# 1. The Groups C<sub>1</sub>, C<sub>s</sub>, C<sub>i</sub>

C <sub>1</sub> (1)	E
A	1

$C_s = C_h$ $(m)$	E	$\sigma_{ m h}$		
A'	1	1	$x, y, R_z$	$x^2$ , $y^2$ , $z^2$ , $xy$
A"	1	-1	$z$ , $R_x$ , $R_y$	yz, xz

$C_{i} = S_{2}$ $(\overline{1})$	E	i		
$A_{g}$	1	1	$R_x$ , $R_y$ , $R_z$	$x^2, y^2, z^2,$
$A_{\rm u}$	1	-1	x, y, z	xy, xz, yz

#### 2. The Groups $C_n$ (n = 2, 3, ..., 8)

$C_2$ (2)	E	$C_2$		
A	1	1	$z$ , $R_z$	$x^2$ , $y^2$ , $z^2$ , $xy$
В	1	-1	$x, y, R_x, R_y$	yz, xz

$$\begin{array}{c|cccc}
C_3 & \mathcal{E} & C_3 & C_3^2 & \varepsilon = \exp(2\pi i/3) \\
\hline
A & 1 & 1 & 1 & z, R_z & x^2 + y^2, z^2 \\
\hline
E & \begin{cases}
1 & \varepsilon & \varepsilon^* \\
1 & \varepsilon^* & \varepsilon
\end{cases} & (x, y)(R_x, R_y) & (x^2 - y^2, 2xy)(yz, xz)
\end{array}$$

C <sub>6</sub> (6)	Е	$C_6$	$C_3$	$C_2$	$C_3^2$	$C_6^5$		$\varepsilon = \exp(2\pi i/6)$
A	1	1	1	1	1	1	$z$ , $R_z$	$x^2 + y^2, z^2$
В	1	-1	1	-1	1	-1		
$E_1$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$-\varepsilon^*$ $-\varepsilon$	-1 -1	$-arepsilon \ -arepsilon^*$	$\left. egin{aligned} arepsilon^* \ arepsilon \end{aligned}  ight.$	$(x, y)  (R_z, R_y)$	(xy, yz)
						$egin{array}{c} -arepsilon \ -arepsilon^* \end{array} igg $		$(x^2-y^2, 2xy)$

## 2. The Groups $C_n$ (n = 2, 3, ..., 8) (cont..)

$C_7$	E C	$C_7$ $C_7^2$ $C_7$	$C_7^3  C_7^4  C_7^5$	$C_7^6$	$\varepsilon = \exp(2\pi i/7)$
A				1 $z, R_z$	$x^2 + y^2, z^2$
$\mathrm{E}_1$	$\begin{cases} 1 & \epsilon \\ 1 & \epsilon \end{cases}$	$arepsilon  arepsilon^2  arepsilon^*  arepsilon^{*2}  arepsilon$	$arepsilon^3  arepsilon^{*3}  arepsilon^*  $	$\left\{\begin{array}{ll} \varepsilon^* & \varepsilon^* \\ \varepsilon & \varepsilon \end{array}\right\} \left(\begin{matrix} (x, y) \\ (R_x, R_y) \end{matrix}\right)$	(xz, yz)
$E_2$	$\begin{cases} 1 & \varepsilon^2 \\ 1 & \varepsilon^* \end{cases}$	$oldsymbol{arepsilon}^2  oldsymbol{arepsilon}^{*3}  oldsymbol{arepsilon}^3  oldsymbol{arepsilon}^3  oldsymbol{arepsilon}^3$	$\varepsilon^*$ $\varepsilon$ $\varepsilon^3$ $\varepsilon^*$ $\varepsilon^{*3}$	$\left. egin{aligned} arepsilon^{*2} \ arepsilon^2 \end{array}  ight\}$	$(x^2-y^2,2xy)$
$\mathrm{E}_3$	$\begin{cases} 1 & \varepsilon^3 \\ 1 & \varepsilon^* \end{cases}$	$arepsilon^{8}  arepsilon^{*}  arepsilon^{}  arepsilon^{} \ arepsilon$	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$egin{array}{c} oldsymbol{arepsilon}^{*3} \ oldsymbol{arepsilon}^3 \end{array} igg\}$	

