probabilistic data structure

average cost —
over n operations

skip list

head

$n/2$

n

Generate height $\Rightarrow$

* geometric distribution

\# bernolli distribution
   Head p   Tail 1-p

\# number of consecutive heads in n bernolli trails

	Number of Heads	Prob
H	1	$\left(\frac{1}{2}\right)$
HH	2	$\left(\frac{1}{4}\right)$
HHH	3	$\left(\frac{1}{8}\right)$
$\vdots$	$\vdots$	
	k	$\left(\frac{1}{2^k}\right)$
H ... H		

↑↑
\# levels in our node

1, 2, 3, 4 - - - - - - . 100 →

↑

25   50   75

Pseudo Random Number Generators
LCG Linear Congruential Generator

Seed $S_0 = 1234$

$$S_1 = (A * S_0 + C) \% M$$

S_0 ———————————◯//////////// S_M

$$X = 15$$

Probability of getting to level
K with n nodes

$$P_K \leq \frac{1}{2^k} + \frac{1}{2^k} + \frac{1}{2^k} \cdots \frac{1}{2^k}$$

$$T_K = \underbrace{2^k \; 2^k \; 2^k \quad 2}_{n}$$

$$P_K \leq \frac{n}{2^k}$$
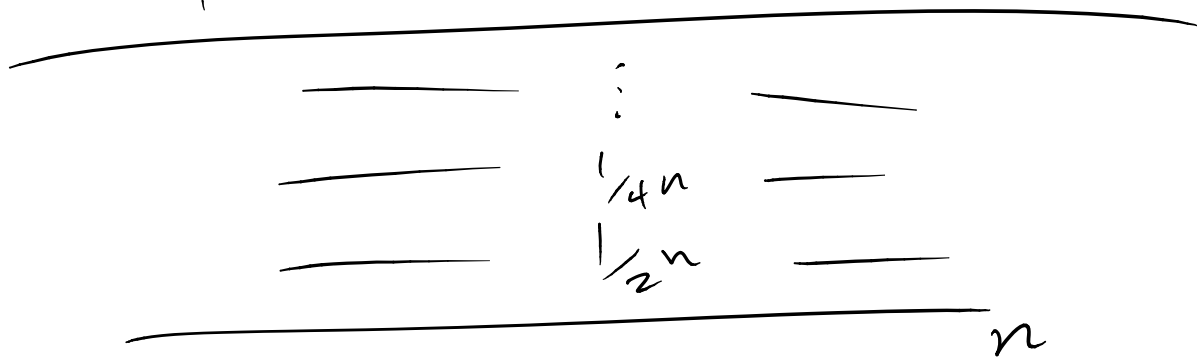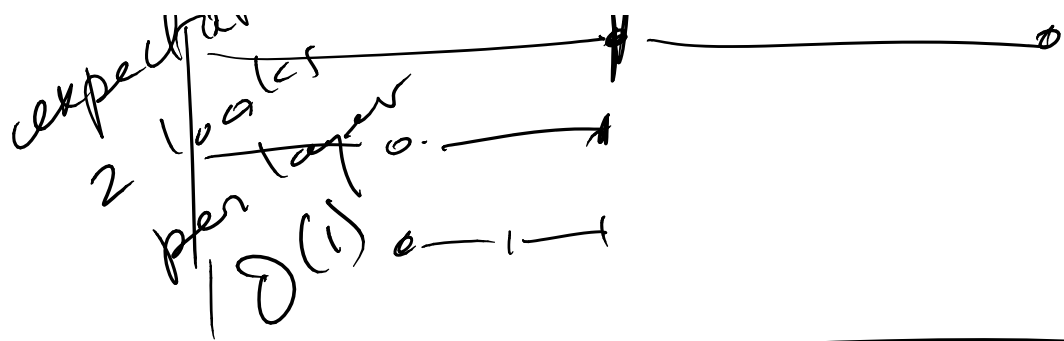
Let $k = c \log_2 n$ $c = constant$

$$P_{c \log_2 n} \leq \frac{n}{2^{c \log_2 n}}$$

$$P \leq \frac{n}{n^c}$$

$$\boxed{P \leq \frac{1}{n^{c-1}}}$$

Seach
Insert $\}$ $O(\log n)$
Remove

expectation

$\sum \frac{k \leq}{2^k} N$ ... → 0

expected

2 lookes
per layer
$O(1)$

$$\frac{1}{4}n$$

$$\frac{1}{2}n$$

n

$\sim 2n$ $O(n)$