Chronomodel

User manuel

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Introduction

Présentation générale du logiciel.
Méthodes bayésiennes, modèles hierarchiques, méthodes MCMC ...
Données archéologiques, Calibration des mesures ...

Chapter 2

Event model

2.1 Event model for dating combination with calibration

2.1.1 Definition of an event

An event is determined by its unknown calendar date.

2.1.2 Mathematical model

Poser le DAG associé, les formules, le lien vers l'article.
Expliquer la gestion des outliers.
Parler des cas particuliers : wiggle matching, mesures répétées ...

2.1.3 Illustrative example

Illustrer la démarche de réflexion à avoir avant d'attaquer la modélisation des mesures.

2.2 Modelling an event with Chronomodel

Détailler les différents paramètres à entrer pour les différents types de mesures.
Parler des courbes de calibrations de chacune des méthodes (+ comment modifier cette courbe en fonction de la région d'étude)
Mettre un mot sur les méthodes MCMC à choisir et faire référence à la partie dédiée au détail de ces méthodes.

2.2.1 Mesure en C14

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2.2.3 Mesure en TL/OSL

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2.3 Modelling statigraphic constraints with Chronomodel

2.4 Example

Montrer un ou plusieurs exemples. A voir ...

Chapter 3

La phase

3.1 Modèle de phase

3.1.1 Définition d'une phase

Qu'est-ce qu'une phase ?

3.1.2 Explication du modèle

Poser le DAG associé, les formules, le lien vers l'article.
Expliquer la gestion des outliers.
Parler des cas particuliers : wiggle matching, mesures répétées ...

3.1.3 Exemple illustratif

Illustrer la démarche de réflexion à avoir avant d'attaquer la modélisation des mesures.

3.2 Modéliser une ou plusieurs phases

3.3 Modéliser des relations d'ordre entre les phases

3.4 Exemple

Montrer un ou plusieurs exemples. A voir ...

Chapter 4

MCMC methods

In general, the posterior distribution does not have an analytical form. However, we can always write down the posterior distribution up to proportionality, by multiplying prior and likelihood $p(\theta|y) \propto p(\theta)p(y|\theta)$. Elaborated algotithms are then required to approximate this posterior distribution only known up to proportionality.
Markov chain Monte Carlo (MCMC) is a general method based on drawing values of θ from approximate distributions and then corrected those draws to better approximate the target posterior distribution $p(\theta|y)$. The sampling is done sequentially, with the distribution of the sampled draws depending on the past value drawn. Indeed, a Markov chain is a sequence of random variables θ^1, θ^2, ..., for which, for any t, the distribution of θ^t given all previous θ's depend only on the recent value, θ^{t-1} [1, 2].

4.1 Choice of the MCMC algorithm

A convenient algorithm useful in many multidimensional problems is the Gibbs sampler (or conditional sampling) [1, 2].
Let's say we want to approximate $f(\theta_1, \theta_2, ..., \theta_d)$. The algorithm starts with a sample of initial values $(\theta_1^{(0)}, \theta_2^{(0)}, ..., \theta_d^{(0)})$ randomly selected . The first step of the algorithm is to update the first value by sampling a candidate value of $\theta_1^{(1)}$ knowing $\theta_2^{(0)}, ..., \theta_d^{(0)}$ from an adapted proposal density. This candidate value will be accepetd with a certain probability and $\theta_1^{(1)} = \theta_1^{(0)}$ otherwise. The next step is to find a candidate value $\theta_2^{(1)}$ knowing $\theta_1^{(1)}, \theta_3^{(0)}, ..., \theta_d^{(0)}$. Again, we need a proposal density, that can be different from the latter one, and this candidate value will be accepted with a probability. And so on... Then the dst step is to find a candidate value for $\theta_d^{(1)}$ knowing $\theta_1^{(1)}, \theta_2^{(1)}, ..., \theta_{d-1}^{(1)}$ from another adapted proposal density. This process is then iteratively repeated.

Starting values : In Chronomodel, the initial values of each Markov chain are

randomly selected out of an interval calculated as follow... A REVOIR

Proposal density : In Chronomodel, two main proposal densities are implemented, the rejection sampling method and the Metropolis Hastings algorithm. These algorithms are briefly explained in the following lines.

- **Rejection sampling**.
  The rejection sampling generates sampling values from a proposal density function by using an instrumental distribution. The instrumental distribution should be easily sampled from.

- **Metropolis-Hastings algorithm**.
  The Metropolis-Hastings algorithm need a proposal density that will generate new candidate values. A common choice is a symmetric density, such as the Gaussian density.

Depending on the type of the parameter, an event, the mean of a calibrated measure, the variance of a calibrated measure, or the boundaries of a phase, different methods are proposed in order to generate a new candidate values at each step of the Gibbs sampler. These methods are described here in turn.

4.1.1 Drawings from the conditionnal posterior distribution of an event

Three different methods can be chosen.

- **Rejection sampling with a Gaussian proposal** (REFERENCE?)

- **Rejection sampling with a Double exponentiel proposal**(REFERENCE?)

Both methods are exact methods. We recommend to use one of these two except when the event considered is involved in statigraphic constraints .

- **Metropolis-Hastings algorithm with an adaptative Gaussian random walk**(REFERENCE?)

This last method should be used when there are statigraphic constraints.

4.1.2 Drawings from the full conditionnal posterior distribution of the mean of a calibrated measure

In this case, three different methods can be choosen.

