Used concepts

Diffraction, diffraction sharpness, Kirchhoff's diffraction formula, measurement precision, local uncertainty, impulse uncertainty, wave-matter duality, de Broglie's relation.

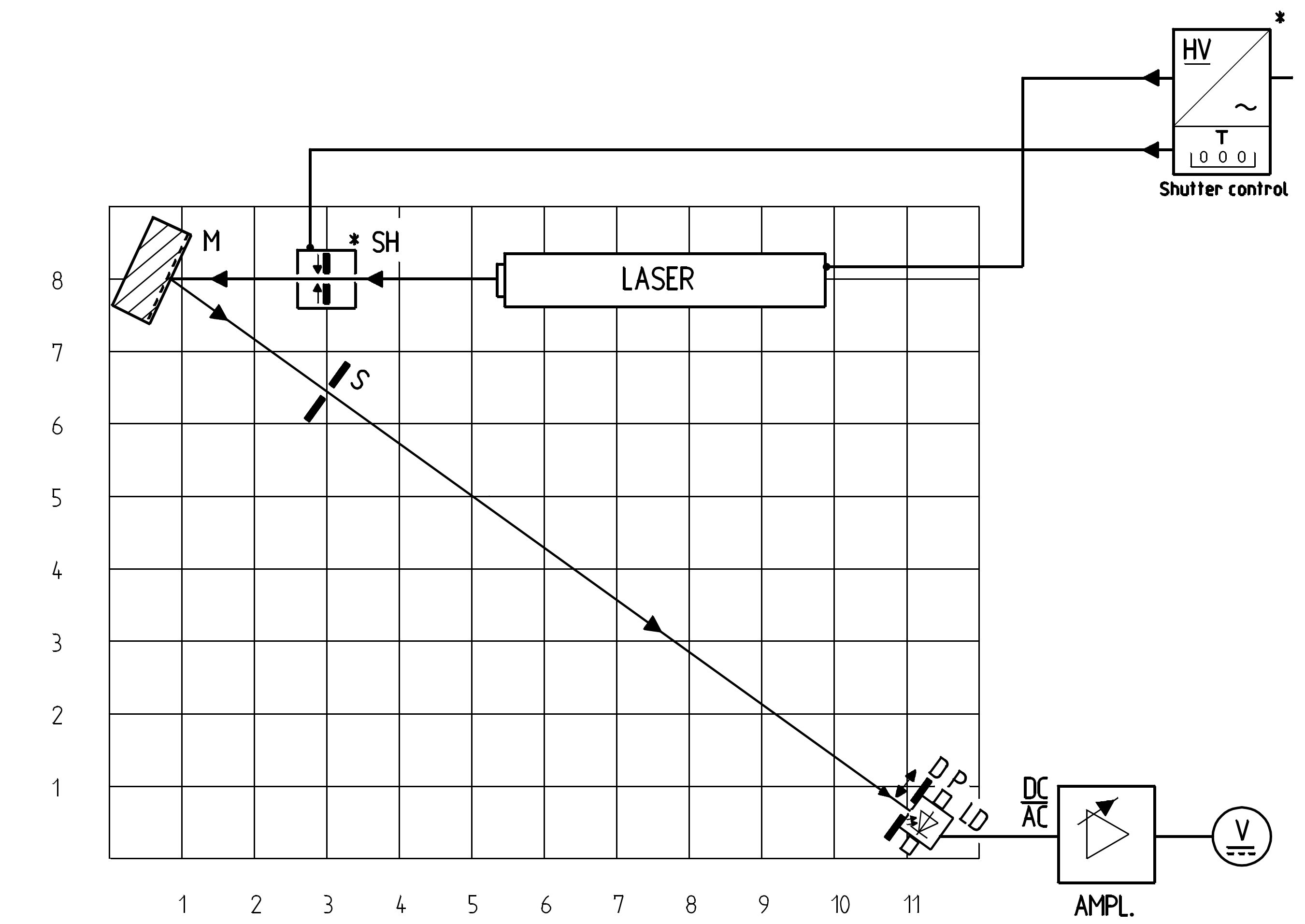
Principle

The intensity distribution in the Fraunhofer diffraction pattern of a slit is measured. Measurement results are evaluated both in the wave representation through comparison with Kirchhoff's diffraction fromula and in the photon representation, in order to verify Heisenberg's uncertainty principle.

