

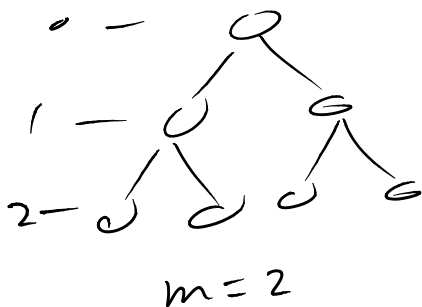
Btree of order m (m is max number of child)

level	nodes		keys	
	min	max	min	max
0	1	1	1	$(m-1)$
1	$2 \lceil \frac{m}{2} \rceil^0$	m	$2(\lceil \frac{m}{2} \rceil - 1)$	$m(m-1)$
2	$2 \lceil \frac{m}{2} \rceil^1$	m^2	$2 \lceil \frac{m}{2} \rceil^1 (\lceil \frac{m}{2} \rceil - 1)$	$m^2(m-1)$
\vdots	\vdots	\vdots	\vdots	\vdots
k	$2 \lceil \frac{m}{2} \rceil^{k-1}$	m^k	$2 \lceil \frac{m}{2} \rceil^{k-1} (\lceil \frac{m}{2} \rceil - 1)$	$m^k(m-1)$
$\sum_{i=0}^n a^i = \frac{a^{n+1} - 1}{a - 1} \Rightarrow \sum_{i=0}^k n_i = \frac{m^{k+1} - 1}{m - 1}$				

max nodes

max keys = $(m-1) \times$ max nodes

$$\text{max keys} = \frac{(m-1) \frac{m^{k+1} - 1}{m - 1}}{(m-1)} = (m^{k+1} - 1)$$



$$m=4$$

$$m^{k+1} - 1$$

$$2^{2+1} - 1 = 7$$

$$4^{2+1} - 1 = 63$$

$$M = 1000$$

$$1000^{2+1} - 1 =$$

$$1,000,000,000 - 1$$

min number of nodes

$$1 + \sum_{i=0}^{k-1} 2 \left\lceil \frac{m}{2} \right\rceil^i$$

$$1 + 2 \sum_{i=0}^{k-1} \left\lceil \frac{m}{2} \right\rceil^i$$

$$1 + 2 \frac{\left(\left\lceil \frac{m}{2} \right\rceil^k - 1 \right)}{\left\lceil \frac{m}{2} \right\rceil - 1}$$

min nodes

min keys

$$1 + 2 \frac{\left(\left\lceil \frac{m}{2} \right\rceil^k - 1 \right)}{\left\lceil \frac{m}{2} \right\rceil - 1} \cdot \left(\left\lceil \frac{m}{2} \right\rceil + 1 \right)$$

$$\textcircled{1} + \textcircled{2} \left\lceil \frac{m}{2} \right\rceil^k - \textcircled{1}$$

$$2 \left\lceil \frac{m}{2} \right\rceil^k - 1$$

$$m = 1000$$

$$2 \left\lceil \frac{1000}{2} \right\rceil^2 - 1$$

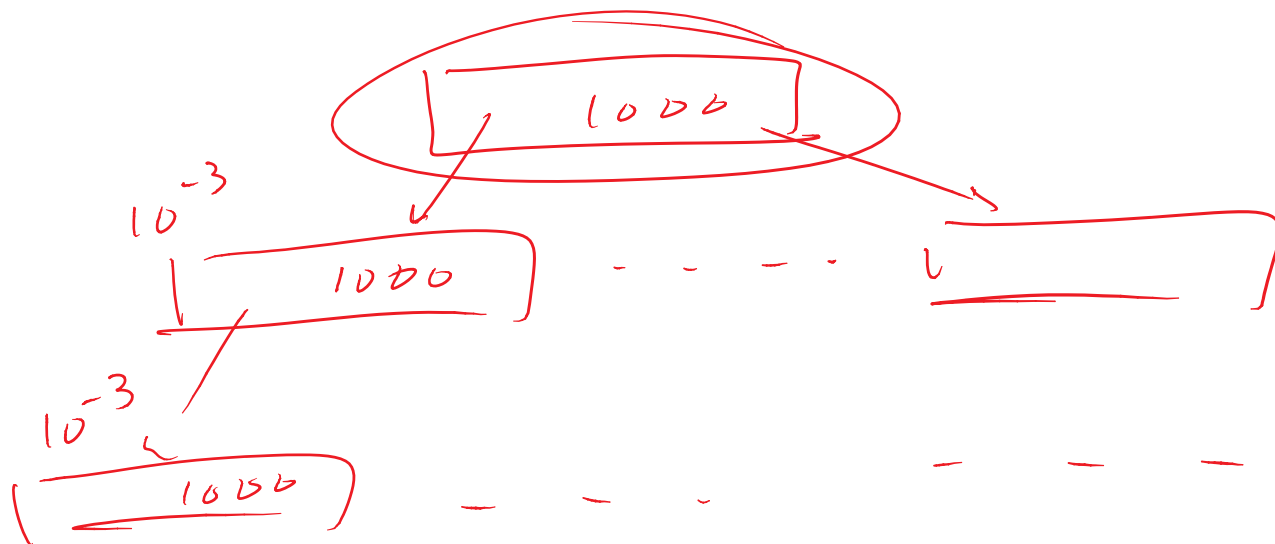
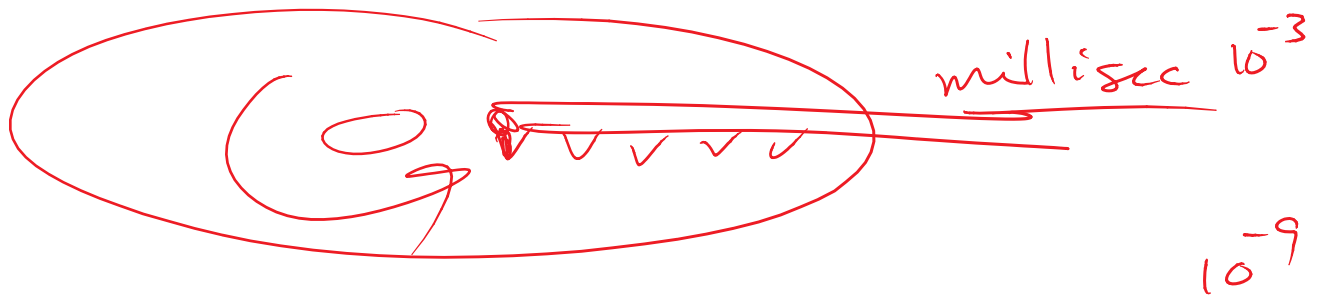
$$2(500 \cdot 500) - 1$$

$$2(250000) - 1$$

$$500,000 - 1$$



Operating System - physical memory
disk



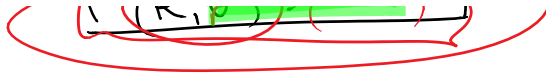
Variation on Btree

B-tree $\lceil \frac{m}{2} \rceil \leftrightarrow m$ $1 \rightarrow 2$

B^* -tree $\lceil \frac{2m}{3} \rceil \leftrightarrow m$ $2 \rightarrow 3$

B^+ -tree





$$O(n)$$

$$O(\log n)$$

$$O(1)$$