- **Metropolis-Hastings algorithm using the posterior distribution of the associated individual calibration ($P(M_i|t_i)$)**
  This method is adapted for calibrated measures, namely C14 or AM but not ref typo, and when densities are multimodal.

- **Metropolis-Hastings algorithm using the parameter prior distribution ($P(t_i|\sigma_i^2, \theta)$)**
  This method is recommanded when no calibration is needed, namely for measures of the type TL/OSL, Gauss or référence typo (TRADUCTION ?).

- **Metropolis-Hastings algorithm using an adaptative Gaussian random walk**
  This method is recommanded when no calibration is needed or when there are statigraphic constraints. This method is adapted when the density to be approximated is unimodal.

4.1.3 Drawings from the conditionnal posterior distribution of the variance of a calibrated measure

In this case, only one method is implemeted in Chronomodel, the uniform skrinkage as explained in Daniels [3]. The full conditional density is unimodal, hence the Metropolis Hastings algorithm can be implemented here. The proposal density involved is an adaptative Gaussian random walk (REFERENCE?). The variance of this proposal density is adapted during the process.
As a variance parameter can only be positive, this step is done on the logarithm of the variance of the calibrated measure.

4.2 Burn in period

The algorithms described above will generate a Markov chain for each parameter, that is a sample of values from the posterior full conditional distribution. Now, wait for all the Markov chains to reach equilibrium. Let's say this occurs at some time T. The time before T is usually called the burn-in period.

How do we determine T ? Unfortunatly, T cannot be determine theoretically. With Chronomodel, the best way to decide when all Markov chains reach equilibrium is to observe plots of these Markov chains usually, called **traces** (See Results section). (METTRE UN EXEMPLE DE TRACE ET INDIQUER LA PERIODE DE CHAUFFE ASSOCIEE) T will be the longest burn-in period of all traces.
It is also recommended to produce several Markov chains for each full conditional distribution, all chains starting from different initial values. Then the traces of these different Markov chains should converge to the same equilibrium after a burn-in period. (METTRE UN EXEMPLE DE TRACES QUI SE REJOIGNENT ET INDIQUER LA PERIODE DE CHAUFFE ASSOCIEE) A t on les statistiques a posteriori ? Variabilité inter chaine / intra chaine ?

Parameters choice : REVOIR LES PARAMETRES DE CHRONOMODEL

4.3 Adaptation period

This period exists if the AGMH method is chosen. Otherwise, it does not mean anything ?? WHAT IS DONE DURING THAT PERIOD IF AGMH IS NOT CHOSEN ?

EXPLIQUER LE PROCESSUS DE CHOIX ET TEST DE LA VARIANCE. PARLER DU TAUX DE REJET? A REVOIR

4.4 Acquisition period

In this period, all Markov chains are supposed to have reach their equilibrium distribution, the target distributions. Of course, this has to be checked and the next section provides useful tools that can help controlling if equilibrium is actually reached. If the equilibrium is reached, Markov chains can be sampled and informations about conditionnal posterior distribution can be extracted.
Sampling from these Markov chains need to be carefully made. Indeed, successive value of a Markov chain are not independent. In order to limit the correlation of the sample, we can choose to thin the sample by only keeping equally spaced values. This can be done by ... REVOIR CHRONOMODEL.

Chapter 5

Results and Interpretations

5.1 Checking the Markov chains

When Markov chains are generated, two points have to be verified : the convergence of the chains and the absence of correlation between successive values. If the Markov chain has not reach its equilibrium, values exctracted from the chains will give inapropriate estimates of the posterior distribution. If high correlation remains between successive values of the chain, then variance of the posterior distribution will be biased. Here are some tools to detect whether a chain has reach its equilibrium and whether successive values are correlated. We also give indications about what can be done in these unformtunate situations.

5.1.1 Is the equilibrium reached ?

As said previously, there is no theoretical way to determine how long will be the burn-in period of a Markov chain. The first thing to do is to observe the trace of the chain and inspect it for signs of convergence. METTRE DES EXEMPLES DE GRAPHES AVEC DES TRACES QUI NE CONVERGENT PAS ET DES TRACES QUI CONVERGENT;
Producing parallel Markov chains, all with different starting values, can help deciding if (and when) a chain has reach its equilibrium. METTRE DES EXEMPLES.

What should be done when equilibrium is not reached ? The only thing to do is to go back to the MCMC interface and ask for a longer burn-in period.

5.1.2 Correlation between successive values ?

A Markov chain is a sequence of random variables θ^1, θ^2, ..., for which, for any t, the distribution of θ^t given all previous θ's depend only on the recent value, θ^{t-1} [1, 2]. Hence, a high correlation between two consecutive values is expected, but not between all values. To check whether the chain is highly correlated, observe the autocorrelation plot. Only the first correlation should be high, the remaining correlations should be negligible.

What should be done is correlation is high ? A good thing to do is to thin the sample by keeping spacer values. REVOIR AVEC LES PARAMETRES DE CHRONOMODEL

5.2 Interpretation

If all Markov chains are checked, then the results section has a real meaning.

5.2.1 Calibrated curves

5.2.2 Posterior densityi

5.2.2.1 Cas des paramètres de date

5.2.2.2 Cas des variances : détection des outliers

5.2.3 Régions à plus haute densité (HPD)

Mettre en garde sur les interprétations à ne pas faire, notamment sur les densités a posteriori marginales.
Traitement des sorties possibles sous R (voir comment exporter les sorties du logiciel).

Chapter 6

Installation and data management

6.1 Installation

6.2 Importation des données au format CSV

6.3 Gestion des projets

6.4 Exportation des sorties

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