# Computational Astrophysics - Assignment 1

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October 2, 2019

## 1 Introduction

For this first assignment for the class named 'Computational Astrophysics' we were asked to investigate gravitational braids. As mentioned in the description of the assignment, it wasn't until Moore (1993), Montgomery (1998) that a stable solution was found for N-body simulations containing three bodies. More recently, Li et al. (2017) presented a paper in which they claim to have found 1223 new periodic orbits for the planar three-body problem with unequal mass and zero angular momentum. Our task was to reproduce one of the braids from this paper and to determine whether or not this braid is really stable. In the method section we will give a short overview of the way we used the AMUSE package Portegies Zwart et al. (2019) to reproduce the braids, then we will present our results followed by a short analysis and conclusion.

## 2 Method

In Li et al. (2017) four parameters were used in order to determine the initial conditions, namely: $(x_1, x_2, v_1, v_2)$. Furthermore they also altered the mass of one of the three bodies. If we number the three bodies from 0 to 2, then the initial conditions are given (in N-body units where $G = 1$) and can be found in Table 1.

Table 1: This table presents the relation between the initial conditions for the three bodies. Note that these values are given in terms of n-body units.

| | **0** | **1** | **2** |
|---|---|---|---|
| m | 1 | 1 | $m_2$ |
| **r** | (0,-1) | (0,1) | (0,0) |
| **v** | $(v_1, v_2)$ | $(v_1, v_2)$ | $(\frac{-2v_1}{m_2}, \frac{2v_2}{m_2})$ |

The stability of the braids can be tested using the return proximity function given by:

$$|\boldsymbol{y}(t) - \boldsymbol{y}(0)| = \sqrt{\sum_{i=1}^{4}(y_i(t) - y_i(0))^2} \tag{1}$$

Where $y_i$ is one of the previously given parameters and $t$ is any given time. In order to find the initial conditions for which stable braids were possible they used an elimination process. They started by selecting initial conditions that gave a proximity value function of less than $10^{-1}$. Then they narrowed down their selection by looking at conditions that returned a proximity value of less than $10^{-3}$ and then $10^{-6}$. At each step they used an increasingly accurate integrator. In order to replicate the calculations we used the n-body simulator named *Hermite* which is available in the AMUSE software package. We took time steps equal to 0.001 in n-body system time units and we used an end time which is a multiple of the period given in the paper, which was different for each braid. We based our code on one of the AMUSE examples give named `sun_venus_earth.py`. The final version of the script for producing the braids and the plots visible in this write-up is named `3_body_braid_final.py`.

## 3 Results

In this section we will present our findings for a braid that we found to be non stable and one that we found to be stable. The initial conditions for both braids can be found in Table 2. In this

Table 2: Starting values of the two braids that we tested.

| Name | $m_2$ | $v_1$ | $v_2$ | Stable? |
|---|---|---|---|---|
| $I.A._4$ | 0.5 | 0.2009656237 | 0.2431076328 | No |
| $I.A._2$ | 4 | 0.9911981217 | 0.7119472124 | Yes |

report we only talk about two specific braids but we replicated many more of the braids shown in the paper. These replications can be found in our GITHUB repository.

### 3.1 Non-stable braid

The first braid that we decided to look at, named $I.A._4(0.5)$, turned out not to have a stable orbit when integrated using the method that we decided on (*Hermite*). As we can see in Figure 1 the orbits become quite chaotic after two periods. In order to see if this was due to the step size we decided to rerun the code with a step-size of 0.0001 but this also gave a chaotic result. Seeing as decreasing the step-size did not give a better result but did make for a much longer running time, we decided not to decrease the step-size even more. In order to see if we could find a stable braid, we decided to test the other initial conditions given in the paper. This gave us the following result.

### 3.2 Stable braid

In Figure 2 we can see the patterns produced by the braid named $I.A._2(4)$ which we found to be fairly stable. We can see that for 1, 4 and 10 periods no significant shift is visible to the naked eye. However, integrating over 100 periods (with a larger step-size) does give some visible shift. This could mean that if we were to integrate for much longer, eventually the orbits may become chaotic. As with the previous braid, we integrated these orbits using the *Hermite* algorithm. In the following section we did some further analysis of the stability of the braids and we will then also draw a conclusion from our findings.

