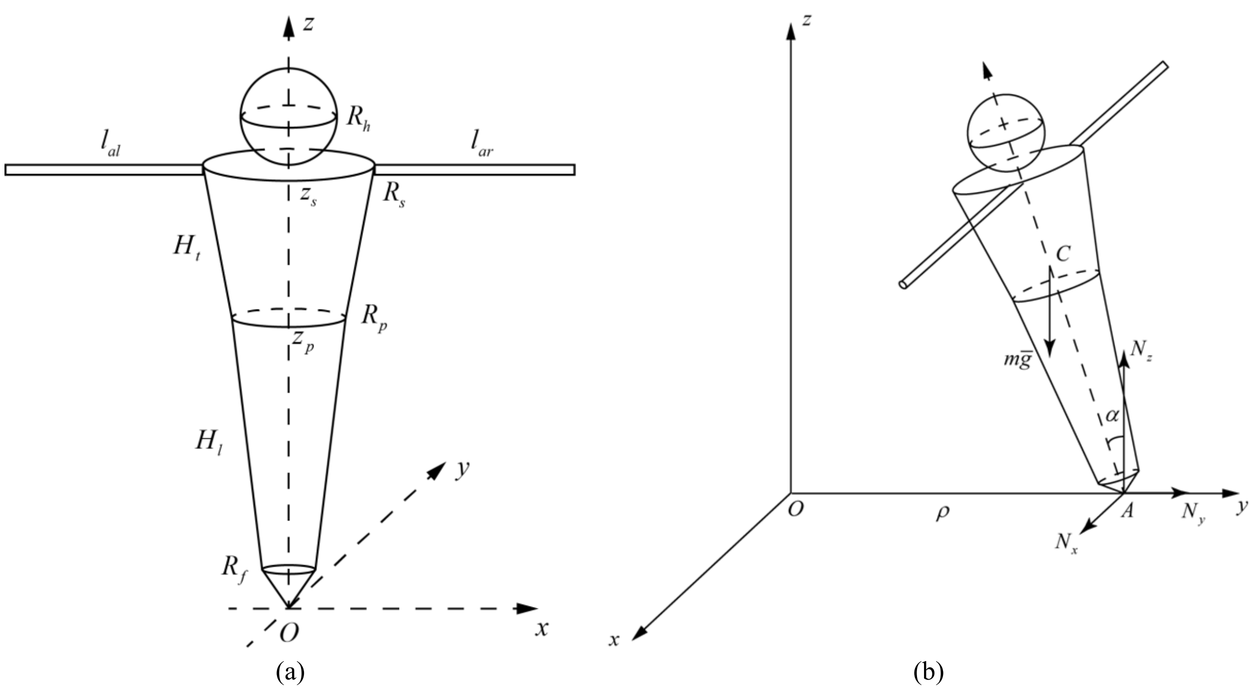

FIGURE 1. Scheme of an athlete's body (a) and a skater, gliding on a curve (b)

Given that the body axis of symmetry is located in the *yOz* plane, moments of inertia are:

$$J_{zx} = 0,\ J_{yz} = \frac{\sin 2\alpha}{2}\left(J_{y_C} - J_{z_C}\right) + m y_C z_C, \tag{2}$$

where J_{y_C} and J_{z_C} are moments of inertia about the axes trough the skater's center of mass. Substitution in eq. (1) yields the following relation between the angular velocity, the radius and the center of mass position:

$$\omega = \sqrt{\frac{mg\left(\rho - y_C\right)}{J_{yz}}}. \tag{3}$$

Coordinates of body's center of mass and its moments of inertia are defined by the lean angle α, masses and sizes of athlete's body parts. Equation (3) shows that the angular velocity grows with increase of the lean angle and decrease of the circle radius. It should be noted that similar models were created earlier [14, 15]. A. Mishin represented a skate's body as a single rod [14]. V. Vinogradova [15] proposed models for different body configurations, but for each configuration it was necessary to calculate the resultant force of inertia, which for more detailed body models could be difficult. The proposed here approach allows to easily change body model, in this case only coordinates of the center of mass and body's moments of inertia will change.

