# Hybrid Beamforming for mmWave Massive MIMO

Haoyue Tang

Department of Electronic Engineering
Tsinghua University

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## Hybrid Analog and Digital Beamforming Structure

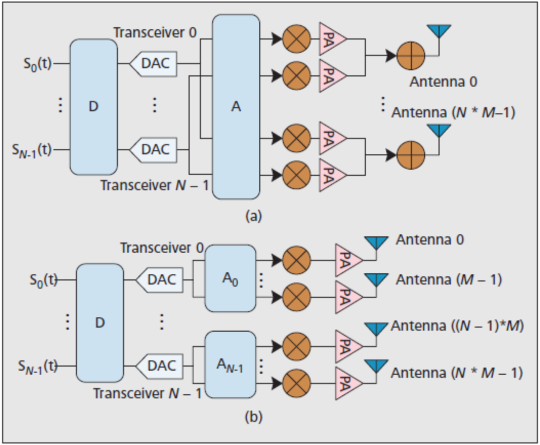


Figure: Hybrid Beamforming for LSAS system

# Hybrid Beamforming Design

Prerequists

$$N_s \leq N_t^{RF} \leq N_t$$

Transmitted signal

$$\mathbf{x} = \mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s}, \|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 = N_s$$

Received Signal

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s} + \mathbf{n}$$

Processed Received Signal

$$\tilde{y} = \sqrt{\rho}\mathbf{W}_{BB}^*\mathbf{W}_{RF}^*\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s} + \mathbf{W}_{BB}^*\mathbf{W}_{RF}^*\mathbf{n}$$

Rate Boundary from Shannon's Theory

$$R = \log_2 \left|\mathbf{I}_{N_s} + \frac{\rho}{N_s}\mathbf{R}_n^{-1}\mathbf{W}_{BB}^*\mathbf{W}_{RF}^*\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^*\mathbf{F}_{RF}^*\mathbf{H}^*\mathbf{W}_{RF}\mathbf{W}_{BB}\right|$$

$$\mathbf{R}_n = \mathbf{W}_{BB}^*\mathbf{W}_{RF}^*\mathbf{W}_{RF}\mathbf{W}_{BB}$$

# Goal

$$(\mathbf{F}_{BB}^{opt}, \mathbf{F}_{RF}^{opt}) = \arg\max \log_2 |\mathbf{I}_{N_s} + \frac{\rho}{N_s \sigma_n^2} \mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^* \mathbf{F}_{RF}^* \mathbf{H}^*|$$
$$\text{s.t} \quad \mathbf{F}_{RF} \in \mathcal{F}_{RF}, \|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 = N_S$$

After Approximations: Objective Functions

$$\mathcal{R}(\mathbf{F}_{BB}, \mathbf{F}_{RF}) = \log_2 |\mathbf{I}_{N_s} + \frac{\rho}{N_s \sigma_n^2} \Sigma_1^2| - (N_s - \|\mathbf{V}_1^* \mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2)$$

Alternate Optimization Problem

$$(\mathbf{F}_{BB}^{opt}, \mathbf{F}_{RF}^{opt}) = \arg\min \|\mathbf{F}_{opt} - \mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2$$
$$\text{s.t} \quad \mathbf{F}_{RF} \in \mathcal{F}_{RF}, \|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 = N_s, \mathbf{F}_{opt} = \mathbf{V}_1$$