$C_8$	Е	$C_8$	$C_4$	$C_2$	$C_4^3$	$C_8^3$	$C_8^5$	$C_8^7$		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	$z$ , $R_z$	$x^2 + y^2, z^2$
В	1	-1	1							
$E_1$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	i i	-1 -1	-і і	$-\varepsilon^*$ $-\varepsilon$	$-arepsilon \ -arepsilon^*$	$\left. egin{aligned} arepsilon^* \ arepsilon \end{aligned}  ight.  ight.$	$(x, y)  (R_x, R_y)$	(xz, yz)
$E_2$	$\begin{cases} 1 \\ 1 \end{cases}$	i i	-1 -1	1 1	-1 -1	-і і	i i	$\begin{bmatrix} -i \\ i \end{bmatrix}$		$(x^2-y^2, 2xy)$
$E_3$	$\begin{cases} 1 \\ 1 \end{cases}$	$-\varepsilon$ $-\varepsilon^*$	i i	-1 -1	-і і	ε* ε	€ €*	$egin{array}{c} -arepsilon^* \ -arepsilon \end{array}  ight\}$		

#### 3. The Groups $D_n$ (n = 2, 3, 4, 5, 6)

$D_2$ (222)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		$x^{2}, y^{2}, z^{2}$
$B_1$	1	1	-1	-1	$z$ , $R_z$	xy
$\mathrm{B}_2$	1	-1	1	-1	$y$ , $R_y$	$\chi_Z$
$B_3$	1	-1	-1	1	$x$ , $R_x$	yz

$D_3$	Е	$2C_3$	$3C_2$		
(32)					
$A_1$	1	1	1		$x^2 + y^2$ , $z^2$
$A_2$	1	1	-1	$z$ , $R_z$	
Е	2	-1	0	$(x, y)(R_x, R_y)$	$(x^2-y^2, 2xy) (xz, yz)$

$D_4$ (422)	E	$2C_4$	$C_2(=C_4^2)$	$2C_{2}^{'}$	$2C_2^{"}$		
$A_1$	1	1	1	1	1		$x^2 + y^2$ , $z^2$
$A_2$	1	1	1	-1	-1	$z$ , $R_z$	
$\mathbf{B}_1$	1	-1	1	1	-1		$x^2 - y^2$
$\mathrm{B}_2$	1	-1	1	-1	1		xy
E	2	0	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

$D_5$	Е	2 <i>C</i> <sub>5</sub>	$2C_5^2$	5C <sub>2</sub>		
$A_1$	1	1	1	1		$x^2 + y^2$ , $z^2$
$A_2$	1	1	1	-1	$z$ , $R_z$	
$E_1$	2	$2\cos 72^{\circ}$	$2\cos 144^{\circ}$	0	$(x, y)(R_x, R_y)$	(xz, yz)
$E_2$	2	2 cos 144°	$2\cos 72^{\circ}$	0		$(x^2-y^2, 2xy)$

$D_6$	F	2C.	$2C_3$	C	3C'	3C"		
(622)	L	206	203	$C_2$	$\mathcal{IC}_2$	$3C_2$		
$A_1$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$z$ , $R_z$	
$\mathbf{B}_1$	1	-1	1	-1	1	-1		
$\mathrm{B}_2$	1	-1	1	-1	-1	1		
$E_1$	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
$E_2$	2	-1	-1	2	0	0		$(x^2 - y^2, 2xy)$

## 4. The Groups $C_{nv}(n = 2, 3, 4, 5, 6)$

$C_{2v}$ $(2mm)$	E	$C_2$	$\sigma_{v}(xz)$	$\sigma'_{v}(yz)$		
$A_1$	1	1	1	1	Z	$x^2, y^2, z^2$
$A_2$	1	1	-1	-1	$R_z$	xy
$\mathbf{B}_1$	1	-1	1	-1	$x, R_y$	$\chi_Z$
$B_2$	1	-1	-1	1	$y, R_x$	yz

$C_{3v}$	E	$2C_3$	$3\sigma_{v}$		
(3m)					
$A_1$	1	1	1	Z	$x^2 + y^2, z^2$
$A_2$	1	1	-1	$R_z$	
E	2	-1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(xz, yz)$

$C_{4v}$ (4mm)	Е	$2C_4$	$C_2$	$2\sigma_{\rm v}$	$2\sigma_{ m d}$		
$A_1$	1	1	1	1	1	Z	$x^2 + y^2$ , $z^2$
$A_2$	1	1	1	-1	-1	$R_z$	
$\mathbf{B}_1$	1	-1	1	1	-1		$x^2-y^2$
$\mathrm{B}_2$	1	-1	1	-1	1		xy
E	2	0	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

$C_{5v}$	Е	$2C_5$	$2C_5^2$	$5\sigma_{\rm v}$		
$A_1$	1	1	1	1	Z	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	$R_z$	
$E_1$	2	2 cos 72°	2 cos 144°	0	$(x, y)(R_x, R_y)$	(xz, yz)
$E_2$	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$

