Move Exam 2

Monday Nov 17th ⟨ in-class / DL 8PM

Irreflexive

Reflexive

Symmetric

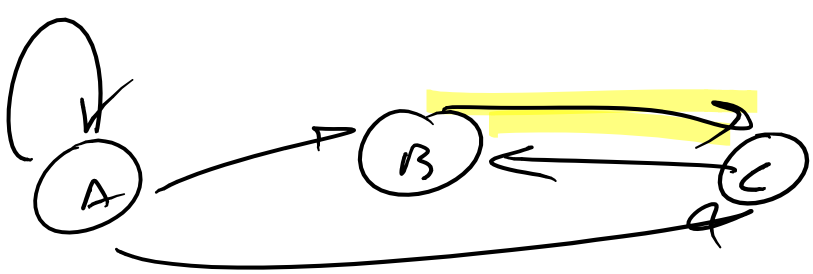
Asymmetric $\qquad DG = \langle V, E \rangle$

Antisymmetric

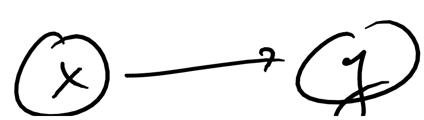
$\forall x \in V$ and $\forall y \in V$

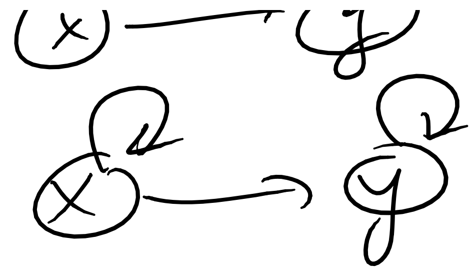
if $\langle x, y \rangle \in E$ then $\langle y, x \rangle \notin E$

unless $x == y$



integers $(x < y)$



integer $(x < y)$ 

$(x \leq y)$

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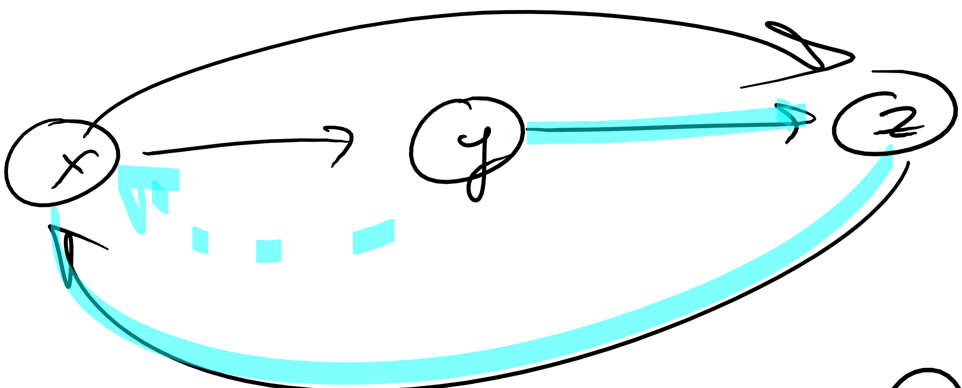
transitive

$\forall x, \forall y, \forall z \in V$

if $\langle x, y \rangle \in E$ and $\langle y, z \rangle \in E$

then $\langle x, z \rangle \in E$



Equivalence Relation

Transitive

Reflexive, Symmetric, Transitive

A == B   A == A

Partial Order

Anti-symmetric, Transitive

$x \leq y$

weak - reflexive $\leq$

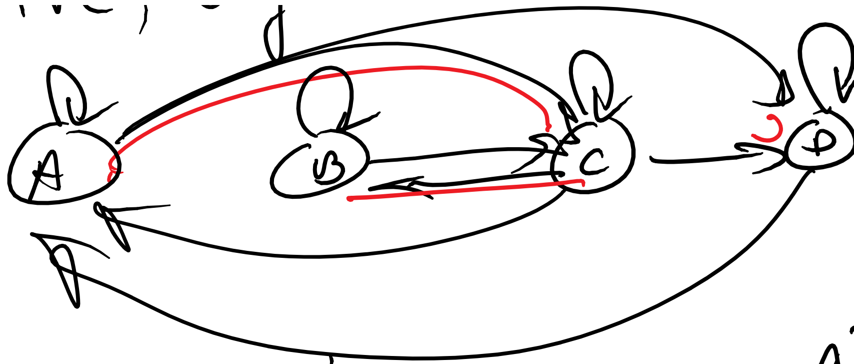
strong - irreflexive $<$

A^1

	A	B	C	D
A	1	0	1	1
B	0	1	1	0
C	1	1	1	0
D	1	0	0	1

Reflexive, Symmetric

Reflexive, ...