Solutions: Spatially Sparse Precoding via Orthogonal Matching Pursuit

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# Structure

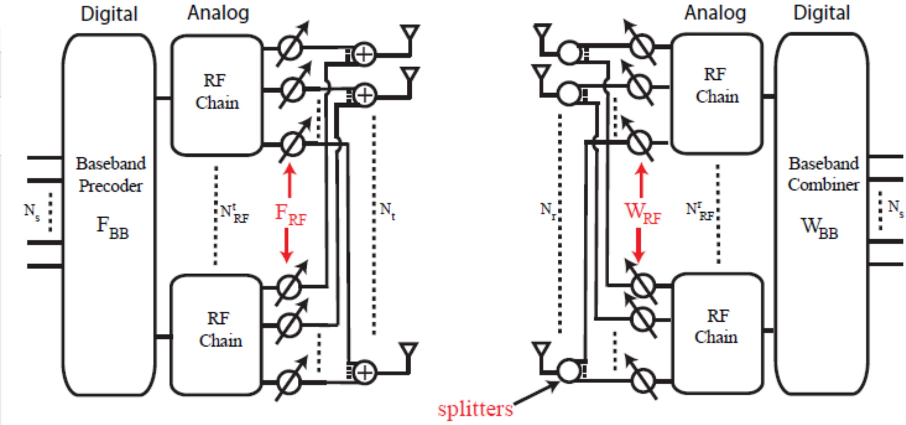


Figure: Block Diagram of Transmitter and Receiver Side

# Receiver Design Criterion

$$(\mathbf{W}_{RF}^*, \mathbf{W}_{BB}^*) = \arg\min \mathbb{E}(\|\mathbf{s} - \mathbf{W}_{RF}\mathbf{W}_{BB}\mathbf{x}\|), \text{s.t} \quad \mathbf{W}_{RF} \in \mathcal{W}_{RF}$$

Objective Function Reformulation

$$\mathcal{J}(\mathbf{W}_{RF}, \mathbf{W}_{BB}) = \|\mathbb{E}[\mathbf{y}\mathbf{y}^{*1/2}](\mathbf{W}_{MMSE} - \mathbf{W}_{RF}\mathbf{W}_{BB})\|_F^2$$

$$\mathbf{W}_{MMSE} = \mathbb{E}[\mathbf{s}\mathbf{y}^*]\mathbb{E}[\mathbf{y}\mathbf{y}^*]^{-1}$$

Solutions: Spatially Sparse MMSE Combining via Orthogonal Matching Pursuit

Similarities: $\|\cdot\|_F$ on Grassman manifolds, optimization for equivalent problem, Greedy matrix formulation

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# SE-EE Relationship

$$C = WN\log_2(1 + MP\eta_{PA}/WN_0)$$

$$\eta_{SE} = \frac{C}{W} = N\log_2(1 + MP\eta_{PA}/WN_0)$$

$$\eta_{EE} = \frac{C}{P_{total}} = \frac{\eta_{SE}}{(2^{\frac{\eta_{SE}}{N}} - 1)\frac{N_0}{\eta_{PA}}\frac{N}{M} + \frac{NP_0 + P_{common} + NMP_{rfcircuit}}{W}}$$

## Relationship between green points EE&SE

$$\eta_{EE} = \left(\frac{n_0 N 2^{\frac{\eta_{SE}^*}{N}}\ln 2}{L\eta_{PA}}\right)^{-1}$$

Uniqueness: For given $L$, $\eta_{SE}$, optimal $N$ for $\eta_{EE}^*$ is unique.

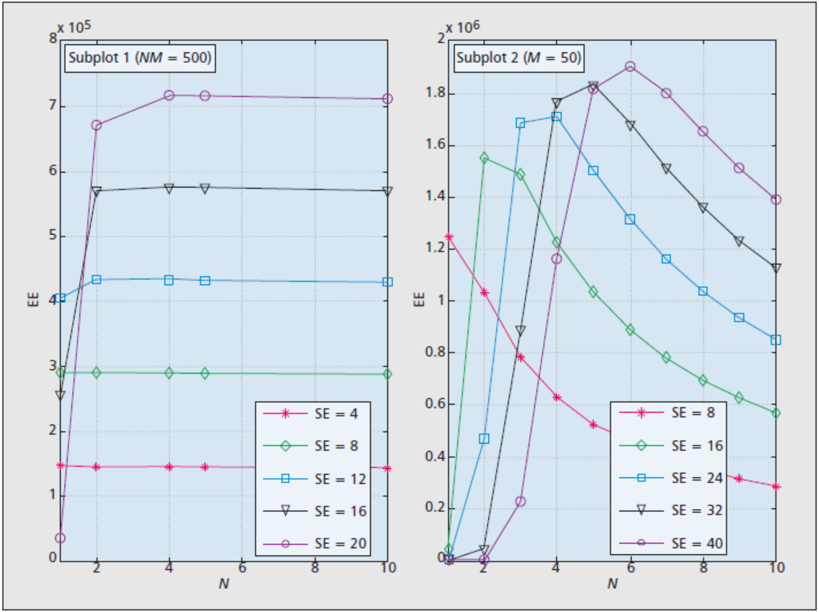


Figure: N-EE for different SE

# References


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