CGRA 151 Introduction to Computer Graphics
Mathematics Worksheet

Give answers to the following twenty mathematics questions. You may handwrite or typeset your answers but you must submit your answers as a PDF le via the ECS submission system.

You are given the following vectors and matrices:

$$\mathbf{a} = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 8 \\ -4 \\ 8 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 8 \\ 0 \\ 6 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 5 & 0 \\ -2 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

1. $\mathbf{a} + \mathbf{b}$ $= \begin{bmatrix} 1+8 \\ 4+-4 \\ 8+8 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 16 \end{bmatrix}$
2. $\mathbf{c} + \mathbf{d}$ $= \begin{bmatrix} 2+8 \\ -2+0 \\ 1+6 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 7 \end{bmatrix}$
3. $3\mathbf{a}$ $= \begin{bmatrix} 3\times1 \\ 3\times4 \\ 3\times8 \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \\ 24 \end{bmatrix}$
4. $-2\mathbf{b}$ $= \begin{bmatrix} -2\times8 \\ -2\times-4 \\ -2\times8 \end{bmatrix} = \begin{bmatrix} -16 \\ 8 \\ -16 \end{bmatrix}$
5. $\mathbf{a} - \mathbf{b}$ $a-b = \begin{bmatrix} 1-8 \\ 4--4 \\ 8-8 \end{bmatrix} = \begin{bmatrix} -7 \\ 8 \\ 0 \end{bmatrix}$
6. $|\mathbf{a}|$ $= \sqrt{1^2+4^2+8^2} = \sqrt{1+16+64} = \sqrt{81} = 9$
7. $|\mathbf{b}|$ $= \sqrt{8^2+(-4)^2+8^2} = \sqrt{64+16+64} = \sqrt{144} = 12$
8. $\mathbf{a} \cdot \mathbf{b}$ $= (1\times8)+(4\times-4)+(8\times8) = 8-16+64 = 56$
9. $\mathbf{c} \cdot \mathbf{d}$ $= (2\times8)+(-2\times0)+(1\times6) = 16+0+6 = 22$
10. What is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$? $|a| = 9$, $|b| = 12$, $a \cdot b = 56$ $\cos\theta = \frac{56}{9\cdot12} \approx 0.5185$ $\theta = \cos^{-1}(0.5185) \approx 58.77$
11. What is the angle between vectors $\mathbf{c}$ and $\mathbf{d}$? $|c| = 3$, $|d| = 10$, $c \cdot d = 22$ $\cos\theta = \frac{22}{3\cdot10} \approx 0.7333\ldots$ $\theta = \cos^{-1}(0.7333) \approx 42.83^\circ$
12. How long is the projection of vector $\mathbf{c}$ onto vector $\mathbf{d}$? $c \cdot d = 22$, $|c| = 3$ $l = \frac{22}{3} = 7.333\ldots$
13. Calculate $\mathbf{e}$, the linear interpolation between $\mathbf{c}$ and $\mathbf{d}$, $\mathbf{e} = (1-t)\mathbf{c} + t\mathbf{d}$, for $t = 0.8$.
14. $\mathbf{Ab}$ $= \begin{bmatrix} 2 & 5 & 0 \\ -2 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -4 \\ 8 \end{bmatrix} = \begin{bmatrix} 2\times8+5\times-4+0\times8 \\ -2\times8+3\times-4+0\times8 \\ -1\times8+0\times-4+2\times8 \end{bmatrix} = \begin{bmatrix} 16-20 \\ -16-12 \\ -8+16 \end{bmatrix} = \begin{bmatrix} -4 \\ -28 \\ 8 \end{bmatrix}$
15. $\mathbf{Bc}$ $= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\times2+0\times-2+0\times1 \\ 0\times2+0\times4+0\times1 \\ 0\times2+0\times-2+1\times1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix}$
16. $\mathbf{A} + \mathbf{B}$ $= \begin{bmatrix} 2 & 5 & 0 \\ -2 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+3 & 5+0 & 0+0 \\ -2+0 & 3+4 & 0+0 \\ -1+0 & 0+0 & 2+1 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 0 \\ -2 & 7 & 0 \\ -1 & 0 & 3 \end{bmatrix}$
17. $\mathbf{AB}$ $= \begin{bmatrix} 2 & 5 & 0 \\ -2 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 20 & 0 \\ -6 & 12 & 0 \\ -3 & 0 & 2 \end{bmatrix}$
18. $\mathbf{BC}$ $BC = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 15 \\ 0 & 4 & 8 \\ 0 & 0 & 1 \end{bmatrix}$
19. What two-dimensional transformation is represented by the 3×3 matrix $\mathbf{C}$? Translate (5, 2)
20. Give a 3×3 matrix that represents a rotation in two-dimensional space of $60°$.

$$\begin{bmatrix} \cos60^\circ & -\sin60^\circ & 0 \\ \sin60^\circ & \cos60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$