MHF3202
HW1

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Question 1

See attached image.

Question 2

See attached image.

Question 3

$$\mathcal{P}(A) = \varnothing, \{\varnothing\}, \{\{1,2\}\}, \{1\}, \{\varnothing, \{1,2\}\}, \{\varnothing, 1\}, \{\{1,2\}, 1\}, \{\varnothing, \{1,2\}, 1\}$$

Question 4

1. $$A \cap B = \{1, \{\{1\}\}\}$$
2. $$A \cup B = \{1, \{1\}, \{\{1\}\}, \{1, \{1\}\}, \varnothing\}$$
3. $$A \setminus B = \{\{1\}, \varnothing\}$$

Question 5

See attached image.

Question 6

The statement $(\mathbb{R} \times \mathbb{Z}) \cap (\mathbb{Z} \times \mathbb{R}) = \mathbb{Z} \times \mathbb{Z}$ is true. Consider $\mathbb{R} \times \mathbb{Z}$, which forms ordered pairs of (x, y) where $x \in \mathbb{R}$ and $y \in \mathbb{Z}$, which when graphed is the set of horizontal lines at every integer on the y-axis. A similar result is reached with $\mathbb{Z} \times \mathbb{R}$, which when graphed is the set of vertical lines at every integer on the x-axis. Taking the intersection of where those intersect would result in a grid of dots at every (x, y) where $x, y \in \mathbb{Z}$, which is $\mathbb{Z} \times \mathbb{Z}$, meaning that the statement is true.

The statement $(\mathbb{R} \times \mathbb{Z}) \cup (\mathbb{Z} \times \mathbb{R}) = \mathbb{R} \times \mathbb{R}$ is false. Note that $\mathbb{R} \times \mathbb{R}$ is $\mathbb{R}^2$, which is the entire xy-plane. However, the union of the set of vertical lines at every integer and the set of horizontal lines at every integer is a grid of lines, which is not the real plane. Therefore the statement is false.

Question 7

1. $$\overline{A} = \{4, 5, 6, 7, 8\}$$
2. $$\overline{A \cup B} = \{4, 6, 7, 8\}$$
3. $$\overline{\overline{A} \cap B} = \overline{\{4, 5, 6, 7, 8\} \cap \{2, 3, 5\}} = \{5\}$$

Question 8

1. $$\bigcup_{n\in\mathbb{N}} A_n = \{x : x \in \mathbb{Z} \setminus \{0\}\}$$

2. $$\bigcup_{n\in\mathbb{N}} B_n = \{2x : x \in \mathbb{Z}\}$$

3. $$\bigcup_{n\in\mathbb{N}} (A_n \cup B_n) = \mathbb{Z}$$

4. $$\bigcup_{n\in\mathbb{N}} (A_n \cap B_n) = \varnothing$$

5. $$\left(\bigcup_{n\in\mathbb{N}} A_n\right) \cup \left(\bigcup_{n\in\mathbb{N}} B_n\right) = \mathbb{Z}$$

6. $$\left(\bigcup_{n\in\mathbb{N}} A_n\right) \cap \left(\bigcup_{n\in\mathbb{N}} B_n\right) = \{2x : x \in \mathbb{Z} \setminus \{0\}\}$$