The main characteristic of every figure skating jump is the number of turns the skate completes in the air. To rotate faster an athlete moves arms and legs close to the axis of rotation. Assuming that the axis of rotation crosses the skater's center of mass, the athlete's rotation in the air is defined by the law of conservation of angular momentum:

$$\frac{d}{dt}\left(\omega_f(t) J_{zf}(t)\right) = 0 \Leftrightarrow \omega_f(t) = \frac{L_{z0}}{J_{zf}(t)}, \tag{4}$$

where $\omega_f(t)$ and $J_{zf}(t)$ are the angular velocity and the moment of inertia during the flight phase, L_{z0} is the initial angular momentum, generated in the entrance and the takeoff phases.

For the present study it is assumed that the skater rotates in the air about the central vertical axis and that he reduces the moment of inertia only by drawing closer his arms. Thus the moment of inertia is:

$$J_{zf} = \frac{2}{5} m_h R_h^2 + \frac{3}{10} m_t \frac{R_s^5 - R_p^5}{R_s^3 - R_p^3} + \frac{3}{10} m_l \frac{R_p^5 - R_f^5}{R_p^3 - R_f^3} + \frac{3}{10} m_f R_f^2 + J_{za}(t), \tag{5}$$

where $J_{za}(t)$ is the moment of inertia of arms, m_h, m_t, m_l, m_f, R_h, R_s, R_p, R_f are masses and sizes of body parts (see Fig. 1a). To describe how the moment of inertia of arms changes with time the position of arms during the flight phase was determined using a motion capture system.

As indicated in eq. (4), the angular velocity in the air also depends on the initial angular momentum. There are several ways to generate it: by gliding on a curve while preparing for the jump, by twisting the upper body during the entrance and takeoff phases and by skidding or using the toe pick just before the take-off [16]. Each type of jumps requires two or more mechanisms, but twisting the upper body is considered to be the main mechanism for the majority of jumps [16]. Gliding on a curve is also very important for the Salchow and the loop. The present study considers both mechanisms.

In the preparation and entrance phases the skater twists his upper body and then just before the takeoff rapidly untwists it to create the initial angular momentum. To calculate this momentum in the present study it is assumed that the angular velocity of torso's twisting changes linearly from zero at the pelvis to $\omega_s(t)$ at the shoulders. It's also assumed, that the head and arms are rotating with the same angular velocity as shoulders and that legs and feet don't rotate. The torso is divided into thin cylindrical plates of equal height. After calculating the angular momentum for every plate and integrating, the angular momentum for entire body is:

$$L_{zt}(t) = \left(\frac{m_t}{20}\frac{R_p^4 + 2R_p^3 R_s + 3R_p^2 R_s^2 + 4R_p R_s^3 + 5R_s^4}{R_p^2 + R_p R_s + R_s^2} + \frac{2m_h R_h^2}{5} + 2m_a\left(R_s^2 + R_s l_a + \frac{l_a^2}{3}\right)\right)\omega_s(t). \tag{6}$$

The resultant initial angular momentum takes into account the angular momentum, generated by rotating around the fixed axis, and the angular momentum, represented by eq. (6).

To find out how arms and shoulders move while the skater executes the jump the experimental results are necessary. For that purpose motion caption systems are usually used. In this research to obtain the above-mentioned data an off-ice tour jump was filmed with a camera, fixed above the athlete. The athlete was wearing markers on shoulders, elbows and wrists, coordinates of these markers were computed [17]. Since the tour jump is widely used for off-ice training and utilizes the same arms and torso movements as all the figure skating jumps, kinematic characteristics of this jump can be applied for the preliminary study of on-ice jumps.

3. RESULTS AND DISCUSSION

Figure 2 shows the data obtained from the experiment with an amateur figure skater, who executed 11 tour jumps. It is demonstrated that the skater uses the same movement pattern and that at the takeoff the angular velocity of shoulders is almost constant for different tries (Fig. 2a). The skater's moment of inertia does not change much in the flight phase (Fig. 2b) since the skater was asked to move arms only in a horizontal plane and could not achieve tight body position. This restriction was imposed due to the use of 2d video analysis.