$C_{6v}$ $(6mm)$	Е	$2C_6$	$2C_3$	$C_2$	$3\sigma_{\rm v}$	$3\sigma_{ m d}$		
$A_1$	1	1	1	1	1	1	Z	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$R_z$	
$\mathbf{B}_1$	1	-1	1	-1	1	-1		
$\mathrm{B}_2$	1	-1	1	-1	-1	1		
$E_1$	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
$E_2$	2	-1	-1	2	0	0		$(x^2-y^2, 2xy)$

## 5. The Groups $C_{nh}$ (n = 2, 3, 4, 5, 6)

$C_{2h} $ $(2/m)$	Е	$C_2$	i	$\sigma_{ m h}$		
$\overline{A_g}$	1	1	1	1	$R_z$	$x^2, y^2, z^2, xy$
$\mathrm{B}_{\mathrm{g}}$	1	-1	1	-1	$R_x$ , $R_y$	xz, yz
$A_{u}$	1	1	-1	-1	$\boldsymbol{z}$	
$B_{u}$	1	-1	-1	1	<i>x</i> , <i>y</i>	

$C_{3h}$ $(\overline{6})$	Е	$C_3$	$C_{3}^{2}$	$\sigma_{h}$	$S_3$	$S_3^5$		$\varepsilon = \exp(2\pi i/3)$
A'	1	1	1	1	1	1	$R_z$	$x^2 + y^2$ , $z^2$
E'	$\begin{cases} 1 \\ 1 \end{cases}$	${\cal E} \ {\cal E}^*$	$oldsymbol{arepsilon}^*$	1 1	${\cal E} \ {\cal E}^*$	$\left. egin{array}{c} \mathcal{E}^{ *} \\ \mathcal{E} \end{array}  ight\}$	(x, y)	$(x^2-y^2,2xy)$
Α"						-1		
Е"	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$oldsymbol{arepsilon}^*$	-1 -1	$-\varepsilon$ $-\varepsilon^*$	$egin{array}{c} -arepsilon^* \ -arepsilon \end{array}  ight\}$	$(R_x, R_y)$	(xz, yz)

$C_{4h}$ $(4/m)$	Е	$C_4$	$C_2$	$C_4^3$	i	$S_4^3$	$\sigma_{h}$	$S_4$		
$\overline{A_{g}}$	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$\mathrm{B_g}$	1	-1	1	-1	1	-1	1	-1		$(x^2 - y^2, 2xy)$
$\mathrm{E}_{\mathrm{g}}$	$\begin{cases} 1 \\ 1 \end{cases}$	i -i	-1 -1	-і і	1 1	i -i	-1 -1	$\begin{bmatrix} -i \\ i \end{bmatrix}$	$(R_x, R_y)$	(xz, yz)
$A_{\mathrm{u}}$	1	1	1	1	-1	-1	-1	-1	z	
$\mathbf{B}_{\mathrm{u}}$	1	-1	1	-1	-1	1	-1	1		
Eu	$\begin{cases} 1 \\ 1 \end{cases}$	i -i	-1 -1	−i i	-1 -1	−i i	1	i - i	(x, y)	

# 5. The Groups $C_{nh}$ (n = 2, 3, 4, 5, 6) (cont...)

$C_{5\mathrm{h}}$	Е	$C_5$	$C_5^2$	$C_5^3$	$C_{5}^{4}$	$\sigma_{h}$	$S_5$	$S_5^7$	$S_5^3$	$S_5^9$		$\varepsilon = \exp(2\pi i/5)$
A′	1	1	1	1	1	1	1	1	1	1	$R_z$	$\frac{1}{x^2+v^2}$ , $z^2$
$E_1'$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$								$\left. egin{array}{c} arepsilon^* \ arepsilon \end{array}  ight.  ight.$	=	. , , -
$\mathrm{E}_2'$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^2 \ arepsilon^{*2}$	$arepsilon^* \ arepsilon$	$arepsilon^*$	$arepsilon^{*2}$ $arepsilon^2$	1 1	$arepsilon^2 \ arepsilon^{*2}$	$arepsilon^* \ arepsilon$	$oldsymbol{arepsilon}^*$	$\left. egin{aligned} arepsilon^{*2} \ arepsilon^2 \end{aligned}  ight\}$	Z	$(x^2-y^2,2xy)$
A"	1	1	1	1	1 -	-1	–1 ·	-1	-1	-1		
E'' <sub>1</sub>	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$arepsilon^2 \ arepsilon^{*2}$	$\varepsilon^{*2}$ $\varepsilon^2$	$arepsilon^* - \ arepsilon \ -$	·1 -	-ε -ε*	$-\varepsilon^2$ $-\varepsilon^{*2}$	$-\varepsilon^{*2}$ $-\varepsilon^{2}$	$egin{array}{c} -arepsilon^* \ -arepsilon \end{array}  ight\}$	$(R_x, R_y)$	(xz, yz)
E <sub>2</sub> "	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^2 \ arepsilon^{*2}$	ε* ε	ε ε ε* ε	*2 — = = = = = = = = = = = = = = = = = =	1 - 1 -	$-\varepsilon^2$ $\varepsilon^{*2}$	$-\varepsilon^*$ $-\varepsilon$	$-\varepsilon$ $-\varepsilon^*$	$egin{array}{c} -arepsilon^{*2} \ -arepsilon^2 \end{array}  ight\}$		

