```
NN solver could be used after data format modification (I first created NN solver with custom vertex class and other login, then was BnC algorithm
        implementation with tsplib. Compatibility is not purpose in this case).
        I would recommend to use %matplotlib widget command (interactive plots) instead of %matplotlib inline.
        Assumptions:

    TSP is symmetrical.

          · Triangle inequality is satisfied
        Terminology is aligned with tsplib library, e.g. tour is a list of node indexes, etc...
        References
          • tsplib documentation
          • base guideline for solution

    <u>lin prog with python</u>

In [ ]: # %matplotlib widget # Recommended
        %matplotlib inline
        import os
        import tsplib95
        from pulp import LpMinimize, LpProblem, LpStatus, LpVariable, LpAffineExpression, GLPK, pulpTestAll, lpSum
        from matplotlib import pyplot as plt
        from collections import Counter
In [ ]: # Make sure it run with no errors
        # Install required sw to proceed
        pulpTestAll()
In [ ]: VERBOSE = False
        PLOT = True
        DATA_SET = 'berlin52'
        TSP_ROOT = './tsp/'
In [ ]: problem = tsplib95.load(os.path.join(TSP_ROOT, f'{DATA_SET}.tsp'))
        assert problem.is_symmetric()
        assert problem.edge_weight_type == 'EUC_2D'
        if PLOT:
             plt.figure(figsize=(20, 10))
             ax = plt.subplot()
            ax.set_title(f'Initial state for {DATA_SET}')
             for x, y in problem.node_coords.values():
                 ax.scatter(x, y, s=5, color='k')
                                                               Initial state for berlin52
         1200
         1000
          800
          600
          400
          200
In [ ]: | n = len(list(problem.get_nodes()))
        model = LpProblem(name="tsp", sense=LpMinimize)
        def get_edge_name(a, b):
             return f'x_{a}_{b}'
        # Build Xij variables - fox each xi to each xj (exclude xi to xi)
        edges = {}
        for xi in problem.get_nodes():
             for xj in problem.get_nodes():
                 if xi != xj:
                     name = get_edge_name(xi, xj)
                     edges[name] = (LpVariable(name, cat='Binary'), xi, xj)
        # Build cost function
        costs = []
        for x, xi, xj in edges.values():
             cost = problem.get_weight(xi, xj)
             costs.append((x, cost))
        model += (LpAffineExpression(costs), f'Cost x_{xi}_{xj}')
        # Build local constraints
        for j in range(1, n + 1):
             for e_x, xi, xj in edges.values():
                 if xj == j:
                     leave_x = []
                     for l_x, xj, xk in edges.values():
                         if xj == j and xk != xi:
                             leave_x.append((l_x, 1))
                     leave_x.append((e_x, 1))
                     model += (LpAffineExpression(leave_x) == 2, f'Visit once x_{xi}_{j}')
        model.solve(solver=GLPK(msg=False))
        print(f"Status: {model.status}, {LpStatus[model.status]}")
        if model.status == 1:
             # Divide by 2 because xi-xj = xj-xi edge
             print(f"Path length: {model.objective.value() / 2}")
             if VERBOSE:
                 # Make sure all nodes are used twice as enter, twice as exit
                 # Each node should lead to other 2 and the same edges but in reverse directions should exist
                 ii = []
                 jj = []
                 for var in model.variables():
                     if var.value():
                         x, i, j = var.name.split('_')
                         ii.append(i)
                         jj.append(j)
                 if VERBOSE:
                     print(f'Enter != 2 (should be empty): {[i for i in dict(Counter(ii)).values() if i != 2]}')
                     print(f'Leave != 2 (should be empty): {[j for j in dict(Counter(jj)).values() if j != 2]}')
            if PLOT:
                 plt.figure(figsize=(20, 10))
                 ax = plt.subplot()
                 ax.set_title(f'Partial solution for {DATA_SET}')
                 for var in model.variables():
                     if var.value():
                         _, xi, xj = edges[var.name]
                         [xi_x, xi_y], [xj_x, xj_y] = problem.get_display(xi), problem.get_display(xj)
                         ax.scatter(xi_x, xi_y, s=5, color='k')
                         ax.scatter(xj_x, xj_y, s=5, color='k')
                         ax.plot([xi_x, xj_x], [xi_y, xj_y], linewidth=1, color='b')
        Status: 1, Optimal
        Path length: 7164.0
                                                              Partial solution for berlin52
         1200
         1000
          800
          600
          400
          200
                                                                                                                          1750
                               250
                                              500
                                                              750
                                                                             1000