Table 1 demonstrates the number of revolutions in the air calculated using presented here mathematical model and experimental results. Values of the horizontal and vertical velocity at the takeoff are taken from experiments, reported by King et al [4, 5] for the single Axel jump. To successfully execute this jump, a skater should rotate in the air 1.5±0.25 revolutions. If the jump is under-rotated by more than 1/4 revolution; it receives a diminished score and if the jump is over-rotated, it is difficult for a skater to land it safely. Thus the presented results for medium velocities give the expected amount of revolution, but under-rotations and over-rotations are also observed (table 1). It is interesting to note that the rise of number of revolutions can be achieved by different combinations of parameters.

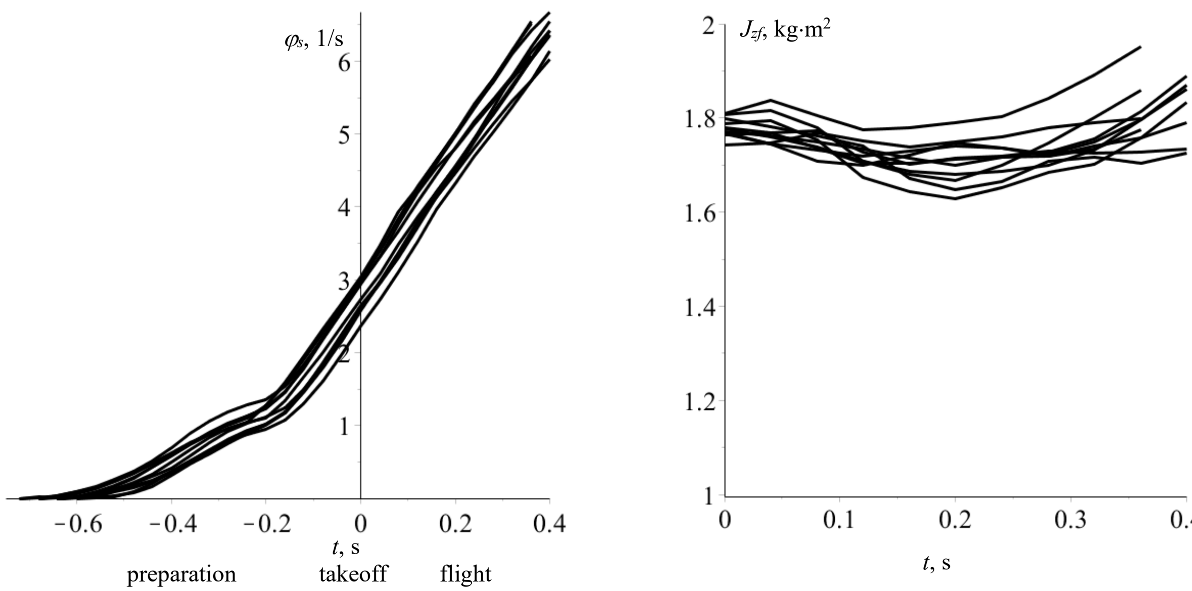

FIGURE 2. Experimental data for shoulders angle during the preparation and flight phases (a) and the moment of inertia in the flight phase (b) for the tour jump

TABLE 1. Number of revolutions in the air for different entrance velocities and trajectory radiuses.

Horizontal velocity, m/s	Vertical velocity, m/s	Number of revolutions radius 1.00 m	Number of revolutions radius 0.75 m	Number of revolutions radius 0.50 m
4.20	1.22	1.21	1.09	1.15
4.20	1.84	1.82	1.64	1.73
4.20	2.46	2.43	2.20	2.32
3.60	1.84	1.78	1.57	1.61
4.80	1.84	1.87	1.74	1.87

Although these results have a preliminary character, the proposed mathematical model can be improved by incorporating data of the 3d-video analysis, detailing body model and taking into consideration the third mechanism of obtaining the initial angular momentum. The presented approach to mathematical modeling of figure skating jumps can be used to analyze and develop an individual jumping technique. In order to perform that kind of analysis the execution of the jump by the skater should be recoded and the position of arms and shoulders during the preparation and flight phase should be determined. Then by applying a personalized model (with the skate's measures and typical movement pattern) the number of revolutions for different parameters (the vertical and horizontal velocity, the lean angle, entrance trajectory radius) can be calculated and the best training strategy can be chosen.