$C_{6h}$ $(6/m)$	E	$C_6$	$C_3$	$C_2$	$C_{3}^{2}$	$C_{6}^{5}$	i	$S_3^5$	$S_6^5$	$\sigma_{h}$	$S_6$	$S_3$		$\varepsilon = \exp(2\pi i/6)$
$A_{g}$	1	1	1	1	1	1	1	1	1	1	1	1		$x^2+y^2, z^2$
													$(R_x, R_y)$	(xz, yz)
$\mathrm{E}_{1\mathrm{g}}$	∫1	${\cal E}$	$-\varepsilon^*$	-1	$-\varepsilon$	$\boldsymbol{\mathcal{E}}^*$	1	${\cal E}$	$-\varepsilon^*$	-1	$-\varepsilon$	$arepsilon^*igg brace$		
$\mathrm{E}_{1g}$	$\lfloor 1$	$\boldsymbol{\mathcal{E}}^*$	$-\varepsilon$	-1	$-arepsilon^*$	${\cal E}$	1	$\boldsymbol{\mathcal{E}}^*$	$-\varepsilon$	-1	$-\varepsilon^*$	$\varepsilon$		
$\mathrm{E}_{2\mathrm{g}}$	$\begin{cases} 1 \\ 1 \end{cases}$	$-\varepsilon^*$ $-\varepsilon$	$-arepsilon \ -arepsilon^*$	1 1	$-arepsilon^* \ -arepsilon$	$-\varepsilon$ $-\varepsilon^*$	1	$-arepsilon^* \ -arepsilon$	$-arepsilon \ -arepsilon^*$	1	$-\varepsilon^*$ $-\varepsilon$	$egin{array}{c} -arepsilon \ -arepsilon^* \end{array}  ight\}$		$(x^2-y^2,2xy)$
$A_{u}$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	Z	
			1											
$E_{1u}$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$-\varepsilon^*$ $-\varepsilon$	-1 -1	$-\varepsilon$ $-\varepsilon^*$	$arepsilon^*$	-1 -1	- <i>ε</i>	· ε΄ ε	* 1 1	$arepsilon^*$	$egin{array}{c} -arepsilon^* \ -arepsilon \end{array}  ight\}$	(x, y)	
$E_{2u}$	$\begin{cases} 1 \\ 1 \end{cases}$	$-arepsilon^* \ -arepsilon$	-ε -ε	1 * 1	$-arepsilon^* \ -arepsilon$	- <i>ε</i> - <i>ε</i>	* _	·1 ε ·1 ε	* E	-1 -1	$egin{array}{cccc} & arepsilon^* \ & arepsilon \end{array}$	$\left. egin{aligned} arepsilon \ arepsilon^* \end{aligned}  ight\}$		

#### **Character Table 6**

## The Groups $D_{nh}$ (n = 2, 3, 4, 5, 6)

$D_{2\mathrm{h}} \ (mmm)$	Е	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	σ(xy)	$\sigma(xz)$	σ(yz)		
$A_{g}$	1	1	1	1	1	1	1	1		$x^2, y^2, z^2$
$\mathrm{B}_{1\mathrm{g}}$	1	1	-1	-1	1	1	-1	-1	$R_z$	xy
$\mathrm{B}_{\mathrm{2g}}$	1	-1	1	-1	1	-1	1	-1	$R_y$	xz
$\mathrm{B}_{3\mathrm{g}}$	1	-1	-1	1	1	-1	-1	1	$R_x$	yz
$A_{\rm u}$	1	1	1	1	-1	-1	-1	-1		
$\mathbf{B}_{1u}$	1	1	-1	-1	-1	-1	1	1	z	
$\mathrm{B}_{\mathrm{2u}}$	1	-1	1	-1	-1	1	-1	1	y	
$B_{3u}$	1	-1	-1	1	-1	1	1	-1	х	

