

**Written Problem 1.**

(a)  $x(t) = x_1t + x_0$   
 $y(t) = y_1t + y_0$   
 $z(t) = z_1t + z_0$

(b)  $u = \frac{fx}{z}, v = \frac{fy}{z}$   
 $u = \frac{f \cdot (x_1t + x_0)}{z_1t + z_0}, v = \frac{f \cdot (y_1t + y_0)}{z_1t + z_0}$

(c) Solving for  $t$  (showing process for only  $u$ ),

$$u(z_1t + z_0) = f(x_1t + x_0)$$

$$uz_1t + uz_0 = fx_1t + fx_0$$

$$t(uz_1 - fx_1) = fx_0 - uz_0$$

$$t = \frac{fy_0 - vz_0}{vz_1 - fy_1}$$

Result for  $v$  (follows same process),

$$t = \frac{fx_0 - uz_0}{uz_1 - fx_1}$$

Setting equal,

$$\frac{fy_0 - vz_0}{vz_1 - fy_1} = \frac{fx_0 - uz_0}{uz_1 - fx_1}$$

$$(fy_0 - vz_0)(uz_1 - fx_1) = (fx_0 - uz_0)(vz_1 - fy_1)$$

$$fy_0uz_1 - vz_0uz_1 - fy_0fx_1 + fx_1vz_0 = fx_0vz_1 - uz_0vz_1 - fx_0fy_1 + fy_1uz_0$$

Eliminating like terms,

$$fy_0uz_1 - fy_0fx_1 + fx_1vz_0 = fx_0vz_1 - fx_0fy_1 + fy_1uz_0$$

Rearranging into implicit form ( $ax + by + c = 0$ ),

$$(y_0z_1 - y_1z_0)fu + (x_1z_0 - x_0z_1)fv + (y_0x_1 - x_0y_1)f^2 = 0$$

In this case, implicit form in image coordinates would be  $au + bv + c = 0$ .

Therefore, a straight line in the world projects onto a straight line in the image.

(d) Since there is 'no'  $z$ -axis (camera is lying on  $z$ -axis), a line on or parallel to the  $z$ -axis would appear as a dot to the viewer.

Here we can set  $x$  and  $y$  values to 0 to eliminate the line's dependence on them,

$$0 \cdot fu + 0 \cdot fv + 0 \cdot f^2 = 0$$