# Assignment_1

January 22, 2019

# 1 Assignment 1

#### 1.0.1 Math 502 - Lamoureux

#### 1.0.2 Due January 31, 2019

## 1.1 Exercise 0

Plot a few Bessel functions of the first kind, using your knowledge of Python. Plot for orders $\alpha = 0, 1, 2, 3$ and choose a sensible range of values for the domain.

(Hint: You might want to look back at our sample code on how we plotted a sine or cosine function, and replicate that for a Bessel function. You may have to look up on Google how to call a Bessel function from some toolbox in Python.)

Please don't work too hard -- I don't want to see your approximate solution as an infinite sum or integral form!! Use functions you can call from Python and its modules.

## 1.2 Exercise 1

Recall in Lecture 4 (Jan 22), we had a model for the traffic jam model, with a jump in the velocity function. Replace the jump function for $v(x)$ with a smooth, monotonic function that has a left limit of $v_{left}$ and a right limit of $v_{right}$. Choose a function where it is easy to calculate the antiderivative of slowness $1/v(x)$.

Do calculate the function $S(x)$ as the antiderivative of slowness $1/v(x)$, for your choice of $v(x)$.

Hint: Rational functions won't work. (Exercise 1a. Why not?) You might want to consider functions like arctan, $\tan^{-1}(x)$ or hyperbolic tan, $\tanh(x)$.

## 1.3 Exercise 2

Adjust the code in Lecture 4 to use your new definition of $v(x)$ and the resulting $S(x)$. Show that it works by making a few illustrative plots.

## 1.4 Exercise 3

Kepler's third law of planetary motion says that the length of time it takes a planet to orbit the sun is proportional to its distance from the sun, raised to some (fractional) power. That is:

$$T = kR^{\alpha},$$

where $T$ is the length of time for one complete orbit, $R$ is the distance between the planet and the sun, $\alpha$ is a fixed power, and $k$ is some univeral constant that works for all the planets around our sun.

Use Dimensional Analysis (Buckingham's Pi Theorem) to derive this result. Tell me what the value of $\alpha$ is.

Don't use calculus! (Although you may have seen this solved via differential equations in a calc or physics class.)

Hint: There is some ambiguity because of two masses involved (sun and planet). Newton knew that the mass of the planet does not matter to $T$, so you can assume this as well. Newton's universal gravitation constant $G$ also enters into the problem -- you can look up what units it is measured in, on Google. Or you can figure it out yourself from the force formula for planetary attraction

$$\text{Force } = G\frac{Mm}{R^2}$$

where $M, m$ are the masses of the sun and planet.

You can also check your answer by looking up Kepler's laws on Google.

### 1.5 Exercise 4

Make a table listing the 8 planets plus Pluto, their distance to the sun, and the period of their orbit.

Make a log-log plot of period versus distance, and check that you get a straight line with slope equal to $\alpha$. (At least approximately.)

i.e. Taking logs of the equation $T = kR^\alpha$ gives

$$\log T = \log k + \alpha \log R,$$

which is the equation of a line in $x = \log R, y = \log T$.

### 1.6 Exercise 5

Nuclear bombs, when exploded in the atmosphere, produce a large fireball that expands in the approximate shape of a sphere of some radius $r = r(t)$, which is a function of elapsed time. The rate at which it expands depends on the energy $E$ released by the bomb, the elapsed time $t$ since the detonation, and the density $\rho$ of the surrounding air.

Curiously, the actual air pressure is irrelevant, as it is so small compared to the explosive blast.

Use dimensional analysis to find $r(t)$ as a function of $E, t, \rho$. (times some fixed constant)

### 1.7 Exercise 6 - for fun.

Can you animate the result in Exercise 5, showing some bomb blasts of various energies? Something pretty?