$D_{3\mathrm{h}}$	Е	$2C_3$	$3C_2$	$\sigma_{\rm h}$	$2S_3$	$3\sigma_{\rm v}$		
$(\overline{6})m2$								
A' <sub>1</sub>	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2'$	1	1	-1	1	1	-1	$R_z$	
E'	2	-1	0	2	-1	0	(x, y)	$(x^2 - y^2, xy)$
$\mathbf{A_1''}$	1	1	1	-1	-1	-1		
$A_2''$	1	1	-1	-1	-1	1	z	
E"	2	-1	0	-2	1	0	$(R_x, R_y)$	(xz, yz)

$D_{4\mathrm{h}} \ (4/mmm)$	Е	$2C_4$	$C_2$	2C' <sub>2</sub>	2C'' <sub>2</sub>	i	$2S_4$	$\sigma_{h}$	$2\sigma_{\rm v}$	$2\sigma_{d}$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	-1	-1	1	1	1	-1	-1	$R_{\rm z}$	
$\mathrm{B}_{\mathrm{1g}}$	1	-1	1	1	-1	1	-1	1	1	-1		$x^2-y^2$
$\mathrm{B}_{\mathrm{2g}}$	1	-1	1	-1	1	1	-1	1	-1	1		xy
$E_{g}$	2	0	-2	0	0	2	0	-2	0	0	$(R_x, R_y)$	(xz, yz)
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	-1	-1	-1	-1	-1	1	1	Z	
$B_{1u}$	1	-1	1	1	-1	-1	1	-1	-1	1		
$\mathrm{B}_{\mathrm{2u}}$	1	-1	1	-1	1	-1	1	-1	1	-1		
$E_{u}$	2	0	-2	0	0	-2	0	2	0	0	(x, y)	

## 6. The Groups $D_{nh}$ (n = 2, 3, 4, 5, 6) (cont...)

$D_{5h}$	E	$2C_5$	$2C_5^2$	$5C_2$	$\sigma_{\text{h}}$	$2S_5$	$2S_{5}^{3}$	$5\sigma_{\rm v}$		
A' <sub>1</sub>	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2'$	1	1	1	-1	1	1	1	-1	$R_z$	
$E_1'$	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	(x, y)	
$E_2'$	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2-y^2,2xy)$
$\mathbf{A_1''}$	1	1	1	1	-1	-1	-1	-1		
$A_2''$	1	1	1	-1	-1	-1	-1	1	z	
$E_1''$	2	2 cos 72°	2 cos 144°	0	-2	−2 cos 72°	-2 cos 144°	0	$(R_x, R_y)$	(xy, yz)
$E_2''$	2	2 cos 144°	2 cos 72°	0	-2	-2 cos 144°	-2 cos 72°	0		

D <sub>6h</sub> (6/ <i>mmm</i> )	Е	$2C_6$	2 <i>C</i> <sub>3</sub>	C <sub>2</sub>	$3C_2'$	3C'' <sub>2</sub>	i	$2S_3$	$2S_6$	$\sigma_{h}$	$3\sigma_{d}$	$3\sigma_{\rm v}$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	-1	-1	1	1	1	1	-1	-1	$R_z$	
$\mathrm{B}_{\mathrm{1g}}$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
$\mathrm{B}_{\mathrm{2g}}$	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1		
$E_{1g}$	2	1	-1	-2	0	0	2	1	-1	-2	0	0	$(R_x - R_y)$	(xz, yz)
$\mathrm{E}_{2\mathrm{g}}$	2	-1	-1	2	0	0	2	-1	-1	2	0	0		$(x^2-y^2, 2xy)$
$A_{1u}$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\boldsymbol{z}$	
$B_{1u}$	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
$B_{2u}$	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
$E_{1u}$	2	1	-1	-2	0	0	-2	-1	1	2	0	0	(x, y)	
$E_{2u}$	2	-1	-1	2	0	0	-2	1	1	-2	0	0		

## 7. The Groups $D_{nd}$ (n = 2, 3, 4, 5, 6)

$D_{2d} = V_{d}$ $(\overline{42})m$	Е	$2S_4$	$C_2$	$2C_2'$	$2\sigma_{d}$		
$A_1$	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$R_z$	
$\mathbf{B}_1$	1	-1	1	1	-1		$x^2-y^2$
$\mathrm{B}_2$	1	-1	1	-1	1	z	xy
E	2	0	-2	0	0	(x, y)	(xz, yz)
						$(R_x, R_y)$	