                                                                                            1250
                                                                                                           1500
In [ ]: def detect_tour(m: LpProblem, verbose=VERBOSE):
             # Start at random point and go from i to j, if original i encountered - return tour
             tour = set()
             tour_variables = []
             tour_next_index = None
             tour_prev_index = None
             tour_orig_index = None
             for var in m.variables():
                 if var.value():
                     x, xi, xj = edges[var.name]
                     if verbose:
                         print(f'start: {xi} -> {xj}')
                     tour_prev_index = tour_orig_index = xi
                     tour_next_index = xj
                     # Add all points that belong to tour
                     tour.add(xi)
                     tour.add(xj)
                     break
             while True:
                 for var in m.variables():
                     if var.value():
                         x, xi, xj = edges[var.name]
                         if xi == tour_next_index and xj != tour_prev_index:
                             if verbose:
                                 print(f'next: {xi} -> {xj}')
                             tour_prev_index = xi
                             tour_next_index = xj
                             tour.add(xi)
                             tour.add(xj)
                             break
                 else:
                    break
                 if xj == tour_orig_index:
                     break
             for var in m.variables():
                 if var.value():
                     x, xi, xj = edges[var.name]
                     if xi in tour or xj in tour:
                         tour_variables.append(x)
            return tour, tour_variables
In [ ]: tour, variables = [], []
        while True:
             tour, variables = detect_tour(model)
            if len(tour) == n:
                 print('No subtours found, abort')
                 break
            tour_str = '_'.join([str(t) for t in tour])
            print(f'Solving fou tour: {tour_str}, length: {len(tour)}')
            leave_from_tour = []
             for i in tour:
                 leave_from_i = []
                 for name, (x, xi, xj) in edges.items():
                     if i == xi and xj not in tour:
                         leave_from_i.append(x)
                 leave_from_tour.extend(leave_from_i)
            model += (lpSum(leave_from_tour) >= 1, f'Leave tour {tour_str}')
             model.solve()
        Solving fou tour: 1_3_4_8_9_10_16_17_18_19_20_21_22_23_25_29_30_31_32_33_41_43_44_45_46_49_50, length: 27
        Solving fou tour: 1_3_4_5_6_8_9_10_15_16_17_18_19_20_21_22_23_24_25_29_30_31_32_33_34_35_36_37_38_39_40_41_43_44_45_4
        6_48_49_50, length: 39
        Solving fou tour: 1_3_4_5_6_8_9_10_11_12_13_14_15_16_17_18_19_20_21_22_23_24_25_26_27_28_29_30_31_32_33_34_35_36_37_3
        8_39_40_41_43_44_45_46_47_48_49_50_51_52, length: 49
        Solving fou tour: 1_2_3_4_7_8_9_10_16_17_18_19_20_21_22_23_25_29_30_31_32_33_41_42_43_44_45_46_49_50, length: 30
        Solving fou tour: 1_2_3_4_5_6_7_8_9_10_15_16_17_18_19_20_21_22_23_24_25_29_30_31_32_33_34_35_36_37_38_39_40_41_42_43_
        44_45_46_48_49_50, length: 42
        No subtours found, abort
In [ ]: print(f"Status: {model.status}, {LpStatus[model.status]}")
        if model.status == 1:
             # Divide by 2 because xi-xj = xj-xi edge
             print(f"Path length: {model.objective.value() / 2}")
                 # Make sure all nodes are used twice as enter, twice as exit
                 # Each node should lead to other 2 and the same edges but in reverse directions should exist
                 ii = []
                 jj = []
                 for var in model.variables():
                     if var.value():
                         x, i, j = var.name.split('_')
                         ii.append(i)
                         jj.append(j)
                 print(f'Enter != 2 (should be empty): {[i for i in dict(Counter(ii)).values() if i != 2]}')
                 print(f'Leave != 2 (should be empty): {[j for j in dict(Counter(jj)).values() if j != 2]}')
            if PLOT:
                 plt.figure(figsize=(20, 10))
                 ax = plt.subplot()
                 for var in model.variables():
                     if var.value():
                         _, xi, xj = edges[var.name]
                         [xi_x, xi_y], [xj_x, xj_y] = problem.get_display(xi), problem.get_display(xj)
                         ax.scatter(xi_x, xi_y, s=5, color='k')
                         ax.scatter(xj_x, xj_y, s=5, color='k')
                         ax.plot([xi_x, xj_x], [xi_y, xj_y], linewidth=1, color='b')
                 ax.set_title(f'Full solution for {DATA_SET}')
        Status: 1, Optimal
        Path length: 7542.0
                                                               Full solution for berlin52
         1200
         1000
          800
          600
          400
```

200

1000

1250

1500

1750

Introduction