## 4 Analysis and conclusion

In order to take a closer look at the stability of the braids we decided to look at the value of the proximity function (Equation 1) as a function of the number of time-steps taken. The result of this is visible in Figure 3. In it we can see that the unstable braid (Figure 3a) remains relatively regular for the first two periods but then becomes quite erratic, this matches the behaviour observed in Figure 1. The graph made for the stable braid (Figure 3b) clearly shows a regular period with troughs that indicate whenever the system nears its initial conditions. What is interesting to note however is that it looks like the value of proximity function after a few periods is not necessarily a good predictor for the stability of the system. This is visible in the fact that the second trough of the unstable braid is much lower than the troughs of the second stable braid. This suggests that the second system is more resilient against deviation from the initial conditions.

The reason that the first braid is not stable and that we observe some drifting in the second braid (which could eventually lead to instability) could be either due to the fact that the initial conditions were not correct or that the integration performed by Li et al. (2017) was done by a more accurate integration method. We believe that the second cause is far more likely since Li et al. (2017) made use of a super-computer whilst we were limited to using a method that could be done in a normal amount of time by the computers at the sterrenwacht.

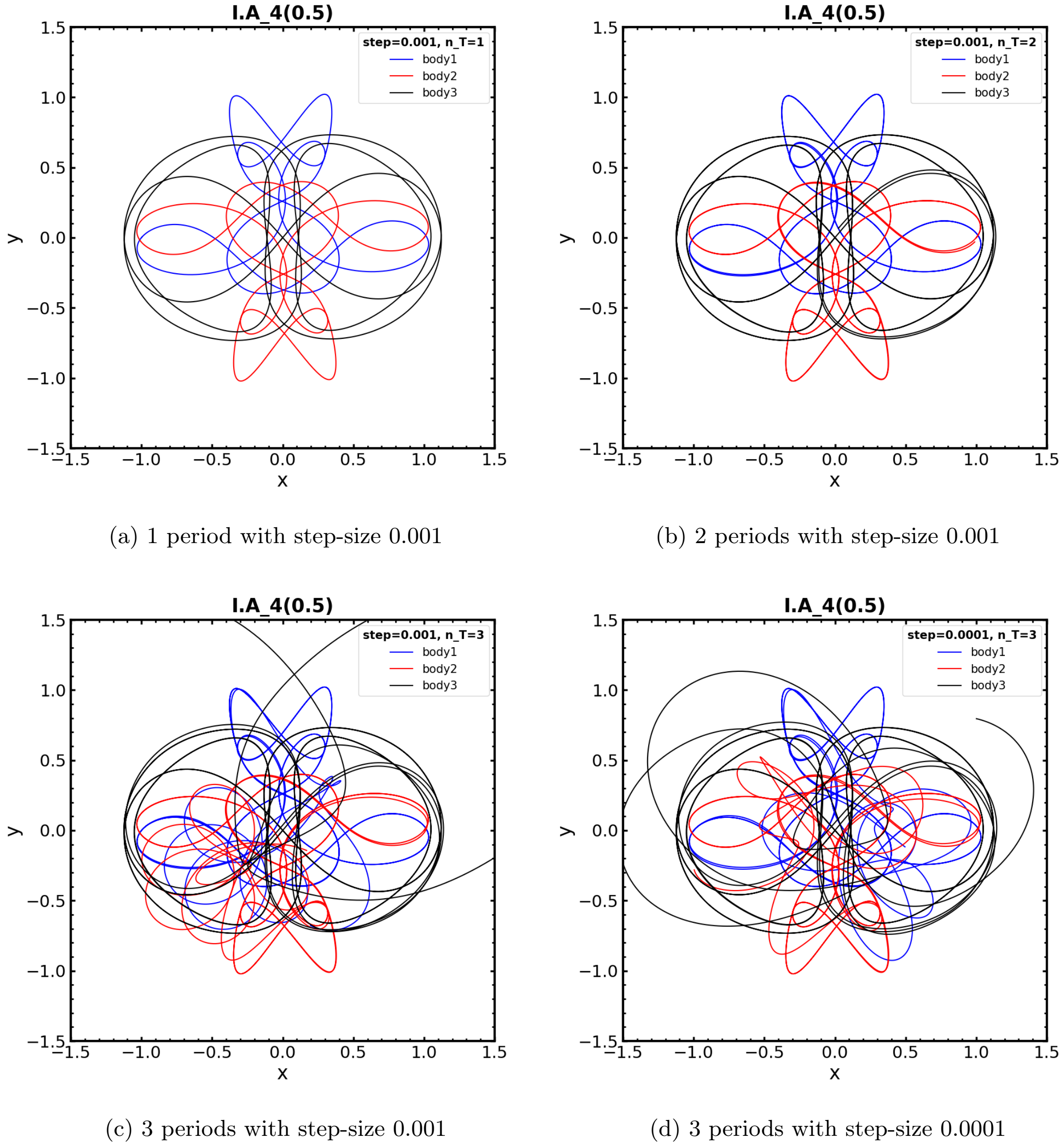
## References

Li X., Jing Y., Liao S., 2017, ] 10.1093/pasj/psy057

Montgomery R., 1998, Nonlinearity, 11, 363

Moore C., 1993, Physical Review Letters, 70, 3675

Portegies Zwart S., et al., 2019, AMUSE: the Astrophysical Multipurpose Software Environment, doi:10.5281/ZENODO.3260650, `https://zenodo.org/record/3260650#.XZMXFeYzaAk`



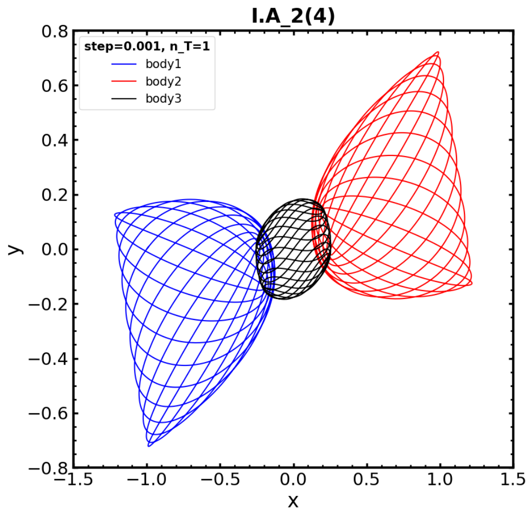

(a) 1 period with step-size 0.001

(b) 2 periods with step-size 0.001

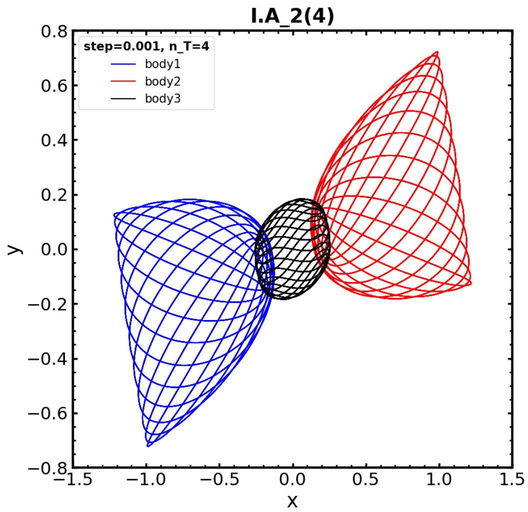
(c) 3 periods with step-size 0.001

(d) 3 periods with step-size 0.0001

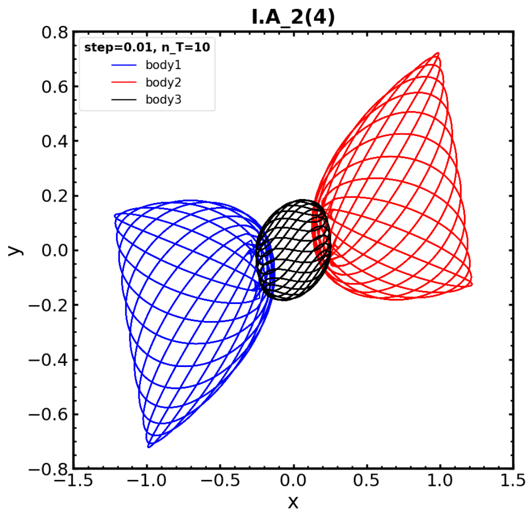
Figure 1: Here we see the same braid for three different numbers of periods. In sub-figure a, b and c we can see 1, 2 and 3 periods respectively. In sub-figure d we can see the orbits after three periods for a step-size of 0.0001. Note the manner in which the orbits become progressively more chaotic after two periods, even with a reduced step-size.