$D_{3d}$	Е	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_{\rm d}$		
$(\overline{3})m$								
$A_{1g}$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	-1	1	1	-1	$R_z$	
$E_{g}$	2	-1	0	2	-1	0	$(R_x, R_y)$	$(x^2 - y^2, 2xy)$ $(xz, yz)$
$A_{1u}$	1	1	1	-1	-1	-1		
$A_{2u}$	1	1	-1	-1	-1	1	z	
$E_{u}$	2	-1	0	-2	1	0	(x, y)	

$D_{ m 4d}$	E	$2S_8$	$2C_4$	$2S_{8}^{3}$	$C_2$	$4C_2'$	$4\sigma_{d}$		
$A_1$	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	-1	-1	$R_z$	
$\mathbf{B}_1$	1	-1	1	-1	1	1	-1		
$\mathrm{B}_2$	1	-1	1	-1	1	-1	1	Z	
$E_1$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	(x, y)	
$E_2$	2	0	-2	0	2	0	0		$(x^2 - y^2, 2xy)$
$E_3$	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	$(R_x, R_y)$	(xz, yz)

# 7. The Groups $D_{nd}$ (n = 2, 3, 4, 5, 6) (cont..)

$D_{5d}$	Е	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^{3}$	$2S_{10}$	$5\sigma_{d}$		
$A_{1g}$	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	-1	1	1	1	-1	$R_{\rm z}$	
$\mathrm{E}_{1g}$	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	$(R_x, R_y)$	(xy, yz)
$\mathrm{E}_{2\mathrm{g}}$	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2-y^2, 2xy)$
$A_{1u}$	1	1	1	1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	-1	-1	-1	-1	1	$\boldsymbol{z}$	
$\mathrm{E}_{1\mathrm{u}}$	2	2 cos 72°	2 cos 144°	0	-2	-2 cos 72°	-2 cos 144°	0	(x, y)	
$E_{2u} \\$	2	2 cos 144°	2 cos 72°	0	-2	−2 cos 144°	−2 cos 72°	0		

$D_{ m 6d}$	Е	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^{5}$	$C_2$	6C' <sub>2</sub>	$6\sigma_{d}$		
$A_1$	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	1	1	-1	-1	$R_z$	
$\mathbf{B}_1$	1	-1	1	-1	1	-1	1	1	-1		
$B_2$	1	-1	1	-1	1	-1	1	-1	1	z	
$E_1$	2	$\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	(x, y)	
$E_2$	2	1	-1	-2	-1	1	2	0	0		$(x^2-y^2, 2xy)$
$E_3$	2	0	-2	0	2	0	-2	0	0		
$E_4$	2	-1	-1	2	-1	-1	2	0	0		
$E_5$	2	$\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	$(R_x, R_y)$	(xy, yz)

## 8. The Groups $S_n$ (n = 4, 6, 8)

$S_4$ $(\overline{4})$	Е	$S_4$	$C_2$	$S_4^3$		
A	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
В	1	-1	1	-1	z	$(x^2-y^2,2xy)$
E	$\begin{cases} 1 \\ 1 \end{cases}$	i –i	-1 -1	-i $i$	$(x,y)(R_x,R_y)$	(xz, yz)

$\frac{S_6}{(\overline{3})}$	Е	$C_3$	$C_3^2$	i	$S_6^5$	$S_6$		$\varepsilon = \exp(2\pi i/3)$
$A_{g}$	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$\mathrm{E}_{\mathrm{g}}$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$arepsilon^*$	1 1	$arepsilon^*$	$\left. egin{array}{c} arepsilon^* \ arepsilon \end{array}  ight.  ight.$	$(R_x, R_y)$	$(x^2 - y^2, 2xy) (xy, yz)$
$\mathbf{A}_{\mathrm{u}}$	1	1	1	-1	-1	-1	Z	
Eu	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$arepsilon^*$	1 1	$\mathcal{E}$	$\left. egin{aligned} arepsilon^* \ arepsilon \end{aligned}  ight.  ight.$	(x, y)	

$S_8$				-		$S_8^5$	-	0		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
В	1	-1	1	-1	1	-1	1	-1	$\boldsymbol{z}$	
$E_1$	$\begin{cases} 1 \\ 1 \end{cases}$	${m arepsilon}^*$	i –i	$-\varepsilon^*$ $-\varepsilon$	-1 -1	$-arepsilon \ -arepsilon^*$	-i i	$\begin{bmatrix} 1 \\ -1 \\ \varepsilon^* \\ \varepsilon \end{bmatrix}$	(x, y)	
$E_2$	$\begin{cases} 1 \\ 1 \end{cases}$	i -i	-1 -1	-i i	1 1	i -i	-1 -1	-i $i$		$(x^2-y^2,2xy)$
E <sub>3</sub>	$\begin{cases} 1 \\ 1 \end{cases}$	$-arepsilon^* \ -arepsilon$	-і і	$\mathcal{E}$	-1 -1	$arepsilon^*$	i –i	$-\mathcal{E} \left. igg  -\mathcal{E}^*  ight.$	$(R_x, R_y)$	(xy, yz)