REFERENCES

1. L. Cabell and E. Bateman, “Biomechanics in figure skating”, in *The Science of Figure Skating*, ed. by J.D. Vescovi and J.L. VanHeest (Routledge, New York, 2018), pp. 20–35.
2. A. Mazurkiewicz, D. Iwańska and C. Urbanik, Biomechanics of the Axel Paulsen Figure Skating Jump, Pol. J. Sport Tourism **25**, 3–9 (2018).
3. K. Schafer, N. Brown and W. Alt, “MISSLE – a New Method to Analyse Performance Parameters of Figure Skating Jumps”, in *34th International Conference on Biomechanics in Sports* (Tsukuba, Japan, 2016), pp. 101–104.
4. D.L. King, A.S. Arnold and S.L. Smith, A kinematic comparison of single, double, and triple axels, J. of Appl. Biomechan. **10(1)**, 51–60 (2016).
5. D.L. King, “Generation of Vertical Velocity in Toe-Pick Figure Skating Jumps”, in *19 Int. Symp. on Biomechan. in Sports*, ISBS Proc. (San Francisco, United States, 2001), pp. 66–69.
6. D.L. King, Performing triple & quadruple figure skating jumps: implications for training, Canad. J. of Exercise Physiol. **30(6)**, 743–753 (2005).
7. K. Knoll and F. Hildebrand, “Effective Use of Angular Momentum for Rotations about the Longitudinal Axis – Examples of Quadruple Jumps in Figure Skating”, in *37th Int. Society of Biomechan. in Sport Conf.*, ISBS Proc. Archive 37(1) (United States, Oxford, OH, 2019), pp. 376–379.
8. K. Knoll and T. Hartel, “Biomechanical Conditions for Stabilizing Quadruple Figure Skating Jumps as a Process of Optimization”, in *23 Int. Symp. on Biomechan. in Sports*, ISBS Proc. (Beijing, China, 2005), pp. 134–137.
9. T. Kepple, Application of Induced Acceleration Analysis and Computer Simulation in Sports, Portuguese J. of Sport Sci. **11(3)**, 19–22 (2011).
10. M. Haguenauer, P. Legreneur and K.M. Monteil, Influence of Figure Skating Skates on Vertical Jumping Performance, J. of Biomechan. **39**, 699–707 (2006).
11. M.E. Kho, *Bone-on-Bone Forces at the Ankle and Knee in Figure Skaters during Loop Jumps*: *Clinical Implications* (Master of Science thesis, University of Waterloo, Canada, 1996).
12. K.L. Lockwood, J.P. Baudin and P.L. Gervais, “Kinematic characteristics of impact absorption during landings of multi-revolution jumps in figure skating,” *13 Int. Symp. on Biomechan. in Sports*, ISBS Proc. (Thunder Bay, Ontario, Canada, 1995), pp. 236–239.
13. K.L. Lockwood, P.L. Gervais and D.R. Mccreary, Landing for success: A biomechanical and perceptual analysis of on-ice jumps in figure skating, Sport Biomechan. **5(2)**, 222–237 (2006).
14. A.N. Mishin, *Biomechanics of Figure Skater's Movements* (Fizkultura i sport, Moscow, 1981), 144 p.
15. V.I. Vinogradova, *Fundamentals of Biomechanics of Jumps in Figure Skating* (Sovetsky sport, Moscow, 2013), 216 p.
16. A.N. Mishin, *Figure Skating Jumps*. (Fizkultura i sport, Moscow, 1976), 104 p.
17. E.A. Liadova, N.S. Shabrykina and S.D. Anferov, “Mathematical Modeling of Human Movement while Performing Jumps in Figure Skating on the Example of a Tour Jump,” in XXVI Russ. School-Conf. for Young Scientists and Students *“Mathematical Modeling in Natural Sciences”*, MMNS Conf. Proc. (Perm National Research Polytechnic University, Perm, Russia, 2017), pp. 48–52.