

MHF3202

HW2

Oliver Deng

February 7, 2025

Question 1

1. P : Aliens abduct cows
 Q : Aliens give cows pats
Translation: $P \wedge Q$
2. P : I know Jack
 Q : I know John
 R : I know Ralph
 S : I know Jacob
Translation: $P \wedge Q \wedge R \wedge (\neg S)$
3. P : I am surly in the morning
 Q : I get caffeine
Translation: $Q \implies (\neg P)$

Question 2

Looking at the 2 truth tables below, the 2 statements are equivalent.

1. Truth table for $\neg(P \wedge Q)$:

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

2. Truth table for $Q \implies (\neg P)$

P	Q	$\neg P$	$Q \implies (\neg P)$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

Question 3

Let $P(x)$ be the statement " x is a rational number" or $x \in \mathbb{Q}$. Then the translation of that statement into symbolic logic is:

$$P(x) \iff P(5x)$$

This statement is true, since if $x \in \mathbb{Q}$, then $x = \frac{p}{q}$, $p, q \in \mathbb{Z}$, $q \neq 0$, and $5x = \frac{5p}{q}$, which is still a rational number since the integers are closed under multiplication.

Also, if $5x \in \mathbb{Q}$, then $5x = \frac{p}{q}$, $p, q \in \mathbb{Z}$, $q \neq 0$, and $x = \frac{p}{5q}$, which is still a rational number since again the integers are closed under multiplication, and $5q \neq 0$ since $q \neq 0$.

Question 4

$$\forall x \forall y, x, y \in \mathbb{R} \text{ s.t. } x * y = 0 \implies (x = 0) \vee (y = 0)$$

Question 5

Looking at the two truth tables below, the statements are **not** equivalent.

1. Truth table for $P \wedge (Q \vee R)$:

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

2. Truth table for $(P \wedge Q) \vee R$

P	Q	R	$P \wedge Q$	$(P \wedge Q) \vee R$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Question 6

Let $H(x)$ mean that " x goes to Heaven" and let $N(x)$ mean that " x goes to Nirvana." Also, let $D(x)$ mean that " x is a dog", and let $C(x)$ mean that " x is a cat."

- 1.

$$(\forall x(D(x) \implies H(x)) \wedge (\exists y(C(y) \wedge N(y)))$$

2. There is a dog that doesn't go to Heaven, or there are no cats that go to Nirvana.

- 3.

$$(\exists x(D(x) \wedge \neg H(x))) \vee (\forall y(C(y) \implies \neg N(y)))$$

Question 7

1. If P and Q are true, then it is not true that either R is true or that S is false.
2. Either Q is false or R is true if and only if P is true.
3. For any rational number x , there is an integer q such that x multiplied by q is a natural number.
4. (Didn't we translate this from English already? In that case I'll generalize the statements.) Either there exists an x such that $D(x)$ is true and $H(x)$ is false, or if for all y , $C(y)$ is true, then $N(y)$ is false.

Question 8

1. No Jedi have clairvoyance.
2. Either there is someone who is mortal or no one is gullible.
3. Either there is an even number not divisible by two, or all primes are not divisible by two.