## 9. The Cubic Groups

<i>T</i> (23)	Ε	4 <i>C</i> <sub>3</sub>	$4C_3^2$ $3C_2$	$\varepsilon = \exp(2\pi i/3)$
A	1	1	1 1	$x^2 + y^2 + z^2$
Е	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$egin{array}{ccc} arepsilon^* & & 1 \ arepsilon & & 1 \ \end{array}$	$(\sqrt{3}(x^2-y^2)2z^2-x^2-y^2)$
T	3	0	0 –1	$(x, y, z)   (xy, xz, yz) $ $(R_x, R_y, R_z)$

$T_{\rm d} \over (\overline{4}3m)$	E	8 <i>C</i> <sub>3</sub>	$3C_2$	$6S_4$	$6\sigma_d$		
$A_1$	1	1	1	1	1		$x^2 + y^2 + z^2$
$A_2$	1	1	1	-1	-1		
E	2	-1	2	0	0		$(2z^2-x^2-y^2, \sqrt{3}(x^2-y^2))$
$T_1$	3	0	-1	1	-1	$(R_x, R_y, R_z)$	
T <sub>2</sub>	3	0	-1	-1	1	(x, y, z)	(xy, xz, yz)

$T_{\rm h}$ $(m3)$	Е	4 <i>C</i> <sub>3</sub>	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_{6}^{2}$	$3\sigma_d$		$\varepsilon = \exp(2\pi i/3)$
$A_{g}$	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
$\mathrm{E}_{g}$	$\begin{cases} 1 \\ 1 \end{cases}$	$arepsilon^*$	$oldsymbol{arepsilon}^* \ oldsymbol{arepsilon}$	1 1	1 1	${oldsymbol{arepsilon}}^*$	$arepsilon^*$	1 $1$		$(2z^2 - x^2 - y^2, \sqrt{3} (x^2 - y^2)$
$T_{g}$	3	0	0	-1	3	0	0	-1	$(R_x, R_y, R_z)$	(xy, yz, xz)
$\mathbf{A}_{\mathrm{u}}$	1	1	1	1	-1	-1	-1	-1		
$E_{u}$	$\begin{cases} 1 \\ 1 \end{cases}$	${oldsymbol{arepsilon}^*}$	$arepsilon^* \ arepsilon$	1 1	-1 -1	$-\varepsilon$ $-\varepsilon^*$	$-arepsilon^* \ -arepsilon$	-1 $-1$		
$T_{\rm u}$	3	0	0	-1	-3	0	0	1	(x, y, z)	

0	Е	$8C_3$	$3C_2$	$6C_{4}$	6C' <sub>2</sub>	
(432)					2	
$A_1$	1	1	1	1	1	$x^2 + y^2 + z^2$
$A_2$	1	1	1	-1	-1	
E	2	-1	2	0	0	$(2z^2 - x^2 - y^2, \sqrt{3} (x^2 - y^2))$
						$\sqrt{3} (x^2 - y^2)$
$T_1$	3	0	-1	1	-1	(x, y, z)
						$(R_x, R_y, R_z)$
$T_2$	3	0	-1	-1	1	(xy, xz, yz)

## 9. The Cubic Groups (cont...)

<i>O</i> <sub>h</sub> ( <i>m</i> 3 <i>m</i> )	Ε	8 <i>C</i> <sub>3</sub>	6C <sub>2</sub>	6 <i>C</i> <sub>4</sub>	$3C_2 = C_4^2$	i	6S <sub>4</sub>	8 <i>S</i> <sub>6</sub>	$3\sigma_h$	6σ <sub>d</sub>		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
$A_{2g}$	1	1	-1	-1	1	1	-1	1	1	-1		
$E_{g}$	2	-1	0	0	2	2	0	-1	2	0		$(2z^2 - x^2 - y^2, \sqrt{3} (x^2 - y^2))$
$T_{1g}$	3	0	-1	1	-1	3	1	0	-1	-1	$(R_x, R_y, R_z)$	
$T_{2g}$	3	0	1	-1	-1	3	-1	0	-1	1		(xy, xz, yz)
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	-1	-1	1	-1	1	-1	-1	1		
$E_{u}$	2	-1	0	0	2	-2	0	1	-2	0		
$T_{1u}$	3	0	-1	1	-1	-3	-1	0	1	1	(x, y, z)	
$T_{2u}$	3	0	1	-1	-1	-3	1	0	1	-1		