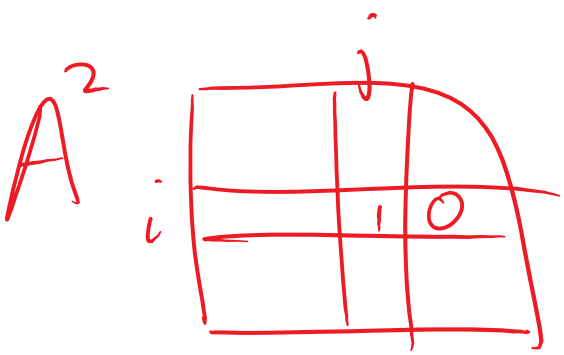
$$A^1 \qquad\qquad\qquad A^1 \qquad\qquad\qquad A^2$$

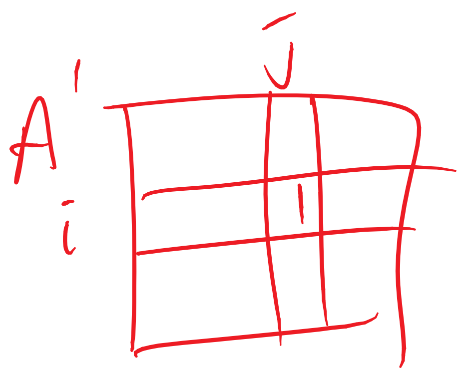
$$\begin{array}{c|cccc} & & & & \\ \hline A & 1 & 0 & 1 & 1 \\ B & 0 & 1 & 1 & 0 \\ C & 1 & 1 & 1 & 0 \\ D & 1 & 0 & 0 & 1 \end{array} \;*\; \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array} \;=\; \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 1 & 1 & 1 \\ B & 1 & 1 & 1 & 0 \\ C & 1 & 1 & 1 & 1 \\ D & 1 & 0 & 1 & 1 \end{array}$$

$C \overset{1}{\Rightarrow} B \qquad\qquad C \overset{2}{\Rightarrow} B$

$$\boxed{A^2 \subseteq A^1}$$

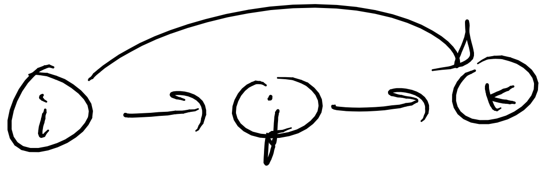
$$\begin{tikzcd} i \arrow[r] \arrow[rr, bend left] & j \arrow[r] & k \end{tikzcd}$$



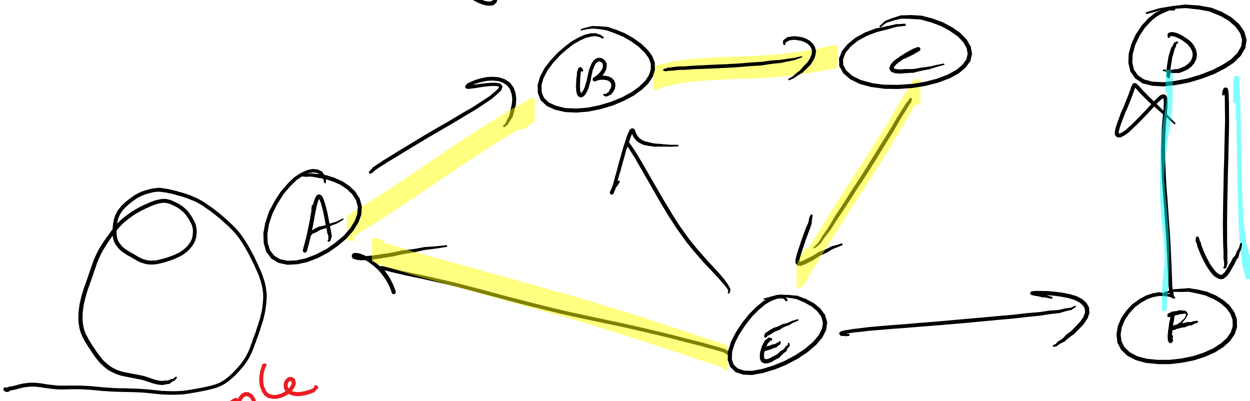


bool isTransitive(

```
vector<vector<bool>> adj )
{
    bool ok = true;
    for (int i = 0; i < adj.size(); i++)
      for (int j = 0; j < adj.size(); j++)
        for (int k = 0; k < adj.size(); k++)
          if (Adj[i][j] && A[j][k])
            ok &&= Adj[i][k];
    return ok;
}
```



Strong Components
(Strongly connected components)