Equipment

Base plate with rubber feet	08700.00	1
HeNe laser	08180.93	1
Adjustable mounting 35 x 35 mm	08711.00	1
Surface mirror 30 x 30 mm	08711.01	1
Photocell, silicone	08734.00	1
Sliding device, horizontal	08713.00	1
Magnet foot	08710.00	3
Aperture screen support	08724.00	1
Aperture with three simple slits	08522.00	1
Voltmeter 0.3...300 V/10...300 V~	07035.00	1
Measurement amplifier, universal	13626.93	1
Connecting cable red, *l* = 500 mm	07361.01	2

Fig. 1: Experimental set up for diffraction through a slit (* only required for 5 mW laser)



Problem

1. The intensity distribution of the Fraunhofer diffraction pattern due to a simple slit is measured. The amplitudes of the peaks and of the minima are calculated according to Kirchhoff’s diffraction formula and compared to measured values.
2. Momentum uncertainty is calculated with the assistance of the diffraction patterns of simple slits of different widths, and Heisenberg’s uncertainty relation is verified.

Set- up and performance

— The experimental set up is shown in fig. 1. The recommended set up height (height of beam path) should be 130 mm.

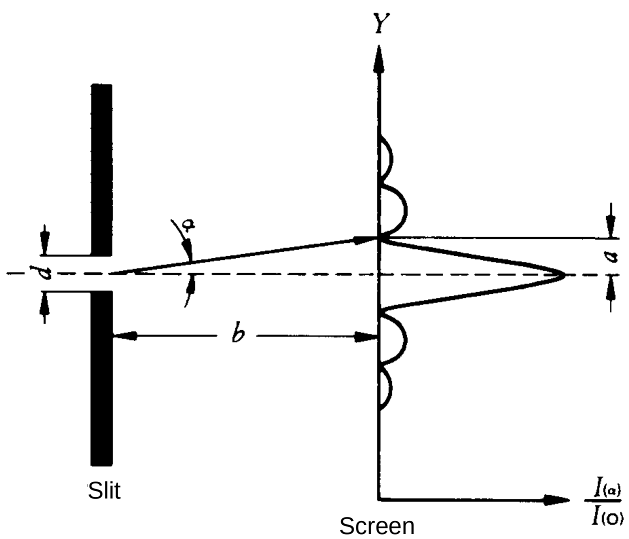
— The laser should be switched on about 1/2 an hour before beginning the experiment, in order to be sure laser light intensity will be constant.

— Laser light is adjusted successively onto slits **S** of different width by means of adjustable mirror **M**.

— Intensity distribution is measured by means of photocell **LD** on the measurement sliding device. Voltage is proportional to the intensity of incident light. Shifting is carried out by means of a micrometer (1 U $\hat{=}$ 500 µm).

— For one width of the slit, the main peak is recorded on one side, together with the first secondary peak. For the other widths of the slit, it is sufficient to record both minima to the right and to the left of the main peak, in order to determine α (fig. 2).

Fig. 2: Farfield diffraction according to Fraunhofer through a slit



— Measurement should be carried out in a dark room or at constant ambient light (setting of zero on the universal measurement amplifier).

Theory and evaluation

1. Consideration according to the wave representation

When a parallel, monochromatic and coherent light beam of wavelength λ goes through a simple slit of width *d*, a diffraction pattern with a main peak and several secondary peaks is obtained on the screen (fig. 2).

According to Kirchhoff’s diffraction formula, intensity, which is a function of the deflection angle α, is:

$$J(\alpha) = J(0) \cdot \left(\frac{\sin\beta}{\beta}\right)^2 \tag{1}$$

where

$$\beta = \frac{\pi d}{\lambda} \cdot \sin\alpha$$

Intensity minima are situated at

$$\alpha_n = \arcsin n \cdot \frac{\lambda}{d}$$

where n = 1, 2, 3, ...

For the intensity peaks, one obtains the angles

$$\alpha_0' = 0$$

$$\alpha_1' = \arcsin 1.430 \cdot \frac{\lambda}{d}$$

$$\alpha_2' = \arcsin 2.459 \cdot \frac{\lambda}{d}$$

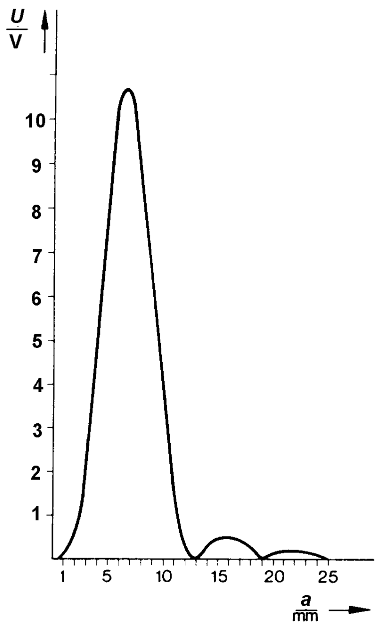
The relative height of the secondary peaks is:

$$J(\alpha_1) = 0.0472 \cdot J(0)$$

$$J(\alpha_1) = 0.0165 \cdot J(0)$$

Measurement values (fig. 3) are compared to the calculated values.