## 10. The Groups I, Ih

I	Е	12 <i>C</i> <sub>5</sub>	$12C_{5}^{2}$	$20C_{3}$	15 <i>C</i> <sub>2</sub>		$\eta^{\pm} = \frac{1}{2} \left( 1 \pm 5^{\frac{1}{2}} \right)$
A	1	1	1	1	1		$x^2+y^2+z^2$
$T_1$	3	$\eta^+$	$\eta^-$	0	-1	$(x, y, z)  (R_x, R_y, R_z)$	
$T_2$	3	$\eta^-$	$\eta^{^+}$	0	-1		
G	4	-1	-1	1	0		
Н	5	0	0	-1	1		$(2z^{2} - x^{2} - y^{2}, \sqrt{3} (x^{2} - y^{2}) xy, yz, zx)$

$I_{\rm h}$ $E$	12 <i>C</i> <sub>5</sub>	$12C_{5}^{2}$	20 <i>C</i> <sub>3</sub>	$15C_{2}$	i	$12S_{10}$	$12S_{10}^{3}$	20 <i>S</i> <sub>6</sub>	5 15 <sub>σ</sub>		$\eta^{\pm} = \frac{1}{2} \left( 1 \pm 5^{\frac{1}{2}} \right)$
A <sub>g</sub> 1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
$T_{1g}$ 3	$\eta^{^+}$	$\eta^-$	0	-1	3	$\eta^-$	$\eta^{^{+}}$	-1	-1	$(R_x,R_y,R_z)$	
$T_{2g}$ 3	$\eta^-$	$\eta^{^{+}}$	0	-1	3	$\eta^{^{+}}$	$\eta^-$	0	-1		
$G_g$ 4	-1	-1	1	0	4	-1	-1	1	0		
H <sub>g</sub> 5	0	0	-1	1	5	0	0	-1	1		$(2z^{2}-x^{2}-y^{2}, \sqrt{3} (x^{2}-y^{2})) (xy, yz, zx)$
$A_u$ 1	1	1	1	1	-1	-1	-1	-1	-1		
$T_{1u}$ 3	$\eta^{^+}$	$\eta^-$	0	-1	-3	$\eta^-$	$\eta^{^+}$	0	1	(x, y, z)	
$T_{2u}$ 3	$\eta^-$	$\eta^{^{+}}$	0	-1	-3	$\eta^{^{+}}$	$\eta^-$	0	1		
$G_u$ 4	-1	-1	1	0	<b>-4</b>	1	1	-1	0		
H <sub>u</sub> 5	0	0	-1	1	-5	0	0	1	-1		

# 11. The Groups $C_{\infty v}$ and $D_{\infty h}$

$C_{\infty_{ m V}}$	E	$C_2$	$2C_{\infty}^{\phi}$	 $\infty \mathbf{Q}^{\mathrm{A}}$		
$A_1 \equiv \sum^+$	1	1	1	 1	Z	$x^2 + y^2, z^2$
$A_2 = \sum_{-}^{-}$	1	1	1	 -1	$R_z$	
$E_1 \equiv \Pi$	2	-2	$2\cos\phi$	 0	$(x,y)(R_x,R_y)$	(xz, yz)
$E_2 = \Delta$	2	2	$2\cos 2\phi$	 0		$(x^2-y^2,2xy)$
Е₃≡Ф	2	-2	$2\cos 3\phi$	 0		

$D_{\infty \mathrm{h}}$	E	$2C_{\infty}^{\phi}$		$\infty \sigma_{\rm v}$	i	$2S_{\infty}^{\phi}$		$\infty C_2$		
$\Sigma_g^+$	1	1		1	1	1		1		$x^2 + y^2, z^2$
$\Sigma_g^-$	1	1		-1	1	1		-1	$R_z$	
$\prod_{g}$	2	$2\cos\phi$		0	2	$-2\cos\phi$		0	$(R_x, R_y)$	(xz, yz)
$\Delta_{ m g}$	2	$2\cos 2\phi$		0	2	$2\cos 2\phi$		0		$(x^2-y^2,2xy)$
• • •	• • •		• • •	• • •	• • •	•••		• • •		
$\Sigma_u^+$	1	1		1	-1	-1		-1	Z	
$\Sigma_u^-$	1	1		-1	-1	-1		1		
$\prod_{\mathrm{u}}$	2	$2\cos\phi$		0	-2	$2\cos\phi$		0	(x,y)	
$\Delta_{\mathrm{u}}$	2	$2\cos 2\phi$		0	-2	$-2\cos 2\phi$		0		
•••			•••	• • •		•••	•••			