simple

vertices

A simple cycle sequence of vertices $v_1\, v_2\, v_3 \cdots v_n$ such that

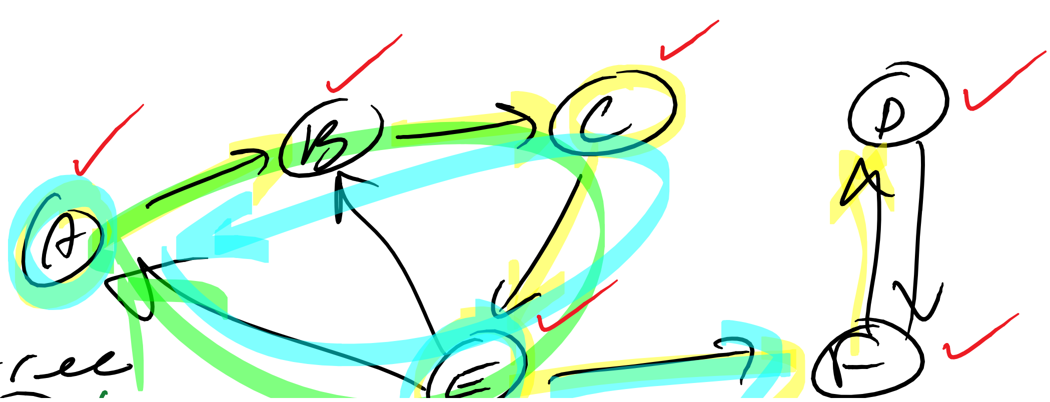
$\langle v_i, v_j \rangle \in E$ for $i, j = 1, 2, \ldots n$

and $v_i \neq v_j$ except for $v_1 == v_n$

A strongly connect component in $G = \langle V, E \rangle$ is a maximal set of vertices in V such that for all $x, y \in V$

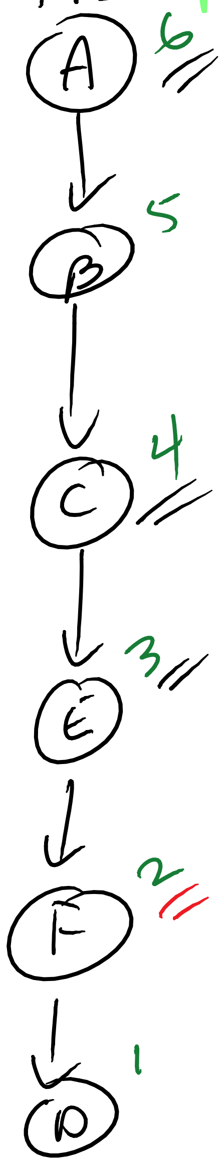
$x \overset{*}{\Longrightarrow} y$ and $y \overset{*}{\Longrightarrow} x$

path of length 1 or more

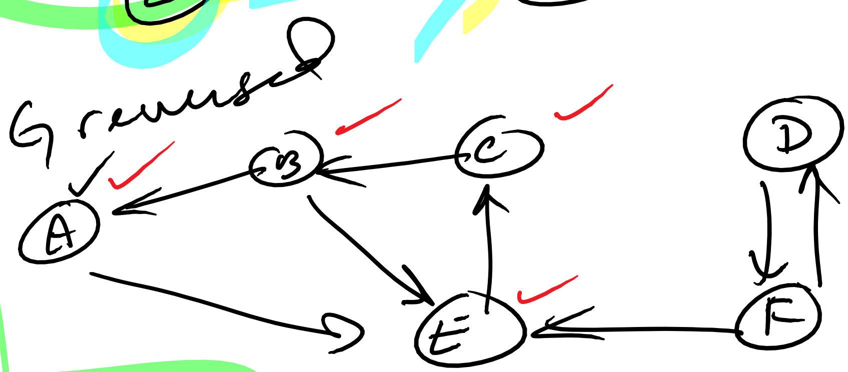


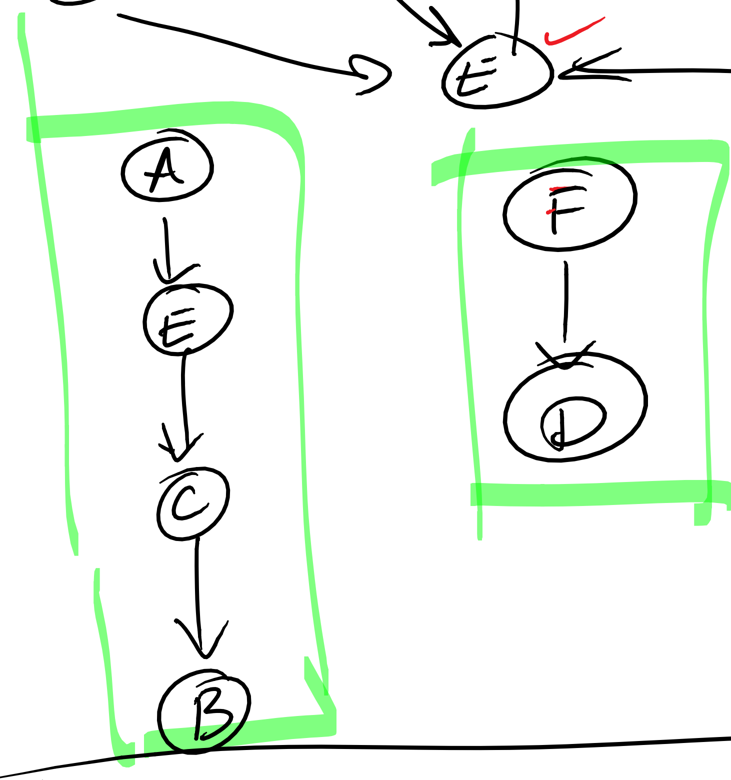
DFS tree

DFS tree



DG reversed





C1: A, E, C, B

C2: F, D

Reduce Graph

equivalence class