Fig. 3: Intensity distribution of the diffraction pattern of a 0.05 mm wide slit, at a distance of 490 mm



Minima

Measurement	Calculation
$\alpha_1 = 0.36^o$	$\alpha_1 = 0.36^o$
$\alpha_2 = 0.72^o$	$\alpha_2 = 0.72^o$
$\alpha_3 = 1.04^o$	$\alpha_3 = 1.07^o$

Maxima

Measurement	Calculation
$\alpha'_1 = 0.52^o$	$\alpha'_1 = 0.51^o$
$\alpha'_2 = 0.88^o$	$\alpha'_2 = 0.88^o$

$$\frac{J(\alpha_1)}{J(0)} = 0.044; \qquad \frac{J(\alpha_1)}{J(0)} = 0.047$$

$$\frac{J(\alpha_2)}{J(0)} = 0.014; \qquad \frac{J(\alpha_2)}{J(0)} = 0.017$$

This allows the verification of Kirchhoff's diffraction formula within the limits of errors.

2. Consideration according to Quantum Mechanics

Heisenberg's uncertainty principle states that two canonically conjugated magnitudes, e. g. location and momentum, cannot be determined exactly simultaneously.

For example, a group of photons are considered, whose sojourn probability is described by the function f_y and whose momentum is described by the function f_p. The uncertainties of location and of momentum are defined by the standard deviations, which yields:

$$\Delta y \cdot \Delta p \ \geq \frac{h}{4\pi} \qquad (2)$$

(h = 6.6262 $\cdot$ 10^{-34} Js, Plancks constant), equality being valid for variables with gaussian distribution.

For a stream of photons which cross a slit of width d, one sets

$$\Delta y = d \ . \qquad (3)$$

Whereas the photons situated before the slit can only move perpendicularly to the plane of the slit (x-direction), they also have a component in the y-direction behind the slit.

The probability density for velocity component v_y is given through the intensity distribution in the diffraction pattern. The first minimum is used to define the velocity uncertainty (figs. 2 and 4).

$$\Delta v_y = c \cdot \sin \alpha_1 \qquad (4)$$

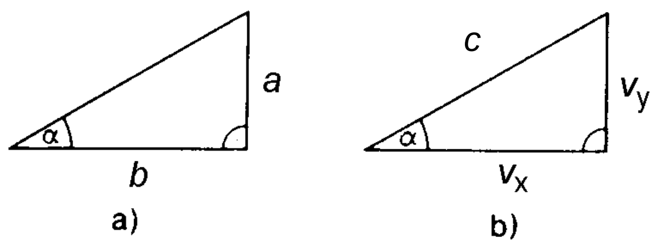
(α_1 = angle of the first minimum).

This yields the momentum uncertainty:

$$\Delta p_y = m \cdot c \cdot \sin \alpha_1 \qquad (5)$$

(m = photon mass, c = velocity of light).

Fig. 4: Geometry for diffraction through a simple slit; a) travelled path; b) velocity components of a photon



Momentum and wavelength of a particle are related through de Broglie's relation:

$$\frac{h}{\lambda} = p = m \cdot c \qquad (6)$$

Thus,

$$\Delta p_y = \frac{h}{\lambda} \cdot \sin \alpha_1 \qquad (7)$$

According to (1), the angle $\alpha 1$ of the first minimum results to be

$$\sin \alpha_1 = \frac{\lambda}{d} \qquad (8)$$

Introducing (8) into (7) and (3), the uncertainty principle yields:

$$\Delta y \cdot \Delta p_y = h \qquad (9)$$

If the width of the slit, Δy, decreases, the first minimum of the diffraction pattern is found for larger angles α_1.
In the experiment, angle α_1 is given by the location of the first minimum (fig. 4a)

$$\tan \alpha_1 = \frac{a}{b} \qquad (10)$$

Setting (10) into (7) yields

$$\Delta p = \sin(\arctan \frac{a}{b}) \qquad (11)$$

Setting (3) and (11) into (9) and dividing through h yields:

$$\frac{d}{\lambda} \sin (\arctan \frac{a}{b}) = 1 \qquad (12)$$

The results of measurements verify (12) within the limits of errors.

Table 1:

Width of slit* d/mm	1^{st} minimum a/mm	$\frac{d}{\lambda} \sin (\arctan \frac{a}{b})$ b = 485mm
0.101	3.05	1.00
0.202	1.50	0.99
0.051	6.06	1.01

* the widths of the slits were measured using a microscope.