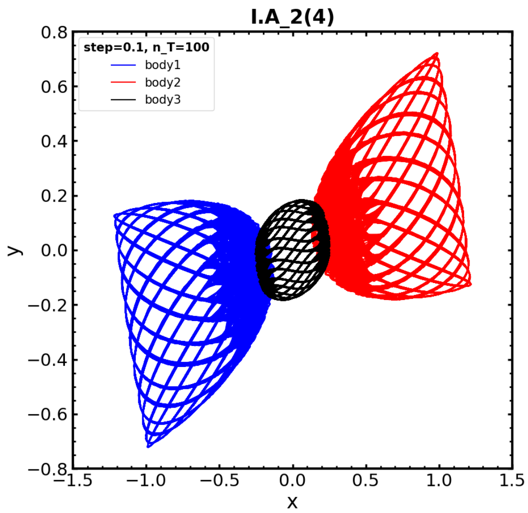

(a) 1 period with step-size 0.001




(b) 4 periods with step-size 0.001

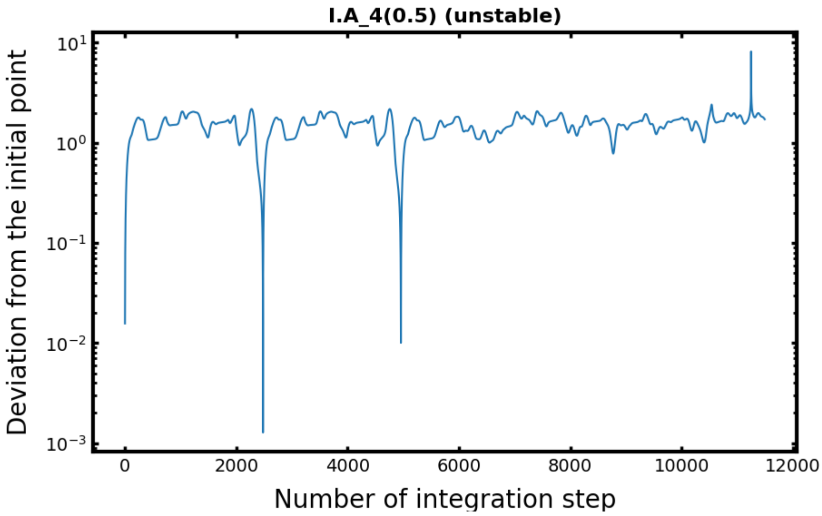



(c) 10 periods with step-size 0.01

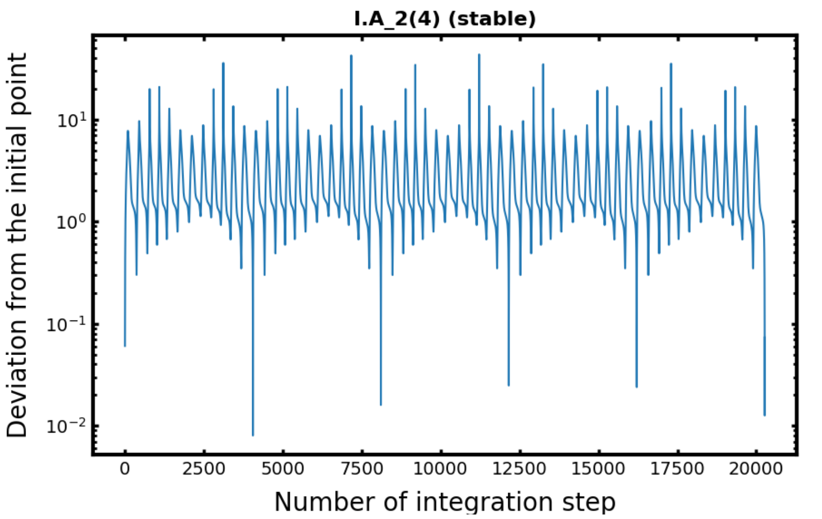



(d) 100 periods with step-size 0.1

Figure 2: Here we see the same braid for 4 different numbers of periods and different time-steps. Note that even after 100 periods with a relatively large time-step the orbits still appear to be stable.




(a)




(b)

Figure 3: Here we see two plots of the value of the proximity function as a function of the number of time-steps taken for 5 periods. Each of the periods is reflected in the valleys of the graphs. The reason that we decided to plot this information on a logarithmic scale is to better see the difference between the minima.